Fixed Field Alternating Gradient (FFAG) Accelerator

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CERN Accelerator School

Accelerators in various fields

Three major uses of particle accelerators

High energy frontier

LHC, ILC

Energy of each particle

- Always a driving force of accelerator development.
- Small emittance is preferred.

Synchrotron light source

ESRF, DIAMOND

Brightness of light

- Ultimate goal is a beam as a point source.
(or zero emittance.)

High power source

μ, *K*, Neutron source, ν factory

Energy carried by a beam

- Large emittance to avoid multi particle effects
- Proton, can be ion, but not electron.

Is a synchrotron the best for high power source machine?

- High energy frontier machine (of electron) is now linac.
- Next generation synchrotron light source will be linac.

- First generation was cyclotron.
- Second generation was either synchrotron or linac plus storage ring.
  - ISIS (UK) uses 800 MeV proton synchrotron.
  - PSR (US) uses 800 MeV linac and storage ring.
- Trend is still the same.
  - J-PARC (Japan) is 3 GeV proton synchrotron.
  - SNS (US) is 1 GeV linac and storage ring.

Beam power of accelerator

Energy of each particle [GeV]

Higher energy is preferable, but size should be moderate

x

Number of particle per beam [ppp]

Enlarge aperture as much as possible

x

Repetition rate [Hz]

Continuous operation is the best, but very high repetition is acceptable

Low loss

Keep accessibility

Reliability

Hardware failures cut integrated beam power

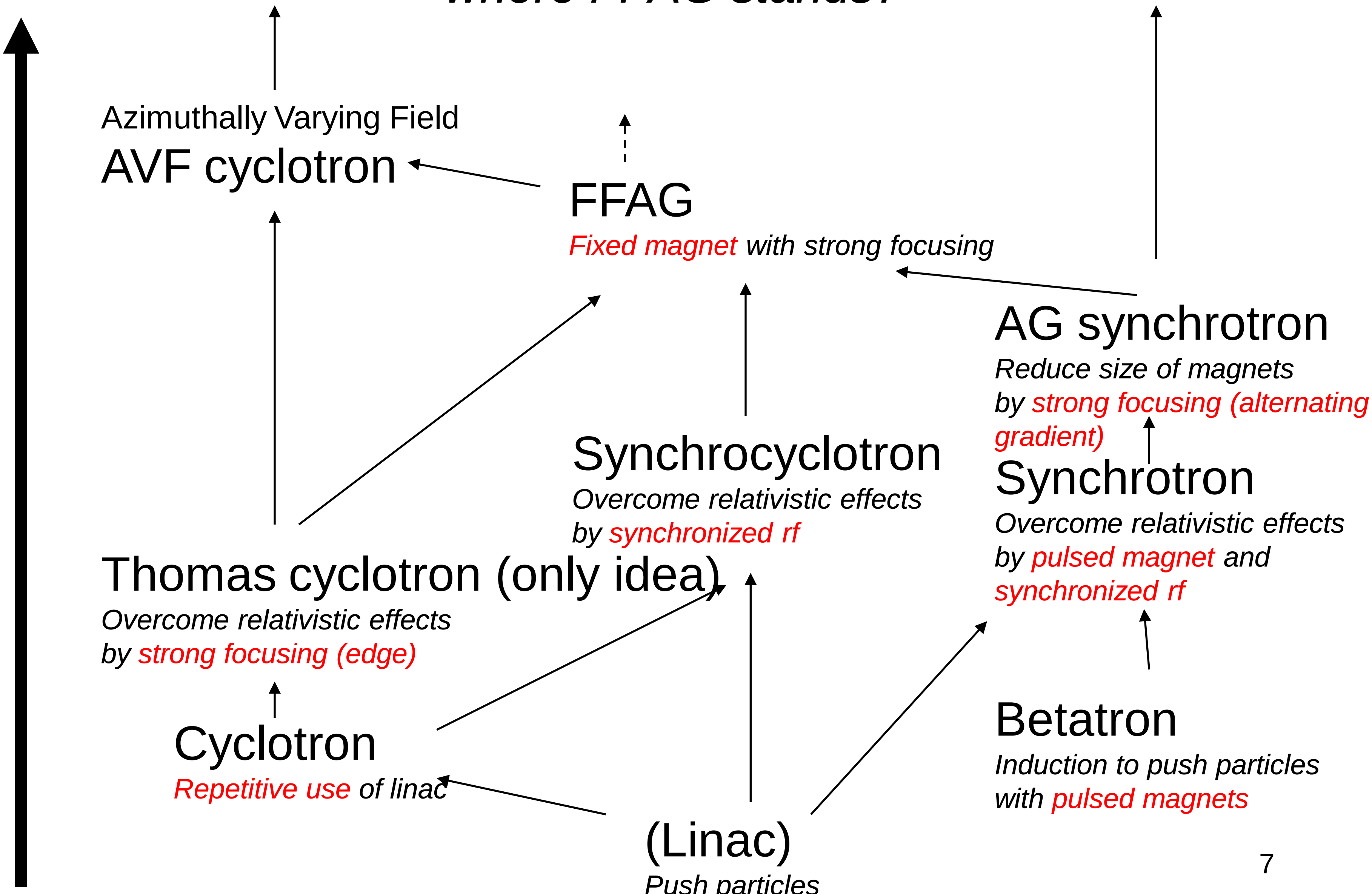
Future of high beam power machine

- Type of accelerators depends on the needs of users, physics limitations, available technologies, and cost.

- Physics limitations
  - Synchrotron radiation
  - Repulsive force among particles
- Technical limitations (example)
  - Maximum strength of magnets (6-10 T)
  - Ramping rate of pulsed magnets (50 Hz)
  - rf field gradient (50 kV/m in a few MHz)
  - Modulation rate of rf frequency (1-10 kHz)
  - Proton emittance from a source (1 π mm mrad)

Evolution of accelerators
where FFAG stands?

time



Accelerator for high beam power (1)

fixed or pulsed B field

Energy of each particle [GeV] x Number of particle per beam [ppp] x Repetition rate [Hz]

- Modulation of B field cannot be fast.
  - e.g. 50 Hz of ISIS is the fastest.
- Modulation of freq. can be 1 kHz or more.

	Variable B (Fixed path)	Fixed B (Variable path)
Fixed frequency	Betatron *e* synchrotron	**Cyclotron** **Thomas cyclotron** **Microtron**
Variable frequency	*p* synchrotron	**Synchrocyclotron** **FFAG**

Accelerator for high beam power (2)

weak or strong focusing

Energy of each particle [GeV]
x
Number of particle per beam [ppp]
x
Repetition rate [Hz]

Strong focusing can accommodate a higher number of particles with limited aperture.

	Variable B (Fixed path)	Fixed B (Variable path)
Weak focusing	WF synchrotron Betatron	Cyclotron Synchrocyclotron Microtron
Strong focusing	SF synchrotron	**FFAG** **Thomas cyclotron**

Accelerator for high beam power (3)
possibility of high energy

Energy of each particle [GeV]
x
Number of particle per beam [ppp]
x
Repetition rate [Hz]

Because of larger orbit shift, higher energy (>1GeV) cyclotron is not realistic.

	Variable B (Fixed path)	Fixed B (Variable path)
Weak focusing	WF synchrotron Betatron	Cyclotron Synchrocyclotron Microtron
Strong focusing	SF synchrotron	**FFAG** Thomas cyclotron

Accelerator for high beam power (4)
other considerations

- FFAG seem to be the best choice for a high beam power machine.

- However, there is a problem.

  – Almost no development of FFAG until recently.
  – No one wants to take a risk.

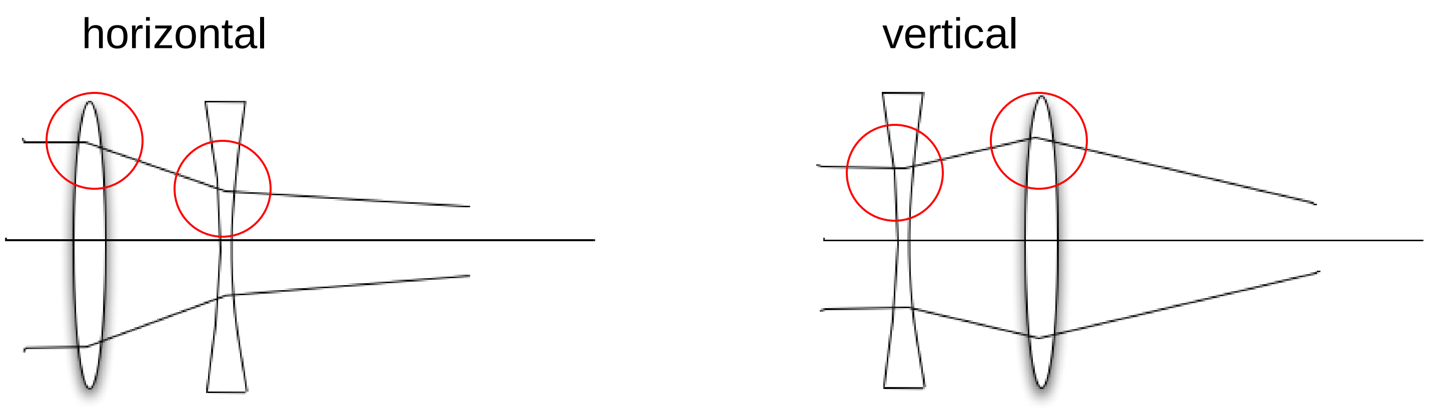
FFAG and early developments

FFAG basics (1)
alternating gradient

- Use “Fixed Field” magnet like cyclotron.
- Gradient is determined by focusing condition, not by isochronous condition.

$$\omega = \frac{eB}{m\gamma} \neq \text{constant}$$

- Alternating gradient is a focusing scheme with both sign of gradient magnets, focusing and defocusing elements.

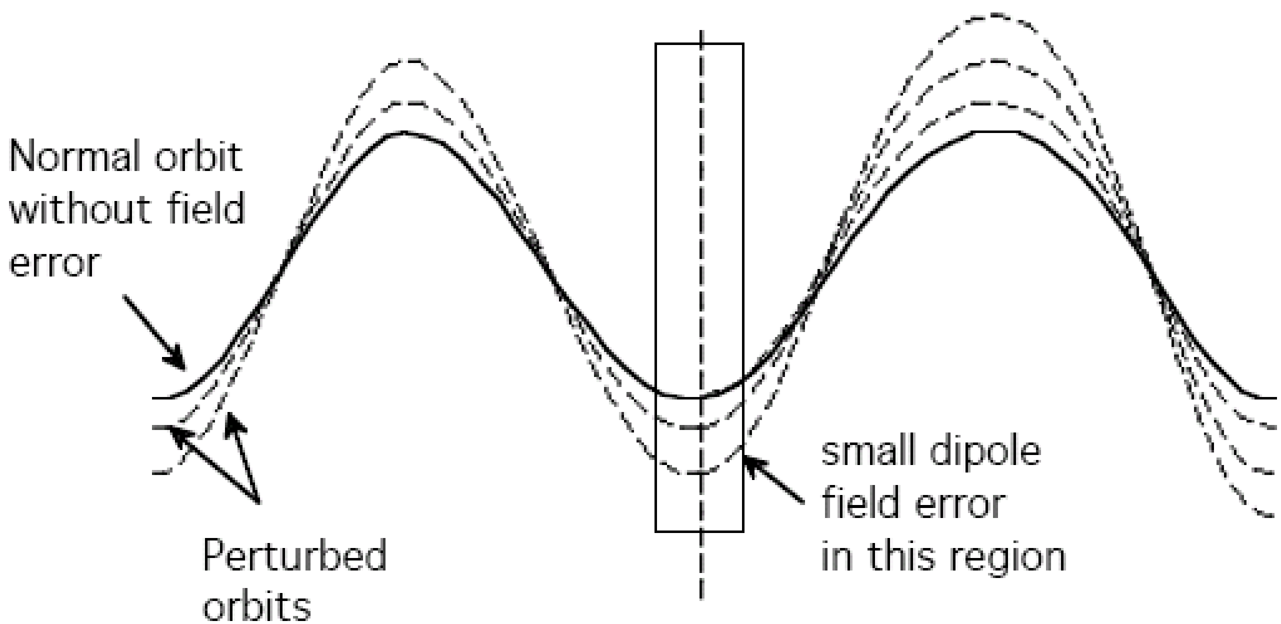


FFAG basics (2)

avoiding resonances

Transverse tune has to be constant during acceleration.

Resonance in accelerator



FFAG basics (3)

constant tune

To impose a constant tune.

Conventional strong focusing synchrotron	FFAG

“cardinal condition”

(next slide)

Same orbit shape

$$\theta = \frac{B(t)L}{p(t)/e}$$

Same focal length

$$\frac{1}{f} = \frac{dB(t)/dx \cdot L}{p(t)/e}$$

FFAG basics (4)

cardinal conditions of a FFAG

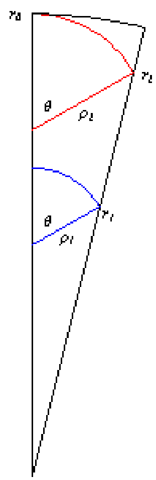
Geometrical similarity

$$\frac{\partial}{\partial p}\left(\frac{\rho}{\rho_0}\right)\Bigg|_{\vartheta=const.} = 0$$

ρ_0 : average curvature

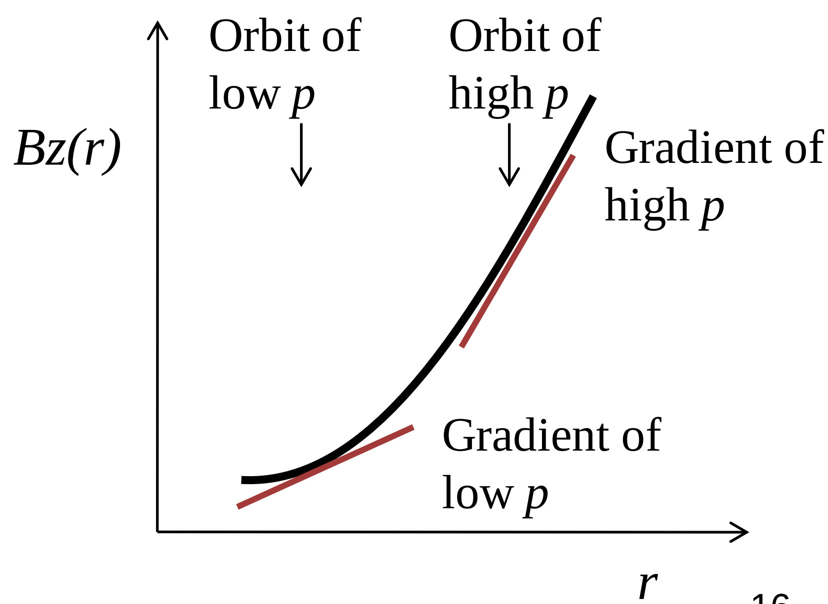
ρ : local curvature

: generalized azimuth



Constancy of k at corresponding orbit points

$$\frac{\partial k}{\partial p}\Bigg|_{\vartheta=const.} = 0 \qquad k = \frac{r}{B}\left(\frac{\partial B}{\partial r}\right)$$

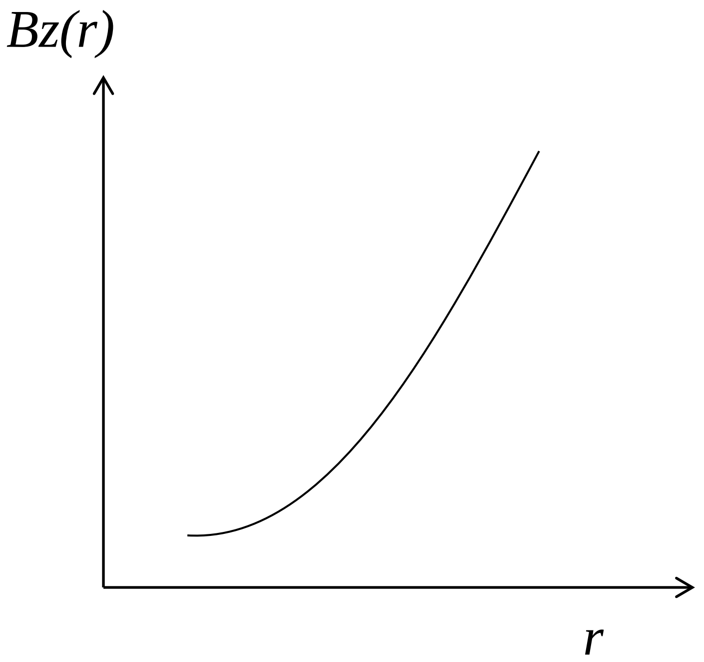


FFAG basics (5)

field profile

If the field profile has the shape of

$$B(r,\theta) = B_0 \left(\frac{r}{r_0} \right)^k F(\vartheta)$$



Cardinal condition is satisfied.

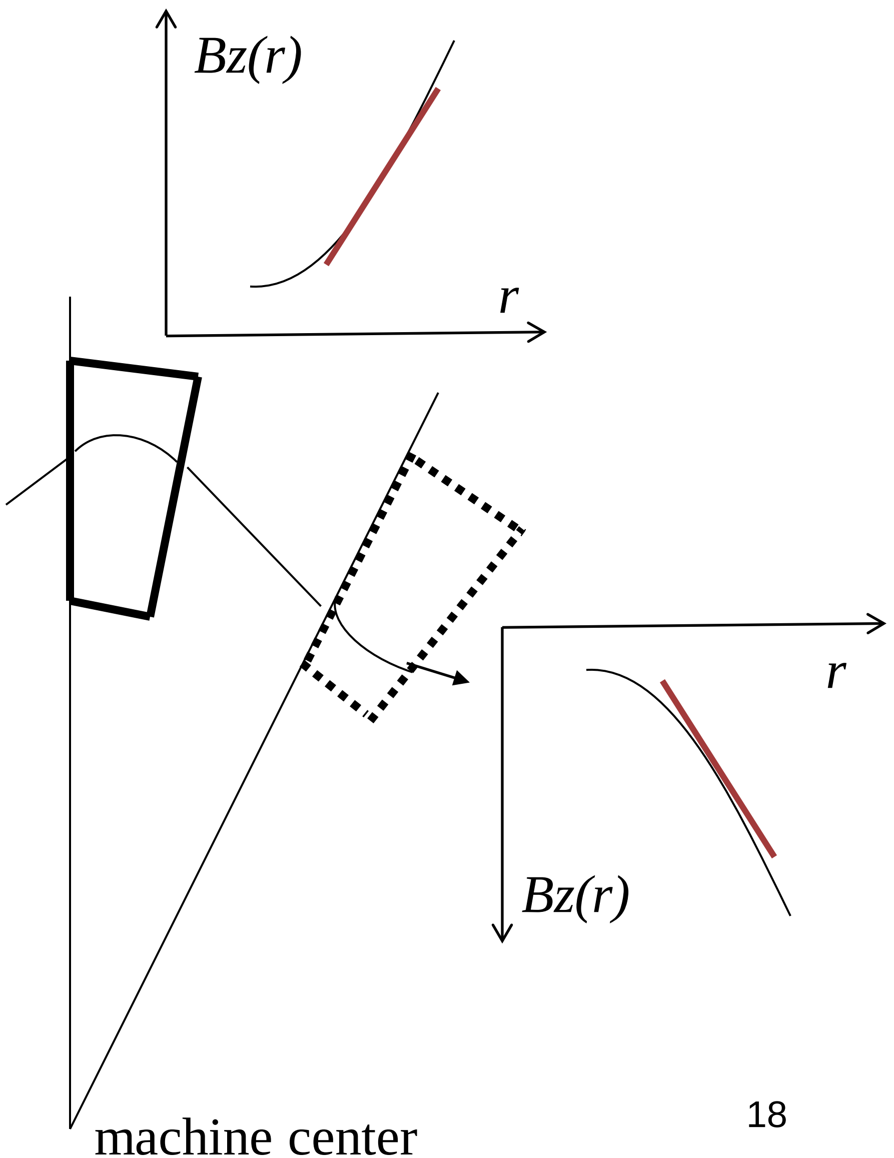
FFAG basics (6)

simple case

$$B(r,\theta) = B_0 \left(\frac{r}{r_0} \right)^k F(\vartheta)$$

$$F(\vartheta) = F(\theta)$$

Radial sector type



Alternating magnet has opposite sign of bending.

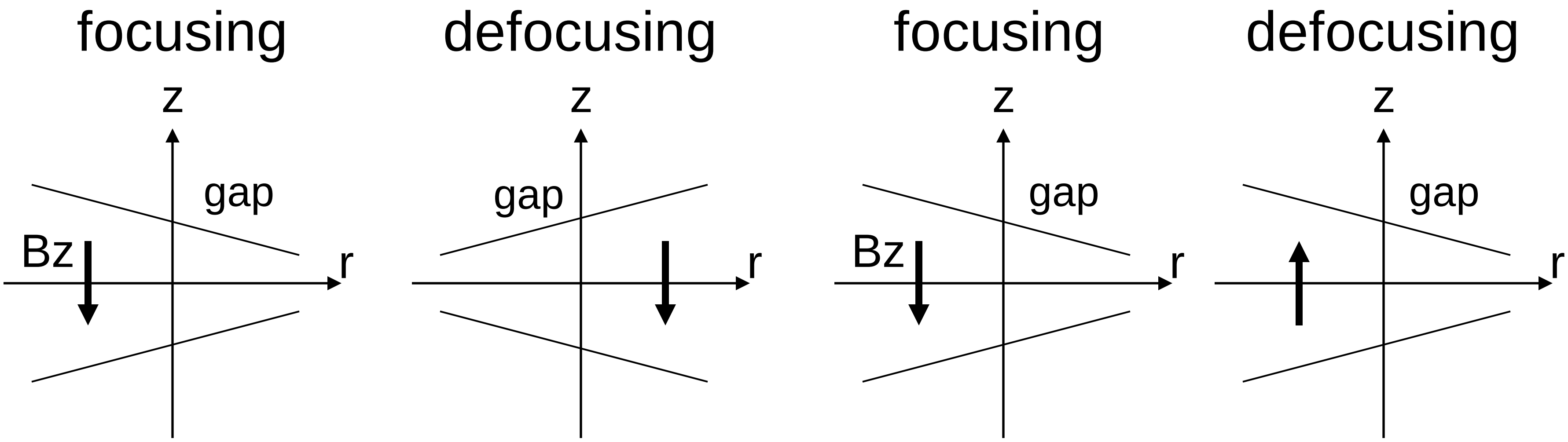
FFAG basics (7)
a way to realize AG

Conventional strong focusing synchrotron

Bending a beam in the same direction.

FFAG

Bending a beam in the opposite direction to change the sign.



FFAG basics (8)
another way

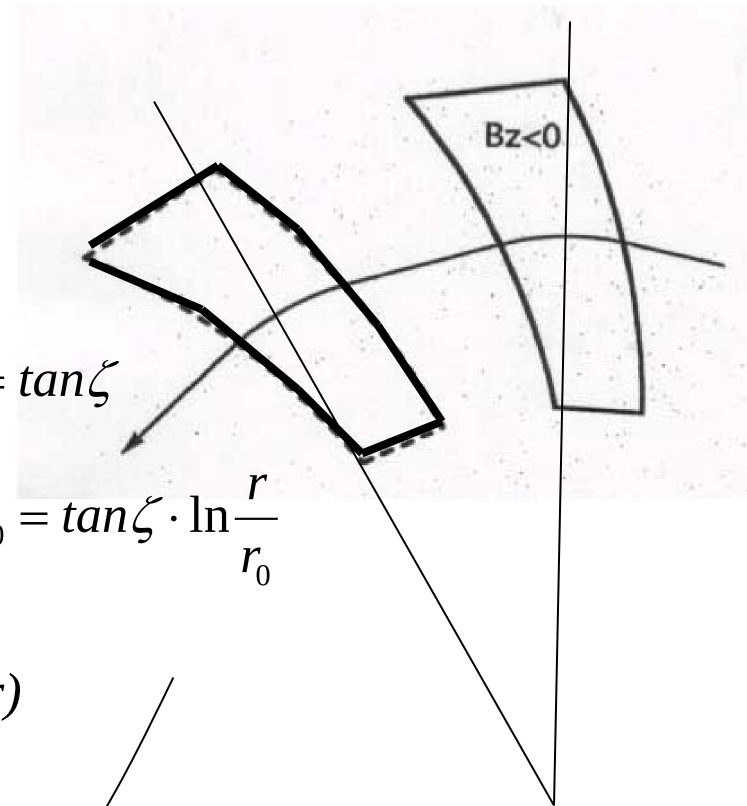
$$B(r,\theta)=B_0\left(\frac{r}{r_0}\right)^k F(\vartheta)$$

$$F(\vartheta)=F\left(\theta-\tan\zeta\cdot\ln\frac{r}{r_0}\right)$$

Spiral sector type

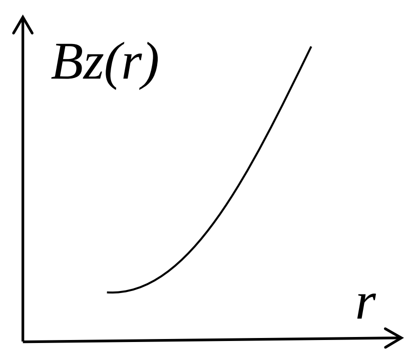
$$\frac{r\,d\theta}{dr}=\tan\zeta$$

$$\theta-\theta_0=\tan\zeta\cdot\ln\frac{r}{r_0}$$



Spiral angle gives strong edge focusing.

$$\therefore \Delta p_z=\frac{e}{v_x}\int_{-\infty}^{\infty}\left(-v_y B_x\right)dx=-eB_{z0}\tan\zeta\cdot z$$



FFAG basics (9)

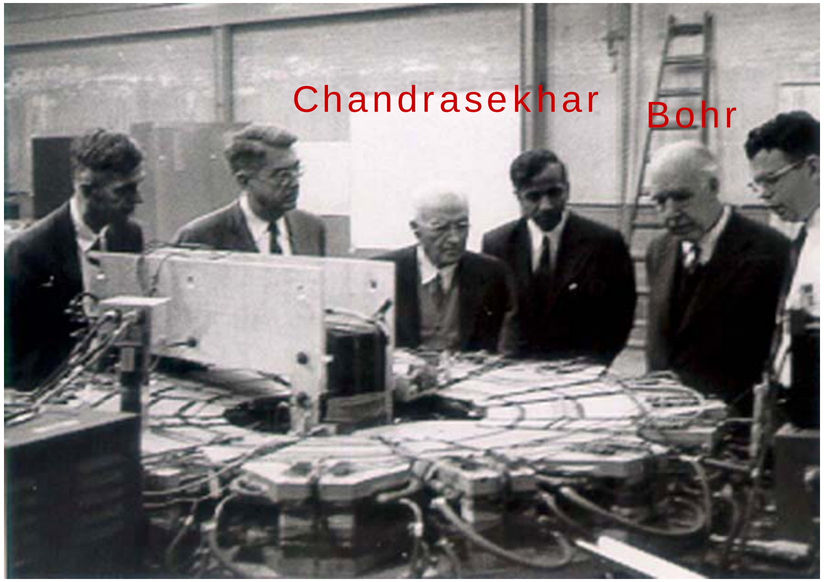
comparison

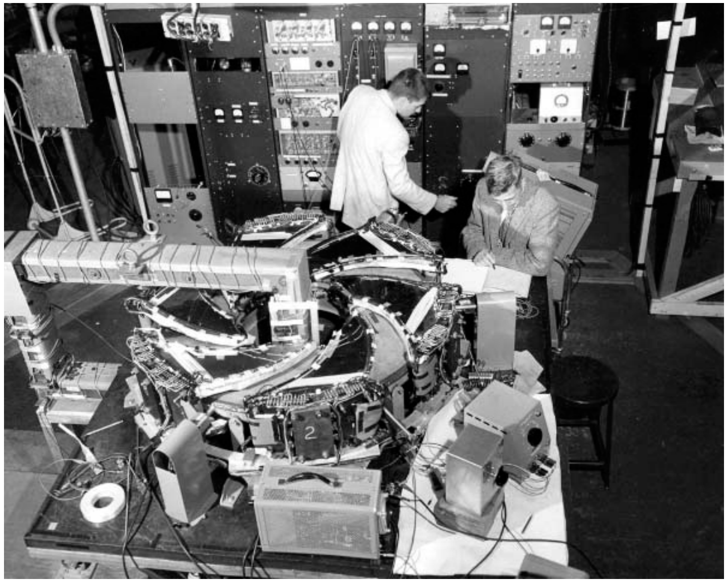
	Thomas (AVF) cyc	FFAG	Strong focus sync
B field	Fixed	Fixed	Pulsed
Strong focus	Body and edge	AG or Body and edge	Alternating Gradient
Tune	Small	Large	Large
Isochronism	Yes	No	No
RF freq.	Fixed	Varied	Varied
Duty	100% (CW)	Large	Small
Longitudinal focusing	No	Yes	Yes
Tune	Change	Constant	Constant

MURA (1)
birth

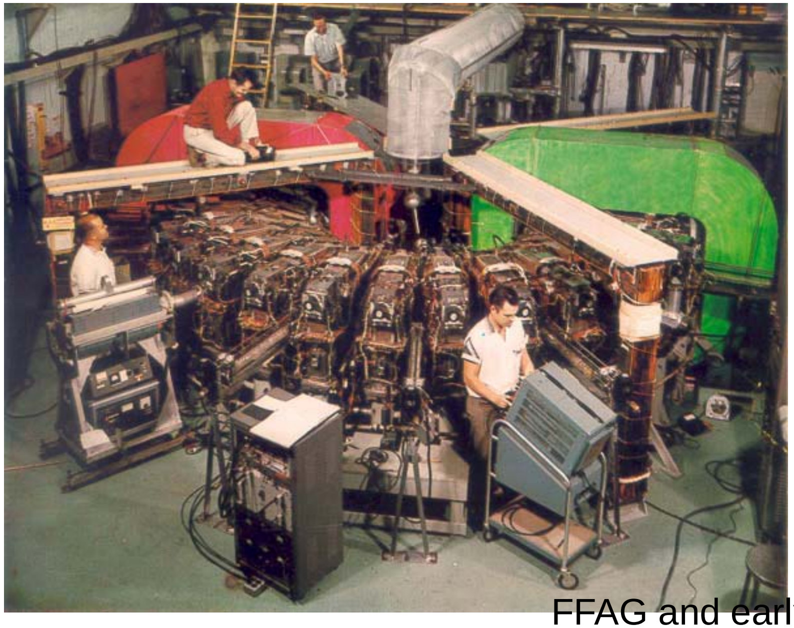
- Three inventors independently discovered the principle.
  - Symon and Kerst in US
  - Ohkawa in Japan
  - Kolomensky in USSR
- Birth of FFAG at Midwestern Universities Research Association (MURA) in 1950's.
- They discussed FFAG as the main accelerator in Midwestern region of US.

MURA (2)





400 keV radial sector



180 keV spiral sector

40 MeV two beam accelerator

All are electron FFAGs.

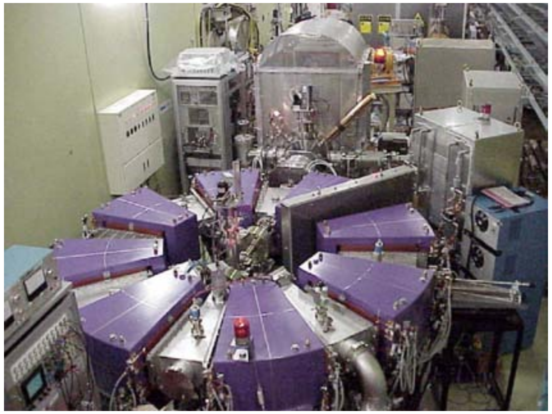
MURA (5)
closure

- Closure of MURA because
  - Synchrotron turned out to be the best choice for high energy frontier machine.
  - Magnet fabrication was complicated. No tool to model 3D magnetic fields.
  - RF cavities was not matured enough for high repetition and high gradient with wide aperture.

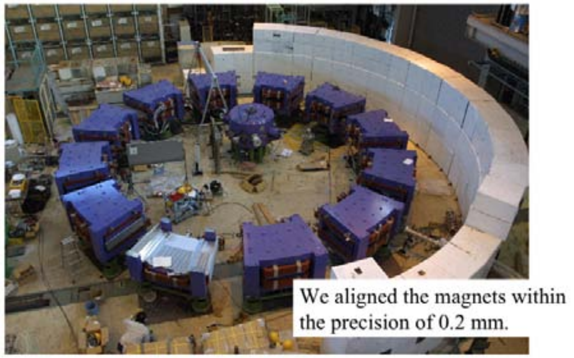
- Activities in 1980's
  - Design study at Argonne (US), Juelich (Germany), and KEK (Japan).

New era of FFAG

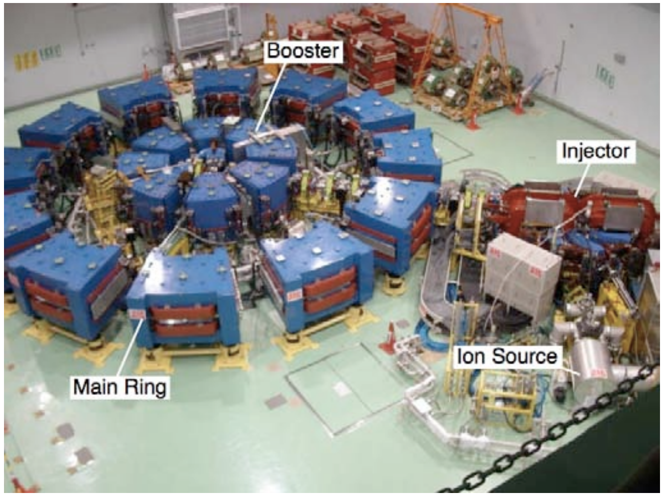
Rebirth of FFAG
with new technology



Proof of principle (proton acceleration with rf cavity) machine was constructed in 1999 and demonstrate rapid acceleration with 1 kHz.



Scale up version of PoP FFAG was constructed as a prototype of proton therapy machine.



3 stage FFAG for ADSR
- 2.5 MeV spiral (ion beta) FFAG with induction cores
- 25 MeV radial (booster) FFAG with RF and flat gap
- 150 MeV radial (main) FFAG with RF and tapered gap

Neutrino factory and FFAG (1)

how it works?

Produce a well collimated neutrino beam to a detector a few 1000 km away.

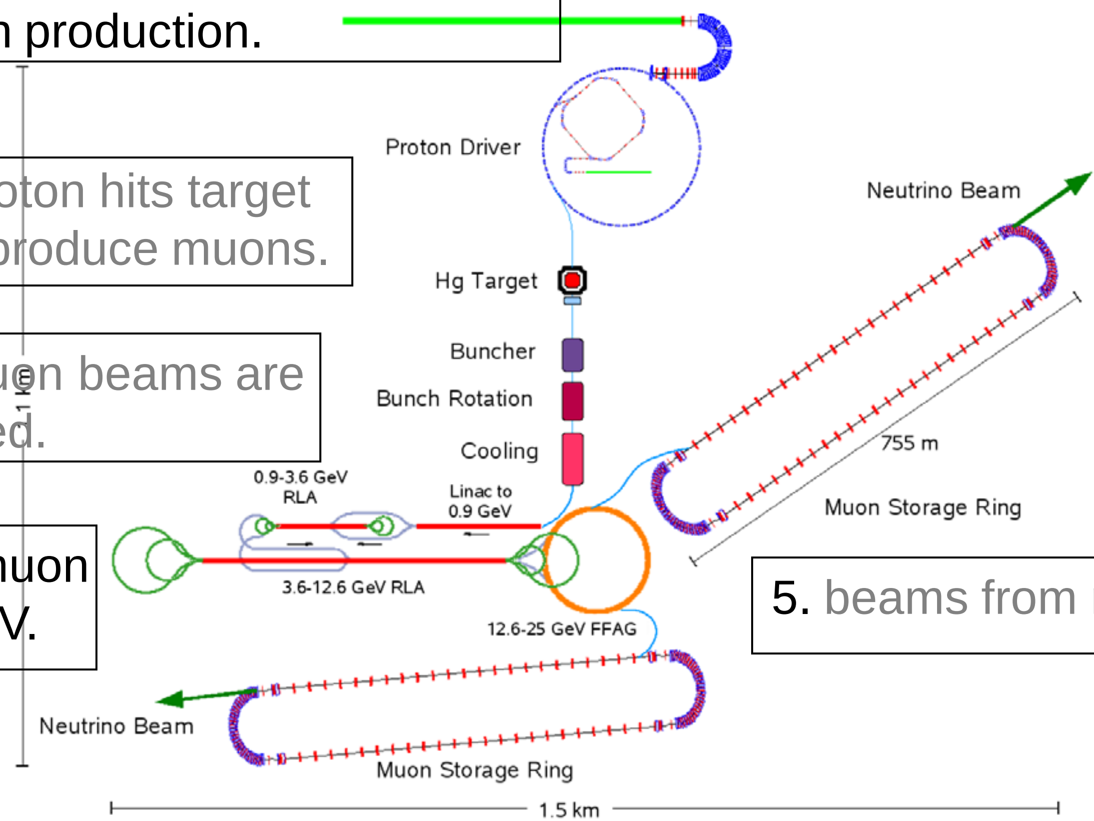
1. High power proton beam for muon production.

2. Proton hits target and produce muons.

3. Muon beams are cooled.

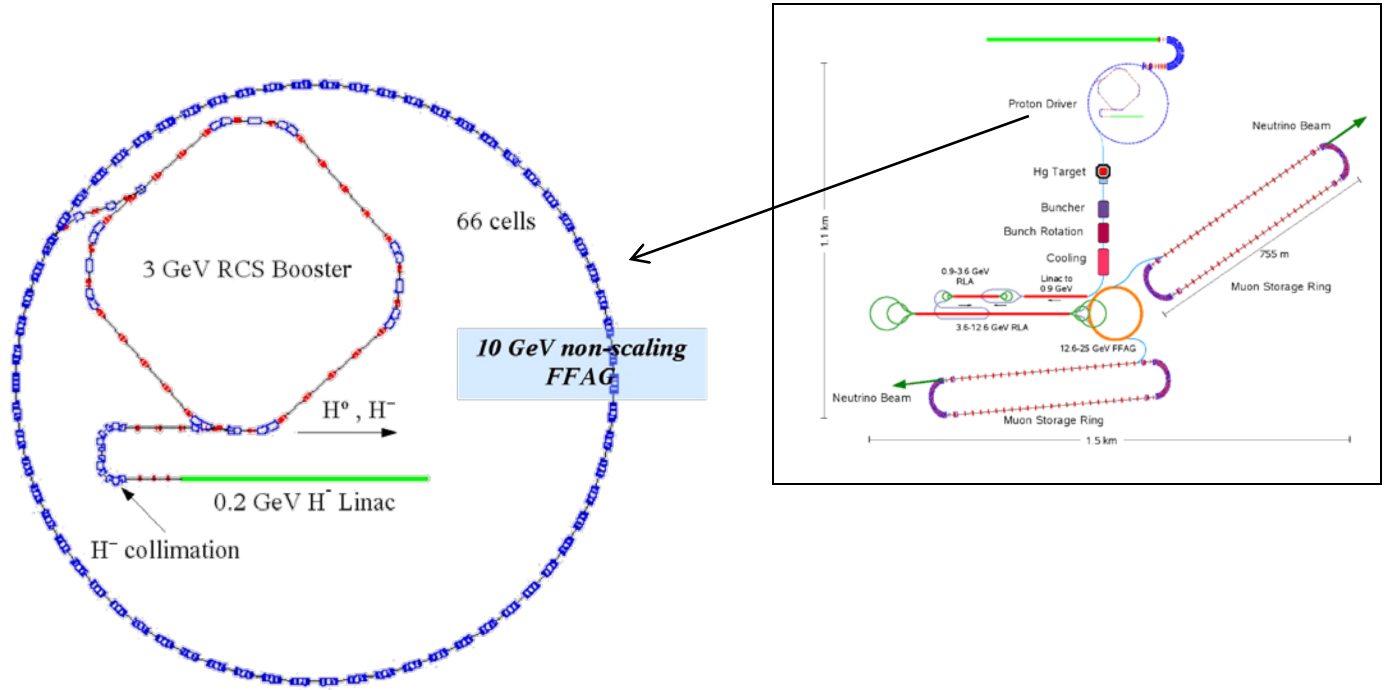
4. Acceleration of muon beams to 20-50 GeV.

5. beams from muon decay.



Neutrino factory and FFAG (2)

FFAG as high power proton source (proton driver)



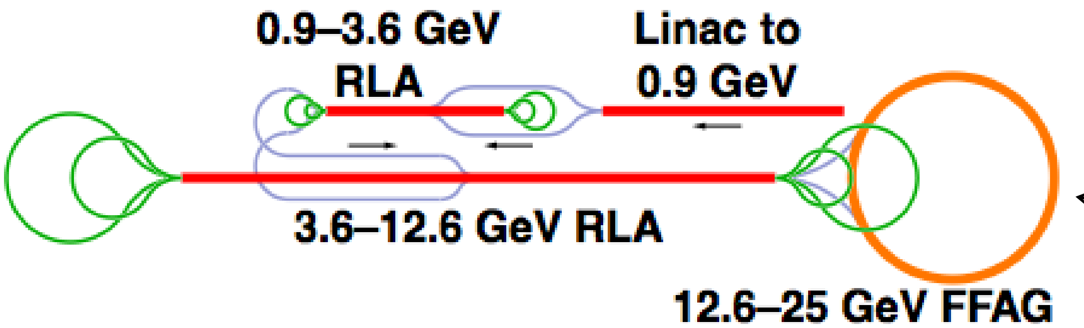
Beam power: 4 MW

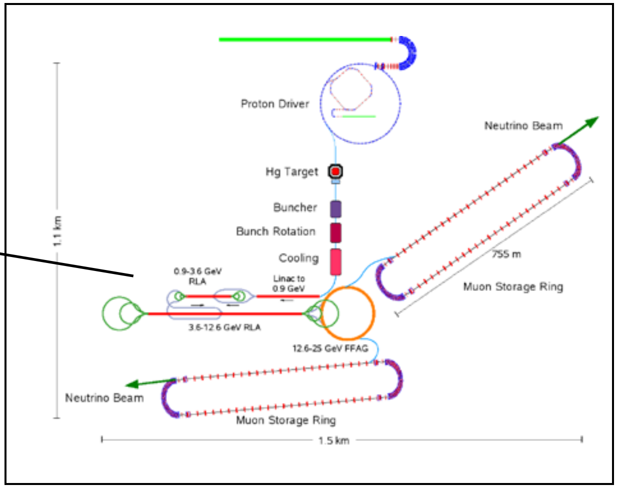
Energy: 10 GeV

c.f. Beam power of SNS is 1.4 MW with 1 GeV.

Neutrino factory and FFAG (3)

requirements for muon acceleration





- Energy up to 20 - 50 GeV
  - It is ultra relativistic.

- Large acceptance
  - Muon emittance is a few tens of thousand π mm mrad even after cooling. (e.g. 30,000 π mm mrad)

- Quick acceleration
  - Muon’s lifetime at rest is 2.2 μs.

Neutrino factory and FFAG (4)
muon and high power beam acceleration

Muon acceleration	High power beam acceleration
Energy up to 20 - 50 [GeV]	Energy of each particle [GeV]
It is ultra relativistic.	Higher energy is preferable, but size should be moderate
x	x
Large acceptance	Number of particle per beam [ppp]
Muon emittance is a few tens of thousand π mm mrad even after cooling. (e.g. 30,000 π mm mrad)	Enlarge aperture as much as possible
x	x
Quick acceleration	Repetition rate [Hz]
Muon's lifetime in rest frame is 2.2 μs.	Continuous operation is the best, but very high repetition is acceptable

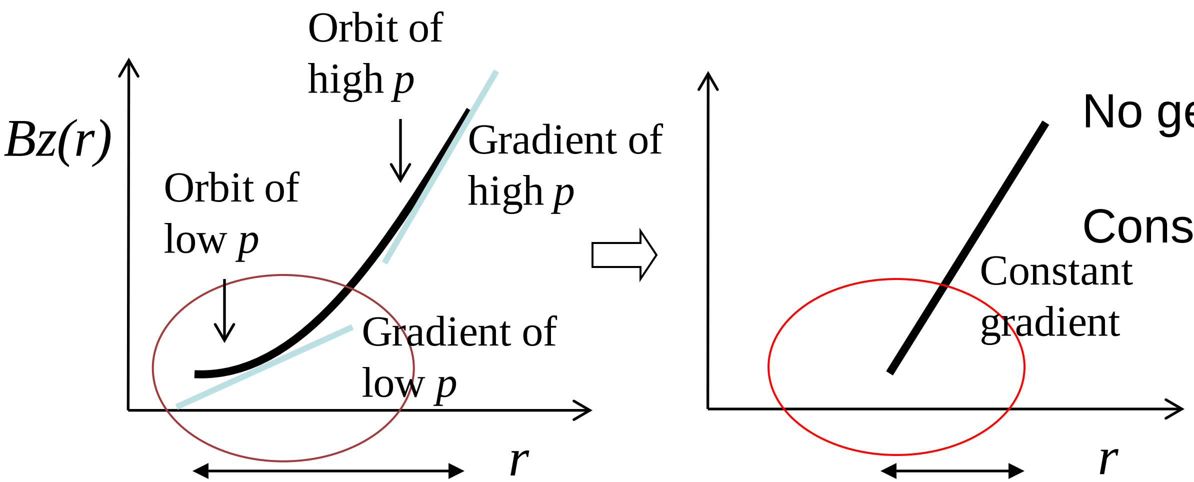
Requirements are similar !

New FFAG development

FFAG for muon acceleration (1)

making the magnet more compact

- If we could break cardinal conditions (scaling law), FFAG would be much simpler and magnet would be smaller.



No gentle slope at low momentum.
- Orbit excursion is shorter.

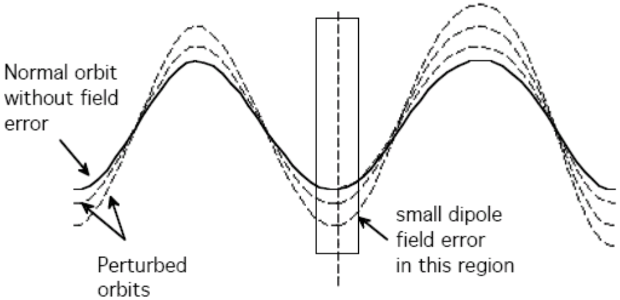
Constant gradient.
- Linear magnet.

- Why we wanted to have cardinal conditions?

To avoid resonance in accelerator.

FFAG for muon acceleration (2)
resonances

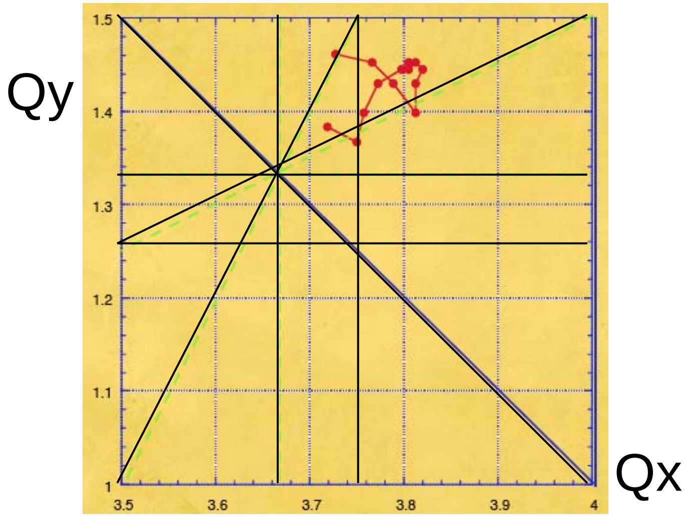
- There are many resonances in tune space. Normally, once a particle hits one of them, it will be lost.



- However, particles can survive after crossing resonances <u>if resonance is weak and crossing is fast.</u>

Furthermore, in reality, tune moves due to imperfections of magnet (red zigzag line).

Tune diagram of 150 MeV FFAG



FFAG for muon acceleration (3)

new idea: nonscaling FFAG

Muons circulate only a few (~10) turns in FFAG.

Is resonance really harmful to a beam?

Maybe not, because

- Resonance has a cumulative effect.
- We can avoid strong resonance such as integer one.

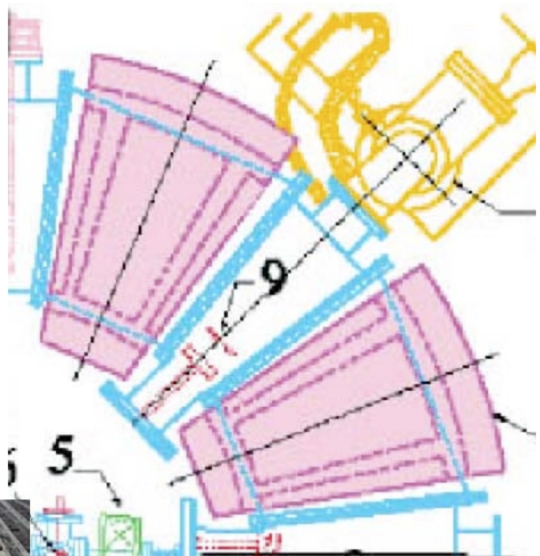
In FFAG for muon acceleration, "cardinal conditions" are not necessary.

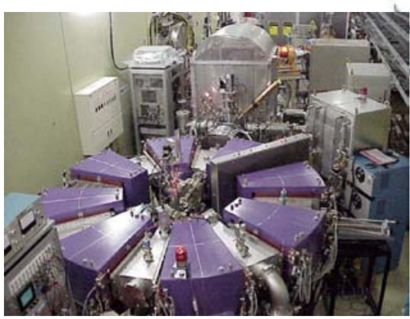
Instead, let us use simple fixed field Bend and Quad magnets to optimize optics and make a compact machine.

Birth of nonscaling FFAG.

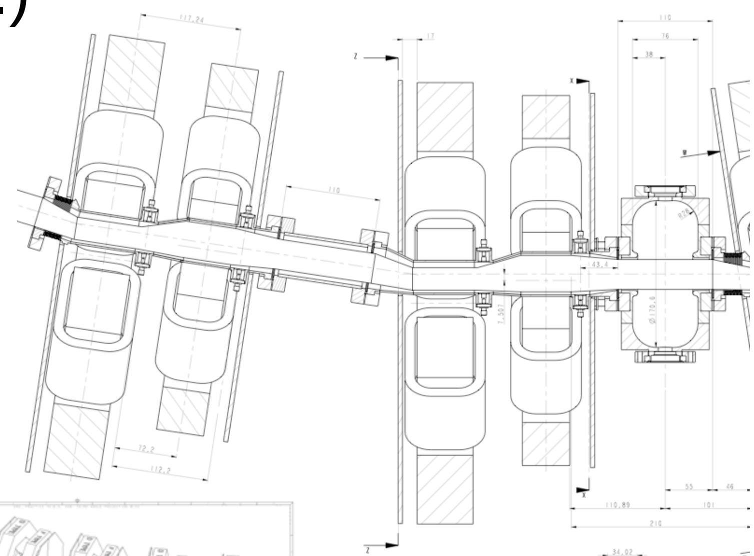
FFAG for muon acceleration (4)
lattice elements

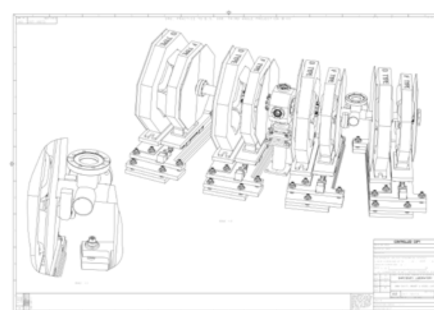
Original FFAG (scaling FFAG) layout which satisfies cardinal conditions.





Nonscaling FFAG uses simple shifted Quad (to produce Bend field.)





FFAG for muon acceleration (5)

conditions

- Lattice with very small momentum compaction α_p

$$\frac{dL}{L} = \alpha_p \frac{dp}{p} \qquad \alpha_p = \frac{1}{C}\oint \frac{D(s)}{\rho} ds$$

  - makes the orbit shift in radial direction small.
  - reduces the magnet size and makes a compact machine.

- Eliminate nonlinear fields.

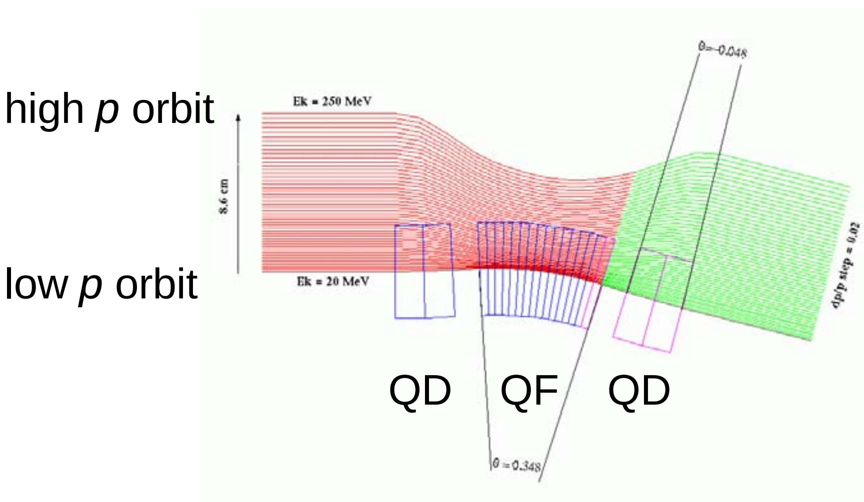
$$\frac{B(r,\theta)}{B_0} = \left(\frac{r}{r_0}\right)^k = \left[1 + \frac{k}{r_0}(r - r_0) + \frac{k(k-1)}{2r_0^2}(r - r_0)^2 + \frac{k(k-1)(k-2)}{3\cdot 2r_0^3}(r - r_0)^3 + \cdots\right]$$

dipole quadrupole sextupole octupole

- This is like a storage ring without chromaticity correction.

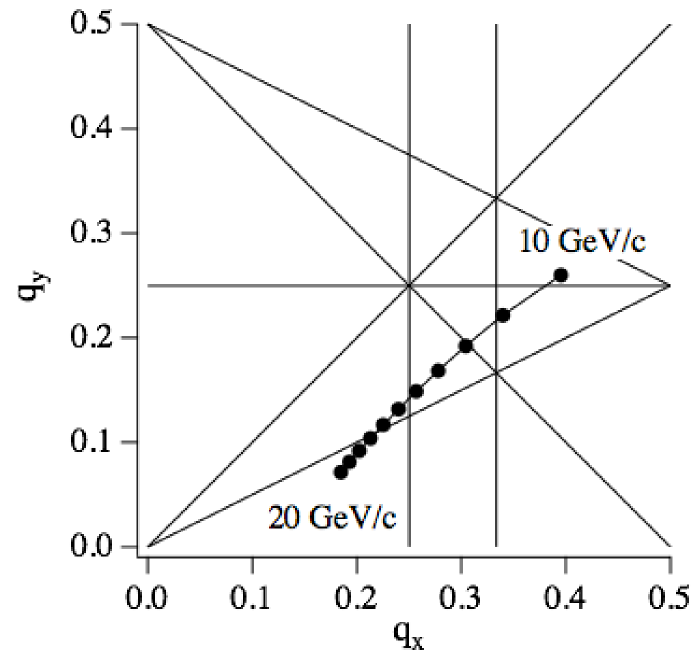
FFAG for muon acceleration (6)
orbit and optics

- Orbits for different momenta are no longer similar.



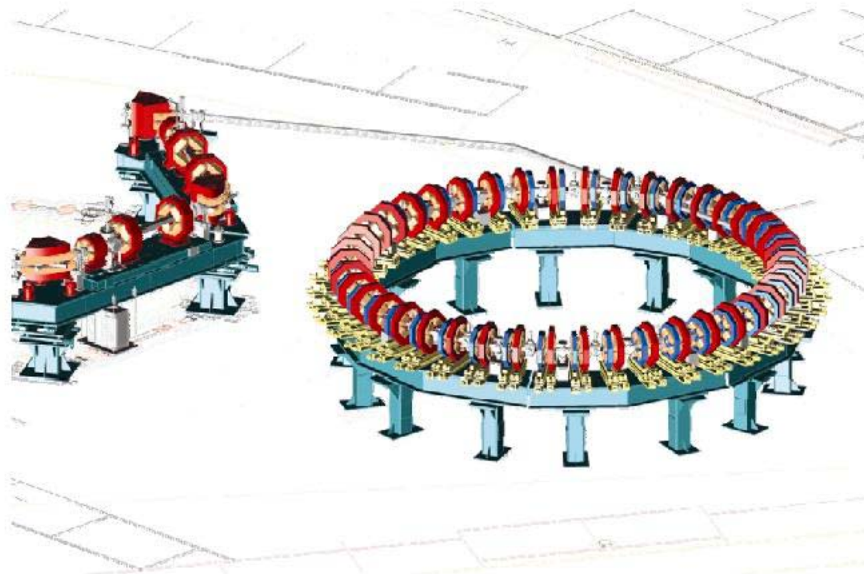
- Focusing force decreases as momentum increases.

$$\frac{1}{f} = \frac{B'}{p/e}$$



EMMA (1)
Electron Model of Muon Acceleration

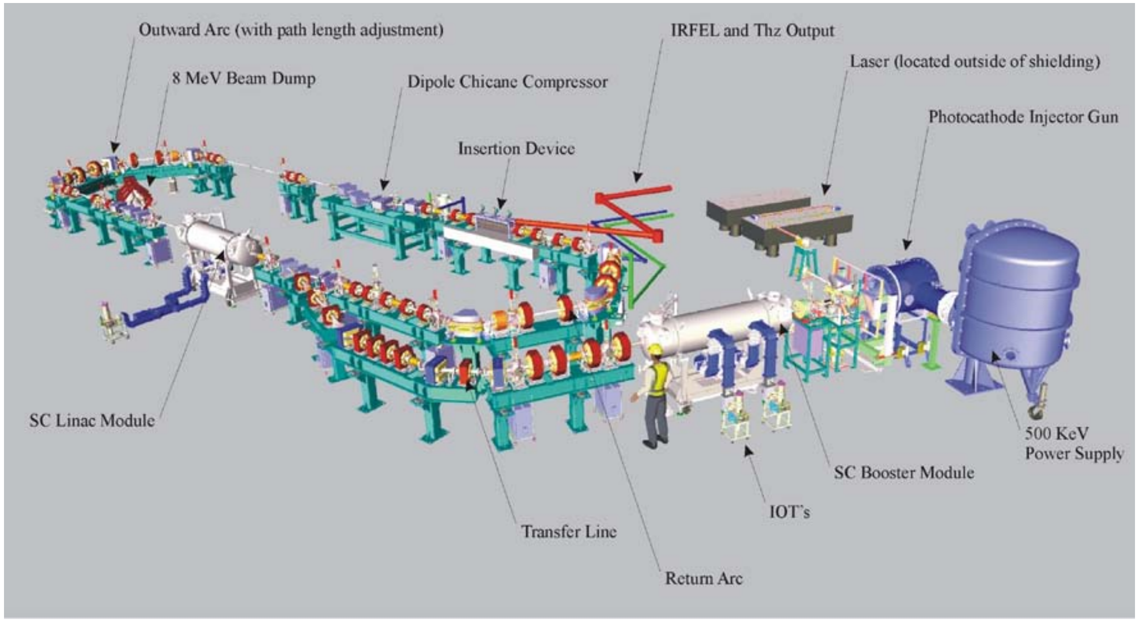
- No one ever made a nonscaling FFAG before.



- Demonstrate that nonscaling FFAG works as expected
  - Study resonance crossing in detail.
  - Prove quick acceleration and large acceptance.
  - Study outside bucket acceleration in detail.

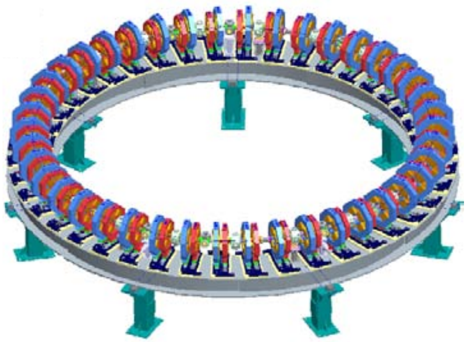
EMMA (3)
injector

- Energy Recovery Linac Prototype at Daresbury Laboratory provides,
  - Variable injection momentum from 10 to 20 MeV.
- 1.3 GHz rf infrastructure.
  - Harmonic number becomes 72.



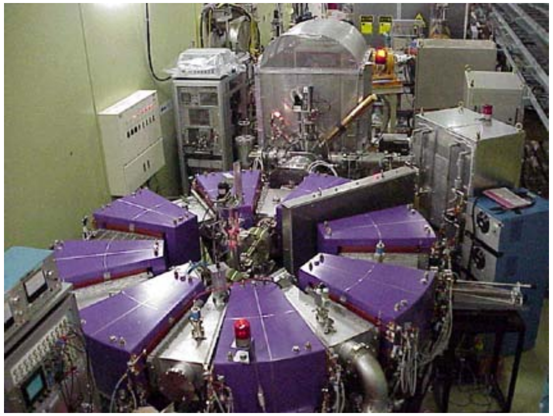
EMMA (4)
periodicity

- Periodicity of 42, that is a half of 10 to 20 GeV muon ring, has been chosen,
  - Enough to simulate a muon ring lattice.
  - Not a tabletop size machine. Circumference is 16.57 m
    - Scaling PoP FFAG's circumference is about 3 m.



Nonscaling counterpart.

42 doublet cells



PoP scaling FFAG (2000)

8 triplet cells

- Space is available next to ERLP.

EMMA (5)
commissioning

- Since this summer, we started commissioning of the world first nonscaling FFAG in Daresbury Laboratory UK.

QuickTime™ and a
decompressor
are needed to see this picture.

QuickTime™ and a
decompressor
are needed to see this picture.

Beam circulating for more than 1000 turns.

EMMA construction

Summary

- As a high power beam accelerator, FFAG has many advantages.
- Concept is old (proposed ~50 years ago), but intensive study has been carried out for last ten years.
- FFAG turns out to be attractive choice for muon acceleration as well.
- For muon acceleration, different type of FFAG (nonscaling FFAG which does not satisfy cardinal condition) has been proposed.
- Commissioning of nonscaling FFAG has just started this summer.

MURA (3)

remarkable outcome in accelerator physics

- Three Electron models were constructed.
- Accelerator physics issues

• Beam stacking.
• Hamiltonian theory of longitudinal motion.
• Useful colliding beams
• Storage ring
• Spiral sector geometry
• Lattice with zero dispersion and low beta section for colliding beams.
• Multiturn injection into a strong focusing lattice.
• First calculations of the effects of nonlinear forces in accelerators.
• First space charge calculations including effects of the beam surroundings.

•First experimental measurement of space charge effects.
• Theory of negative mass and other collective instabilities and correction systems.
• The use of digital computation in design of orbits, magnets and rf structures.
• Proof of the existence of chaos in digital computation
• Synchrotron radiation rings.

[1] F. T. Cole, "A Memoir of the MURA years." http://accelconf.web.cern.ch/AccelConf/c01/cyc2001/extra/Cole.pdf

MURA (4)
collider

- Concept of a beam collider
- In one ring, a particle can rotate in both directions.

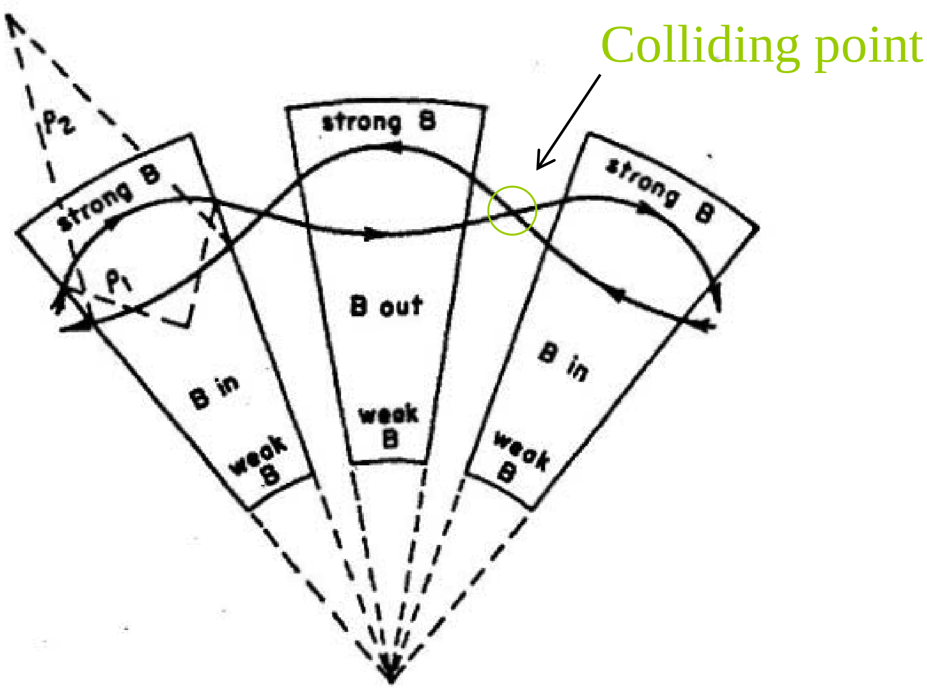


Fig. 13-21. Orbits in a two-beam accelerator.

……
Head-On Accelerator. Another team of dealers in magnetic fields. Dr. Lawrence W. Jones of the University of Michigan and Tihiro Ohkawa of Tokyo University, told their colleagues about a new and cataclysmic kind of atom smasher. The most powerful one in operation at present is the Bevatron at Berkeley (6 billion electron volts), and a 25-Bev monster is under construction at Brookhaven National Laboratory on Long Island. These are rather puny little gadgets, think Jones and Ohkawa. The way to get real power is to force head-on collisions between high-speed particles. ……

TIME, February 11, 1957

http://www.time.com/time/magazine/article/0,9171,809067-1,00.html

Remark: Particle and beam

- A beam is a group of particles confined in the small volume in 6-D phase space.
- Our machine is a “particle beam” accelerator, not a “particle” accelerator.
  - “particle” acceleration mechanism in the universe.
  - Laser driven acceleration: from a particle accelerator to a beam accelerator.

Remark: Stable and unstable particles

- Accelerators used to accelerate stable (or long life) particles such as proton and electron.
- New challenge is to accelerate unstable (or short life) particles.

- For example, acceleration of muons.
  – Muon collider: high energy frontier
  – Neutrino factory: high power (intensity) source

Why high energy frontier machine has been always a synchrotron?

- Repetitive use of rf cavities to increase energy.
  - Must be a circular type.
  - Linac is not efficient.
- Relatively small magnets.
  - Number of magnets tends to be large.
  - Cyclotron is out of question.
- High energy proton machine should be still a synchrotron (with superconducting magnet.)

- However
  - Synchrotron radiation from electron becomes significant and the next electron machine should be linear.

Why synchrotron light source machine has been always a synchrotron?

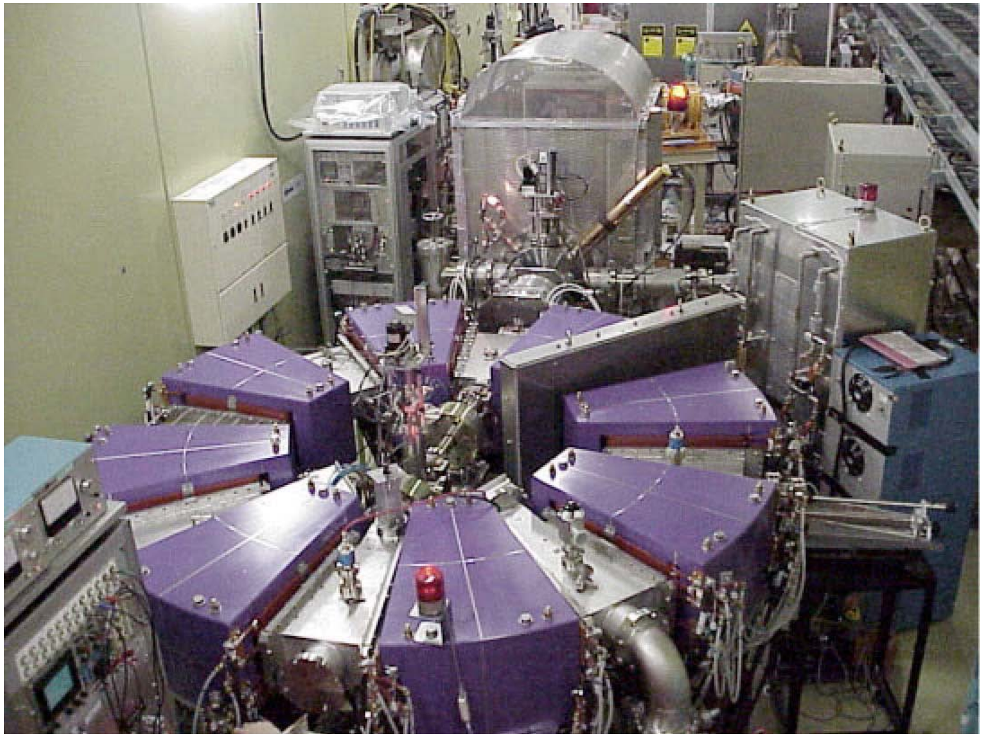
- Repetitive use of rf cavities to maintain energy level.
  - Must be a circular type.
  - Need bending magnet to produce light
- Electron is ultra relativistic.
  - Cyclotron is out of question.

- However
  - Emittance blows up in a synchrotron even if emittance at the source is very small.
  - Deterioration of emittance suggests use of linac.
  - Instead of using rf cavities repetitively, use “rf energy” many time, which is Energy Recovery Linac (ERL).

Development at KEK (1)
rebirth

Proof of principle (proton acceleration with rf cavity) machine was constructed in 2000.

Demonstrate acceleration from 50 keV to 500 keV in 1 ms (or 1 kHz operation).



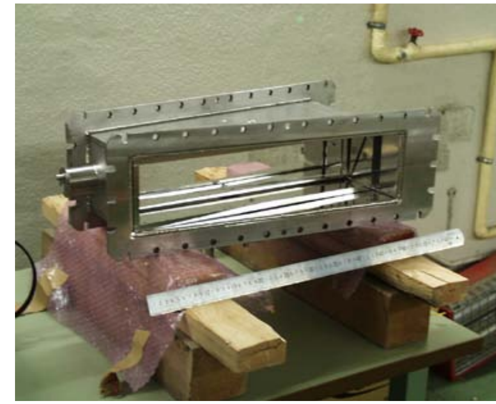
Development at KEK (2)

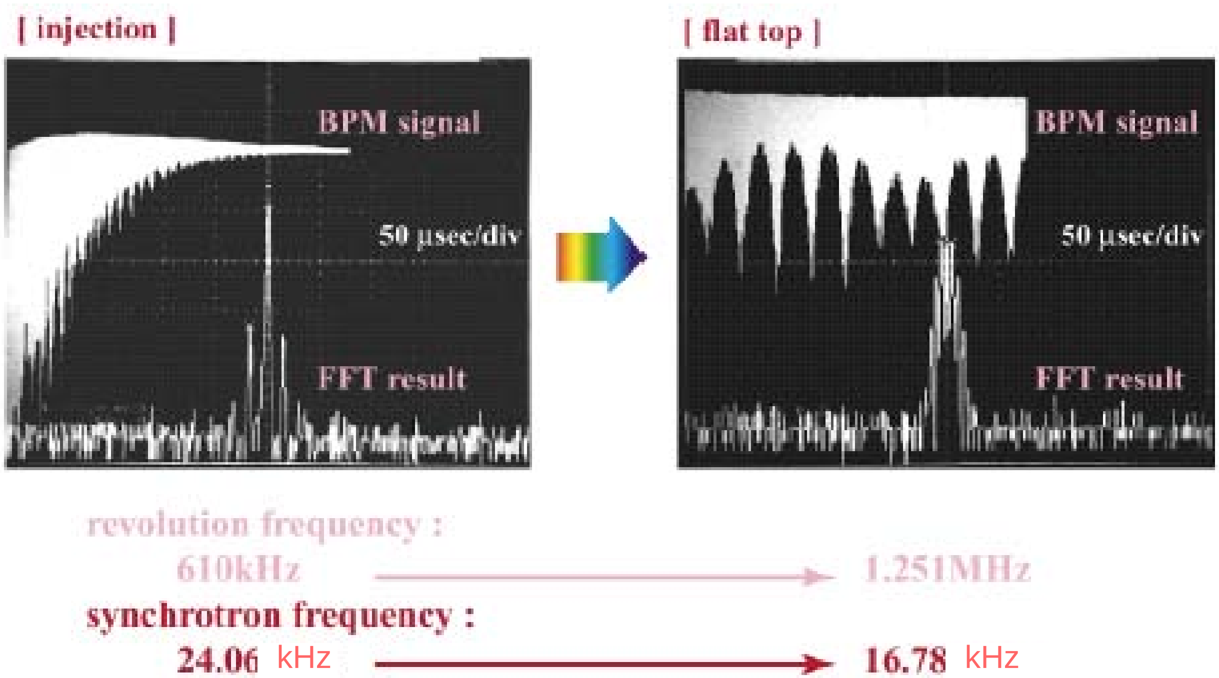
Beam commissioning

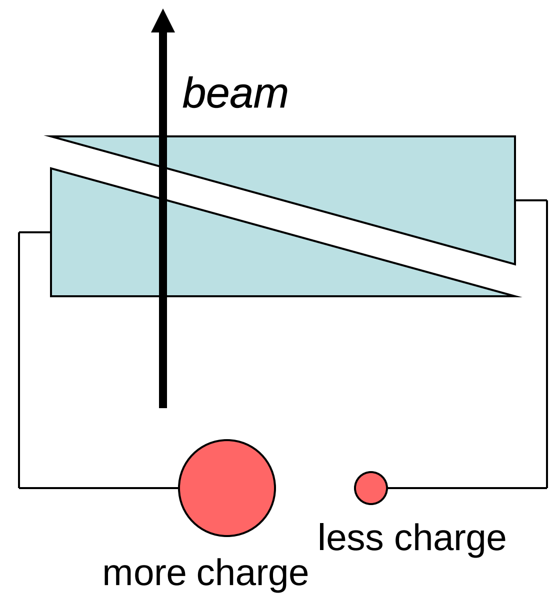
Beam position monitor tells

- total charge
- position
- time structure

Frequency analysis gives tune.

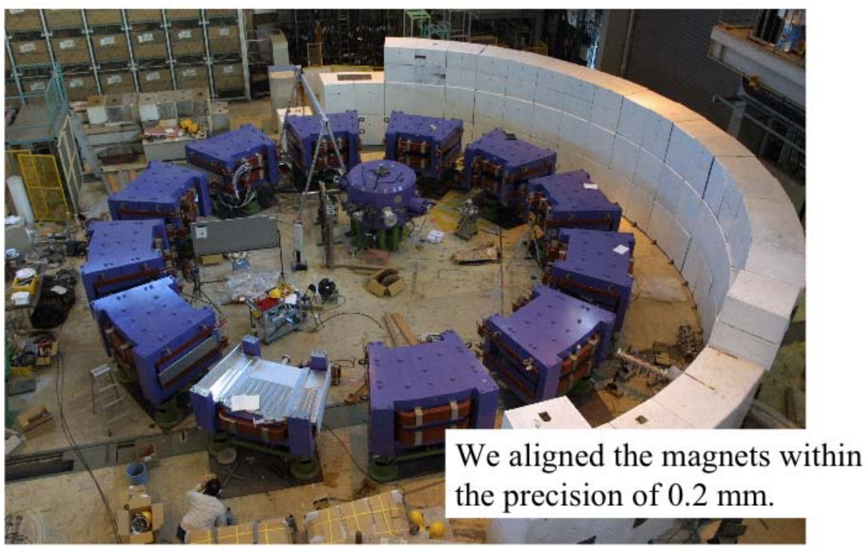




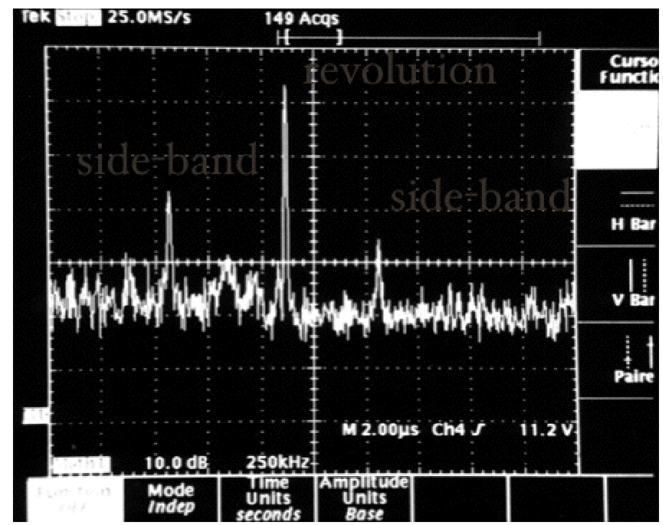


Development at KEK (3)
150 MeV machine for proton therapy

- High repetition rate makes it possible to spot scanning technique for proton therapy.
- As a prototype of proton therapy machine, 150 MeV proton FFAG was constructed.



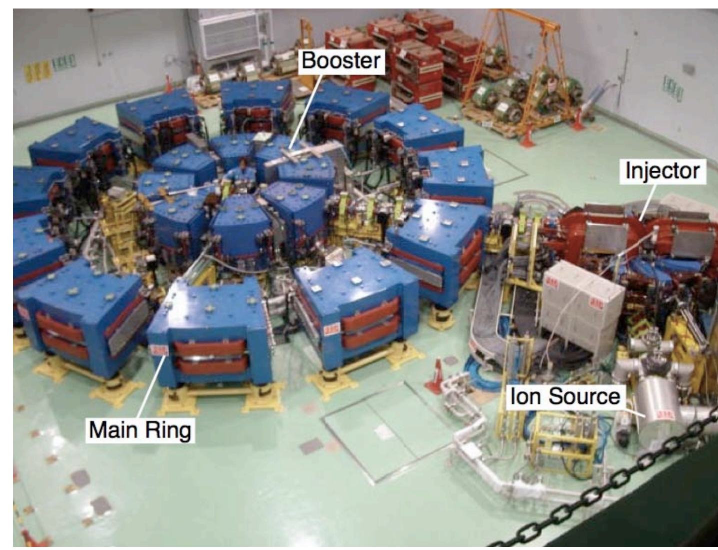
Frequency spectrum



Development at KEK (4)
machine for ADSR

- Accelerator driven subcritical reactor (ADSR) needs high power and reliable accelerator.
- This is a field where FFAG accelerator has advantage.

- 3 stage FFAG
  - 2.5 MeV spiral (ion beta) FFAG with induction cores
  - 25 MeV radial (booster) FFAG with RF and flat gap
  - 150 MeV radial (main) FFAG with RF and tapered gap



EMMA (2)

difficulties of demonstration of nonscaling FFAG

- Nonscaling FFAG is for a high energy muon accelerator.
  - The beam is supposed to be relativistic already at injection.
  - Electron beam of 10 MeV (γ=20) is needed.

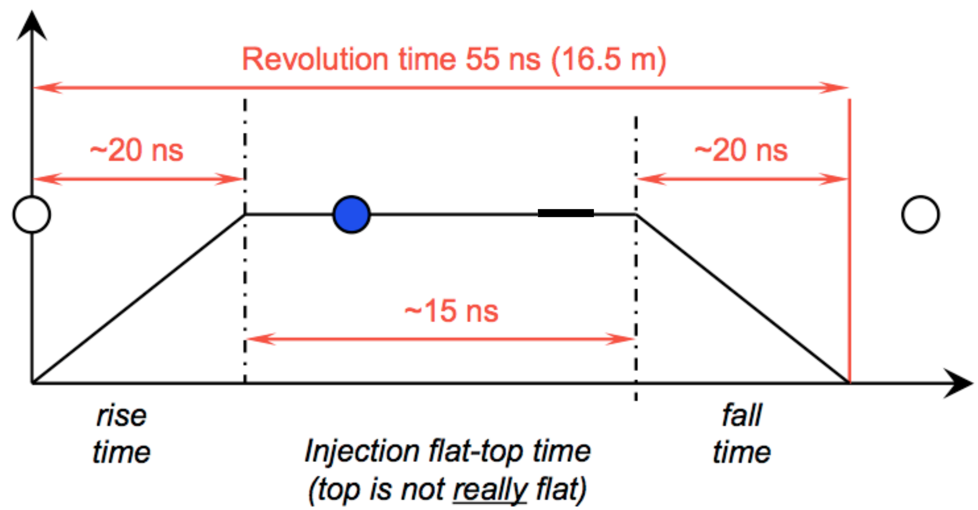
- Beam dynamics rely on a high periodicity lattice.
  - Proposed muon ring has 84 cells (periodicity).
  - Electron model should have the similar number of periodicity.

- Beams stay in the ring for only 10 turns.
  - Diagnostics for single path measurements.
  - Inject a beam with full momentum range to scan.

It turns out that Daresbury is the ideal place.

EMMA (5)
single bunch

- Single bunch parameters of ERLP.
  - 80 pC
  - 3 to 10 π mm mrad (normalized)
  - 2 ps rms pulse length
  - 10 to 20 Hz repetition



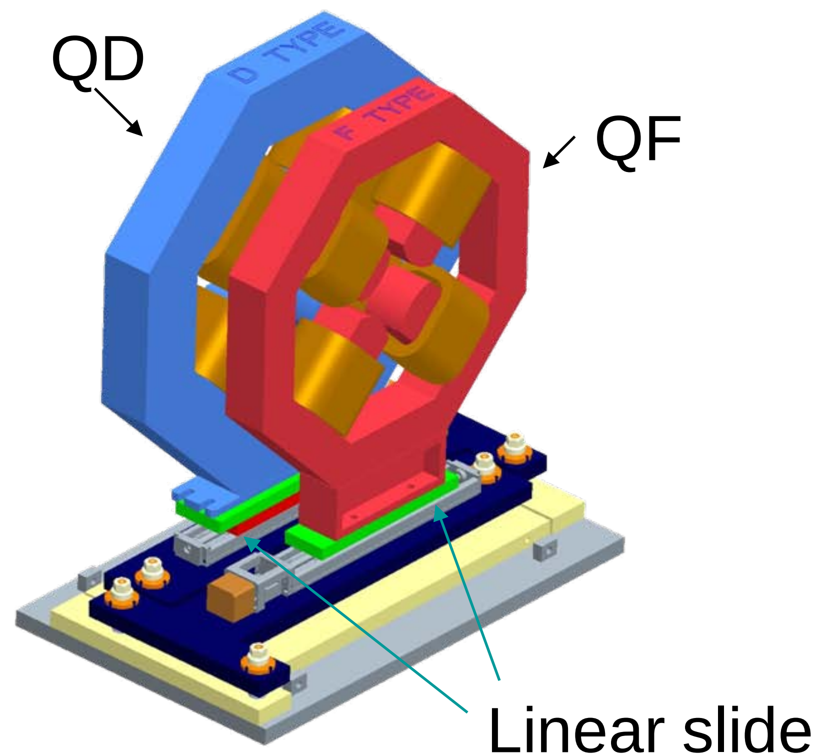
- Enough for single path diagnostics and phase space scanning.
  - In fact, it is a little too bright !

EMMA (6)
lattice

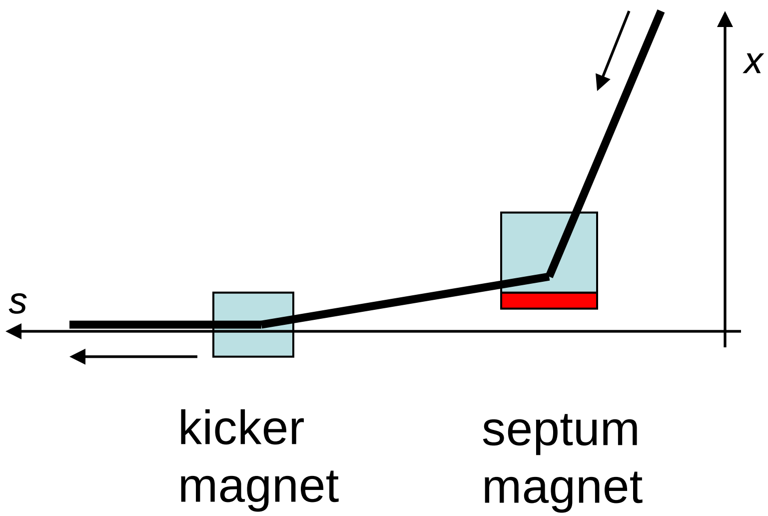
- Double focusing lattice (QF and QD).
- Bend fields are created by shifting Quad.

- 4 knobs
  - QF and QD strength
  - QF and QD position (hor.)

- 4 parameters to fit
  - Qx and Qy
  - ToF shape and offset

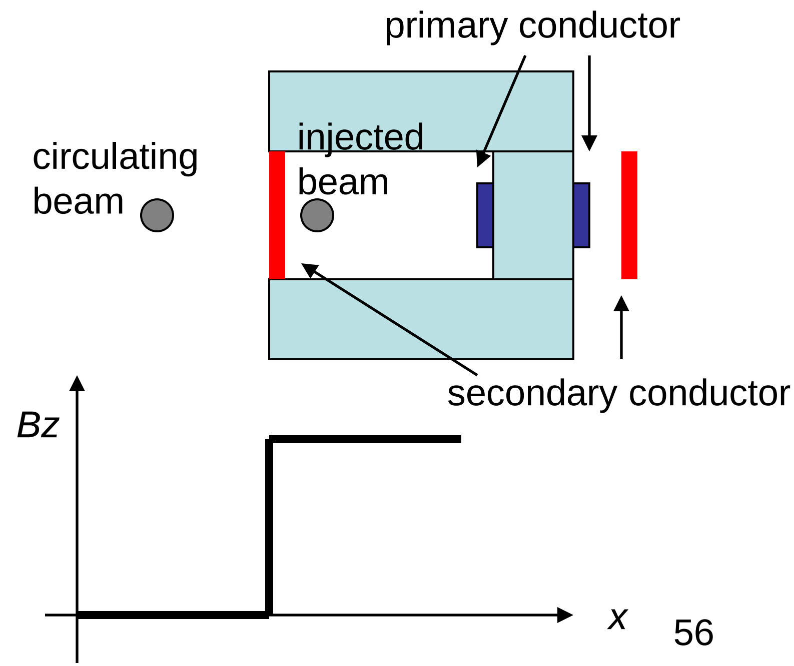


EMMA (7)
injection (and extraction)



- Septum magnet kicks a beam with large angle (~1T).
- Septum magnet can be DC operation because circulating beam goes outside.

- Kicker magnet is placed over injected and circulating beam orbit.
- Kicker magnet cannot be strong (~0.05T) because it is pulsed.
- Rise time of kicker is order of bunch spacing (~100ns).



Different kinds of accelerator (8)
fixed field alternating gradient (FFAG)

Alternating gradient synchrotron can be operated with fixed field magnet !

–Synchrocyclotron with alternating gradient focusing or

–AVF cyclotron with frequency modulation.

- What is advantage?
  - DC magnet is easier to operate.
  - Repetition rate can be high so the current become higher.
  - Beam current of synchrotron in early days was not enough.
- Price we have to pay
  - Larger magnets to accommodate orbit shift in horizontal direction.
  - Orbit shift can be minimized by using a high gradient magnet.

FFAG basics (2)

remark: alternating gradient and strong focusing

- Strong focusing <-> weak focusing ($-1<n<0$)

- Thomas (AVF) cyclotron uses a combination of focusing (from the body) and defocusing (from the edge) elements to realize net focusing.
- Strong focusing synchrotron by Courant, Livingston and Snyder uses large gradient magnets with alternating sign.

- Alternating gradient is one way to realize strong focusing.
  - However, they are used in the same meaning.
  - There is even a FFAG without alternating gradient magnets !

FFAG basics (8)

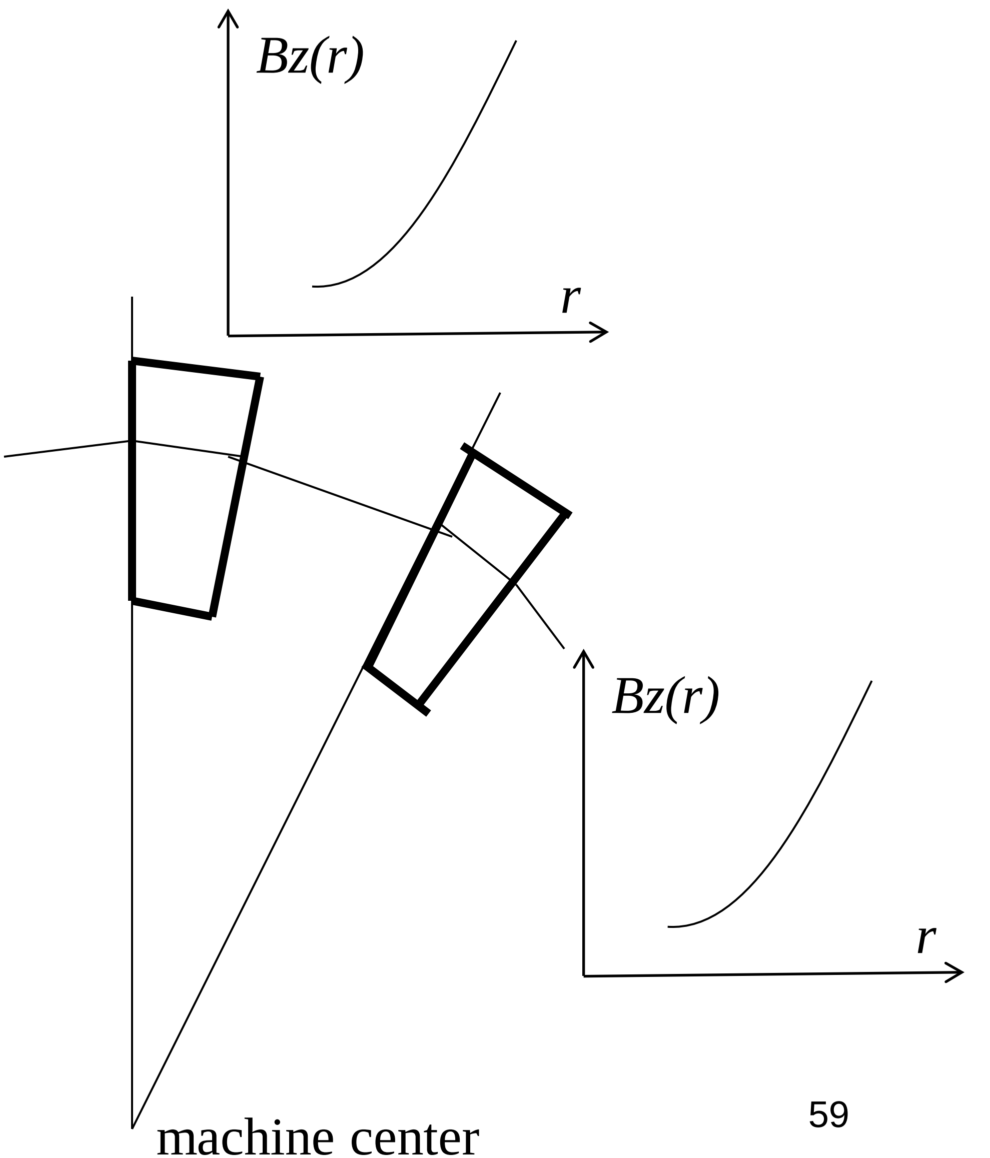
another way (1)

$$B(r,\theta) = B_0 \left(\frac{r}{r_0}\right)^k F(\vartheta)$$

$F(\vartheta) = F(\theta)$

Radial sector type with edge focusing.

In practice, edge focusing is not strong enough.

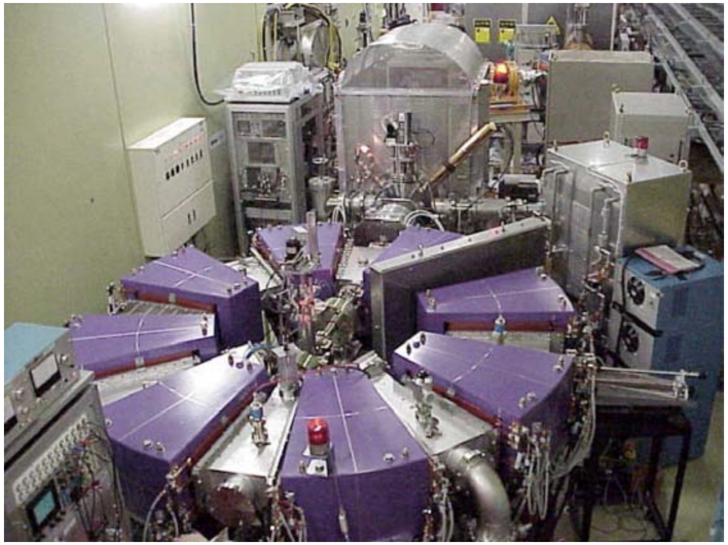


Commissioning (4)
remark

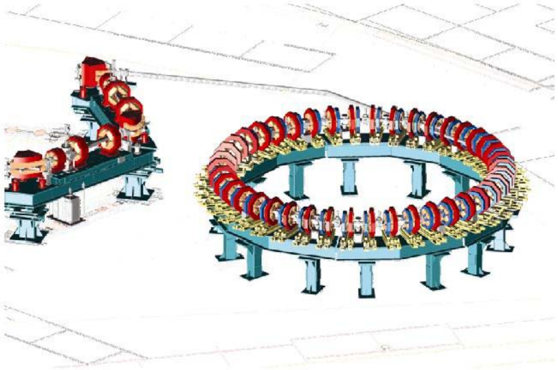
- It is nice to have such a small and “homemade” accelerator.
  - Easy to operate but still have all aspects of accelerator.

- EMMA at Daresbury will be a gadget every one can play with !

PoP FFAG at KEK

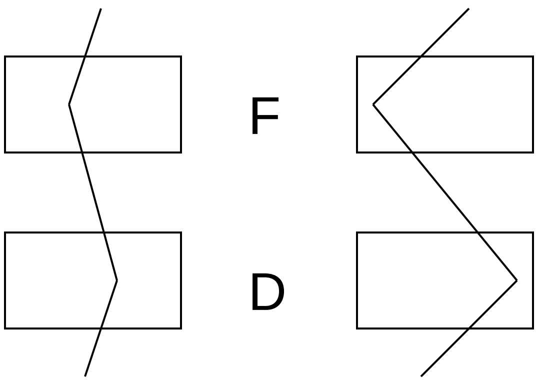


EMMA

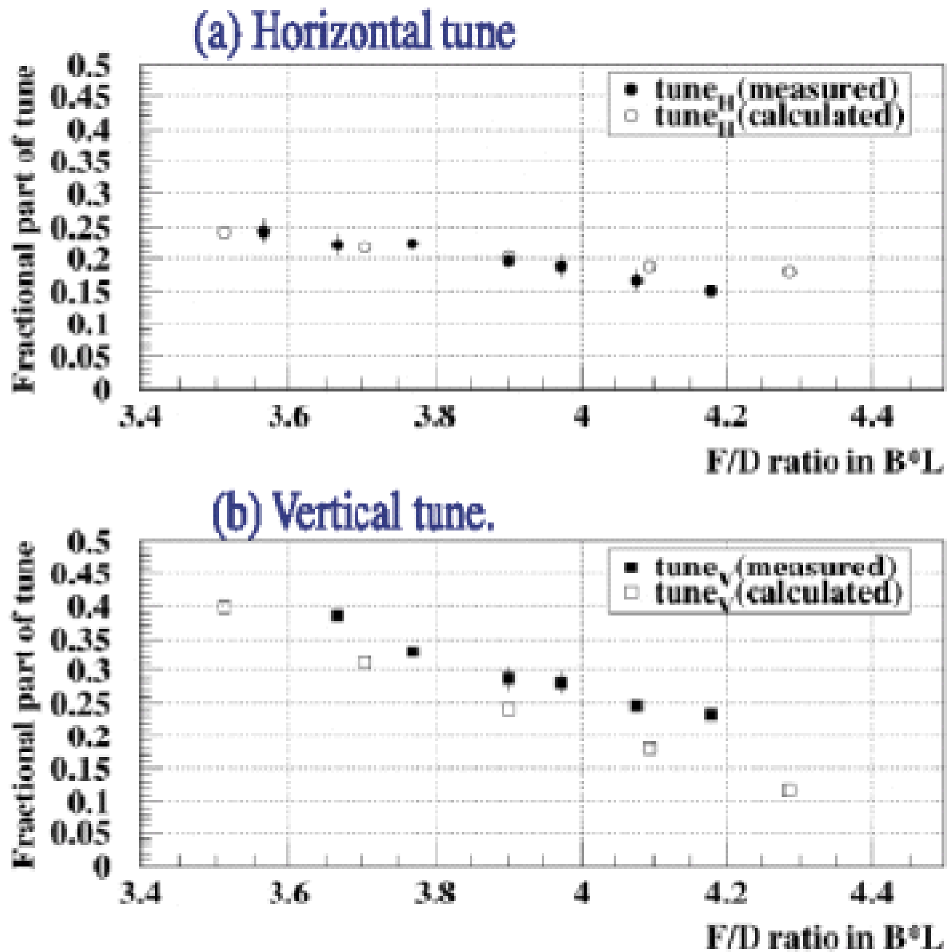


Commissioning (2)
tune measurement

Keeping the total bending angle same, change the strength of F and D.



Vertical focusing should be sensitive to F/D because of edge effects.

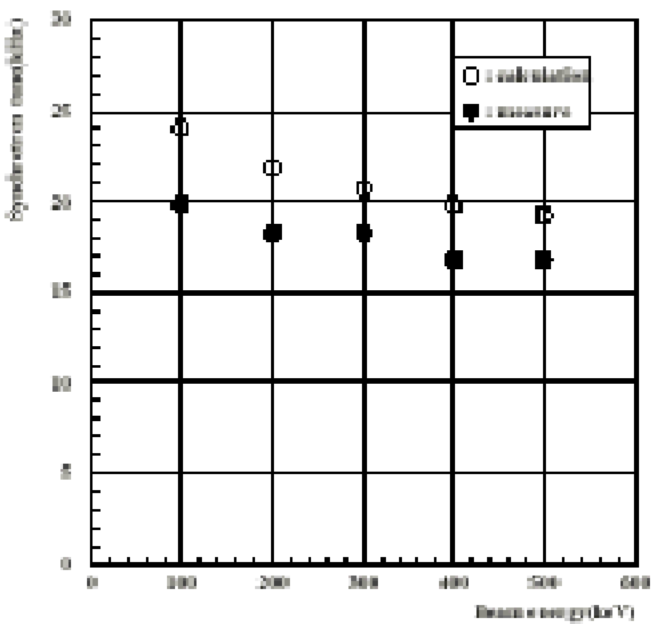


Commissioning (3)

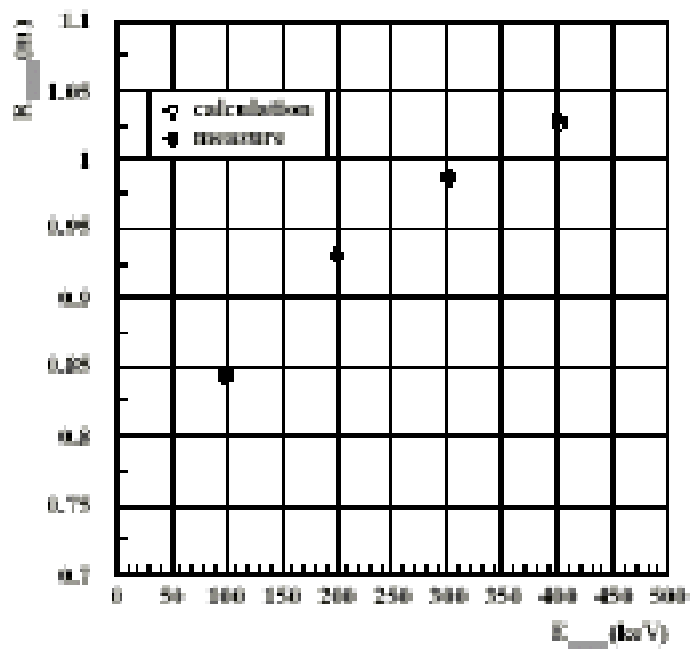
synchrotron tune and beam position

With acceleration, synchrotron oscillation frequency and radial position move.

synchrotron oscillation frequency



radius position



FFAG development in UK and in the world

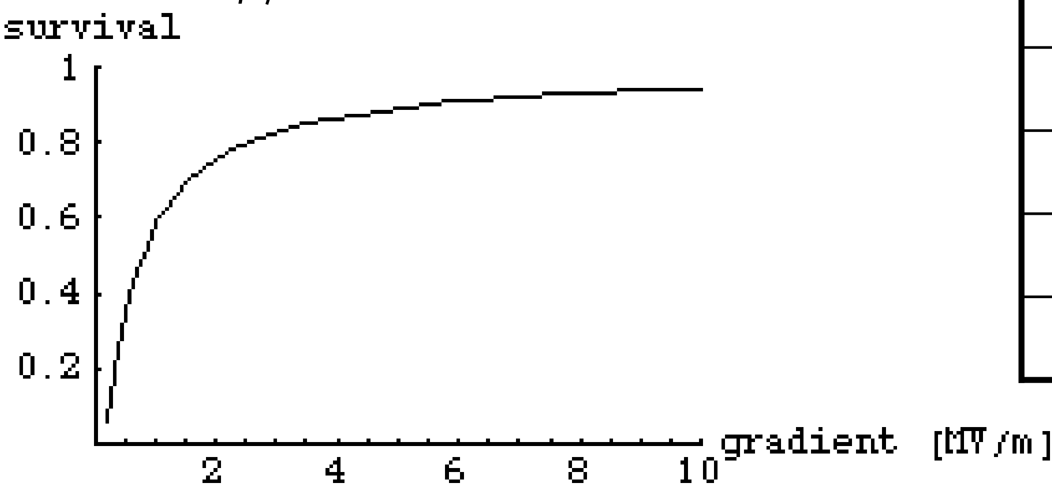
Neutrino factory and FFAG (5)
how quick should acceleration be?

Muon survival

$$\frac{n_f}{n_i} = \left(\frac{\gamma_f}{\gamma_i}\right)^{-\frac{m_\mu c^2}{c\tau g}}$$

- Where τ is muon life time at rest, g is field gradient.
- Once γ_f / γ_i is fixed, survival ratio is determined by field gradient.

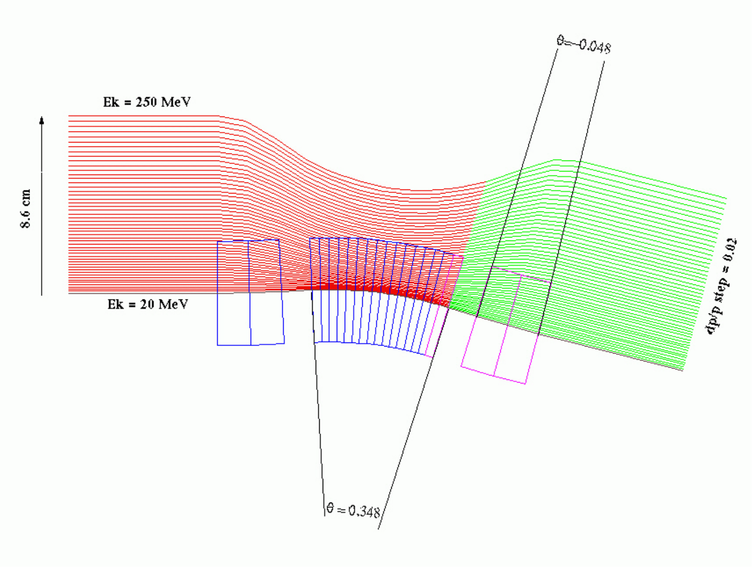
$\gamma_f / \gamma_i = 40$ (0.5 to 20 GeV)



MV/m	%	μs
1	57	65
2	76	33
5	89	13
10	95	6.5

FFAG for muon acceleration (7)
path length (1)

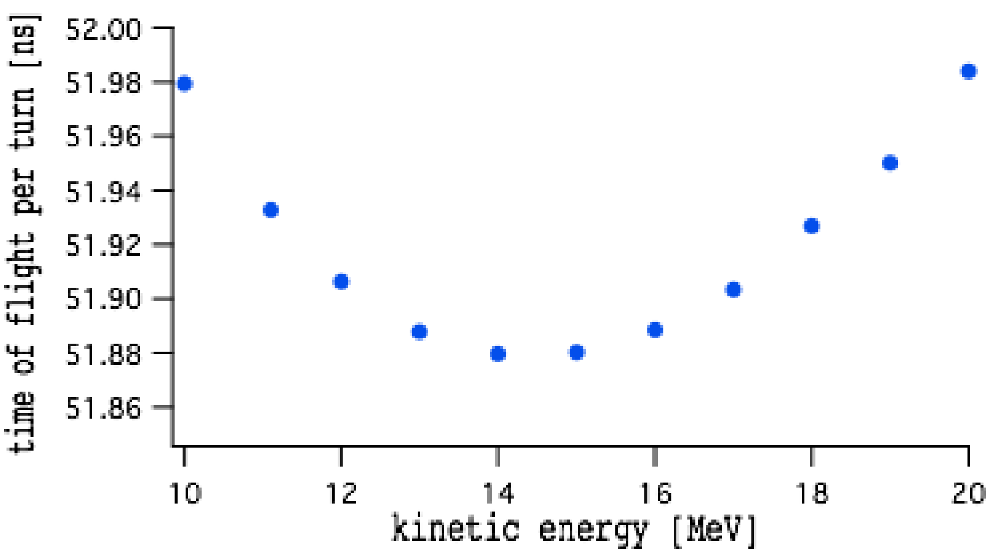
- Acceleration is so quick that RF frequency cannot be synchronized with revolution frequency of muons.
- Revolution frequency changes because orbit shifts and path length changes.
- Speed of mouns is already at the speed of light.
- If you look at orbits carefully, path length at the central momentum is shortest.



FFAG for muon acceleration (7)

path length (2)

- In a first half of a cycle, path length becomes shorter and revolution time becomes shorter.
- In a second half of a cycle, path length becomes longer and revolution time becomes longer.



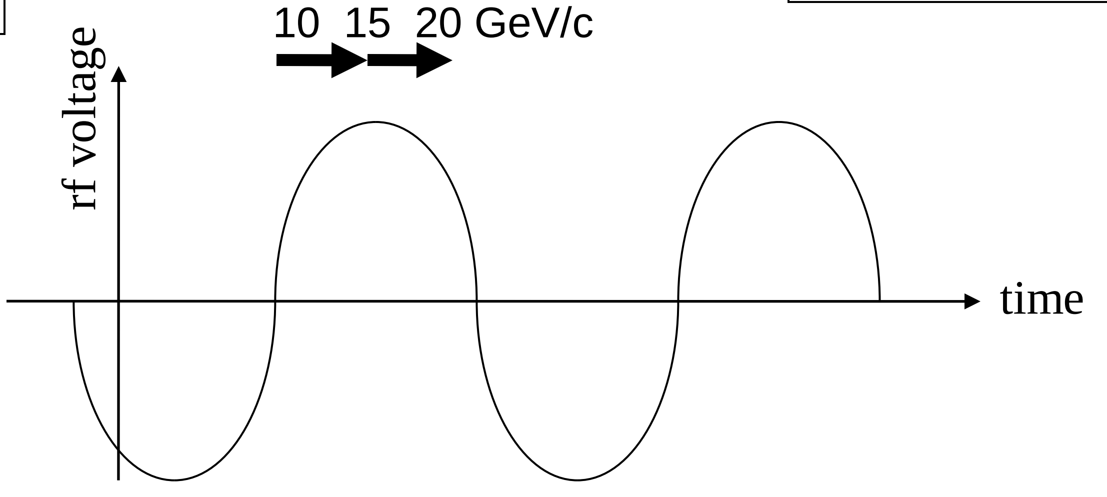
FFAG for muon acceleration (8)
acceleration (1)

- Suppose we choose rf frequency that is synchronized with revolution frequency at the center.

In the first half of a cycle, a particle lags behind the rf.

At the center momentum, a particle is synchronized with rf.

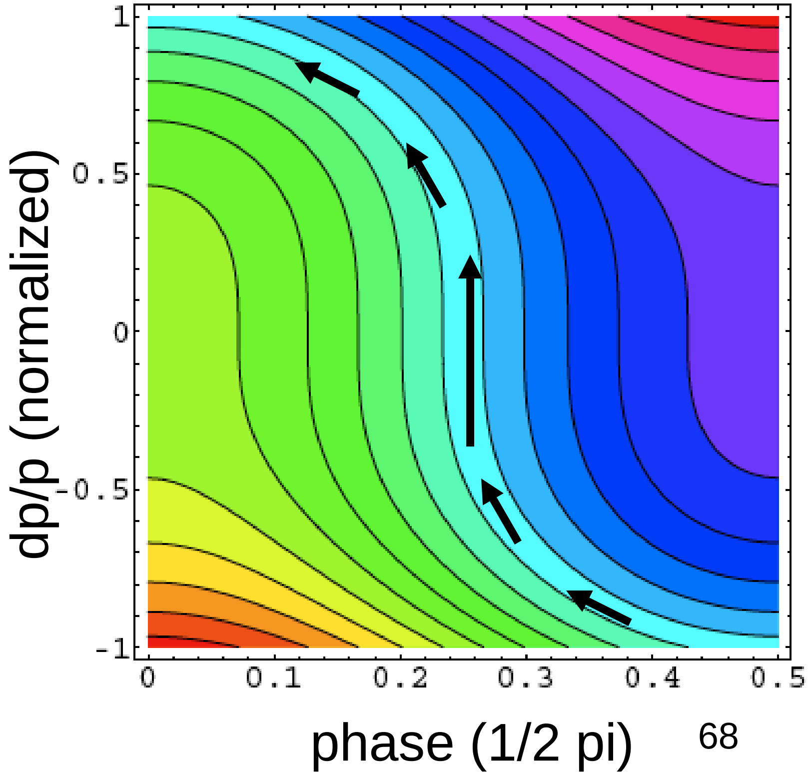
In the second half, a particle lags again.



- If the total time lag is less than a half of rf cycle, a beam has net energy gain.

FFAG for muon acceleration (8)
acceleration (2)

- In the longitudinal phase space, a particle follows the path with constant color below.
- If there is enough RF voltage, a particle can be accelerated to the top energy.
- This is called “gutter” or “serpentine” acceleration.
- Similar to longitudinal motion in cyclotron without accurate isochronism.



FFAG for muon acceleration (9)
choice of rf frequency

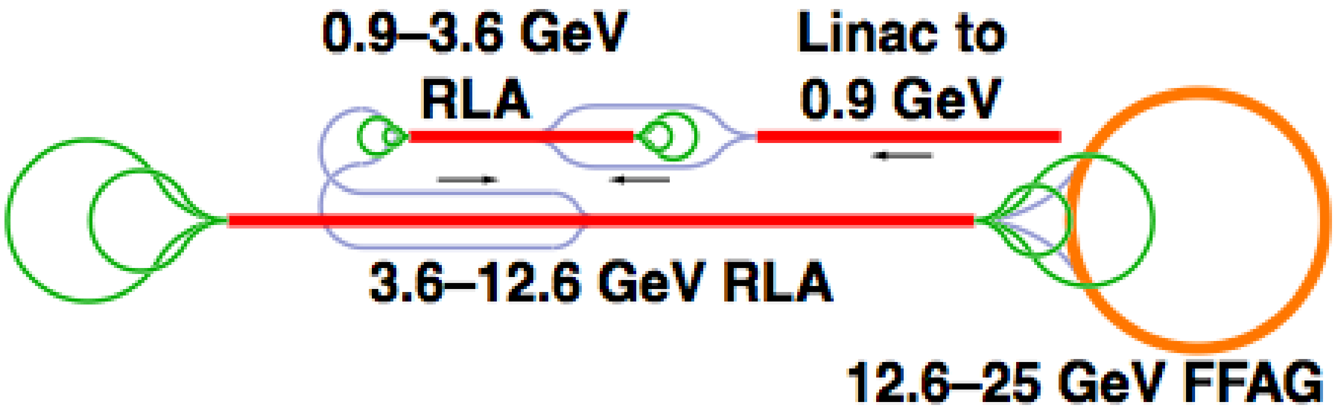
- Lower frequency gives relatively small phase lag.
- Higher frequency rf gives more field gradient.
  - A rule of thumb is

$$E \propto \sqrt{f}$$

- Muon ring chose 200 MHz.
- EMMA ring chose 1.3 GHz.

FFAG for muon accelerations (10)

muon acceleration scheme with FFAG



From 0.9 to 12 GeV: Recirculating linear accelerator
From 12 GeV to 25 GeV: nonscaling FFAG

Circumference	409.667 m
Number of cell	84
Quad gradient	23.70 (F)/-29.03 (D) T/m
Quad length	1.451 (F)/0.925 (D) m
Drift length	2.0 (long)/0.5 (short) m