Circular Hadron Collider Beam Dynamics II

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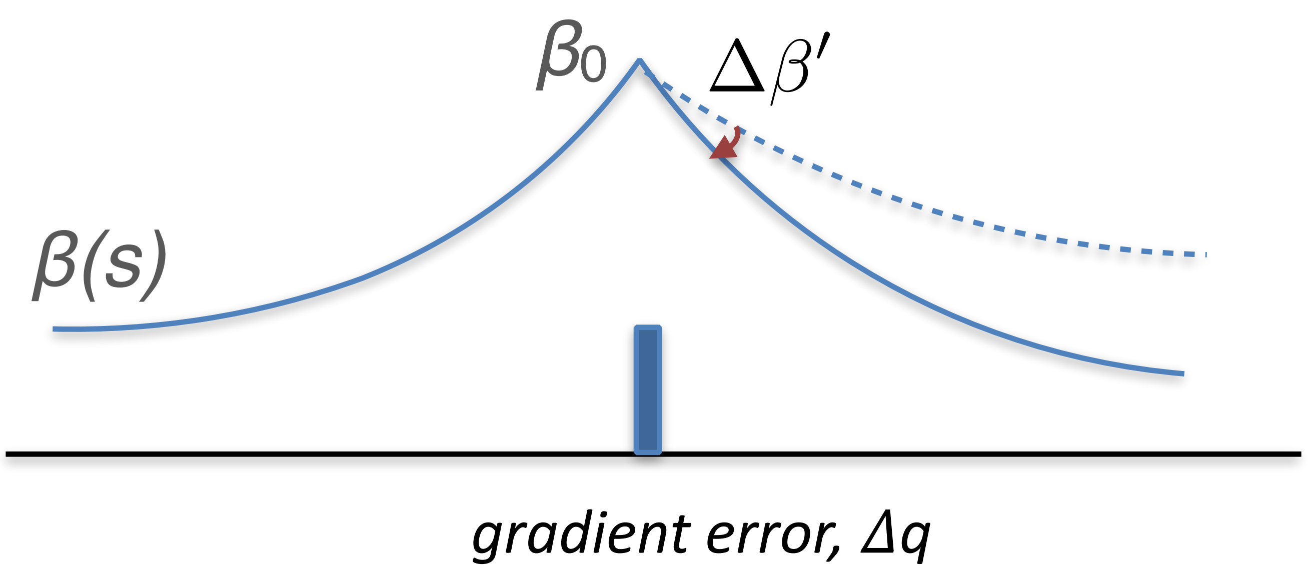
Outline

- Part I
  - Luminosity
  - Review
    - Longitudinal Dynamics
    - Transverse Dynamics
  - Collider Optical Design
    - FODOs, Insertions
    - Interaction Regions
    - Dispersion Suppression
  - Errors and Adjustments I
    - Linear Errors
- Part II
  - Errors and Adjustments II
    - Linear Errors
    - Nonlinear Effects
  - Space Charge and the Beam-Beam Interaction
  - Emittance Control and Luminosity
  - Effects due to Synchrotron Radiation
  - Optimization of Luminosity

Focusing (quadrupole) Errors

- β, α distortions and "beta-beat"



$$\Delta\alpha = -\frac{1}{2}\Delta\beta'$$

if ideal gradient produces strength $q = B'\ell/(B\rho)$,
then a gradient error will produce $\Delta q = \Delta B'\ell/(B\rho)$
and the slope of β will change according to

$$\Delta\alpha = \beta_0 \Delta q$$

Downstream, the distortion will propagate:

$$\frac{\Delta\beta}{\beta}(s) \approx -\Delta q \beta_0 \sin 2\psi_0(s)$$

β Distortion in a Synchrotron

dipole error:

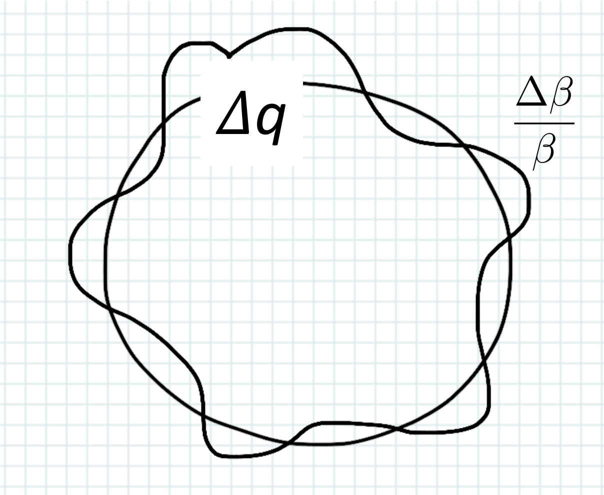
$$x(s) = \Delta\theta\sqrt{\beta_0\beta(s)}\sin\Delta\psi$$

quadrupole error:

$$\frac{\Delta\beta}{\beta}(s) \approx -\Delta q\beta_0 \sin 2\psi_0(s)$$

- In a circular accelerator, the closed solution of the amplitude function(s) will be altered by the gradient error. With analysis similar to the situation for a closed *orbit* distortion, the gradient error will produce a closed *β*-distortion all around the ring which, for small errors, will be given by:

$$\frac{\Delta\beta}{\beta}(s) \approx -\frac{\Delta q\beta_0}{2\sin 2\pi\nu}\cos(2|\Delta\psi| - 2\pi\nu)$$



Focusing (quadrupole) Errors

- Phase/tune shift
  - A gradient error will distort the amplitude function, and therefore distort the development of the phase advance downstream. As the β distortion will oscillate about the ideal β function, the phase advance will slightly increase and decrease along the way. This is particularly important in a ring where the betatron tune, ν, might need fine control.
  - A small gradient error *q* at location with amplitude function b will create a change in tune:

$$\Delta\nu \approx \frac{1}{4\pi}\beta_0 q$$

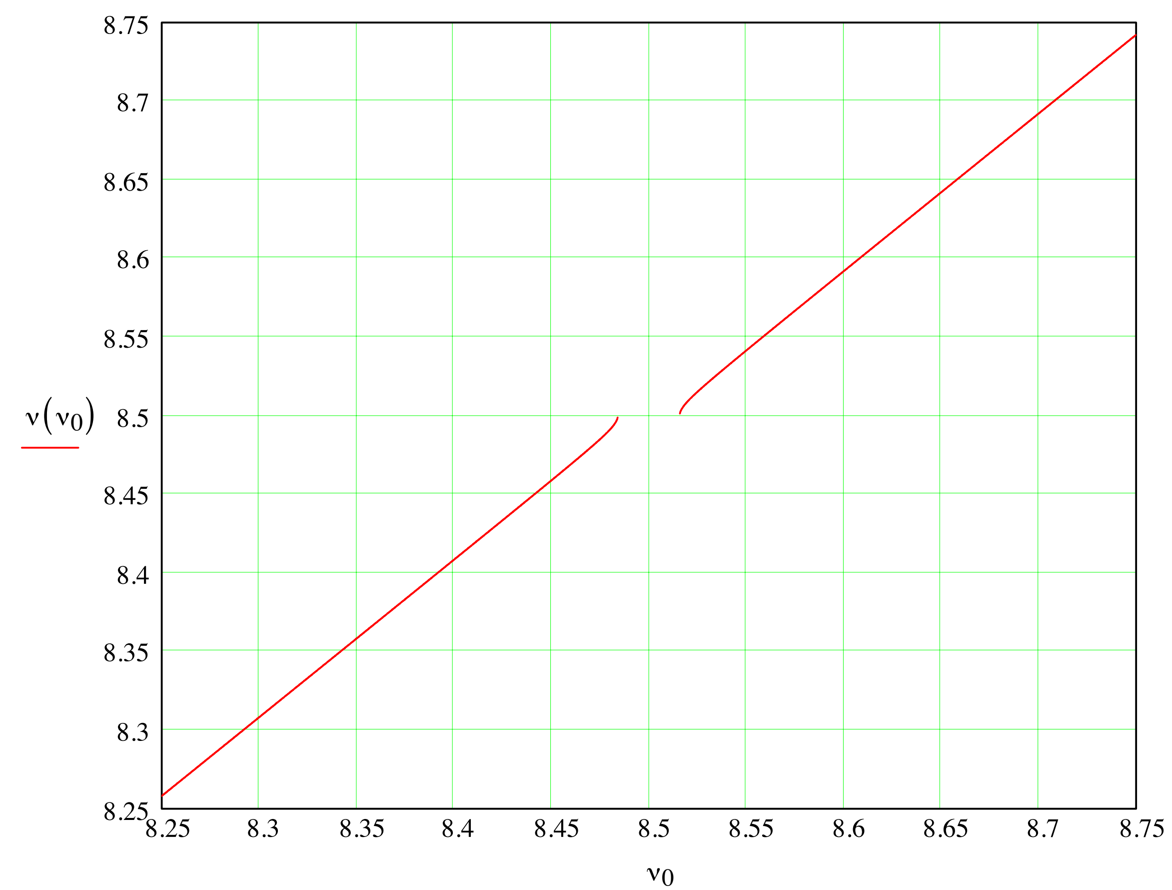
Focusing (quadrupole) Errors

- What happens if the gradient error is ***too*** big?

$$\cos 2\pi\nu = \cos 2\pi\nu_0 - \frac{1}{2} q\beta_0 \sin 2\pi\nu_0$$

if too large, $|\cos 2\pi\nu|$ can become >1, thus unstable!

- Half-integer stop band:



- Beta distortion:

$$\frac{\Delta\beta}{\beta}(s) \approx -\frac{\Delta q \beta_0}{2\sin 2\pi\nu} \cos(2|\Delta\psi| - 2\pi\nu)$$

as ν —> **integer/2**, distortions enhanced a resonance!

Beta-Mismatch Invariant

- We noted that a local gradient error will produce a distortion in the amplitude function (in its slope, in particular):

$$\Delta\alpha = \beta_0 \Delta q$$

- In the absence of further gradient errors,
  - the quantity $\Delta\beta\Delta\gamma - \Delta\alpha^2$ is an invariant

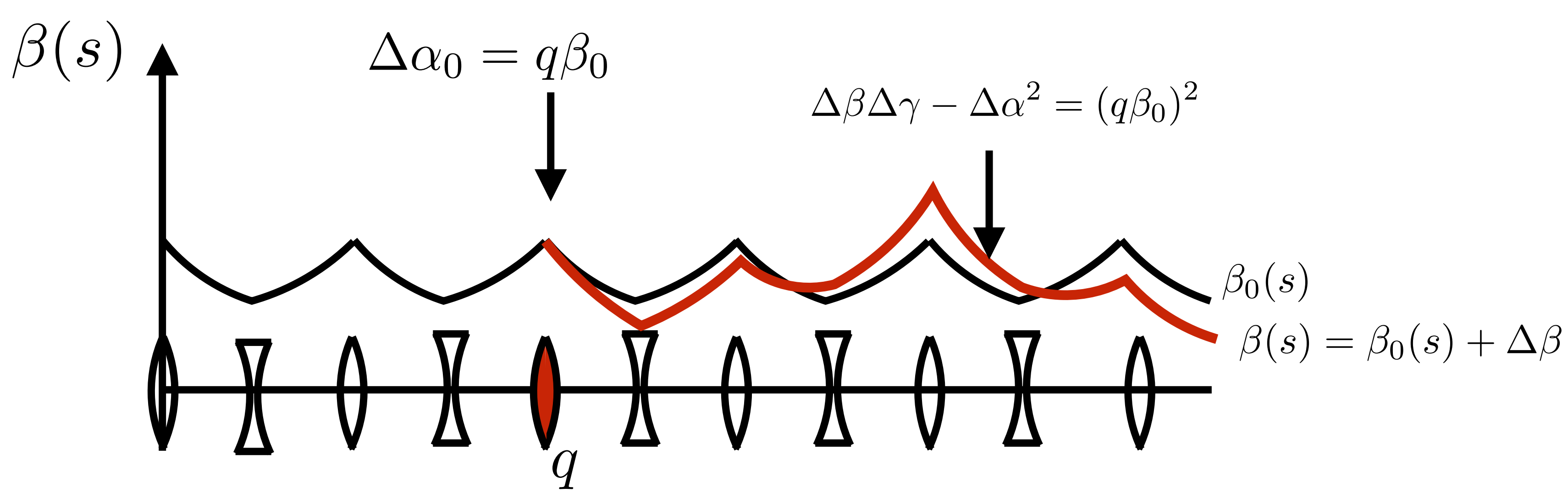
Check:

$$\Delta J = M \Delta J_0 M^{-1}$$

$$\det(\Delta J) = \det M \det(\Delta J_0) \det M^{-1}$$

$$\det(\Delta J) = \det(\Delta J_0)$$

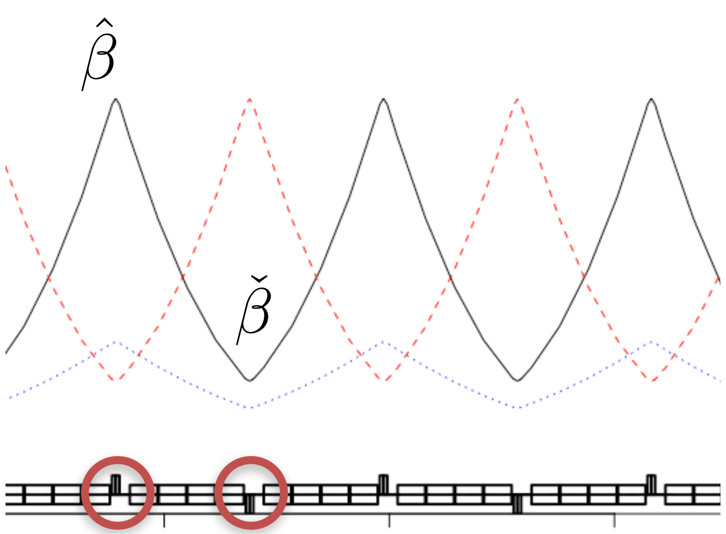
$$\Delta\beta\Delta\gamma - \Delta\alpha^2 = invariant$$



Tune correction/adjustment

- Suppose we have a FODO arrangement, and we put adjustable quadrupoles near every "main" quadrupole (N = # quads):

$$\Delta\nu_x = \frac{N}{4\pi}\left[\hat{\beta}\Delta q_1 + \check{\beta}\Delta q_2\right]$$
$$\Delta\nu_y = -\frac{N}{4\pi}\left[\check{\beta}\Delta q_1 + \hat{\beta}\Delta q_2\right]$$



- The quadrupoles can be wired in two separate circuits, and thus the two tunes can be independently adjusted by any (reasonable) amount desired.

Chromaticity of a Circular Accelerator

- Chromaticity -- change in the betatron tune, ν, with respect to relative momentum deviation ($\Delta p/p$):

$$x'' + K(s)x = x'' + \frac{qB'(s)}{p}x = 0$$

$$\xi \equiv \frac{\Delta\nu}{\Delta p/p}$$

- There will be a different chromaticity value for each degree of freedom:

$$\xi_x = \frac{\Delta\nu_x}{\Delta p/p}$$

$$\xi_y = \frac{\Delta\nu_y}{\Delta p/p}$$

How to estimate the scale of the effect?

The Natural Chromaticity

- There will be a "natural" dependence of tune on momentum from the fact that the various spring constants $K(s) \sim 1/p$, hence dK/K = - dp/p

- Starting from $\Delta\nu = \dfrac{1}{4\pi}\beta\Delta q$ for a single gradient error,

$$\Delta q \equiv \frac{\Delta B'\ell}{B\rho}$$

$$\Delta\nu = \int \frac{1}{4\pi}\beta(s)\left[-\frac{B'(s)}{B\rho}\frac{\Delta p}{p}\right] ds$$

$$\xi = -\frac{1}{4\pi}\int \beta(s)K(s)ds$$

$$\xi \equiv \frac{\Delta\nu}{\Delta p/p}$$

Can show that for a FODO-style lattice, $\xi \approx -\nu$

Chromatic Corrections

- Sextupole Field:

$$B_y = \frac{1}{2}B''(x^2 - y^2)$$
$$B_x = B''xy$$

$$\frac{\partial B_y}{\partial x} = \frac{\partial B_x}{\partial y} = B''x$$ *gradient*

So here, if "x" is due to Dispersion: $x = D\frac{\Delta p}{p}$

$$\Delta\nu = \frac{1}{4\pi}\beta/f$$

then, $$\frac{1}{f} = \frac{(\partial B_y/\partial x)\ell}{B\rho} = \frac{B''\ell}{B\rho}\cdot D\,\frac{\Delta p}{p}$$

$$\Delta\xi = \frac{1}{4\pi}\beta\; D\;\frac{B''\ell}{B\rho}$$

ℓ = length of the sextupole field

Note: since $\frac{\partial B_y}{\partial x} = \frac{\partial B_x}{\partial y} \propto D\cdot\frac{\Delta p}{p}$, provides focusing in one plane, defocusing in the other plane

Thus, need 2 sextuples (or 2 families of sextuples) for optimal independent corrections/adjustment of ξ_x, ξ_y.

Also Note: introduces (intentionally!) a non-linear field!!

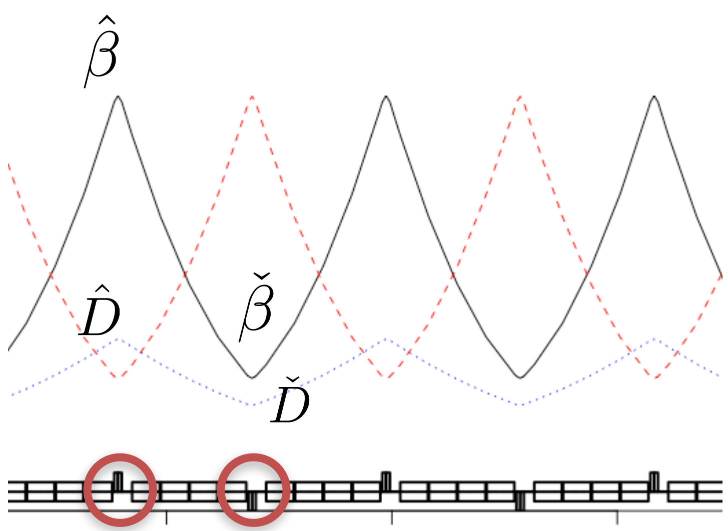
Correction/Adjustment of Chromaticity

- Suppose we have a FODO arrangement, and we put adjustable sextuple magnets near every "main" quadrupole (N = # sextupole magnets):

$$S \equiv \frac{B''\ell}{2B\rho}$$

$$\Delta\xi_x = \frac{N}{4\pi}\left[\hat{\beta}\hat{D}\Delta S_1 + \check{\beta}\check{D}\Delta S_2\right]$$

$$\Delta\xi_y = -\frac{N}{4\pi}\left[\check{\beta}\hat{D}\Delta S_1 + \hat{\beta}\check{D}\Delta S_2\right]$$



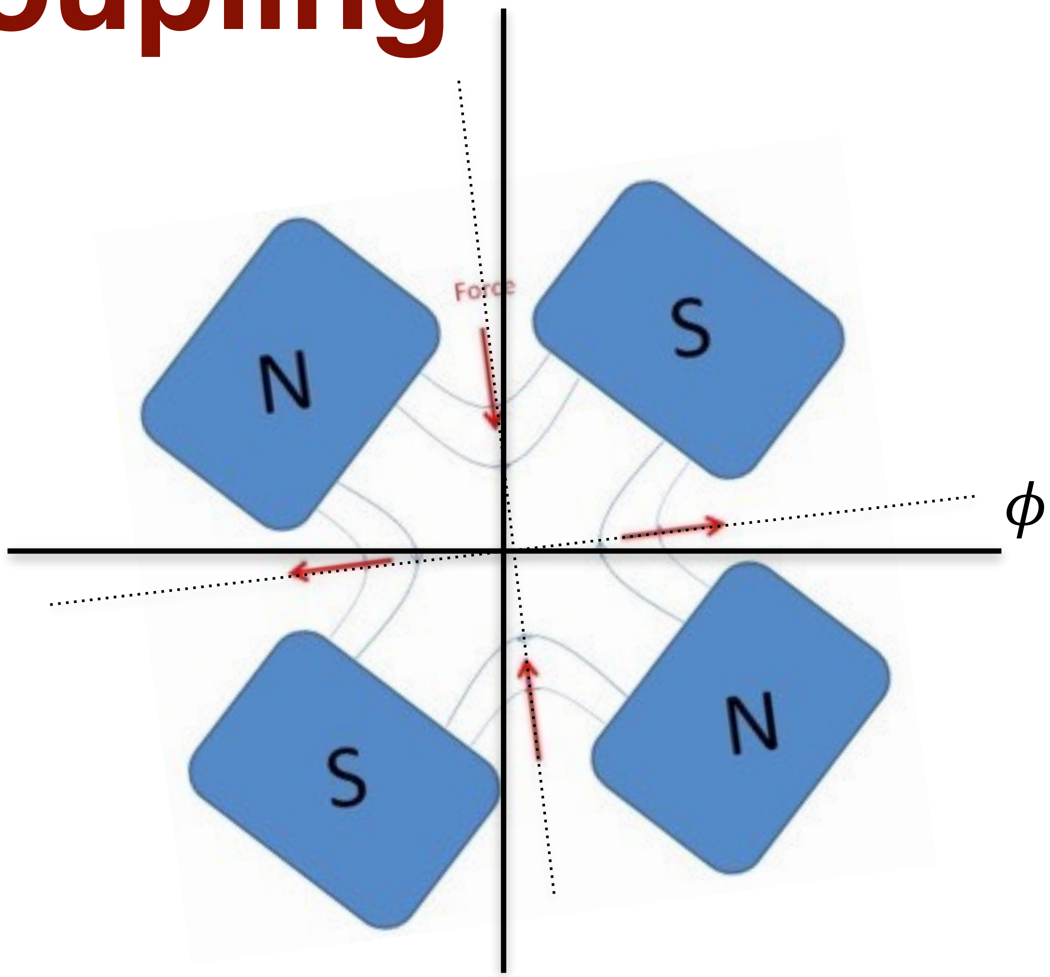
- The sextupoles can be wired in two separate circuits, and thus the two chromaticities can be independently adjusted by any (reasonable) amount desired.

Quick Questions?

Errors Creating Linear Coupling

- Rotated quadrupole magnet

$$B_x = B' \cos 2\phi \; x + B' \sin 2\phi \; y$$
$$B_y = -B' \sin 2\phi \; x + B' \cos 2\phi \; y$$



a normal quad, rotated by a small angle:

$$B_y = B' \; x + 2\phi \; B' \; y$$
$$B_x = B' \; y - 2\phi \; B' \; x$$

normal *skew quad, strength:* $\frac{\Delta B'\ell}{B\rho} = 2\phi \frac{B'\ell}{B\rho} \equiv k$

Clearly, skew quad field couples the horizontal and vertical motion: $\Delta x' = \frac{B_y \ell}{B\rho} = \frac{\Delta B' \ell}{B\rho} y$

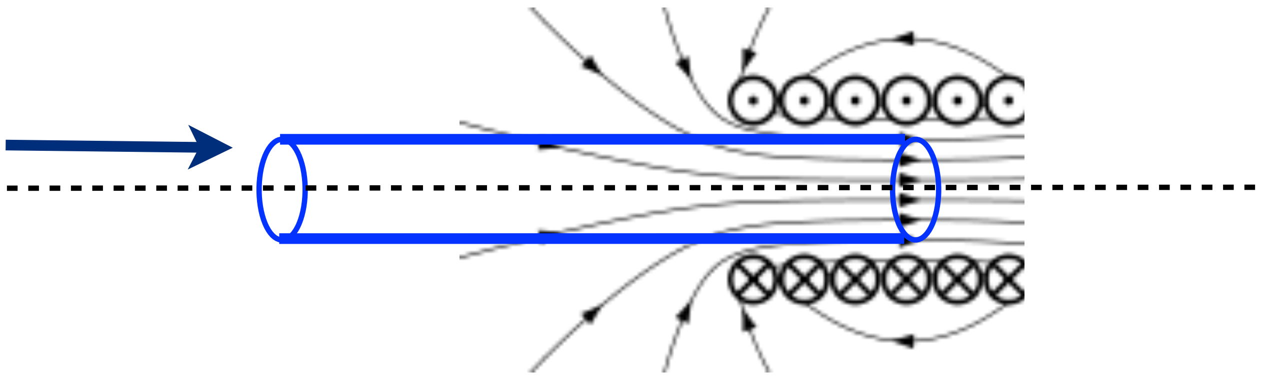
Linear Coupling From Solenoid Fields

$$\Delta p_\theta \approx q \int_{-\infty}^{0} (\vec{v} \times \vec{B})_\theta dt \quad = -\frac{qB_0}{2}\, r$$

upon entrance:

(opposite signs upon exit)

$$\Delta x' = \frac{B_0}{2B\rho} y$$
$$\Delta y' = -\frac{B_0}{2B\rho} x$$



through central region:

$$\begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix} = \begin{pmatrix} 1 & \rho \sin\theta & 0 & \rho(1-\cos\theta) \\ 0 & \cos\theta & 0 & \sin\theta \\ 0 & -\rho(1-\cos\theta) & 1 & \rho\sin\theta \\ 0 & -\sin\theta & 0 & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix}_0$$

$$\theta = \frac{B_0 \ell}{B\rho} \qquad \rho = \frac{B\rho}{B_0}$$

The motion in each plane depends upon the trajectory in *both* planes

Beam Transport through Coupled Systems

- We've just seen the possible introduction of a "4x4" matrix approach to analyzing coupled motion
- If we look at 4x4 transport matrices that operate on (*x,x',y,y'*) vectors, then the transport of covariance matrices works just as before:

$$\Sigma = M\Sigma_0 M^T$$

$$\Sigma = \begin{pmatrix} \langle x^2\rangle & \langle xx'\rangle & \langle xy\rangle & \langle xy'\rangle \\ \langle x'x\rangle & \langle x'^2\rangle & \langle x'y\rangle & \langle x'y'\rangle \\ \langle yx\rangle & \langle yx'\rangle & \langle y^2\rangle & \langle yy'\rangle \\ \langle y'x\rangle & \langle y'x'\rangle & \langle y'y\rangle & \langle y'^2\rangle \end{pmatrix}$$

4x4 matrices now

can also extend to 6x6, which includes *W-t* (or *z-z'* or *z-dp/p*, or…)

Eigen-frequencies of Coupled Oscillator

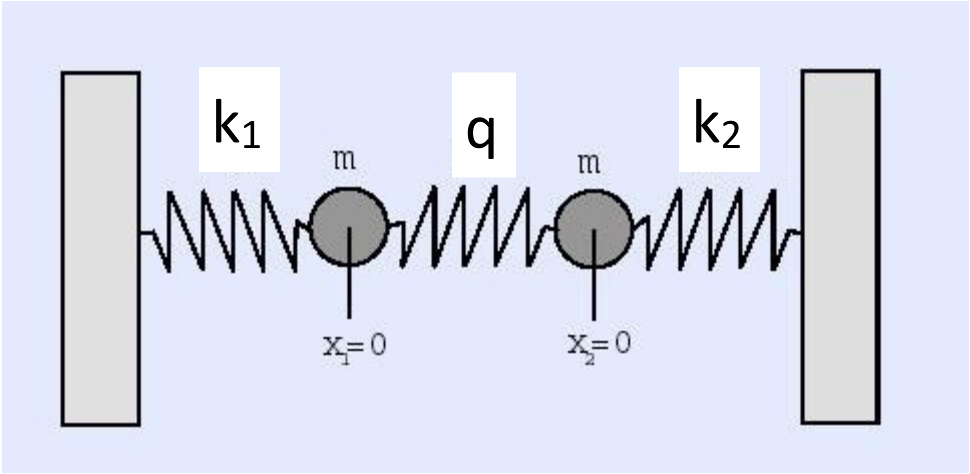
Coupled harmonic oscillator has two eigenvalues

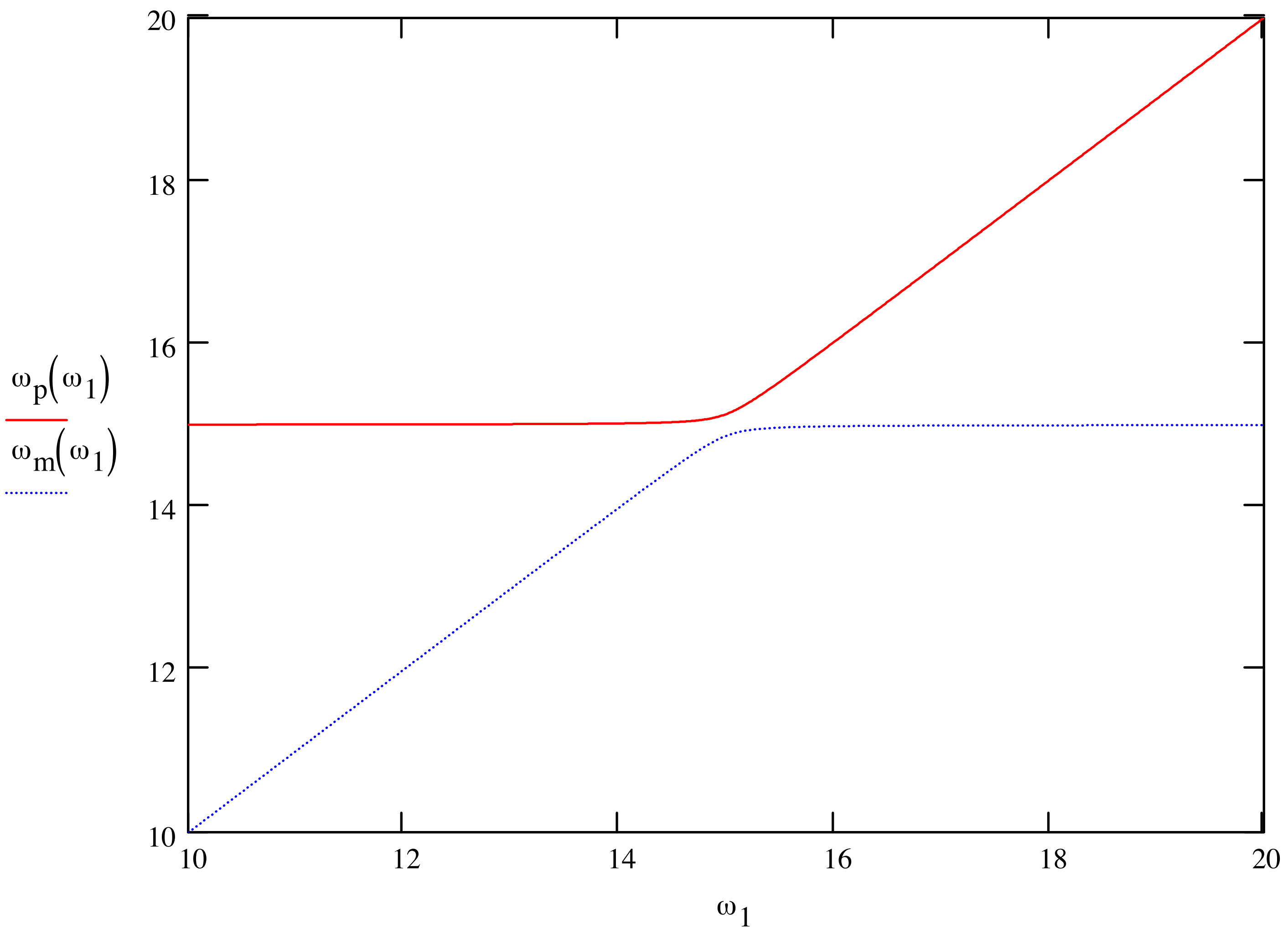
$q := 2 \qquad \omega_2 := 15$

$\omega_1 := 10, 10.01 .. 20$

$$\omega_m(\omega_1) := \sqrt{\frac{\omega_1^2 + \omega_2^2 - \sqrt{\left(\omega_2^2 - \omega_1^2\right)^2 + 4 \cdot q^4}}{2}}$$

$$\omega_p(\omega_1) := \sqrt{\frac{\omega_1^2 + \omega_2^2 + \sqrt{\left(\omega_2^2 - \omega_1^2\right)^2 + 4 \cdot q^4}}{2}}$$





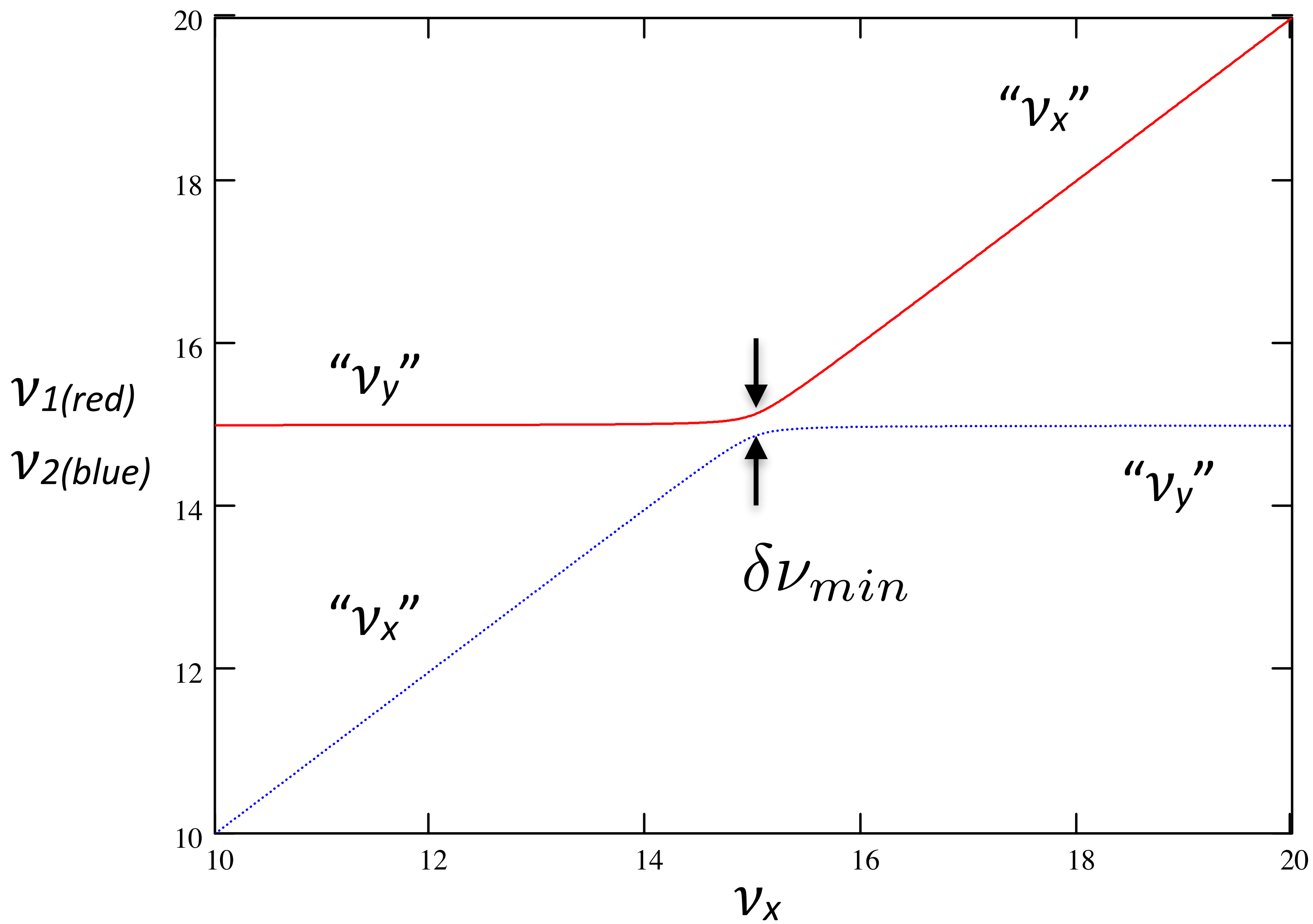
Eigen-frequencies of Coupled Synchrotron

Coupled harmonic oscillator has two eigenvalues

When the natural frequencies in the horizontal and vertical tunes are far apart, they behave rather independently. However, they can never be made identical. So, by varying the quadrupoles in a ring to vary the tunes, the minimum separation observed is a measure of the amount of coupling, in a global sense, between the two planes

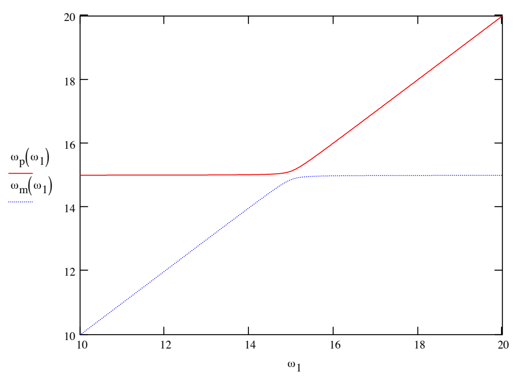
Adjust horizontal tune, say, and measure both horizontal and vertical tunes w/ FFT of BPM data. For a single rotated quadrupole,

$$\delta\nu_{min} = \frac{|\phi|}{\pi}\frac{\sqrt{\beta_x\beta_y}}{F}$$

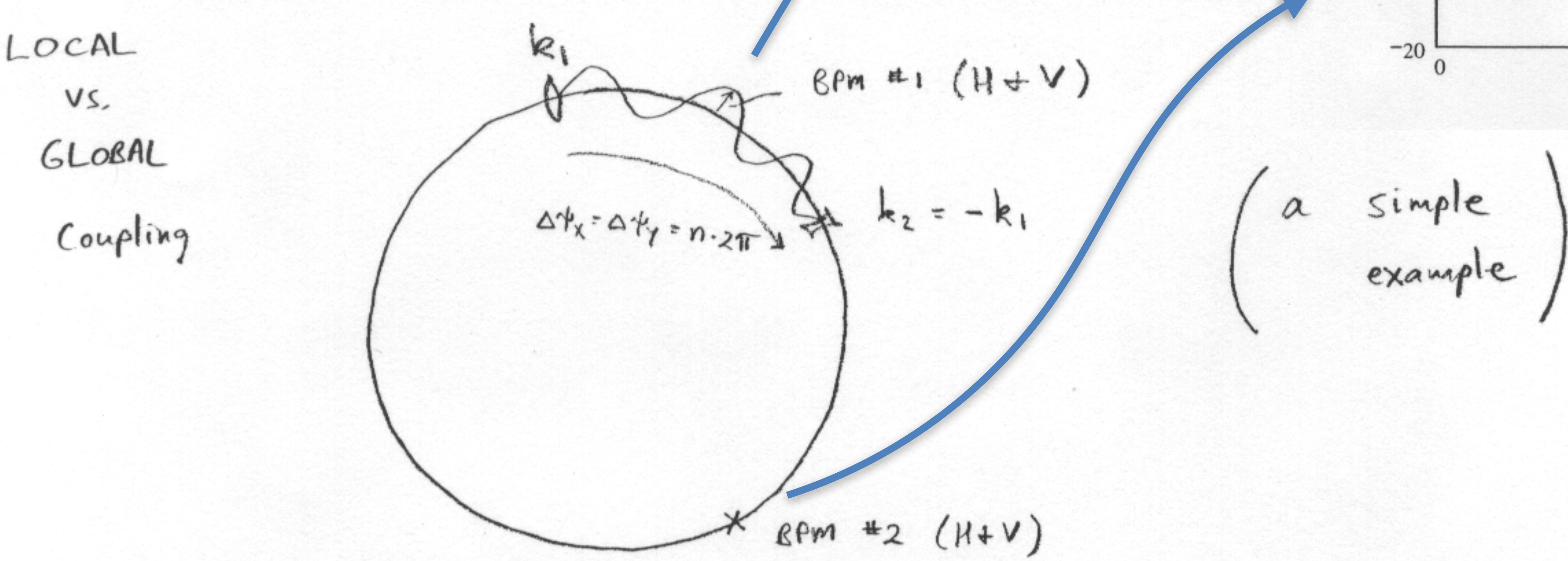


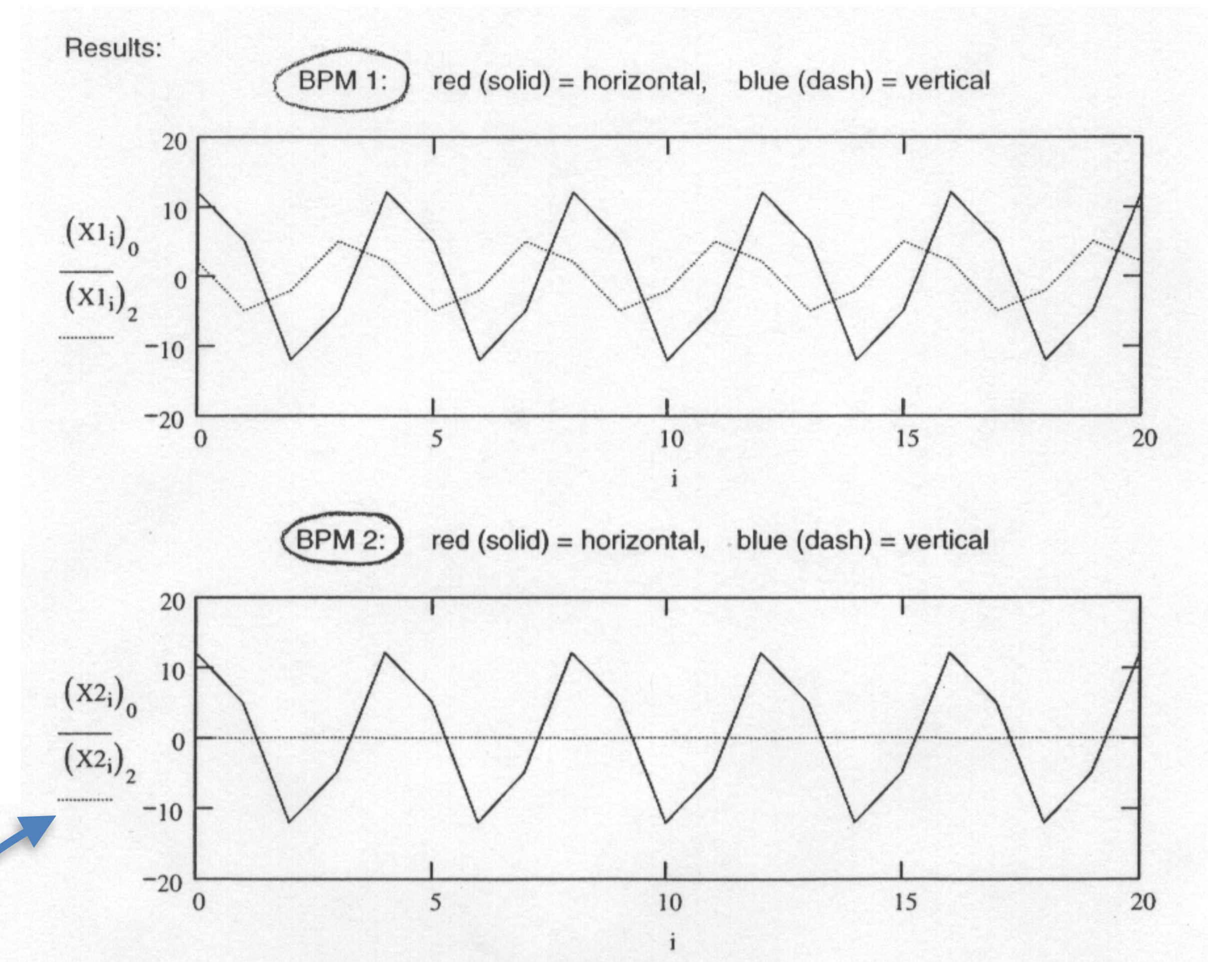
Global vs. Local

- Global Coupling



- Local Coupling





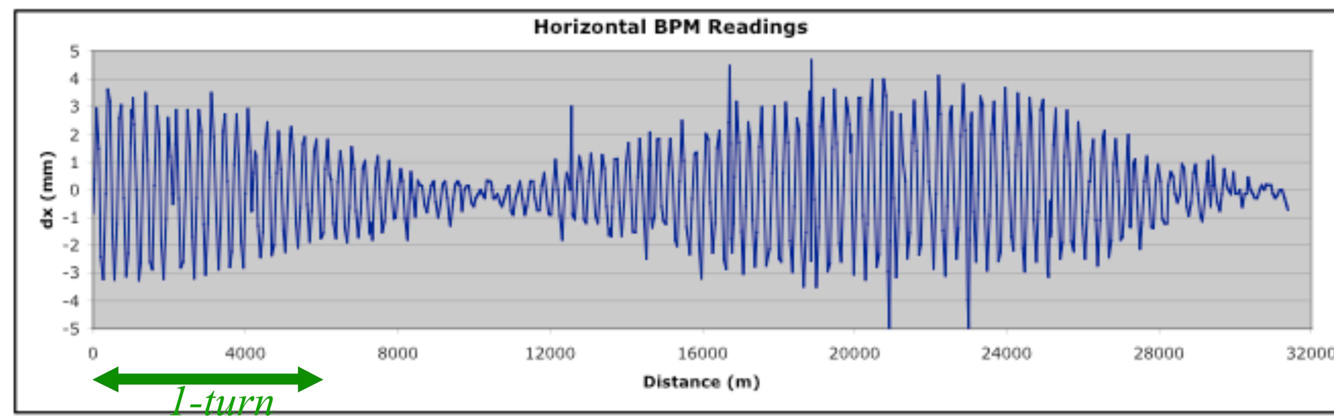
An Extreme Case: The Coupled Tevatron

In early 2000's, Tevatron measurements of the minimum tune split showed that there was a strong source of coupling somewhere. Where?? So, went looking for the source(s)…

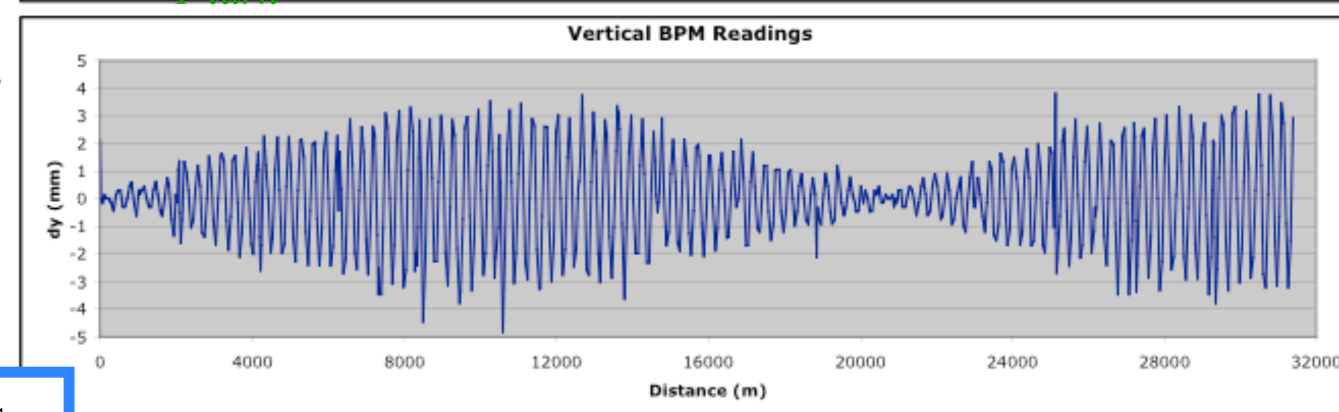
The Measurement …

Inject with horizontal oscillation and look for source of vertical oscillation…

Hor BPM's



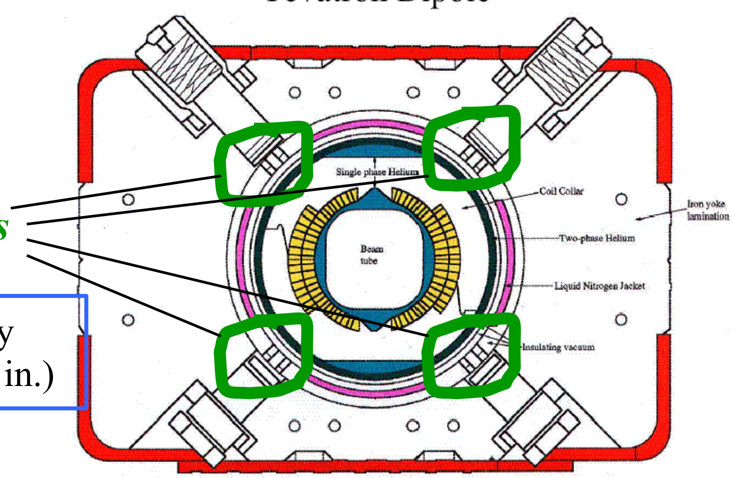
Ver BPM's



The source is *everywhere*!

Syphers/APS Apr04 5

Tevatron Dipole

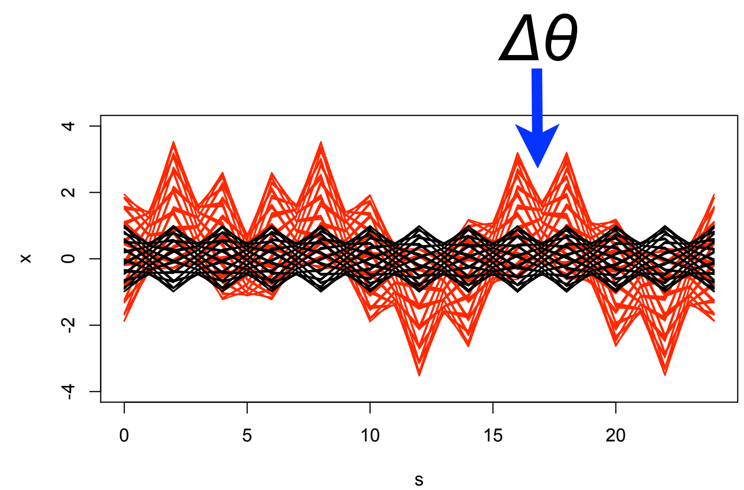


G-11 spacers

Compressed by 6 mil (0.006 in.)

Linear Resonances in Circular Accelerators

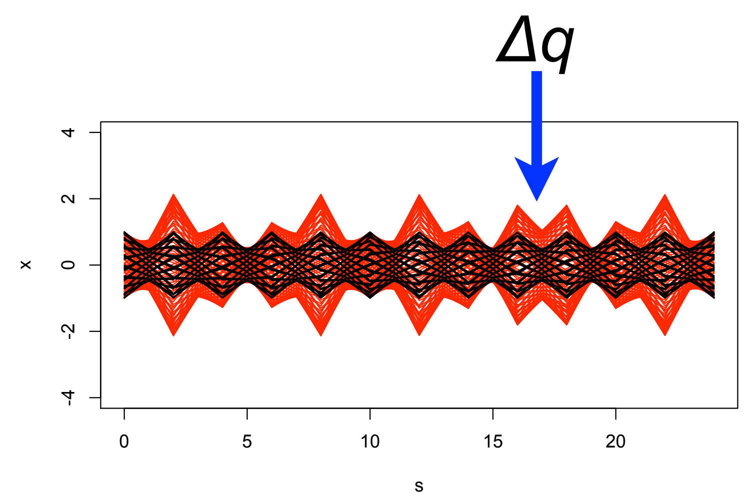
- Guide-field errors
  - the 'closed' trajectory about the synchrotron will become distorted -- average beam trajectory must be adjusted using corrector magnets
- Focusing field errors
  - distortions of the beam envelope
- Thus, avoid tune values = *integer*, *integer*/2



black = ideal
red = distortion

$$\Delta x \sim \frac{1}{\sin \pi \nu}$$

Orbit distortion due to single dipole field error



$$\Delta\beta/\beta \sim \frac{1}{\sin 2\pi \nu}$$

Envelope Error (Beta-beat) due to gradient error

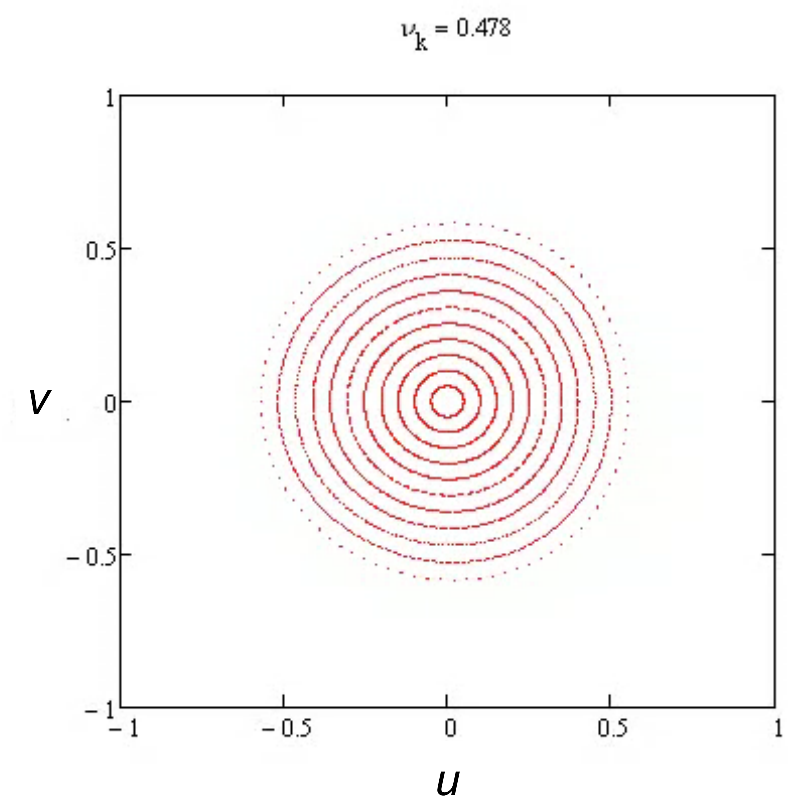
Effect on Phase Space due to Single Sextuple

- Track the trajectory of a particle around an ideal ring, but include the kick from a single sextupole every revolution:

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{n+1} = \begin{pmatrix} \cos\mu + \alpha\sin\mu & \beta\sin\mu \\ -\gamma\sin\mu & \cos\mu - \alpha\sin\mu \end{pmatrix} \begin{pmatrix} x \\ x' - Sx^2 \end{pmatrix}_n \qquad \mu = 2\pi\nu$$

  - transform to new coordinates: $u \equiv \beta S x, \quad v \equiv \beta S(\alpha x + \beta x')$

$$\begin{pmatrix} u \\ v \end{pmatrix}_{n+1} = \begin{pmatrix} \cos 2\pi\nu & \sin 2\pi\nu \\ -\sin 2\pi\nu & \cos 2\pi\nu \end{pmatrix} \begin{pmatrix} u \\ v - u^2 \end{pmatrix}_n$$



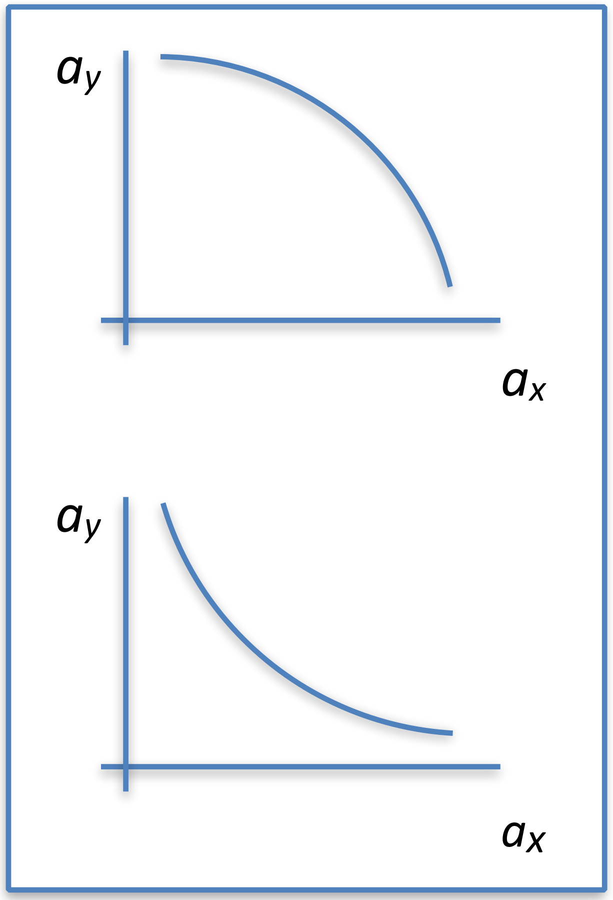
- The topology of the phase space here only depends upon the choice of tune, ν.
- With nonlinear fields present, must avoid rational tunes:
  - integer, integer/2, integer/3, ...

Coupling Resonances

- We've seen that coupling produces conditions where the motion in one plane (x) can depend upon the motion in the other plane (y) and *vice versa*. When the frequencies of the coupled motion create integer relationships, then coupling resonances can occur:

$$m\,\nu_x \pm n\,\nu_y = k$$

  - In general, a "difference" resonance will simply exchange the energy between the two planes, back and forth, but the motion remains bounded
  - A "sum" resonance will exchange energy, but the overall motion can become unbounded



avoid ALL rational tunes???

Tune Diagram

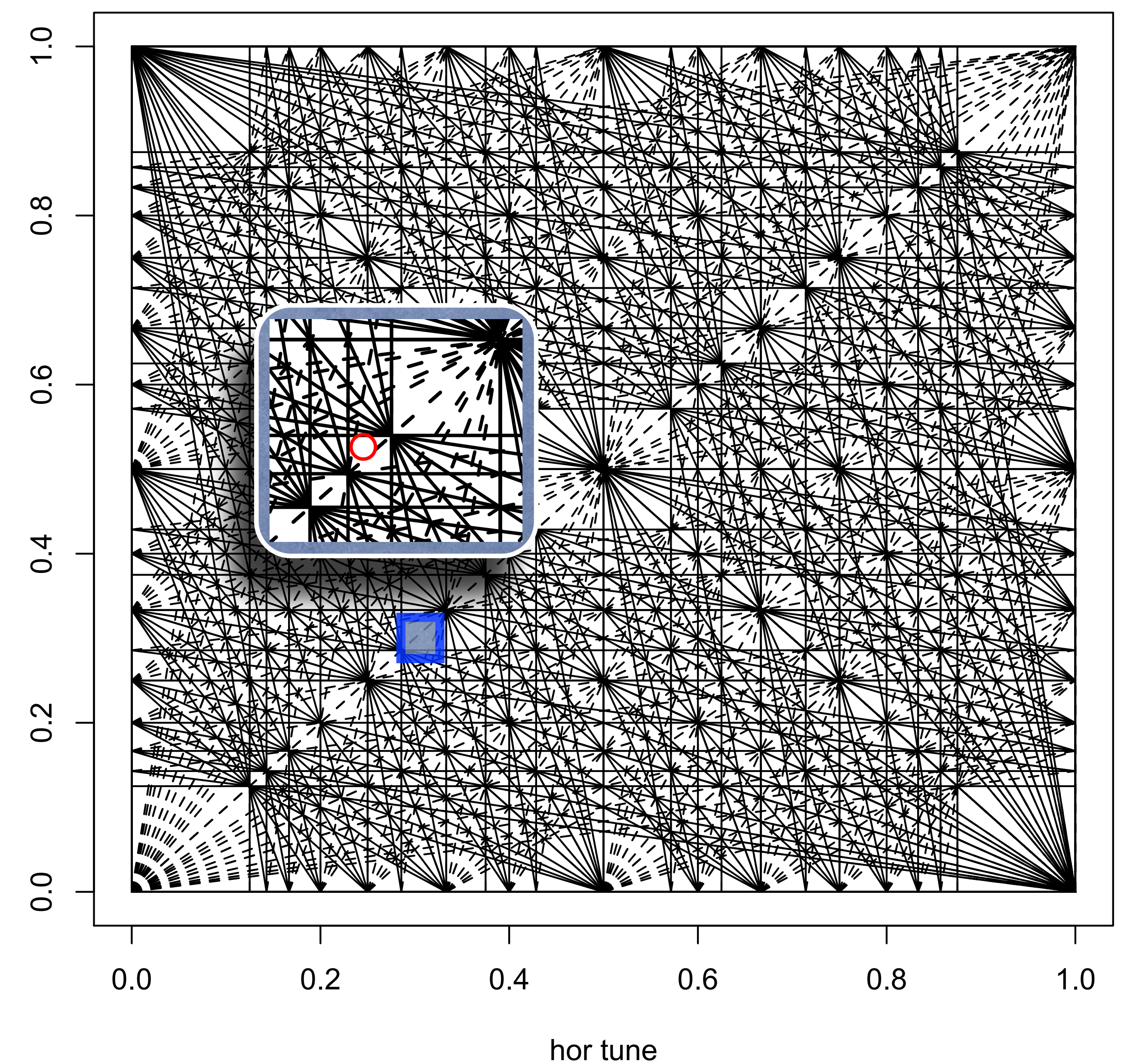
lines of $m\,\nu_x \pm n\,\nu_y = k$

Through order k =2

Through order k =3

Through order k =5

Through order k =8



width ~ 0.025

Note:
In LHC,
~60 oscillations
per revolution

Dynamic Aperture and Design Criteria

- Computations of dynamic aperture began in earnest during the Tevatron design studies
- SSC Design Study, LHC design study:

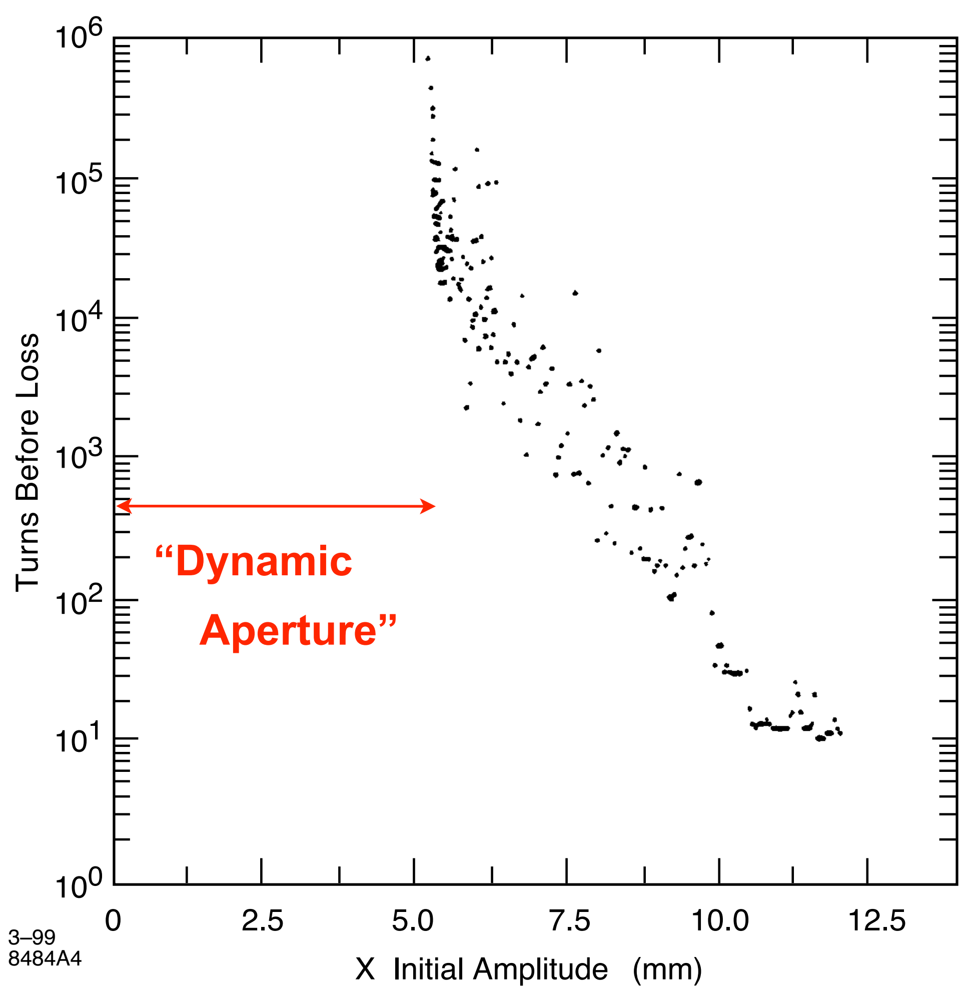


Figure 3. Survival plot for an SSC model.

Y. Yan, *et al.*,SSCL Report 303 (1990)

- Generate a model of the accelerator with error fields
  - systematic
  - random
  - "Track" particles with various initial oscillation amplitudes; record survival times

Space Charge Force for Gaussian Beam

Gauss:

$$2\pi r E \ell = \frac{QeN'\ell}{2\pi\sigma^2\epsilon_0}\int_0^{2\pi}\int_0^{r} e^{-\frac{r^2}{2\sigma^2}}\, r dr d\theta$$

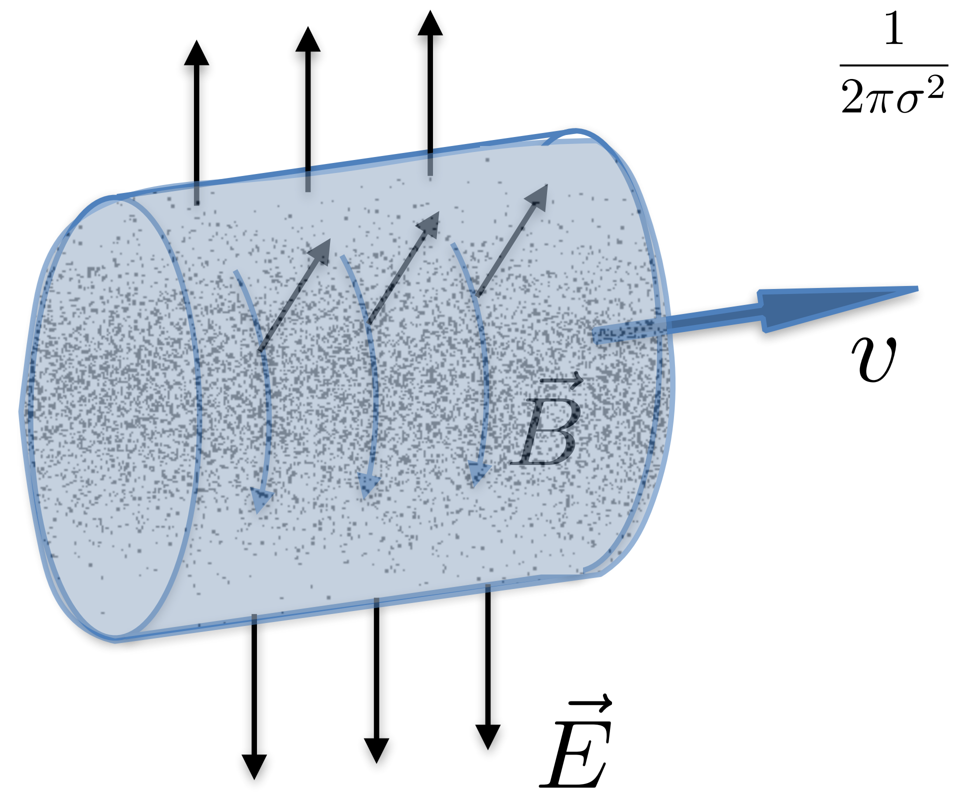
$$= \frac{QeN'\ell}{\epsilon_0}\left(1 - e^{-\frac{r^2}{2\sigma^2}}\right)$$

$$F_E = \frac{Q^2e^2N'}{2\pi\epsilon_0}\,\frac{1 - e^{-\frac{r^2}{2\sigma^2}}}{r}$$

Ampere: $$F_B = -\frac{Q^2e^2N'}{2\pi\epsilon_0}\,\frac{v^2}{c^2}\,\frac{1 - e^{-\frac{r^2}{2\sigma^2}}}{r}$$

each particle of charge Qe

$$\frac{1}{2\pi\sigma^2}\, e^{-\frac{r^2}{2\sigma^2}}$$

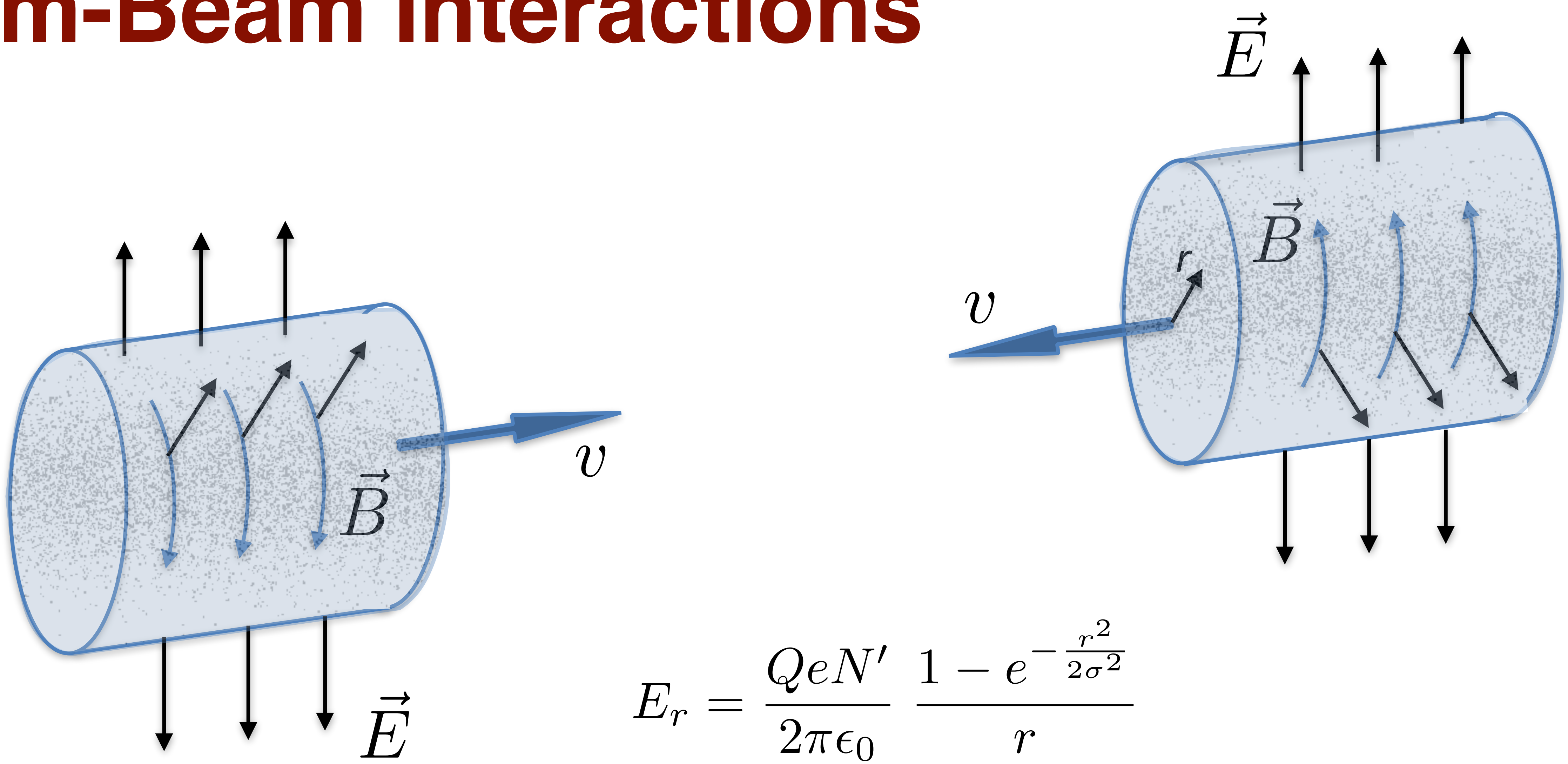


and so, …

$$F_{tot} = \frac{Q^2e^2N'}{2\pi\epsilon_0\gamma^2}\,\frac{1 - e^{-\frac{r^2}{2\sigma^2}}}{r} \longrightarrow \frac{Q^2e^2N'}{4\pi\epsilon_0\gamma^2\sigma^2}\, r$$

for $r \ll \sigma$

Beam-Beam Interactions



$$E_r = \frac{QeN'}{2\pi\epsilon_0}\,\frac{1-e^{-\frac{r^2}{2\sigma^2}}}{r}$$

$$B_\theta = \frac{\mu_0 QeN'v}{2\pi}\,\frac{1-e^{-\frac{r^2}{2\sigma^2}}}{r} = \frac{QeN'v}{2\pi\epsilon_0 c^2}\,\frac{1-e^{-\frac{r^2}{2\sigma^2}}}{r}$$

Here, *v* of the "test" particle is opposite that of the other beam

Beam-Beam Interaction in a Collider

- Previous Space Charge Calculation:

small, for very high γ

$$\vec{F} = Qe[\vec{E} + \vec{v} \times \vec{B}] \quad \rightarrow \quad F = \frac{Q^2 e^2 N'}{4\pi\epsilon_0 \sigma^2} \left(1 - v^2/c^2\right) r \quad \rightarrow \quad = \frac{Q^2 e^2 N'}{4\pi\epsilon_0 \gamma^2 \sigma^2} \, r$$

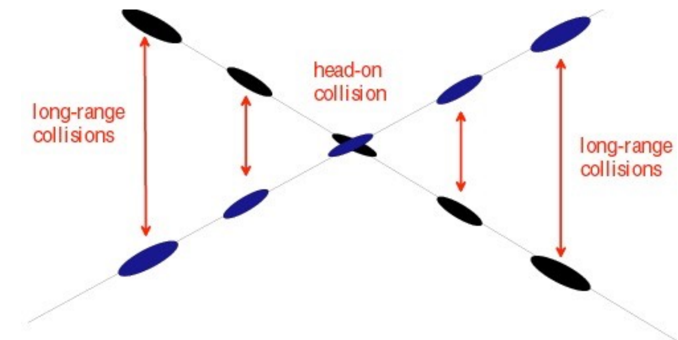
for $r << \sigma$

- For the Beam-Beam Interaction

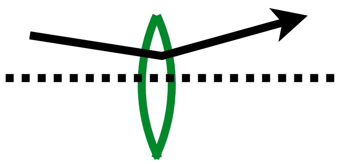
$$\vec{F} = Qe[\vec{E} + \vec{v} \times \vec{B}] \quad \rightarrow \quad F = \frac{Q^2 e^2 N'}{4\pi\epsilon_0 \sigma^2} \left(1 + v^2/c^2\right) r \quad \rightarrow \quad = \frac{Q^2 e^2 N'}{2\pi\epsilon_0 \sigma^2} \, r$$

  - for like-charges, the force is radially out, hence *defocusing*
    - for the central particles in the bunch, acts like a defocusing lens

Beam-Beam Interaction



- Thin lens approximation, round beams:



$$\Delta r' = \frac{\Delta p_r}{p} = \frac{F_r \cdot \ell_{int}/v}{p} = \frac{1}{pv}\frac{Q^2 e^2}{2\pi\epsilon_0\sigma^2}\, r \int_0^{\ell_{int}} N' ds$$

[redacted] $= \frac{1}{2}N$ *N* = no. particles / bunch

So,

$$\Delta r' = \frac{Q^2 e^2 N}{4\pi\epsilon_0 mc^2\gamma\beta^2\sigma^2}\, r \qquad \frac{1}{f} = -\frac{Q^2 e^2 N}{4\pi\epsilon_0 mc^2\gamma\beta^2\sigma^2} = -\frac{Q^2 r_0 N}{\gamma\sigma^2}$$

r_0 = "classical radius"

- tune shift

$$\Delta\nu = \frac{1}{4\pi}\beta^* \cdot \frac{1}{f} = -\frac{Q^2 r_0 N}{4}\cdot\frac{\beta^*}{\gamma\pi\sigma^2} \equiv -\xi \qquad (Q=1)$$

$$\boxed{\Delta\nu_{bb} = -\frac{Q^2 r_0 N}{4\epsilon_N}}$$

Typical range of values for ξ:
~0.01, proton colliders
~0.1, electron colliders

Note: *if the colliding particles have **opposite** signs, will be a **focusing** effect rather than defocusing*

Beam-Beam Interaction

- The beam-beam interaction will also alter the amplitude function, β, at the IP and around the ring

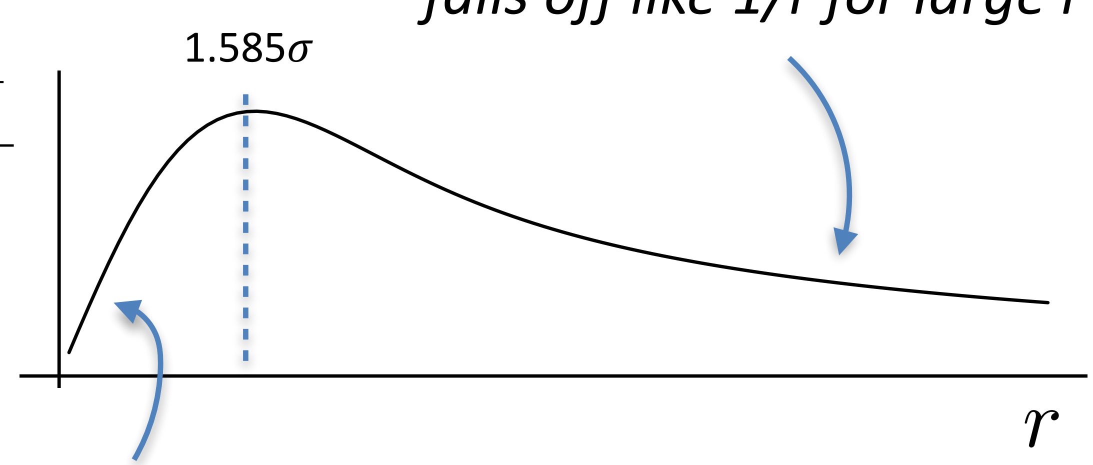
$$\frac{\Delta\beta}{\beta}(s) \approx -\frac{\Delta q \beta_0}{2\sin 2\pi\nu}\cos(2|\Delta\psi| - 2\pi\nu) \quad \text{then} \quad \frac{\Delta\beta^*}{\beta^*} \approx -2\pi\xi\cot(2\pi\nu)$$

(can be *several* %)

- Force is actually nonlinear —> $F_{tot} = \frac{Q^2 e^2 N'}{\pi\epsilon_0}\,\frac{1 - e^{-\frac{r^2}{2\sigma^2}}}{r}$
  - thus, a tune *spread*
  - nonlinear resonances
  - tune space

falls off like 1/r for large r

$$\frac{1 - e^{-\frac{r^2}{2\sigma^2}}}{r}$$

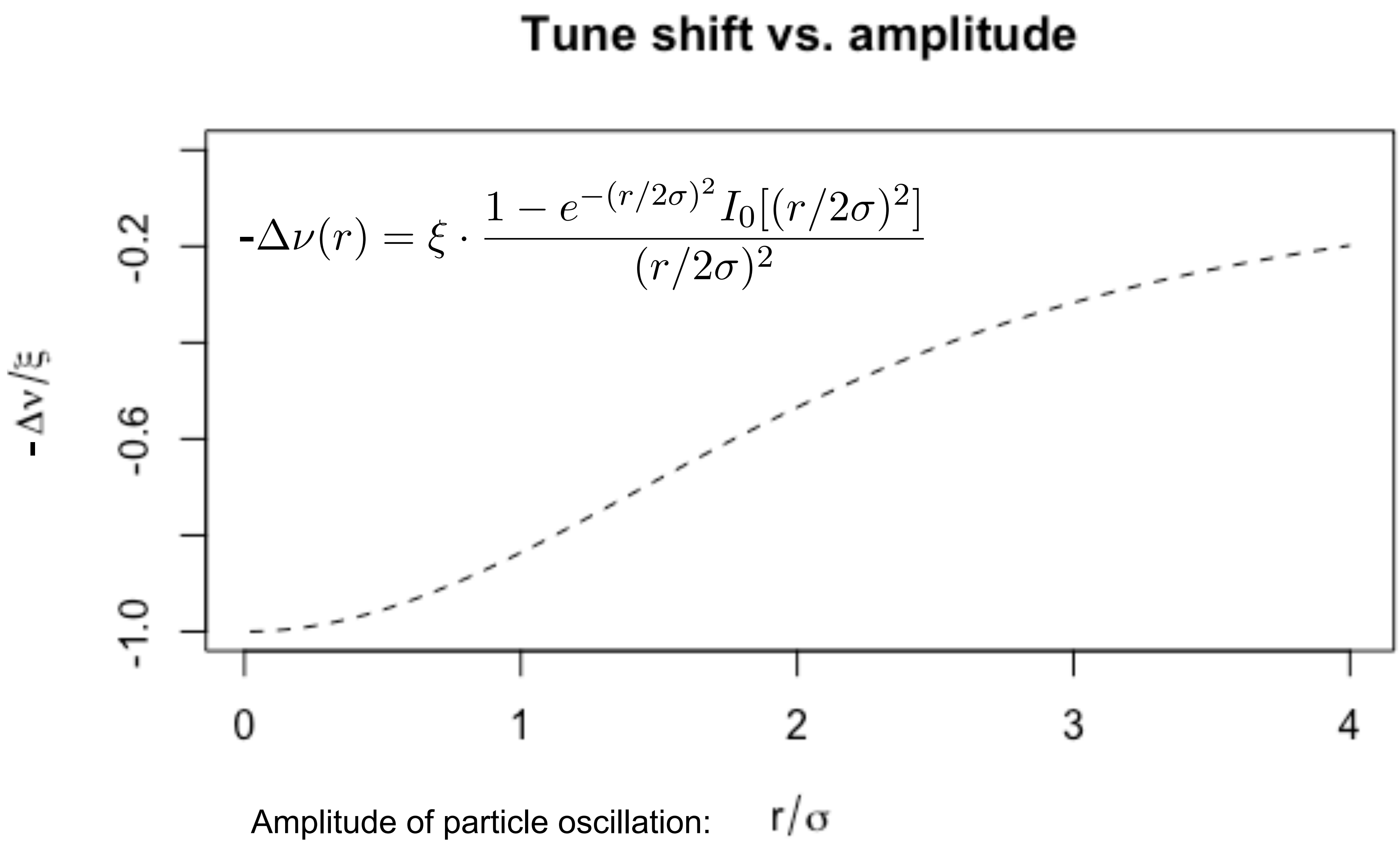


linear defocusing force

The Beam-Beam Tune "shift"

- Due to nonlinear nature of the perturbation, ``the tune" will depend upon the particle's betatron amplitude

$$\xi = \frac{r_0 N}{4\epsilon_N}$$

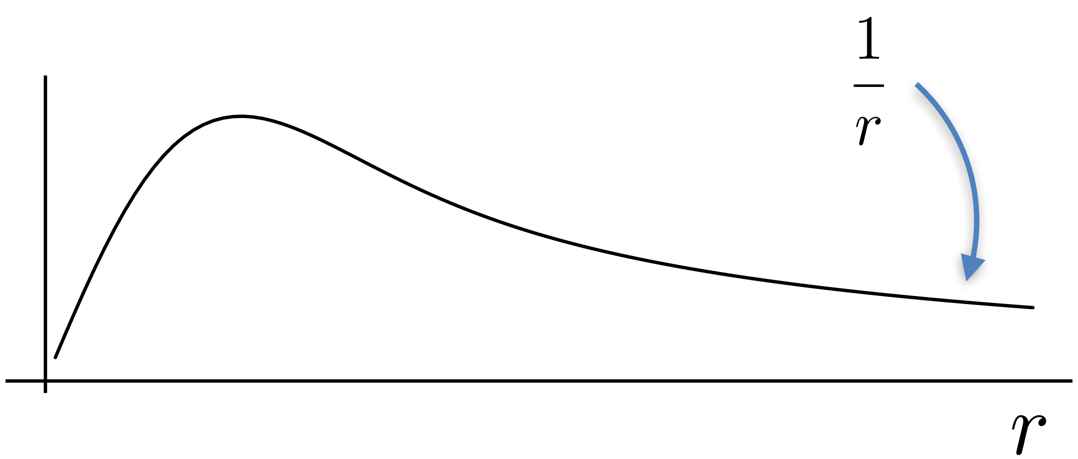


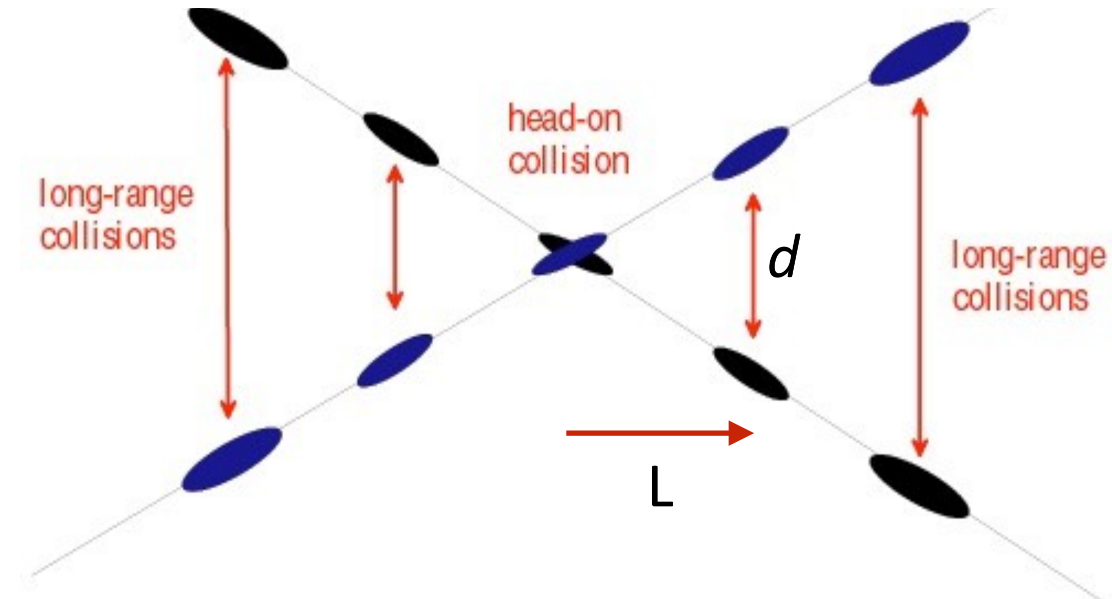
r_0 = "classical radius"
= 1.5×10^{-18} m (proton)

Long-Range Beam-Beam Interactions

- Long-range force: $r >> \sigma$

$$\frac{1-e^{-\frac{r^2}{2\sigma^2}}}{r}$$





If have a central head-on tune spread of $\Delta\nu_{bb} = \frac{Q^2 r_0 N}{4\epsilon_N}$

then **each** long-range interaction will generate a tune shift of

$$\Delta\nu_{LR} = \frac{2\Delta\nu_{bb}}{(d/\sigma)^2}$$

crossing angle, θ ~ d/L

and want $d \sim n\sigma = n\sigma^* (\beta/\beta^*)^{1/2}$

$\beta \sim L^2/\beta^*$, so $(\beta/\beta^*)^{1/2} \sim L/\beta^*$

Thus, the crossing angle:

$$\theta \sim n\ \sigma^*/\beta^*$$

typically choose n ~ 10-12

Luminosity in Accelerator Terms

- Can now express in terms of beam physics parameters; ex.: for short, round beams...

$$\mathcal{L} = \frac{f_0 B N^2}{4\pi\sigma^{*2}} = \frac{f_0 B N^2 \gamma}{4\epsilon\beta^*}$$

+ *x-angle, etc., …*

- If different bunch intensities, different transverse beam emittances for the two beams,

$$\mathcal{L} = \frac{f_0 B N_1 N_2}{2\pi(\sigma_1^{*2} + \sigma_2^{*2})} = \frac{f_0 B N_1 N_2 \gamma}{2\beta^*(\epsilon_1 + \epsilon_2)}$$

and assorted other variations...

In Terms of Beam-Beam Parameter

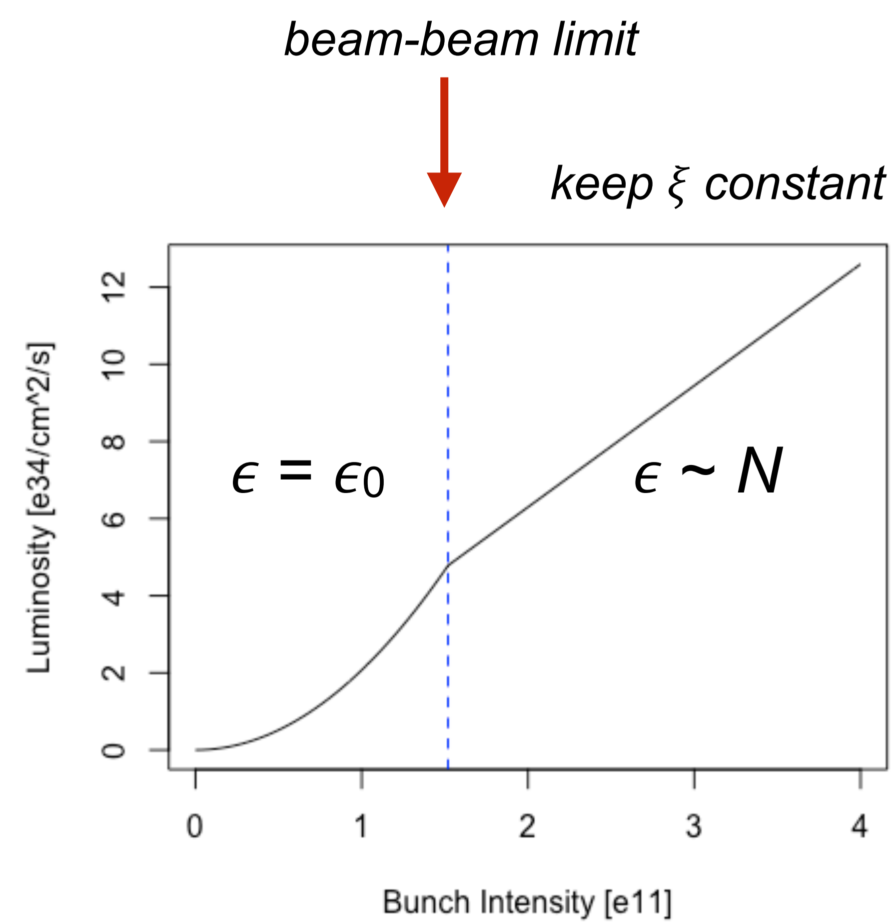
when run at the “beam-beam limit”, …

$$\mathcal{L} = \frac{fN^2}{4\pi\sigma^2} = \frac{f_0 B N^2 \gamma}{4\beta^* \epsilon_N} = \frac{N^2\gamma}{4 t_b \beta^* \epsilon_N} = \frac{\xi\gamma N}{t_b r_0 \beta^*}$$

t_b = bunch spacing, r_0 = 1.58x10^{-16} cm (protons)

LHC Example:

$$\mathcal{L} = \frac{(0.01)(7000)(10^{11})}{(25\times 10^{-9}\text{s})(1.58\times 10^{-16}\text{cm})(50\text{ cm})} \sim 10^{34}$$



Integrated Luminosity Revisited

- Luminosity lifetime not just dictated by reduction of the beam intensity but also by dynamical variation of emittance, *etc*., that also play role into the instantaneous luminosity

$$\mathcal{L} = \frac{f_0 B N^2 \gamma}{4\beta^* \epsilon_N} \cdot \frac{1}{1 + (\alpha\sigma_z/2\sigma_0)^2}$$

  - can have *N* decrease due to collisions, but also
    - $d\epsilon/dt$ due to scattering, noise, synch rad, etc.
    - $d\sigma_z/dt$ due to intrabeam scattering, noise, etc
  - can generate purposeful variations in β^*, x-angle to optimize luminosity lifetime

Emittance Control

- Electrons radiate extensively at high energies; combined with energy replenishment from RF system, small equilibrium emittances are a result
- For Hadron Colliders, emittance at collision energy determined by proton source, and its control through the injectors, acceleration in Collider, low-β squeeze, etc.

- larger emittance -- smaller instantaneous luminosity
- larger emittance growth rates during collisions result in particle loss
  - thus, lower integrated luminosity

Non-adiabatic Disturbances
Example: Discharge of a beam kicker in a synchrotron

- Initially, the distribution is simply "displaced" by the action of the kick:
- Nonlinearities will yield:
  - tune vs. amplitude
  - decoherence
  - *filamentation*
  - emittance growth

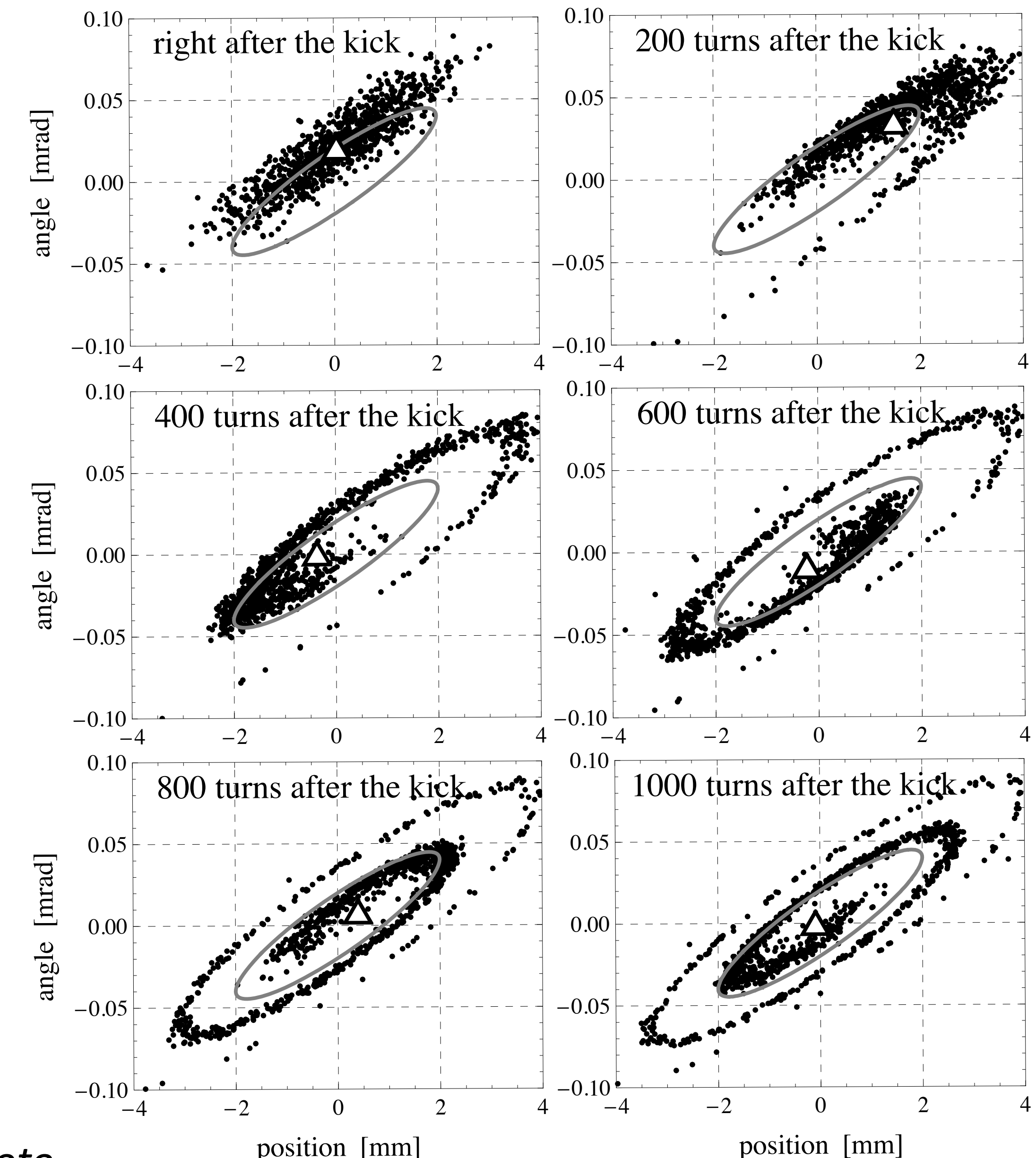
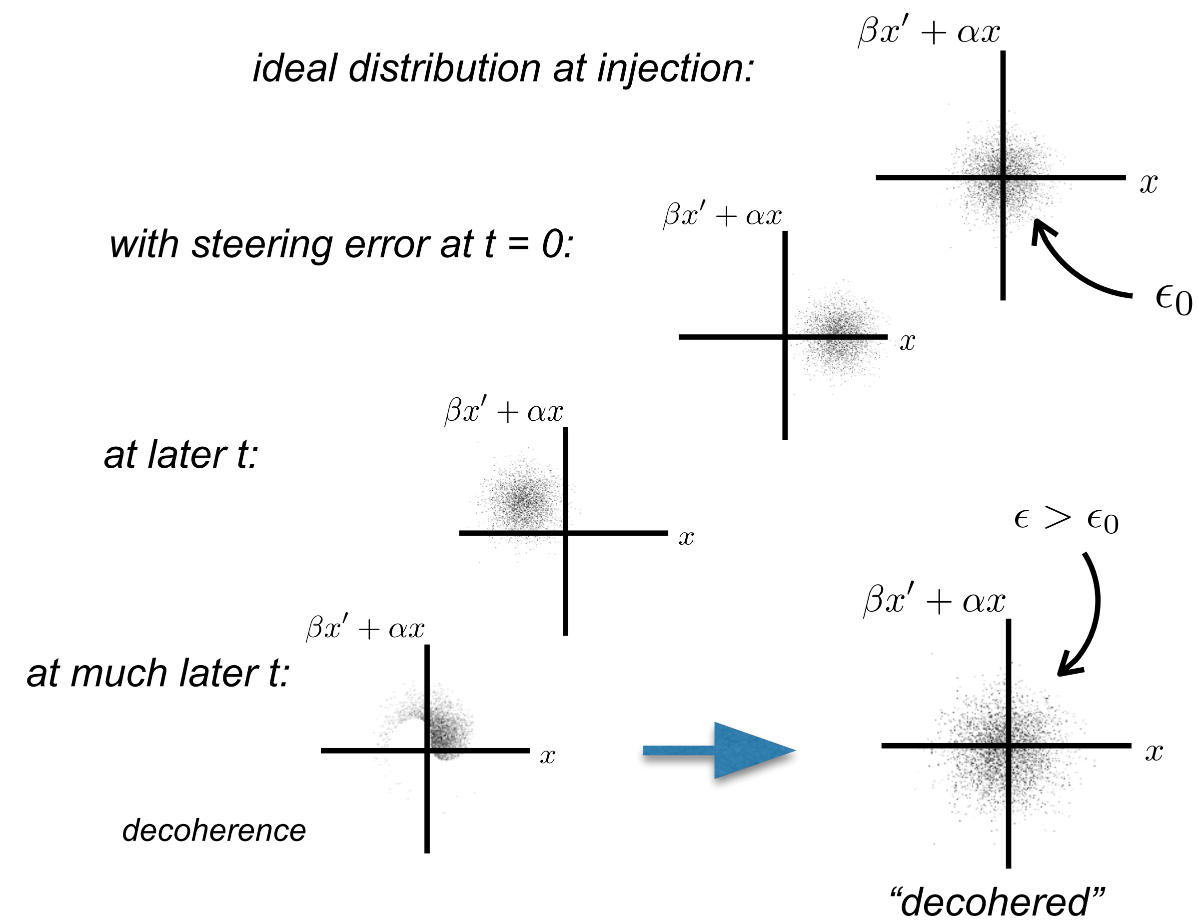


figure by R. Miyamoto

Accelerator Model

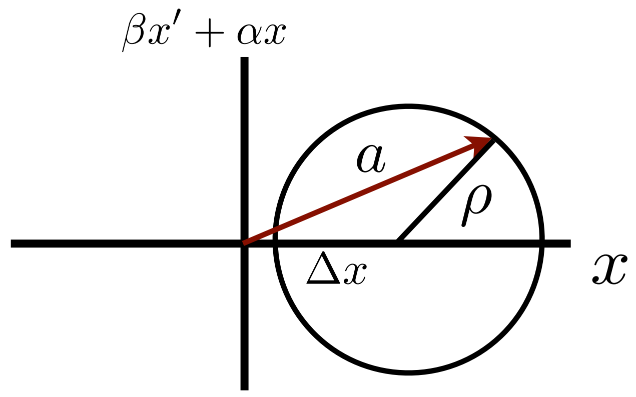
- So we will model these effects by assuming the distribution will oscillate about the closed orbit, and that the oscillation frequencies of the particles will depend upon the amplitude of their oscillations
  - typically: $\nu \approx \nu_0 + ka^2$

    (nonlinear tune shift)
  - coherent at first,
  - then “decoheres”
    - leads to filamentation
  - eventually larger emittance



Example: Injection Steering Mismatch

How does rms evolve?



if $\Delta x = 0$, would have $\langle x^2 \rangle = \frac{1}{2}\langle \rho^2 \rangle \equiv \sigma_0^2$

but here, $$a^2 = \rho^2 + \Delta x^2 - 2\Delta x \rho \cos\phi$$

average over all particles: $$\langle a^2 \rangle = \langle \rho^2 \rangle + \Delta x^2 - 2\Delta x \langle \rho \cos\phi \rangle$$

~ 0

$$\longrightarrow \langle a^2 \rangle = \langle \rho^2 \rangle + \Delta x^2 \quad \text{after decoherence}$$

$$\longrightarrow \langle x^2 \rangle = \sigma_0^2 + \frac{1}{2}\Delta x^2$$

if $\epsilon \equiv \dfrac{\pi \sigma_0^2 \gamma}{\beta(s)}$

$$\Delta \epsilon_N = \frac{\pi \gamma}{2} \frac{\Delta x_e^2}{\beta(s)}$$

or,

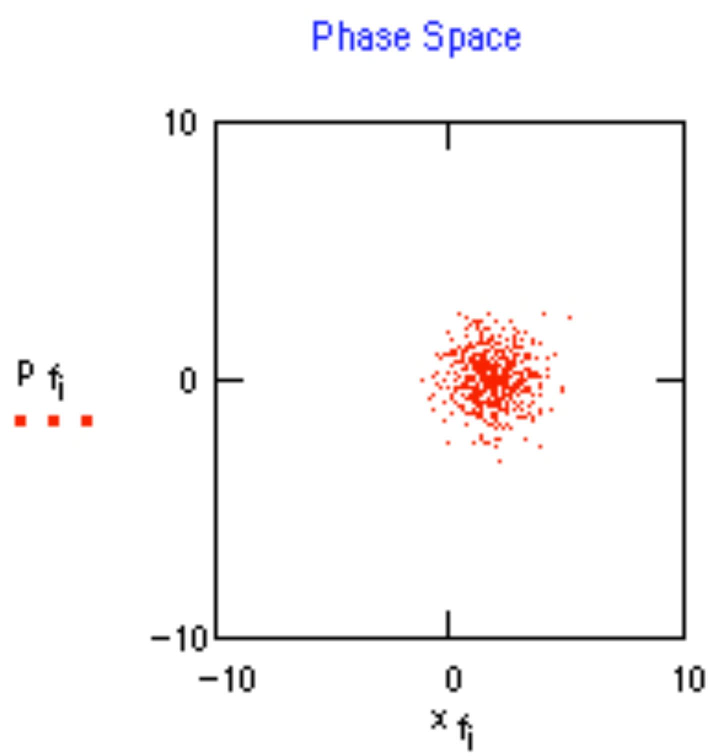
$$\epsilon/\epsilon_0 = 1 + \frac{1}{2}\left(\frac{\Delta x_e}{\sigma_0}\right)^2$$

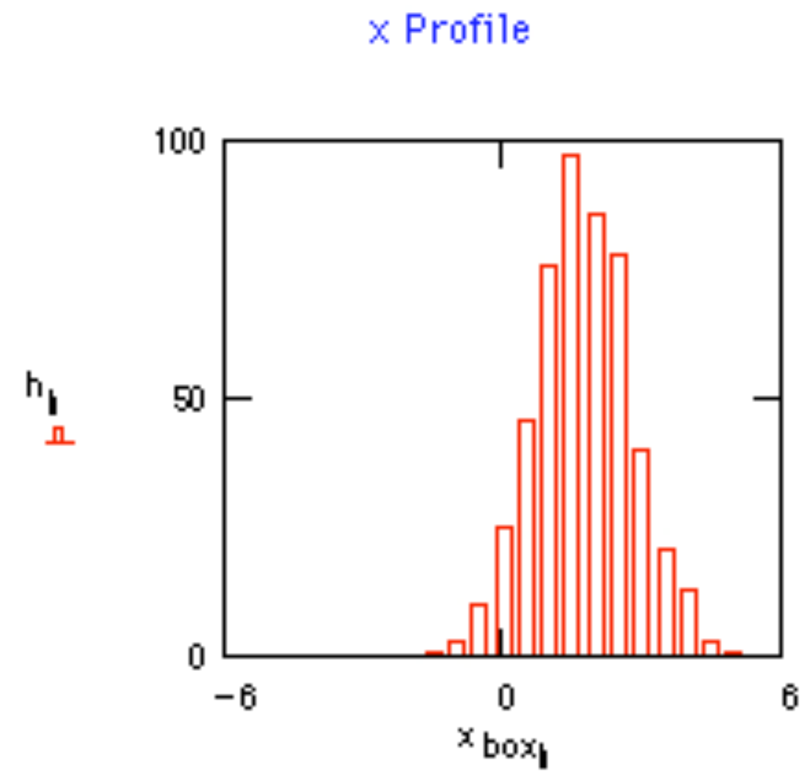
β @ location of Δx

$$\Delta x_e \equiv \sqrt{\Delta x^2 + (\beta_0 \Delta x' + \alpha_0 \Delta x)^2}$$

Note: emittance growth ~ γ; thus, more important at higher energies

Injection Steering Mismatch





$\text{mean}(x_f) = 1.985$ $\text{stdev}(x_f) = 1.039$

Emittance Increase: $\text{stdev}(x_f)^2 = 1.08$

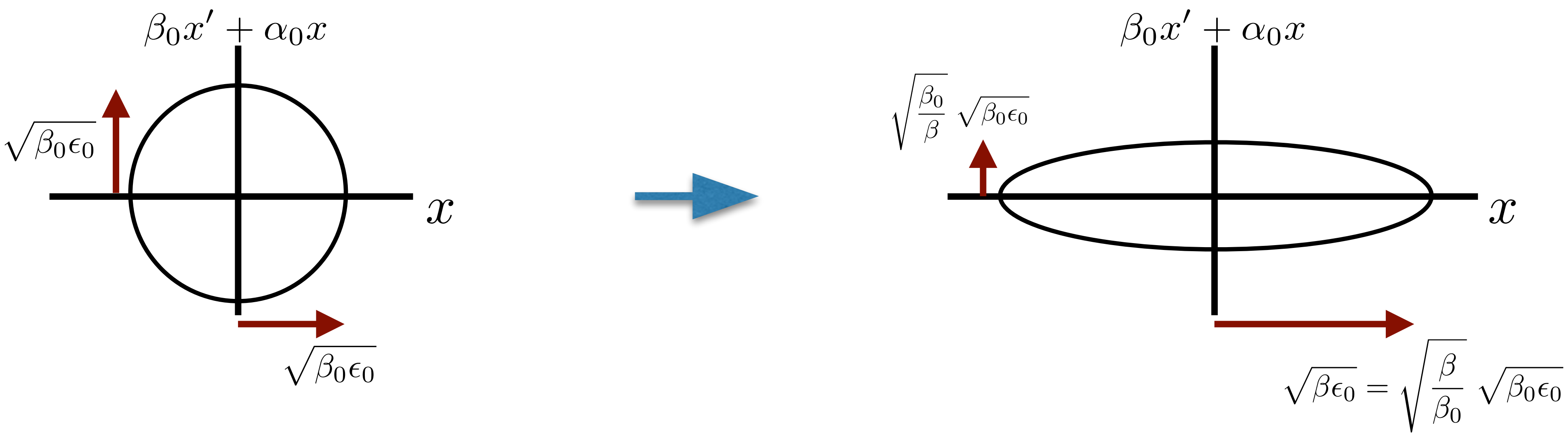
Predicted "typical" values: (Steering Mismatch) $1 + \frac{1}{2}\cdot\Delta x^2 = 3$

FRAME = 0 (Amplitude function Mismatch) $\frac{r_\beta^2 + 1}{2\cdot r_\beta} = 1$

Injection "Beta" Mismatch

- We imagine a ring with an ideal amplitude function, β, at an injection point. But, suppose the beam line transporting beam from an upstream injector delivers the wrong β function…

but keep α = α₀ = 0



β_0, α_0: periodic functions of the synchrotron
β, α: functions delivered by the beam line

$$b \equiv \beta/\beta_0$$

Then, after the distribution tumbles and filaments in phase space, the emittance will have grown…

$$\langle a^2\rangle = 2\sigma^2 = (\sqrt{b}\,\sigma_0)^2 + (\frac{1}{\sqrt{b}}\,\sigma_0)^2 \qquad 2\sigma^2 = b\,\sigma_0^2 + \frac{1}{b}\,\sigma_0^2$$

$$\boxed{\epsilon/\epsilon_0 = \frac{1+b^2}{2b}} \approx 1 + \frac{1}{2}(\Delta\beta/\beta_0)^2$$

Injection "Beta" Mismatch

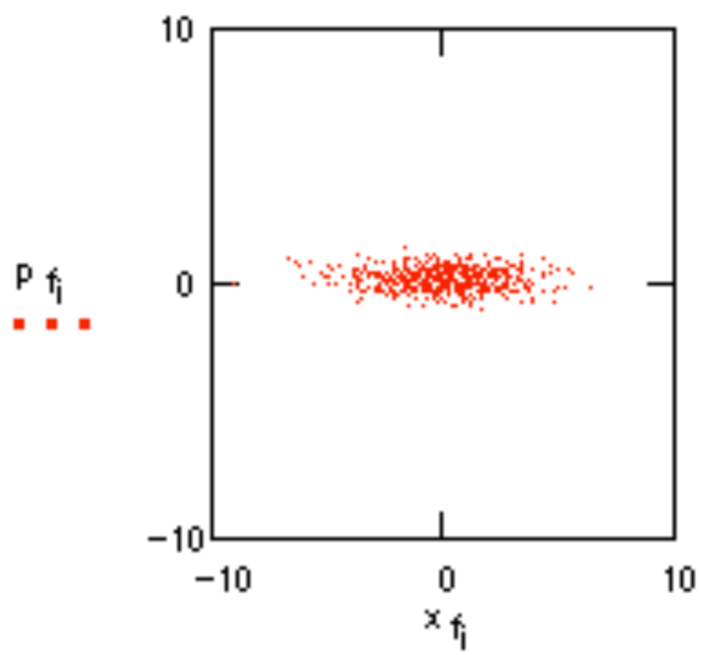
- Can write a more general result in terms of the "mismatch" invariant mentioned earlier:
  - $| \det(\Delta J) | = | \Delta\beta\Delta\gamma - \Delta\alpha^2 | =$ *invariant*
- If inject with *"beam"* parameters α, β, γ, whereas the ring has periodic parameters $\alpha_0, \beta_0, \gamma_0$, then…
- … after filamentation, the final emittance will be given by

$$\Delta J = \begin{pmatrix} \Delta\alpha & \Delta\beta \\ -\Delta\gamma & -\Delta\alpha \end{pmatrix}$$

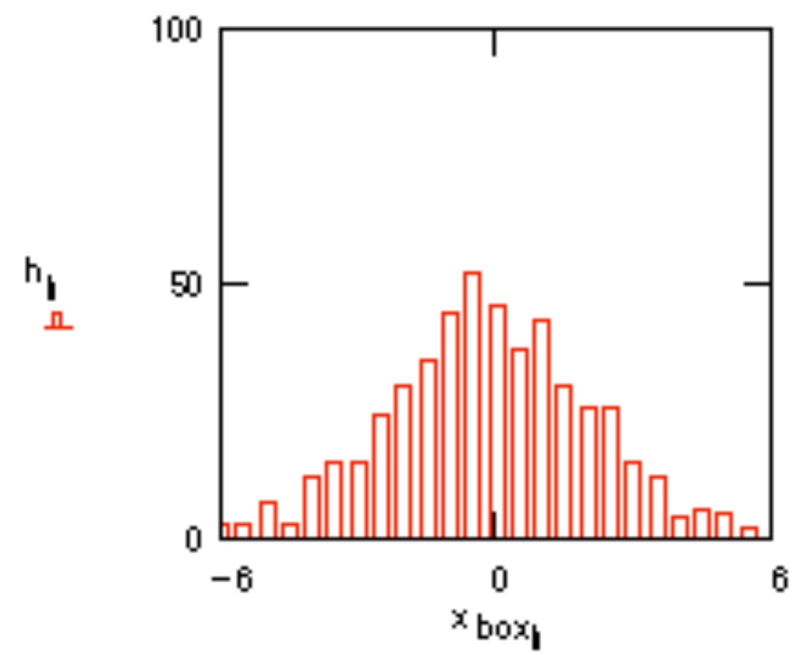
$$\boxed{\epsilon/\epsilon_0 = 1 + \frac{1}{2}|\det \Delta J|}$$

Injection Beta Mismatch

Phase Space



x Profile



$\text{mean}\left(x_f\right) = 0.056$ $\quad$ $\text{stdev}\left(x_f\right) = 2.283$

Emittance Increase: $\text{stdev}\left(x_f\right)^2 = 5.21$

Predicted "typical" values:

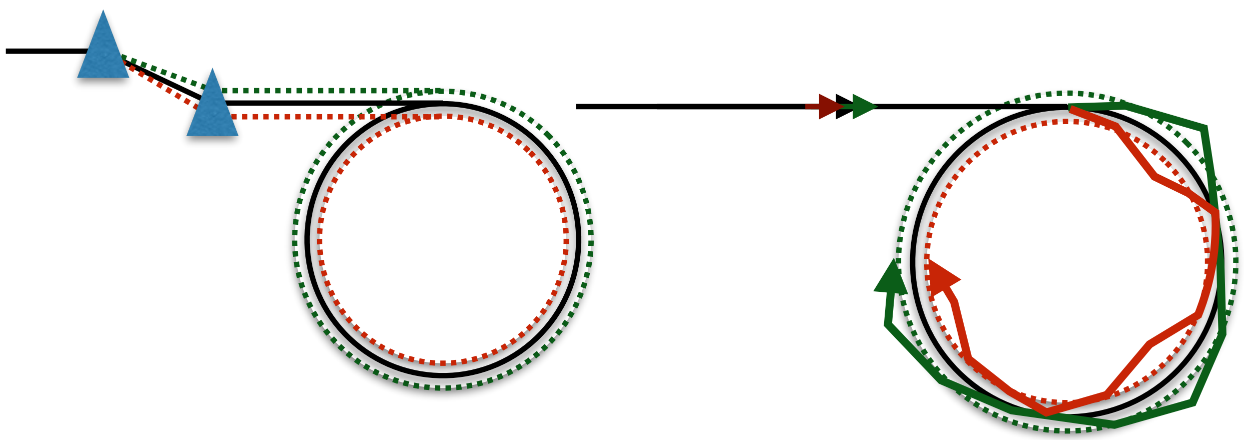
(Steering Mismatch) $1 + \frac{1}{2}\cdot\Delta x^2 = 1$

FRAME = 0

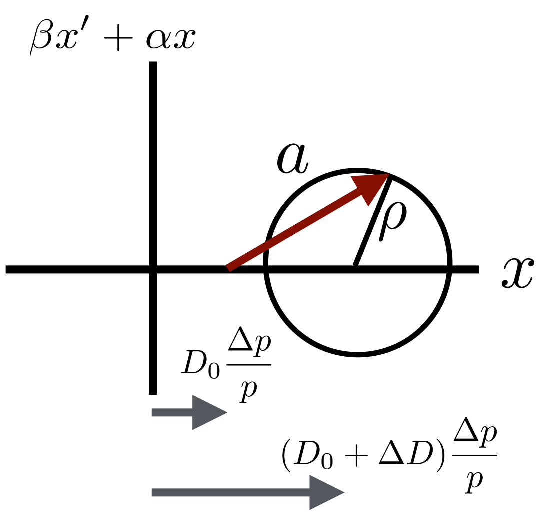
(Amplitude function Mismatch) $1 + \frac{1}{2}\cdot\frac{\delta\beta^2}{1+\delta\beta} = 2.6$

Mismatch of the Dispersion Function

- Can also imagine having the dispersion function entering the accelerator from a beam line having the wrong value
  - amounts to an injection steering error for an off-momentum particle — similar analysis as before



$$\Delta\epsilon_N = \frac{\pi\gamma}{2}\frac{\Delta D_e^2}{\beta}\left(\frac{\sigma_p}{p}\right)^2$$



each particle of momentum $\Delta p/p$ will have an injection steering error of amplitude $\Delta x_p = \Delta D\, \Delta p/p$

$$\langle a^2\rangle = \langle \rho^2\rangle + \Delta D^2 \langle (\frac{\Delta p}{p})^2\rangle$$

especially important if the incoming beam has a large momentum spread

$$\Delta D_e \equiv \sqrt{\Delta D^2 + (\beta_0 \Delta D' + \alpha_0 \Delta D)^2}$$

Diffusion

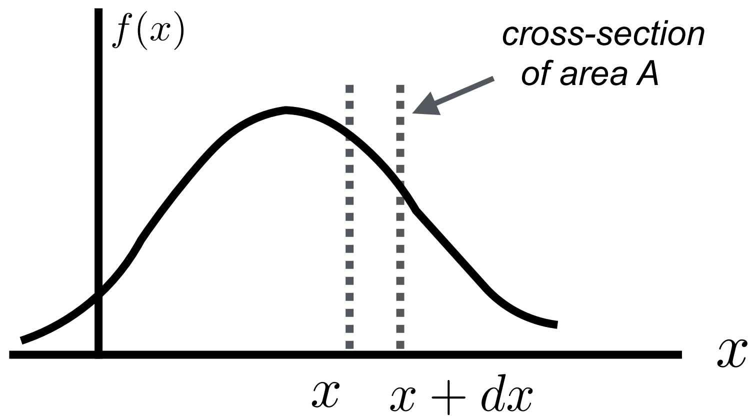
- Random sources (power supply noise; beam-gas scattering in vacuum tube; ground motion) will alter the oscillation amplitudes of individual particles

  - in simplest cases will grow like $\sqrt{N}$, amplitudes of the particle oscillations will eventually reach the limiting aperture

- Thus, beam lifetime will develop, affecting beam intensity, emittance, and thus luminosity

The Diffusion Equation

first, look at 1-D case:



$$\frac{\partial}{\partial t}(f \cdot A \cdot dx) = A \cdot J(x) - A \cdot J(x + dx)$$

 # into *# out of*

J = average number of particles per unit time passing position x

$$\Longrightarrow \frac{\partial f}{\partial t} = -\frac{\partial J}{\partial x}$$

if f uniform, then $J = 0$; otherwise, $J \propto -\frac{\partial f}{\partial x}$

$$\frac{\partial f}{\partial t} = C \cdot \frac{\partial^2 f}{\partial x^2} \qquad C = constant$$

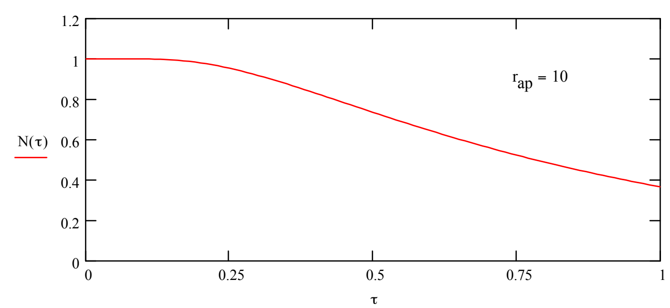
more general 3-D case:

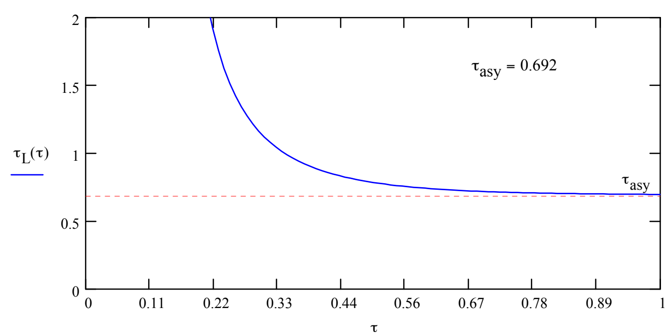
$$\frac{\partial f}{\partial t} = C\, \nabla^2 f$$

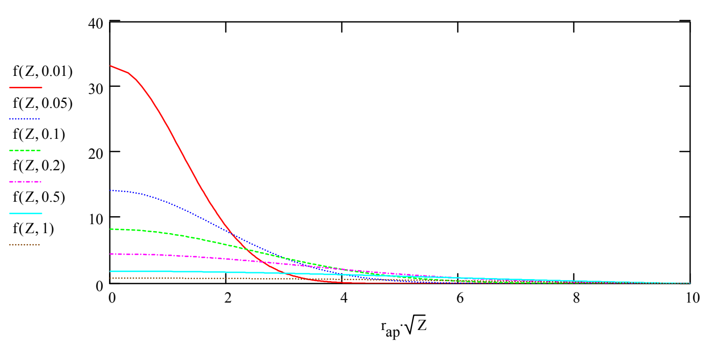
- particle velocities are randomly altered
- particles will move from one region into another
- the rate at which particles cross into or out of a region depends on the *slope* of the distribution function

The Diffusion Equation

- Analytical Calculations:
  - solve, and make plots:







- Numerical Simulations
  - give particles random kicks over time, track in phase space, and plot distribution, etc.

$$\begin{pmatrix} x_{n+1} \\ x'_{n+1} \end{pmatrix} = M_{2\pi\nu} \begin{pmatrix} x_n \\ x'_n + \Delta\theta_n \end{pmatrix}$$

random number each time, determined by the process

Transverse Diffusion — Scattering

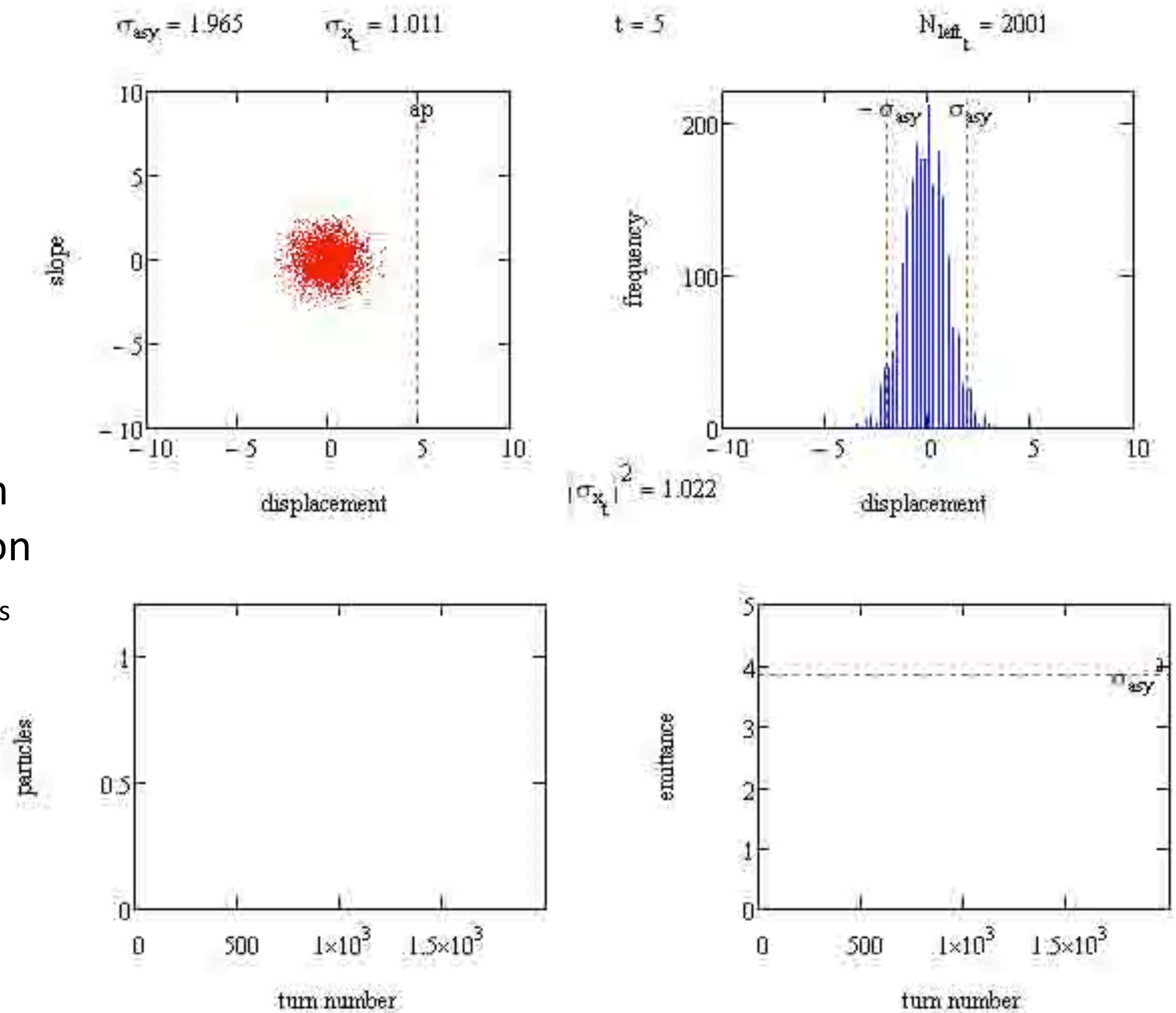
Phase Space

Beam Profile

each particle gets a random "kick" in x' each turn, taken from a Gaussian distribution with rms value of θ_{rms}

Beam Intensity

Beam Emittance



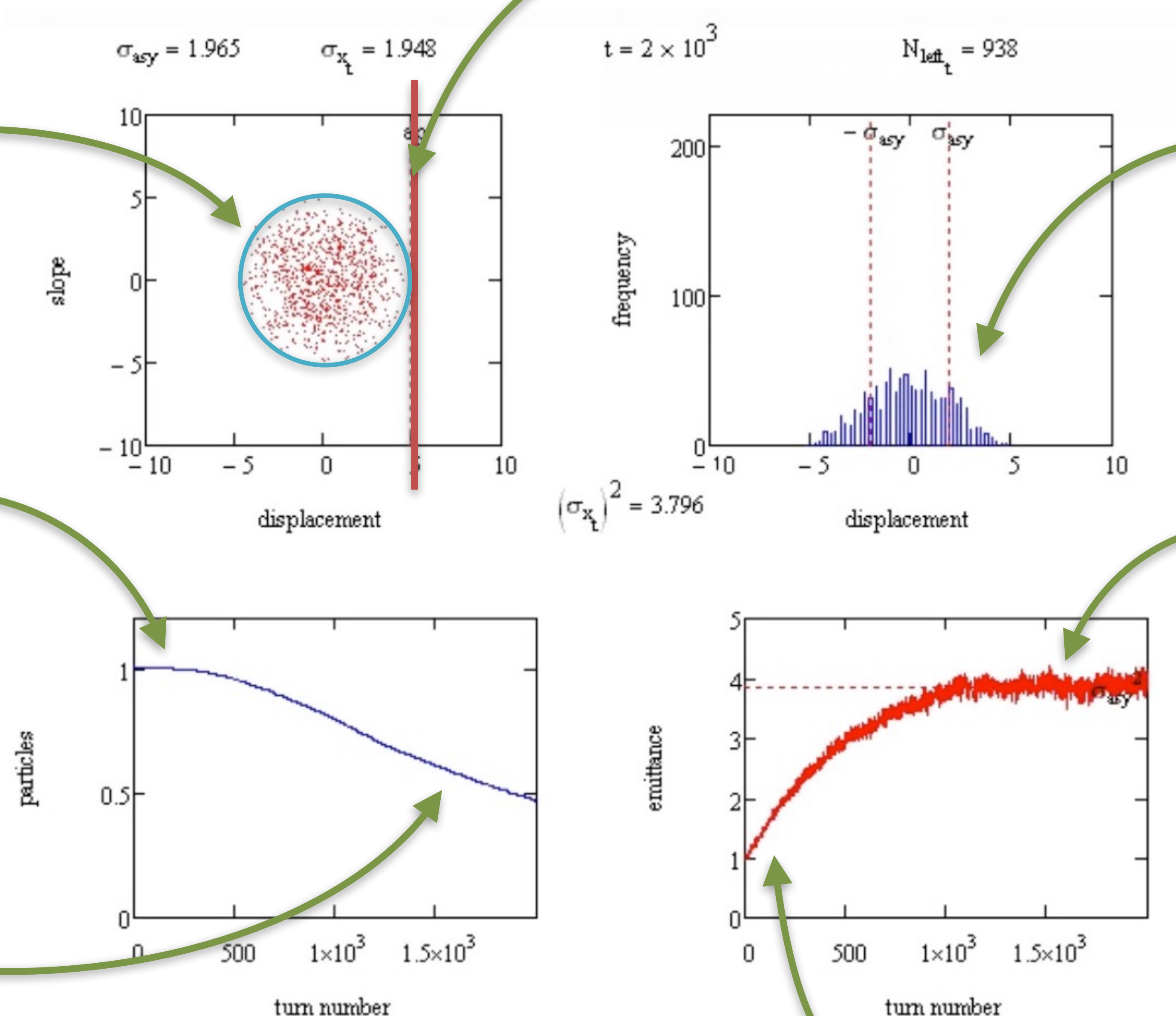
Transverse Diffusion

aperture at x = a

$W = \pi a^2/\beta$

final equilibrium distribution is well-defined

beam has not reached the aperture yet



no *apparent* emittance growth, but particle amplitudes are indeed growing, and particles are being lost

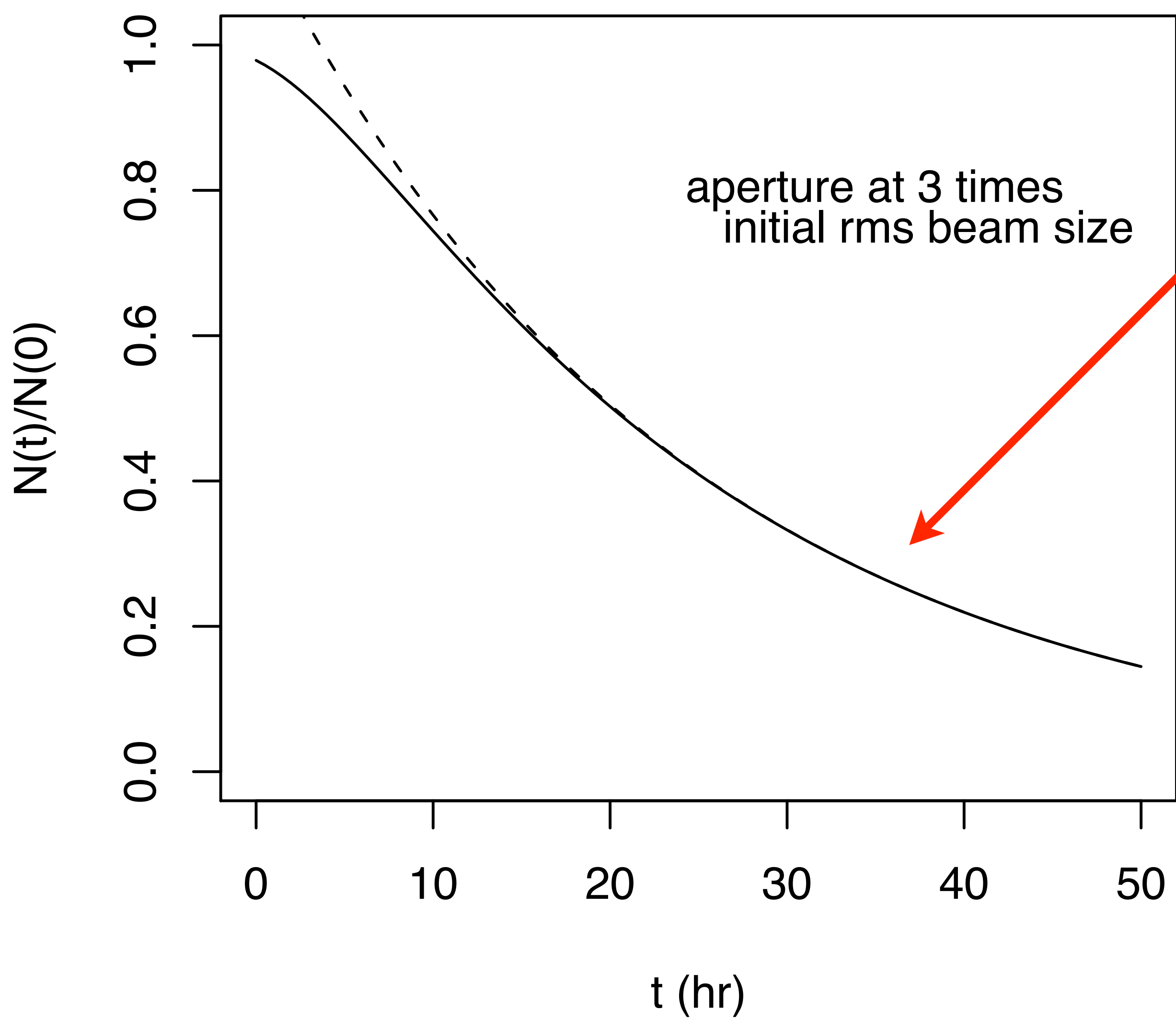
final $N(t) \sim e^{-t/\tau}$

$\tau = (4/2.405^2)(W/R)$

$R = d\epsilon/dt$ = constant *(given by mechanism)*

Effects on Luminosity ...

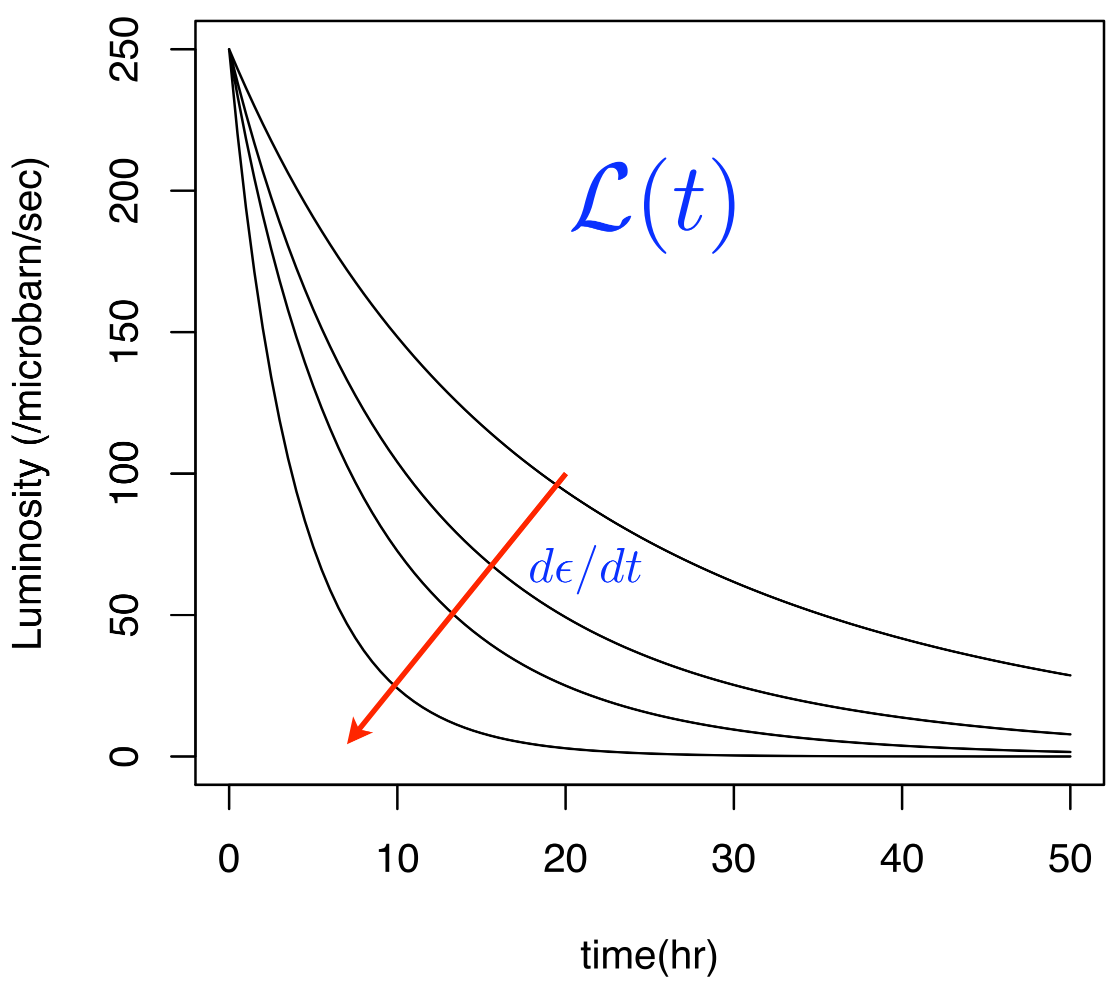
here, no collisions!

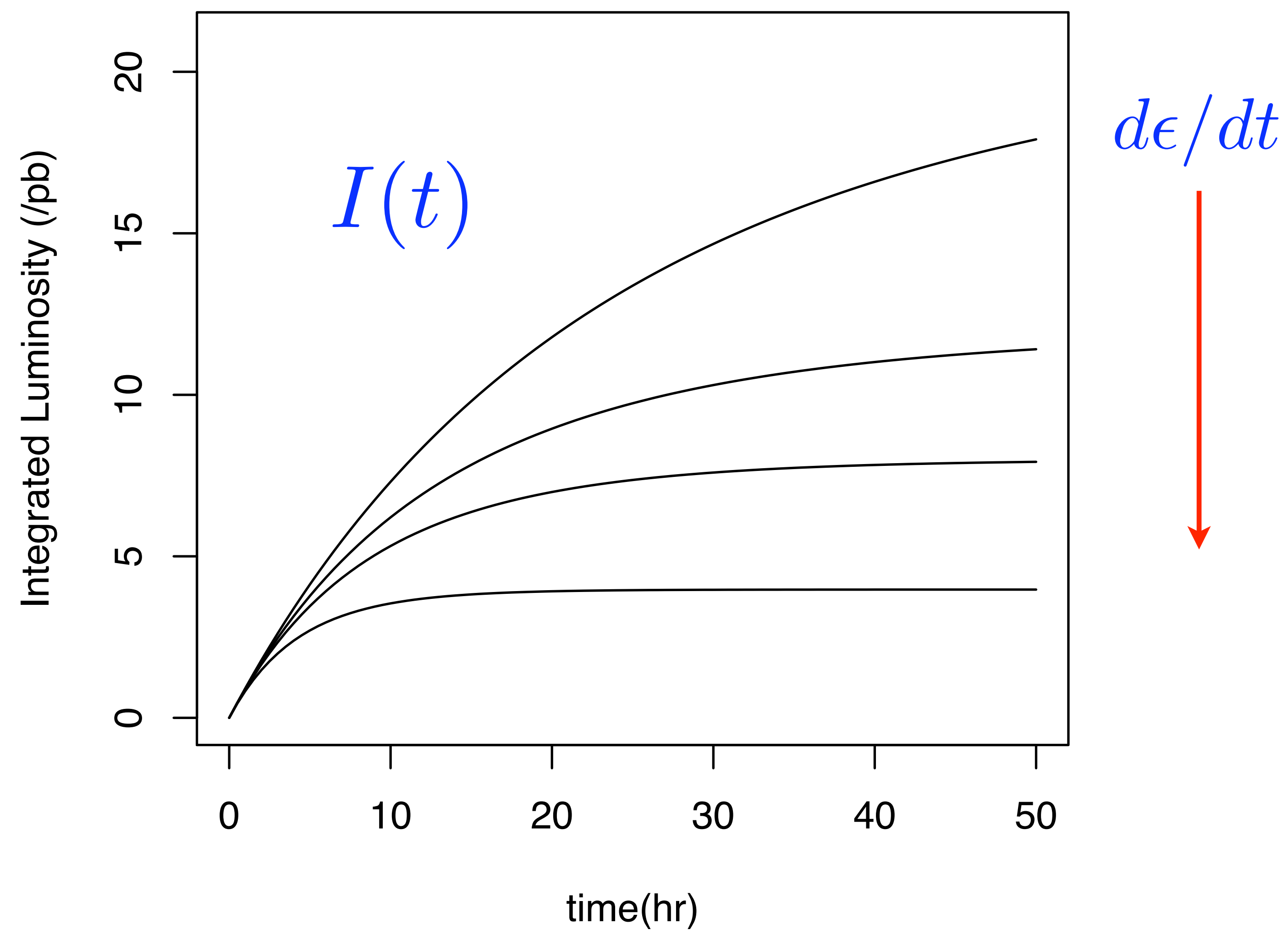


$$\tau = \frac{2a^2}{\lambda_1^2 d\langle x^2\rangle/dt} \approx \frac{2\hat{\epsilon}}{\dot{\epsilon}}$$

- Diffusion of transverse particle amplitudes leads to beam loss at locations other than at the IP
- In absence of luminosity interactions, beam attains an equilibrium lifetime
  - if beam initially nearly fills the aperture, this lifetime is achieved early

... and, on Integrated Luminosity

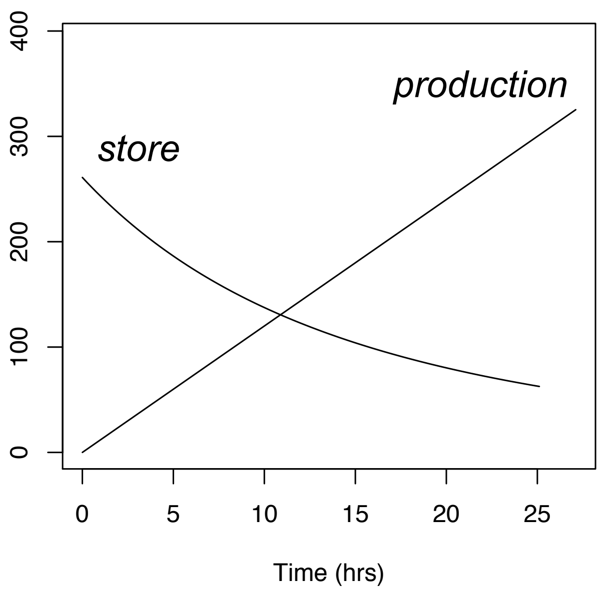


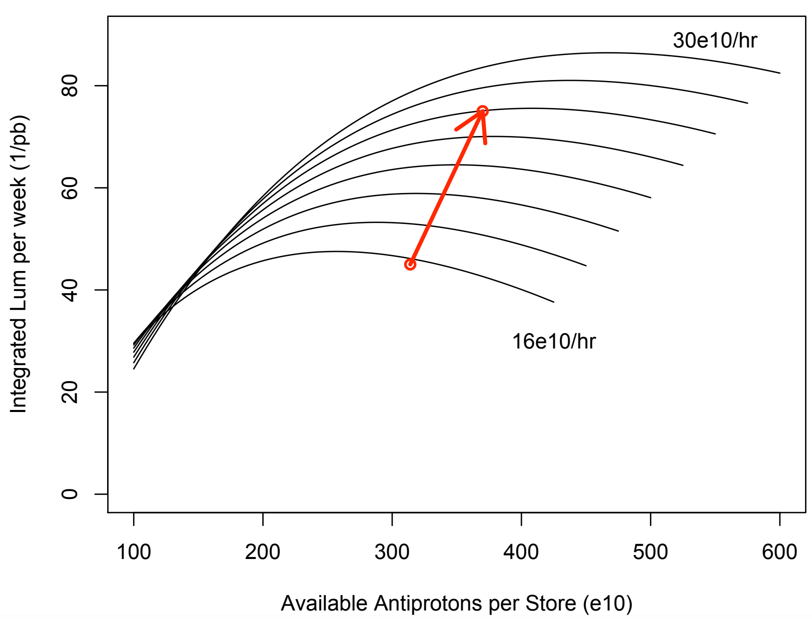


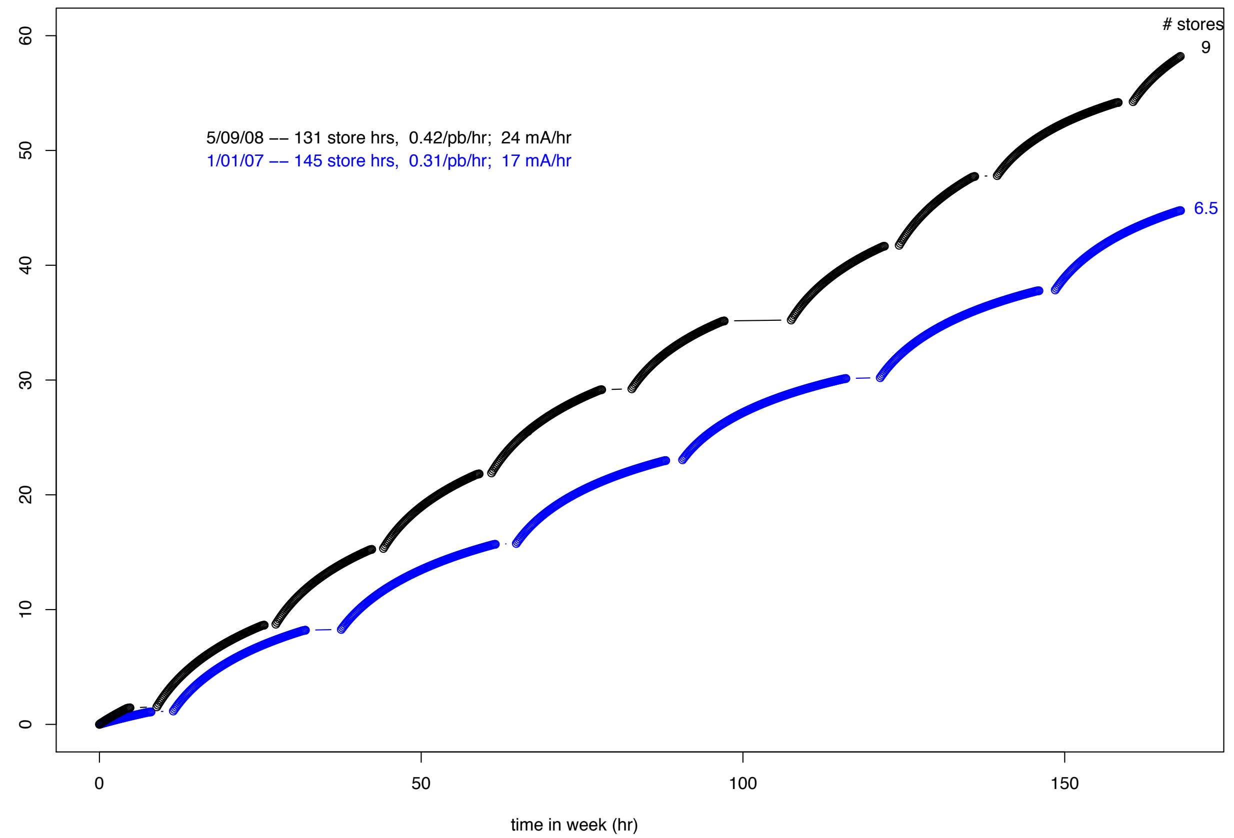
Tevatron conditions, in this example

Optimization of Integrated Luminosity

- The ultimate goal for the accelerator -- provide largest **total** number of collisions possible
- So, optimize initial luminosity, according to turn-around time, emittance growth rates, *etc.* to produce most **integrated luminosity** per week (say)
- More straightforward for LHC than it was for Tevatron
  - in Tevatron operation, needed to balance the above with the production rate of antiprotons, longer turn-around times, to find optimum running conditions

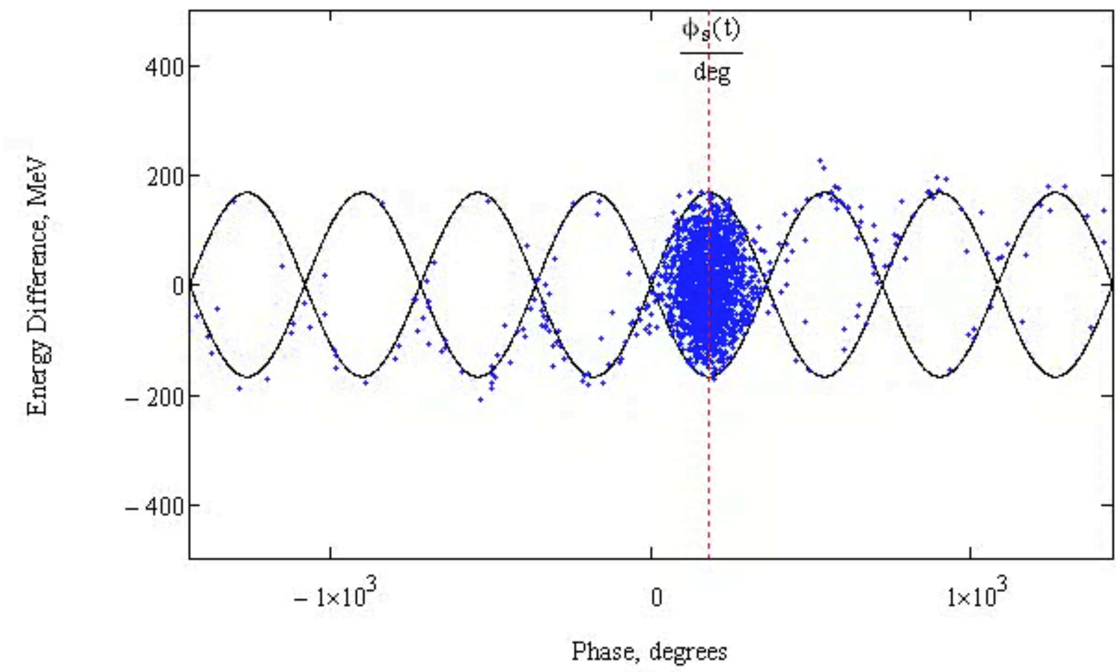


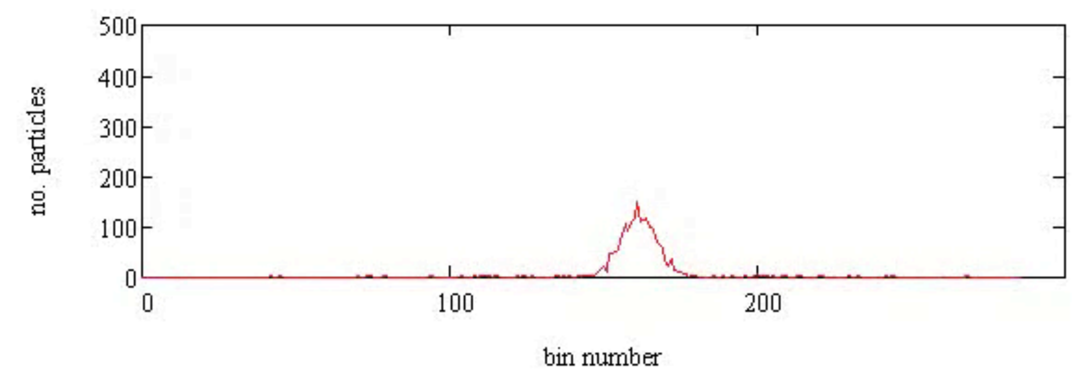


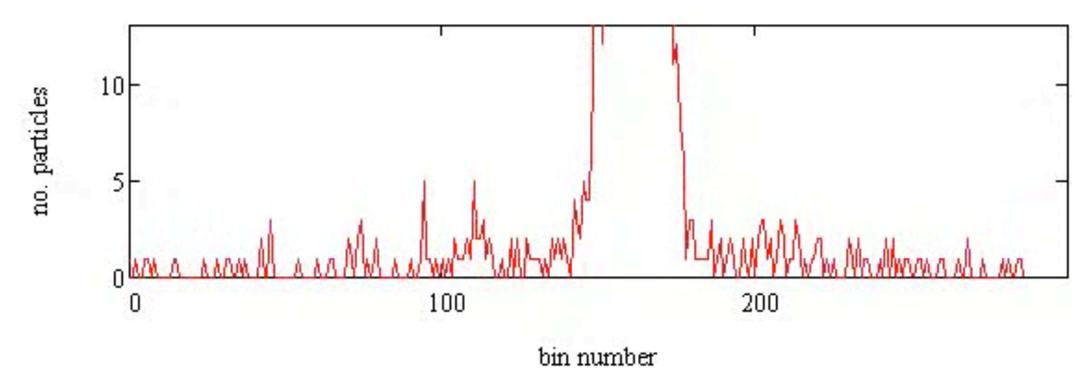


Longitudinal Diffusion — DC Beam

- Noise from RF system (phase noise, voltage noise) will increase the beam *longitudinal* emittance
- Particles will "leak" out of their original bucket, and circulate around the circumference out of phase with the RF
  - "DC Beam"
- Hence, collisions can occur between nominal bunch crossings; can be of concern for the experiments
- Perhaps more important, must remove DC beam that wanders into the abort gap(s) to permit clean removal of stored beams
  - typically "cleaned up" using fast, low-amplitude kicker magnets, electron lens deflectors, etc.







The Role of Synchrotron Radiation

- As hadron energies get higher, synchrotron radiation will no longer be just a nuisance, but will actually enhance performance as in a lepton collider
- Damping of oscillations toward an equilibrium emittance will increase the luminosity of the collider, also make the system less forgiving of emittance errors and mismatches along the injector chain

- Review some important concepts…

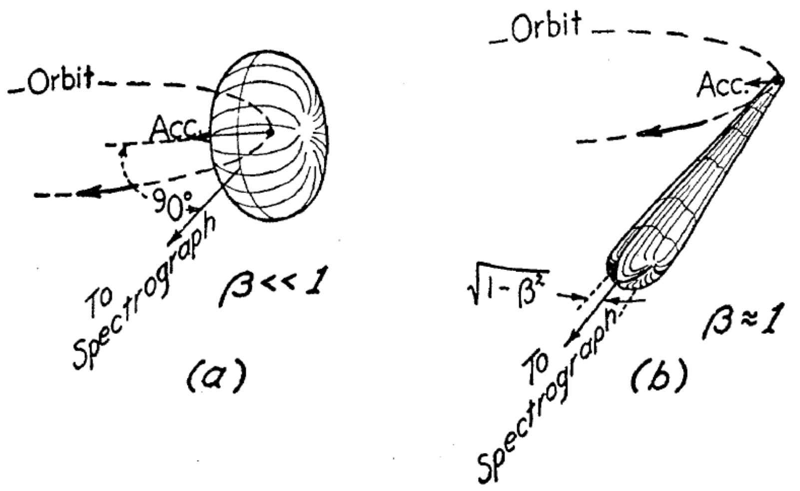
Synchrotron Radiation

- Energy loss per revolution:

electron : $C_\gamma = 8.85 \times 10^{-5} \frac{\text{m}}{\text{GeV}^3}$

proton : $C_\gamma = 7.8 \times 10^{-12} \frac{\text{km}}{\text{TeV}^3}$

$$U_0 = C_\gamma \frac{E^4}{\rho}$$



D. H. Tomboulian and P. L. Hartman, Phys. Rev., 102, 1423 (1956).

- Average Synchrotron Radiation Power:

$$\langle P \rangle = f_0 U_0 = \frac{cU_0}{2\pi R} = \frac{cC_\gamma E^4}{2\pi R \rho^2}$$

- Local power loss per meter:
  - (scale to get "per magnet", for instance)

$$\frac{dP}{ds} = \frac{cC_\gamma E^4}{4\pi^2 \rho^3}$$

Synchrotron Radiation

- As in our example of adiabatic damping during acceleration, the release of photon radiation is along the direction of motion of the particle (x', y'), while the acceleration is in the ideal direction, *s*.
  - thus, returning energy to the particle will damp the oscillations
- However, the spontaneous emission of a photon is a discrete, random process and hence will instantly alter the betatron amplitude (emittance) of the particle
- These competing effects will result in an equilibrium transverse emittance and an equilibrium momentum spread

Damping Times and Equilibrium Emittance

- Characteristic Damping Time: $\tau_0 = \dfrac{E}{\langle P \rangle}$
- Betatron amplitude damping time: $\tau_x = \tau_y = 2\tau_0$ *for hh collider-style lattices*
- Emittance damping time: $\tau_e = \tau_y/2$

$$\omega_0 \equiv c/\rho$$

$$w_c = \frac{3}{2}\gamma^3 \hbar\omega_0$$

critical photon energy

$$\langle w \rangle = \frac{8}{15\sqrt{3}} w_c$$

average photon energy

$$\langle w^2 \rangle = \frac{11}{27} w_c^2$$

photon energy variance

- Equilibrium Emittance:
  - horizontal — $\epsilon_x = \gamma\sigma_x^2/\beta_x \longrightarrow \frac{1}{2}\langle \mathcal{H} \rangle \frac{\langle w^2 \rangle}{mc^2 \langle w \rangle}$ $\mathcal{H} \equiv \frac{D^2 + (\alpha_x D + \beta_x D')^2}{\beta_x}$
  - vertical — $\epsilon_y = 0$ maybe $\sim 1\%\ \epsilon_x$
- Equilibrium Momentum Spread: $\sigma_E/E \longrightarrow \frac{1}{2}\sqrt{\frac{\langle w^2 \rangle}{E\langle w \rangle}}$

Luminosity Enhancement at Higher Energies

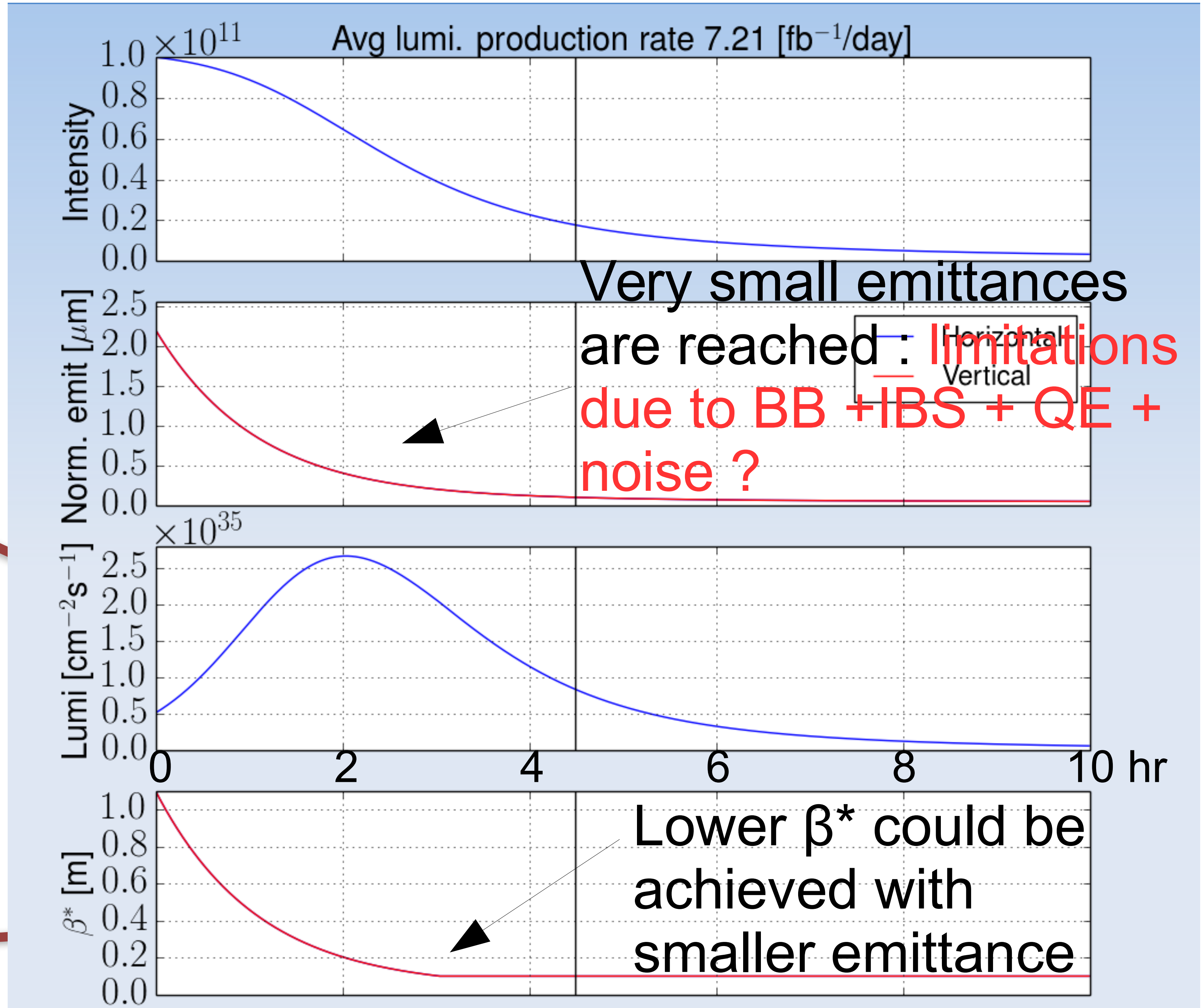
- Synchrotron Radiation at high energies will
  - Enhance instantaneous luminosity
  - Enhance integrated luminosity
  - Forgive (somewhat) numerous injection errors
- As momentum spread damps, local charge density increases, enhancing the space charge effects and the “intra-beam” scattering of particles
  - can lead to transverse emittance growth, possible instabilities
- Thus, will likely wish to control longitudinal emittance damping rate *via* RF noise source, for example

Beam Parameter Evolution — an Example

early FCC example…

luminosity rises, falls as in the SSC design

actively vary the final focus optics to mitigate beam-beam interaction effects



X. Buffat

Luminosity Optimization

$$\mathcal{L} = \frac{f_0 B N^2 \gamma}{4\beta^* \epsilon_N} \cdot \frac{1}{1 + \pi\gamma\alpha^2\sigma_z^2/(4\beta^*\epsilon_N)}$$

- As luminosity will increase, plateau, and decrease, may wish to control the instantaneous luminosity through various means, such as
  - varying the final focus (β^*)
  - varying the crossing angle (α)
  - controlling the damping rates through intentional noise sources: $\dot{\epsilon}_N, \quad \dot{\sigma}_z$

in addition to the natural (and sometimes un-natural) development of the emittance, etc.

Final Summary

- Have provided a glimpse of the basic physics of particle accelerators and particle beam dynamics relevant to colliders
- Hope this will enhance your future explorations into
  - Wake fields, impedance, coherent instabilities
  - Beam cooling techniques
  - RF manipulations
  - Energy deposition, collimation techniques
  - Magnet, cavity design considerations
  - Beam Instrumentation and diagnostics
  - …

References

- D. A. Edwards and M. J. Syphers, *An Introduction to the Physics of High Energy Accelerators*, John Wiley & Sons (1993)
- E. J. N. Wilson, *An Introduction to Particle Accelerators*, Oxford University Press (2001)
- M. Conte and W. MacKay, *An Introduction to the Physics of Particle Accelerators*, 2nd Edition, World Scientific (2008)
- S. Y. Lee, *Accelerator Physics*, 3rd Edition, World Scientific (2012)
- and many others…

- See also:
  - Physical Review Accelerators and Beams (PRAB)
    - previously called *PR Special Topics —AB*
    - prominent peer-reviewed journal for the field
  - Nuclear Instruments and Methods - A (NIM-A)
    - many peer-reviewed accelerator articles
  - Joint Accelerator Conferences Website (JACoW)
    - on-line proceedings of major accelerator conferences