

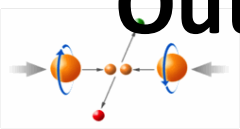
Physics of Polarized Protons/Electrons in Accelerators

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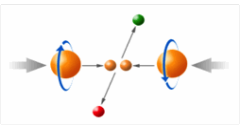
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Outline



- Introduction
 - What is polarized proton/electron beam?
 - Why high energy polarized beams?
- Physics of polarized protons in accelerators
 - Spin dynamics
 - Challenges in accelerating polarized protons to high energy
 - Brief history of high energy polarized proton beams development
- Brief introduction of polarized electrons in accelerators
- Summary

Polarized Proton/electron Beam



- Proton/electron, as spin half particle
 - **Spin vector**

$S = \langle \psi | \sigma | \psi \rangle$; Here, ψ is spin state of the particle

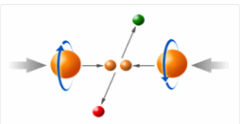
- **Intrinsic magnetic moment**

$$\vec{\mu} = \frac{g}{2} \frac{q}{m} \vec{S}; \quad \text{and} \quad \frac{d\vec{S}}{dt} = \vec{\mu} \times \vec{B} \text{ in the particle's frame}$$

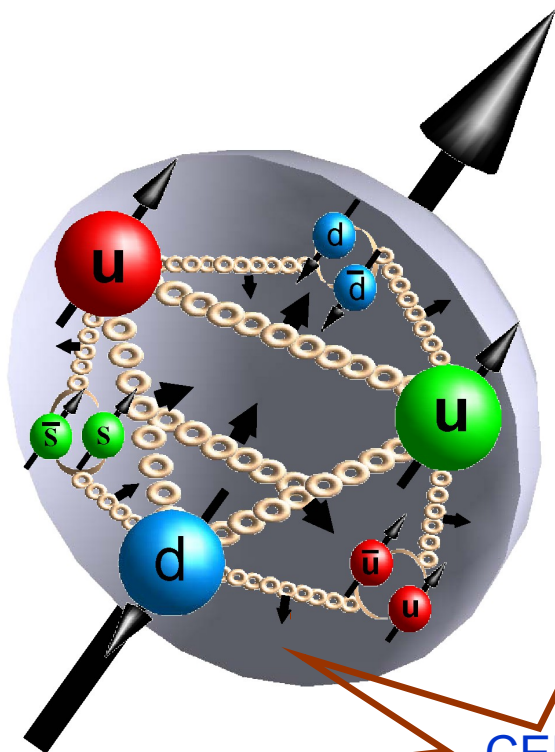
- Polarized proton/electron beam
 - **Beam polarization**, with N_{\pm} is the number of particles in the state of ψ_{+} (up state) and ψ_{-} (down state), respectively

$$P = \frac{N_{+} - N_{-}}{N_{+} + N_{-}}$$

Why Polarized Beams?



- Study proton spin structure



Spin contribution from quarks

Spin contribution from all the gluons

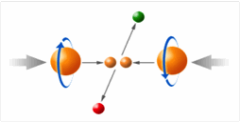
$$S = \frac{1}{2} = \frac{1}{2} \Delta\Sigma + \Delta g + L_q + L_g$$

Orbital angular momentum of quarks and gluons

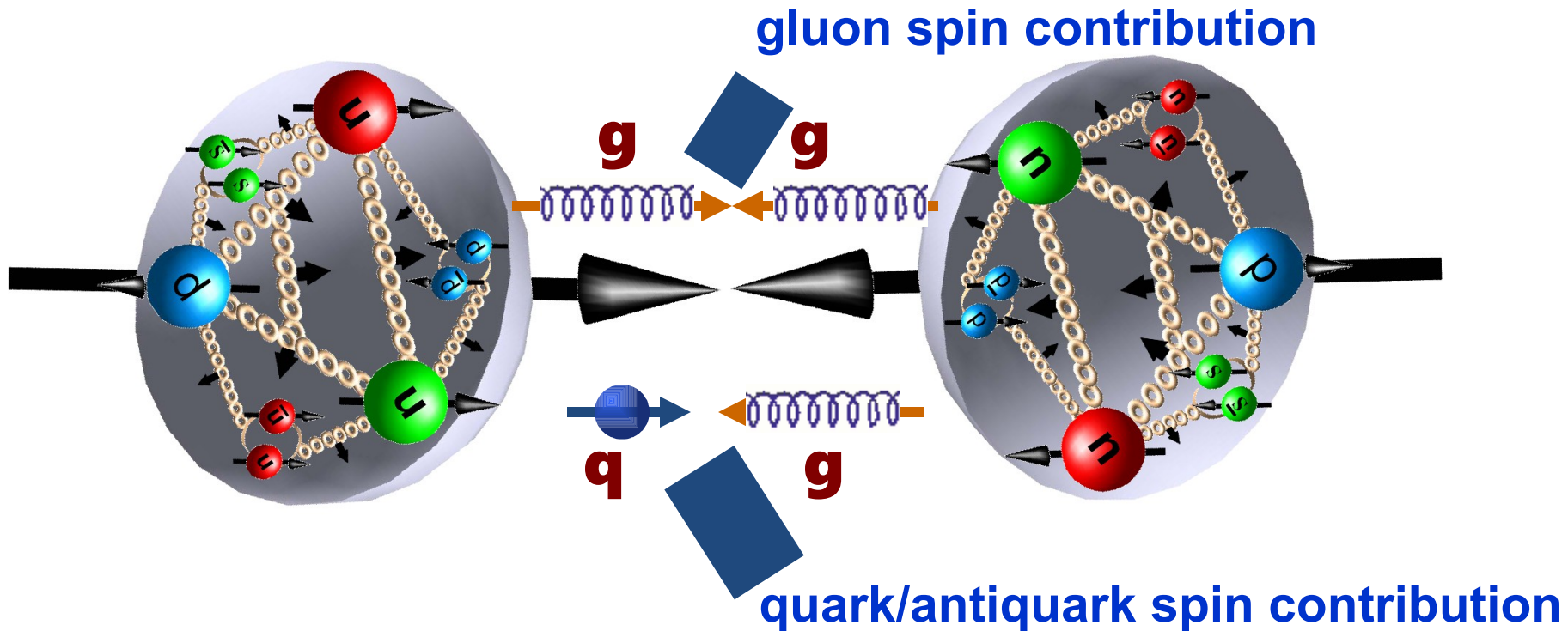
CERN EMC and SLAC SMC:

$\Delta\Sigma \sim 20\%$

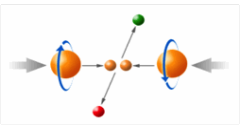
Why high energy polarized protons?



High energy proton proton collisions:
gluon gluon collision and gluon quark collision



Why Polarized Beams?

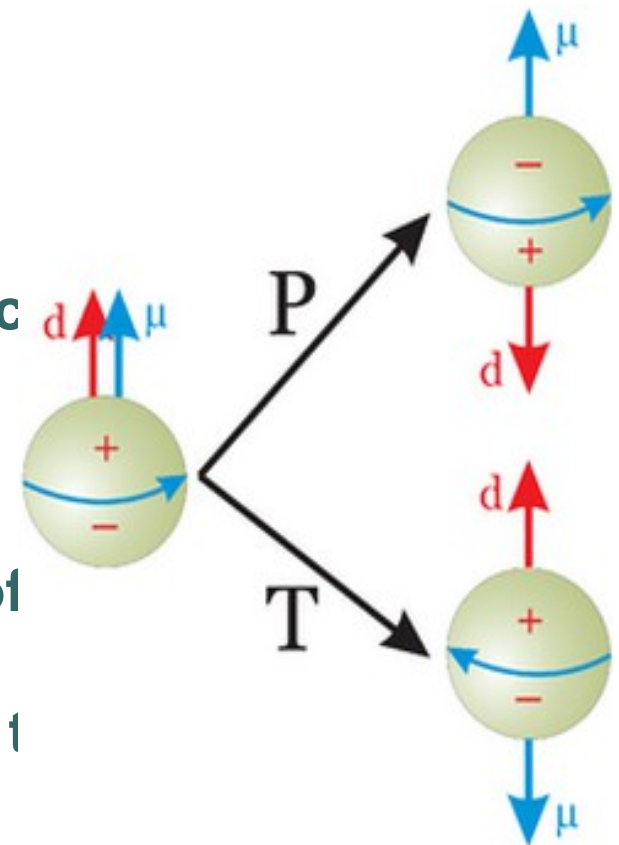


- Search for Electric Dipole Moment

Describes the positive and negative charge distribution inside a particle

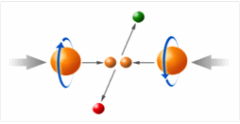
It aligns along the spin axis of the particle and violates both Parity and Time Reversal.

Hence, significant EDM measurement of fundamental particles is an effective probe of CP-violation, could be the key to explain the asymmetry between matter and antimatter



“Deuteron & proton EDM Experiment”, Yannis K. Semertzidis, BNL

Spin motion in a circular accelerator



Thomas BMT Equation: (1927, 1959)

L. H. Thomas, *Phil. Mag.* **3**, 1 (1927); V. Bargmann, L. Michel, V. L. Telegdi, *Phys. Rev. Lett.* **2**, 435 (1959)

$$\frac{d\vec{S}}{dt} = \vec{\Omega} \times \vec{S} = -\frac{e}{\gamma m} [G\gamma\vec{B}_{\perp} + (1+G)\vec{B}_{\parallel}] \times \vec{S}$$

Spin vector in particle's rest frame

➤ G is the anomalous g-factor, for proton,

$$G=1.7928474$$

➤ γ : Lorenz factor

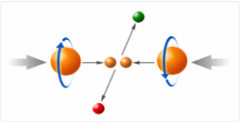
Magnetic field along the direction of the particle's velocity

Magnetic field perpendicular to the particle's velocity

Spin tune Q_s : number of precessions in one orbital revolution:

$$Q_s = G\gamma$$

Spinor



- Thomas-BMT equation

$$\frac{d\vec{S}}{ds} = \vec{n} \times \vec{S} = \left[G\gamma \hat{y} + (1 + G\gamma) \frac{B_x}{B\rho} \hat{x} + (1 + G) \frac{B_{//}}{B\rho} \hat{s} \right] \times \vec{S}; \quad ds = \rho d\theta$$

$$\vec{S} = \langle \psi | \vec{\sigma} | \psi \rangle; \quad \text{with } \psi = \begin{pmatrix} u \\ d \end{pmatrix}$$

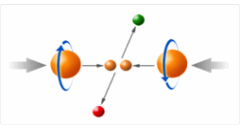
- Equation of motion of spinor

$$\frac{d\psi}{d\theta} = -\frac{i}{2} (\vec{\sigma} \cdot \vec{n}) \psi = -\frac{i}{2} H \psi$$

- Spinor transfer matrix M

$$\psi(\theta_2) = e^{-\frac{i}{2} H(\theta_2 - \theta_1)} \psi(\theta_1) = M(\theta_2, \theta_1) \psi(\theta_1)$$

Spinor Transfer Matrix



- A dipole

$$n = G\gamma\hat{y} \quad M(\theta_2, \theta_1) = e^{-iG\gamma(\theta_2 - \theta_1)\sigma_3/2}$$

- A thin quadrupole

$$\vec{n} = (1 + G\gamma)\left(\frac{\partial B_x}{\partial y} l / B\rho\right)y\hat{x} = (1 + G\gamma)kly\hat{x} \quad M = e^{-i(1+G\gamma)kly\sigma_1/2}$$

- A spin rotator which rotates spin vector by a precession of χ around an axis of \hat{n} , $M = e^{-i\chi\hat{n}\cdot\hat{\sigma}}$

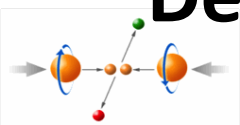
- One turn matrix of a ring with a localized spin rotation at θ

$$\text{OTM} = e^{-\frac{i}{2}2\pi Q_s \hat{n}_{co} \cdot \vec{\sigma}} = e^{-\frac{i}{2}G\gamma(2\pi - \theta)\sigma_3} e^{-\frac{i}{2}\chi\hat{n}_e \cdot \vec{\sigma}} e^{-\frac{i}{2}G\gamma\theta\sigma_3}$$

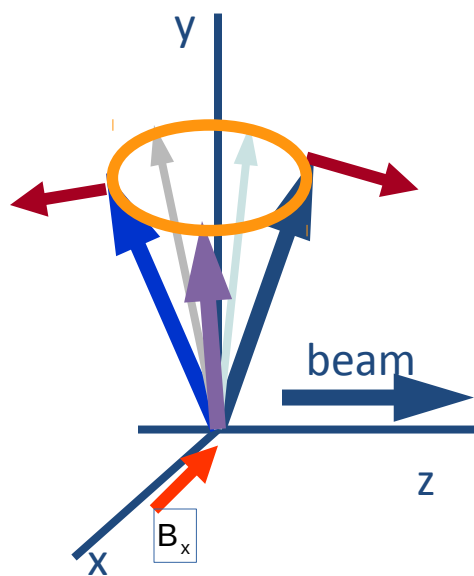
Spin tune becomes,

$$\cos \pi Q_s = \cos G\gamma\pi \cos \frac{\chi}{2} - \sin G\gamma\pi \sin \frac{\chi}{2} (\hat{n}_e \cdot \hat{y})$$

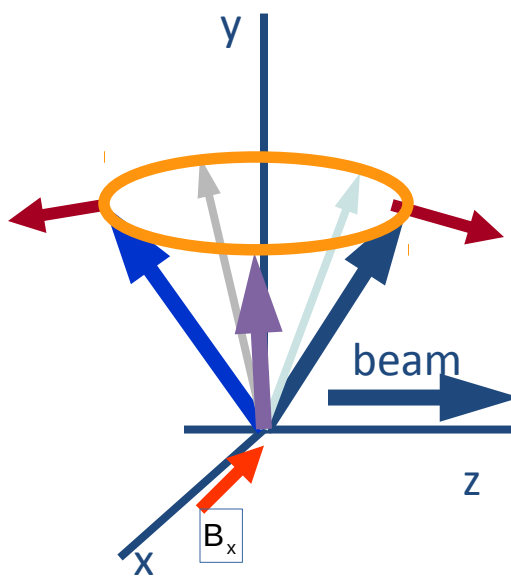
Depolarizing mechanism in a synchrotron



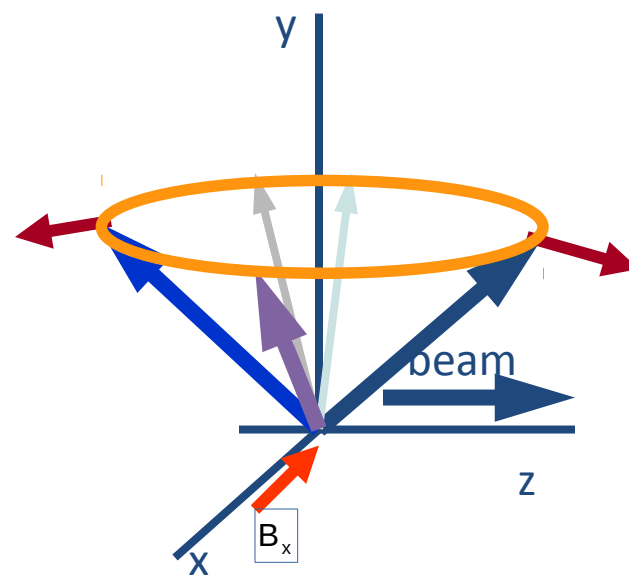
- horizontal field kicks the spin vector away from its vertical direction, and can lead to polarization loss
 - dipole errors, misaligned quadrupoles, imperfect orbits
 - betatron oscillations
 - other multipole magnetic fields
 - other sources



Initial

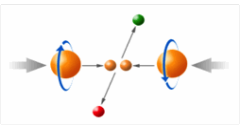


1st full betatron
Oscillation period



2nd full betatron
Oscillation period

Depolarizing Resonance



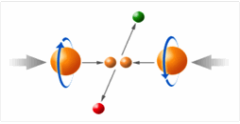
○ Imperfection resonance:

- Source: dipole errors, quadrupole misalignments
- Resonance location:
 $G\gamma = k$, k is an integer
- Resonance strength:
 - Proportional to the size of the vertical closed orbit distortion

❖ For protons, imperfection spin resonances are spaced by 523 MeV

❖ Between RHIC injection and 250 GeV, a total of 432 imperfection resonances

Depolarizing Resonance



○ Intrinsic resonance:

- Focusing field due to the intrinsic betatron oscillation
- Location:

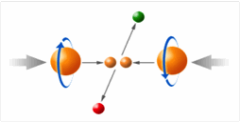
$$G\gamma = kP \pm Q_y$$

P: super periodicity of the accelerator,

Q_y : vertical betatron tune

- Resonance strength:
 - Proportional to the size of the betatron oscillation
 - When crossing an isolated intrinsic resonance, the larger the beam is, the more the polarization loss is. This is also known as the polarization profile

Stable Spin Direction



- an invariant direction that spin vector aligns to when the particle returns back to the same phase space, i.e.

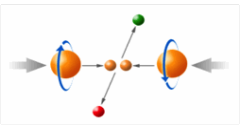
$$\hat{n}_{co}(I_z, \phi_z, \theta) = \hat{n}_{co}(I_z, \phi_z + 2\pi, \theta)$$

Here, I_z and ϕ_z are the 6-D phase-space coordinates.

- For an ideal machine, i.e. the closed orbit is zero, the stable spin direction is along the direction of the guiding field
- The stable spin direction \hat{n}_0 for a particle on the closed orbit is the eigenvector of its one turn spin transfer matrix

$$M(\theta + 2\pi, \theta) = e^{-\frac{i}{2} 2\pi Q_s \hat{n}_0 \cdot \vec{\sigma}}$$

Stable Spin Direction



- $\hat{n}_{co}(I_z, \phi_z, \theta)$ is function of phase space
- For particles on closed orbit, stable spin direction can be computed through one-turn spin transfer matrix. \hat{n}_{co} is also known as \hat{n}_0
- For particles not on closed orbit, since in general the betatron tune is non-integer, the stable spin direction is no longer the eigen vector of one turn spin transfer matrix. Algorithms like SODOM[1,2], SLIM[3], SMILE[4] were developed to compute the stable spin direction

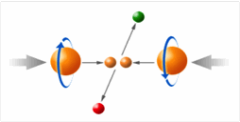
[1] K. Yokoya, Non-perturbative calculation of equilibrium polarization of stored electron beams, KEK Report 92-6, 1992

[2] K. Yokoya, An Algorithm for Calculating the Spin Tune in Accelerators, DESY 99-006, 1999

[3] A. Chao, Nucl. Instr. Meth. 29 (1981) 180

[4] S. R. Mane, Phys. Rev. A36 (1987) 149

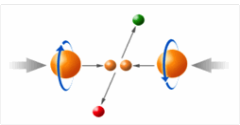
Stable Spin Direction



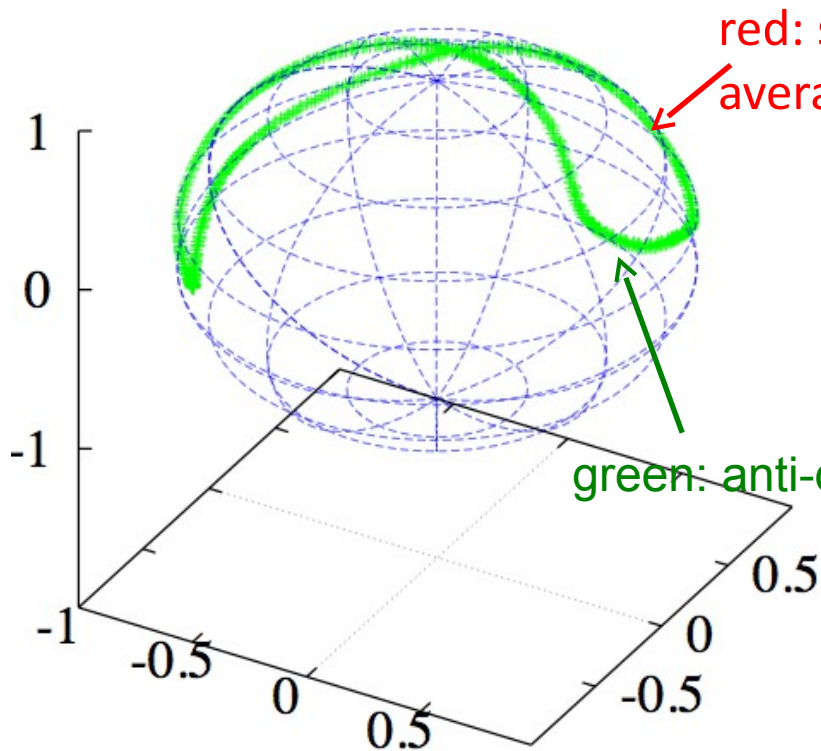
- $\hat{n}_{co}(I_z, \phi_z, \theta)$ is function of phase space
- It can also be calculated numerically with stroboscopic averaging, a technique developed by Heinemann, Hoffstaetter from DESY[1]
- One can also compute \hat{n}_{co} through numerical tracking with adiabatic anti-damping technique, i.e. populate particles on closed orbit first with their spin vectors aligned with \hat{n}_0 . The particles are then adiabatically excited to the phase space during which spin vector should follow the stable spin direction as long as it is far from a spin resonance

[1] K. Heinemann, G. H. Hoffstaetter, Tracking Algorithm for the Stable Spin Polarization Field in Storage Rings using Stroboscopic Averaging, PRE, Vol. 54, Number 4

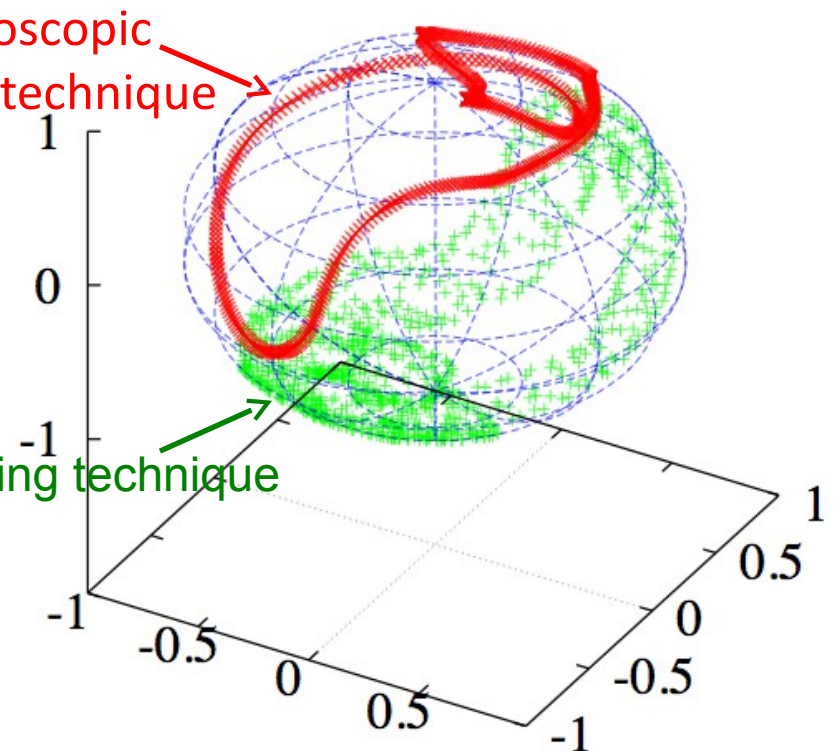
Stable Spin Direction



- Particles on a 20π mm-mrad phase space

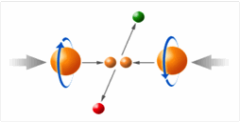


- Particles on a 40π mm-mrad phase space



D. P. Barber, M. Vogt, The Amplitude Dependent Spin Tune and The Invariant Spin Field in High Energy Proton Accelerators, Proceedings of EPAC98

Resonance Crossing



- In a planar ring, for a single isolated resonance at

$$G\gamma = K$$

- Froissart-Stora formula[1]: 1960

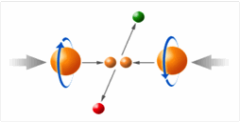
$$p_f = p_i (2e^{-\pi|\varepsilon_K|^2/\alpha} - 1) \text{ with } \alpha = d(G\gamma) / d\theta$$

and resonance strength is

$$\varepsilon_K = \frac{1}{2\pi} \int_{\square} [(1 + G\gamma) \frac{\Delta B_x}{B\rho} + (1 + G) \frac{\Delta B_{//}}{B\rho}] e^{iK\theta} ds$$

[1] Froissart-Stora, Depolarisation d'un faisceau de protons polarises dans un synchrotron, NIM (1960)

Resonance Crossing



- For an imperfection

$$\varepsilon_K \propto G\gamma \sqrt{\langle y_{co}^2 \rangle}$$

- No depolarization dependence on the betatron amplitude

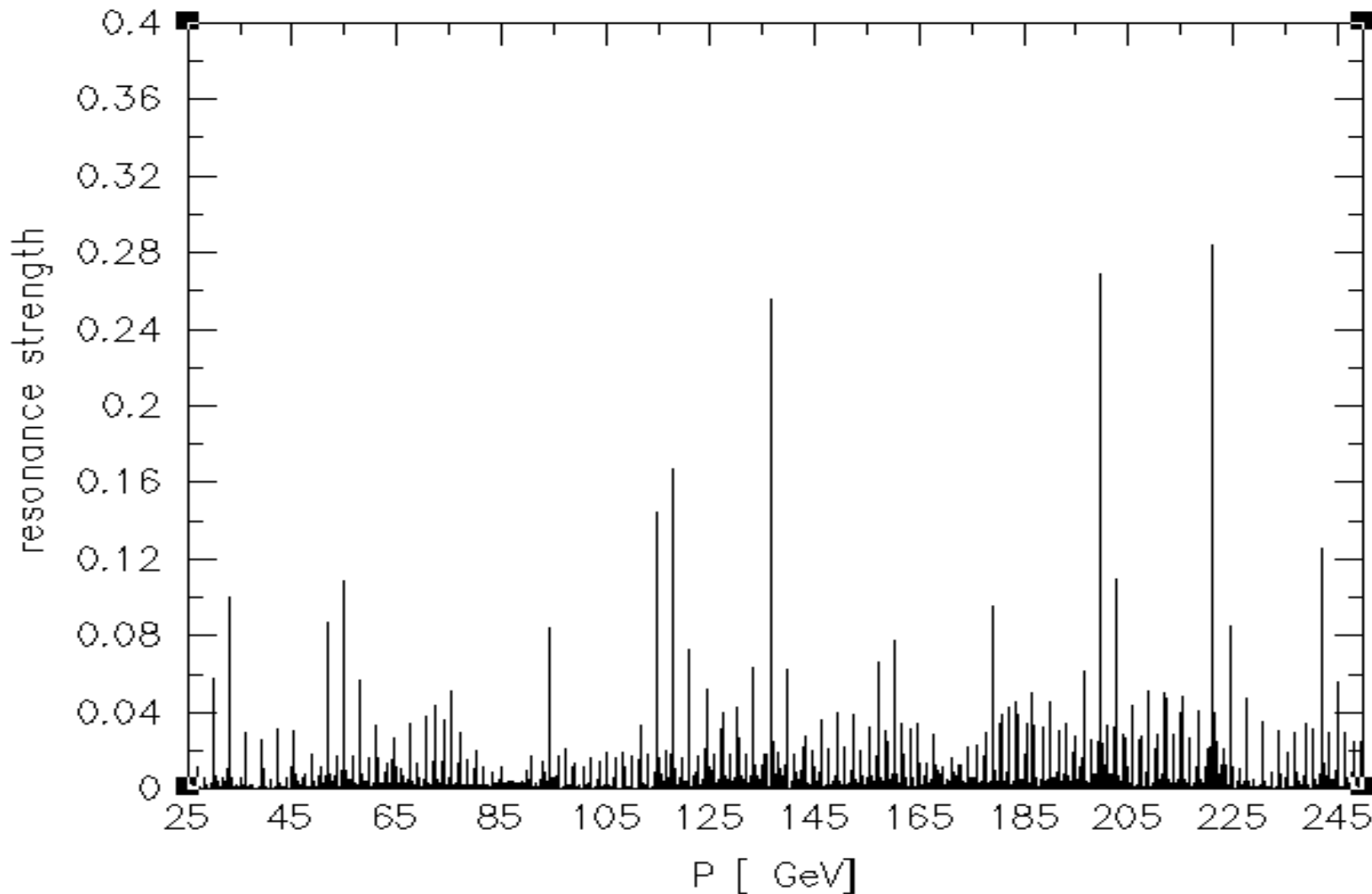
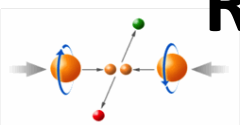
- For an intrinsic resonance

$$\varepsilon_K \propto G\gamma \sqrt{\varepsilon_{y,N} / \beta\gamma}$$

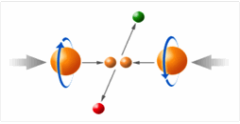
- Source of **polarization profile**, i.e. polarization depends on the particle's betatron amplitude in a beam
- For a Gaussian beam,

$$p_f = p_i \frac{1 - \pi |\epsilon_{K,rms}|^2 / \alpha}{1 + \pi |\epsilon_{K,rms}|^2 / \alpha}$$

RHIC Intrinsic Spin Depolarizing Resonance



Overcoming Depolarizing Resonance

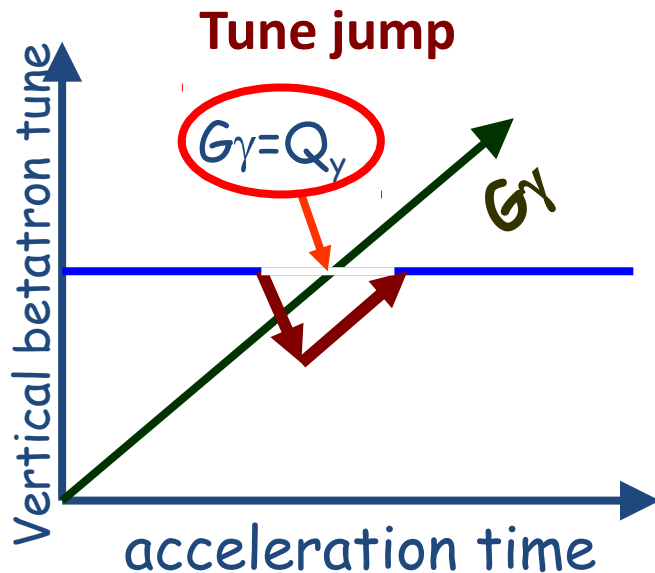


○ Harmonic orbit correction

○ to minimize the closed orbit distortion at all imperfection resonances

○ Operationally difficult for high energy accelerators

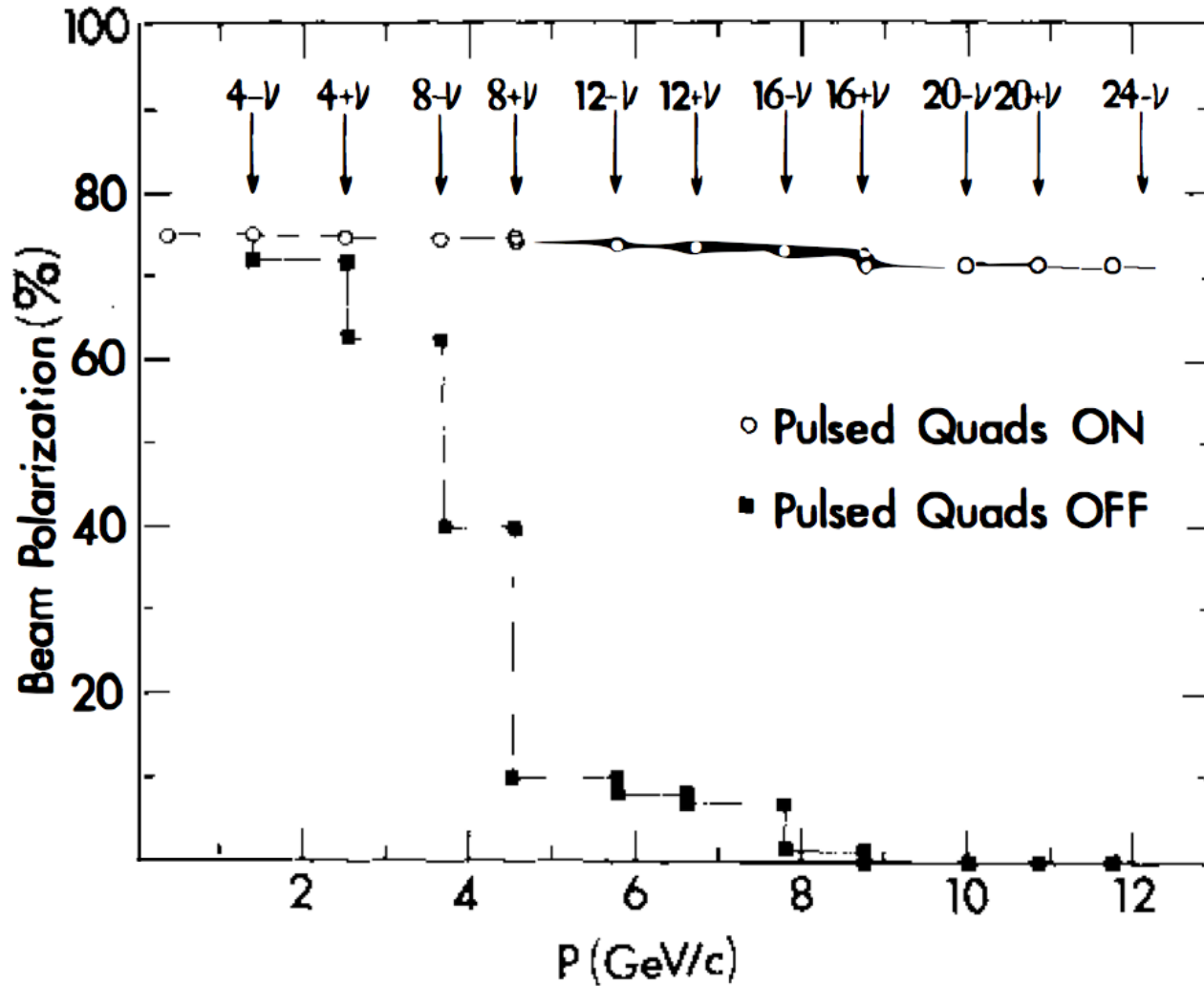
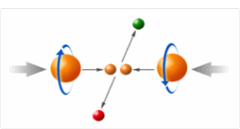
○ Tune Jump



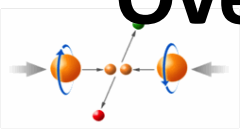
• Operationally difficult because of the number of resonances

• Also induces emittance blowup because of the non-adiabatic beam manipulation

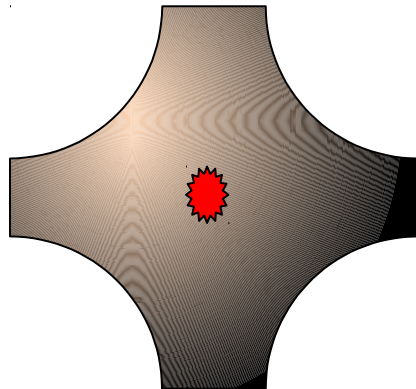
Zero Gradient Synchrotron Tune Jump



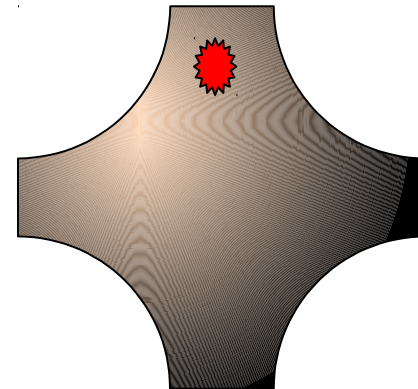
Overcome Intrinsic Resonance w. RF Dipole



- Adiabatically induces a vertical coherent betatron oscillation
 - Drive all particles to large amplitude to enhance the resonance strength
 - full spin flip with normal resonance crossing rate
 - Easy to control and avoid emittance blowup
 - Employed for the AGS polarized proton operation from 1998-2005



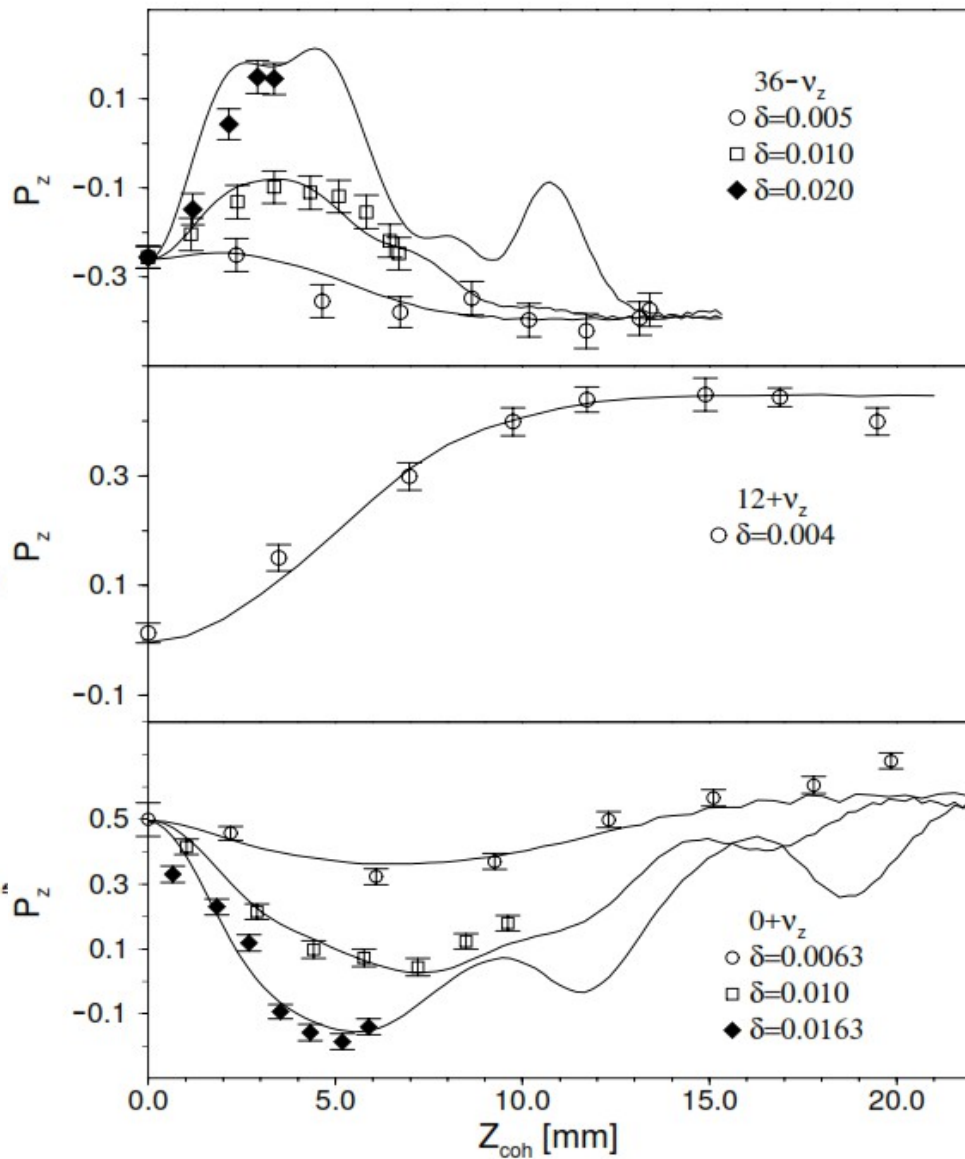
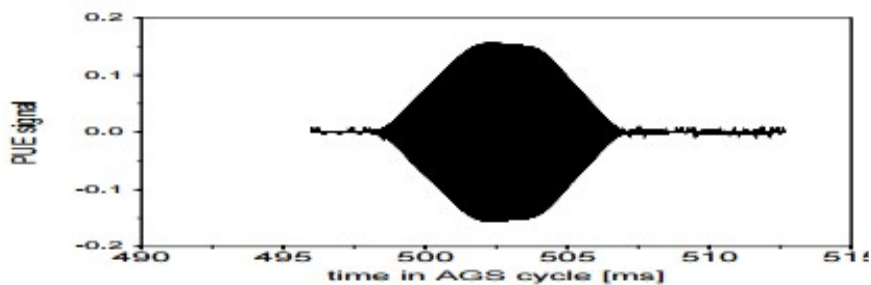
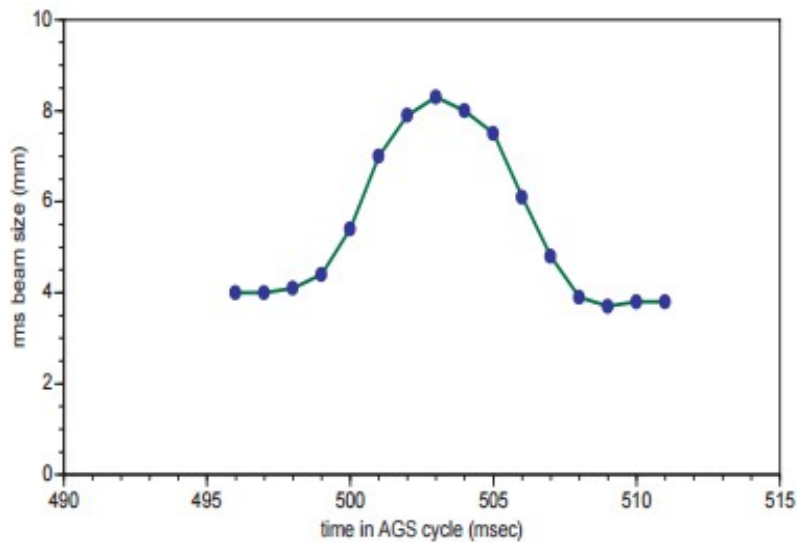
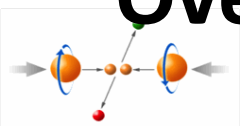
Without coherent oscillation



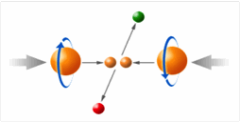
With coherent oscillation

- Can only be applied to strong intrinsic spin resonances

Overcome Intrinsic Resonance w. RF Dipole

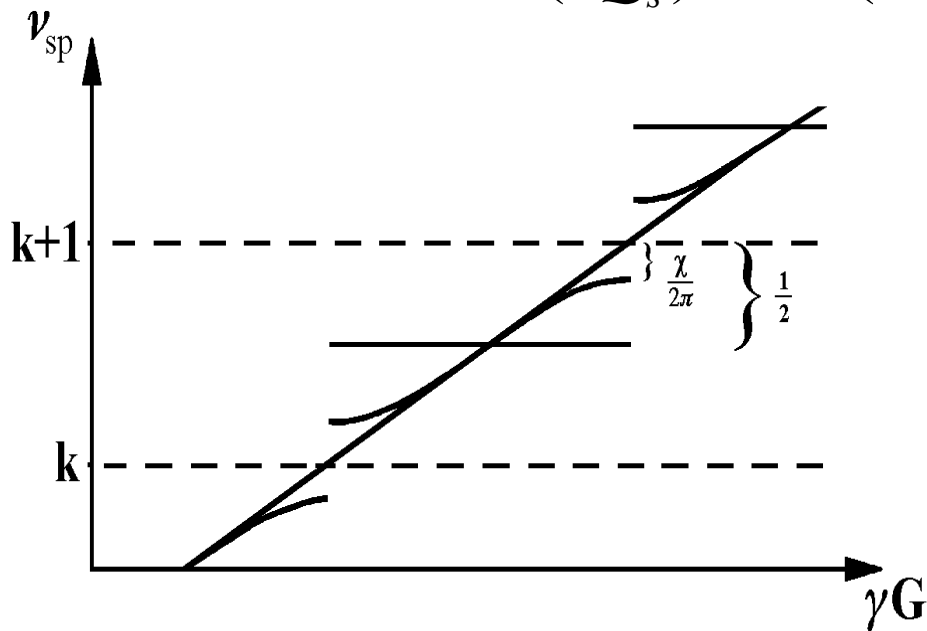


Partial Siberian Snake

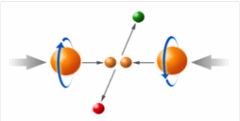


- rotates spin vector by an angle of $\psi < 180^\circ$
- Keeps the spin tune away from integer
- Primarily for avoiding imperfection resonance
- Can be used to avoid intrinsic resonance as demonstrated at the AGS, BNL.

$$\cos(\pi Q_s) = \cos(G\gamma\pi) \cos\left(\frac{\Psi}{2}\right)$$



Dual partial snake configuration

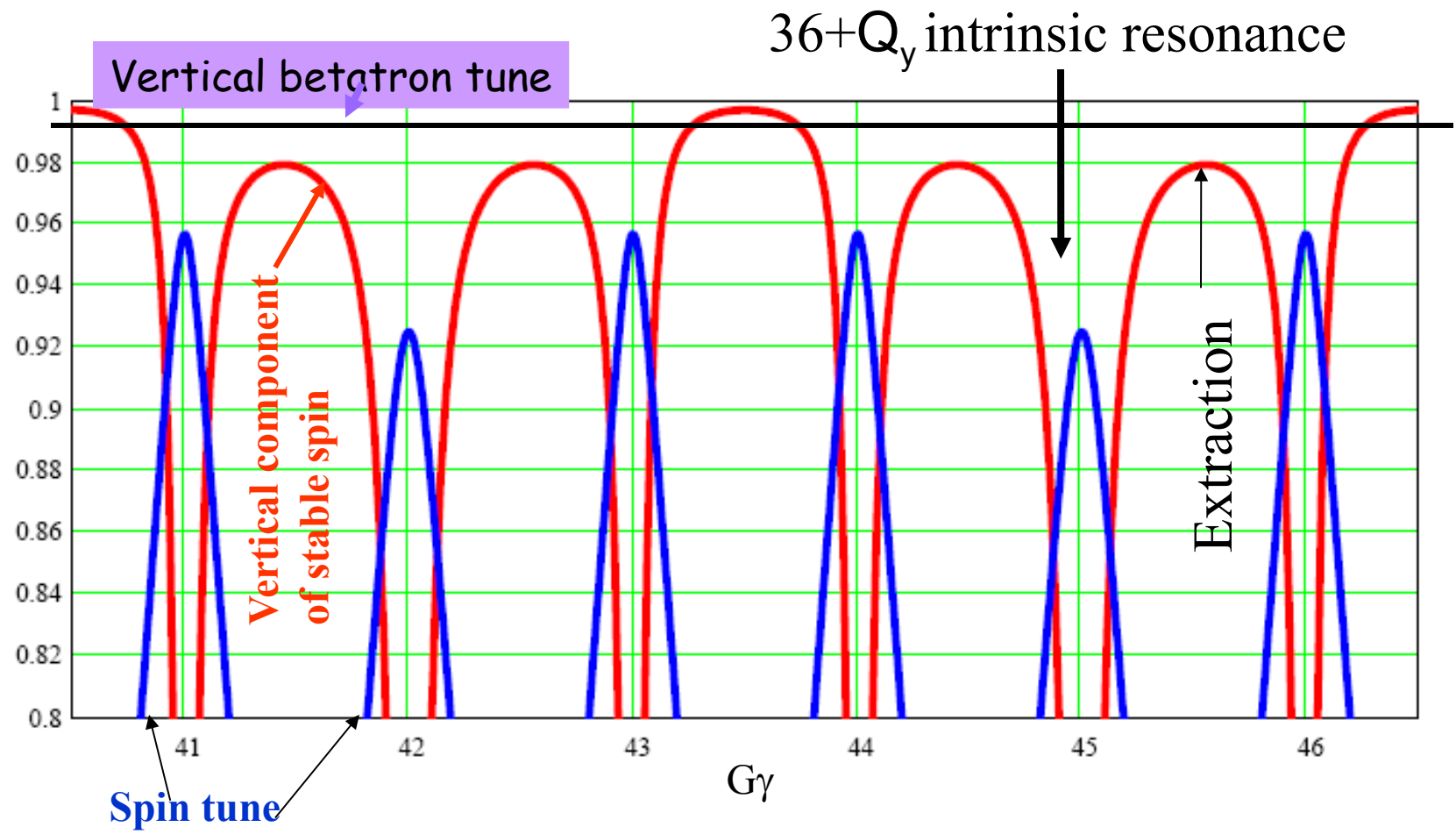
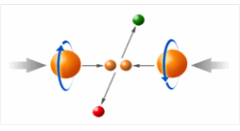


- For two partial snakes apart from each other by an angle of ϑ , spin tune the becomes

$$\cos \pi Q_s = \cos G\gamma\pi \cos \frac{\psi_1}{2} \cos \frac{\psi_2}{2} - \cos(G\gamma(\pi - \theta)) \sin \frac{\psi_1}{2} \sin \frac{\psi_2}{2}$$

- Spin tune is no-longer integer, and stable spin direction is also tilted away from vertical
- The distance between spin tune and integer is modulated with $\text{Int}[360/\vartheta]$. For every integer of $\text{Int}[360/\vartheta]$ of $G\gamma$, the two partial snakes are effectively added. This provides a larger gap between spin tune and integer, which can be wide enough to have the vertical tune inside the gap to avoid both intrinsic and imperfection resonance
- Stable spin direction is also modulated

Spin tune with two partial snakes



$36+Q_y$ intrinsic resonance

Vertical betatron tune

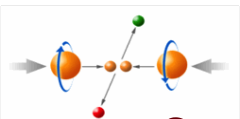
Vertical component of stable spin

Extraction

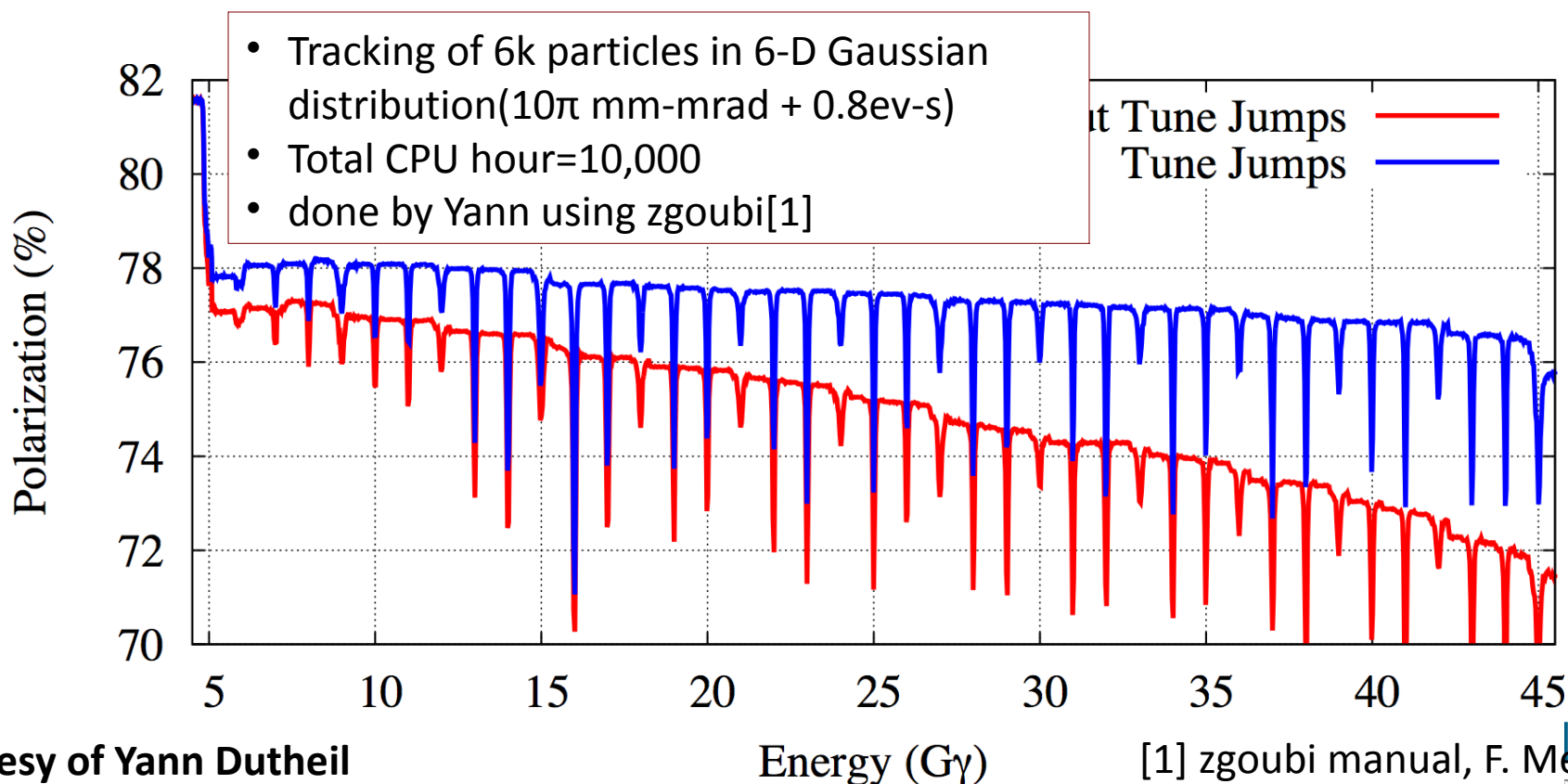
Spin tune

$$\cos\pi Q_s = \cos G\gamma\pi \cos \frac{\Psi_w}{2} \cos \frac{\Psi_c}{2} - \cos G\gamma \frac{\pi}{3} \sin \frac{\Psi_w}{2} \sin \frac{\Psi_c}{2}$$

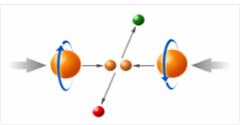
Horizontal Resonance



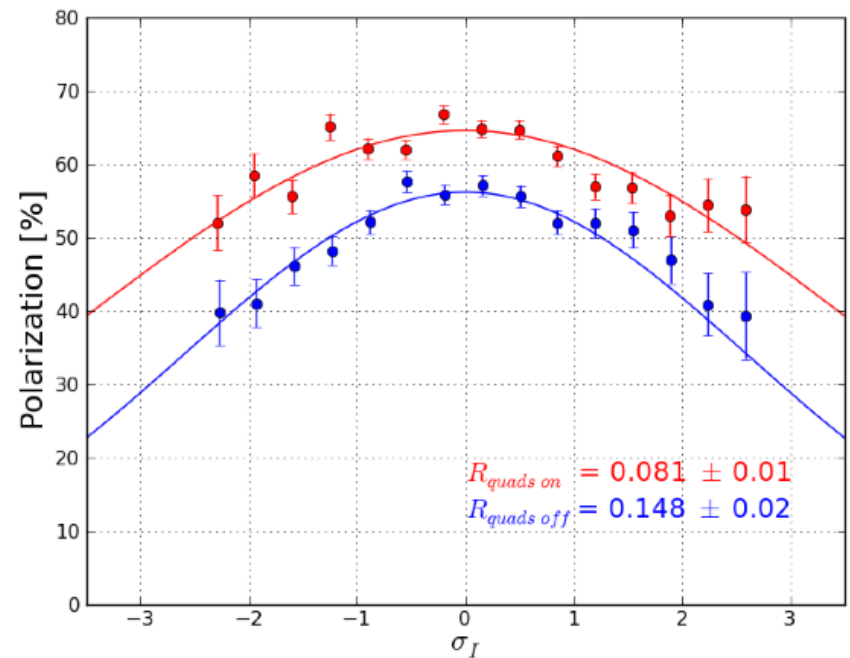
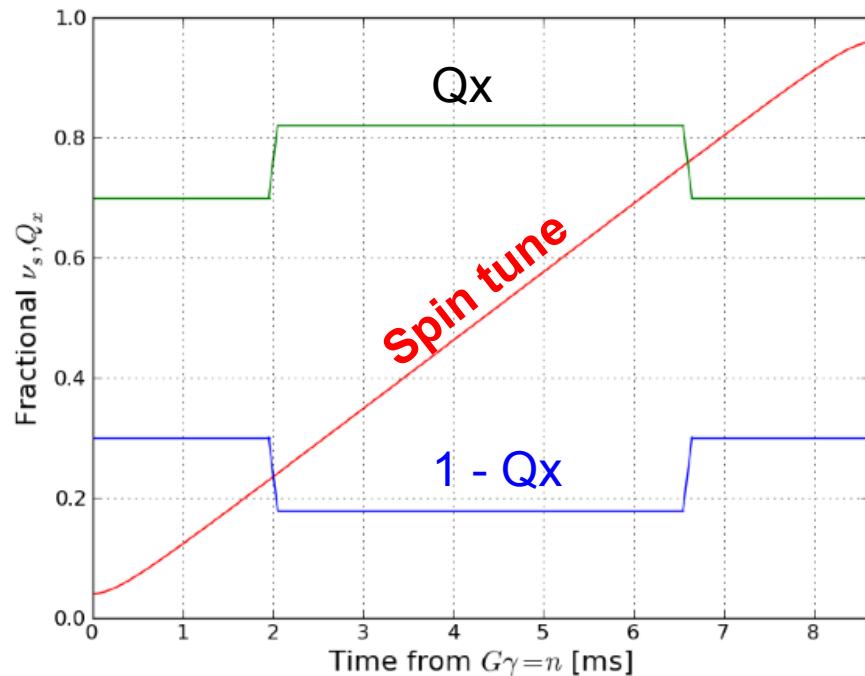
- Stable spin direction in the presence of two partial snakes is no longer along vertical direction
 - vertical fields due to horizontal betatron oscillation can drive a resonance at $G\gamma = kP \pm Q_x$
 - Each is weak, and can be cured by tune jump



Overcome Horizontal Resonance

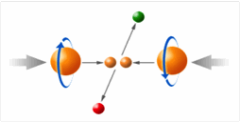


- AGS horizontal tune jump quadrupoles to overcome a total of 80 weak horizontal spin resonances during the acceleration

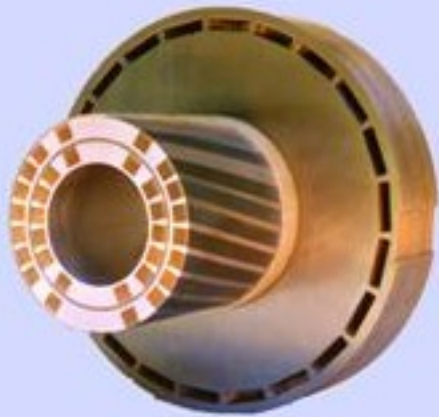
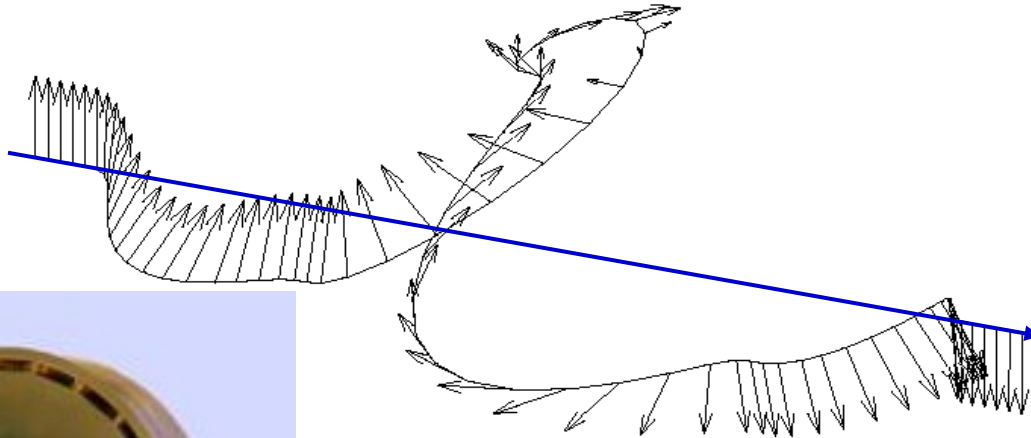


V. Schoefer *et al*, INCREASING THE AGS BEAM POLARIZATION WITH 80 TUNE JUMPS, Proceedings of IPAC2012, New Orleans, Louisiana, USA

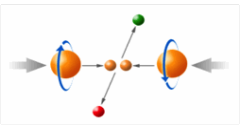
Full Siberian Snake



- A magnetic device to rotate spin vector by 180°
- Invented by Derbenev and Kondratanko in 1970s [*Polarization kinematics of particles in storage rings, Ya.S. Derbenev, A.M. Kondratanko* (Novosibirsk, IYF) . Jun 1973. Published in Sov.Phys.JETP 37:968-973,1973, Zh.Eksp.Teor.Fiz 64:1918-1929]
- Keep the spin tune independent of energy



Snake Depolarization Resonance



- Condition

$$mQ_y = Q_s + k$$

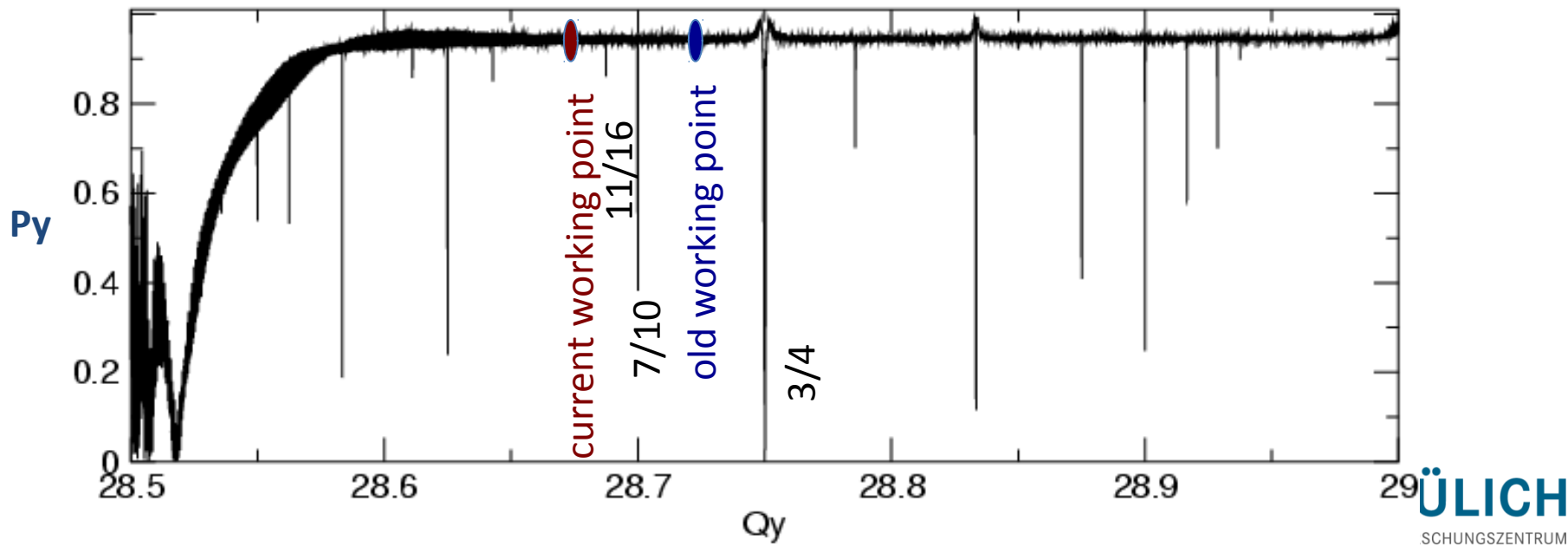
- S. Y. Lee, Tepikian, Phys. Rev. Lett. 56 (1986) 1635
- S. R. Mane, NIM in Phys. Res. A. 587 (2008) 188-212

- even order resonance

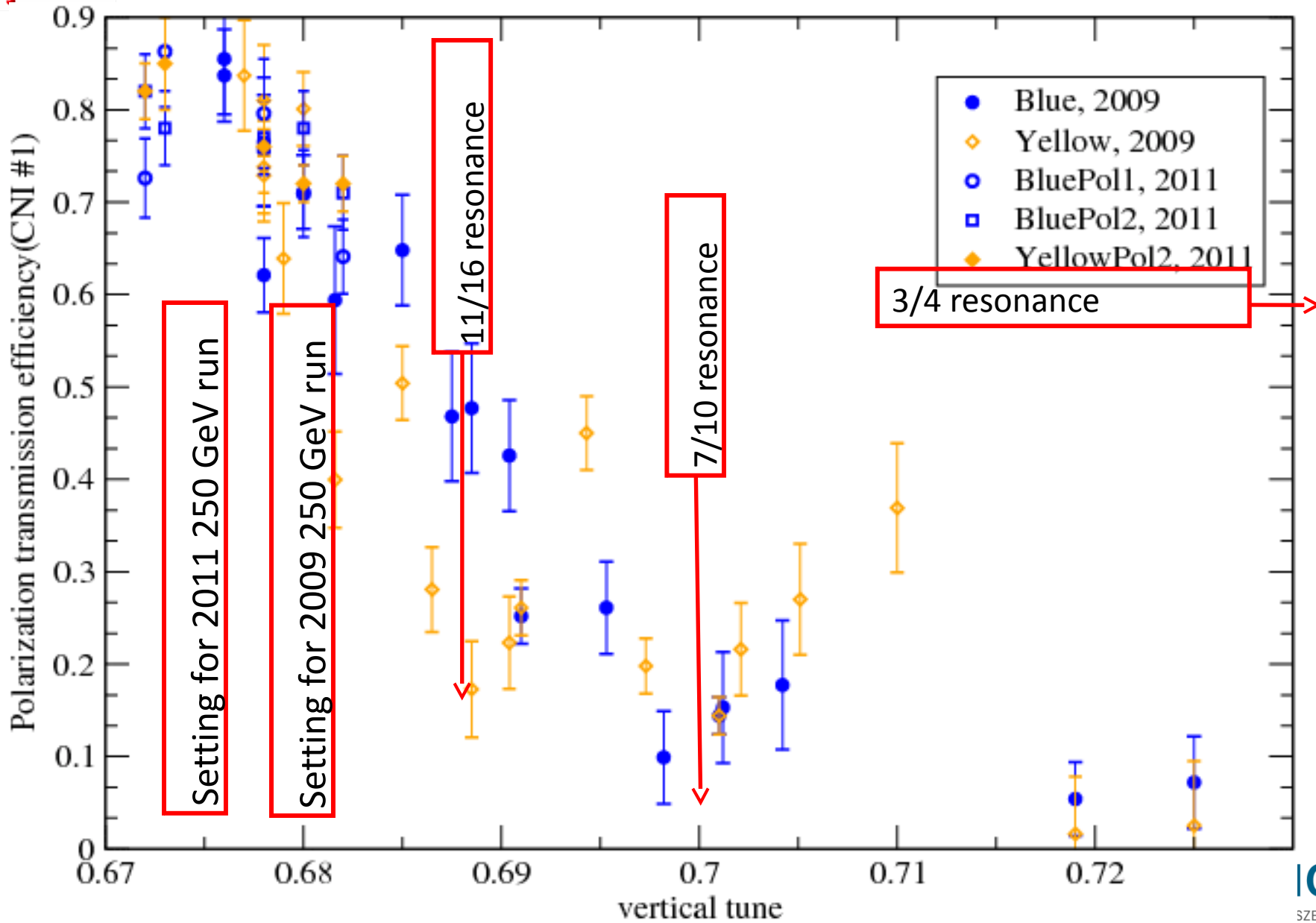
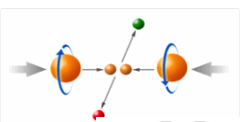
- Disappears in the two snake case if the closed orbit is perfect

- odd order resonance

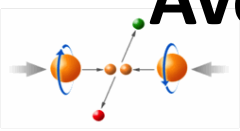
- Driven by the intrinsic spin resonances



Snake resonance observed in RHIC



Avoid polarization losses due to snake resonance



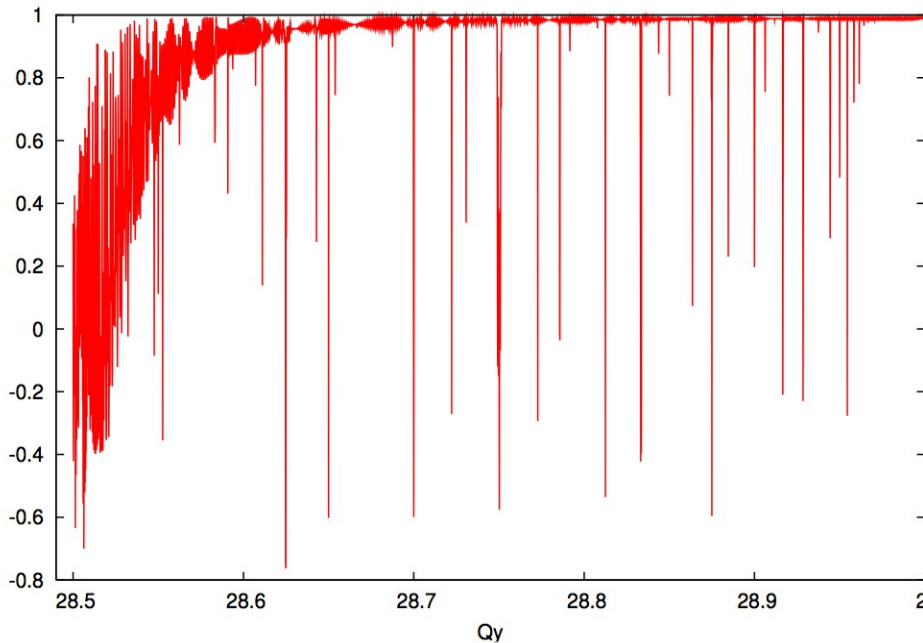
- Adequate number of snakes

$$N_{snk} > 4 \left| \varepsilon_{k,max} \right| \quad Q_s = \sum_{k=1}^{N_{snk}} (-1)^k \phi_k$$

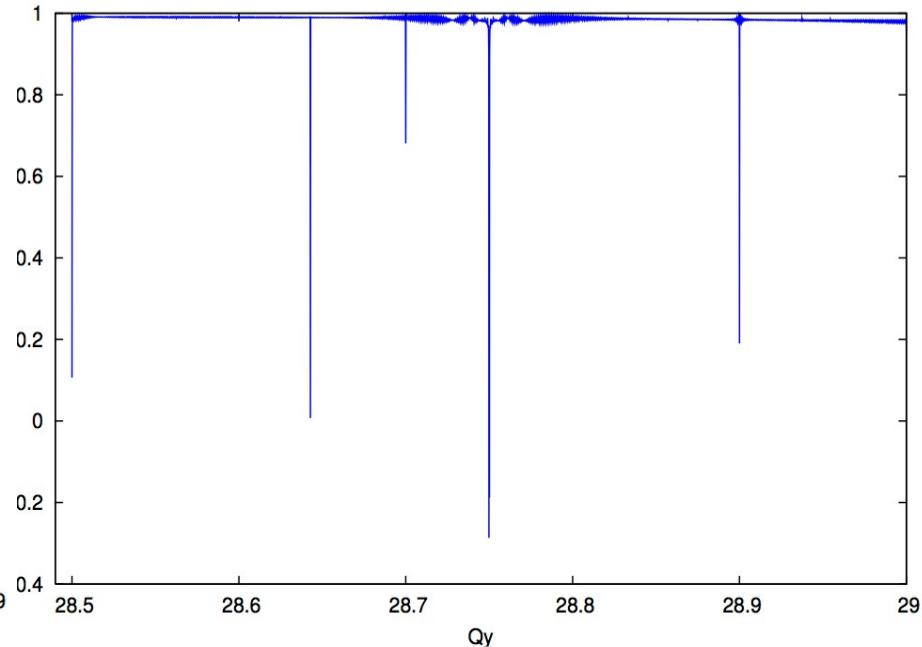
ϕ_k is the snake axis relative to the beam direction

- Minimize number of snake resonances to gain more tune spaces for operations

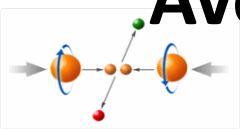
He-3 with dual snake



He-3 with six-snake



Avoid polarization losses due to snake resonance



- Adequate number of snakes

$$N_{snk} > 4 \left| \epsilon_{k,max} \right| \quad Q_s = \sum_{k=1}^{N_{snk}} (-1)^k \phi_k$$

ϕ_k is the snake axis relative to the beam direction

- Keep spin tune as close to 0.5 as possible

- **Source of spin tune deviation**

- Snake configuration

- Local orbit at snakes as well as other spin rotators. For RHIC,

angle between two snake axes

$$\Delta Q_s = \frac{|\Delta \phi|}{\pi} + (1 + G\gamma) \frac{\Delta \theta}{\pi}$$

H orbital angle between two snakes

- **Source of spin tune spread**

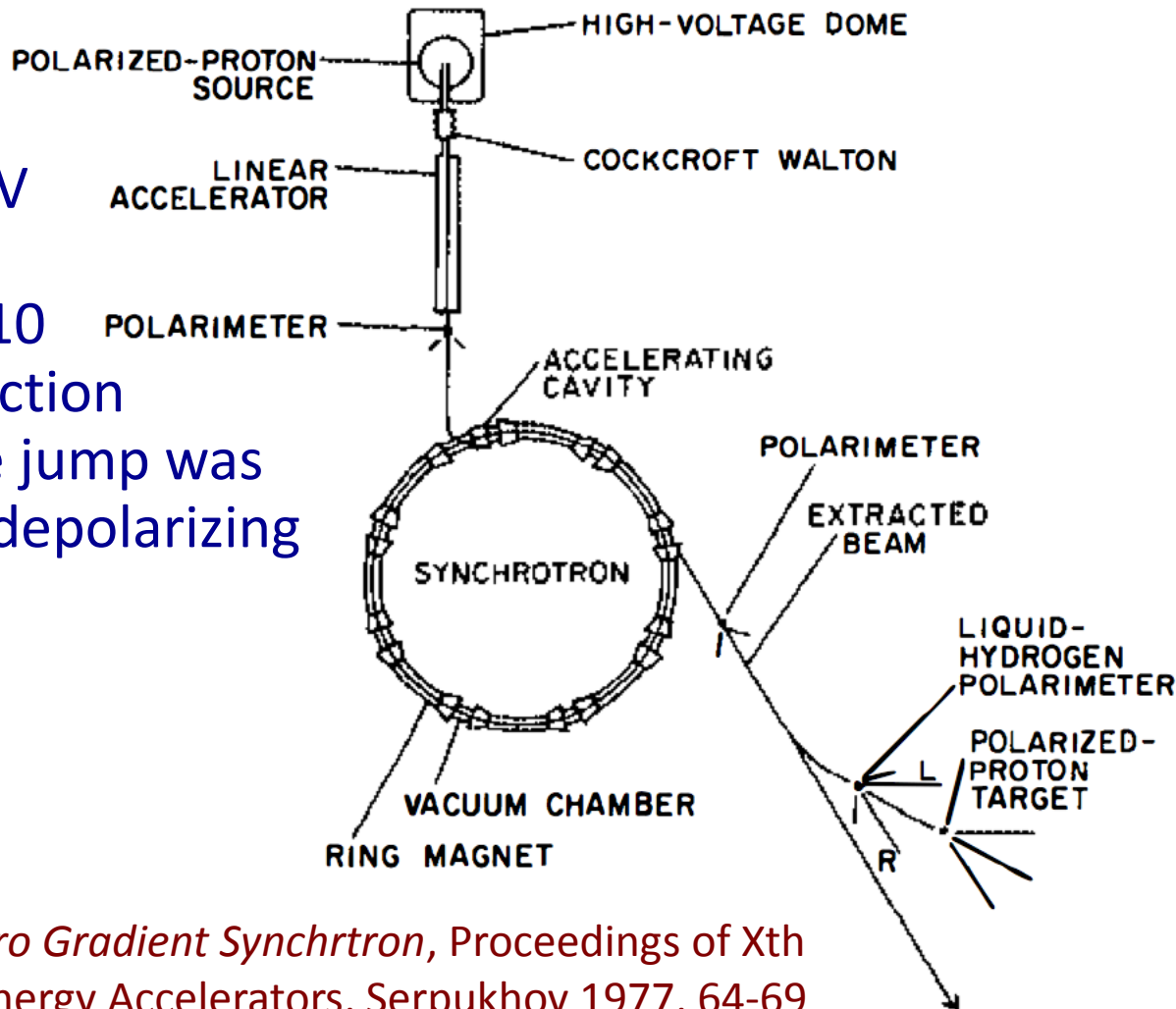
- momentum dependence due to local orbit at snakes
- betatron amplitude dependence

History of High Energy Polarized Proton Beams

ZGS at Argonne National Lab

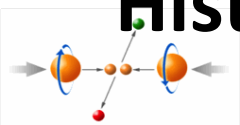
- 1969~1973:

- proton energy 1-12 GeV
- Polarization 71%
- Beam intensity: 9×10^{10}
- Orbital harmonic correction together with fast tune jump was used to overcome the depolarizing resonances



L. G. Ratner, *Polarized Protons at Zero Gradient Synchrotron*, Proceedings of Xth International Conference On High Energy Accelerators, Serpukhov 1977, 64-69

History of High Energy Polarized Proton Beams

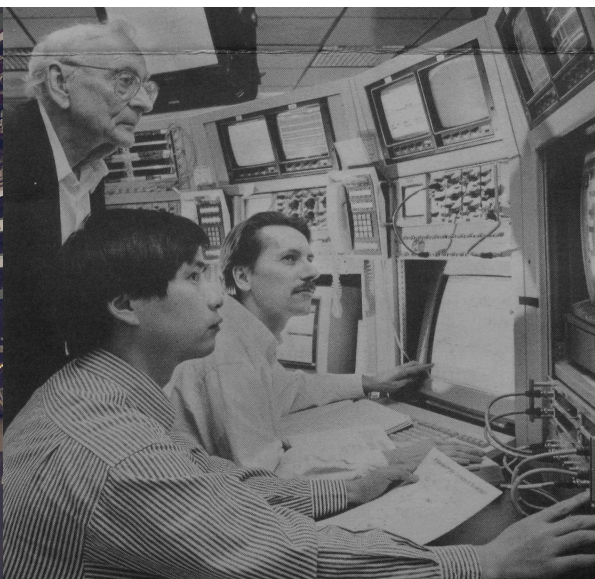


Brookhaven AGS : 1974~present

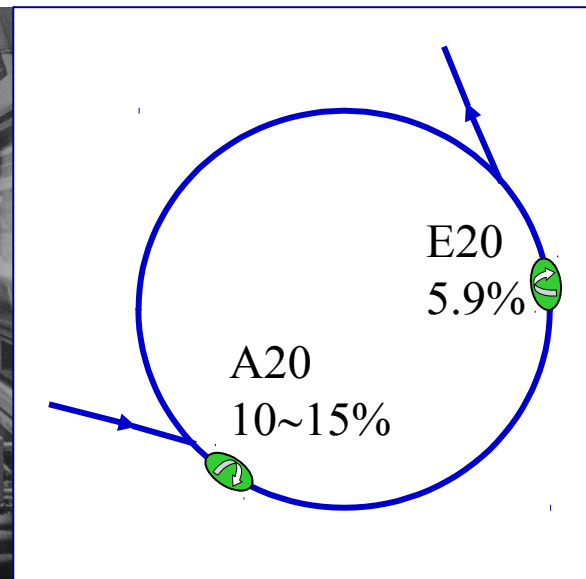
1980s



1990s



2006 - now



Alan Krisch and Larry Ratner in the AGS MCR.

~ 40% polarization at 22 GeV, 7 weeks dedicated time for setup

5% snake +RF dipole

~ 2 weeks setup parasitic to RHIC Ion program

50% at 24 GeV

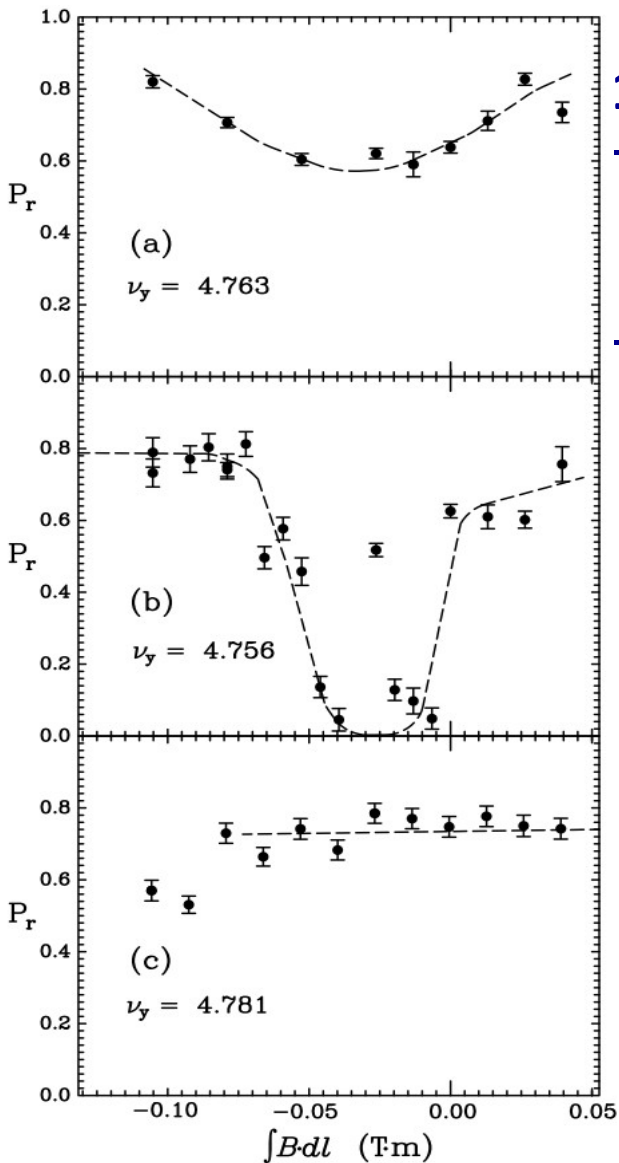
6% warm helical snake +10% cold helical snake

~2 weeks setup

65%-70% at 24 GeV

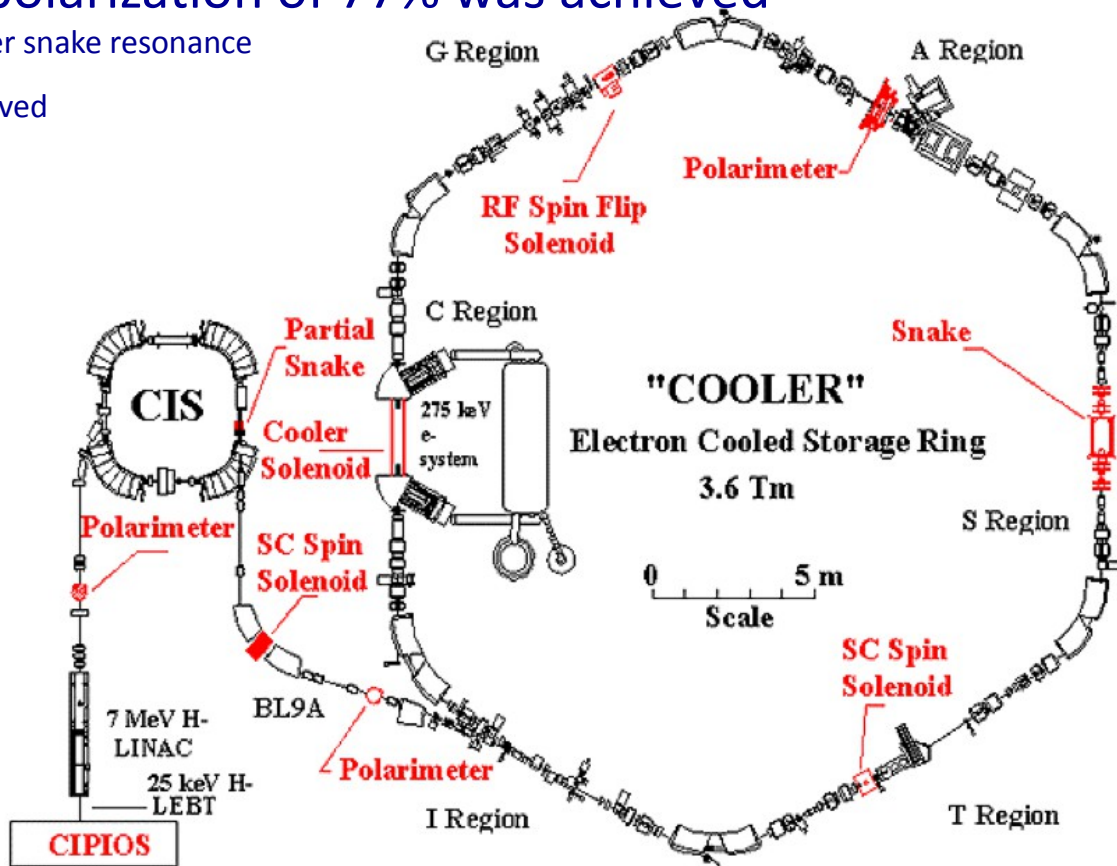
History of High Energy Polarized Proton Beams

Cooler Ring at Indiana University Cyclotron Facility



1985 -- 2002:

- Successfully accelerated polarized protons up to 200MeV with a super-conducting solenoid snake. Best polarization of 77% was achieved
- 2nd order snake resonance was observed

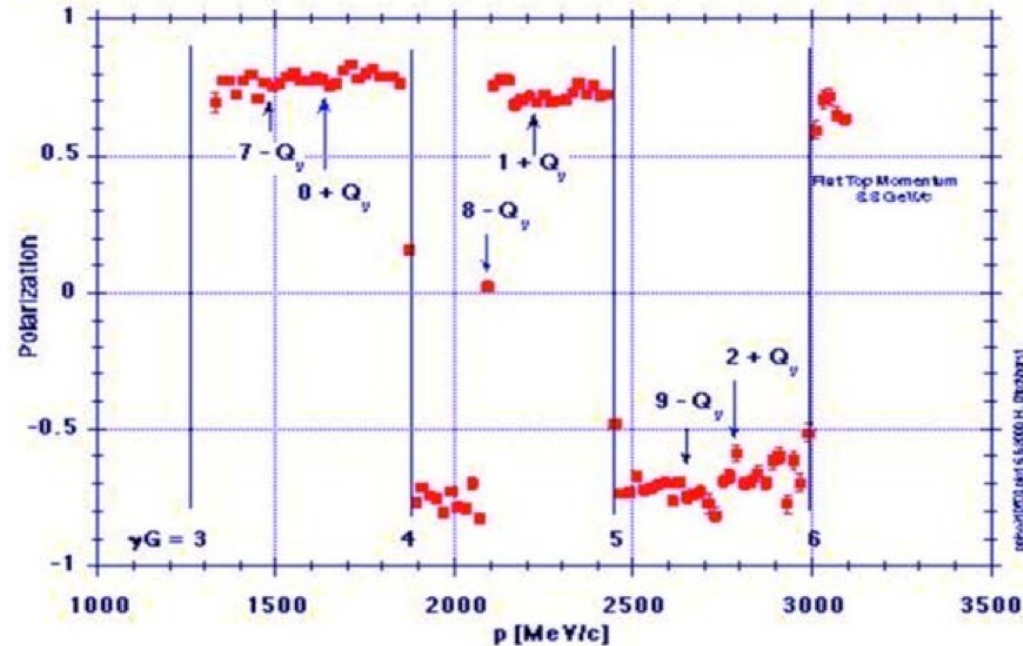
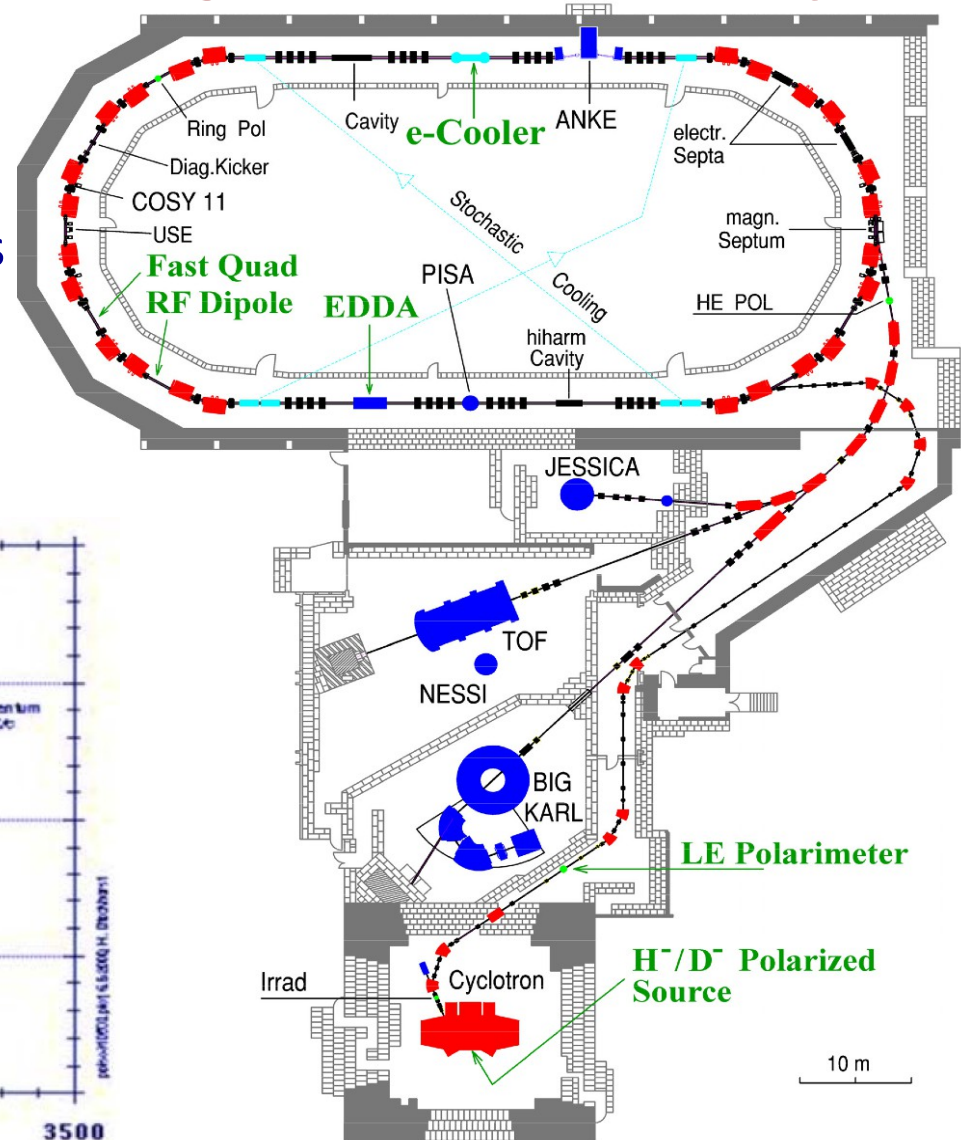


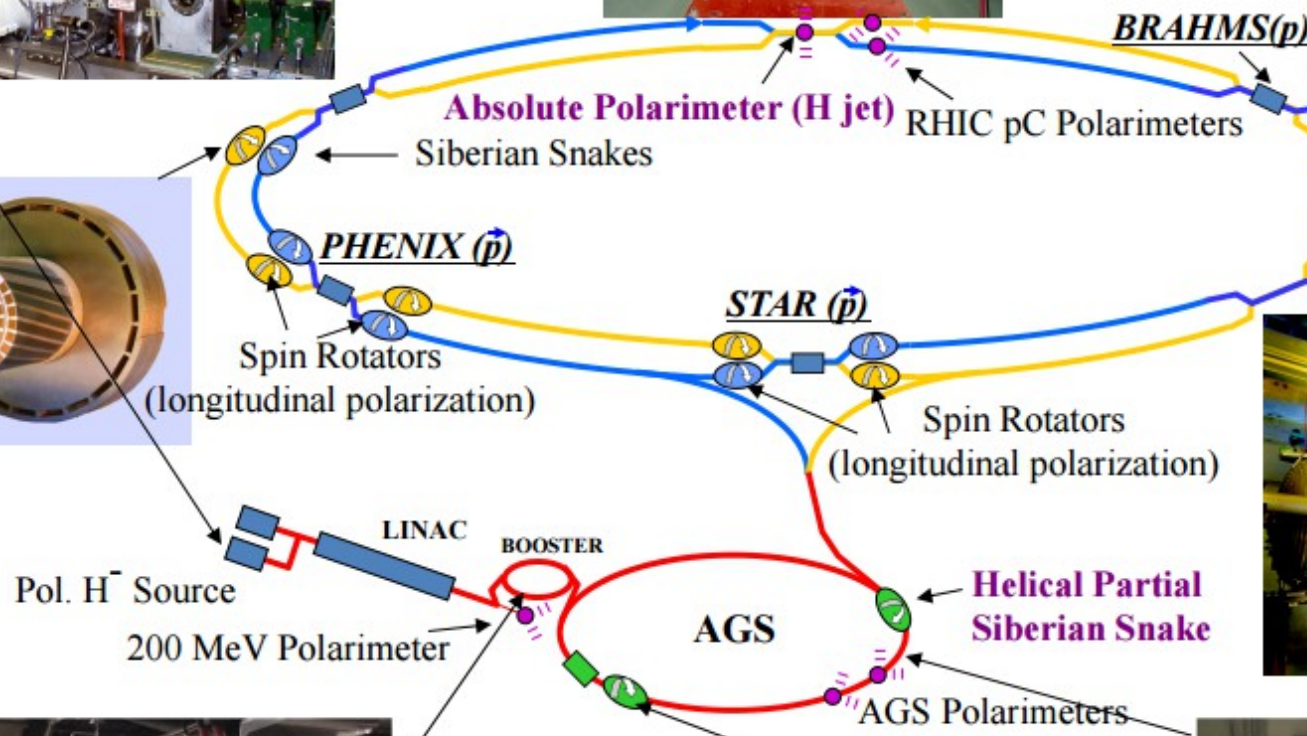
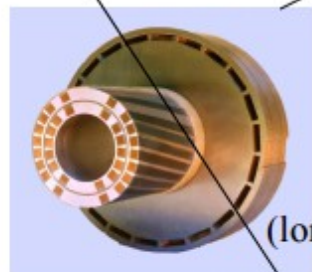
History of High Energy Polarized Proton Beams

COSY (Cooler Synchrotron ring) at Julich, Germany

- 1985 -- present:

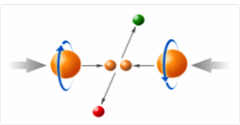
- proton energy: 3 GeV/c
- Full spin flip at each imperfection resonance with vertical correctors
- Fast tune jump with an air-core quadrupole at each intrinsic spin resonance





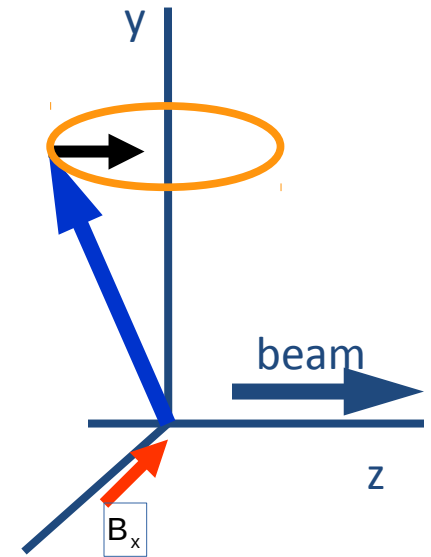
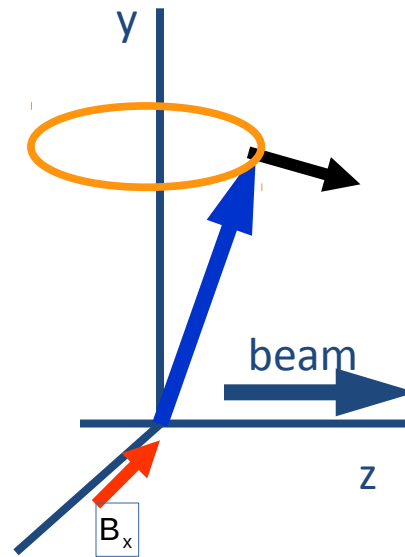
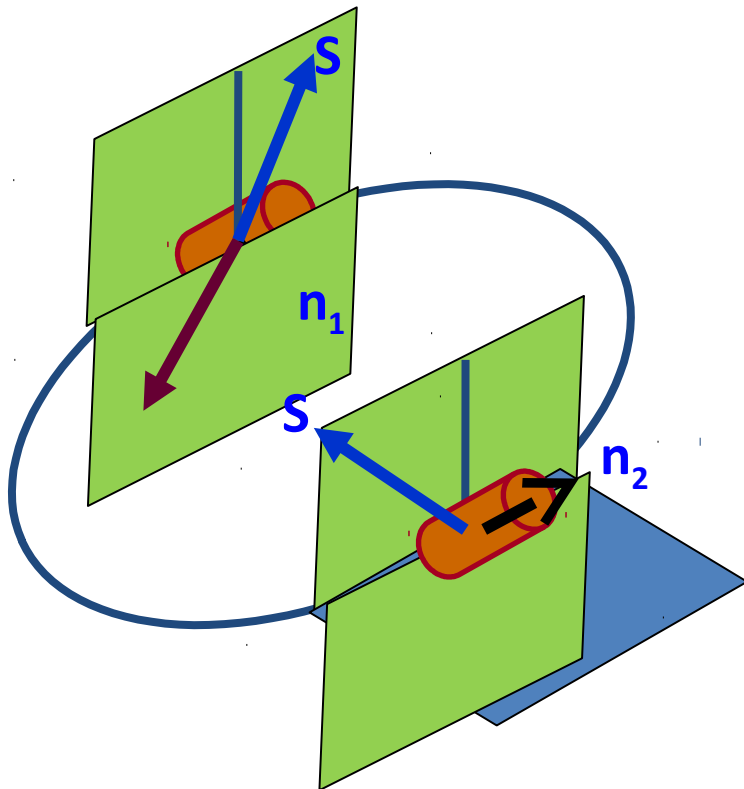
SpinFest, August 7, 2006

Dual Snake Set-up

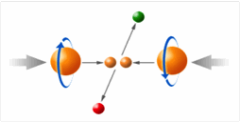


- Use one or a group of snakes to make the spin tune to be at $\frac{1}{2}$

- Break the coherent build-up of the perturbations on the spin vector

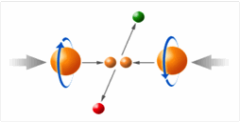


How to avoid a snake resonance?

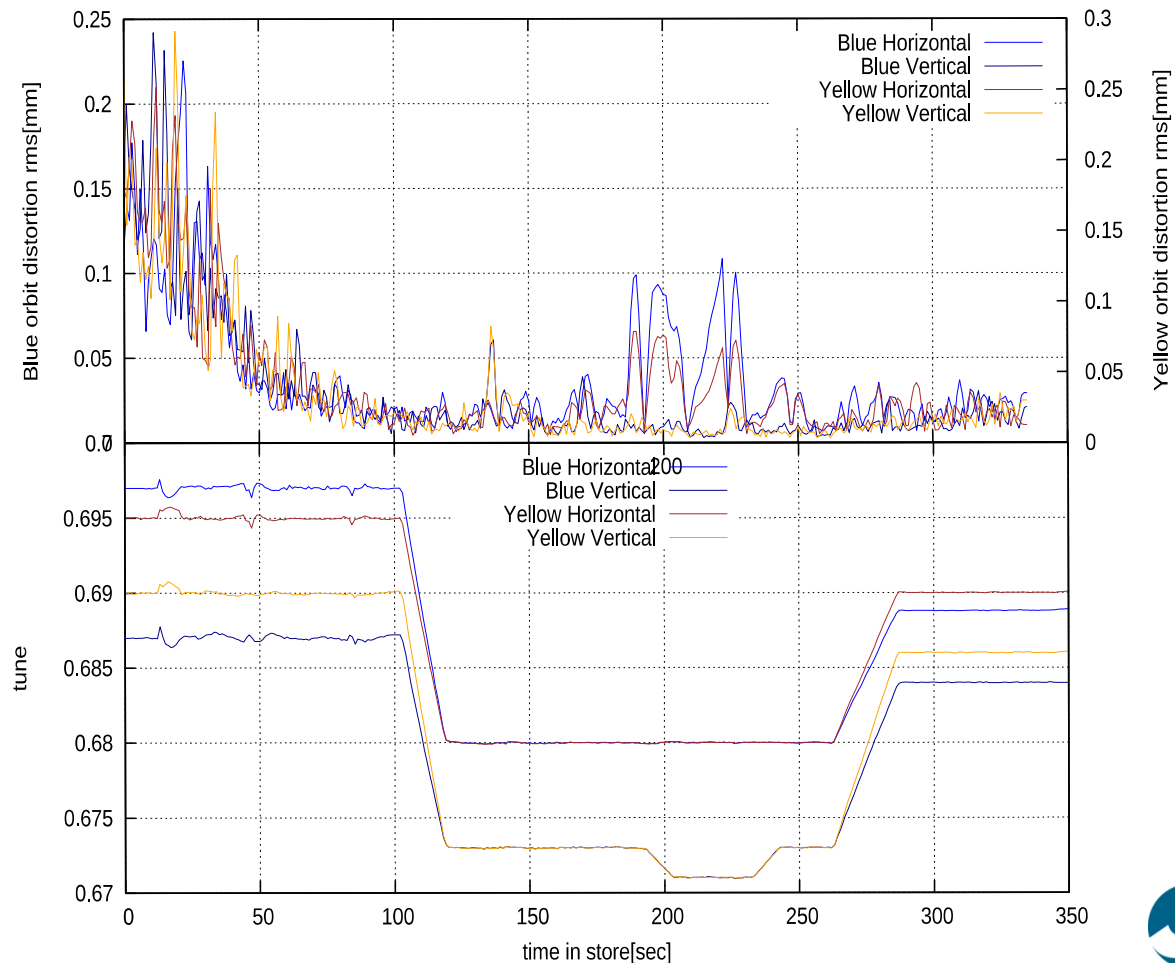


- Adequate number of snakes
- Keep spin tune as close to 0.5 as possible
- Precise control of the vertical closed orbit
- Precise optics control
 - Choice of working point to avoid snake resonances
 - near 3rd order resonance. Current RHIC operating tune is chosen to be $Q_y=0.673$ for acceleration beyond 100 GeV
 - near integer tune, much weaker snake resonances
 - However, it requires very robust linear optics correction
 - Minimize the linear coupling to avoid the resonance due to horizontal betatron oscillation

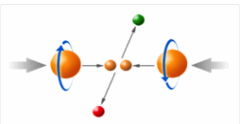
Precise Beam Control



- Tune/coupling feedback system: acceleration close to $2/3$ orbital resonance
- Orbit feedback system: rms orbit distortion less than 0.1mm



Beam-beam Effect on Polarization

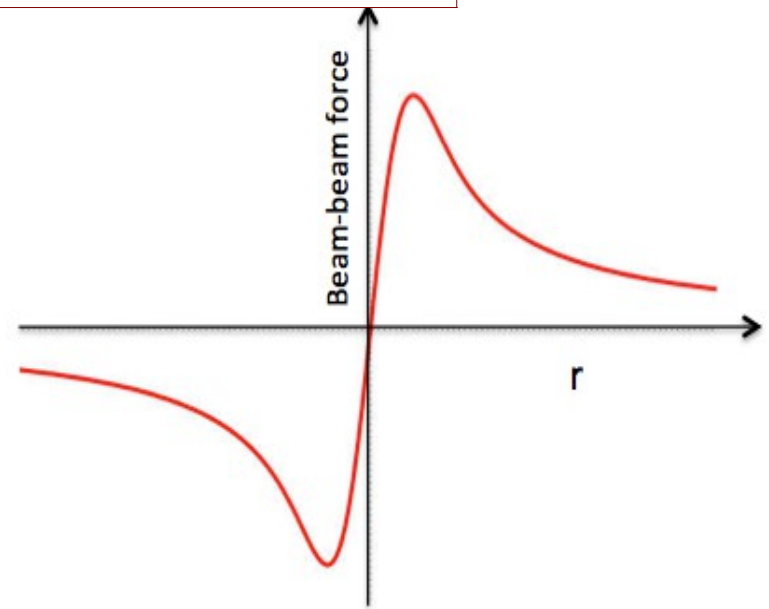
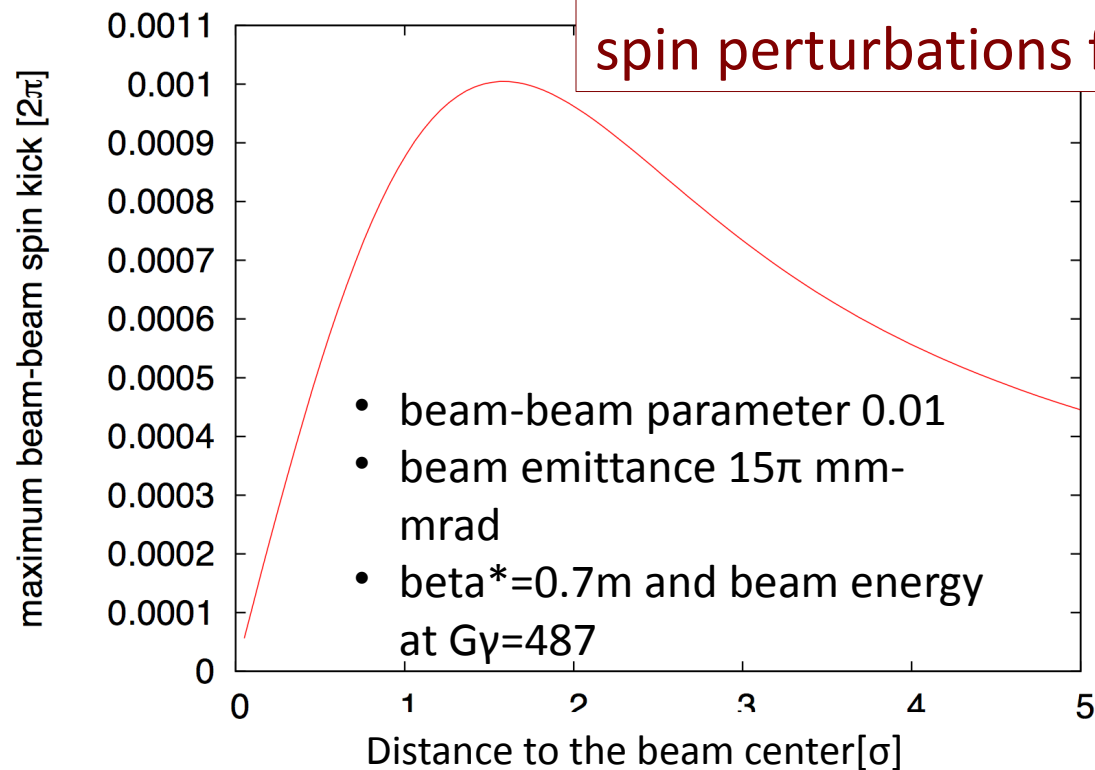


- Beam-Beam force on spin motion
- For a Gaussian round beam, particle from the other beam sees

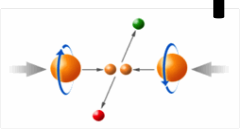
$$\vec{E} = \frac{qN}{2\pi\epsilon_0 lr} \left[1 - \exp\left(-\frac{r^2}{2\sigma^2}\right) \right] \hat{r}$$

$$\vec{B} = \frac{1}{c} \vec{\beta} \times \vec{E}$$

The effect is much weaker than the spin perturbations from the lattice



Polarization Performance and Beam-beam

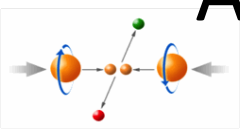


- Beam-Beam induces tune shift of

$$\xi = \frac{Nr_0\beta^*}{4\pi\gamma\sigma^2}$$

- It also induces an incoherent tune spread, which can populate particles on
 - orbital resonances, and causes emittance growth
 - snake resonances, and result in polarization loss during collision

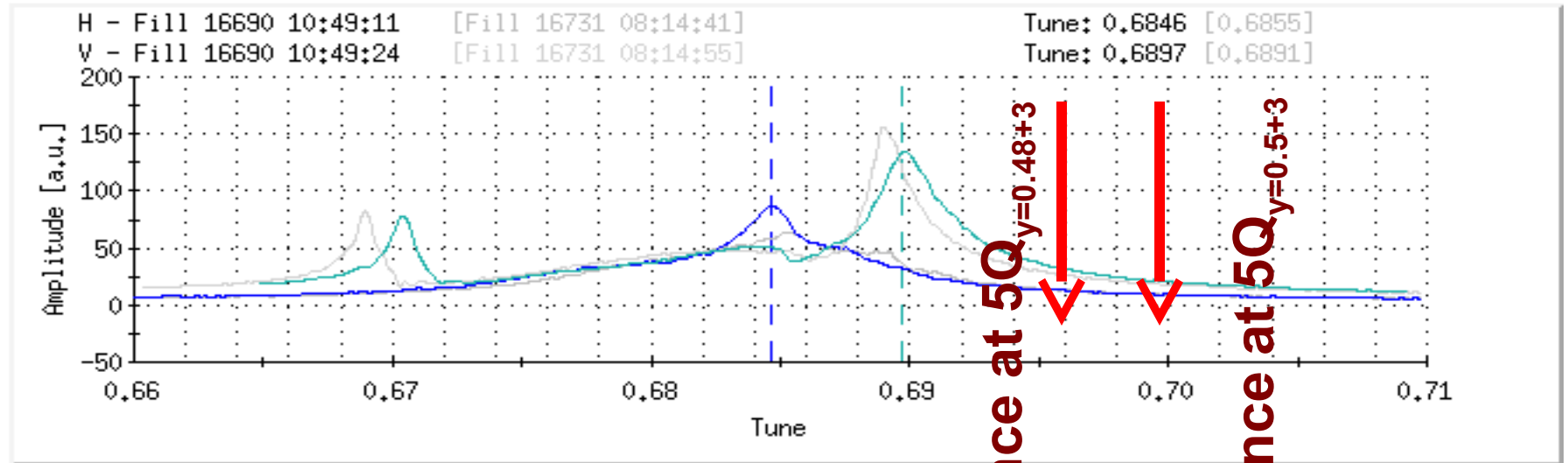
A Typical BTF of RHIC Beam in Collision



Hor A P T C

BLUE

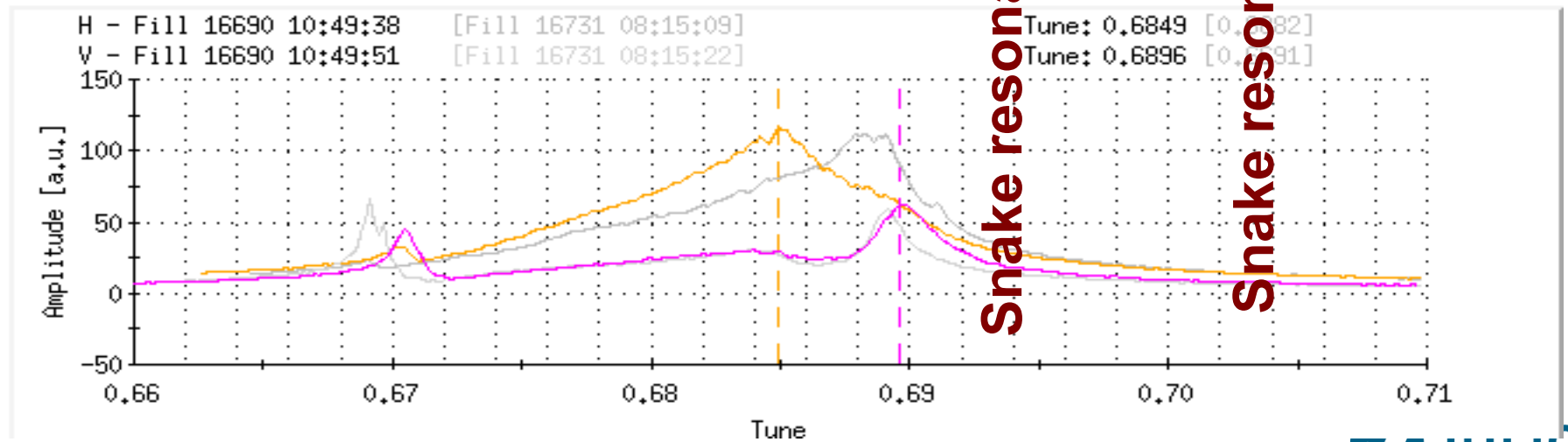
Ver A P T C



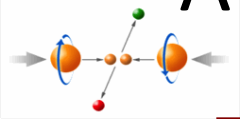
Hor A P T C

YELLOW

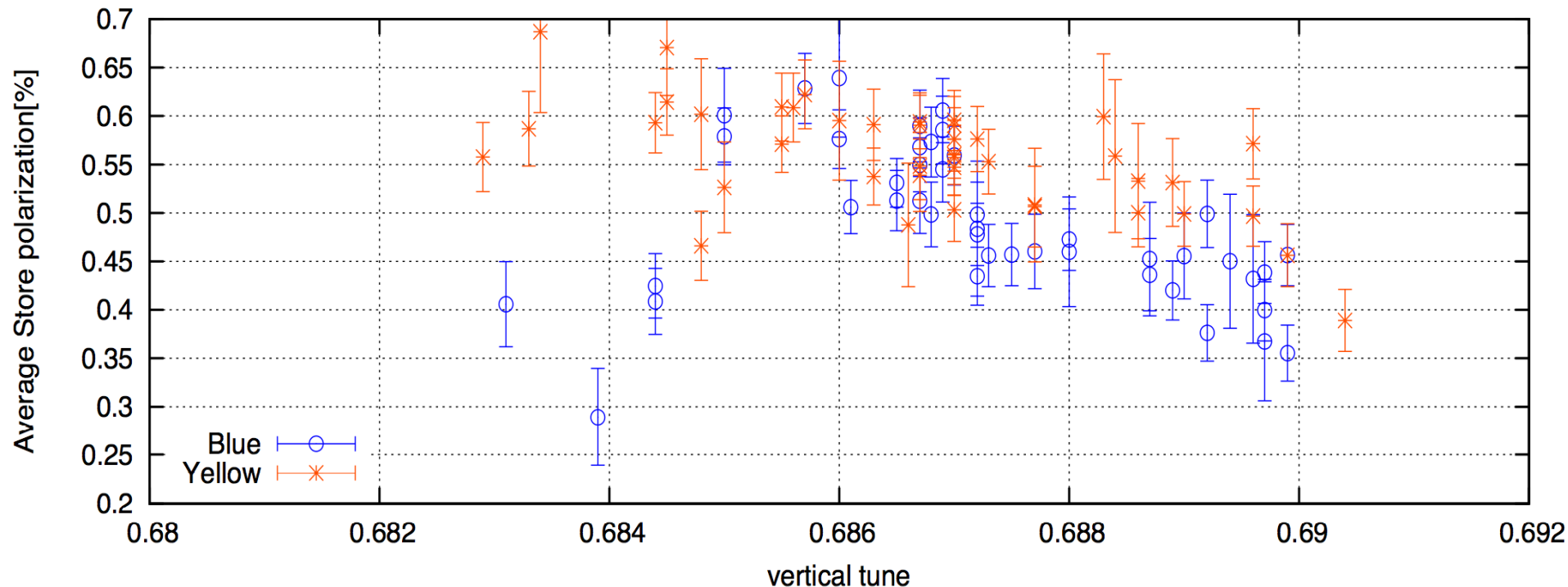
Ver A P T C



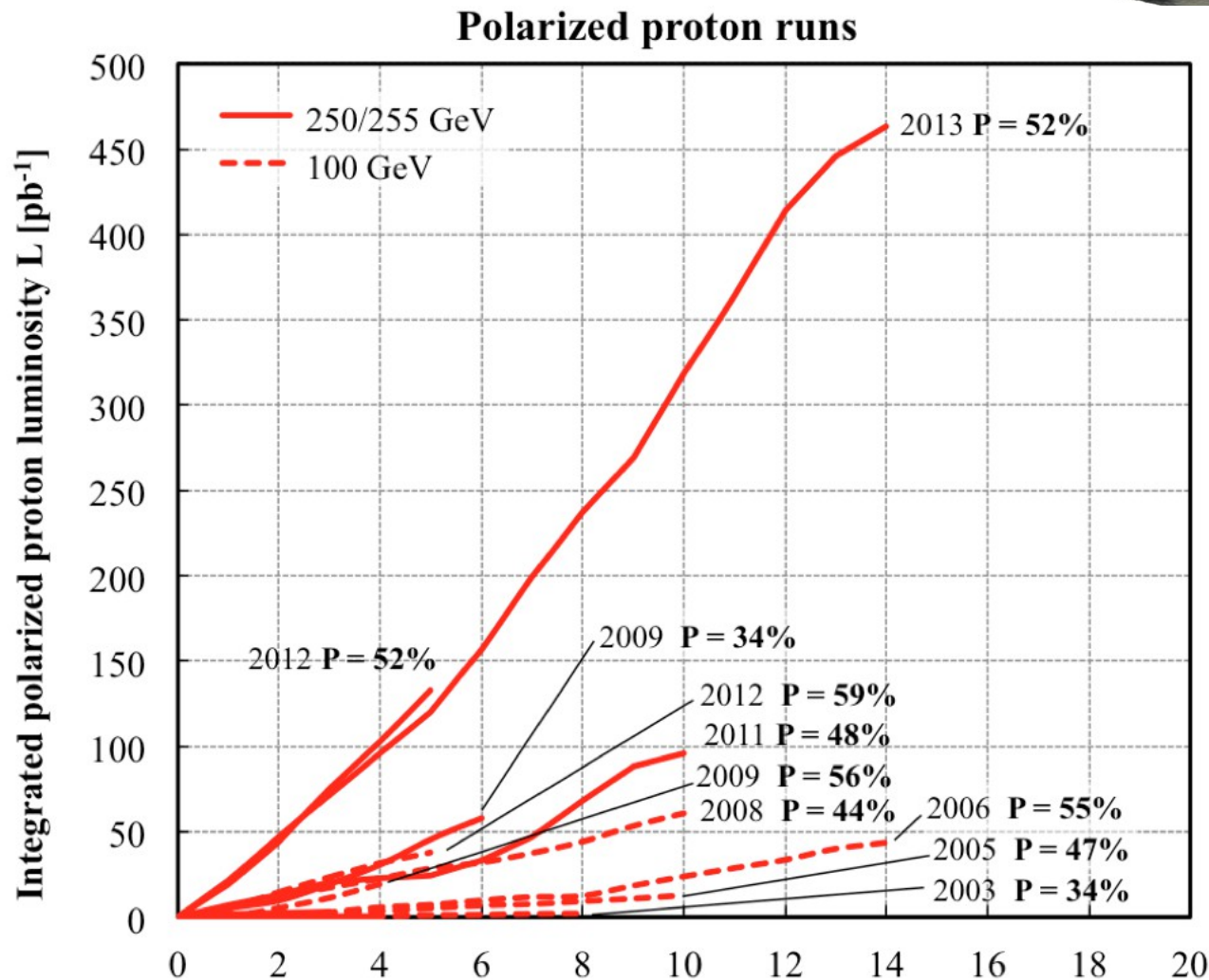
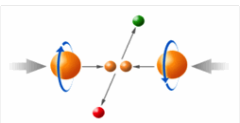
Average Store Polarization vs. vertical tune



- ❑ The closer the vertical tune towards 0.7, the lower the beam polarization
- ❑ The data also shows that the direct beam-beam contribution to polarization loss during store is weak



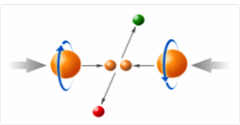
RHIC Polarized Proton Performance



Courtesy of W. Fischer

Polarization as measured by H Jet target, average of the entire beam distribution. For 250(255) GeV, sharper polarization profile was observed and hence, effective polarization is $\sim 20\%$ higher

Polarized Electrons



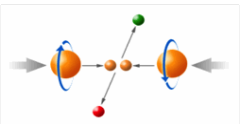
- High energy polarized electrons, on the other hand, is quite different due to Sokolov-Ternov effect,
 - Discovered by Sokolov-Ternov in 1964
 - Emission of synchrotron radiation causes spontaneous spin flip



- The difference of probability between the two scenarios allows the radiative polarization build up $P(t) = P_{max} (1 - e^{-t/\tau_{pol}})$, where $P_{ST} = 8/5 \sqrt{3}$ and polarization build up time is

$$\tau_{pol}^{-1} = 5 \frac{\sqrt{3}}{8} \frac{e^2 \hbar \gamma^5}{m^2 c^2 \rho^3} = 5 \frac{\sqrt{3}}{8} c \lambda_e r_e \frac{\gamma^5}{\rho^3}$$

Polarized Electrons



- For electron, rule of thumb of polarization build up time

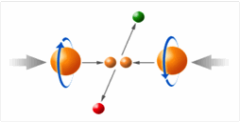
$$\tau_{pol}^{-1} = 3654 \frac{R/\rho}{B[T]^3 E[GeV]^2}$$

S. Mane et al, Spin-polarized charged particle bams

	VEPP[10]	VEPP2-M[11]	ACO[8,9]	BESSY[44]	SPEAR[45]	VEPP4[46]
$E(\text{GeV})$	0.640	0.625	0.536	0.800	3.70	5.0
$\tau_p(\text{min})$	50	70	160	150	15	40
$P(\%)$	52	90	90	>75	>70	80
	DORIS II[47]	CESR[48]	PETRA[49]	HERA[19]	TRISTAN[50]	LEP[51]
$E(\text{GeV})$	5.0	4.7	16.5	26.7	29	46.5
$\tau_p(\text{min})$	4	300	18	40	2	300
$P(\%)$	80	30*	80**	70**	75**	57**

- What's the polarization buildup time at RHIC@250GeV and LHC@1TeV?

In a planar circular accelerator

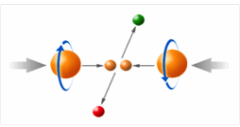


- where the magnetic field is distributed piece-wisely

$$P_{\infty} = \frac{8}{5\sqrt{3}} \frac{\langle |\rho^{-3}| \hat{n} \cdot \hat{b} \rangle}{\langle |\rho^{-3}| \left[1 - \frac{2}{9} (\hat{\beta} \cdot \hat{n})^2 \right] \rangle}$$
$$\tau_p^{-1} = \frac{5\sqrt{3}}{8} c \lambda_c r_e \gamma^5 \langle |\rho^{-3}| \left[1 - \frac{2}{9} (\vec{\beta} \cdot \vec{n})^2 \right] \rangle$$

- Clearly, a single snake or other configurations which lays the stable spin direction in the horizontal plane, can cancel the S-T radiative polarization build-up

Now, let's add in spin diffusion

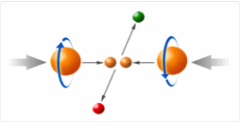


- An emission of a photon yields a sudden change of the particle's energy, as well as its spin phase

$$P_{\infty} = \frac{8}{5\sqrt{3}} \frac{\left\langle \left| \rho^{-3} \left[\hat{b} \cdot \left[\hat{n} - \gamma \frac{\partial \hat{n}}{\partial \gamma} \right] \right] \right\rangle}{\left\langle \left| \rho^{-3} \left[1 - \frac{2}{9} (\hat{\beta} \cdot \hat{n})^2 + \frac{11}{18} \left| \gamma \frac{\partial \hat{n}}{\partial \gamma} \right|^2 \right] \right\rangle}$$

$$\tau_p^{-1} = \frac{5\sqrt{3}}{8} c \lambda_c r_e \gamma^5 \left\langle \left| \rho^{-3} \left[1 - \frac{2}{9} (\vec{\beta} \cdot \vec{n})^2 + \frac{11}{18} \left| \gamma \frac{\partial \hat{n}}{\partial \gamma} \right|^2 \right] \right\rangle$$

Synchrotron Sideband



- Spin tune is modulated due to synchrotron oscillation

$$\gamma = \gamma_0 + \Delta\gamma \cos \psi \quad \text{with} \quad \psi = \nu_s \theta + \phi_0$$

$$\nu = G\gamma = \nu_0 + G\Delta\gamma \cos \psi \quad \text{with} \quad \nu_0 = G\gamma_0$$

- Hence, the spin-orbit coupling factor averaged over all synchrotron phase becomes

$$\left\langle \left| \vec{\Gamma} \right|^2 \right\rangle = \left| \gamma \frac{\partial \hat{n}}{\partial \gamma} \right|^2 = \nu_0^2 \epsilon_K^2 \sum_m \frac{J_m^2(\Delta\nu / \nu_s)}{\left[\left((\nu_0 - K)^2 \right) - \nu_s^2 \right]^2}$$

Depolarizing Resonance @ SPEAR

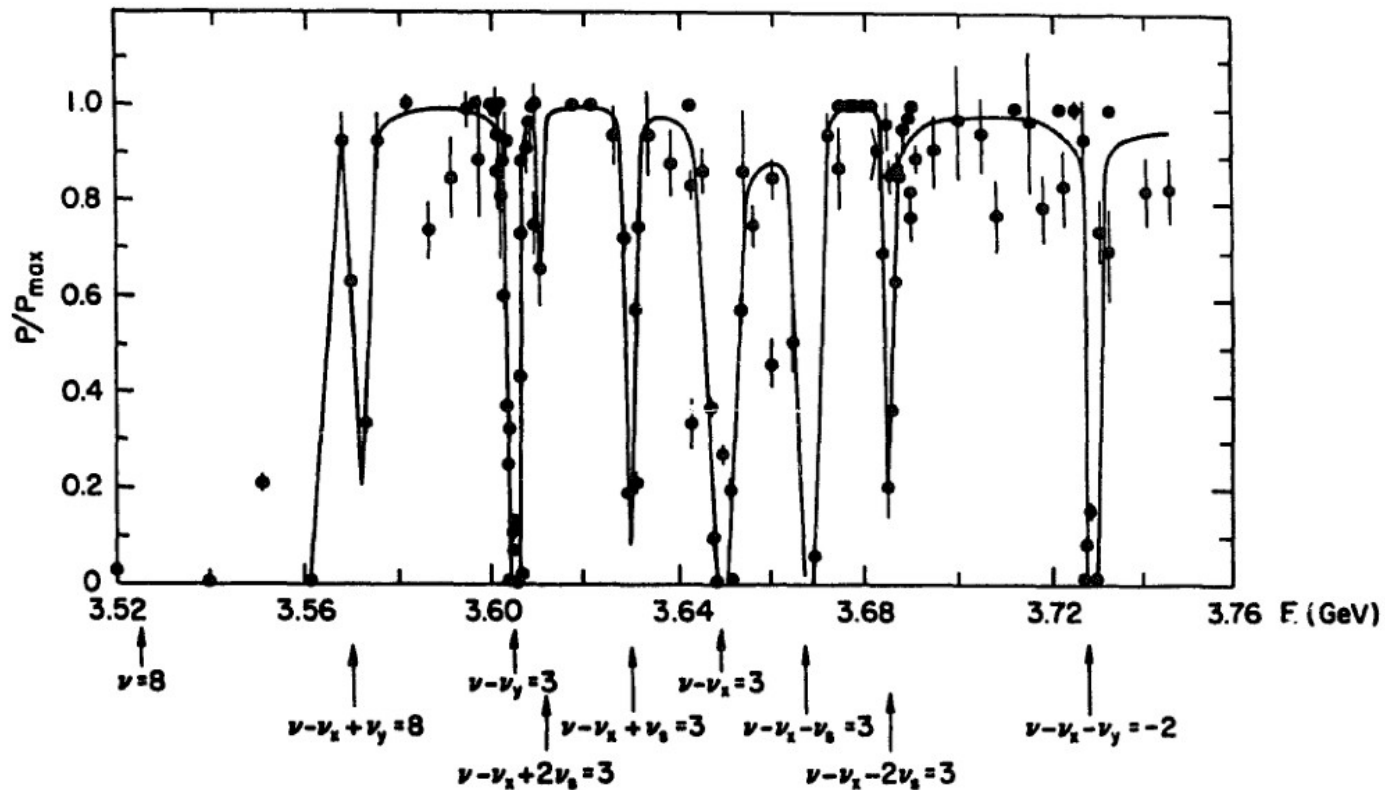
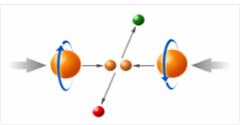
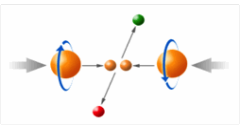
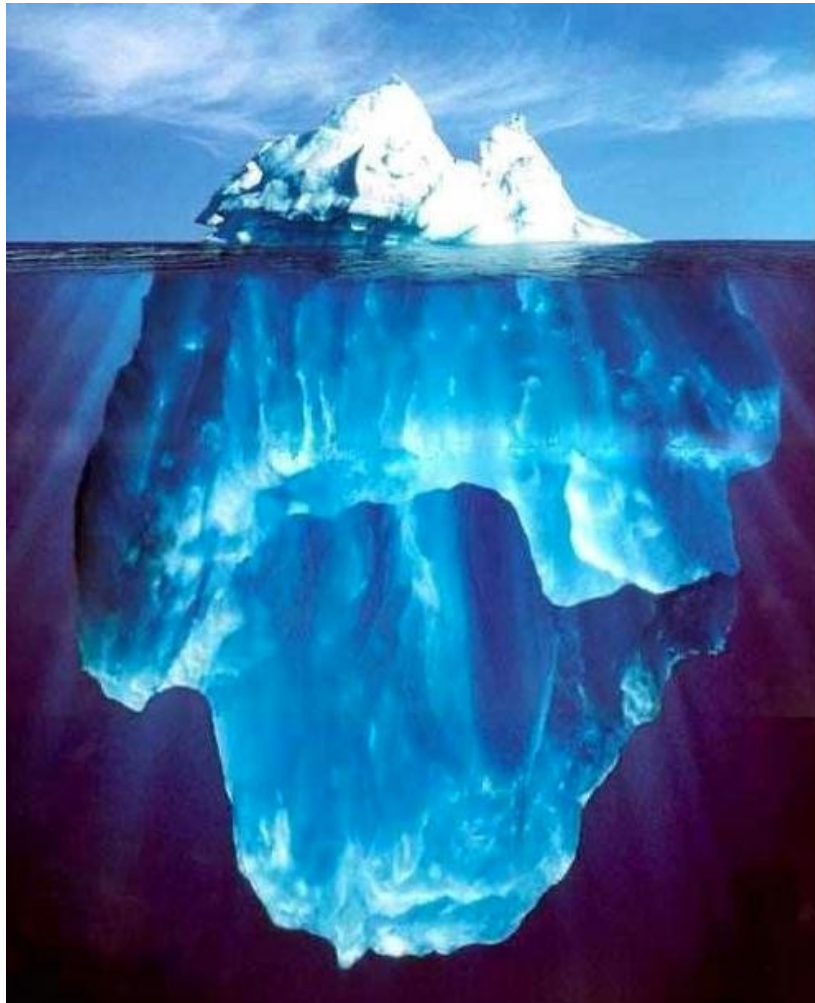


Fig. 1. Polarization measurements at SPEAR (from ref. [2]). The quantity P_{\max} is $8/(5\sqrt{3}) \approx 92.4\%$. The curve is a guide for the eye, not a theoretical calculation. Various resonances have been identified in the data. The orbital tunes are called $\nu_{x,y,s}$ instead of $Q_{x,y,s}$. The spin tune is ν . A single beam of positrons was circulated when making measurements. The graph is not a single experiment, but a compilation of many runs.

What's Missing in this talk



The iceberg 😊



Linear spin dynamics

- 1st order depolarizing resonance
- Techniques for preserving polarization

Non-linear spin dynamics

- High order depolarizing resonance

Spin tracking

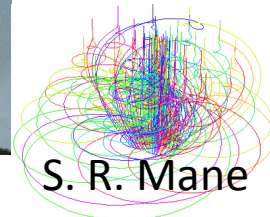
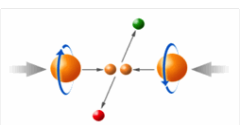
- Robustness and modern architect
- Optimization, spin matching

Spin manipulation

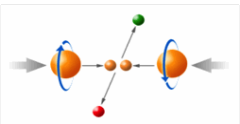
- Spin flipping
- Spin tune-meter

Polarimetry

To the great minds who pioneered



Achieved Performance and Projection



p↑ - p↑ operation	2009	2012	2013	2015
Energy	GeV	100/250	100/255	100
No of collisions	...	107	107	107
Bunch intensity	10^{11}	1.3/1.1	1.3/1.8	1.85
Beta*	m	0.7	0.85/0.65	0.65
Peak L	$10^{30}\text{cm}^{-2}\text{s}^{-1}$	50/85	46/165	115
Average L	$10^{30}\text{cm}^{-2}\text{s}^{-1}$	28/55	33/105	63
Polarization P	%	56/35	59/52	56/57.4

- Polarization quoted here is from Absolute Polarimeter using polarized H Jet