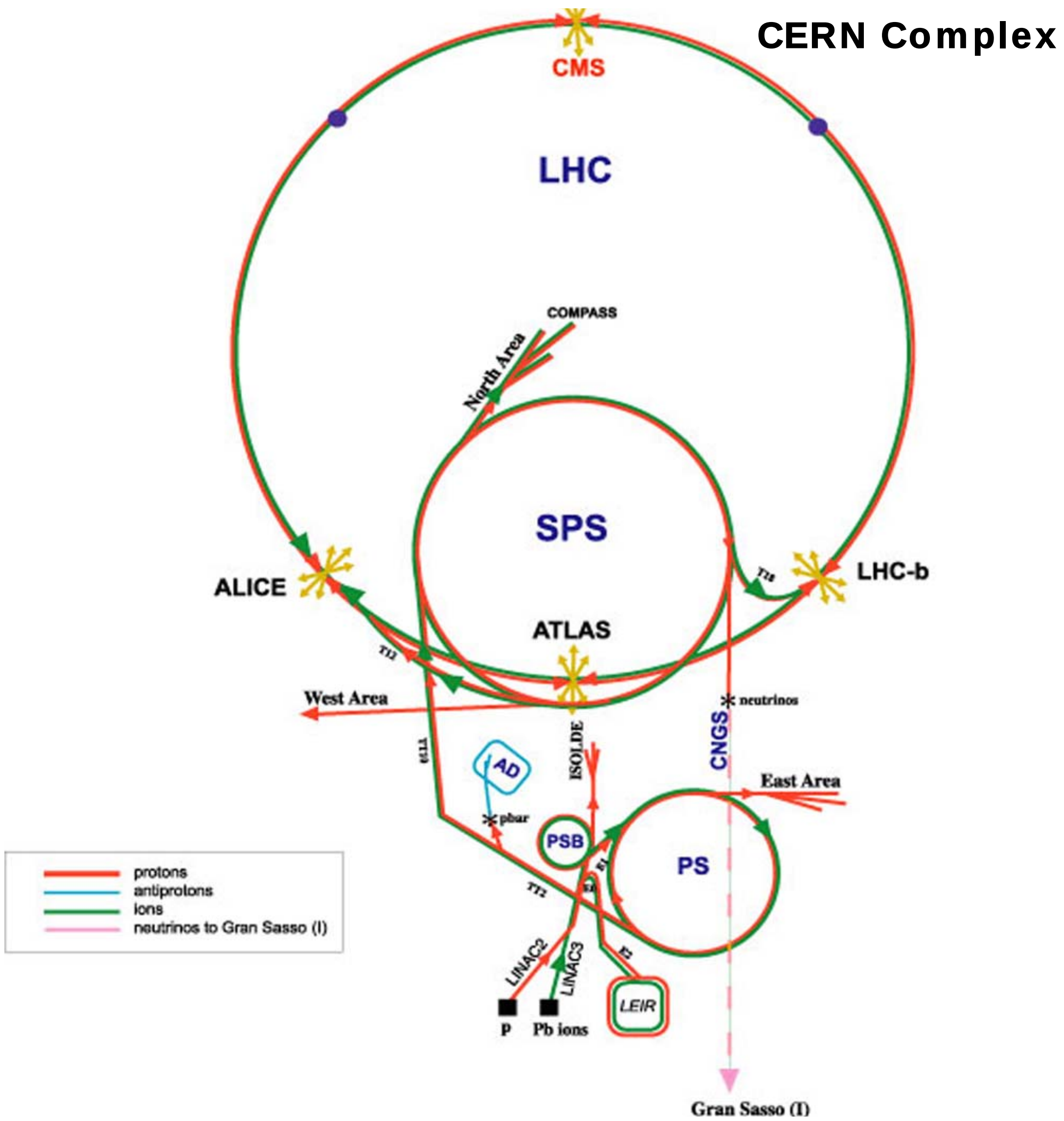
Injection, extraction and transfer

- An accelerator has limited dynamic range.
- Chain of stages needed to reach high energy
- Periodic re-filling of storage rings, like LHC
- External experiments, like CNGS

Transfer lines transport the beam between accelerators, and onto targets, dumps, instruments etc.

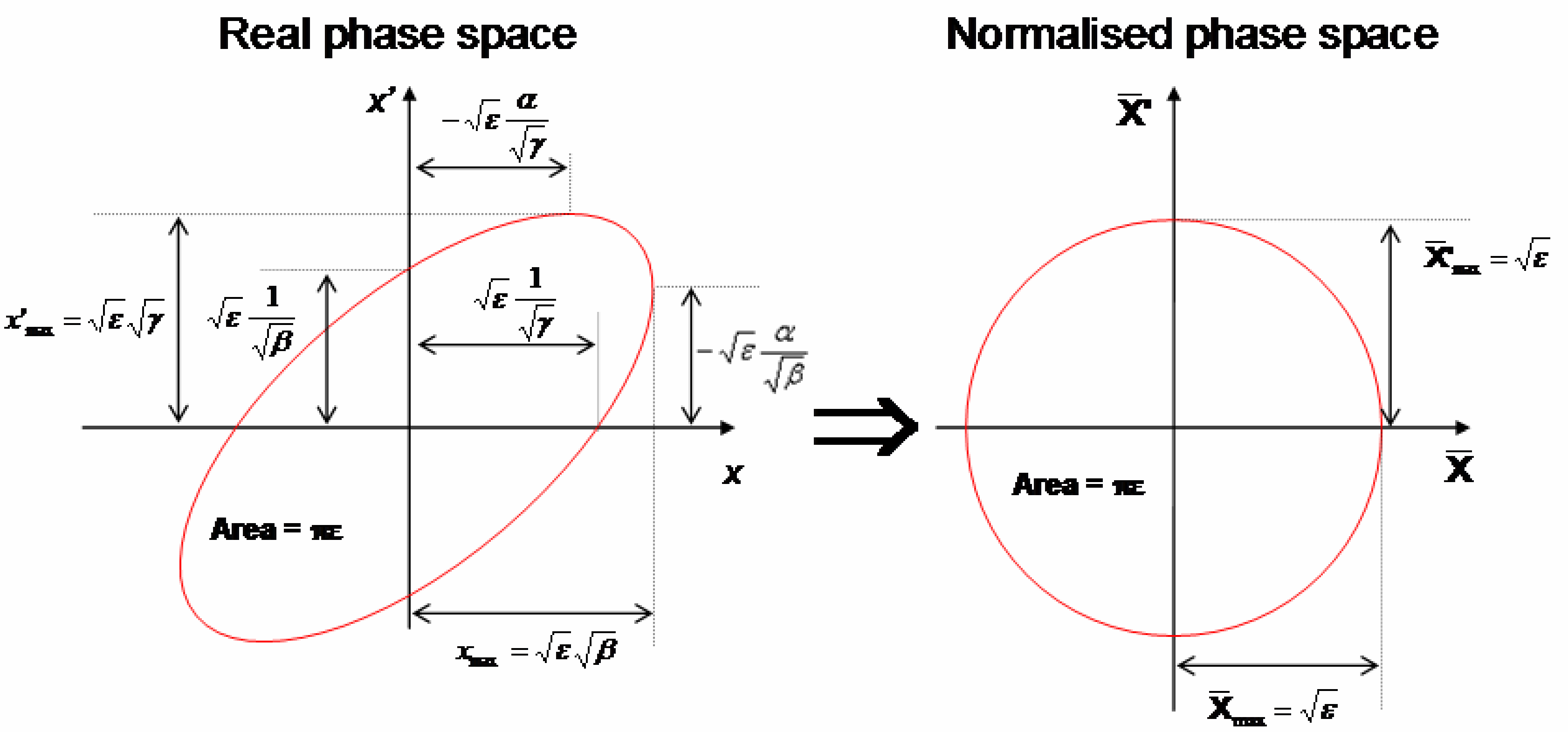


LHC:	Large Hadron Collider
SPS:	Super Proton Synchrotron
AD:	Antiproton Decelerator
ISOLDE:	Isotope Separator Online Device
PSB:	Proton Synchrotron Booster
PS:	Proton Synchrotron
LINAC:	LINear Accelerator
LEIR:	Low Energy Ring
CNGS:	CERN Neutrino to Gran Sasso

Beam Transfer lines

- Distinctions between transfer lines and circular machines
- Linking circular machines
- Trajectory correction
- Emittance and mismatch measurement
- Delivery precision and errors
- Blow-up from betatron mismatch
- Thin screens: blow-up and charge stripping

Normalised phase space

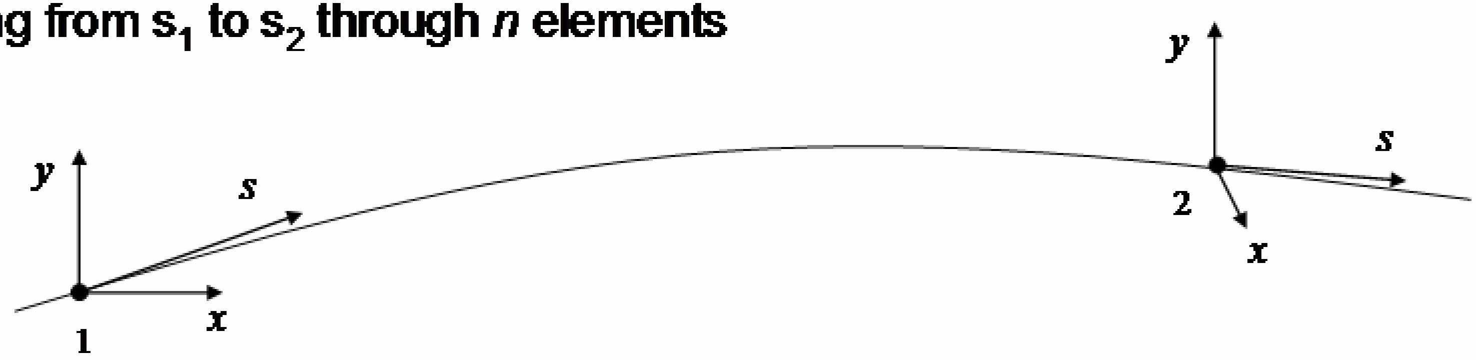


$$\varepsilon = \gamma \cdot x^2 + 2\alpha \cdot x \cdot x' + \beta \cdot x'^2$$

$$\varepsilon = \overline{X}^2 + \overline{X}'^2$$

Distinction between Transfer Lines and Circular Machines

Moving from s_1 to s_2 through *n* elements



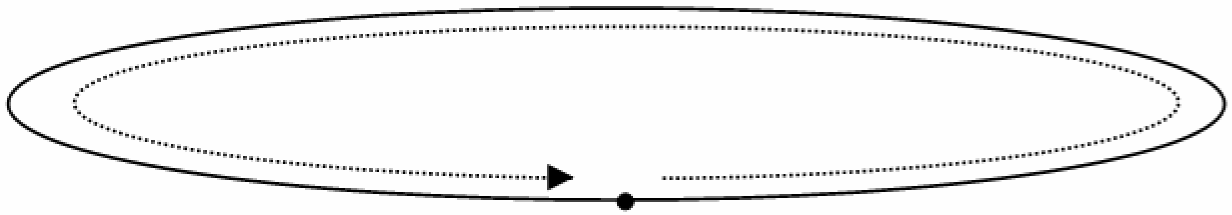
$$\begin{bmatrix} x_2 \\ x_2' \end{bmatrix} = \mathbf{M}_{1\to 2} \cdot \begin{bmatrix} x \\ x' \end{bmatrix} = \begin{bmatrix} C & S \\ C' & S' \end{bmatrix} \cdot \begin{bmatrix} x \\ x' \end{bmatrix}$$

$$\mathbf{M}_{1\to 2} = \prod_{i=1}^{n} \mathbf{M}_n$$

Circular Machine

Twiss parameterisation
$$\mathbf{M}_{1\to 2} = \begin{bmatrix} \sqrt{\beta_2/\beta_1}\left(\cos\Delta\mu + \alpha_1 \sin\Delta\mu\right) & \sqrt{\beta_1\beta_2}\sin\Delta\mu \\ \sqrt{1/\beta_1\beta_2}\left[(\alpha_1-\alpha_2)\cos\Delta\mu - (1+\alpha_1\alpha_2)\sin\Delta\mu\right] & \sqrt{\beta_1/\beta_2}\left(\cos\Delta\mu - \alpha_2\sin\Delta\mu\right) \end{bmatrix}$$

Circumference = L

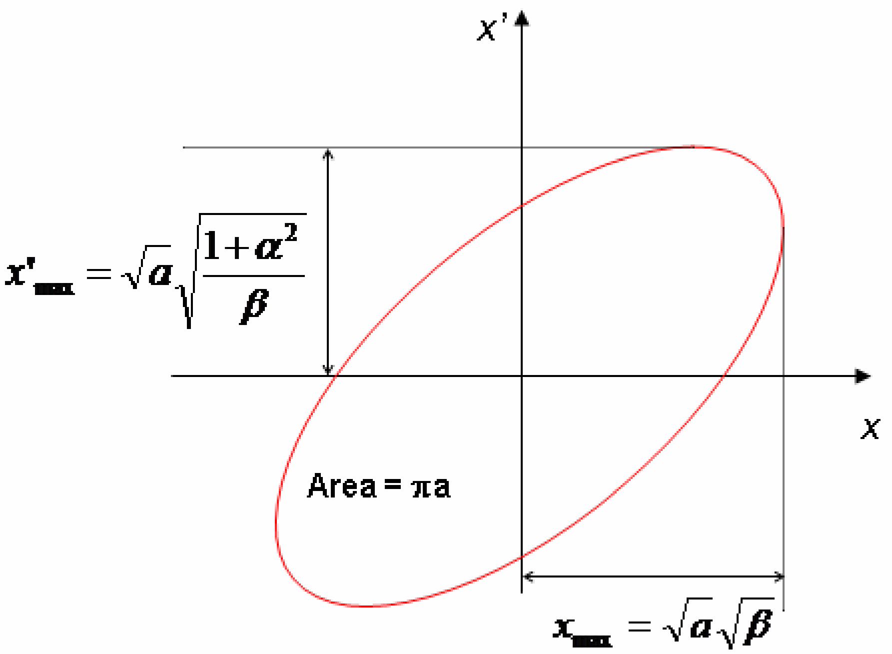


One turn
$$\mathbf{M}_{1\to 2} = \mathbf{M}_{0\to L} = \begin{bmatrix} \cos 2\pi Q + \alpha \sin 2\pi Q & \beta \sin 2\pi Q \\ -1/\beta\left(1+\alpha^2\right)\sin 2\pi Q & \cos 2\pi Q - \alpha \sin 2\pi Q \end{bmatrix}$$

- Periodicity condition for one turn (closed ring) imposes $\alpha_1 = \alpha_2$, $\beta_1 = \beta_2$
- This condition *uniquely* determines $\alpha(s)$, $\beta(s)$ and $\mu(s)$ around the whole ring

Circular Machine

- Map the coordinates of a particle on each turn.
  - Periodicity of the structure leads to regular motion
  - At any location in the ring, particle motion over many turns describe an ellipse in phase space, defined by one set of α and β values $\Rightarrow$ Matched Ellipse

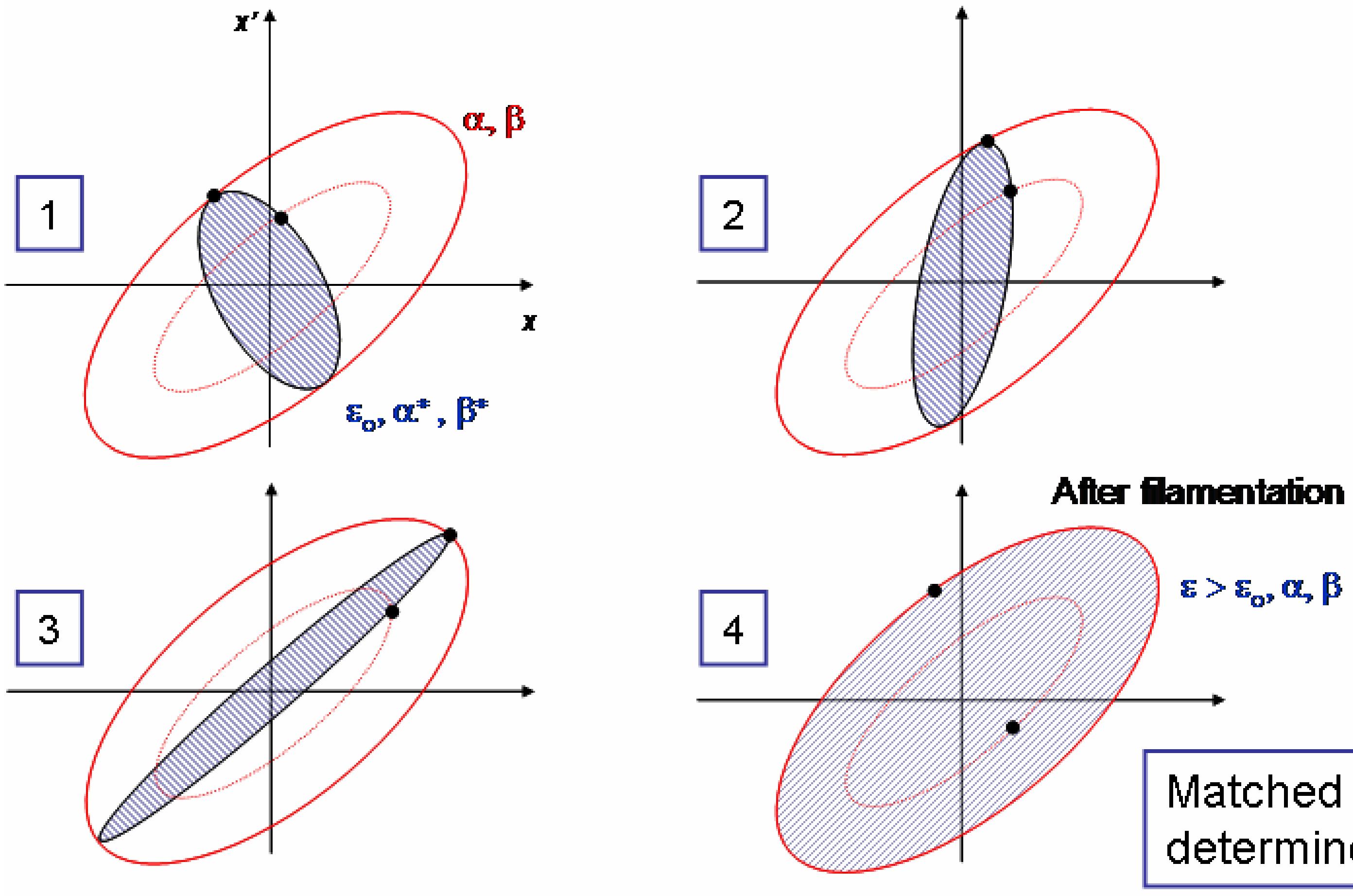


$$a = \gamma \cdot x^2 + 2\alpha \cdot x \cdot x' + \beta \cdot x'^2$$

$$\gamma = \frac{1+\alpha^2}{\beta}$$

Circular Machine

- A beam injected with emittance ε, characterised by a different ellipse (α^*, β^*) generates (via filamentation) a large ellipse with the original α, β, but larger ε



Matched ellipse determines beam shape

Transfer line

One pass $\begin{bmatrix} x_2 \\ x_2' \end{bmatrix} = \mathbf{M}_{1\to 2} \cdot \begin{bmatrix} x \\ x' \end{bmatrix}$

$$\begin{bmatrix} x_1 \\ x_1' \end{bmatrix} \longrightarrow \begin{bmatrix} x_2 \\ x_2' \end{bmatrix}$$

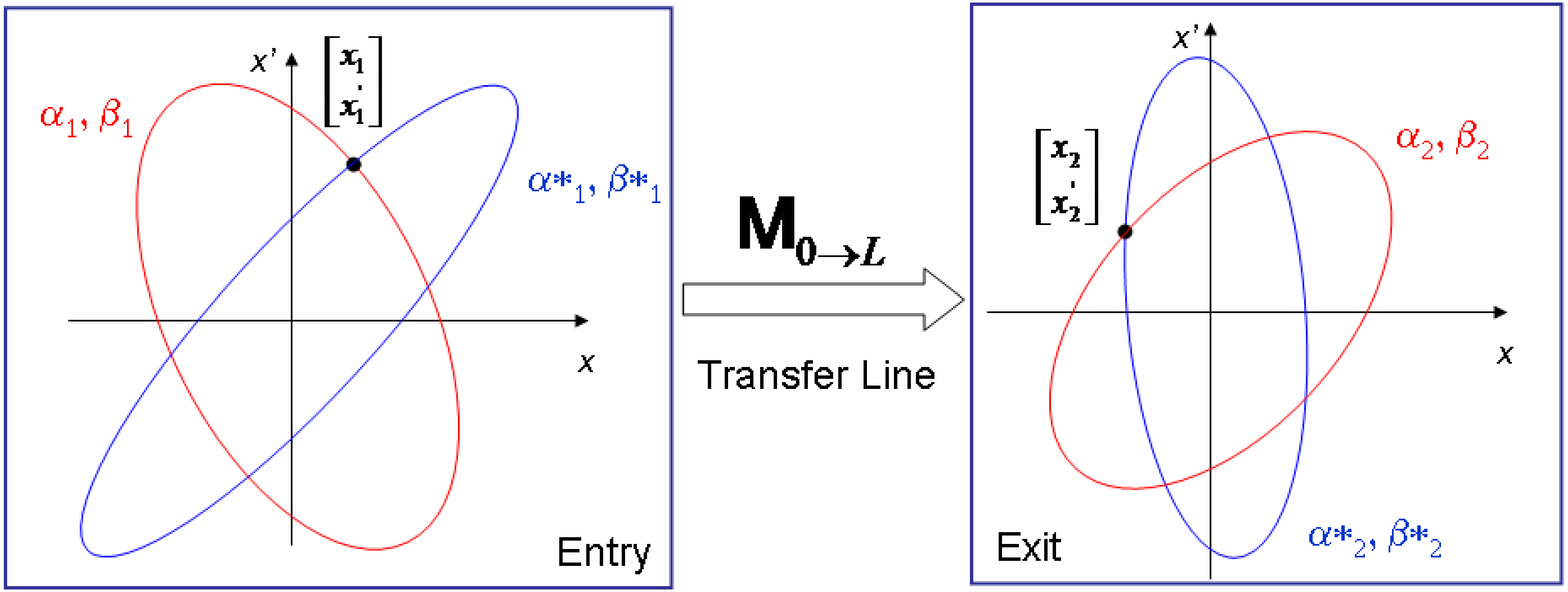
$$\mathbf{M}_{1\to 2} = \begin{bmatrix} \sqrt{\beta_2 / \beta_1}\,(\cos\Delta\mu + \alpha_1 \sin\Delta\mu) & \sqrt{\beta_1\beta_2}\,\sin\Delta\mu \\ \sqrt{1 / \beta_1\beta_2}\,[(\alpha_1 - \alpha_2)\cos\Delta\mu - (1 + \alpha_1\alpha_2)\sin\Delta\mu] & \sqrt{\beta_1 / \beta_2}\,(\cos\Delta\mu - \alpha_2 \sin\Delta\mu) \end{bmatrix}$$

- **No periodic condition exists**
- Twiss parameters are propagated from beginning to the end of the line
- At any point in the line, $\alpha(s)$ $\beta(s)$ are functions of α_1 β_1

Transfer line

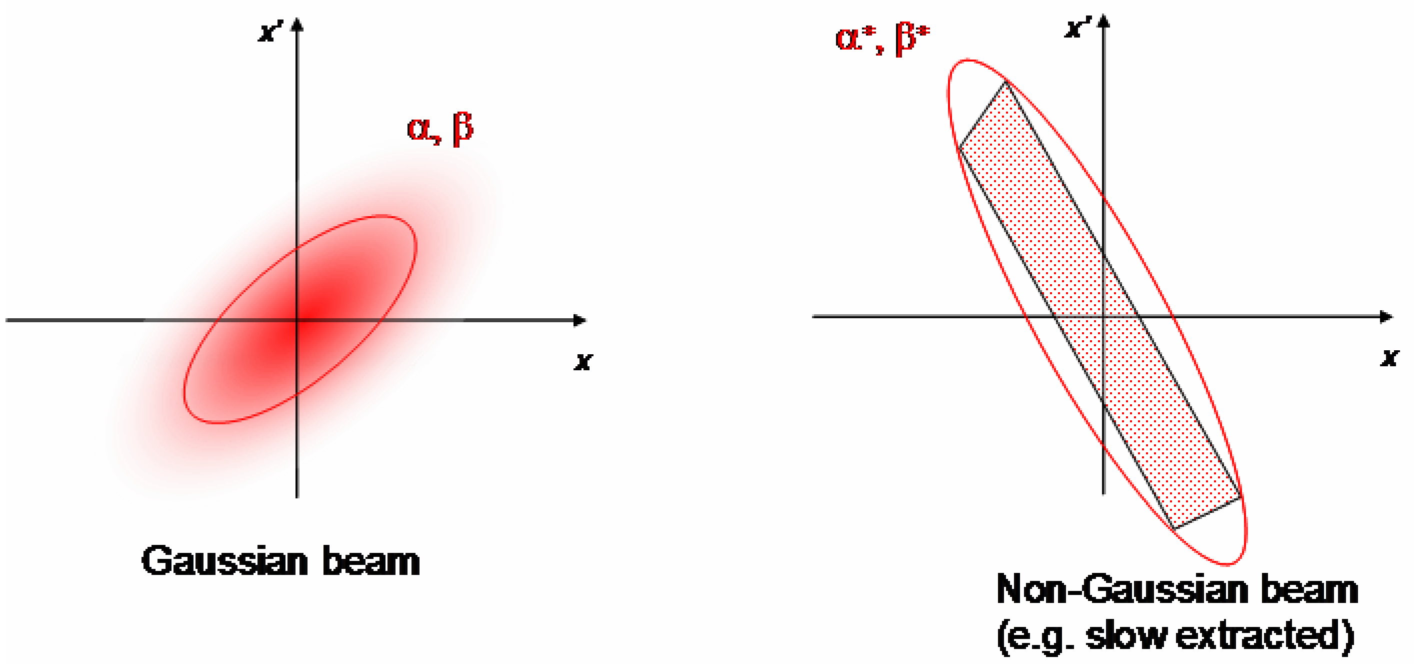
- Map single particle coordinates at entrance and exit.
- Infinite number of possible starting ellipses…

  …transported to *infinite number* of final ellipses!



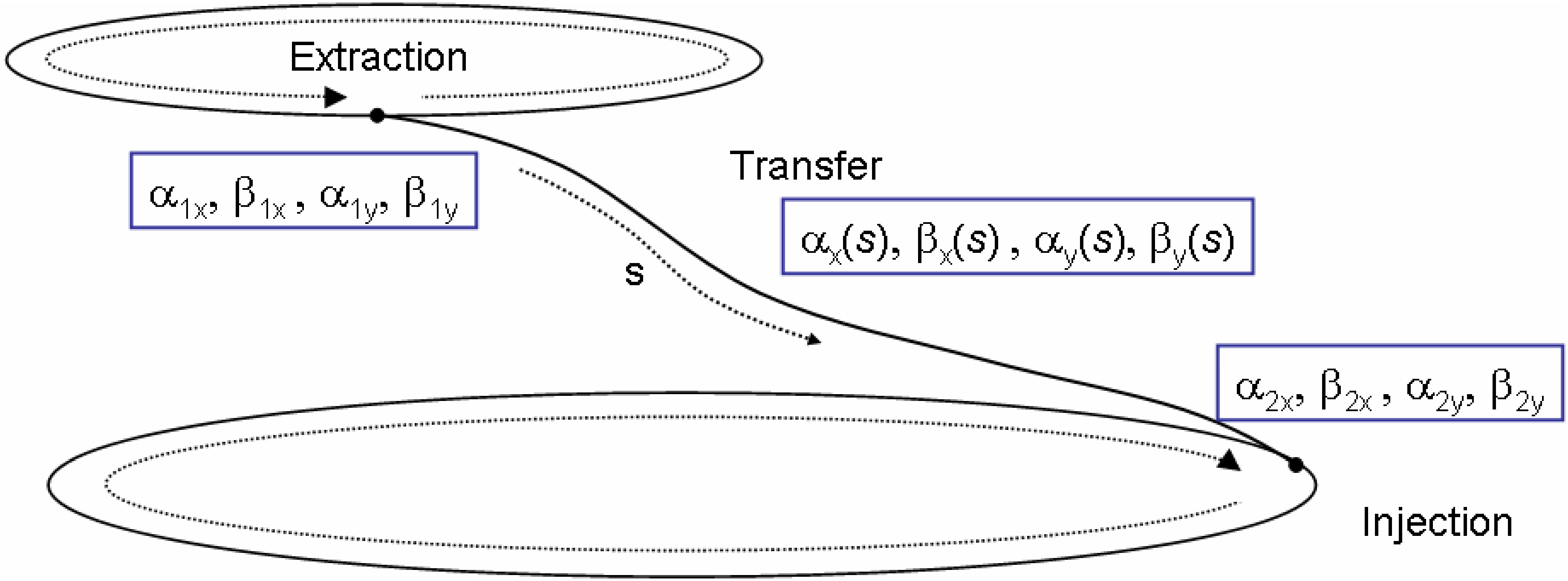
Transfer Line

- Initial α, β are defined for a transfer line by the beam shape at the entrance



- Propagation of this beam ellipse depends on the line
- Line optics is different for different input beams!

Linking Circular Machines



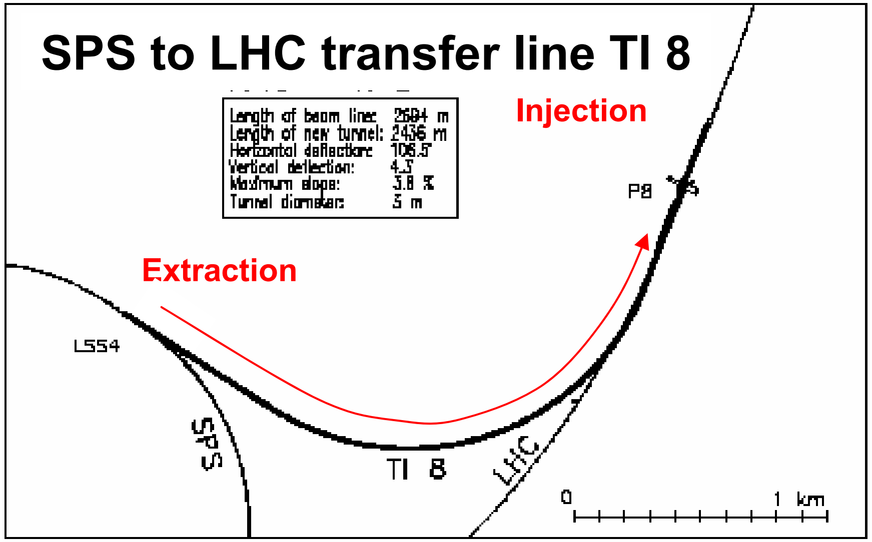
The Twiss parameters can be propagated when the transfer matrix **M** is known

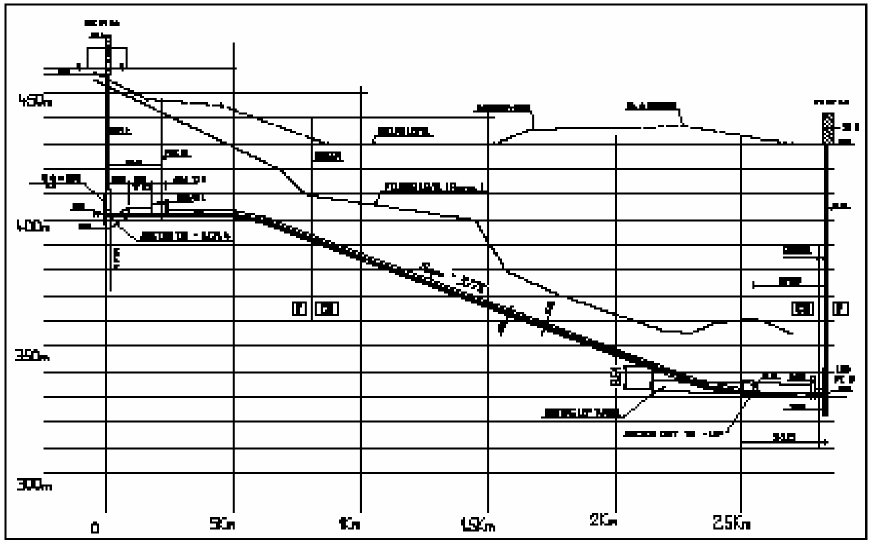
$$\begin{bmatrix} x_2 \\ x_2' \end{bmatrix} = \mathbf{M}_{1\to2} \cdot \begin{bmatrix} x \\ x' \end{bmatrix} = \begin{bmatrix} C & S \\ C' & S' \end{bmatrix} \cdot \begin{bmatrix} x \\ x' \end{bmatrix}$$

$$\begin{bmatrix} \beta_2 \\ \alpha_2 \\ \gamma_2 \end{bmatrix} = \begin{bmatrix} C^2 & -2CS & S^2 \\ -CC' & CS'+SC' & -SS' \\ C'^2 & -2C'S' & S'^2 \end{bmatrix} \cdot \begin{bmatrix} \beta_1 \\ \alpha_1 \\ \gamma_1 \end{bmatrix}$$

Linking Circular Machines

- Constraints include
  - Matching the trajectories
  - Minimum bend radius
  - Magnet aperture
  - Cost
  - Geology





Linking Circular Machines

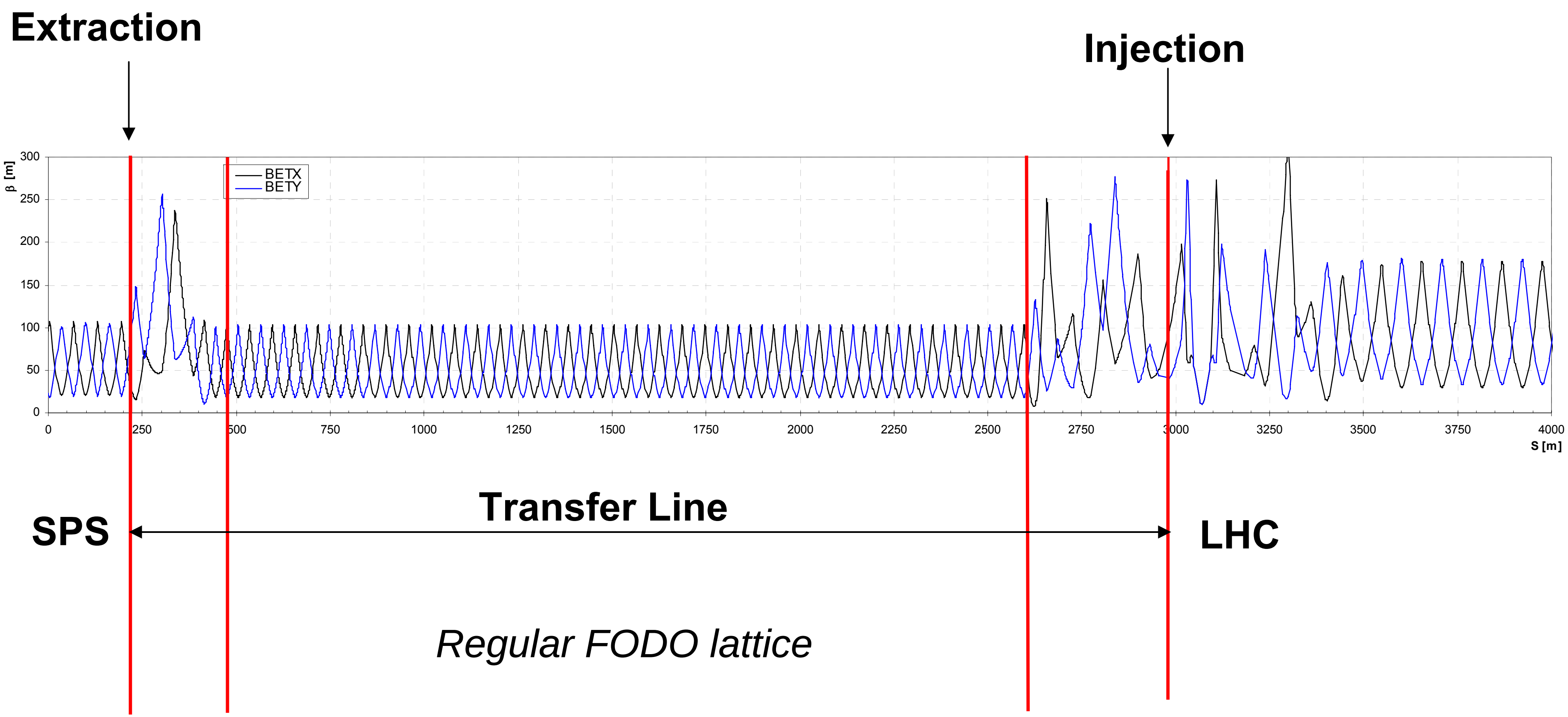
- Matching the optics is a non-trivial process
  - Parameters at start of line have to be propagated to matched parameters at the end of the line
  - Need in theory to match 8 variables (α_x β_x D_x D'_x and α_y β_y D_y D'_y)
  - Maximum β and D values are imposed by magnet apertures
  - Other constraints can exist
    - phase conditions for collimators,
    - insertions for special equipment like stripping foils
- Need to use a number of independently powered quadrupoles

Linking Circular Machines

- For long transfer lines we can simplify the problem by designing the line in 3 separate sections
  - Regular central section – e.g. FODO with F and D quads at regular spacing, (+ bending dipoles)
  - Initial and final matching sections – independently powered quadrupoles, with sometimes irregular spacing.

Linking Circular Machines

- SPS to LHC transfer line TI 8 – beta functions

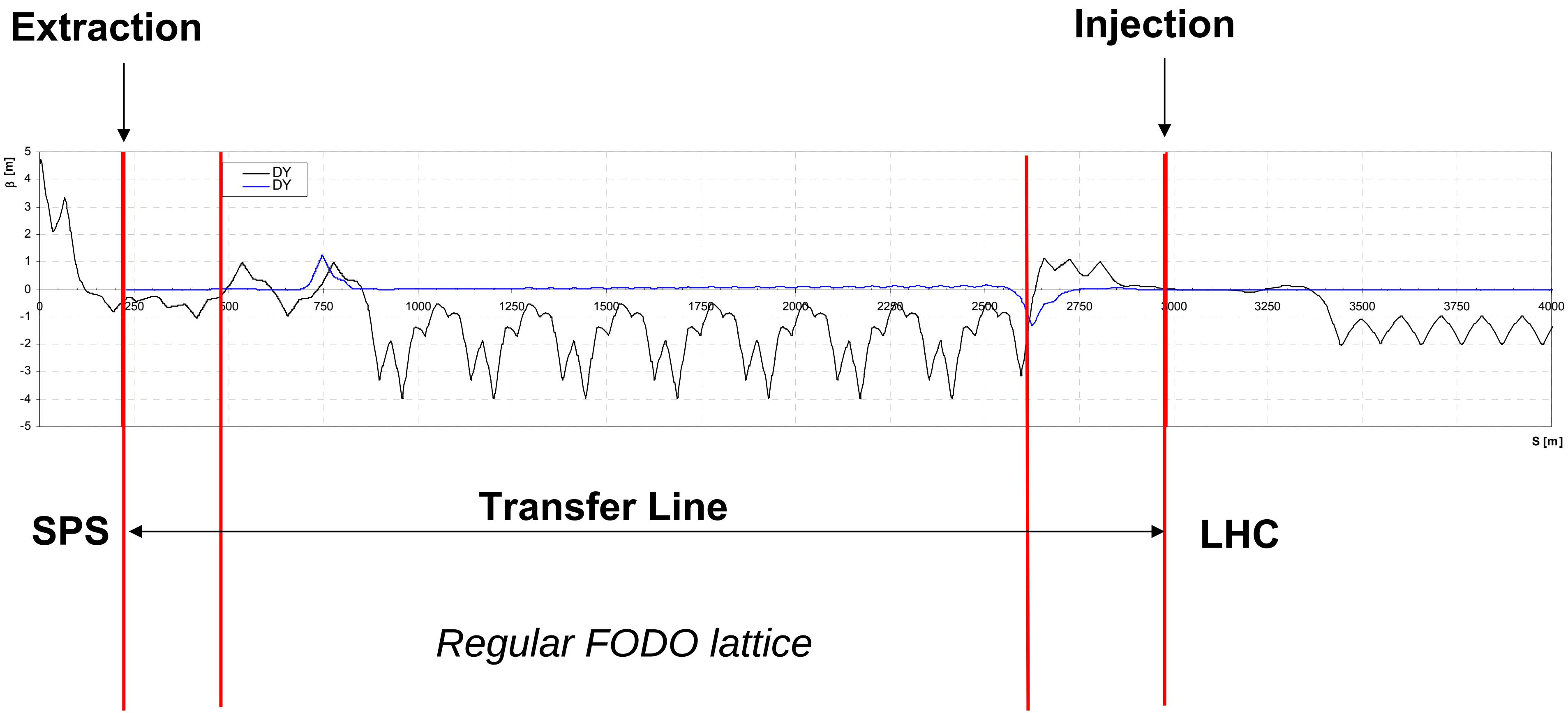


Initial matching section

Final matching section

Linking Circular Machines

- SPS to LHC transfer line TI 8 – dispersion functions

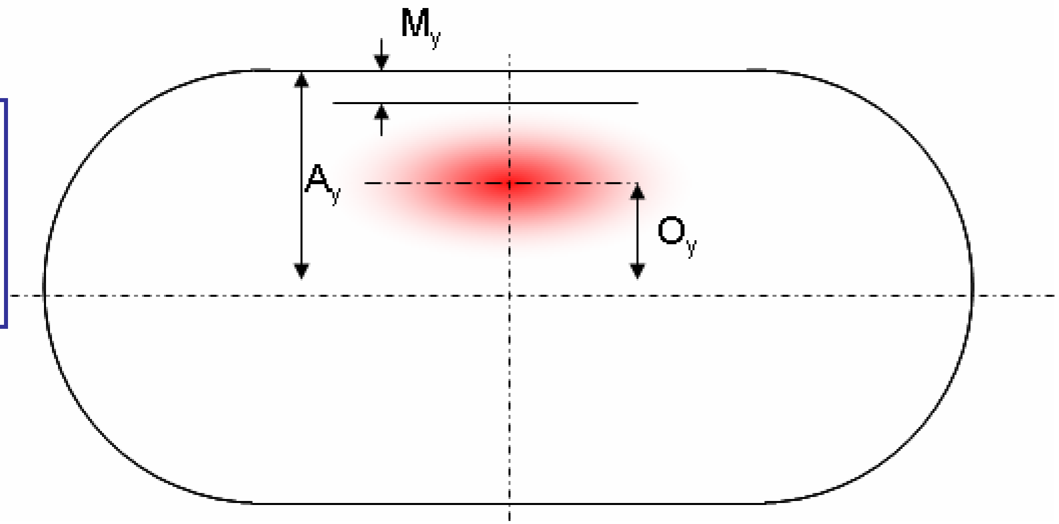


Initial matching section *Final matching section*

Aperture

- Available aperture for the beam depends on:
  - optics (β, D), trajectory O, mechanical and alignment tolerance M, magnet aperture A, energy spread $\Delta p/p$ and emittance ε.
- Can use a general expression to evaluate number of beam σ which can be accommodated (k = factor to allow for optical errors, typically 1.1)

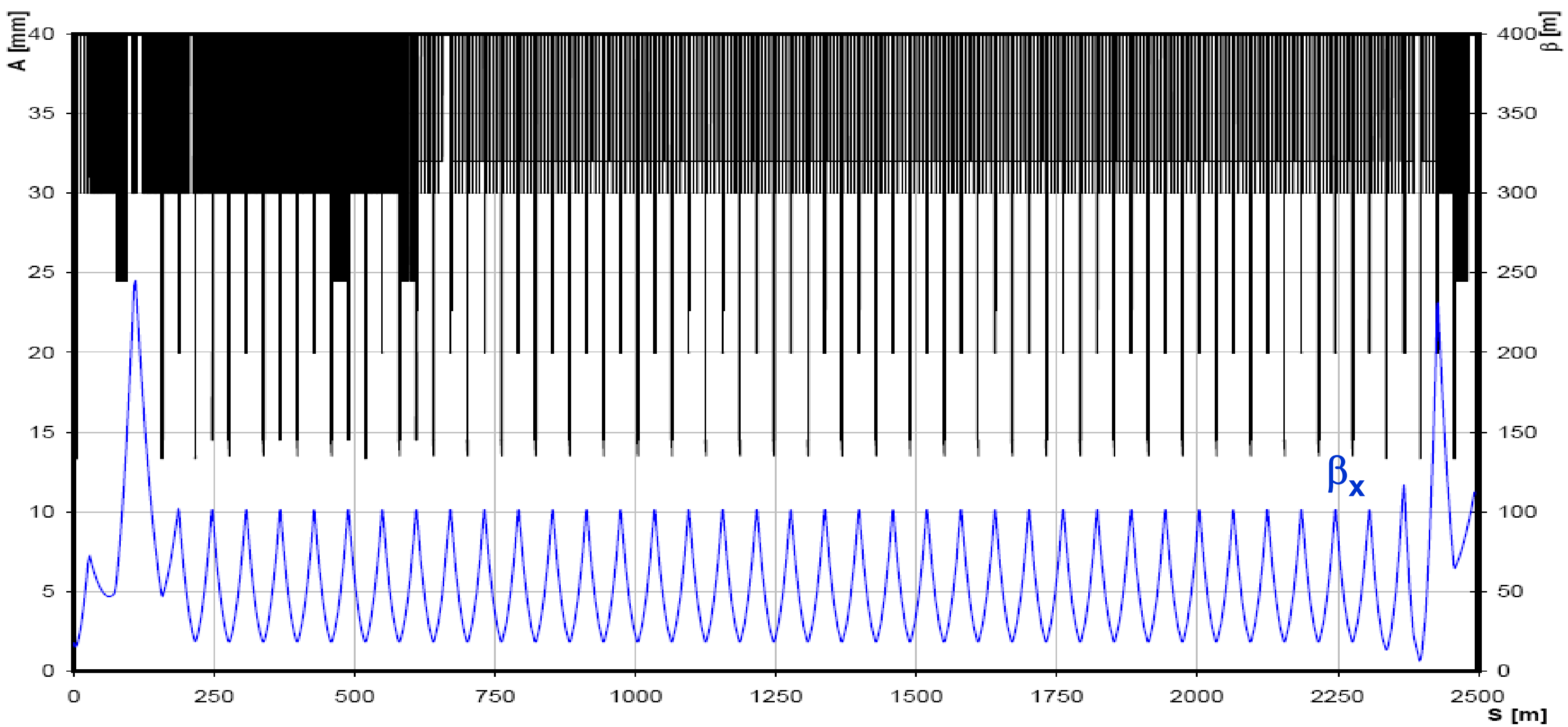
$$\boxed{n\sigma_y = \frac{A_y - O_y - M_y}{k\sqrt{\beta_y \varepsilon_y + \left(D_y \left|\Delta p / p\right.\right)^2}}}$$



Aperture

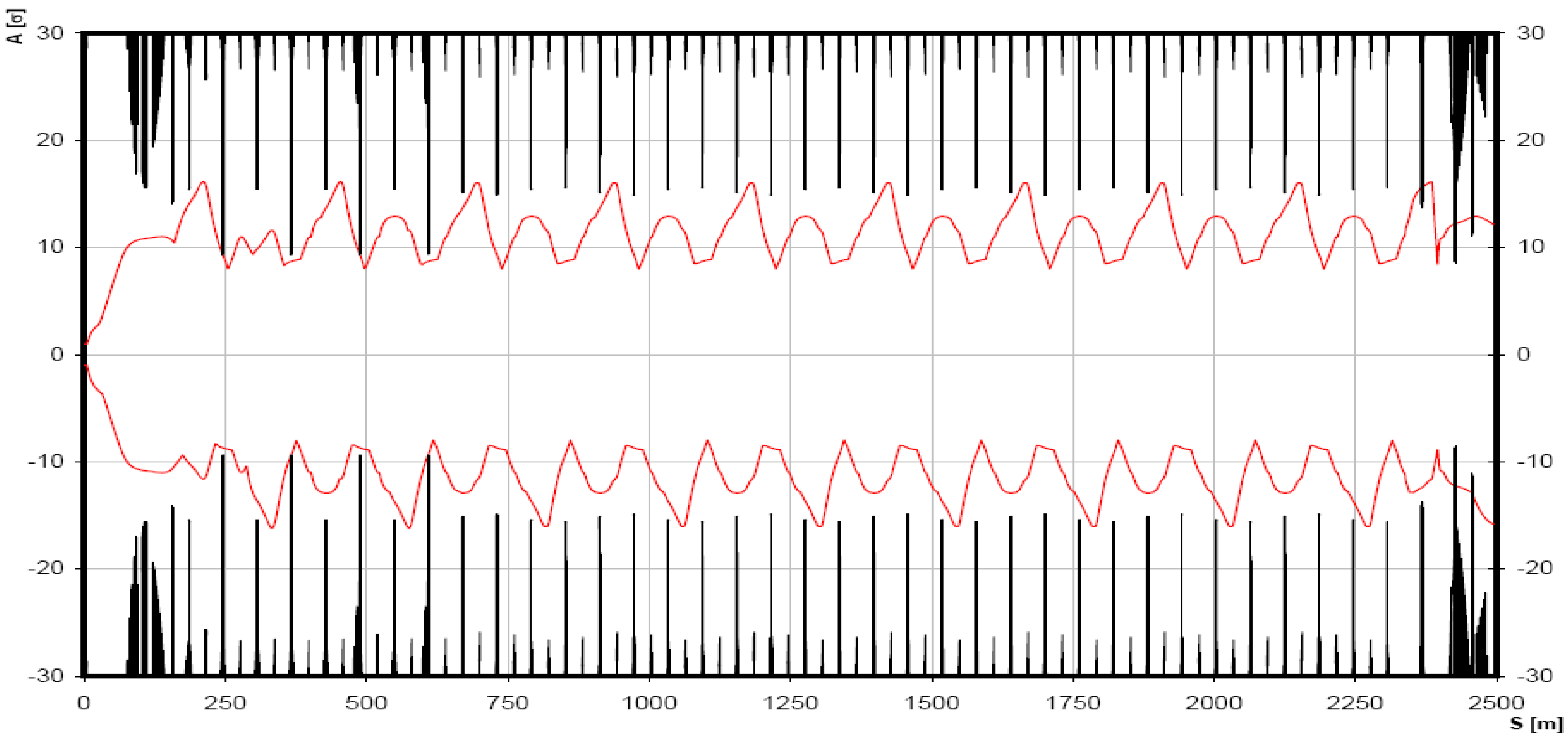
- Aperture can be evaluated with optics and physical line description
- Critical areas can be fitted with extra instruments or correctors.

SPS to LHC transfer line TI8



Aperture measurement

- Deflect beam with corrector magnets
- Measure transmission as function of kick



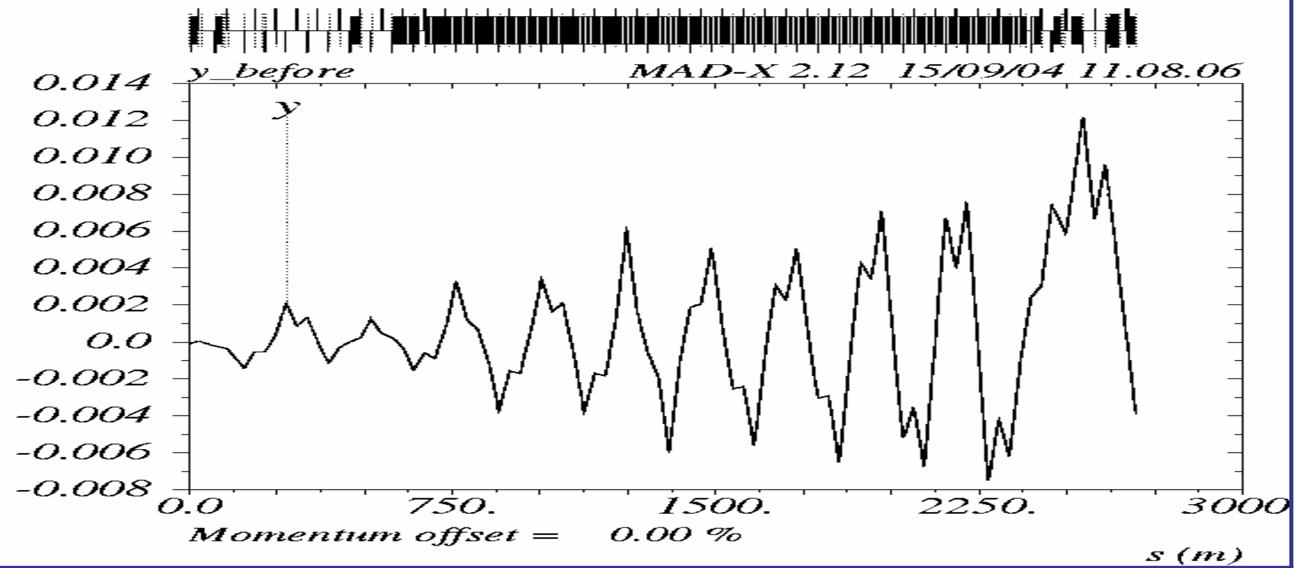
Trajectory correction

- Magnet misalignments, field and powering errors cause the trajectory to deviate from the design
- Use small independently powered dipole magnets (correctors) to steer the beam.
- Measure the response using monitors (pick-ups) downstream of the corrector ($\pi/2$, $3\pi/2$, …).
- Separate horizontal and vertical elements.
- H-correctors and pick-ups located at F-quadrupoles (large β_x).
- V-correctors and pick-ups located at D-quadrupoles (large β_y).
- In long lines, not all quadrupoles are equipped…

Trajectory correction

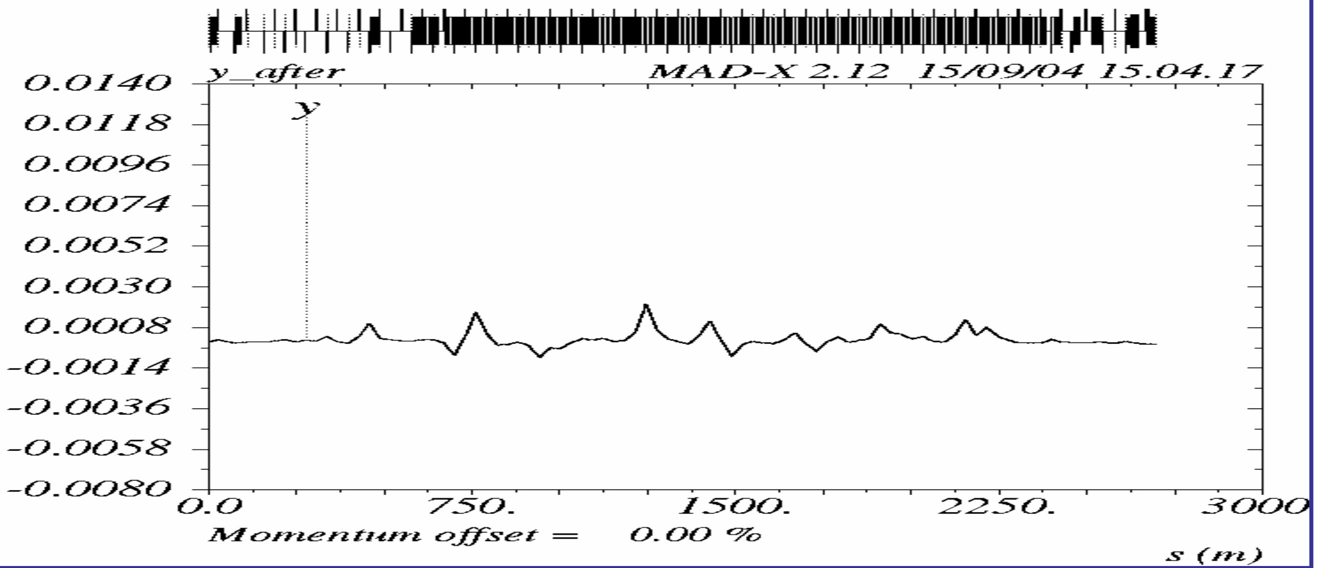
Uncorrected trajectory, with *y* growing as a result of random errors in the line.

The RMS at the BPMs is 3.4 mm, and y_{max} is 12.0mm



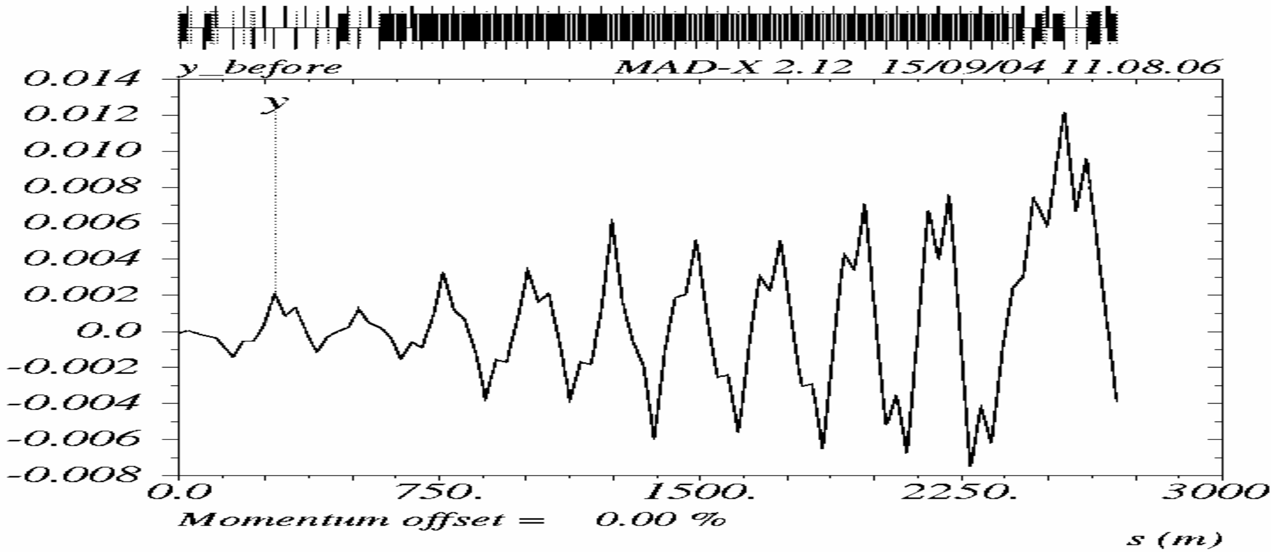
Corrected trajectory.

The RMS at the BPMs is 0.3mm and y_{max} is 1mm



Trajectory correction

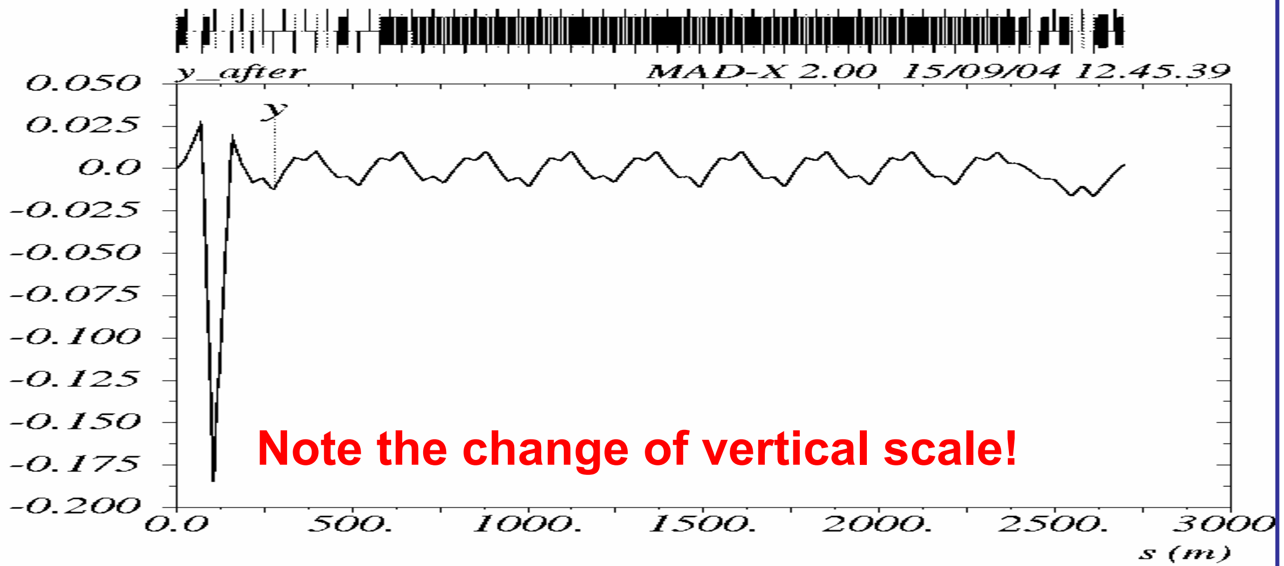
Uncorrected trajectory, with *y* growing as a result of random errors in the line.



Correction with some monitors disabled

If the BPM phase sampling is poor, the correction algorithm can make the trajectory very bad, while all the monitor readings being ~zero….

… in this case 185mm y_{max} !



Trajectory correction

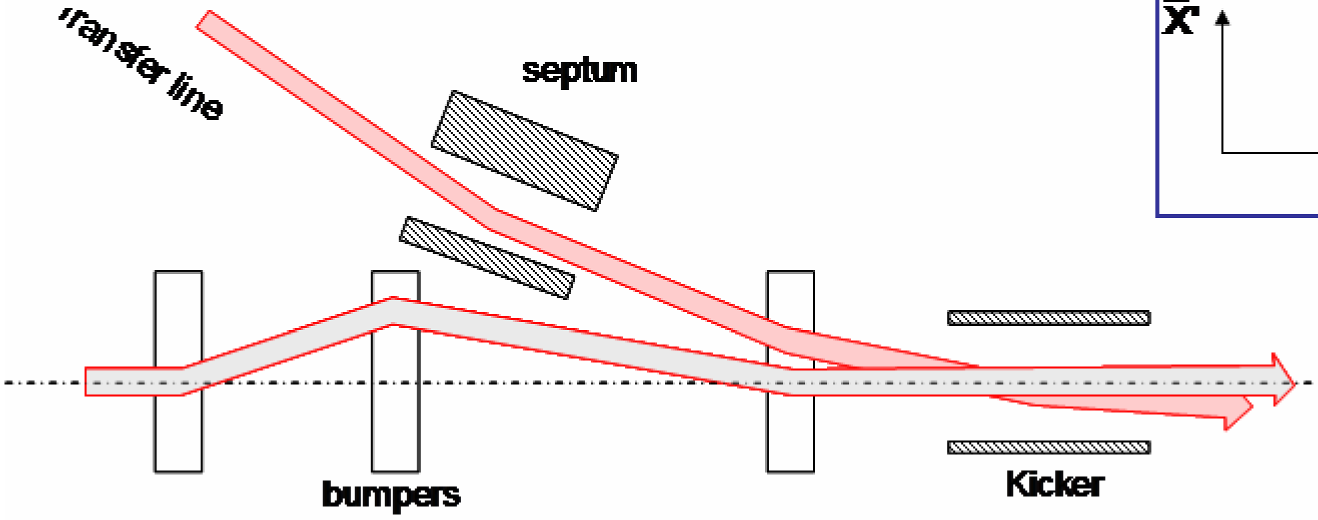
- Global correction can be used which attempts to minimise the RMS offsets at the BPMs, using all or some of the available corrector magnets.
- Steering in matching sections, extraction and injection region requires particular care
  - D and β functions can be large $\rightarrow$ bigger beam size
  - Often very limited in aperture
  - Injection offsets can be detrimental for performance

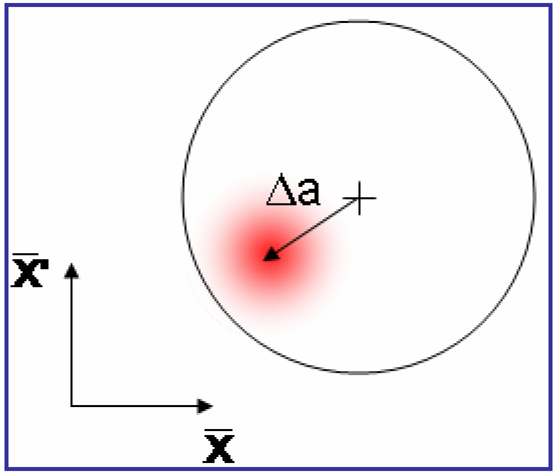
Delivery precision

- Precise delivery of the beam is important.
  - To avoid injection oscillations and emittance growth in rings
  - For stability on secondary particle production targets
- Express injection error in σ

$$\Delta a = \sqrt{(X^2 + X'^2)} = \sqrt{(\gamma x^2 + 2\alpha x x' + \beta x'^2)}$$

$$\Delta a/\sigma = \sqrt{(\beta a/\beta\varepsilon)} = \sqrt{[(\gamma x^2 + 2\alpha x x' + \beta x'^2)/\varepsilon]}$$

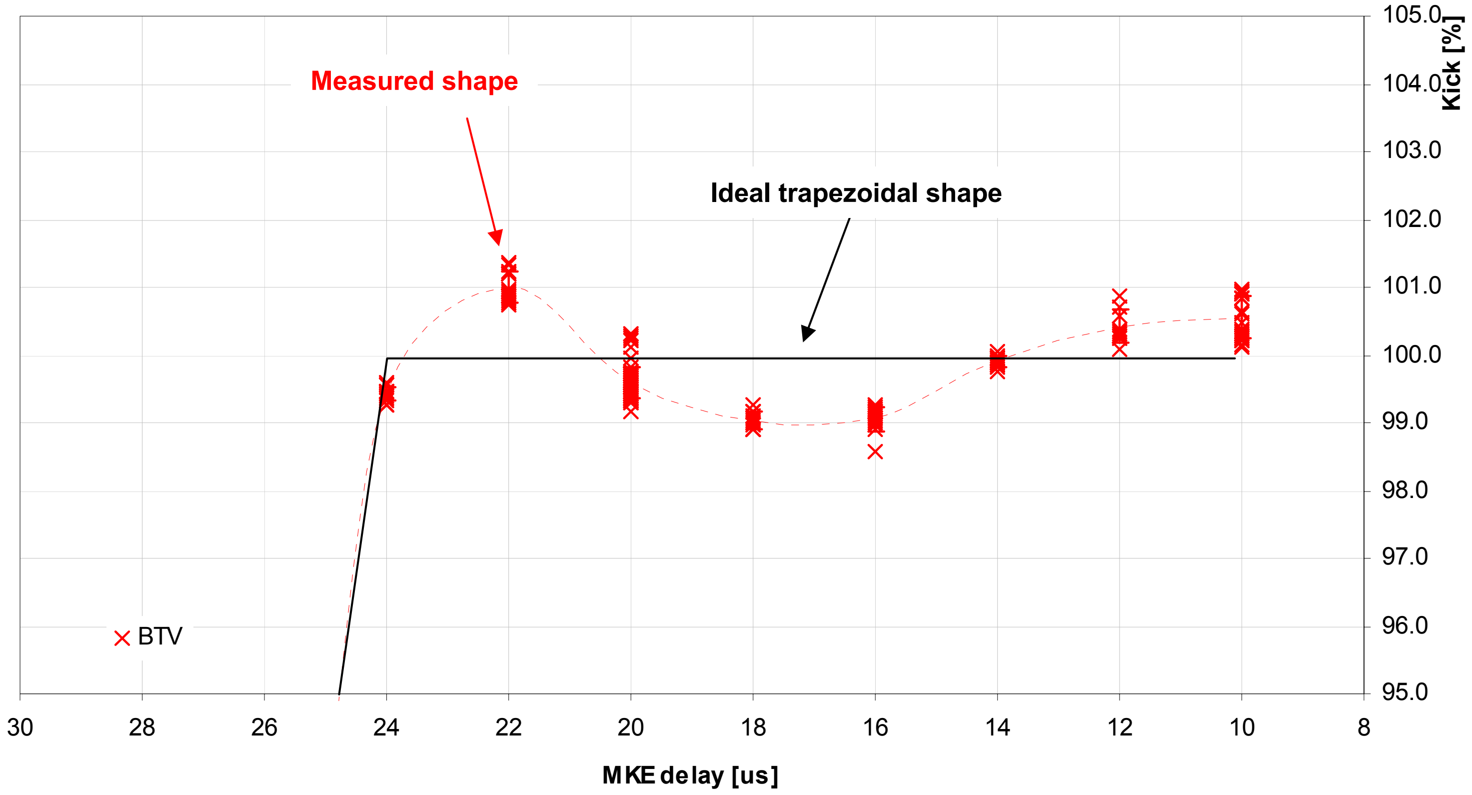




Delivery precision

- Static effects (e.g. from errors in alignment, field, calibration, …) are dealt with by trajectory correction (steering).
- But there are also dynamic effects, from:
  - Power supply ripples
  - Temperature variations
  - Non-trapezoidal kicker waveforms
- These dynamic effect produce a variable injection offset which can vary from batch to batch, or even within a batch.
- Important for performance and for transverse damper specification

Delivery precision



Measurement with beam of extraction kicker waveform
- this example would lead to ±0.5 σ offset for some bunches

Delivery precision

- The errors in a line superimpose:
  - for uncorrelated errors the effects can be added quadratically
  - for correlated errors the effects must be added linearly
- Typical patterns of errors can be generated with a Monte-Carlo simulation
- Worst-case errors in families of magnets can be calculated analytically by introducing field errors into the Accelerator design code and calculating the resulting $\Delta a/\sigma$

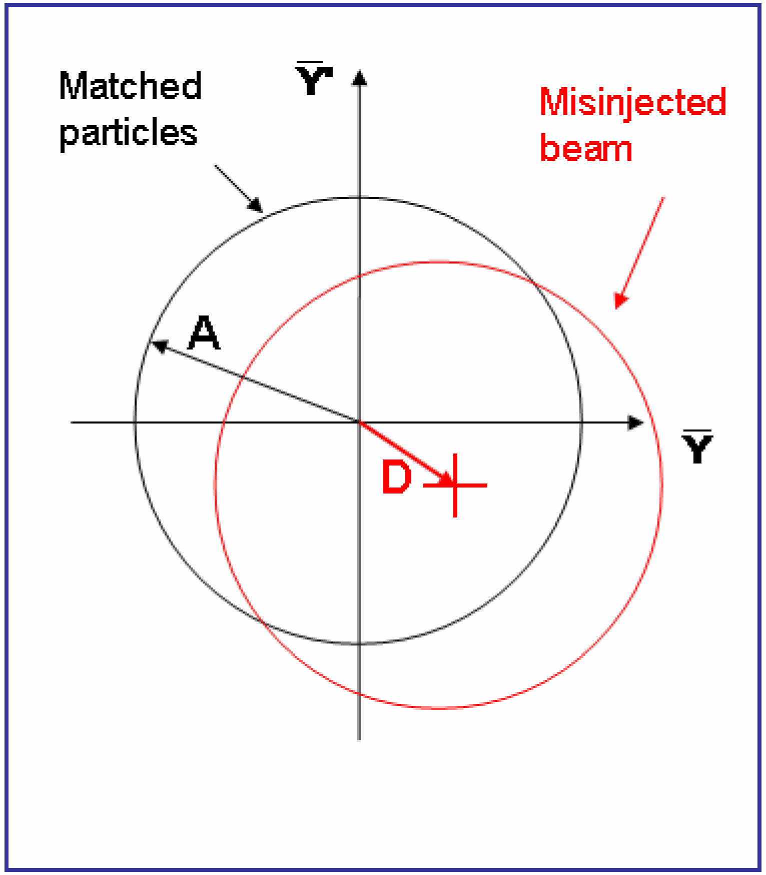
Delivery precision

- Example –power supply ripples in SPS to LHC line TI 8.

Family / element	rms ±ΔI/Imax	x mm	x' mrad	y mm	y' mrad	$\Delta a_x/\sigma$	$\Delta a_y/\sigma$
Quadrupole MQF	5.0E-05	0.0006	0.0000	0.0001	0.0000	0.004	0.002
Quadrupole MQD	5.0E-05	0.0006	0.0000	0.0001	0.0000	0.004	0.002
Bumper MPLH	2.8E-04	0.0043	-0.0005	0.0000	0.0000	0.022	0.000
Septum MSE	1.3E-04	0.0282	0.0015	0.0000	0.0000	0.130	0.000
Dipole BH1	5.0E-05	0.0096	-0.0010	0.0000	0.0000	0.055	0.000
Dipole BH2	5.0E-05	0.0501	-0.0015	0.0000	0.0000	0.083	0.000
Dipole BH3	5.0E-05	-0.0008	0.0002	0.0000	0.0000	0.014	0.000
Dipole BH4	5.0E-05	-0.0030	-0.0010	0.0000	0.0000	0.088	0.000
Dipole MBI	2.5E-05	-0.0509	0.0013	0.0088	0.0005	0.091	0.035
Dipole BV1	5.0E-05	0.0000	0.0000	-0.0001	0.0002	0.000	0.021
Dipole BV2	5.0E-05	0.0000	0.0000	0.0187	-0.0005	0.000	0.084
Septum BH5A	5.0E-05	-0.0059	-0.0002	0.0000	0.0000	0.033	0.000
Septum BH5B	5.0E-05	-0.0140	-0.0002	0.0000	0.0000	0.058	0.000
Kicker MKE	2.5E-04	0.0035	-0.0003	0.0000	0.0000	0.012	0.000
Kicker MKI	2.5E-04	0.0000	0.0000	-0.0023	-0.0002	0.000	0.018
					rms	**0.220**	**0.095**
					linear sum	**0.594**	**0.162**

Blow-up from injection error

- Consider a collection of particles with amplitudes A
- The beam can be injected with a error in angle and position.
- For an injection error Δa_y (in units of sigma = $\sqrt{\beta\varepsilon}$) the mis-injected beam is offset by D, where $|D| = \Delta a_y \sqrt{\varepsilon}$



Blow-up from injection error

- The new particles coordinates in normalised phase space are

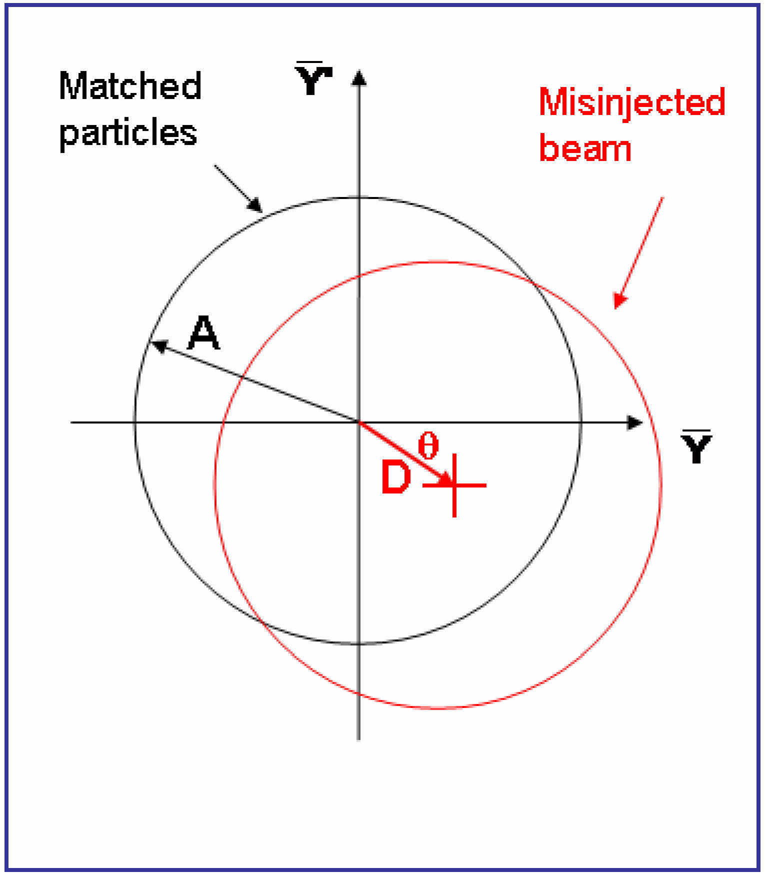
$$\overline{Y}_{new} = \overline{Y}_0 + D\cos\theta$$

$$\overline{Y}'_{new} = \overline{Y}'_0 + D\sin\theta$$

- For a general particle distribution, where A denotes amplitude in normalised phase space

$$A^2 = \overline{Y}^2 + \overline{Y}'^2$$

$$\varepsilon = \langle A^2 \rangle / 2$$



Blow-up from injection error

- So if we plug in the new coordinates….

$$\mathbf{A}^2_{new} = \overline{\mathbf{Y}}^2_{new} + \overline{\mathbf{Y}}'^2_{new} = \left(\overline{\mathbf{Y}}_0 + \mathbf{D}\cos\theta\right)^2 + \left(\overline{\mathbf{Y}}'_0 + \mathbf{D}\sin\theta\right)^2$$

$$= \overline{\mathbf{Y}}_0^2 + \overline{\mathbf{Y}}_0'^2 + 2\mathbf{D}\left(\overline{\mathbf{Y}}_0\cos\theta + \overline{\mathbf{Y}}'_0\sin\theta\right) + \mathbf{D}^2$$

$$\left\langle \mathbf{A}^2_{new} \right\rangle = \left\langle \overline{\mathbf{Y}}_0^2 \right\rangle + \left\langle \overline{\mathbf{Y}}_0'^2 \right\rangle + \left\langle 2\mathbf{D}\left(\overline{\mathbf{Y}}_0\cos\theta + \overline{\mathbf{Y}}'_0\sin\theta\right) \right\rangle + \left\langle \mathbf{D}^2 \right\rangle$$

$$= 2\varepsilon_0 + 2\mathbf{D}\left(\cos\theta \cancelto{0}{\left\langle \overline{\mathbf{Y}}_0 \right\rangle} + \sin\theta \cancelto{0}{\left\langle \overline{\mathbf{Y}}'_0 \right\rangle}\right) + \mathbf{D}^2$$

$$= 2\varepsilon_0 + \mathbf{D}^2$$

- Giving for the emittance increase

$$\varepsilon_{new} = \left\langle \mathbf{A}_{new}{}^2 \right\rangle / 2 = \varepsilon_0 + \mathbf{D}^2 / 2$$

$$= \varepsilon_0\left(1 + \Delta \mathbf{a}^2 / 2\right)$$

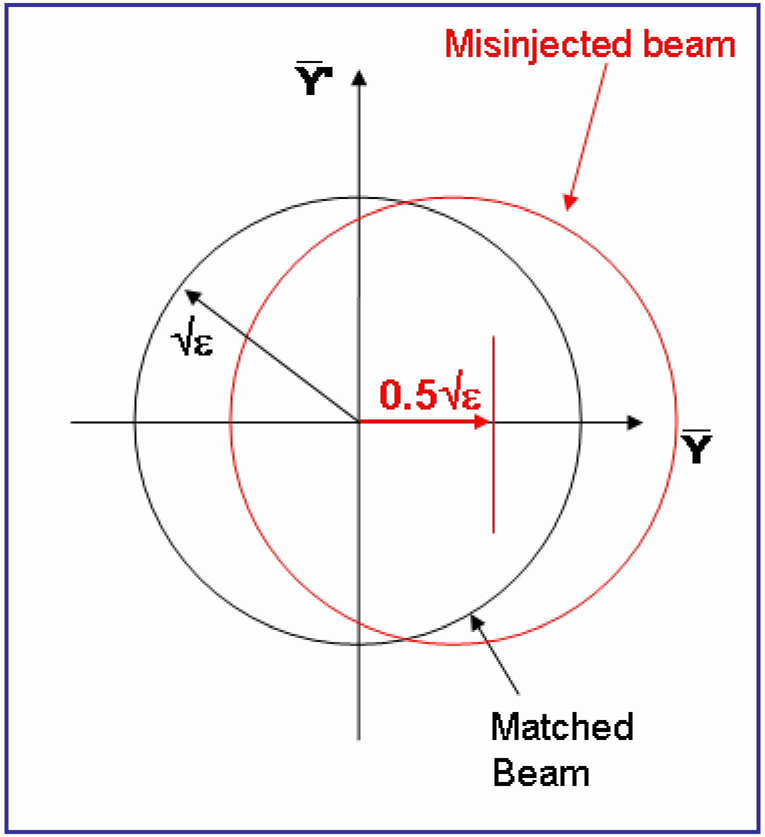
Blow-up from injection offset

A numerical example….

Consider an offset Δa of 0.5 sigma for injected beam

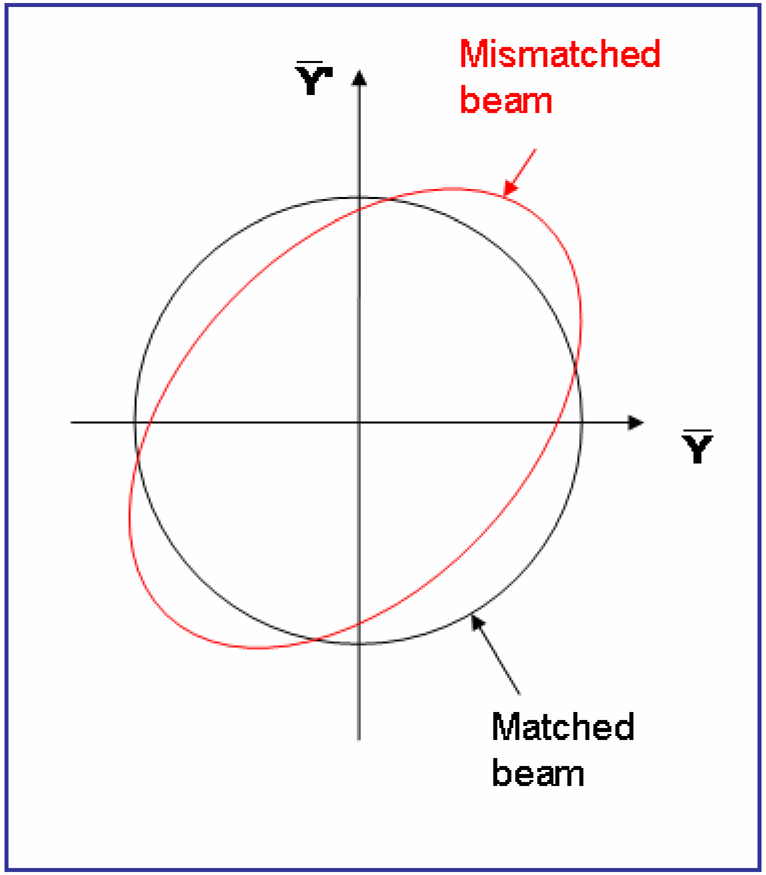
$$\varepsilon_{new} = \varepsilon_0 \left(1 + \Delta a^2 / 2\right)$$

$$= 1.125\varepsilon_0$$



Blow-up from betatron mismatch

- Optical errors occur in transfer line and ring, such that the beam can be injected with a mismatch.
- Filamentation will produce an emittance increase.
- In normalised phase space, consider the matched beam as a circle, and the mismatched beam as an ellipse.



Blow-up from betatron mismatch

General betatron motion

$$y_2 = \sqrt{\varepsilon_2 \beta_2}\,\sin(\phi + \phi_o)$$

$$y'_2 = \sqrt{\varepsilon_2 / \beta_2}\left[\cos(\phi + \phi_o) - \alpha_2 \sin(\phi + \phi_o)\right]$$

applying the normalising trans-
formation for the matched beam

$$\begin{bmatrix} \overline{\mathbf{Y}}_2 \\ \overline{\mathbf{Y}}'_2 \end{bmatrix} = \sqrt{\frac{1}{\beta_1}} \cdot \begin{bmatrix} 1 & 0 \\ \alpha_1 & \beta_1 \end{bmatrix} \cdot \begin{bmatrix} y_2 \\ y'_2 \end{bmatrix}$$

an ellipse is obtained in <u>normalised</u> phase space

$$A^2 = \overline{\mathbf{Y}}_2^2 \left[\frac{\beta_1}{\beta_2} + \frac{\beta_2}{\beta_1} \left(\alpha_1 - \alpha_2 \frac{\beta_1}{\beta_2} \right)^2 \right] + \overline{\mathbf{Y}}'^2_2 \frac{\beta_2}{\beta_1} - 2 \overline{\mathbf{Y}}_2 \overline{\mathbf{Y}}'_2 \left[\frac{\beta_2}{\beta_1} \left(\alpha_1 - \alpha_2 \frac{\beta_1}{\beta_2} \right) \right]$$

characterised by γ_{new}, β_{new} *and* α_{new}, where

$$\alpha_{new} = \frac{-\beta_2}{\beta_1} \left(\alpha_1 - \alpha_2 \frac{\beta_1}{\beta_2} \right), \qquad \beta_{new} = \frac{\beta_2}{\beta_1}, \quad \gamma_{new} = \frac{\beta_1}{\beta_2} + \frac{\beta_2}{\beta_1} \left(\alpha_1 - \alpha_2 \frac{\beta_1}{\beta_2} \right)^2$$

Blow-up from betatron mismatch

From the general ellipse properties

$$a = \frac{A}{\sqrt{2}}\left(\sqrt{H+1} + \sqrt{H-1}\right), \qquad b = \frac{A}{\sqrt{2}}\left(\sqrt{H+1} - \sqrt{H-1}\right)$$

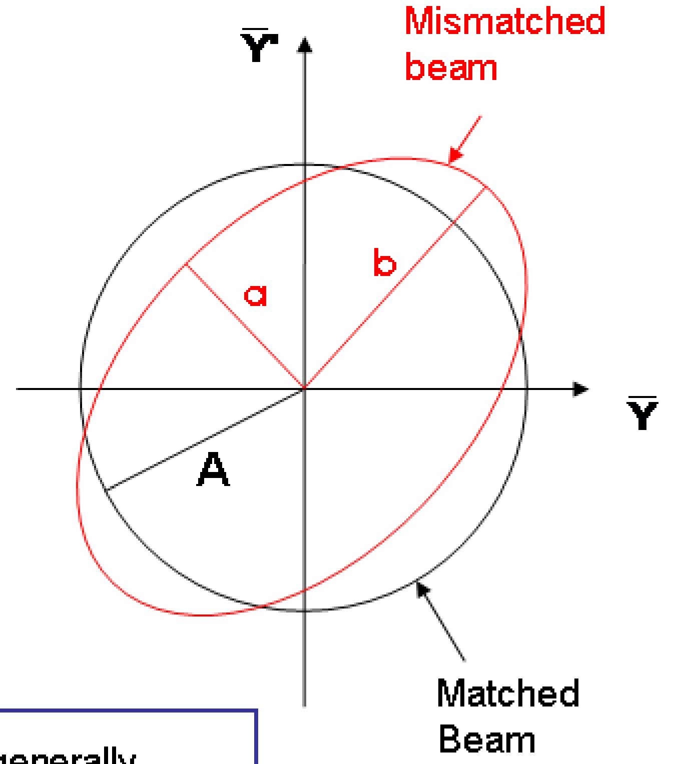
where

$$H = \frac{1}{2}\left(\gamma_{new} + \beta_{new}\right)$$

$$= \frac{1}{2}\left(\frac{\beta_1}{\beta_2} + \frac{\beta_2}{\beta_1}\left(\alpha_1 - \alpha_2\frac{\beta_1}{\beta_2}\right)^2 + \frac{\beta_2}{\beta_1}\right)$$

and thus

$$\lambda = \frac{1}{\sqrt{2}}\left(\sqrt{H+1} + \sqrt{H-1}\right), \qquad \frac{1}{\lambda} = \frac{1}{\sqrt{2}}\left(\sqrt{H+1} - \sqrt{H-1}\right)$$



generally

$$a = A/\lambda$$
$$b = A \cdot \lambda$$

Blow-up from betatron mismatch

We can evaluate the square of the distance of a particle from the origin as

$$\mathbf{A}_{new}^2 = \overline{\mathbf{Y}}_{new}^2 + \overline{\mathbf{Y}}'^2_{new} = \lambda^2 \cdot \mathbf{A}_0^2 \sin^2(\phi + \phi_1) + \frac{1}{\lambda^2} \mathbf{A}_0^2 \cos^2(\phi + \phi_1)$$

The new emittance is the average over all phases

$$\varepsilon_{new} = \frac{1}{2}\left\langle \mathbf{A}_{new}^2 \right\rangle = \frac{1}{2}\left(\lambda^2 \left\langle \mathbf{A}_0^2 \sin^2(\phi + \phi_1) \right\rangle + \frac{1}{\lambda^2} \left\langle \mathbf{A}_0^2 \cos^2(\phi + \phi_1) \right\rangle \right)$$

$$= \frac{1}{2}\left\langle \mathbf{A}_0^2 \right\rangle \left(\lambda^2 \left\langle \sin^2(\phi + \phi_1) \right\rangle + \frac{1}{\lambda^2} \left\langle \mathbf{A}_0^2 \right\rangle \left\langle \cos^2(\phi + \phi_1) \right\rangle \right)$$

$$= \frac{1}{2}\varepsilon_0 \left(\lambda^2 + \frac{1}{\lambda^2} \right)$$

If we like, we can substitute back for λ to give

$$\varepsilon_{new} = \frac{1}{2}\varepsilon_0 \left(\lambda^2 + \frac{1}{\lambda^2} \right) = H\varepsilon_0 = \frac{1}{2}\varepsilon_0 \left(\frac{\beta_1}{\beta_2} + \frac{\beta_2}{\beta_1}\left(\alpha_1 - \alpha_2 \frac{\beta_1}{\beta_2} \right)^2 + \frac{\beta_2}{\beta_1} \right)$$

where subscript 1 refers to the matched ellipse, 2 refers to the mismatched ellipse.

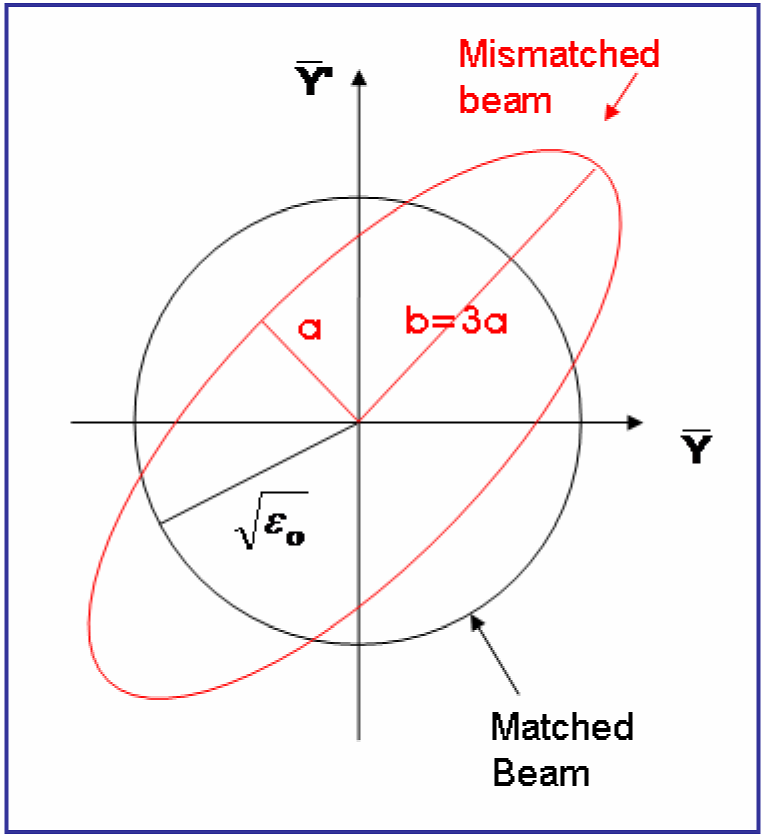
Blow-up from betatron mismatch

A numerical example….

Consider b = $3a$ for the mismatched ellipse

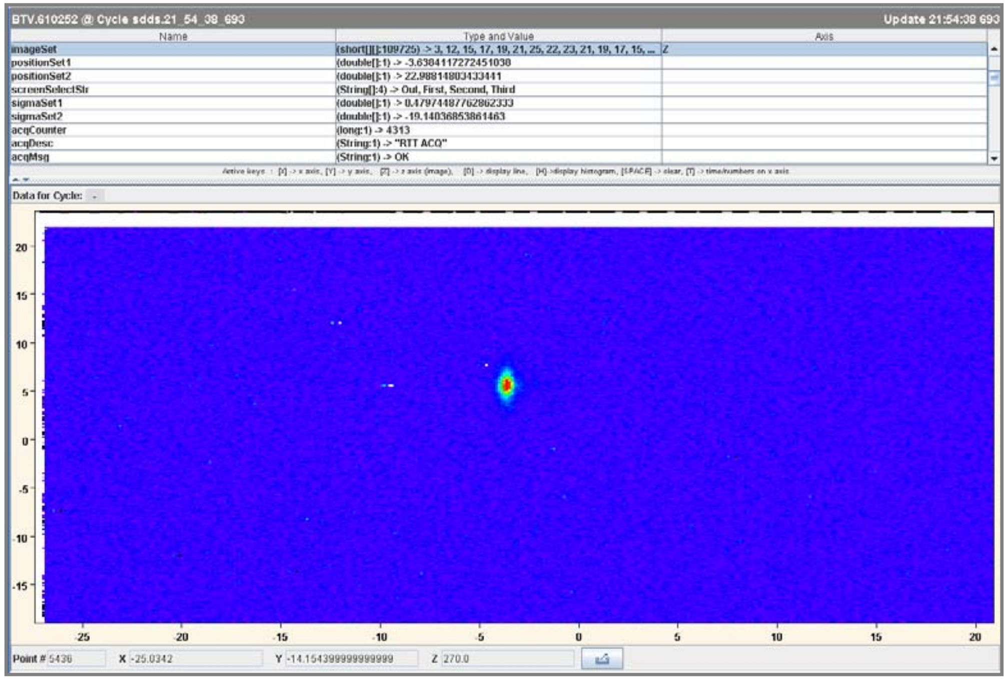
$$\lambda = \sqrt{b/a} = \sqrt{3}$$

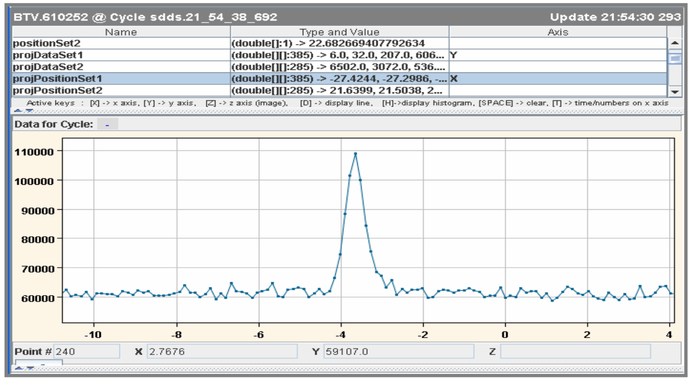
$$\varepsilon_{new} = \frac{1}{2}\varepsilon_0\left(\lambda^2 + 1/\lambda^2\right)$$
$$= 1.67\varepsilon_0$$

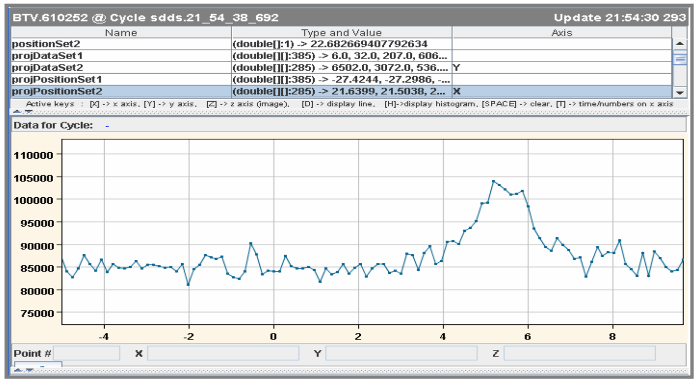


Emittance and mismatch measurement

- Beam screen provides density profile of the beam
- Profile fit gives transverse beam sizes σ.

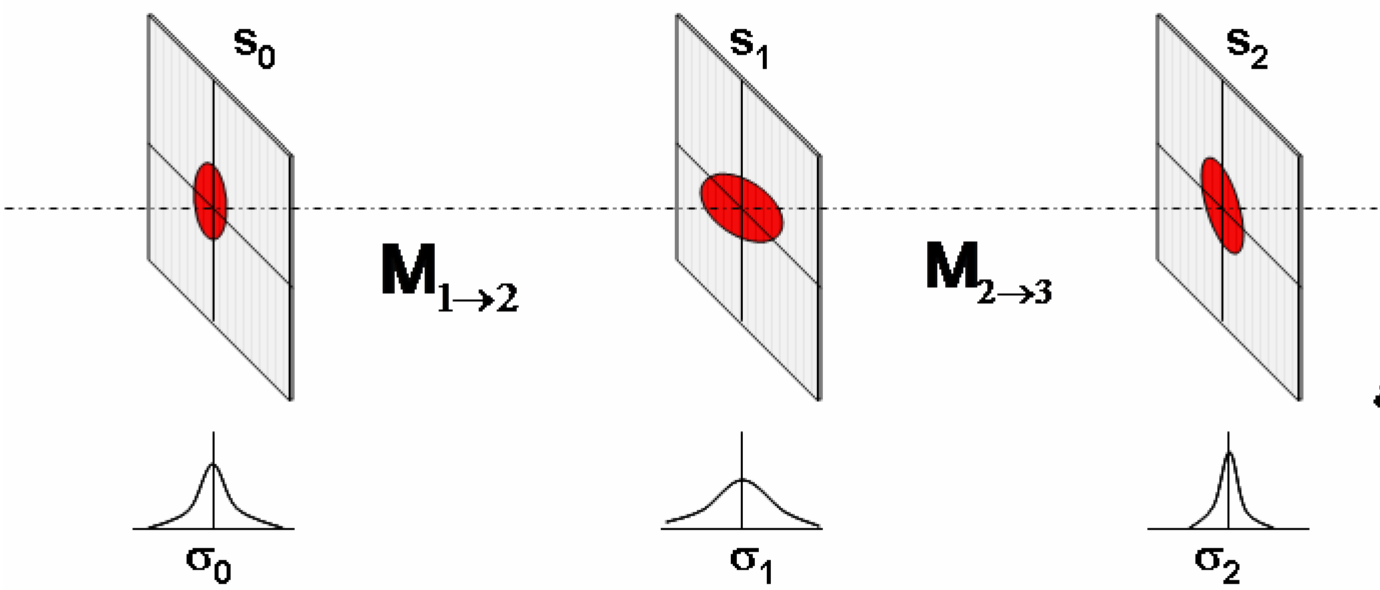






Emittance and mismatch measurement

- In a ring, β is 'known' so ε can be calculated from a single screen
- In a line, need measurements at 3 screens, plus the two transfer matrices M_{01} and M_{12}
- 3 measurement locations allows determination of ε, α and β



$$\varepsilon = \frac{\sigma_0^2}{\beta_0} = \frac{\sigma_1^2}{\beta_1} = \frac{\sigma_2^2}{\beta_2}$$

Emittance and mismatch measurement

We have $$\begin{bmatrix}\beta_1\\ \alpha_1\\ \gamma_1\end{bmatrix}=\begin{bmatrix}C^2 & -2CS & S^2\\ -CC' & CS'+SC' & -SS'\\ C'^2 & -2C'S' & S'^2\end{bmatrix}\cdot\begin{bmatrix}\beta_0\\ \alpha_0\\ \gamma_0\end{bmatrix}$$

so that $$\beta_1=C_1^2\beta_0-2C_1S_1\alpha_0+\frac{S_1^2}{\beta_0}\left(1+\alpha_0^2\right),\quad \beta_2=C_2^2\beta_0-2C_2S_2\alpha_0+\frac{S_2^2}{\beta_0}\left(1+\alpha_0^2\right)$$

using $$\beta_0=\frac{\sigma_0^2}{\varepsilon},\quad \beta_1=\left(\frac{\sigma_1}{\sigma_0}\right)^2\beta_0,\quad \beta_2=\left(\frac{\sigma_2}{\sigma_0}\right)^2\beta_0$$

we find $$\alpha_0=\frac{1}{2}\beta_0\mathbf{W}$$

where $$\mathbf{W}=\frac{\left(\sigma_2/\sigma_0\right)^2/S_2^2-\left(\sigma_1/\sigma_0\right)^2/S_1^2-\left(C_2/S_2\right)^2+\left(C_1/S_1\right)^2}{\left(C_1/S_1\right)-\left(C_2/S_2\right)}$$

Emittance and mismatch measurement

with $$\beta_1 = C_1^2\beta_0 - 2C_1S_1\alpha_0 + \frac{S_1^2}{\beta_0}\left(1+\alpha_0^2\right), \quad \beta_2 = C_2^2\beta_0 - 2C_2S_2\alpha_0 + \frac{S_2^2}{\beta_0}\left(1+\alpha_0^2\right)$$

and $$\beta_0 = \frac{\sigma_0^2}{\varepsilon}, \quad \beta_1 = \left(\frac{\sigma_1}{\sigma_0}\right)^2\beta_0, \quad \beta_2 = \left(\frac{\sigma_2}{\sigma_0}\right)^2\beta_0$$

and $$\mathbf{W} = \frac{(\sigma_2/\sigma_0)^2/S_2^2 - (\sigma_1/\sigma_0)^2/S_1^2 - (C_2/S_2)^2 + (C_1/S_1)^2}{(C_1/S_1) - (C_2/S_2)}$$

Some algebra with the three above equations gives

$$\beta_0 = 1\Big/\left|\sqrt{(\sigma_2/\sigma_0)^2/S_2^2 - (C_2/S_2)^2 + \mathbf{W}(C_2/S_2)^2 - \mathbf{W}^2/4}\right|$$

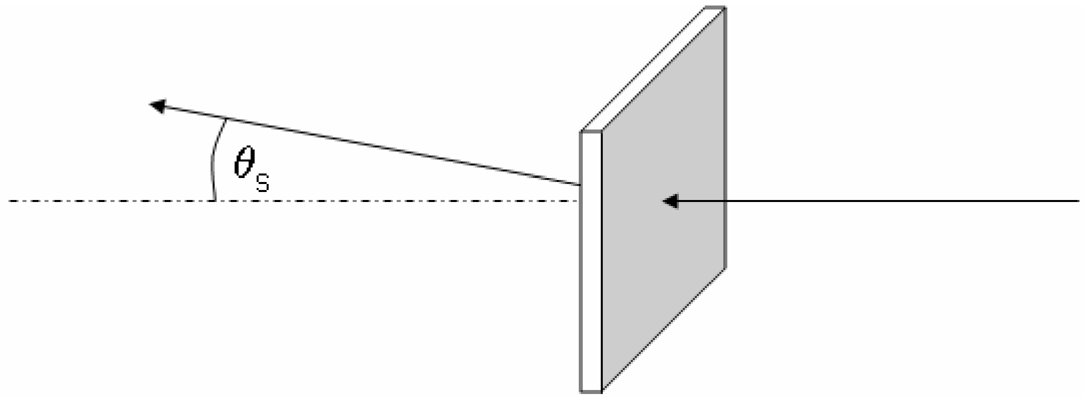
And finally we can evaluate

$$\varepsilon = \sigma_0^2\beta_0$$

Comparing measured α_o, β_0 with expected values gives measurement of mismatch

Blow-up from thin scatterer

- Thin beam screens (Al_2O_3,Ti) used to generate profiles.
- Thin metal windows used to separate vacuum of transfer lines from vacuum in circular machines.
- The emittance of the beam increases when it passes through, due to multiple Coulomb scattering.



rms angle increase: $$\sqrt{\langle \theta_s^2 \rangle}[mrad] = \frac{14.1}{\beta_c p[MeV/c]} Z_{inc} \sqrt{\frac{L}{L_{rad}}} \left(1 + 0.11 \cdot \log_{10} \frac{L}{L_{rad}}\right)$$

$\beta_c = v/c$, p = momentum, Z_{inc} = particle charge /e, L = target length, L_{rad} = radiation length

Blow-up from thin scatterer

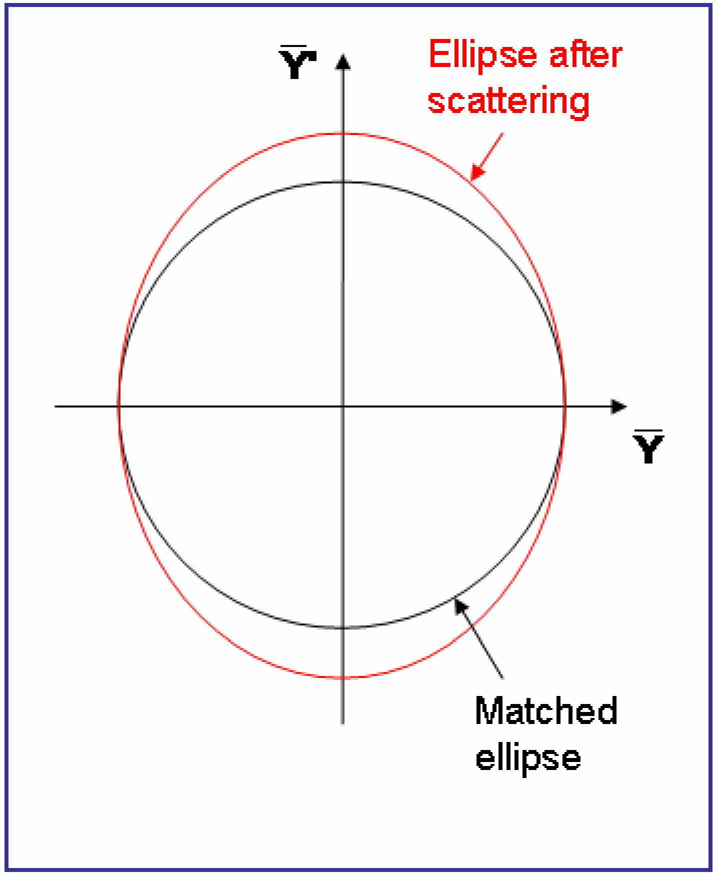
Each particles gets a random angle change θ_s but there is initially no effect on the positions

$$\overline{\mathbf{Y}}_{new} = \overline{\mathbf{Y}}_0$$

$$\overline{\mathbf{Y}}'_{new} = \overline{\mathbf{Y}}'_0 + \theta_s$$

After filamentation the particles have different amplitudes and the beam has a larger emittance

$$\varepsilon = \left\langle \mathbf{A}^2_{new} \right\rangle / 2$$



Blow-up from thin scatterer

$$A^2_{new} = \bar{Y}^2_{new} + \bar{Y}'^2_{new}$$

$$= \bar{Y}_0^2 + \left(\bar{Y}'_0 + \sqrt{\beta}\,\theta_s\right)^2$$

uncorrelated

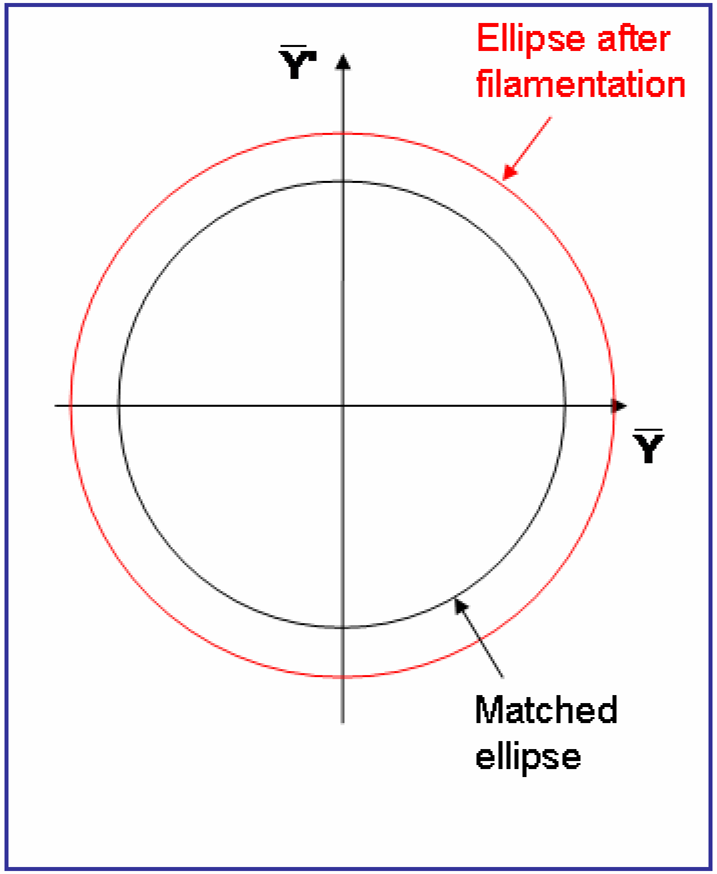
$$= \bar{Y}_0^2 + \bar{Y}'^2_0 + 2\sqrt{\beta}\left(\bar{Y}'_0\theta_s\right) + \beta\theta_s^2$$

$$\left\langle A^2_{new}\right\rangle = \left\langle \bar{Y}_0^2\right\rangle + \left\langle \bar{Y}'^2_0\right\rangle + 2\sqrt{\beta}\left\langle \bar{Y}'_0\theta_s\right\rangle + \beta\left\langle\theta_s^2\right\rangle$$

$$= 2\varepsilon_0 + 2\sqrt{\beta}\,\overset{0}{\cancel{\left\langle \bar{Y}'_0\right\rangle}}\left\langle\theta_s\right\rangle + \beta\left\langle\theta_s^2\right\rangle$$

$$= 2\varepsilon_0 + \beta\left\langle\theta_s^2\right\rangle$$

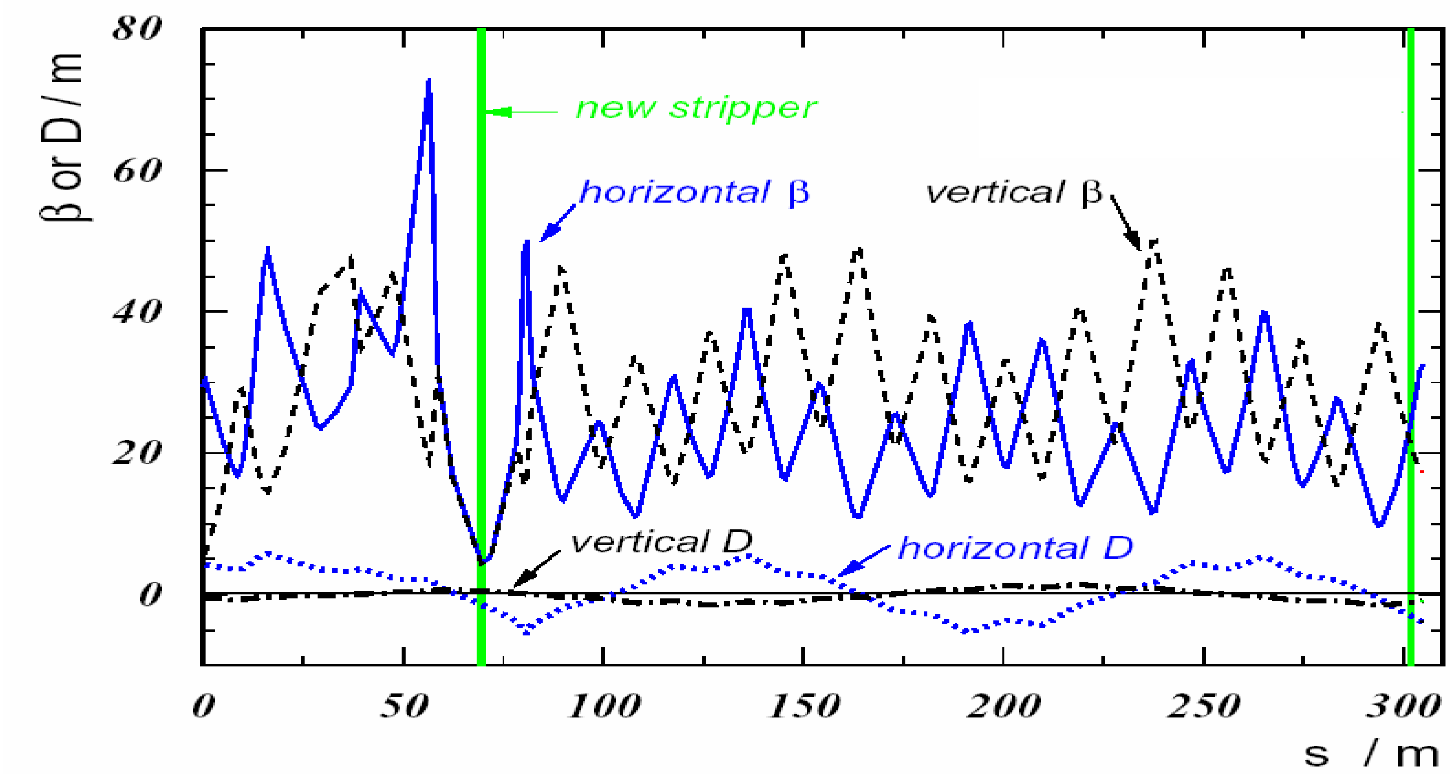
$$\boxed{\varepsilon_{new} = \varepsilon_0 + \frac{\beta}{2}\left\langle\theta_s^2\right\rangle}$$



Need to keep β small to minimise blow-up

An example - charge stripping

- For LHC heavy ions, Pb^{53+} is stripped to Pb^{82+} at 4.25GeV/u using a 0.8mm thick Al foil, in the PS to SPS line
- $\Delta\varepsilon$ minimised with low-β insertion in the transfer line
- Emittance increase expected is about 8%



Summary

- Transfer lines present interesting challenges and differences from circular machines
  – No periodic condition mean optics is defined by initial beam ellipse and transfer line lattice
  – Matching at the extremes is non-trivial
  – Performance criteria include aperture and emittance blow-up