

Introduction to RF Linear Accelerators

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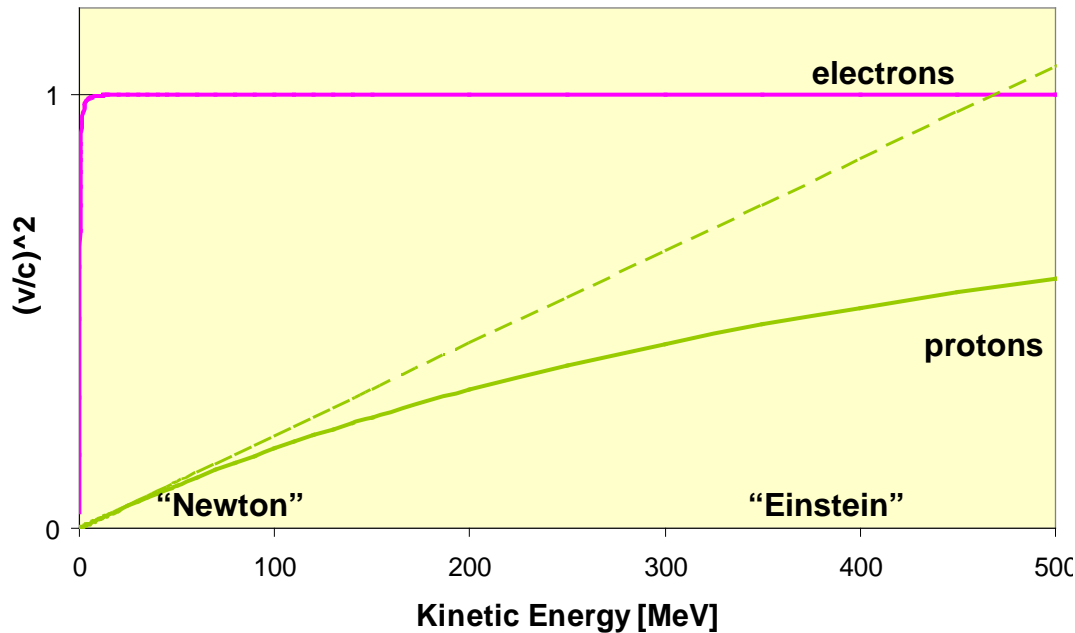
Varna 2010

1. Why linear accelerators - basic concepts
2. Acceleration in Periodic Structures
3. Overview of linac structures
4. Basics of linac beam dynamics
5. More on periodic structures
6. The Radio Frequency Quadrupole (RFQ)
7. Linac Technology



Linear Accelerators are used for:

1. Low-Energy acceleration (injectors to synchrotrons or stand-alone):
for protons and ions, linacs can be **synchronous with the RF fields** in the range where **velocity increases with energy**. When velocity is ~constant, synchrotrons are more efficient (multiple crossings instead of single crossing).
Protons : $\beta = v/c = 0.51$ at 150 MeV, 0.95 at 2 GeV.
2. High-Energy acceleration in the case of:
 - Production of high-intensity proton beams
in comparison with synchrotrons, linacs can go to **higher repetition rate**, are less affected by **resonances** and have more **distributed beam losses**. Higher injection energy from linacs to synchrotrons leads to **lower space charge effects** in the synchrotron and allows increasing the beam intensity.
 - High energy linear colliders for leptons, where the main advantage is the **absence of synchrotron radiation**.



$\beta^2 = (v/c)^2$ as function of kinetic energy T for protons and electrons.

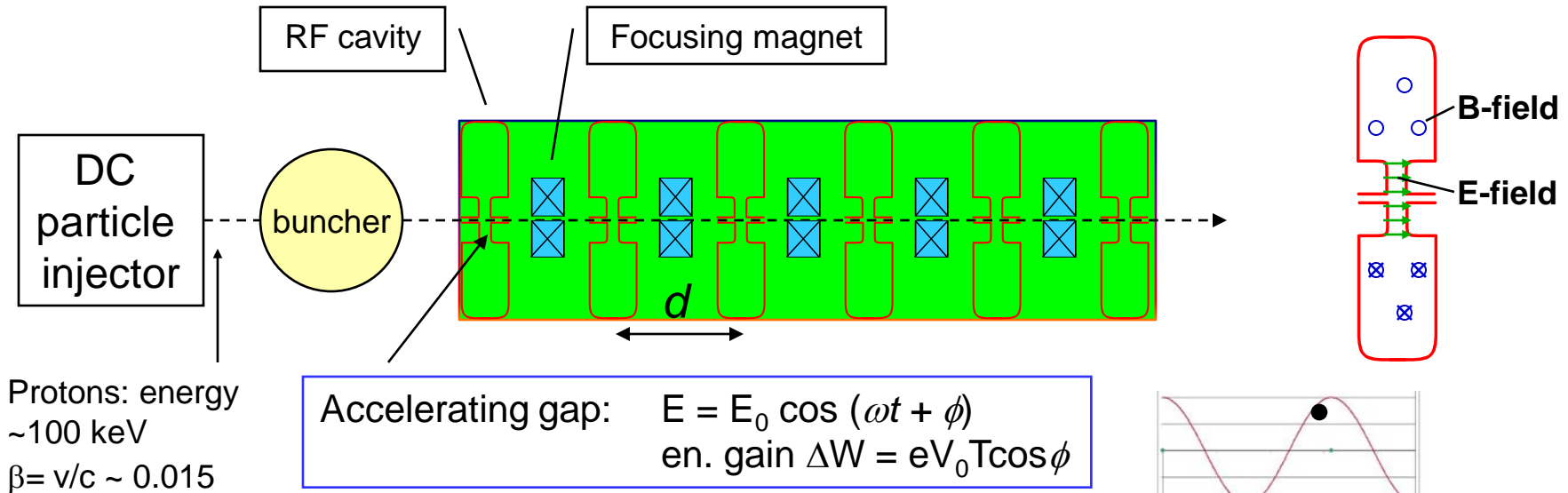
Classic (Newton) relation:

$$T = m_0 \frac{v^2}{2}, \quad \frac{v^2}{c^2} = \frac{2T}{m_0 c^2}$$

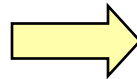
Relativistic (Einstein) relation:

$$\frac{v^2}{c^2} = 1 - \frac{1}{\sqrt{1 + T/m_0 c^2}}$$

- **Protons** (rest energy 938.3 MeV): follow "Newton" mechanics up to some **tens of MeV** ($\Delta v/v < 1\%$ for $W < 15$ MeV) then slowly become relativistic ("Einstein"). From the **GeV range** velocity is nearly constant ($v \sim 0.95c$ at 2 GeV) → linacs can cope with the increasing particle velocity, synchrotrons cover the range where v nearly constant.
- **Electrons** (rest energy 511 keV, 1/1836 of protons): relativistic from the **keV range** ($v \sim 0.1c$ at 2.5 keV) then increasing velocity up to the **MeV range** ($v \sim 0.95c$ at 1.1 MeV) → $v \sim c$ after few meters of acceleration in a linac (typical gradient 10 MeV/m). ³

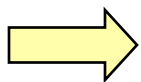


Acceleration \rightarrow the beam has to pass in each cavity on a phase ϕ near the crest of the wave



1. The beam must be bunched at frequency ω
2. distance between cavities and phase of each cavity must be correlated

Phase change from cavity i to $i+1$ is $\Delta\phi = \omega t = \omega \frac{d}{\beta c} = 2\pi \frac{d}{\beta\lambda}$

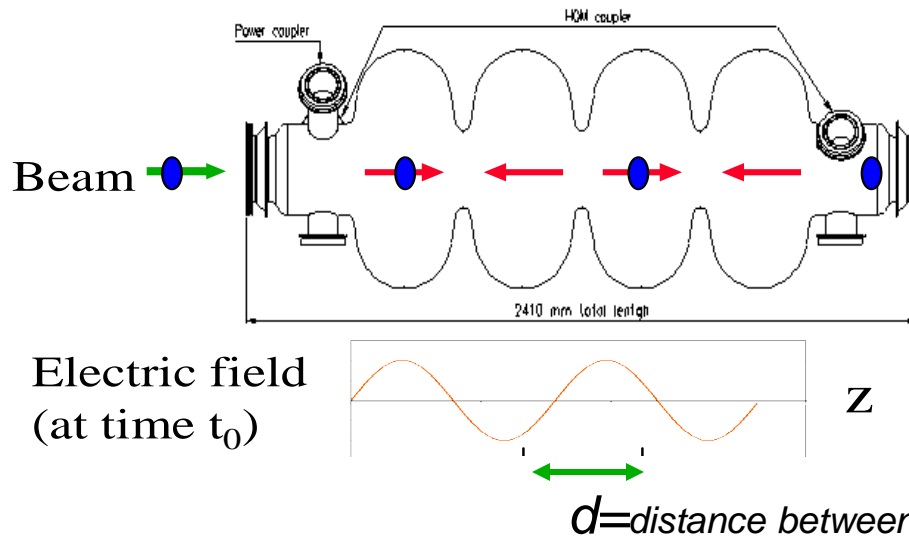


For the beam to be synchronous with the RF wave ("ride on the crest") phase must be related to distance by the relation:

$$\frac{\Delta\phi}{d} = \frac{2\pi}{\beta\lambda}$$

... and on top of acceleration, we need to introduce in our "linac" some focusing elements
 ... and on top of that, we will couple a number of gaps in an "accelerating structure"

Typical linac case: multi-cell accelerating cavity with $d=\text{constant}$ and phase difference between cells $\Delta\phi=\pi$ given by the electric field distribution.

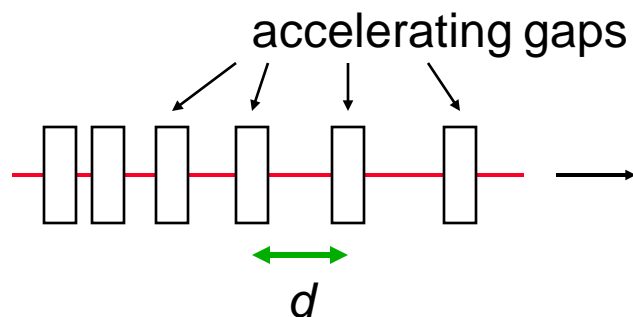


Example: a linac superconducting 4-cell accelerating structure

Synchronism condition bw. particle and wave
 t (travel between centers of cells) = $T/2$

$$\frac{d}{\beta c} = \frac{1}{2f} \quad \rightarrow \quad d = \frac{\beta c}{2f} = \frac{\beta \lambda}{2}$$

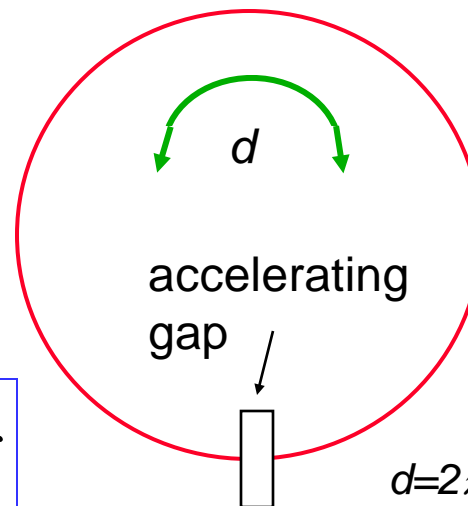
1. In an ion linac cell length has to increase (up to a factor 200 !) and the linac will be made of a **sequence of different accelerating structures** (changing cell length, frequency, operating mode, etc.) matched to the ion velocity.
2. For electron linacs, $\beta=1$, $d=\lambda/2 \rightarrow$ An electron linac will be made of an **injector** + a **series of identical accelerating structures**, with cells all the same length



$$d = \beta\lambda/2 = \text{variable}$$

$$f = \text{constant}$$

$$d = \frac{\beta c}{2f} = \frac{\beta\lambda}{2}, \quad \beta c = 2df$$



$$d = 2\pi R = \text{constant}$$

$$f = \beta c / 2d = \text{variable}$$

Linear accelerator:

Particles accelerated by a sequence of gaps (all at the same RF phase).

Distance between gaps increases proportionally to the particle velocity, to keep synchronicity.

Used in the range where β increases.
"Newton" machine

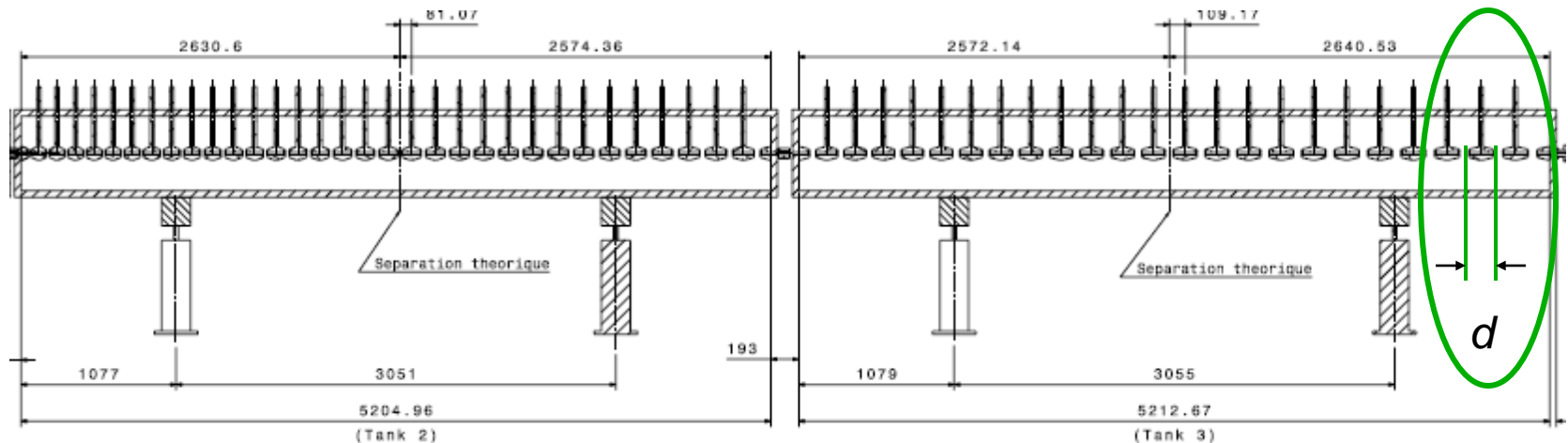
Circular accelerator:

Particles accelerated by one (or more) gaps at given positions in the ring.

Distance between gaps is fixed. Synchronicity only for $\beta \sim \text{const}$, or varying (in a limited range!) the RF frequency.

Used in the range where β is nearly constant.
"Einstein" machine

Example 1: gap spacing in a Drift Tube Linac (low β)

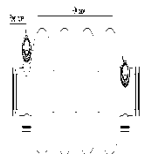


Tank 2 and 3 of the new Linac4 at CERN:
 Beam energy from 10 to 50 MeV
 Beta from 0.145 to 0.31
 Cell length from 12.3 cm to 26.4 cm (factor 2!)

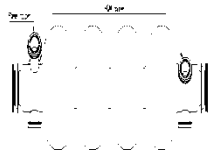
This arrangement works only for one type of particles and one range of energies!

Example 2: cavities in a superconducting linac (medium β)

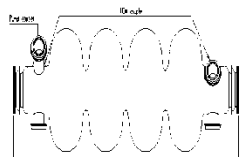
The same superconducting cavity design can be used for different proton velocities. The linac has different sections, each made of cavities with cell length matched to the average beta in that section. At "medium energy" (>150 MeV) we are not obliged to dimension every cell or every cavity for the particular particle beta at that position, and we can accept a slight "asynchronicity".



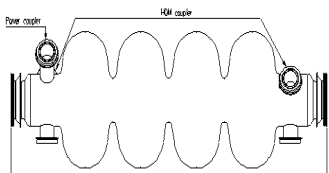
$\beta=0.52$



$\beta=0.7$

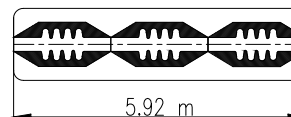


$\beta=0.8$

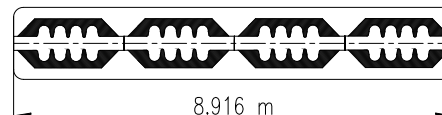


$\beta=1$

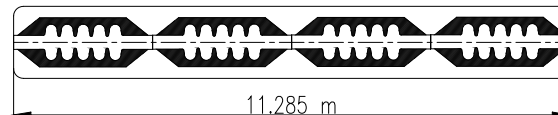
A). $\beta=0.52$



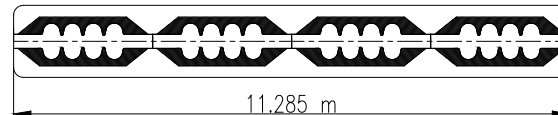
B). $\beta=0.7$



C). $\beta=0.8$, LEP cryostat

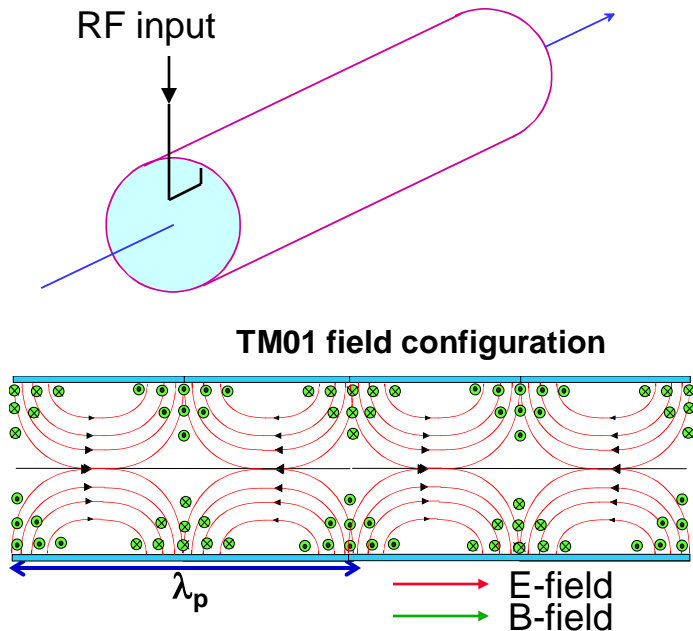


D). $\beta=1$, LEP cryostat

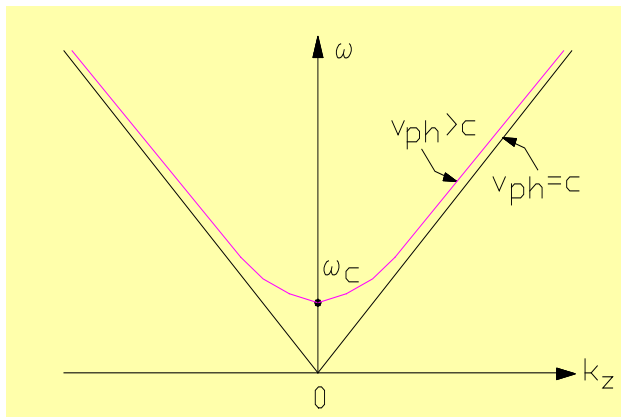


2 - Acceleration in Periodic Structures

Wave propagation in a cylindrical pipe



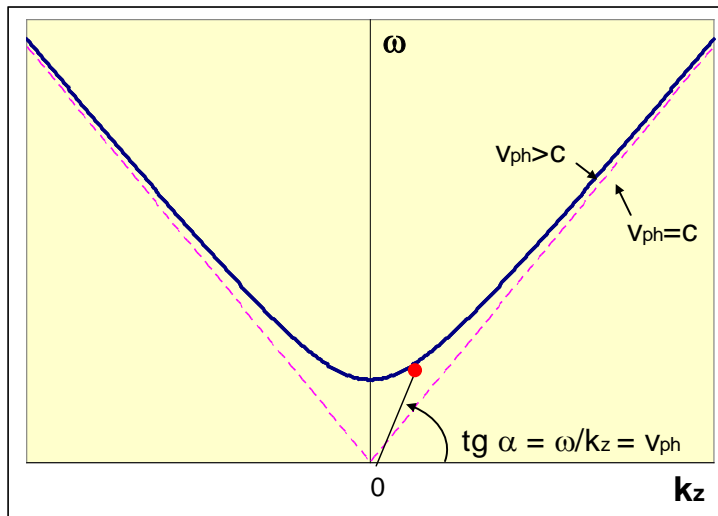
- In a cylindrical waveguide different **modes** can propagate (=Electromagnetic field distributions, transmitting power and/or information). The field is the superposition of waves reflected by the metallic walls of the pipe → velocity and wavelength of the modes will be different from free space (c, λ)
- To accelerate particles, we need a mode with longitudinal E-field component on axis: a TM mode (Transverse Magnetic, $B_z=0$). The simplest is TM01.
- We inject RF power at a frequency exciting the TM01 mode: sinusoidal E-field on axis, wavelength λ_p depending on frequency and on cylinder radius. Wave velocity (called "phase velocity") is $v_{ph} = \lambda_p / T = \lambda_p f = \omega / k_z$ with $k_z = 2\pi / \lambda_p$
- The relation between frequency ω and propagation constant k is the **DISPERSION RELATION** (red curve on plot), a fundamental property of waveguides.



The dispersion relation $\omega(k)$ can be calculated from the theory of waveguides:

$$\omega^2 = k^2 c^2 + \omega_c^2$$

Plotting this curve (hyperbola), we see that:



$$k = 2\pi/\lambda_p$$

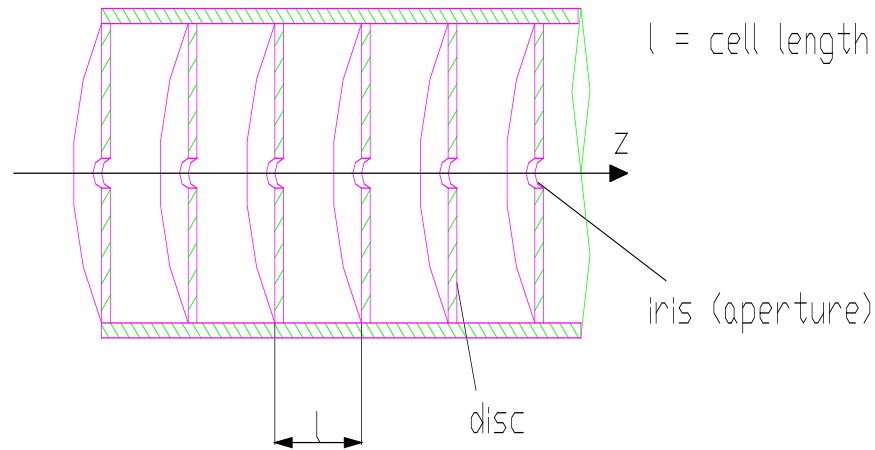
$$v_{ph} = \omega/k = (c^2 + \omega_c^2/k^2)^{1/2}$$

$$v_g = d\omega/dk$$

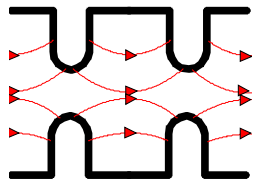
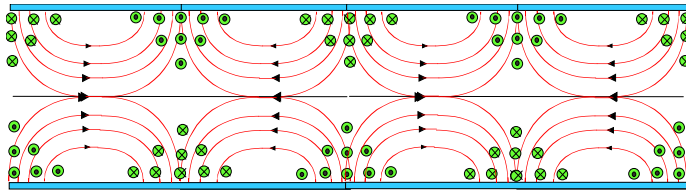
- 1) There is a "cut-off frequency", below which a wave will not propagate. It depends on dimensions ($\lambda_c = 2.61a$ for the cylindrical waveguide).
- 2) At each excitation frequency is associated a **phase velocity**, the velocity at which a certain phase travels in the waveguide. $v_p = \infty$ at $k=0$, $\omega = \omega_c$ and then decreases towards $v_p = c$ for $k, \omega \rightarrow \infty$.
- 3) To see at all times an accelerating E-field a particle traveling inside our cylinder has to travel at $v = v_{ph} \rightarrow v > c$!!!

Are we violating relativity? **No**, energy (and information) travel at **group velocity** $d\omega/dk$, always between 0 and c .

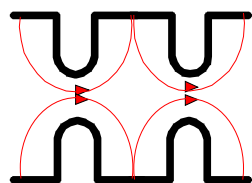
To use the waveguide to accelerate particles, we need a "trick" to slow down the wave.



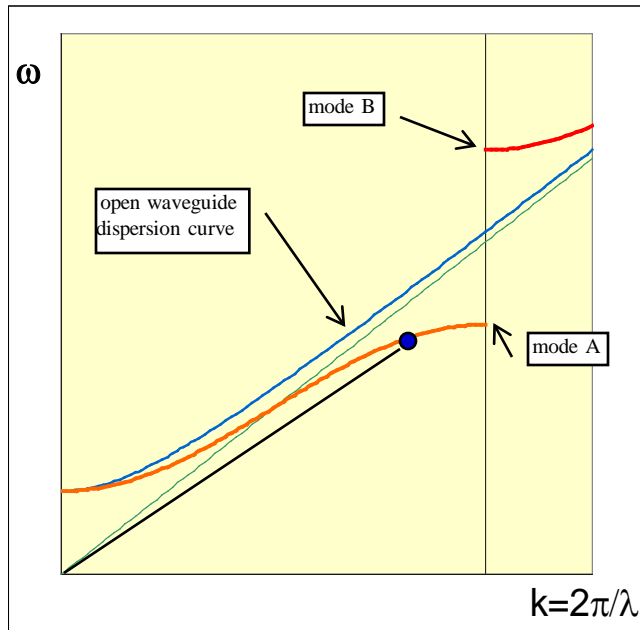
Discs inside the cylindrical waveguide, spaced by a distance ℓ , will induce multiple reflections between the discs.



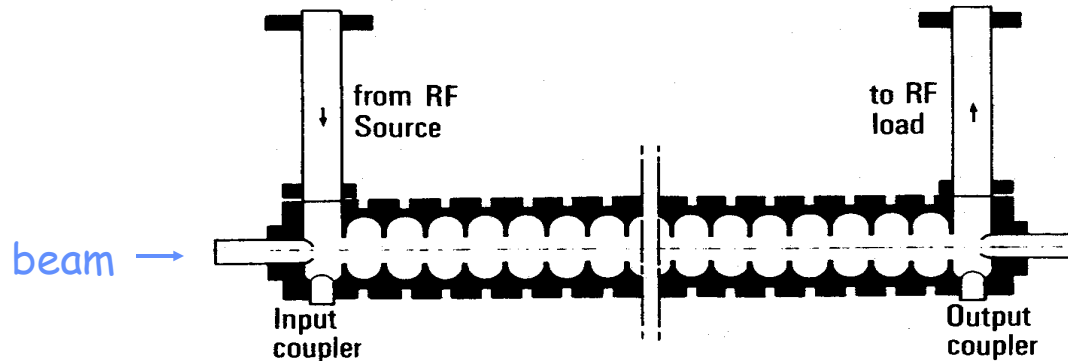
electric field pattern - mode A



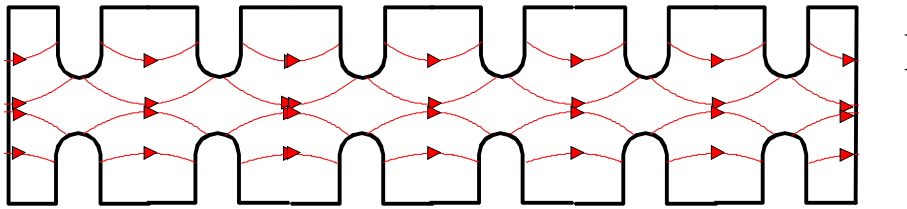
electric field pattern - mode B



- Wavelengths with $\lambda_p/2 \sim \ell$ will be most affected by the discs. On the contrary, for $\lambda_p=0$ and $\lambda_p=\infty$ the wave does not see the discs \rightarrow the dispersion curve remains that of the empty cylinder.
- At $\lambda_p/2 = \ell$, the wave will be confined between the discs, and present 2 "polarizations" (mode A and B in the figure), 2 modes with same wavelength but different frequencies \rightarrow the dispersion curve splits into 2 branches, separated by a **stop band**.
- In the disc-loaded waveguide, the lower branch of the dispersion curve is now "distorted" in such a way that we can find a range of frequencies with $v_{ph} = c \rightarrow$ we can use it to accelerate a particle beam!
- We have built a linac for $v \sim c \rightarrow$ a **TRAVELING WAVE (TW) ELECTRON LINAC**



- Disc-loaded waveguide designed for $v_{ph}=c$ at a given frequency, equipped with an input and an output coupler.
- RF power is introduced via the input coupler. Part of the power is dissipated in the structure, part is taken by the beam (beam loading) and the rest is absorbed in a matched load at the end of the structure. Usually, structure length is such that $\sim 30\%$ of power goes to the load.
- The "traveling wave" structure is the standard linac for **electrons from $\beta \sim 1$** .
- Can **not** be used for protons at $v \ll c$:
 1. constant cell length does not allow synchronism
 2. structures are long, without space for transverse focusing

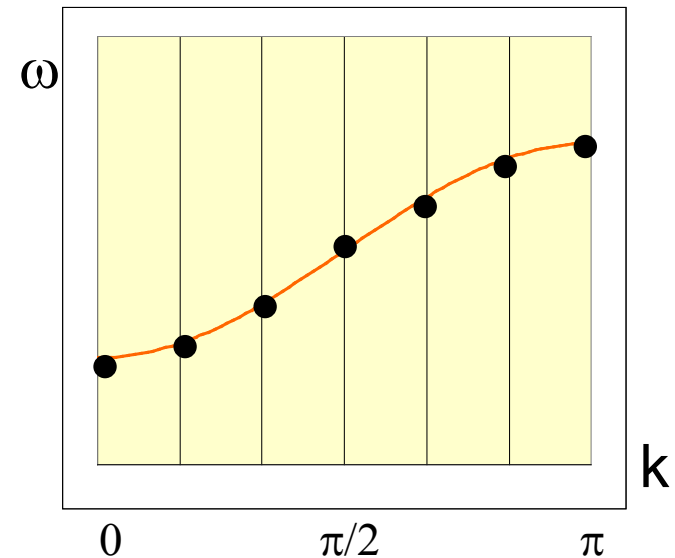


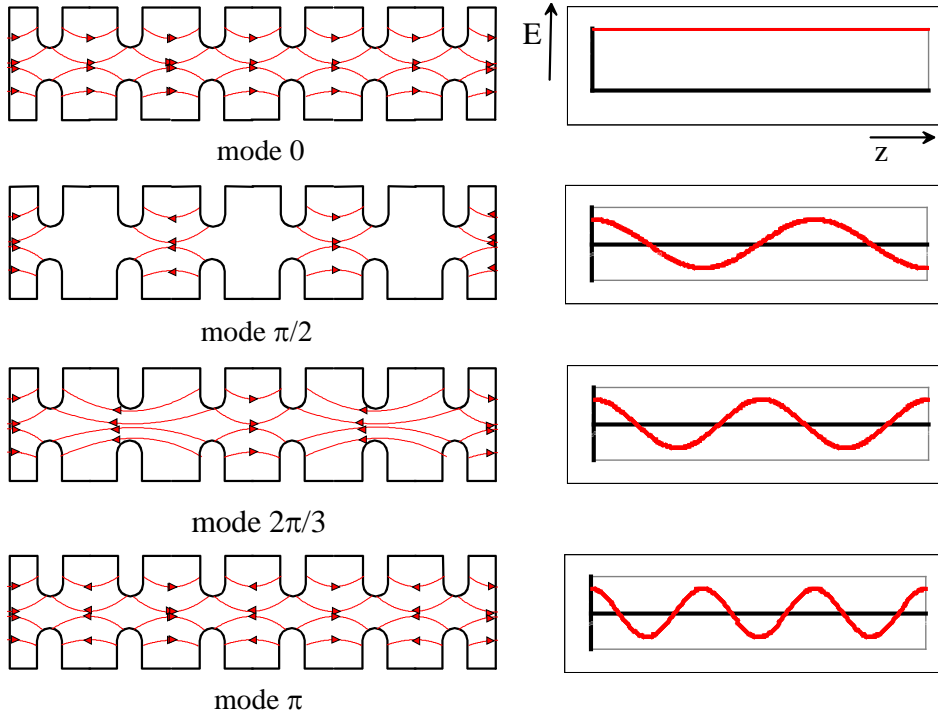
To obtain an accelerating structure for protons we close our disc-loaded structure at both ends with metallic walls → multiple reflections of the waves.

Boundary condition at both ends is that electric field must be perpendicular to the cover → Only **some modes on the disc-loaded dispersion curve** are allowed → only some frequencies on the dispersion curve are permitted.

In general:

1. the modes allowed will be equally spaced in k
2. The number of modes will be identical to the number of cells (N cells → N modes)
3. k represents the phase difference between the field in adjacent cells.





- **STANDING WAVE MODES** are generated by the sum of 2 waves traveling in opposite directions, adding up in the different cells.
- For acceleration, the particles must be in phase with the E-field on axis. We have already seen the π mode: synchronism condition for cell length $l = \beta\lambda/2$.
- Standing wave structures can be used for **any β** (→ ions and electrons) and their cell length can increase, to follow the **increase in β** of the ions.

Standing wave modes are named from the phase difference between adjacent cells: in the example above, mode 0, $\pi/2$, $2\pi/3$, π .

In standing wave structures, cell length can be matched to the particle velocity!

Synchronism conditions:

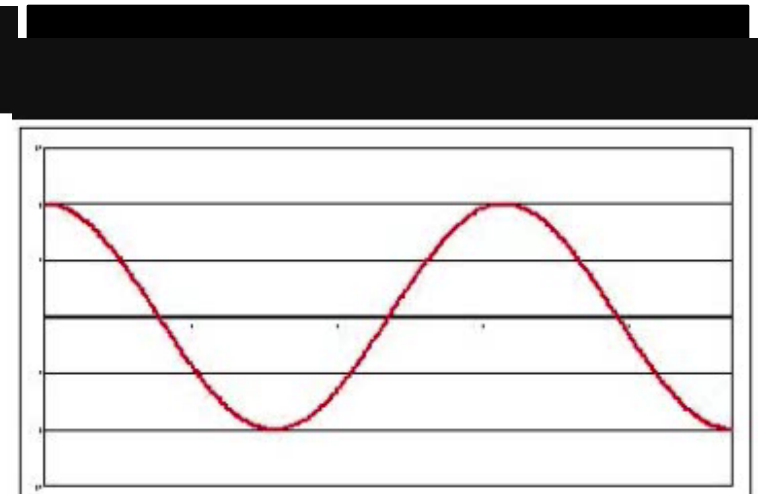
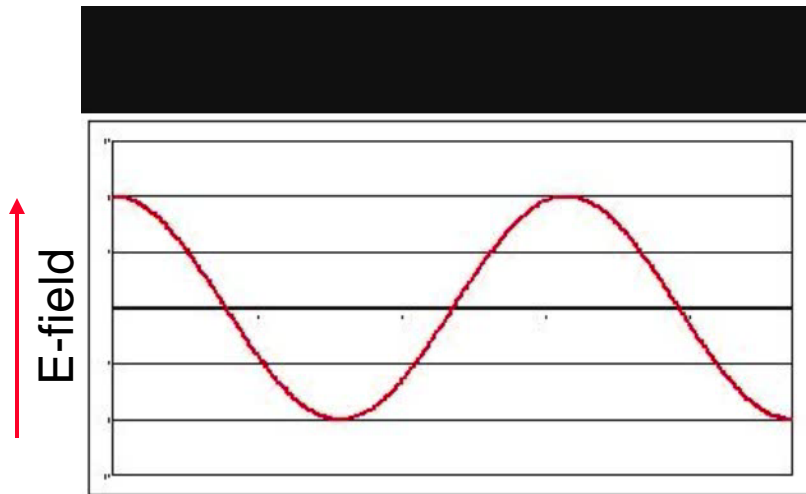
0-mode : $l = \beta\lambda$

$\pi/2$ mode: $2l = \beta\lambda/2$

π mode: $l = \beta\lambda/2$

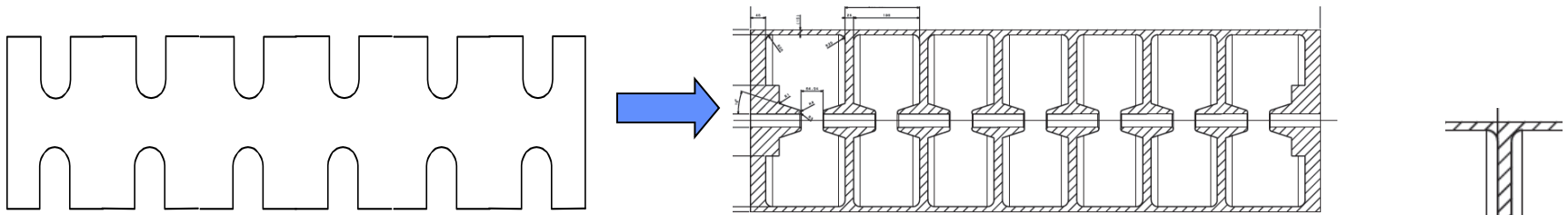
TRAVELING Wave

STANDING Wave



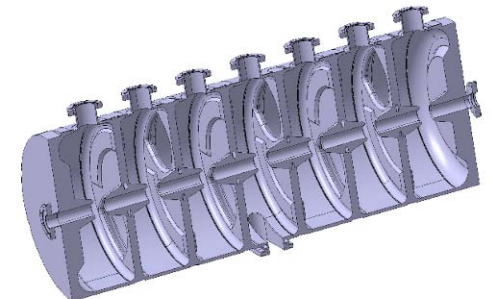
→ position z

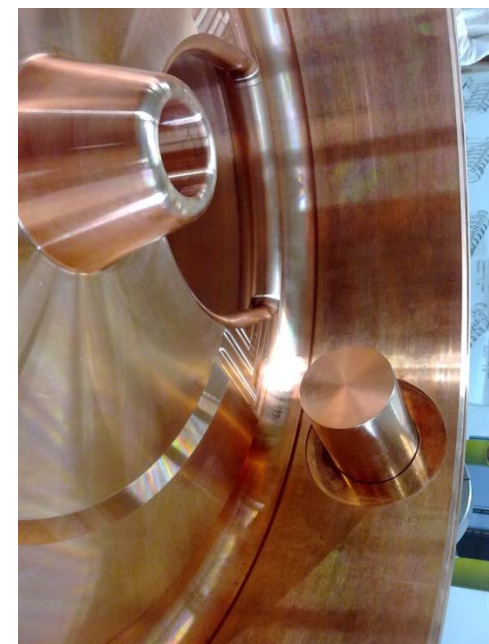
→ position z



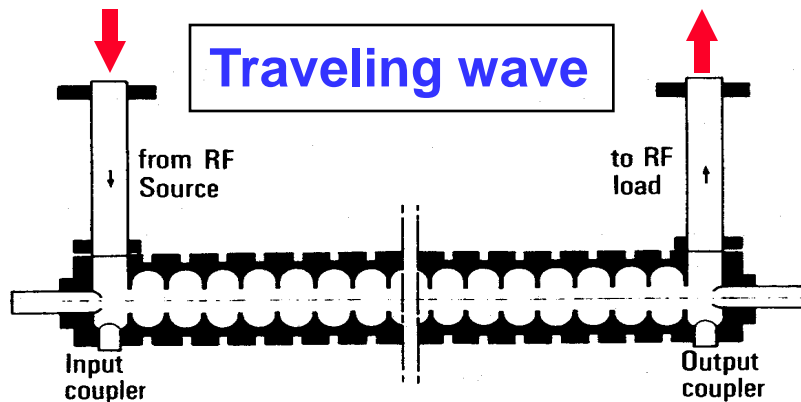
From **disc-loaded structure** to a **real cavity** (Linac4 PIMS, Pi-Mode Structure)

1. To increase acceleration efficiency (=shunt impedance ZT^2 !) we need to concentrate electric field on axis ($Z \uparrow$) and to shorten the gap ($T \uparrow$) → introduction of "noses" on the openings.
2. The smaller opening would not allow the wave to propagate → introduction of "coupling slots" between cells.
3. The RF wave has to be coupled into the cavity from one point, usually in the center.





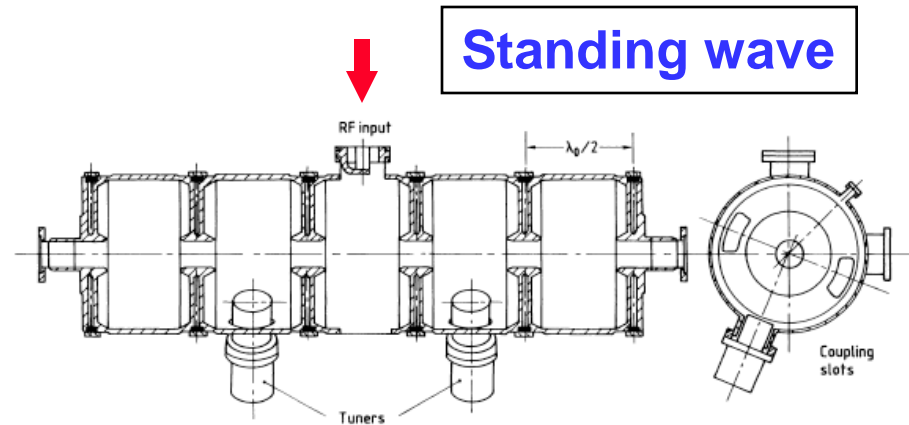
Comparing traveling and standing wave structures



Chain of coupled cells in TW mode
Coupling bw. cells from on-axis aperture.
RF power from input coupler at one end,
dissipated in the structure and on a load.

Short pulses, High frequency (≥ 3 GHz).
Gradients 10-20 MeV/m

Used for Electrons at $v \sim c$



Chain of coupled cells in SW mode.
Coupling (bw. cells) by slots (or open). On-axis aperture reduced, higher E-field on axis and power efficiency.
RF power from a coupling port, dissipated in the structure (ohmic loss on walls).

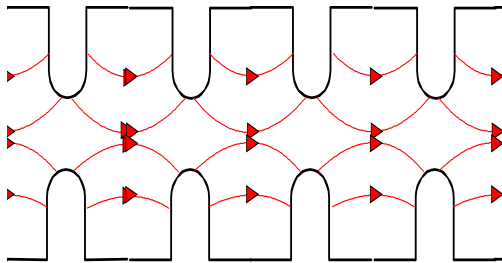
Long pulses. Gradients 2-5 MeV/m

Used for Ions and electrons, all energies

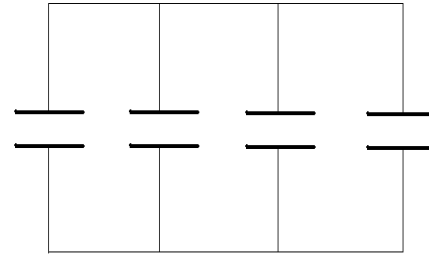
Comparable RF efficiencies

3 - Examples of linac accelerating structures:

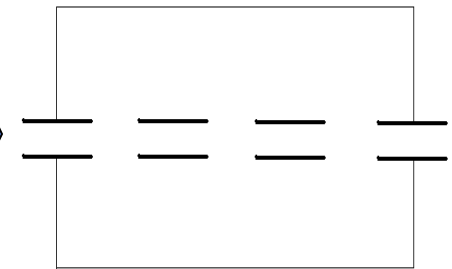
- a. protons,
- b. electrons,
- c. heavy ions



Disc-loaded structures operating in 0-mode



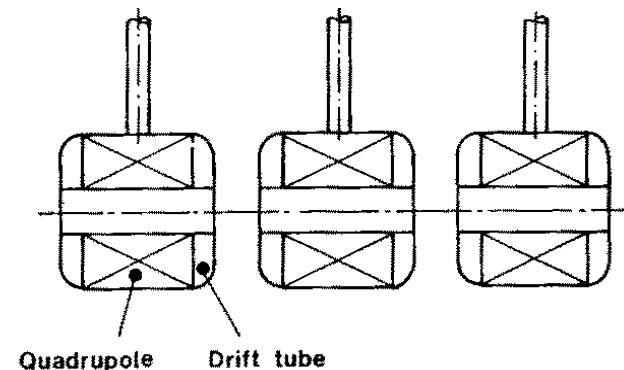
Add tubes for high shunt impedance

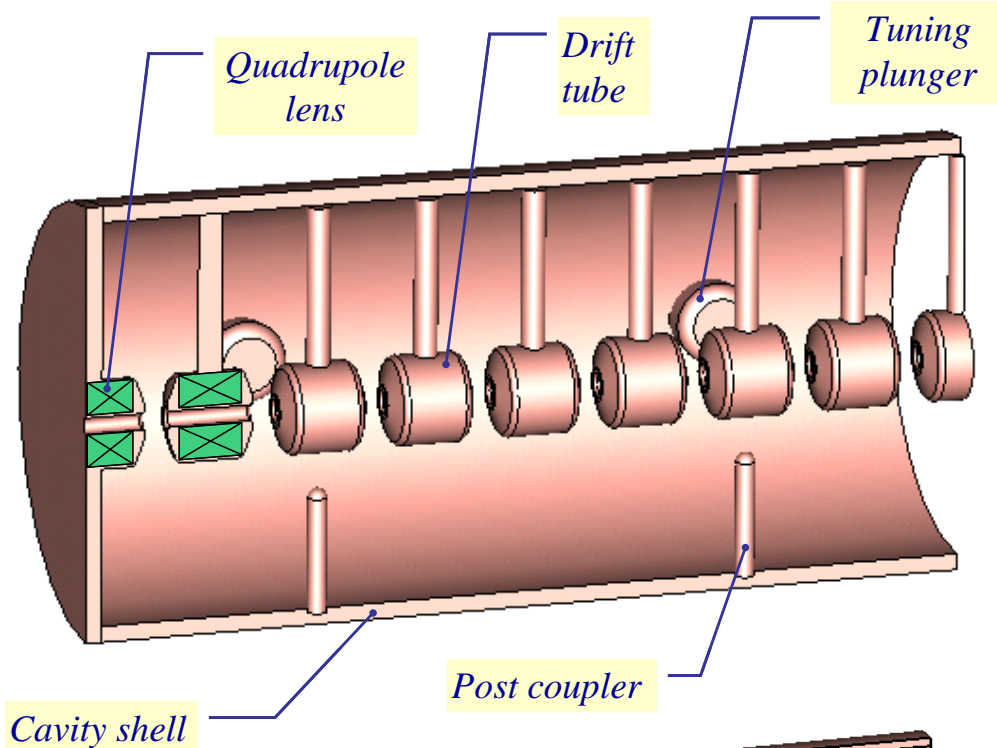


Maximize coupling between cells → remove completely the walls

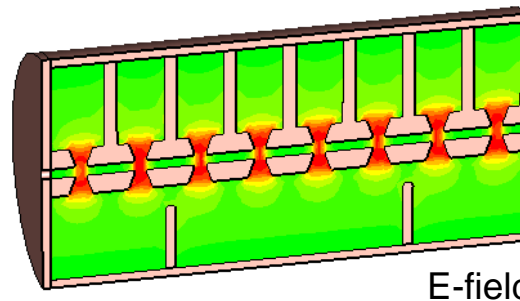
2 advantages of the 0-mode:

1. the fields are such that if we eliminate the walls between cells the fields are not affected, but we have less RF currents and higher power efficiency ("shunt impedance").
2. The "drift tubes" are long ($\sim 0.75 \beta \lambda$). The particles are inside the tubes when the electric field is decelerating, and we have space to introduce focusing elements (quadrupoles) inside the tubes.

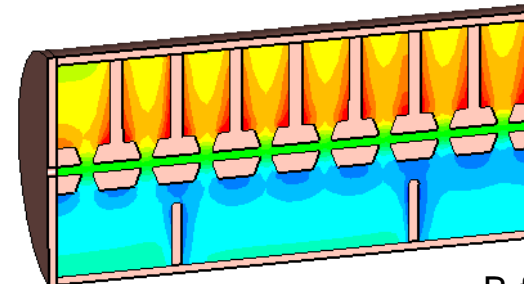




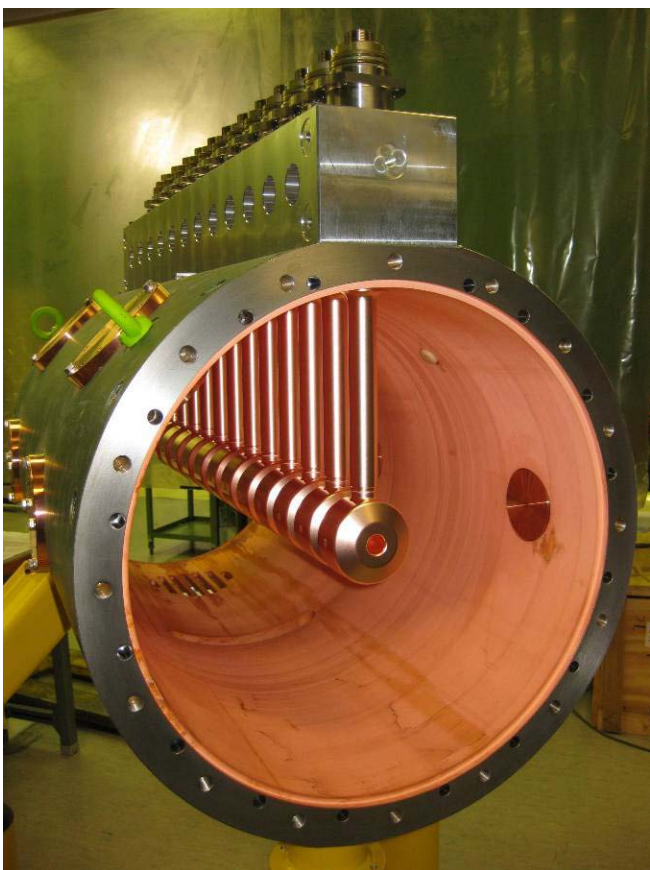
Standing wave linac structure for protons and ions,
 $\beta=0.1-0.5$, $f=20-400$ MHz
 Chain of coupled cells, completely open (no walls), maximum coupling.
 Operating in 0-mode, cell length $\beta\lambda$.
 Drift tubes are suspended by stems (no net current)
 Drift tubes contain focusing quadrupoles.



E-field

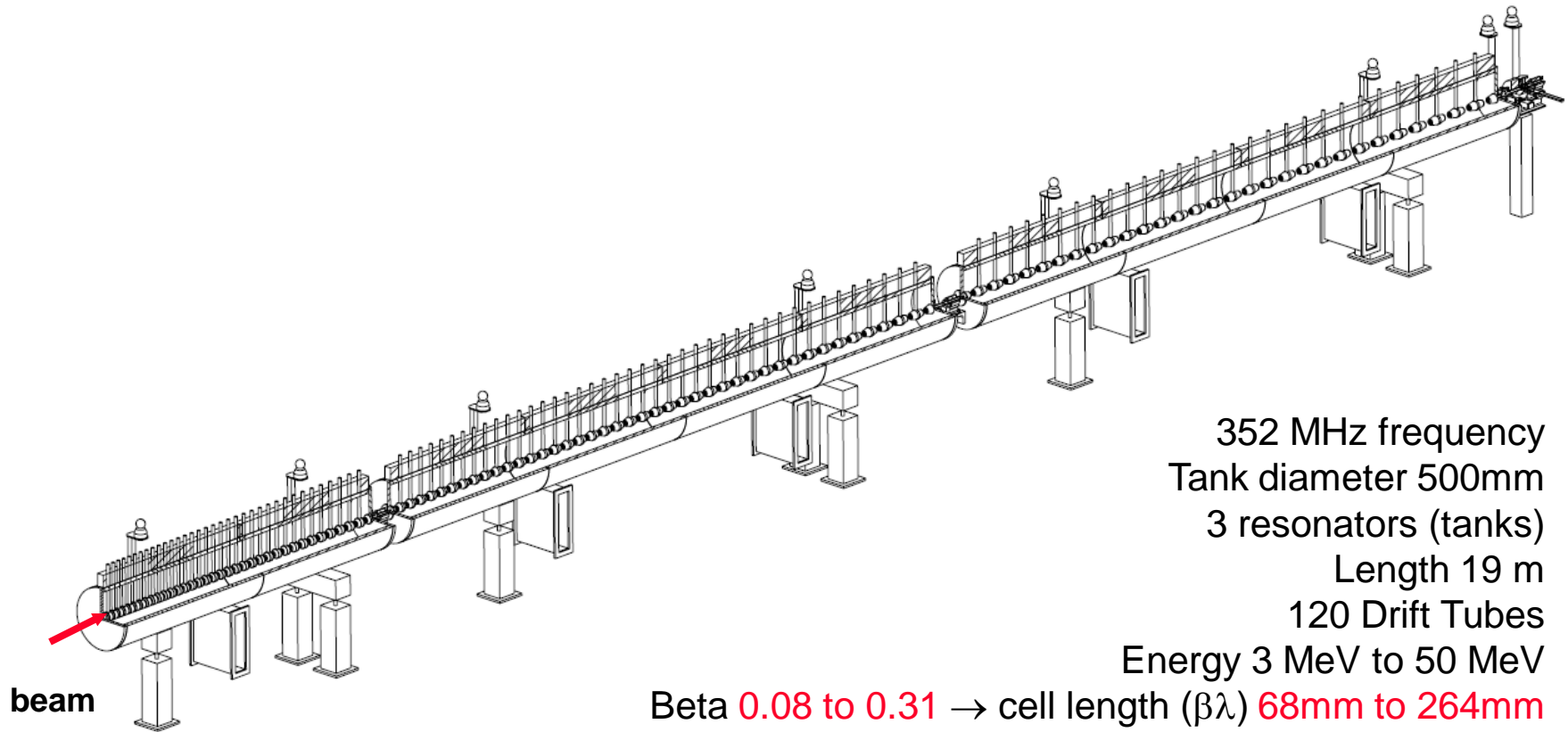


B-field



Top; CERN Linac2 Drift Tube Linac accelerating tank 1 (200 MHz). The tank is 7m long (diameter 1m) and provides an energy gain of 10 MeV.

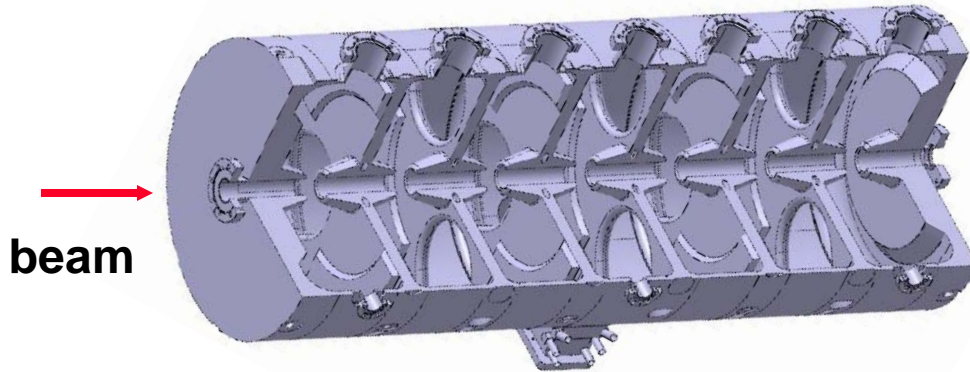
Left: DTL prototype for CERN Linac4 (352 MHz). Focusing is provided by small permanent quadrupoles inside drift tubes. Length of drift tubes (cell length) increases with proton velocity.



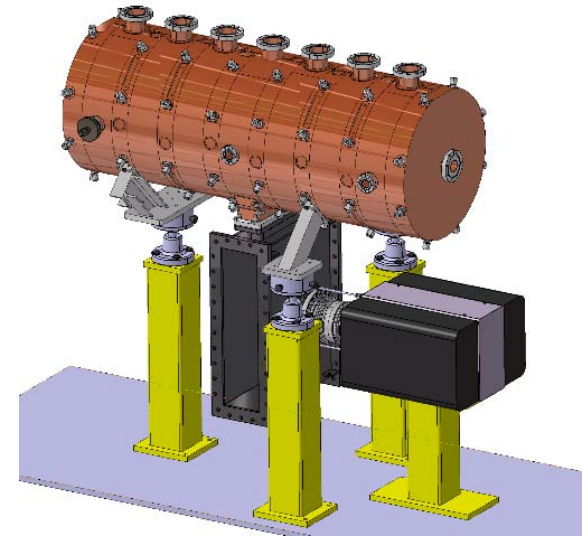
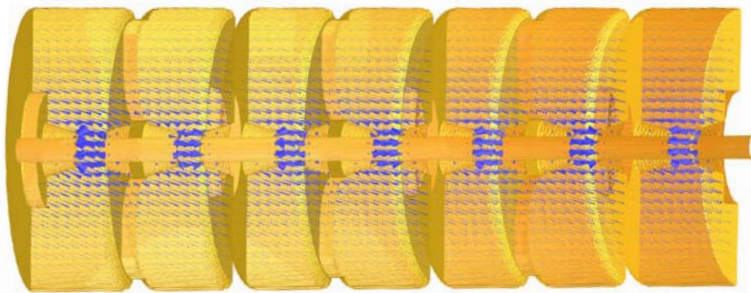
352 MHz frequency
 Tank diameter 500mm
 3 resonators (tanks)
 Length 19 m
 120 Drift Tubes
 Energy 3 MeV to 50 MeV

Beta 0.08 to 0.31 → cell length ($\beta\lambda$) 68mm to 264mm
 → factor 3.9 increase in cell length

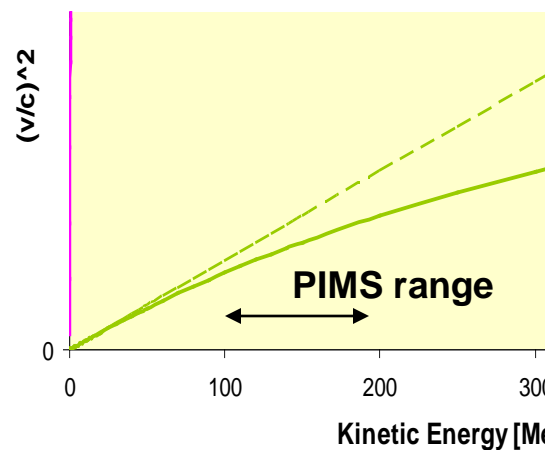
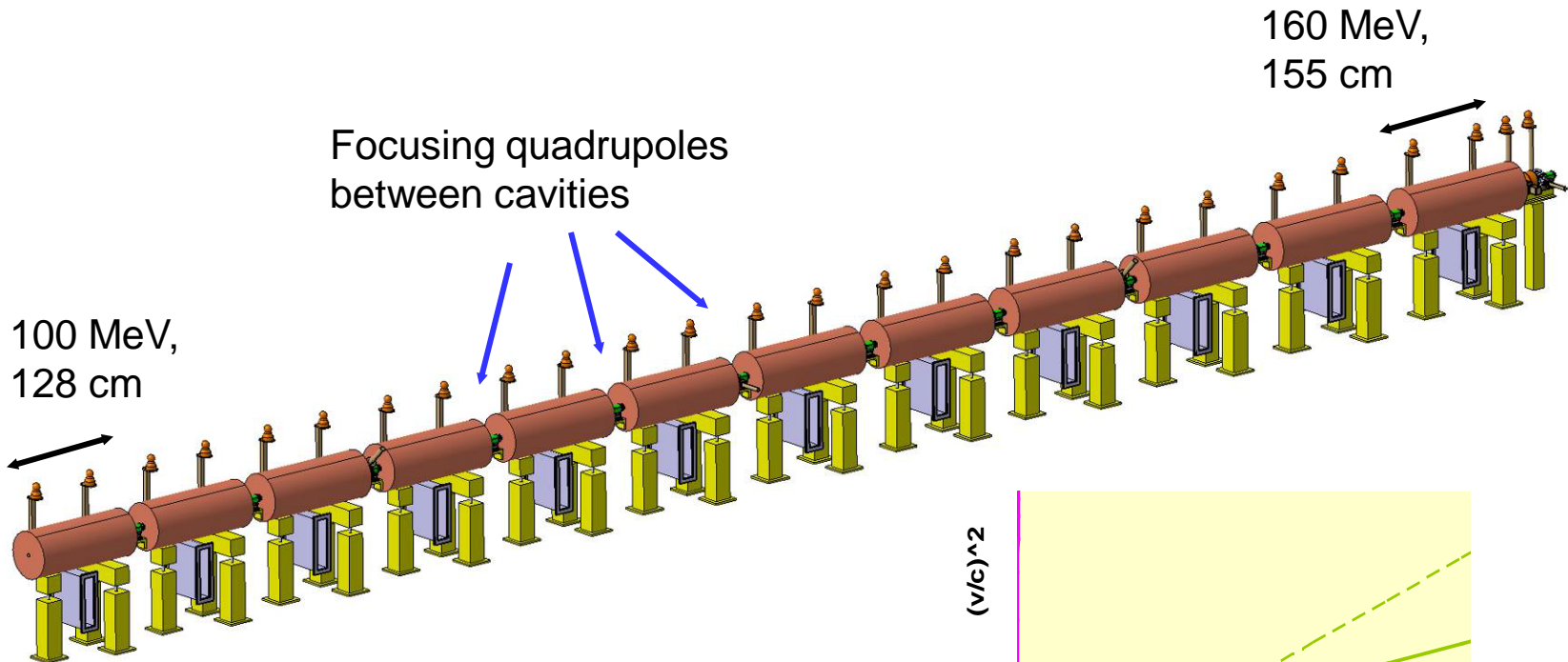
We can increase the cell length from one cell to the next without perturbing the 0-mode if the individual frequency of each cell remain constant (inductance \uparrow , capacitance \downarrow)



PIMS=PI Mode Structure
Standing wave linac structure for
protons, $\beta > 0.4$
Frequency 352 MHz
Chain of coupled cells with coupling
slots in walls.
Operating in π -mode, cell length
 $\beta\lambda/2$.

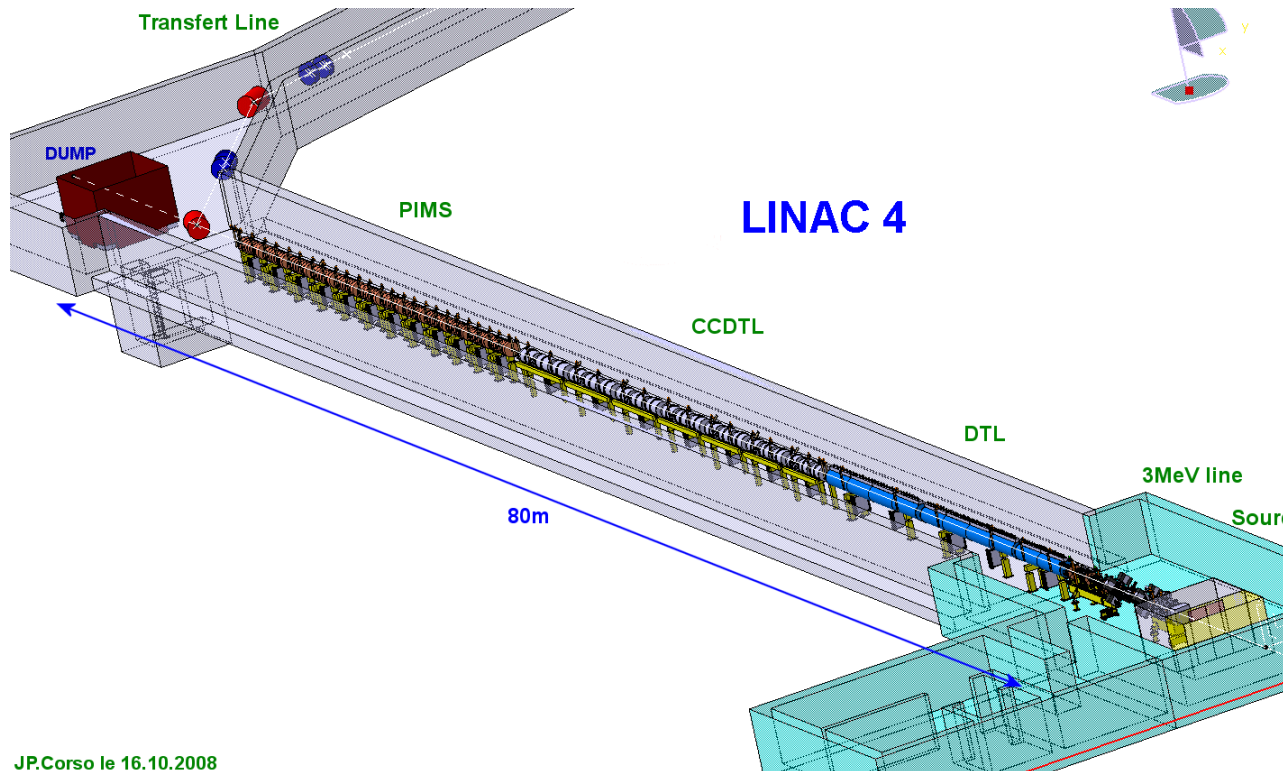


Cells have same length inside a cavity (7 cells) but increase from one cavity to the next. At high energy (>100 MeV) beta changes slowly and phase error ("phase slippage") is small.



EXAMPLE: the **Linac4 project at CERN**. H⁻, 160 MeV energy, 352 MHz.
A 3 MeV injector + 22 multi-cell standing wave accelerating structures of 3 types

DTL: every cell is different, focusing quadrupoles in each drift tube
CCDTL: sequences of 2 identical cells, quadrupoles every 3 cells
PIMS: sequences of 7 identical cells, quadrupoles every 7 cells

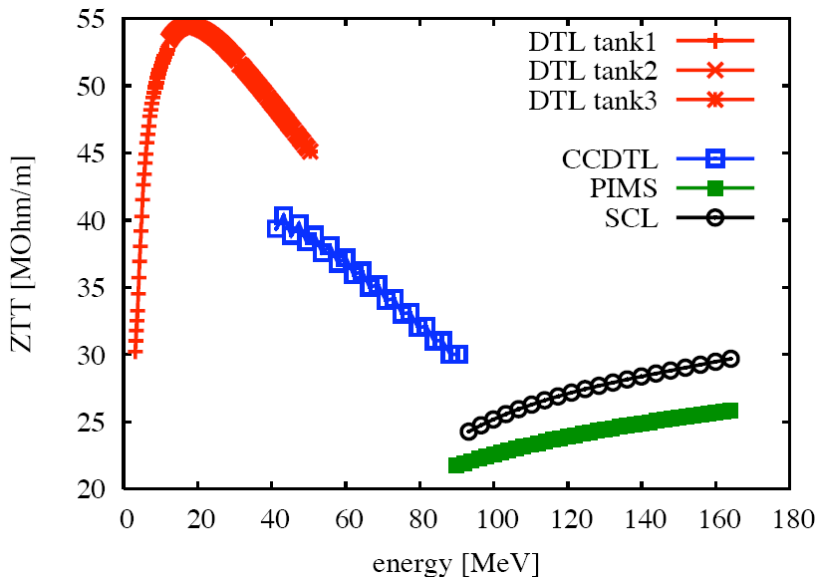


Two basic principles to remember:

1. As beta increases, phase error between cells of identical length becomes small → we can have **short sequences of identical cells** (lower construction costs).
2. As beta increases, the **distance between focusing elements can increase** (more details in 2nd lecture!).

A third basic principle:

Every proton linac structure has a characteristic curve of shunt impedance (=acceleration efficiency) as function of energy, which depends on the mode of operation.



| 3MeV | 50MeV | 100MeV | 160MeV |
|------------------|-------------------------------|-------------------|--------|
| DTL | CCDTL | PIMS | |
| Drift Tube Linac | Cell-Coupled Drift Tube Linac | Pi-Mode Structure | |
| 18.7 m | 25 m | 22 m | |
| 3 tanks | 7 tanks | 12 tanks | |
| 3 klystrons | 7 klystrons | 8 klystrons | |

The choice of the best accelerating structure for a certain energy range depends on **shunt impedance**, but also on **beam dynamics** and construction **cost**.

Effective shunt impedance ZT^2 : ratio between voltage (squared) seen by the beam and RF power.

It corresponds to the parallel resistance of the equivalent circuit (apart a factor 2) 29

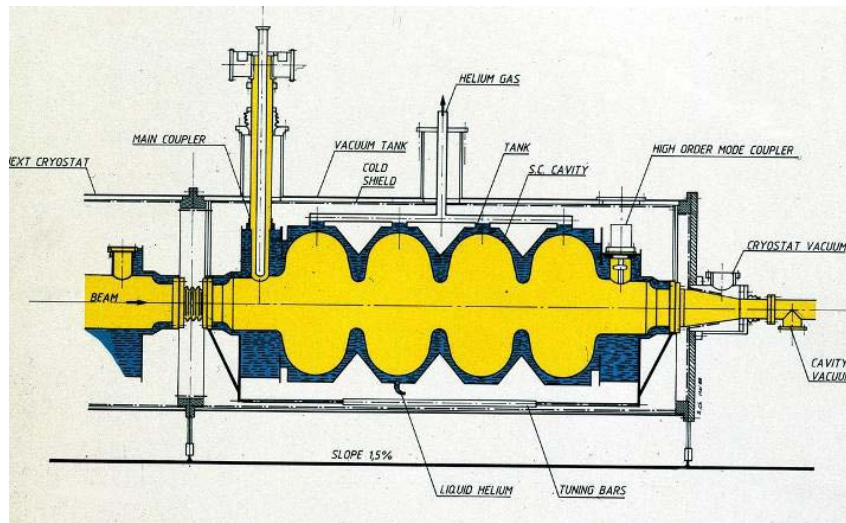
$$\Delta W = eE_0 T \cos \varphi$$

$$ZT^2 = \frac{V_{eff}^2}{P} = \frac{(E_0 T)^2}{P}$$

Multi-gap Superconducting linac structures (elliptical)



Standing wave structures for particles at $\beta > 0.5-0.7$, widely used for protons (SNS, etc.) and electrons (ILC, etc.)
 $f = 350-700$ MHz (protons),
 $f = 350$ MHz - 3 GHz (electrons)
 Chain of cells electrically coupled, large apertures (ZT² not a concern).
 Operating in π -mode, cell length $\beta\lambda/2$
 Input coupler placed at one end.



Other linac structures (the superconducting zoo)

Spoke (low beta)
[FZJ, Orsay]



4 gaps

CH (low/medium beta)
[IAP-FU]



10 gaps

QWR (low beta)
[LNL, etc.]



2 gaps

HWR (low beta)
[FZJ, LNL, Orsay]



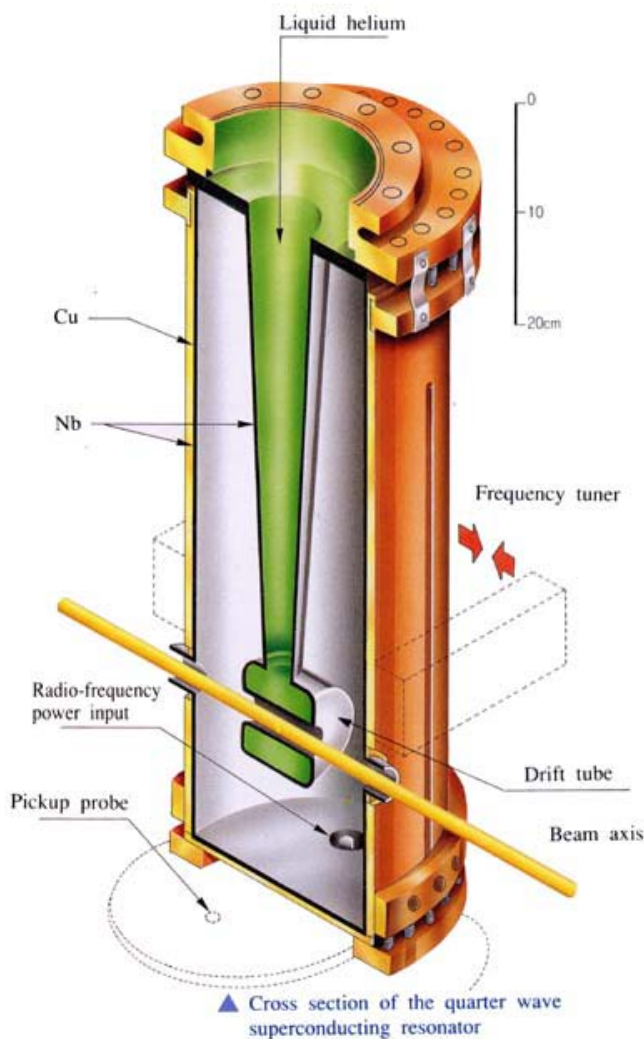
2 gaps

Re-entrant
[LNL]



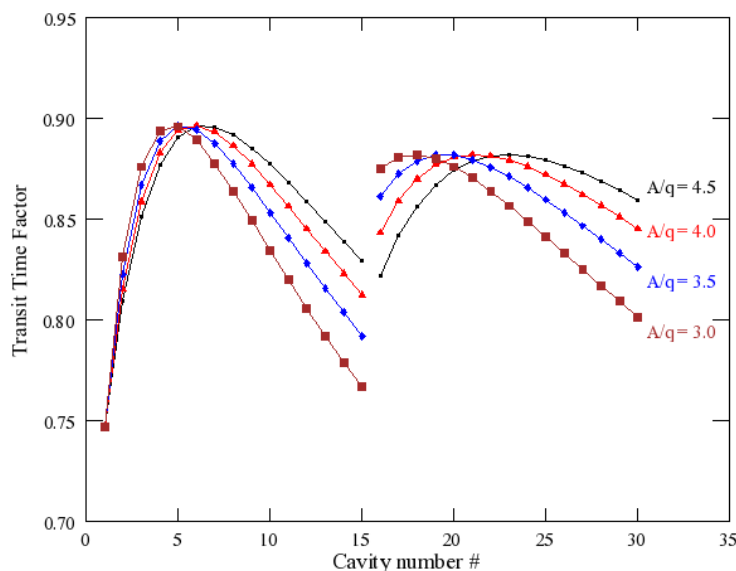
1 gap

Superconducting linacs for low and medium beta ions are made of multi-gap (1 to 4) individual cavities, spaced by focusing elements. Advantages: can be individually phased → linac can accept different ions
Allow more space for focusing → ideal for low β CW proton linacs



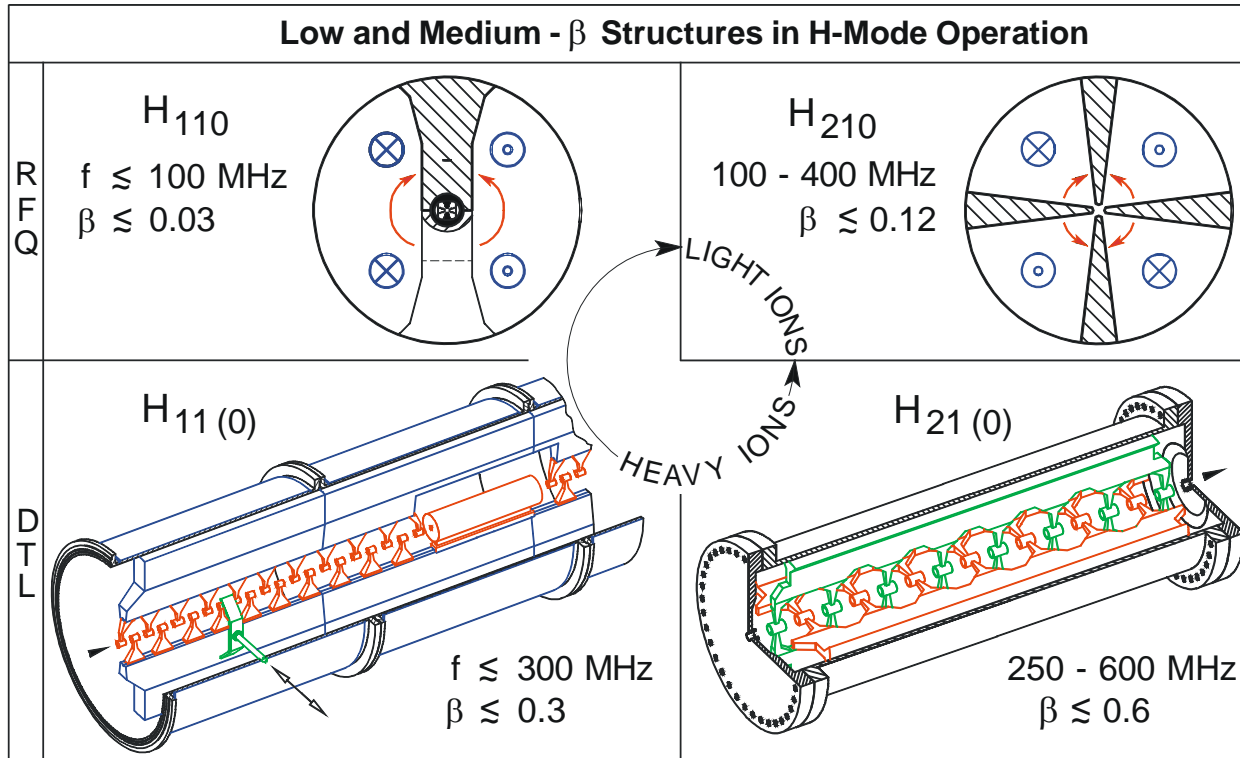
Simple 2-gap cavities commonly used in their superconducting version (lead, niobium, sputtered niobium) for low beta protons or ion linacs, where \sim CW operation is required.

Synchronicity (distance $\beta\lambda/2$ between the 2 gaps) is guaranteed only for one energy/velocity, while for easiness of construction a linac is composed by series of identical QWR's \rightarrow reduction of energy gain for "off-energy" cavities, Transit Time Factor curves as below:
 "phase slippage"



$$T = \frac{V_{eff}}{V_0}$$

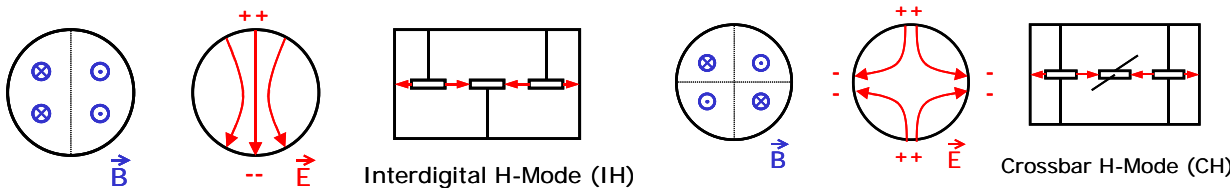
Transit time factor T is the ratio between voltage seen by the beam (because of finite velocity) and actual voltage in the gap



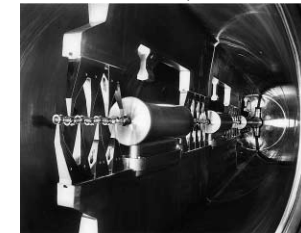
Interdigital-H Structure Operates in TE₁₁₀ mode
 Transverse E-field "deflected" by adding drift tubes
 Used for ions, $\beta < 0.3$

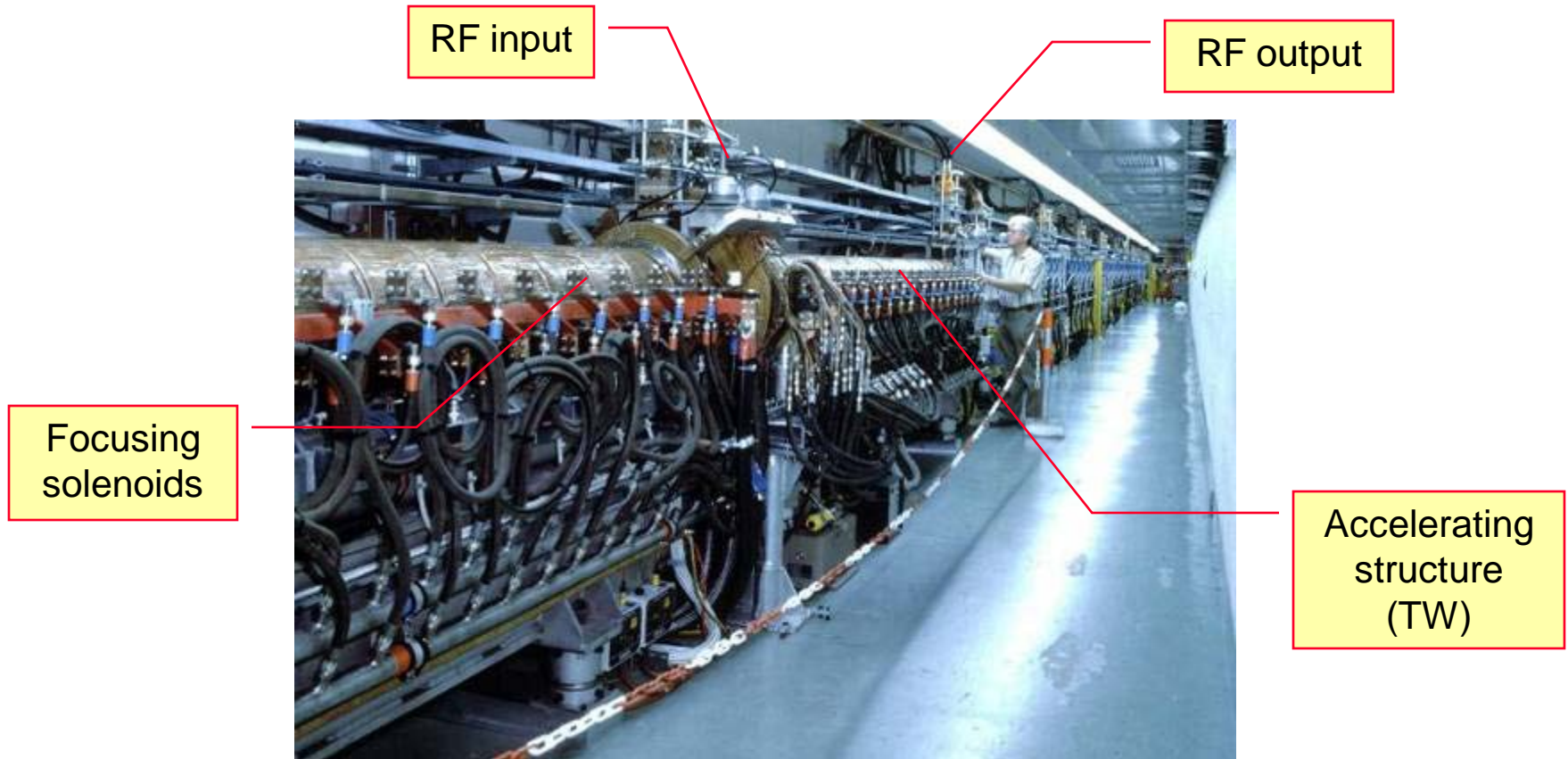
CH Structure operates in TE₂₁₀, used for protons at $\beta < 0.6$

High ZT^2 but more difficult beam dynamics (no space for quads in drift tubes)



HSI - IH DTL, 36 MHz





The old CERN LIL (LEP Injector Linac) accelerating structures (3 GHz). The TW structure is surrounded by focusing solenoids, required for the positrons.

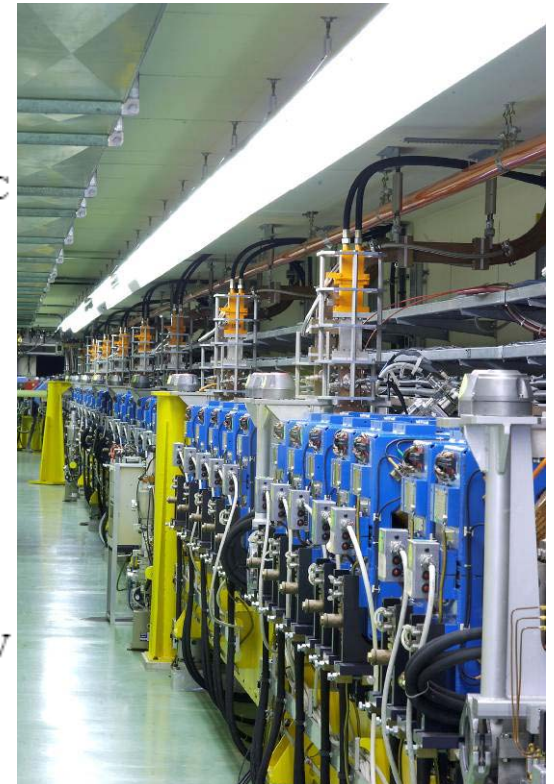
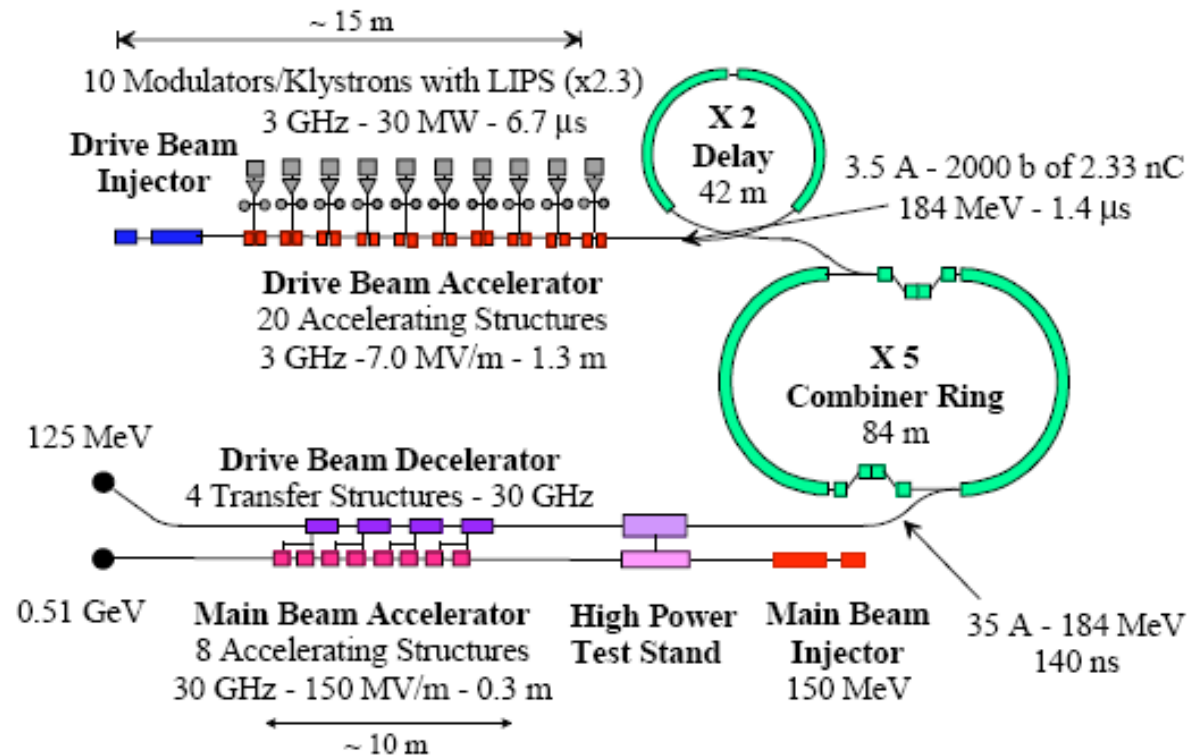


A 3 GHz LIL accelerating structure used for CTF3. It is 4.5 meters long and provides an energy gain of 45 MeV. One can see 3 quadrupoles around the RF structure.

CAS Electron linac architecture



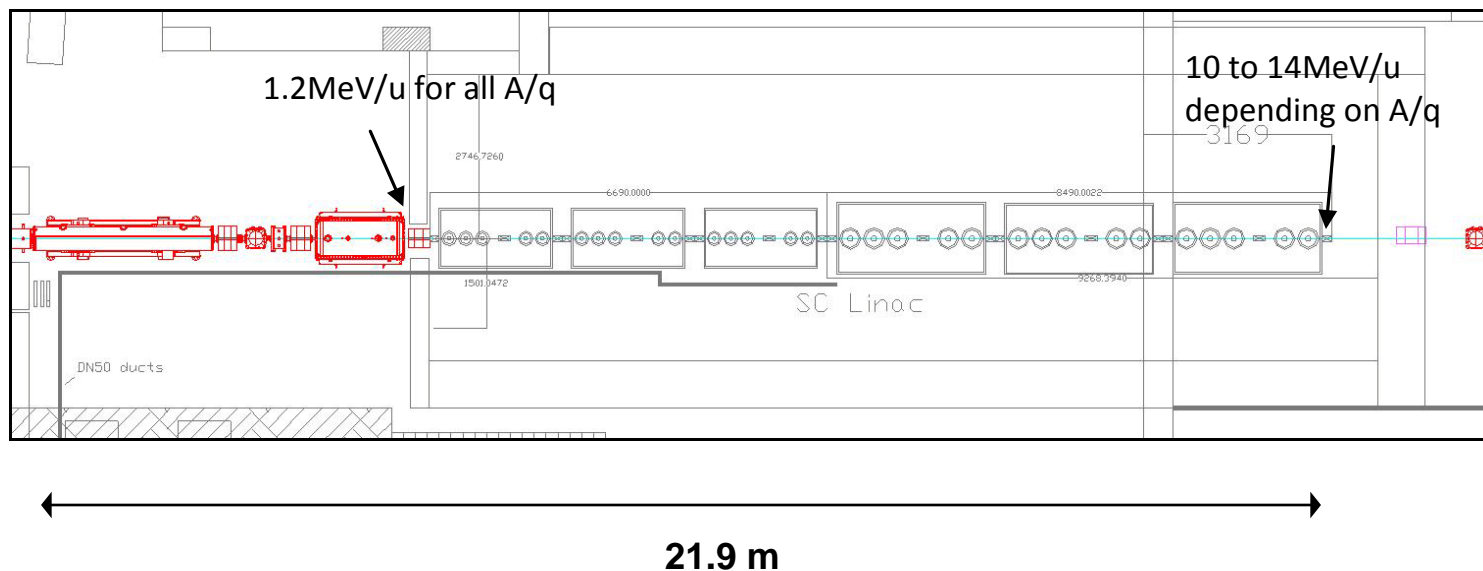
EXAMPLE: the **CLIC Test facility (CTF)** at **CERN**: drive linac, 3 GHz, 184 MeV.
An injector + a sequence of 20 identical multi-cell traveling wave accelerating structures.
Main beam accelerator: 8 identical accelerating structures at 30 GHz, 150-510 MeV



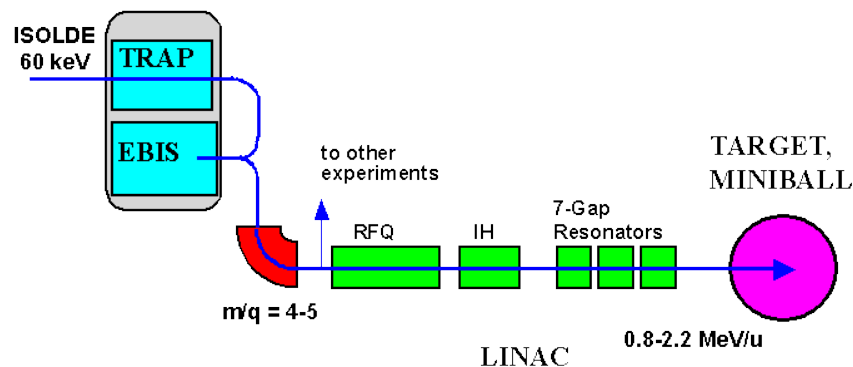
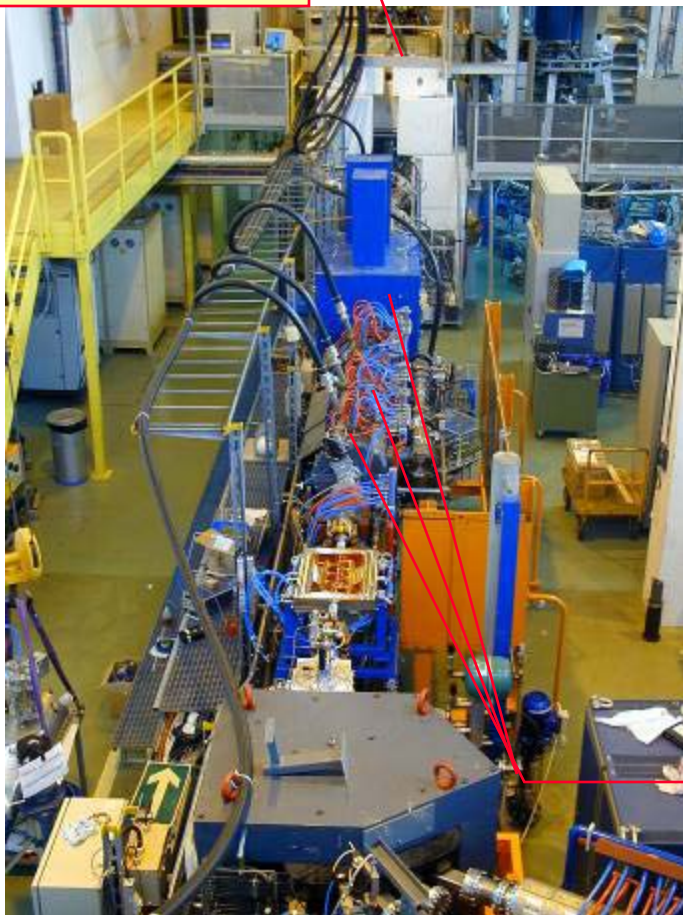
CAS Heavy Ion Linac Architecture



EXAMPLE: the **REX upgrade project at CERN-ISOLDE**. Post-acceleration of radioactive ions with different A/q up to energy in the range 2-10 MeV. An injector (source, charge breeder, RFQ) + a sequence of short (few gaps) standing wave accelerating structures at frequency 101-202 MHz, normal conducting at low energy (Interdigital, IH) and superconducting (Quarter Wave Resonators) at high energy \rightarrow mix of NC-SC, different structures, different frequencies.

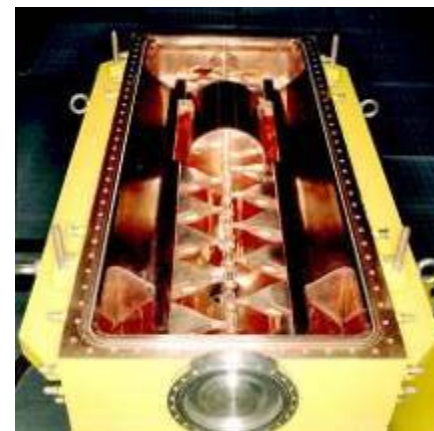


Particle source

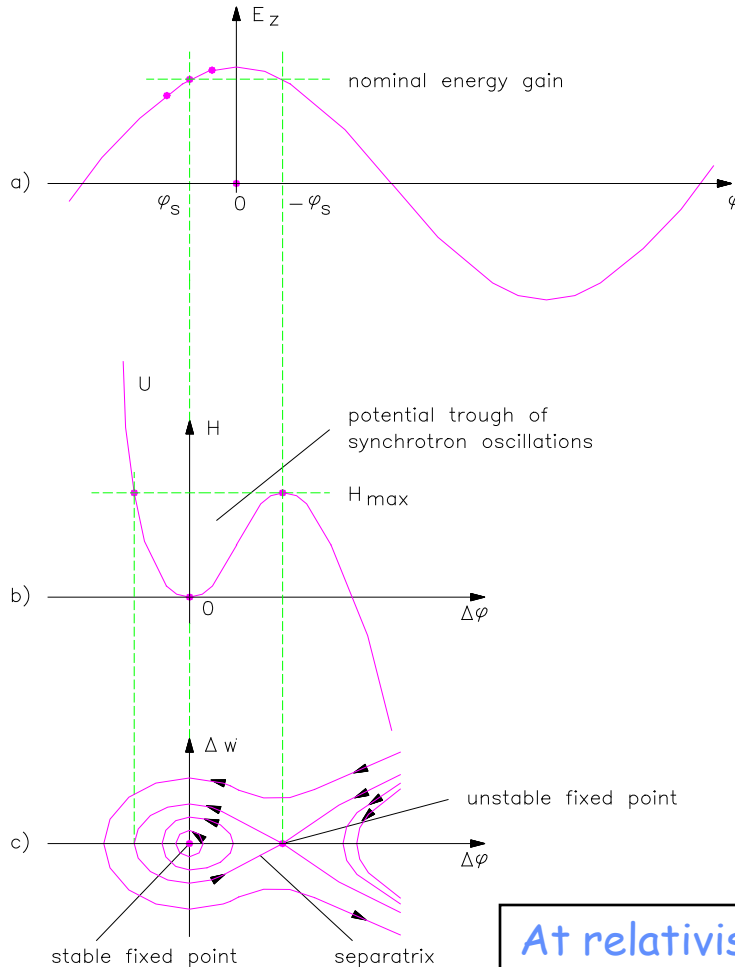


The REX heavy-ion post accelerators at CERN. It is made of 5 short standing wave accelerating structures at 100 MHz, spaced by focusing elements.

Accelerating structures



4 - Beam Dynamics of Ion and Electron Linacs



→ Ions are accelerated around a (negative = linac definition) synchronous phase.

→ Particles around the synchronous one perform oscillations in the longitudinal phase space.

→ Frequency of small oscillations:

$$\omega_l^2 = \omega_0^2 \frac{qE_0 T \sin(-\varphi)\lambda}{2\pi mc^2 \beta\gamma^3}$$

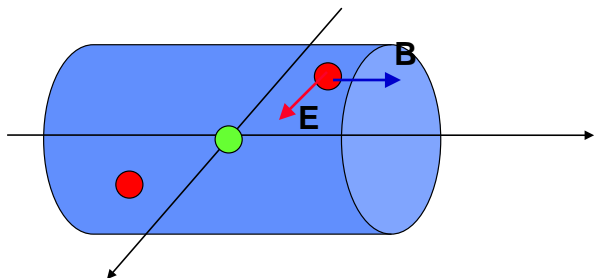
→ Tends to zero for relativistic particles $\gamma \gg 1$.

→ Note phase damping of oscillations:

$$\Delta\varphi = \frac{const}{(\beta\gamma)^{3/4}} \quad \Delta W = const \times (\beta\gamma)^{3/4}$$

At relativistic velocities phase oscillations stop, and the beam is compressed in phase around the initial phase. The crest of the wave can be used for acceleration.

- Large numbers of particles per bunch ($\sim 10^{10}$).
- Coulomb repulsion between particles (space charge) plays an important role.
- But **space charge forces $\sim 1/\gamma^2$ disappear at relativistic velocity** : the magnetic attraction compensates exactly for the coulomb repulsion!

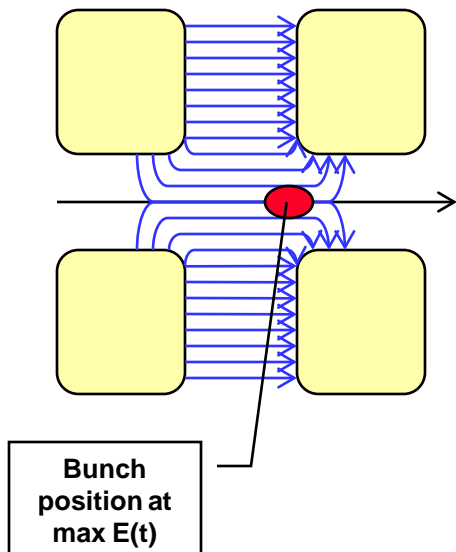


Force on a particle inside a long bunch with density $n(r)$ traveling at velocity v :

$$E_r = \frac{e}{2\pi\epsilon r} \int_0^r n(r) r dr \quad B_\phi = \frac{\mu}{2\pi} \frac{ev}{r} \int_0^r n(r) r dr$$

$$F = e(E_r - vB_\phi) = eE_r \left(1 - \frac{v^2}{c^2}\right) = eE_r (1 - \beta^2) = \frac{eE_r}{\gamma^2}$$

Note that the expression for space charge forces in a bunch can be vary complicated (and linac beam dynamics in the space charge regime is a science in itself!)



- RF defocusing experienced by particles crossing a gap on a longitudinally stable phase.
- In the rest frame of the particle, only electrostatic forces → no stable points (maximum or minimum) → radial defocusing.
- Lorentz transformation and calculation of radial momentum impulse per period (from electric and magnetic field contribution in the laboratory frame):

$$\Delta p_r = -\frac{\pi e E_0 T L r \sin \varphi}{c \beta^2 \gamma^2 \lambda}$$

- Transverse defocusing $\sim 1/\gamma^2$ disappears at relativistic velocity (transverse magnetic force cancels the transverse RF electric force).
- Important consequence: in a proton linac, transverse and longitudinal dynamics are coupled!

The equilibrium between external focusing force and internal defocusing forces defines the **frequency of beam oscillations**.

Oscillations are characterized in terms of *phase advance per focusing period* σ_+ or *phase advance per unit length* k_+ .

Ph. advance = Ext. quad focusing - RF defocusing - space charge - Instabilities

$$k_t^2 = \left(\frac{\sigma_t}{N\beta\lambda} \right)^2 = \left(\frac{qGl}{2mc\beta\gamma} \right)^2 - \frac{\pi q E_0 T \sin(-\phi)}{mc^2 \lambda \beta^3 \gamma^3} - \frac{3qI\lambda(1-f)}{8\pi\epsilon_0 r_0^3 mc^3 \beta^2 \gamma^3} - \dots$$

Approximate expression valid for:

FODO lattice, smooth focusing approximation, space charge of a uniform 3D ellipsoidal bunch.

G =quadrupole gradient, ϕ =synchronous phase, I =beam current, f =bunch form factor, r =average beam radius

Electron Linac:

Ph. advance = Ext. focusing + ~~RF defocusing~~ + ~~space charge~~ + Instabilities

For $\gamma \gg 1$ (electron linac): RF defocusing and space charge disappear, *phase advance* $\rightarrow 0$.

External focusing is required only to control the emittance and to stabilize the beam against instabilities (as **wakefields** and **beam breakup**).

Focusing provided by quadrupoles (but solenoids for low β !).

Different **distance between focusing elements** (=1/2 length of a FODO focusing period)! For the main linac accelerating structure (after injector):

Protons, (high beam current and high space charge) require short distances:

- $\beta\lambda$ in the DTL, from $\sim 70\text{mm}$ (3 MeV, 352 MHz) to $\sim 250\text{mm}$ (40 MeV),
- can be increased to $4-10\beta\lambda$ at higher energy (>40 MeV).
- longer focusing periods require special dynamics (example: the IH linac).

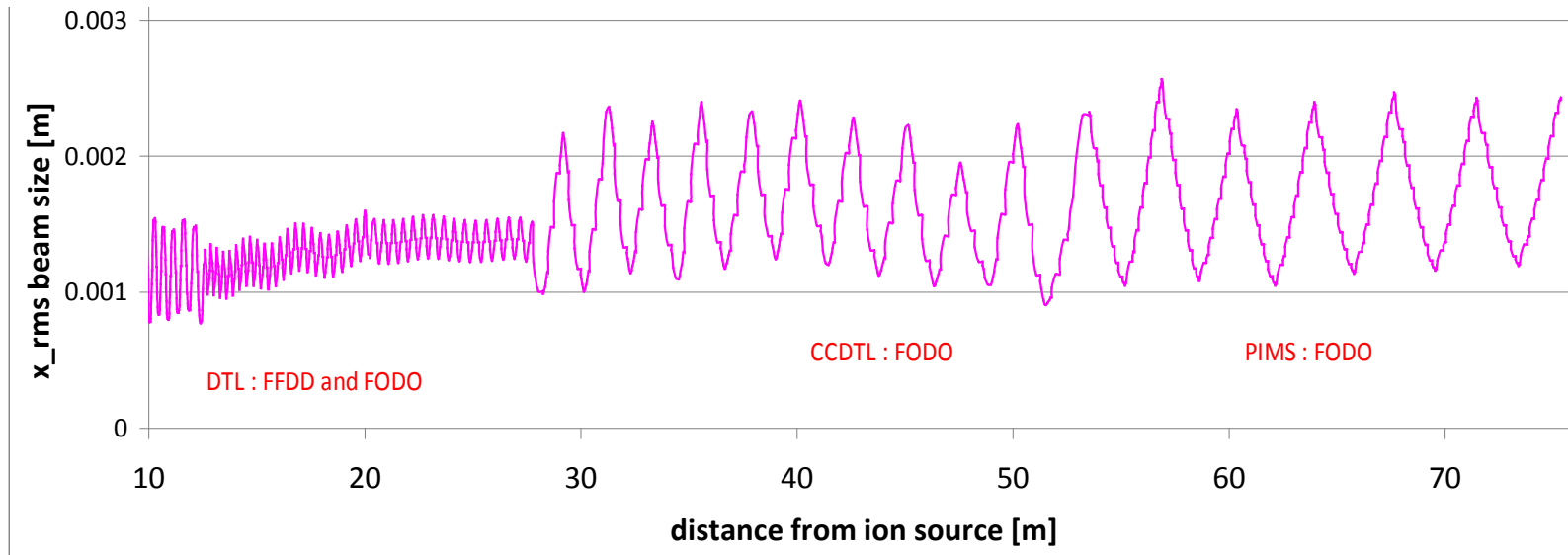
Heavy ions (low current, no space charge):

$2-10 \beta\lambda$ in the main linac ($>\sim 150\text{mm}$).

Electrons (no space charge, no RF defocusing):

up to several meters, depending on the required beam conditions. Focusing is mainly required to control the emittance.

Transverse (x) r.m.s. beam envelope along Linac4



Example: beam dynamics design for Linac4@CERN.

High intensity protons (60 mA bunch current, duty cycle could go up to 5%), 3 - 160 MeV

Beam dynamics design minimising emittance growth and halo development in order to:

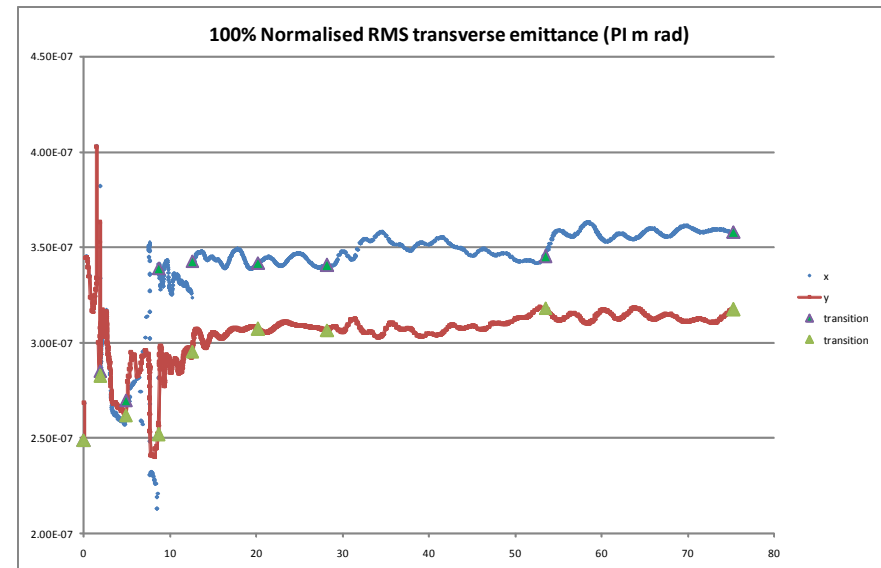
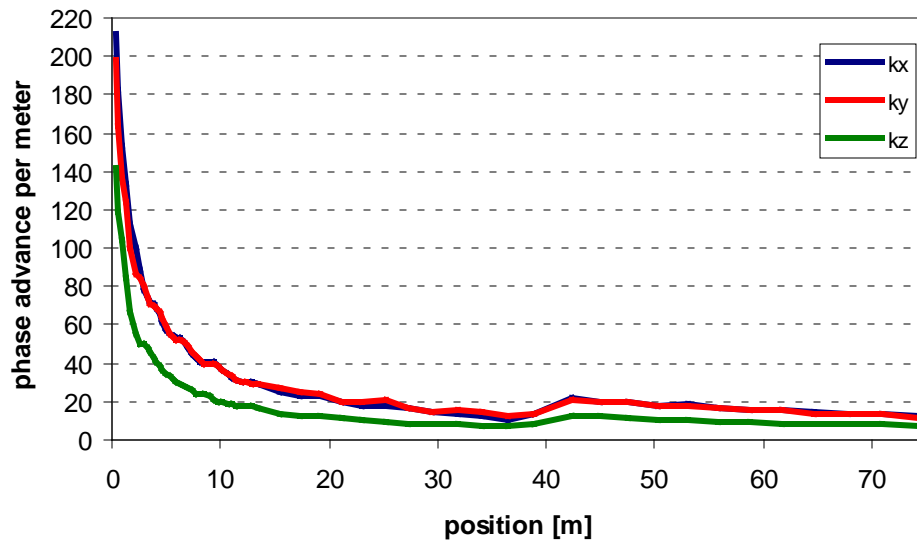
1. **avoid uncontrolled beam loss** (activation of machine parts)
2. **preserve small emittance** (high beam brightness in the following accelerators)

CAS Beam Optics Design Guidelines



Prescriptions to **minimise emittance growth and halo formation**:

1. Keep zero current phase advance always below 90° , to avoid resonances
2. Keep longitudinal to transverse phase advance ratio 0.5-0.8, to avoid emittance exchange
3. Keep a smooth variation of transverse and longitudinal phase advance per meter.
4. Keep sufficient safety margin between beam radius and aperture



Transverse r.m.s. emittance and phase advance along Linac4 (RFQ-DTL-CCDTL-PIMS)

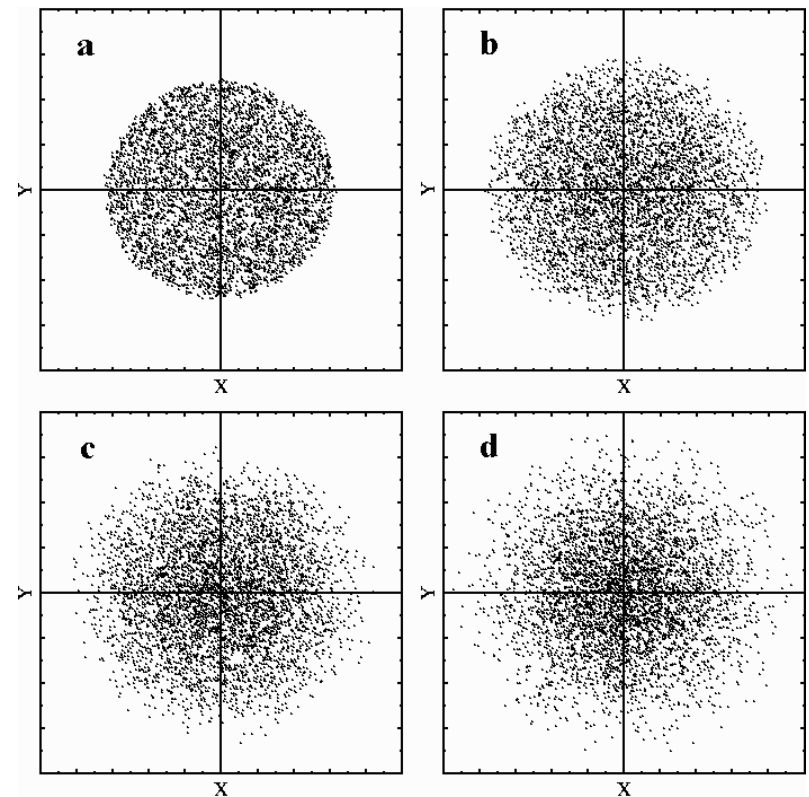
Additional challenge on beam dynamics:

Beam loss has to be avoided not because it reduces current, but because activation due to beam loss could come to levels preventing access to the machine.

Commonly accepted loss limit for hands-on maintenance is 1 W/m.

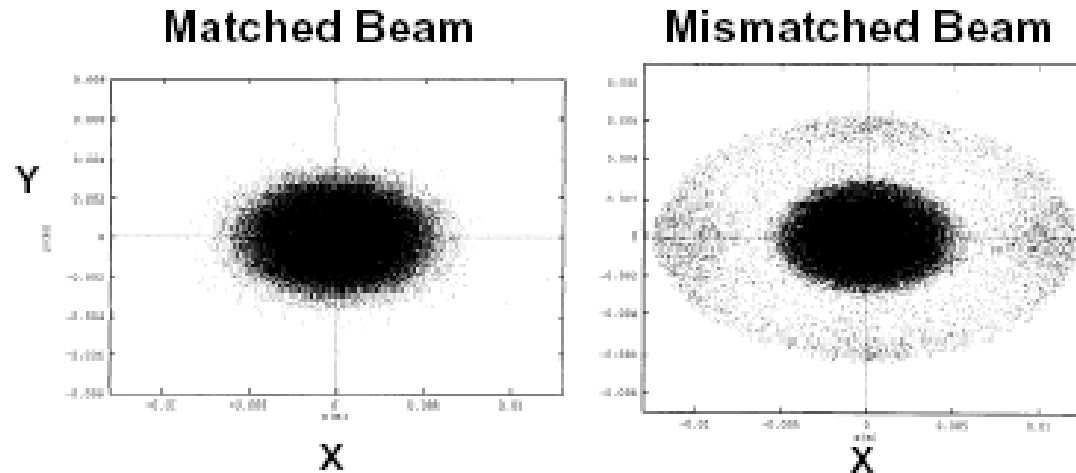
For example, in the case of the SPL design at CERN (5 GeV, 20 mA, 5% duty cycle) this corresponds to a maximum loss at 5 GeV of 0.2×10^{-6} /m, or 4 nA/m !!

In usual linacs, the beam distribution can be quite complicated, and present a core surrounded by a “halo”. Halo formation has to be studied and controlled!



What is beam halo? Example from simulation of mismatched beam in quadrupole-focusing channel.

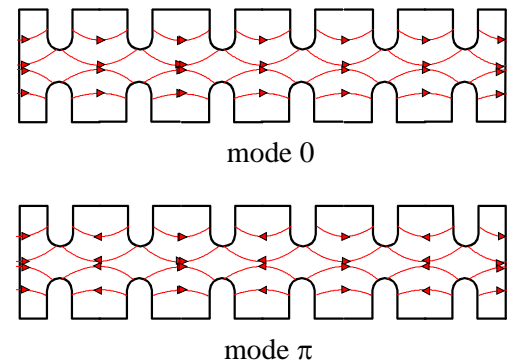
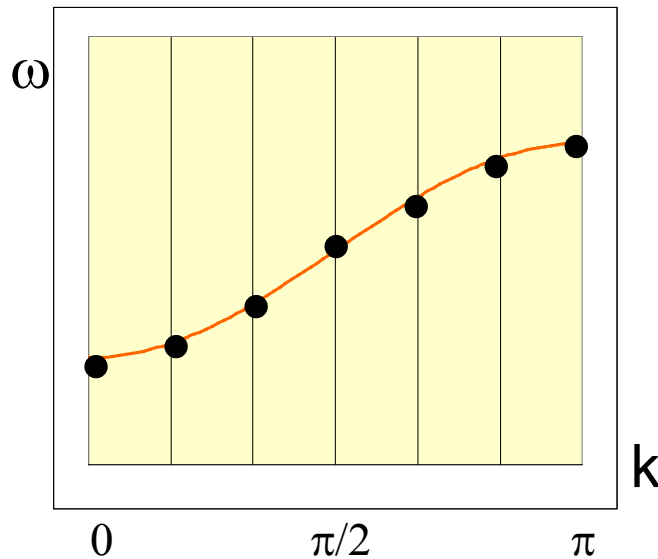
Mismatched beam develops larger amplitudes than matched beam.



Matching = adjust focusing gradients in such a way that (α, β) at the exit of a period are the same as at the entrance

5. Double periodic accelerating structures

- ➔ To reduce RF cost, linacs use high-power RF sources feeding a large number of **coupled cells** (DTL: 30-40 cells, other high-frequency structures can have >100 cells).
- ➔ Long linac structures operating in the 0 or π modes are extremely **sensitive to mechanical errors**: small machining errors in the cells can induce large differences in the accelerating field between cells.

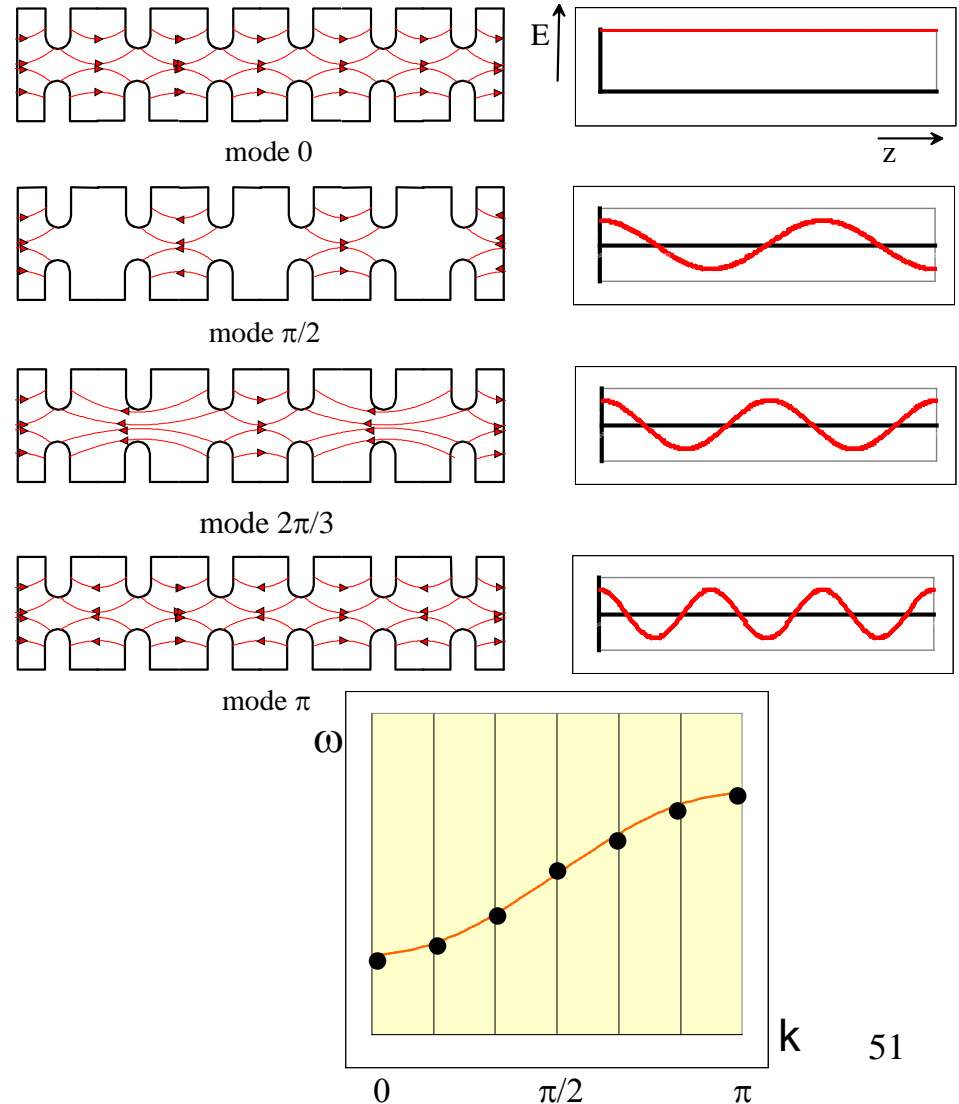


Mechanical errors \rightarrow differences in frequency between cells \rightarrow to respect the new boundary conditions the electric field will be a linear combination of all modes, with weight

$$\frac{1}{f^2 - f_0^2}$$

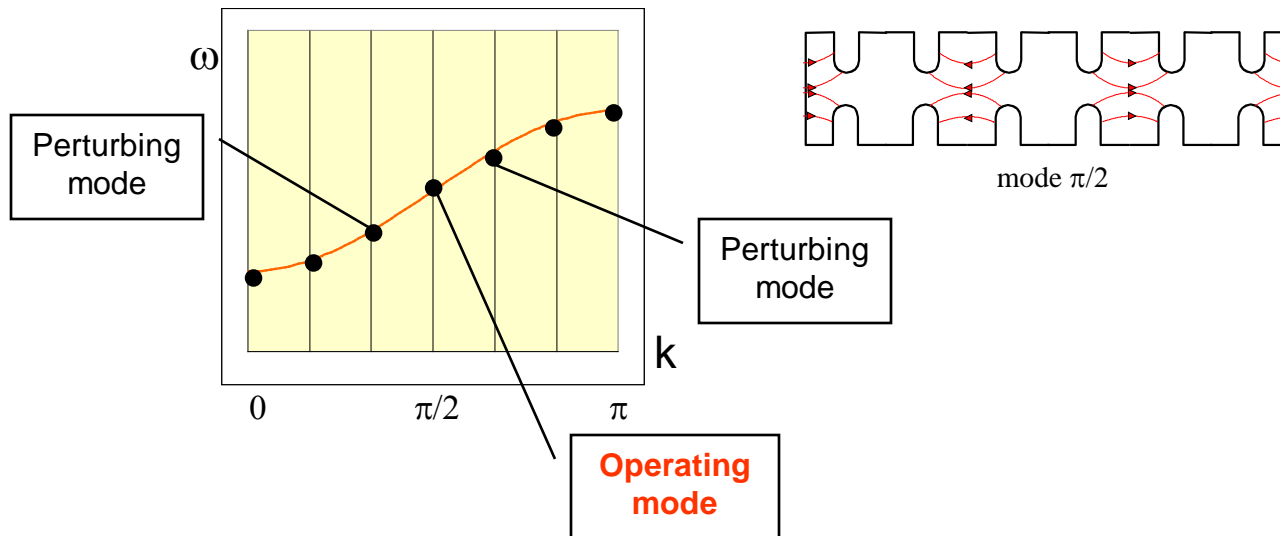
(general case of small perturbation to an eigenmode system, the new solution is a linear combination of all the individual modes)

The nearest modes have the highest effect, and when there are many modes on the dispersion curve (number of modes = number of cells !) the difference in E-field between cells can be extremely high.



Solution:

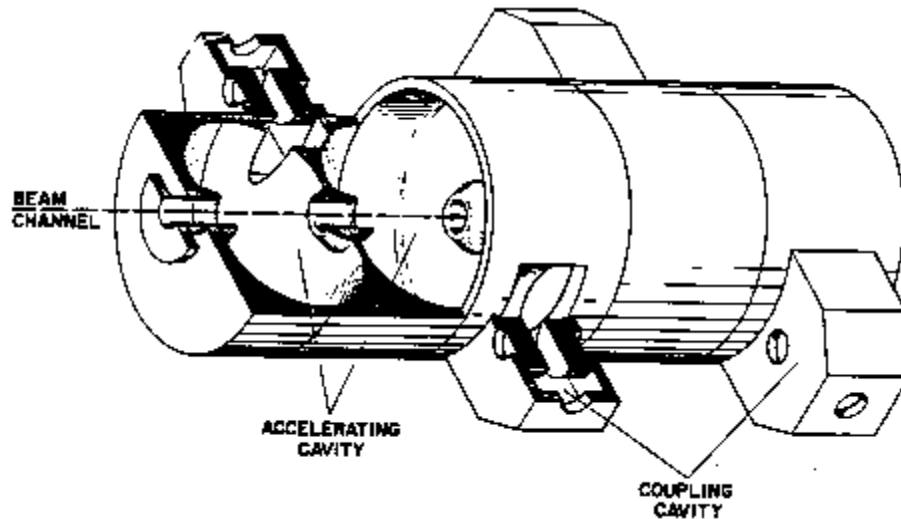
Long chains of linac cells are operated in the $\pi/2$ mode, which is **intrinsically insensitive** to differences in the cell frequencies.



Contribution from adjacent modes proportional to $\frac{1}{f^2 - f_0^2}$ **with the sign !!!**

Contribution from equally spaced modes in the dispersion curve will cancel each other.

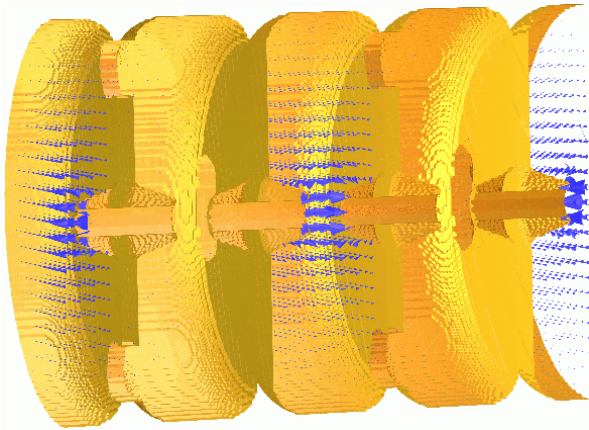
To operate efficiently in the $\pi/2$ mode, the cells that are not excited can be removed from the beam axis \rightarrow they become coupling cells, as for the **Side Coupled Structure**.



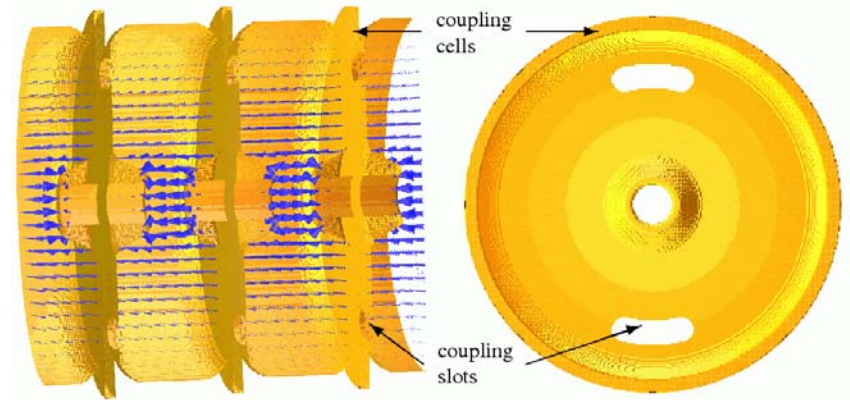
Example: the Cell-Coupled Linac at SNS, >100 cells/module

multi-cell Standing Wave structure in $\pi/2$ mode
frequency 800 - 3000 MHz
for protons ($\beta=0.5 - 1$)

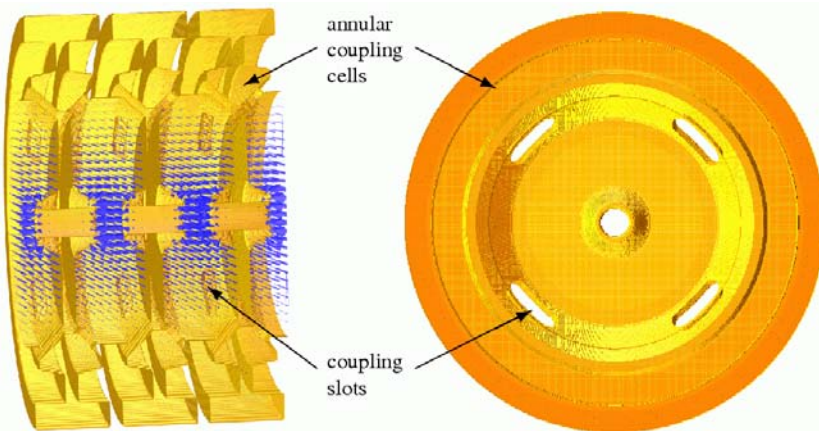


$\pi/2$ -mode in a coupled-cell structure

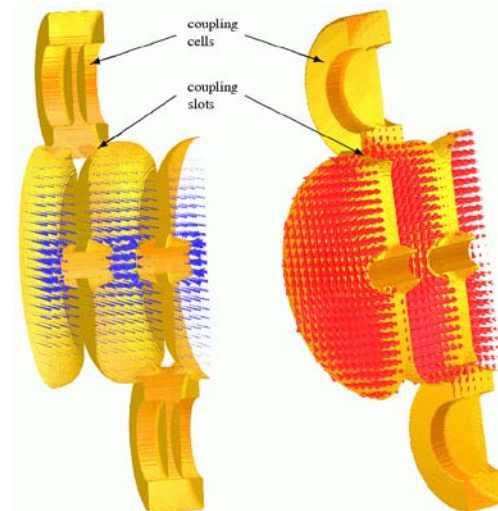
On axis Coupled Structure (OCS)

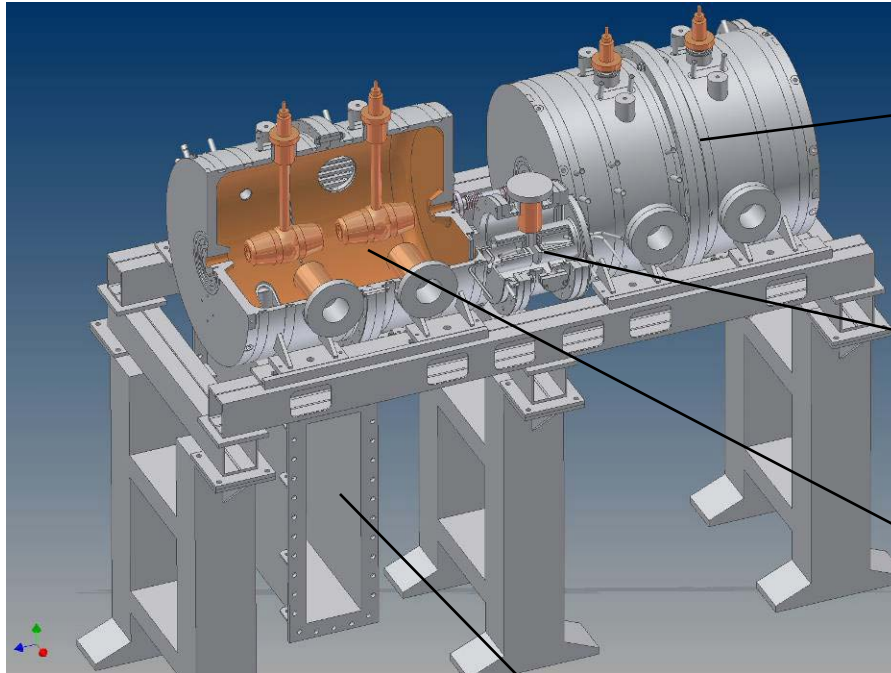


Annular ring Coupled Structure (ACS)



Side Coupled Structure (SCS)





DTL-like tank
(2 drift tubes)

Coupling cell

DTL-like tank
(2 drift tubes)

Series of DTL-like tanks (0-mode), coupled by coupling cells ($\pi/2$ mode)

352 MHz, will be used for the CERN Linac4 in the range 40-100 MeV.

Quadrupoles between tanks → easier alignment, lower cost than standard DTL



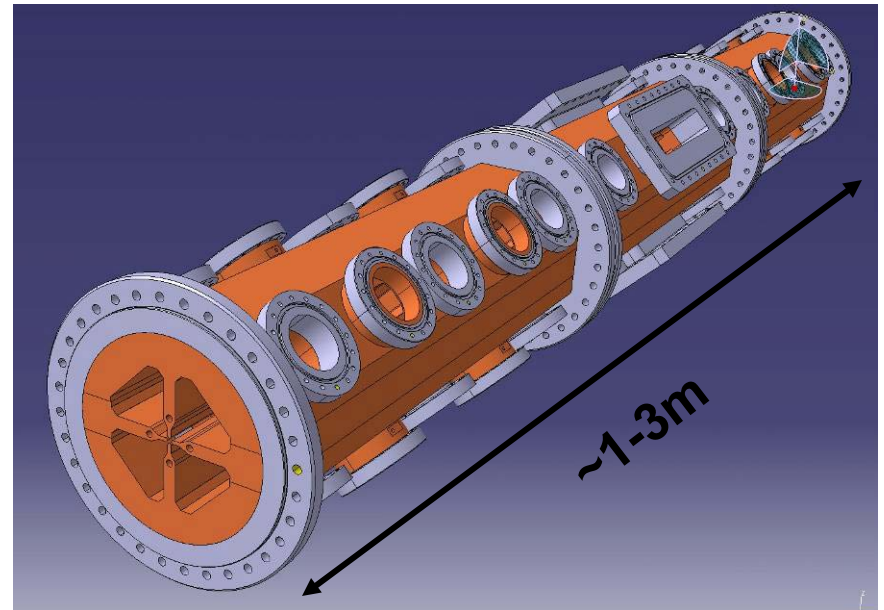
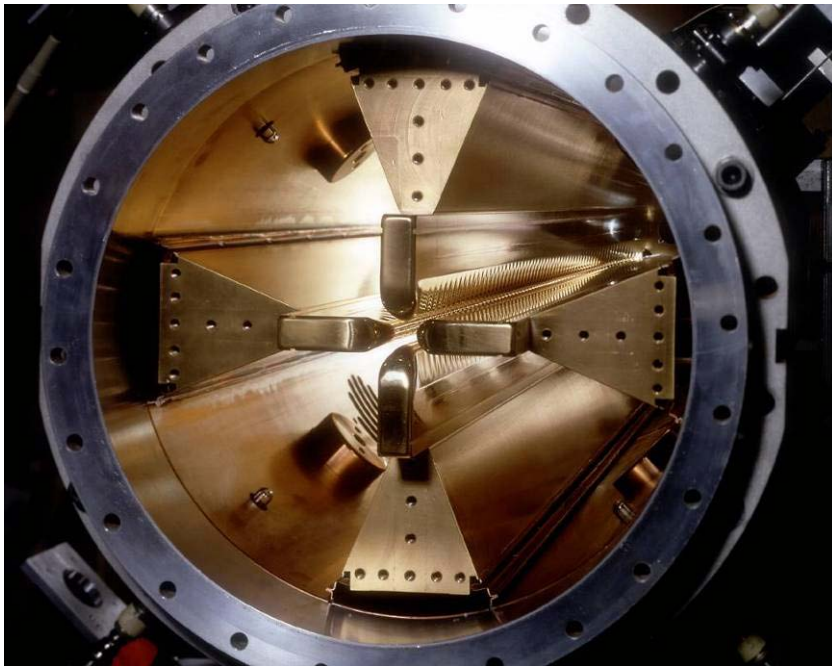
Waveguide
input coupler

6. The Radio Frequency Quadrupole

The Radio Frequency Quadrupole (RFQ)



At low proton (or ion) energies, space charge defocusing is high and quadrupole focusing is not very effective, cell length becomes small → conventional accelerating structures (Drift Tube Linac) are very inefficient → use a (relatively) new structure, the Radio Frequency Quadrupole.

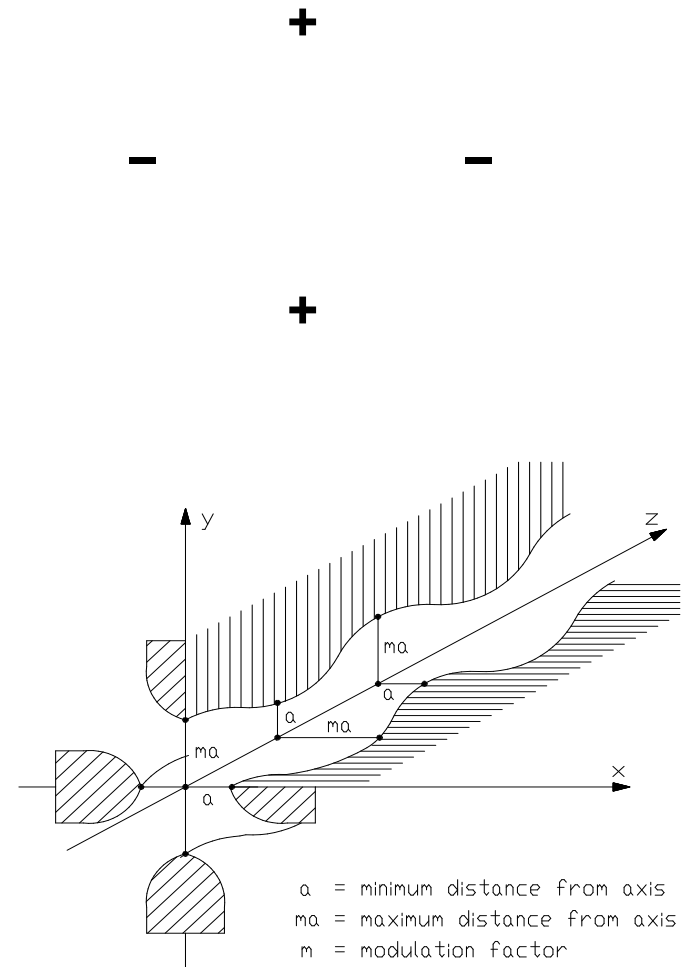


RFQ = Electric quadrupole focusing channel + bunching + acceleration

- Four electrodes (vanes) between which we excite an RF Quadrupole mode (TE₂₁₀)
 → **Electric focusing channel**, alternating gradient with the period of the RF. Note that electric focusing does not depend on the velocity (ideal at low β !)
- The vanes have a **longitudinal modulation** with period = $\beta\lambda$ → this creates a longitudinal component of the electric field. The modulation corresponds exactly to a series of RF gaps and can provide acceleration.

Opposite vanes (180°)

Adjacent vanes (90°)

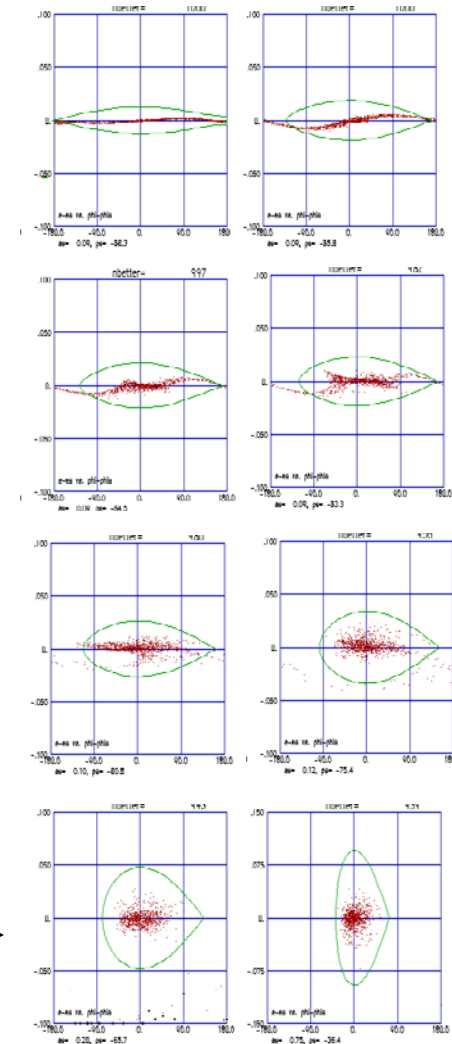


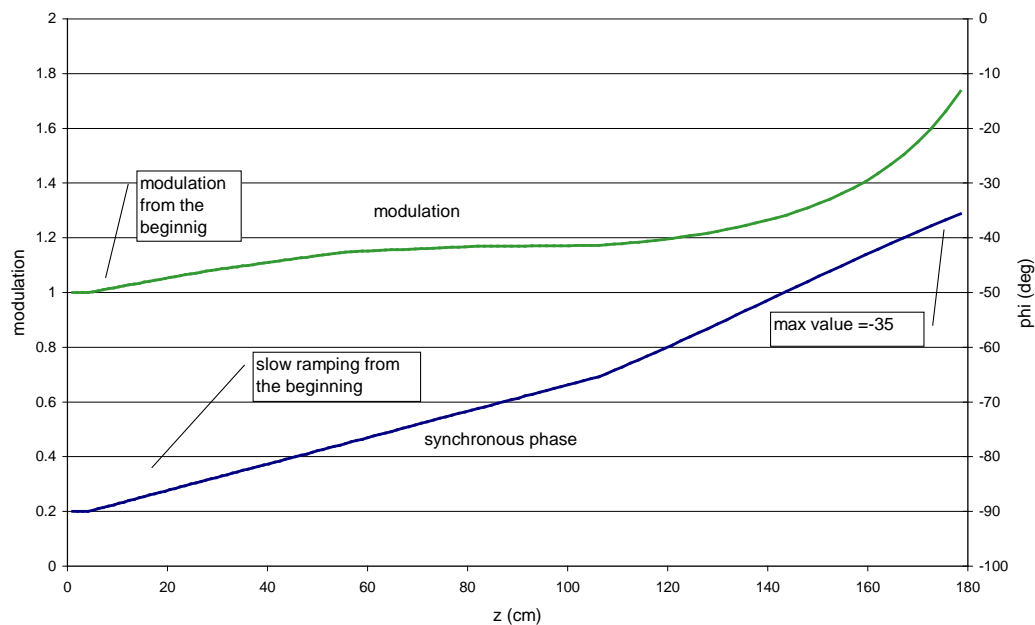
3. The modulation period (distance between maxima) can be slightly adjusted to change the phase of the beam inside the RFQ cells, and the amplitude of the modulation can be changed to change the accelerating gradient → we can start at -90° phase (linac) with some bunching cells, progressively bunch the beam (adiabatic bunching channel), and only in the last cells switch on the acceleration.

☞ An RFQ has 3 basic functions:

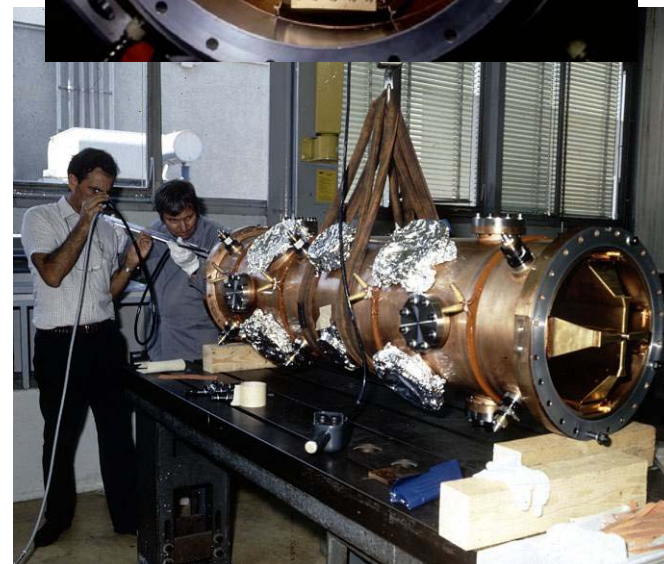
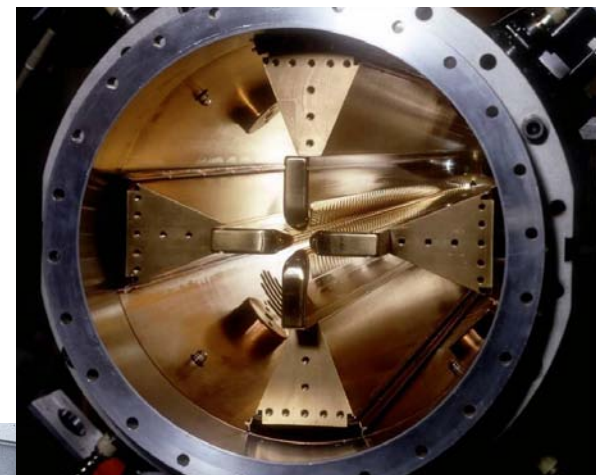
1. Adiabatically bunching of the beam.
2. Focusing, on electric quadrupole.
3. Accelerating.

Longitudinal beam profile of a proton beam along the CERN RFQ2: from a continuous beam to a bunched accelerated beam in 300 cells. →

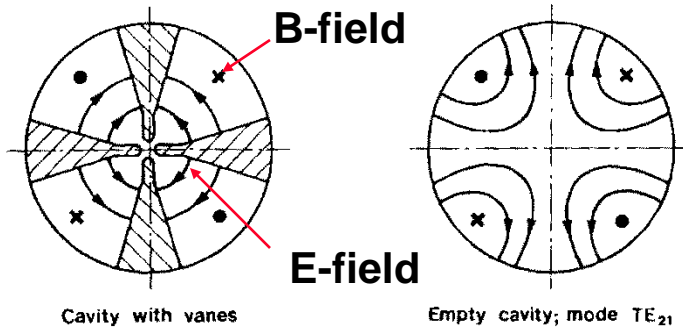




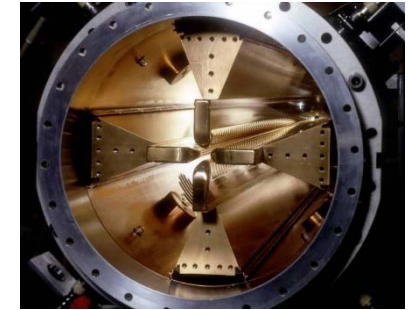
CERN High intensity RFQ
(RFQ2, 200 mA, 1.8m length, 90 – 750 keV)



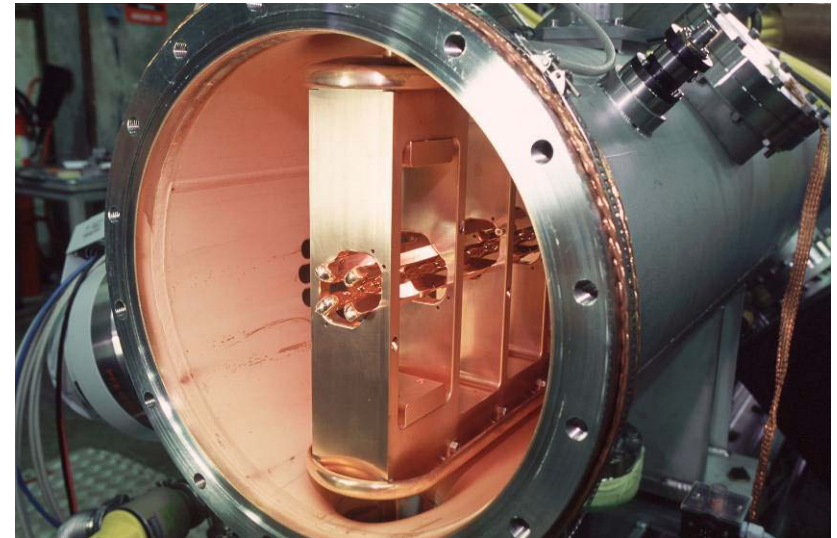
How to create a quadrupole RF mode ?



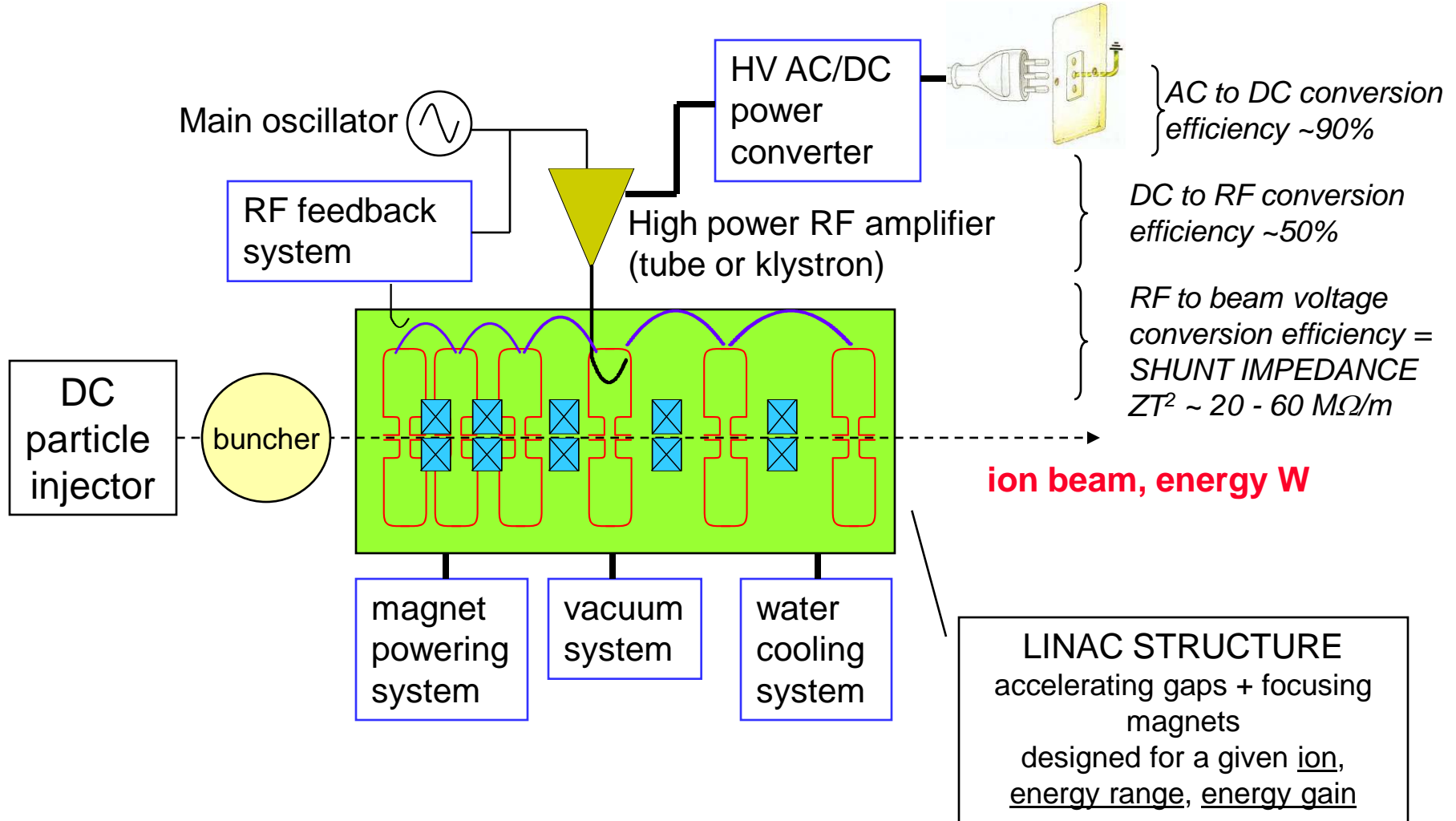
The TE_{210} mode in the “4-vane” structure and in the empty cavity.



Alternative resonator design: the “4-rod” structure, where an array of $\lambda/4$ parallel plate lines loads four rods, connected in such a way as to provide the quadrupole field.



7. Linac Technologies



Electron sources:

give energy to the free electrons inside a metal to overcome the potential barrier at the boundary.

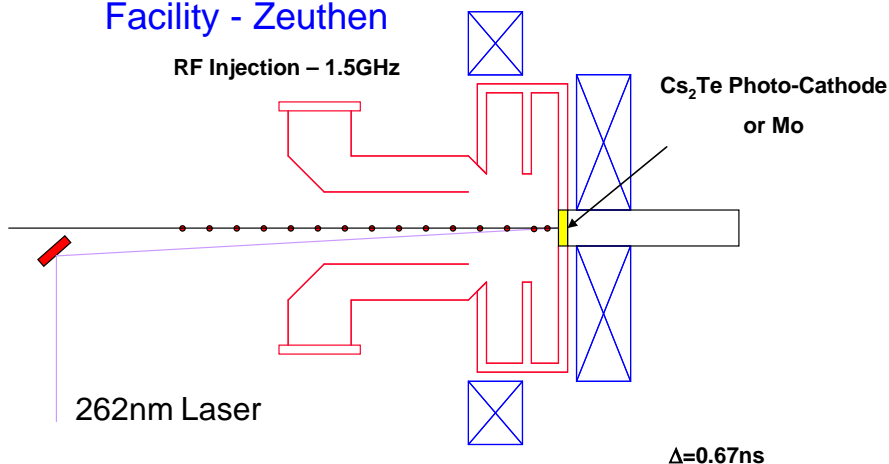
Used for electron production:

- thermoionic effect
- laser pulses
- surface plasma

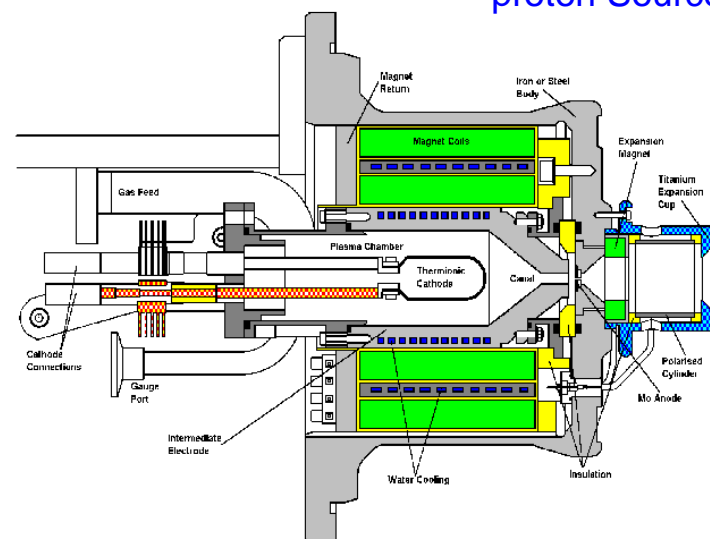
Ion sources:

create a plasma and optimise its conditions (heating, confinement and loss mechanisms) to produce the desired ion type. Remove ions from the plasma via an aperture and a strong electric field.

Photo Injector Test Facility - Zeuthen



CERN Duoplasmatron proton Source



Ion injector (CERN Linac1)



Electron injector (CERN LIL)



3 common problems for protons and electrons after the source, up to ~ 1 MeV energy:

1. large space charge defocusing
2. particle velocity rapidly increasing
3. need to form the bunches

Solved by a special **injector**

Ions: RFQ bunching, focusing and accelerating.

Electrons: Standing wave bunching and pre-accelerating section.

☞ For all particles, the injector is where the emittance is created!

- Type of **RF power source** depend on frequency:
 - ☞ Klystrons (>350 MHz) for electron linacs and modern proton linacs. RF distribution via waveguides.
 - ☞ RF tube (<400 MHz) or solid state amplifiers for proton and heavy ion linacs. RF distribution via coaxial lines.
- **Construction technology** depends on dimensions (\rightarrow on frequency):
 - ☞ brazed copper elements (>500 MHz) commonly used for electron linacs.
 - ☞ copper or copper plated welded/bolted elements commonly used for ion linacs (<500 MHz).



**3 GHz klystron
(CERN LPI)**

**200 MHz triode amplifier
(CERN Linac3)**

approximate scaling laws for linear accelerators:

| | |
|--------------------------------------|------------------------------|
| ⇒ RF defocusing (ion linacs) | ~ frequency |
| ⇒ Cell length ($=\beta\lambda/2$) | ~ (frequency) ⁻¹ |
| ⇒ Maximum surface electric field | ~ (frequency) ^{1/2} |
| ⇒ Shunt impedance (power efficiency) | ~ (frequency) ^{1/2} |
| ⇒ Accelerating structure dimensions | ~ (frequency) ⁻¹ |
| ⇒ Machining tolerances | ~ (frequency) ⁻¹ |

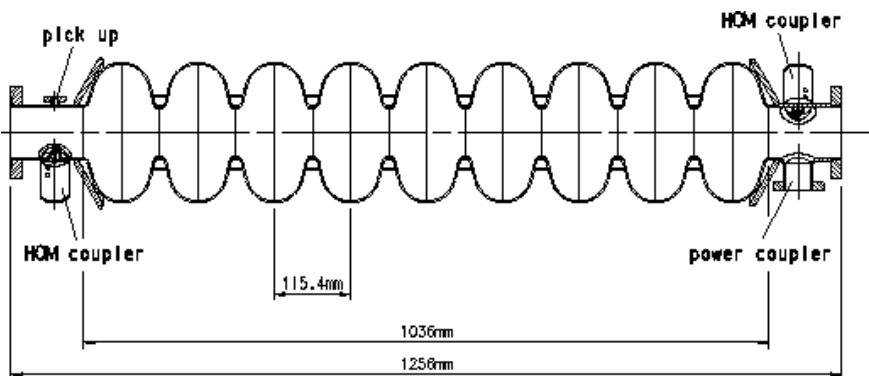
- **Higher frequencies** are economically convenient (shorter, less RF power, higher gradients possible) but limitation comes from **mechanical precision** in construction (tight tolerances are expensive!) and **beam dynamics** for ion linacs at low energy.
- **Electron linacs** tend to use **higher frequencies** (0.5-12 GHz) than ion linacs. Standard frequency 3 GHz (10 cm wavelength). No limitations from beam dynamics, iris in TW structure requires less accurate machining than nose in SW structure.
- **Proton linacs** use **lower frequencies** (100-800 MHz), increasing with energy (ex.: 350 - 700 MHz): compromise between focusing, cost and size.
- **Heavy ion linacs** tend to use **even lower frequencies** (30-200 MHz), dominated by the low beta in the first sections (CERN lead ion RFQ at 100MHz, 25 keV/u: $\beta\lambda/2=3.5\text{mm}$!)

What is new (& hot) in the field of linacs?

1. Frequencies are going up for both proton and electron linacs (←less expensive precision machining, efficiency scales roughly as \sqrt{f}). Modern proton linacs start at 350-400 MHz, end at 800-1300 MHz. Modern electron linacs in the range 3-12 GHz.
2. Superconductivity is progressing fast, and is being presently used for both electron and ion linacs → multi-cell standing wave structures in the frequency range from ~100 MHz to 1300 MHz.

Superconductivity is now **bridging the gap between electron and ion linacs**.

The 9-cell TESLA/ILC SC cavities at 1.3 GHz for electron linear colliders, are now proposed for High Power Proton Accelerators (Fermilab 8 GeV linac) !



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