Beam dynamics for cyclotrons

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OUTLINE

- History
- Cyclotron review
- Transverse dynamics
- Longitudinal dynamics
- Injection/extraction
- Computation
- Cyclotron as a mass separator
- Few cyclotrons examples

Avant-propos

• Fortunately, the beam dynamics in cyclotrons obeys to the same laws than for the other accelerators.

• The courses from A. Lombardi, J. Le Duff, D. Möhl, N. Pichoff etc … are obviously to keep entirely and to be applied to the cyclotrons. Which makes more than 11h of courses !!!

• In this following 2h, I will admit the previous lessons as understood and will attach more importance to the application of the formalism (focalisation, stability, acceleration …) to the cyclotron case.

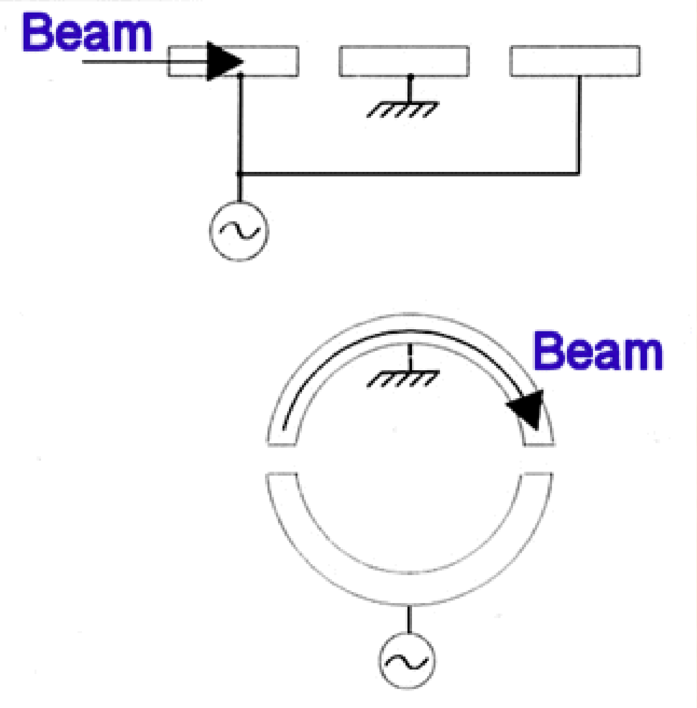
HISTORY

Ion acceleration need (E. Rutherford)

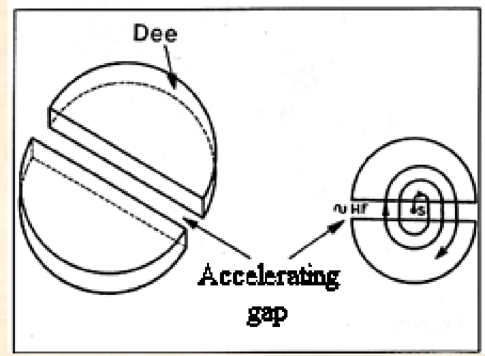
- Idea 1 : Large potential difference (Cockroft-Walton, Van de Graaf). But high voltage limit around 1.5 MV (Breakdown)

- Idea 2: Linacs (Wideröe). Successive drift tubes with alternative potential (sinusoidal). Large dimensions

- Idea 3 : Another genius idea (E. Lawrence, Berkeley, 1929). The device is put into a magnetic field, curving the ion trajectories and only one electrode is used



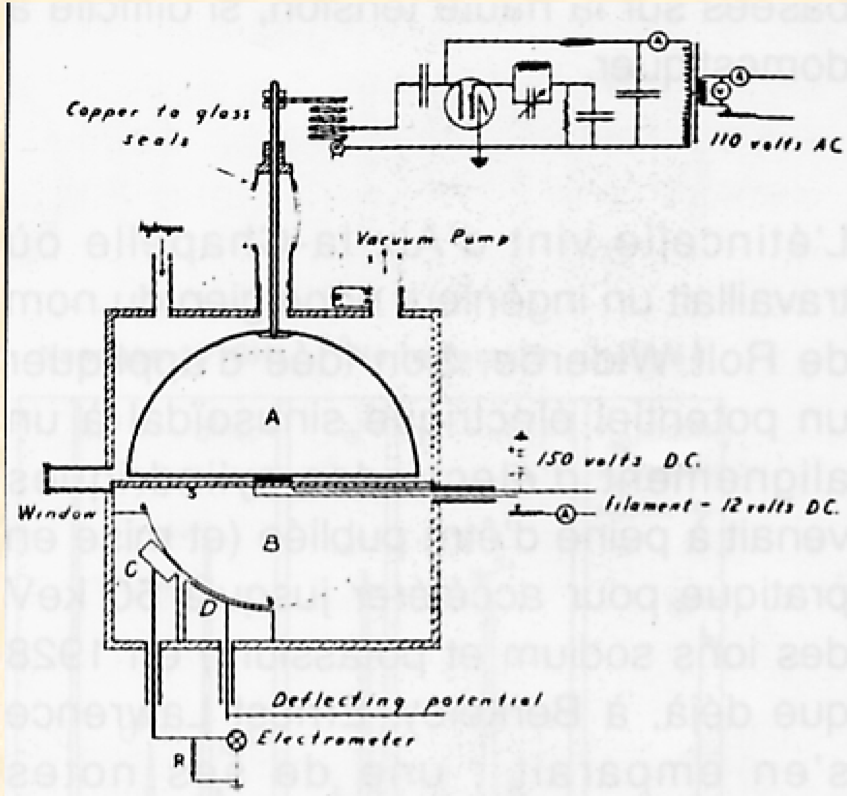
HISTORY II



- A circular copper box is cut along the diameter: a half is at the ground and the other (« Dee ») is connected to an AC generator

- Insert all under vacuum (the first vessel was in glass) and slip it into the magnet gap

- At the centre, a heated filament ionise an injected gas: This is the ion source.



Cyclotrons

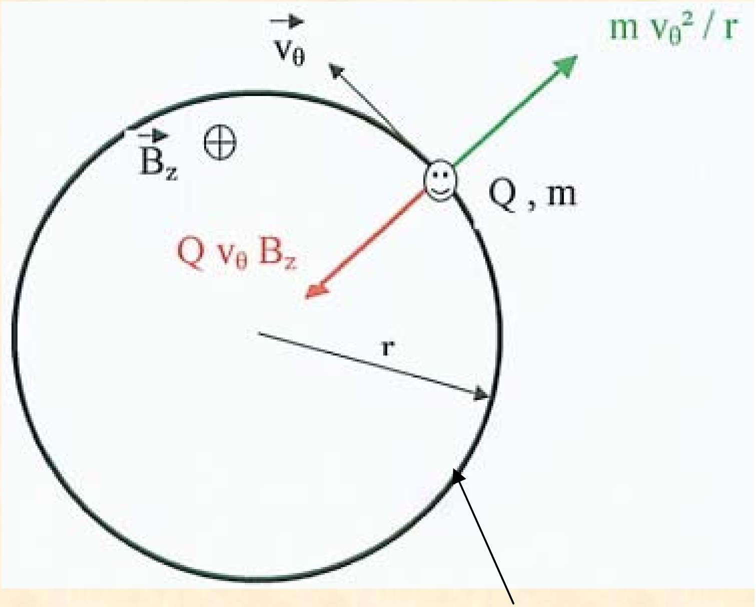
1. Uniform field cyclotron
2. Azimuthally Varying Field (AVF) cyclotron
3. Separated sector cyclotron
4. Spiral cyclotron
5. Superconducting cyclotron
6. Synchrocyclotron

Classic cyclotron

- An ion (Q, m) with a speed v_θ, under an uniform magnetic field B_z, has a circle as trajectory with a radius r:

Centrifugal force=Magnetic force

$$\boxed{\frac{mv_\theta^2}{r} = Qv_\theta B_z}$$



Closed orbit

- Angular velocity: $\boxed{\omega_{rev} = \frac{d\theta}{dt} = \frac{v_\theta}{r} = \frac{QB_z}{m}}$

(Larmor)

- The magnetic rigidity: $\boxed{Br = \frac{P}{Q}}$

Classic cyclotrons means *non relativistic* cyclotrons

low energy ⇨ $\gamma \sim 1$ ⇨ $m / m_0 \sim 1$

In this domain $$\boxed{\omega_{rev} = \frac{QB_z}{m} = const}$$

We can apply between the Dees a RF <u>accelerating</u> voltage:

$$\boxed{V = V_0 \cos \omega_{RF} t}$$

with

$$\boxed{\omega_{RF} = h \omega_{rev}}$$

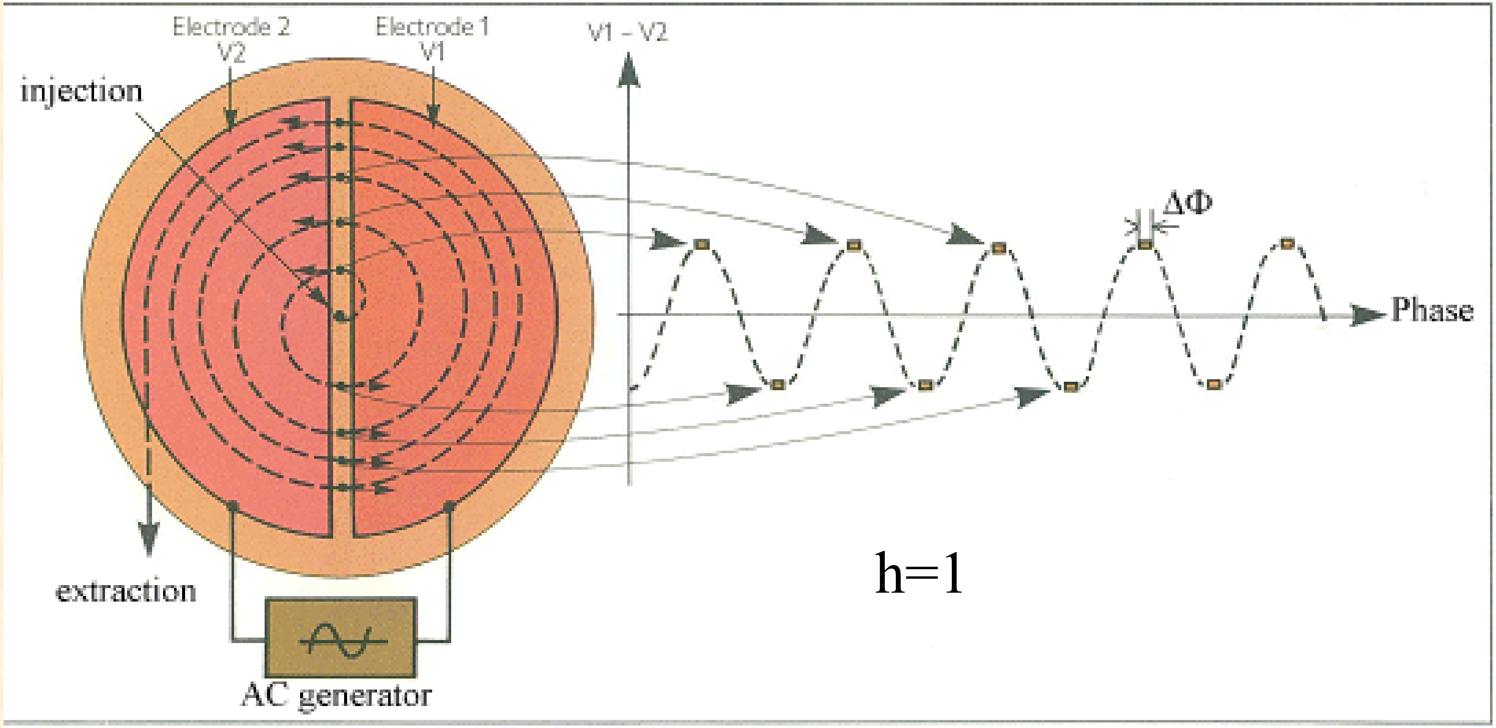
h = 1, 2, 3, … called the RF harmonic

Isochronism condition: The particle takes the same amount of time to travel one turn

and

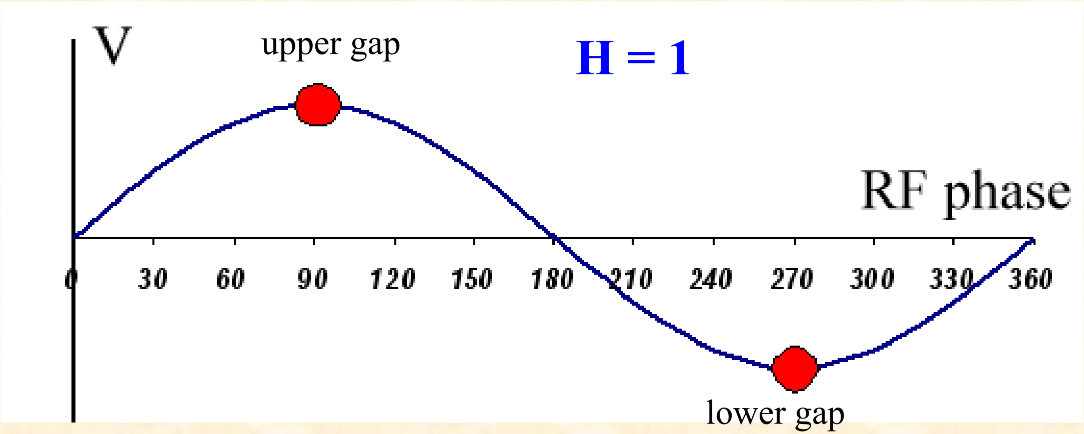
with $\omega_{rf} = h\ \omega_{rev}$, the particule is synchron with the RF wave.

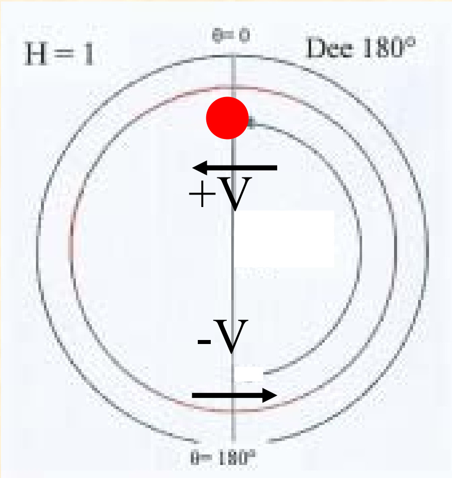
In other words, the particle arrives always at the same RF phase in the middle of the accelerating gap.



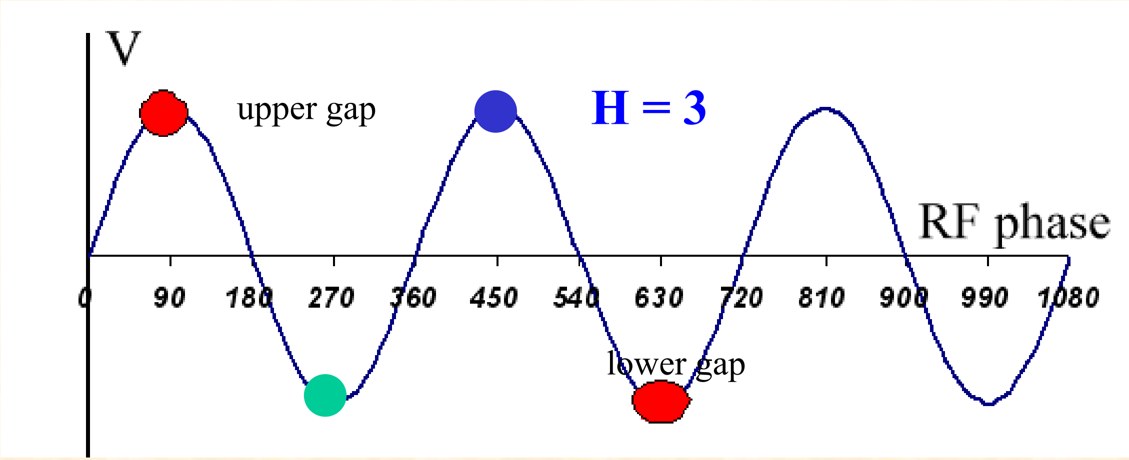
Harmonic notion

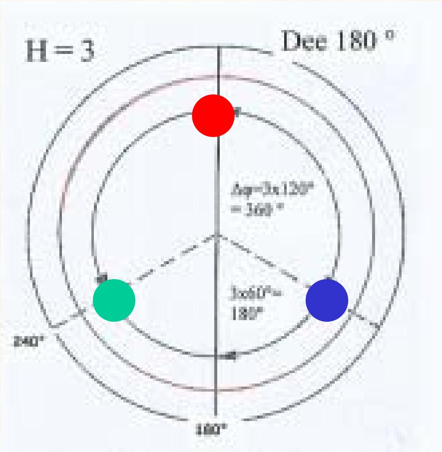
1 beam by turn $\omega_{rf} = \omega_{rev}$





3 beams by turn $\omega_{rf} = 3\ \omega_{rev}$



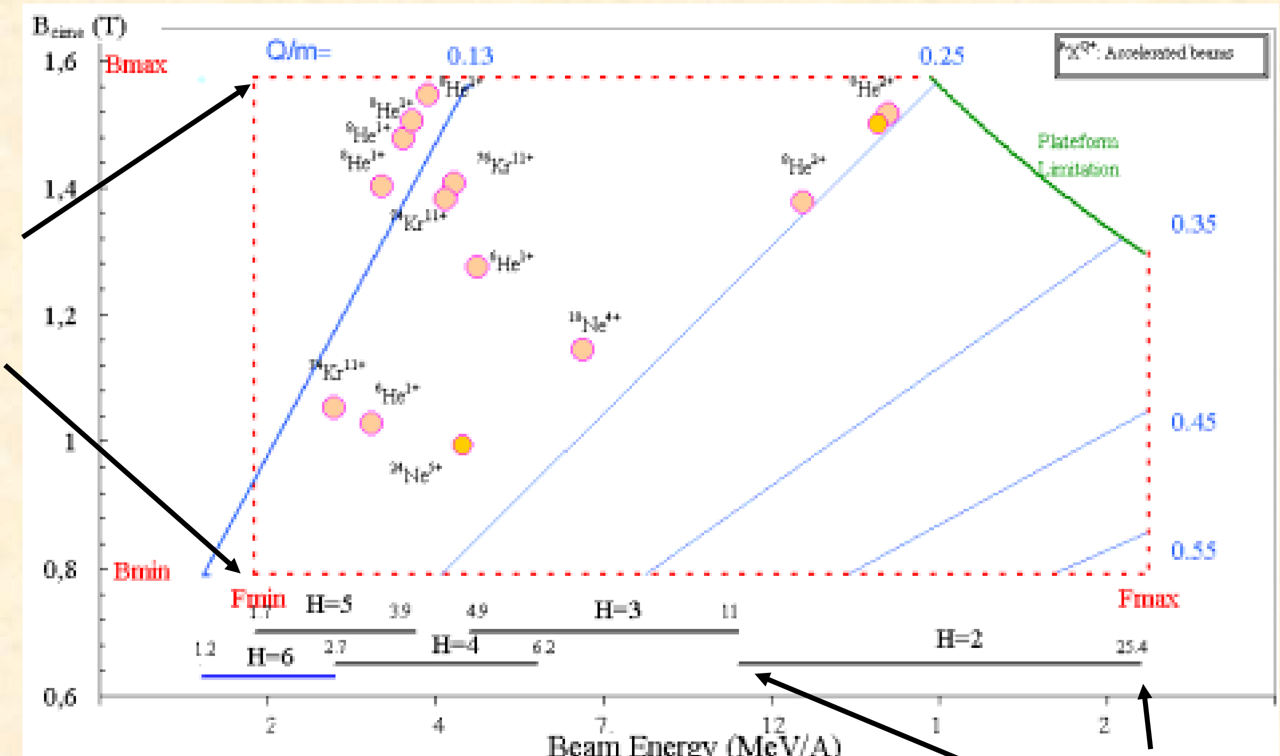


For the same RF frequency, the beam goes 3 times slower

Why harmonic is important

- RF cavities has a fixed and reduced frequency range
- The energy is proportionnal to f^2 $(W=1/2mv^2=1/2m(\omega R)^2 \propto f^2)$
- Working on various harmonic extend the energy range of the cyclotron

Magnetic field limit



Frequency/energy limit for h=2

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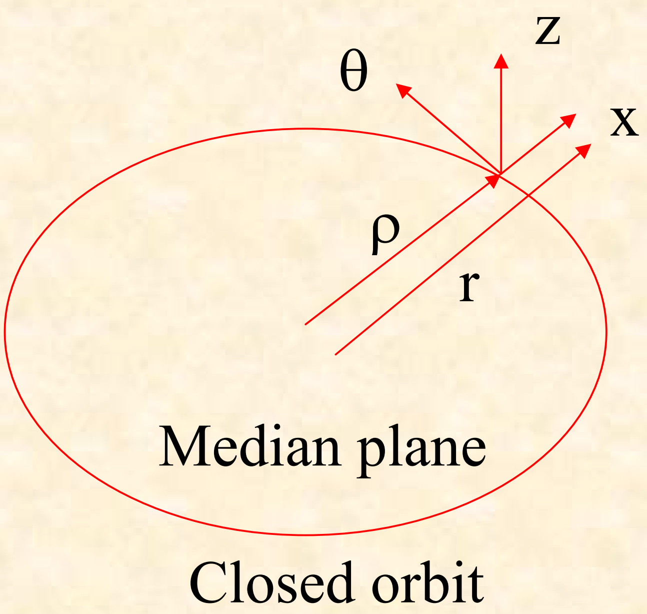
Transverse dynamics

Steenbeck 1935, Kerst and Serber 1941

Horizontal stability : cylindrical coordinates **(r,θ,z)**

and

define **x** a small orbit deviation



$$r = \rho + x = \rho\left(1 + \frac{x}{\rho}\right)$$

$$x \ll \rho \quad \text{(Gauss conditions)}$$

• Taylor expansion of the field around the median plane:

$$B_z = B_{0z} + \frac{\partial B_z}{\partial x} x = B_{0z}\left(1 + \frac{\rho}{B_{0z}} \frac{\partial B_z}{\partial x} \frac{x}{\rho}\right) = B_{0z}\left(1 - n\frac{x}{\rho}\right)$$

with $n = -\frac{\rho}{B_{0z}} \frac{\partial B_z}{\partial x}$ the field index

• Horizontal restoring force = Centrifugal force - Magnetic force

$$F_x = \frac{mv_\theta^2}{r} - Q\, v_\theta B_z \quad \Longrightarrow \quad F_x = \frac{mv_\theta^2}{\rho}\left(1 - \frac{x}{\rho}\right) - Q\, v_\theta B_{0z}\left(1 - n\frac{x}{\rho}\right)$$

$$F_x = \frac{mv_\theta^2}{\rho}\left(1 - \frac{x}{\rho}\right) - Qv_\theta B_{0z}\left(1 - n\frac{x}{\rho}\right)$$

$$F_x = -\frac{mv_\theta^2}{\rho}\frac{x}{\rho}(1 - n)$$

Motion equation under the restoring force $F_x = m\ddot{x}$

$$\ddot{x} + \frac{v_\theta^2}{\rho^2}(1-n)x = 0 \quad \Rightarrow \quad \boxed{\ddot{x} + \omega^2 x = 0}$$

Harmonic oscillator with the frequency $\boxed{\omega = \sqrt{1-n}\ \omega_0}$

Horizontal stability condition : $\boxed{n < 1}$

Vertical stability

Vertical restoring force requires B_x : $F_z = m\ddot{z} = Q\ v_\theta\ B_x$

(no centrifugal force)

Because $\nabla \times B = 0$ $\dfrac{\partial B_x}{\partial z} - \dfrac{\partial B_z}{\partial x} = 0$ $B_x = -n\dfrac{B_{oz}}{\rho} z$

Motion equation $\boxed{\ddot{z} + \omega^2 z = 0}$

Harmonic oscillator with the frequency $\boxed{\omega = \sqrt{n}\,\omega_0}$

Vertical stability condition : $\boxed{n > 0}$

Betatron oscillation

A selected particular solution in the median plane:

$$x(t) = x_0 \cos(\nu_r \omega_0 t)$$

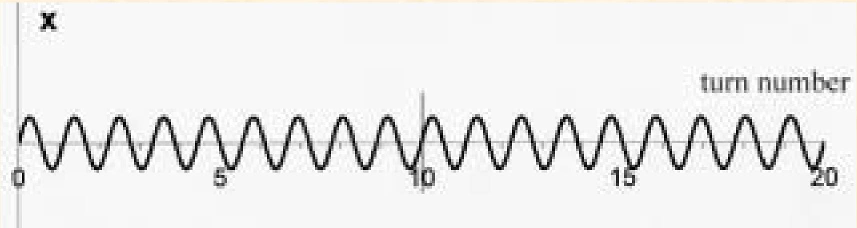
$$\boxed{\nu_r = \sqrt{1-n}}$$

The oscillations around the median plane:

$$z(t) = z_0 \cos(\nu_z \omega_0 t)$$

$$\boxed{\nu_z = \sqrt{n}}$$

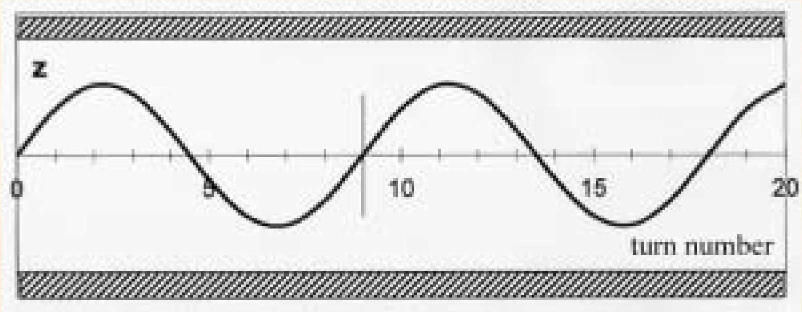
Horizontal oscillation



9 horizontal oscillations for 10 turns in the cyclotron

$(9/10 = 0{,}9 = \nu_r)$

Vertical oscillation



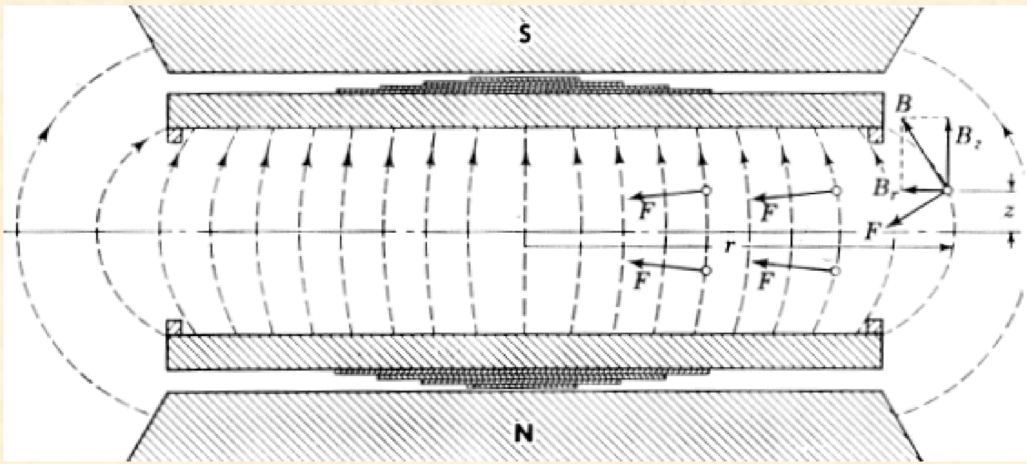
1 vertical oscillation for 9 turns in the cyclotron $(1/9 = 0{,}11 = \nu_z)$

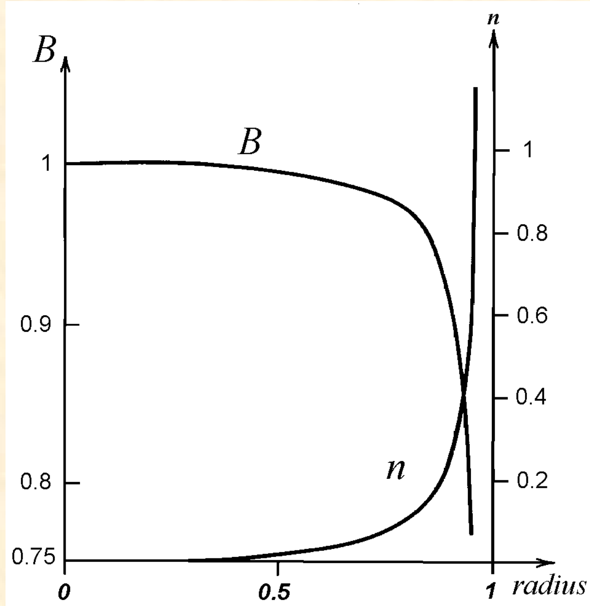
Weak focusing

Simultaneous radial and axial focusing : Weak focusing

$$\boxed{0 \le n \approx -\frac{\partial B_z}{\partial x} \le 1}$$

slightly decreasing field





<u>Horizontal focusing n < 1 means</u> :

- 0 < n < 1 Bz can slightly decrease
- n < 0 Bz can increase as much as wanted

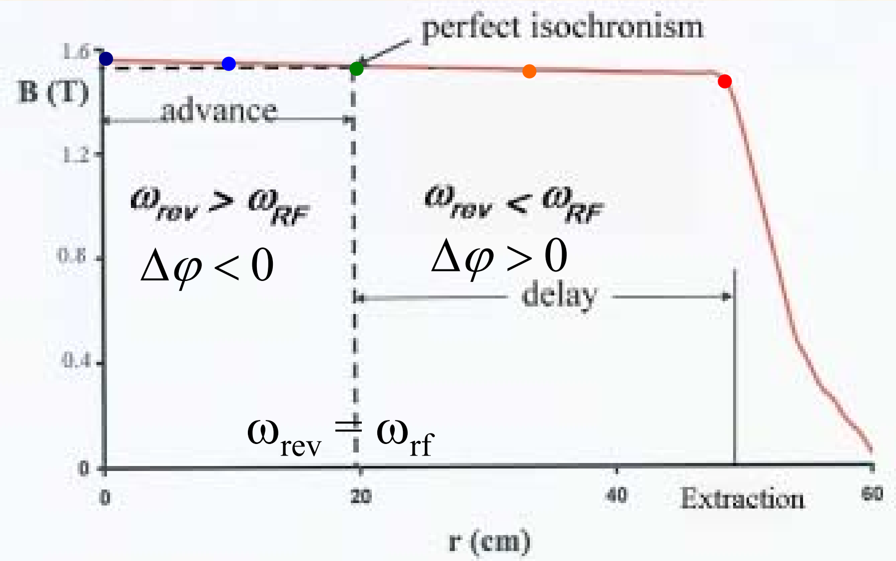
<u>Vertical focusing n > 0 means</u> :

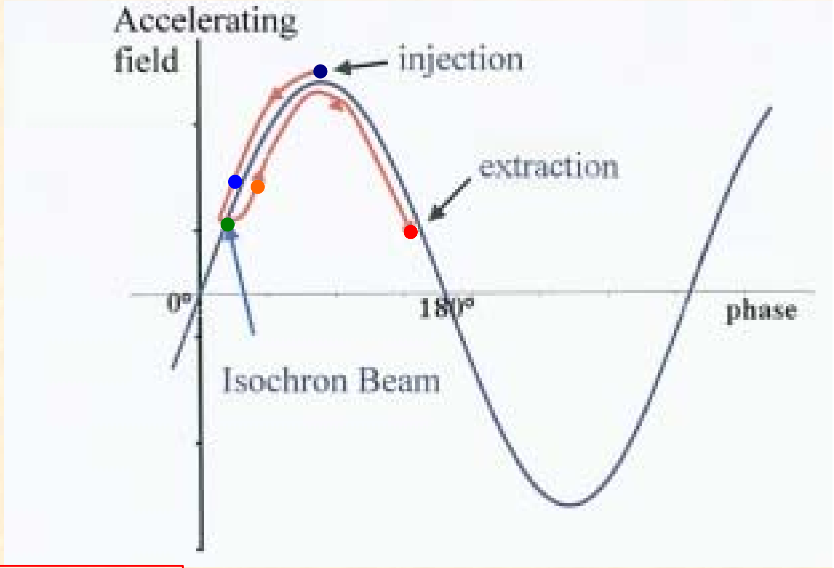
- Bz should decrease with the radius

First limit

Decreasing field* (0 < n < 1) *for vertical focusing gives* 1 point *with a perfect isochronism

$$\omega_{RF} = \omega_{rev}$$





$$\omega_{rev} = \frac{QB_z}{m}$$

$$\Delta\varphi = \pi \cdot \left(\frac{\omega_{RF}}{\omega_{rev}} - 1 \right)$$

Cf : J. Leduff cyclotron (2)

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Limited acceleration

Relativistic case

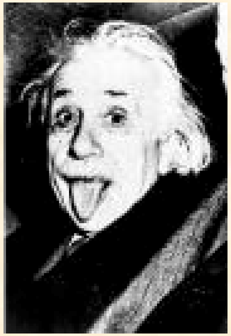
Isochronism and Lorentz factor

$$m = \gamma m_0 = \frac{m_0}{\sqrt{1-\beta^2}} \quad , \quad \beta = \frac{v}{c}$$

$$\boxed{\omega_{rev} = \frac{QB(r)}{\gamma(r) m_0}}$$

ω_{rev} constant if $B(r)=\gamma(r)B_0$ $\nearrow$ increasing field (n < 0)

Not compatible with a decreasing field for vertical focusing



Vertical focusing

AVF or Thomas focusing (1938)

We need to find a way to increase the vertical focusing :

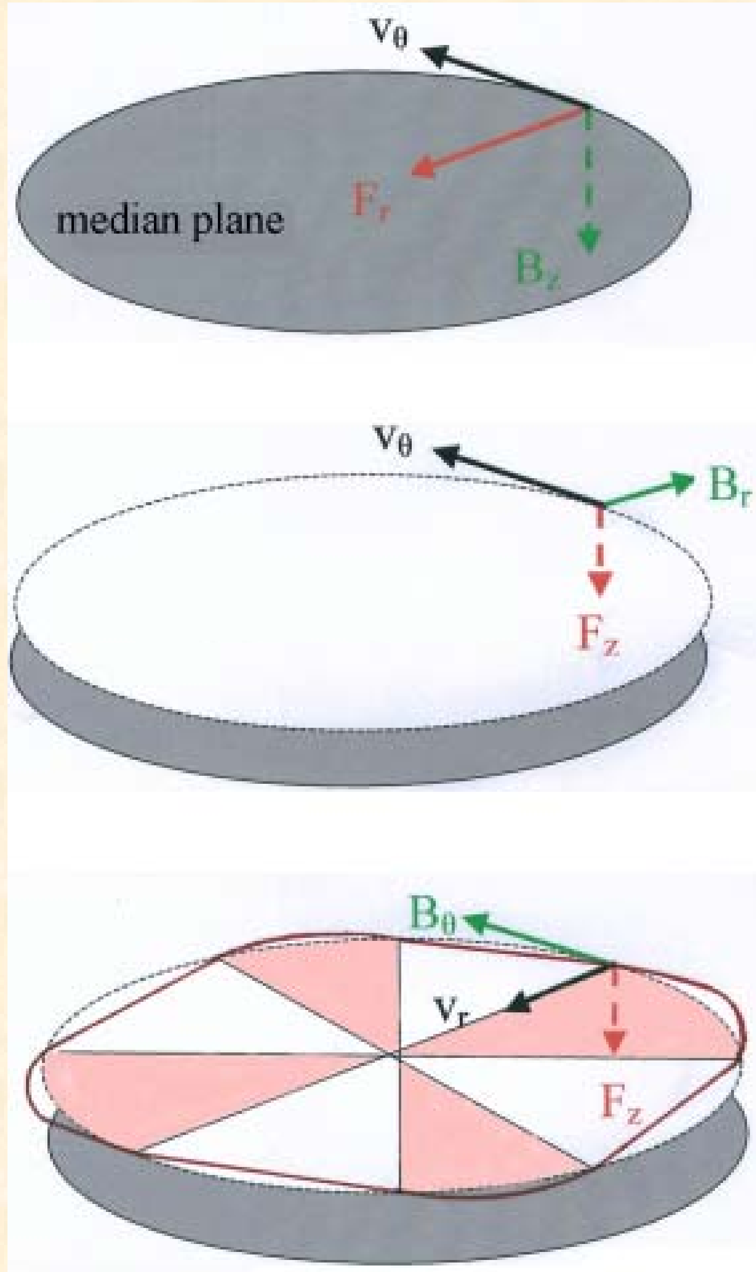
• F_r v_θ B_z : ion on the circle

• F_z v_θ B_r : vertical focusing (not enough)

<u>Remains</u>

• F_z with v_r , B_θ : one has to find an azimuthal component $\mathbf{B_\theta}$ and a radial component $\mathbf{v_r}$ (meaning a non-circle trajectory)

➡ Sectors



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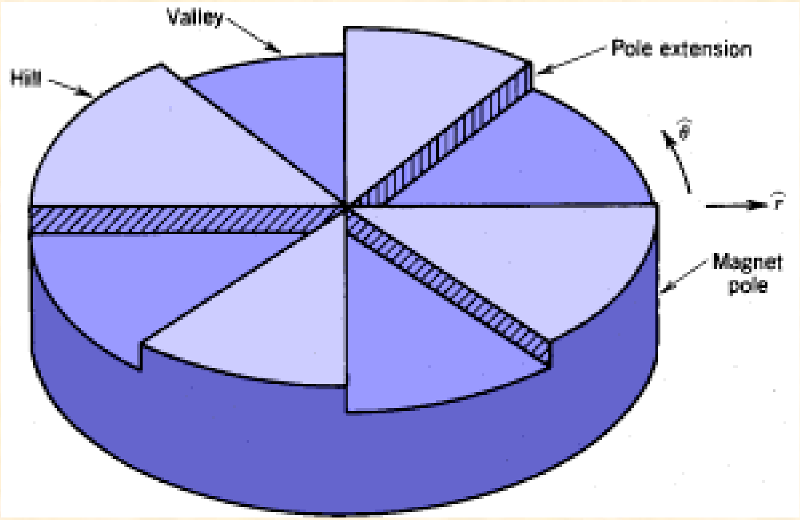
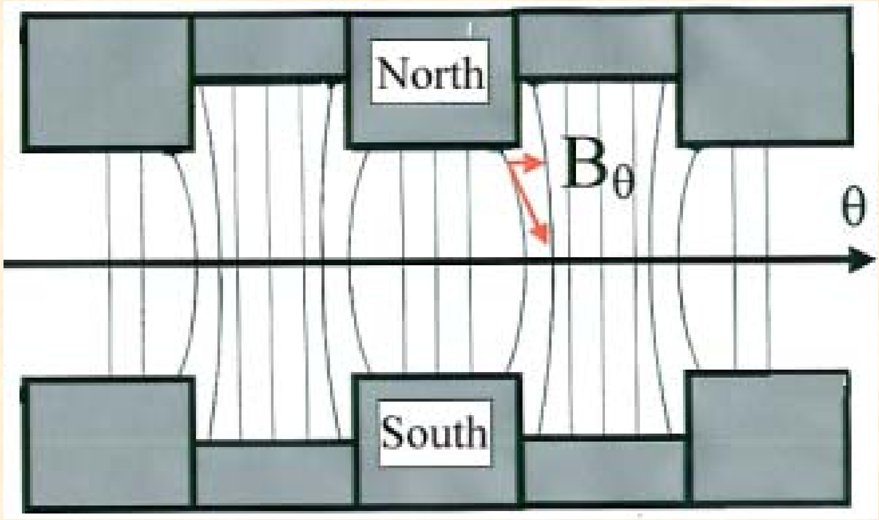
Azimuthally varying Field (AVF)

B_θ created with :

- Succession of high field and low field regions
- B_θ appears around the median plane
    - Valley : large gap, weak field
    - Hill : small gap, strong field

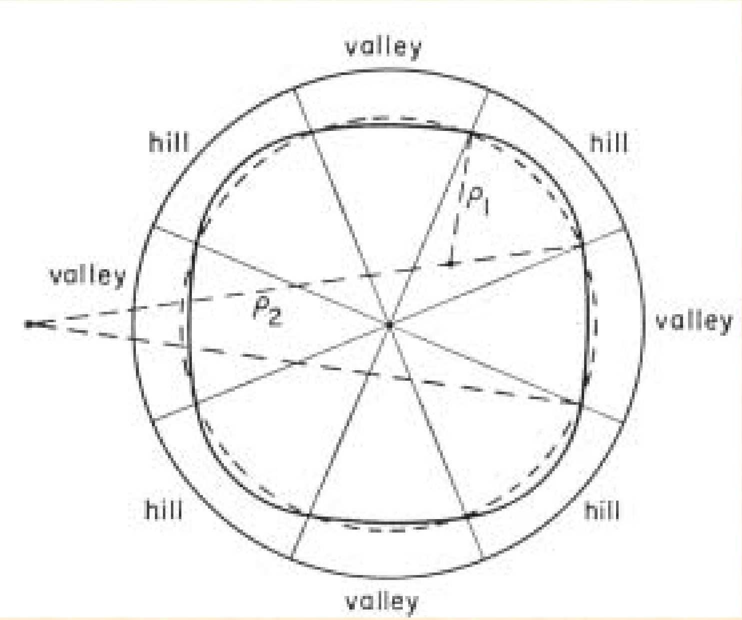




V_r created with :

- Valley : weak field, large trajectory curvature
- Hill : strong field, small trajectory curvature
- Trajectory is not a circle
- Orbit not perpendicular to hill-valley edge

⇨ Vertical focusing $F_z \propto v_r \,.\, B_\theta$



Vertical focusing and isochronism

2 conditions to fulfil

- **Vertical focusing :** $F_z \sim \nu_z^2$

  - Field modulation: or flutter

$$\boxed{F = \frac{\langle B^2 \rangle - \langle B \rangle^2}{\langle B \rangle^2} \approx \frac{\left(B_{hill} - B_{val}\right)^2}{8 \langle B \rangle^2}}$$

where <B> is the average field over 1 turn

  - we can derive the betatron frequency:

$$\boxed{\nu_z^2 = n + \frac{N^2}{N^2 - 1} F + \ldots > 0}$$

- **Isochronism condition :**

$$\overline{B}_z(r) = \gamma(r)\overline{B}_z(0) \Rightarrow \frac{\partial B_z}{\partial r} > 0 \Rightarrow \boxed{n = 1 - \gamma^2 < 0}$$

The focusing limit is:

$$\boxed{\frac{N^2}{N^2-1} F > -n = \gamma^2 - 1}$$

Separated sector cyclotron

Focusing condition limit:

$$\boxed{\frac{N^2}{N^2-1} F > -n = \gamma^2 - 1}$$

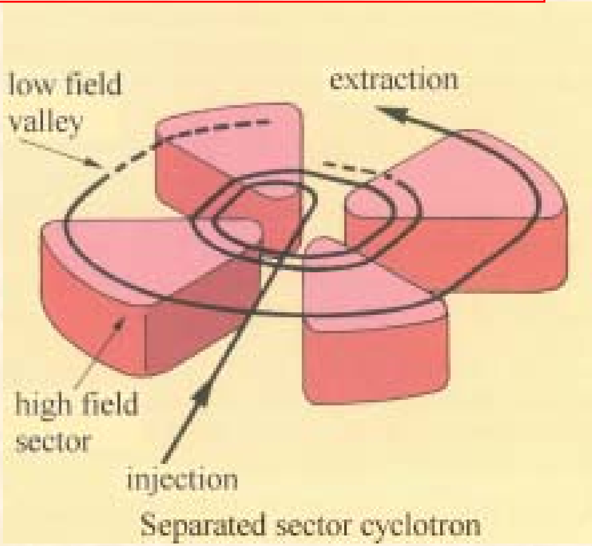
If we aim to high energies:

γ ↗ then -n >> 0

➢ Increase the flutter F, using separated sectors where $B_{val} = 0$

$$\boxed{F = \frac{(B_{hill} - B_{val})^2}{8 \langle B \rangle^2}}$$

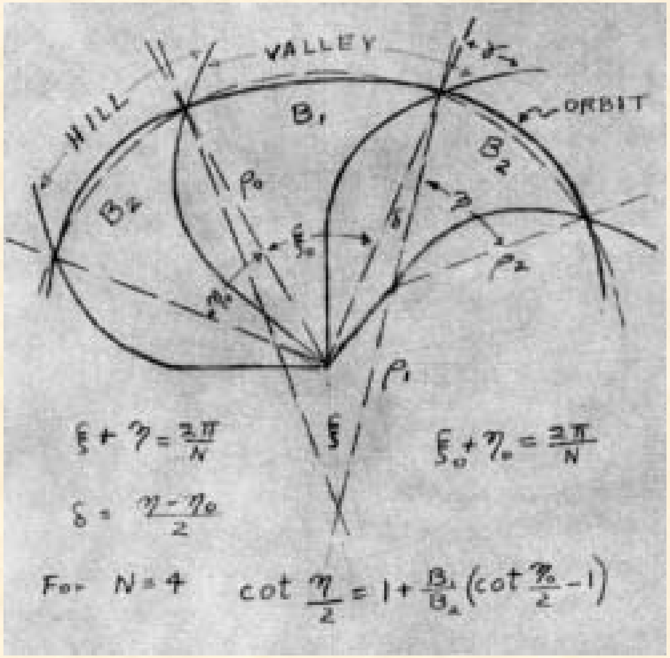
⇨ High energies



Spiralled sectors

In 1954, Kerst realised that the sectors need not be symmetric. By tilting the edges (ξ angle) :

- The valley-hill transition became more focusing
- The hill-valley less focusing.



But by the strong focusing principle (larger betatron amplitude in focusing, smaller in defocusing), the net effect is focusing (cf F +D quadrupole).

$$\nu_z^2 = n + \frac{N^2}{N^2 - 1} F \left(1 + 2 \tan^2 \xi\right)$$

Superconducting cyclotron (1985)

- Most existing cyclotrons utilize room temperature magnets Bmax =2T (iron saturation)
- Beyond that, superconducting coils must be used: $B_{hill} \sim 6$ T
  1. Small magnets for high energy
  2. Low operation cost

Energy and focusing limits

1. For conventional cyclotron, F increases for small hill gap (B_{hill} ↗) and deep valley (B_{val} ↘) but <u>does not depend</u> on the magnetic field level:

$$\boxed{F = \frac{\left(B_{hill} - B_{val}\right)^2}{8\langle B\rangle^2}}$$

2. For superconducting cyclotron, the iron is saturated, the term $(B_{hill}-B_{val})^2$ is constant, hence $F \propto 1/\langle B\rangle^2$

⇨ consequences on W_{max}

Energy max for conventionnal cyclotrons

A cyclotron is characterised by its K_b factor giving its max capabilities

$$\boxed{W_{\max}(MeV/nucleon) = K_b \left\{\frac{Q}{A}\right\}^2} \quad \text{with } K_b = 48{,}244 \left(\langle B \rangle r_{ej}\right)^2$$

- $W \propto r^2$: iron volume as r^3 ! → for compact $r_{extraction} \sim 2$ m.
- For a same ion or isobar A=cst, W_{max} grows with Q^2 (great importance of the ion sources cf P. Spädtke)

Energy max for superconducting cyclotrons

Because of the focusing limitation due to the Flutter dependance on the B field:

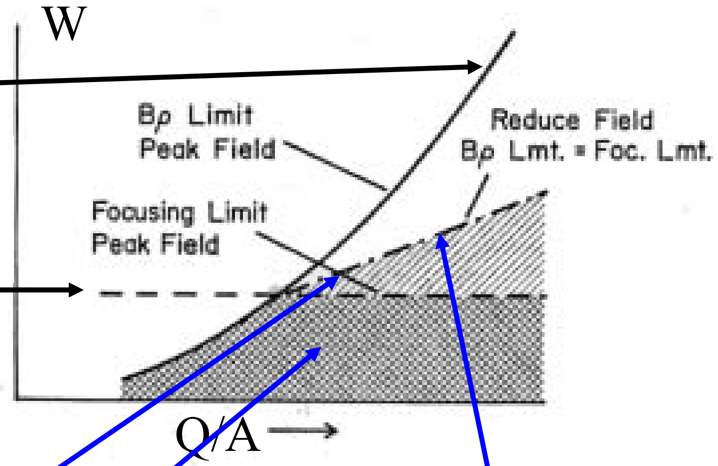
$$\boxed{W_{\max}\left(MeV\ /\ nucleon\right) = K_f\left\{\frac{Q}{A}\right\}}$$

Flutter and focusing

(Blosser 1974)

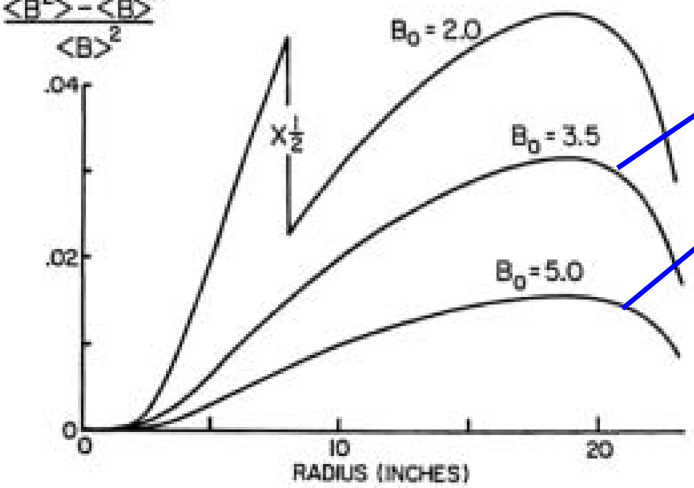
$$W_{\max} = K_b \left(\frac{Q}{A}\right)^2$$

Focusing condition limit: $\dfrac{N^2}{N^2-1} F = \gamma^2 - 1$



Superconducting cyclotron

$$F = \frac{\langle B^2\rangle - \langle B\rangle^2}{\langle B\rangle^2}$$



$$W_{\max} = K_f \left(\frac{Q}{A}\right)$$

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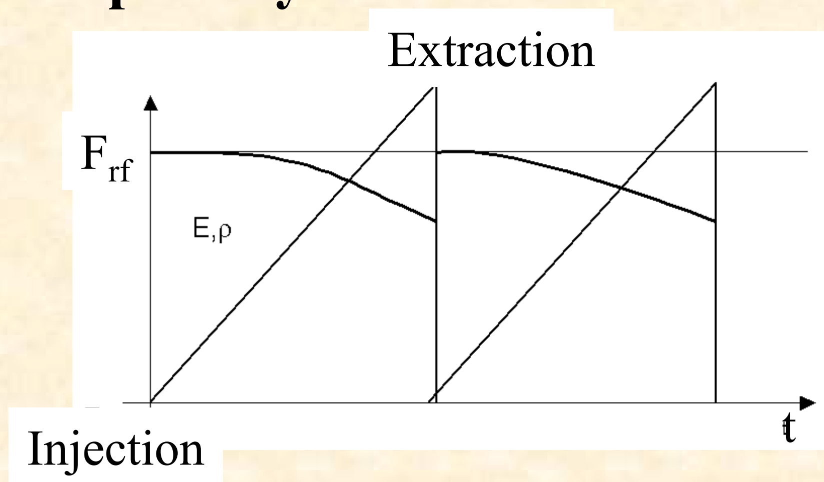
Synchrocyclotron

• Machine : n > 0 & uniform magnetic field.

• **The <u>RF frequency is varied</u> to keep the synchronism between the beam and the RF**

$$\omega_{rev} = QB/\gamma m_0 = \omega_{rf} \searrow$$

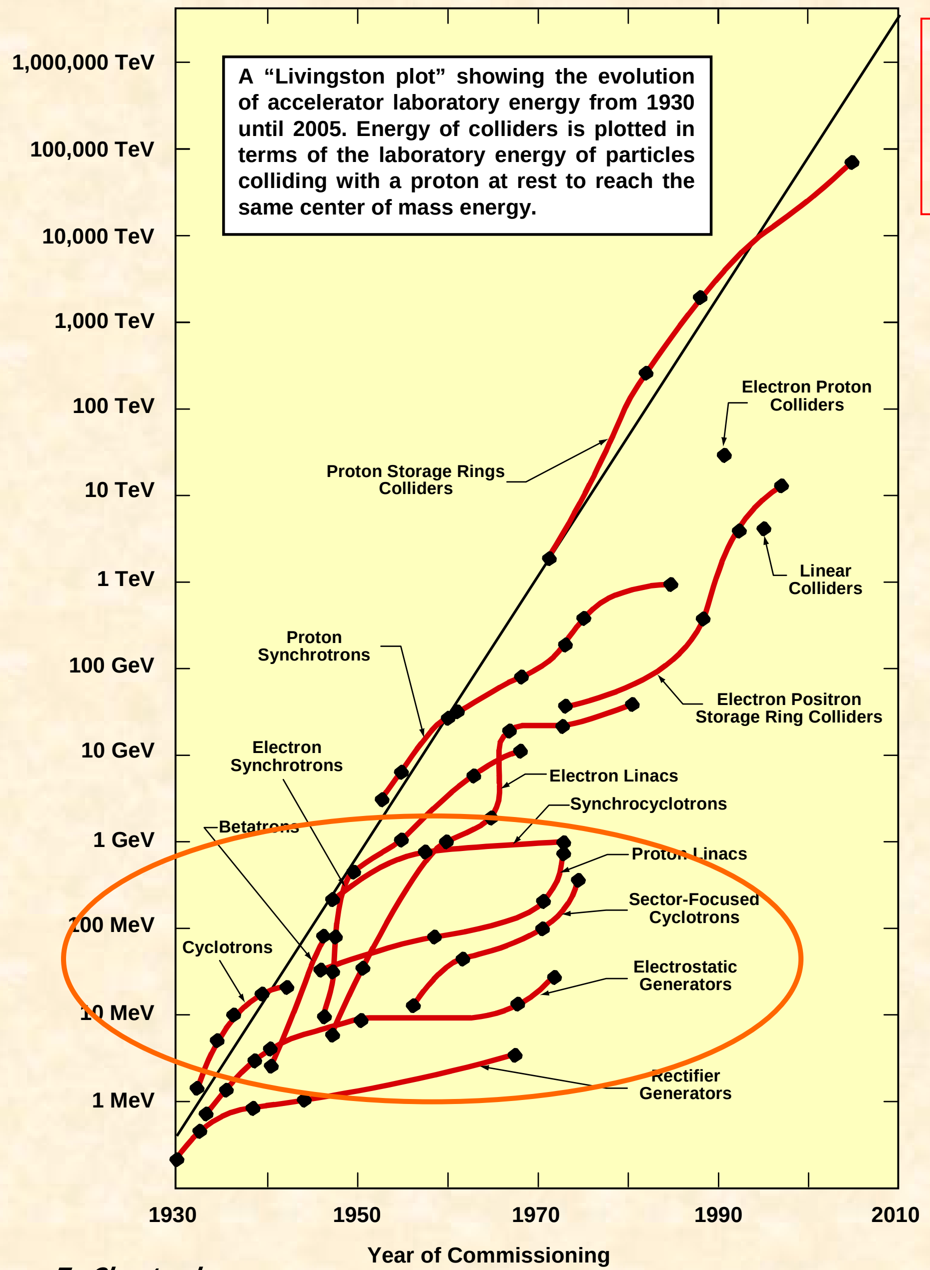


• Cycled machine
(continuous for cyclotrons)

• 10000 to 50000 turns (RF variation speed limitation) ➔ low Dee voltage ➔ small turn separation

• W ~ few MeV to GeV

Livingston chart



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Longitudinal dynamics

Longitudinal matching : A cyclotron can accelerate only a portion of a RF cycle

The acceptance is ± 20° RF (out of 360°).

The external source, such as ECR ou EBIS etc… delivers DC-beams compared to the cyclotron RF frequency.

A buncher located upstream the cyclotron injection will accelerate particles which would come late to the first accelerating gap and decelerates the ones coming too early. Then, more particles can be accelerated in the cyclotron within the ± 20° RF acceptance. <u>Increase the efficiency by a factor 4-6</u>

Acceleration

- The final energy is independent of the accelerating potential

$V = V_0 \cos\varphi$, if V_0 varies, the number of turn varies.

- The energy gain per turn depends on the crest potential V_0, but is constant, if the cyclotron is isochronous (φ =const):

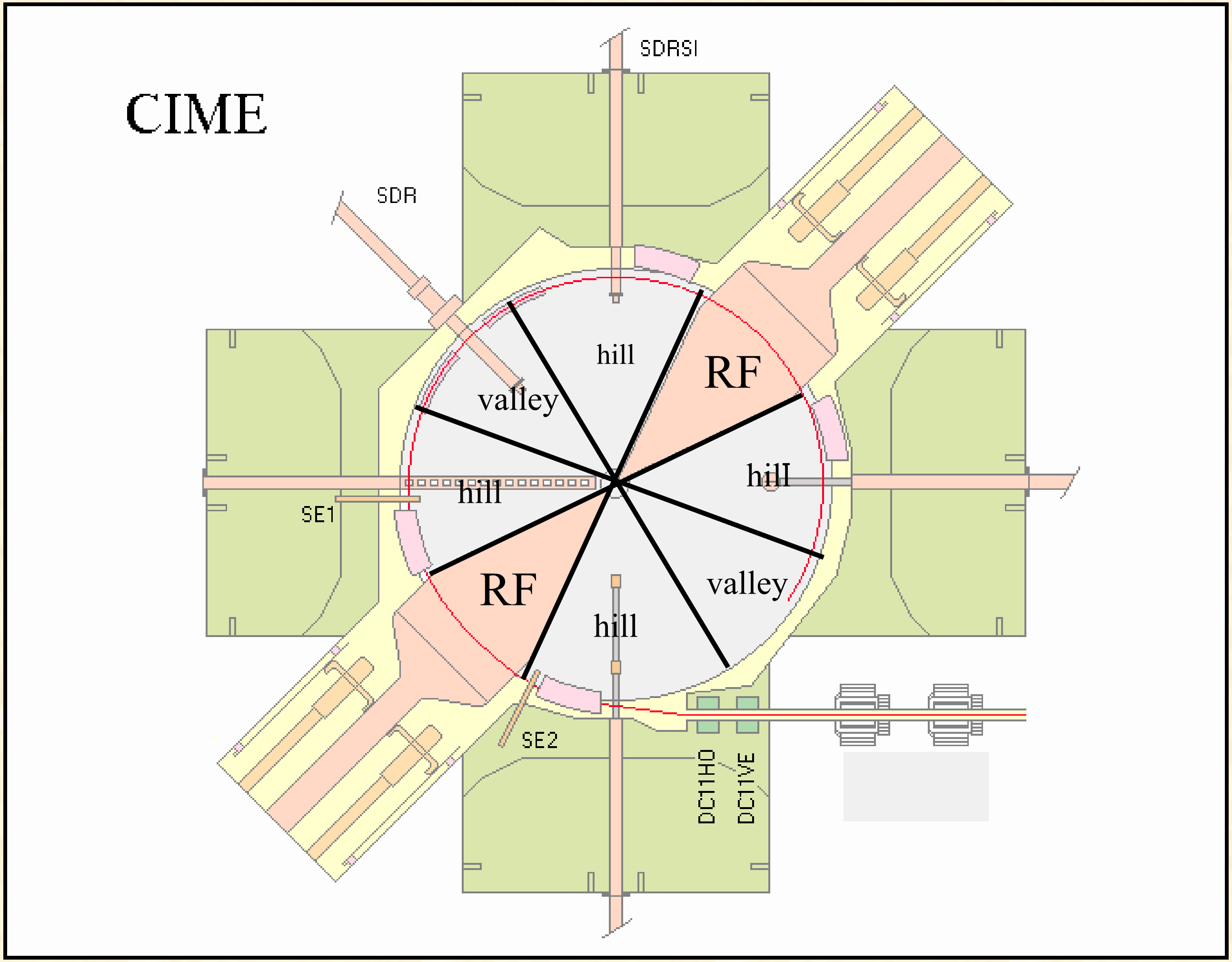
$$\delta W = N_g \; Q \; V_0 \cos\varphi$$ N_g : number of gaps

- The radial separation turn between two turns varies as 1/r ($\gamma \sim 1$) :

$$\frac{\delta r}{r} = \frac{1}{2}\frac{\delta W}{W} = \frac{Q V_0 \cos\varphi}{2 \; W} \propto \frac{1}{r^2}$$

$$\boxed{\delta r \propto \frac{1}{r}}$$

RF Cavities (not Dees)



RF Cavities (not Dees)

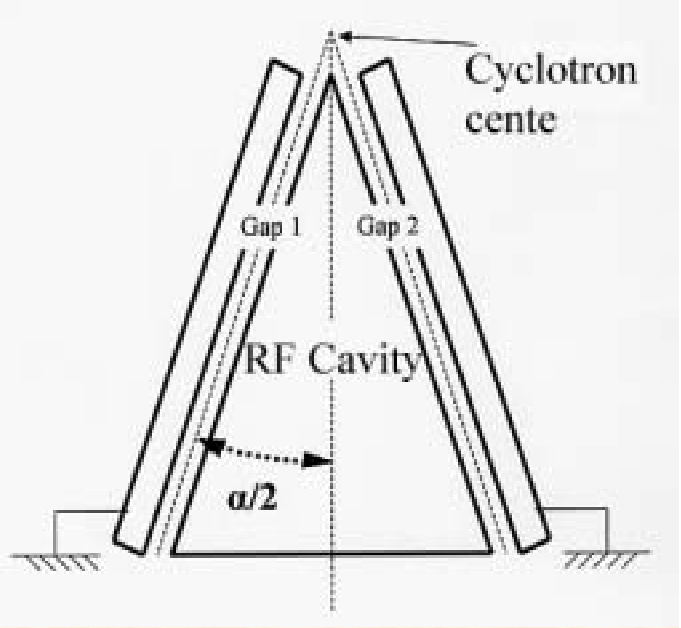
CIME cyclotron with two RF cavities :

$$\delta W = QV_0 \sin\left(\frac{h\alpha}{2}\right)\cos\varphi$$

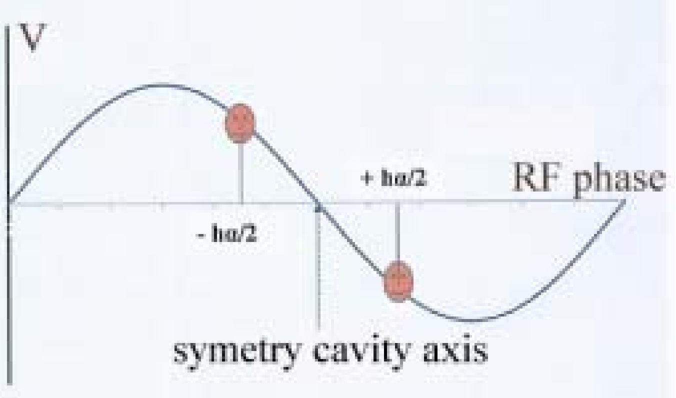
Their azimuthal apertures are α = 40° :

- For a maximum energy gain, the particle passes the symmetry cavity axis when φ = 0° (cosφ = 1)
- Energy gain per gap for the various harmonic mode

$$\delta W = QV_0 \sin\left(\frac{h\alpha}{2}\right)$$



h	2	3	4	5	6	7	8
Sin(hα/2)	0,64	0,87	0,98	0,98	0.86	0.64	0.34



All the modes accelerate the particles but for h > 7 the efficiency is too low.

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Accelerating gap

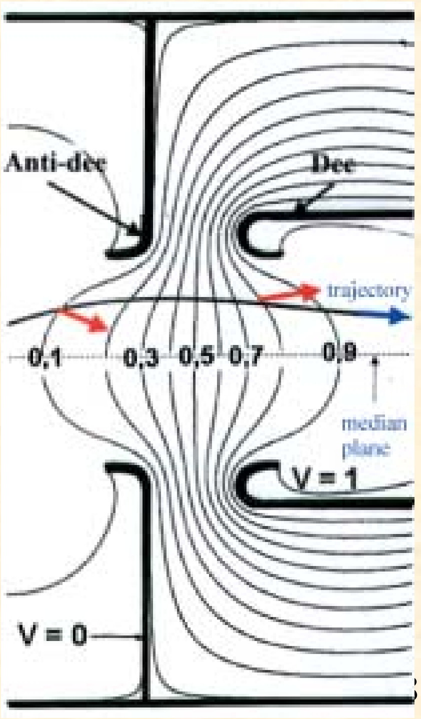
The formula $\delta W = QV_0 \sin(\frac{h\alpha}{2})$ corresponds to small accelerating gaps

Because of the gap geometry, the efficiency of the acceleration through the gap is modulated by the transit time factor τ:

$$\tau = \frac{\sin\left\{\frac{hg}{2r}\right\}}{\frac{hg}{2r}} < 1$$

$$\boxed{\delta W = QV_0\, \tau \sin(\frac{h\alpha}{2})}$$

(Cf N. Pichoff)



Introduction of pillars into the cavity to reduce the azimuthal field extension (seen in the § injection)

Few K_b

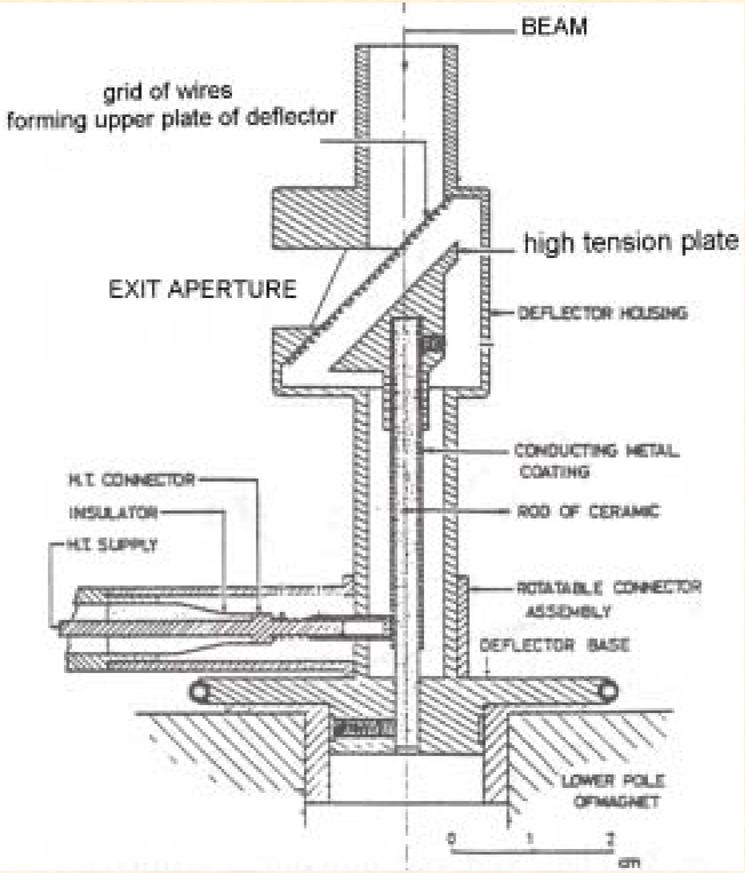
Laboratories	Cyclotron name/type	K (MeV/n) (or proton energy Q/A =1)	$R_{extraction}$(m)
GANIL(FR)	C0	28	0,48
NAC (SA)	SSC	220	4.2
GANIL (FR)	CIME	265	1,5
GANIL (FR)	SSC2	380	3
RIKEN (JP)	RING	540	3.6
PSI (CH)	Ring	592	4,5
DUBNA (RU)	U400	625	1.8
MSU (USA)	K1200(cryo)	1200 (k_f=400)	1

42 more http://accelconf.web.cern.ch/accelconf/c01/cyc2001/ListOfCyclotrons.html

Axial injection

1. The electrostatic mirror

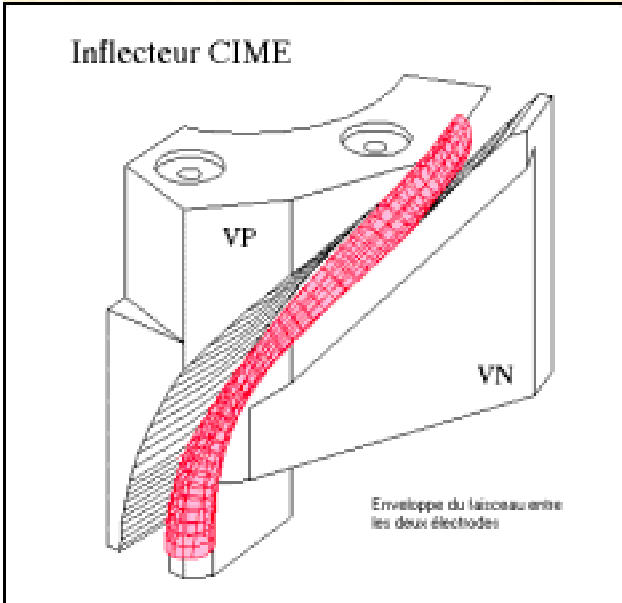
- Simpliest: A pair of planar electrodes which are at an angle of 45° to the incoming beam. The first electrode is a grid reducing transmission (65% efficiency).
- smallest
- High voltage

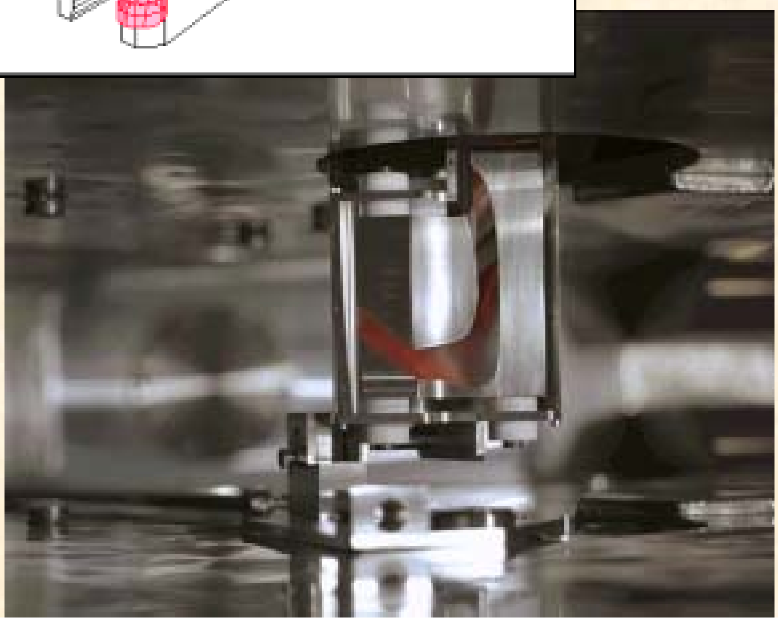


Axial injection

1. The electrostatic mirror
   - Simpliest: A pair of planar electrodes which are at an angle of 45° to the incoming beam. The first electrode is a grid reducing transmission (65% efficiency).
   - smallest
   - High voltage
2. Spiral inflector (or helical channel)
   - analytical solution
3. The hyperboloid inflector
   - Simplier to construct because of revolution surface
   - No free parameters and bigger than a Spiral inflector
   - No transverse correletion. Easy beam matching
4. The parabolic inflector: not use in actual cyclotron, similar to hyperboloid
5. Axial hole

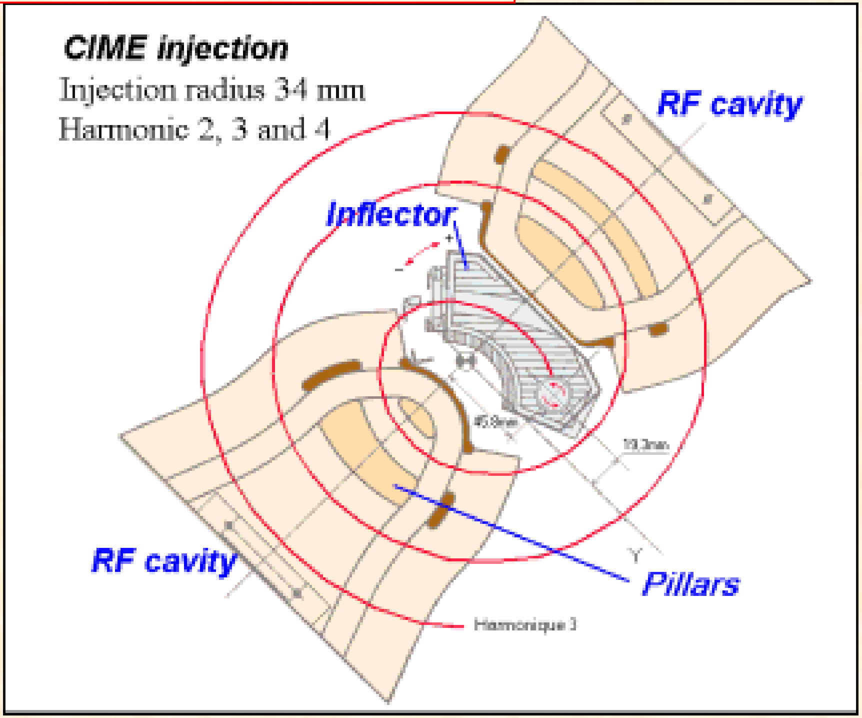
Spiral inflector





- First used in Grenoble (J.L. Pabot J.L. Belmont)
- Consists of 2 cylindrical capacitors which have been twisted to take into account the spiralling of the ion trajectory from magnet field.
- $\vec{v}_{beam} \perp \vec{E}$: central trajectory lies on an equipotential surface. Allows lower voltage than with mirrors.
- 2 free parameters (spiral size in z and xy) giving flexibility for central region design
- 100 % transmission

Central region



- Beam created by:

  -Internal source (PIG)

  -External source 1962 (ECR): The beam is injected vertically through the cyclotron yoke and reaches the horizontal trajectory with a spiralled inflector

  ➢ Dynamics problems encountered especially when running the machine for various harmonics.

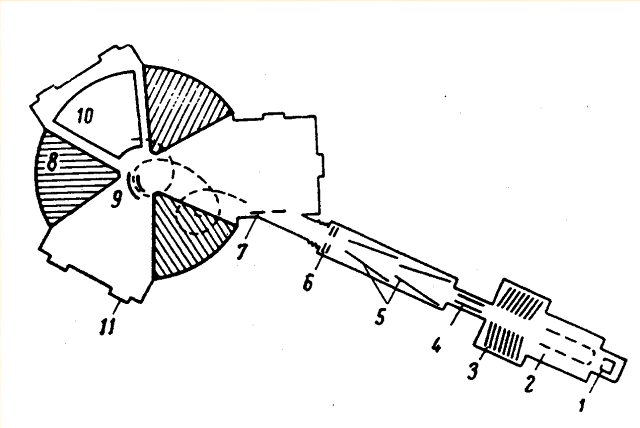
➢ Goal : put the beam on the « good orbit » with the proper phase.

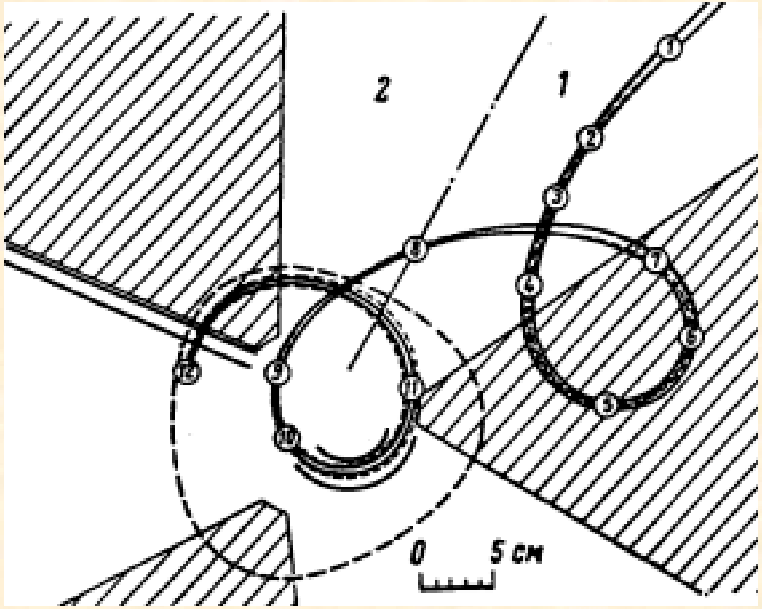
➢ The initial gaps are delimited with pillars reducing the transit time and the vertical component of the electric field.

➢ The potential map are computed (in 3D if necessary)

Radial injection (1)

1. Trochoidal (Lebedev Institut in Moscow)
   - Field difference between hill-valley to send the beam on a trochoidal trajectory to the central region. (300 keV)
   - Not used today



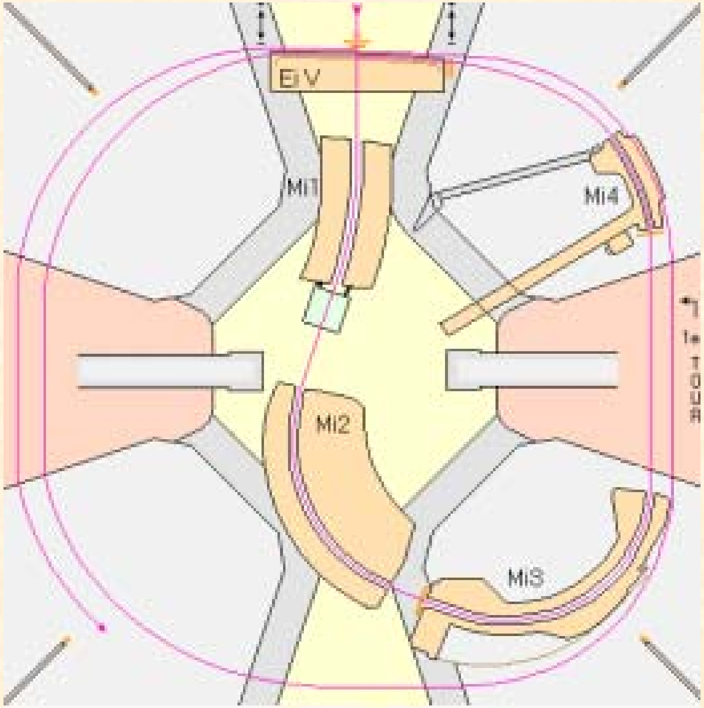


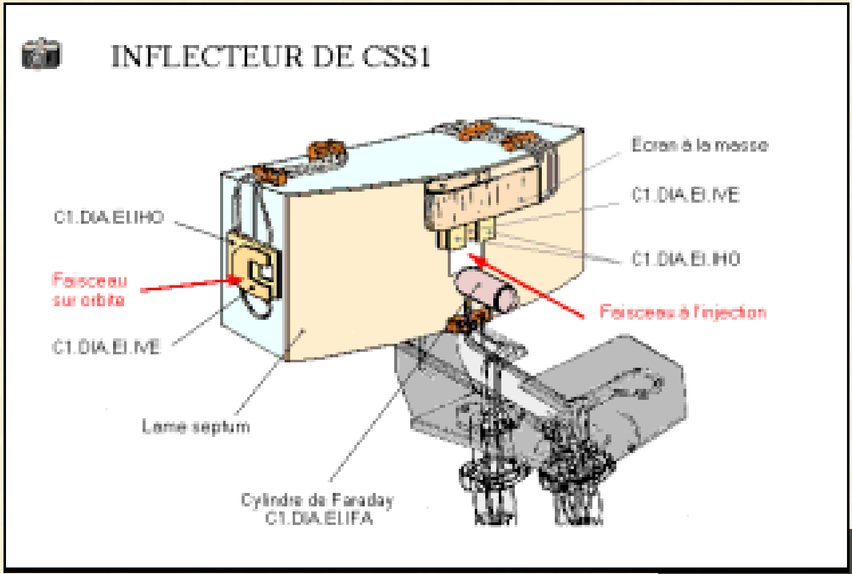
Radial injection (2)

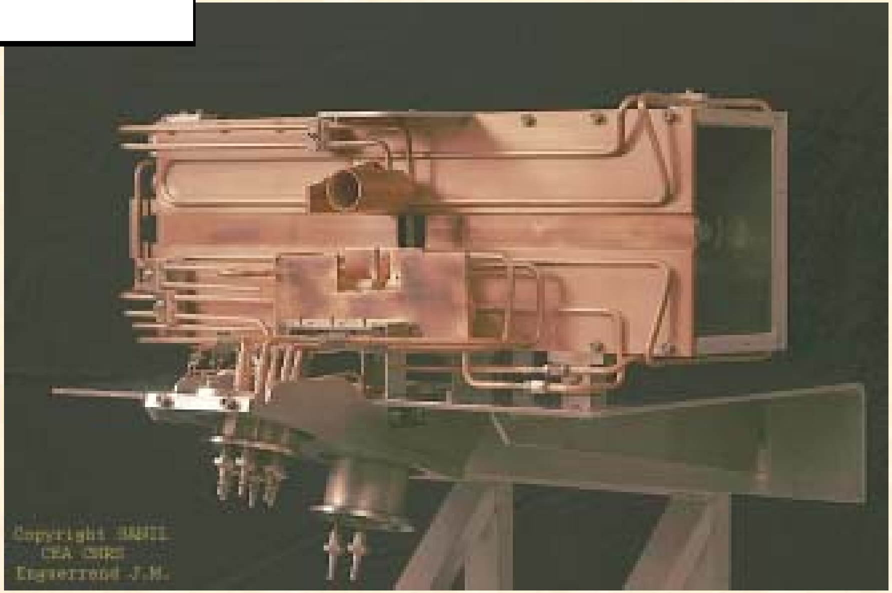
1. Trochoidal (Lebedev Institut in Moscow)
   - Field difference between hill-valley to send the beam on a trochoidal trajectory to the central region. (300 keV)
   - Not used today
2. Electric field cancelling magnetic field (Saclay, 1965)
   - system of electrodes shaped to provide horizontal electric field to cancel the force for the magnetic field to focus the beam on its path to the cyclotron centre.
   - Poor transmission (few percent)
3. Injection from another accelerator
   - Tandem + stripping + cyclotron : Oak Ridge, Chalk River
   - Matching between magnetic rigidity of the injected beam and the first cyclotron orbit rigidity
4. Injection into separated sector cyclotron
   - More room for injection pieces and excellent transmission

Injection in SSC

- More room to insert bending elements.
- The beam coming from the pre-injector enters the SSC horizontally.
- It is guided by 4 magnetic dipoles to the "good trajectory", then an electrostatic inflector deflect the beam behind the dipole yokes.




INFLECTEUR DE CSS1
Ecran à la masse
C1.DIA.EI.IVE
C1.DIA.EI.IHO
C1.DIA.EI.IHO
Faisceau sur orbite
Faisceau à l'injection
C1.DIA.EI.IVE
Lame septum
Cylindre de Faraday
C1.DIA.EI.IFA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Extraction

Goal : High extraction rate by achieving a radial orbit increase

1. <u>Extraction by acceleration</u> $$\delta r = r \times \frac{\delta W}{W} \times \frac{\gamma}{\gamma + 1} \times \frac{1}{\nu_r^2}$$
   - Cyclotron with large radius
   - Energy gain per turn as high as possible
   - Accelerate the beam to fringing field where ν_r drops $\nu_r = \sqrt{1 - n}$
2. <u>Resonant extraction</u>
   - If turn separation not enough then magnetic perturbations are used. Particles are forced to oscillate around their equilibrium orbit with a magnetic bump
3. <u>Stripping extraction</u>

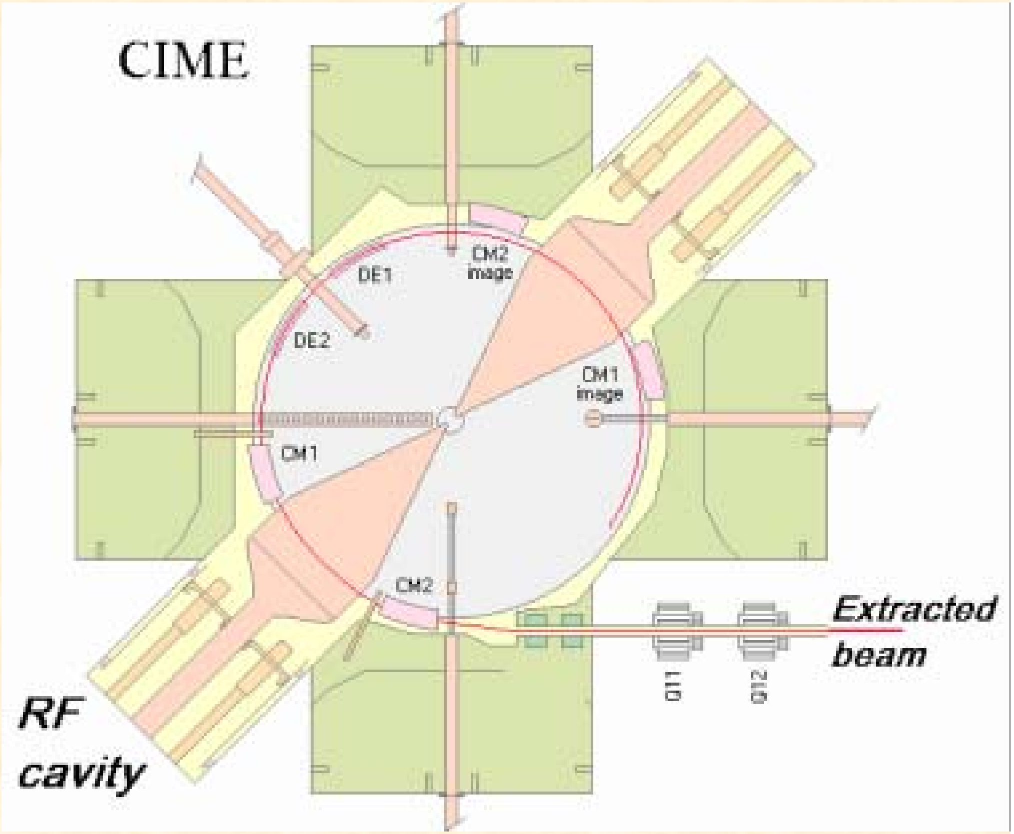
Compact cyclotron

- The last turn passes first through two electrostatic septa (< 90 kV) in order to deviate to beam towards the ejection channel.

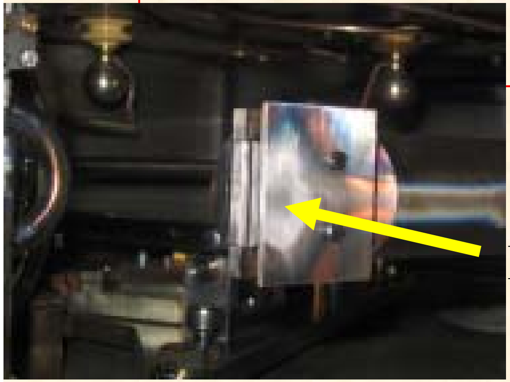
- Two movable magneto static dipoles drive the beam across the last cavity.

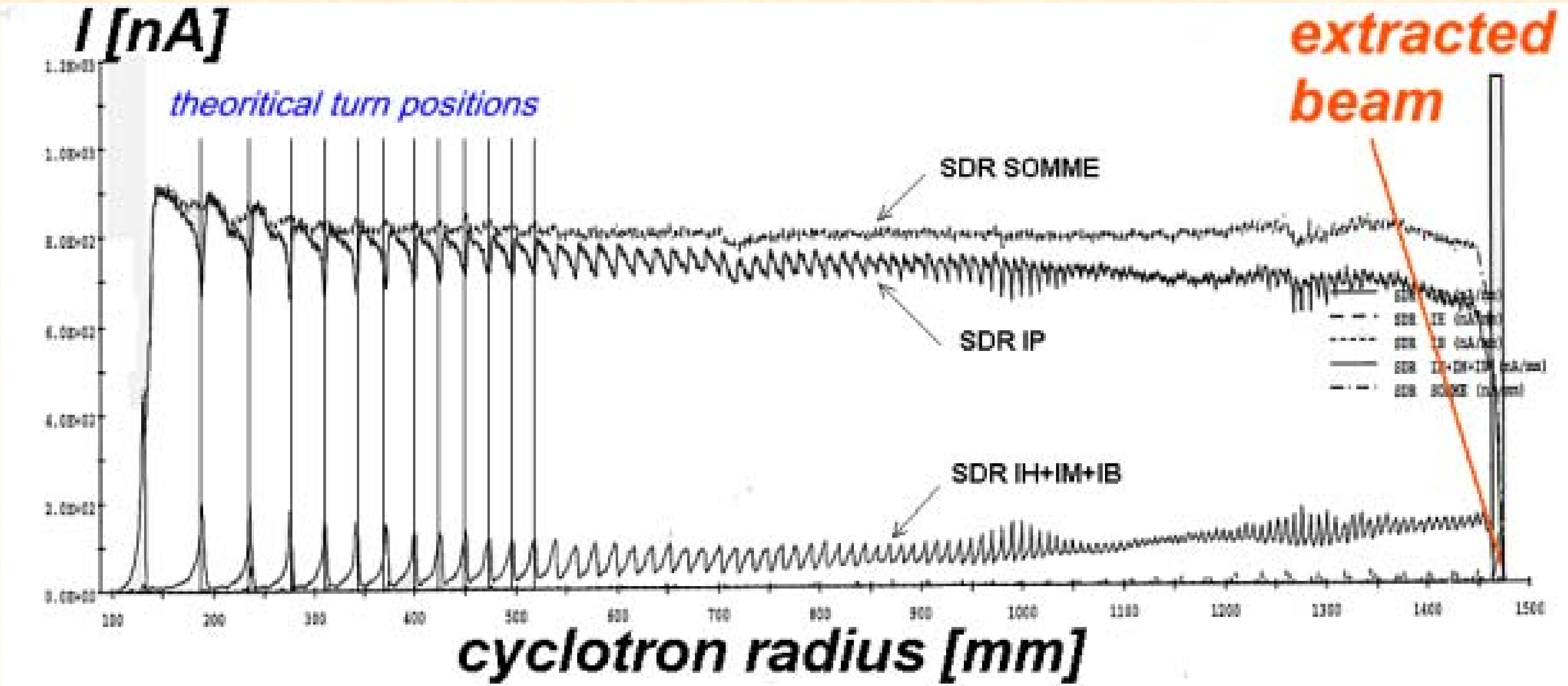
- Despite the strong fringing field along the extracted beam trajectory, the simulations (confirmed by experiments) showed that the beam dynamics (envelops and alignment) can be done with a 90% efficiency

- overlapped turns ⇨ bunch extracted over several turns



Turn visualisation


Beam



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Separated sector cyclotron

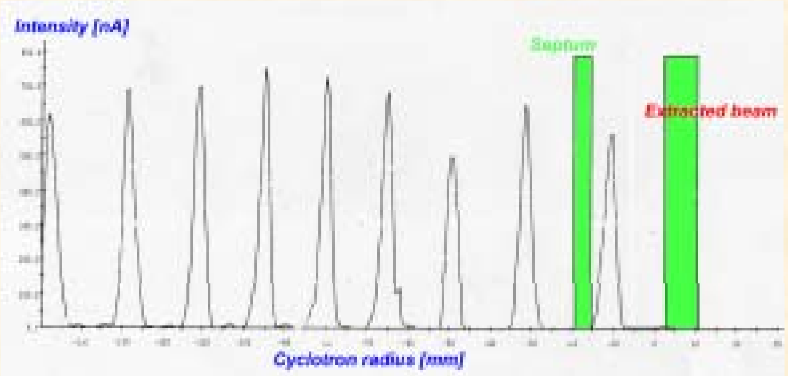
➢As for the injection, dipoles and deflectors are placed in the cyclotron sector to deviate the beam trajectory.

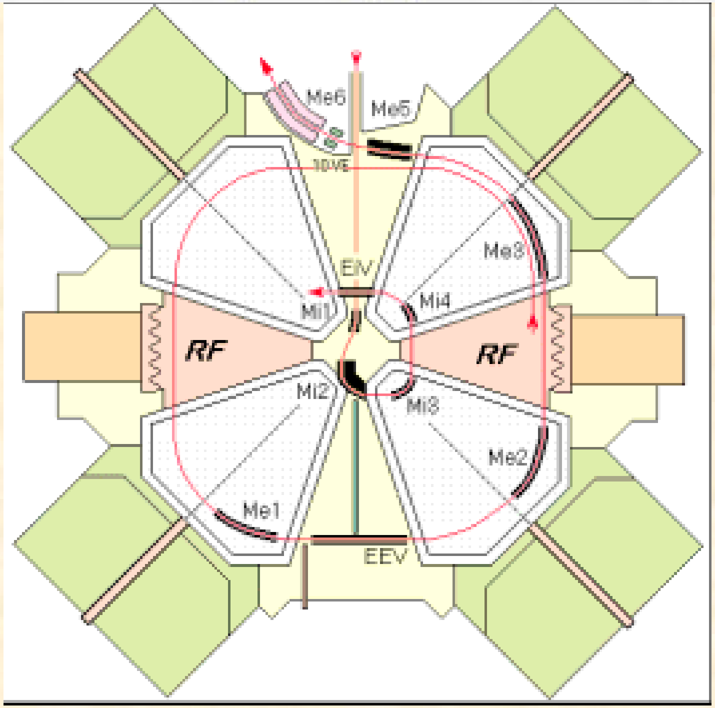
➢For large radii, the turn separation become narrower : $\delta r \propto \frac{1}{r}$

Find a way to increase the separation to avoid the interaction between the beam and deflector (extraction efficiency ⬀):

➢Electrostatic deflector (EV) gives a small angle (<u>precession</u>)

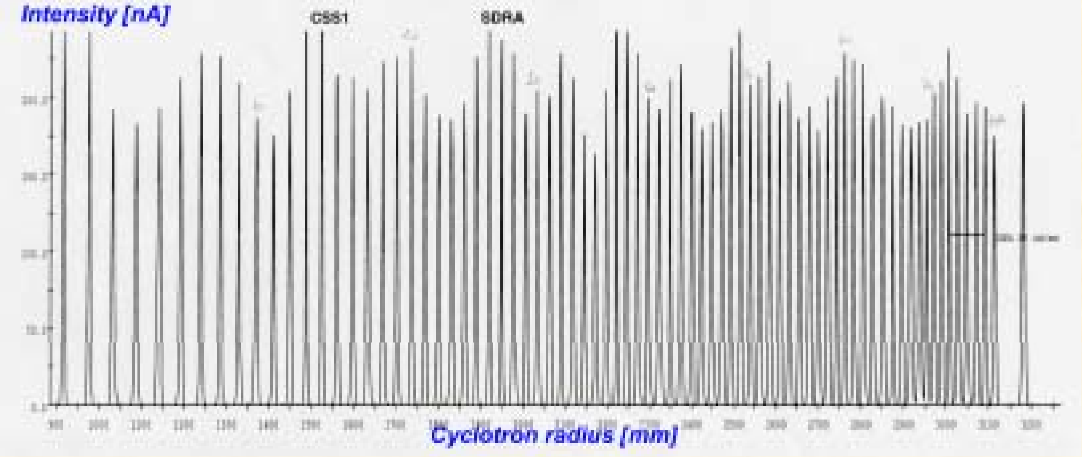
➢Extraction channel : EEV, Me1-6



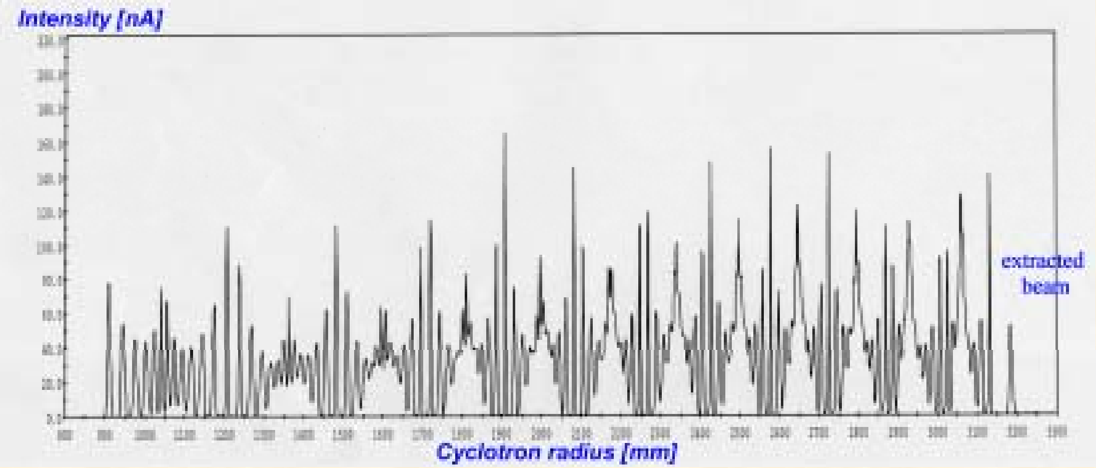


Extraction from SSC

Well centered beam orbits CSS1



Precession for optimized extraction CSS2



1 period for

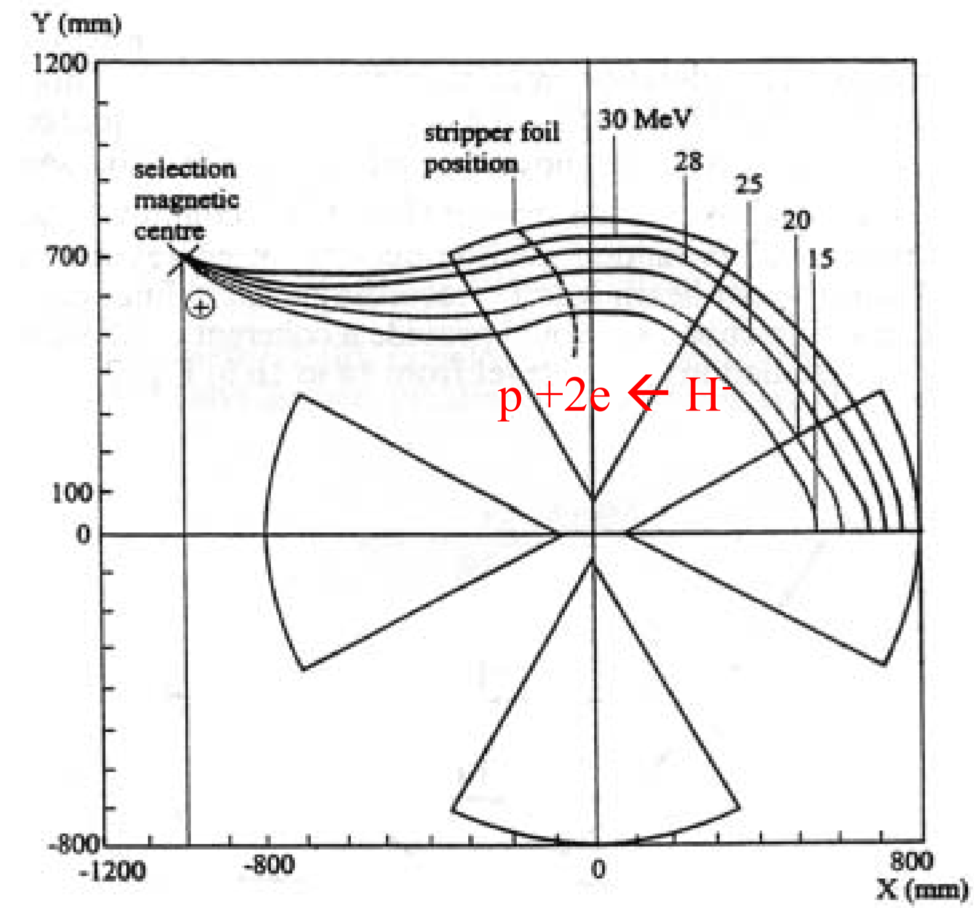
13 turns ⇨

$\nu_r = 1.08$

Extraction by stripping

- Method :
  - H- ions are changed into protons (H+ ions) by stripping the electrons off, on thin stripping foil (µm carbon). Since the protons are positively charged, they then curve the opposite way from the negatively charged circulating beam ions. Thus, the protons curve out of the cyclotron into the primary beamlines.
  - All, or just a fraction, of the negatively charged circulating beam passes through a thin extraction foil, looses its electrons and comes out as a positive proton beam. (Triumf, Louvain)

Extraction by stripping



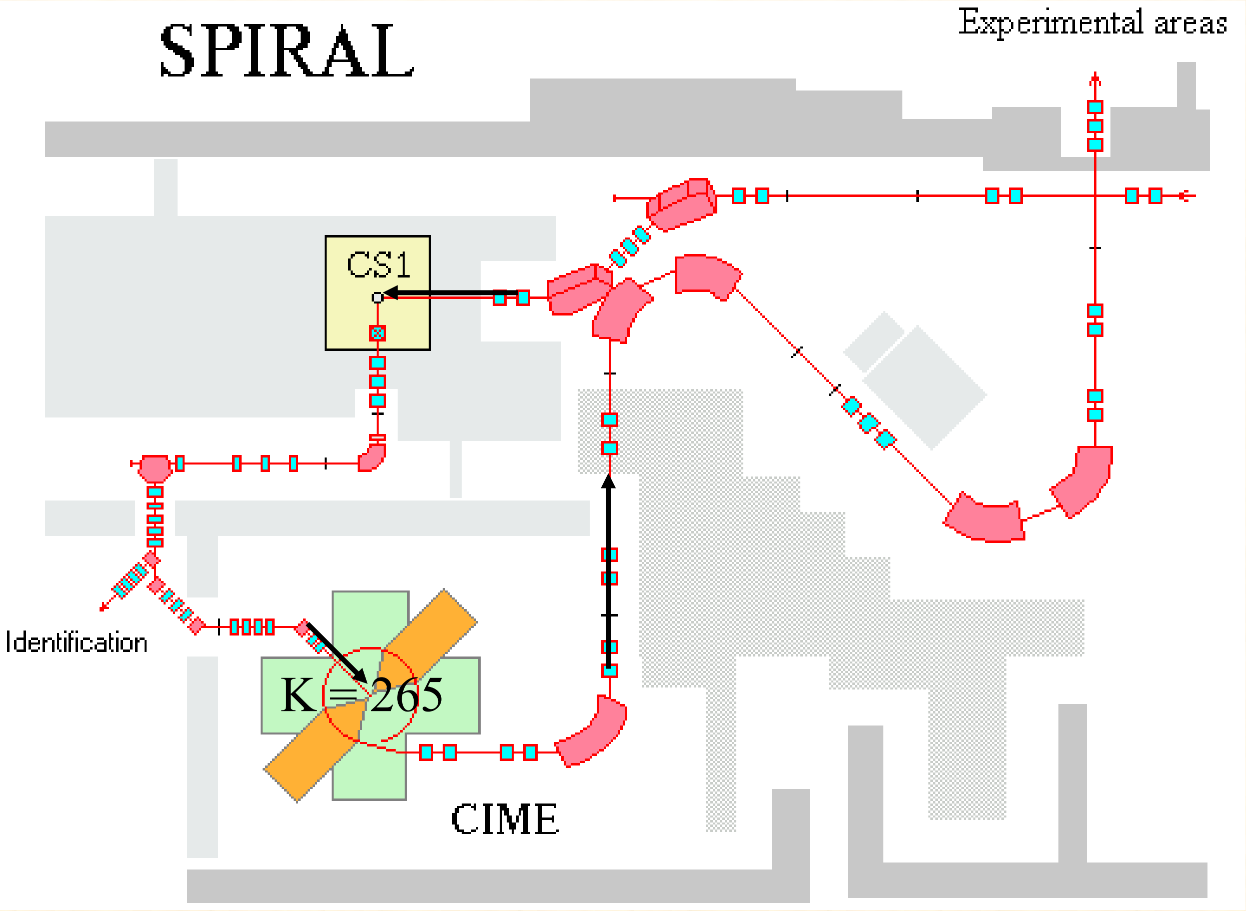
Extraction orbits in the IBA Cyclone 30

Computation

Putting dipoles and drift into a transport code is not going to work. We do not know *a priori* where the orbit is for any momentum neither the edge angles or the field index in that region.

The only realistic solution is to get the field map and the equation of motion.

SPIRAL cyclotron example



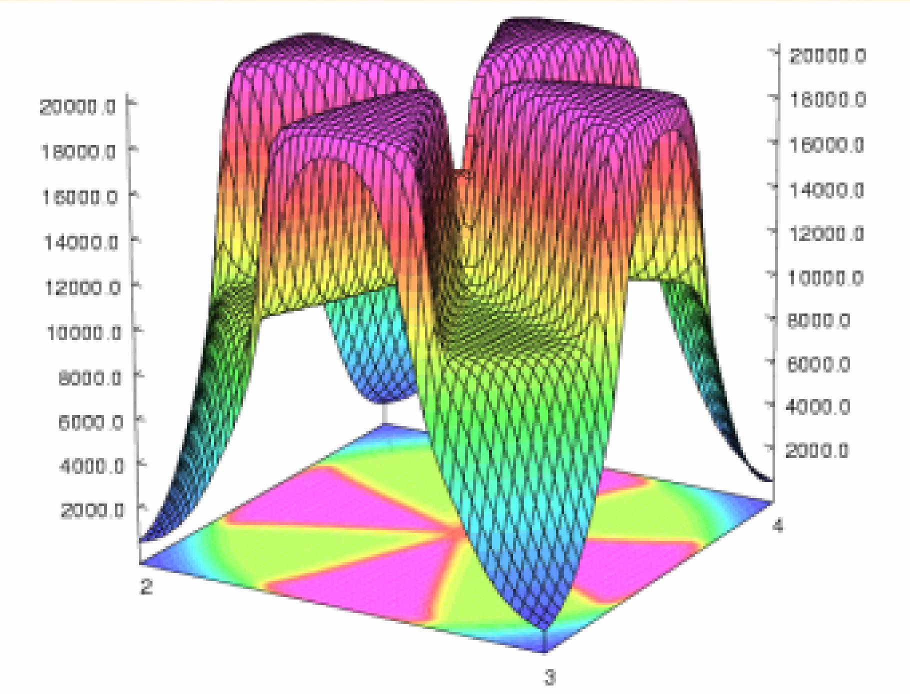
SPIRAL cyclotron example

Cyclotron modelisation

- Magnetic configuration: Computed field maps (Tosca …) or measured field maps at various field level (10 field levels)
- RF cavity field models (for 6 harmonics)
- Multiparticle computation codes

⇨ find a tuning for the whole working diagram

Field map

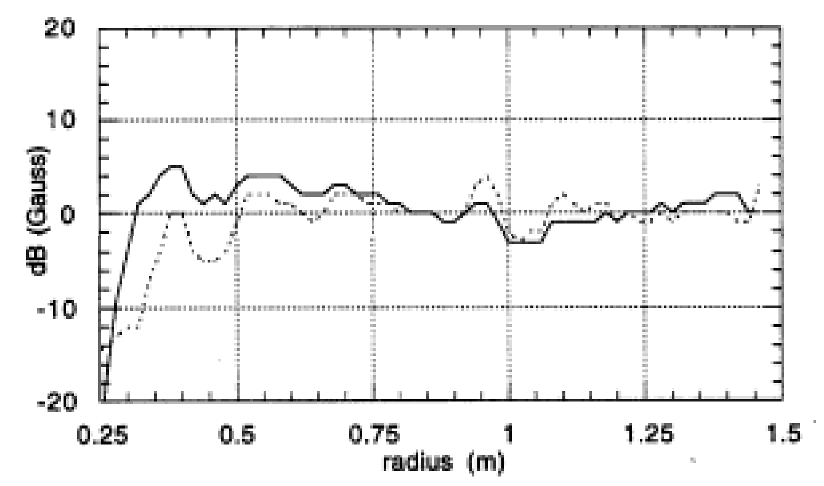


The use of codes such as TOSCA allows the determination of a magnet field map in 3D.

The computation figures are remarkably close to the measurements.

The transport of particles through the 3D field map will predict the behaviour of the beam during the acceleration.

One can rely on modelisation even for large machine.



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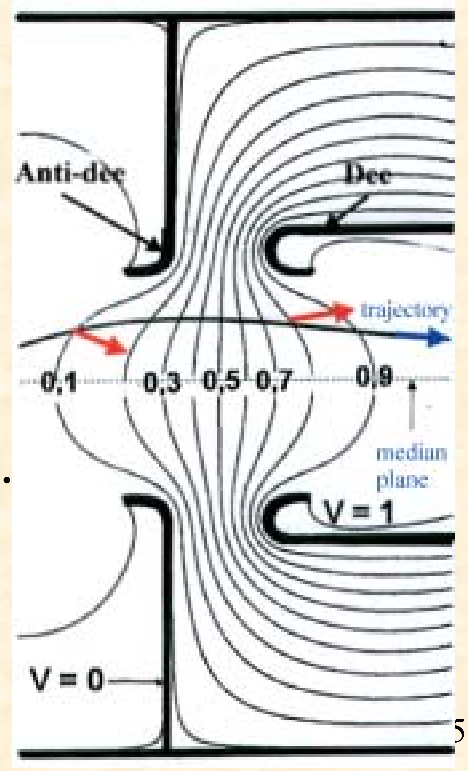
Accelerating gaps

The transport of the particle through the accelerating gaps depends on its vertical Z-position. One has to take into account the real equipotential distribution. Especially in the central region when the energy is low.

- The gap length has an equivalent length
- The transit time factor varies

as a function of z

- The vertical beam focusing is affected as well.



Trajectories and matching recipes

- Find a central trajectory (1 particle)
  - For a isochronous field level and a given frequency

    ➩Start from a closed orbit at large radius (no RF field)

    ➩ Then turn on RF field to decelerate the central particle to the injection.

    ➩ Tune the RF and the magnetic field at the injection to join the inflector trajectory.
- Find a matched beam in the cyclotron (multiparticles)

    ➩ Start with a matched beam at large radius around the central trajectory (6D matching)

    ➩Again in backward tracking determine the 6D phase-space at the injection
- Forward tracking

    ➩ confirm the matching to the extraction

    ➩ tune the isochronism

    ➩ and if the matching at the injection is not feasible by the injection line predict the new beam envelope and extraction
- Ejection

Iterative process

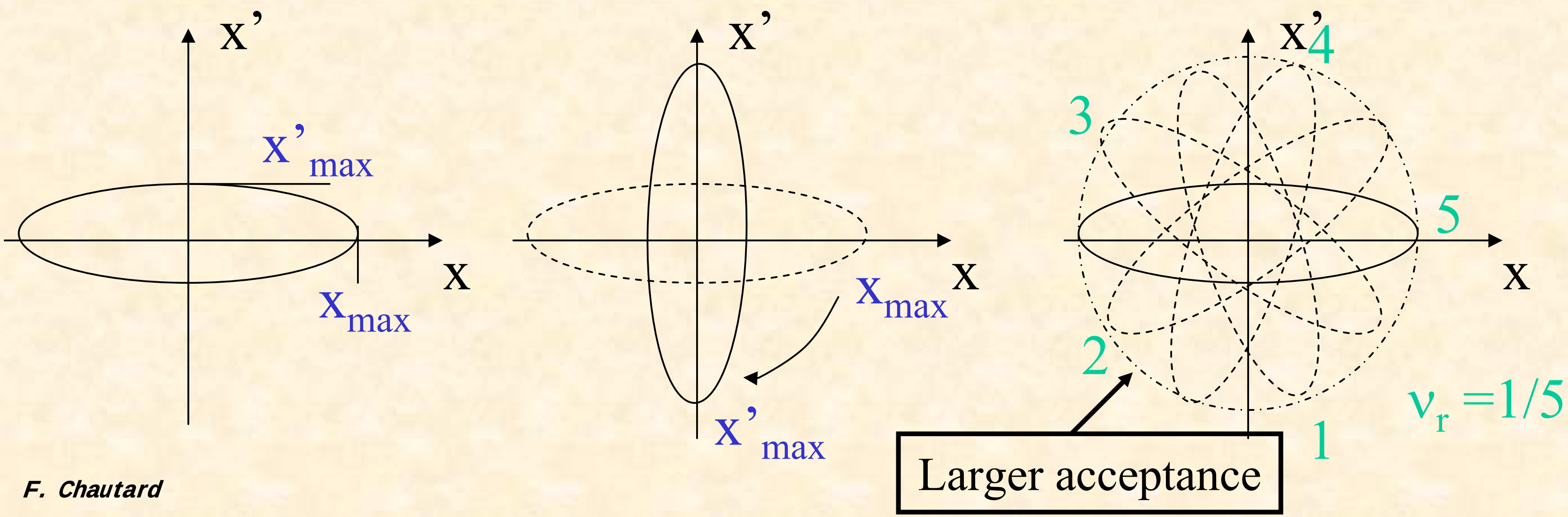
Transverse phase space (x)

We define a closed orbit ⇨ without acceleration

$$\begin{cases} x(t) = x_{max} \cos(\nu_r \omega_0 t) \\ x'(t) = x'_{max} \sin(\nu_r \omega_0 t) \end{cases}$$

Emittance area : $\varepsilon = \pi\, x_{max}.x'_{max}$ (and $\varepsilon = \pi\, z_{max}.z'_{max}$)

Betatron oscillation for mismatched beam



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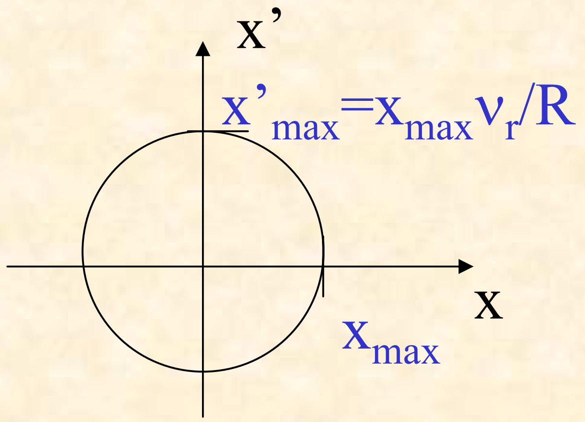
Transverse phase space (x)

$$\begin{cases} x(t) = x_{max}\cos(\nu_r\omega_0 t) \\ x'(t) = dx/ds = dx/R\omega_0 dt = -(x_{max}\nu_r/R)\sin(\nu_r\omega_0 t) \end{cases}$$

$$|x'_{max}| = |x_{max}\nu_r/R| \text{ and } \varepsilon = \pi\, x_{max}.x'_{max} = x_0^2\nu_r / R$$

- ⇨ Initial beam conditions depending of the cyclotron field (νr)
- ⇨ betatron oscillation disappear
- ⇨ Matched beam
- ⇨ Minumum of acceptance

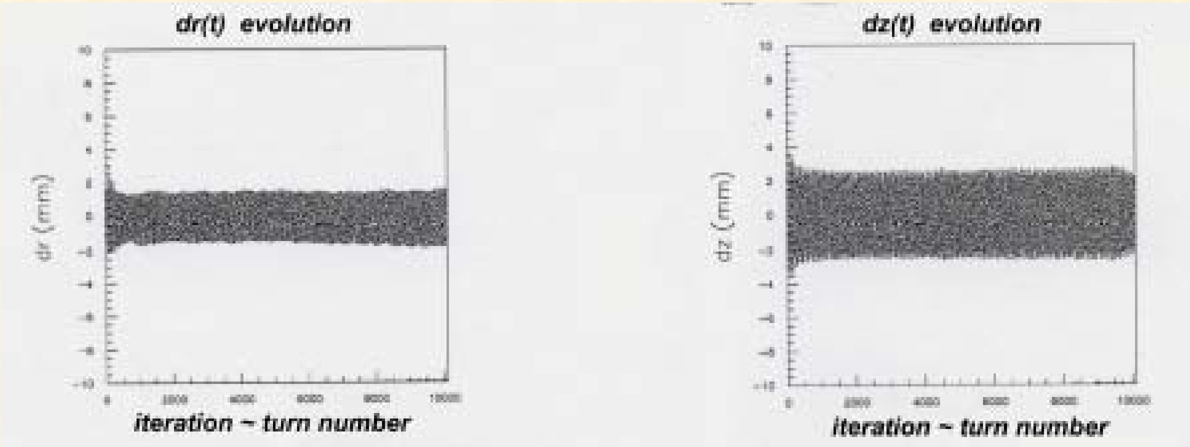


Beam matching

Liouville: Under acceleration and taking into account relativistic mass increase the normalized emittances are contant.

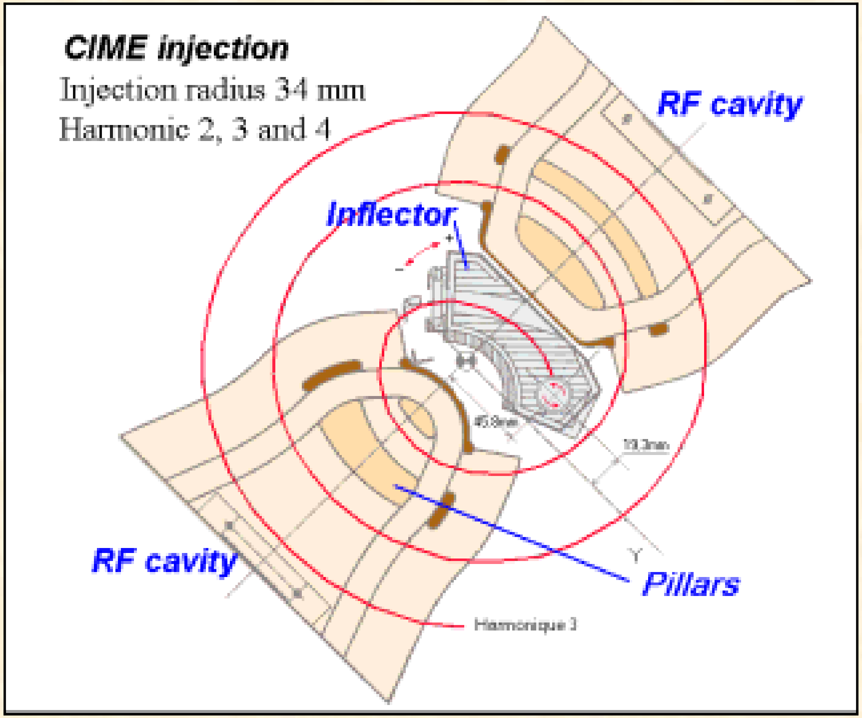
$X_{max} \sim 1/\gamma$ constant

$Z_{max} \sim 1/\nu_z\gamma$ can be large with weak focusing (ν_z small)

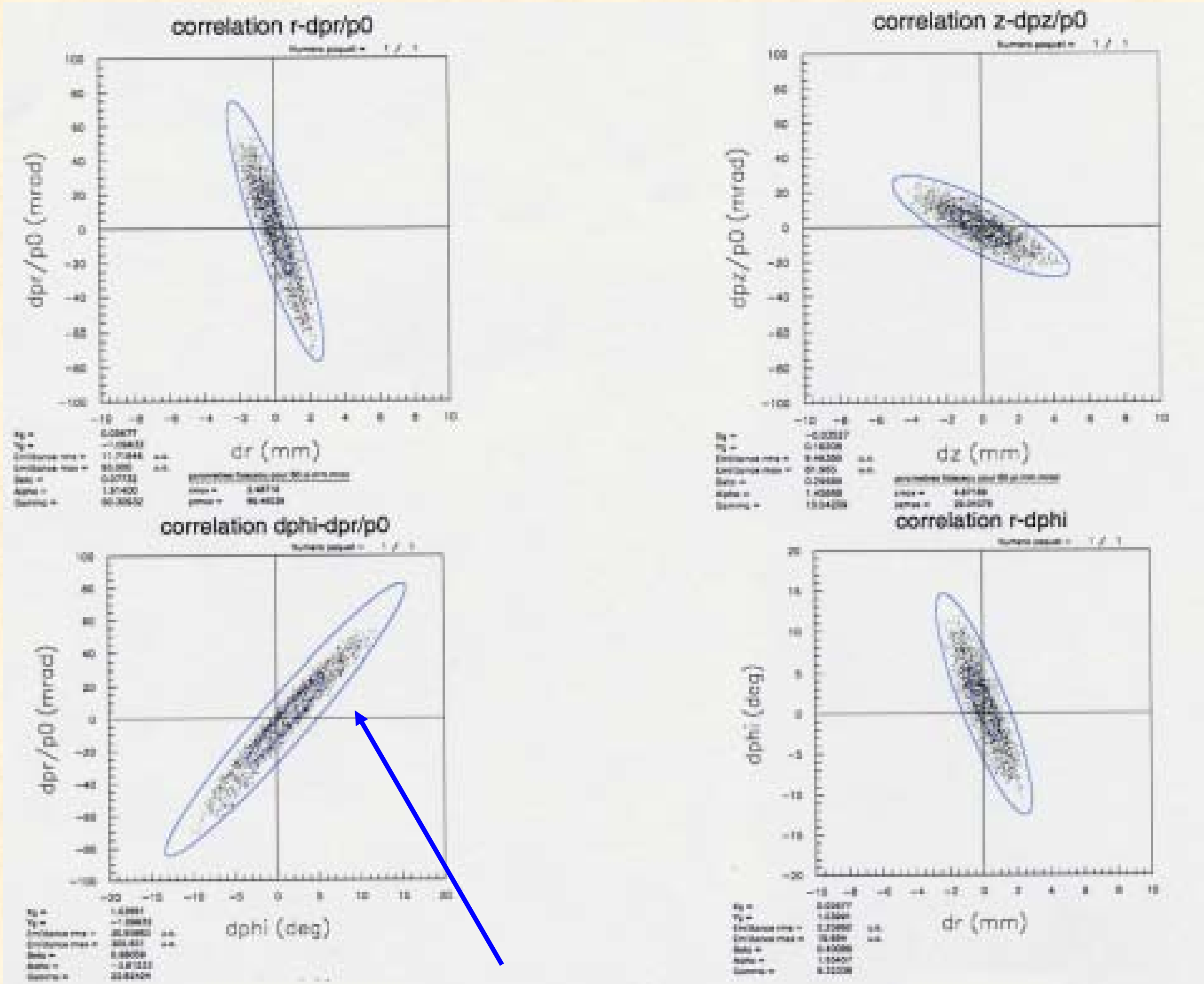


Geometric beam size

Backward 6D matching

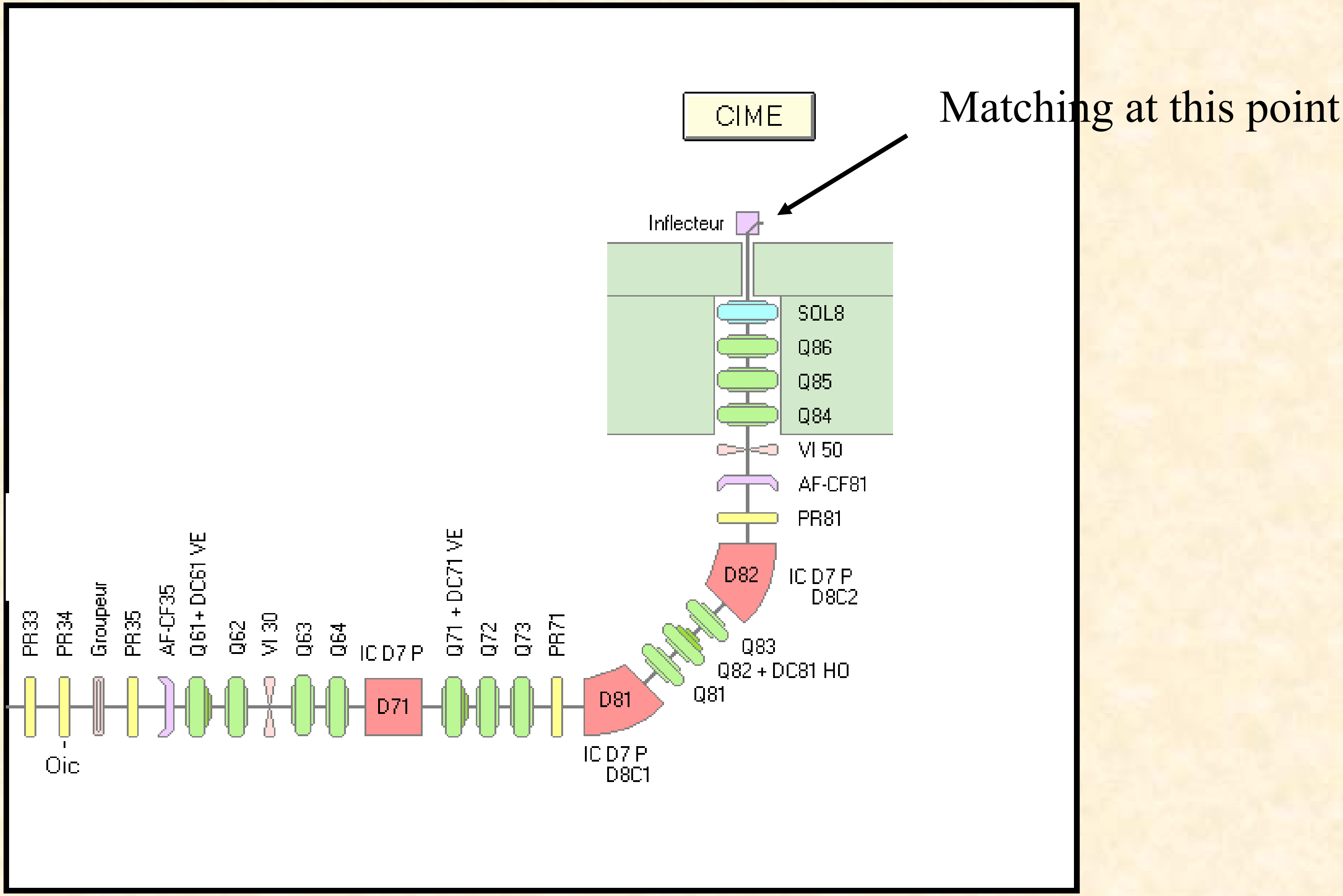


Backward 6D matching



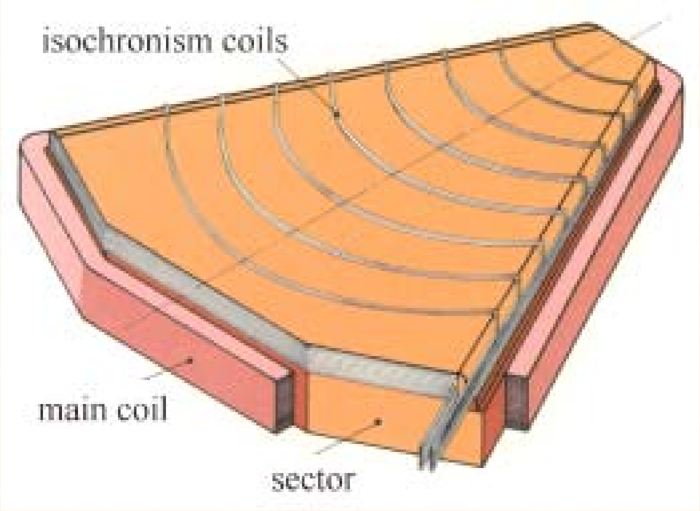
Not well represented by a gaussian beam ⇨ mismatch in forward

Backward 6D matching

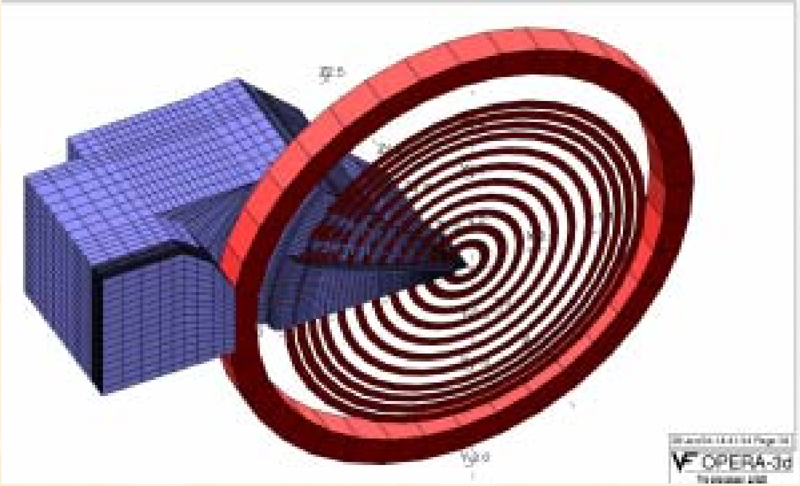


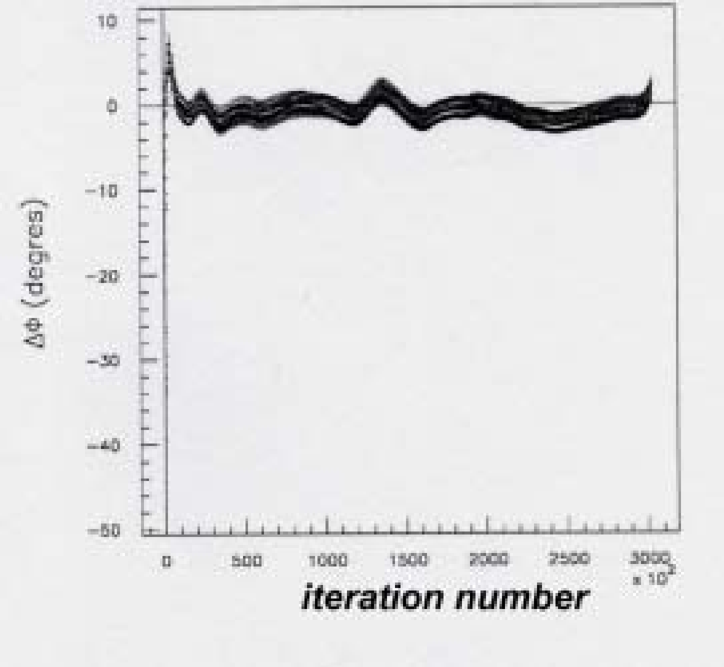
Isochronism $B(r) = \gamma(r)B_0$

<u>sectors</u>



<u>Compact</u>

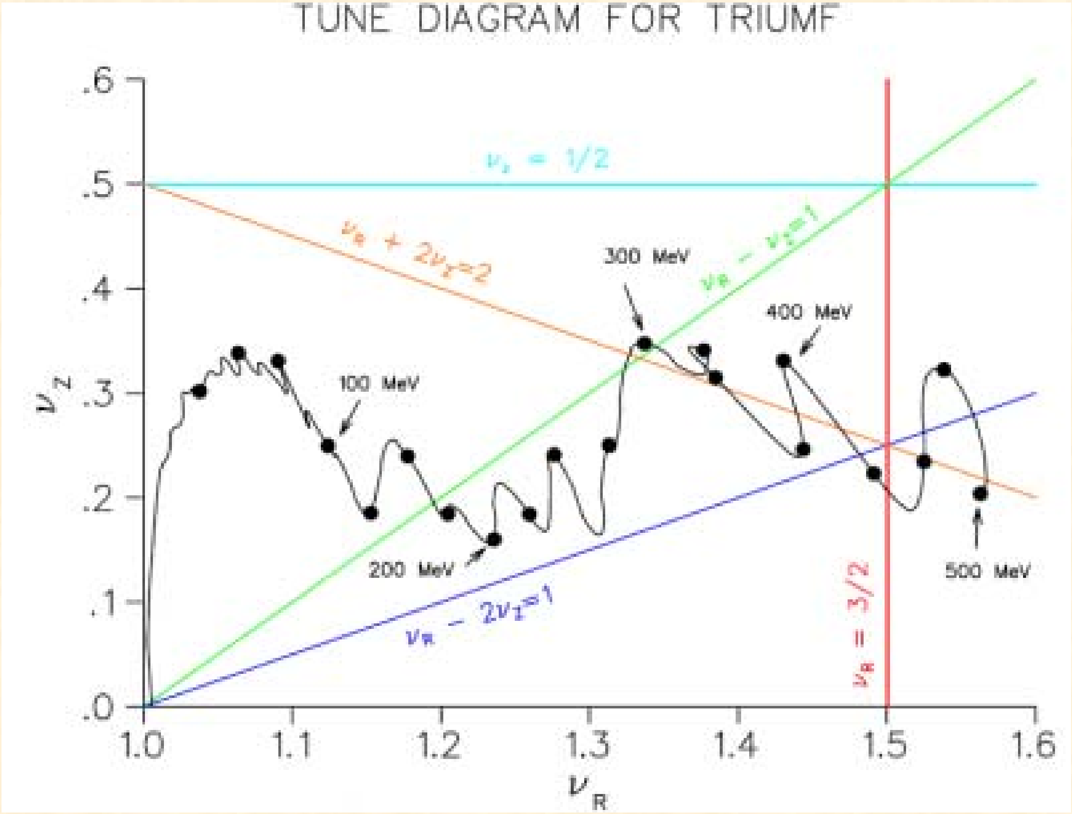




Tunes

$$\mathbf{K.\nu_r + L.\nu_z = P}$$

- K, L and P integer
- |K| + |L| is called the resonance order (1, 2, 3 …)



$W \propto r^2$

Cyclotron as a separator

For an isochronous ion (Q_0, m_0): $\omega_{rev} = \dfrac{Q_0 B(r)}{m_0 \gamma}$

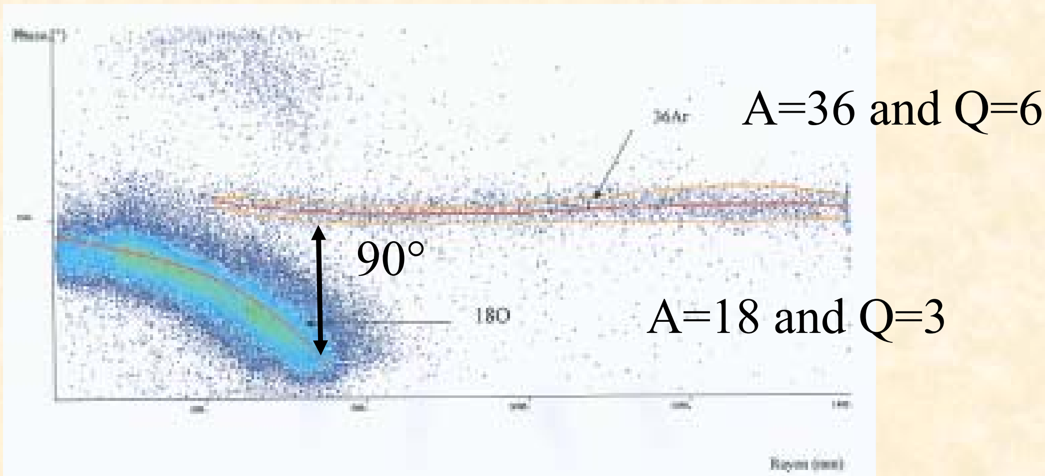
Constant energy gain per turn: $\delta T \approx QV_0 \cos(\varphi)$

For ions with a Q/m different from the isochronous beam Q_0/m_0, $\omega \neq \omega_{rev}$

There is a phase shift of this ion compared to the RF field during acceleration

when the phase φ reaches 90°, the beam is decelerated and lost.

The phase shifting : $\boxed{\Delta\varphi = 2\pi Nh \dfrac{1}{\gamma^2} \dfrac{\Delta(m/Q)}{m_0/Q_0}}$



Cyclotron resolution

An important figure for heavy ion cyclotrons is its mass resolution.

There is the possibility to have out of the source not only the desired ion beam (m_0,Q_0) but also polluant beams with close Q/m ratio.

If the mass resolution of the cyclotron is not enough, both beams will be accelerated and sent to the physics experiments.

Mass resolution:

$$\boxed{R = \frac{\Delta\left(\frac{m}{Q}\right)}{\frac{m_0}{Q_0}} = \frac{1}{2\pi h N}}$$

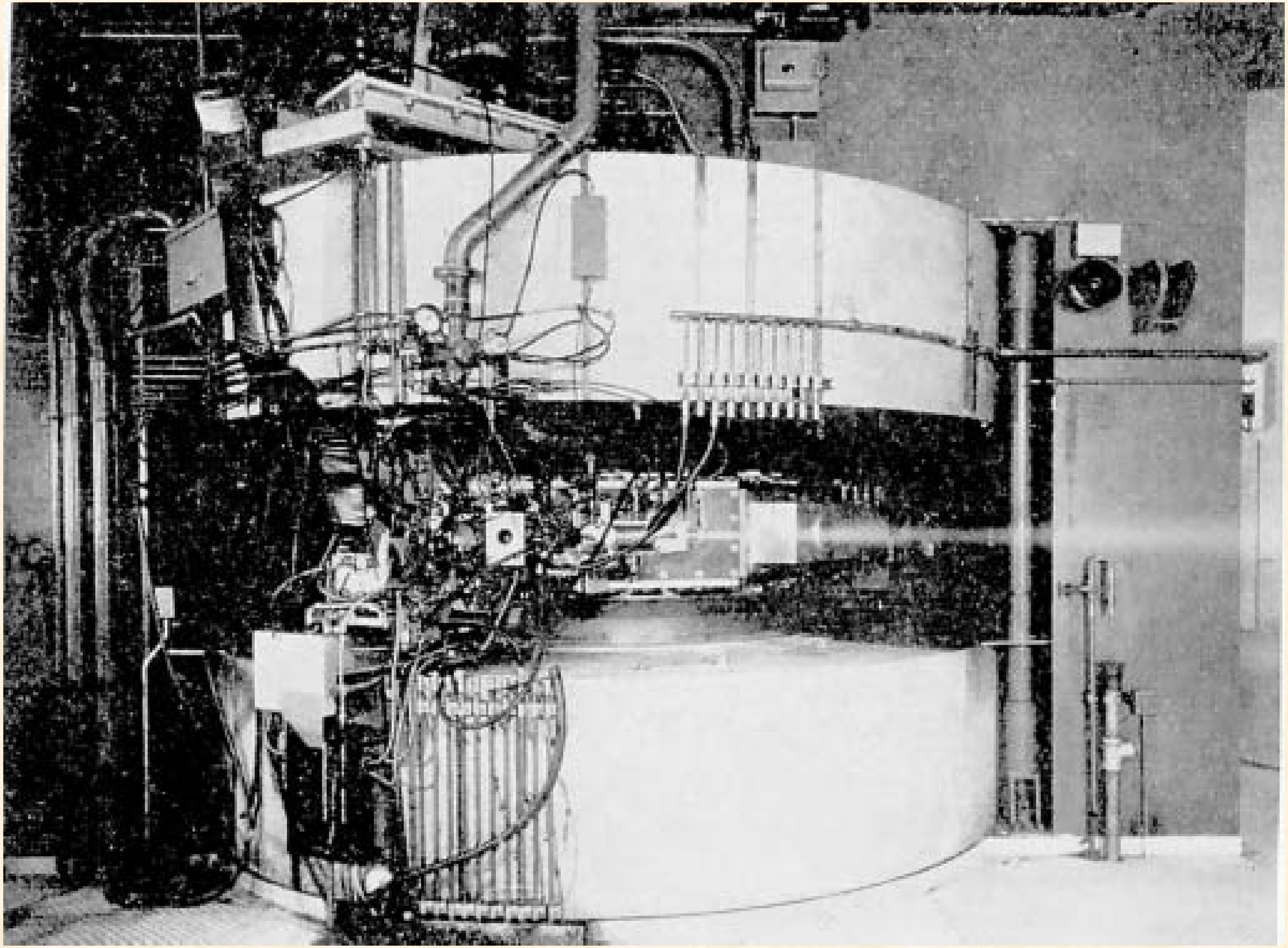
We want R small ⇨separation of close ion polluants

To have R small for a given harmonic h, the number of turn needs to be increase ⇨lowering the accelerating voltage ⇨small turn separation ⇨poor injection and/or extraction.

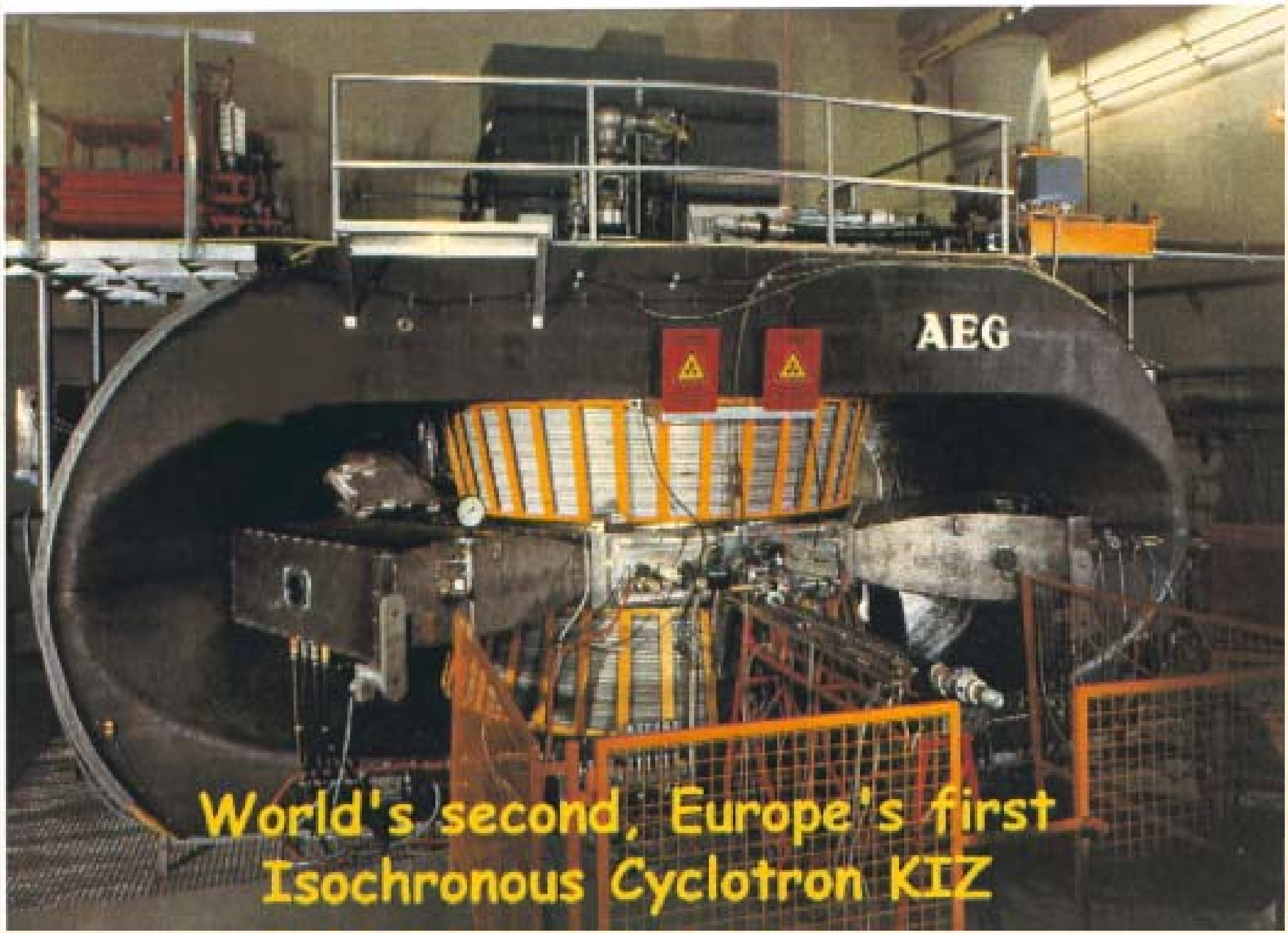
CIME example: h=6, N = 280 ⇨ $R = 10^{-4}$

Meaning that ions with a $m/Q > 1.0001 \times m_0/Q_0$ will not be extracted (great problems for new exotics beam machines : isobar and contamination for new machine…)

Few cyclotrons



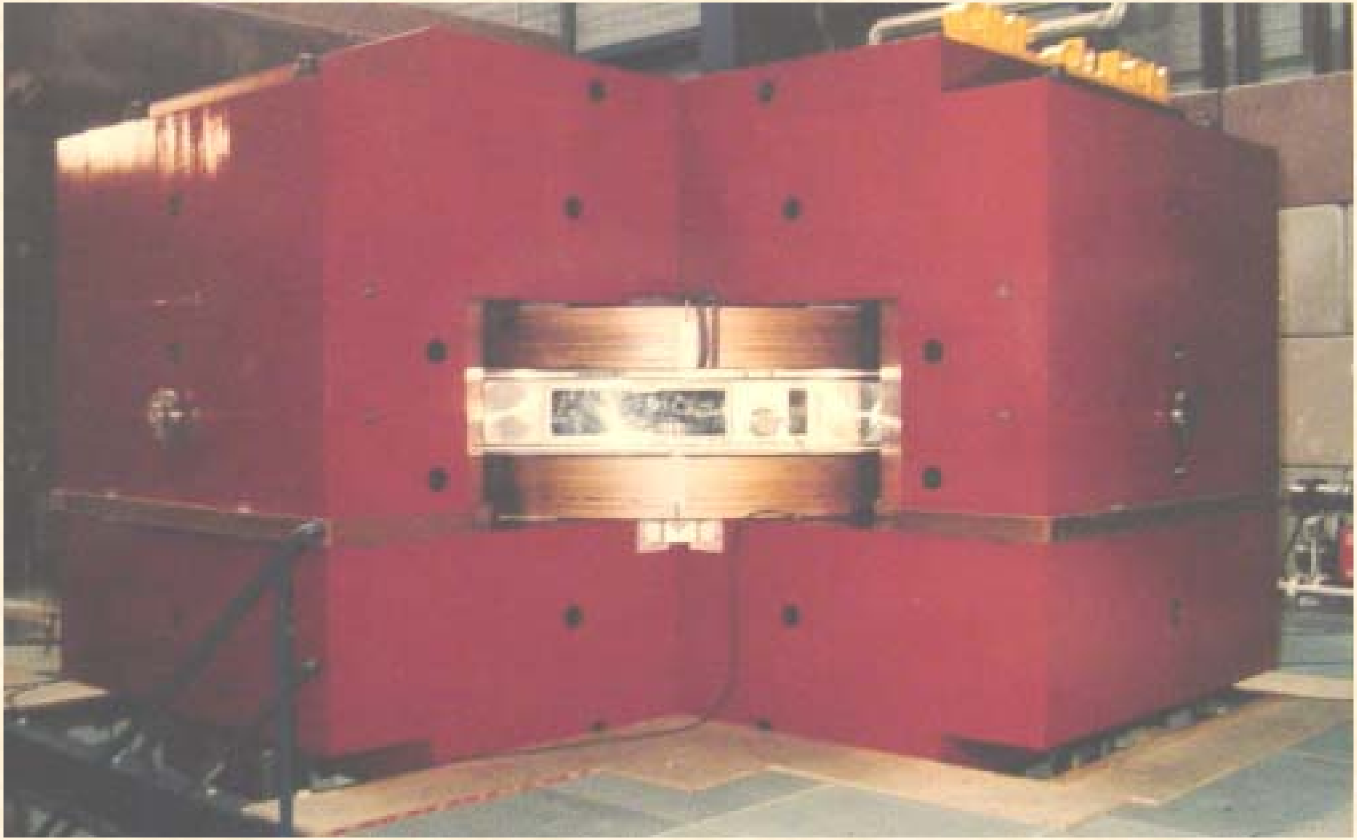
Argonne 60 inches cyclotron (deutons 21,6 MeV deuton beam out of an aluminium foil)



Karlsruhe cyclotron.

CYCLONE 30 (IBA) : H^- 15 à 30 MeV

primarily designed for industrial and medical applications



CIME cyclotron (yoke and coils only)

520 MeV proton, Triumf, Canada

The diameter of the machine is about 18 m

Lower half of the Main Magnet poles

END