The Physics of Accelerators

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Contents

- Basic concepts in the study of Particle Accelerators
- Methods of acceleration
  - Linacs and rings
- Controlling the beam
  - Confinement, acceleration, focusing
- Electrons and protons
  - Synchrotron radiation
  - Luminosity

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Pre-requisites

- Basic knowledge for the study of particle beams:
  - Applications of relativistic particle dynamics
  - Classical theory of electromagnetism (Maxwell's equations)
- More advanced studies require
  - Hamiltonian mechanics
  - Optical concepts
  - Quantum scattering theory, radiation by charged particles
  - Computing ability

Applications of Accelerators

- Based on
  - directing beams to hit specific targets or colliding beams onto each other
  - production of thin beams of synchrotron light
- Particle physics
  - structure of the atom, standard model, quarks, neutrinos, CP violation
- Bombardment of targets used to obtain new materials with different chemical, physical and mechanical properties
- Synchrotron radiation covers spectroscopy, X-ray diffraction, x-ray microscopy, crystallography of proteins. Techniques used to manufacture products for aeronautics, medicine, pharmacology, steel production, chemical, car, oil and space industries.
- In medicine, beams are used for Positron Emission Tomography (PET), therapy of tumours, and for surgery.
- Nuclear waste transmutation – convert long lived nucleides into short-lived waste
- Generation of energy (Rubbia's energy amplifier, heavy ion driver for fusion)

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Basic Concepts I

- Speed of light $c = 2.99792458 \times 10^8 \ \mathrm{m\,sec^{-1}}$
- Relativistic energy $E = mc^2 = m_0 \gamma c^2$
- Relativistic momentum $p = mv = m_0 \gamma \beta c$

$$\beta = \frac{v}{c} \qquad \gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} = \frac{1}{\sqrt{1-\beta^2}}$$

- E-p relationship $\frac{E^2}{c^2} = p^2 + m_0^2 c^2$

$$\text{ultra-relativistic particles} \quad \beta \approx 1, \quad E \approx pc$$

- Kinetic energy $T = E - m_0 c^2 = m_0 c^2 (\gamma - 1)$
- Equation of motion under Lorentz force

$$\frac{d\vec{p}}{dt} = \vec{f} \quad \Rightarrow \quad m_0 \frac{d}{dt}(\gamma \vec{v}) = q\left(\vec{E} + \vec{v} \wedge \vec{B}\right)$$

Basic Concepts II

- Electron charge $e = 1.6021\times10^{-19}$ Coulombs
- Electron volts $1\,\text{eV} = 1.6021\times10^{-19}$ joule
- Energy in eV $E[\text{eV}] = \dfrac{mc^2}{e} = \dfrac{m_0\gamma c^2}{e}$
- Energy and rest mass $1\,\text{eV}/c^2 = 1.78\times10^{-36}$ kg
  - Electron $m_0 = 511.0\,\text{keV}/c^2 = 9.109\times10^{-31}$ kg
  - Proton $m_0 = 938.3\,\text{MeV}/c^2 = 1.673\times10^{-27}$ kg
  - Neutron $m_0 = 939.6\,\text{MeV}/c^2 = 1.675\times10^{-27}$ kg

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Motion in Electric and Magnetic Fields

- Governed by Lorentz force $$\frac{d\vec{p}}{dt} = q\left[\vec{E} + \vec{v} \times \vec{B}\right]$$

$$E^2 = \vec{p}^{\,2} c^2 + m_0^2 c^4$$

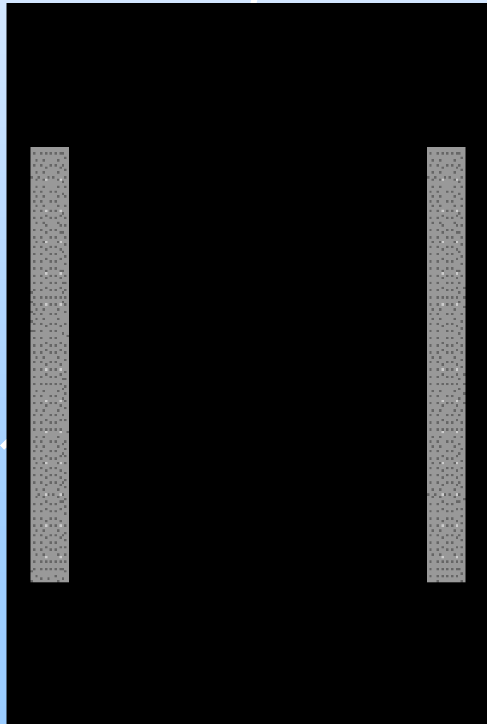
$$\Rightarrow \quad E\frac{dE}{dt} = c^2 \vec{p} \cdot \frac{d\vec{p}}{dt}$$

$$\Rightarrow \quad \frac{dE}{dt} = \frac{qc^2}{E}\vec{p} \cdot \left(\vec{E} + \vec{v} \wedge \vec{B}\right) = \frac{qc^2}{E}\vec{p} \cdot \vec{E}$$

A magnetic field does not alter a particle's energy. Only an electric field can do this.

- Acceleration along a uniform electric field (B=0)

$$\left.\begin{aligned} z &\approx vt \\ x &\approx \frac{eE}{2m\gamma_0} t^2 \end{aligned}\right\} \quad \text{parabolic path for } v << c$$



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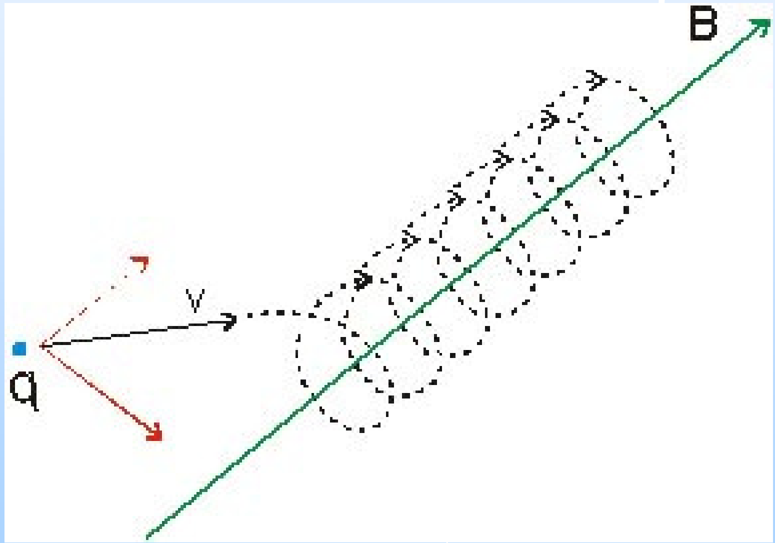
Behaviour under constant B-field, E=0

- Motion in a uniform, constant magnetic field
  - Constant energy with spiralling along a uniform magnetic field

$$\frac{m_0 \gamma v^2}{\rho} = qvB \quad \Rightarrow$$

$$(a) \qquad \rho = \frac{m_0 \gamma v}{qB}$$

$$(b) \qquad \omega = \frac{v}{\rho} = \frac{qB}{m_0 \gamma}$$



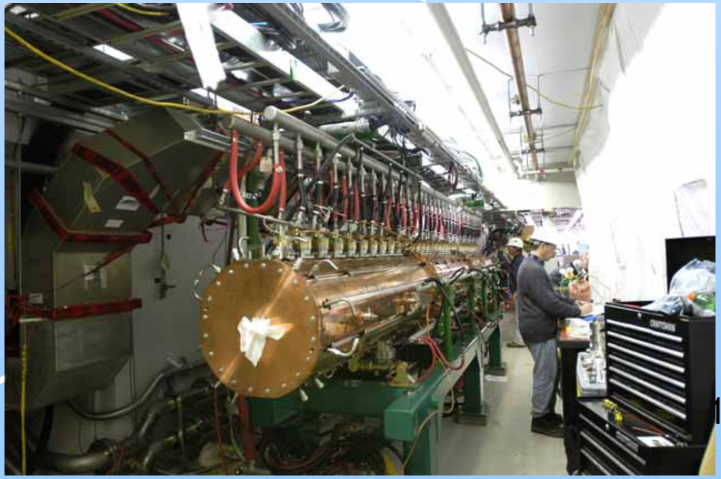
$$\rho = \left| \frac{p}{qB} \right|$$

$$\omega = \frac{qBc^2}{E} = \frac{v}{\rho}$$

Method of Acceleration: Linear

- Simplest example is a vacuum chamber with one or more DC accelerating structures with the *E*-field aligned in the direction of motion.
  - *Limited to a few MeV*
- To achieve energies higher than the highest voltage in the system, the *E*-fields are alternating at RF cavities.
  - *Avoids expensive magnets*
  - *No loss of energy from synchrotron radiation (q.v.)*
  - *But requires many structures, limited energy gain/metre*
  - *Large energy increase requires a long accelerator*

SLAC linear accelerator



SNS Linac, Oak Ridge

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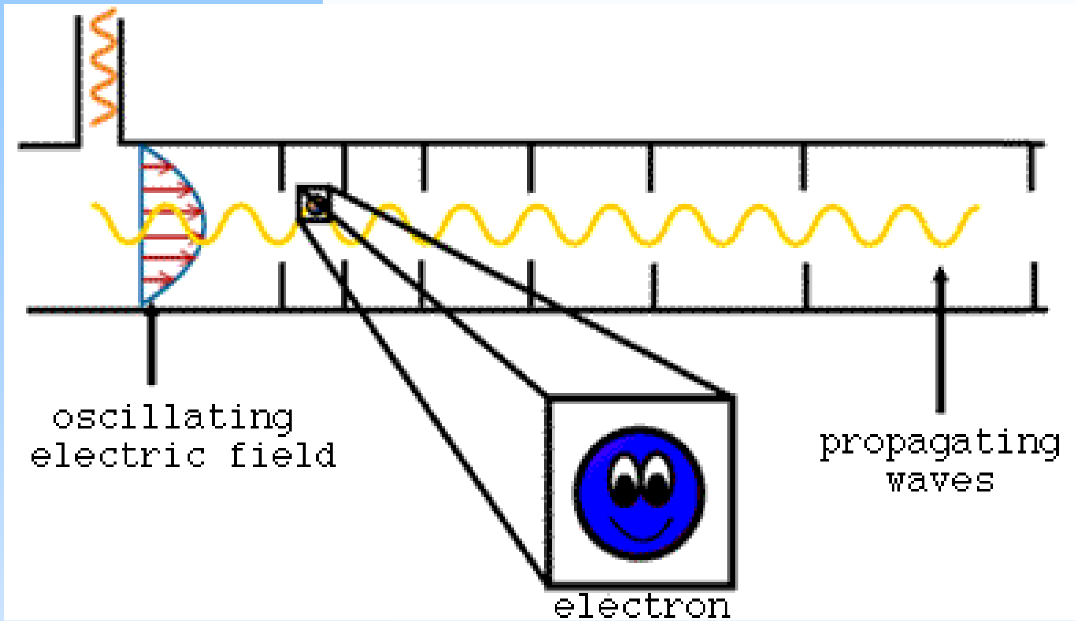
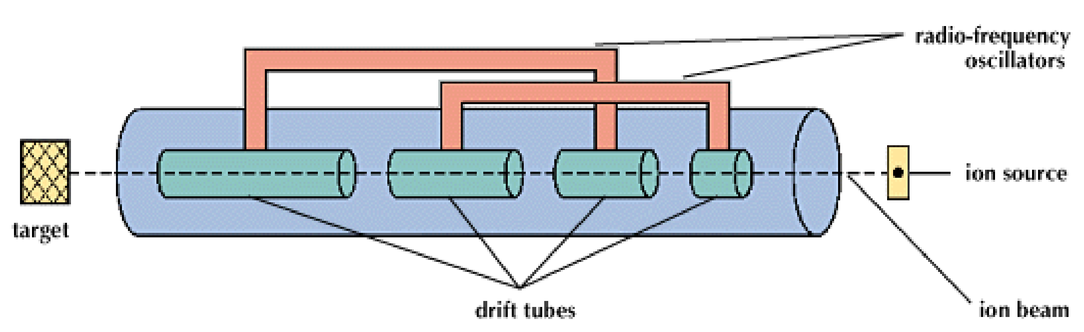


Structure 1:

- Travelling wave structure: particles keep in phase with the accelerating waveform.
- Phase velocity in the waveguide is greater than c and needs to be reduced to the particle velocity with a series of irises inside the tube whose polarity changes with time.
- In order to match the phase of the particles with the polarity of the irises, the distance between the irises increases farther down the structure where the particle is moving faster. But note that electrons at 3 MeV are already at 0.99c.

Structure 2:

- A series of drift tubes alternately connected to high frequency oscillator.
- Particles accelerated in gaps, drift inside tubes .
- For constant frequency generator, drift tubes increase in length as velocity increases.
- Beam has pulsed structure.



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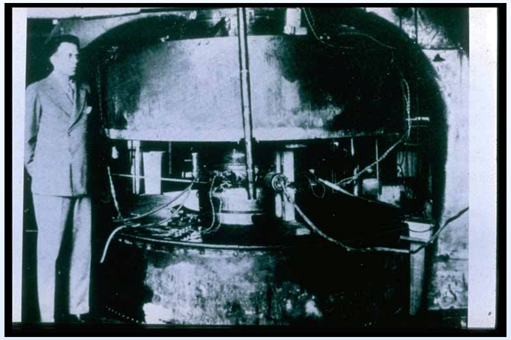
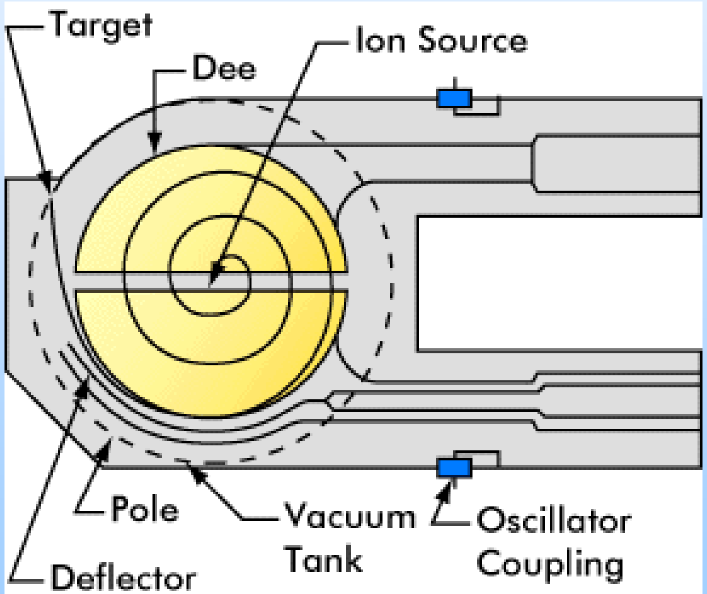
Methods of Acceleration: Circular

- Use magnetic fields to force particles to pass through accelerating fields at regular intervals
- **Cyclotrons**
  - Constant B field
  - Constant accelerating frequency f
  - Spiral trajectories
  - For synchronism $f = n\omega$, which is possible only at low energies, $\gamma \sim 1$.
  - Use for heavy particles (protons, deuterons, α-particles).



George Lawrence and cyclotron



$$\rho = \left|\frac{p}{qB}\right|$$

$$\omega = \frac{qBc^2}{E} = \frac{v}{\rho}$$

Methods of Acceleration: Circular

$$\rho = \left|\frac{p}{qB}\right|$$

$$f = n\omega$$

$$\omega = \frac{qBc^2}{E} = \frac{v}{\rho}$$

- Higher energies => relativistic effects => ω no longer constant.
- Particles get out of phase with accelerating fields; eventually no overall acceleration.
- **Isochronous cyclotron**
  - Vary B to compensate and keep f constant.
  - For stable orbits need both radial (because ρ varies) and azimuthal *B*-field variation
  - Leads to construction difficulties.
- **Synchro-cyclotron**
  - Modulate frequency f of accelerating structure instead.
  - In this case, oscillations are stable (McMillan & Veksler, 1945)

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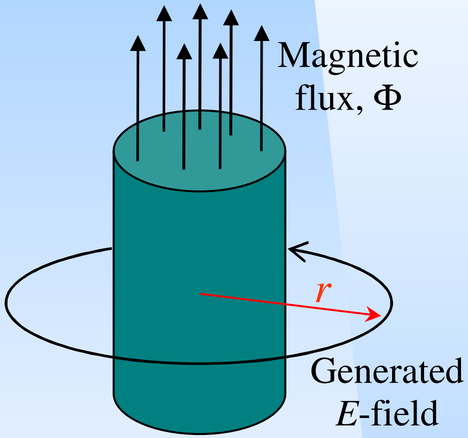
Betatron

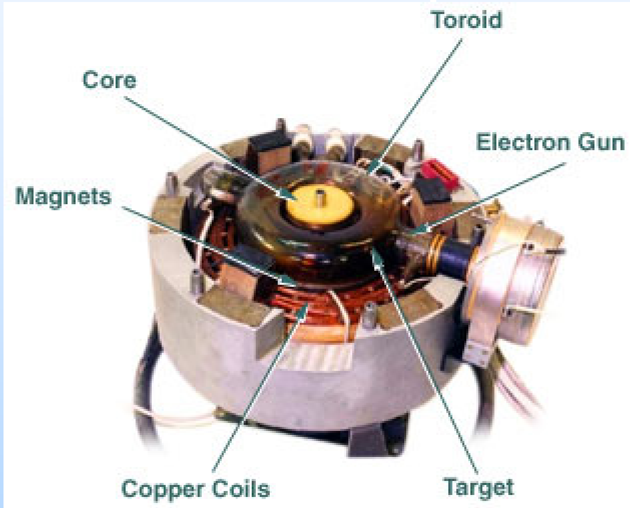
- Particles accelerated by the rotational electric field generated by a time varying magnetic field.



$$\nabla \wedge \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\Leftrightarrow \quad \oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \iint \vec{B} \cdot d\vec{S}$$

$$\Rightarrow \quad 2\pi r E = -\frac{d\Phi}{dt}$$



- In order that particles circulate at constant radius:

Oscillations about the design orbit are called betatron oscillations

$$B = -\frac{p}{qr}$$

$$\Rightarrow \quad \dot{B}(r,t) = -\frac{\dot{p}}{qr} = -\frac{E}{r} = \frac{1}{2\pi r^2} \frac{d\Phi}{dt}$$

$$\Rightarrow \quad B(r,t) = \frac{1}{2} \frac{1}{\pi r^2} \iint B \, dS$$

B-field on orbit is one half of the average *B* over the circle. This imposes a limit on the energy that can be achieved. Nevertheless the constant radius principle is attractive for high energy circular accelerators.

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Methods of Acceleration: Circular

Synchrotron

$$\rho = \left|\frac{p}{qB}\right|$$

$$f = n\omega$$

$$\omega = \frac{qBc^2}{E} = \frac{v}{\rho}$$

- Principle of frequency modulation but in addition variation in time of B-field to match increase in energy and keep revolution radius constant.
- Magnetic field produced by several bending magnets (*dipoles*), increases linearly with momentum. For $q=e$ and high energies:

$$B\rho = \frac{p}{e} \approx \frac{E}{ce} \text{ so } E[\text{GeV}] \approx 0.3\, B[\text{T}]\, \rho[\text{m}] \text{ per unit charge}$$

- Practical limitations for magnetic fields => high energies only at large radius

e.g. **LHC** **E = 8 TeV, B = 10 T, ρ = 2.7 km**

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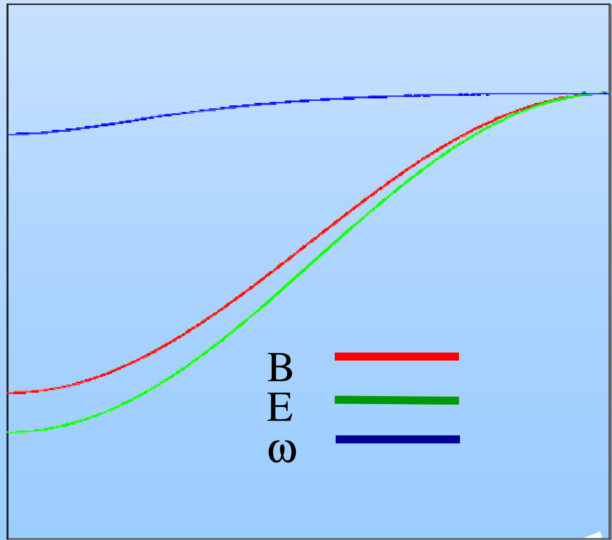
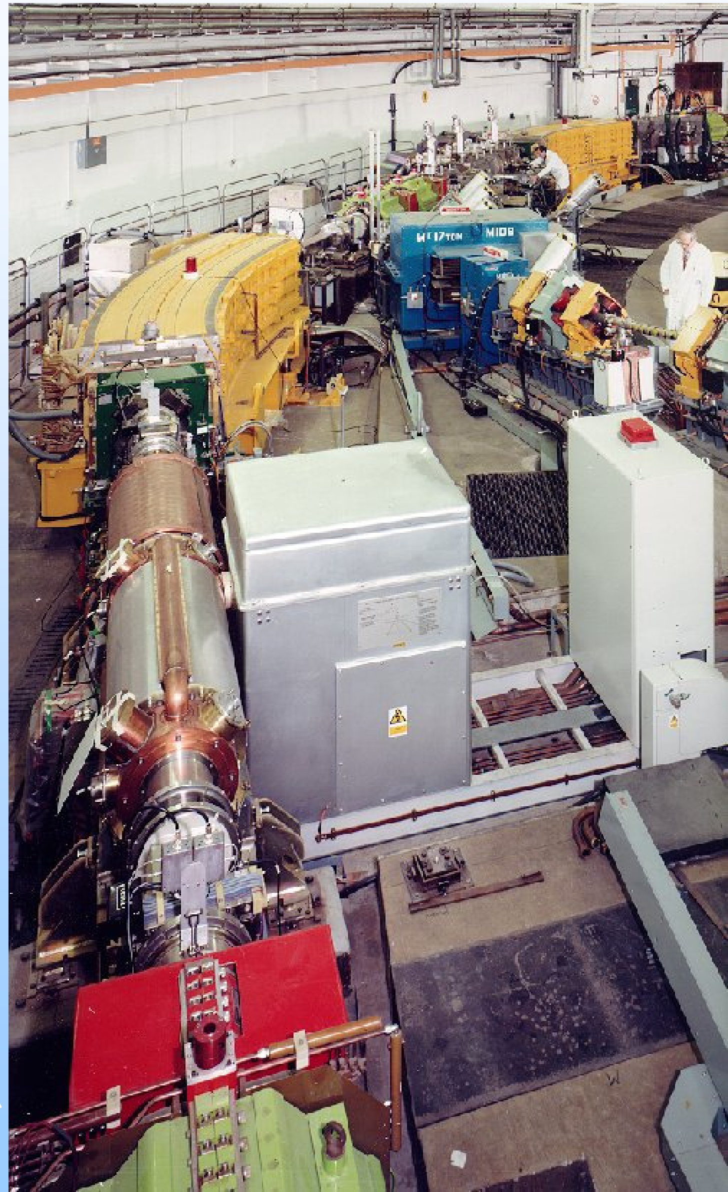
Types of Synchrotron

- Storage rings: accumulate particles and keep circulating for long periods; used for high intensity beams to inject into more powerful machines or synchrotron radiation factories.
- Colliders: two beams circulating in opposite directions, made to intersect; maximises energy in centre of mass frame.

Variation of parameters with time in the ISIS synchrotron:

$B = B_0 - B_1 \cos(2\pi f t)$





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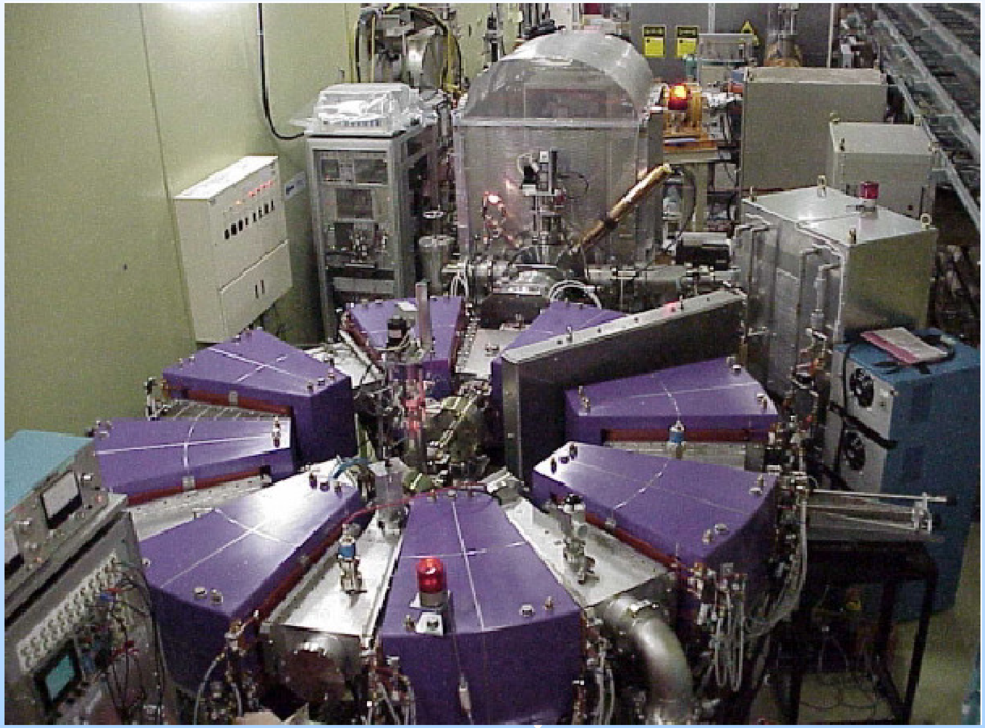
Fixed Field Alternating Gradient Circular Machines (FFAG)

- An old idea, dating from 1950's, given a new lease of life with the development of new magnetic alloy cavities.
- Field constant in time, varies with radius according to a strict mathematical formula.
- Wide aperture magnets and stable orbits.
- High gradient accelerating cavities combine with fixed field for rapid acceleration.
- Good for particles with short half-lives (e.g. muons).



Prototype FFAG, accelerating protons from 50 keV to 500 keV, was successfully built and tested at the KEK laboratory in Japan, 2000.

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Summary of Circular Machines

Machine	**RF frequency** f	**Magnetic Field** B	**Orbit Radius** ρ	**Comment**
Cyclotron	constant	constant	increases with energy	Particles out of synch with RF; low energy beam or heavy ions
Isochronous Cyclotron	constant	varies	increases with energy	Particles in synch, but difficult to create stable orbits
Synchro-cyclotron	varies	constant	increases with energy	Stable oscillations
Synchrotron	varies	varies	constant	Flexible machine, high energies possible
FFAG	varies	constant in time, varies with radius	increases with energy	Increasingly attraction option for 21st century designs

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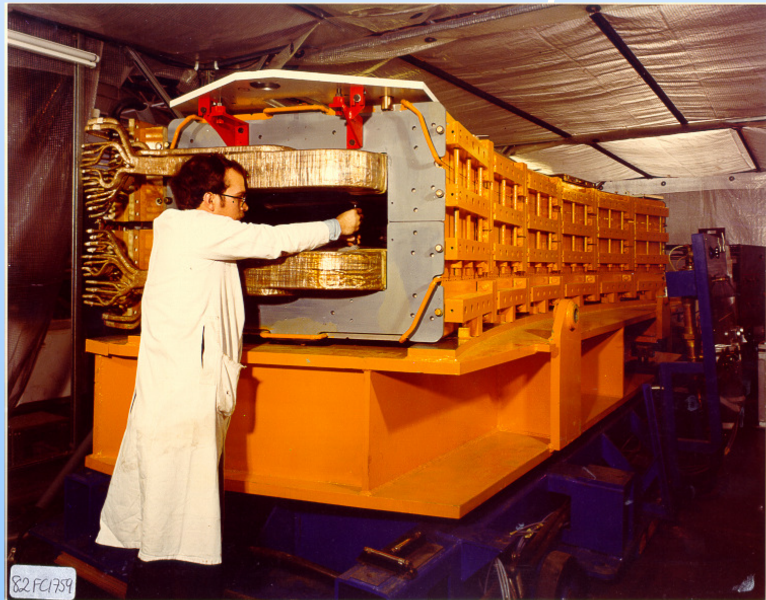
$$\rho = \left|\frac{p}{qB}\right|$$

$$\omega = \frac{qBc^2}{E} = \frac{v}{\rho}$$

Confinement, Acceleration and Focusing of Particles

- By increasing E (hence p) and B together in a synchrotron, it is possible to maintain a constant radius and accelerate a beam of particles.
- In a synchrotron, the confining magnetic field comes from a system of several magnetic dipoles forming a closed arc. Dipoles are mounted apart, separated by straight sections/vacuum chambers including equipment for focusing, acceleration, injection, extraction, collimation, experimental areas, vacuum pumps.

$$\rho = \left| \frac{p}{qB} \right|$$



ISIS dipole

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Ring Layout

- Mean radius of ring

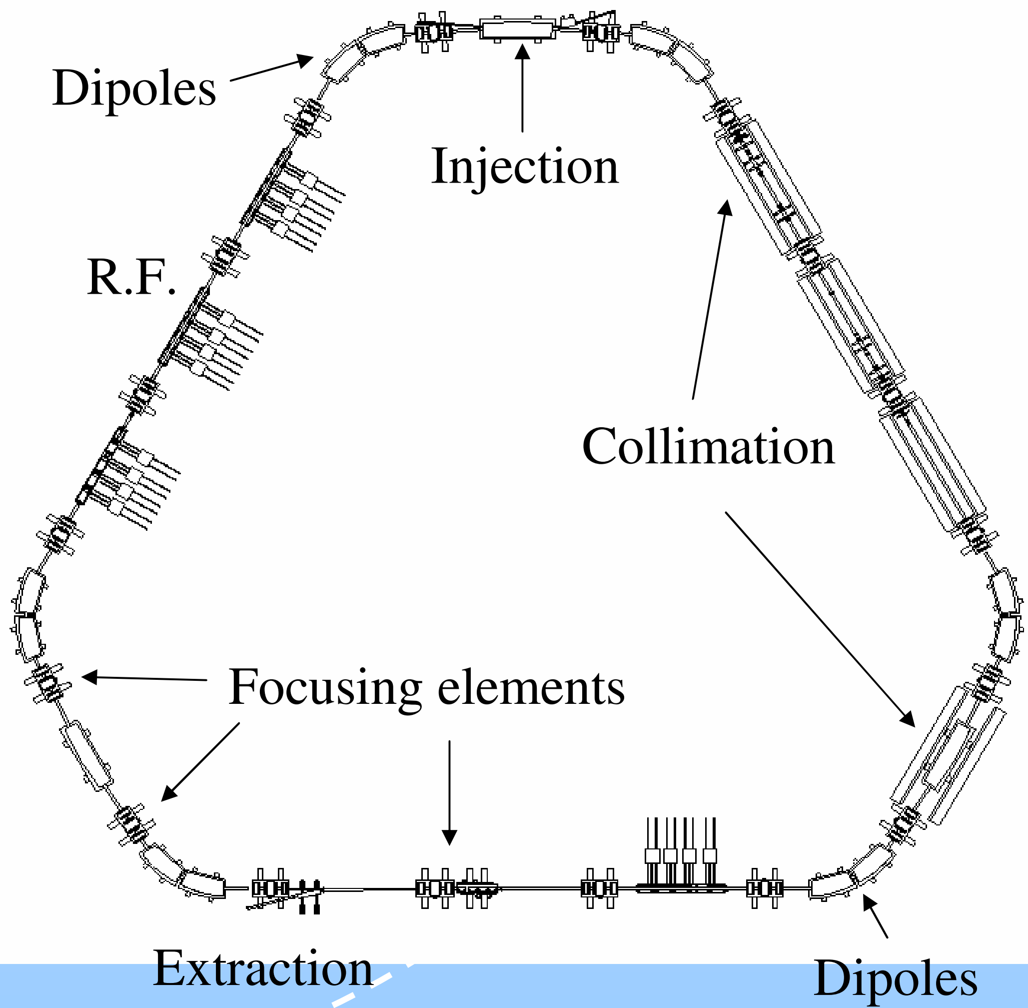
  $R > \rho$

- e.g. CERN SPS

  $R = 1100\ \text{m},\ \rho = 225\ \text{m}$

- Can also have large machines with a large number of dipoles each of small bending angle.
- e.g. CERN SPS

  744 dipole magnets, 6.26 m long, angle $\theta = 0.48^{\circ}$



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Ring Concepts

$$\tau = \frac{2\pi R}{v} \approx \frac{L}{c}$$

$$\frac{\omega}{2\pi} = \frac{1}{\tau} \approx \frac{c}{L}$$

$$\omega_{rf} = h\omega \approx \frac{hc}{L}$$

$$\rho = \left|\frac{p}{qB}\right|$$

- Important concepts in rings:
  - Revolution period τ
  - Revolution (angular) frequency ω
- If several bunches in a machine, introduce RF cavities in straight sections with fields oscillating at a multiple *h* of the revolution frequency.
  - h is the harmonic number.
- For synchrotrons, energy increase ΔE when particles pass RF cavities $\Rightarrow$ can increase energy only so far as can increase B-field in dipoles to keep constant ρ.

Magnetic Rigidity

$$B\rho = \frac{p}{q}$$

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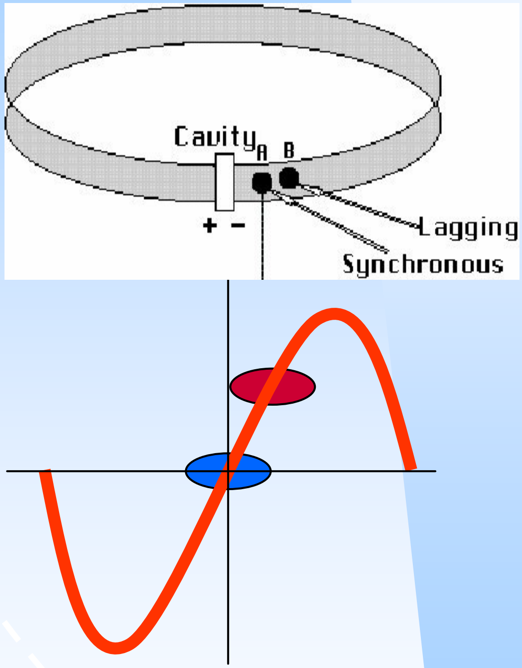
Effect on Particles of an RF Cavity



Bunching Effect

- Cavity set up so that particle at the centre of bunch, called the synchronous particle, acquires just the right amount of energy.
- Particles see voltage $V_0 \sin 2\pi\omega_{rf} t = V_0 \sin \varphi(t)$
- In case of no acceleration, synchronous particle has $\varphi_s = 0$
  - Particles arriving early see $\varphi < \varphi_s$
  - Particles arriving late see $\varphi > \varphi_s$
  - energy of those in advance is decreased relative to the synchronous particle and vice versa.
- To accelerate, make $0 < \varphi_s < \pi$ so that synchronous particle gains energy $\Delta E = qV_0 \sin \varphi_s$

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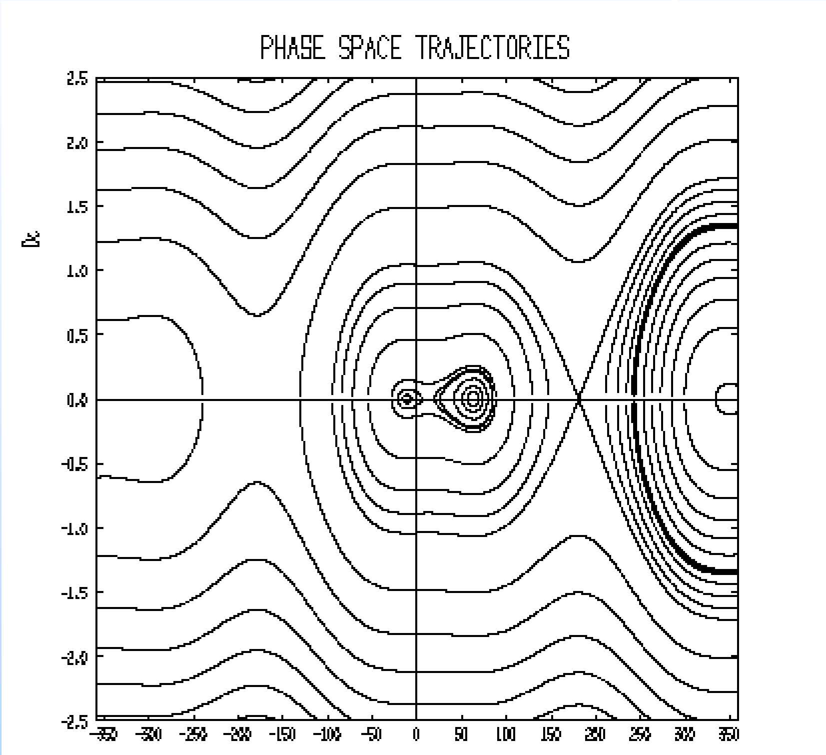
animation

Animation

Limit of Stability

- Phase space is a useful idea for understanding the behaviour of a particle beam.
- Longitudinally, not all particles are stable. There is a limit to the stable region (the separatrix or "bucket") and, at high intensity, it is important to design the machine so that all particles are confined within this region and are "trapped".



Example of longitudinal phase space trajectories under a dual harmonic voltage

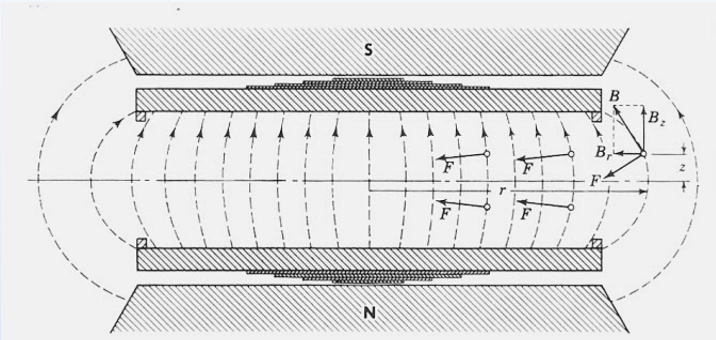
$$V(\varphi,t) = V_0(t)\sin\varphi + V_1(t)\sin(2\varphi+\theta)$$

$$\text{with} \quad \theta = -60^\circ, \quad \varphi_s = 28^\circ, \quad V_1 : V_0 = 0.6$$

Note that there are two stable oscillation centres inside the bucket

Transverse Control: Weak Focusing

- Particles injected horizontally into a uniform, vertical, magnetic field follow a circular orbit.
- Misalignment errors and difficulties in perfect injection cause particles to drift vertically and radially and to hit walls.
  - severe limitations to a machine
- Require some kind of stability mechanism.
- Vertical focusing from non-linearities in the field (fringing fields). Vertical stability requires negative field gradient. But radial focusing is reduced, so effectiveness of the overall focusing is limited.



$$qBv = \frac{m_0 \gamma v^2}{\rho} > \frac{m_0 \gamma v^2}{r} \quad if \quad r > \rho$$

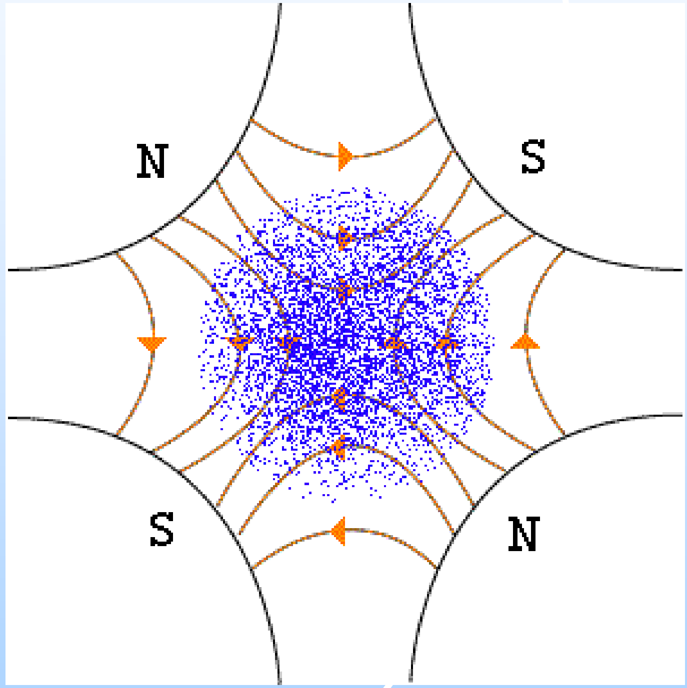
i.e. horizontal restoring force is towards the design orbit.

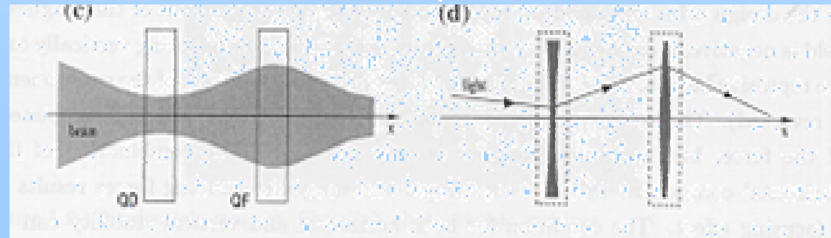
Stability condition: $$0 < n = -\frac{\rho}{B}\frac{dB}{d\rho} < 1$$

Weak focusing: if used, scale of magnetic components of a synchrotron would be large and costly

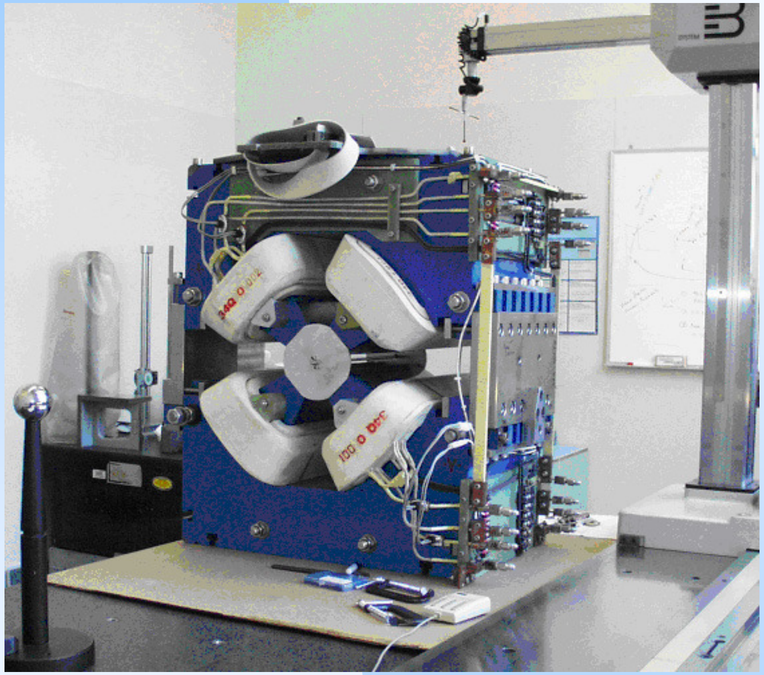
Strong Focusing: Alternating Gradient Principle

- A sequence of focusing-defocusing fields provides a stronger net focusing force.
- Quadrupoles focus horizontally, defocus vertically or vice versa. Forces are linearly proportional to displacement from axis.
- A succession of opposed elements enable particles to follow stable trajectories, making small (betatron) oscillations about the design orbit.
- Technological limits on magnets are high.

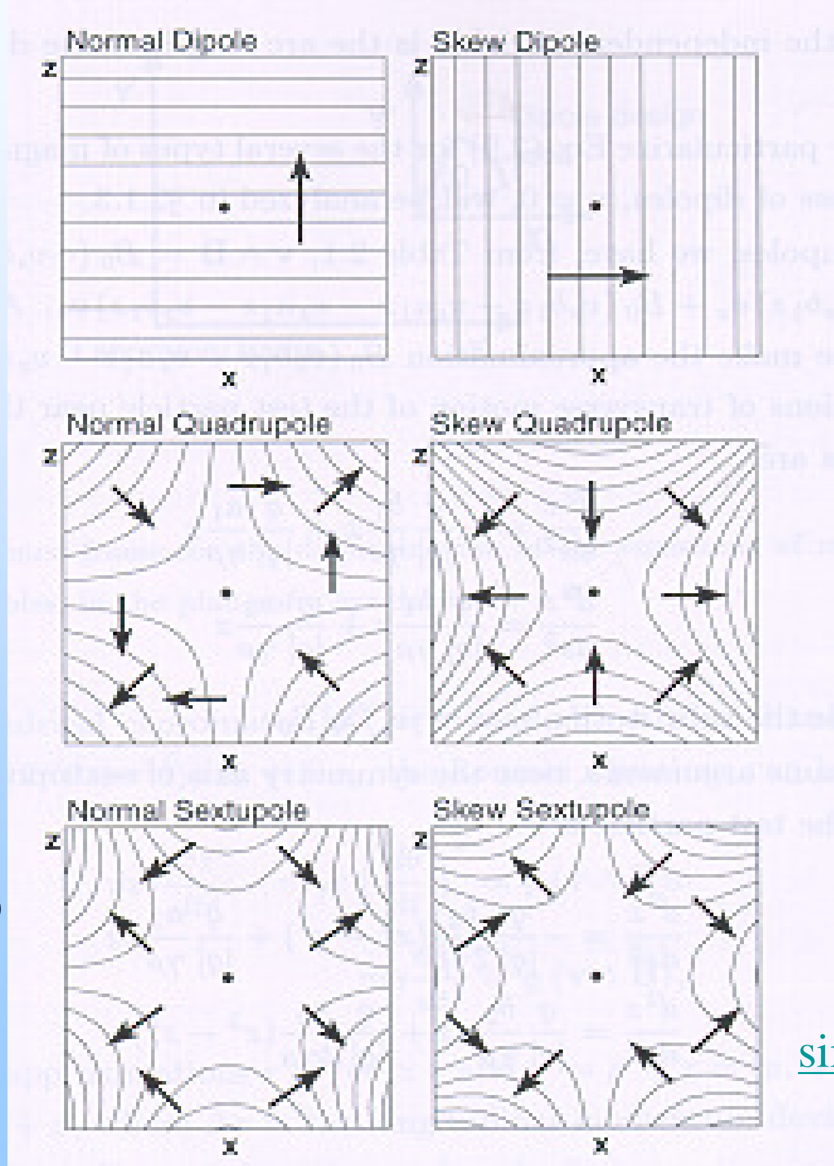




Focusing Elements



SLAC quadrupole

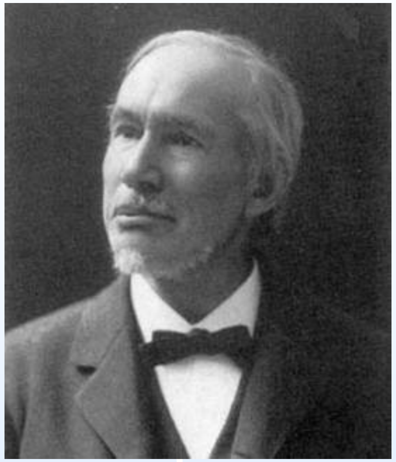


Sextupoles are used to correct longitudinal momentum errors.

Simulations

simulations

Hill's Equation


George Hill

- Equation of transverse motion
  - Drift: $x'' = 0, \quad y'' = 0$
  - Solenoid: $x'' + 2k\,y' + k'\,y = 0, \quad y'' - 2k\,x' - k'\,x = 0$
  - Quadrupole: $x'' + k\,x = 0, \quad y'' - k\,y = 0$
  - Dipole: $x'' + \dfrac{1}{\rho^2}x = 0, \quad y'' = 0$
  - Sextupole: $x'' + k\left(x^2 - y^2\right) = 0, \quad y'' - 2kxy = 0$
- Hill's Equation: $x'' + k_x(s)x = 0, \quad y'' + k_y(s)y = 0$

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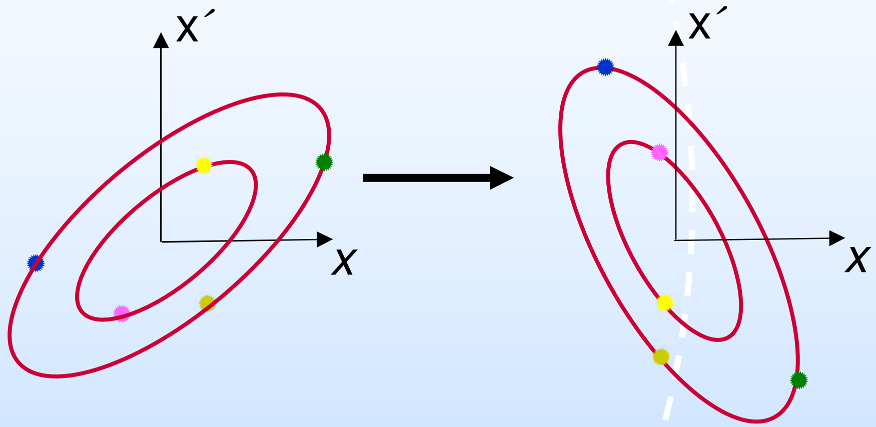
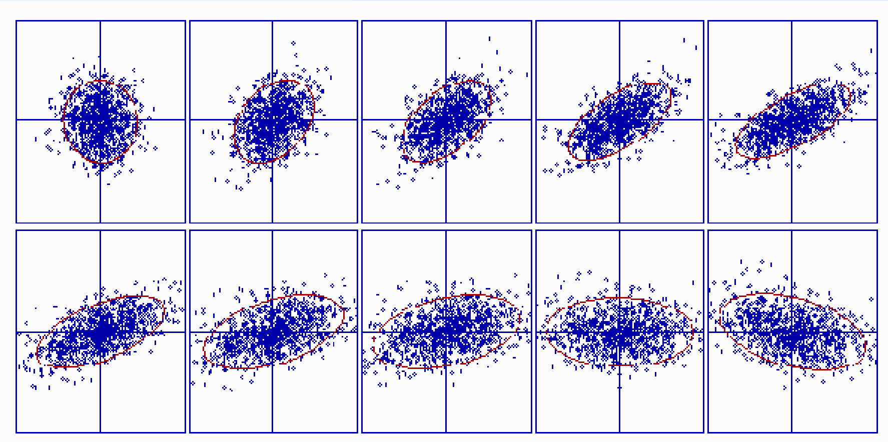
Transverse Phase Space

- Under linear forces, any particle moves on an ellipse in phase space (x,x´).
- Ellipse rotates in magnets and shears between magnets, but its area is preserved: *Emittance*





- General equation of ellipse is

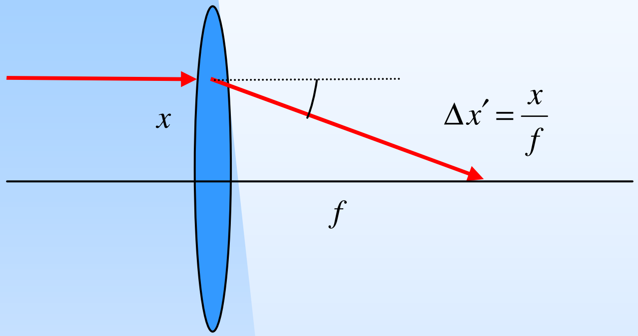
$$\beta x'^2 + 2\alpha x x' + \gamma x^2 = \varepsilon$$

- α, β, γ are functions of distance (Twiss parameters), and ε is a constant. Area = πε.
- For non-linear beams can use 95% emittance ellipse or RMS emittance

$$\varepsilon_{rms} = \sqrt{\langle x^2 \rangle \langle x'^2 \rangle - \langle x x' \rangle^2}$$

(statistical definition)

Thin Lens Analogy of AG Focusing



Effect of a thin lens can be represented by a matrix

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{\text{out}} = \begin{pmatrix} 1 & 0 \\ -\dfrac{1}{f} & 1 \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}_{\text{in}}$$

In a drift space of length ℓ, x' is unaltered but $x \rightarrow x + \ell x'$

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{\text{out}} = \begin{pmatrix} 1 & l \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}_{\text{in}}$$

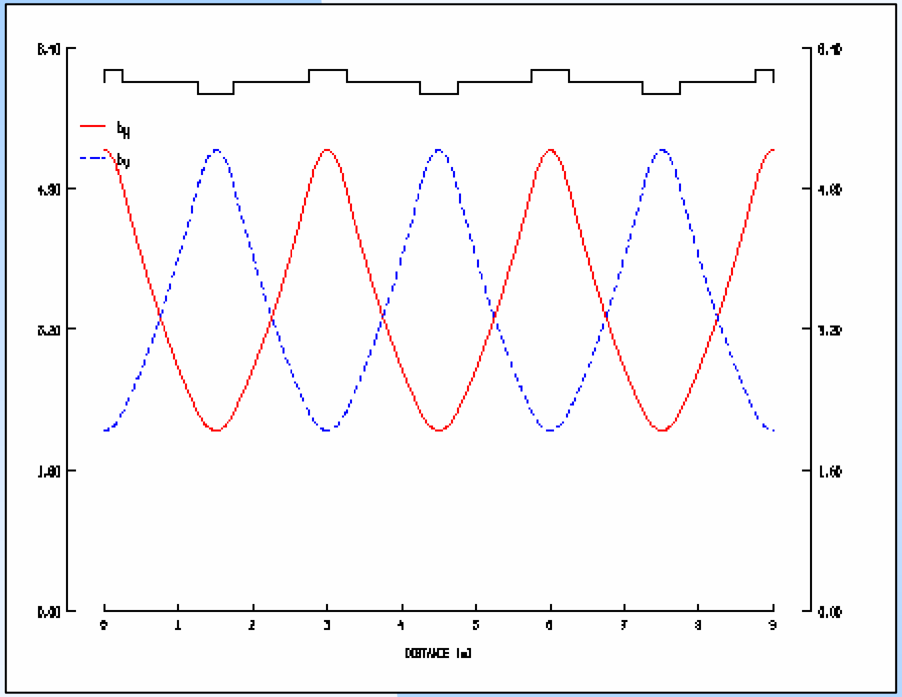
In an F-drift-D system, combined effect is

$$\begin{pmatrix} 1 & 0 \\ 1/f & 1 \end{pmatrix} \begin{pmatrix} 1 & l \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix} = \begin{pmatrix} 1 - l/f & l \\ -l/f^2 & 1 + l/f \end{pmatrix}$$

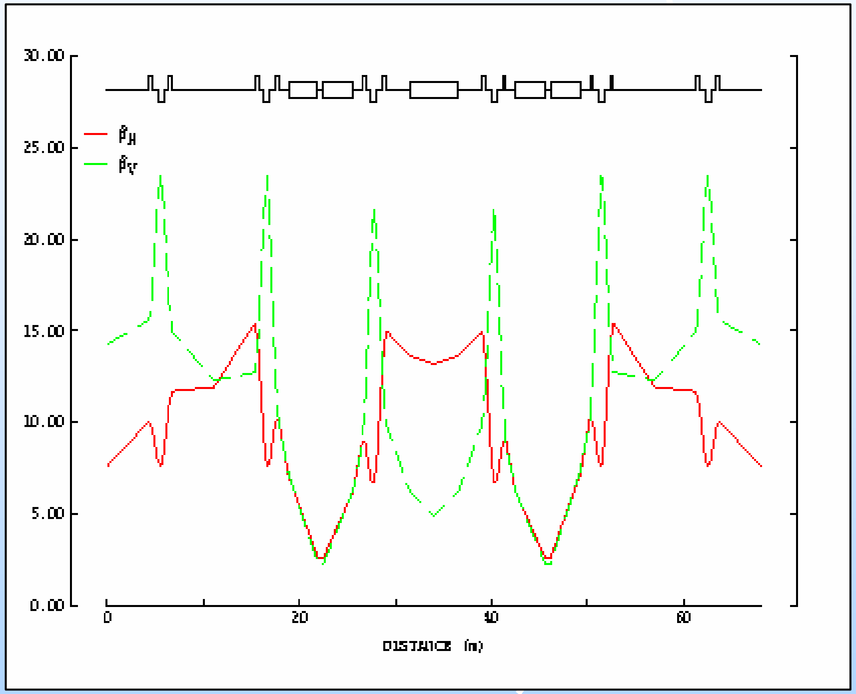
$$\begin{pmatrix} x \\ 0 \end{pmatrix}_{\text{in}} \Rightarrow \begin{pmatrix} \left(1 - l/f\right) x \\ \left(-l/f^2\right) x \end{pmatrix}_{\text{out}}$$

Thin lens of focal length f^2/ℓ, focusing overall, if $\ell < f$. Same for D-drift-F ($f \rightarrow -f$), so system of AG lenses can focus in both planes simultaneously

Examples of Transverse Focusing



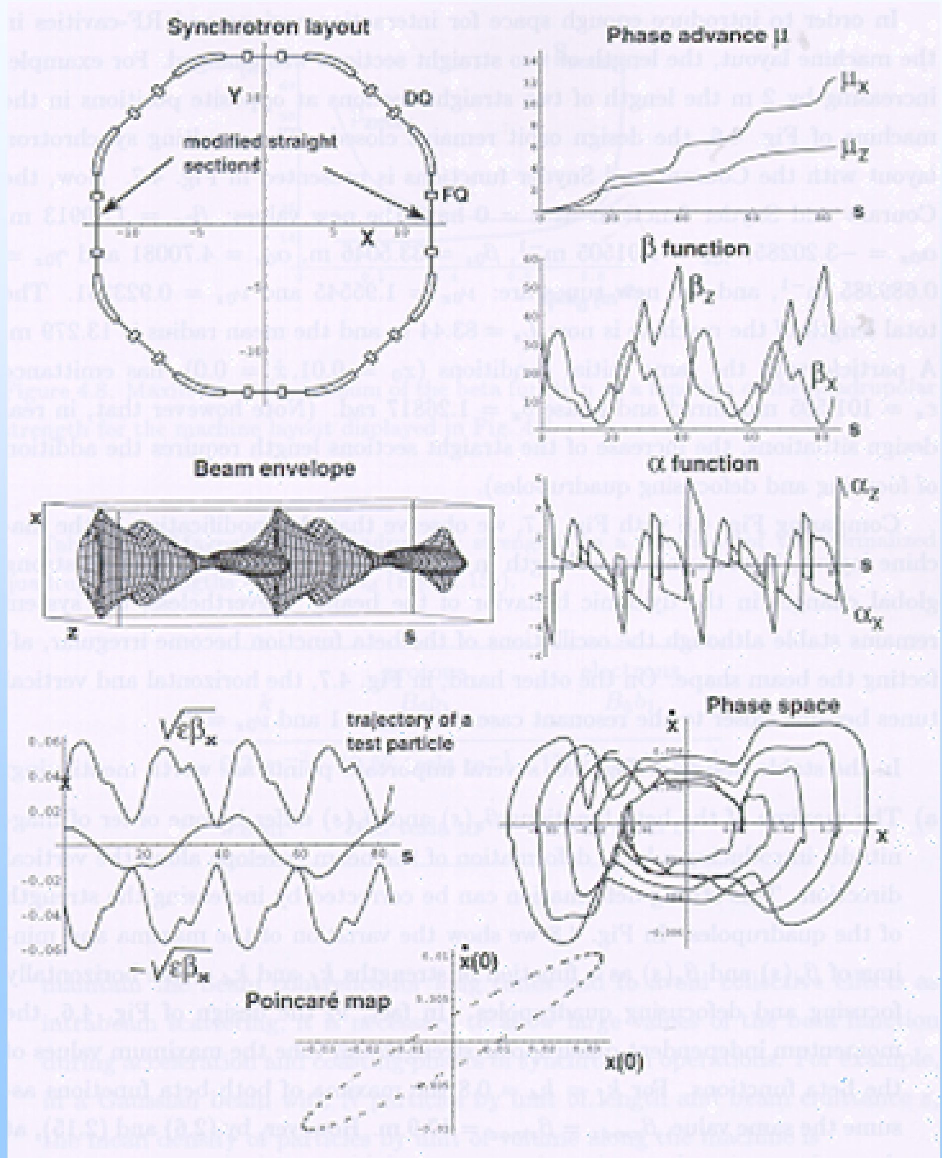
Matched beam oscillations in a simple FODO cell



Matched beam oscillations in a proton driver for a neutrino factory, with optical functions designed for injection and extraction

Ring Studies

- Typical example of ring design
  - *Basic lattice*
  - *Beam envelopes*
  - *Phase advances, resonances*
  - *Matching*
  - *Analysis of phase space non-linearities*
  - *Poincaré maps*



Electrons and Synchrotron Radiation

- Particles radiate when they are accelerated, so charged particles moving in the magnetic dipoles of a lattice in a ring (with centrifugal acceleration) emit radiation in a direction tangential to their trajectory.

- After one turn of a circular accelerator, total energy lost by synchrotron radiation is

$$\Delta E\,[\mathrm{GeV}] = \frac{6.034 \times 10^{-18}}{\rho\,[\mathrm{m}]} \left(\frac{E\,[GeV]}{m_0\,[\mathrm{GeV}/c^2]} \right)^4$$

- Proton mass : electron mass = 1836. For the same energy and radius,

$$\Delta E_e : \Delta E_p \approx 10^{13}$$

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Synchrotron Radiation

- In electron machines, strong dependence of radiated energy on energy.
- Losses must be compensated by cavities
- Technological limit on maximum energy a cavity can deliver $\Rightarrow$ upper band for electron energy in an accelerator:

$$E_{\max}\left[\mathrm{GeV}\right]=10\left(\rho[\mathrm{m}]\,\Delta E_{\max}\right)^{1/4}$$

- Better to have larger accelerator for same power from RF cavities at high energies.
- To reach twice a given energy with same cavities would require a machine 16 times as large.
- e.g. LEP with 50 GeV electrons, ρ =3.1 km, circumference =27 km:
  - Energy loss per turn is 0.18 GeV per particle
  - Energy is halved after 650 revolutions, in a time of 59 ms.

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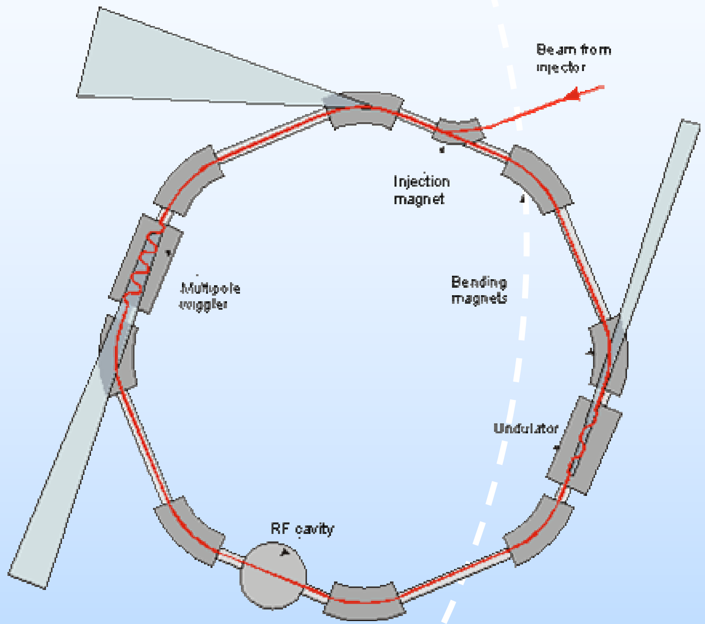
Synchrotron Radiation

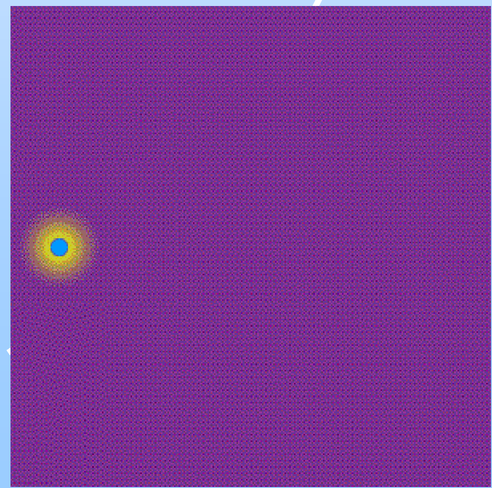
- Radiation is produced within a light cone of angle

$$\theta \approx \frac{1}{\gamma} = \frac{511}{E[\mathrm{keV}]} \quad \text{for speeds close to } c$$

- For electrons in the range 90 MeV to 1 GeV, θ is in the range 10^{-4} - 10^{-5} degs.

- Such collimated beams can be directed with high precision to a target - many applications, for example, in industry.





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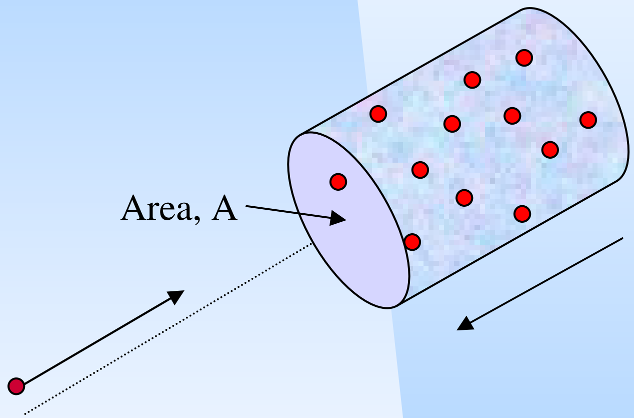
Diamond Light Source, RAL: April 2006

Luminosity

- Measures interaction rate per unit cross section - an important concept for colliders.
- Simple model: Two cylindrical bunches of area *A*. Any particle in one bunch sees a fraction $N\sigma/A$ of the other bunch. (σ=interaction cross section). Number of interactions between the two bunches is $N^2\sigma/A$.



Interaction rate is $R = f\,N^2\sigma/A$, and

- Luminosity $$L = f\,\frac{N^2}{A}$$
- CERN and Fermilab p-pbar colliders have $L \sim 10^{30}\ \mathrm{cm^{-2}s^{-1}}$. SSC was aiming for $L \sim 10^{33}\ \mathrm{cm^{-2}s^{-1}}$

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Reading

- E.J.N. Wilson: *Introduction to Accelerators*
- S.Y. Lee: *Accelerator Physics*
- M. Reiser: *Theory and Design of Charged Particle Beams*
- D. Edwards & M. Syphers: *An Introduction to the Physics of High Energy Accelerators*
- M. Conte & W. MacKay: *An Introduction to the Physics of Particle Accelerators*
- R. Dilao & R. Alves-Pires: *Nonlinear Dynamics in Particle Accelerators*
- M. Livingston & J. Blewett: *Particle Accelerators*

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