Basic Principles of RF Superconductivity and Superconducting Cavities

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I. Superconducting cavities for particle acceleration

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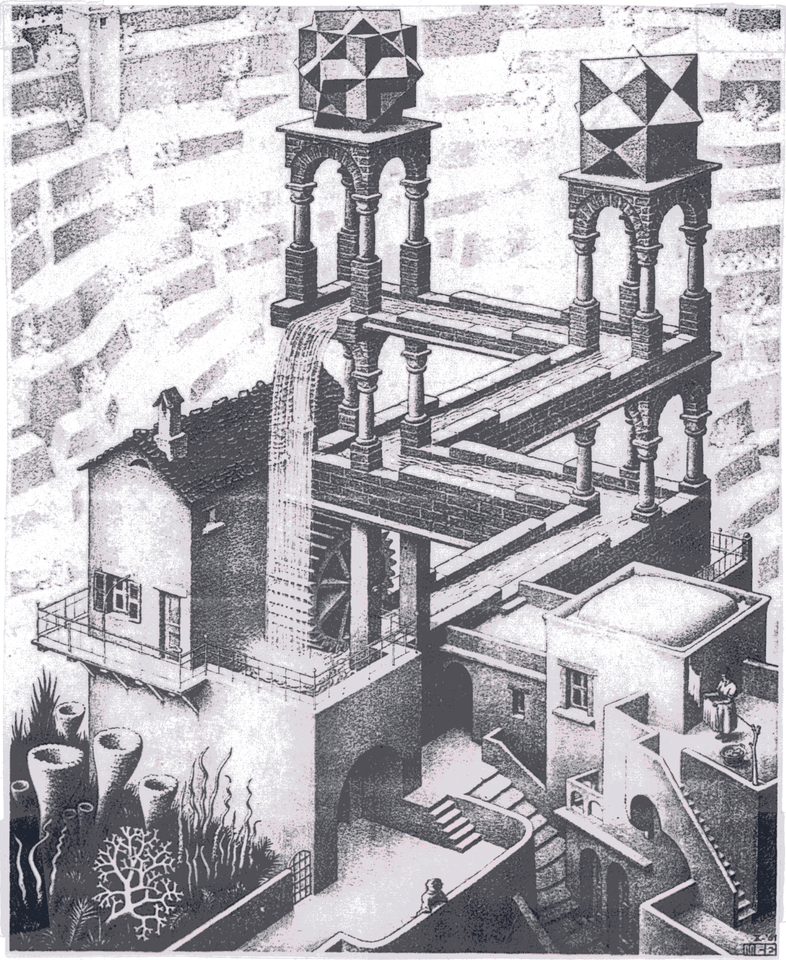
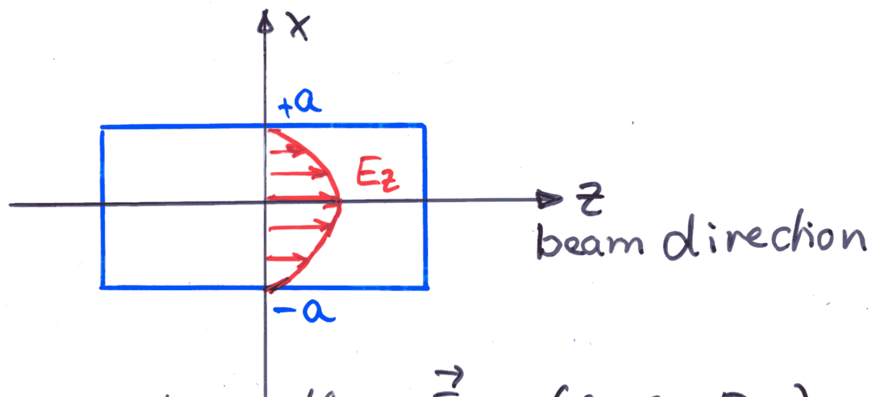


The Circular accelerator of M. C. Escher

In the reel world we need radio frequency cavities.

Rectangular cavity



wave number $|\vec{k}| = \frac{2\pi}{\lambda}$

eigenfrequency

$\omega_0 = 2\pi f_0 = c\,|\vec{k}|$

eigenmode with $\vec{E} = (0, 0, E_z)$

wave equation: $\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \frac{\partial^2 E_z}{\partial z^2} = \frac{1}{c^2}\frac{\partial^2 E_z}{\partial t^2}$

boundary conditions: $E_z = 0$ at $x = \pm a$, $y = \pm a$

simplest solution

$$E_z(x, y, z, t) = E_0 \cos(k_1 x)\cos(k_2 y)\cos(\omega_0 t)$$

bound. cond. $\Rightarrow k_1, k_2 = \frac{\pi}{2a}$

$\vec{k}^2 = k_1^2 + k_2^2 = \frac{\pi^2}{2a^2}$

resonant frequency

$\omega_0 = c\,\frac{\pi}{\sqrt{2}\,a}$

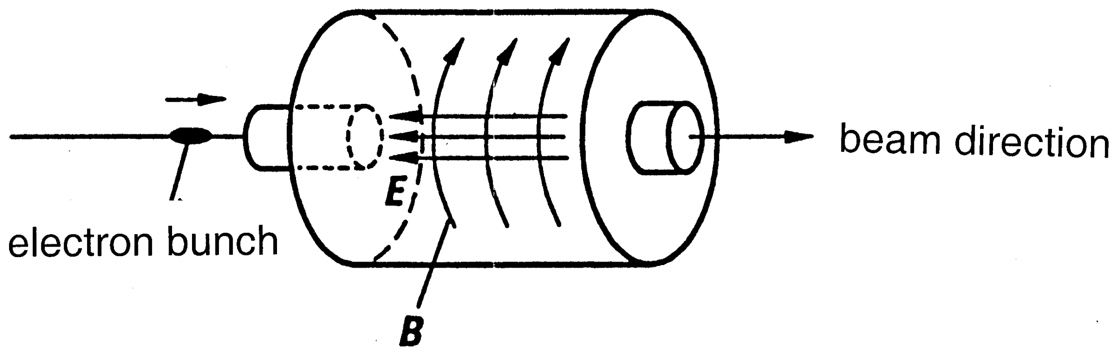
Pill Box Cavity

Simplest practical model of accelerating cavity: hollow cylinder

Neglect beam pipes, then field pattern inside resonator and all cavity parameters can be calculated analytically

Field pattern: for particle acceleration we need longitudinal electric field on the axis

$\Rightarrow$ choose TM (transverse magnetic) eigenmode of cavity



Electric and magnetic field in a pillbox cavity for the accelerating mode TM_{010}

Use cylindrical coordinates (r, θ, z), search for eigenmode with cylindrical symmetry (independence of θ) and with longitudinal electric and azimuthal magnetic field

Wave equation for electric field

$$\frac{\partial^2 E_z}{\partial r^2} + \frac{1}{r}\frac{\partial E_z}{\partial r} = \frac{1}{c^2}\frac{\partial^2 E_z}{\partial t^2} \tag{1}$$

For harmonic time dependence $E_z(r)\cos(\omega t)$

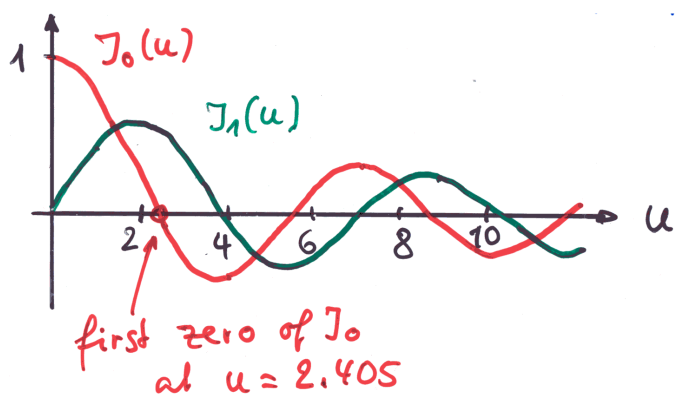
$$\frac{\partial^2 E_z}{\partial u^2} + \frac{1}{u}\frac{\partial E_z}{\partial u} + E_z(u) = 0 \quad \text{with} \quad u = \frac{\omega\, r}{c} \tag{2}$$

Bessel equation of zero order, solution $J_0(u)$

$$E_z(r) = E_0 J_0\left(\frac{\omega r}{c}\right) \tag{3}$$

For perfectly conducting cylinder of radius R_c: $E_z(R_c) = 0 \;\Rightarrow\; J_0(\omega R_c/c) = 0$
First zero of $J_0(u)$ is at $u = 2.405$. This defines the frequency of the lowest eigenmode
fundamental mode or accelerating mode

$$f_0 = \frac{2.405c}{2\pi R_c}\,, \quad \omega_0 = \frac{2.405c}{R_c} \tag{4}$$



In cylindrical cavity: frequency independent of length L_c

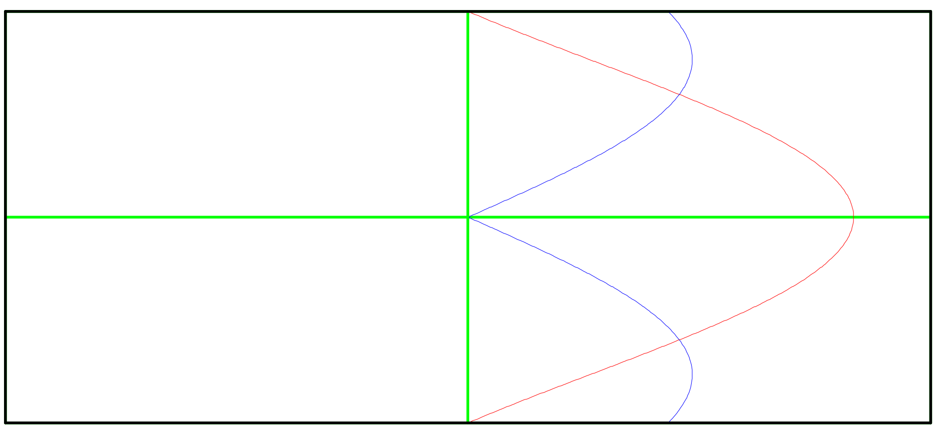
Magnetic field is computed from

$$\frac{\partial E_z}{\partial r} = \mu_0 \frac{\partial H_\theta}{\partial t}$$

Fields in the fundamental TM_{010} mode

$$\begin{aligned} E_z(r,t) &= E_0 J_0\left(\frac{\omega_0 r}{c}\right)\cos(\omega_0 t)\ , \\ H_\theta(r,t) &= -\frac{E_0}{\mu_0 c} J_1\left(\frac{\omega_0 r}{c}\right)\sin(\omega_0 t)\ . \end{aligned} \tag{5}$$

Electric and magnetic fields are 90° out of phase. Magnetic field vanishes on the axis, maximum value close to cavity wall



red curve: E_z

blue curve: B_θ

Stored energy

Integrate energy density $(\varepsilon_0/2)E^2$ (at time $t = 0$) over volume of cavity

$$U = \frac{\varepsilon_0}{2} 2\pi L_c E_0^2 \int_0^{R_c} J_0^2(\frac{\omega_0 r}{c}) r dr$$
$$= \frac{\varepsilon_0}{2} 2\pi L_c E_0^2 \left(\frac{c}{\omega_0}\right)^2 \int_0^a J_0^2(u) u du \quad (6)$$

where $a = 2.405$ is the first zero of J_0. Using
$\int_0^a J_0^2(u) u du = 0.5(aJ_1(a))^2$

$$U = \frac{\varepsilon_0}{2} E_0^2 (J_1(2.405))^2 \pi R_c^2 L_c \quad (7)$$

Power dissipation

Consider first copper cavity:

- rf electric field vanishes at cavity wall, hence no losses
- magnetic field penetrates into wall with exponential attenuation, induces currents within skin depth

$$\delta = \sqrt{\frac{2}{\mu_0 \omega \sigma}} \quad (8)$$

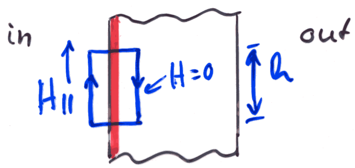
Here σ is the conductivity of the metal

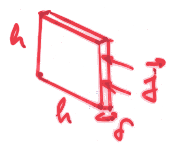
Copper at room temperature and 1 GHz $\delta = 2\mu$m

The current density in the skin depth is

$$j = \frac{H_\theta}{\delta}$$

consider small section of the conductor surface (square of height h)





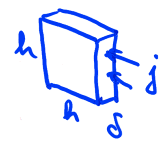
$\oint \vec{H} \cdot d\vec{s} = I$

$H_\parallel \cdot h = j \cdot h \cdot \delta$

current density in skin layer

$j = \frac{H_\parallel}{\delta}$

dissipated power in pad of area $\Delta A = h^2$



current $I = j \cdot h\delta$

resistance $R_p = \frac{1}{\sigma} \cdot \frac{h}{h \cdot \delta} = \frac{1}{\sigma \cdot \delta}$

$\Delta P = \frac{1}{2} R_p I^2$

from time average

$\Delta P = \frac{1}{2} \frac{1}{\sigma\delta} \cdot j^2 \delta^2 \Delta A$

dissipated power per unit area

$$\boxed{\frac{\Delta P}{\Delta A} = \frac{1}{2} R_{surf} \cdot H_\parallel^2 \qquad R_{surf} = \frac{1}{\sigma\delta}}$$

surface resistance

Dissipated power per unit area

$$\frac{dP_{diss}}{dA} = \frac{1}{2\sigma\delta} H_\theta^2 = \frac{1}{2} R_{surf} H_\theta^2 \qquad (9)$$

Important parameter of rf cavities: surface resistance

$$R_{surf} = \frac{1}{\sigma \delta} \tag{10}$$

The power per unit area has to be integrated over the inner surface of cavity

Integration is straightforward for the cylindrical mantle where $H_\theta = const.$ To compute power dissipation in the circular end plates evaluate $\int_0^a (J_1(u))^2 u du = a^2 (J_1(a))^2/2$ with $a = 2.405$

Total power dissipation in cavity walls

$$P_{diss} = R_{surf} \cdot \frac{E_0^2}{2\, \mu_0^2\, c^2} (J_1(2.405))^2\, 2\pi R_c\, L_c\, (1 + R_c/L_c) \tag{11}$$

Quality Factor Q_0

Very important parameter of a resonating cavity

The quality factor is defined as the number of cycles needed to dissipate the stored energy (except for factor of 2π)

alternative definition: resonance frequency f_0 divided by half width Δf of resonance curve

$$Q_0 = 2\pi \cdot \frac{U\, f_0}{P_{diss}} = \frac{f_0}{\Delta f} \tag{12}$$

Using (7) and (11) we get:

$$Q_0 = \frac{G}{R_{surf}} \quad \text{with} \quad G = \frac{2.405\,\mu_0\,c}{2(1 + R_c/L_c)} \tag{13}$$

geometry constant G depends only on cavity shape but not on the material, typical value is $G = 300\ \Omega$.

Accelerating field, peak electric and magnetic fields

A relativistic particle needs a time c/L_c to travel through the cavity. During this time the longitudinal electric field changes

The accelerating field E_{acc} is the average field seen by particle

$$E_{acc} = \frac{1}{L_c}\int_{-L_c/2}^{L_c/2} E_0 \cos(\omega_0 z/c) dz\,, \quad V_{acc} = E_{acc}\,L_c\,. \tag{14}$$

Choosing a cell length of one half the rf wavelength, $L_c = c/(2f_0)$, we get $E_{acc} = 0.64\,E_0$ for a pill box cavity

Peak electric field E_{peak} at the cavity end plate, here $E_{peak} = E_0$

Peak magnetic field B_{peak} is near cylindrical wall

For a pillbox cavity

$$E_{peak}/E_{acc} = 1.57\,,\ B_{peak}/E_{acc} = 2.7\,\mathrm{mT/(MV/m)}\,. \tag{15}$$

Pillbox cavity with beam pipes: peak fields increase by $20 - 30\%$

Shunt Impedance

Represent the cavity by a parallel LCR circuit, parallel Ohmic resistor is called shunt impedance
Relation between peak voltage in equivalent circuit and accelerating field in cavity:

$$V_0 = V_{acc} = E_{acc} L_c$$

Dissipated power in LCR circuit

$$P_{diss} = \frac{V_0^2}{2R_{shunt}}$$

Identify this with the dissipated power in the cavity, eq. (11)
Then the shunt impedance of the pillbox cavity is[1]

$$R_{shunt} = \frac{2L_c^2 \mu_0^2 c^2}{\pi^3 (J_1(2.405))^2 R_c (R_c + L_c)} \cdot \frac{1}{R_{surf}}$$

The ratio of shunt impedance to quality factor is an important cavity parameter

$$(R/Q) \equiv \frac{R_{shunt}}{Q_0} = \frac{4L_c \mu_0 c}{\pi^3 (J_1(2.405))^2 2.405 R_c} \tag{16}$$

(R/Q) is independent of the material, depends only on the shape
Typical value for 1-cell cavity $(R/Q) = 100\ \Omega$

[1] R_{shunt} is often defined by

$$P_{diss} = \frac{V_0^2}{R_{shunt}}$$

then (R/Q) is a factor of 2 larger

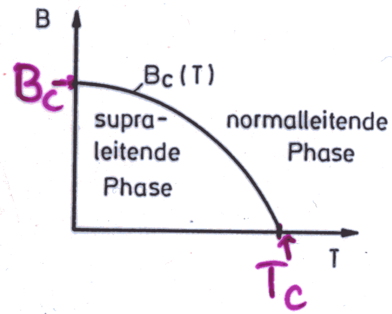
Superconductivity Basics

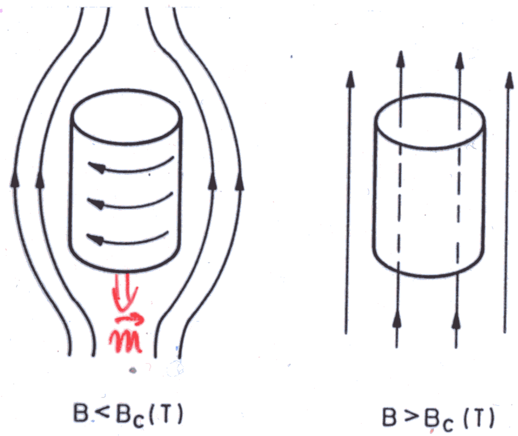
Short introduction into:

- Type I and type II superconductors
- Hard and soft superconductors
- Superconductors in microwave fields

Type I Superconductors

pure elements: lead, indium, tin, .. but not niobium!



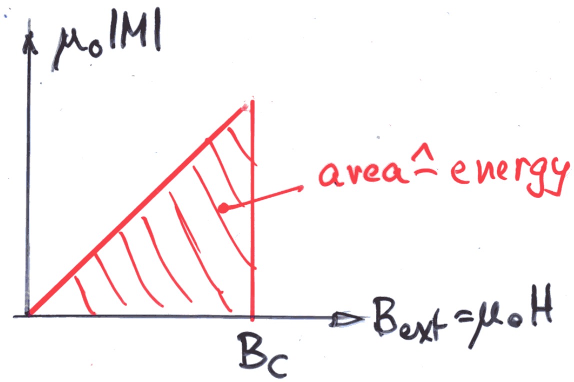


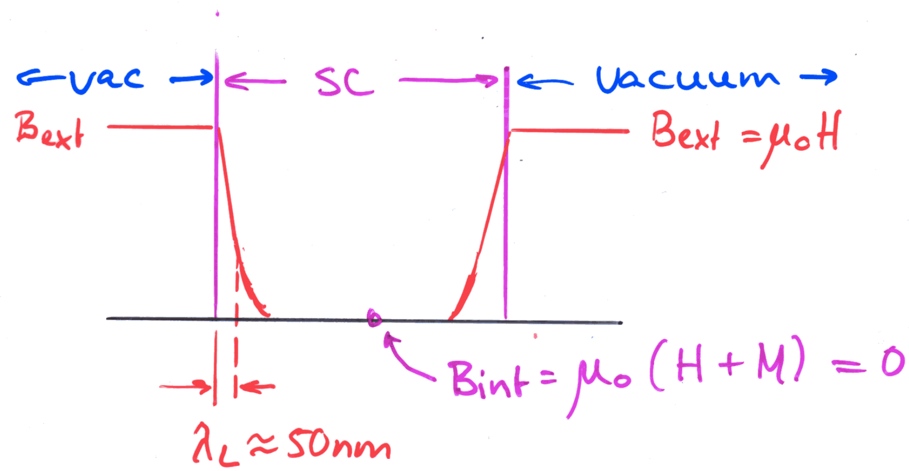
Meissner effect:

$B = \mu_0 (H + M) = 0$

inside supercond.

except for thin surface layer

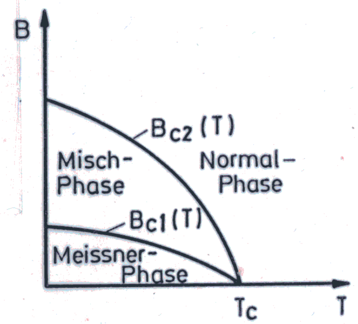


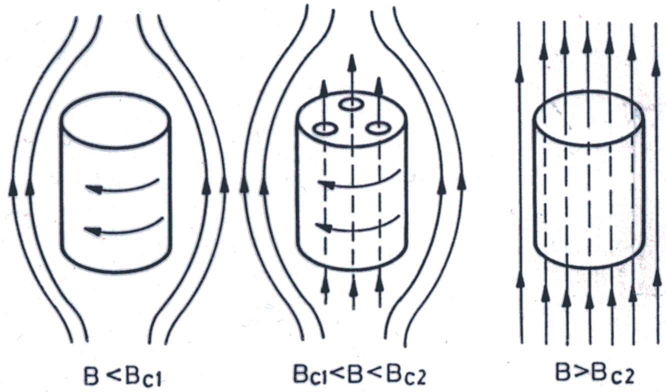


$M = -H$ for $H < H_c$

Type II Superconductors

niobium, all alloys ($NbTi$, Nb_3Sn...)

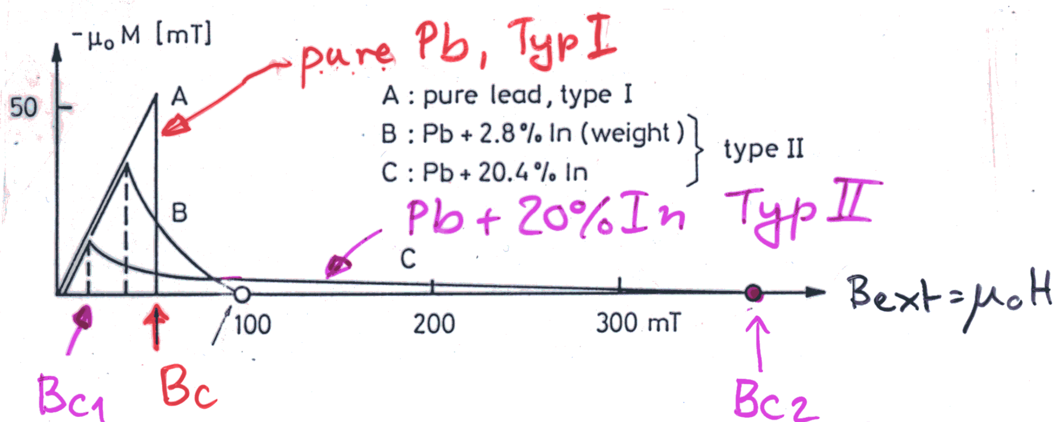




2 critical fields

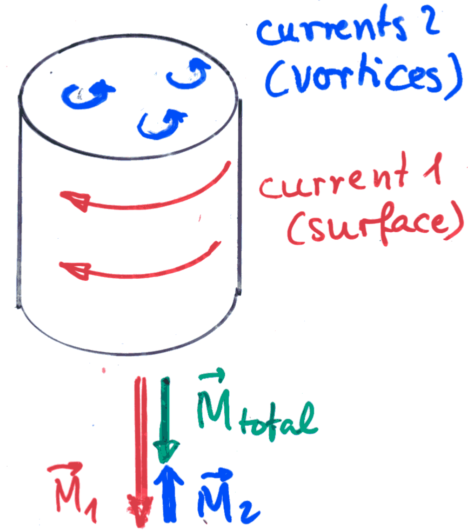
$B < B_{c1}$ Meissner phase

$B_{c1} < B < B_{c2}$ mixed phase



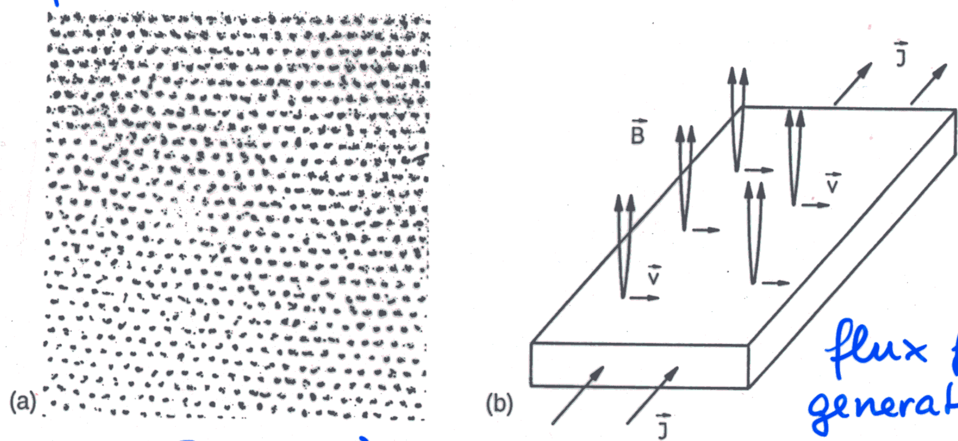
high B_{c2} : excellent for magnets

lower magnetization in mixed phase



Hard Superconductors

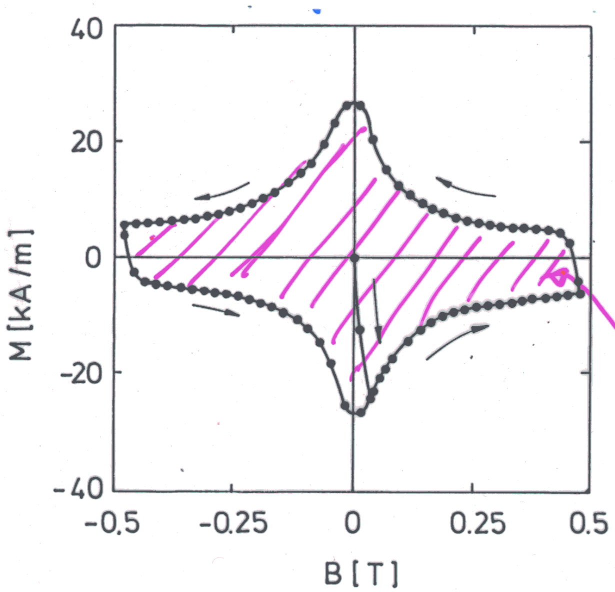
flux tubes in Nb



(U. Essmann)

flux flow generates heat

force $\sim \vec{j} \times \vec{B}$



produced heat is

$\mu_0 \oint M(H)\, dH$

area enclosed by the loop

hysteresis in NbTi

For sc-magnets: flux-flow inbited by pinning centers

Not good for micro-wave cavities: magn. hysteris causes energy dissipation

→ use "soft" supercond.

Requirements on Superconductor:

general : high critical temper. T_c

(but not "High-T_c" ceramic S.C.)

Accelerator magnets

B_c large $\Rightarrow$ only type II, alloys

strong flux pinning $\Rightarrow$ lattice defects etc

$\Rightarrow$ need "dirty" superconductor

NbTi $T_c = 9.2K$, $B_{c2} \approx 14T$

very ductile, easily extruded with Cu

the "Standard conductor"

Nb_3Sn $T_c = 18K$, $B_{c2} \approx 20T$

brittle material, difficult to use in accelerator magnets

Microwave cavities

high heat conductivity, no flux pinning

$\Rightarrow$ need very pure superconductor

Pb $T_c = 7.2K$ $B_c = 80mT$

Nb $T_c = 9.2K$ $B_c^{th} = 200mT$

Superconductors in microwave fields

Copper cavity: surface resistance is given by

$$R_{surf} = \frac{1}{\delta\sigma} \tag{17}$$

In case of superconductor: skin depth δ must be replaced with London penetration depth

$$\lambda_L \approx 50\ \mathrm{nm}\ \ll\ \delta \approx 2000\ \mathrm{nm}$$

Big question: what is the conductivity σ? If we take $\sigma \rightarrow \infty$ we get $R_{surf} = 0$. That would be nice but is wrong!

According to the Bardeen-Cooper-Schrieffer (BCS) theory of superconductivity the supercurrent is carried by Cooper pairs

Response of a superconductor to an ac field described by Two-Fluid Model :

- Cooper pairs are superfluid
- Unpaired electrons are normal fluid, yield conductivity σ_n

Study response of two fluids to a periodic electric field

normal current obeys Ohm's law (and dissipates power)

$$J_n = \sigma_n\, E_0\, \exp(-i\omega t) \tag{18}$$

Cooper pairs are accelerated $m_c\,\dot{v}_c = -2e\,E_0\,\exp(-i\omega t)$
Supercurrent density

$$J_s = i\,\frac{n_c\,2\,e^2}{m_e\omega}E_0\,\exp(-i\omega t) \tag{19}$$

Supercurrent is 90° out of phase with electric field, hence no power dissipation
Write for total current density $J = J_n + J_s = \sigma E_0\exp(-i\omega t)$
with a complex conductivity:

$$\sigma = \sigma_n + i\sigma_s \quad \text{with} \quad \sigma_s = \frac{2\,n_c e^2}{m_e\,\omega} = \frac{1}{\mu_0\lambda_L^2\,\omega} \tag{20}$$

Surface resistance: real part of the complex surface impedance

$$R_{surf} = Re\left(\frac{1}{\lambda_L(\sigma_n + i\sigma_s)}\right) = \frac{1}{\lambda_L}\cdot\frac{\sigma_n}{\sigma_n^2+\sigma_s^2} \tag{21}$$

Important observation: $\sigma_n^2 \ll \sigma_s^2$ at microwave frequencies hence disregard σ_n^2 in the denominator
$\Rightarrow R_{surf} = \sigma_n/(\lambda_L\sigma_s^2)$
Surprising result: The microwave surface resistance is proportional to the normal-state conductivity
Conductivity of normal metal given by classic Drude expression

$$\sigma_n = n_n e^2\ell/(m_e v_F$$

- n_n density of single (unpaired) electrons

- ℓ mean free path of single electrons
- v_F the Fermi velocity

Unpaired electrons are created by thermal breakup of Cooper pairs
Energy gap $E_g = 2\Delta$ between the superconducting (BCS) ground state and the free electron states
By analogy with the conductivity of an intrinsic (undoped) semiconductor we get

$$n_n \propto \exp(-E_g/(2k_B T))$$

and hence

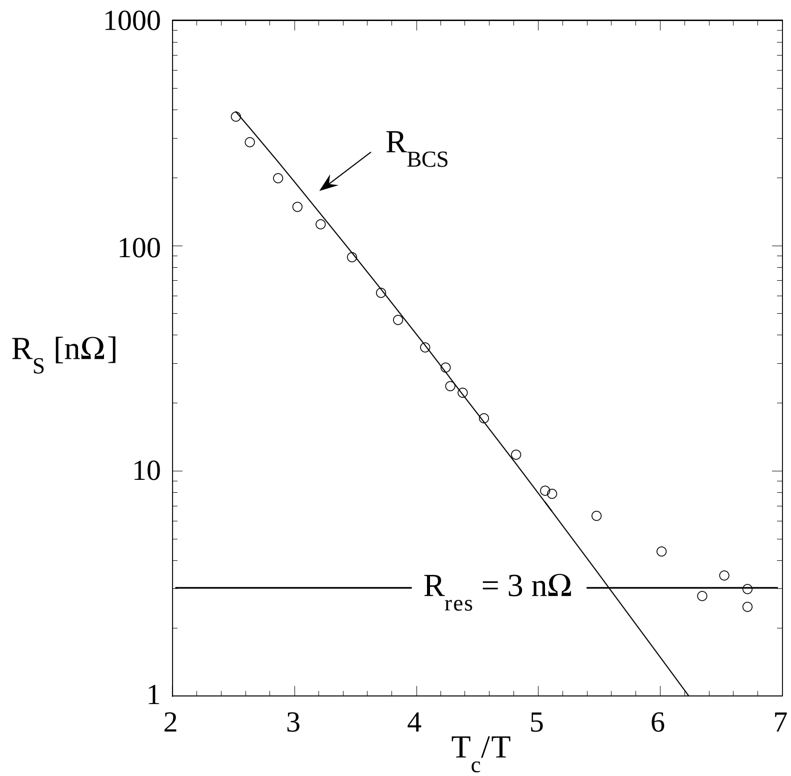
$$\sigma_n \propto \ell \exp(-\Delta/(k_B T)) \ . \tag{22}$$

Using $1/\sigma_s = \mu_0 \lambda_L^2 \omega$ and $\Delta = 1.76 k_B T_c$ we finally obtain for the BCS surface resistance

$$\boxed{R_{BCS} \propto \lambda_L^3 \, \omega^2 \, \ell \exp(-1.76 \, T_c/T)} \tag{23}$$

This formula displays two important aspects of microwave superconductivity
- the surface resistance depends exponentially on temperature
- it is proportional to the square of the rf frequency

For niobium: small correction needed, replace λ_L by $\Lambda = \lambda_L \sqrt{1 + \xi/\ell}$



Surface resistance of a 9-cell TESLA cavity plotted as a function of T_c/T. The residual resistance of 3 nΩ corresponds to a quality factor $Q_0 = 10^{11}$

Residual resistance

Residual resistance caused by impurities, frozen-in magnetic flux or lattice distortions

$$R_{surf} = R_{BCS} + R_{res} \tag{24}$$

R_{res} is temperature independent, amounts to a few nΩ for a clean niobium surface

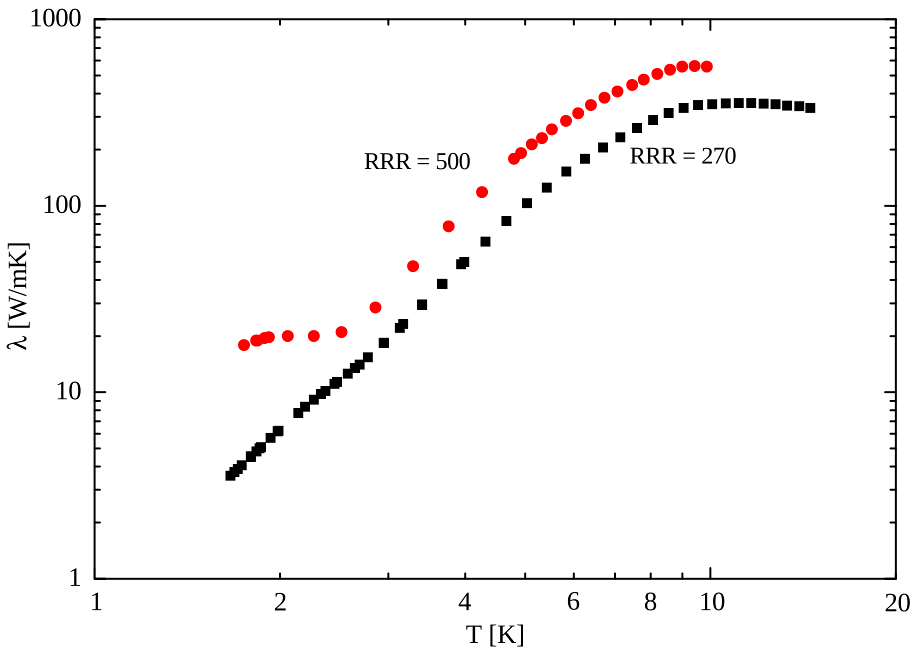
Heat conduction in niobium and heat transfer to liquid helium

Heat produced at inner cavity surface must be guided through cavity wall to liquid helium bath

Thermal conductivity of Nb drops strongly at $T \longrightarrow 0$

Very pure Nb with large residual resistivity ratio RRR needed

$$RRR = R(300K)/R(10K)$$



Measured heat conductivity in niobium with $RRR = 270$ resp. 500 as a function of temperature

Low frequency cavities (350-500 MHz): small BCS surface resistance at 4.2 K, effective cooling by normal liquid helium
Due to f^2 dependence of BCS resistance: at higher frequency, cooling with superfluid helium at 1.8 - 2 K is better
Note: Kapitza resistance at superfluid helium-niobium interface leads to temperature jump

Maximum Field in SC Cavities

Magnetic field of microwave must stay below the critical magnetic field of superconductor
Situation is clear for a type I superconductor such as lead:
at $T = 2$ K one has $B_c = 80$ mT $\quad \rightarrow E_{acc} \leq 20$ MV/m
For type II superconductors the situation is not that clear. Magnetic flux moving in and out of the sc produces heat. Flux pinning is undesirable since the magnetic hysteresis again leads to heat generation in a microwave field.
Consequence: a hard superconductor like NbTi or Nb_3Sn is not well suited for rf cavities, at the large B_{c2} of 10 - 20 Tesla the heat generation would be untolerable.

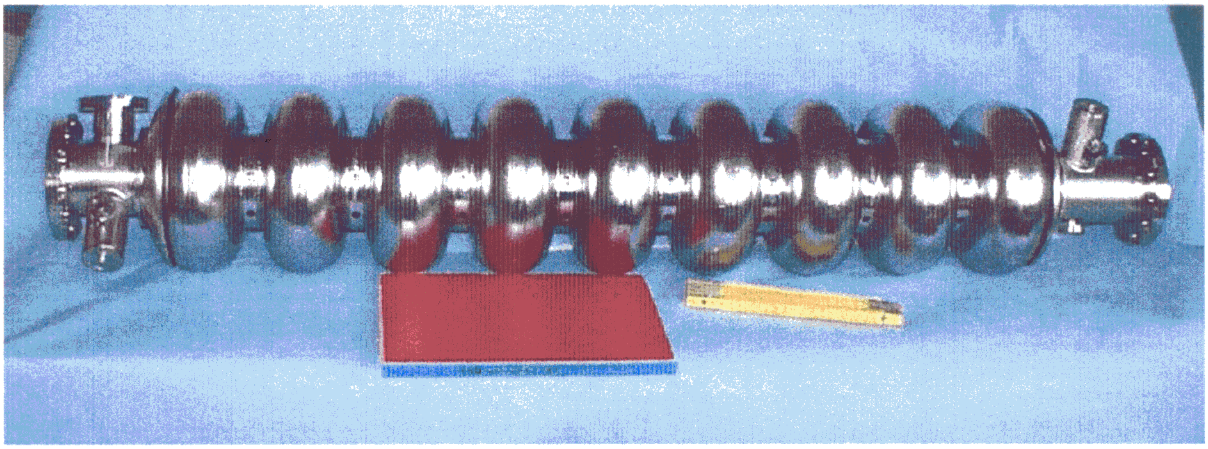
What about niobium? This superconductor is of type II, but close to the boundary of type I. The critical fields at 2 Kelvin are approximately

- $B_{c1} \approx 160$ mT
- $B_c^{therm} \approx 200$ mT
- $B_{c2} \approx 350$ mT

Very safe limit: $B < B_{c1} \quad \Rightarrow E_{acc} \leq 40$ MV/m : no magnetic flux enters the sc
Experimental observation $E_{acc} > 40$ MV/m has been achieved repeatedly, the best value was 45 MV/m

Question: how far above B_{c1} can one go?

Hint
The following pages contain further interesting material on sc cavities which, however, is beyond the scope of a 1-hour lecture at CAS



9-zelliger TESLA - Resonator

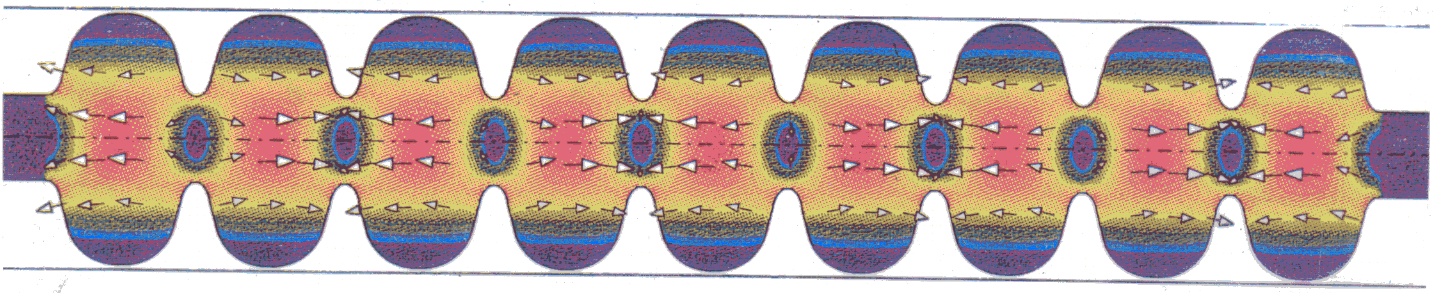
Cornell Univ (USA)

DESY, Univ Hamburg

Saclay (Frankreich)

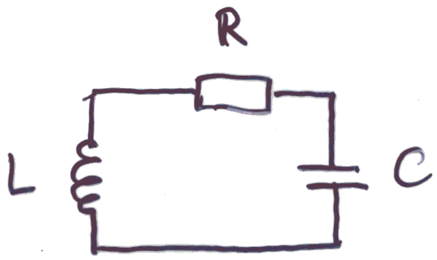
INFN (Italien)

MAFIA



Frequenz 1.3 GHz

Temperatur 2 Kelvin (-271 °C)



R ≈ Oberflächenwiderstand

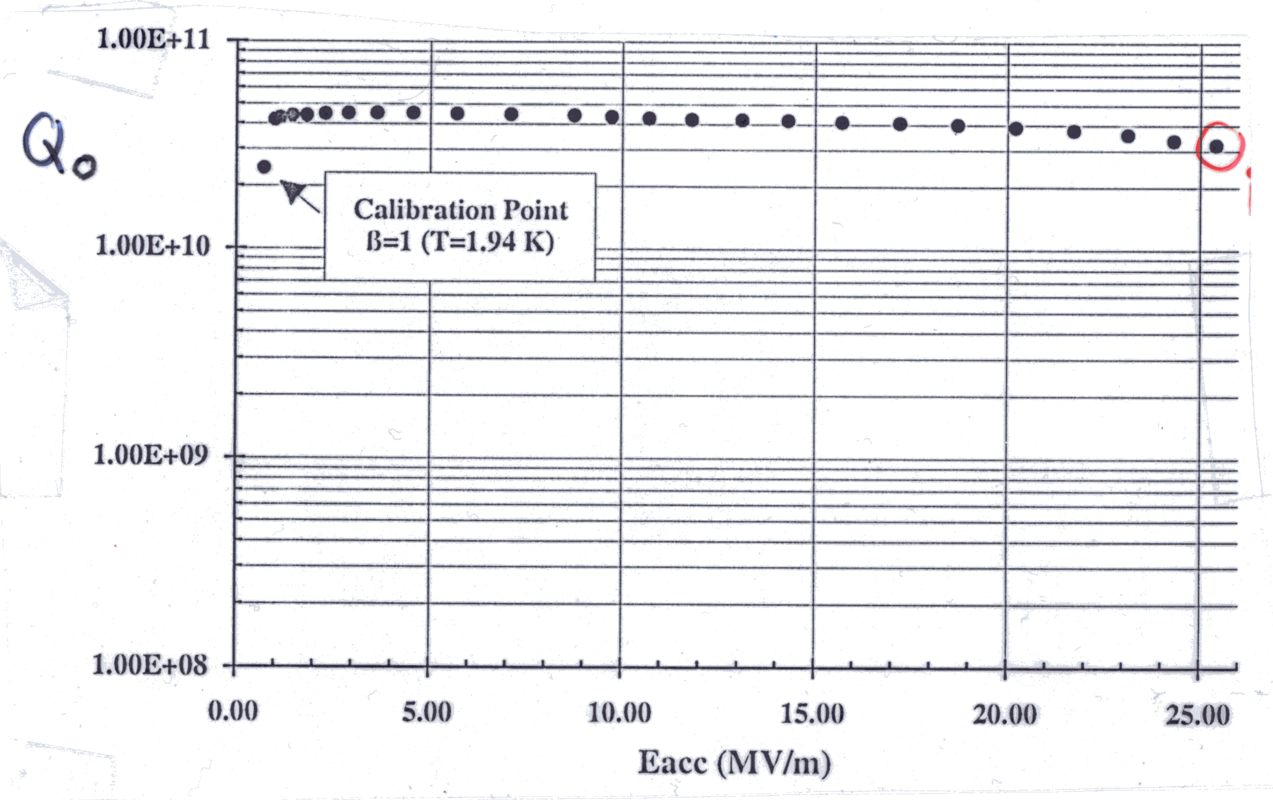
ca $10^{-8}\,\Omega$ in Niob bei 2 Kelvin

Warum supraleitende Kavitäten?

- sehr hohe Gütefaktoren

$$Q_0 = \frac{f_0}{\Delta f} > 10^{10}$$

Kupfer: Q_0 einige 10^4



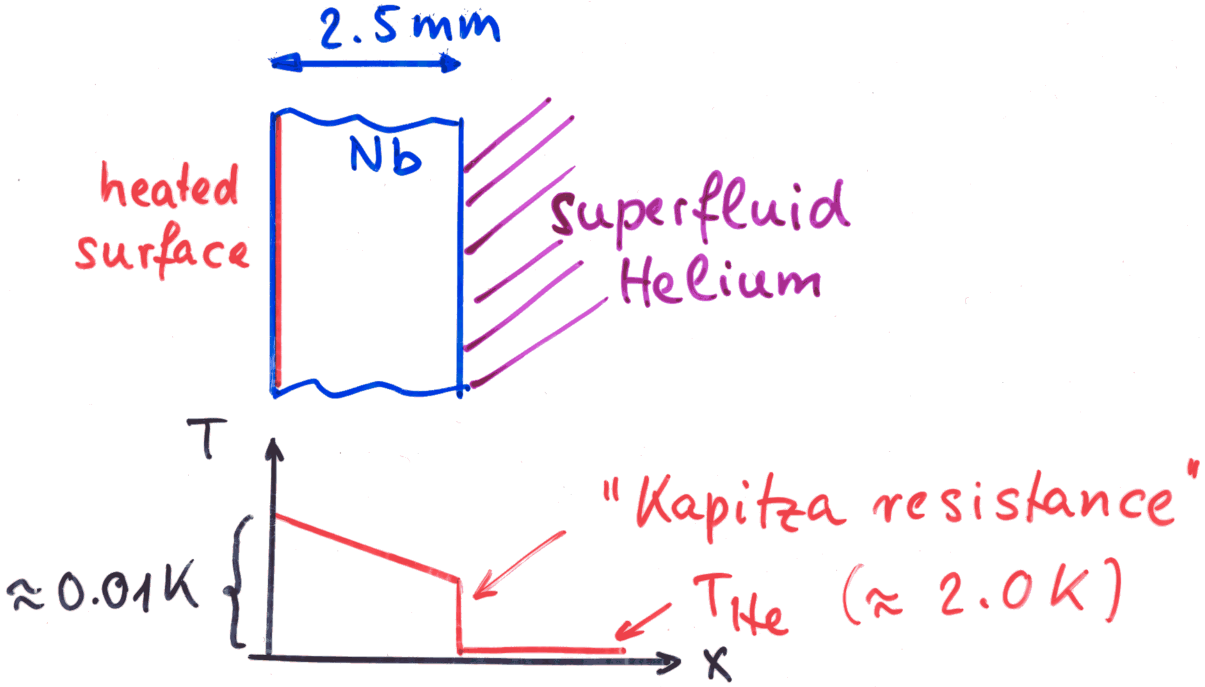
25 Millionen eV auf 1 m Länge

TESLA: Strahlstrom 8 mA

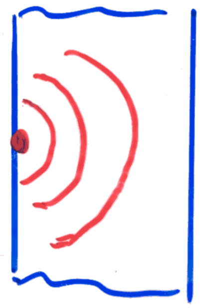
$\Rightarrow$ 200 kW Leistung pro Kavität wird auf Strahl übertragen
nur ca 20 - 40 W geht ins Helium

Cu: $\approx$ 200 kW auf Strahl
$\approx$ 200 kW in das Cu $\rightarrow$ Kühlwasser

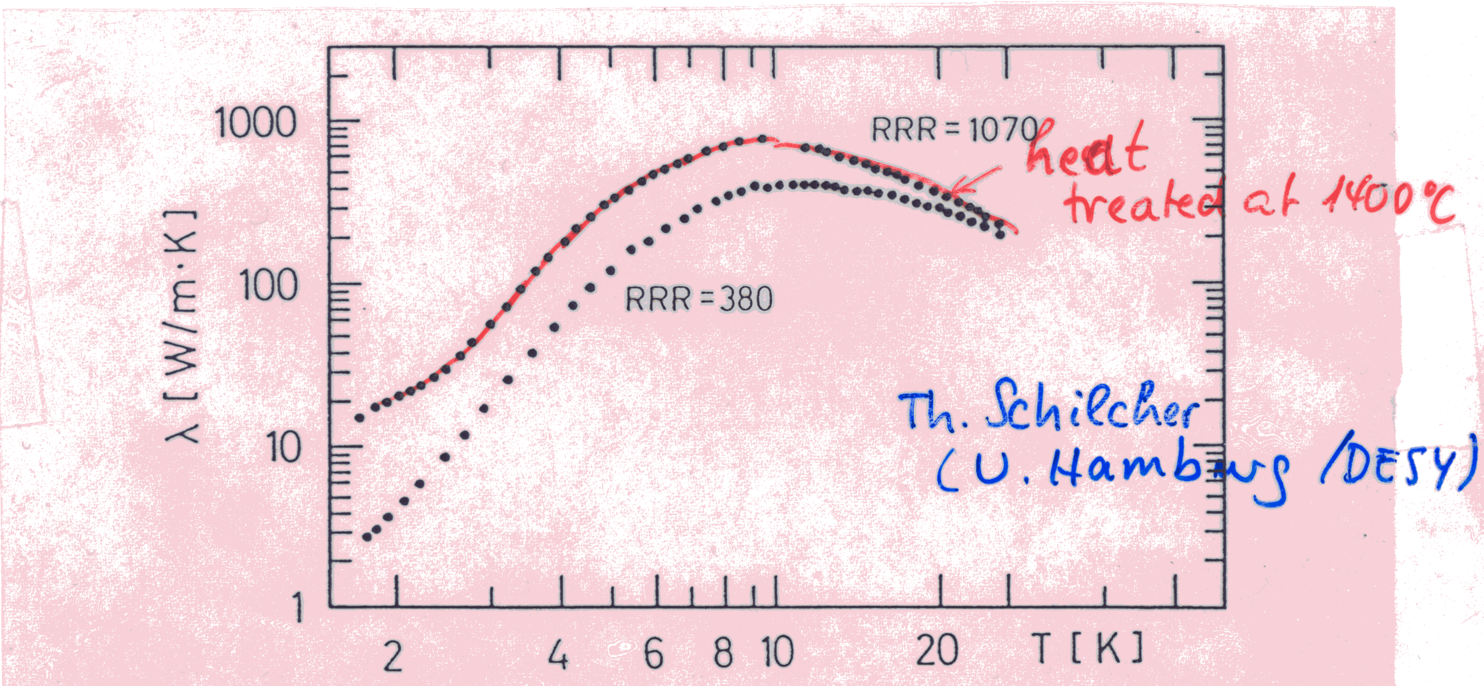
Heat transfer to liquid He



Much stronger heating due to small normal-conducting defects



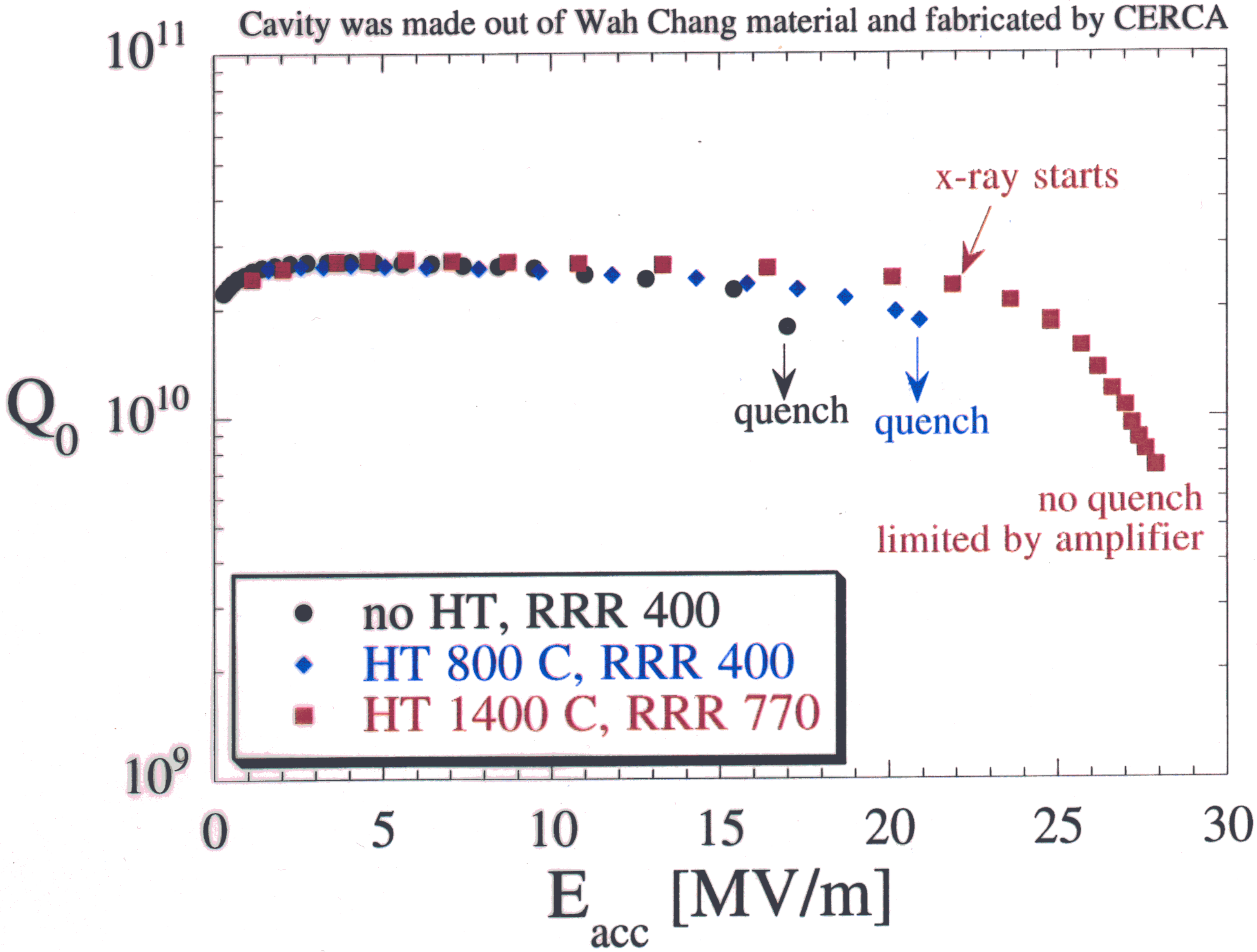
area of defect
< 10^{-6} area of cavity



heat conductivity of very pure Nb

quality factor as a function of accelerating field

Cavity C21

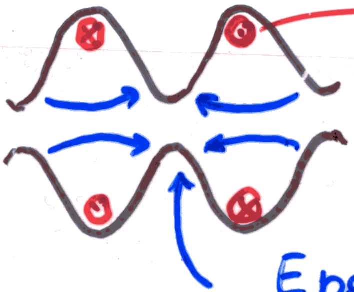


residual resistivity ratio

$$RRR = \frac{R(300K)}{R(10K)}$$

good measure of purity

Performance limit of cavity?



$B_{peak}/E_{acc} \approx 4\,mT/(MV/m)$
at equator $\Rightarrow$ thermal breakdown

$E_{peak} \approx 2 E_{acc}$ at iris
$\Rightarrow$ danger of field emission

Superconductivity breaks down if the rf magnetic field exceeds the critical field of the superconductor.

Complication in case of niobium:
type II superconductor with
3 critical fields: B_{c1}, $B_c^{th.}$, B_{c2}
• surface field $B_{c3} = 1.69\, B_{c2}$

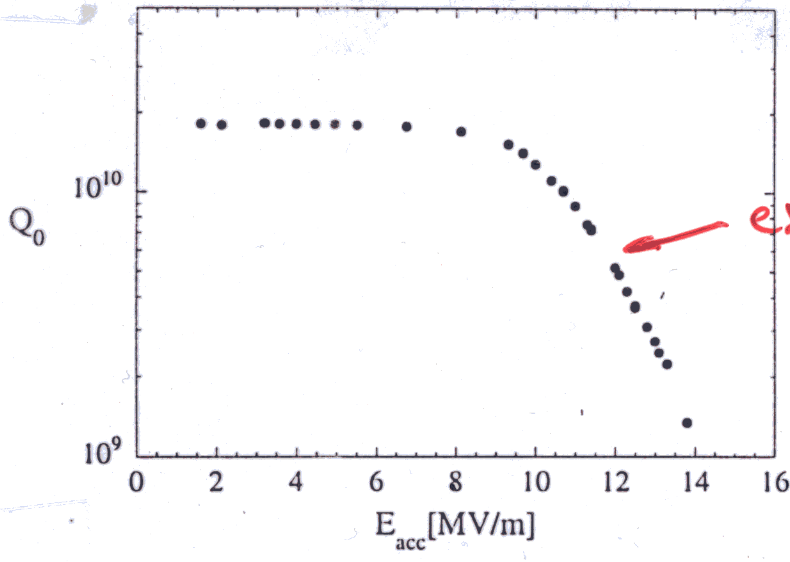
Conservative estimate:
$B < B_{c1}$ ($\approx 160\,mT$ at $2K$)

$\Rightarrow E_{acc}$ max. $\lessapprox 40\,MV/m$

Probably some "superheating" possible up to $B_c^{th} \approx 190\,mT \;\hat{=}\; 48\,MV/m$

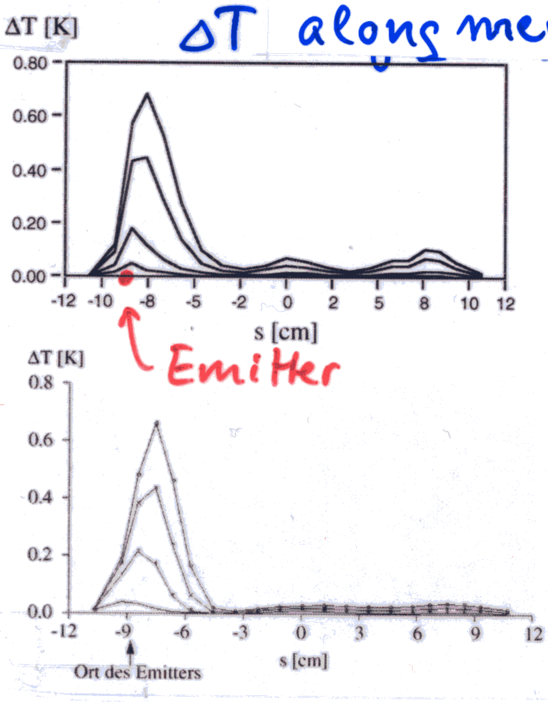
Flux penetration into the s.c. must be avoided since this leads to hysteresis and losses.

Field emission of electrons



exponential decay of quality factor

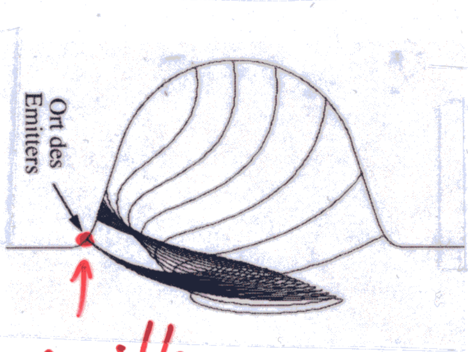
ΔT along meridian



Emitter

← measurement

← simulation

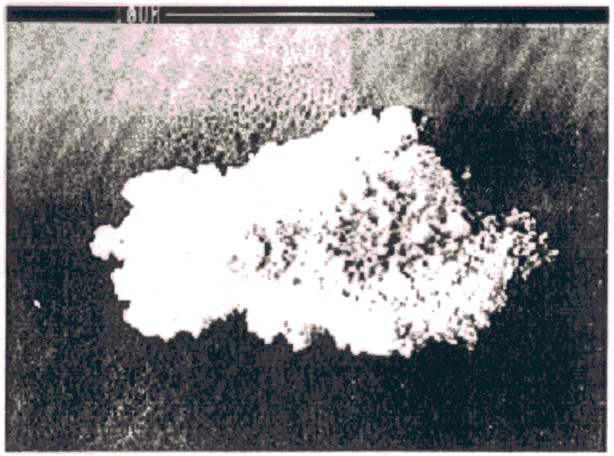


emitter

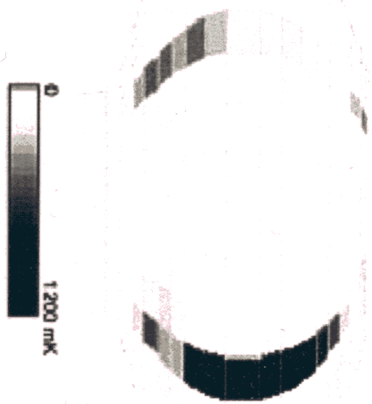
computed trajectories (for various rf phases)

Field Emission

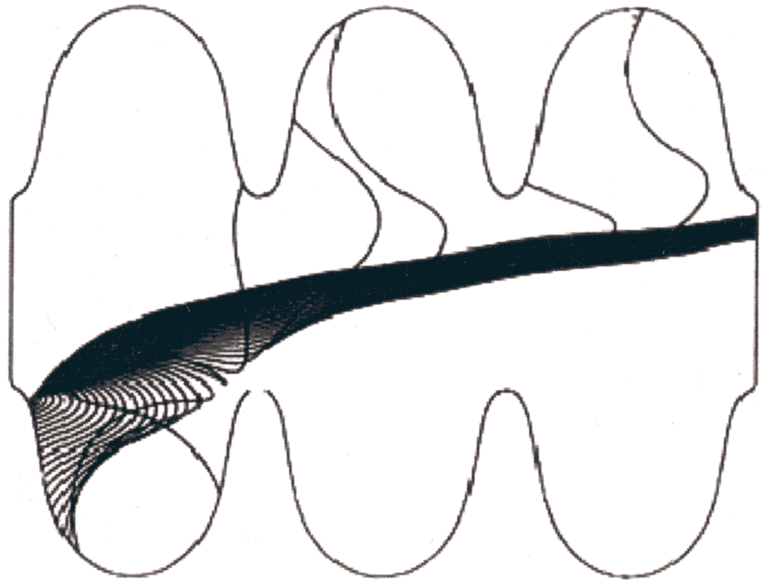
Pictures taken from: H. Padamsee, Supercond. Sci. Technol., 14 (2001), R28 –R51



Particle causing field emission



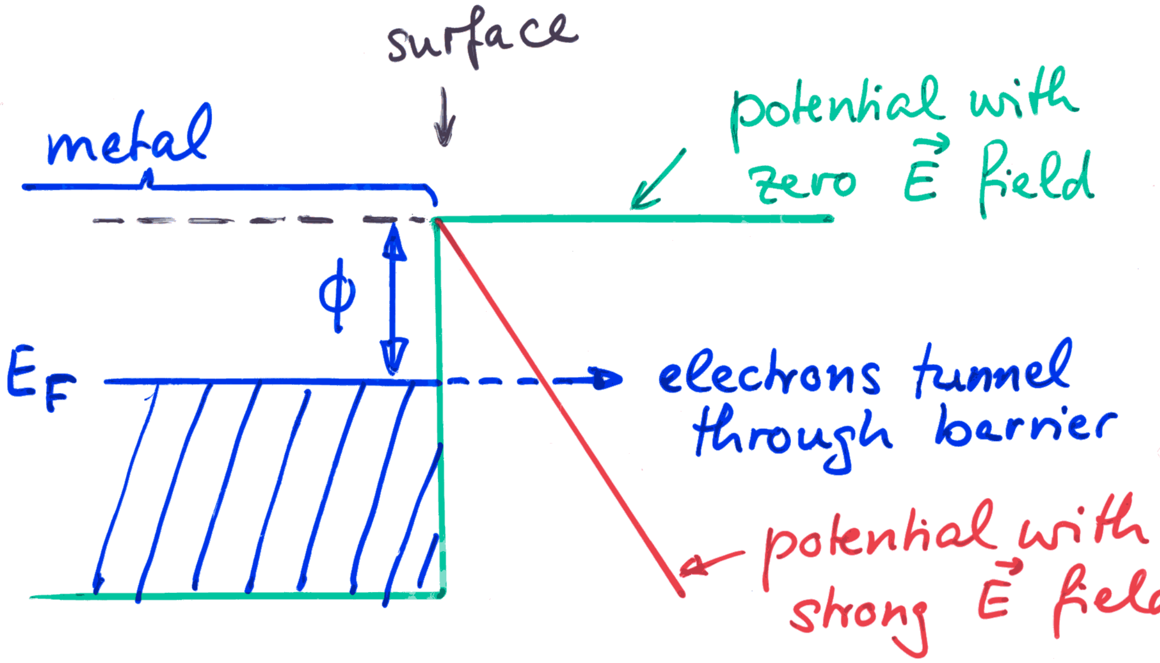
Temperature map of a field emitter



Simulation of electron trajectories in a cavity

What is field emission?

Extraction of electrons from a metal via the quantum mechanical tunnel effect



current density given by Fowler-Nordheim equation

$$j(E) = \frac{A}{\phi} (\beta_{FN} \cdot E)^2 \exp\left(- \frac{B \cdot \phi^{3/2}}{\beta_{FN} \cdot E}\right)$$

ϕ: work function of metal

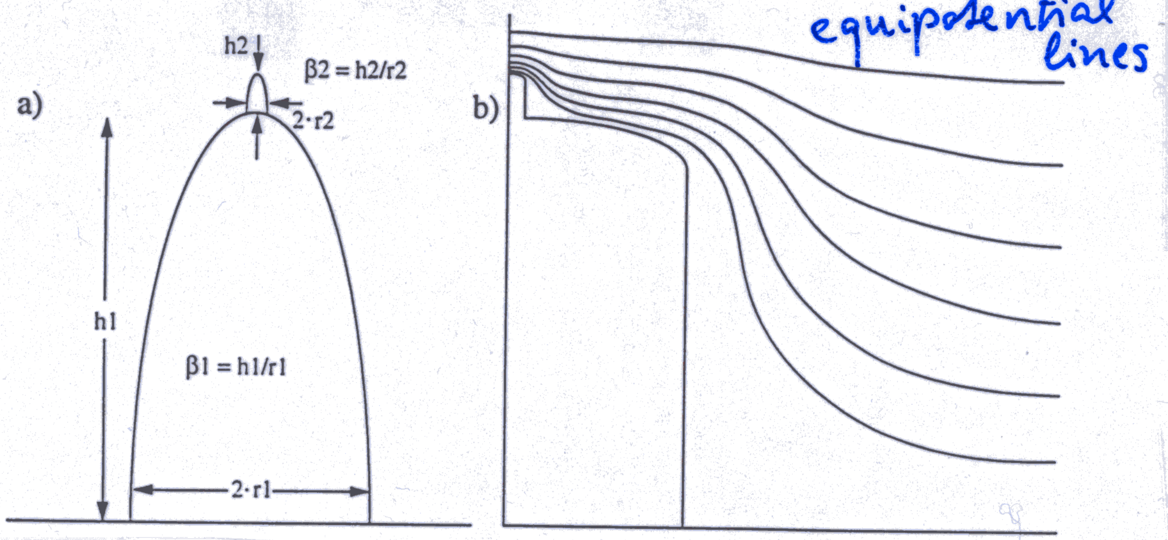
β_{FN} : empirical field enhancement factor

flat surface : expect field emission only at extremely large electric field (10 GV/m)

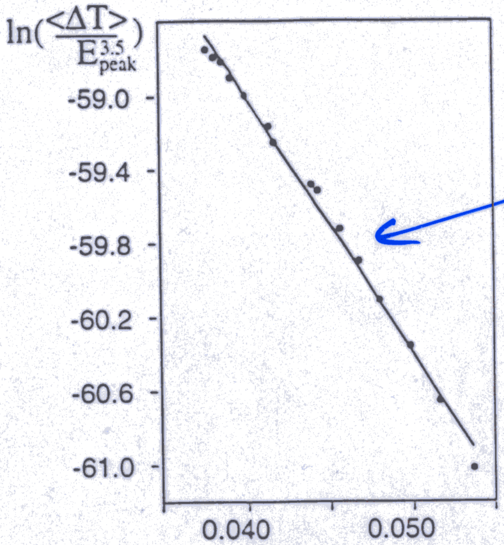
observation in cavities: field emission starts often at $\gtrsim$ 20 MV/m

$\beta_{FN} \gtrsim 100$ needed

tip on tip - model : $\beta_{FN} = \beta_1 \cdot \beta_2$



"Fowler - Nordheim" plot

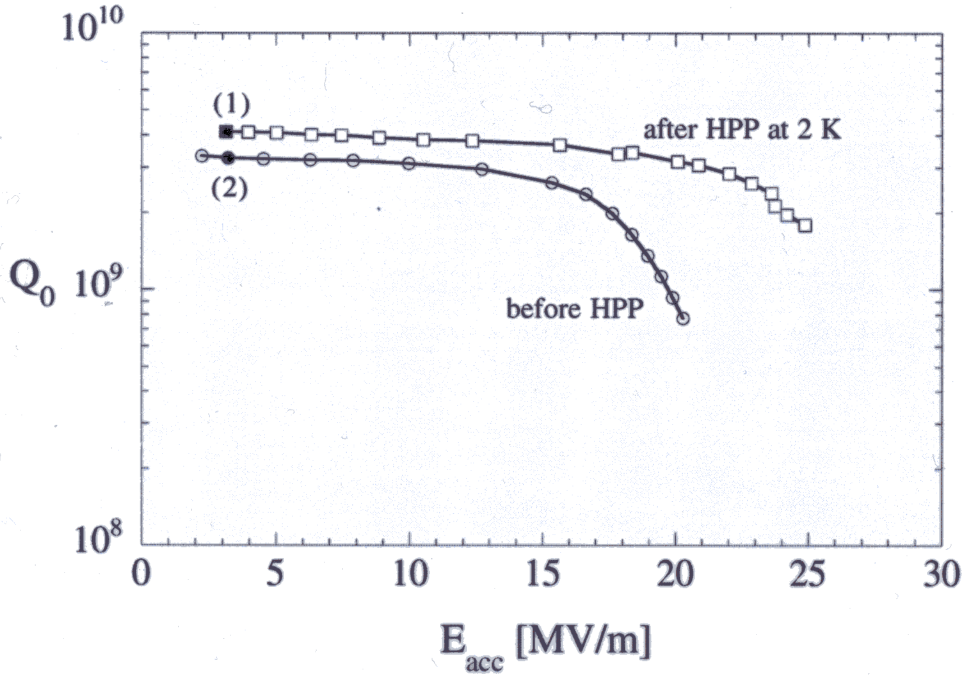


fit yields $\beta_{FN} \approx 300$

Destruction of field emitters by High Power Processing (HPP)

Cornell Univ., H. Padamsee

Apply short (≳ 100μs) rf pulses of several 100 kW instantaneous power



dissertation Michael Pekeler
U. of Hamburg

The rf pulse is so short that most of the cavity remains superconducting.

Emitting tip melts or explodes

Destruction of field emitters by high peak power (HPP) processing (several 100 kW for ≈ 100 μs)

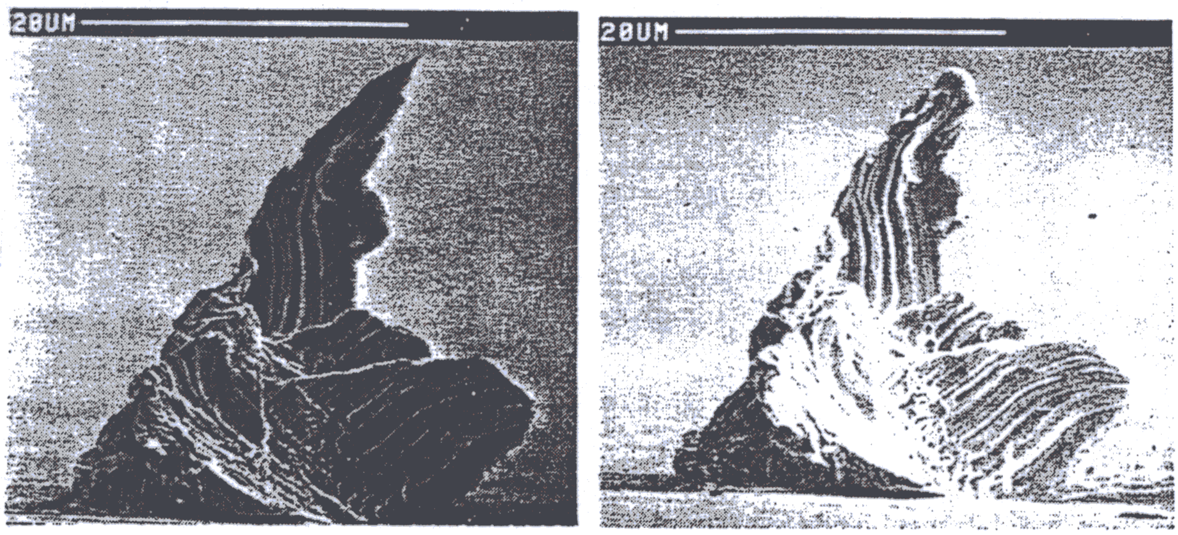
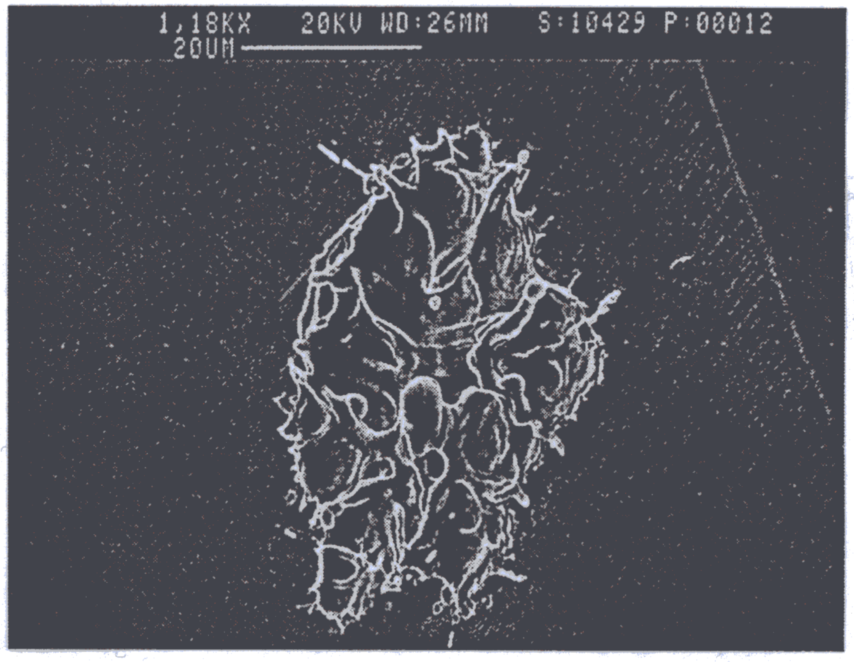
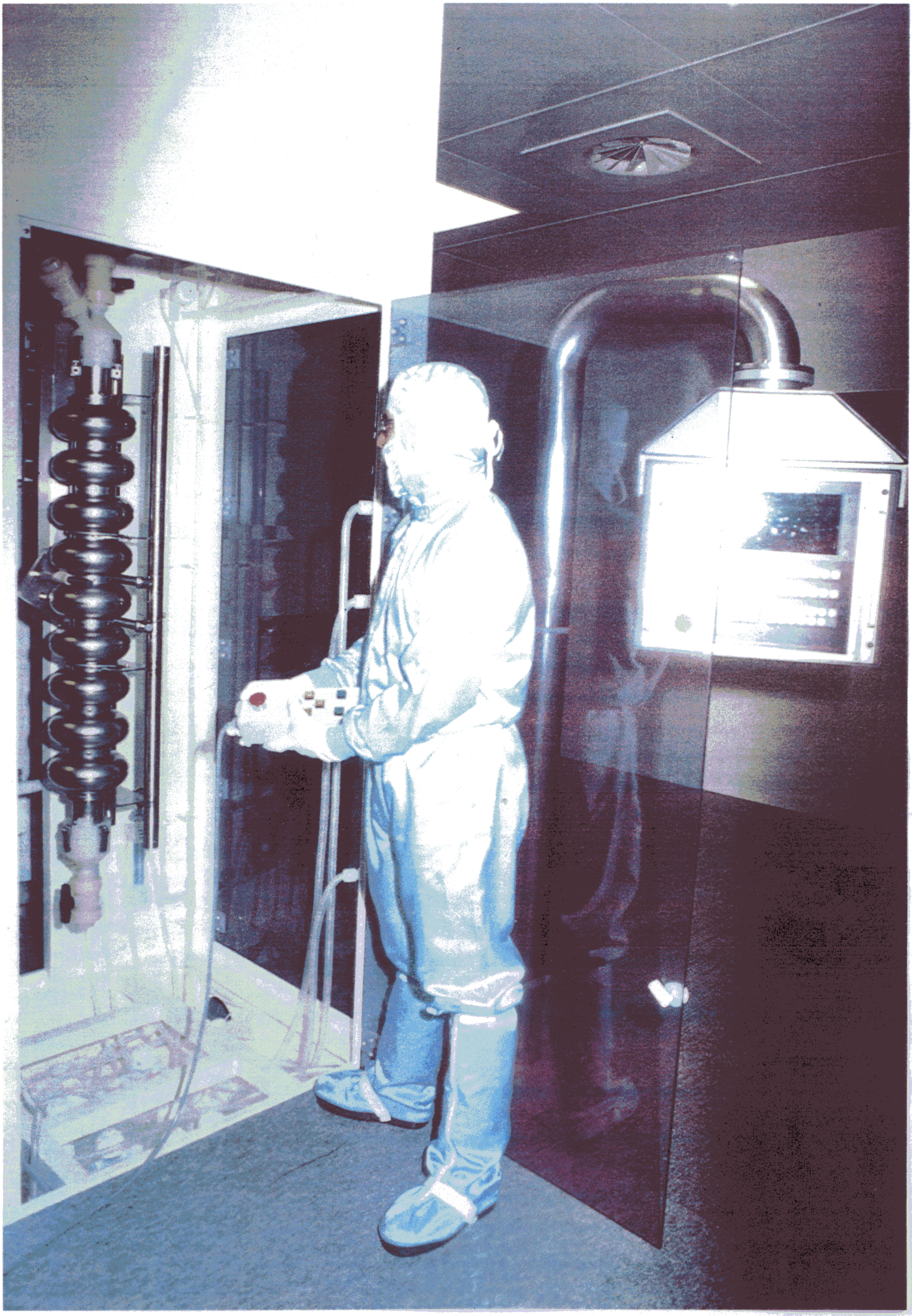


Fig. 6 Microphotograph of an emitting site: a) before emission; b) after emission. Note the apex melting.

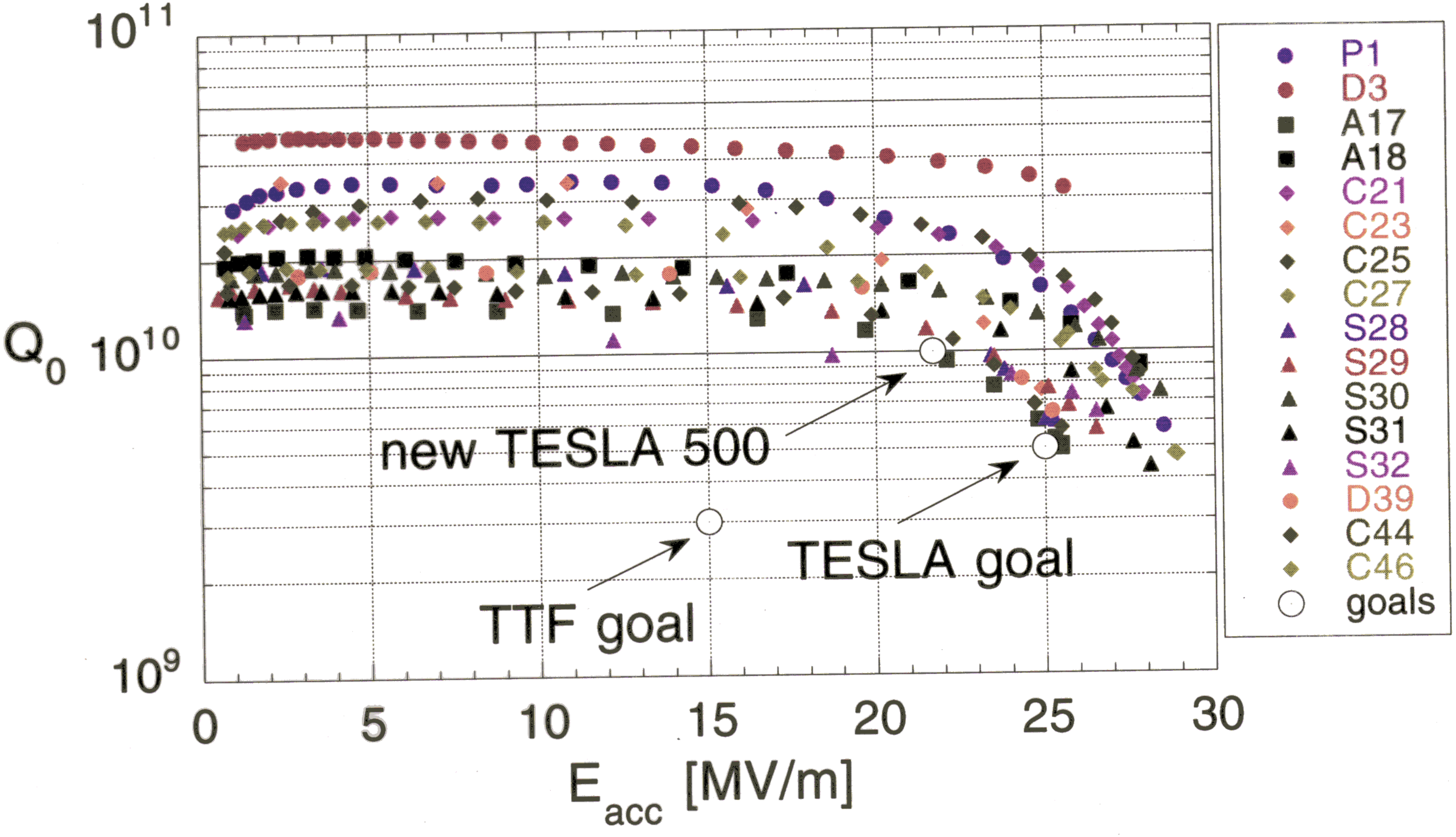
B. Bonin (Saclay): melting of a sharp tip by HPP



D. Moffat (Cornell): remnant of an exploded emitter clean - room treatment



TESLA 9-cell cavities exceeding 25 MV/m



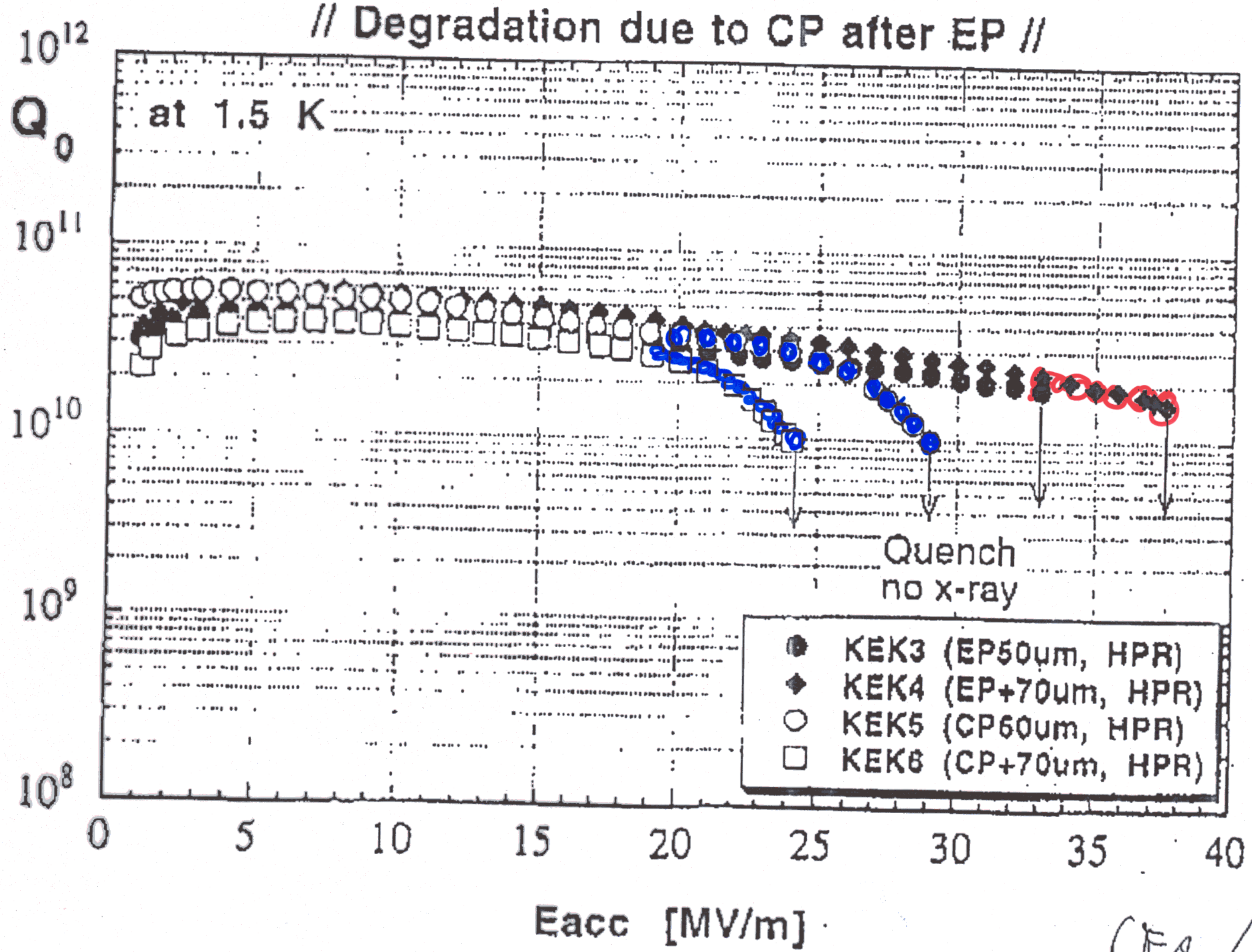
49

KEK (Japan), Saclay (Frankreich)

☆ New Results

- chem. Beizung
- elektrolytische Politur

S-3 / C103 Cavity



Quest for ultimate gradients (all 1-cell cavities)

CEA / KEK

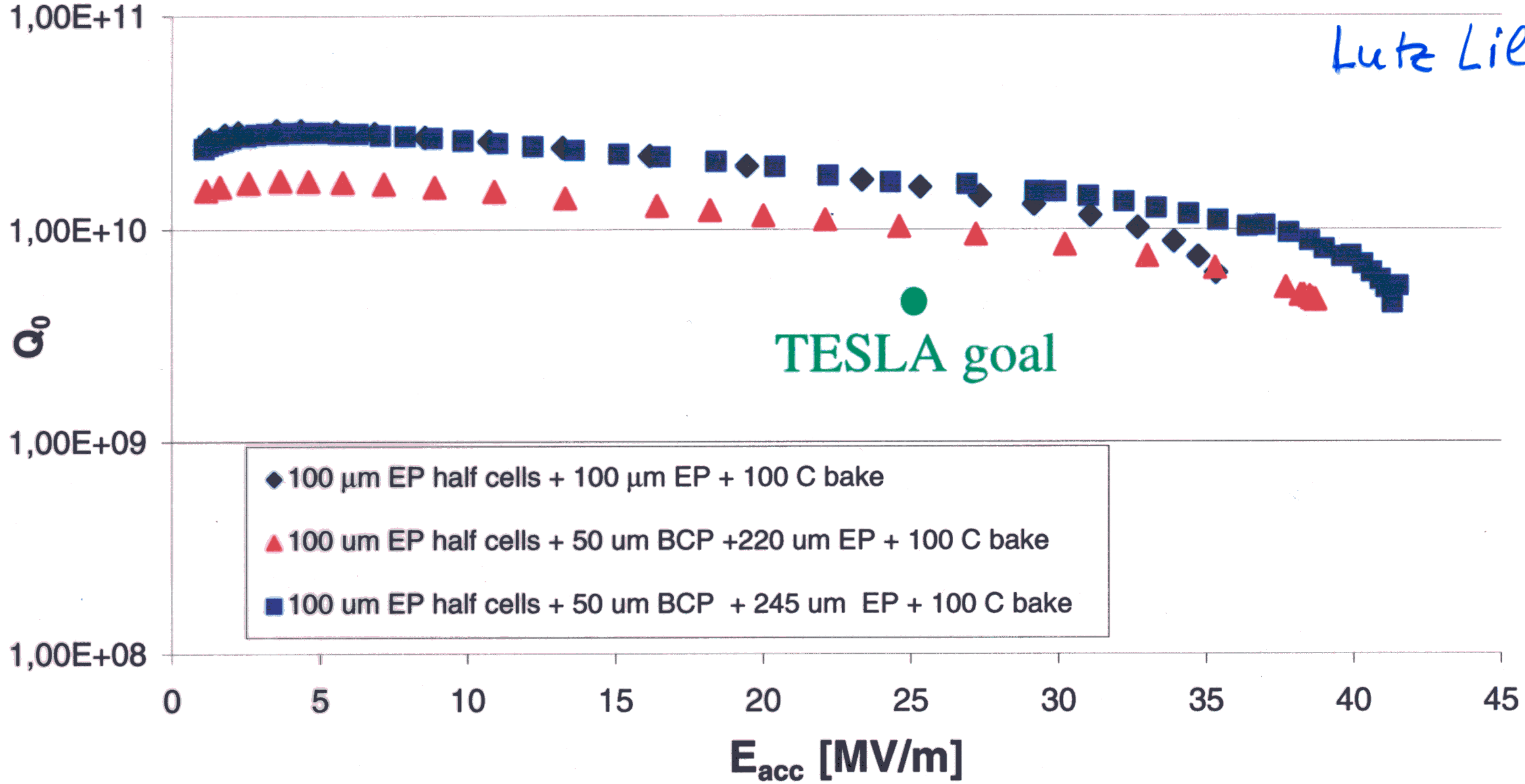
'99 06/02 00:54 FAX 0298 64 3182 加速器研究施設 ☑001

Electropolished 1-cell 1.3 GHz cavities

27

- EP done at CERN, measurements at CEA, CERN, KEK at DESY
- KEK-style electropolishing used for 1-cell cavities

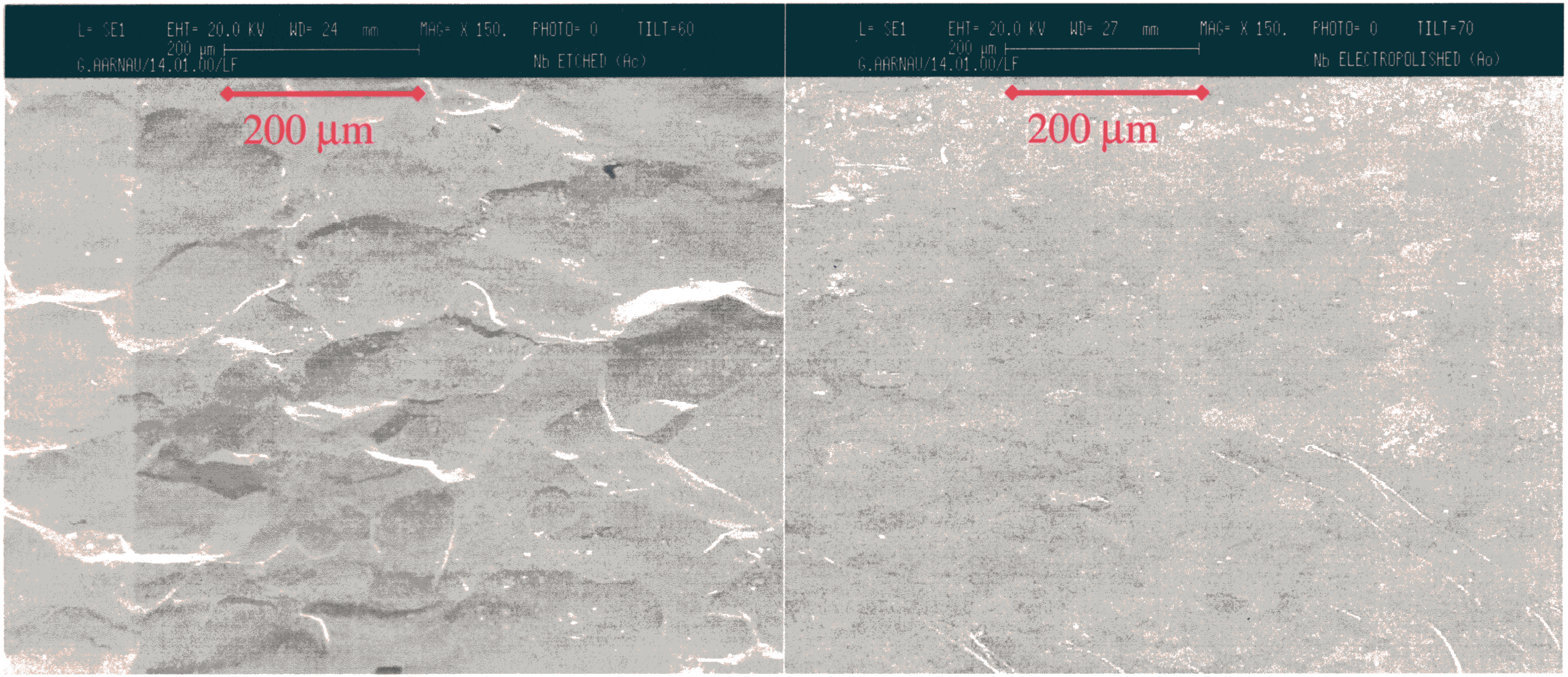
Lutz Lilje (U. HH)



⇒ electropolishing of 9-cell cavities

20

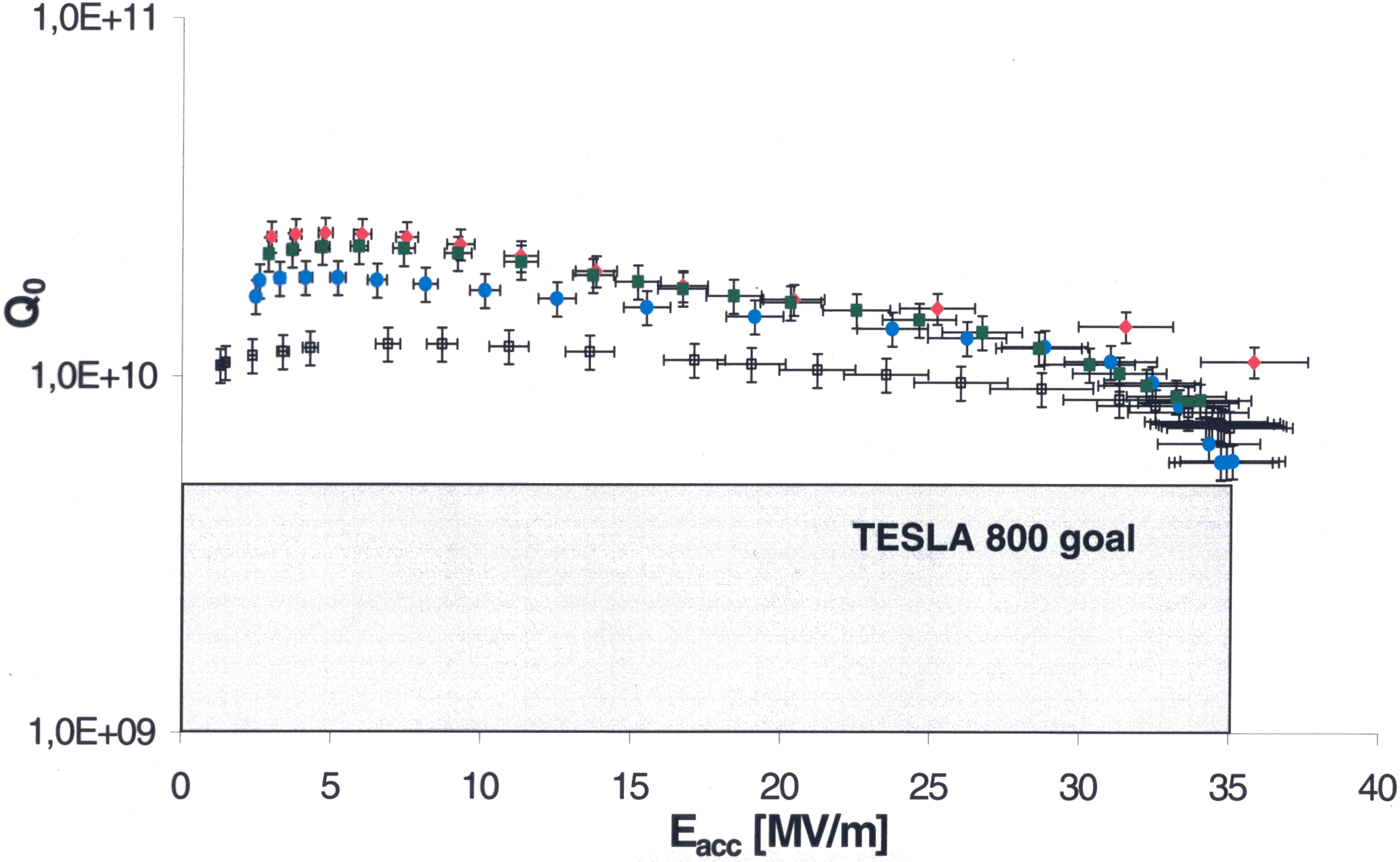
Niobium surfaces



- Etching (Buffered chemical polish)
  - HF, HNO_3, H_3PO_4
- Electropolishing

31

Cavities for TESLA-800



Lutz Lilje DESY

TESLA

14.04.03