Transverse Dynamics II : Emittances

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Emittance Definition

Betatron equation (linear!):

$$x(s)'' + K(s)\,x(s) = 0$$

Solution for particle "i"
(with A_i and δ_i constants given by the initial conditions for particle "i"; β_x (s) and ψ(s) follow from K(s))

$$x = A_i \sqrt{\beta_x}\,\cos(\psi + \delta_i)$$

$$x' = -A_i \sqrt{1/\beta_x}\,\{\alpha_x \cos(\psi + \delta_i) + \sin(\psi + \delta_i)\}$$

'normalised' solution

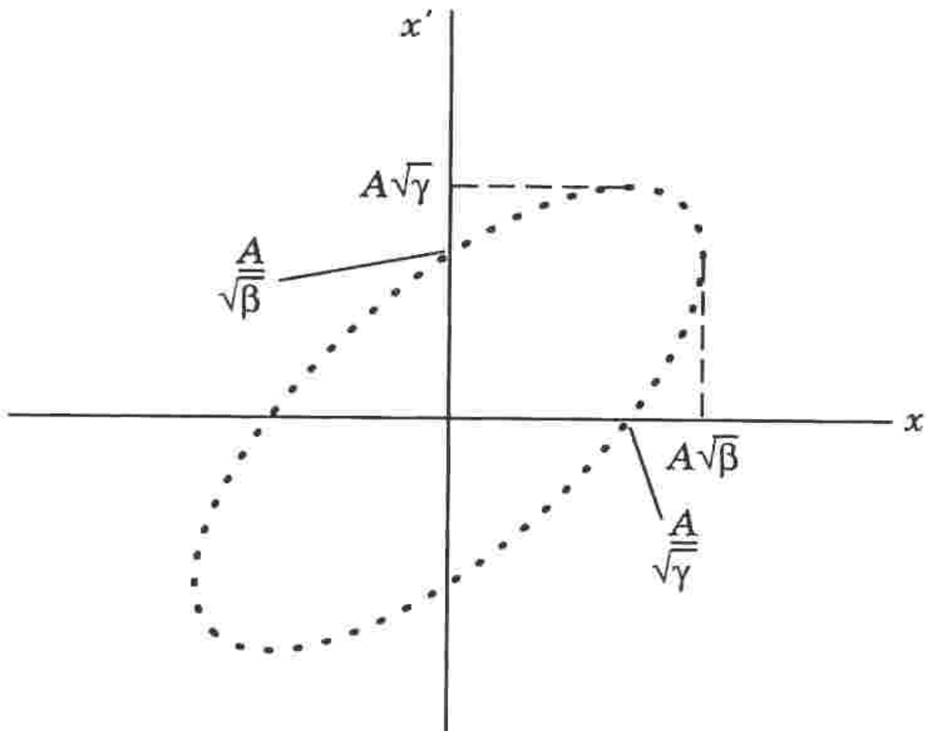
$$x = A_i \sqrt{\beta_x}\,\cos(\psi + \delta_i)$$

$$p_x = \alpha_x x + \beta_x x' = -A_i \sqrt{\beta_x}\,\sin(\psi + \delta_i)$$

Letting ψ(s) vary from 0 to 2π one see can see that the motion of the particle describes an ellipse in (x, x') [circle in (x, p_x)] space! The "single particle emittance" is defined as:

$$\varepsilon_i \equiv A_i^2 = \text{area of ellipse in } (x, x')\text{-space} = \text{area}/\beta_x \text{ of circle in } (x, p_x)\text{-space}$$

Single particle (x x')-phase-space trajectory



K(s)--> [Courant&Snyder transform (analytic or by codes)] -->β(s)

$\alpha = -\beta(s)'/2$

β(s)--> $\gamma = (1+\alpha^2)/\beta$

$\psi = \int ds/\beta \approx 2\pi Q\, s/L$

$Q = \frac{1}{2\pi}\int_0^L ds/\beta$ = nr. of oscillations per length L (per turn in circular machine if L is the circumference)

'single particle emittance' (also called : Courant&Snyder –invariant) :

= area of ellipse [in units π m rad]: $\varepsilon_i \equiv A_i^2 = x_{max}^2 / \beta_x$

Further remark on β-function

It follows from the differential equation ("envelope equation"):

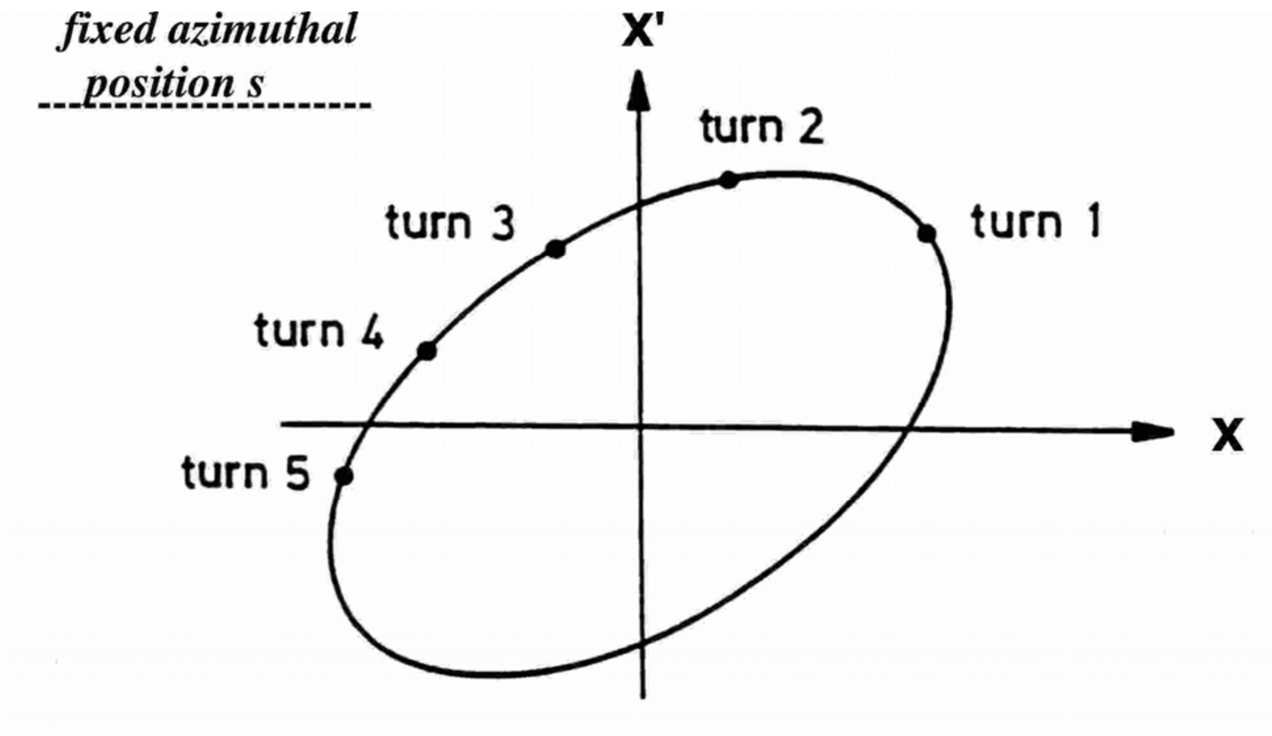
$$w''(s) + K(s)\,w(s) - \frac{1}{w^3(s)} = 0 \qquad where \quad w = \sqrt{\beta}$$

To obtain a unique solution, we have to specify two boundary condition. For a *circular* machine, these are automatically given by the cyclic condition

$$(\beta, \beta')_{s=0} = (\beta, \beta')_{s=2\pi R}$$ **(β and β' repeat after one turn).**

For a *linear* machine or beam line, one usually specifies β and β' at the entrance (to match the beam coming from the previous stage). Then the β function for the whole line is also uniquely determined.

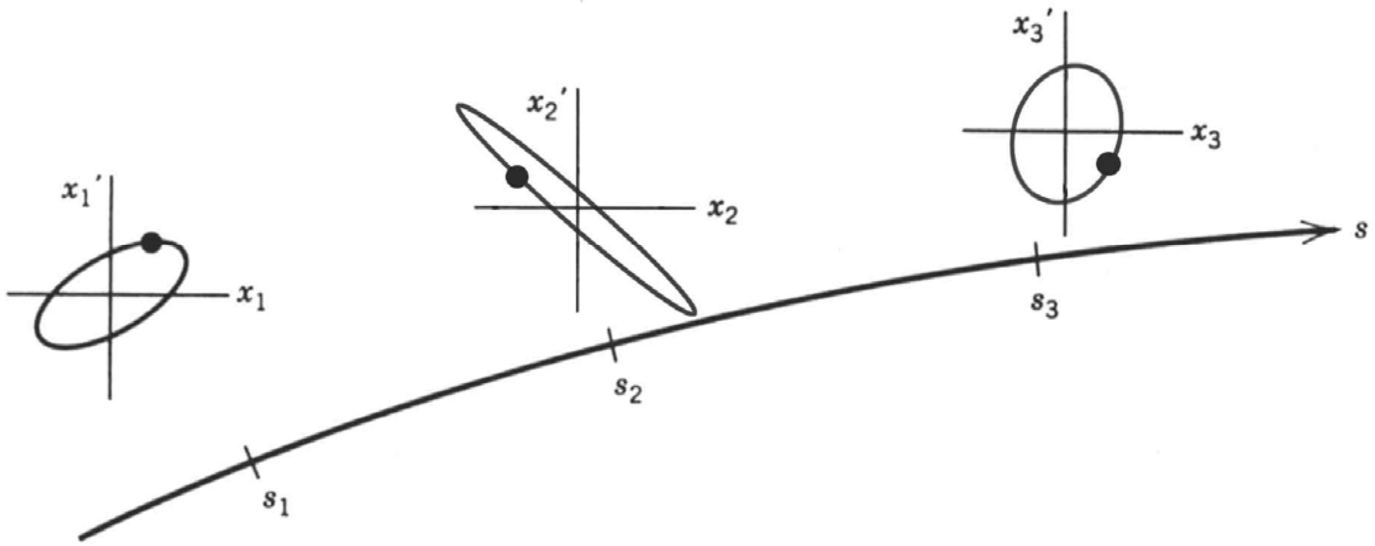
Phase space position of a particle at different turns



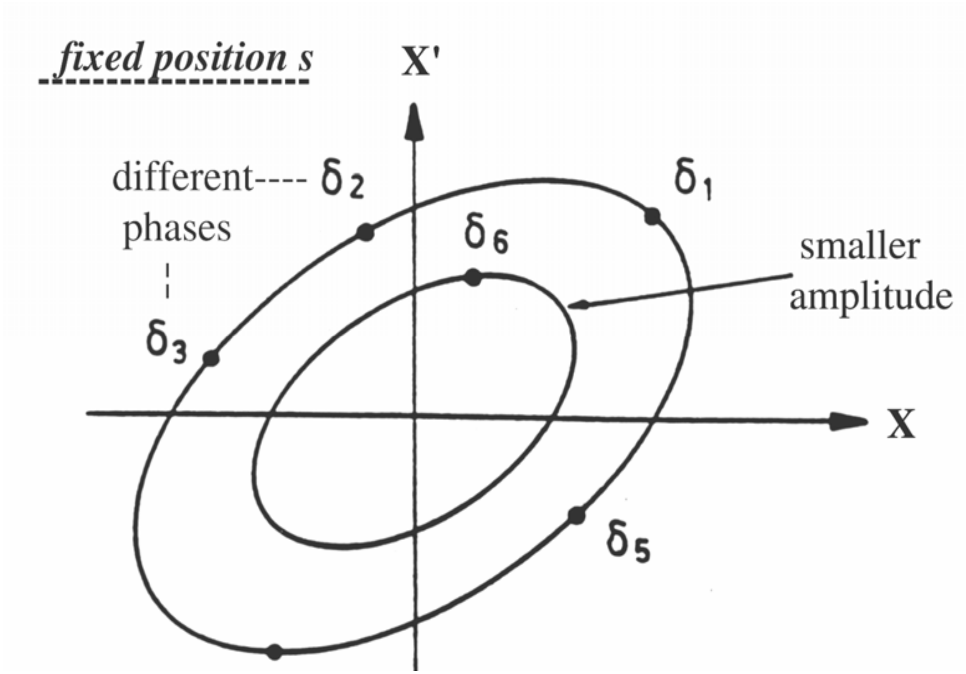
Observed turn by turn at a fixed azimutal position (s) in a circular machine, the phase space coordinates of a particle trace the ellipse. During each revolution the phase ψ advances by 2πQ.

Change of phase space trajectories along a beam channel

As the beta function changes along the channel (line, ring…) the ellipse pattern strongly varies. But the area of the ellipses is the same (as a consequence of Liouville's theorem, to be discussed later)

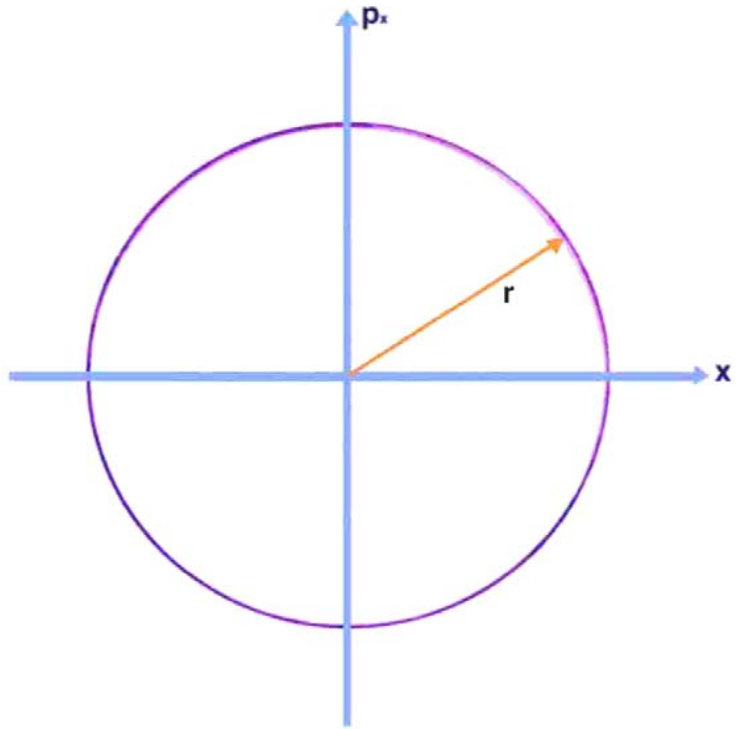


Trajectory of particles with different phases and amplitudes



Viewed at fixed position (s) and time, the phase space coordinates of particles with the same 'amplitude' A but different initial phases δ trace the ellipse. Particles with different amplitude lie on different concentric ellipses.

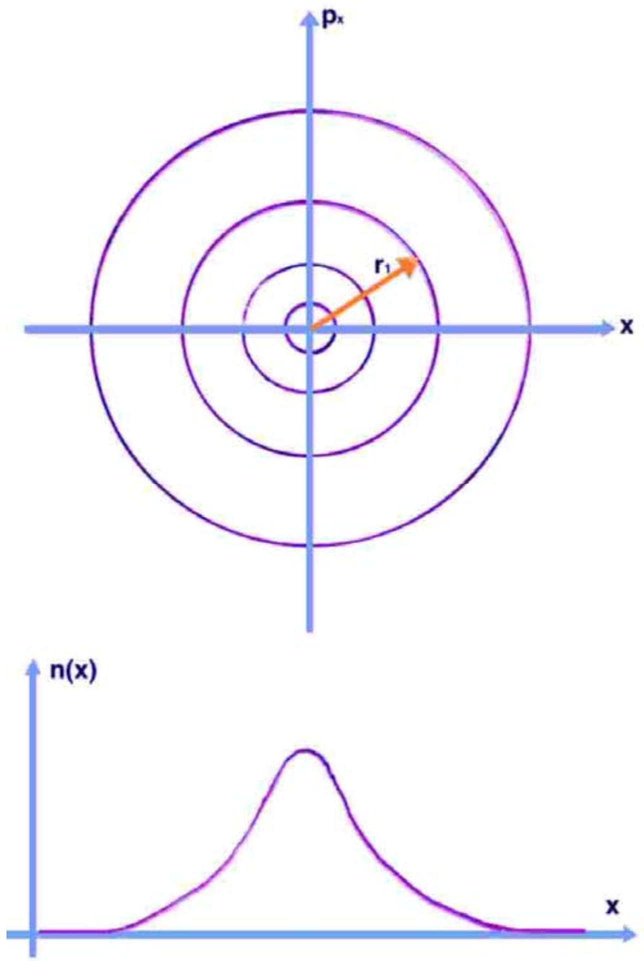
Single particle trajectory in normalized phase-space



In $(x\,,\ p_x = \alpha_x\, x + \beta_x\, x')$**-phase space, particle trajectories are circles**

'single particle emittance' =area of circle/βx : $\varepsilon_i \equiv A_i^2 = r^2 / \beta_x$

Many Particle Trajectories and Projected Density



Beam emittance definitions

--- Beam emittance = " some average" of $\varepsilon_{\mathbf{i}}$ over all beam

Definition referring to the a fixed fraction of particles:

The beam emittance ($\boldsymbol{\varepsilon}_{\%}$) is the area/$\boldsymbol{\beta}_{\mathbf{x}}$ of the circle in (x, p_x) space that contains the motion of a given fraction (F) of the particles. Frequently one refers to F = 39% or 86% or 95% of the beam.

$\varepsilon_{\%}$ is (sometimes) called: "geometrical emittance"

Definition referring to the standard deviation of the projected distribution:

Let $\boldsymbol{\sigma}_{\mathbf{x}}$ be the standard deviation of the particle density projected on the x-axis (i.e. the "rms beam size" as measured e.g. on a profile detector). Then the k-rms emittance is defined as the area/$\boldsymbol{\beta}_{\mathbf{x}}$ in (x, p_x) space with radius $k\sigma_x$.
Usually one choses k=1 or 2 or 2.5.

$\varepsilon_{k\sigma} = (k\sigma)^2/\beta_x = ½\, k^2 \langle \mathbf{A_i}^2 \rangle$ is called 'k -rms emittance '

Gaussian distribution

Suppose that the distribution in transverse coordinate (x) is Gaussian (independend of time)

$$n(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{x^2}{2\sigma^2}}$$

Then the fraction of particles that have their motion contained in a circle of radius "a" in normalised phase space (emittance $\varepsilon = a^2/\beta$) can be shown to be

$$F_{Gauss} = \int_0^a \frac{1}{\sigma_x^2} e^{-\frac{r^2}{2\sigma_x^2}} r dr = 1 - e^{-\frac{a^2}{2\sigma_x^2}} = 1 - e^{-\frac{k^2}{2}}$$

$k = a/\sigma_x$	$\varepsilon_{k\sigma}$	F_{Gauss}	F_{unif} *)
1	$(1\,\sigma_x)^2/\ \beta_x$	39.3 %	50%
2	$(2\,\sigma_x)^2/\ \beta_x$	86.4 %	100%
2.5	$(2.5\,\sigma_x)^2/\ \beta_x$	95.6 %	(100 %)

Beam temperature

Kinetic theory of gases defines temperature (in each direction and global) as

$$k\,T_{x,y,s} = m < v^2_{x,y,s} >, \qquad T = \tfrac{1}{3}(T_x + T_y + T_s) \qquad (\tfrac{1}{2}mv^2 = \tfrac{3}{2}kT)$$

k: Boltzmann constant, m: mass of molecules, $v_{x,y,\,s}$: velocity components of molecules

Definition of beam temperature in analogy:

$$k\,T_{beam,x,y,s} = m_0 < v^2_{x,y,s} >,$$

where $v_{x,y,\,s}$ are the velocity spreads <u>in the system moving with the beam</u>.

The transverse velocity spread in the beam system is given by the r.m.s emittance:

$$< v_x^2 > = (\beta\gamma c)^2 < (x')^2 > = (\beta\gamma c)^2 \gamma_x \cdot \varepsilon_{x,r.m.s} \qquad \text{similar for y direction}$$

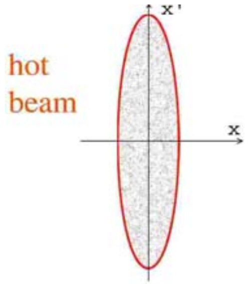
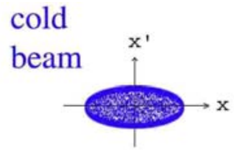
βc: longitudinal beam velocity β, γ : relativistic parameter, $\gamma_x \approx 1/\beta_x$: Twiss (lattice) parameter

Hence

$$===> \quad k\,T_{beam,x,y} = m_0 c^2 (\beta\gamma)^2 \gamma_{x,y} \cdot \varepsilon_{x,y;rms}$$

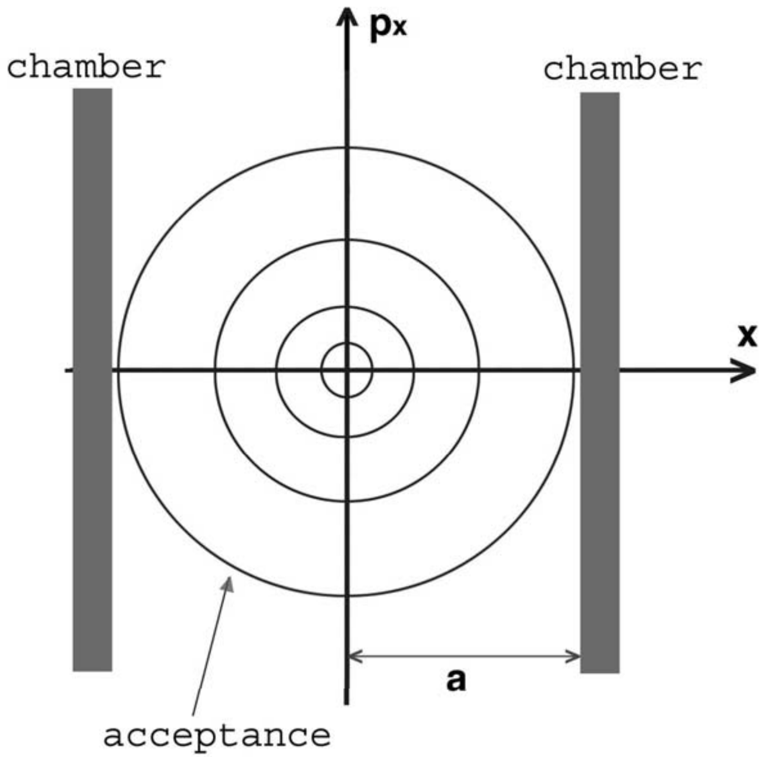
Beam temperature, II

$$\Longrightarrow \quad k\,T_{beam,x,y} = m_0 c^2 (\beta\gamma)^2 \gamma_{x,y} \cdot \varepsilon_{x,y;rms}$$

Property	Hot beam	Cold beam
ion mass (m_o)	heavy ion	light ion
ion energy ($\beta\gamma$)	high energy	low energy
beam emittance (ε)	large emittance	small emittance
lattice properties ($\gamma_{x,y} \approx 1/\beta_{x,y}$)	strong focus (low β)	high β
phase space portrait		

Electron Cooling: Temperature relaxation by mixing a hot ion beam with co-moving cold (light) electron beam.

Acceptance



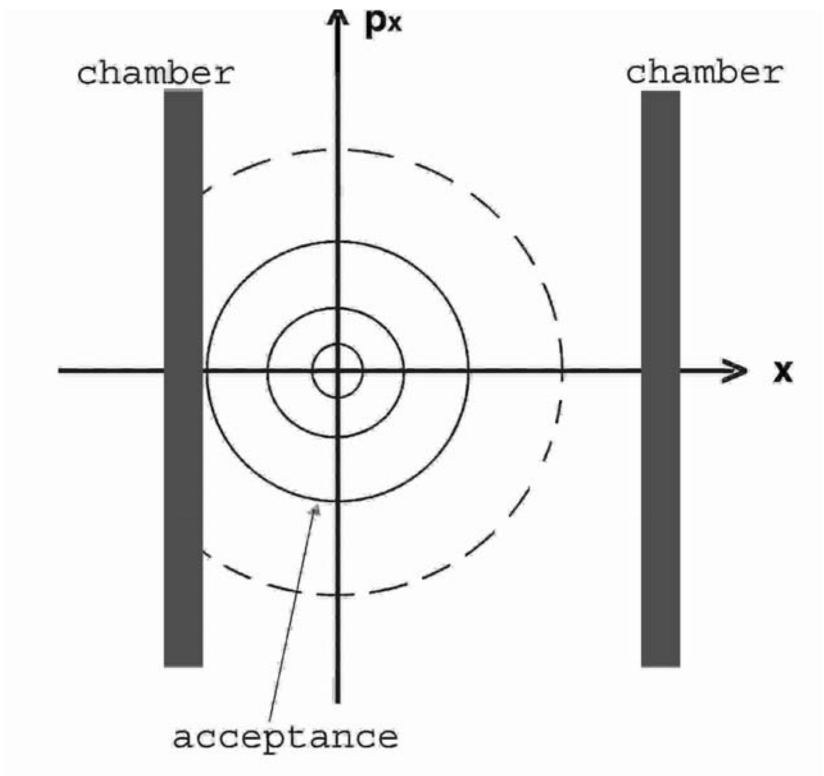
The acceptance (or admittance), A_c , of a beam channel is the maximum single particle emittance that can be transmitted.

$$A_c = a^2 / \beta_x$$

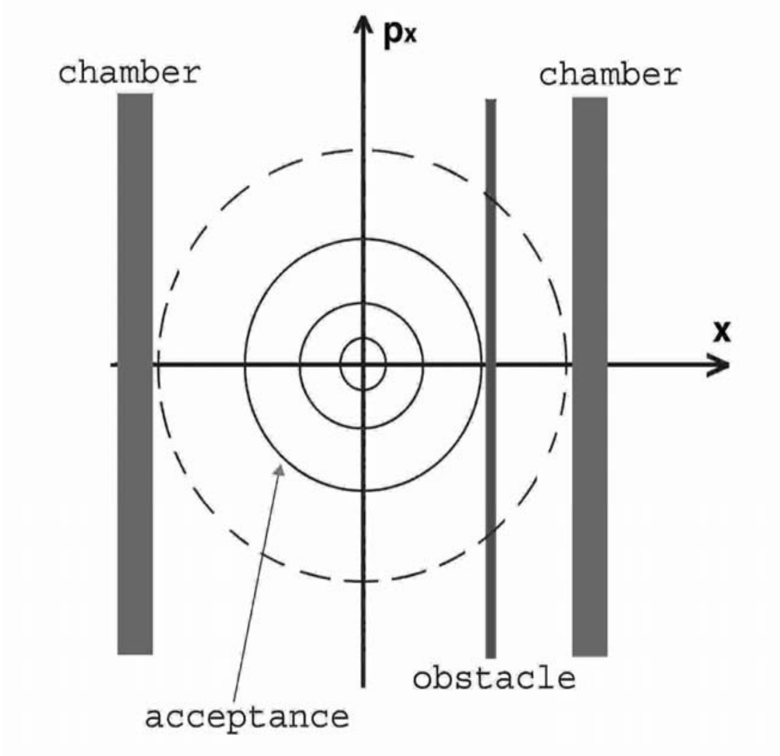
As both aperture, a, and β-function vary along the channel, the minimum determines the acceptance.

To avoid excessive loss, one limits the r.m.s emittance to ~1/6 A_c (96% emittance ~ A_c) for p and ions. For electrons one frequently limits $\varepsilon_{r\,ms}$ to ~1/50A_c÷ 1/100 A_c (σ < 1/7a ÷1/10a)

Reduction of acceptance



mis-centred beam

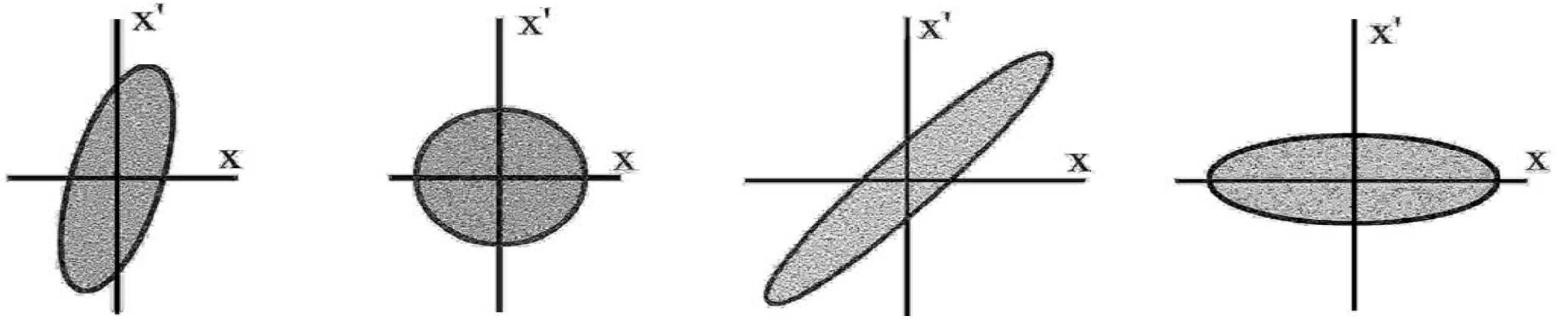


obstacle

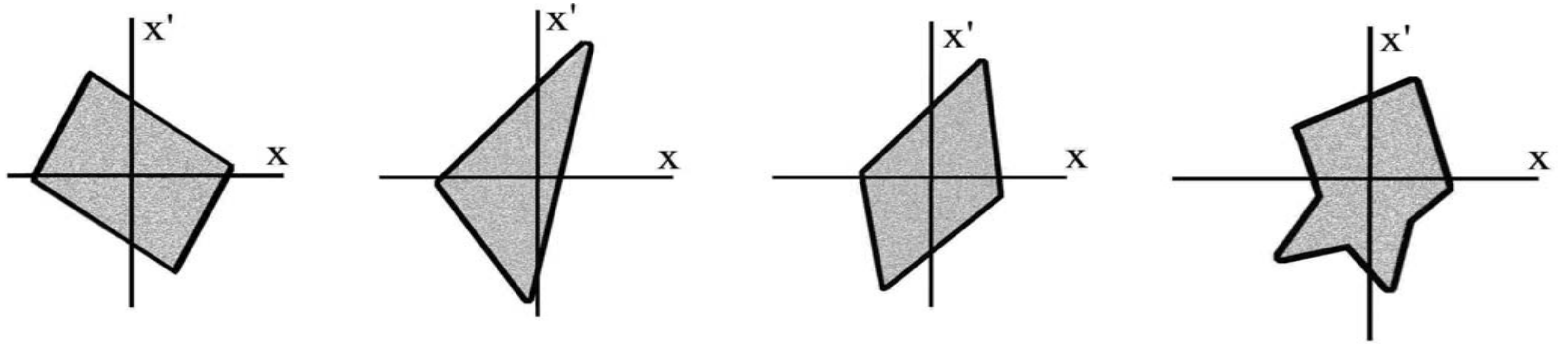
Mis-steering of the beam and obstacles can greatly reduce the acceptance.

(Consequences of) Liouville's theorem

Under certain conditions on the fields, which are thought to be satisfied in accelerators:
The phase space area (emittance) occupied by a particle beam is an invariant.



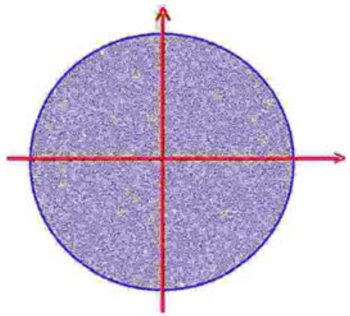
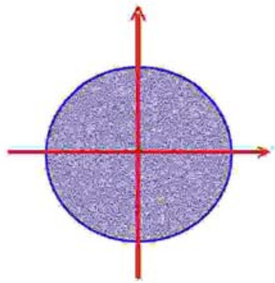
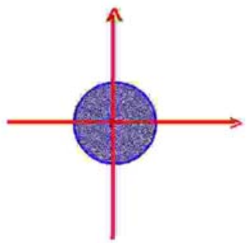
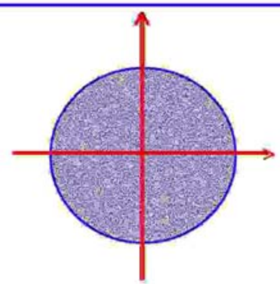
Phase space 'footprints' of the same beam



Corollaries to Liouville

When a beam is accelerated, its emittance decreases such that:

The normalised emittance $\varepsilon^* = \varepsilon\, \beta\gamma$ is (ideally!) invariant.

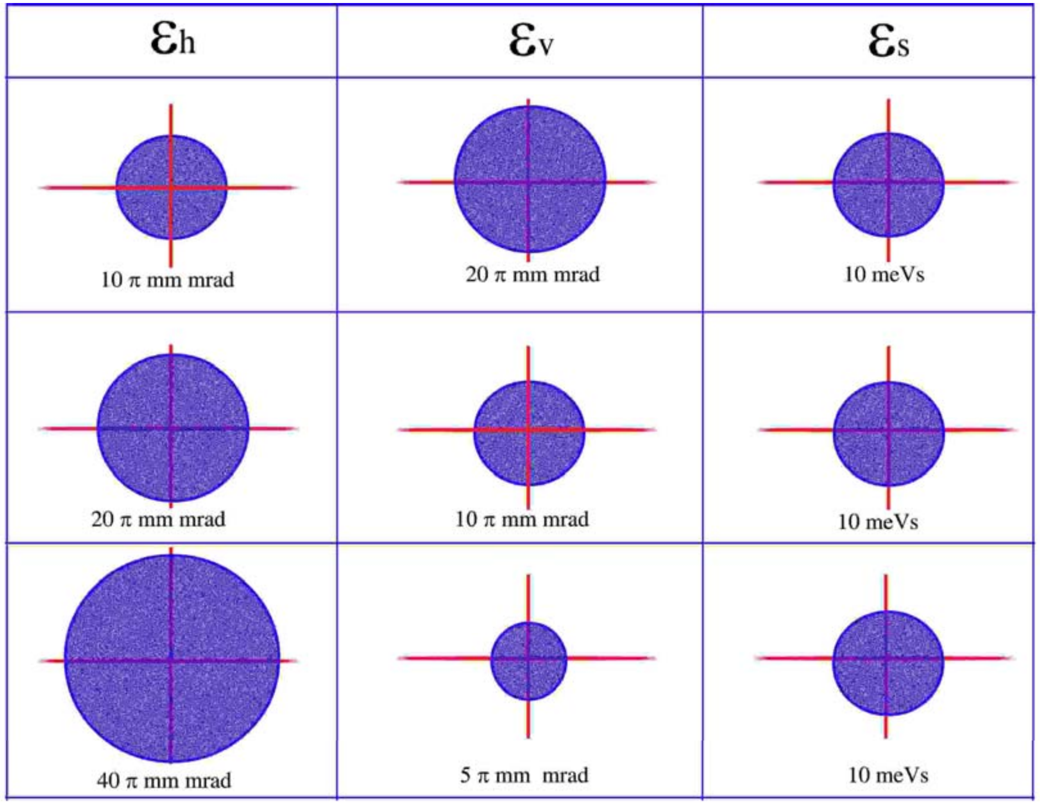
	ε	ε^*
low energy $\beta\gamma < 1$		
high energy $\beta\gamma > 1$		

Corollaries to Liouville, II

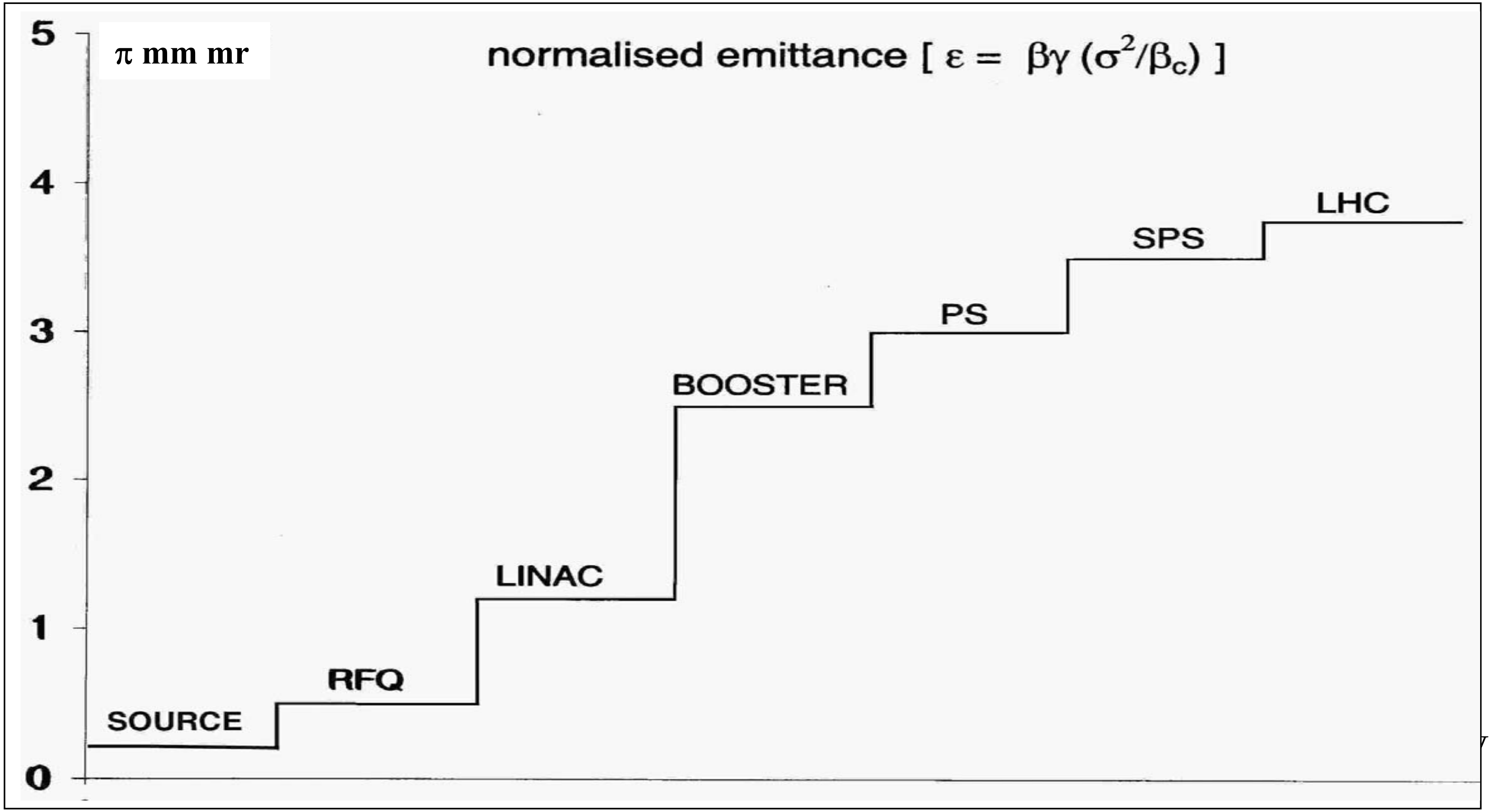
With coupling, the emittance in one plain (e.g. vertical) can decrease at the expense of the emittance in another plain (e.g. horizontal).

The '3-dimentional emittance' $\varepsilon_h \times \varepsilon_v \times \varepsilon_s$ remains invariant (at constant N)

Example:
trade of h → v
phase space volume



Ideal and reality: Emittance history for LHC



Emittance and beam density

The number (N) of particle per unit emittance defines the phase space density (P) of a beam.

1 dimensional: $P_x = N/\varepsilon_x$, $P_y = N/\varepsilon_y$, $P_s = N/\varepsilon_s$

transverse: $P_t = N/(\varepsilon_x \varepsilon_y)$

3 dimensional: $P_3 = N/(\varepsilon_x \varepsilon_y \varepsilon_s)$

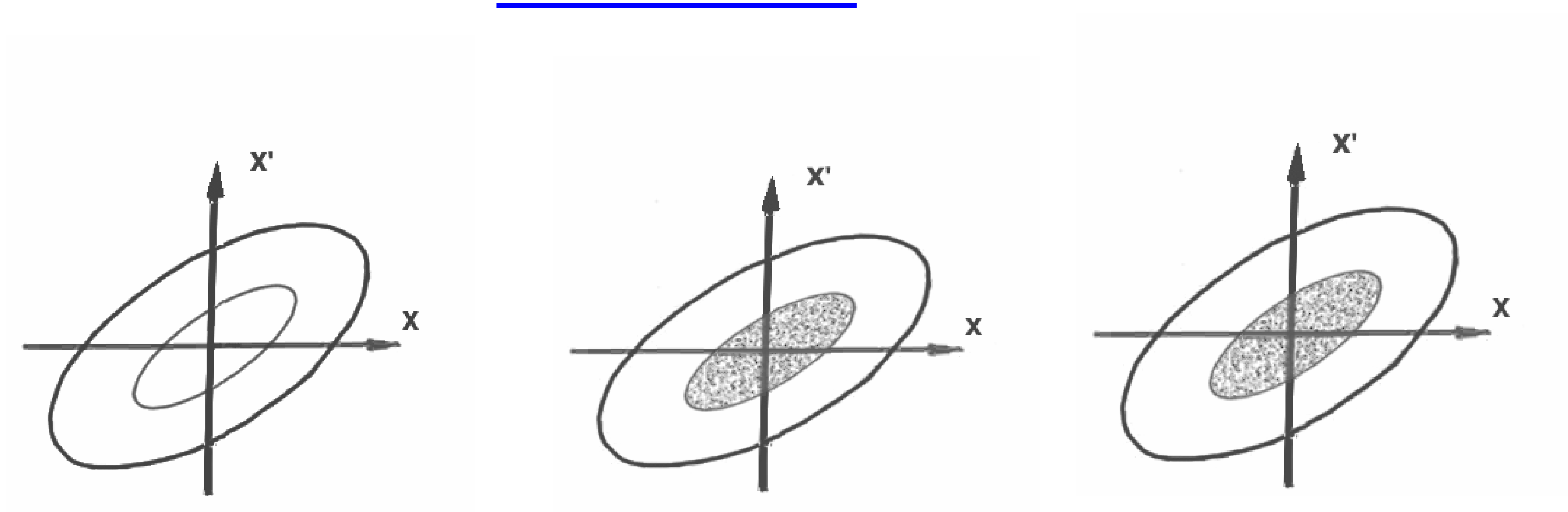
E.g.: for experiments using a beam colliding with a target or another similar beam, the real space tranverse density $\rho_t = N/(\sigma_x \sigma_y)$ is important.
Since $\sigma^2 = \varepsilon_{r\,m\,s}\, \beta^*$ (where: β^* = focussing function at the interaction point)

$$\rho_t \sim \{ (P_x P_y)^{1/2} \} \times \{ 1/(\beta^*_x \beta^*_y)^{1/2} \}$$

property of: beam (first factor), focussing system (second factor)

---> Phase space density is a figure of merit of a beam

Matched beams



Machine phase space | Matched beam injected | Matched beam after long time

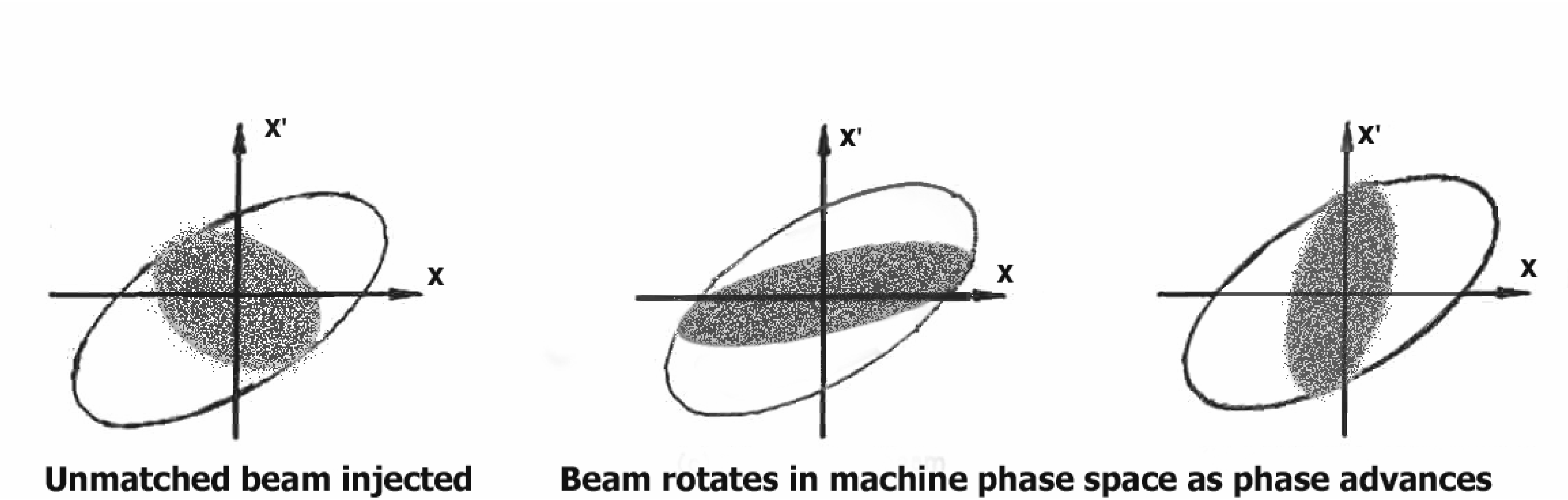
Suppose a beam with a (β,β')-function (given e.g. by that of the preceding stage) is injected ito a machine. It is matched (in phase space) if the β-function of the beam (at the injection point) is the same as that of the machine: $(\beta,\beta')_{beam}=(\beta,\beta')_{machine}$.

Matched beams preserve their shape.

The acceptance required is determined by the incoming beam emittance (optimum!)

Mismatched beams

Suppose a beam matched to the (β,β')-function of the preceding stage is injected into a machine adjusted for a different (β,β')-function at the injection point. This mismatched beam will start to rotate in the machine phase space (its shape/width oscillate).



Mismatched beams oscillate in width! Extra aperture required

Necessary acceptance determined by machine ellipse that encloses the injected emittance.

Does the beam define a beta-function?

The β-function is a property of the focussing system. A matched beam 'conforms' to this (β, β')-function. Suppose we could determine the statistical properties of the beam

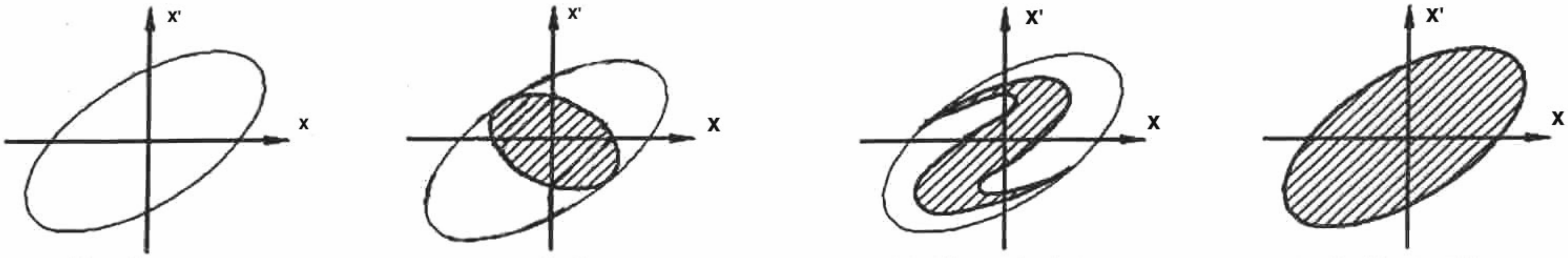
$$\sigma_x^2 = \langle x^2 \rangle, \qquad \sigma_{x'}^2 = \langle x'^2 \rangle \qquad \sigma_{xx'}^2 = \langle xx' \rangle$$

The β-function for this beam to be matched and the corresponding r.m.s. emittance can be determined as (see J. Buon, CAS 1990)

$$\beta_{\text{matched}} = \sigma_x^2 / \varepsilon_{\text{rms}} \qquad \beta'_{\text{matched}} = 2\sigma_{xx'}^2 / \varepsilon_{\text{rms}} \qquad \varepsilon_{\text{rms}} = \sqrt{\sigma_x^2 \sigma_{x'}^2 - \sigma_{xx'}^4}$$

Quantity	emittance	β-function	matched β-function
Property of	beam	focussing system	beam

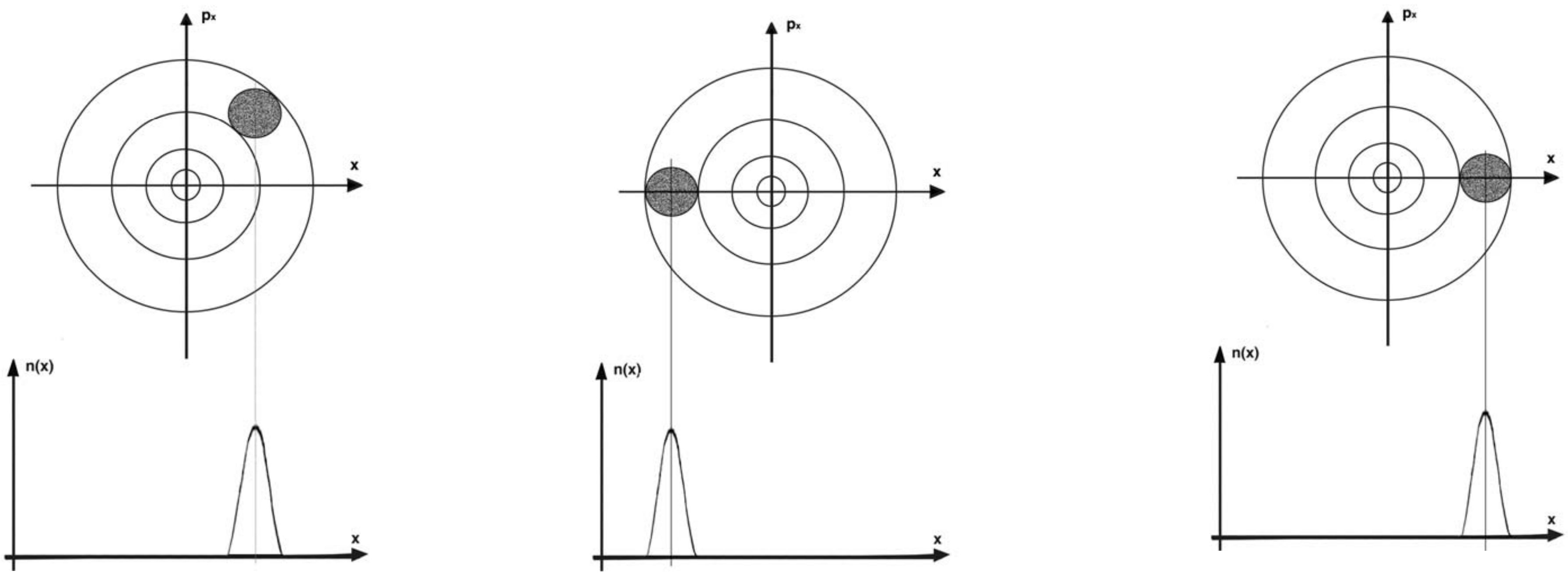
Filamentation (in circular machines)



Nonlinearity of betatron oscillation causes a dependence of betatron frequency on amplitude ---> particles go around in phase space at (slightly) different speed. Over sufficiently long time ($\Delta\omega t >> 1$) a mismatched beam 'smears out' and the larger phase space becomes filled out. Typical in cicular machines. In a beamline, nr. of betatron oscillations is not sufficient, but mismatch (evtl. filamentation) occurs in subsequent stage.

Filamentation = Randomisation of betatron phases (in a mismatched beam)
---> emittance dilution (apparent blow up)

Steering error at injection



(1) (2) (3)

A beam injected with position/angular error (1) rotates in phase space (2), (3) -->

Mis-steered beams oscillate in position ('center of charge')

Extra aperture ! (required acceptance determined by outer circle).

Damping of oscillations at injection

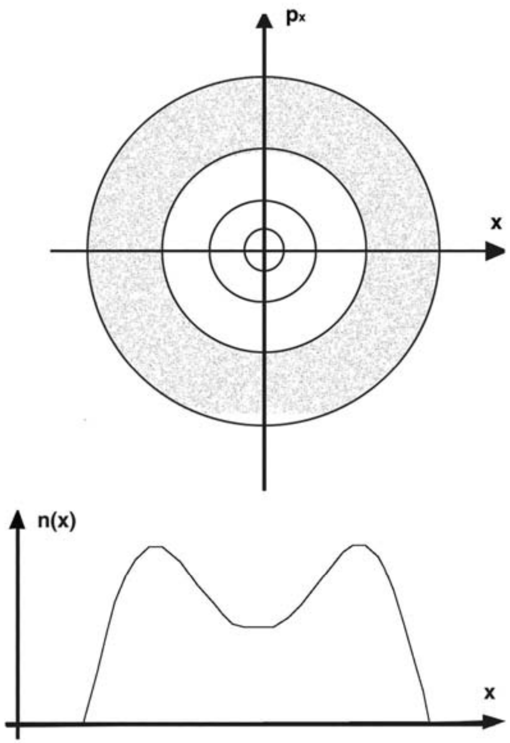
In circular machines, the oscillation due to mis-steering at injection can (in principle) be damped by a feedback system: A position pick up detects the beam's position (Δx) and a kicker electrode (a suitable betatron phase advance Δψ downstream) gives a kick proportional to Δx which tends to bring the beam to the centre of phase space.

Due to limited strength of the (rf-) kicker this takes many tens or hundreds of turns in the machine. O.k. if damping time << filamentation time. Emittance increase can then be avoided but the large aperture (to accommodate the oscillations) remains necessary.

A similar system, but with quadrupolar electrodes to detect the beam width and to damp it, could be used to damp the oscillations following the focussing (β-function) mismatch regarded before.

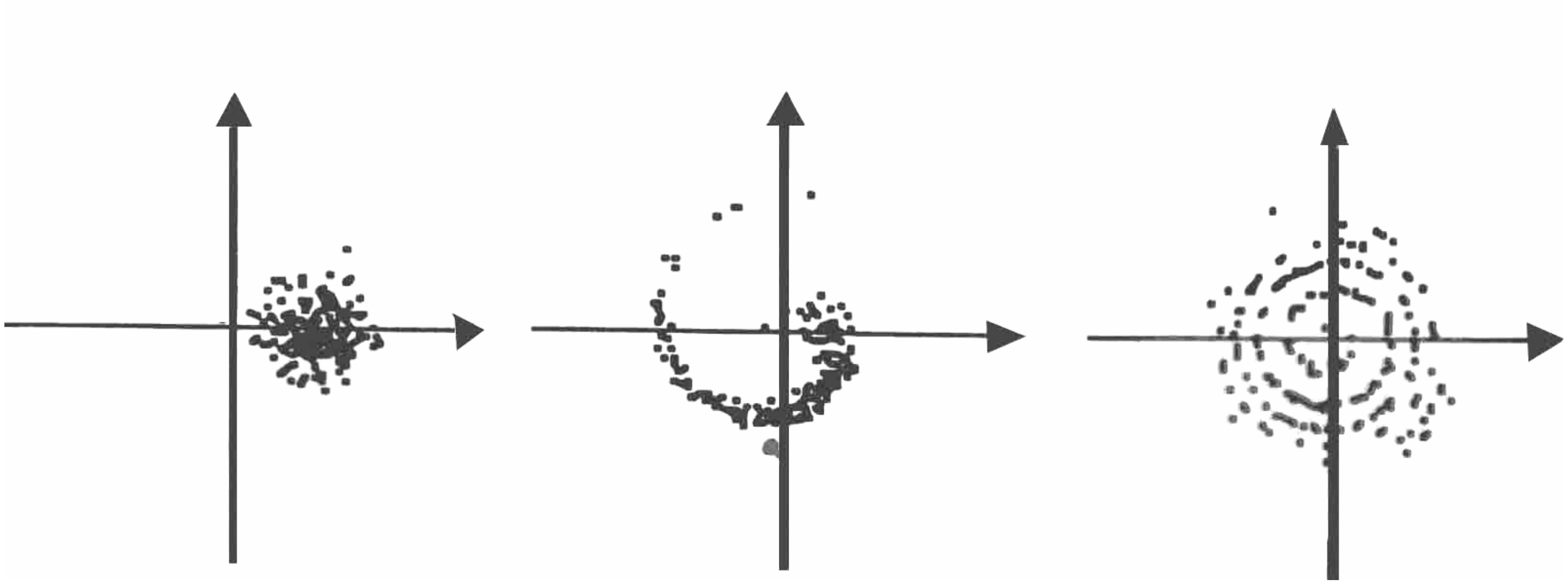
Beam position ('dipolar') dampers are successfully used in many small machines (e.g. PS-booster); qudrupolar dampers have been discussed but not yet built.

Steering error after filamentation



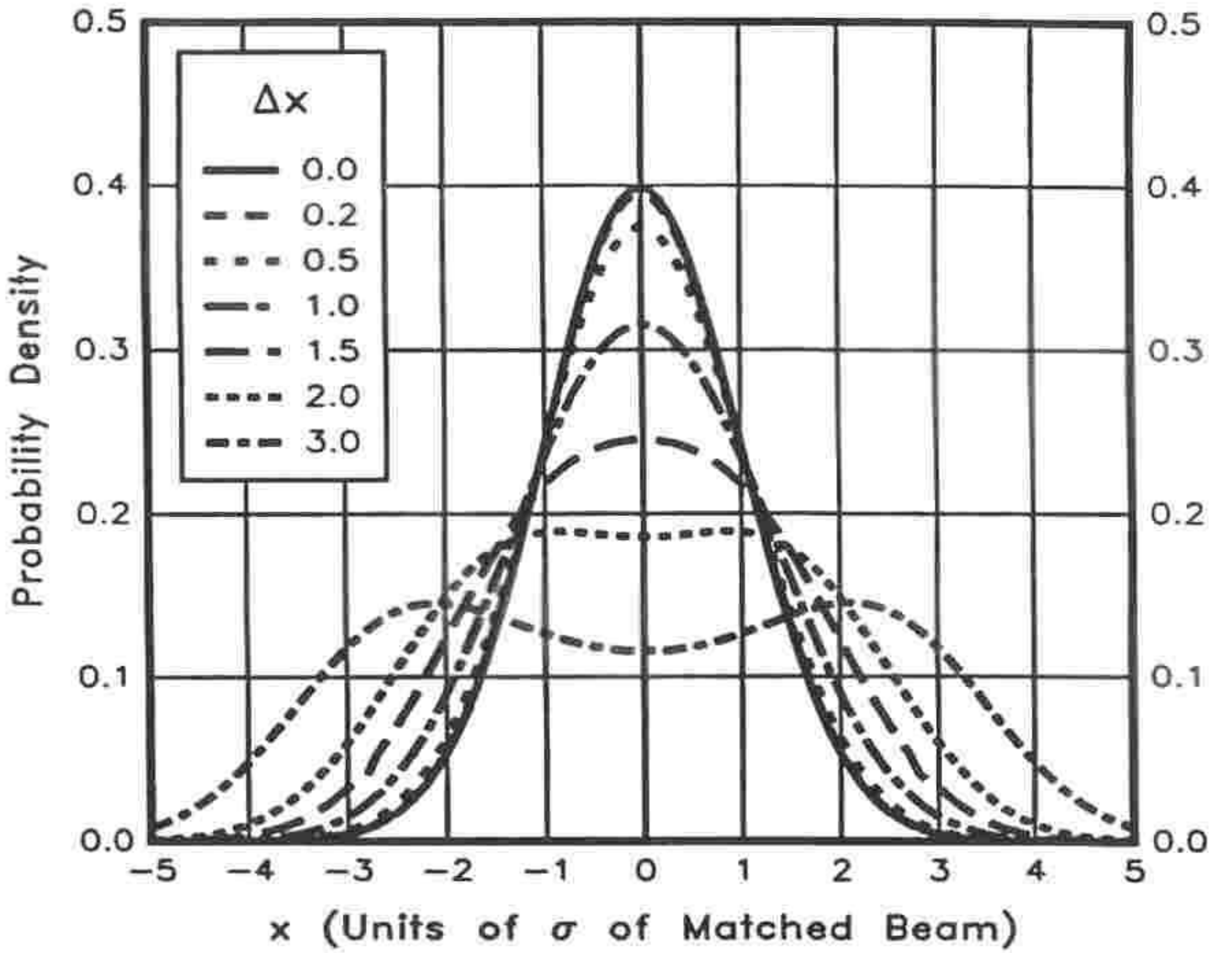
In a circular machine (and without damper), the beam injected with a steering error ‘smears out’ over the annular region. For large error the resulting projected distribution is ‘double humped’.

Simulation of filamentation



injection after some time after long time

Distribution after filamentation as function of the injection error

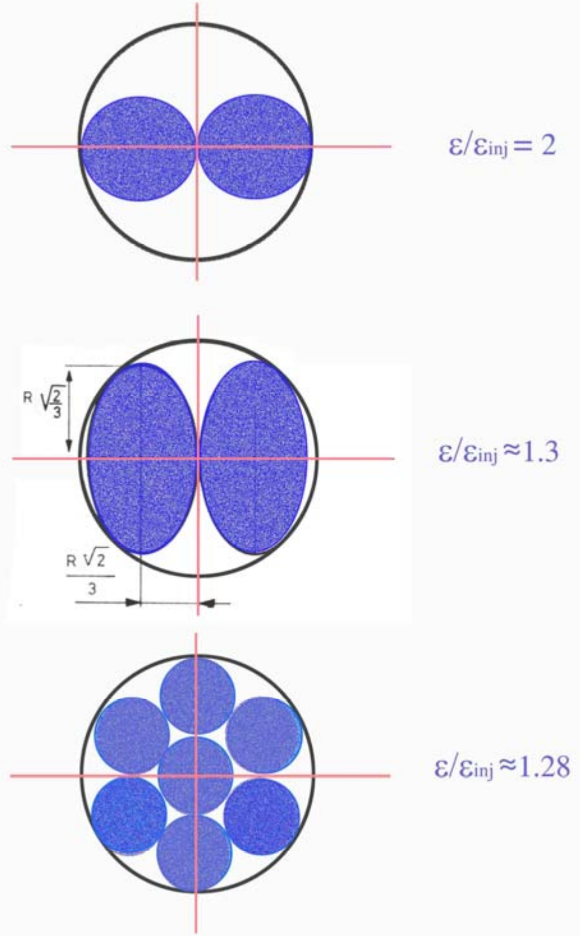


Phase space representation of multiturn injection

(Another example of the beauty of phase space plots)

When the acceptance is much bigger than the incoming beam emittance, several turns can be injected. The emittance increase $\varepsilon/\varepsilon_{inj}$ can be deduced from simple geometrical considerations (filling a circle with smaller circles or ellipses).

Examples depicted: 2 turn, non optimum and optimum, and 7 turn injection



Scattering in a foil or a window

A particle (of charge number q_p (=1 for proton), momentum p [MeV/c], velocity $\beta = v_p/c$) traversing a foil (thickness L, material of 'radiation length' L_{rad} undergoes multiple Coulomb scattering. The rms scattering angle in each of the transverse planes is given by

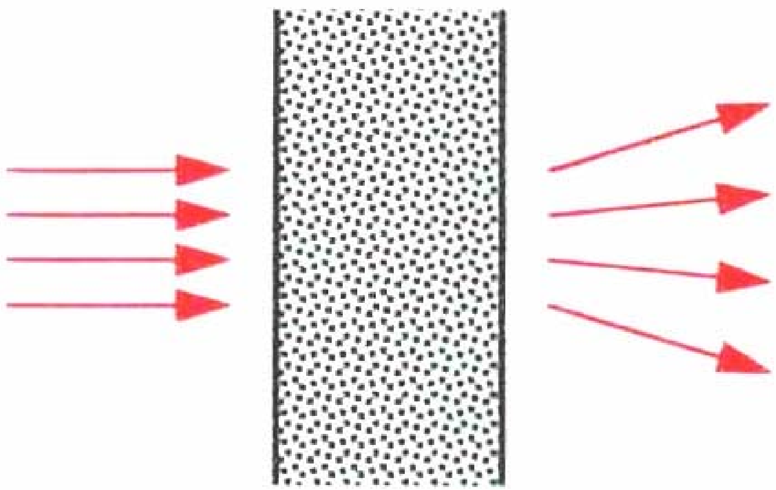
$$\theta_{rms} = \frac{14\,\mathrm{MeV/c}}{p\,\beta}\; q_p \sqrt{\frac{L}{L_{rad}}}$$

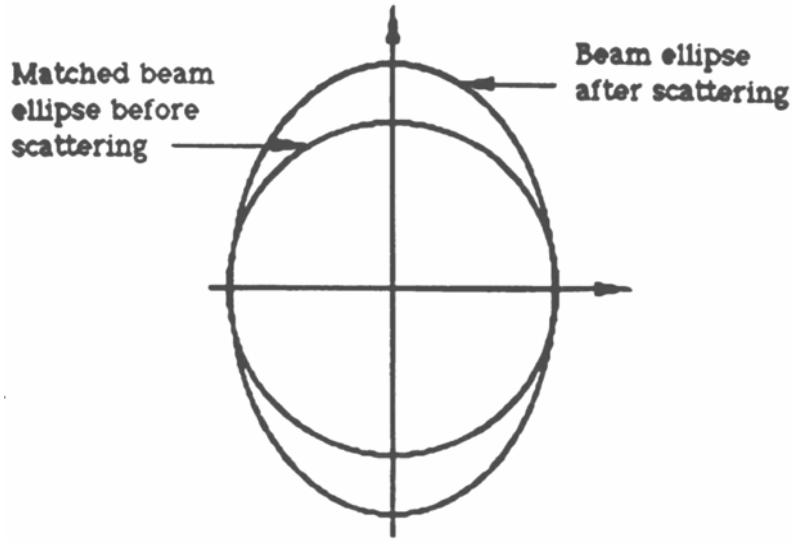
This causes an emittance increase which can readily be calculated by using the corresponding phase space plot.

The blow up of r.m.s emittance turns out to be

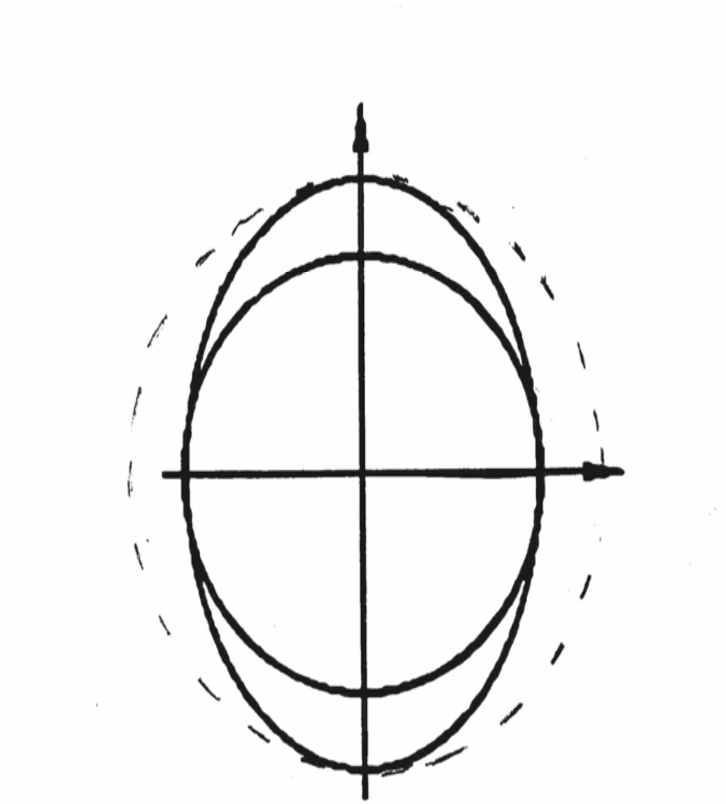
$$\Delta\varepsilon_{\sigma} = \tfrac{1}{2}\theta_{rms}^{2}\,\beta_x \qquad (*)$$

Real space and phase space plot upon transition through a foil





Phase space after scattering at the foil and filamentation



Multiple Coulomb scattering on the residual gas

This is treated in many papers of which I find the one by W. Hardt (CERN ISR-300/GS/68-11) especially instructive. Here we can use our previous results immediately by taking the residal gas atmosphere as a "thin distributed scatterer".

For pure Nitrogen (N_2) at pressure P the radiation length is $L_{radN2} \approx 305\ m/(P/760\ torr)$ and the thickness traversed by the beam in time t is $L=\beta ct$. Then from the scattering blow up Eq.(*) of a previous slide (#30) we get the blow-up of the kσ –emittance (Hardt's formula) as

$$\Delta\varepsilon_{k\sigma} \approx \frac{1}{2} k^2 \frac{q_p^2}{A_p^2}\ 0.3\,\bar{\beta}_x \frac{P\,t}{\beta^3\gamma^2}$$ ([π rad m], P in [torr] t in [sec])

Here $\bar{\beta}_x$ is the average beta function as scattering occurs everywhere around the ring; p=932 MeV/c*A_p *$\beta\gamma$, A_p and q_p : mass number and charge number of the ion (A_p and q_p both1 for protons.

This relation is widely used to determine the vacuum requirement in a storage ring. For a synchrotron one has to integrate $\frac{dt}{\beta^2\gamma}$ over the acceleration cycle to get the blow up of the normalised emittance . For an atmosphere with different gases of partial pressures P_i we can define the N_2 equivalent P for multiple Coulomb scattering as $P_{N2equ} = \sum P_i \left(L_{rad,N2} / L_{rad,i} \right)$

Hardt's classical internal paper (ISR-300/GS/68-11)

EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

ISR-300/GS/68-11

A FEW SIMPLE EXPRESSIONS FOR CHECKING VACUUM REQUIREMENTS
IN A PROTON SYNCHROTRON

by

W. Hardt

Geneva - 14 March, 1968

PS/6445

Multiple Coulomb scattering, orders of magnitude

In LEAR the vacuum pressure (N2 eqivalent for scattering) is of the order of $P=10^{-12}$ torr.
Then: $L_{rad} = 2.3\ 10^{17}$ m (about 25 light years).

At p=100MeV/c ($\beta_p \approx 0.1$) an rms scattering angle of $\theta_{rms}=5\ 10^{-3}$ rad (which is about the acceptance limit of LEAR, s. below) is reached after a path length $L= 1.3\ 10^{-5} \times L_{rad} = 3\ 10^{12}$ m (about 2.7 light hours).

With the speed $\beta_p c = 3\ 10^7$ m/s the beam traverses this distance in ≈ 27 h ($\approx$ one day).
With the circumference of $C \approx 80$ m this corresponds to about $4\ 10^{10}$ revolutions.

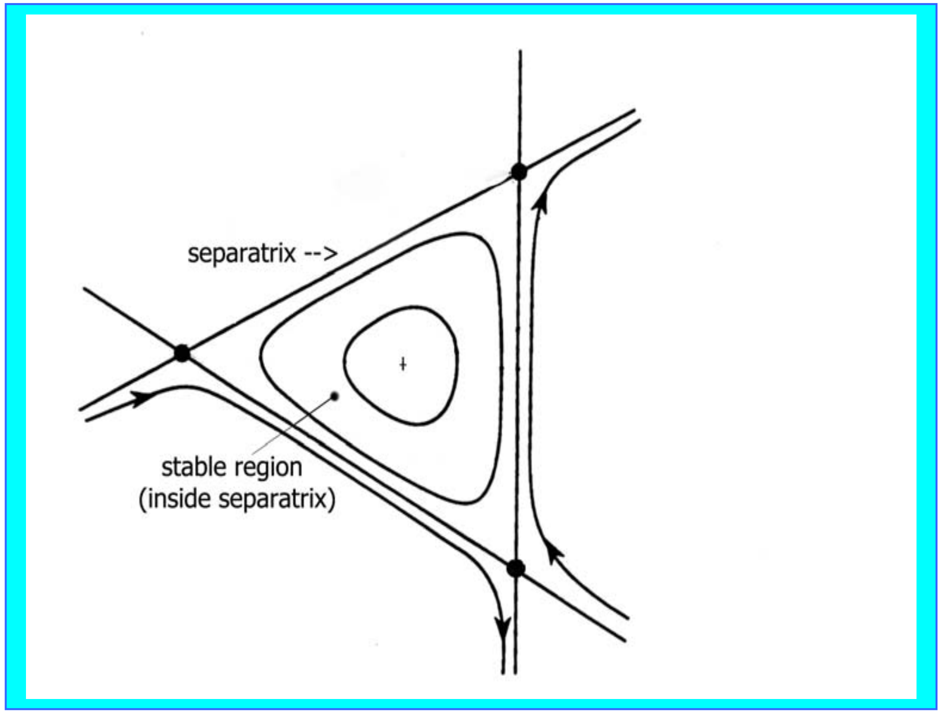
For an average beta function of 10m, the rms scattering angle of $5\ 10^{-3}$ rad corresponds to an encrease of the 1σ -emmittance by $\Delta\varepsilon_{1\sigma} = 125\ \pi$ mm mrad .

With an acceptance of 125 π mm mrad 60 % of a Gaussian beam would be lost in 27 h.

Mismatch due to nonlinearity (space-charge)

Nonlinearity distorts the phase space (example given in the picture).
Space-charge introduces linear and nonliner detuning. Even if the linear term is compensated matching to the nonlinear pattern is difficult.
---> blow up at injection into space-charge limited machines

Phase space in the presence of a 3-rd order nonlinearity. Innermost trajectory of the stable area are normalized



Resonance crossing

Dangerous, low order resonances are usually avoided by choosing an appropriate working point (Qx, Qy). However high order resonances may be touched and traversed due to small, unavoidable or programmed tune changes. For a rapid traversal of a resonance pQ=integer the amplitude increase (for small $\Delta a/a$) is given by

$$\Delta a / a \approx \frac{\pi \ \Delta e}{p \sqrt{\Delta Q_t}} \approx \frac{10^{-3}}{p \sqrt{\Delta Q_t}} \text{ typically}$$

(p: order, Δe : width of the resonance, ΔQ_t: tune change per turn). The emittance growth after filamentation ($\Delta\varepsilon/\varepsilon = \frac{1}{2}\Delta a^2/a^2 \approx \Delta a/a$) is given by the same expression. Hence only few transitions can be tolerated even of high order resonances. For repeated random crossings the amplitude growth is multiplied by the square root of the number crossings.

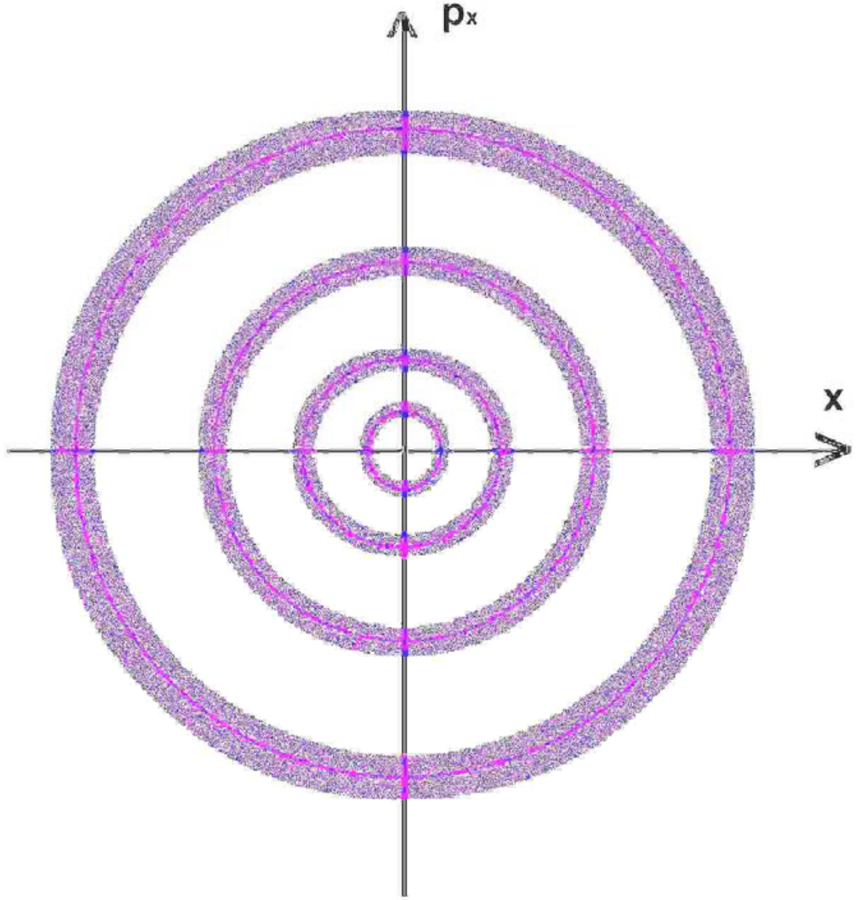
For slow tune variation, particles can be trapped in resonance 'bands' which move them outwards, eventually even to the aperture limit. This can happen through a momentum diffusion (e.g. due to residual gas scattering s. below)

leading to a tune diffusion via the chromaticity ξ: $\Delta Q/Q = \xi \ \Delta p/p$.

------> A very high 'stability' of the tune is essential

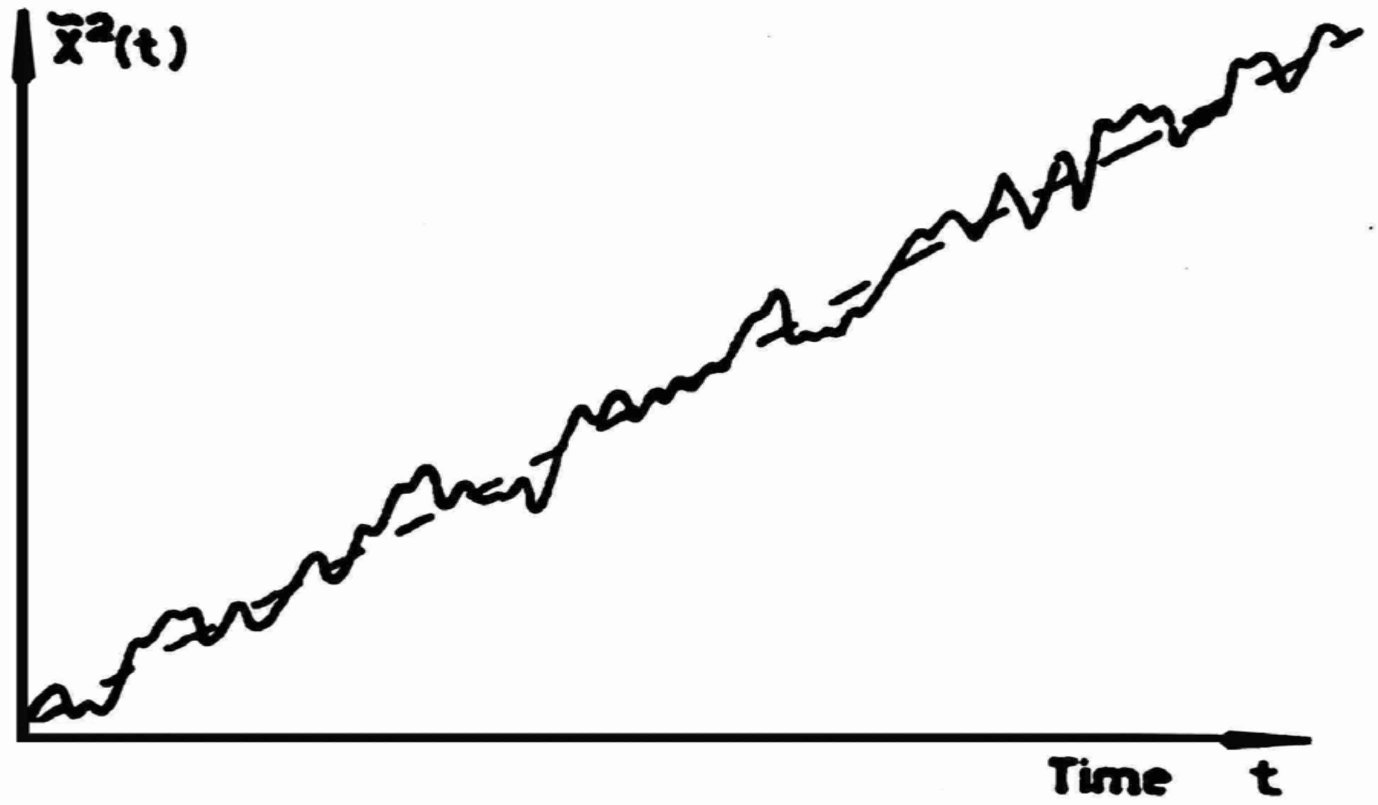
Power supply noise

Noise in the bending fields lead to a ripple of the orgin of the phase space portraits. Noise in the focussing fields leads to wiggling trajectories. Both effects lead to emittance diffusion



Fussy trajectories due to noise in focussing fields

Noise components with frequencies near the betatron sidebands $(n \pm Q)\omega_{rev}$ lead to a linear increase of the mean squared betatron amplitudes (and emittance)



Hereward and Johnson's (yellow) report on noise (CERN/60-38)

C.E.R.N 60-38
Proton Synchrotron Division
4th October, 1960

ORGANISATION EUROPÉENNE POUR LA RECHERCHE NUCLÉAIRE
CERN EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

The Effect of Radio-Frequency Programme
Noise on the Phase-stable Acceleration Process

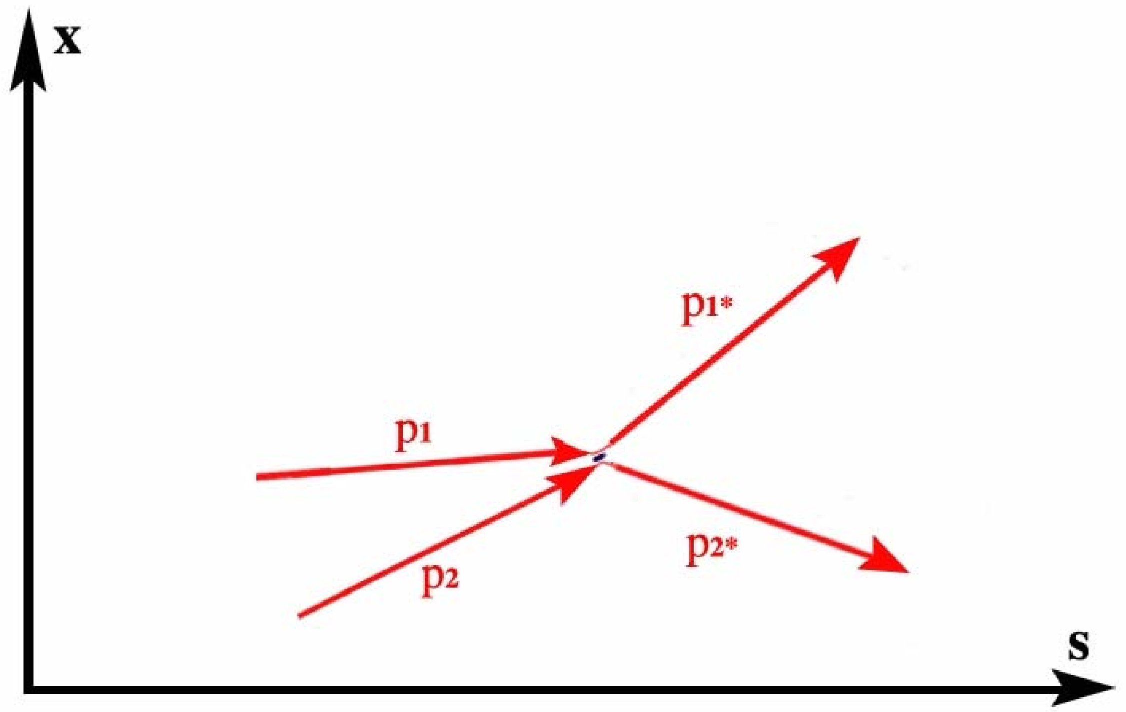
by

H.G. Hereward and K. Johnsen

G E N E V E

Intra-beam scattering

Small angle (multiple) Coulomb scattering between beam particles can lead to blow up.
In the collisions, energy transfer: longitudinal <----> horizontal <---->vertical occurs



Outline of the calculation

(due to A. Piwinski, 1974)

- **Transform the momenta of the two colliding particle into their centre of mass system.**
- **Calculate the change of the momenta using the Rutherford cross-section.**
- **Transform the changed momenta back into the laboratory system.**
- **Calculate the change of the emittances due to the change of momenta at the given location of the collision.**
- **Take the average over all possible scattering angles (impact parameters from the size of the nucleus to the beam radius) and the collision probability (from the beam density)**
- **Assume a 'Gaussian beam' (in all three planes). Take the average over momenta and transverse position of the particles at the given location on the ring circumference.**
- **Finally calculate the average around the circumference (taking the lattice function of the ring into account) to determine the change per turn.**

Particularities of IBS.

The sum of the three emittances

- **For constant lattice functions and below transition energy, the sum of the three emittances is constant (the beam behaves like a 'gas. in a box').**
- **Above transition the sum of the emittances always grows (due to the negative mass effect, i.e. particles 'being pushed go around slower').**
- **In any strong focussing lattice the sum of the emittances always grows (also below transition because of the 'friction' due to the derivatives of the lattice functions).**
- **The increase of the 6-dimentional phase space volume can be explained by transfer of energy from the common longitudinal motion into transverse energy spread.**
- **Although the sum grows there can be strong transfer of emittance and theoretically even reduction in one at the expense of fast growth in another plane (in practise the reduction in one plane has not been observed).**

Particularities of IBS, scaling.

The exact IBS growth rates have to be calculated by computer codes. One determines "form factors" F giving $1/\tau_{x,y,l} = 1/\tau_0 * F_{x,y,l}$. Note that the factors F can strongly vary (e.g. with energy). The quantity $1/\tau_0$ is given by

$$1/\tau_0 = \frac{N_b r_0^2 \left(\frac{q^2}{A}\right)^2}{4\gamma\,\varepsilon_x^*\varepsilon_y^*\varepsilon_s^*/E_0} \propto \frac{N_b \left(\frac{q^2}{A}\right)^2}{\gamma\,\varepsilon_x^*\varepsilon_y^*\varepsilon_s^*}$$

N_b : number of particles per bunch, **r_0 :** classical proton radius,

β, γ **:** relativistic factors, $\varepsilon_{x,y}^* = \beta\gamma\sigma_{x,y}^2 / \beta_{x,y}$, $\varepsilon_s^* = \beta\gamma\sigma_s\sigma_{\Delta p/p}(E_0/c)$

: normalised 1 σ emittances of bunch, **E_0** : proton rest mass

From this (neglecting the variations of the "form factors") one notes:

- **Strong dependence on ion charge (q^4/A^2)**
- **Linear dependence on (normalised) phase space density ($N_b/(\varepsilon_x{*}\varepsilon_y{*}\varepsilon_l{*})$)**
- **Weak dependence on energy**

Conclusion

- Emittance and phase space density are usefull concepts to express the quality of a particle beam. Their optimisation is an important goal in the *design and operation* of beam lines, accelerators and storage rings.

- A couple of invariants, all related to Liouville's theorem, apply to emittance/phase space density. 'Non-Liouvillian' effects, like filament-tation and diffusion tend to reduce the density; beam cooling (not treated in this lecture) aims to increase it.

- Phase space plots are a handy tool to illustrate many common beam 'manipulations' such as injection, beam matching, damping of coherent oscillations and cooling, as well as degrading mechanisms like filamentation, obstacles, influence of noise, diffusion ….

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