Small Synchrotrons

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Development of synchrotrons

- **Historically, the development of accelerators was always driven by the demands of physics research for higher beam energies.**

- **Last type of machines developed were synchrotrons (~1950ies):**
  - To overcome synchronisation problems (relativistic particles and RF) and technological limitations (huge dipole magnets) of the cyclotron.
  - To use the accelerating structure more efficiently than in a Linac consequences of single use of RF as in a linac (efficiency, size, etc.).

- ***A* synchrotron is a circular accelerator:**
  - Design orbit is fixed at a given radius independent of the beam energy (ρ = constant) in contrast to the cyclotron.
  - Beam is accelerated during many revolutions passing through the same accelerating structure (cavity) in contrast to the linac.
  - Accelerating RF is synchronised with particle revolution frequency "SYNCHROTRON".

“Large“ and “small“ synchrotrons

- **Most of the HE physics research accelerators are synchrotrons.**
  - The largest machine is the LHC collider, start operation in 2007.
  - 27 km circumference and 7 + 7 TeV proton beam energy.

- **3 main groups of “small” synchrotrons (circumference ~100 m):**

Application	**Particles**	**Energy**	**Machines**
Synchrotron light sources	e- or e+	few GeV	~ 50
Neutron spallation sources	p	~ GeV	~ 5
Hadrontherapy synchrotrons	p and C, O	250MeV / 450MeV/n	~ 10

- **Presently increasing interest in synchrotrons for hadrontherapy (tumour irradiation with protons or light ions).**
  - Discuss technical aspects of “small” proton and ion synchrotrons.
  - Show the MedAustron synchrotron as an example.

“Limitations“ for p/ion linacs

- **Accelerating structures and RF power sources are expensive and used only during a single beam passage.**

- **For higher energies the Linacs will become long**
  - Typical gradient of ~MV/m (includes space for focusing, etc.).
  - 200 MeV protons with average gradient ~2 MeV/m = 100 m.

- **Beam energy cannot be changed.**
  - Fixed relation: structure geometry – rf frequency – particle velocity.

- **Particles with different Z/A are difficult to accelerate.**
  - Design structure (RF power) for worst (smallest) Z/A.

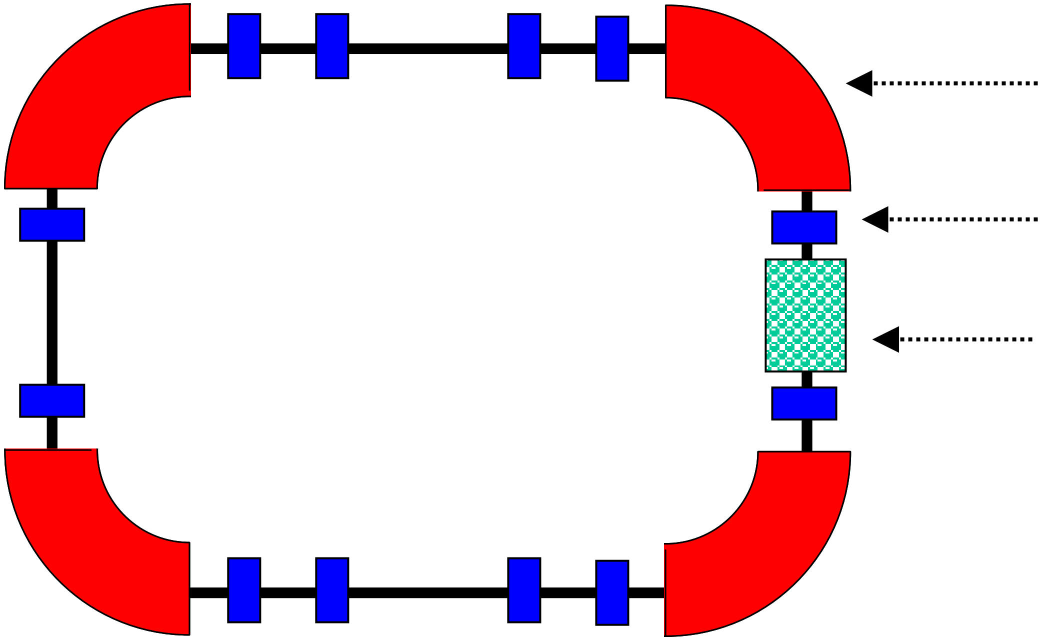
- **Obviously the beam can't be stored in the machine.**

- **Linacs can provide a quasi dc-beam when designed for that.**

“Limitations” for cyclotrons

- **For higher energies the magnets become unfeasibly large.**
  - Build superconducting cyclotrons.
- **For relativistic particles the synchronous condition is lost.**
  - Build synchro-cyclotrons.
- **Even with improvements clearly limited in energy ($p < 1$ GeV).**
- **Beam energy is fixed or difficult to change.**
- **Obviously the beam can’t be stored in the machine!**
- **Cyclotrons provide a quasi dc-beam.**

Main components of a synchrotron



Dipole magnets to bend the beam on the circular orbit.

Quadrupole magnets to focus the beam.

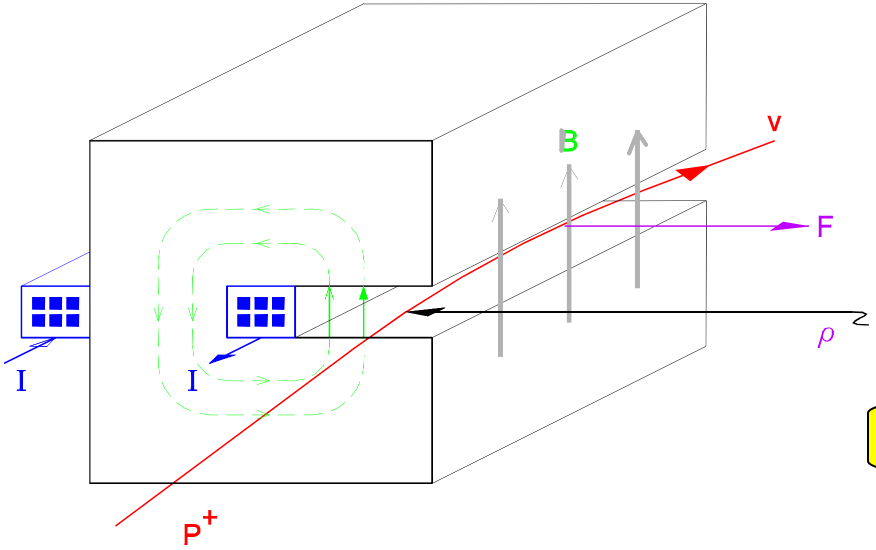
RF cavity to accelerate.

Magnets can be relatively small, the “good field” needs to cover only the region around fixed design orbit.

- In contrast to Linac and cyclotron the beam can be stored for longer period in the synchrotron.
- Particles with different Z/A can be accelerated, the extraction energy can easily be changed.

Keep particles on orbit: Dipole

- **A dipole produces a homogeneous (vertical) magnetic field B.**
  - Equilibrium between Lorentz force and centripetal force to bend the beam on the design orbit.



$F_{\text{radial direction}}$: $evB = mv^2/\rho$

$B\rho = mv/e = m_0 c\beta\gamma/e = p/e$

Magnetic rigidity ∝ **Relativistic momentum**

- **Difference between ions (Z,A) and protons:**
  - For protons (charge e, mass m_0): $B\rho = m_0 c\beta\gamma/e$
  - For ions (charge Ze, mass Am_0): $B\rho = (A/Z)\cdot m_0 c\beta\gamma/e$
  - For identical momentum/nucleon, the ion beam is stiffer and more difficult to bend.

Keep particles on orbit: Dipole

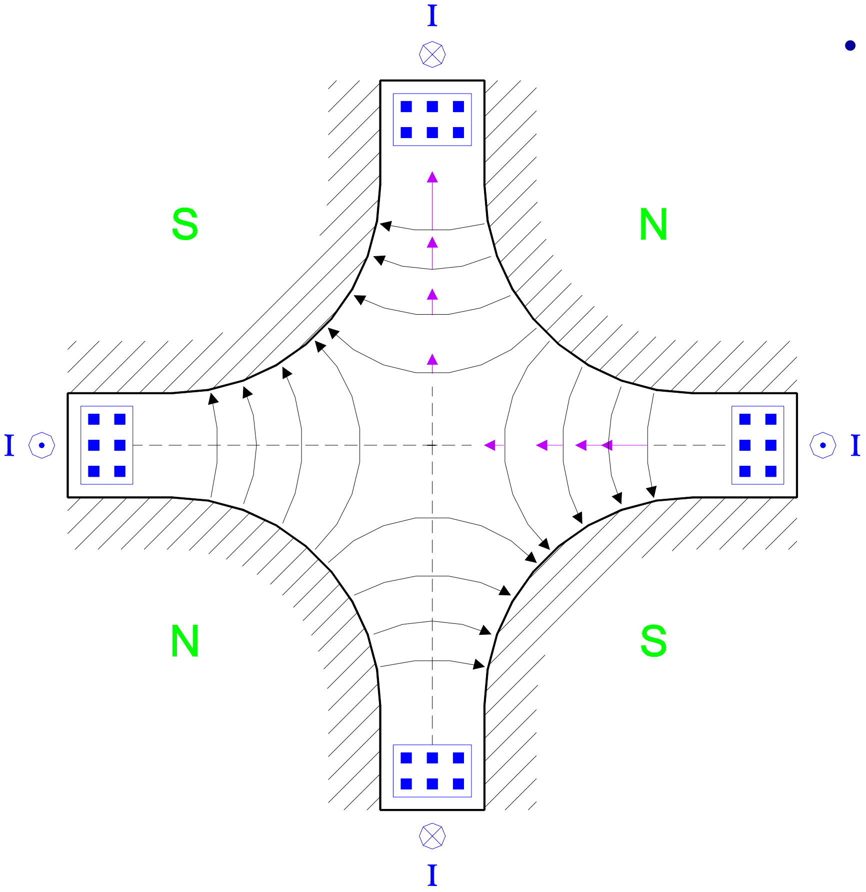
- During acceleration the momentum increases and with $B\rho = p/e$ the magnetic field B has to be increased accordingly to keep the particles on orbit (ρ does NOT change for the synchrotron).
  - In contrast to the cyclotron where B is constant and ρ increases.

- As a consequence the synchrotron CANNOT provide a dc-beam, the beam time structure reflects the magnet ramping (~Hz).

- Dipoles define the "design" orbit of the synchrotron and an "ideal" particle would circulate there forever but…
  - The beam consists of many particles and all have slight deviations from the design orbit (injection errors, energy errors, etc.).
  - The dipole magnets have field imperfection and are misaligned.
  - Gravitation also acts on the beam, etc…

- Therefore we need focusing to stabilise the beam and keep it for long time in the machine.

Focusing particles: Quadrupoles



- **Quadrupole produces a constant gradient g=-dBz/dx.**
  - Focusing forces increase linearly with displacement (Lorentz force):

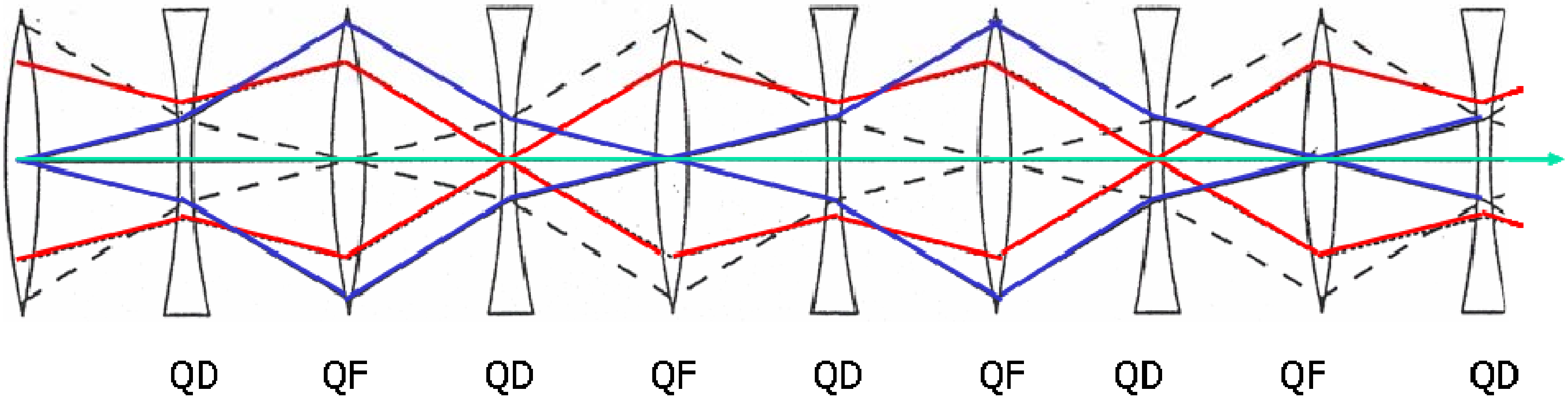
$$\boxed{F_x = -gx \cdot ev \text{ and } F_z = gz \cdot ev}$$

  - **Force is focusing in one plane while defocusing in the other.**
  - Important: no coupling of horizontal and vertical motions.
  - Again the gradient has to increase like the momentum when accelerating

- **Optical lenses are either focusing (concave) or defocusing (convex).**
- **Magnetic lenses focus in one plane but are defocusing in the orthogonal plane (from Maxwell's equations) – but that is not a problem…because**

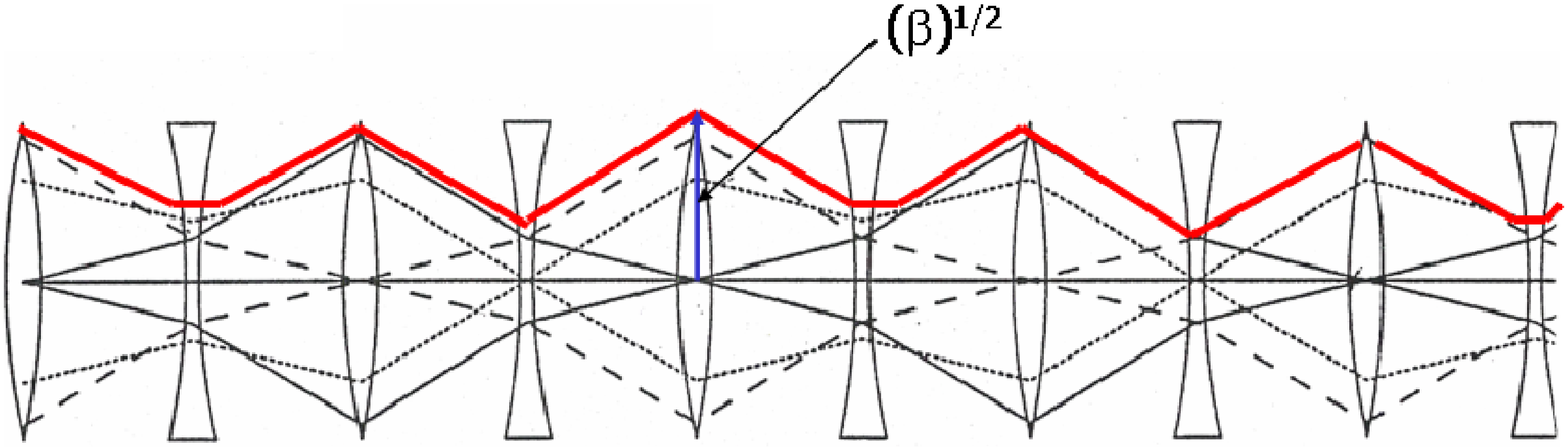
Alternate gradient focusing

- **Alternating focusing and defocusing quadrupole magnets (in correct way) allows to obtain an overall focusing effect.**
  - Intuitive view: beam size increases when passing a defocusing element. This gives a larger beam – and stronger forces – when passing the following focusing element.



- **“Non-ideal” particles perform oscillations about the design orbit.**

Describing the envelope – Beta Function

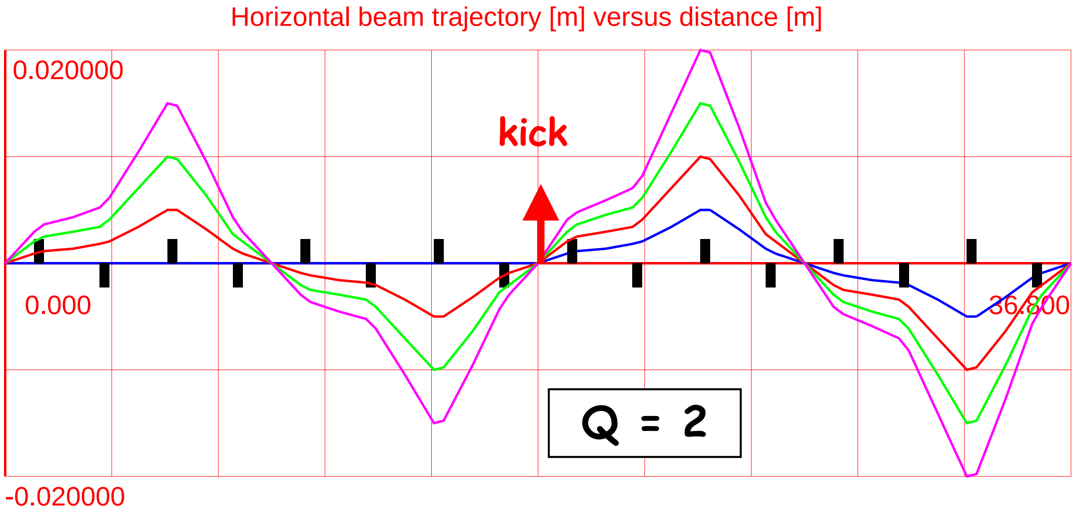


- **When looking at all particle oscillations, the envelope function is proportional to sqrt of the betatron amplitude function:**
  - The shape of the beta-function is determined by the lattice (sequence of quadrupoles and dipoles).
  - The oscillations of the beam particles are called "betatron oscillations".
  - The number of oscillations per revolution in the synchrotron is called the betatron TUNE Q (Q_x horizontal and Q_z vertical). The betatron tune is controlled with the quadrupole magnets.
- **Choice of tune and good control are essential for performance.**

Forbidden tune values - Resonances

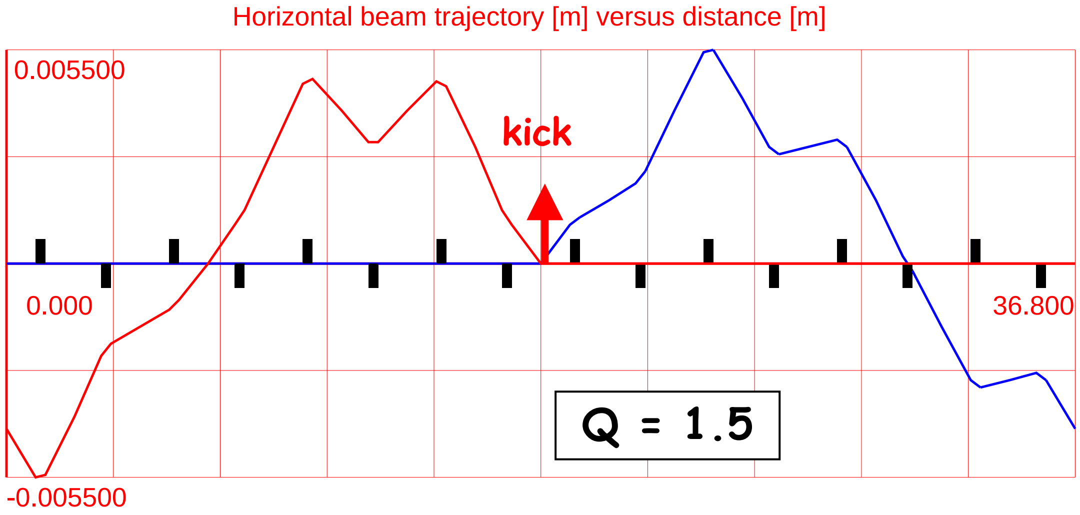
- **Integer Tune Q=N**
- **Dipole error**
  - Kick has always same sign independent of position.

- **FORBIDDEN**
- **Perturbation adds up**


Horizontal beam trajectory [m] versus distance [m]

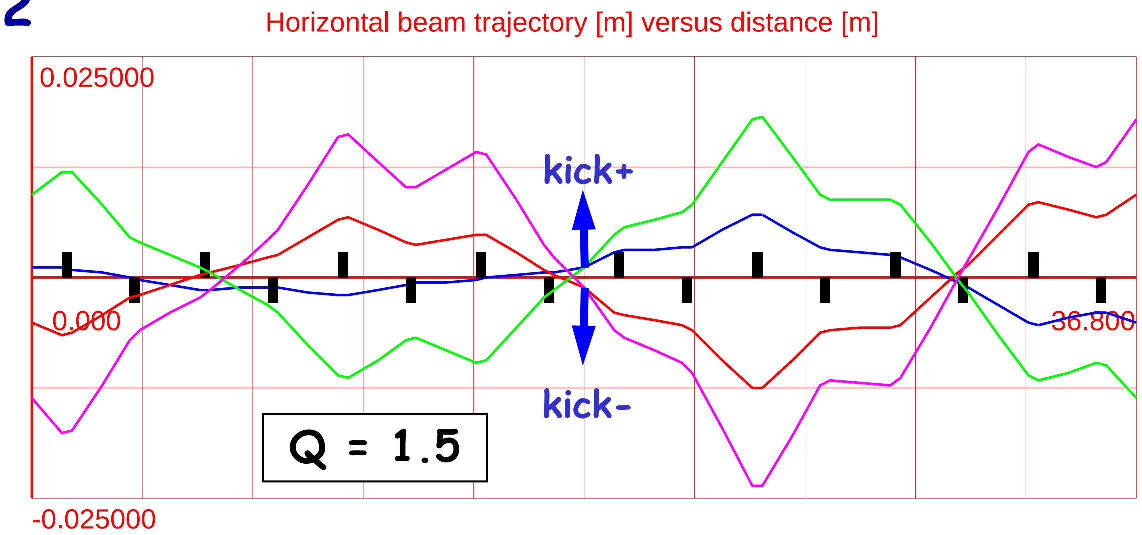
- **1/2 Integ.Tune Q=N+1/2**

- **OK for dipole error**
- **Perturbation cancels after each turn**


Horizontal beam trajectory [m] versus distance [m]

Forbidden tune values - Resonances

- **1/2 Integ.Tune Q=N+1/2**
- **Quadrupole error**
  - Kick depends on position and changes sign.



- **FORBIDDEN**
- **Oscillation amplitude is steadily increasing.**

- **Similar problems for 1/3-integer tune, etc…**
- **All low order resonances are dangerous and have to be avoided.**

$$\boxed{NQx + MQz = P}$$ **with small N and M**

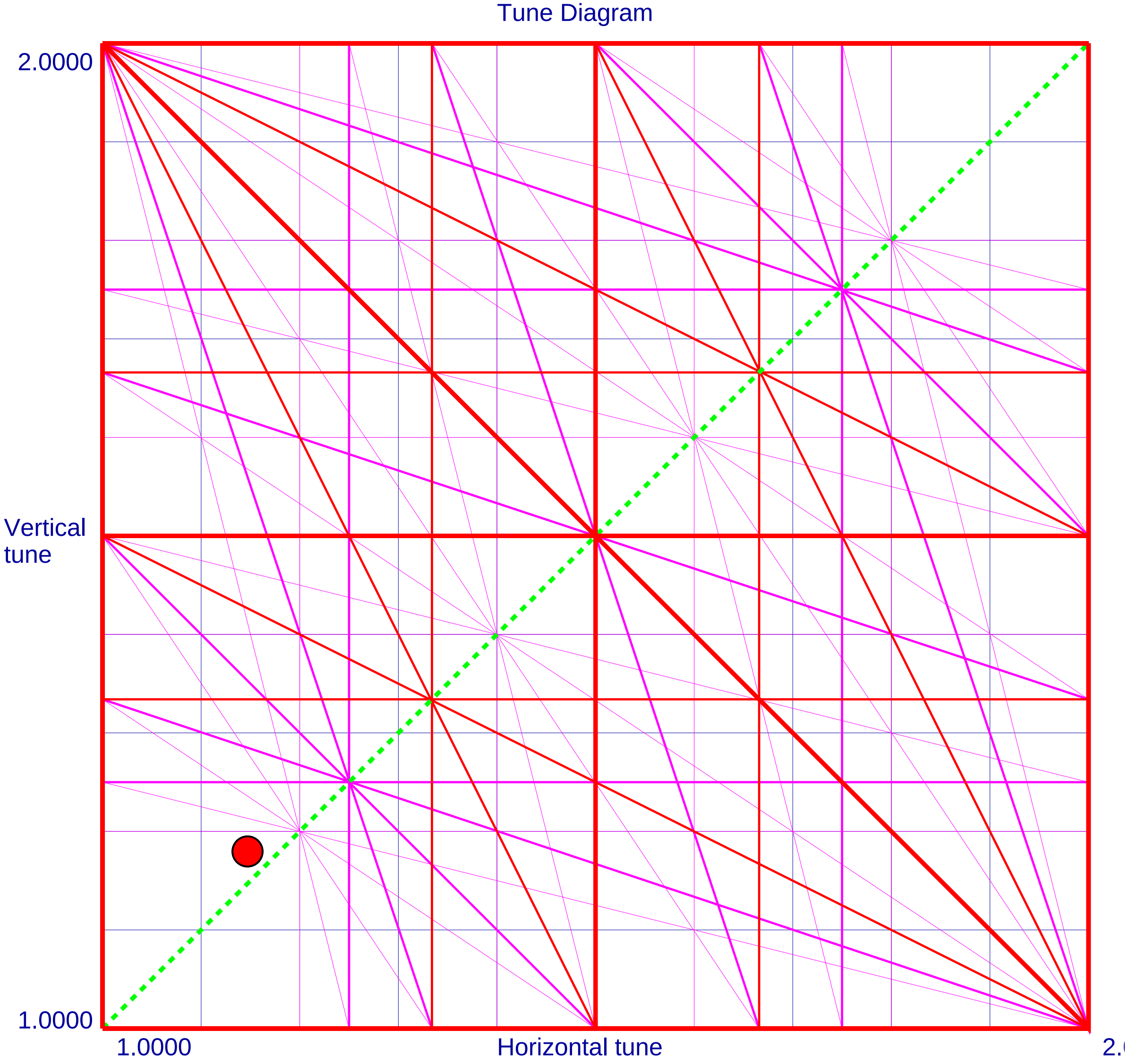
- **The synchrotron working point (Q_x, Q_z) has to be carefully chosen.**

Working point and tune diagram

- **Tune diagram**
  - Qz over Qx
  - Resonances up to fifth order (N+M = P = 5).

- **Choose working point (Q_x, Q_z) not too close to resonances.**

- **Example lower tunes close to diagonal e.g. 1.15/1.18**

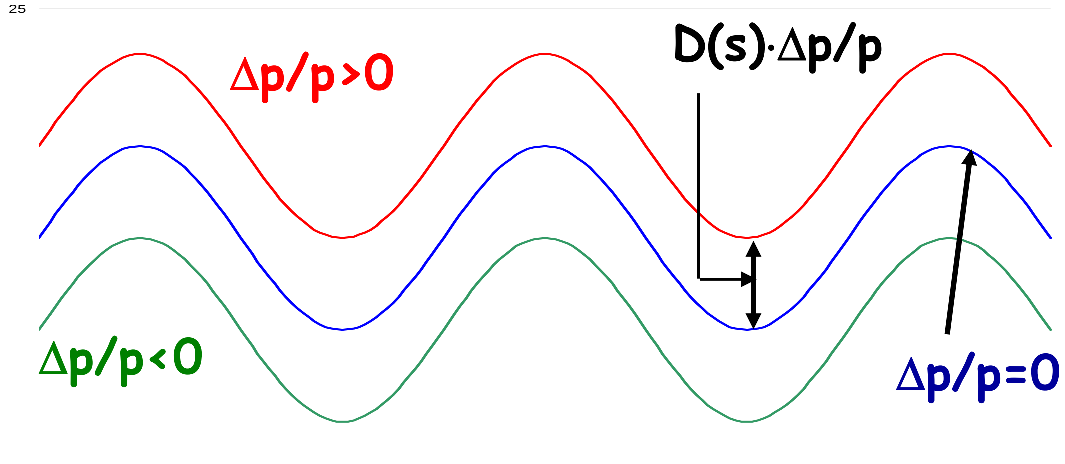


Off momentum particles ($\Delta p/p \neq 0$)

- **Effect from dipoles:**
  - For $\Delta p/p > 0$, the beam is stiffer (higher rigidity) and particles are less bent in the dipoles ➔ should spiral outwards?
  - For $\Delta p/p < 0$, the beam is less rigid and particles are more bent in the dipoles ➔ should spiral inwards?

- **NO, due to the restoring force of quadrupoles there is an equilibrium "Dispersion" orbit for off-momentum particles.**
- **The dispersion function D(s) defines central orbit for off-momentum particles.**

$$\boxed{CO(\Delta p/p) = D(s)\cdot\Delta p/p}$$

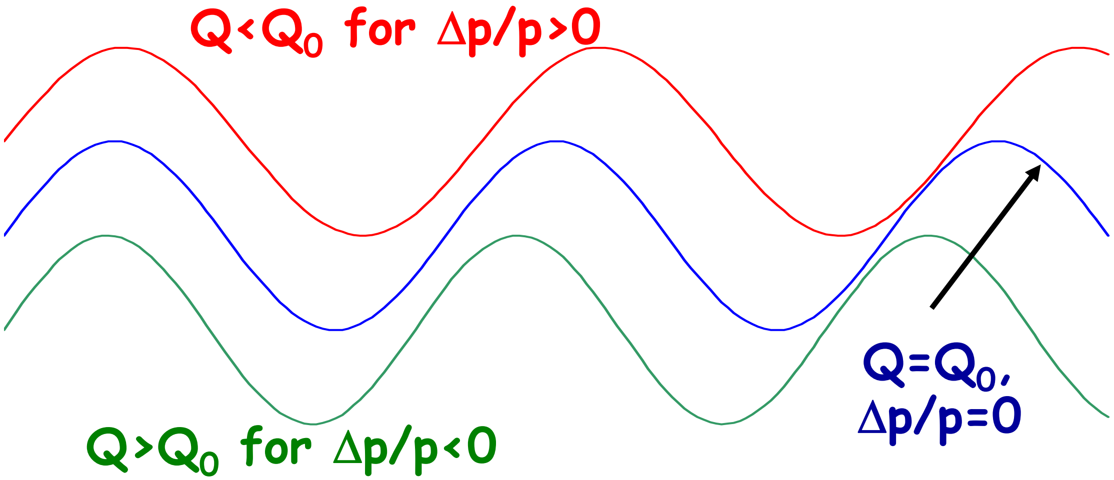


Off momentum particles ($\Delta p/p \neq 0$)

- **Effect from quadrupoles:**
  - For $\Delta p/p > 0$, the beam is stiffer (higher rigidity) and particles are less focused by the quadrupoles ➔ they will have a lower tune Q.
  - For $\Delta p/p < 0$, the beam is less rigid and particles are more focused by the quadrupoles ➔ they will have a higher tune Q.

- **Particles with different momenta will have different tunes.**
- **We need to foresee the required "space" in the tune diagram.**

$Q = f(\Delta p/p)$

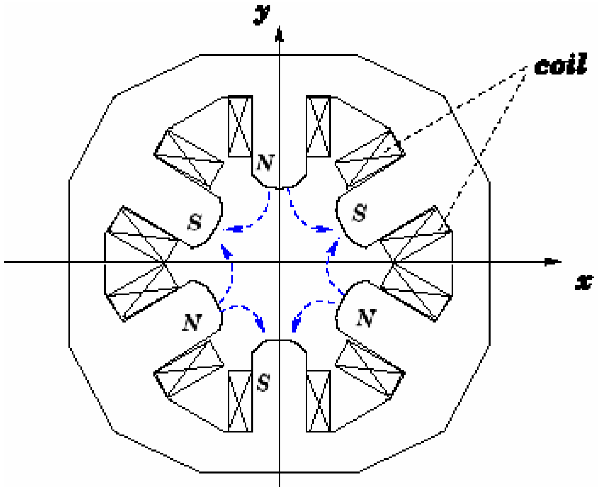


Chromaticity Q'

- **Dependency of the betatron tune on momentum is called chromaticity:**

$$Q' = \Delta Q/(\Delta p/p)$$

- **Chromaticity is fundamental for the stability of the machine:**
  - For non-relativistic particles (i.e. small proton/ion synchrotrons) the chromaticity must be negative $Q' < 0$.
  - For relativistic particles (i.e. electron synchrotrons or high energy proton/ion synchrotrons) the chromaticity must be positive $Q' > 0$.



- **The natural chromaticity (only dipoles and quadrupoles) is usually negative.**
- **Chromaticity can be controlled and adjusted with sextupole magnets ($\Delta x' \propto x^2$) that are positioned in dispersion regions.**

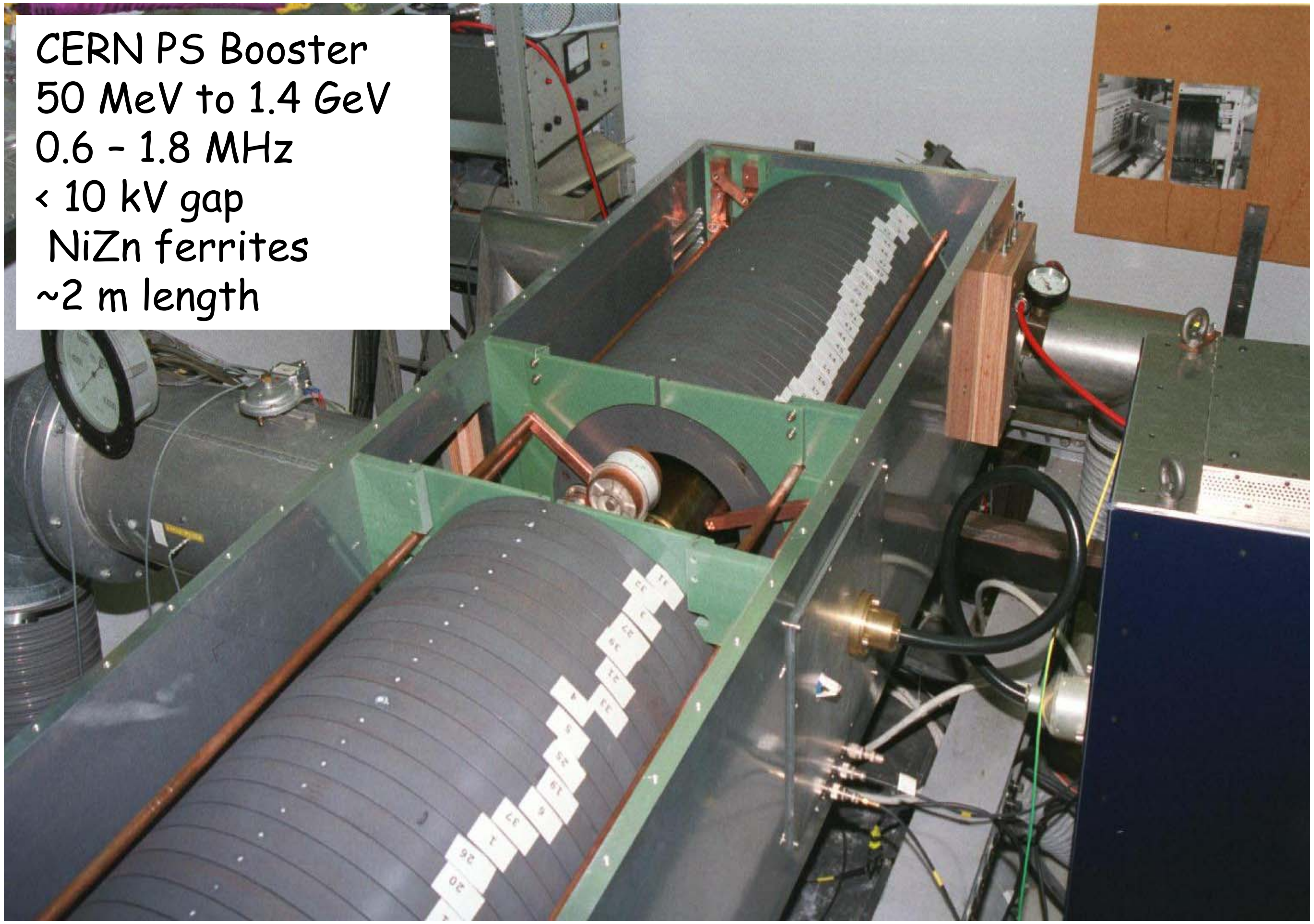
Acceleration – RF systems

- **The beam can be accelerated over many turns (medical machine typically 10^6) so the rf voltage can be relatively moderate.**

- **In small proton and ion synchrotrons the beams are not fully relativistic, therefore velocity and revolution frequency will change significantly during acceleration.**

- **Typical rf system parameters:**
  - Accelerating voltages 1 – 10 kV.
  - Frequency range ~MHz.
  - Frequency swing up to factor 5 (to follow beam velocity).
  - Ferrite loaded RF cavities (tuneable).

Acceleration – RF systems

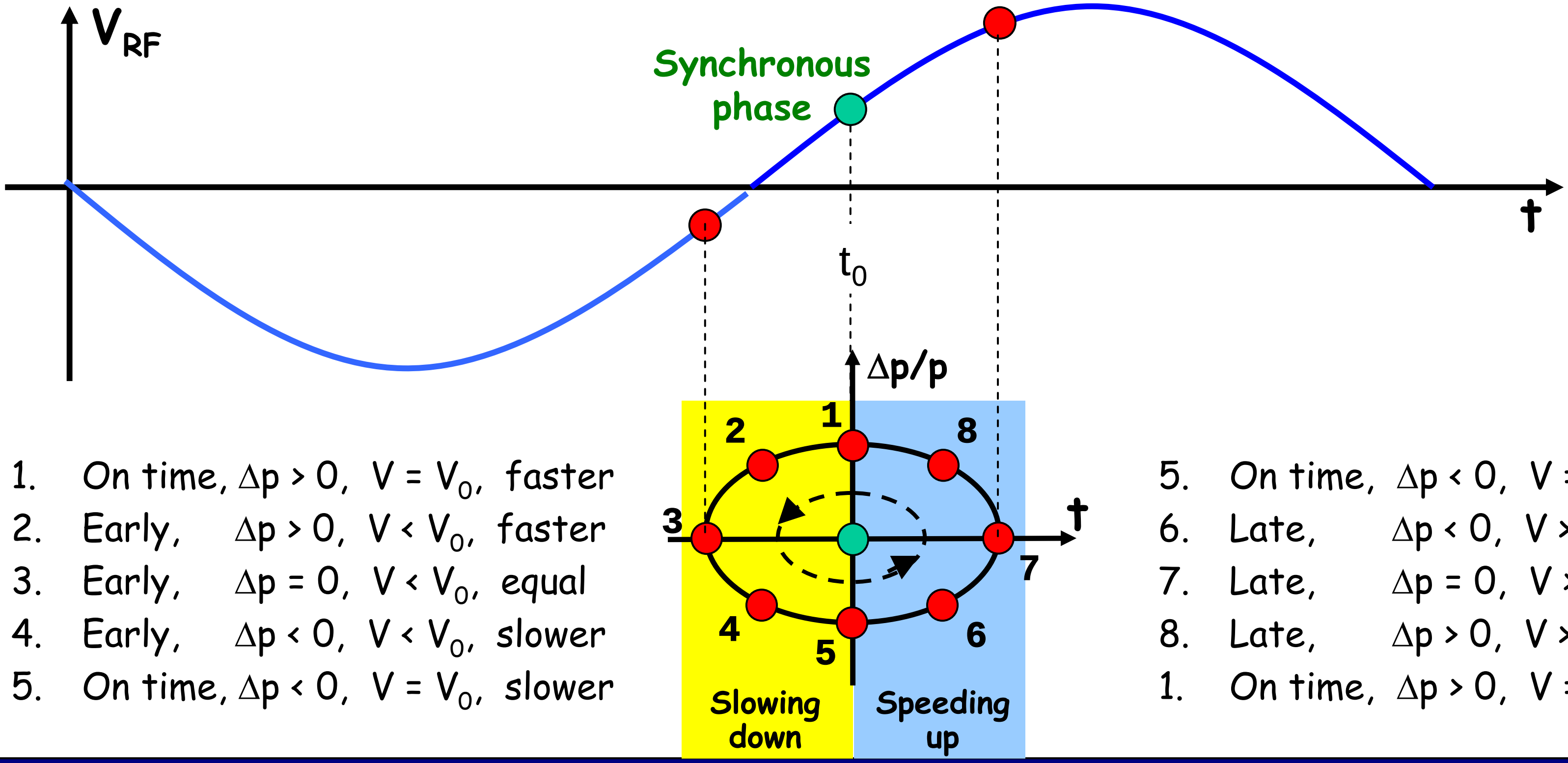


CERN PS Booster
50 MeV to 1.4 GeV
0.6 – 1.8 MHz
< 10 kV gap
NiZn ferrites
~2 m length

- **LC – resonant circuit, change resonant frequency via inductance change by magnetising the ferrites.**

Longitudinal motion – synchrotron oscillations

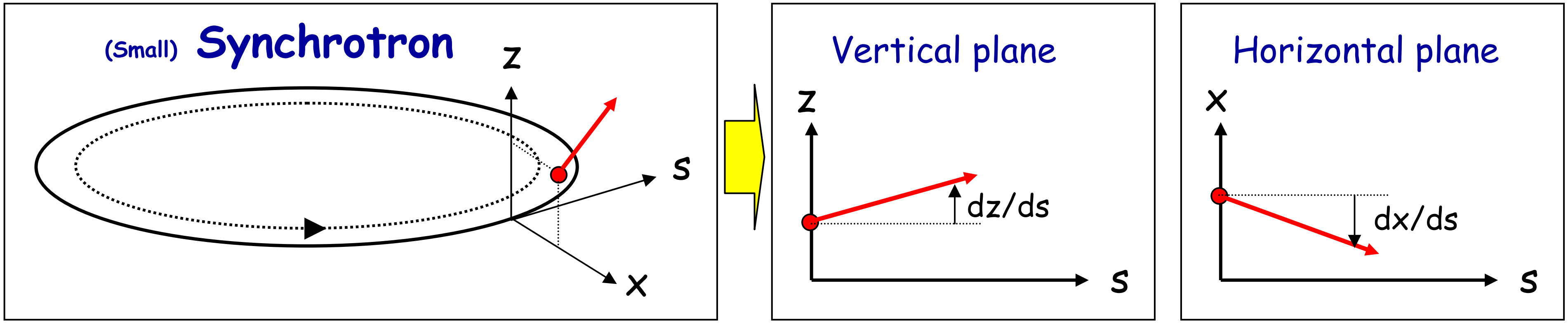
- **Small proton/ion synchrotrons operate below transition**
  - Particle with higher momentum is faster and has shorter revolution time) (in contrast to electron machines and HE p).
  - The ideal particle should arrive always at the same RF phase at the cavity after each revolution (synchronous particle).

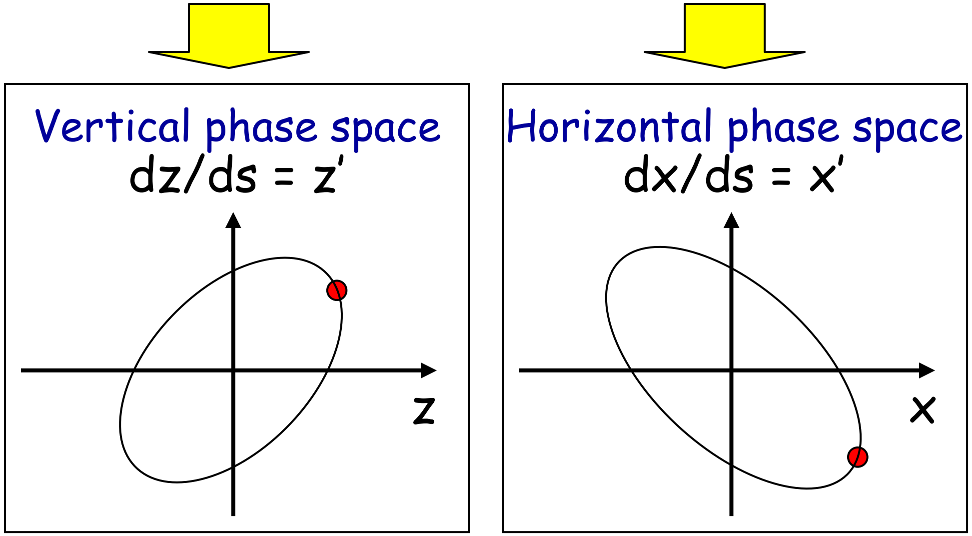


1. On time, $\Delta p > 0$, $V = V_0$, faster
2. Early, $\Delta p > 0$, $V < V_0$, faster
3. Early, $\Delta p = 0$, $V < V_0$, equal
4. Early, $\Delta p < 0$, $V < V_0$, slower
5. On time, $\Delta p < 0$, $V = V_0$, slower

5. On time, $\Delta p < 0$, $V = V_0$, slower
6. Late, $\Delta p < 0$, $V > V_0$, slower
7. Late, $\Delta p = 0$, $V > V_0$, equal
8. Late, $\Delta p > 0$, $V > V_0$, faster
1. On time, $\Delta p > 0$, $V = V_0$, faster

Transverse phase space

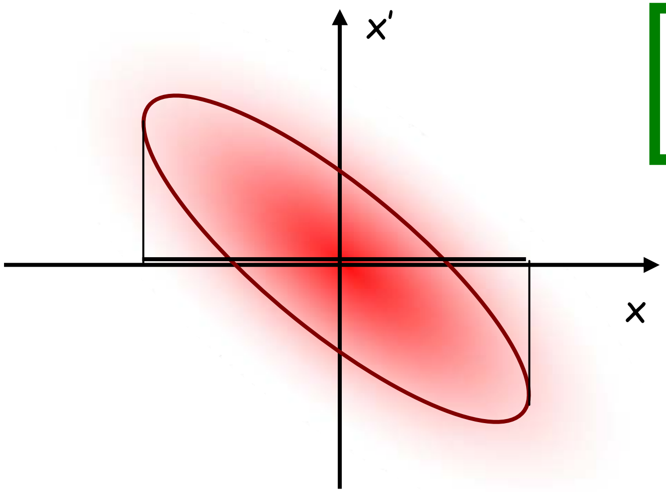




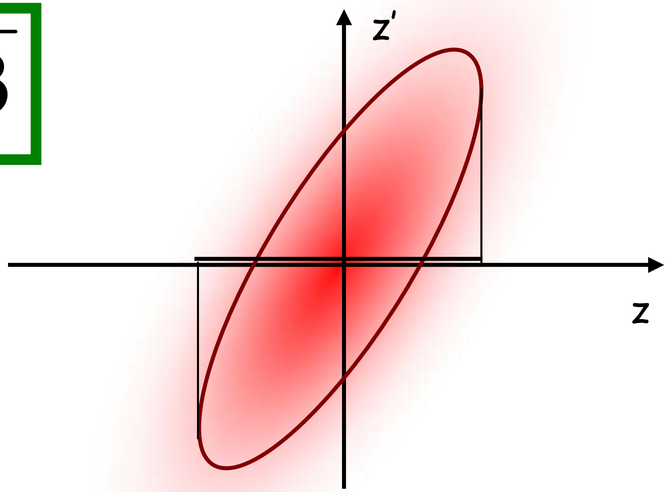
- **Description of single particles**
  - Start from projection onto horizontal and vertical planes.
  - Phase space coordinates (x,x'), (z,z').
- **Observing over many turns**
  - Describes an ellipse in phase space.
  - Imposed by boundary conditions of the circular machine.
  - Observing at a different position s along the machine, the ellipse changes shape but it contains the same phase space area!

Transverse emittance

- **The beam consists of many particles…**
  - All particles describe similar ellipses in phase space.
- **The elliptical phase space area containing (a certain amount of) the beam is the transverse emittance, ε.**
  - The area is constant but the ellipse changes shape around the machine.
- **Beam size is the projection of the ellipse on horizontal/vertical axis.**



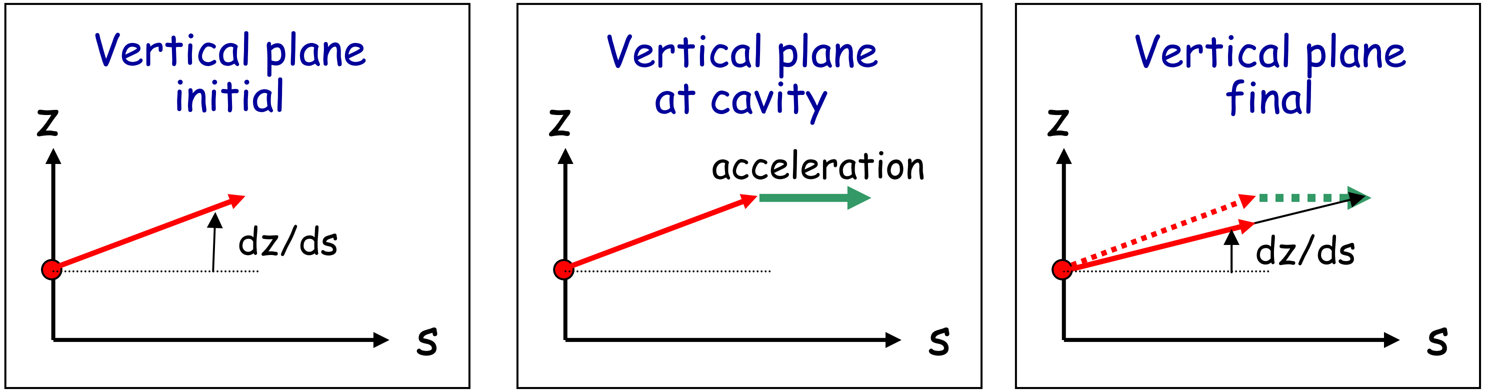
$$\boxed{\sigma = \sqrt{\varepsilon\beta}}$$



- **Beta function: machine property (determined by magnet optics).**
- **Emittance: beam property.**

Adiabatic damping of emittance

- **Acceleration adds longitudinal momentum** **to the particles while leaving the transverse momentum unchanged (first order).**
- **As a result the "angular spread" reduces – and the emittance decreases.**



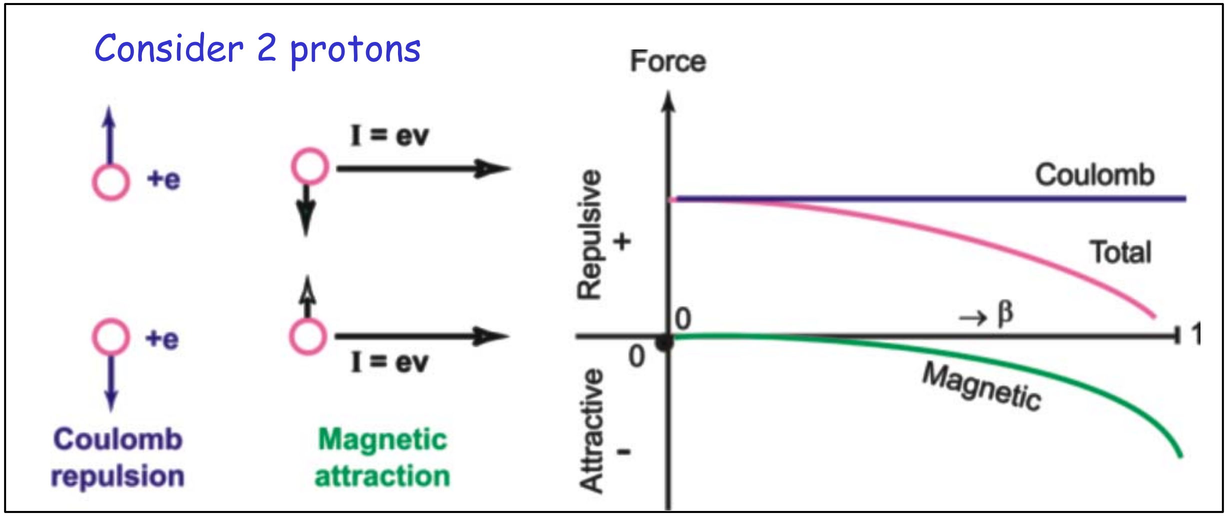
- **This is adiabatic damping, inversely proportional to momentum increase.**

$$\boxed{p(\gamma) = m_0 c \cdot (\beta\gamma) \quad \Rightarrow \quad \varepsilon_{\text{geometrical}}(\gamma) = \frac{\varepsilon_{\text{normalized}}}{\beta\gamma}}$$

- **LHC beam emittance is defined at injection in the PS Booster (50 MeV). Emittance shrinks by a factor 1500 until injection into LHC (450 GeV).**

Space charge

- **Space charge effect:**
  - Electrical force, Coulomb interaction, repulsive.
  - Magnetic force of parallel currents, attractive.



- **Overall force is repulsive but decreases with energy.**
- **Cancellation of forces for v = c**

$$\boxed{F_{\text{rad}} \propto \frac{1}{\beta\gamma^2}}$$

- **Space charge effects are problematic at low energy.**
- **Space charge force has a defocusing effect on the beam.**

Space charge tune spread

- The defocusing effect of space charge reduces the tune and leads to a tune spread ΔQ in the beam:

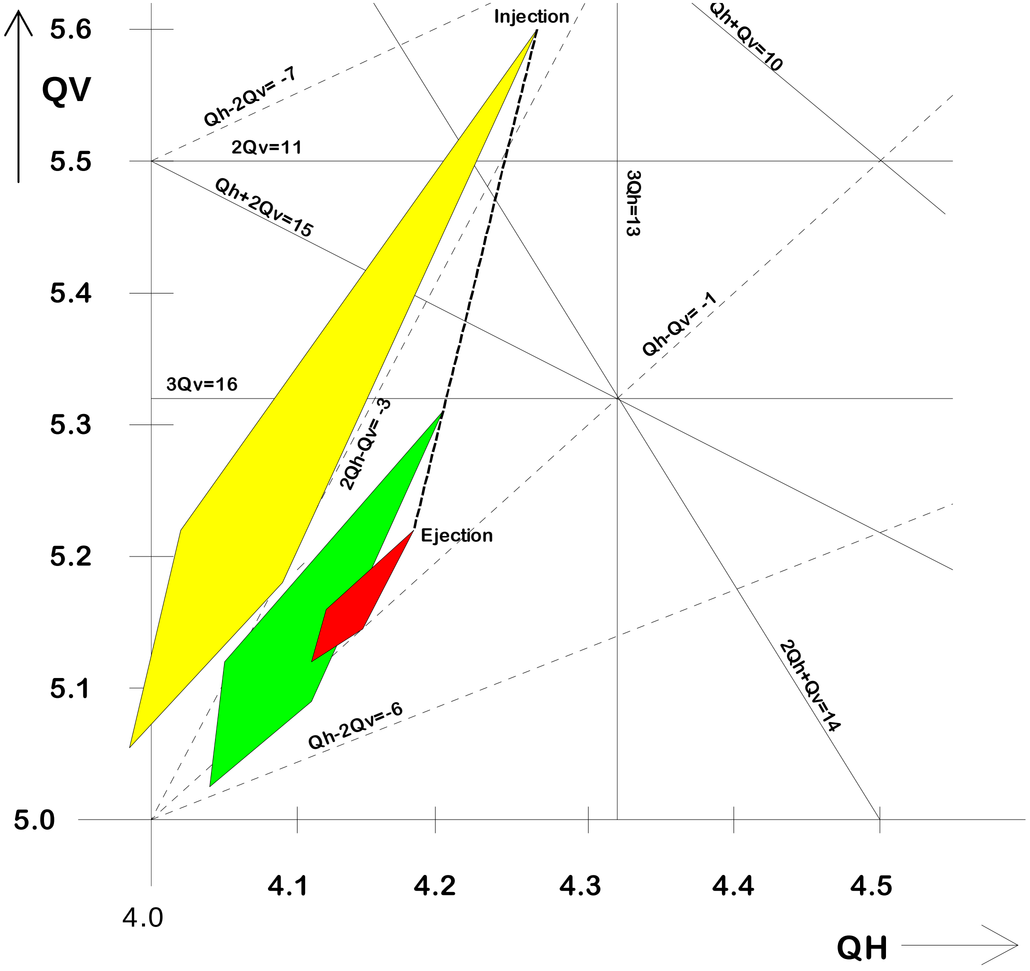
$$\boxed{\Delta Q \propto -\frac{N}{\varepsilon_n}\cdot\frac{1}{\beta\gamma^2}}$$

- Once ΔQ becomes too big there will be always particles fulfilling a resonance condition and these will be lost.

- THE major problem for high intensity at low energy.

CERN PS Booster Synchrotron

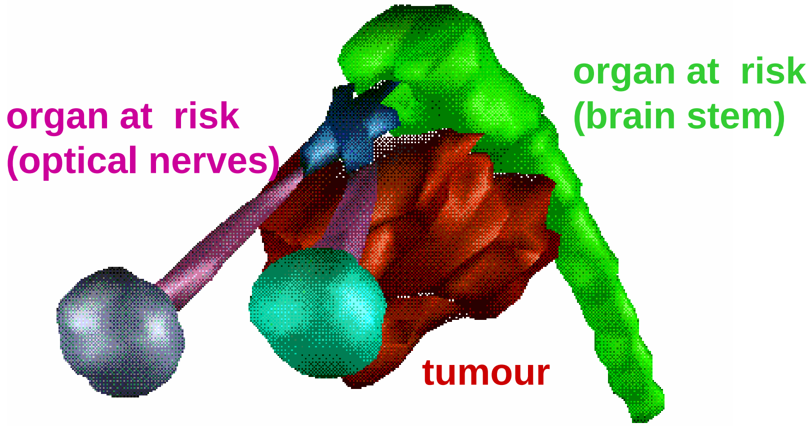
Injection 50 MeV, Ejection 1400 MeV

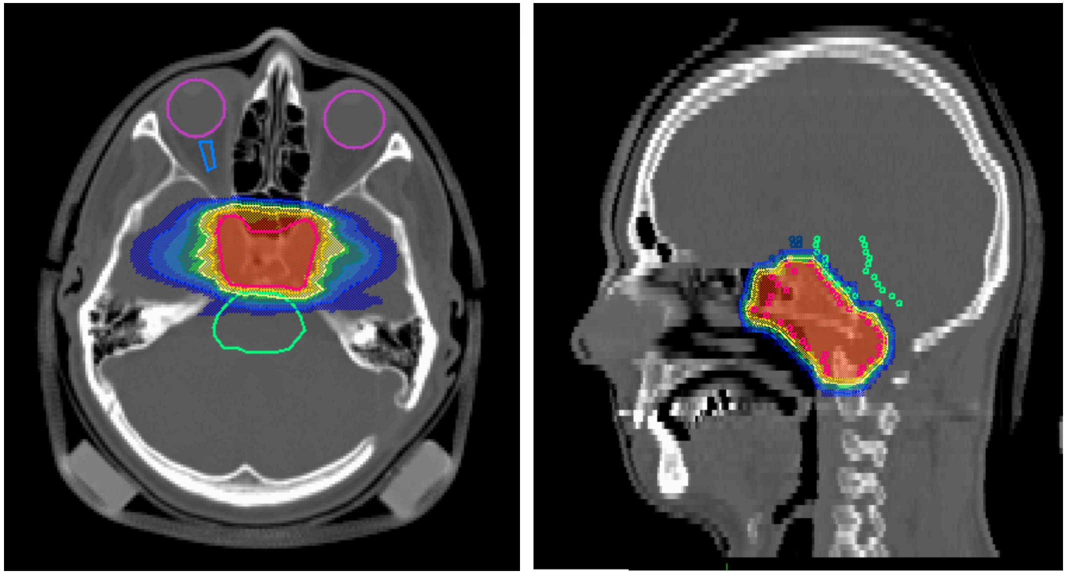


Radio therapy and hadron therapy

- **Goal**
  - Deliver a high radiation dose to the target area to kill all tumour cells.
  - Spare out healthy tissue and organs at risk.
  - Tumour conformal dose distribution.

- **Radiation type**
  - Hadron therapy: protons, light ions
  - Conventional therapy: electrons, photons
  - More exotic: neutrons, pions

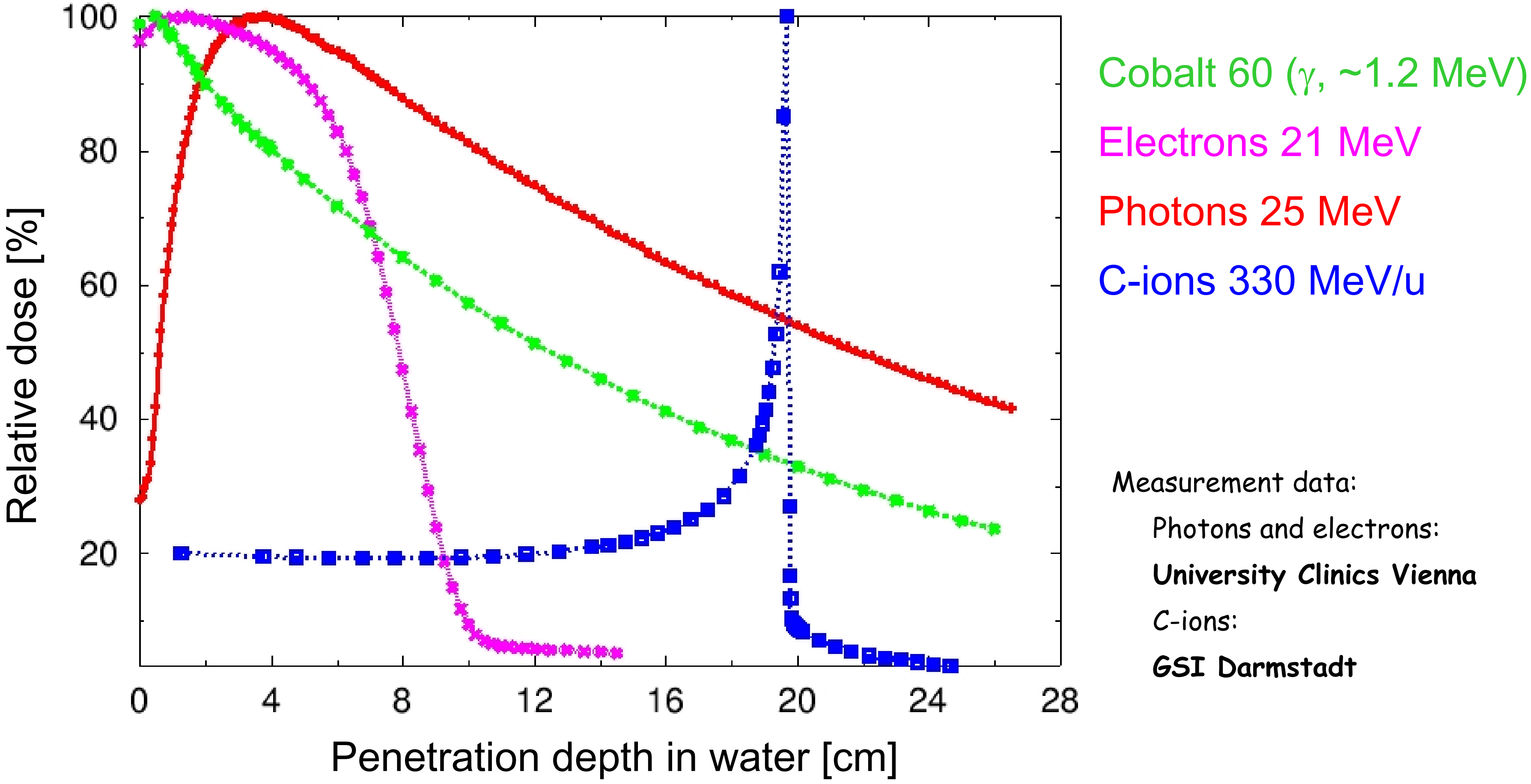




Courtesy GSI

"Bragg-Peak" behaviour of hadrons

Water phantom measurements (~tissue equivalent)



Measurement data:

Photons and electrons:

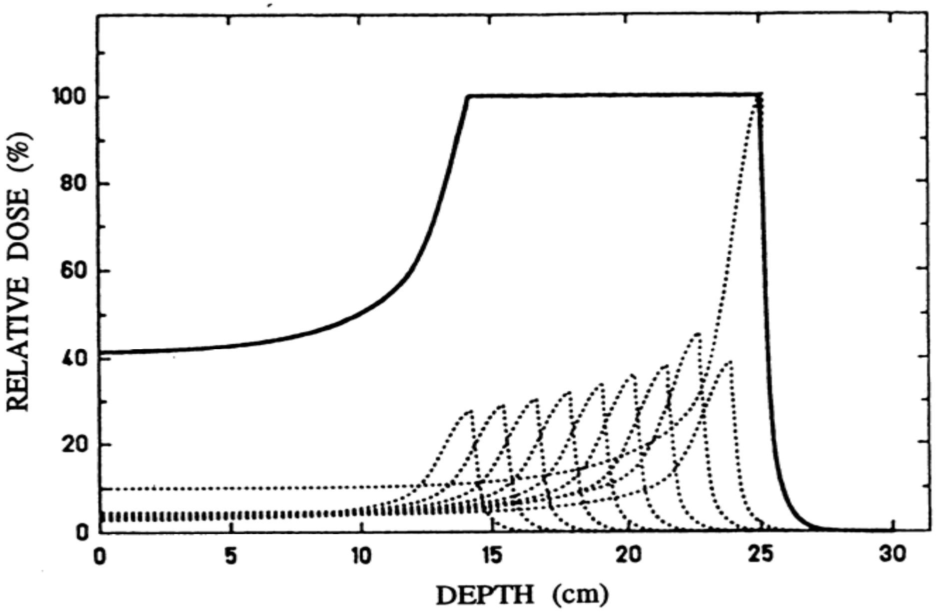
University Clinics Vienna

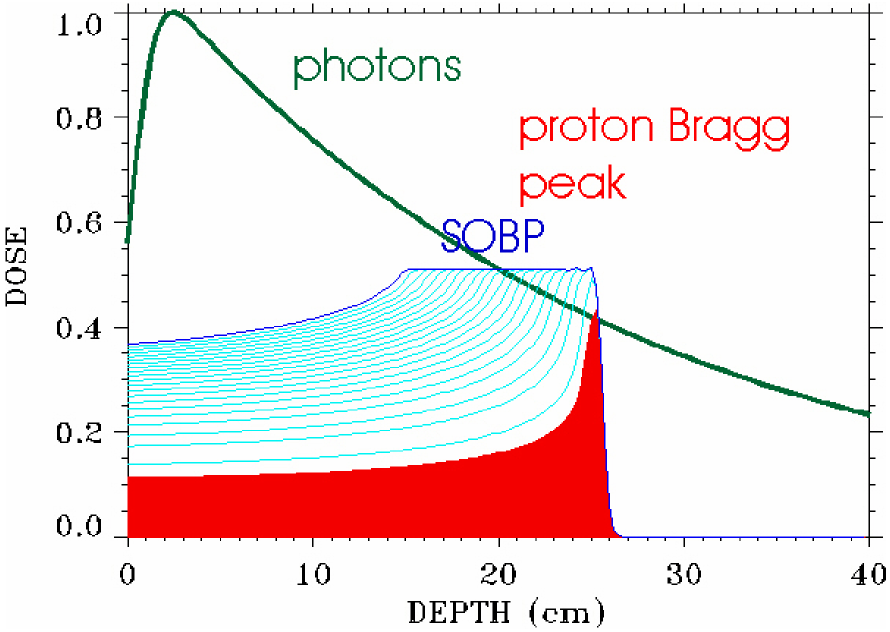
C-ions:

GSI Darmstadt

“Spread out Bragg-Peak” (SOBP)

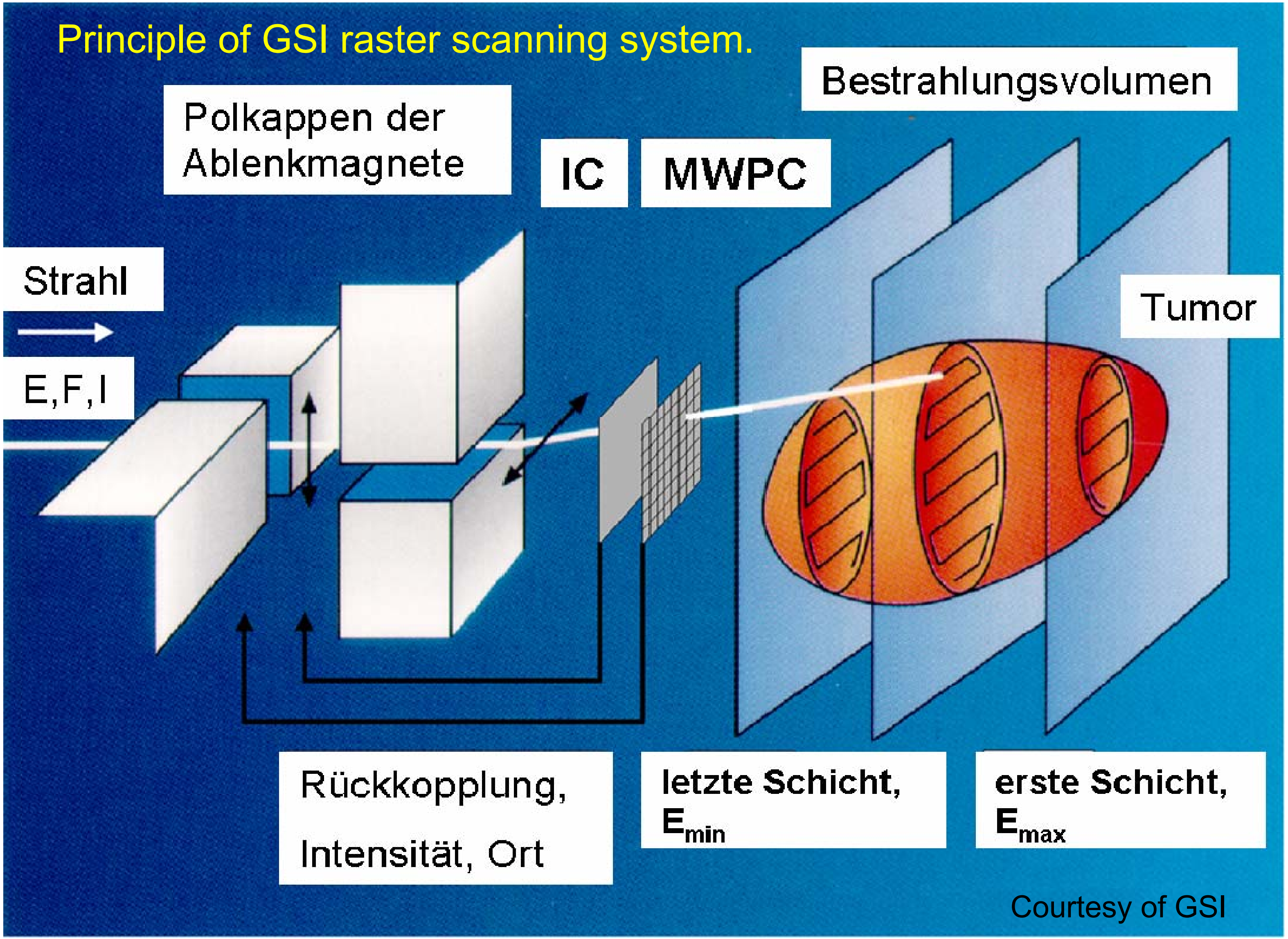
- **Bragg peak has to be widened to cover tumour thickness**
  - Overlapping of beams with different energies.
  - Active energy variation from cycle to cycle with the synchrotron.





- **Beam must also cover transverse tumour cross section**
  - Transverse scanning by fast magnetic scanning.

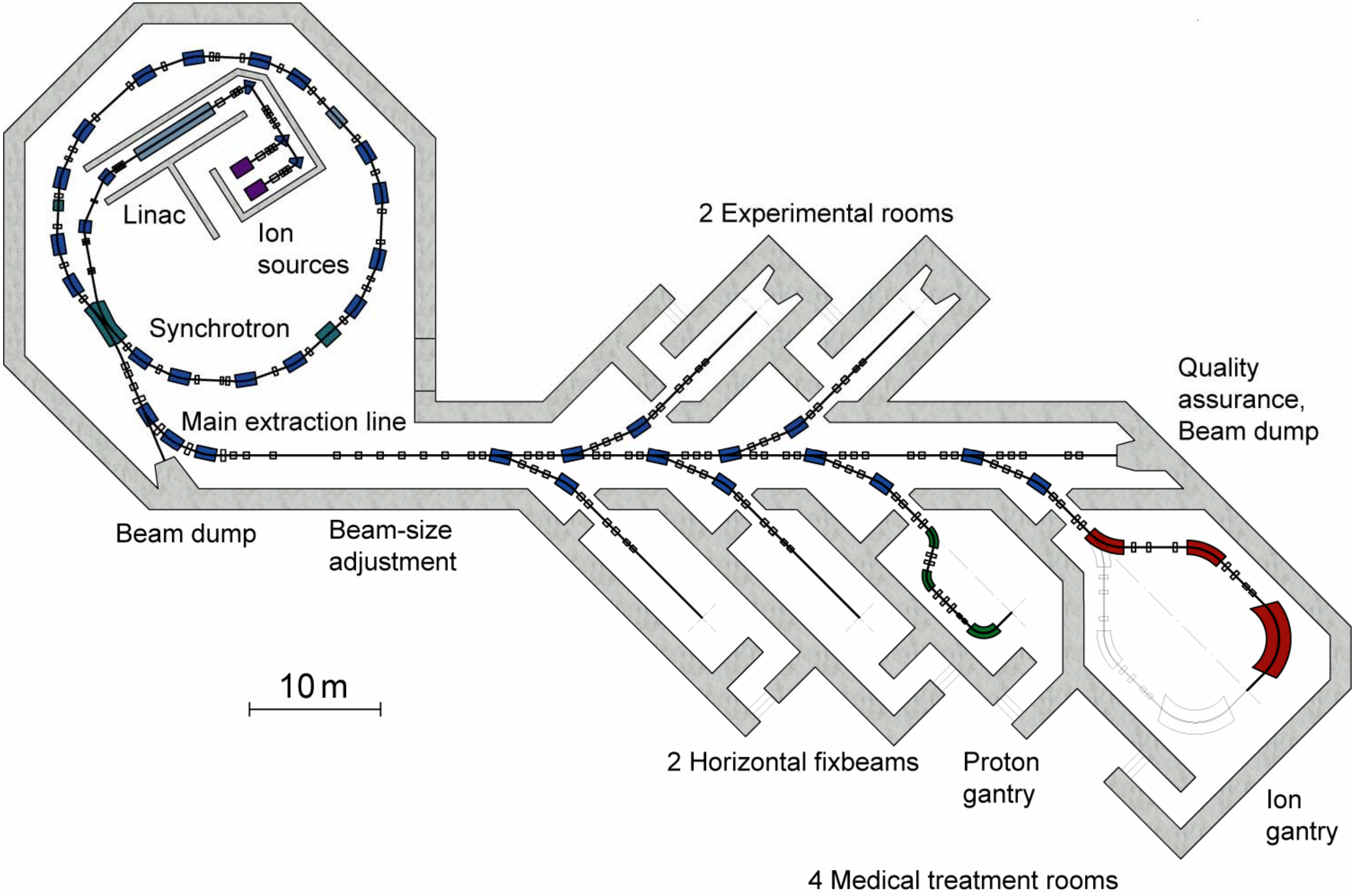
Active Beam Delivery



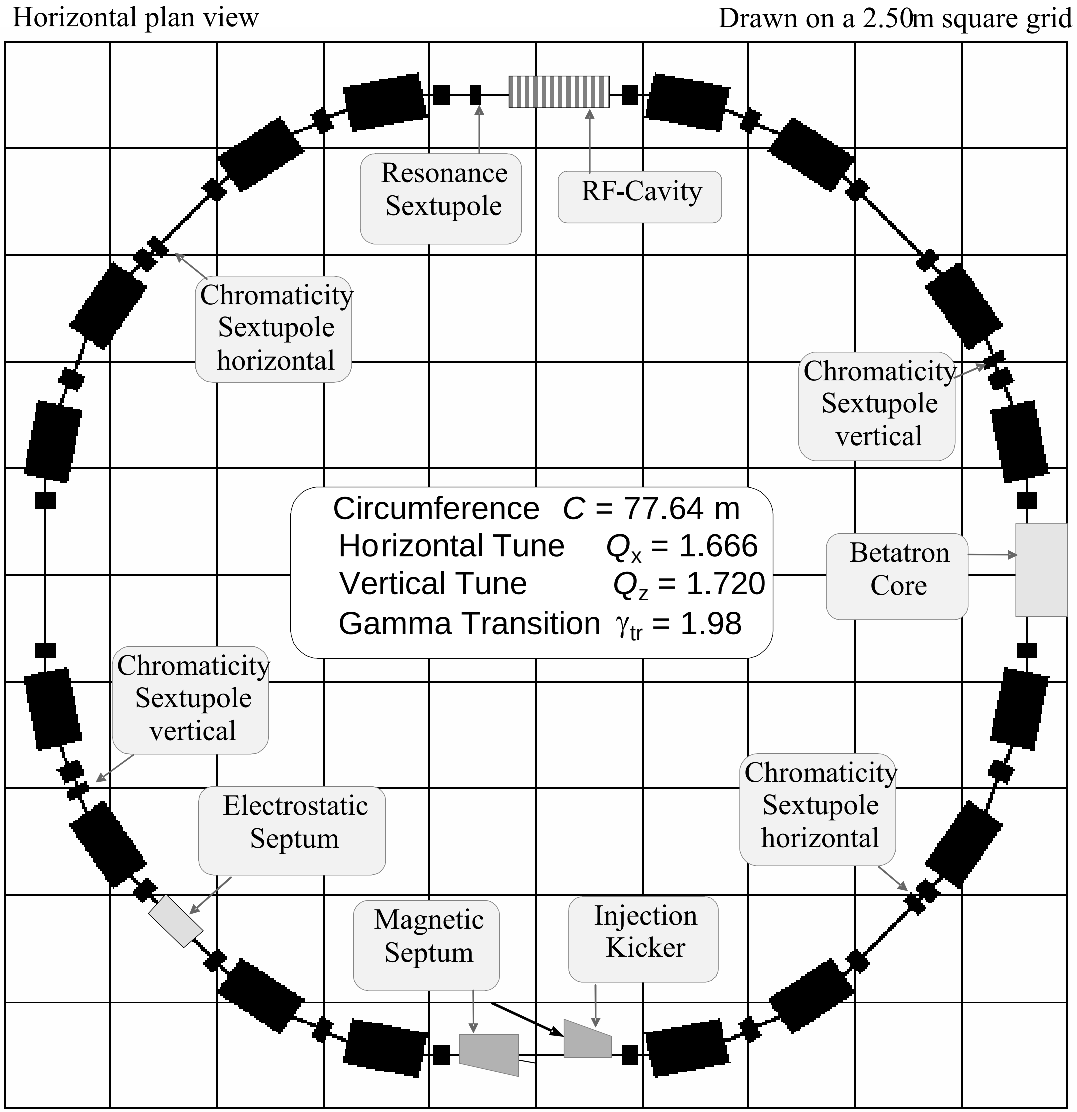
Principle of GSI raster scanning system.

Courtesy of GSI

MedAustron Layout



Synchrotron lattice



Circumference C = 77.64 m
Horizontal Tune Q_x = 1.666
Vertical Tune Q_z = 1.720
Gamma Transition γ_{tr} = 1.98

- **CERN- PIMMS design, further optimized by TERA**

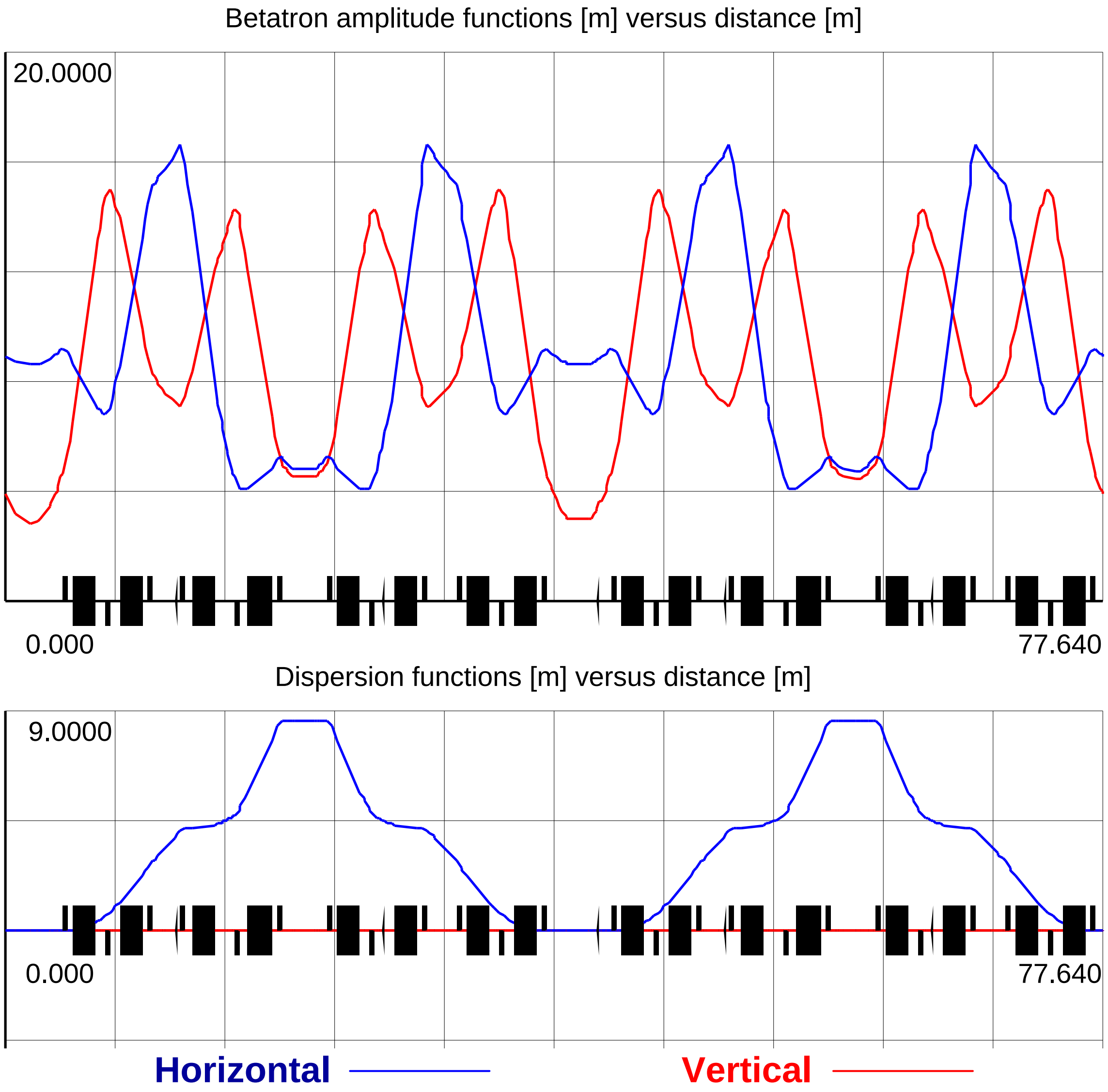
Injection energy: 7 MeV/n

Extraction energy range:
- Protons: 60 to 250 MeV
- C-ions: 120 to 400 MeV/n

Beam intensity:
- Protons: $\leq 1 \times 10^{10}$
- C-ions: $\leq 4 \times 10^{8}$

- Multi-turn injection (≤10turns)
- Slow resonant extraction.
- Spill time ~1s to 10 s
- “Orthogonal” control of resonance and chromaticity

MedAustron lattice functions


Betatron amplitude functions [m] versus distance [m]

Dispersion functions [m] versus distance [m]

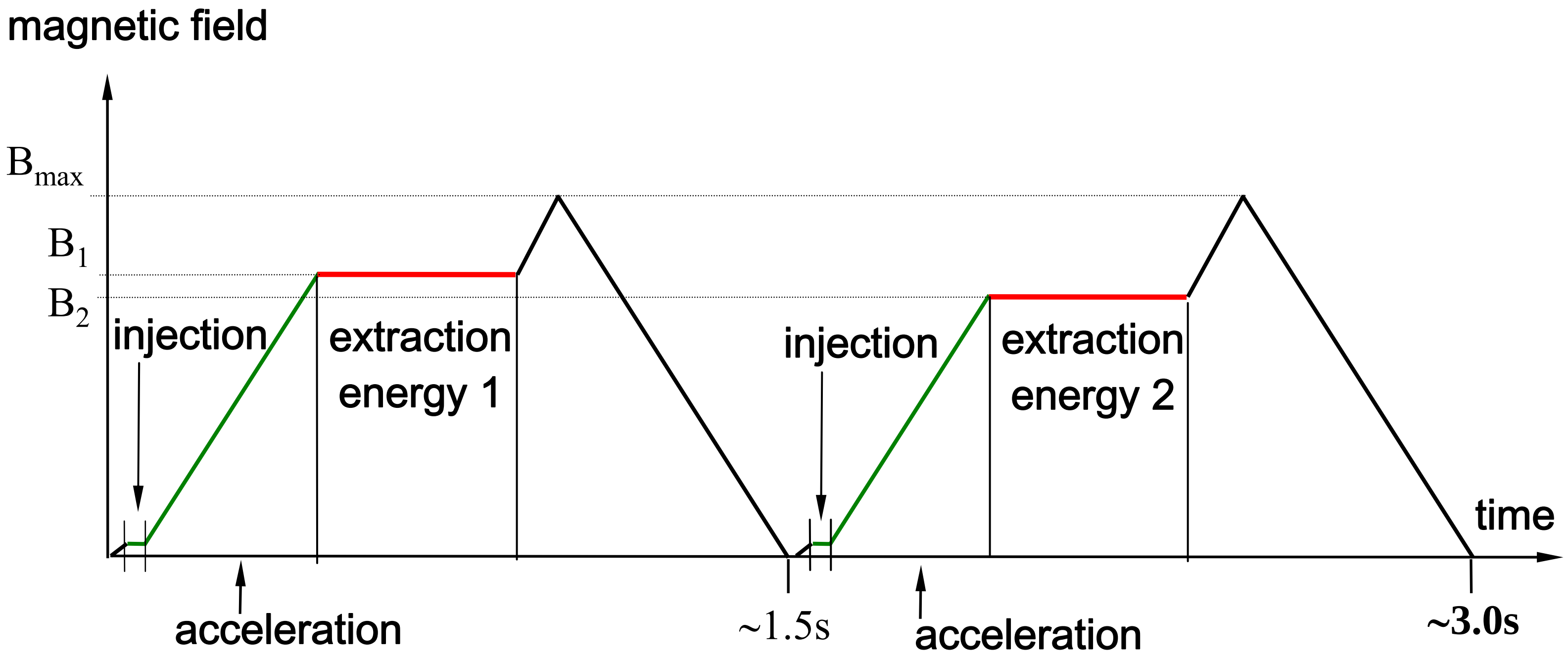
Lattice structure:

- Periodicity 2 with mirror symmetry in each period.
- Split FODO structure FODOF to give "constant" betas in drift spaces.
- Three quadrupole families (i.e. quadrupoles combined in 3 groups).
- No regular cell structure, typical for small machines customised design.

Synchrotron operation

- Change extraction energy from cycle to cycle to create actively a SOPB according to tumour thickness.

- Change intensity from cycle to cycle according to requirements of treatment plan.



- Flexibility of synchrotron fully exploited.