

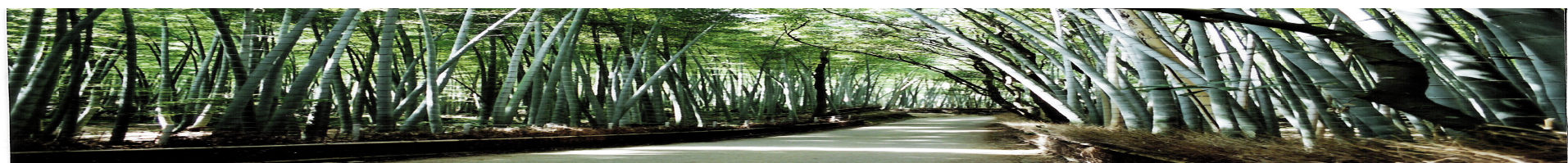
Beam-Beam Effects in Colliders

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Hadron Collective Effects

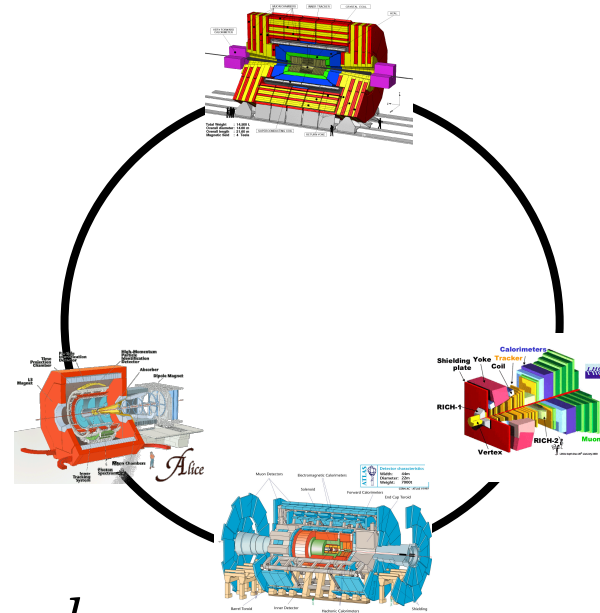
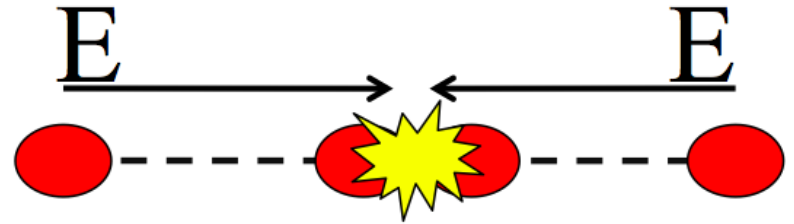


Hadron Colliders

$$E^* \approx 2 \times E$$

$$N_{event}/s = L \cdot \sigma_{event}$$

$$L \propto \frac{N_p^2}{\sigma_x \sigma_y} \cdot n_b \cdot f_{rev}$$



Bunch intensity:

$$N_p = 1.15 - 1.65 \cdot 10^{11} \text{ ppb}$$

Transverse Beam size:

$$\sigma_{x,y} = 16 - 30 \mu m$$

Number of bunches

$$1370 - 2808$$

Revolution frequency

$$11 \text{ kHz}$$

When do we have beam-beam effects?

➤ They occur when two beams get closer and collide

➤ Two types

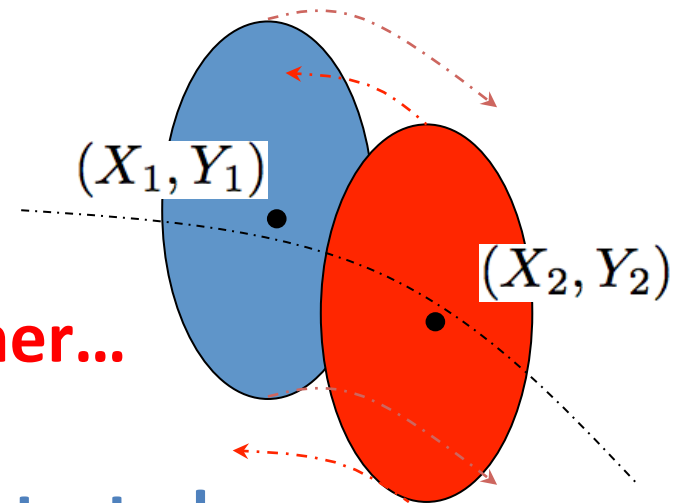
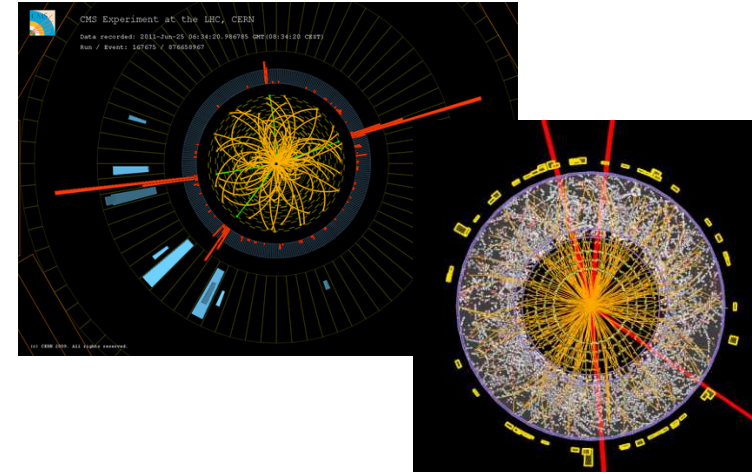
➤ High energy collisions between two particles (**wanted**)

➤ Distortions of beam by electromagnetic forces (**unwanted**)

➤ **Unfortunately: usually both go together...**

➤ 0.001% (or less) of particles collide

➤ 99.999% (or more) of particles are distorted



Beam-beam effects: overview

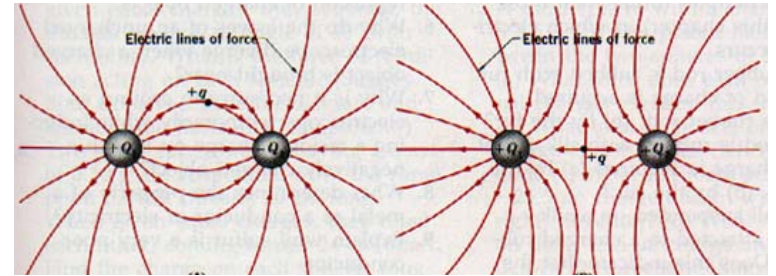
- **Circular Colliders:** interaction occurs at every turn
 - Many effects and problems
 - Try to understand some of them
- Overview of selected effects (single particle and multi-particle effects)
- Qualitative and physical picture of effects
- Observations
- Mathematical derivations and more info in References [1,3,4] or at

Beam-beam webpage <http://lhc-beam-beam.web.cern.ch/lhc-beam-beam/>

And CAS Proceedings

Beams EM potential

- Beam is a collection of charges
- Beam is an electromagnetic potential for other charges

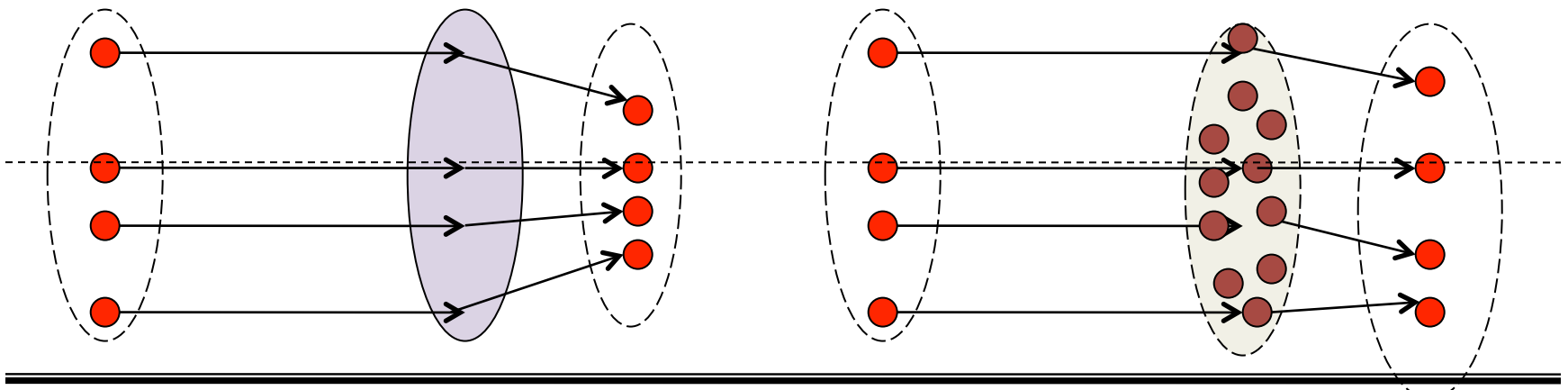


Force on itself (**space charge**) and opposing beam (**beam-beam effects**)

Single particle motion and whole bunch motion **distorted**

Focusing quadrupole

Opposite Beam



A beam acts on particles like an electromagnetic lens, but...

Beam-beam Mathematics

General approach in electromagnetic problems Reference[5] already applied to beam-beam interactions in Reference[1,3, 4]

$$\Delta U = -\frac{1}{\epsilon_0} \rho(x, y, z)$$

Derive potential from Poisson equation for charges with distribution ρ

Solution of Poisson equation

$$U(x, y, z, \sigma_x, \sigma_y, \sigma_z) = \frac{1}{4\pi\epsilon_0} \int \int \int \frac{\rho(x_0, y_0, z_0) dx_0 dy_0 dz_0}{\sqrt{(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2}}$$

$$\vec{E} = -\nabla U(x, y, z, \sigma_x, \sigma_y, \sigma_z)$$

Then compute the fields

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

From Lorentz force one calculates the force acting on test particle with charge q

Making some assumptions we can simplify the problem and derive analytical formula for the force...

Round Gaussian distribution:

Gaussian distribution for charges

Round beams:

Very relativistic, Force has only radial component :

$$\sigma_x = \sigma_y = \sigma$$
$$\beta \approx 1 \quad r^2 = x^2 + y^2$$

$$F \propto \frac{N_p}{\sigma} \cdot \frac{1}{r} \cdot \left[1 - e^{-\frac{r^2}{2\sigma^2}} \right]$$

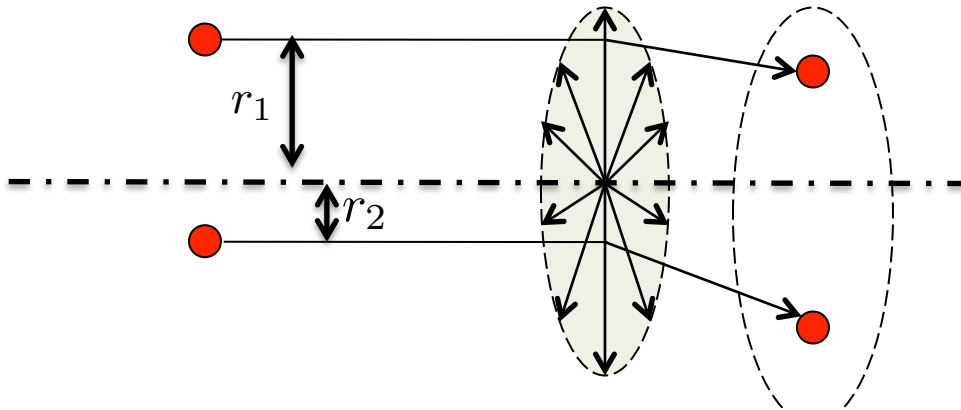
Beam-beam Force

$$\Delta r' = \frac{1}{mc\beta\gamma} \int F_r(r, s, t) dt$$

$$\Delta r' = -\frac{N_p r_0}{r} \cdot \frac{r}{r^2} \left[1 - e^{-\frac{r^2}{2\sigma^2}} \right]$$

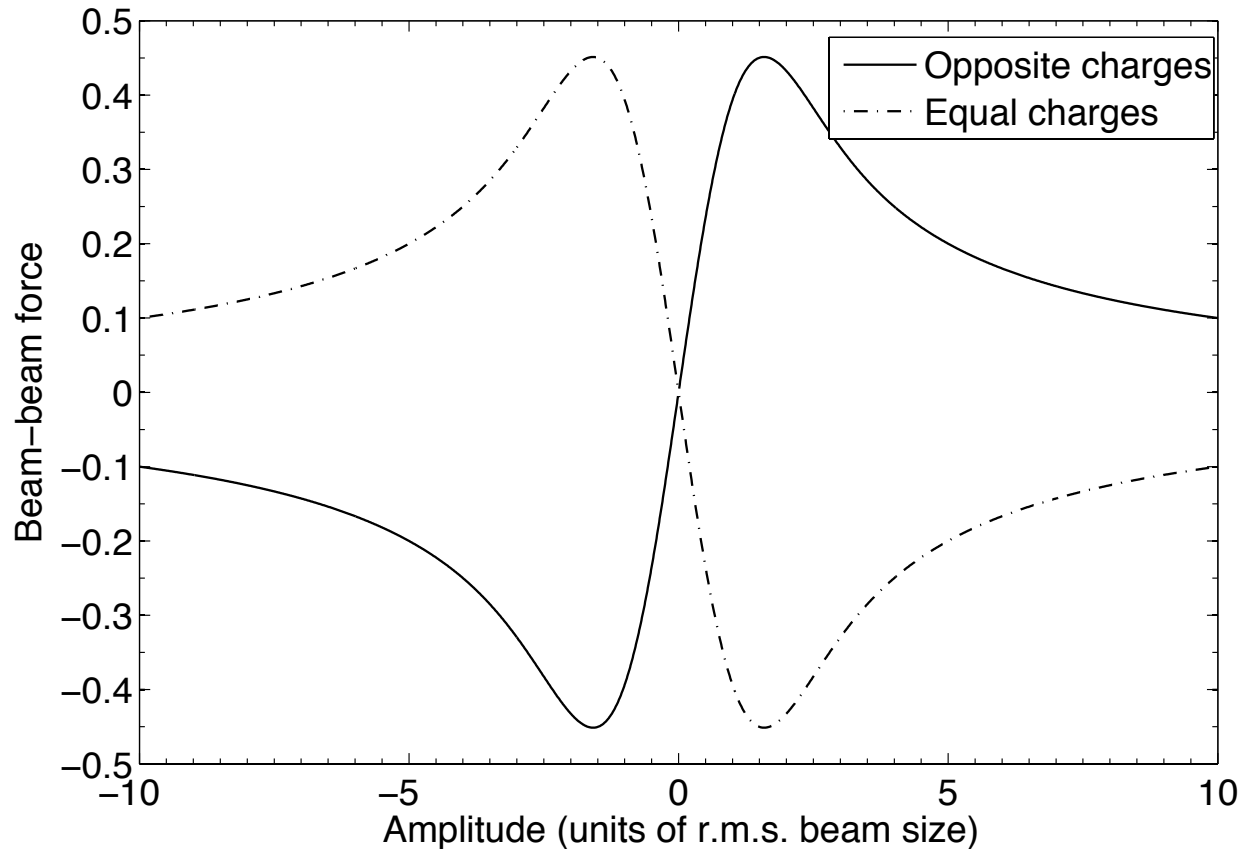
Beam-beam kick obtained
integrating the force over the
collision (i.e. time of passage)

Only radial component in
relativistic case



How does this force looks
like?

Beam-beam Force



$$F_r(r) = \pm \frac{ne^2(1 + \beta_{rel}^2)}{2\pi\epsilon_0} \frac{1}{r} \left[1 - \exp\left(-\frac{r^2}{2\sigma^2}\right) \right]$$

Why do we care?

Pushing for luminosity means stronger beam-beam effects

$$\mathcal{L} \propto \frac{N_p^2}{\sigma_x \sigma_y} \cdot n_b$$

$$F \propto \frac{N_p}{\sigma} \cdot \frac{1}{r} \cdot \left[1 - e^{-\frac{r^2}{2\sigma^2}} \right]$$

Strongest non-linearity in a collider YOU CANNOT AVOID!

Physics fill lasts for many hours 10h – 24h

The screenshot shows the Tribune de Genève website interface. At the top, it displays the date 'Mercredi 4 juillet 2012' and navigation links. The main content area features a physics article titled 'Une nouvelle particule a été découverte' with a green particle detector image. To the right, there is a 'Bourse' section with market data for SMI, Stoxx50, and DJIA. Below the market data is a 'Les plus lus' section with a list of five articles. At the bottom right, there is a blue banner with the text 'Cet été, bien informé rime avec mobilité'.

Bourse	INFORMATION
SMI	6'191.43 -0.04%
Stoxx50	2'437.70 -0.21%
DJIA	12'943.82 +0.56%

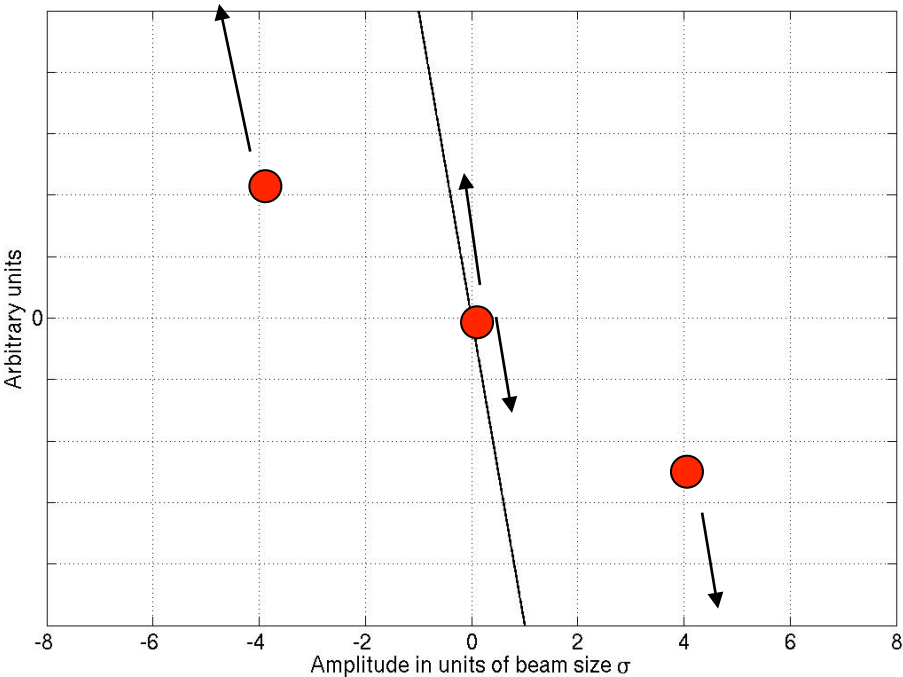
- 1 Le Conseil d'Etat a fait valser ses cadres. Une première
- 2 Les fontaines de Dubaï pleurent Whitney Houston
- 3 Champ-Dollon: pour s'être plaint sur Facebook, un gardien est puni
- 4 La pilote Maria De Villota grièvement blessée
- 5 Sentier des Toblerones: Apple censure la ville de Gland

Two main questions:

- What happens to a single particle?
- What happens to the whole beam?

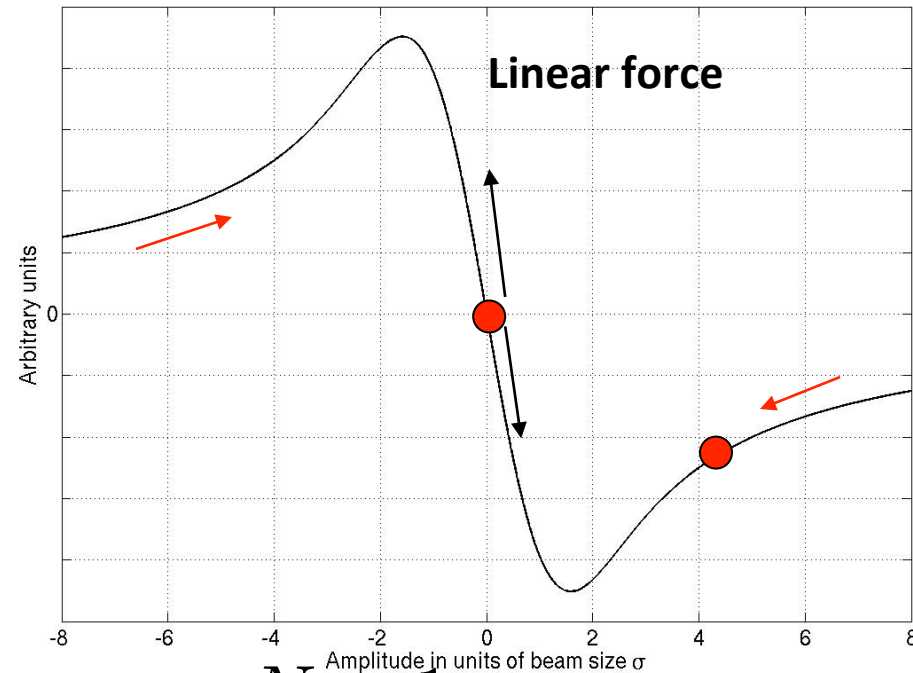
Beam-Beam Force: single particle...

Lattice defocusing quadrupole



$$F = -k \cdot r$$

Beam-beam force



$$F \propto \frac{N_p}{\sigma} \cdot \frac{1}{r} \cdot \left[1 - e^{-\frac{r^2}{2\sigma^2}} \right]$$

For small amplitudes: linear force

For large amplitude: very non-linear

The beam will act as a strong non-linear electromagnetic lens!

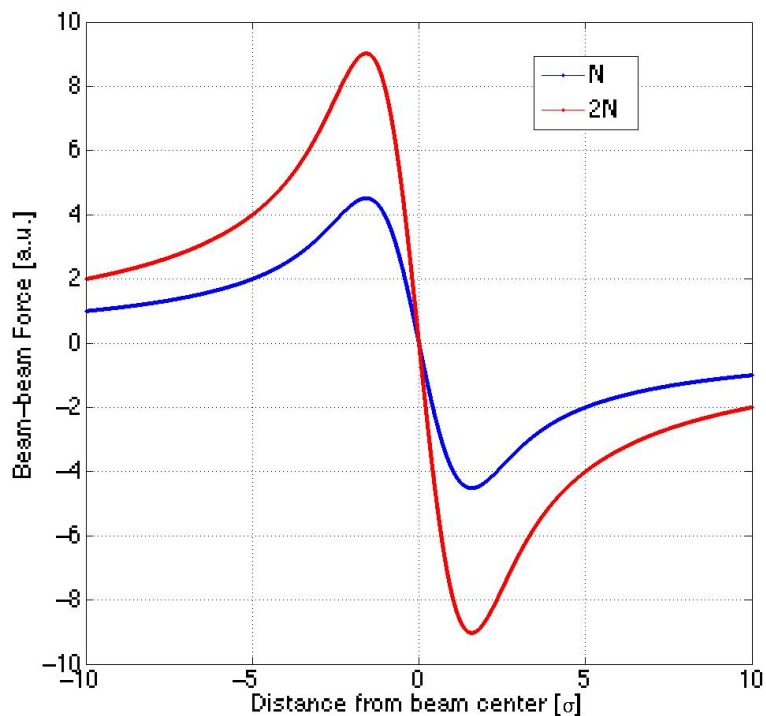
Can we quantify the beam-beam strenght?

Quantifies the strength of the force
but does NOT reflect the nonlinear
nature of the force

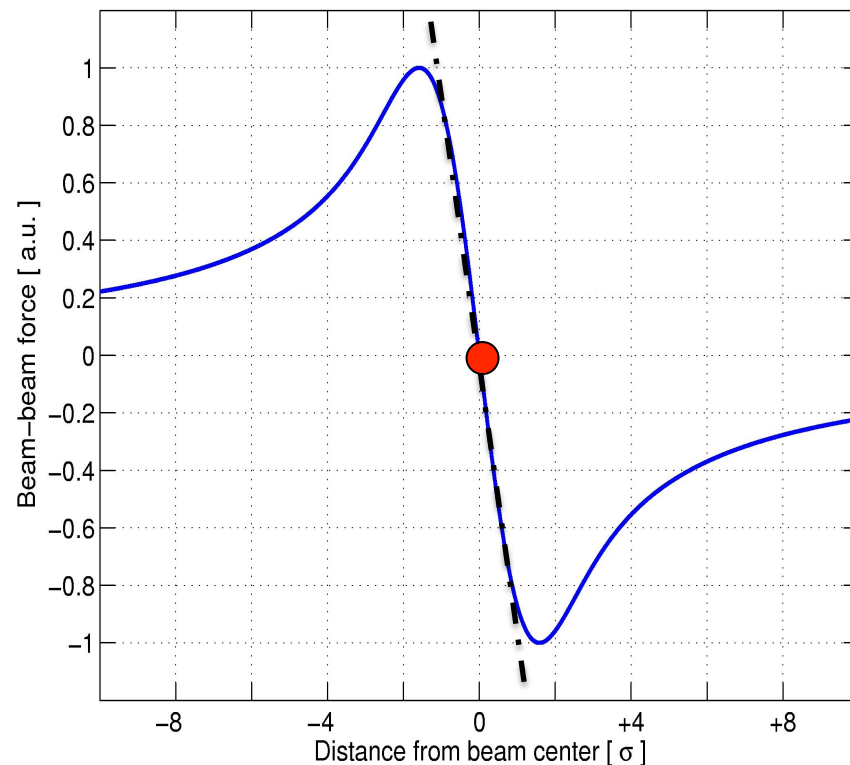
For small amplitudes: linear force

$$F \propto -\xi \cdot r$$

The slope of the force gives you
the **beam-beam parameter** ξ



Beam-beam force



$$\Delta r' = -\frac{N_p r_0}{r} \cdot \frac{r}{r^2} \cdot \left[1 - e^{-\frac{r^2}{2\sigma^2}} \right]$$

$$\Delta r' = \frac{2N_p r_0}{\gamma} \cdot \frac{1}{r} \cdot \left[1 - \left(1 - \frac{r^2}{2\sigma^2} + \dots \right) \right]$$

Beam-Beam parameter:

For round beams:

$$\xi = \frac{\beta^*}{4\pi} \cdot \frac{\delta(\Delta r')}{\delta r} = \frac{Nr_0\beta^*}{4\pi\gamma\sigma^2}$$

For non-round beams:

$$\xi_{x,y} = \frac{Nr_0\beta_{x,y}^*}{2\pi\gamma\sigma_{x,y}(\sigma_x + \sigma_y)}$$

Examples:

Parameters	LEP (e ⁺ e ⁻)	LHC(pp)
Intensity N _{p,e} /bunch	4 10 ¹¹	1.15 10 ¹¹
Energy GeV	100	7000
Beam size H	160-200 μm	16.6 μm
Beam size V	2-4 μm	16.6 μm
β _{x,y} * m	1.25-0.05	0.55-0.55
Crossing angle μrad	0	285
ξ_{bb}	0.07	0.0037

Beam-Beam parameter:

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Examples:

Parameters	LEP (e ⁺ e ⁻)	LHC(pp)	LHC 2012
Intensity N _{p,e} /bunch	4 10 ¹¹	1.15 10 ¹¹	1.7 10 ¹¹
Energy GeV	100	7000	7000
Beam size H	160-200 μm	16.6 μm	18 μm
Beam size V	2-4 μm	16.6 μm	18 μm
β _{x,y} * m	1.25-0.05	0.55-0.55	0.6-0.6
Crossing angle μrad	0	285	290
ξ_{bb}	0.07	0.0037	0.009

LHC 2012 had a Beam-Beam Parameter

Linear Tune shift

For small amplitudes beam-beam can be approximated as linear force as a quadrupole

$$F \propto -\xi \cdot r$$

Focal length:

$$\frac{1}{f} = \frac{\Delta x'}{x} = \frac{Nr_0}{\gamma\sigma^2} = \frac{\xi \cdot 4\pi}{\beta^*}$$

Beam-beam matrix:

$$\begin{pmatrix} 1 & 0 \\ -\frac{\xi \cdot 4\pi}{\beta^*} & 1 \end{pmatrix}$$

Perturbed one turn matrix with perturbed tune ΔQ and beta function at the IP β^* :

$$\begin{pmatrix} \cos(2\pi(Q + \Delta Q)) & \beta^* \sin(2\pi(Q + \Delta Q)) \\ -\frac{1}{\beta^*} \sin(2\pi(Q + \Delta Q)) & \cos(2\pi(Q + \Delta Q)) \end{pmatrix} \\ = \begin{pmatrix} 1 & 0 \\ -\frac{1}{2f} & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos(2\pi Q) & \beta_0^* \sin(2\pi Q) \\ -\frac{1}{\beta_0^*} \sin(2\pi Q) & \cos(2\pi Q) \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ -\frac{1}{2f} & 1 \end{pmatrix}$$

Linear tune

Solving the one turn matrix one can derive the tune shift ΔQ and the perturbed beta function at the IP β^* :

Tune is changed

$$\cos(2\pi(Q + \Delta Q)) = \cos(2\pi Q) - \frac{\beta_0^* \cdot 4\pi\xi}{\beta^*} \sin(2\pi Q)$$

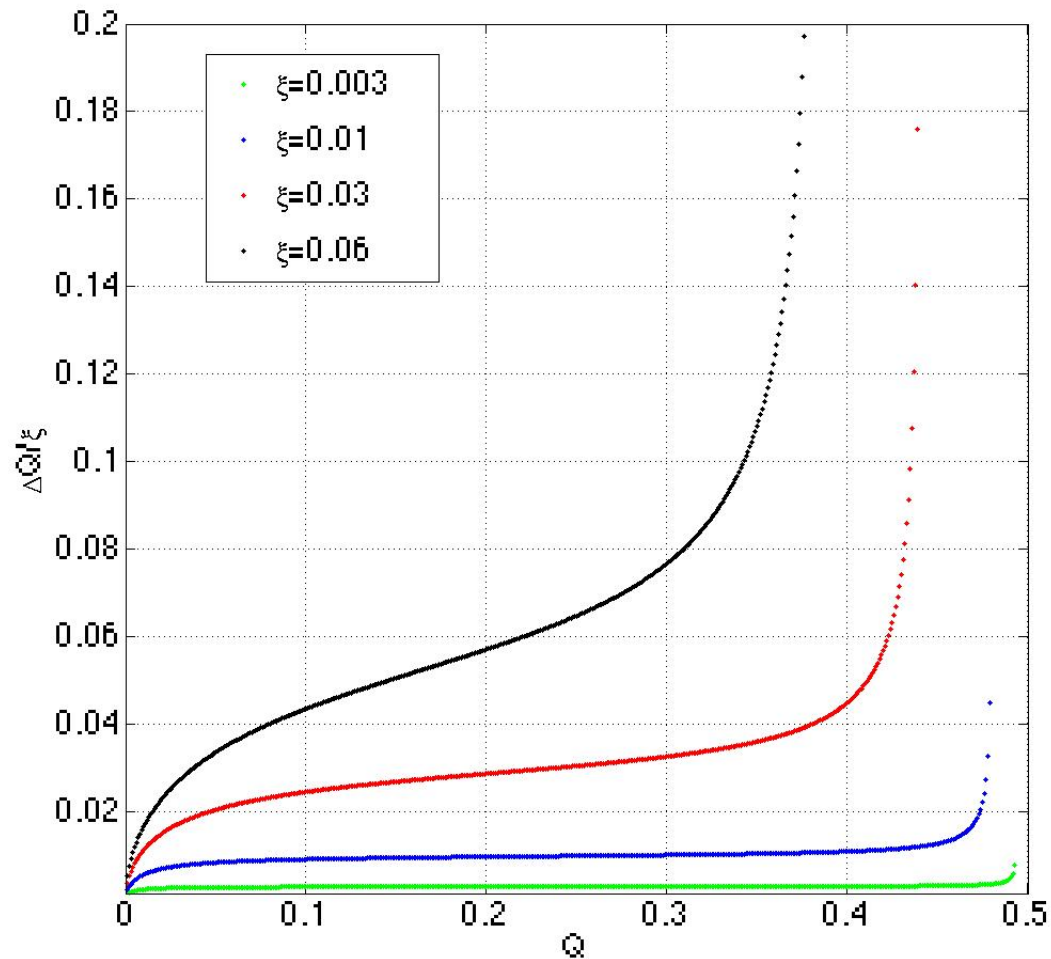
β -function is changed:

$$\frac{\beta^*}{\beta_0^*} = \frac{\sin(2\pi Q)}{\sin(2\pi(Q + \Delta Q))}$$

...how do they change?

Tune dependence of tune shift and dynamic beta

Tune shift as a function of tune



Larger ξ

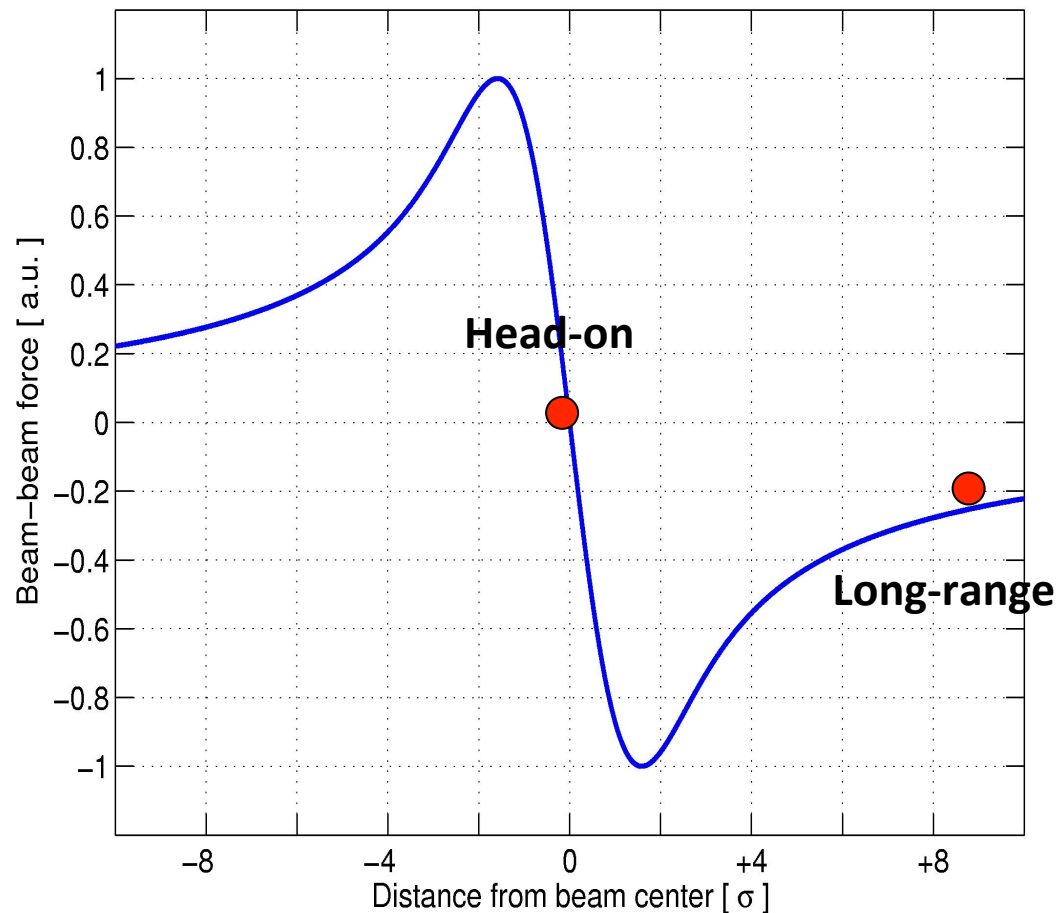


Strongest variation with Q

Head-on and Long-range interactions

Beam-beam force

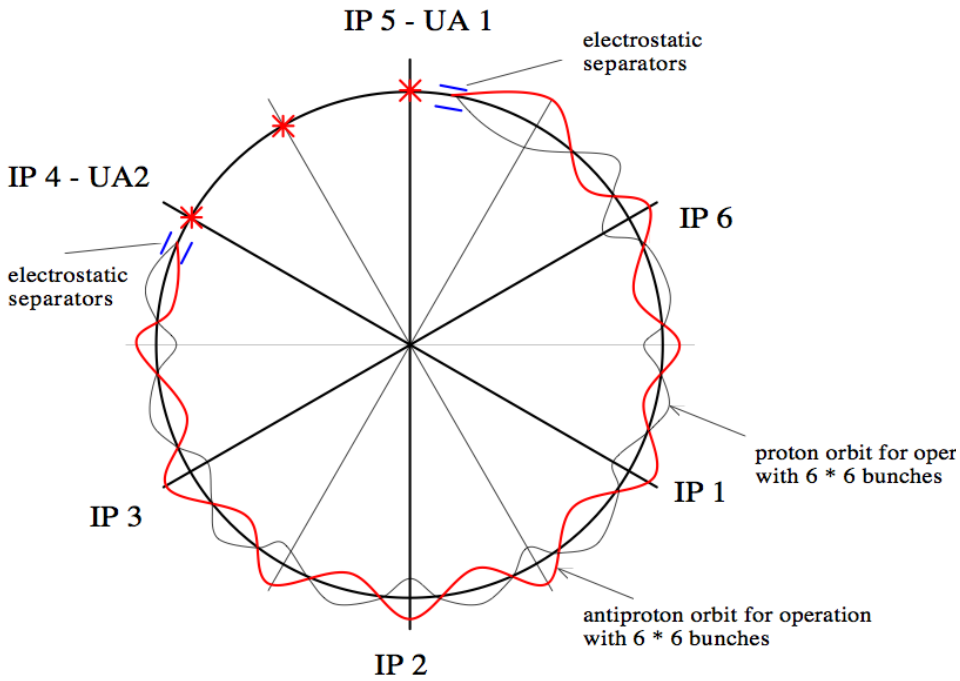
$$L \propto \frac{N_p^2}{\sigma_x \sigma_y} \cdot n_b \cdot f_{rev}$$



Other beam passing in the center force: **HEAD-ON** beam-beam interaction

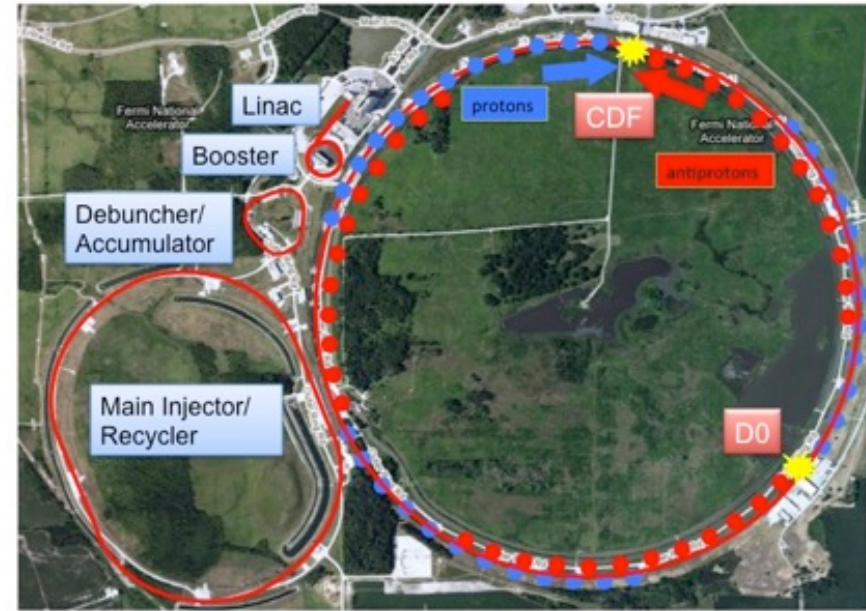
Other beam passing at an offset of the force: **LONG-RANGE** beam-beam interaction

**SPS collider: 6 bunches
3 HO and 9 LR**



Circular colliders HO and LR

$$L \propto \frac{N_p^2}{\sigma_x \sigma_y} \cdot n_b \cdot f_{rev}$$



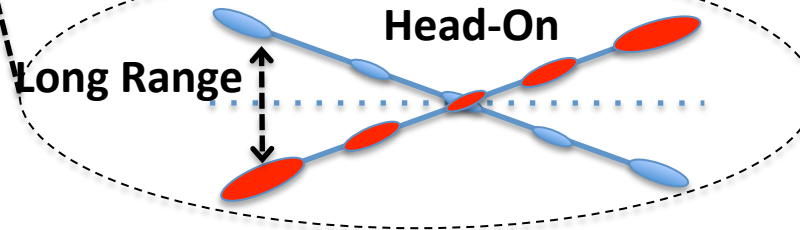
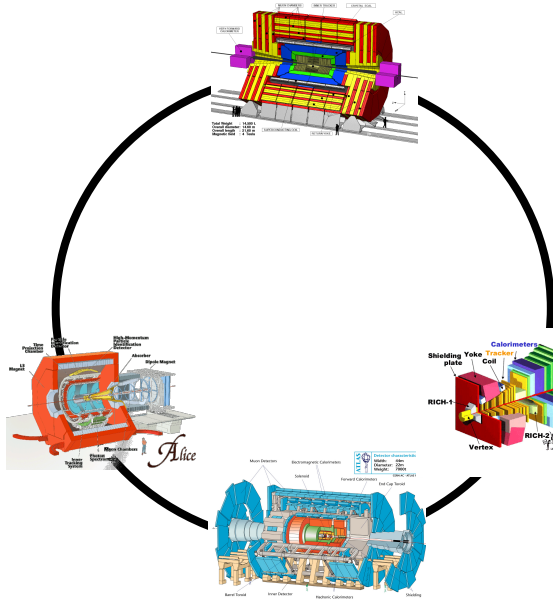
**Tevatron: 36 bunches
2 BBIs Head-on and 72 Long-range**



**RHIC: 110 bunches
2 BBIs Head-on**

LHC, KEKB... colliders

- Crossing angle operation
- High number of bunches in train structures



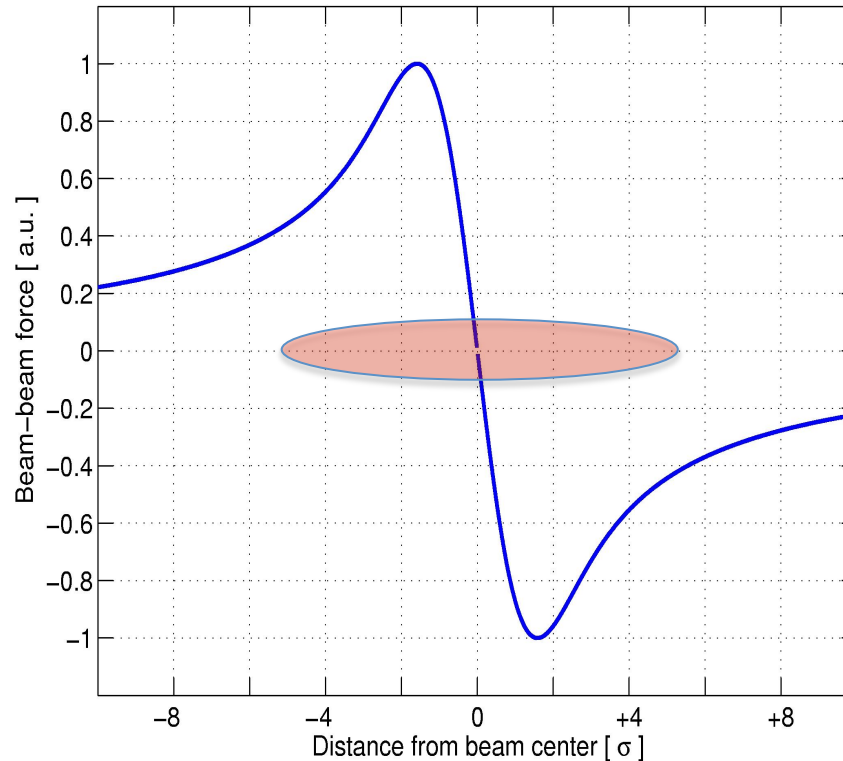
72 bunches

|||||

	SppS	Tevatron	RHIC	LHC
Number Bunches	6	36	109	2808
LR interactions	9	70	0	120/40
Head-on interactions	3	2	2	4

A beam is a collection of particles

Beam-beam force

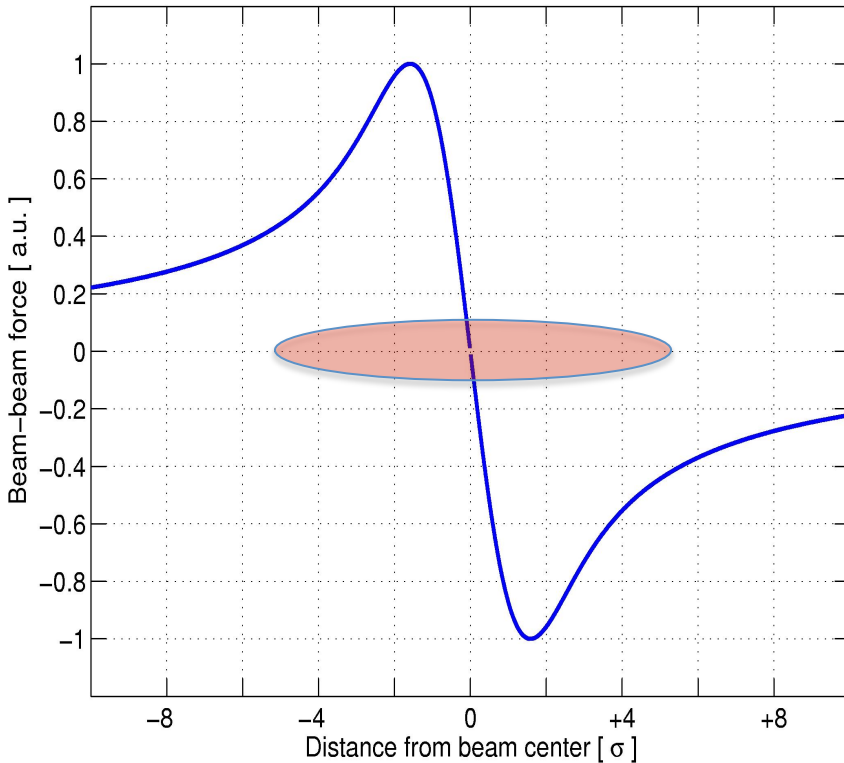


**Beam 2 passing in the center of force produce by Beam 1
Particles of Beam 2 will experience different ranges of the beam-beam forces**

**Tune shift as a function of amplitude (detuning with amplitude or
tune spread)**

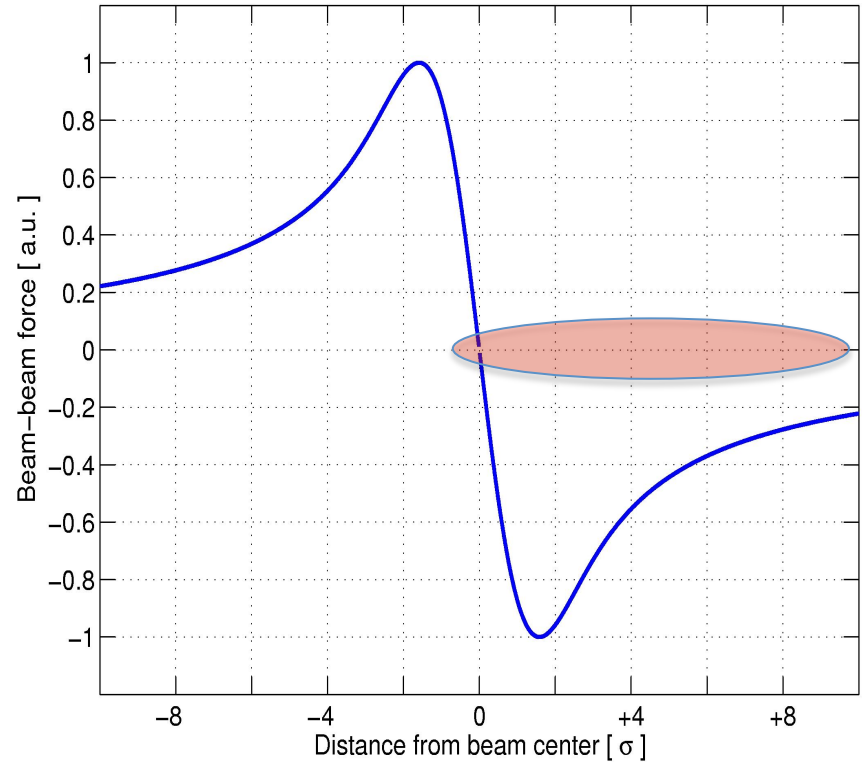
A beam will experience all the force range

Beam-beam force



Second beam passing in the center
HEAD-ON beam-beam interaction

Beam-beam force

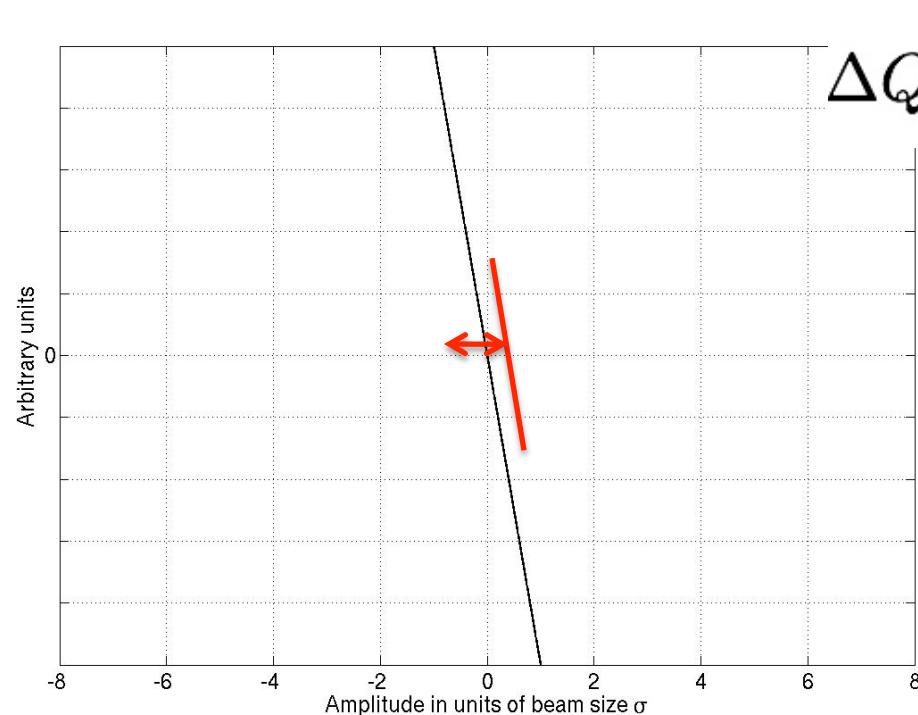


Second beam displaced offset
LONG-RANGE beam-beam interaction

Different particles will see different force

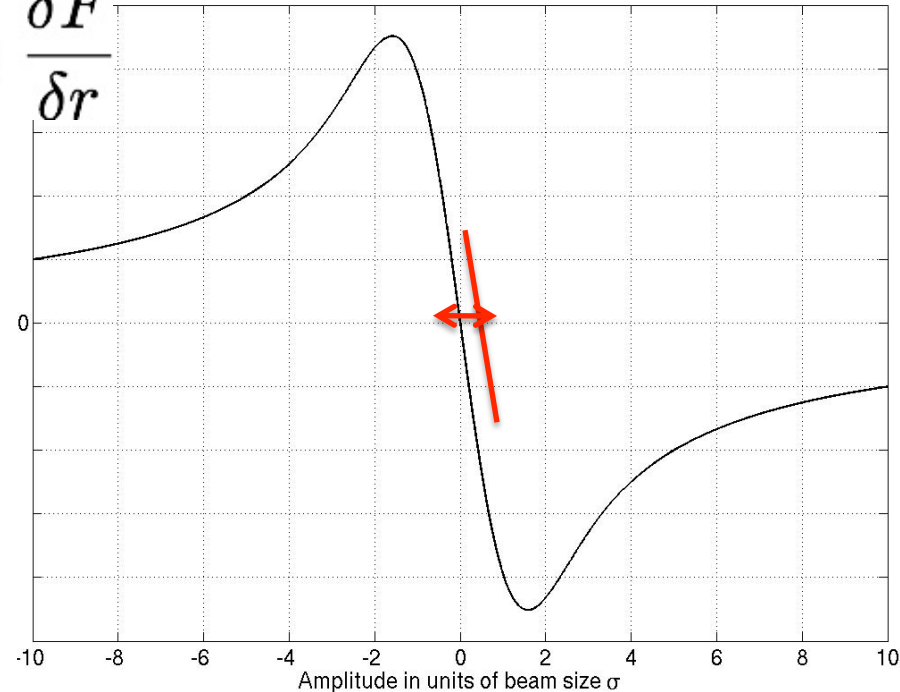
Detuning with Amplitude for head-on

Instantaneous tune shift of test particle when it crosses the other beam is related to the derivative of the force with respect to the amplitude



$$\Delta Q_{quad} = const$$

$$\Delta Q \propto \frac{\delta F}{\delta r}$$



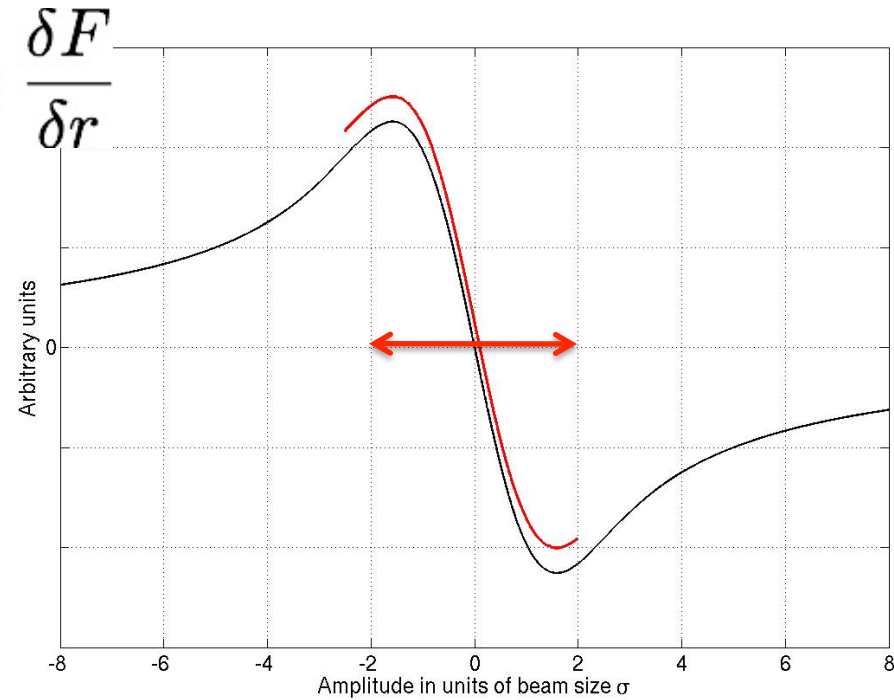
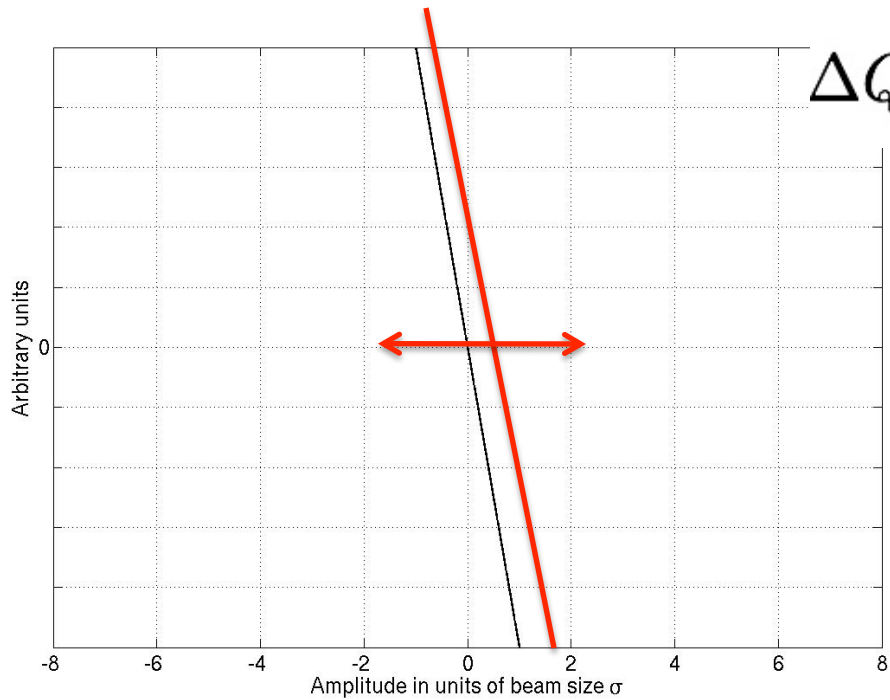
$$\Delta Q_{bb} \approx const$$

For small amplitude test particle
linear tune shift

$$\lim_{r \rightarrow 0} \Delta Q(r) = -\frac{Nr_0\beta^*}{4\pi\gamma\sigma^2} = \xi$$

Detuning with Amplitude for head-on

Beam with many particles this results in a tune spread



$$\Delta Q_{quad} = const$$

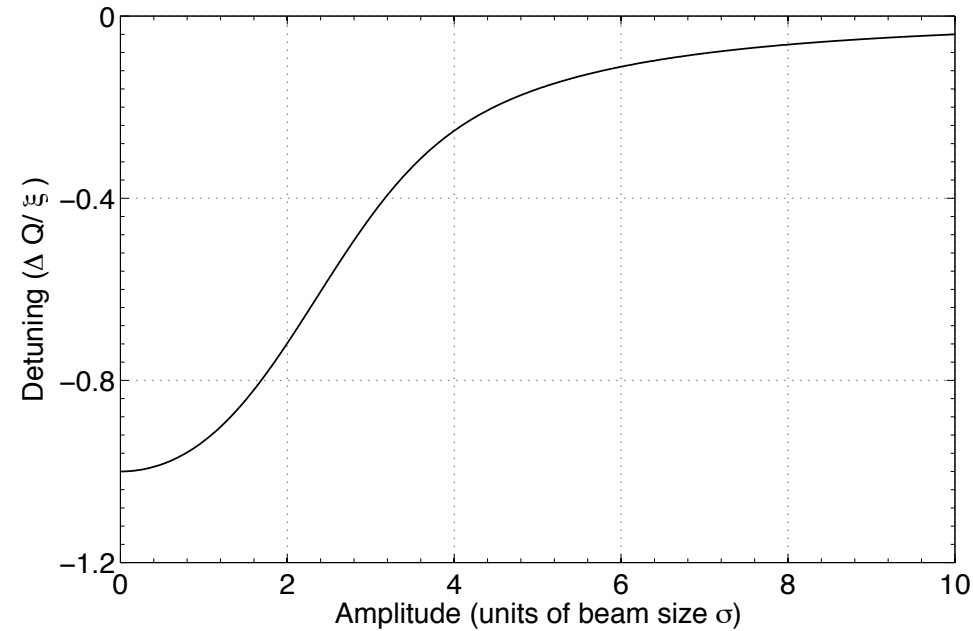
$$\Delta Q_{bb} \neq const$$

$$\Delta Q(x) = \frac{Nr_0\beta}{4\pi\gamma\sigma^2} \cdot \frac{1}{(\frac{x}{2})^2} \cdot (\exp -(\frac{x}{2})^2 I_0 (\frac{x}{2})^2 - 1)$$

Mathematical derivation in Ref [3] using Hamiltonian formalism and in Ref [4] using Lie Algebra

Head-on detuning with amplitude and footprints

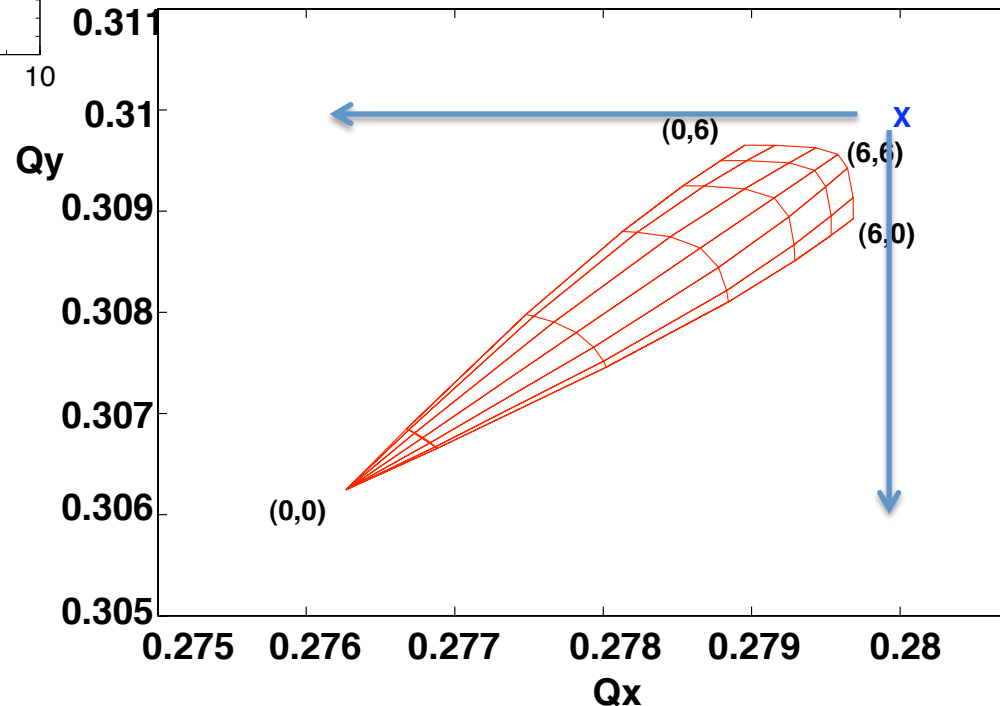
1-D plot of detuning with amplitude



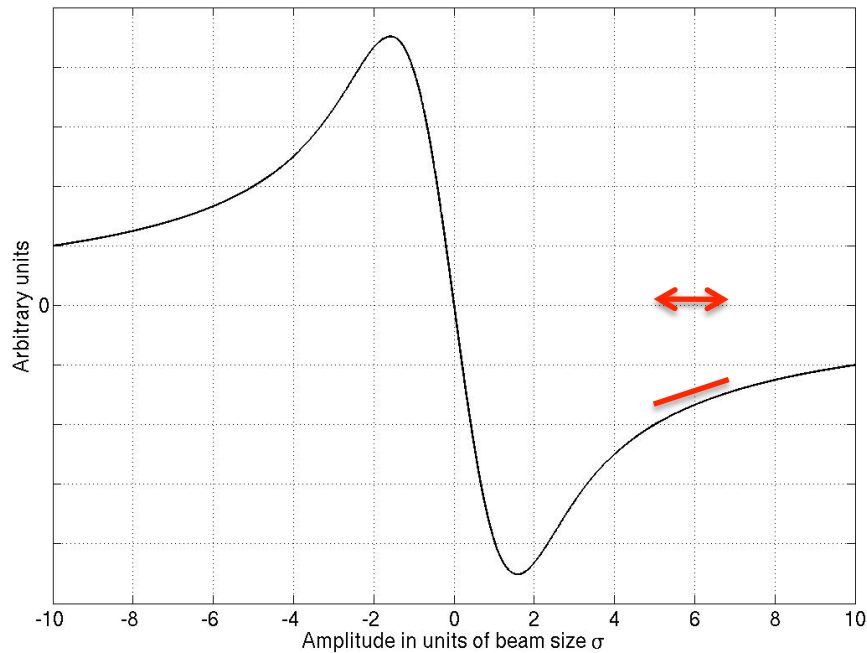
FOOTPRINT
2-D mapping of the detuning with
amplitude of particles

And in the other plane?
THE SAME DERIVATION
same tune spread

Tune footprint for head-on collision



And for long-range interactions?

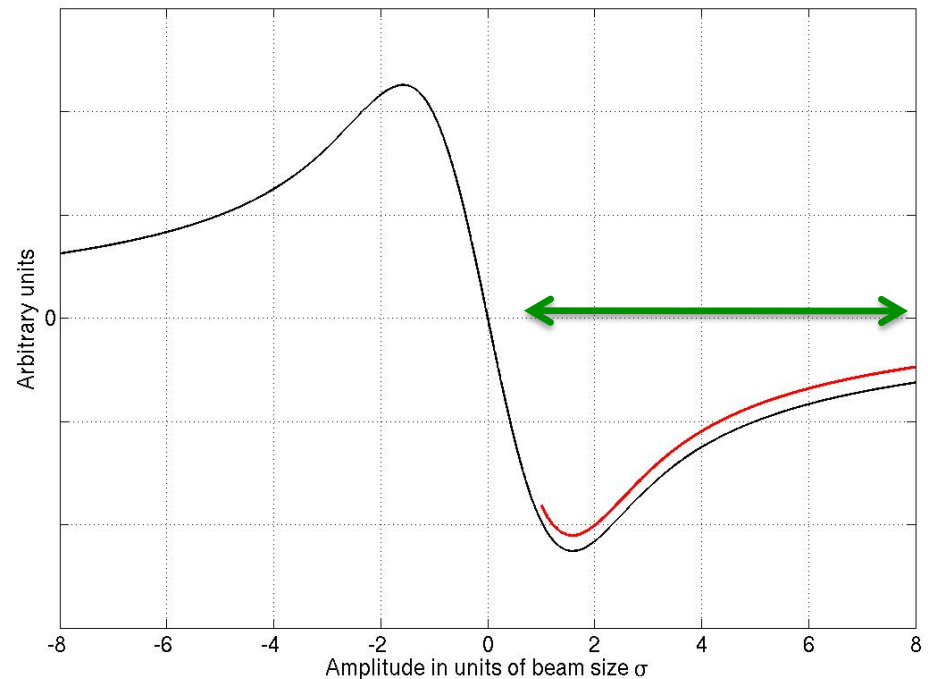


Second beam centered at d (i.e. 6σ)

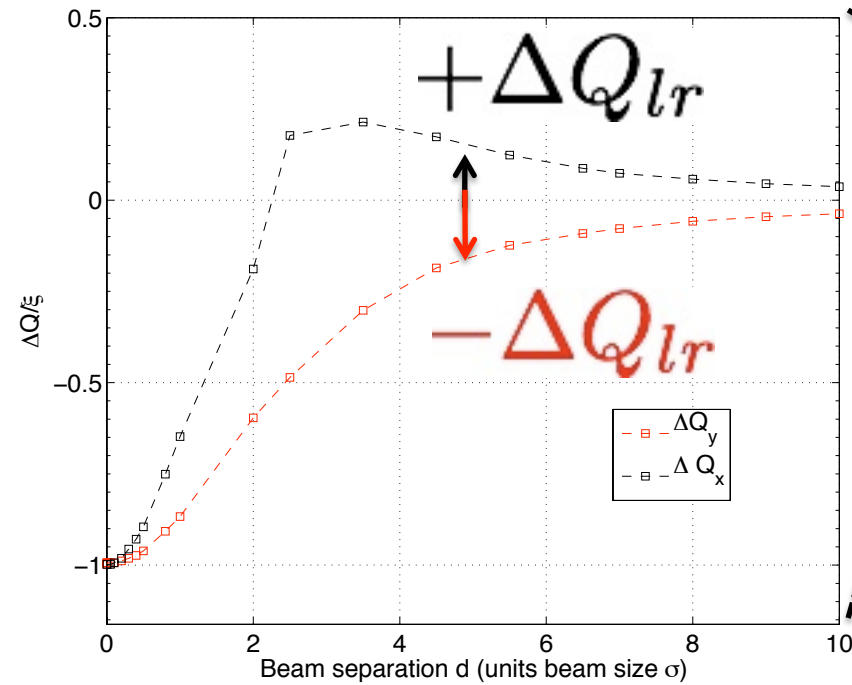
- Small amplitude particles **positive tune shifts**
- Large amplitude can go to **negative tune shifts**

Long range tune shift scaling for distances $d > 6\sigma$

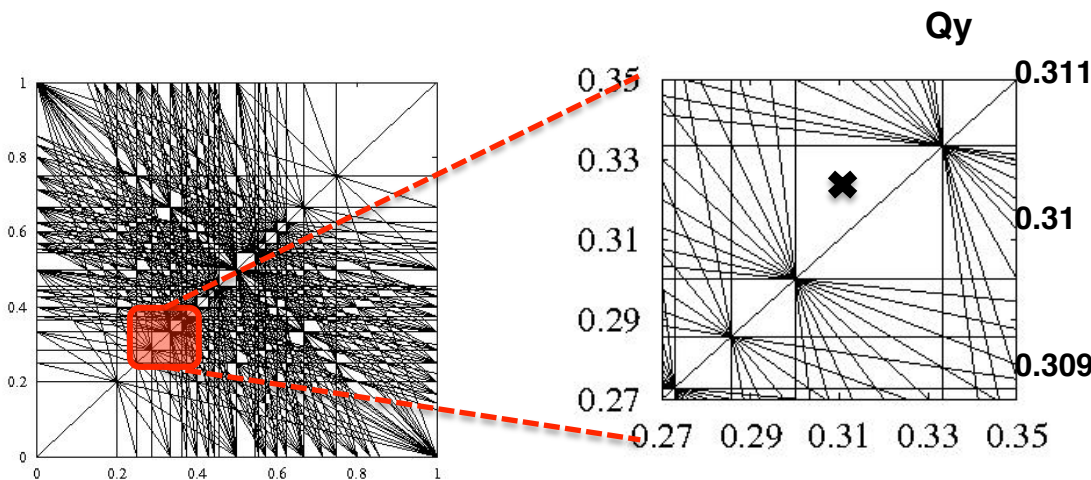
$$\Delta Q_{lr} \propto -\frac{N}{d^2}$$



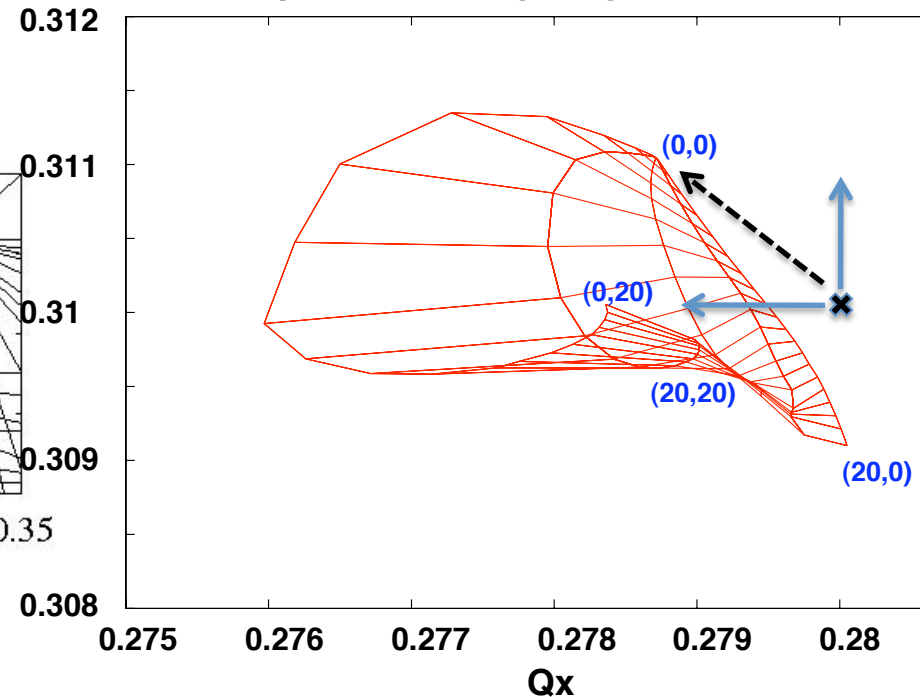
Beam-beam tune shift and speed: 1HO + 1LR



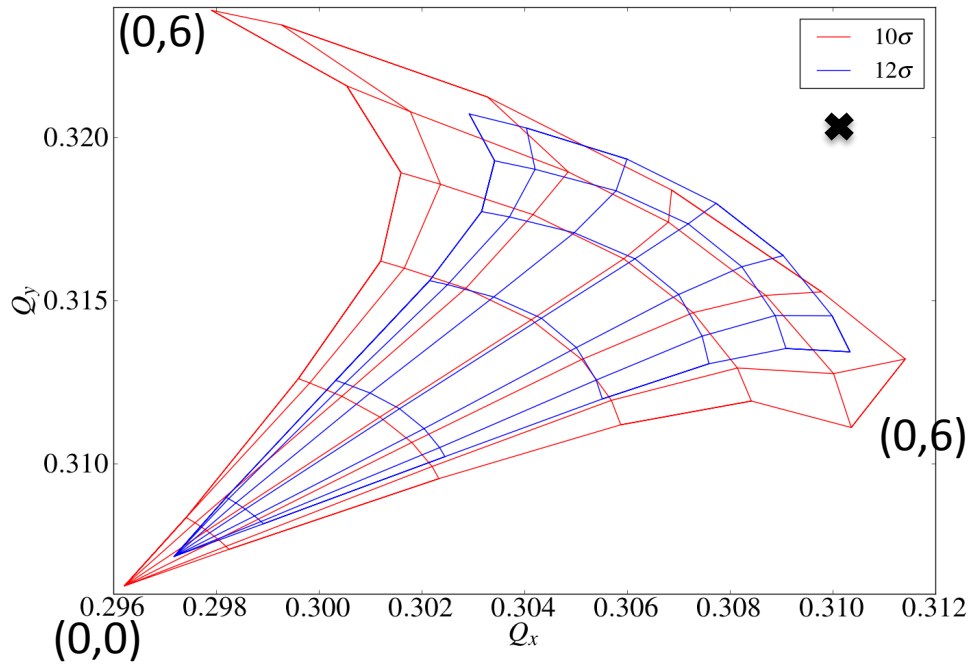
Tune shift as a function of separation in horizontal plane
In the horizontal plane long range tune shift
In the vertical plane opposite sign!



footprint from long range interactions



Beam-beam tune shift and spread

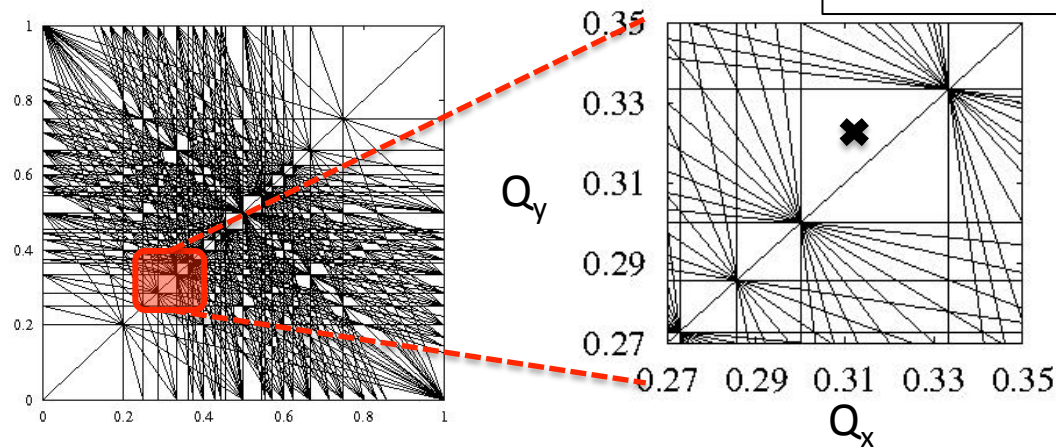


Footprints depend on:

- number of interactions (124 per turn)
- Type (Head-on and long-range)
- Separation
- Plane of interaction

Very complicated depending on collision scheme

Pushing luminosity increases this area while we need to keep it small to avoid resonances and preserve the stability of particles



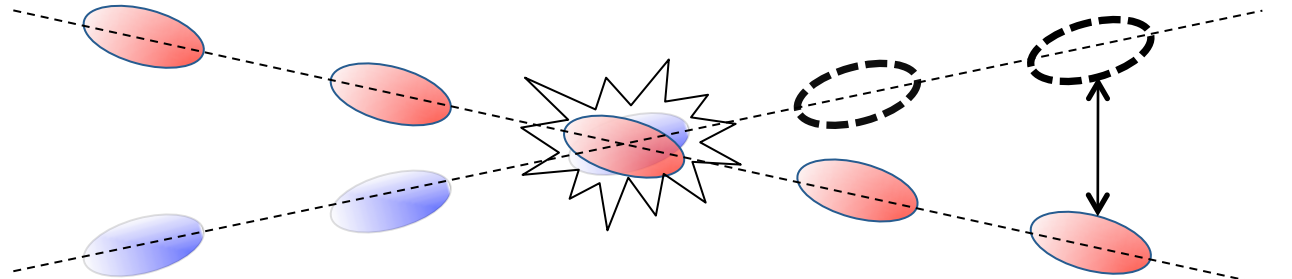
Complications

PACMAN and SUPER PACMAN bunches



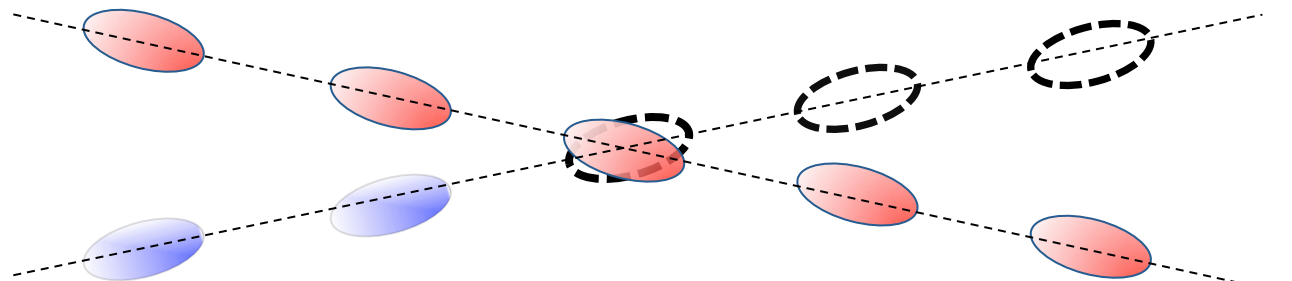
Pacman:

miss long range BBI
(120-40 LR interactions)



Super Pacman:

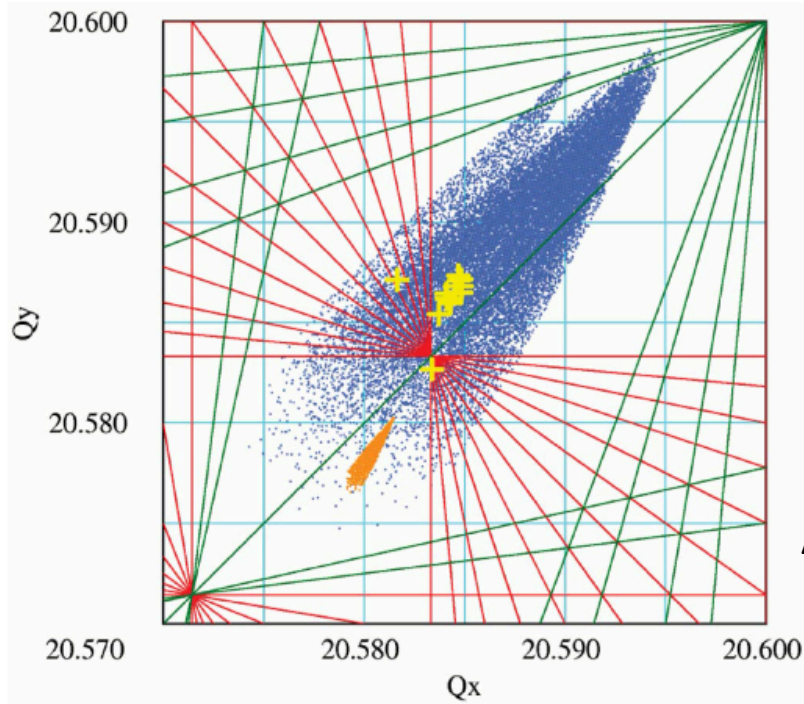
miss head-on BBI
IP2 and IP8 depending on filling scheme



Different bunch families: Pacman and Super Pacman

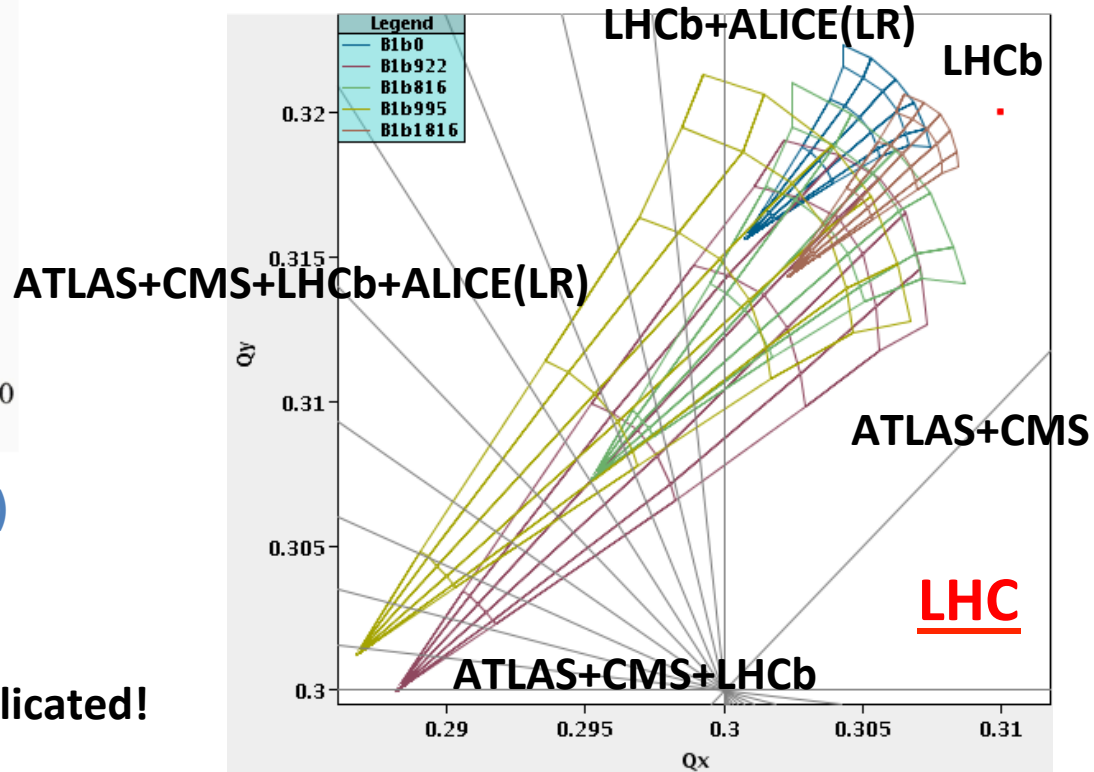
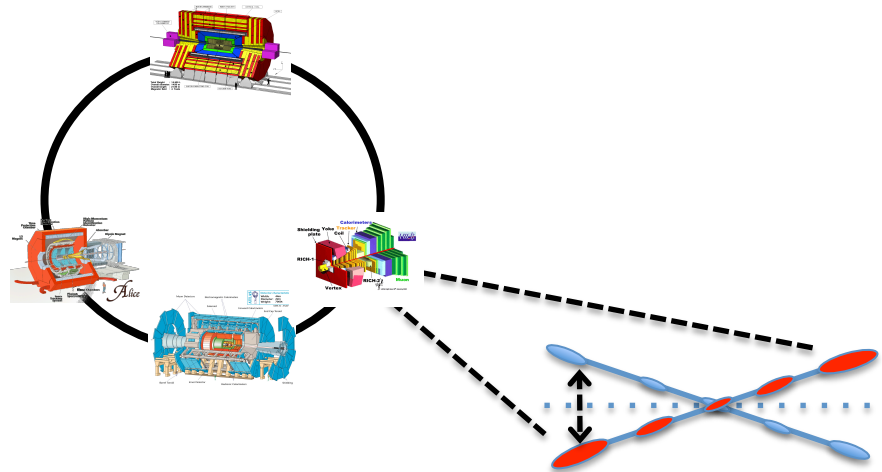
Pacman and Super-pacman

Tevatron



Antiproton bunches footprint (blue)
Proton bunches footprint (orange)

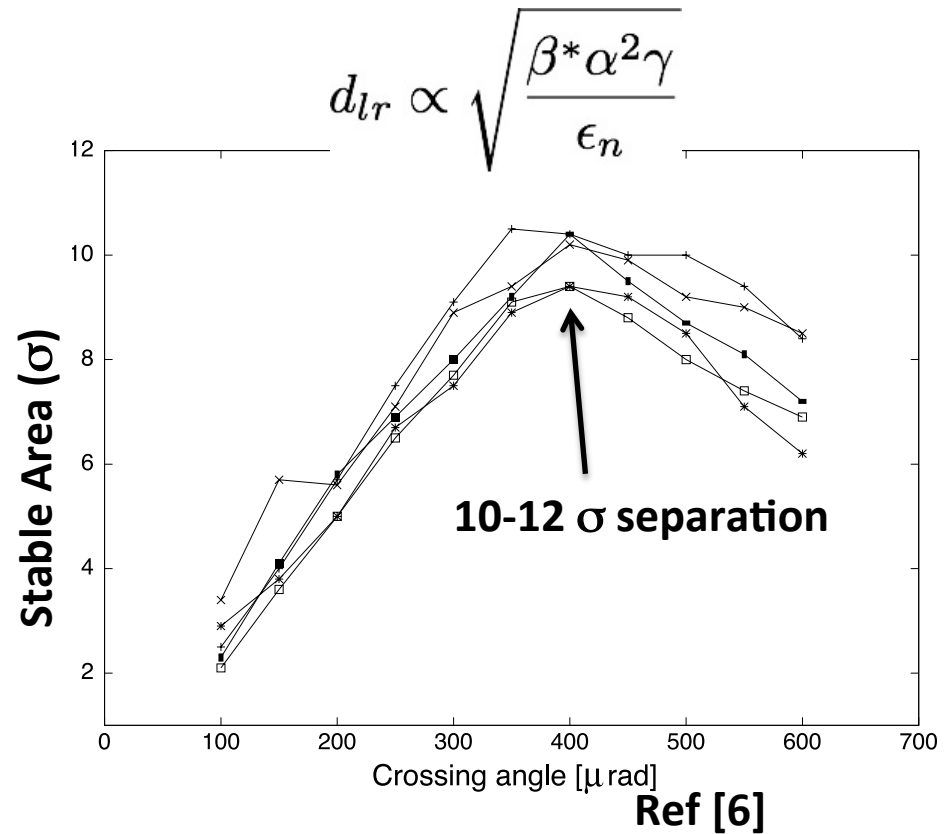
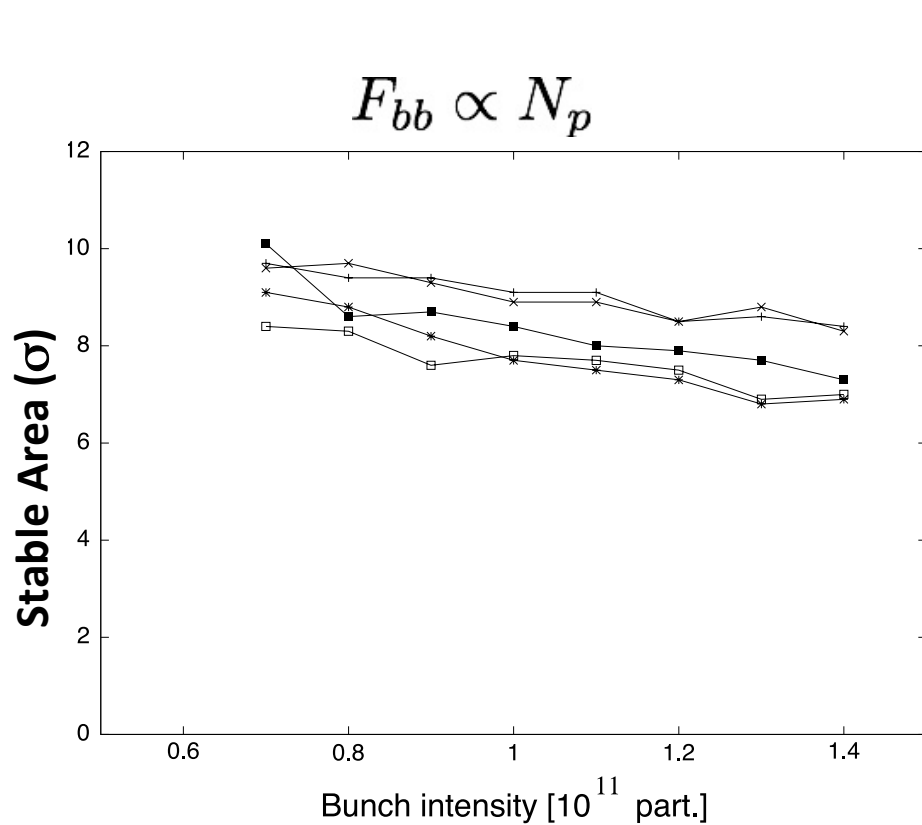
...operationally it is even more complicated!
...intensities, emittances...



Particle Losses

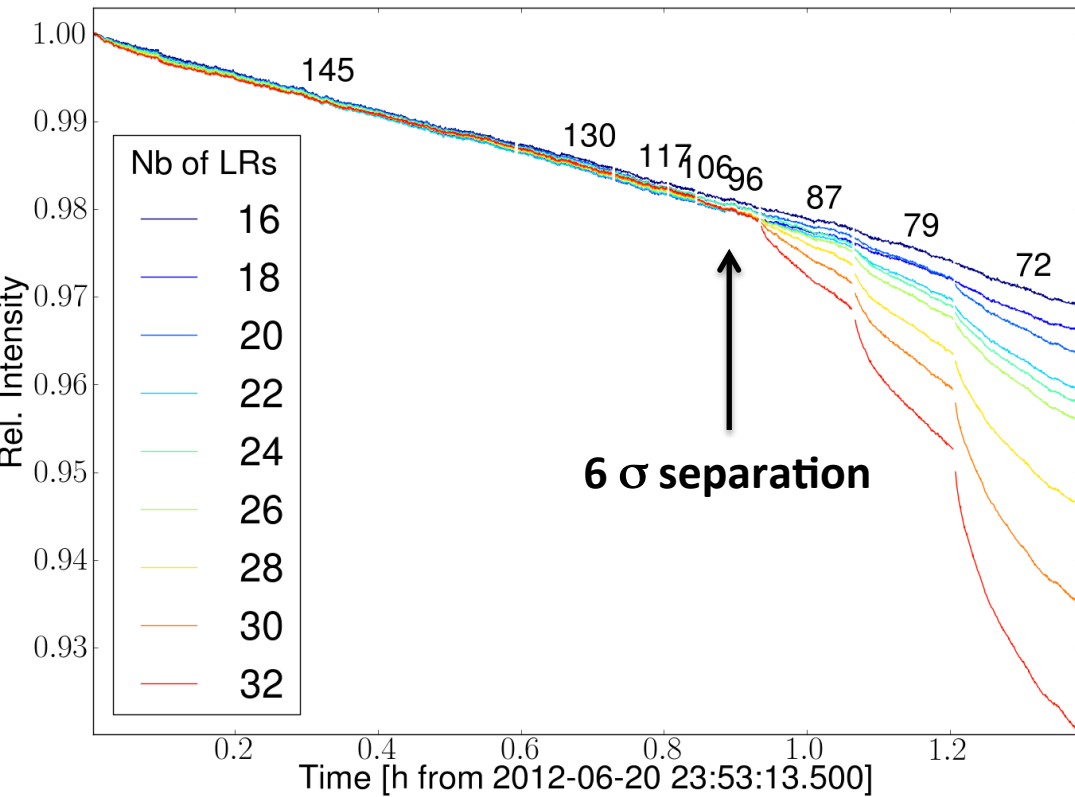
Dynamic Aperture: area in amplitude space with stable motion

Stable area of particles depends on beam intensity and crossing angle



Stable area depends on beam-beam interactions therefore the choice of running parameters (crossing angles, β^* , intensity) is the result of careful study of different effects!

DO we see the effects of LR in the LHC?



Small crossing angle = small separation

If separation of long range too small particles become unstable and get lost

Particle losses follow number of Long range interactions

Observations in Leptons:

From our known formulas:

$$L = \frac{N_1 N_2 f n_b}{4\pi\sigma_x\sigma_y} \quad \xi_{x,y} = \frac{Nr_0\beta_{x,y}^*}{2\pi\gamma\sigma_{x,y}(\sigma_x + \sigma_y)}$$

Increasing bunch population N_1 and N_2 :

- luminosity should increase N_2
- beam-beam parameter linearly

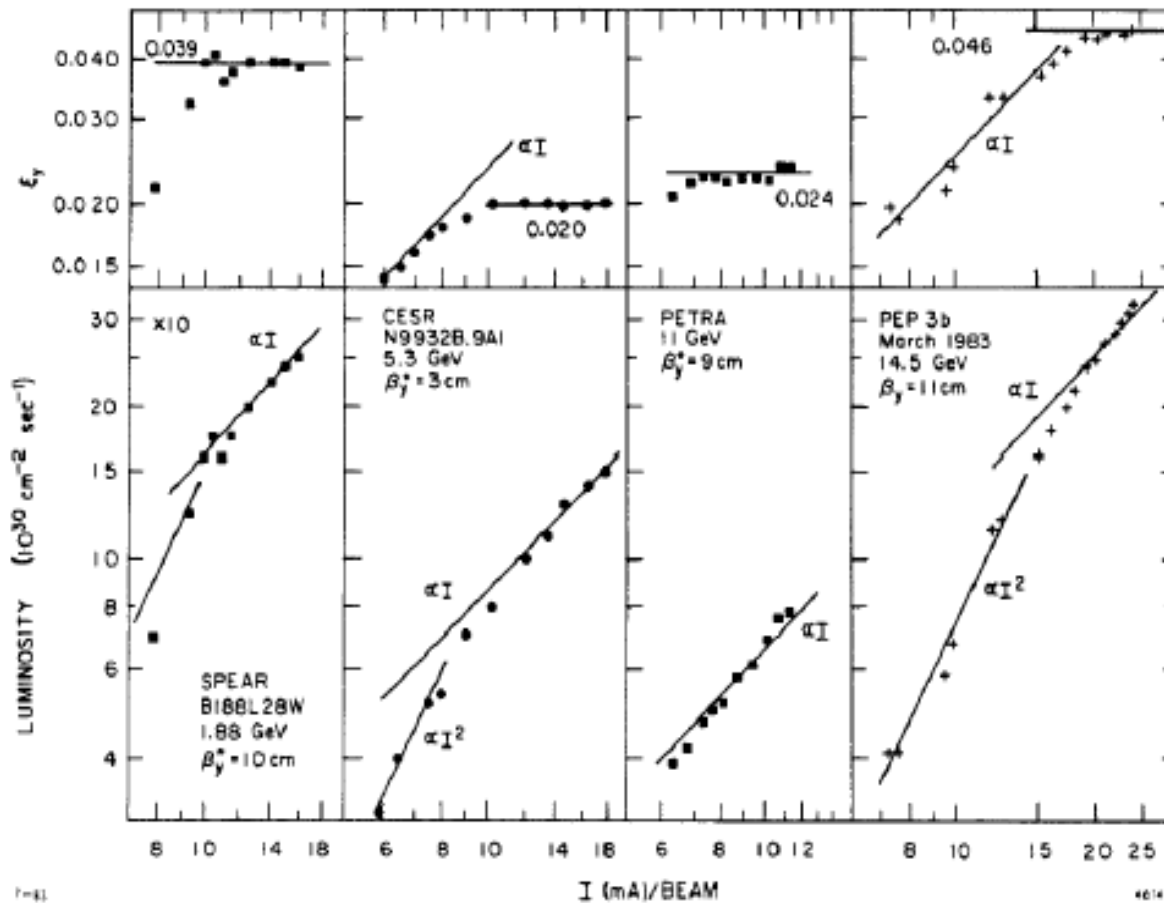
$$L \propto N_1 N_2$$

$$\xi \propto N_{1,2}$$

But...

Leptons beam-beam limit

First beam-beam limit (J. Seeman, 1983)



$$\xi \propto \text{const}$$

$$\xi \propto N_{1,2}$$

$$L \propto N_{1,2}$$

$$L \propto N_1 N_2$$

Luminosity and vertical tune shift parameter vs. beam current for SPEAR, CESR, PETRA & PEP.

What is happening?

Again....

$$\xi_{x,y} = \frac{Nr_0\beta_{x,y}^*}{2\pi\gamma\sigma_{x,y}(\sigma_x + \sigma_y)}$$

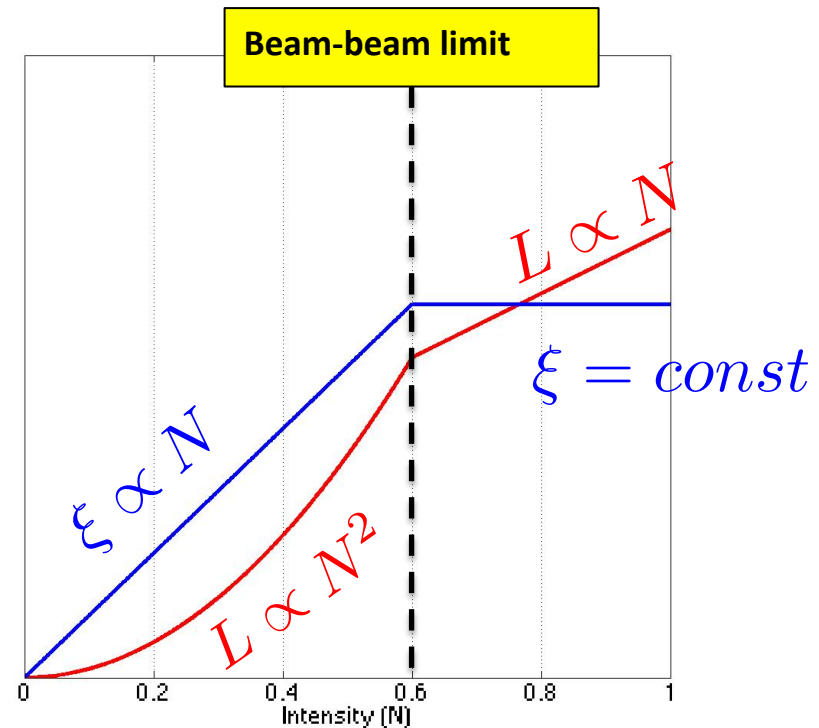
$$L = \frac{N^2fn_b}{4\pi\sigma_x\sigma_y}$$

Lepton colliders $\sigma_x \gg \sigma_y$

$$\xi_y \approx \frac{r_0\beta_y^*}{2\pi\gamma\sigma_x} \left(\frac{N}{\sigma_y} \right)$$

$$L = \frac{Nfn_b}{4\pi\sigma_x} \left(\frac{N}{\sigma_y} \right)$$

As to be constant!



Above beam-beam limit:

σ_y increases when N increases to keep ξ constant

Equilibrium emittance

1. Synchrotron radiation: vertical plane damped, horizontal plane excited!
2. Horizontal beam size normally much larger than vertical (LEP 200 - 4 μm)
3. Vertical beam-beam effect depends on horizontal (larger) amplitude
4. Coupling from horizontal to vertical plane

$$\xi_{x,y} = \frac{Nr_0\beta_{x,y}^*}{2\pi\gamma\sigma_{x,y}(\sigma_x + \sigma_y)}$$

**Equilibrium between horizontal excitation and vertical damping
determines ξ_{limit}**

Long-range BB and Orbit Effects

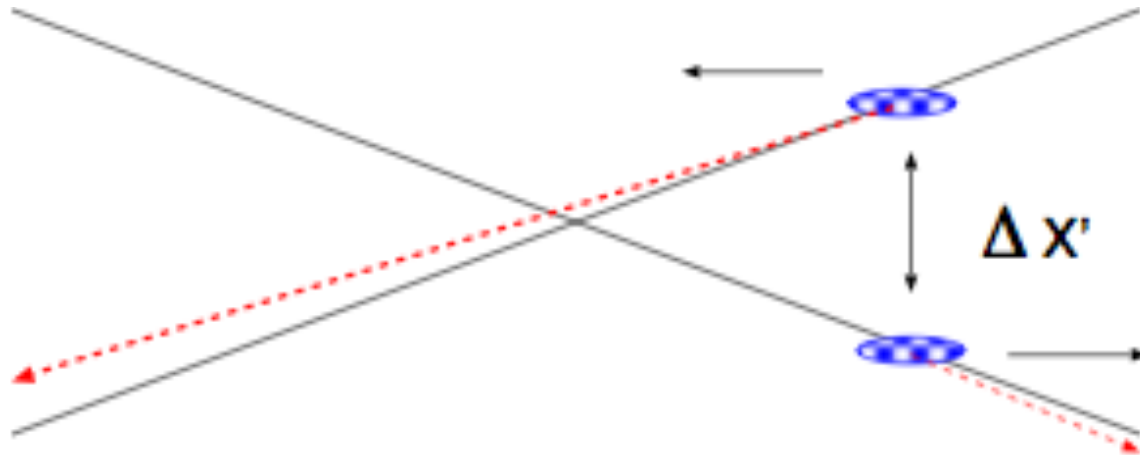
Long Range Beam-beam interactions lead to orbit effects

Long range kick
$$\Delta x'(x + d, y, r) = -\frac{2Nr_0}{\gamma} \frac{(x + d)}{r^2} \left[1 - \exp\left(-\frac{r^2}{2\sigma^2}\right)\right]$$

For well separated beams $d \gg \sigma$

The force has an amplitude independent contribution: **ORBIT KICK**

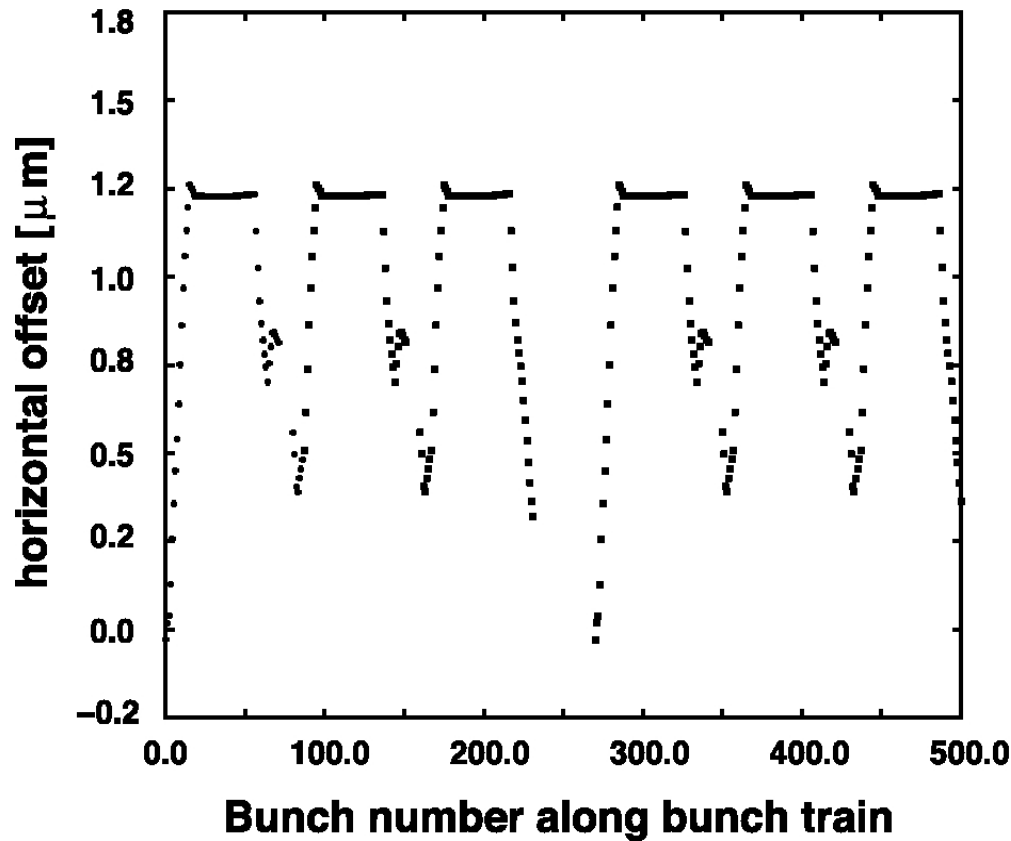
$$\Delta x' = \frac{\text{const}}{d} \left[1 - \frac{x}{d} + O\left(\frac{x^2}{d^2}\right) + \dots\right]$$



Orbit can be corrected but we should remember PACMAN effects

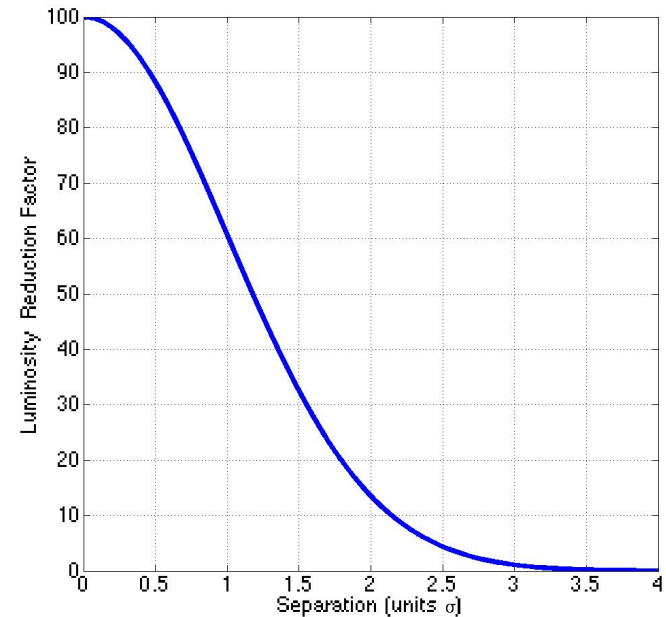
LHC orbit effects

Orbit effects different due to Pacman effects and the many long-range add up giving a non negligible effect



$$L = L_0 \cdot e^{-\frac{d^2}{4\sigma_x^2}}$$

↑
↓
d = 0 - 0.4 units of beam size



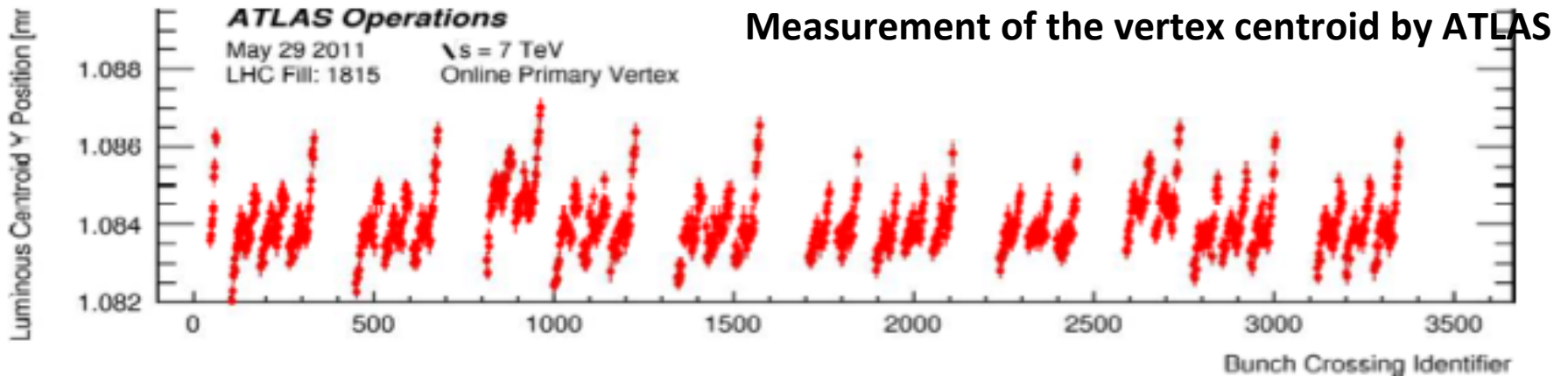
Long range orbit effect

Long range interactions leads to orbit offsets at the experiment a direct consequence is deterioration of the luminosity

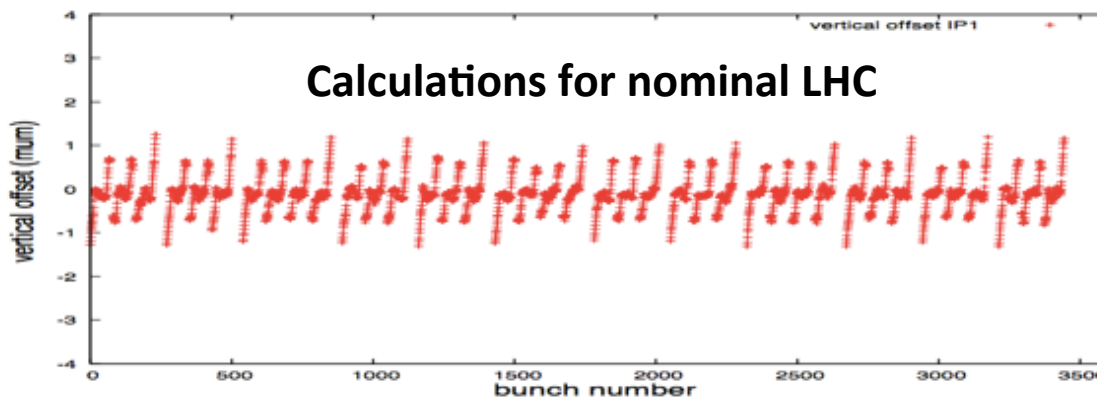
2011-07-05

file:///afs/cern.ch/user/z/zwe/Desktop/PNG/bcid_vs_posY_pm_posYErr.png

#1



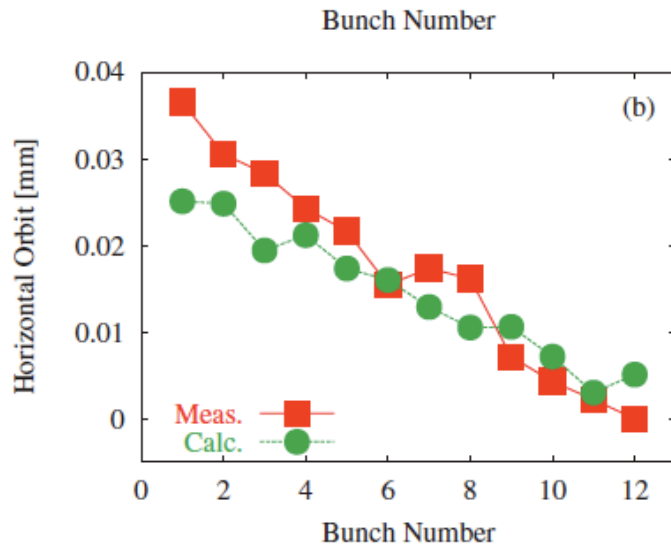
Courtesy W. Kozanecki



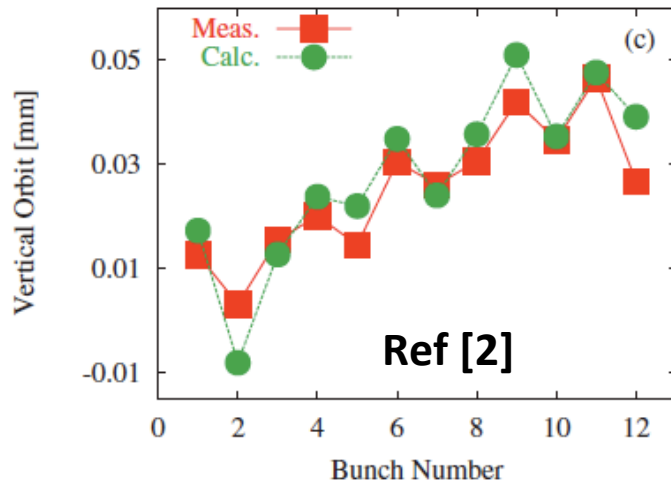
Effect is already visible with reduced number of interactions

Ref [7]

Tevatron orbit effects



Beam-beam single bunch orbit can be well reproduced and measured also in LEP



Effects can become important
(1 σ offset not impossible)

LUMINOSITY Deterioration

Coherent dipolar beam-beam modes

Coherent beam-beam effects arise from the forces which an exciting bunch exerts on a **whole test bunch** during collision

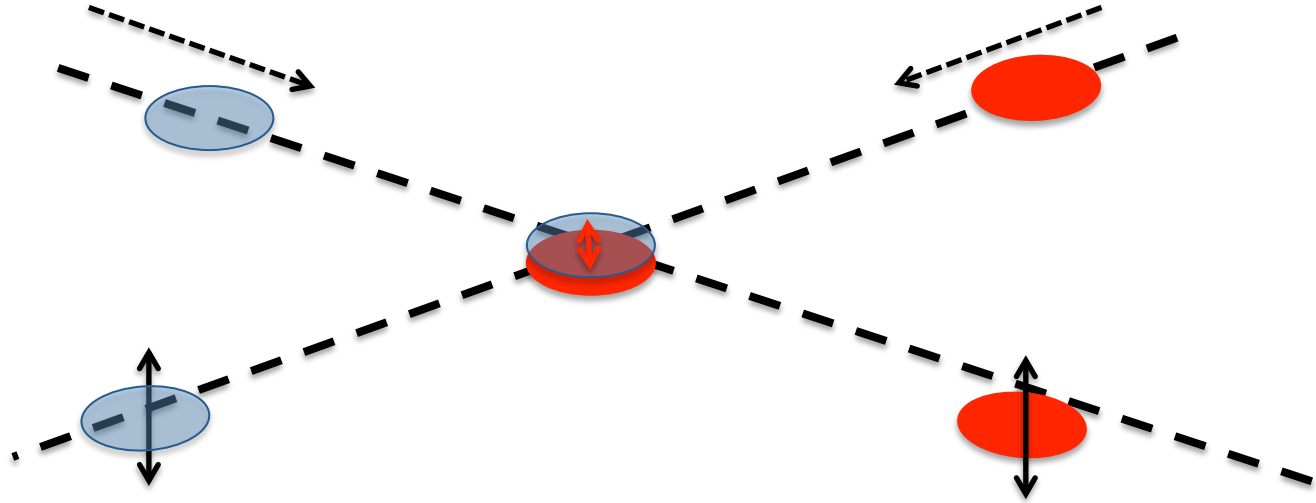
We study the **collective behaviour** of all particles of a bunch

Coherent motion requires an **organized behaviour** of all particles of the bunch

Coherent beam-beam force

- Beam distributions Ψ_1 and Ψ_2 mutually changed by interaction
- Interaction depends on distributions
 - Beam 1 Ψ_1 solution depends on beam 2 Ψ_2
 - Beam 2 Ψ_2 solution depends on beam 1 Ψ_1
- Need a **self-consistent** solution

Coherent beam-beam effects



- Whole bunch sees a kick as an entity (**coherent kick**)
- Coherent **kick seen by full bunch** different from single particle kick
- Requires **integration** of individual kick over particle distribution

$$\Delta r' = -\frac{N_p r_0}{r} \cdot \frac{r}{r^2} \cdot \left[1 - e^{-\frac{r^2}{4\sigma^2}} \right]$$

- Coherent kick of separated beams can excite coherent **dipolar oscillations**
- All bunches couple because each bunch “sees” many opposing bunches(LR): **many coherent modes** possible!

Coherent effects

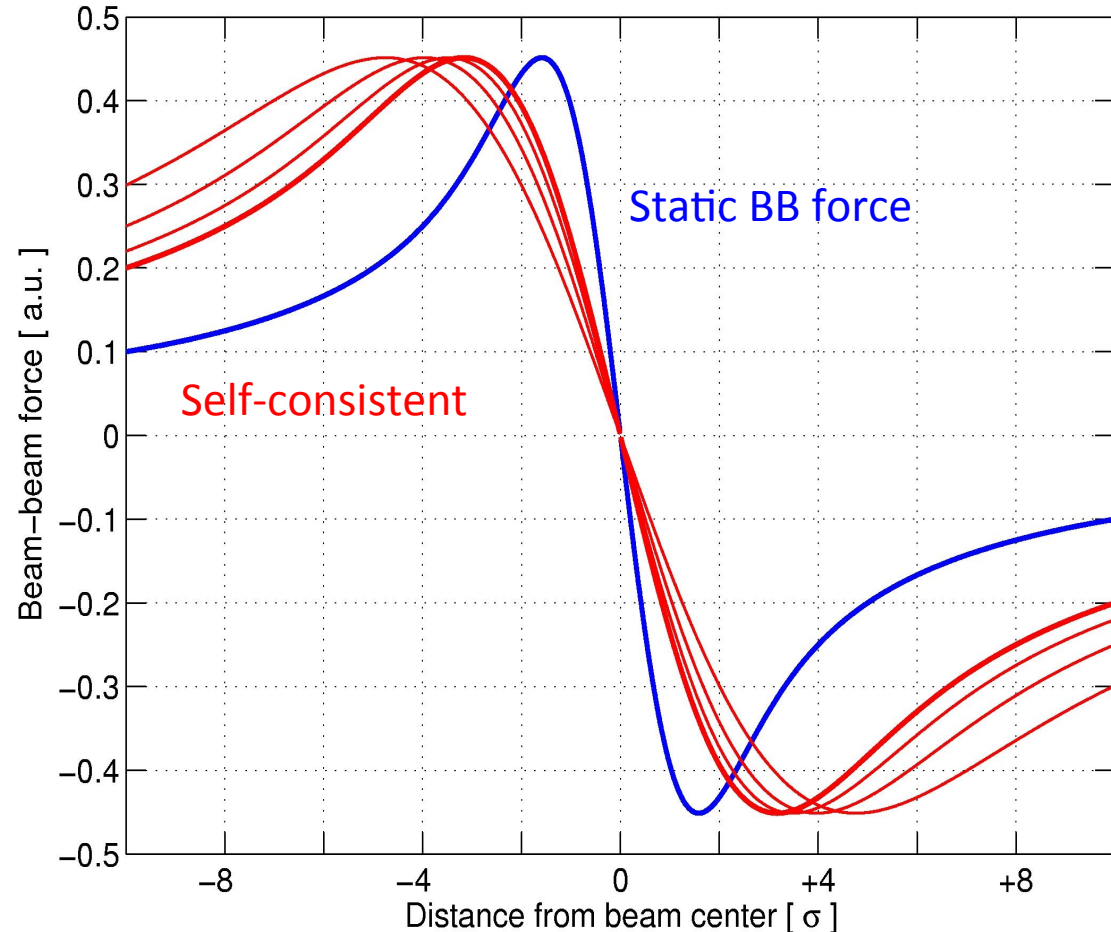
Self-consistent treatment needed

Perturbative methods

static source of distortion:
example magnet

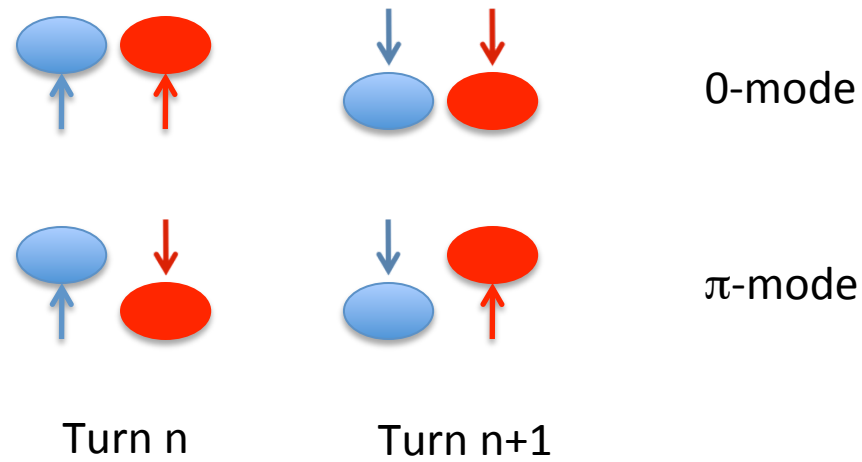
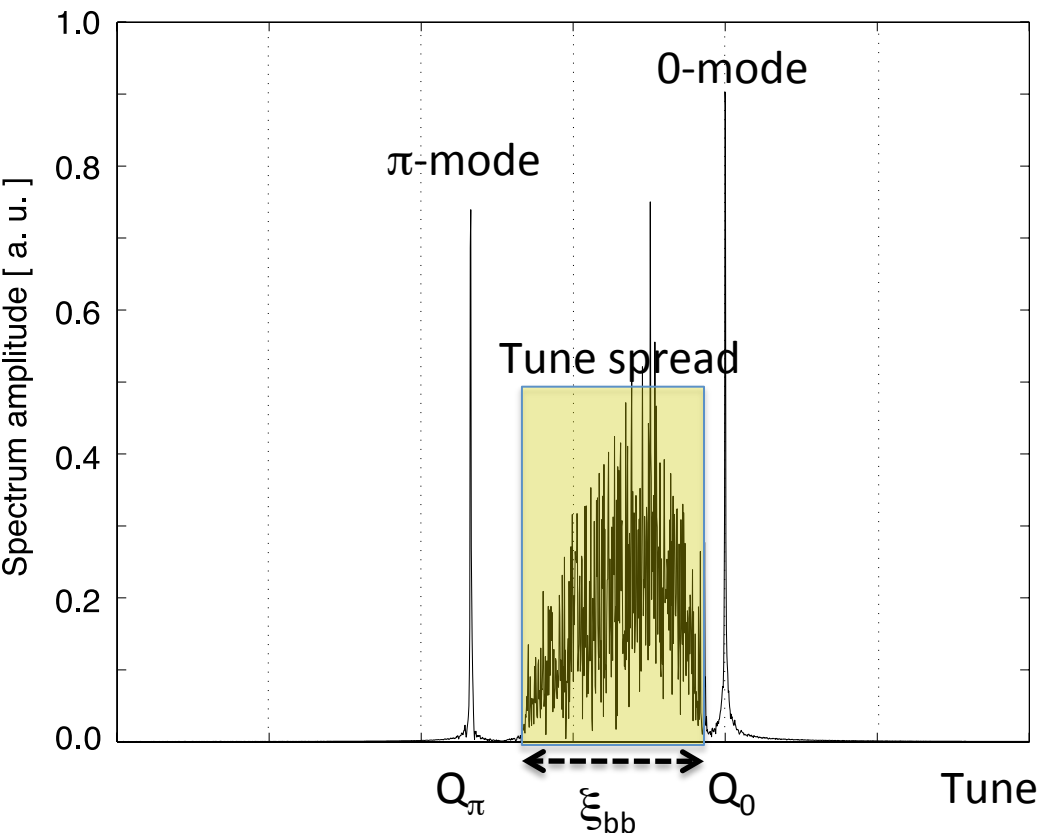
Self-consistent method

source of distortion changes
as a result of the distortion



For a complete understanding of BB effect a self-consistent treatment is necessary

Simple case: one bunch per beam



MOVIE

0-mode at unperturbed tune Q_0

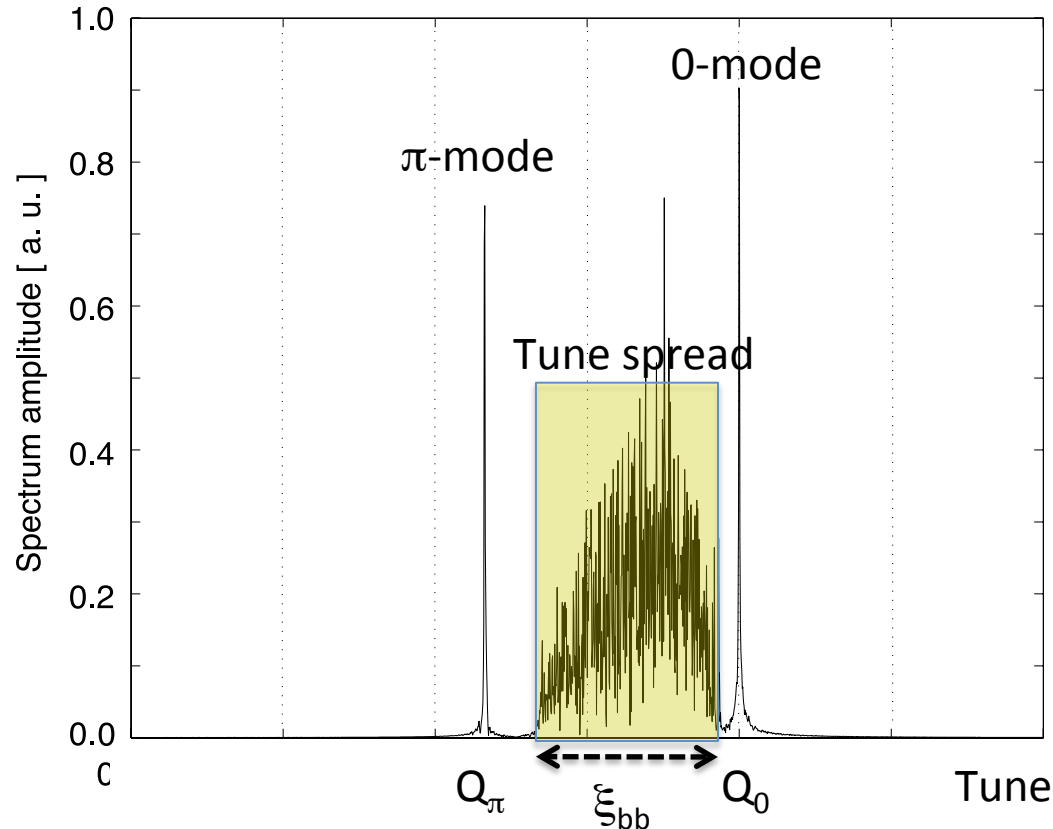
π -mode is shifted at $Q_\pi = 1.1-1.3 \xi_{bb}$

Incoherent tune spread range $[0, -\xi]$

$$\Delta Q = Y \cdot \xi$$

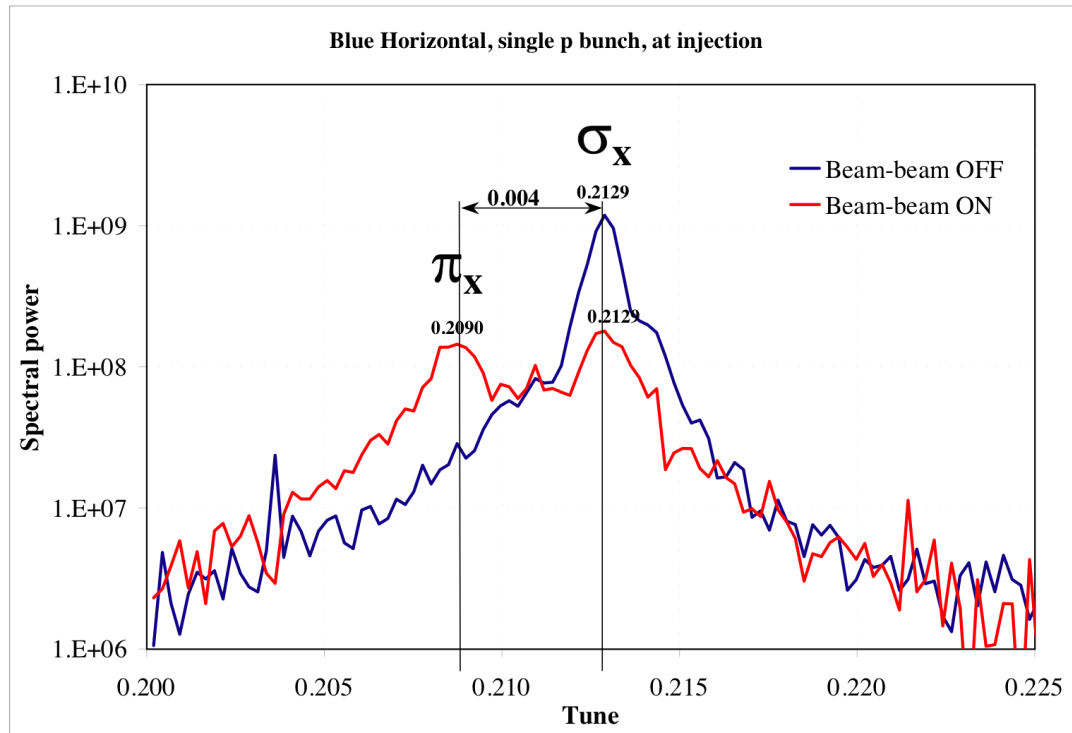
- Coherent mode: two bunches are “locked” in a coherent oscillation
- 0-mode is stable (mode with NO tune shift)
- π -mode can become unstable (mode with largest tune shift)

Simple case: one bunch per beam and Landau damping



- Incoherent tune spread is the Landau damping region any mode with frequency laying in this range should not develop**
- π -mode has frequency out of tune spread (Y) so it is not damped!

Coherent modes at RHIC

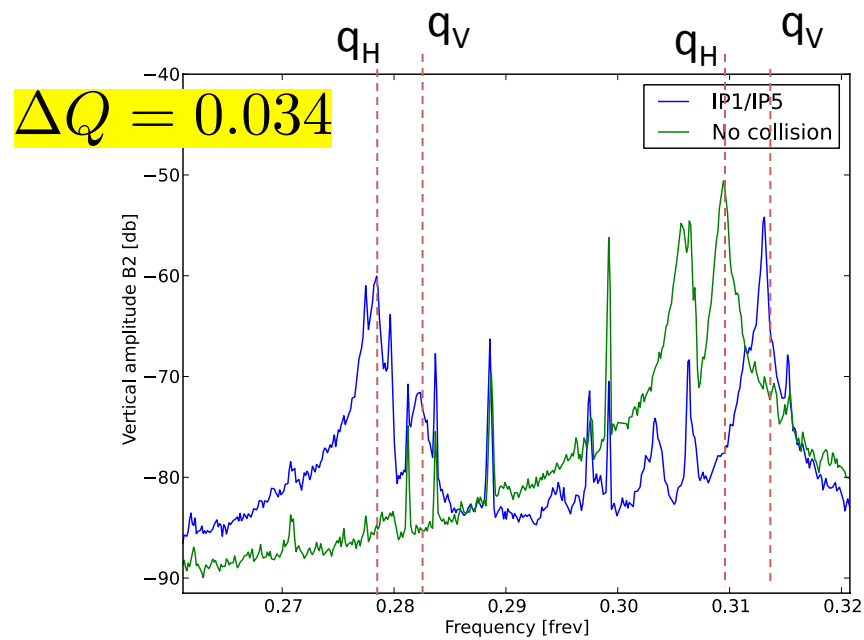
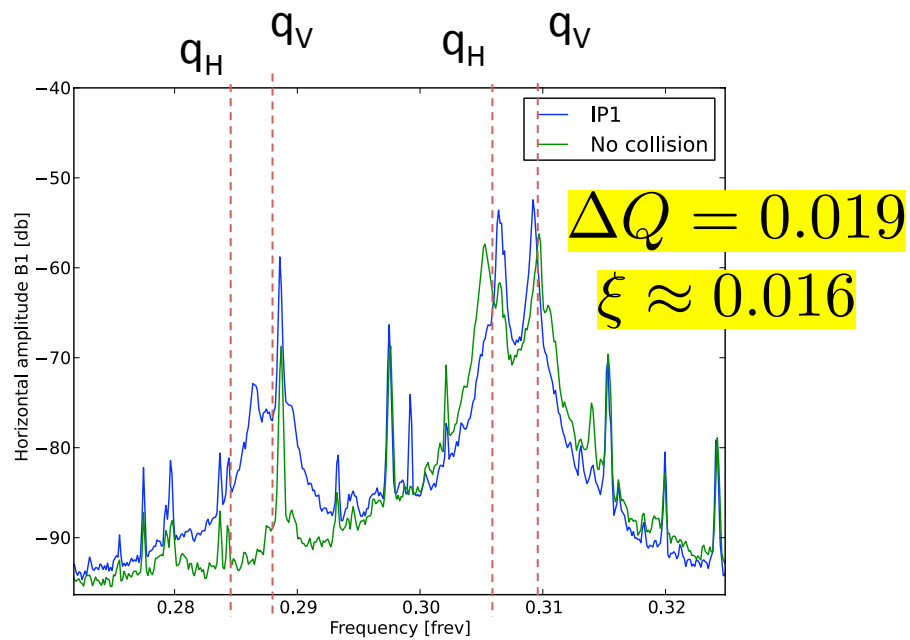
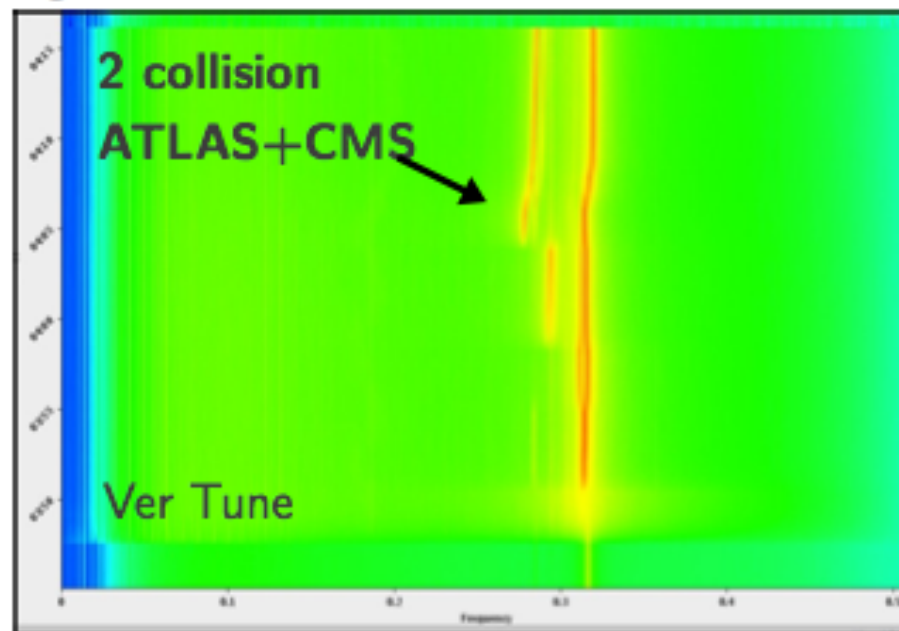
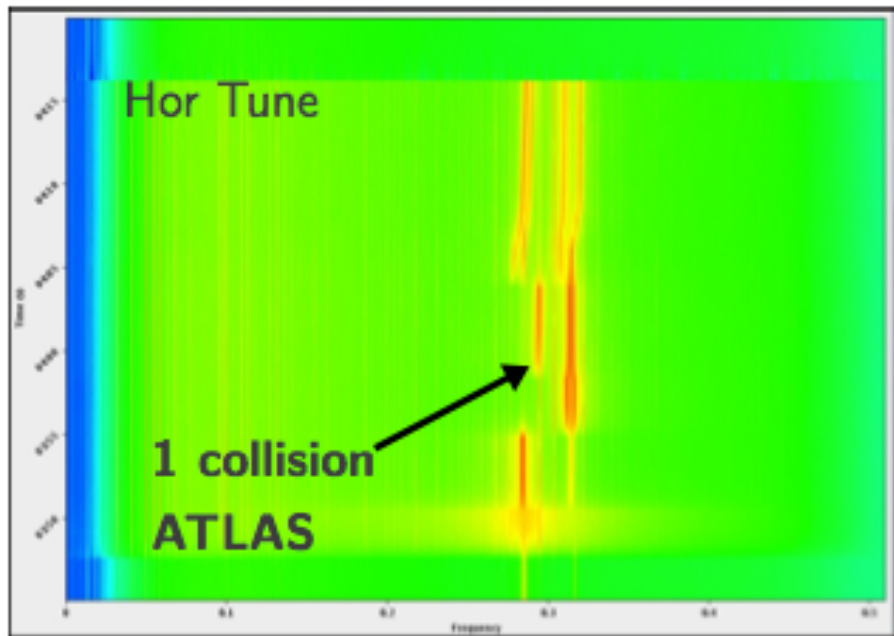


Courtesy W. Fischer (BNL)

Tune spectra before collision and in collision two modes visible

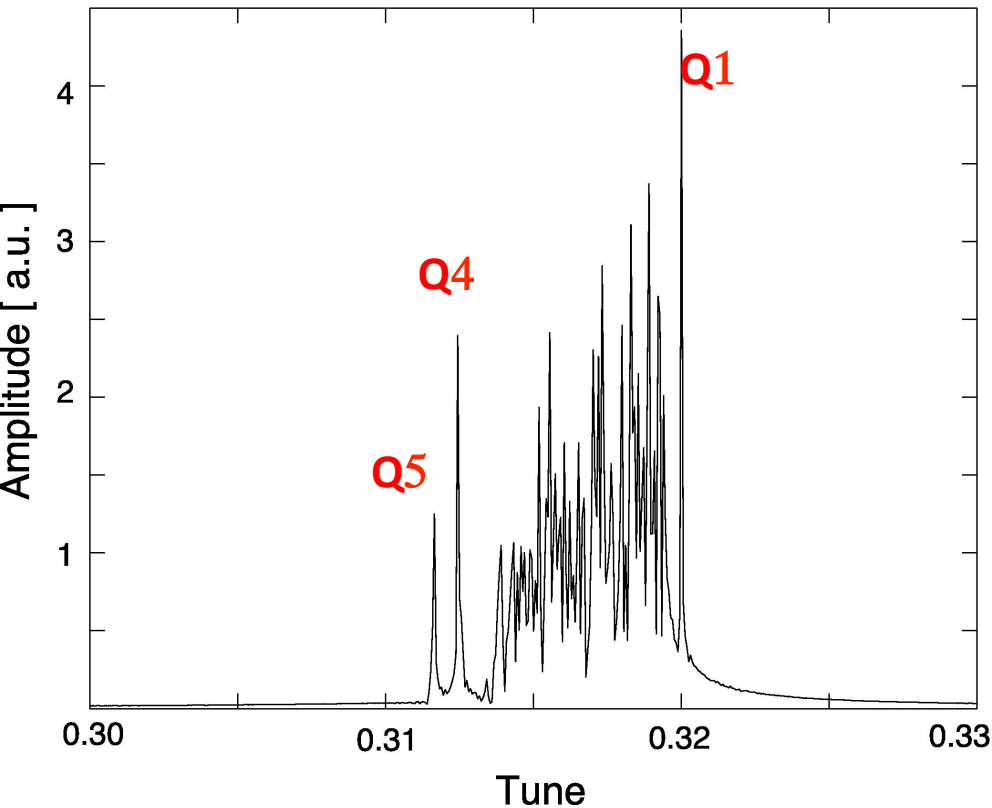
Head-on beam-beam coherent mode: LHC

BBQ Signals

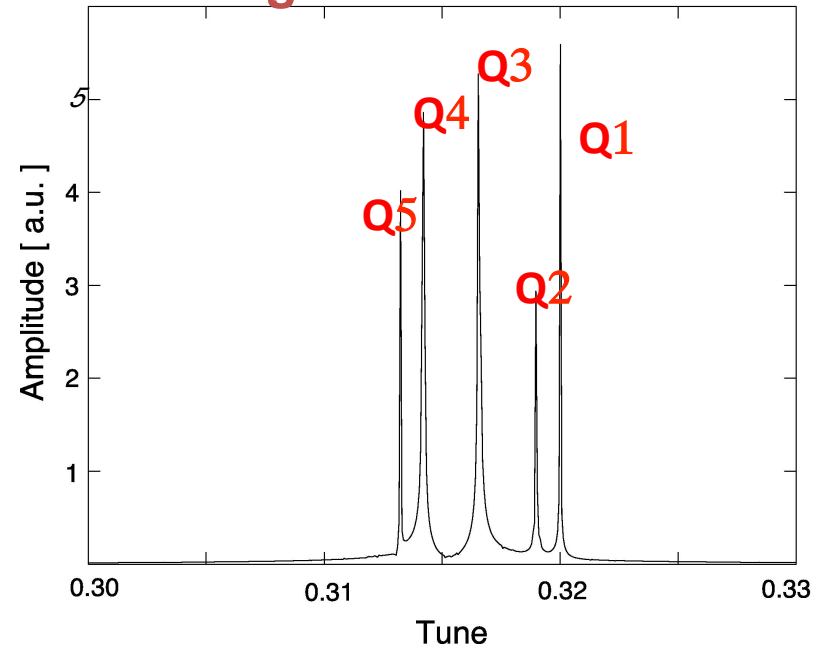


If we have more HO interactions

Multi particle simulations



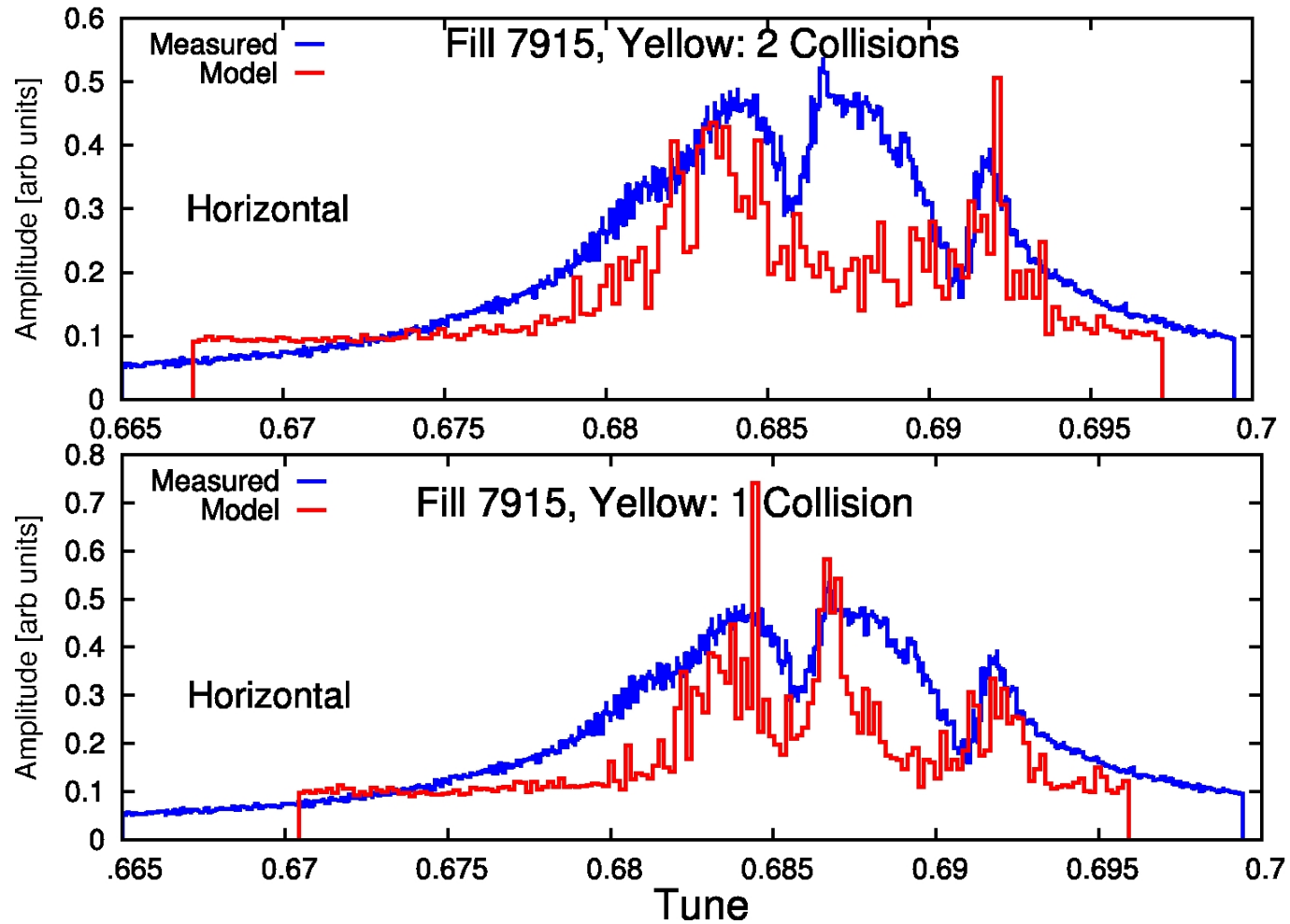
Rigid bunch model



- Bunches couple via the beam-beam interaction
- Additional modes
- Undesirable situation

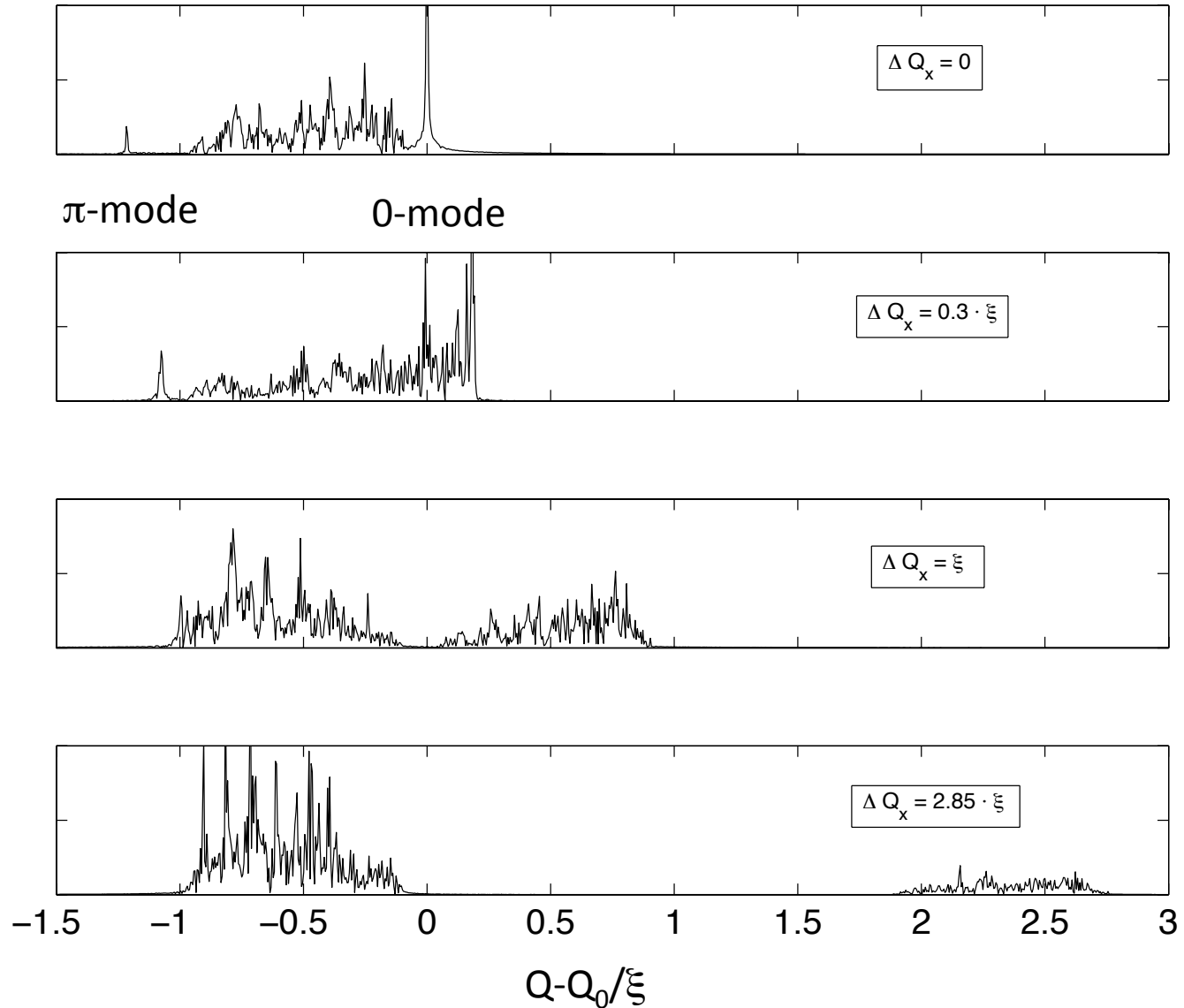
Landau damping of intermediate modes important ingredient

Beam-beam coherent modes and Landau Damping



Pacman effect on coherent modes

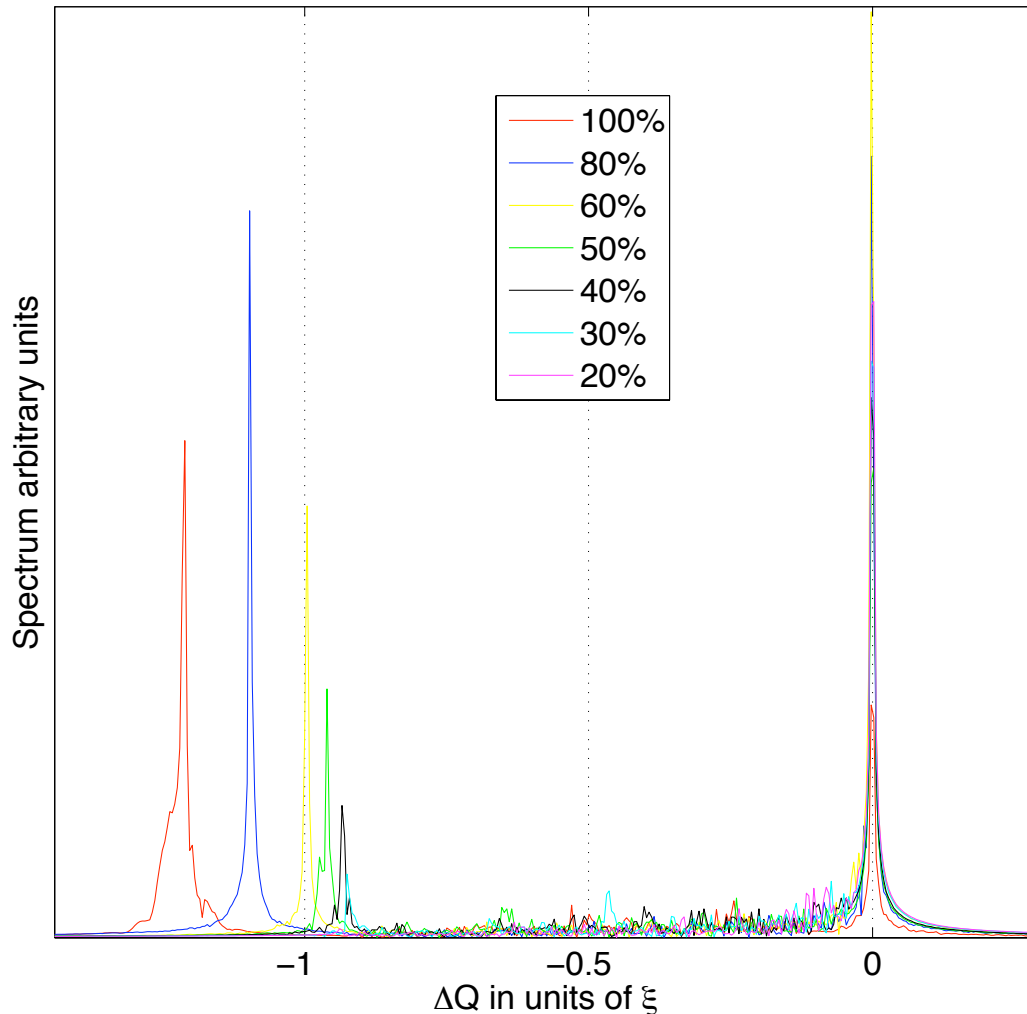
Different Tunes



Tune split breaks symmetry and coherent modes disappear

Analytical calculations in Reference [8]

Different bunch intensities



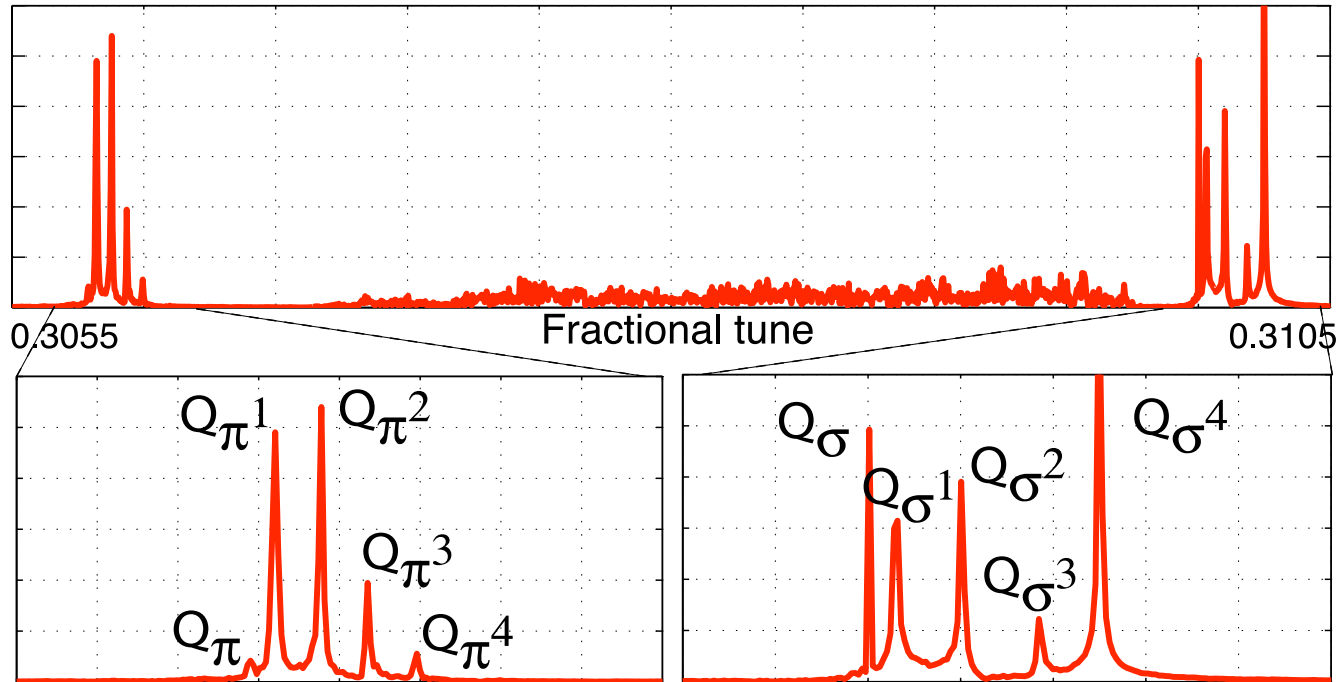
For two bunches colliding head-on in one IP the coherent mode disappears if intensity ratio between bunches is 55% Reference[9]

We assumed:

- equal emittances
- equal tunes
- NO PACMAN effects
(bunches will have different tunes)

For coherent modes the key is to break the symmetry in your coupled system...(tunes, intensities, collision patters...)

Bunch trains and more interactions



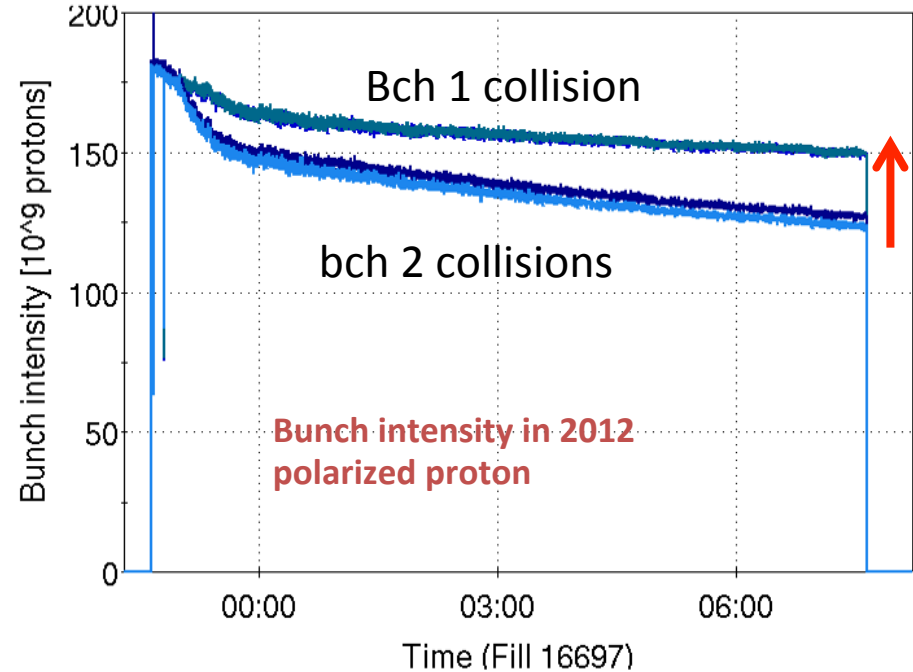
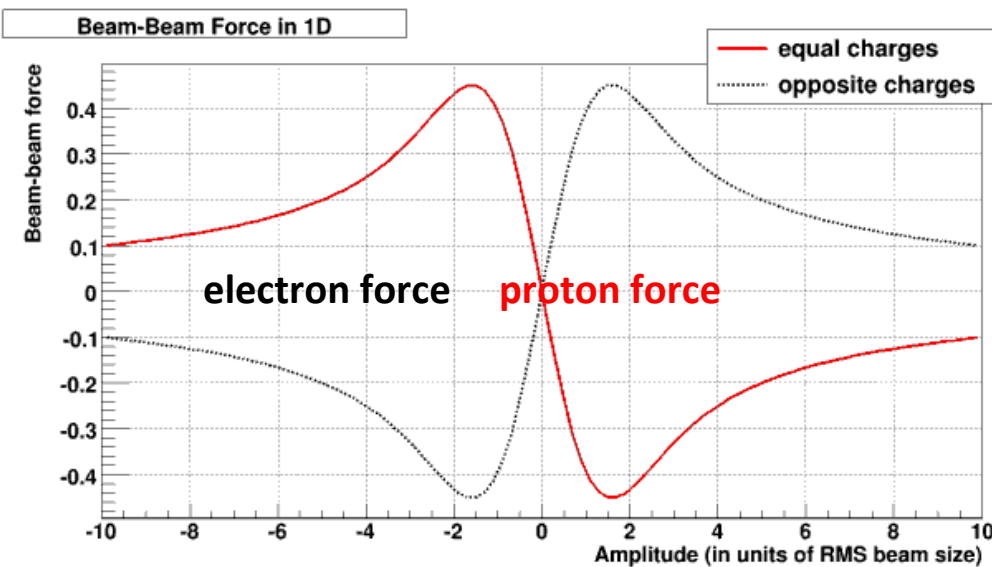
Long-range coherent modes frequencies are sidebands of HO
Pacman bunch will have different number of modes

No Landau damping of long range modes

Beam-beam compensations:

Head-on

- Linear e-lens, suppress shift
- Non-linear e-lens, suppress tune spread

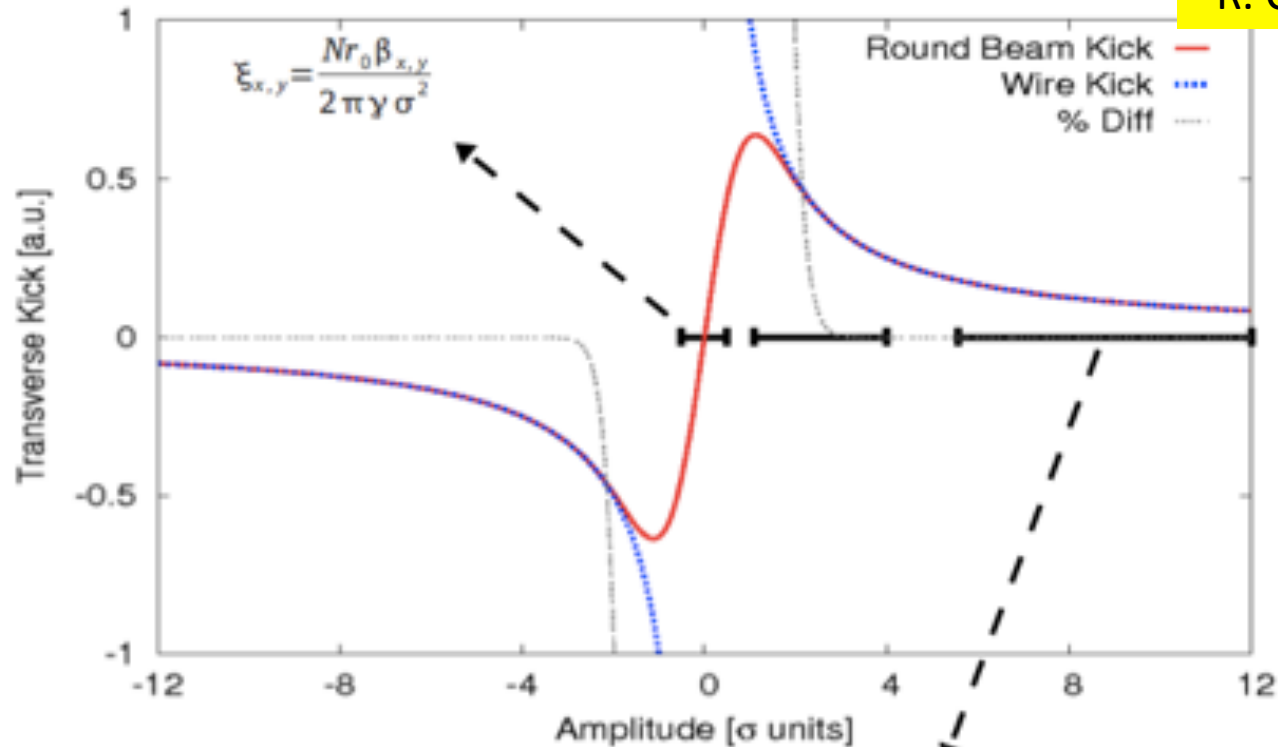


- Past experience: at Tevatron linear and non-linear e-lenses, also hollow....
- Present: test for half compensation at RHIC with non-linear e-lens

Beam-beam compensations: long-range

Beam-beam wire compensation

R. Calaga



$$\sigma \ll d: \quad \Delta x'(x,d) = -\frac{K}{d} \cdot \left(1 + \frac{x}{d} + \frac{x^2}{d^2} + \dots\right)$$

- Past experience: at RHIC several tests till 2009...
- Present: simulation studies on-going for possible use in HL-LHC...

...not covered here...

- *Linear colliders special issues*
- *Asymmetric beams effects*
- *Coasting beams*
- *Beamstrahlung*
- *Synchrotron coupling*
- *Beam-beam experiments*
- *Beam-beam and impedance*
- ...

Questions?

References:

- [1] http://cern.ch/Werner.Herr/CAS2009/proceedings/bb_proc.pdf
- [2] V. Shiltsev et al, "Beam beam effects in the Tevatron", *Phys. Rev. ST Accel. Beams* 8, 101001 (2005)
- [3] Lyn Evans "The beam-beam interaction", CERN 84-15 (1984)
- [4] Alex Chao "Lie Algebra Techniques for Nonlinear Dynamics" SLAC-PUB-9574 (2002)
- [5] J. D. Jackson, "Classical Electrodynamics", John Wiley & Sons, NY, 1962.
- [6] H. Grote, F. Schmidt, L. H. A. Leunissen, "LHC Dynamic Aperture at Collision", LHC-Project-Note 197, (1999).
- [7] W. Herr, "Features and implications of different LHC crossing schemes", LHC-Project-Note 628, (2003).
- [8] A. Hofmann, "Beam-beam modes for two beams with unequal tunes", CERN-SL-99-039 (AP) (1999) p. 56.
- [9] Y. Alexahin, "On the Landau damping and decoherence of transverse dipole oscillations in colliding beams ", *Part. Acc.* 59, 43 (1996).
- [10] R. Assmann et al., "Results of long-range beam-beam studies - scaling with beam separation and intensity "

...much more on the LHC Beam-beam webpage:

<http://lhc-beam-beam.web.cern.ch/lhc-beam-beam/>