

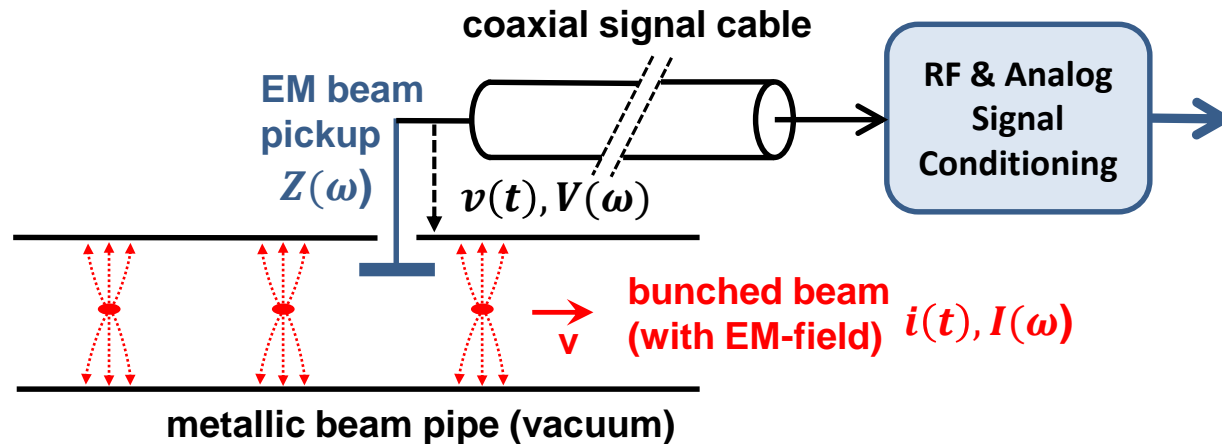
RF Measurement Techniques

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Beam Instrumentation
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Contents

- ◆ Introduction
- ◆ RF signal measurements – in time and frequency domain
- ◆ Signals of bunched beam current and modulated RF
- ◆ The super-heterodyne concept and its application(s)
- ◆ Transmission-lines and Voltage Standing Wave Ratio (VSWR)
- ◆ Introduction to Scattering-parameters (S-parameters)
- ◆ Some hints for operating a Vector Network Analyzer (VNA)
- ◆ Appendices and backup slides

An Electro-Magnetic Beam Monitor



◆ EM beam pickup, e.g.

- beam intensity (toroids and wall-current monitors)
- beam position and tune (button, stripline, cavity, and other BPM pickups)

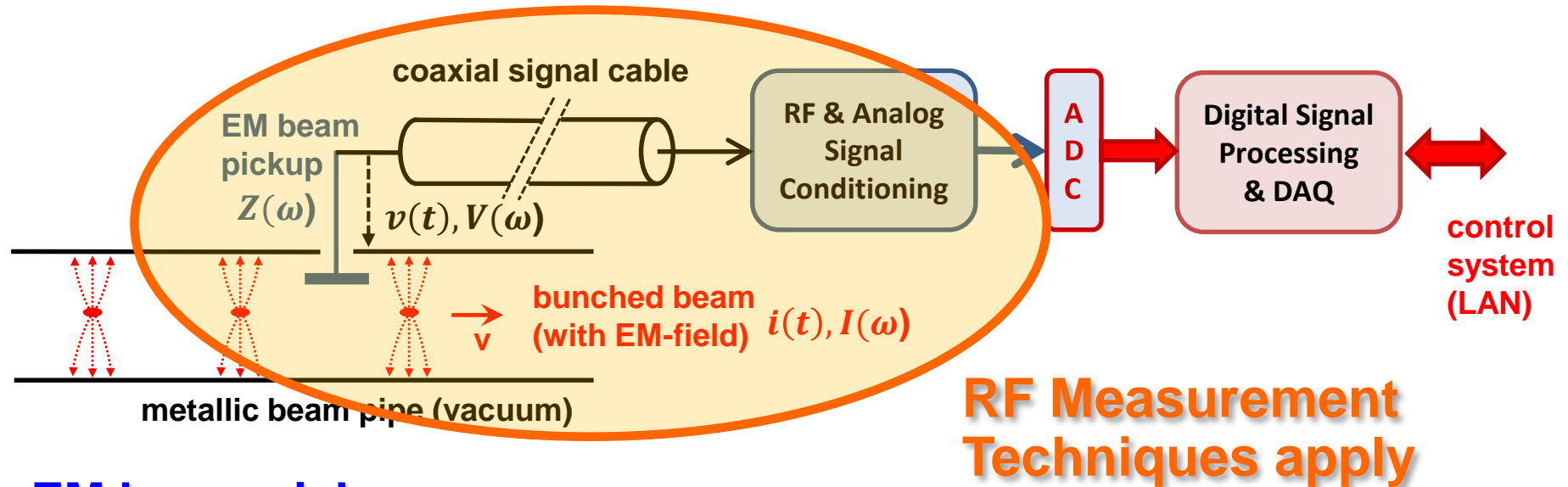
◆ Transmission-lines

- Often coaxial cables for beam and timing (trigger, CLK) signals

◆ RF signal processing

- amplifiers, attenuators, filters, hybrids, couplers, RF diodes & limiters, RF signal splitters & combiners, RF transformers & baluns, etc.

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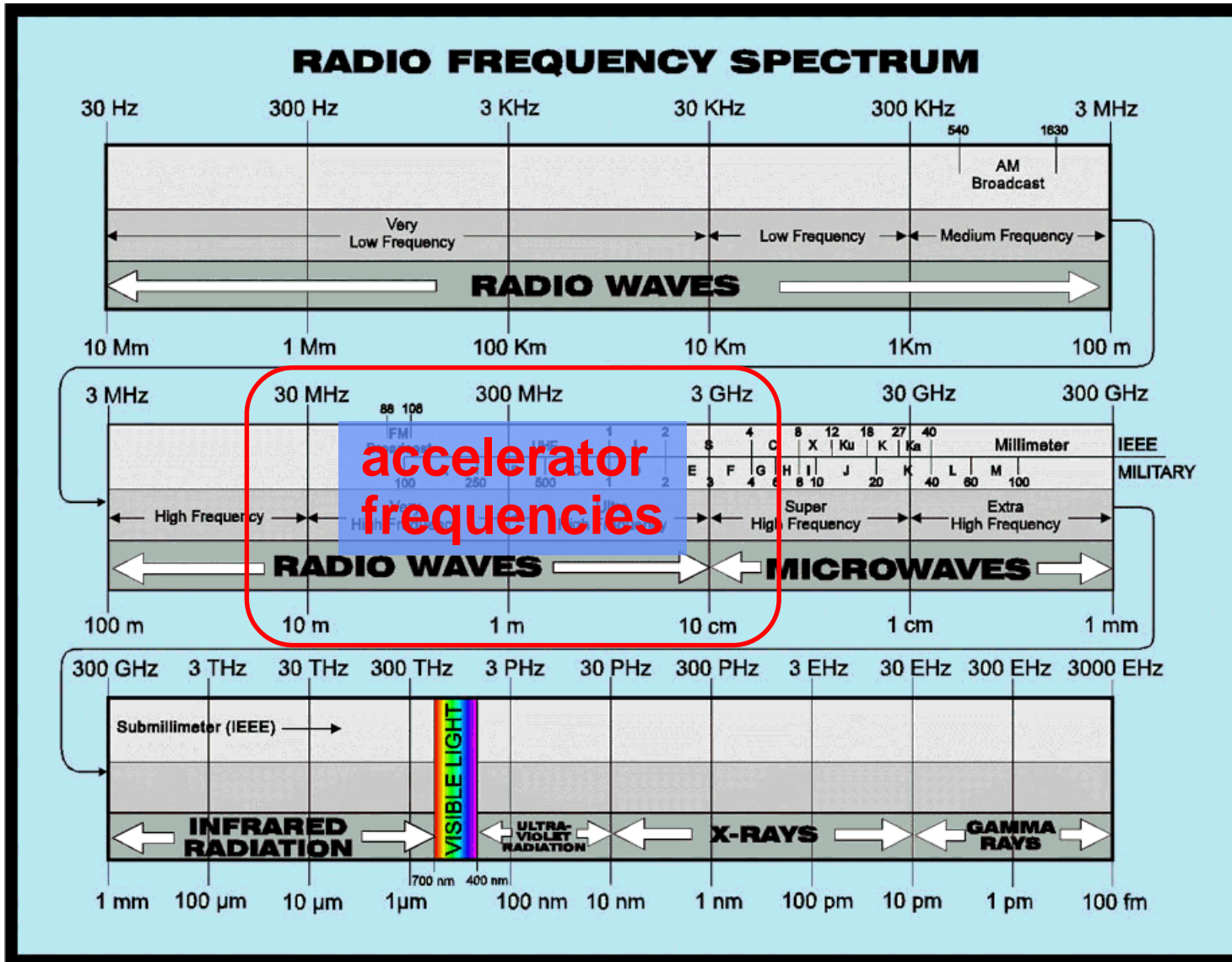
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What are Radio Frequencies?



Free space wavelength:

$$\lambda = \frac{c_0}{f}$$

We care about RF concepts if the physical dimensions of an apparatus is $> \lambda/10$

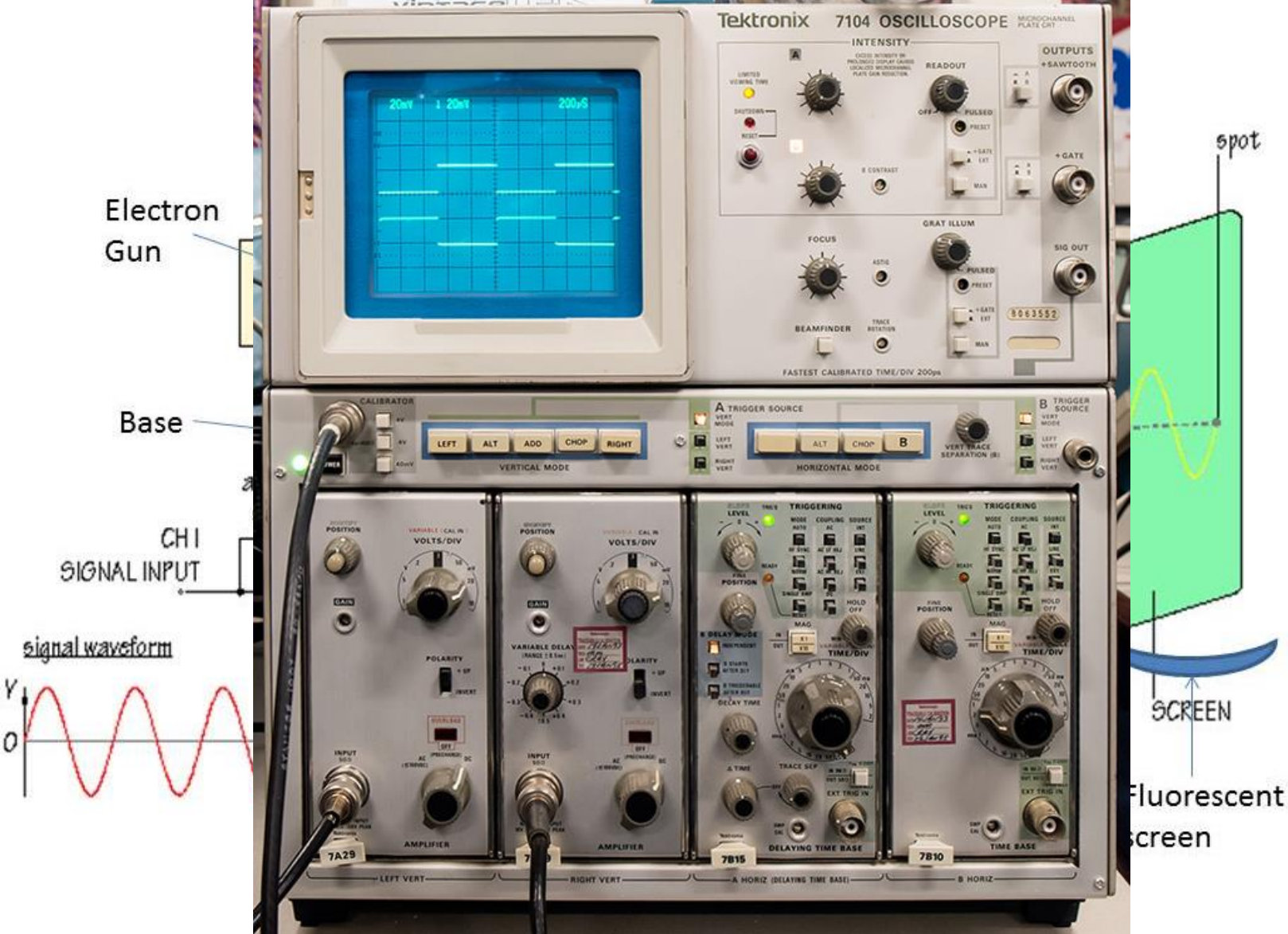
RF Measurements Methods (1)

There are many ways to observe RF or broadband beam signals
Here some typical tools:

◆ Oscilloscope: to observe signals in time domain

- periodic signals
- burst and transient signals with arbitrary waveforms
- application: direct observation of signals from a beam pick-up, from a test generator, or from other sources
- visualizes the shape of a waveform, etc.
- limited performance for the evaluation of non-linear effects.

Cathode Ray Tube (CRT) Oscilloscope



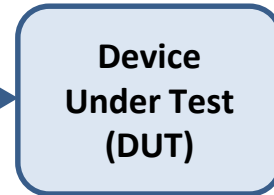
Today: Digital Storage Oscilloscope (DSO)

...and digital signal generator
(AWG: arbitrary waveform generator)

100 GHz bandwidth
240 GS/s oscilloscope
(LeCroy)



50 GS/s, 10-bit AWG (Tektronix)



◆ Signal processing based on fast ADCs and DACs

- Similar “look and feel” as analog oscilloscopes, but better performance
 - 8...12-bit multi-GS/s ADCs, still, be aware of aliasing effects!
 - Fast sampling oscilloscope require sufficient memory resources.

◆ AWG or pulse generator & digital oscilloscope: time domain (TD) test setup

- Device under test (DUT) characterization and trouble shooting
 - Impulse, step, or arbitrary waveform (e.g. beam signal) as stimulus signal
 - High impedance probe for measurements on the printed circuit board (PCB)

RF Measurements Methods (1)

◆ RF detection (Schottky) diode (RF power meter)

- Supplies a rectified (video) output signal proportional to the RF signal level
- Delivers no frequency or phase information, but operates over a very broad frequency range few MHz to many GHz, and up to 90 dB dynamic range.

◆ Spectrum analyzer: to observe signals in frequency domain

- sweeps in equidistant steps through a given frequency range
- application: observation of spectrum from the beam, or from a signal generator or RF source, or the spectrum emitted from an antenna to locate EMI issues in the accelerator tunnel, etc.
- Requires periodic signals
- **Large dynamic range!**

RF Measurements Methods (2)

- ◆ **Vector signal analyzer (VSA), sometimes called FFT analyzer**
 - Acquires the signal, often after down-conversion, in time domain by fast sampling
 - Further numerical treatment in digital signal processors (DSPs)
 - Spectrum calculated using Fast Fourier Transform (FFT)
 - Combines **features of an oscilloscope and a spectrum analyzer**: Signals can be observed directly in time or in frequency domain
 - Contrary to the SA, also the spectrum of non-periodic signals and transients can be measured
 - Application: Observation of tune sidebands, transient behavior of a phase locked loop, single pass beam signal spectrum, etc.
 - **Digital oscilloscopes** and **FFT analyzers** share similar technologies, i.e. fast sampling and digital signal processing, and therefore can provide similar measurement options

RF Measurements Methods (2)

Tools to characterize RF components and sub-systems:

◆ Slotted coaxial (or waveguide) measurement transmission-line

- For study and illustration purposes only – not anymore used in today's RF laboratory environment.

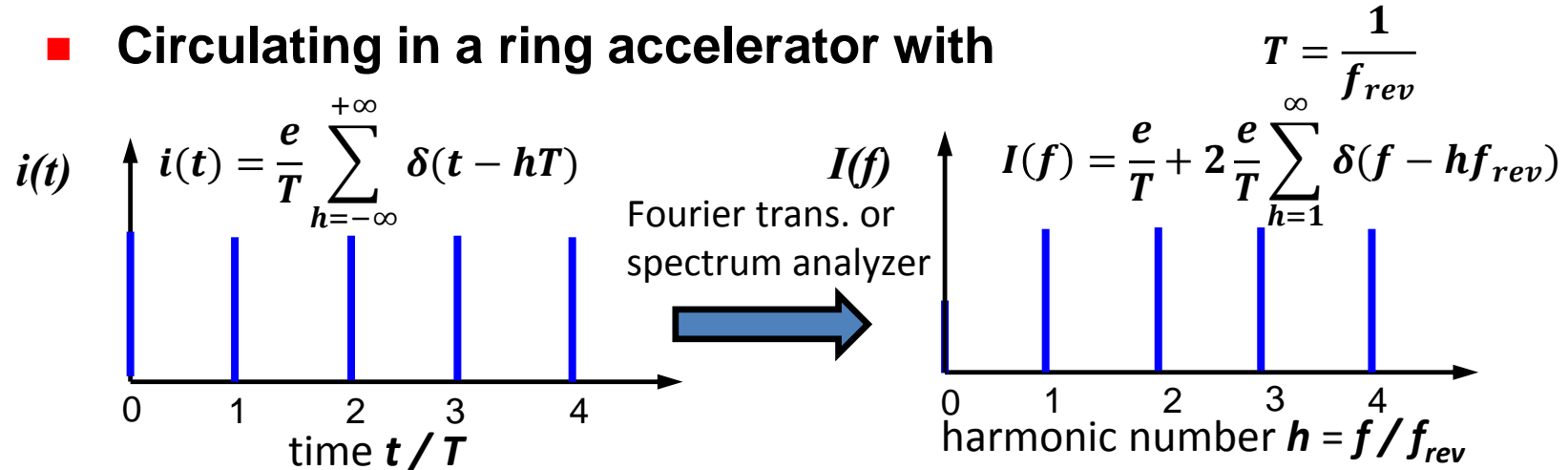
◆ Vector Network Analyzer (VNA)

- Combines the functions of a vector spectrum analyzer (FFT analyzer), a RF sweep generator, and a S-parameter test set (directional coupler)
- Excites a *Device Under Test* (DUT, e.g. circuit, antenna, amplifier, etc.) network at a given *Continuous Wave* (CW) frequency, and measures the response in magnitude and phase => **determines the S-parameters**
 - **What are S-parameters?!**
- Covers a selectable frequency range by measuring step-by-step at subsequent frequency points (similar to the spectrum analyzer)
- Applications: characterization of passive and active RF components, *Time Domain Reflectometry* (TDR) by Fourier transformation of the reflection or transmission response, etc.
- **The VNA is the most versatile and comprehensive tool in the RF laboratory!**

Beam Current Signals

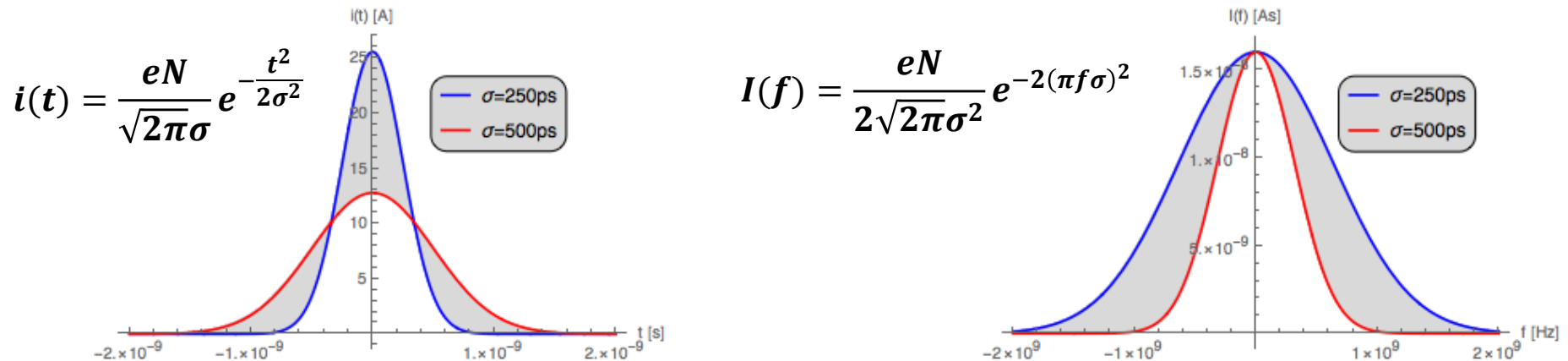
◆ Single charge e in time and frequency domain

- Circulating in a ring accelerator with



◆ Many particles (protons, $N=10^{11}$) in a Gaussian distribution

- Single bunch in time and frequency domain



Beam Current Signals

- Normalized representation in logarithmic amplitude scale

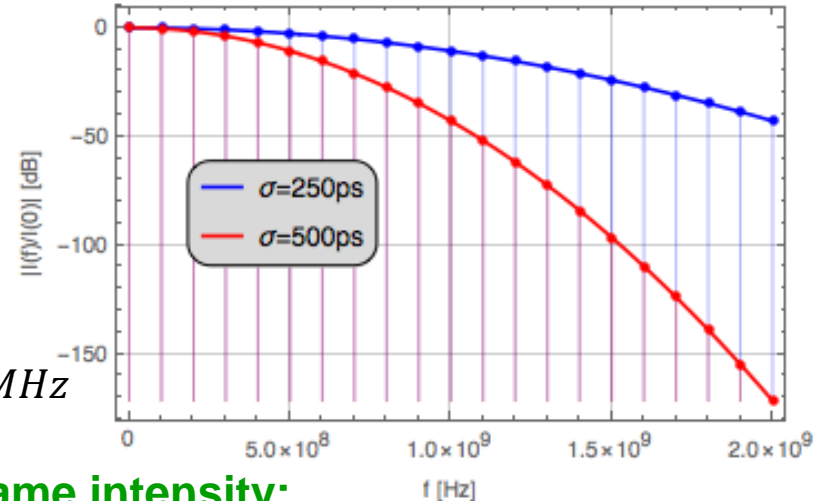
- Typical magnitude spectrum, as it would be observed on a spectrum or network analyzer

$$\left| \frac{I(f)}{I(0)} \right|_{dB} = 20 \log_{10} \left[e^{-2(\pi f \sigma)^2} \right]$$

$$I(0) = \frac{eN}{2\sqrt{2\pi}\sigma^2}$$

revolution harmonics, here

indicated at: $\frac{1}{T} = f_{rev} = 100 \text{ MHz}$



- Spectrum of repetitive bunches of same intensity:
Fourier series expansion

$$i(t) = \frac{eN}{T} + 2 \frac{eN}{T} \sum_{h=1}^{\infty} e^{-\frac{(h\omega\sigma)^2}{2}} \cos(h\omega t) \quad \omega = 2\pi f$$

- Beam bunches have different distribution functions and bunch length

- Electron bunches are typical 100...1000x shorter than proton bunches
- Ion bunches can be 10...1000 longer than relativistic proton bunches
- Longitudinal particle distributions: Gaussian, parabolic, Cos², Square, etc.

“dB” or not to be...

◆ dezi-Bel: 1 dB = 0.1 B (Bel)

- Logarithmic scaling to compare large, e.g. power ratios:

$$P_{dB} = 10 \log_{10} \left(\frac{P_1}{P_2} \right)$$

- or large ratios of other quantities, e.g.:

$$V_{dB} = 20 \log_{10} \left(\frac{V_1}{V_2} \right)$$

dB ratio	P_1/P_2	V_1/V_2
n x 10 dB	10^n	$10^{n/2}$
40 dB	10000	100
20 dB	100	10
10 dB	10	~3.16
6 dB	~4	~2
3 dB	~2	~1.41
0 dB	1	1
-3 dB	~0.5	~0.71
-20 dB	0.01	0.1

$$I_{dB} = 20 \log_{10} \left(\frac{I_1}{I_2} \right)$$

$$\frac{P_1}{P_2} = 10^{\left(\frac{P_{dB}}{10}\right)} \quad \frac{V_1}{V_2} = 10^{\left(\frac{V_{dB}}{20}\right)}$$

← The 3 dB ratio (half power) is a common specification for the bandwidth

“dBm” is not “dB”

◆ dBm is a logarithmic power unit

- based on dB and $P_{ref} = 1 \text{ mW}$

$$P_{dBm} = 10 \log_{10} \left(\frac{P}{P_{ref}} \right)$$

◆ dBm can also be used as logarithmic voltage unit

- e.g. for $Z_0 = 50 \Omega$: $V_{ref} = 0.2236 \text{ V}$

$$V_{dBm} = 20 \log_{10} \left(\frac{V}{V_{ref}} \right)$$

dBm	P	V (RMS)
30 dBm	1 W	7.07 V
20 dBm	100 mW	2.24 V
10 dBm	10 mW	707 mV
6 dBm	4.0 mW	446 mV
0 dBm	1.0 mW	224 mV
-20 dBm	10 μ W	22.4 mV
-60 dBm	1.0 nW	224 μ V
-120 dBm	1.0 fW	224 nV
- 174 dBm	4.0e-21 W	0.446 nV

$$P = P_{ref} 10^{\left(\frac{P_{dBm}}{10}\right)}$$

$$V = V_{ref} 10^{\left(\frac{V_{dBm}}{20}\right)}$$

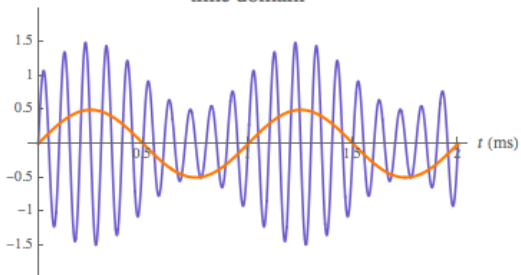
← Noise power in a bandwidth $B = 1 \text{ Hz}$ at room temperature

RF Signals & Modulation

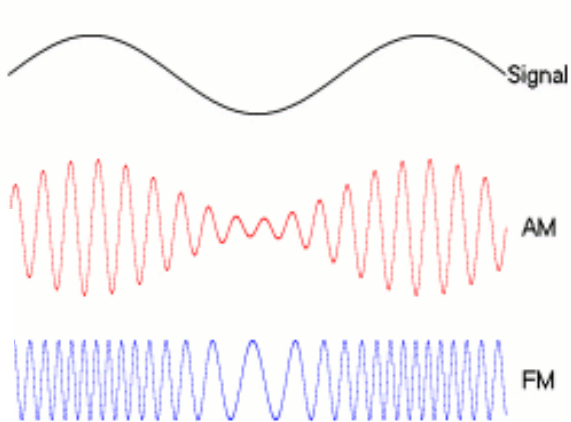
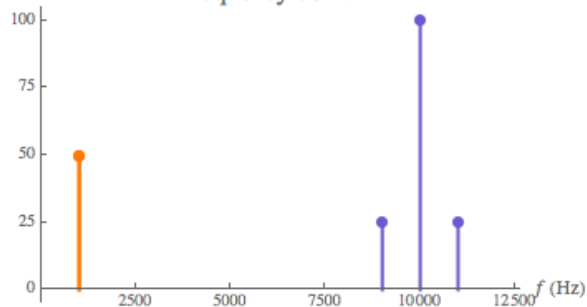
- ◆ Beam current signals: transient (pulse-like) signals
- ◆ RF signals: Sinusoidal signals (CW: continuous wave)
 - High frequency carrier modulated with low frequency information
 - In ring accelerators:

AM: Betatron oscillations

time domain

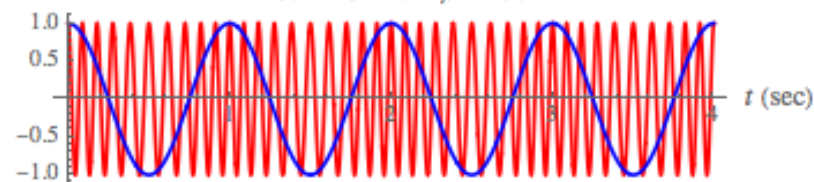


frequency domain



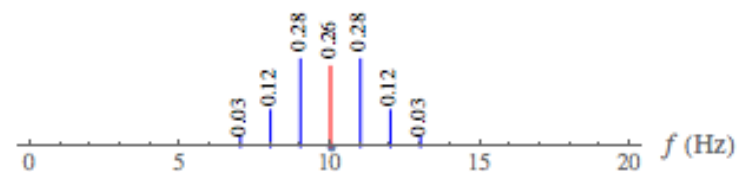
FM: Synchrotron oscillations

FM modulated carrier
modulation index $\beta = 1.50$

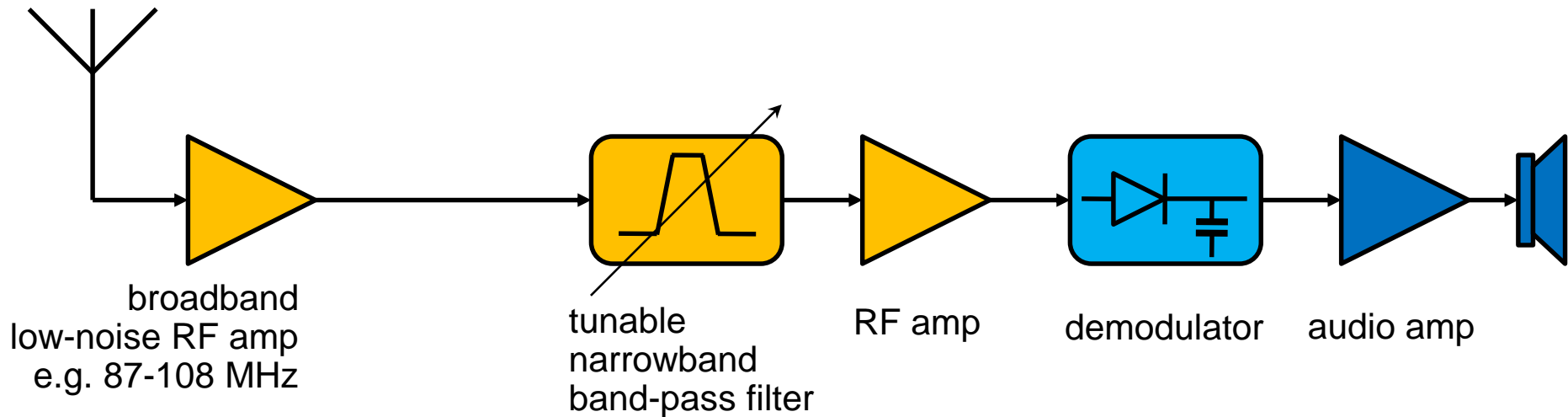


modulated carrier spectra (magnitude)

bandwidth (98% of power) = 5.00 Hz number of sidebands = 6



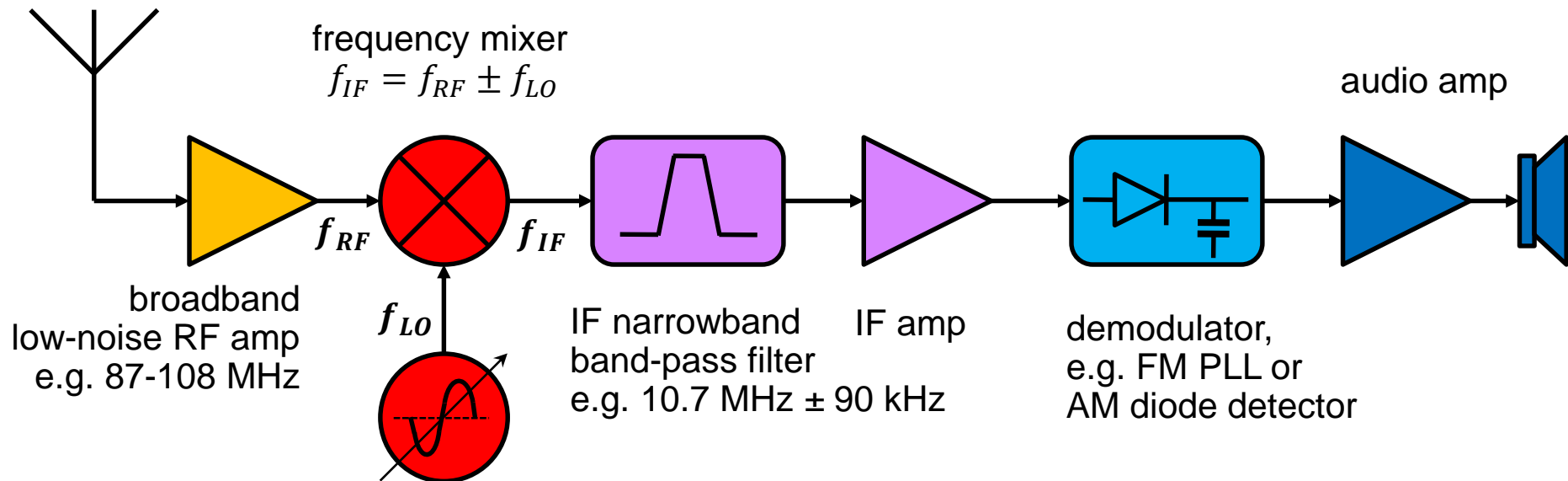
A (too) Simple Radio Receiver



◆ ...or: How does a "traditional" analog radio works?

- It was, and still is, difficult to make precisely tunable narrowband, band-pass filters for high frequencies (~100 MHz)!!
- high frequency low-noise amplifiers are expensive!
- high frequency demodulators are not trivial.
- **direct detection of radio or RF signals is challenging!**

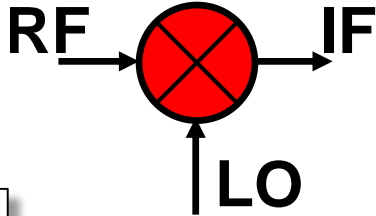
The Super-Heterodyne Receiver



◆ **Introduce a non-linear element: the (frequency) mixer!**

- "down-convert" the RF band to a fixed "intermediate" frequency (IF) $f_{IF} = f_{RF} \pm f_{LO}$
- requires a tunable local oscillator (LO)
- well manageable IF section:
 - narrowband band-pass filter(s) (BPF) and amplifier(s)
- RF telecommunication standard
- Often multiple mixing stages are used in modern RF instruments (spectrum analyzer, vector network analyzer)

The RF Mixer as Downconverter

$$y_{RF}(t) = A_{RF} \sin(\omega_{RF}t + \varphi_{RF})$$


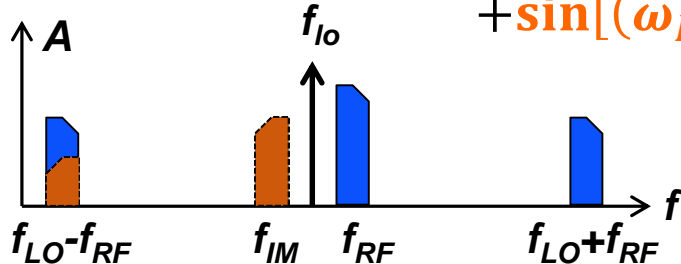
$$y_{IF}(t) = y_{RF}(t)y_{LO}(t)$$

$$f_{IF} = f_{RF} \pm f_{LO}$$

$$y_{LO}(t) = A_{LO} \sin(\omega_{LO}t + \varphi_{LO})$$

◆ Ideal mixer:

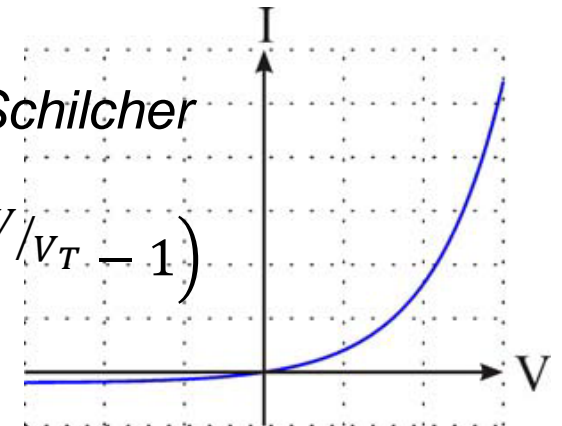
$$y_{IF}(t) = \frac{1}{2} A_{LO} A_{RF} \left\{ \begin{array}{l} \sin[(\omega_{RF} - \omega_{LO})t + (\varphi_{RF} - \varphi_{LO})] \quad \text{upper sideband} \\ + \sin[(\omega_{RF} + \omega_{LO})t + (\varphi_{RF} + \varphi_{LO})] \quad \text{lower sideband} \end{array} \right\}$$



$I = f(V)$ of a **Schottky diode**

courtesy T. Schilcher

$$I = I_0 \left(e^{V/V_T} - 1 \right)$$



◆ Frequency conversion

- $f_{RF} \neq f_{LO}$: heterodyne receiver
- $f_{RF} = f_{LO}$: homodyne, demodulator

◆ Real-world mixer:

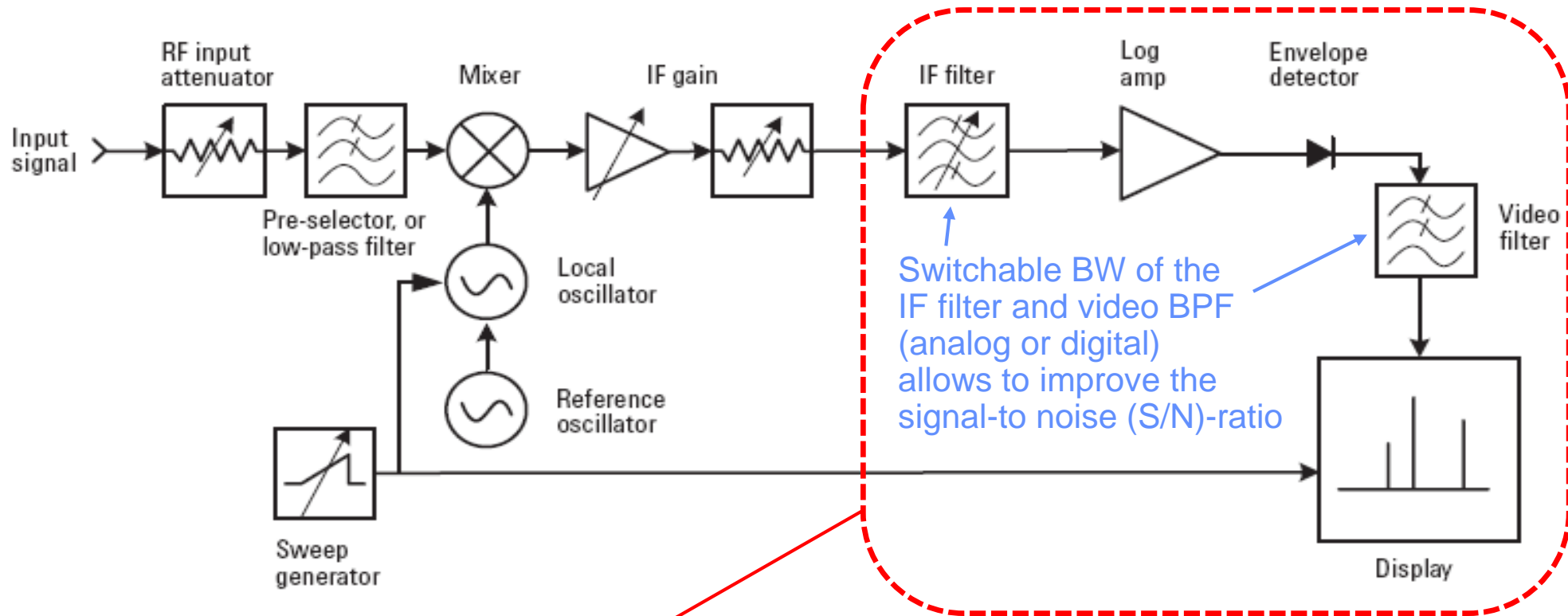
$$f_{IF} = m f_{RF} \pm n f_{LO}$$

- Image frequency: $f_{IM} = f_{LO} - f_{IF}$

$$\Delta I = I_0 e^{V/V_T} \left[\frac{\Delta V}{V_T} + \frac{1}{2} \left(\frac{\Delta V}{V_T} \right)^2 + \frac{1}{6} \left(\frac{\Delta V}{V_T} \right)^3 + \dots \right]$$

Simplified Spectrum Analyzer

- ◆ based on the super-heterodyne principle



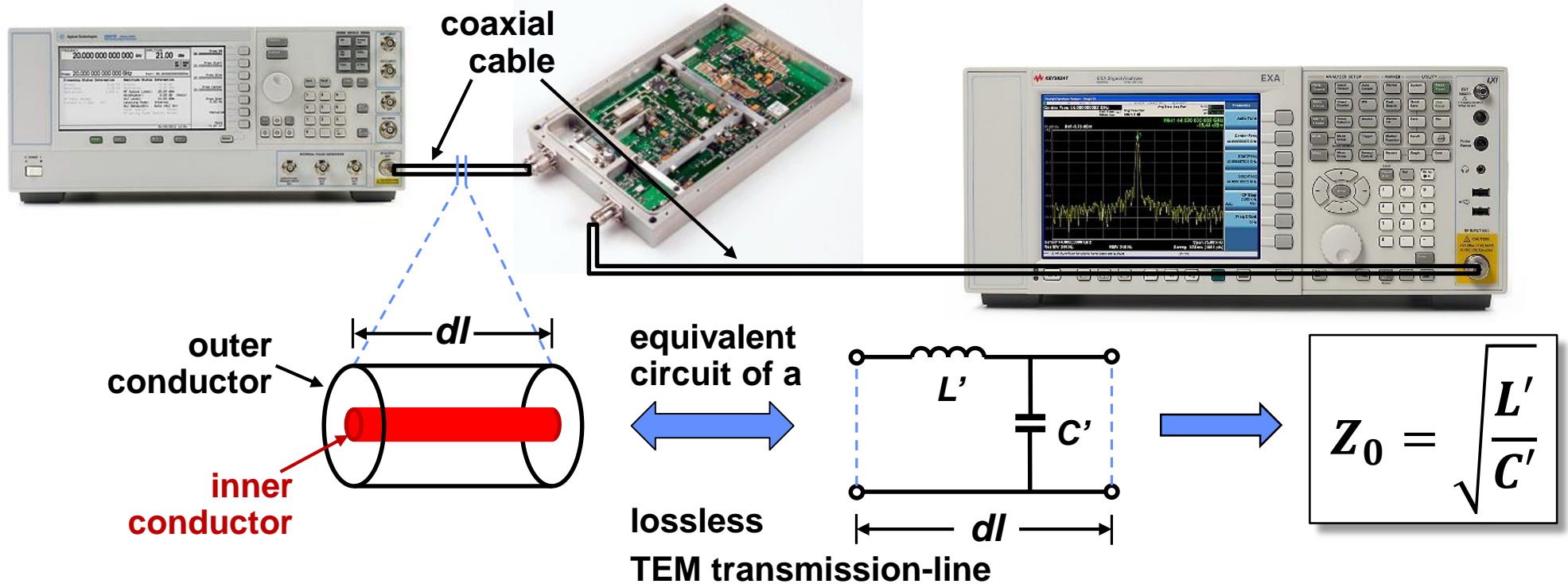
Today, the IF, demodulation, video and display sections of a spectrum analyzer are realized **digitally**

- Requires an analog-digital converter (ADC) with sufficient dynamic range

Modern Spectrum (RF Signal) Analyzer

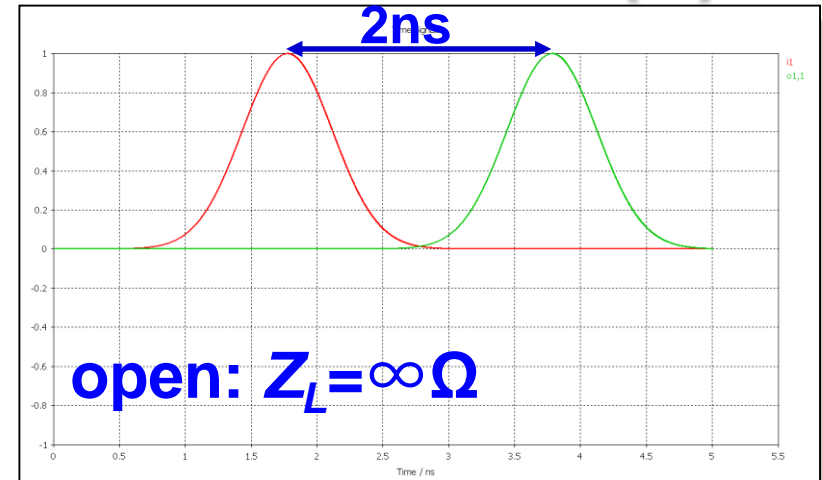
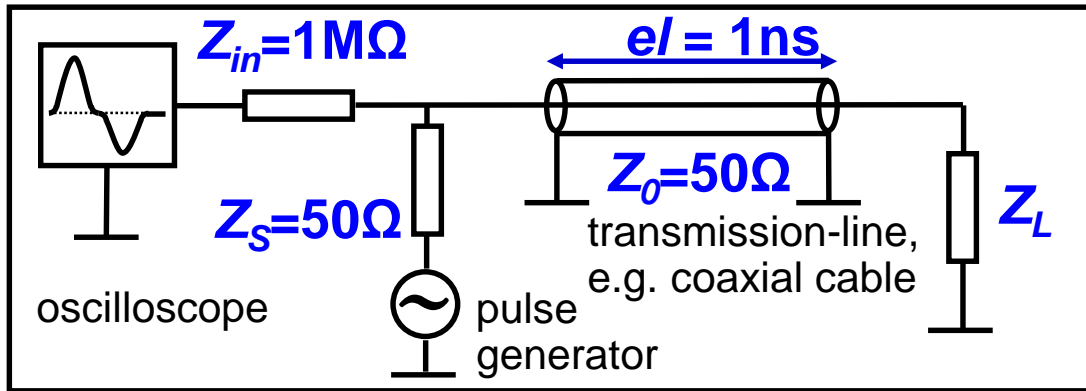


Characteristic Impedance

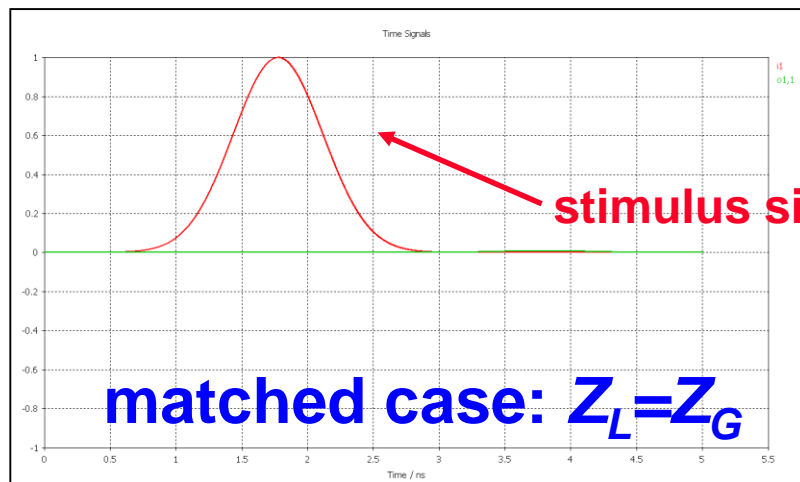


- ◆ The reference impedance Z_0 in a RF system is defined by the characteristic impedance of the interconnect cables
 - often coaxial cables of $Z_0 = 50 \Omega$ (compromise: high voltage / high power handling)
- ◆ The characteristic impedance of a TEM transmission-line is defined by the cross-section geometry
 - Ratio of H- and E-field, represented by L' [H/m] and C' [F/m] in the equivalent circuit of a line segment dl
 - The characteristic impedance Z_0 has the unit Ohm [Ω]

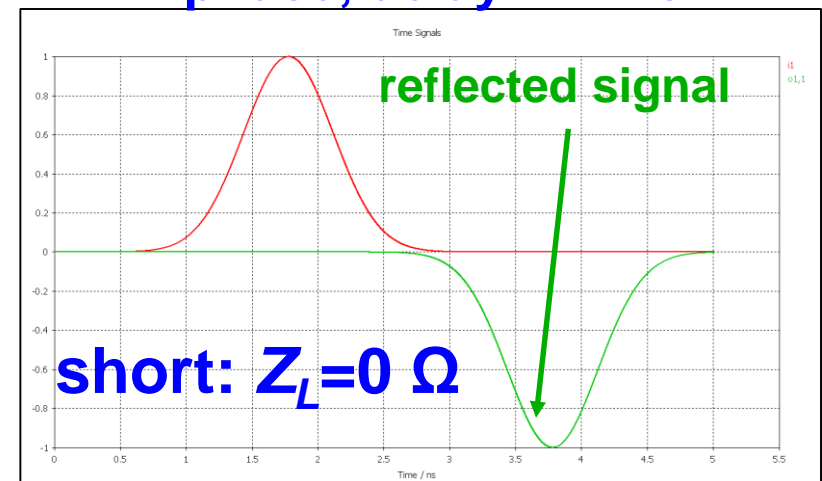
Transmission-lines in Time Domain (2)



total reflection; reflected signal in phase, delay 2×1 ns.

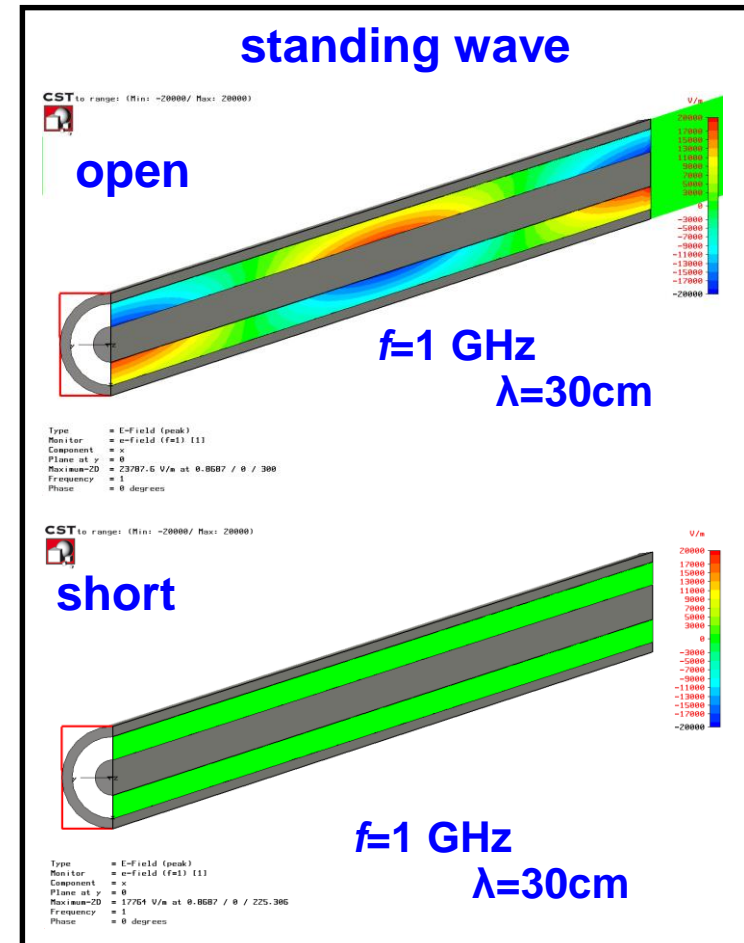
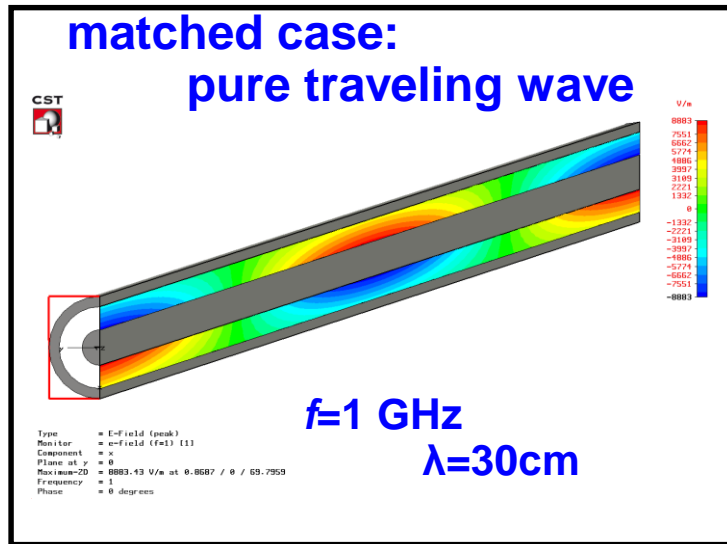


no reflection



total reflection; reflected signal in contra phase

Transmission-lines in Frequency Domain



◆ Standing and traveling waves:

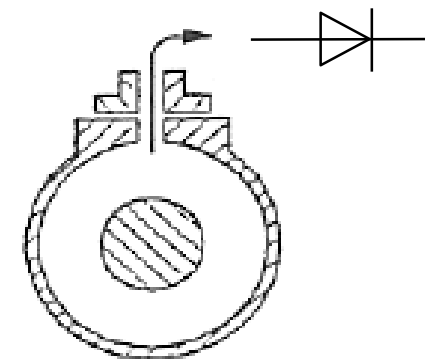
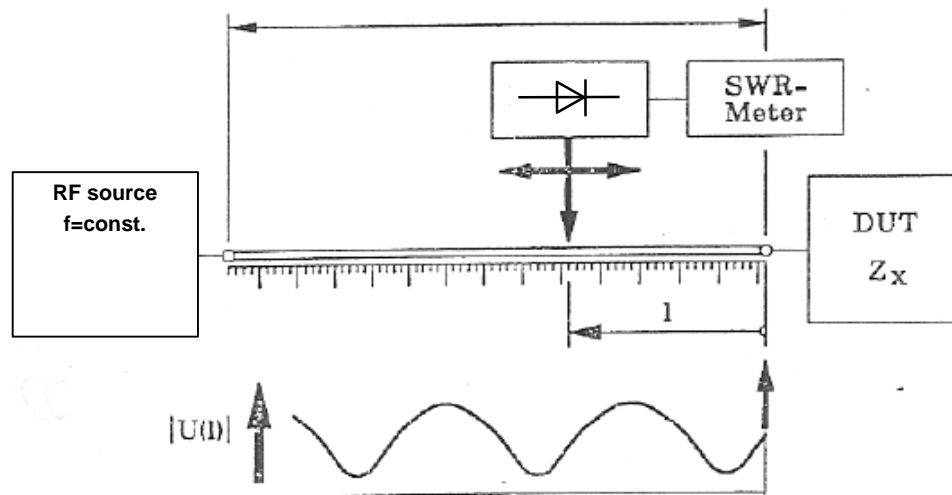
- The patterns for the short and open case are equal; only the phase is opposite, which correspond to different position of nodes.
- In case of perfect matching:
 - traveling wave only.
- Otherwise:
 - mixture of traveling and standing waves.

Caution: the color coding corresponds to the radial electric field strength – these are not scalar equipotential lines, which are anyway not defined for time varying fields

Voltage Standing Wave Ratio VSWR (1)

- ◆ On a transmission-line – single frequency (continuous wave, CW):
 - Superposition of **forward a (E^{inc})** and **backward b (E^{refl})** traveling waves
⇒ **standing waves**
- ◆ **Slotted coaxial air-line is used as standing wave detector**
 - Probes the radial **electric field** along the slotted line.
 - Probes the **minima's E_{min}** and **maxima's E_{max}** of the E-field with a diode detector, sliding along the line, thus detects $|V_{min}|$ and $|V_{max}|$ along the line.
 - Evaluate the **reflection coefficient Γ** of a **DUT of unknown $Z_x=Z_L$** at the end of the line

$$\Gamma = \frac{E^{refl}}{E^{inc}} = \frac{Z_L - Z_0}{Z_L + Z_0}$$



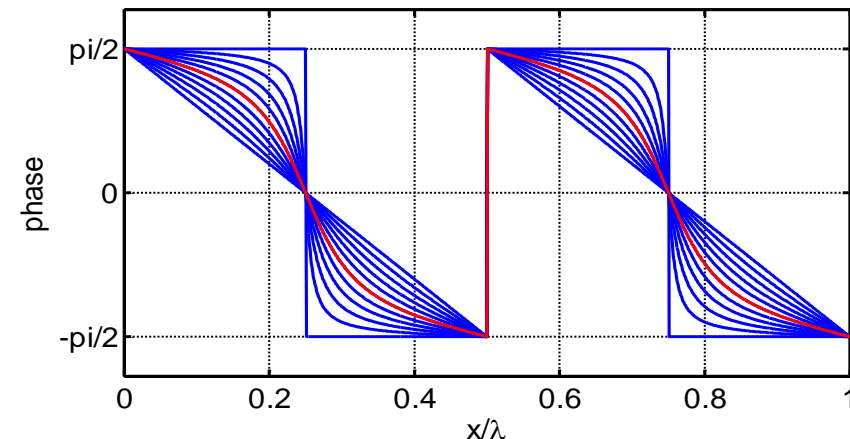
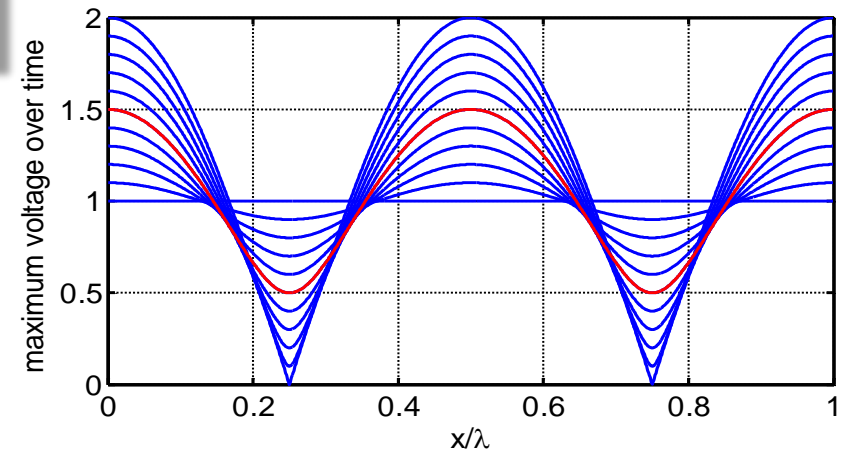
Voltage Standing Wave Ratio VSWR (2)

◆ The VSWR is defined as:

$$VSWR = \frac{|V_{max}|}{|V_{min}|} = \frac{|a| + |b|}{|a| - |b|} = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

- The **phase** of the detected E-field along the **lossless coaxial line** is purged by the diode detection.
 - **Requires a mixer as detector!**

Γ	Return Loss [dB]	$VSWR = Z_L/Z_0$	Refl. Power $1- \Gamma ^2$
0.0	∞	1.00	1.00
0.1	20	1.22	0.99
0.2	14	1.50	0.96
0.3	10	1.87	0.91
0.4	8	2.33	0.84
0.5	6	3.00	0.75
0.6	4	4.00	0.64
0.7	3	5.67	0.51
0.8	2	9.00	0.36
0.9	1	19	0.19
1.0	0	∞	0.00



S-Parameters – Introduction (1)

- ◆ **Light falling on a car window:**
 - Some parts of the incident light is reflected (you see the mirror image)
 - Another part of the light is transmitted through the window (you can still see inside the car)
- ◆ **Optical reflection and transmission coefficients of the window glass define the ratio of reflected and transmitted light**
- ◆ **Similar:**
Scattering (S-) parameters of an n -port electrical network (DUT) characterize reflected and transmitted (power) waves



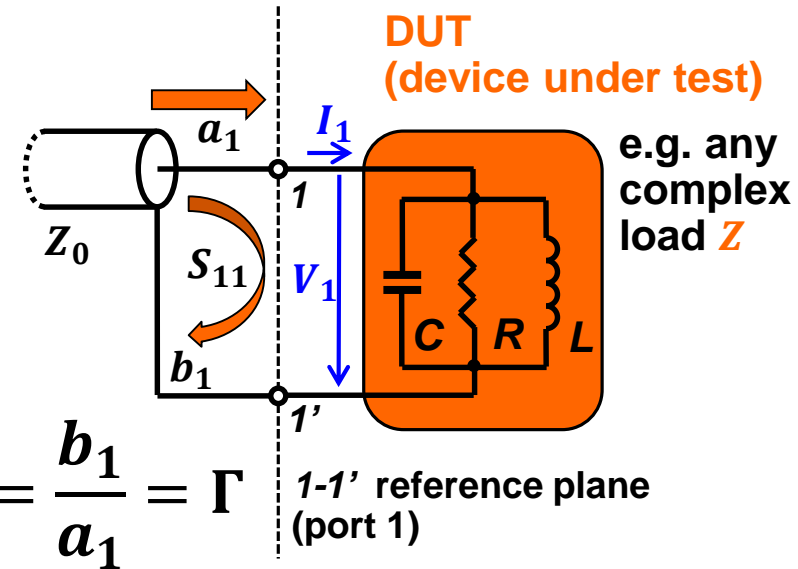
S-Parameters – Introduction (2)

◆ Electrical networks

- 1... n -ports circuits
- Defined by **voltages** $V_n(\omega)$ or $v_n(t)$ and **currents** $I_n(\omega)$ or $i_n(t)$ at the ports
- Characterized by circuit matrices, e.g. ABCD (chain), Z, Y, H, etc.

◆ RF networks

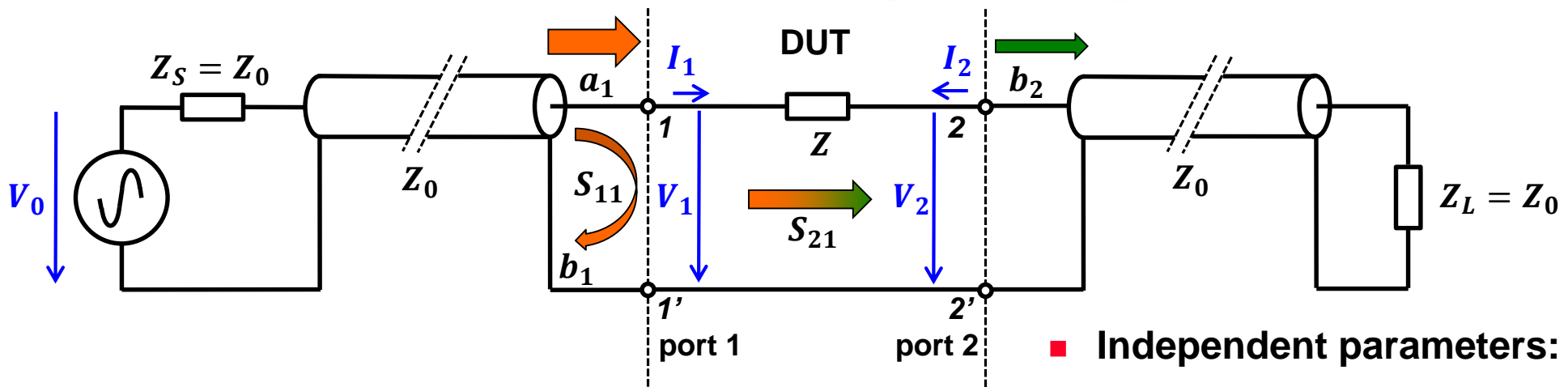
- 1... n -port RF DUT circuit or subsystem, e.g. filter, amplifier, transmission-line, hybrid, circulator, resonator, etc.
- Defined by **incident waves** $a_n(\omega, s)$ and **reflected waves** $b_n(\omega, s)$ at a **reference plane** s (**physical position**) at the ports
- Characterized by a scattering parameter (S-parameter) matrix of the reflected and transmitted power waves
- Normalized to a **reference impedance** $\sqrt{Z_0}$ of typically $Z_0 = 50 \Omega$



1-port DUT example

- ◆ S-Parameters allow to characterize the DUT with the measurement equipment to be located at some distance
- ◆ All high frequency effects of distributed elements are taken into account with respect to the reference plane

S-Parameters – Example: 2-port DUT



◆ Analysis of the forward S-parameters:

$$S_{11} = \left. \frac{b_1}{a_1} \right|_{a_2=0} \equiv \text{input reflection coefficient}$$

$(Z_L = Z_0 \Rightarrow a_2 = 0)$

$$S_{21} = \left. \frac{b_2}{a_1} \right|_{a_2=0} \equiv \text{forward transmission gain}$$

- Examples of 2-ports DUT: filters, amplifiers, attenuators, transmission-lines (cables), etc.
- **ALL ports ALWAYS need to be terminated in their characteristic impedance!**

- Independent parameters:

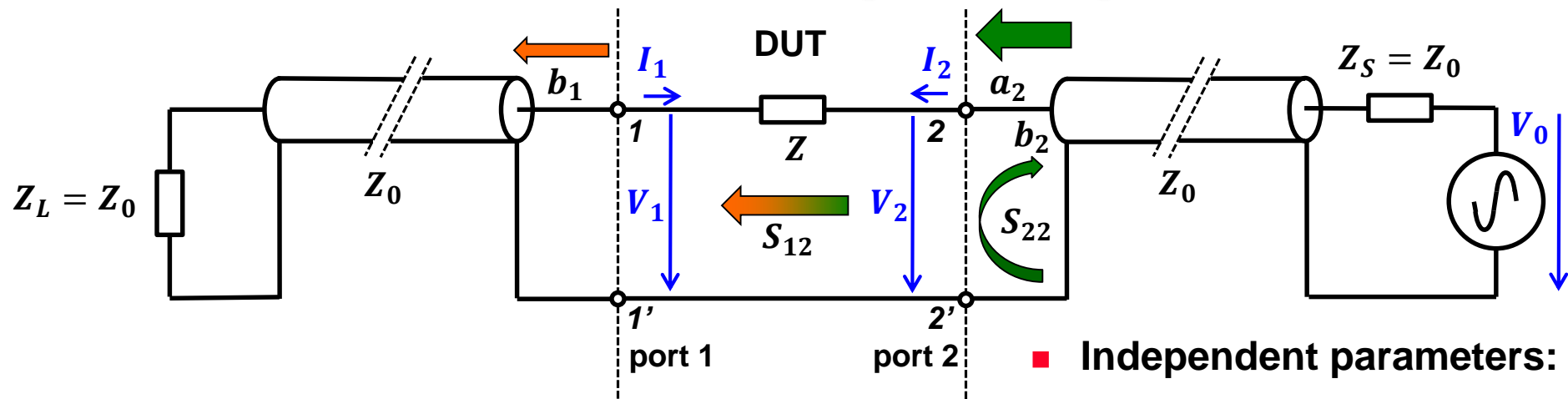
$$a_1 = \frac{V_1^{inc}}{\sqrt{Z_0}} = \frac{V_1 + I_1 Z_0}{2\sqrt{Z_0}}$$

- Dependent parameters:

$$b_1 = \frac{V_1^{refl}}{\sqrt{Z_0}} = \frac{V_1 - I_1 Z_0}{2\sqrt{Z_0}}$$

$$b_2 = \frac{V_2^{refl}}{\sqrt{Z_0}} = \frac{V_2 - I_2 Z_0}{2\sqrt{Z_0}}$$

S-Parameters – Example: 2-port DUT



◆ Analysis of the reverse S-parameters:

$$S_{22} = \left. \frac{b_2}{a_2} \right|_{a_1=0} \equiv \text{output reflection coefficient} \\ (Z_L = Z_0 \Rightarrow a_1 = 0)$$

$$S_{12} = \left. \frac{b_1}{a_2} \right|_{a_1=0} \equiv \text{backward transmission gain}$$

$$a_2 = \frac{V_2^{inc}}{\sqrt{Z_0}} = \frac{V_2 + I_2 Z_0}{2\sqrt{Z_0}}$$

■ Dependent parameters:

$$b_1 = \frac{V_1^{refl}}{\sqrt{Z_0}} = \frac{V_1 - I_1 Z_0}{2\sqrt{Z_0}}$$

$$b_2 = \frac{V_2^{refl}}{\sqrt{Z_0}} = \frac{V_2 - I_2 Z_0}{2\sqrt{Z_0}}$$

- *n*-port DUTs still can be fully characterized with a 2-port VNA, but again: **don't forget to terminate unused ports!**

S-Parameters – Definition (1)

◆ Linear equations for the 2-port DUT:

$$\begin{aligned}b_1 &= S_{11}a_1 + S_{12}a_2 \\b_2 &= S_{21}a_1 + S_{22}a_2\end{aligned}$$

■ with:

$$S_{11} = \left. \frac{b_1}{a_1} \right|_{a_2=0} \equiv \text{input reflection coefficient}$$

$$S_{22} = \left. \frac{b_2}{a_2} \right|_{a_1=0} \equiv \text{output reflection coefficient}$$

$$S_{21} = \left. \frac{b_2}{a_1} \right|_{a_2=0} \equiv \text{forward transmission gain}$$

$$S_{12} = \left. \frac{b_1}{a_2} \right|_{a_1=0} \equiv \text{backward transmission gain}$$

impedance
measurements

transmission
measurements

S-Parameters – Definition (2)

- ◆ Reflection coefficient and impedance at the n^{th} -port of a DUT:

$$S_{nn} = \frac{b_n}{a_n} = \frac{\frac{V_n}{I_n} - Z_0}{\frac{V_n}{I_n} + Z_0} = \frac{Z_n - Z_0}{Z_n + Z_0} = \Gamma_n$$

$$Z_n = Z_0 \frac{1 + S_{nn}}{1 - S_{nn}} \text{ with } Z_n = \frac{V_n}{I_n} \text{ being the input impedance at the } n^{\text{th}} \text{ port}$$

- ◆ Power reflection and transmission for a n -port DUT

$$|S_{nn}|^2 = \frac{\text{power reflected from port } n}{\text{power incident on port } n}$$

$$|S_{nm}|^2 = \text{transmitted power between ports } n \text{ and } m$$

with all ports terminated in their characteristic impedance Z_0
and $Z_S = Z_0$

Here the US notion is used, where power = $|a|^2$.
European notation (often): power = $|a|^2/2$
These conventions have no impact on the S-parameters,
they are only relevant for absolute power calculations

The Scattering Matrix (1)

- ◆ Waves traveling towards the n -port: $(a) = (a_1, a_2, a_3, \dots a_n)$
- ◆ Waves traveling away from the n -port: $(b) = (b_1, b_2, b_3, \dots b_n)$

- ◆ The relation between a_i and b_i ($i = 1..n$) can be written as a system of n linear equations

(a_i = the independent variable, b_i = the dependent variable)

one - port	$b_1 = S_{11}a_1 + S_{12}a_2 + S_{13}a_3 + S_{14}a_4 + K$
two - port	$b_2 = S_{21}a_1 + S_{22}a_2 + S_{23}a_3 + S_{24}a_4 + K$
three - port	$b_3 = S_{31}a_1 + S_{32}a_2 + S_{33}a_3 + S_{34}a_4 + K$
four - port	$b_4 = S_{41}a_1 + S_{42}a_2 + S_{43}a_3 + S_{44}a_4 + K$

- ◆ In compact matrix form follows

$$(b) = (S)(a)$$

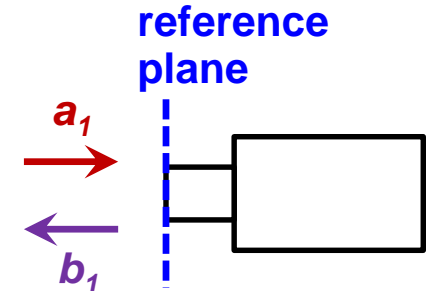
The Scattering Matrix (2)

- ◆ The simplest form is a passive **1-port (2-pole)**

$$(S) = S_{11} \Rightarrow b_1 = S_{11} a_1$$

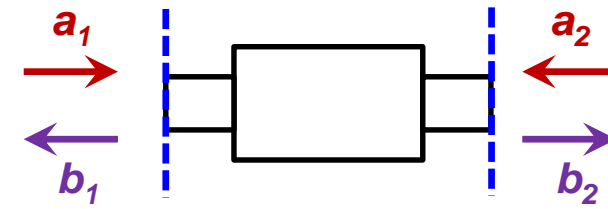
- with the **reflection coefficient**:

$$\Gamma = S_{11} = \frac{b_1}{a_1}$$



- ◆ **2-port (4-pole) DUT:**

$$(S) = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \Rightarrow \begin{aligned} b_1 &= S_{11} a_1 + S_{12} a_2 \\ b_2 &= S_{21} a_1 + S_{22} a_2 \end{aligned}$$



- An unmatched load, present at port 2 with a **reflection coefficient** Γ_{load} transfers to the input port as:

$$\Gamma_{in} = S_{11} + \frac{S_{21} \Gamma_{load} S_{12}}{1 - S_{22} \Gamma_{load}}$$

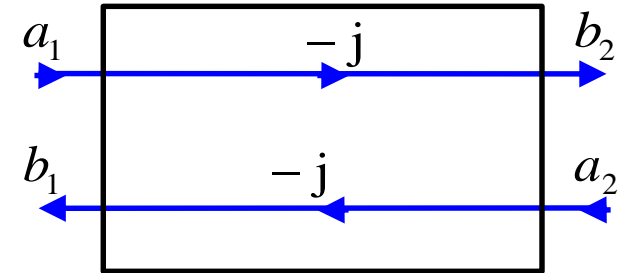
2-Port Examples

Transmission-line of $Z=50\Omega$, length $l=\lambda/4$

$$(S) = \begin{bmatrix} 0 & -j \\ -j & 0 \end{bmatrix} \quad \begin{aligned} b_1 &= -j a_2 \\ b_2 &= -j a_1 \end{aligned}$$

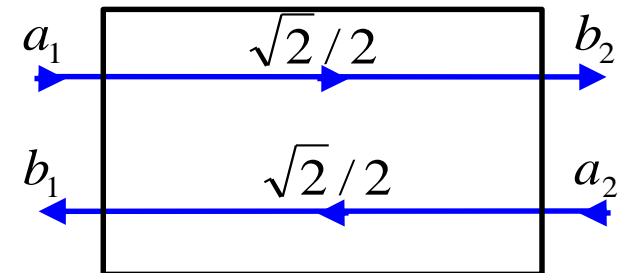
Port 1:

Port 2:



Attenuator 3dB, i.e. half output power

$$(S) = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \begin{aligned} b_1 &= \frac{1}{\sqrt{2}} a_2 = 0.707 a_2 \\ b_2 &= \frac{1}{\sqrt{2}} a_1 = 0.707 a_1 \end{aligned}$$



RF Transistor, amplifier

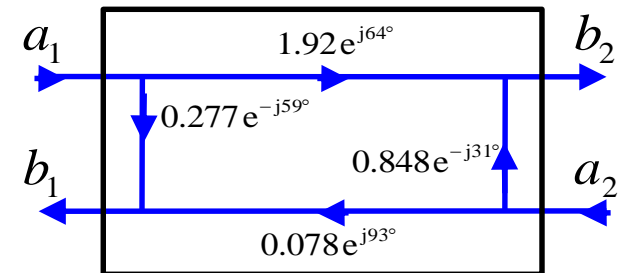
$$(S) = \begin{bmatrix} 0.277 e^{-j59^\circ} & 0.078 e^{j93^\circ} \\ 1.92 e^{j64^\circ} & 0.848 e^{-j31^\circ} \end{bmatrix}$$

backward transmission

forward transmission

non-reciprocal since $S_{12} \neq S_{21}$!

=different transmission forwards and backwards



Examples of 3-ports (1)

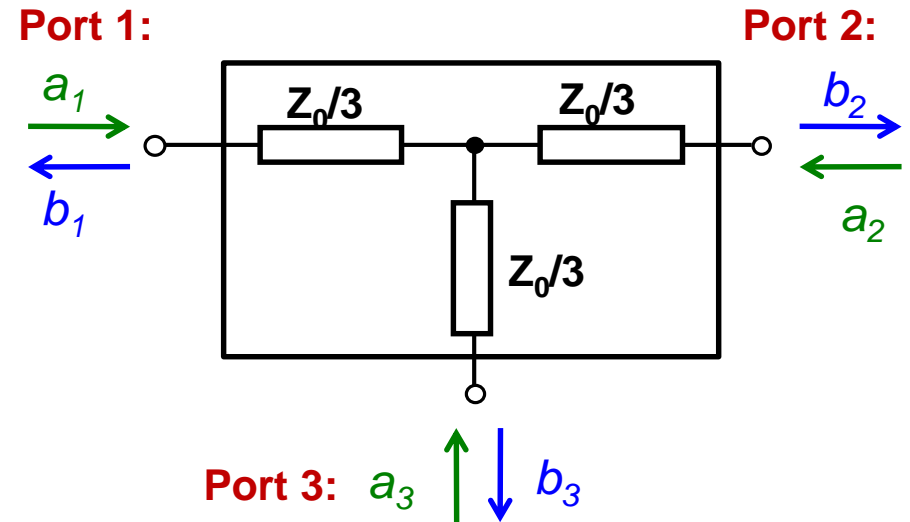
Resistive power divider

$$(S) = \frac{1}{2} \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$b_1 = \frac{1}{2}(a_2 + a_3)$$

$$b_2 = \frac{1}{2}(a_1 + a_3)$$

$$b_3 = \frac{1}{2}(a_1 + a_2)$$



3-port circulator

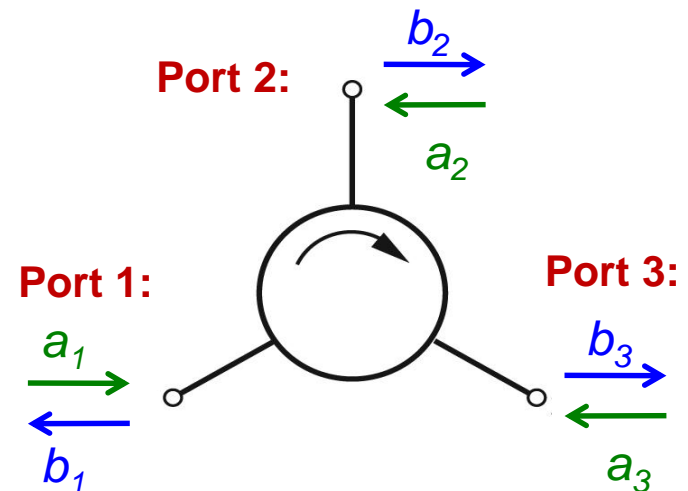
$$(S) = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$b_1 = a_3$$

$$b_2 = a_1$$

$$b_3 = a_2$$

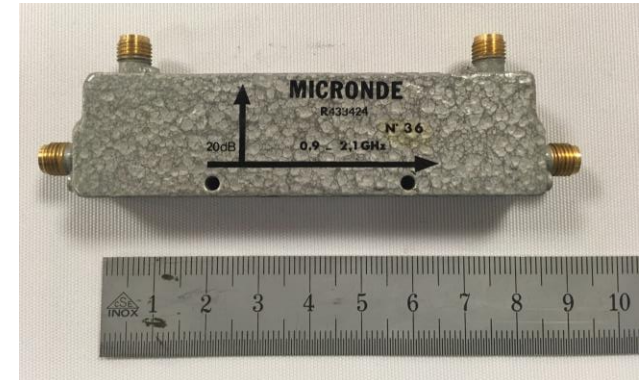
The ideal circulator is lossless, matched at all ports, but not reciprocal. A signal entering the ideal circulator at one port is transmitted exclusively to the next port in the sense of the arrow.



4-Port Examples (1)

Ideal directional coupler

$$(S) = \begin{bmatrix} 0 & jk & \sqrt{1-k^2} & 0 \\ jk & 0 & 0 & \sqrt{1-k^2} \\ \sqrt{1-k^2} & 0 & 0 & jk \\ 0 & \sqrt{1-k^2} & jk & 0 \end{bmatrix} \text{ with } k = \left| \frac{b_2}{a_1} \right|$$

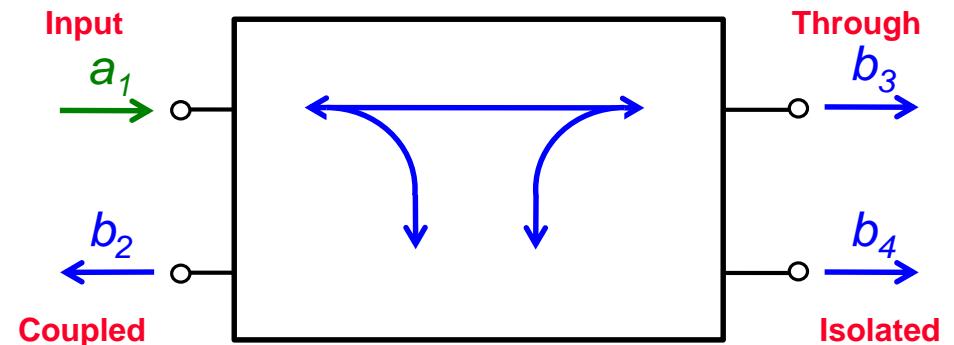


To characterize directional couplers, three important figures are used:

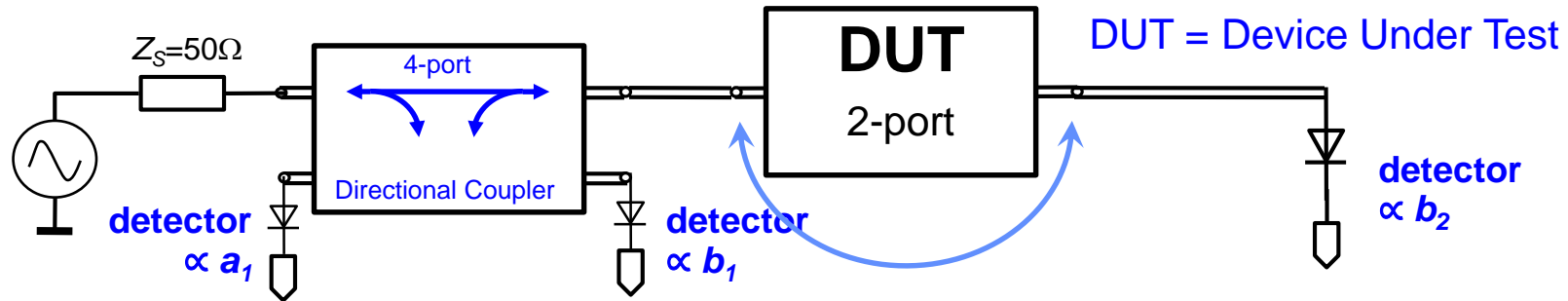
the coupling $C = -20 \log_{10} \left| \frac{b_2}{a_1} \right|$

the directivity $D = -20 \log_{10} \left| \frac{b_4}{b_2} \right|$

the isolation $I = -20 \log_{10} \left| \frac{a_1}{b_4} \right|$



How to measure S-Parameters?



◆ Performed in the frequency domain

- Single or swept frequency generator, stand-alone or as part of a VNA or SA
- Requires a **directional coupler** and RF detector(s) or receiver(s)

◆ Evaluate S_{11} and S_{21} of a 2-port DUT

- Ensure $a_2=0$, i.e. the detector at port 2 offers a well matched impedance
- Measure incident wave a_1 and reflected wave b_1 at the directional coupler ports and compute for each frequency
- Measure transmitted wave b_2 at DUT port 2 and compute

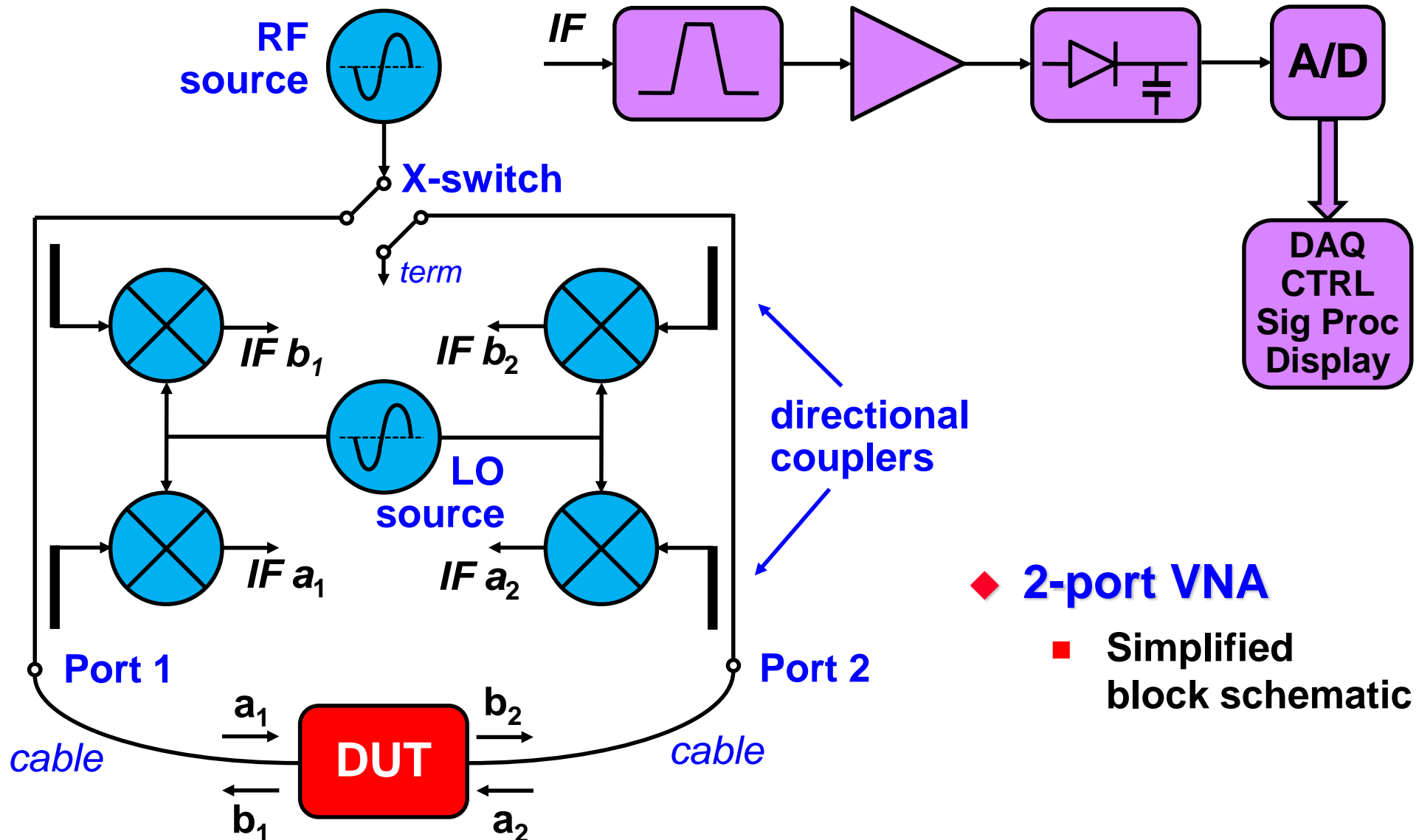
$$S_{11} = \left. \frac{b_1}{a_1} \right|_{a_2=0}$$

$$S_{21} = \left. \frac{b_2}{a_1} \right|_{a_2=0}$$

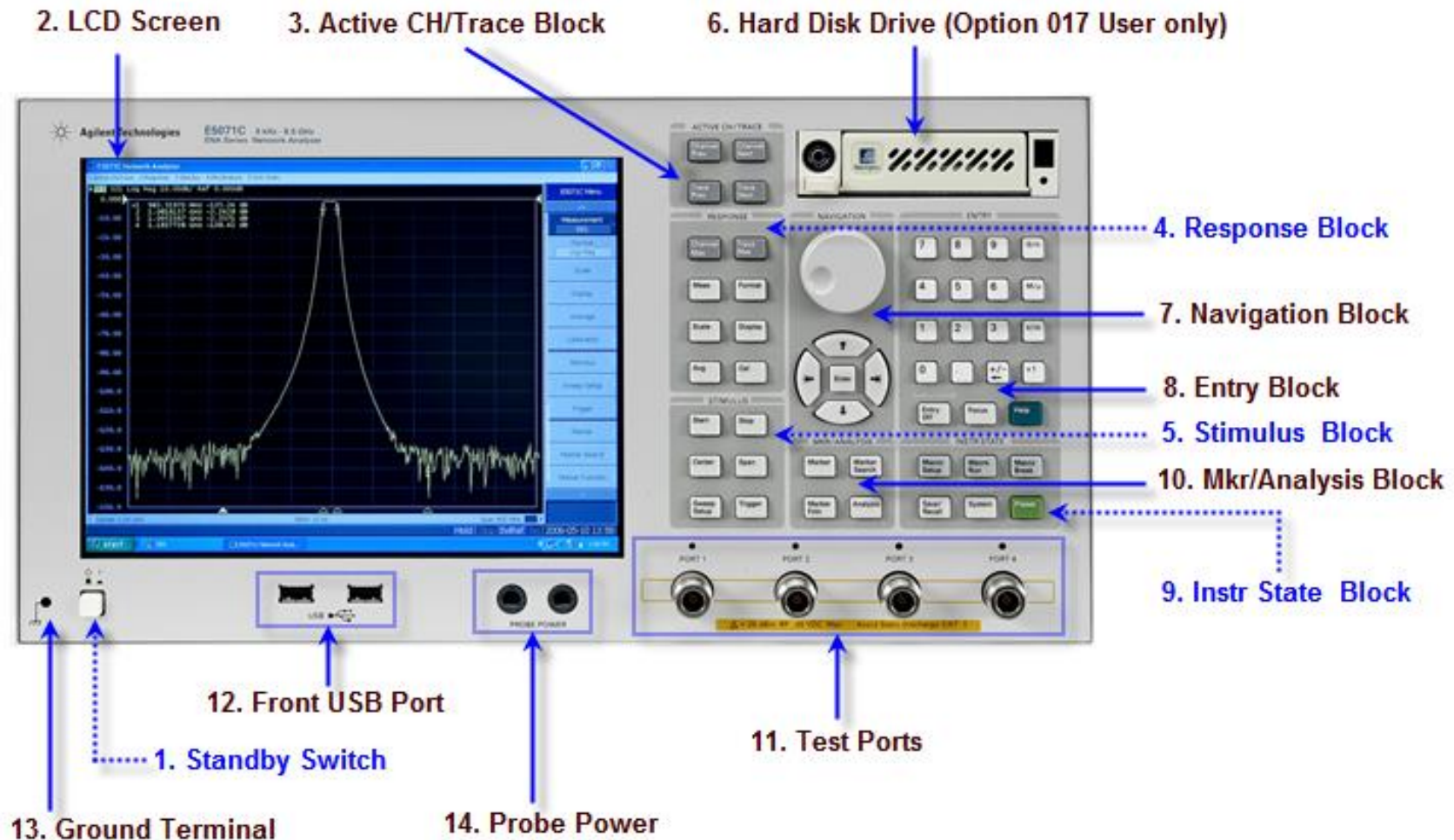
◆ Evaluate S_{22} and S_{12} of the 2-port DUT

- Perform the same methodology as above by exchanging the measurement equipment on the DUT ports

Vector Network Analyzer (VNA)



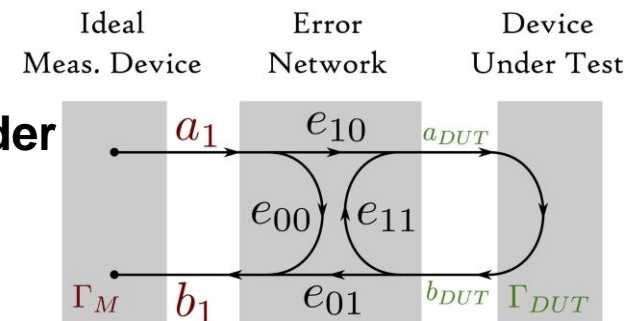
Fun with the VNA!



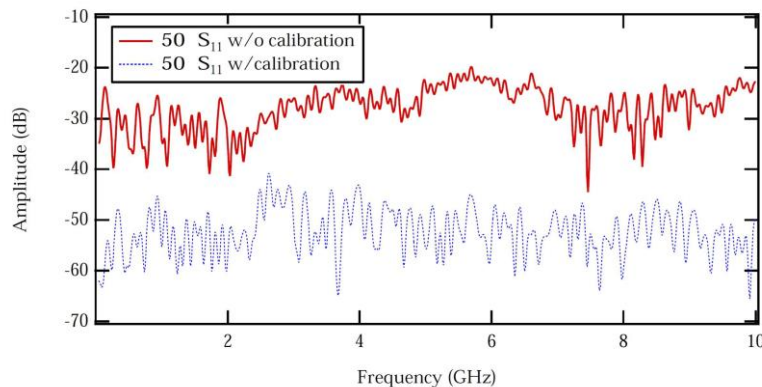
- ◆ **VNAs vary between manufacturers and models**
 - Concepts and operation are quite similar

VNA Calibration

- ◆ Calibration is important to minimize imperfections of the VNA including the connection cables to the DUT
 - Calibration is not necessary for pure frequency or phase measurements
- ◆ Before calibrating the VNA measurement setup, perform a brief measurement and chose appropriate VNA settings:
 - Frequency range (center, span or start, stop)
 - Number of frequency points
 - Can be sometimes increased by rearranging the VNA memory (# of channels)
 - IF filter bandwidth
 - Output power level
- ◆ Calibrate the setup, preferable with an electronic calibration system if more than 2 ports are used!
 - Each port and combination needs to be calibrated, with the cables attached
 - Choose the appropriate connector type and gender
 - The instrument establishes a correction matrix and displays the "CAL" status.



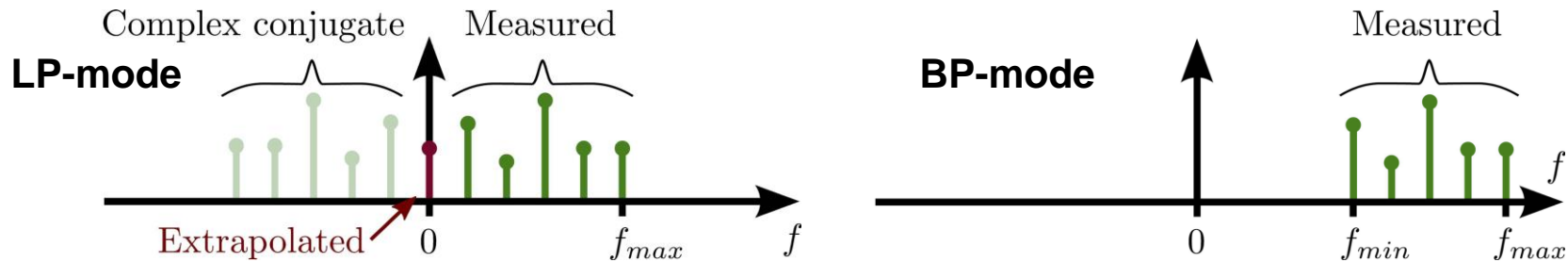
VNA Calibration



- ◆ **Calibration improves the measurement performance**
 - Return loss improvement by typically 20 dB. Enables mdB accuracy measurements
 - Full 2-port or 4-port calibration with manual calibration kits is prone to errors, better use electronic calibration systems.
 - Change VNA settings will cause the instrument to inter- and extrapolate, and the calibration status becomes uncertain.
- ◆ **Cables are included in the calibration**
 - However, changing coaxial connector types not.
 - Special VNA cables allows the adaption of different connector types and gender without requiring a re-calibration of the setup!
 - There are also various methods to de-embed cables, transmission-lines, connectors, etc. which cannot be calibrated (port extension, gating)

Synthetic Pulse TD Measurements (1)

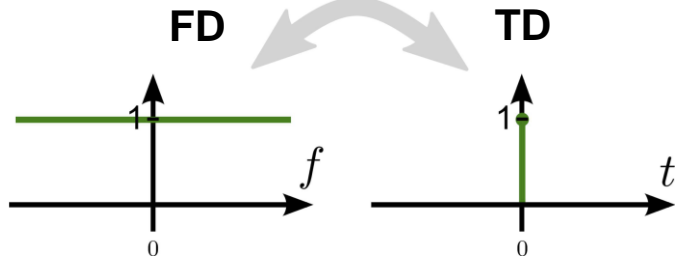
- ◆ Based on an inverse discrete *Fourier* transformation (iDFT) option in the VNA



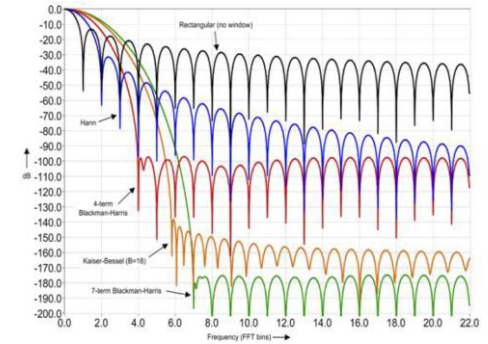
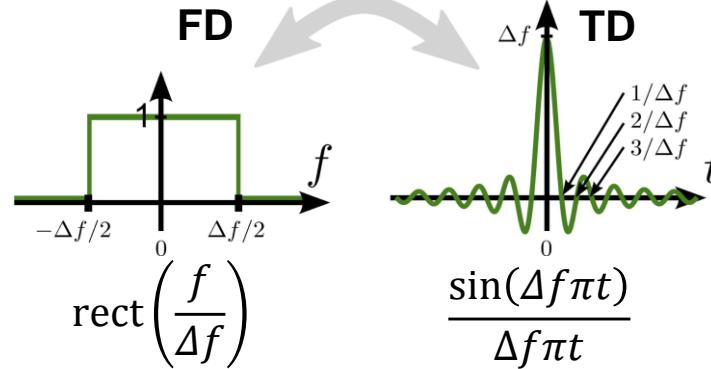
- Low-pass mode: Impulse or step response, relying on equidistant samples over the extrapolated (to DC) frequency range.
 - **The VNA does not measure at DC!**
 - **Manually match frequency range and # of points for DC extrapolation, e.g. 1...1000MHz -> 1001 points, to enable extrapolation exactly to DC, or let the instrument chose the extrapolation settings automatically**
- Band-pass response (no DC extrapolation)
- Enables time-domain reflectometry (TDR)
 - **Very useful on portable VNAs, troubleshooting RF cable problems**
- Allows time-domain gating and de-embedding of non-resonant sub-systems, e.g. on a PCB
- Limited to linear systems
- **Select the "real" format for S11 or S21 for time-domain transformations!**
 - **or dB magnitude to detect small reflections in TDR analysis**

Synthetic Pulse TD Measurements (2)

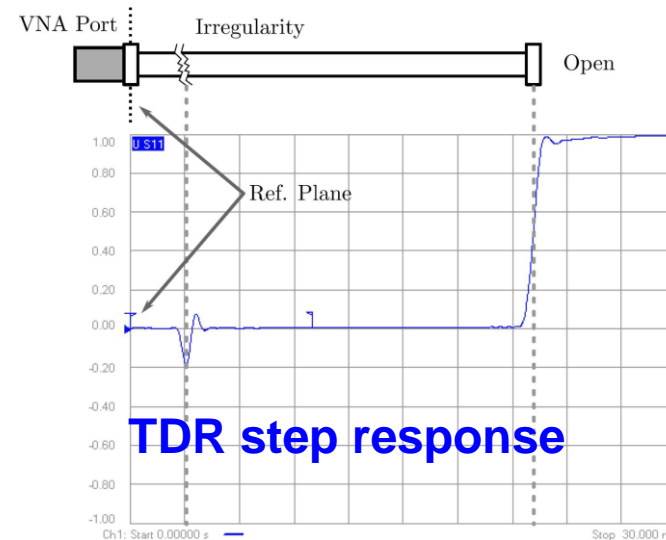
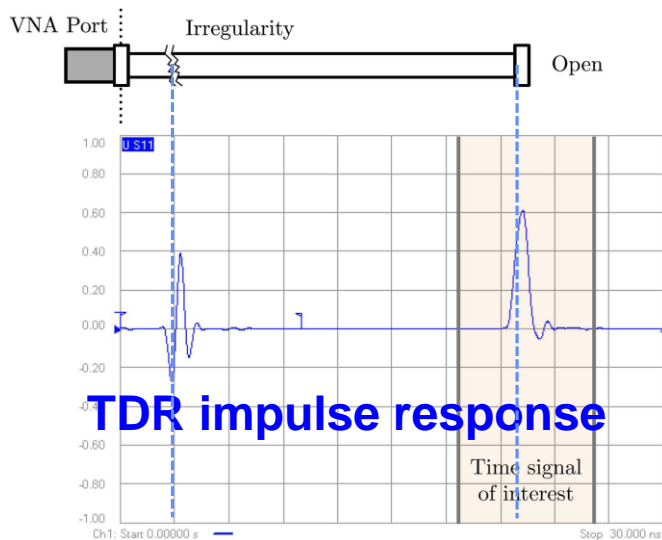
unlimited frequency range



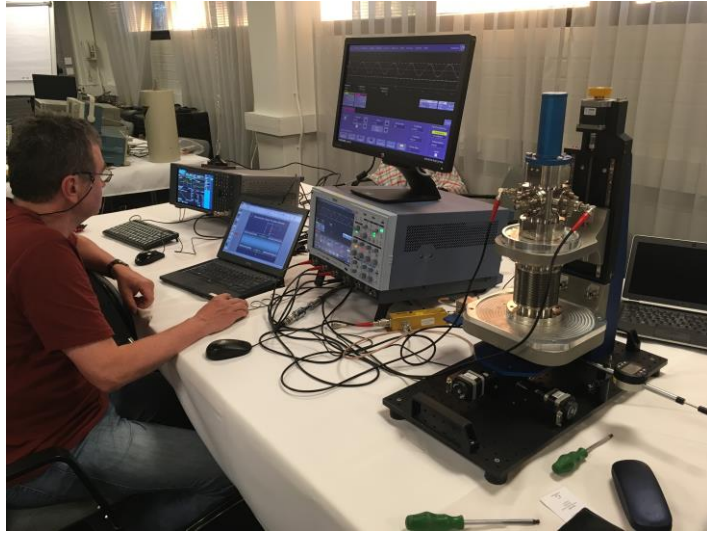
truncated frequency range



smoothing window functions

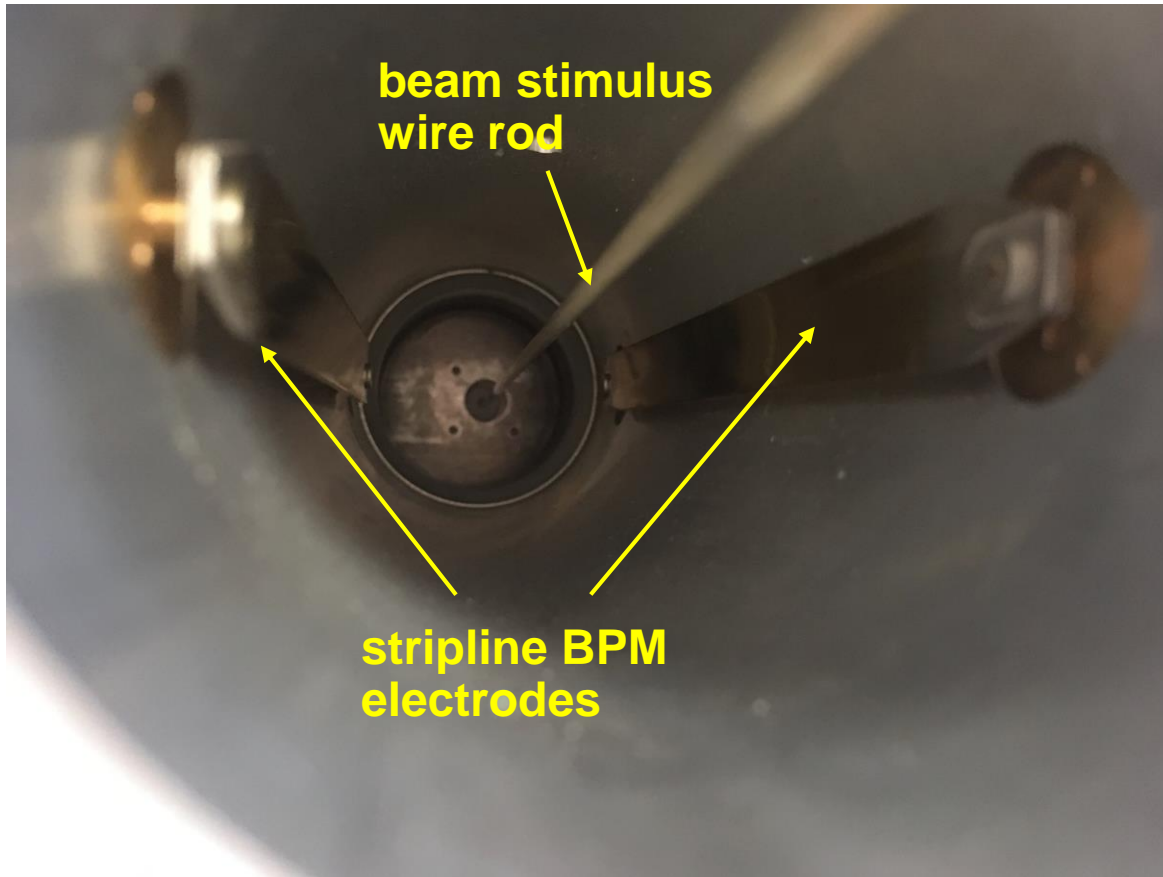


Wait awaits you in the RF lab...?!



...lots of FUN!

◆ 1 ton of RF and BI equipment and “goodies”



- RF measurements on BPMs and current monitors
- Use wires or metal rods with a pulse or RF signal as beam stimulus
 - TEM fields
- 6 experiments, all setup in room “AALTO”
 - 4 students per experiment
 - Max 1 hour / experiment
- “Learning by doing”
 - Not everything is perfectly prepared
 - Room for improvisation, creativity and improvements

I could not cover everything...

- ◆ **Smith chart**
 - A effective map that links impedance and reflection planes
- ◆ **Beam pickups and RF components**
 - Nothing on BPM pickups or wall current monitors, ...yet.
 - Could not cover RF components, but some you will discover in the afternoon RF lab session
- ◆ **Wakefields and beam impedance bench measurements**
 - Stretched wire method
 - HOM analysis
- ◆ **Other VNA goodies...**
 - 4-port VNA with "virtual" ports, e.g. differential S-parameters
 - Power sweep for 1-dB compression point RF amplifier analysis
- ◆ **Spectrum analysis**
 - Modulation, demodulation
 - Intermodulation measurements (IP3) and other non-linear effects
 - Noise and noise figure analysis
- ◆ **More information is given in the Appendix**

Appendix A: Definition of the Noise Figure

$$F = \frac{S_i / N_i}{S_o / N_o} = \frac{N_o}{GN_i} = \frac{N_o}{GkT_0B} = \frac{GN_i + N_R}{GkT_0B} = \frac{GkT_0B + N_R}{GkT_0B}$$

- ◆ F is the *Noise factor* of the receiver
- ◆ S_i is the available signal power at input
- ◆ $N_i = kT_0B$ is the available noise power at input
- ◆ T_0 is the absolute temperature of the source resistance
- ◆ N_o is the available noise power at the output, including amplified input noise
- ◆ N_r is the noise added by receiver
- ◆ G is the available receiver gain
- ◆ B is the effective noise bandwidth of the receiver
- ◆ If the noise factor is specified in a logarithmic unit, we use the term *Noise Figure (NF)*

$$NF = 10 \lg \frac{S_i / N_i}{S_o / N_o} \text{ dB}$$

Measurement of Noise Figure (using a calibrated Noise Source)

Calibrated T_H, T_C Source

DUT

Power Meter

$kT_H B \rightarrow$

$kT_C B \rightarrow$

$N_{OH} = FGkT_O B + (T_H - T_O)kBG$

$N_{OC} = FGkT_O B + (T_C - T_O)kBG$

$$\therefore Y = \frac{N_{OH}}{N_{OC}} = \frac{FT_O + T_H - T_O}{FT_O + T_C - T_O}; \quad F = \frac{\left(\frac{T_H}{T_O} - 1\right) - Y\left(\frac{T_C}{T_O} - 1\right)}{Y - 1}$$

Example:
 $T_H = 10,290^\circ\text{K}$ (argon source), $T_C = 300^\circ\text{K}$
 Measured Y factor: $Y = 9 \text{ dB}$ (7.94:1)
 Then,

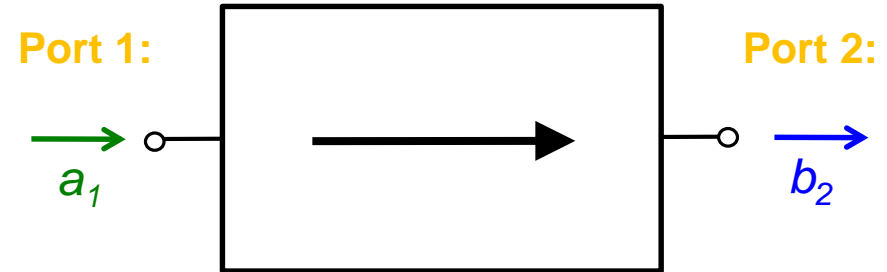
$$F = \frac{\frac{10290}{290} - 1 - 7.94\left(\frac{300}{290} - 1\right)}{7.94 - 1} = 4.94; \quad NF(\text{dB}) = 10 \log(4.94) = 6.9 \text{ dB}$$

Examples of 2-ports (2)

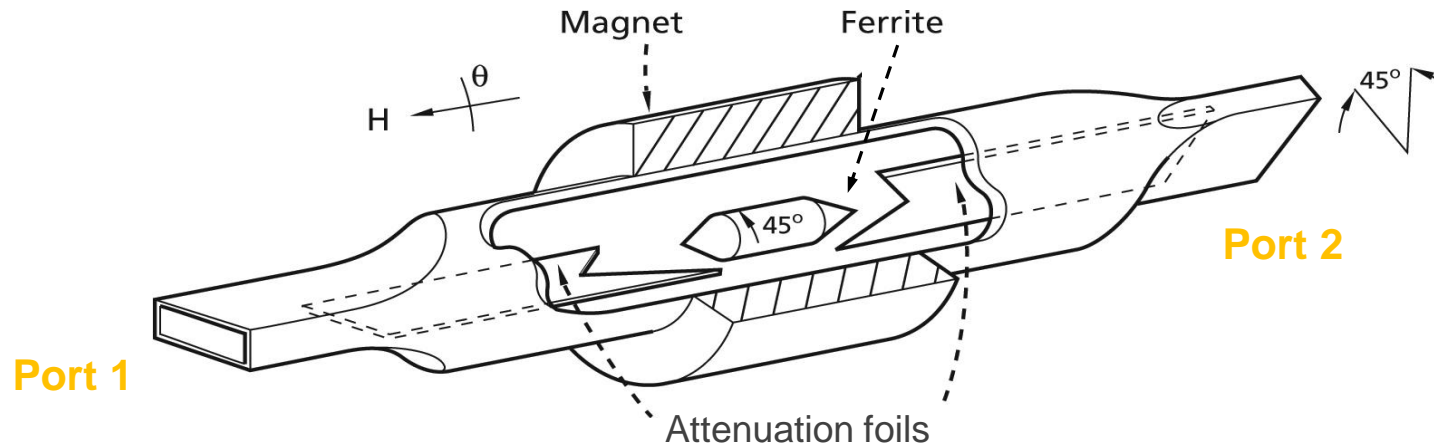
Ideal Isolator

$$(S) = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \quad b_2 = a_1$$

only forward transmission



Faraday rotation isolator



The left waveguide uses a TE_{10} mode (=vertically polarized H field). After transition to a circular waveguide, the polarization of the mode is rotated counter clockwise by 45° by a ferrite. Then follows a transition to another rectangular waveguide which is rotated by 45° such that the forward wave can pass unhindered. However, a wave coming from the other side will have its polarization rotated by 45° clockwise as seen from the right hand side.

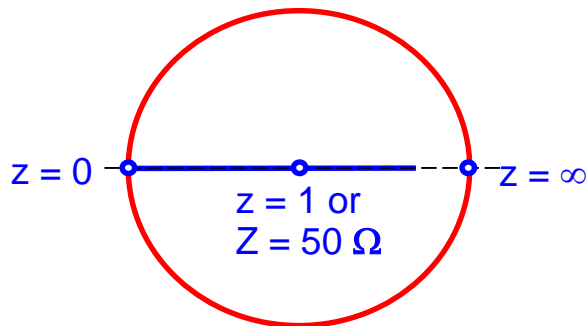
Pathing through a 2-port (1)

In general:

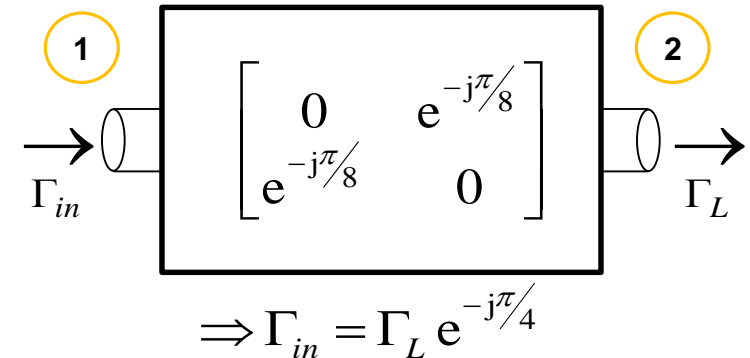
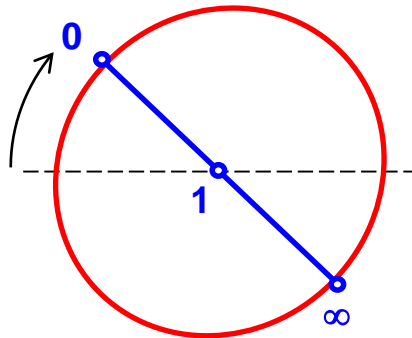
$$\Gamma_{in} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L}$$

where Γ_{in} is the reflection coefficient when looking through the 2-port and Γ_{load} is the load reflection coefficient.

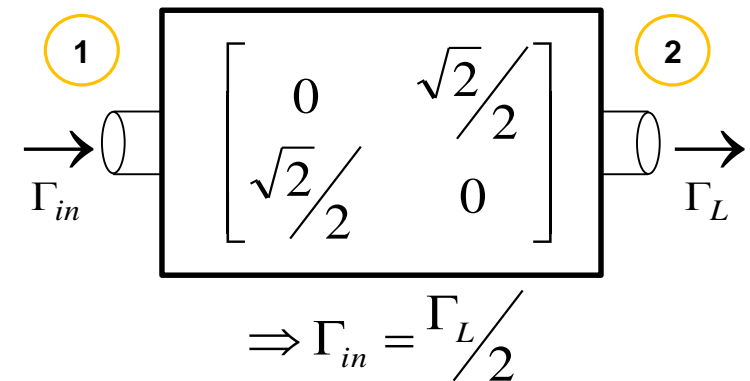
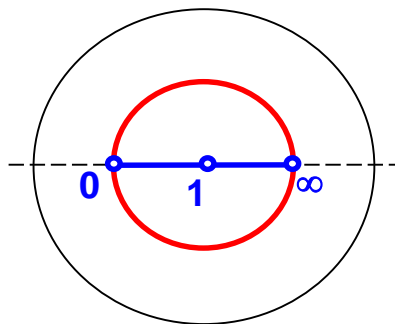
The outer circle and the real axis in the simplified Smith diagram below are mapped to other circles and lines, as can be seen on the right.



Line $\lambda/16$:

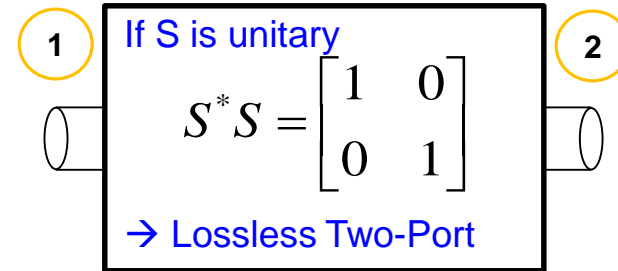
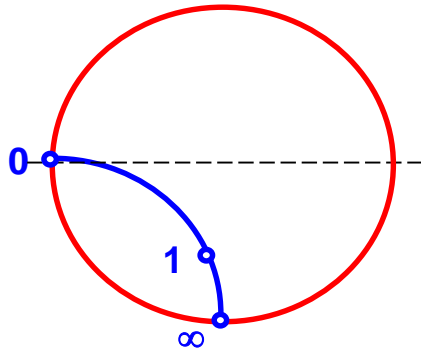


Attenuator 3dB:

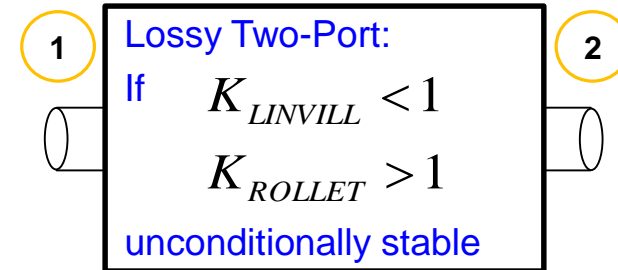
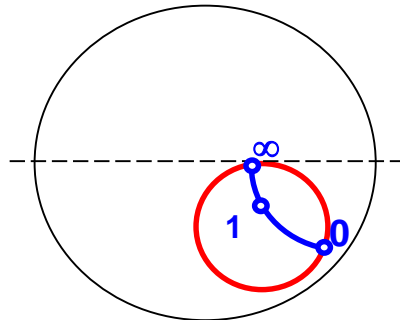


Pathing through a 2-port (2)

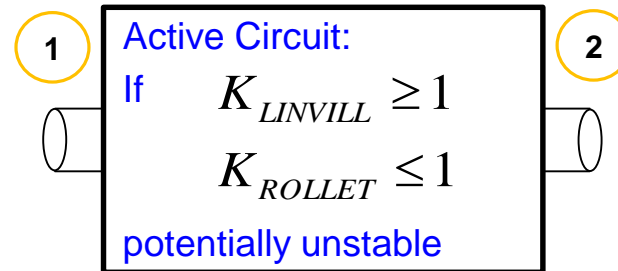
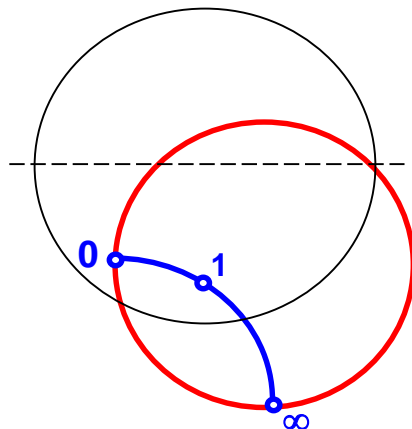
Lossless
Passive
Circuit



Lossy
Passive
Circuit



Active
Circuit



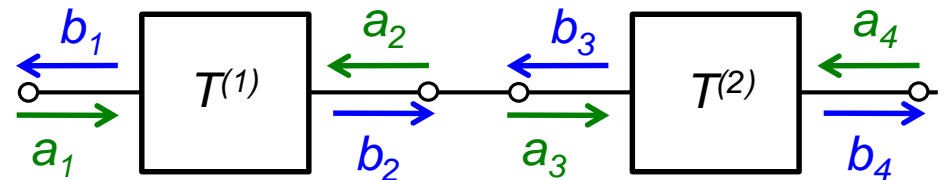
Appendix C: T matrix

The T-parameter matrix is related to the incident and reflected normalised waves at each of the ports.

$$\begin{pmatrix} b_1 \\ a_1 \end{pmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{pmatrix} a_2 \\ b_2 \end{pmatrix}$$

T-parameters may be used to determine the effect of a cascaded 2-port networks by simply multiplying the individual T-parameter matrices:

$$[T] = [T^{(1)}][T^{(2)}] \dots [T^{(N)}] = \prod_N [T^{(i)}]$$



T-parameters can be directly evaluated from the associated S-parameters and vice versa.

From S to T:

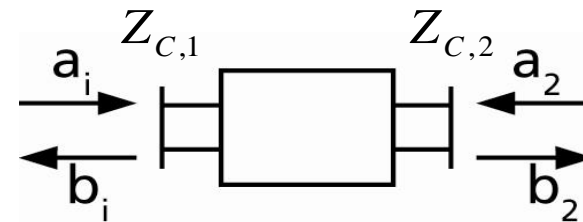
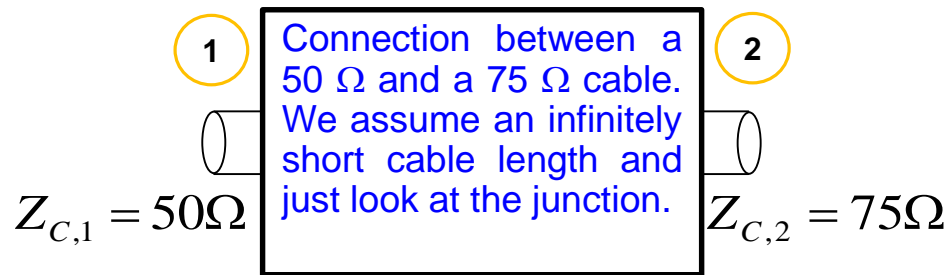
$$[T] = \frac{1}{S_{21}} \begin{bmatrix} -\det(S) & S_{11} \\ -S_{22} & 1 \end{bmatrix}$$

From T to S:

$$[S] = \frac{1}{T_{22}} \begin{bmatrix} T_{12} & \det(T) \\ 1 & -T_{21} \end{bmatrix}$$

Appendix D: A Step in Characteristic Impedance (1)

Consider a connection of two coaxial cables, one with $Z_{C,1} = 50 \Omega$ characteristic impedance, the other with $Z_{C,2} = 75 \Omega$ characteristic impedance.



Step 1: Calculate the reflection coefficient and keep in mind: all ports have to be terminated with their respective characteristic impedance, i.e. 75Ω for port 2.

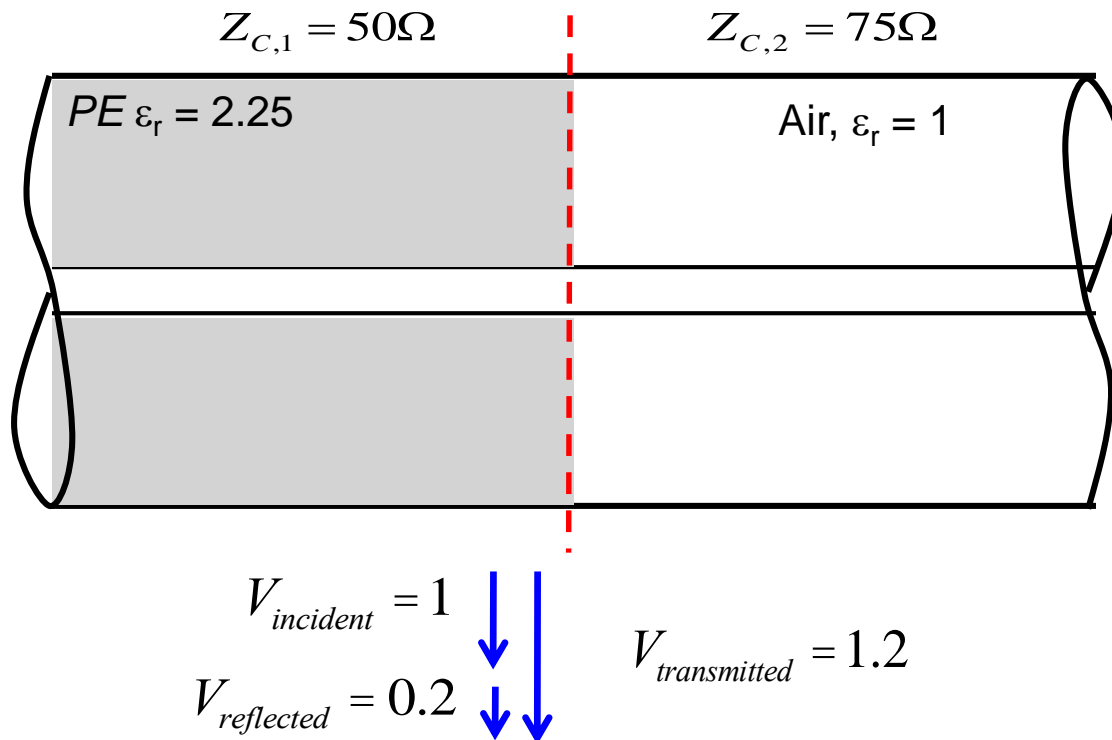
$$\Gamma_1 = \frac{Z - Z_{C,1}}{Z + Z_{C,1}} = \frac{75 - 50}{75 + 50} = 0.2$$

Thus, the voltage of the reflected wave at port 1 is 20% of the incident wave, and the reflected power at port 1 (proportional Γ^2) is $0.2^2 = 4\%$. As this junction is lossless, the transmitted power must be 96% (conservation of energy). From this we can deduce $b_2^2 = 0.96$. **But: how do we get the voltage of this outgoing wave?**

Example: a Step in Characteristic Impedance (2)

Step 2: Remember, a and b are **power-waves**, and defined as voltage of the forward- or backward traveling wave normalized to $\sqrt{Z_c}$.

The tangential electric field in the dielectric in the $50\ \Omega$ and the $75\ \Omega$ line, respectively, must be continuous.



t = voltage transmission coefficient, in this case: $t = 1 + \Gamma$

This is counterintuitive, one might expect $1 - \Gamma$. Note that the voltage of the transmitted wave is higher than the voltage of the incident wave. But we have to normalize to $\sqrt{Z_c}$ to evaluate the corresponding S-parameter. $S_{12} = S_{21}$ via reciprocity! But $S_{11} \neq S_{22}$, i.e. the structure is NOT symmetric.

Example: a Step in Characteristic Impedance (3)

Once we have determined the voltage transmission coefficient, we have to normalize to the ratio of the characteristic impedances, respectively. Thus we get for

$$S_{12} = 1.2 \sqrt{\frac{50}{75}} = 1.2 \cdot 0.816 = 0.9798$$

We know from the previous calculation that the reflected power (proportional Γ^2) is 4% of the incident power. Thus 96% of the power are transmitted.

Check done $S_{12}^2 = 1.44 \frac{1}{1.5} = 0.96 = (0.9798)^2$

$$S_{22} = \frac{50 - 75}{50 + 75} = -0.2 \quad \text{To be compared with } S_{11} = +0.2!$$

Example: a Step in Characteristic Impedance (4)

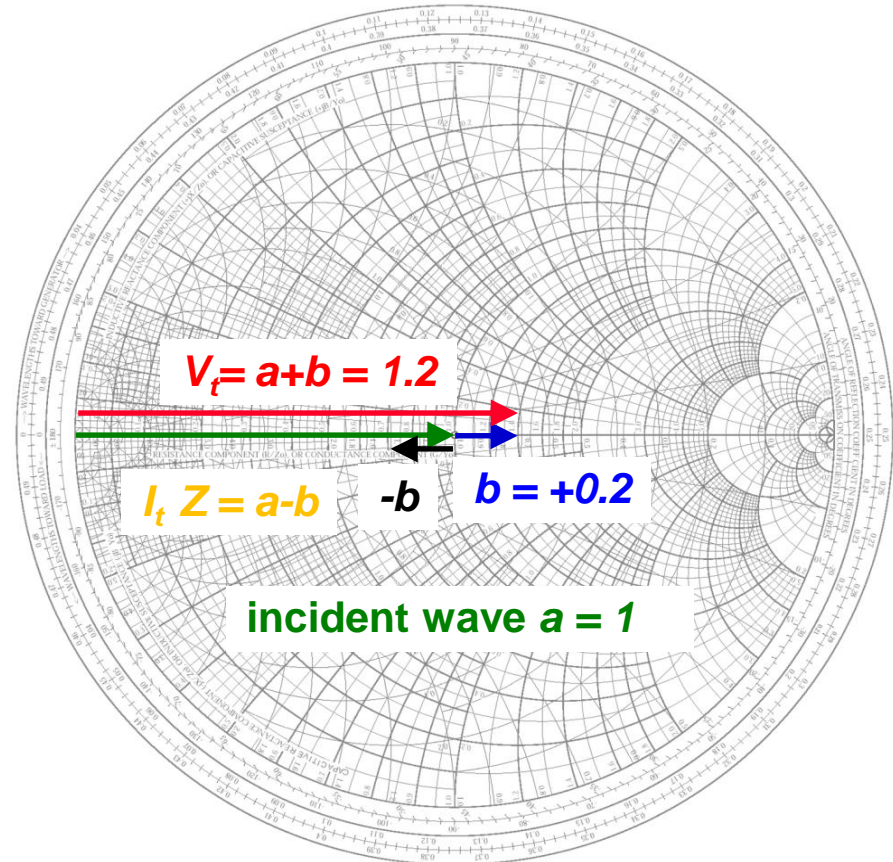
Visualization in the Smith chart:

As shown in the previous slides the voltage of the transmitted wave is

$V_t = a + b$, with $t = 1 + \Gamma$
and subsequently the current is

$I_t Z = a - b$.

Remember: the reflection coefficient Γ is defined with respect to voltages. For currents the sign inverts. Thus a positive reflection coefficient in the normal definition leads to a subtraction of currents or is negative with respect to current.

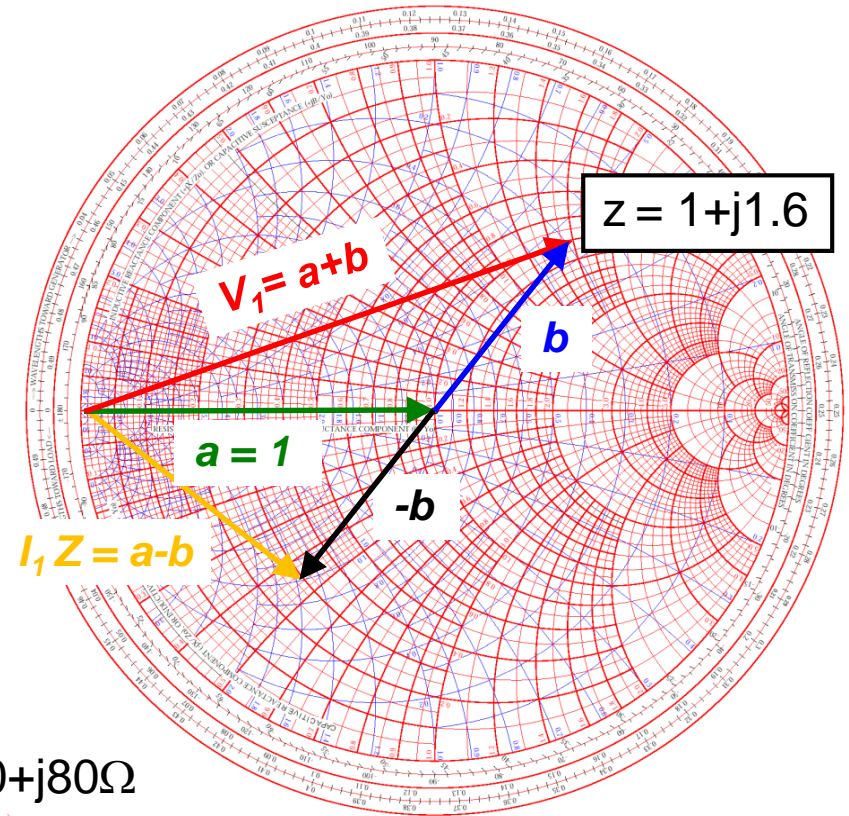
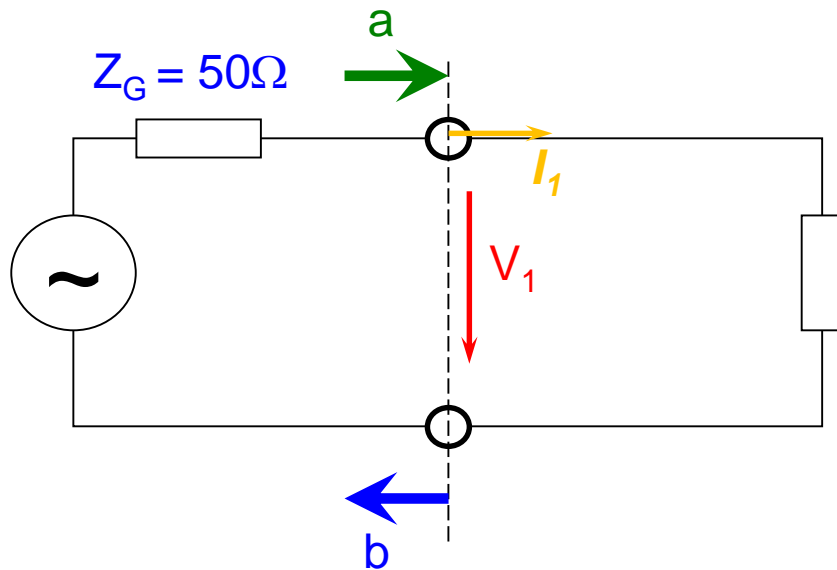


Note: here Z_{load} is real

Example: a Step in Characteristic Impedance (5)

General case:

Thus we can read from the Smith chart immediately the amplitude and phase of voltage and current on the load (of course we can calculate it when using the complex voltage divider).



Appendix E: Navigation in the Smith Chart (1)

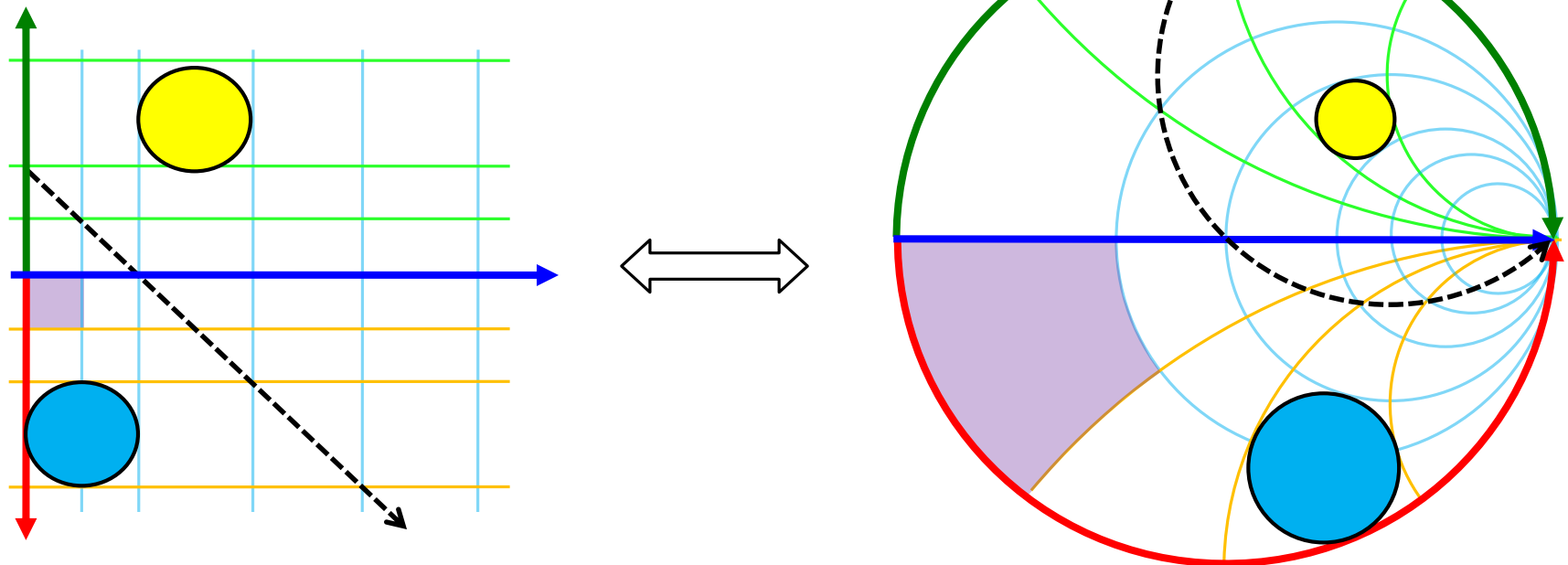
This is a “bilinear” transformation with the following properties:

- generalized circles are transformed into generalized circles
 - circle \rightarrow circle
 - straight line \rightarrow circle
 - circle \rightarrow straight line
 - straight line \rightarrow straight line
- angles are preserved locally

a straight line is nothing else than
a circle with infinite radius

a circle is defined by 3 points

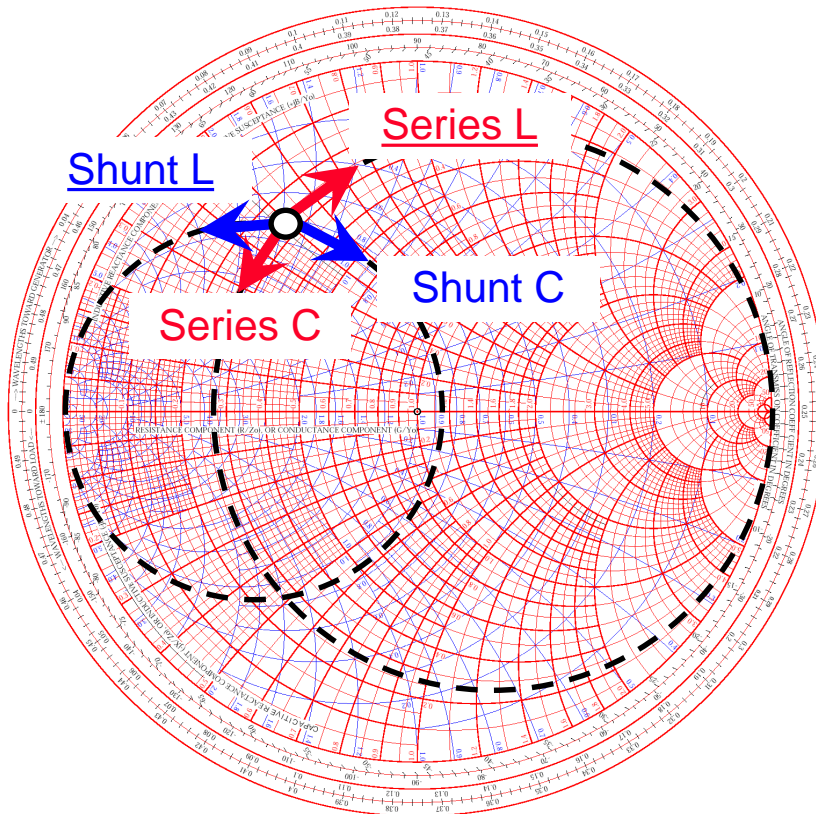
a straight line is defined by 2
points



Navigation in the Smith Chart (2)

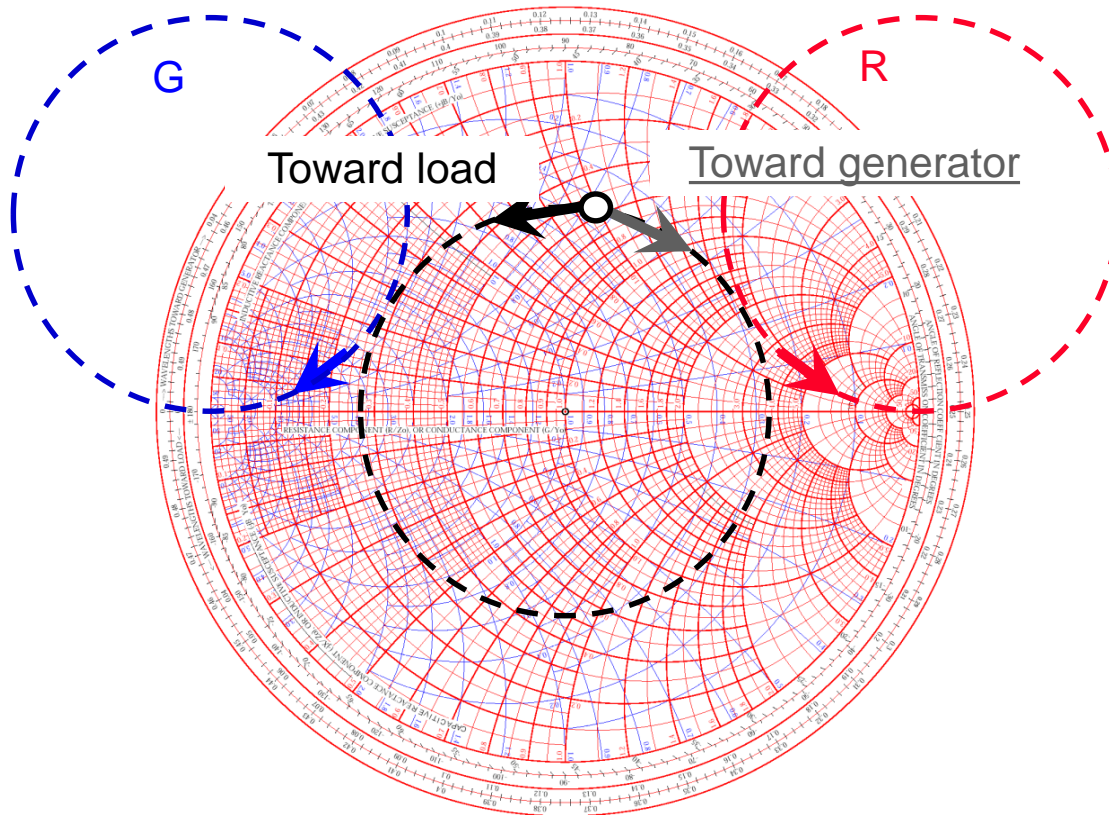
in blue: Impedance plane ($=Z$)

in red: Admittance plane ($=Y$)



	<u>Up</u>	Down
Red circles	<u>Series L</u>	Series C
Blue circles	<u>Shunt L</u>	Shunt C

Navigation in the Smith Chart (3)



Red arcs	Resistance R
Blue arcs	Conductance G
Concentric circle	Transmission line going Toward load <u>Toward generator</u>

Appendix F: The RF diode (1)

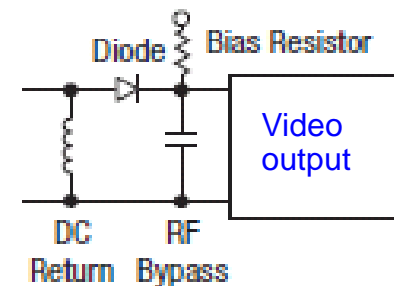
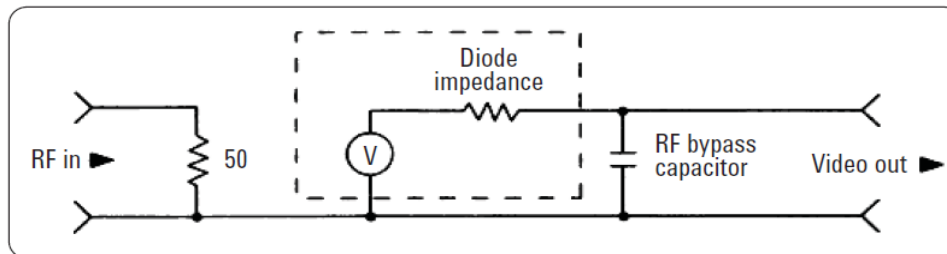
- ◆ We are not discussing the generation of RF signals here, just the detection
- ◆ Basic tool: fast RF* diode (= Schottky diode)
- ◆ In general, Schottky diodes are fast but still have a voltage dependent junction capacity (metal – semi-conductor junction)



A typical RF detector diode

Try to guess from the type of the connector which side is the RF input and which is the output

Equivalent circuit:



*Please note, in this lecture we will use RF (radio-frequency) for both, the RF and the microwave range, since there is no defined borderline between the RF and microwave regime.

The RF diode (2)

◆ Characteristics of a diode:

The current as a function of the voltage for a barrier diode can be described by the Richardson equation:

$$I = AA^{**} \exp\left(-\frac{q\phi_B}{kT}\right) \left[\exp\left(\frac{qV}{nKT}\right) - 1\right]$$

where

A = area (cm²)

A^{**} = modified Richardson constant (amp/oK)²/cm²)

k = Boltzman's Constant

T = absolute temperature (°K)

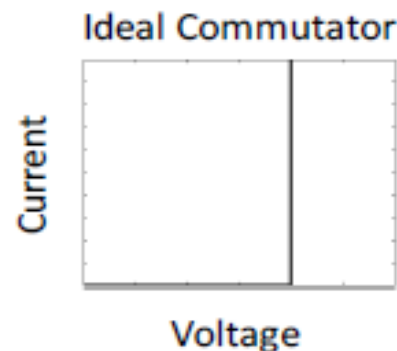
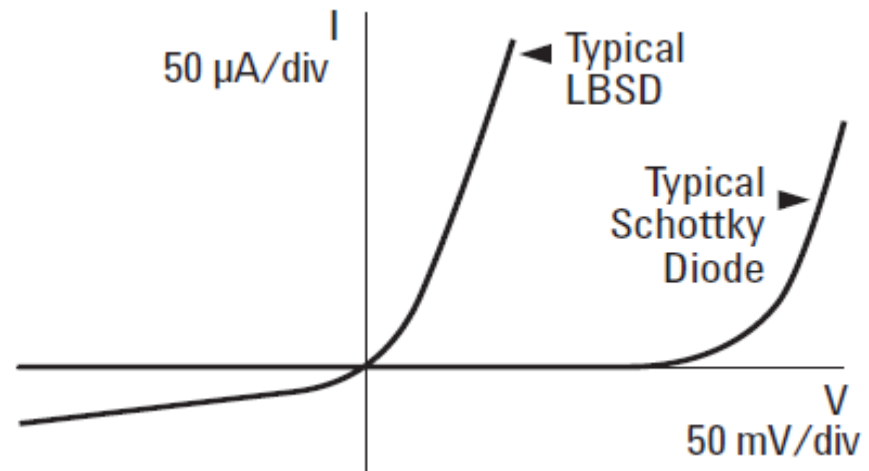
φ_B = barrier heights in volts

V = external voltage across the depletion layer
(positive for forward voltage) - V - IR_S

R_S = series resistance

I = diode current in amps (positive forward current)

n = ideality factor



◆ The RF diode is NOT an ideal commutator for small signals! We cannot apply big signals otherwise burnout

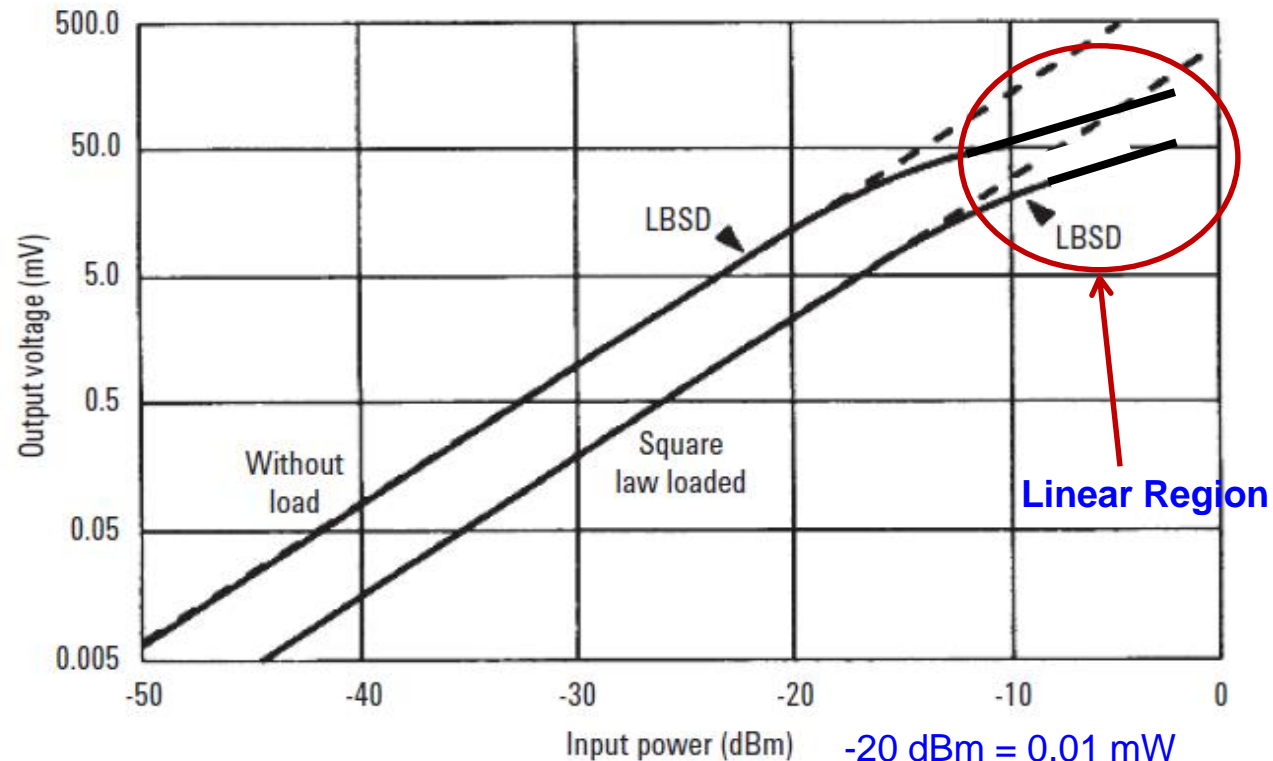
The RF diode (3)

- ◆ This diagram depicts the so called square-law region where the output voltage (V_{Video}) is proportional to the input power

Since the input power is proportional to the square of the input voltage (V_{RF}^2) and the output signal is proportional to the input power, this region is called square-law region.

In other words:

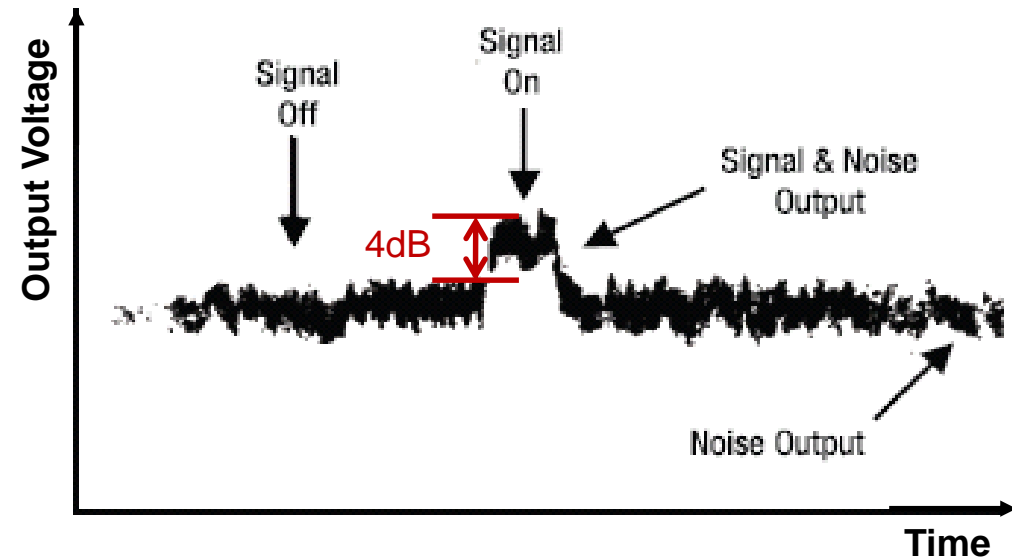
$$V_{\text{Video}} \sim V_{\text{RF}}^2$$



- ◆ The transition between the linear region and the square-law region is typically between -10 and -20 dBm RF power (see diagram).

The RF diode (5)

- ◆ Due to the square-law characteristic we arrive at the thermal noise region already for moderate power levels (-50 to -60 dBm) and hence the V_{Video} disappears in the thermal noise
- ◆ This is described by the term *tangential signal sensitivity* (TSS) where the detected signal (Observation BW, usually 10 MHz) is 4 dB over the thermal noise floor



Appendix G: The RF mixer (1)

- ◆ For the detection of very small RF signals we prefer a device that has a linear response over the full range (from 0 dBm (= 1mW) down to thermal noise = -174 dBm/Hz = $4 \cdot 10^{-21}$ W/Hz)
- ◆ It is called “RF mixer”, and uses 1, 2 or 4 diodes in different configurations (see next slide)
- ◆ Together with a so called LO (local oscillator) signal, the mixer works as a signal multiplier, providing a very high dynamic range since the output signal is always in the “linear range”, assuming the mixer is not in saturation with respect to the RF input signal (For the LO signal the mixer should always be in saturation!)
- ◆ The RF mixer is essentially a multiplier implementing the function

$f_1(t) \cdot f_2(t)$ with $f_1(t)$ = RF signal and $f_2(t)$ = LO signal

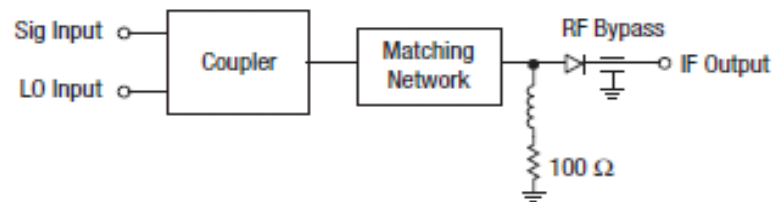
$$a_1 \cos(2\pi f_1 t + \varphi) \cdot a_2 \cos(2\pi f_2 t) = \frac{1}{2} a_1 a_2 [\cos((f_1 + f_2)t + \varphi) + \cos((f_1 - f_2)t + \varphi)]$$

- ◆ Thus we obtain a response at the IF (intermediate frequency) port as sum and difference frequencies of the LO and RF signals

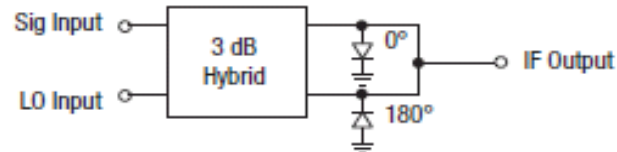
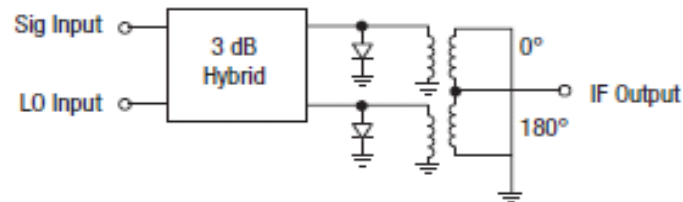
The RF mixer (2)

◆ Examples of different mixer configurations

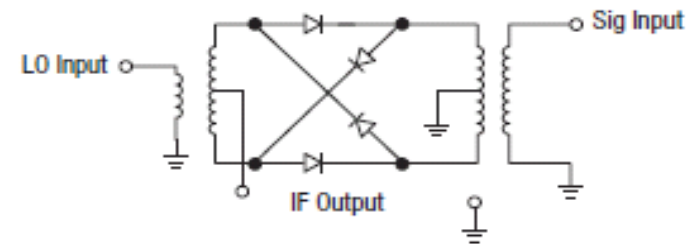
A. Single-Ended Mixer



B. Balanced Mixers



C. Double-Balanced Mixer



◆ A typical coaxial mixer (SMA connector)

The RF mixer (3)

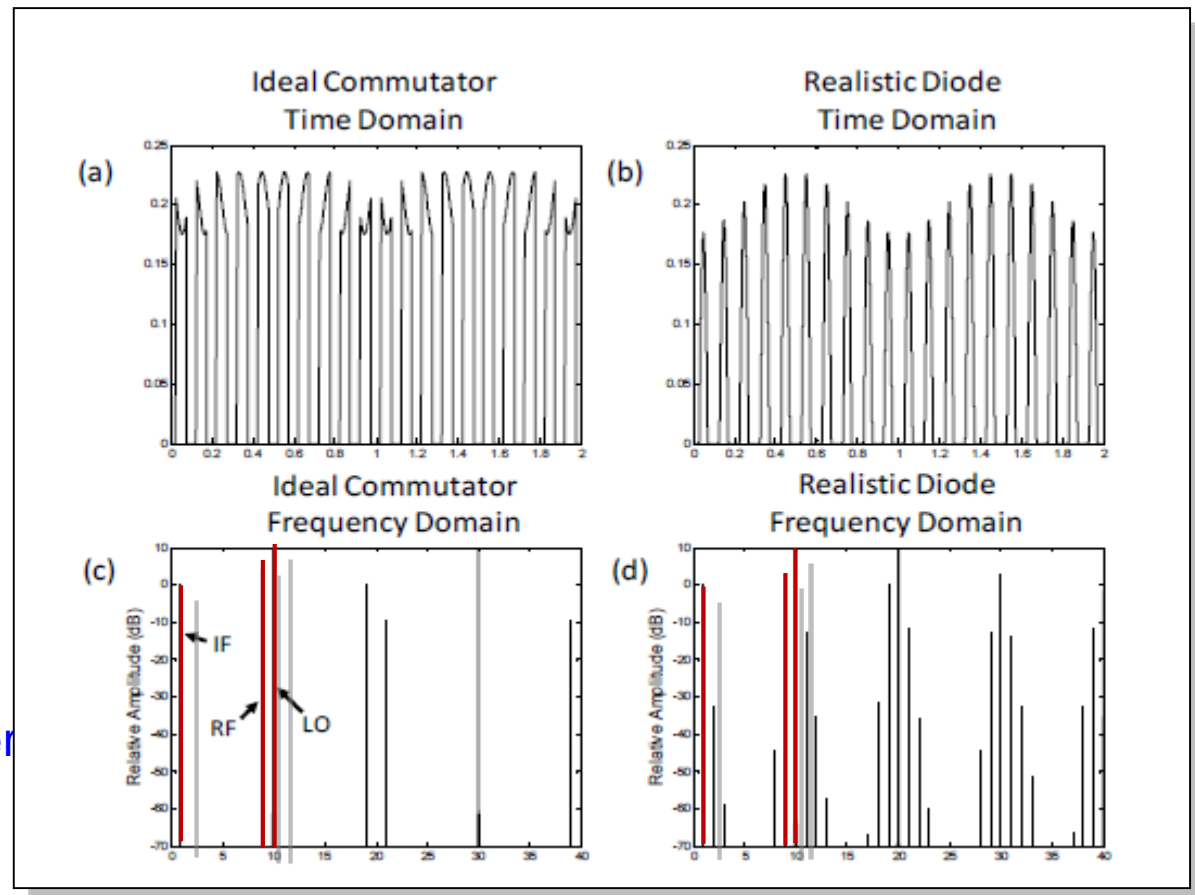
◆ Response of a mixer in time and frequency domain:

◆ Input signals here:

◆ LO = 10 MHz

◆ RF = 8 MHz

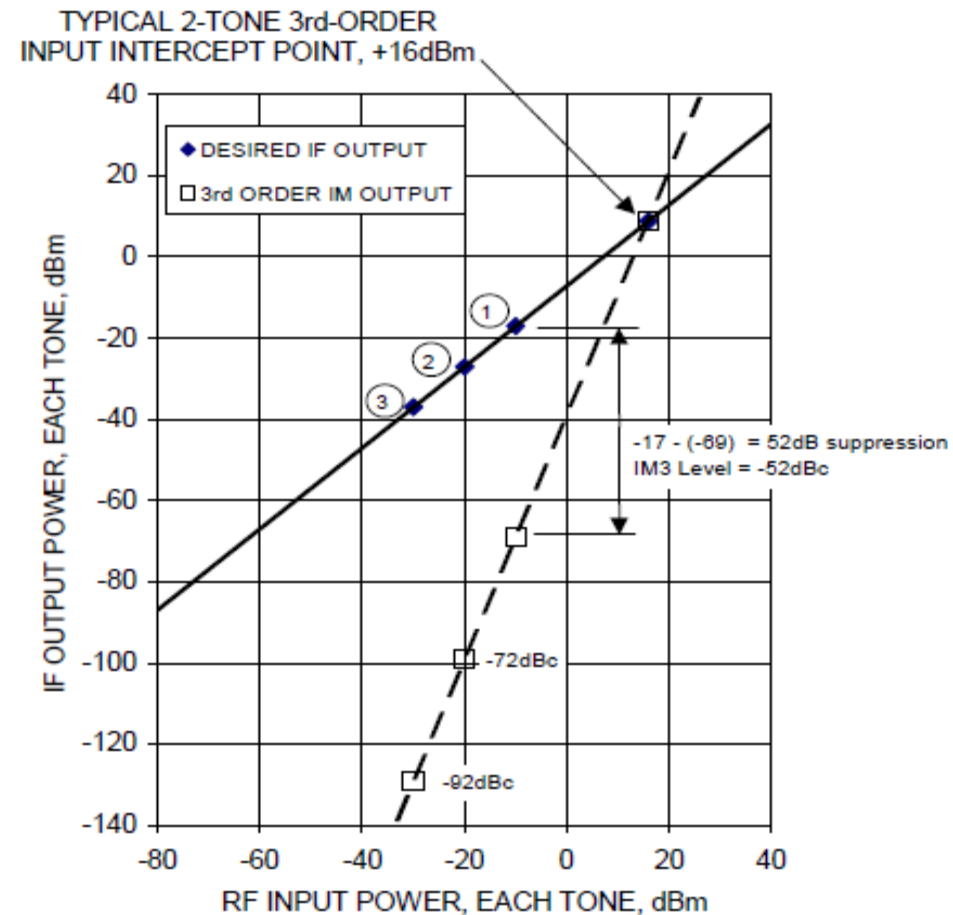
◆ Mixing products at 2 and 18 MHz and higher order terms at higher frequencies



The RF mixer (4)

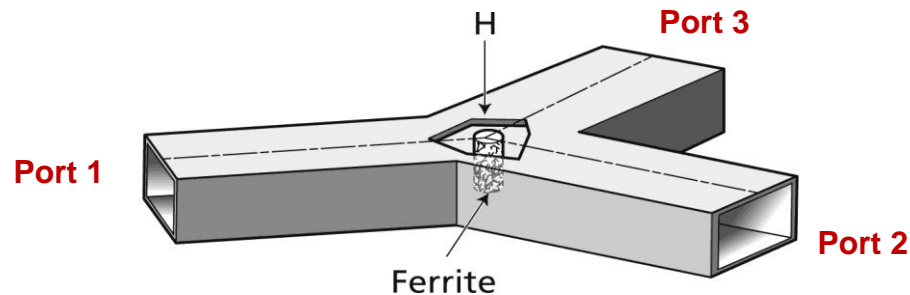
Dynamic range and IP3 of an RF mixer

- ◆ The abbreviation IP3 stands for *third order intermodulation point*, where the two lines shown in the right diagram intersect. Two signals ($f_1, f_2 > f_1$) which are closely spaced by Δf in frequency are simultaneously applied to the DUT. The intermodulation products appear at $+\Delta f$ above f_2 and at $-\Delta f$ below f_1 .
- ◆ This intersection point is usually not measured directly, but extrapolated from measurement data at much lower power levels to avoid overload and/or damage of the DUT.

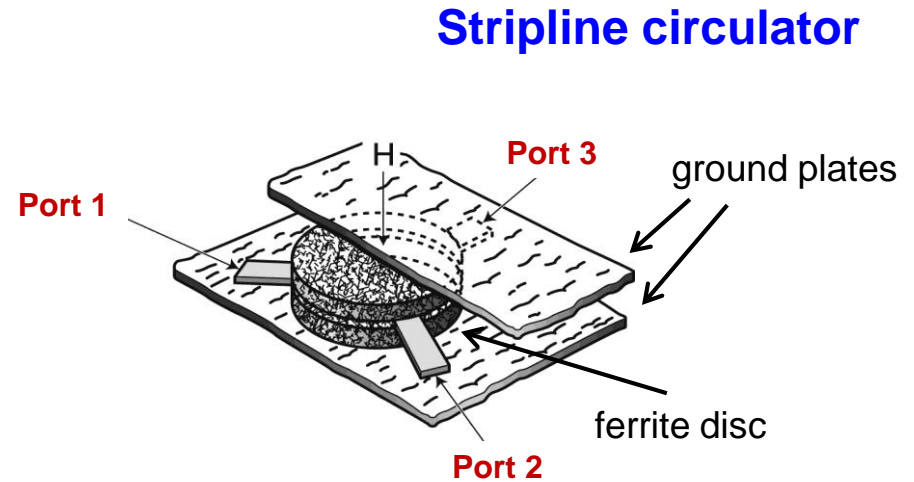


Backup Slides: 3-Port Examples (2)

Practical implementations of circulators:



Waveguide circulator



Stripline circulator

A circulator contains a volume of ferrite. The magnetically polarized ferrite provides the required non-reciprocal properties, thus power is only transmitted from port 1 to port 2, from port 2 to port 3, and from port 3 to port 1.

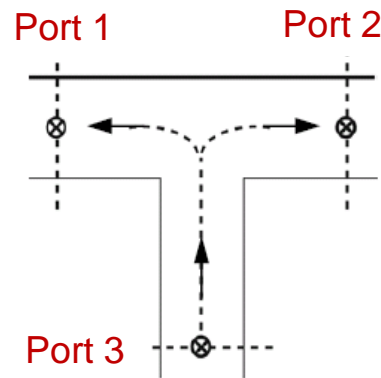
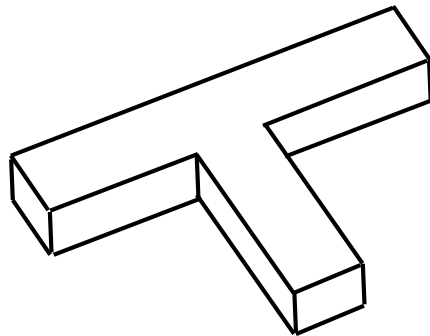
3-Port Examples (3)

- ◆ The T splitter is reciprocal and lossless but not matched at all ports. Using the losslessness condition and symmetry considerations one finds for E and H plane splitters



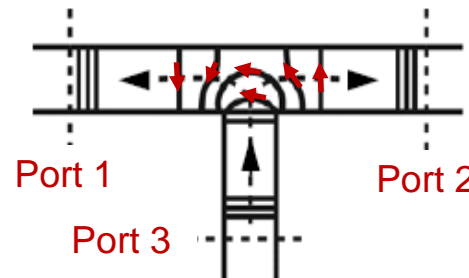
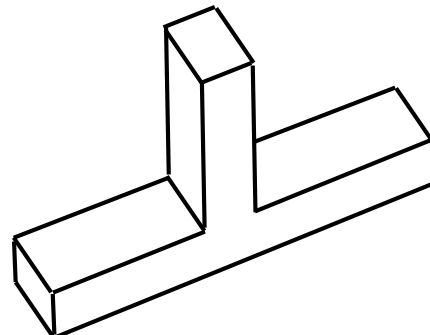
E plane waveguide

H plane waveguide



H-plane splitter

$$S_H = \frac{1}{2} \begin{pmatrix} 1 & -1 & \sqrt{2} \\ -1 & 1 & \sqrt{2} \\ \sqrt{2} & \sqrt{2} & 0 \end{pmatrix}$$



Note: change in sign of wave going left or right

E-plane splitter

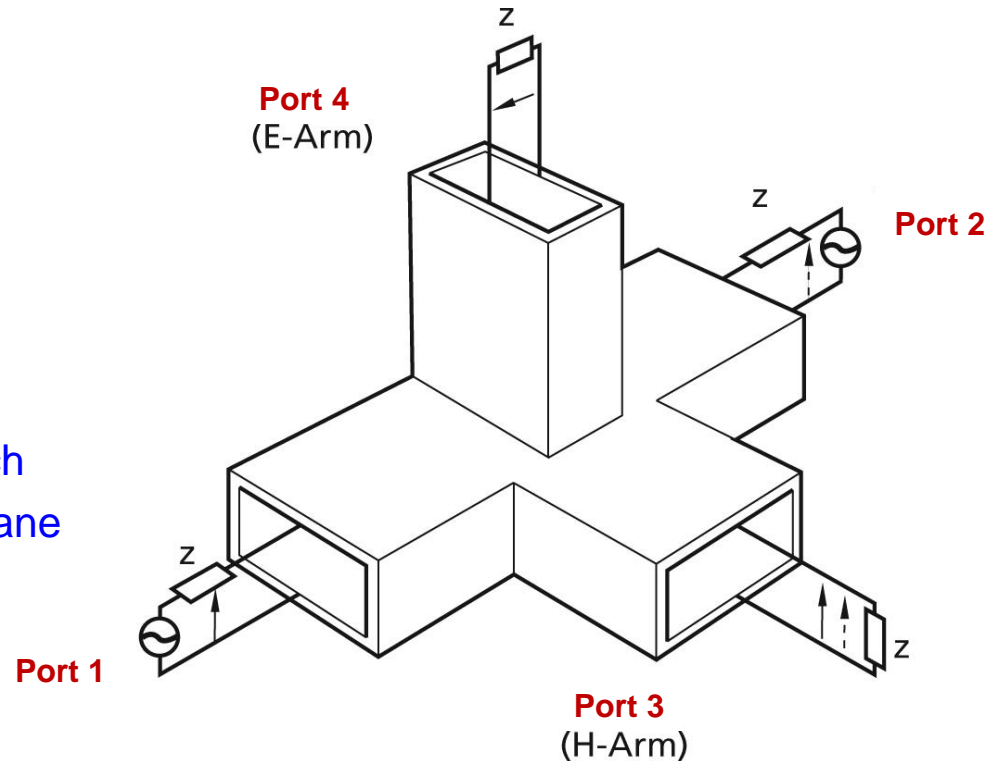
$$S_E = \frac{1}{2} \begin{pmatrix} 1 & 1 & \sqrt{2} \\ 1 & 1 & -\sqrt{2} \\ \sqrt{2} & -\sqrt{2} & 0 \end{pmatrix}$$

4-Port Examples (2)

Magic-T also referred to as 180° hybrid:

$$(S) = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \\ 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \end{bmatrix}$$

The H-plane is defined as a plane in which the magnetic field lines are situated. E-plane correspondingly for the electric field.



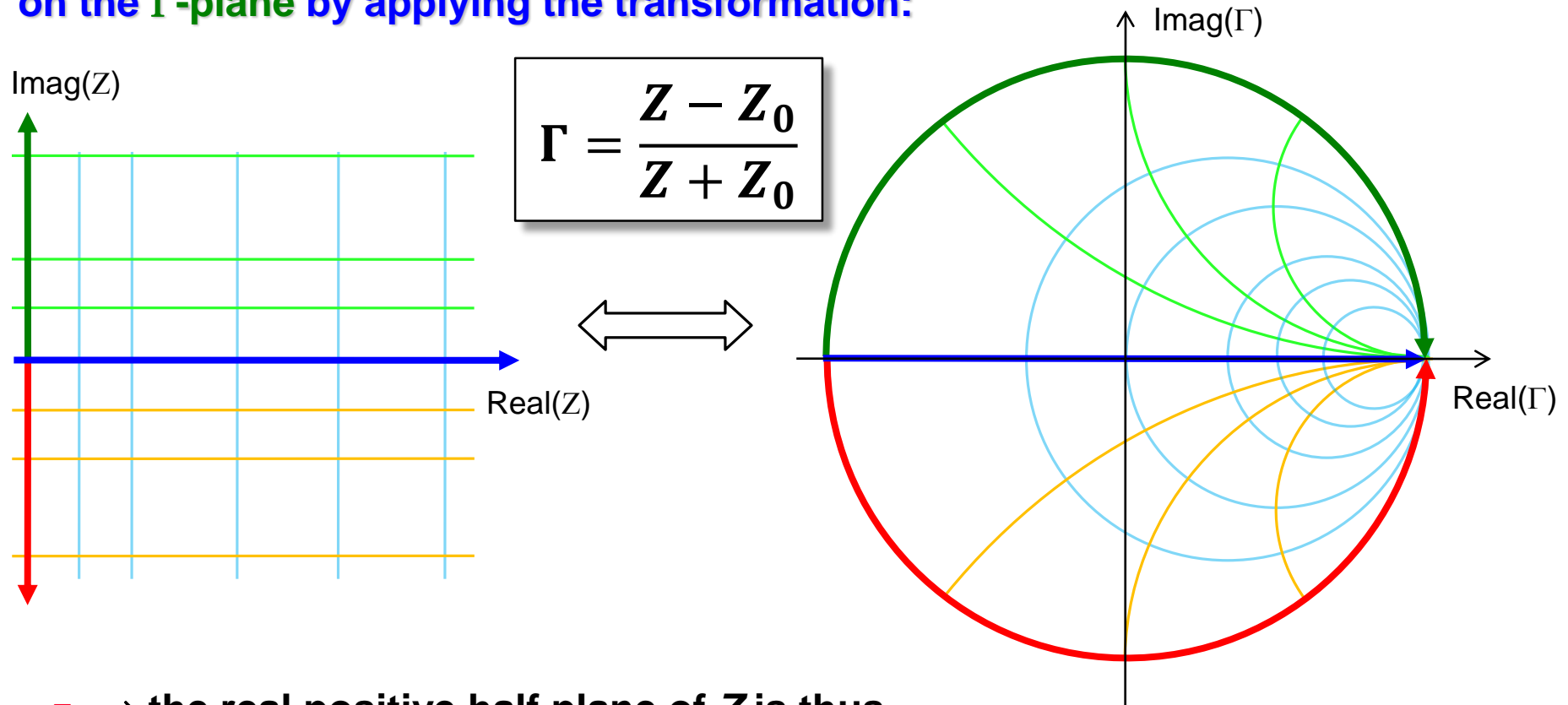
Can be implemented as waveguide or coaxial version. Historically, the name originates from the waveguide version where you can “see” the horizontal and vertical “T”.

S-Parameters – Summary

- ◆ Scattering parameters (S-parameters) characterize an RF component or system (DUT) by a matrix.
 - $n \times n$ matrix for n -port device
 - Based on incident (a_n) and reflected (b_n) power waves
- ◆ ALL ports need to be terminated in their characteristic (reference) impedance Z_0
 - For a proper S-parameter measurement or numerical computation all ports of the Device Under Test (DUT), including the generator port, must be terminated with their characteristic impedance to assure, waves traveling away from the DUT (b_n -waves) are not reflected twice or multiple times, and convert into a_n -waves.
(cannot be stated often enough...!)
- ◆ Typically the S-parameters, and therefore the DUT is "measured" and characterized in the frequency domain
 - S-parameters, as well as DUT circuit elements are described in **complex notation** with the frequency variable $\omega = 2\pi f$
 - Frequency transformation (iDFT) allows time domain measurements with a "modern" vector network analyzer (VNA).

The *Smith Chart* (1)

- ◆ The *Smith Chart* (in impedance coordinates) represents the complex Γ -plane (in polar coordinates) within the unit circle.
- ◆ It is a conformal mapping of the complex Z -plane on the Γ -plane by applying the transformation:



- \Rightarrow the real positive half plane of Z is thus transformed (*Möbius*) into the interior of the unit circle!

The *Smith Chart* (2)

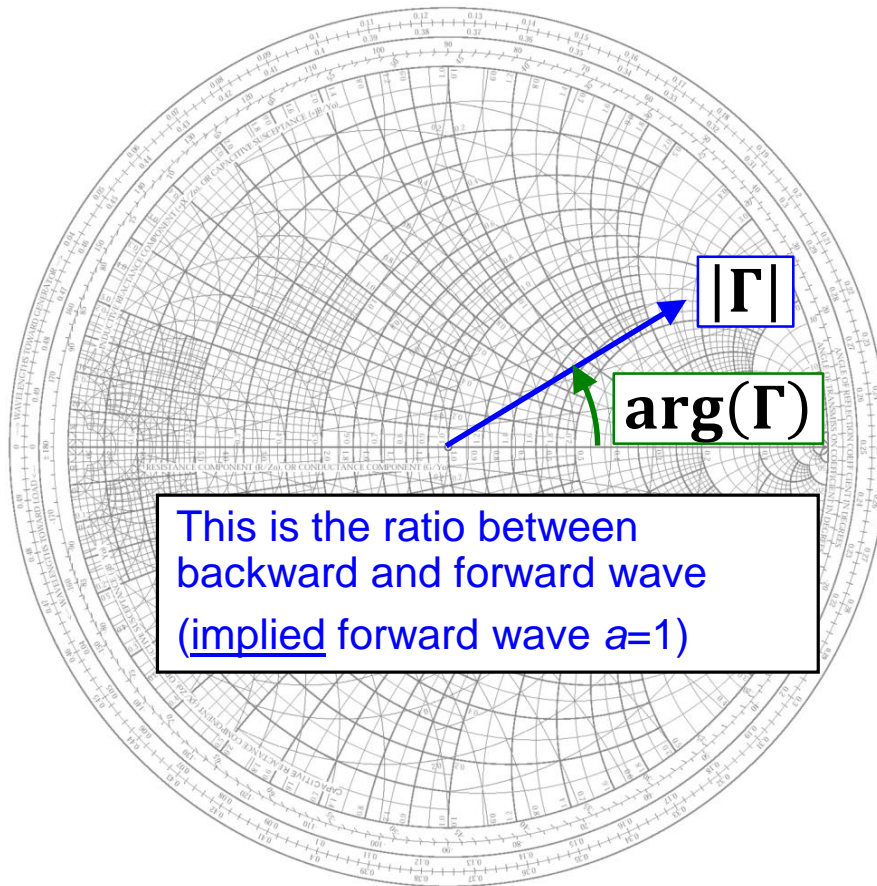
- ◆ The Impedance Z is usually normalized to a reference impedance Z_0 , typically the characteristic impedance of the coaxial cables of $Z_0=50\Omega$.
$$z = \frac{Z}{Z_0}$$
- ◆ The normalized form of the transformation follows then as:

$$\Gamma = \frac{z - 1}{z + 1} \quad \text{resp.} \quad \frac{Z}{Z_0} = z = \frac{1 + \Gamma}{1 - \Gamma}$$

This mapping offers several practical advantages:

- ◆ The diagram includes all “passive” impedances, i.e. those with positive real part, from zero to infinity in a handy format.
 - Impedances with negative real part (“active device”, e.g. reflection amplifiers) would be outside the (normal) *Smith* chart.
- ◆ The mapping converts impedances or admittances into reflection factors and vice-versa. This is particularly interesting for studies in the radiofrequency and microwave domain where electrical quantities are usually expressed in terms of “incident” or “forward”, and “reflected” or “backward” waves.
 - This replaces the notation in terms of currents and voltages used at lower frequencies.
- ◆ Also the reference plane can be moved very easily using the *Smith* chart.

The *Smith Chart* (3)



- ◆ In the *Smith chart*, the complex reflection factor

$$\Gamma = |\Gamma|e^{j\varphi} = \frac{b}{a}$$

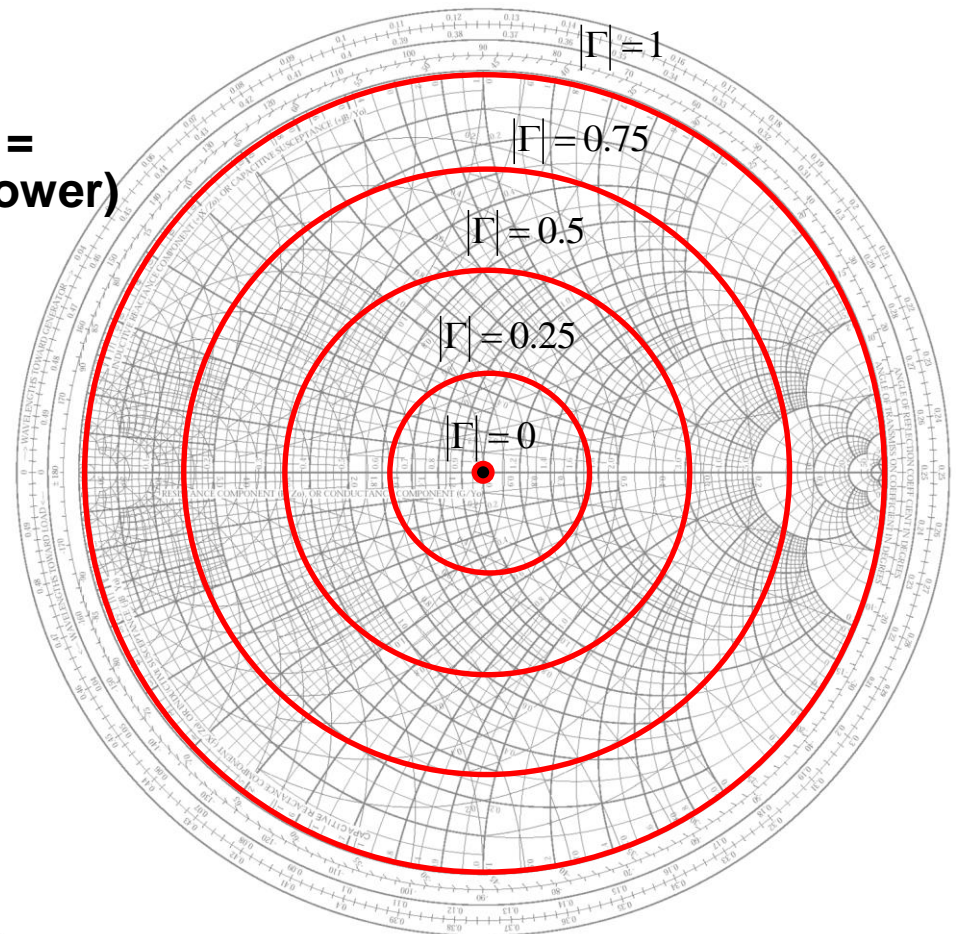
is expressed in linear cylindrical coordinates, representing the ratio of backward vs. forward traveling waves.

The Smith Chart (4)

- ◆ The distance from the center of the diagram to the magnitude of the reflection factor $|\Gamma|$, and permits an easy visualization of the matching performance.
 - In particular, the perimeter of the diagram represents total reflection: $|\Gamma|=1$.
 - (power dissipated in the load) = (forward power) – (reflected power)

$$\begin{aligned} P &= |a|^2 - |b|^2 \\ &= |a|^2 (1 - |\Gamma|^2) \end{aligned}$$

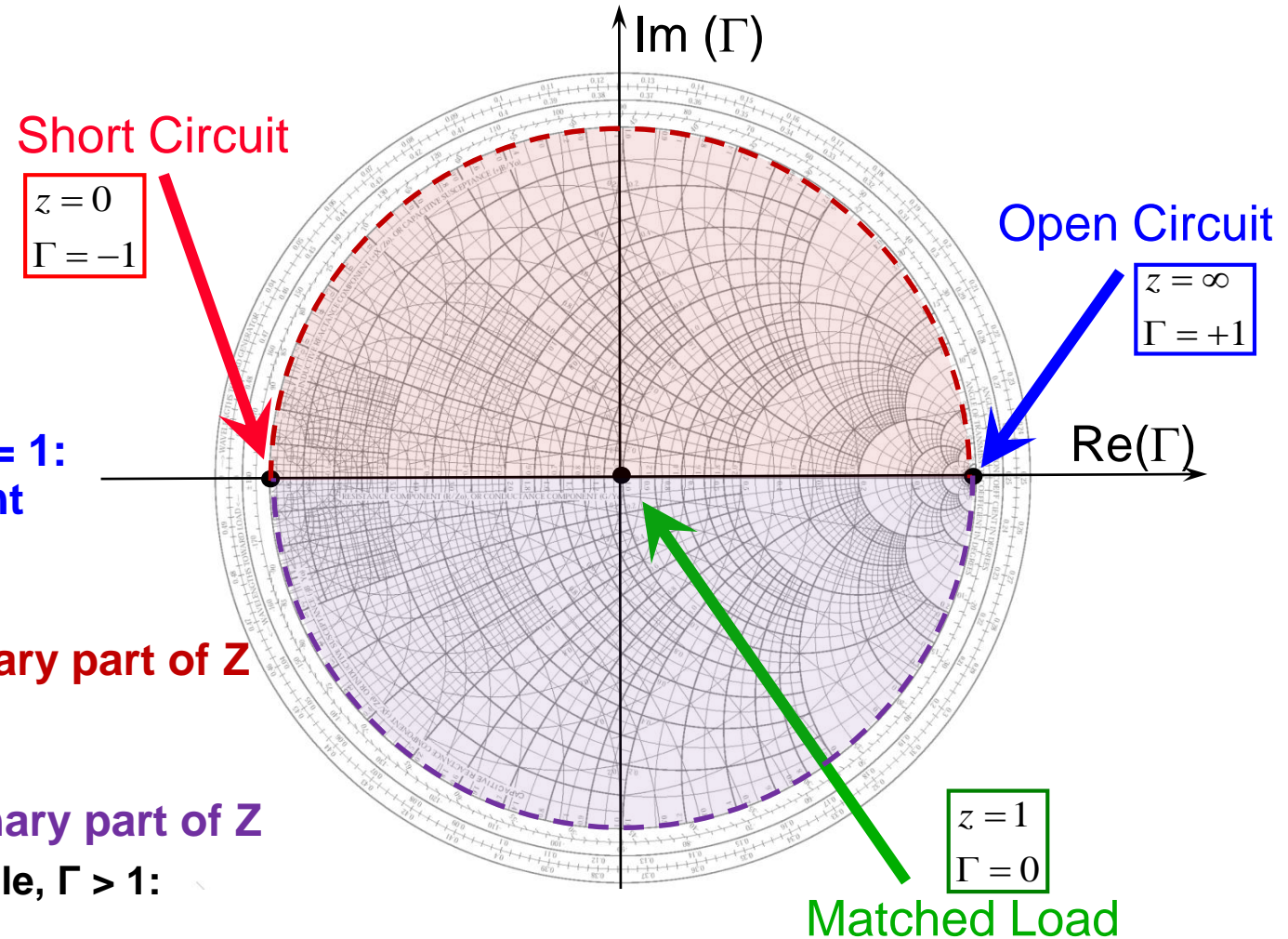
available source power mismatch losses



The *Smith Chart* – “Important Points”

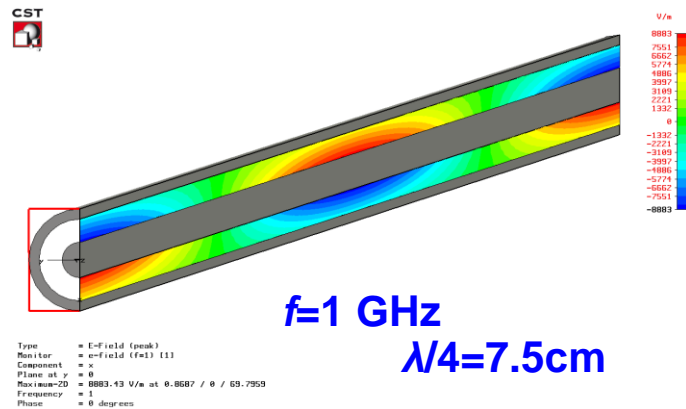
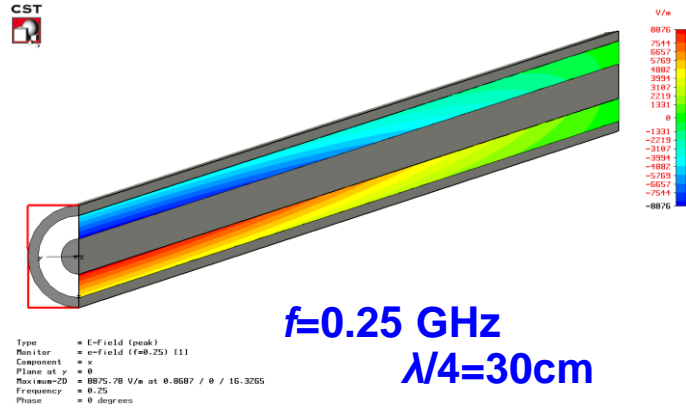
Important Points:

- ◆ **Short Circuit**
 $\Gamma = -1, z = 0$
- ◆ **Open Circuit**
 $\Gamma = +1, z \rightarrow \infty$
- ◆ **Matched Load**
 $\Gamma = 0, z = 1$
- ◆ **On the circle $\Gamma = 1$:**
lossless element
- ◆ **Upper half:**
”inductive” =
positive imaginary part of Z
- ◆ **Lower half:**
”capacitive” =
negative imaginary part of Z
- **Outside the circle, $\Gamma > 1$:**
active element,
for instance tunnel diode reflection amplifier

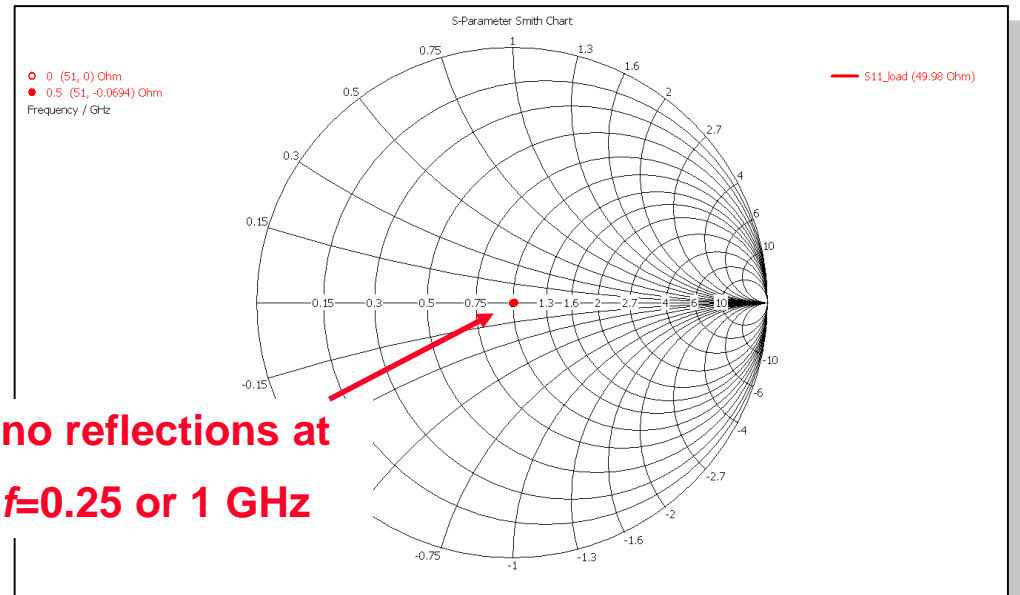


Coming back to our Example...

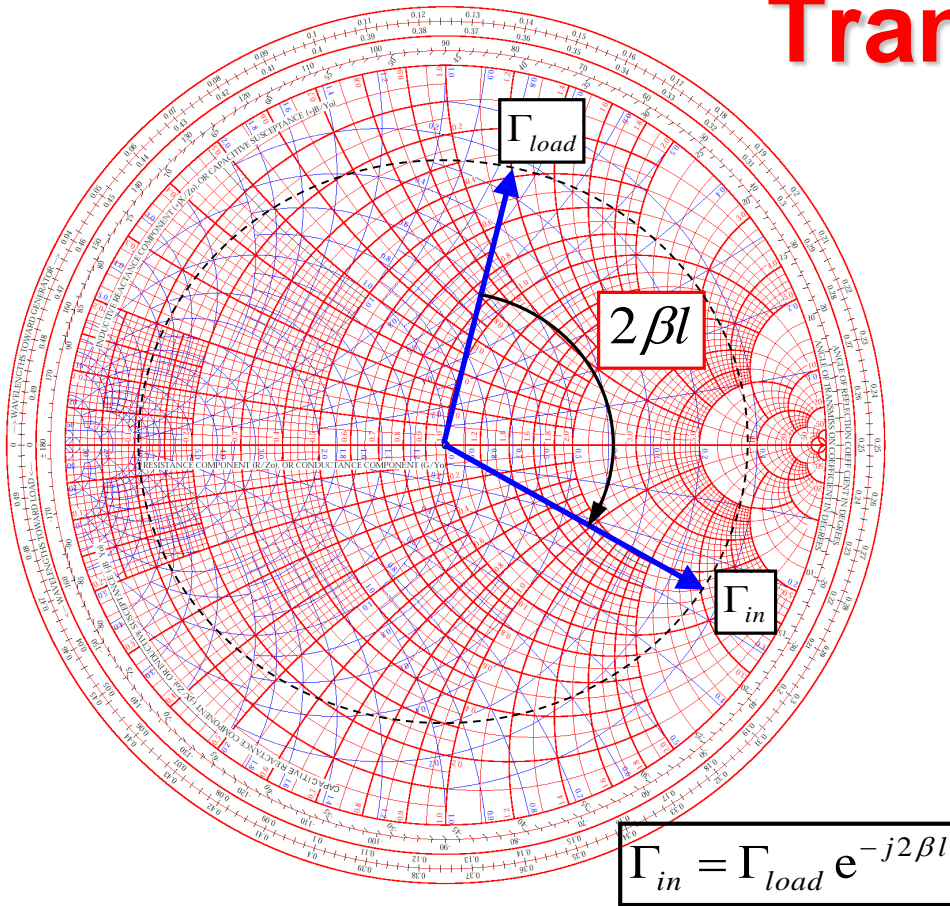
matched case:
pure traveling wave=> no reflection



Coax cable with vacuum or air
with a length of 30 cm



Impedance Transformation using Transmission-lines

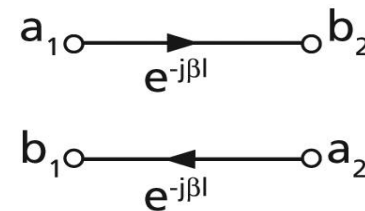


The S-matrix for an ideal, lossless transmission line of length l is given by

$$\mathbf{S} = \begin{bmatrix} 0 & e^{-j\beta l} \\ e^{-j\beta l} & 0 \end{bmatrix}$$

where $\beta = 2\pi / \lambda$

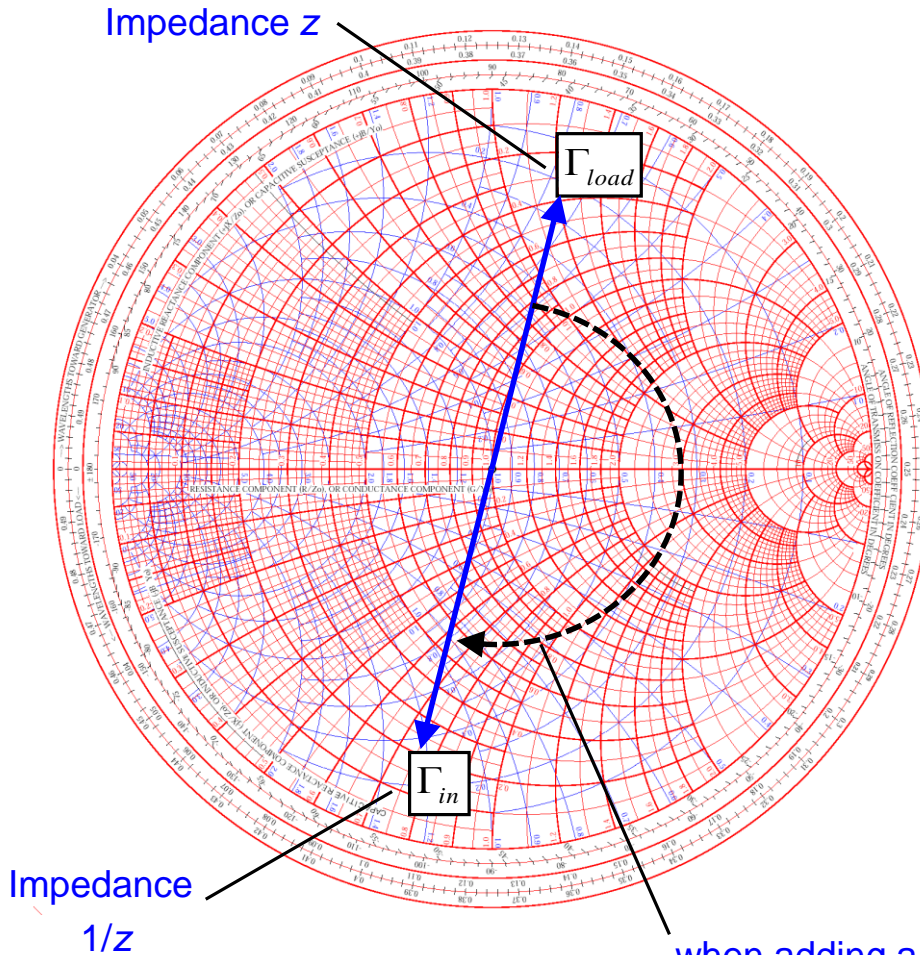
is the propagation coefficient at the wavelength λ (this refers to the wavelength on the line containing some dielectric).



How to remember when adding a section of transmission line, we have to turn clockwise: assume we are at $\Gamma = -1$ (short circuit) and add a short piece of e.g. coaxial cable. We actually introduced an inductance, thus we are in the upper half of the *Smith-Chart*.

N.B.: The reflection factors are evaluated with respect to the characteristic impedance Z_0 of the line segment.

$\lambda/4$ -line Transformations



A transmission line of length

$$l = \lambda / 4$$

transforms a load reflection Γ_{load} to its input as

$$\Gamma_{in} = \Gamma_{load} e^{-j2\beta l} = \Gamma_{load} e^{-j\pi} = -\Gamma_{load}$$

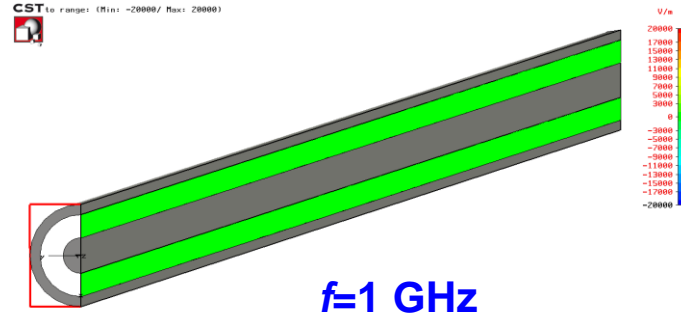
This results, a normalized load impedance z is transformed into $1/z$.

In particular, a short circuit at one end is transformed into an open circuit at the other. This is the principle of $\lambda/4$ -resonators.

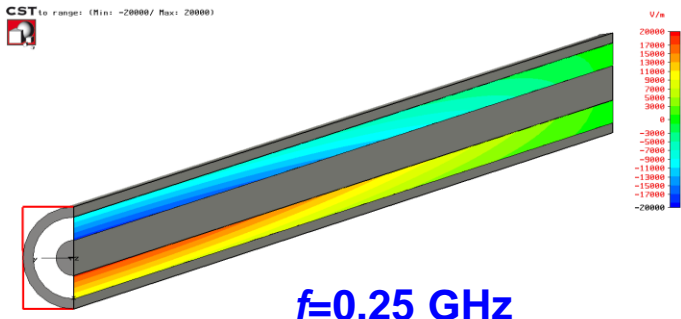
when adding a transmission line to some terminating impedance we rotate clockwise through the *Smith-Chart*.

Again our Example: Short at the end

short : standing wave

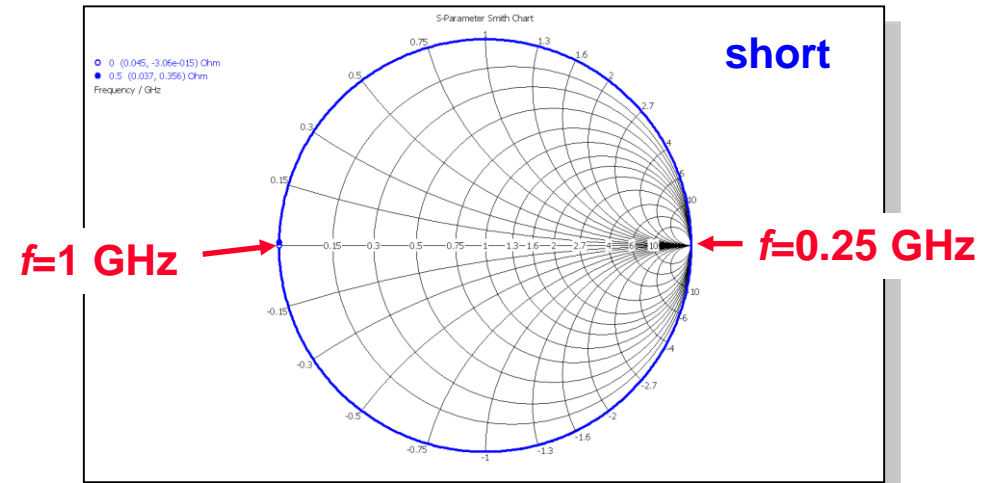


Type = E-Field (peak)
Monitor = e-field (f=1) 111
Component = x
Plane at y = 0
Maximum-ZD = 17764 V/m at 0.8687 / 0 / 225.306
Frequency = 1
Phase = 0 degrees



Type = E-Field (peak)
Monitor = e-field (f=0.25) 111
Component = x
Plane at y = 0
Maximum-ZD = 17736 V/m at 0.8687 / 0 / 0.816327
Frequency = 0.25
Phase = 0 degrees

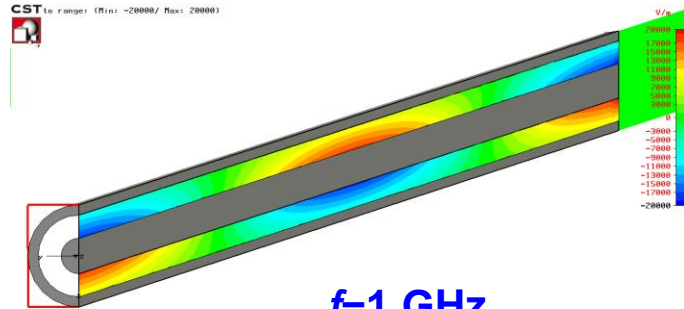
Coax cable with vacuum or air with a length of $l=30 \text{ cm}$



- ◆ If length of the transmission line changes by $\lambda/4$ a short circuit at one side is transformed into an open circuit at the other side.

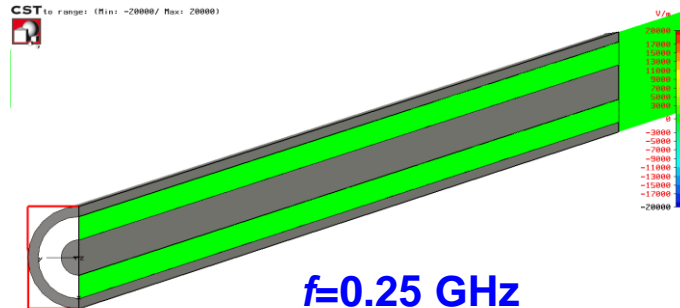
Again our Example: Open end

open : standing wave



$f=1$ GHz
 $\lambda/4=7.5$ cm

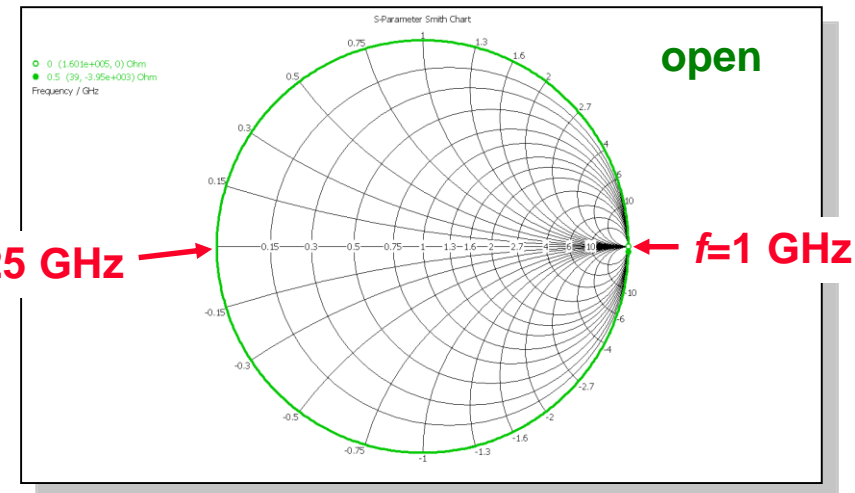
Type = E-field (peak)
Monitor = e-field (f=1) (11)
Component = x
Plane at $r = 0$
Maximum-ZD = 23787.6 V/m at 0.8687 / 0 / 300
Frequency = 1
Phase = 0 degrees



$f=0.25$ GHz
 $\lambda/4=30$ cm

Type = E-field (peak)
Monitor = e-field (f=0.25) (11)
Component = x
Plane at $r = 0$
Maximum-ZD = 23839 V/m at 0.8687 / 0 / 300
Frequency = 0.25
Phase = 0 degrees

Coax cable with vacuum with a length of 30 cm



$f=0.25$ GHz

open

$f=1$ GHz

- ◆ The patterns for the short and open terminated case appear similar; However, the phase is shifted which correspond to a different position of the nodes.
- ◆ If the length of a transmission line changes by $\lambda/4$, an open becomes a short, and vice versa!

Fun with the *Smith Chart*...

◆ Download the Smith 4.0 software (Windows)

- <http://www.fritz.dellsperger.net/smith.html>

◆ Home exercise:

- Find the values of two elements to match to $Z_{in}=50\Omega$ at $f=500\text{MHz}$

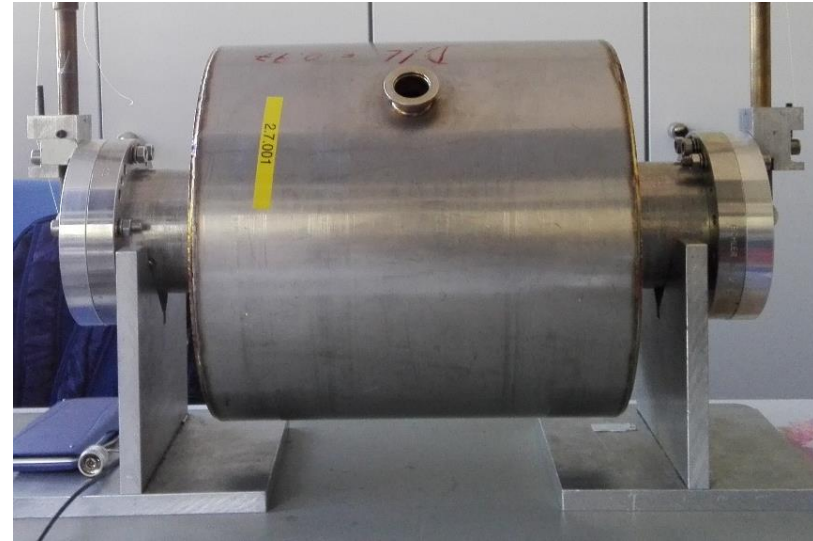
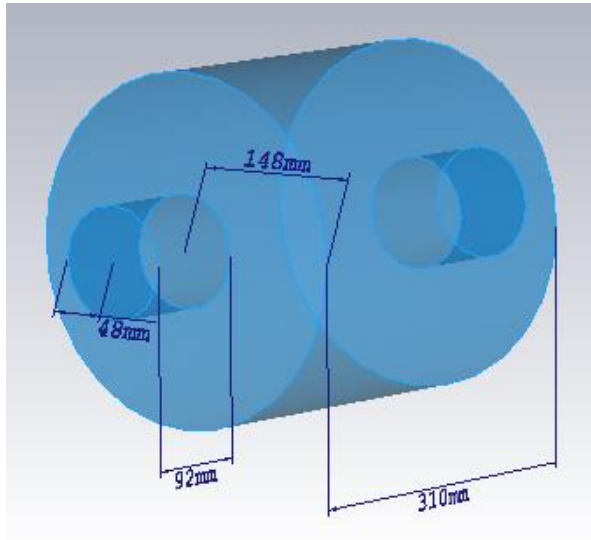
Z_L	C Series	L Series	R Series
$Z = (50 + j25) \Omega$			
$Z = (50 - j25) \Omega$			
$Z = (4 + j21) \Omega$			
$Z = (20 - j50) \Omega$			

Z_L	C Shunt	L Shunt	R Shunt
$Z = (50 + j25) \Omega$			
$Z = (50 - j25) \Omega$			
$Z = (4 + j21) \Omega$			
$Z = (20 - j50) \Omega$			

- Find two reactive elements to match:
(two solutions are possible)

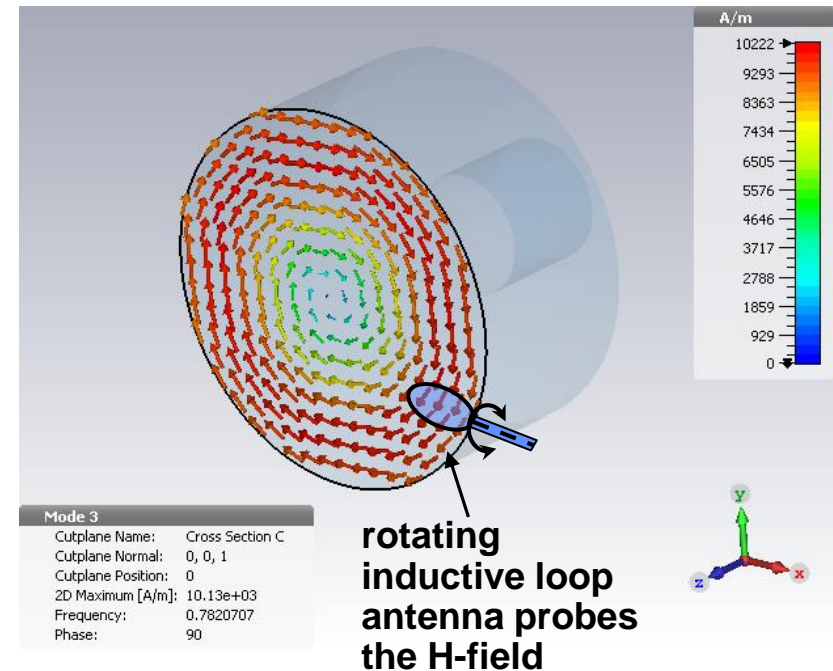
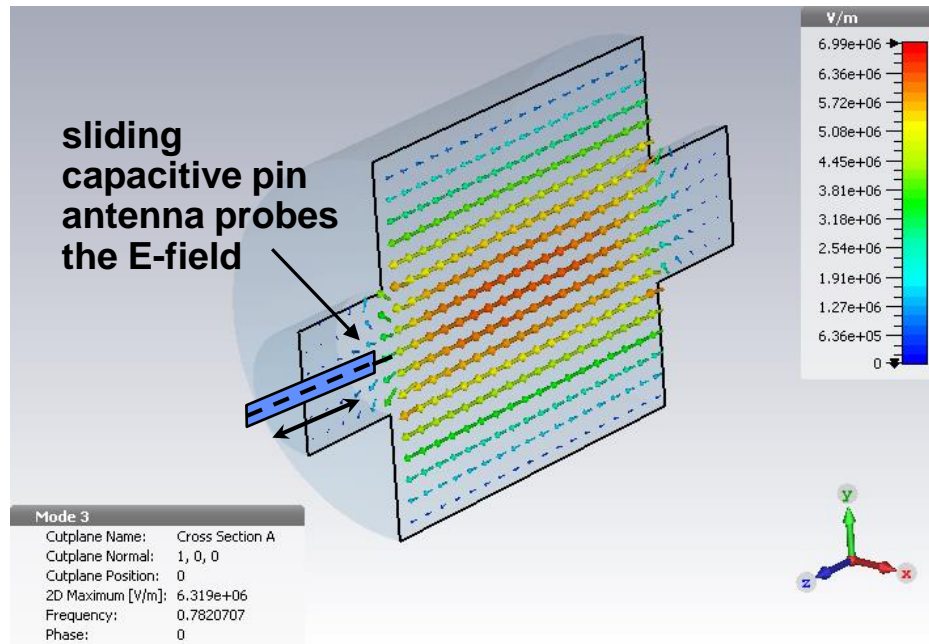
Z_L	C Series	L Series	C Shunt	L Shunt
$Z = (32 - j66) \Omega$				
$Z = (13 - j9) \Omega$				
$Z = (37 + j34) \Omega$				
$Z = (78 + j78) \Omega$				

An Example – Pillbox* TM010 Eigenmode



- ◆ **Characterize the accelerating TM010 mode of a cylindrical cavity with beam ports**
 - The TM010 does not have to be the lowest frequency mode
 - ◆ **Compare the measured values of f_{res} , Q_0 and R/Q**
 - with an analytical analysis of a perfect cylinder (no beam ports)
 - with a numerical analysis
- * normal conducting!**

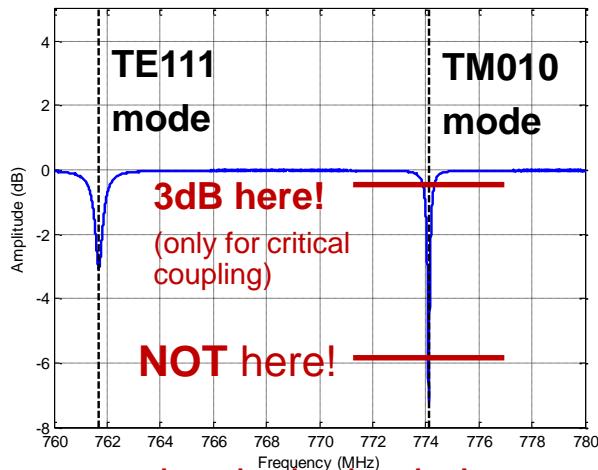
Probe the Mode by Measuring S11



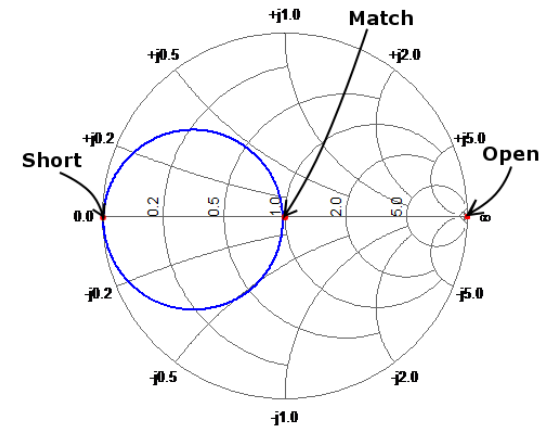
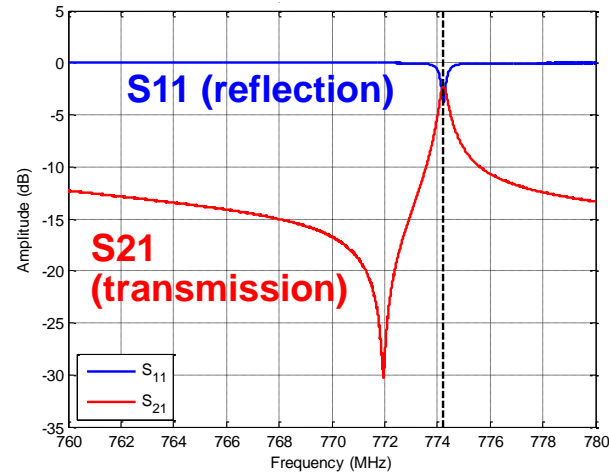
S11 measurement with tunable coupling antenna

- E-field on z-axis using a capacitive coupling pin
 - Center pin, e.g. of semi-rigid coaxial cable
- H-field on the cavity rim using a inductive coupling loop
 - Bend the center conductor to a closed loop connected to ground

Measurement of Frequency and Q-value

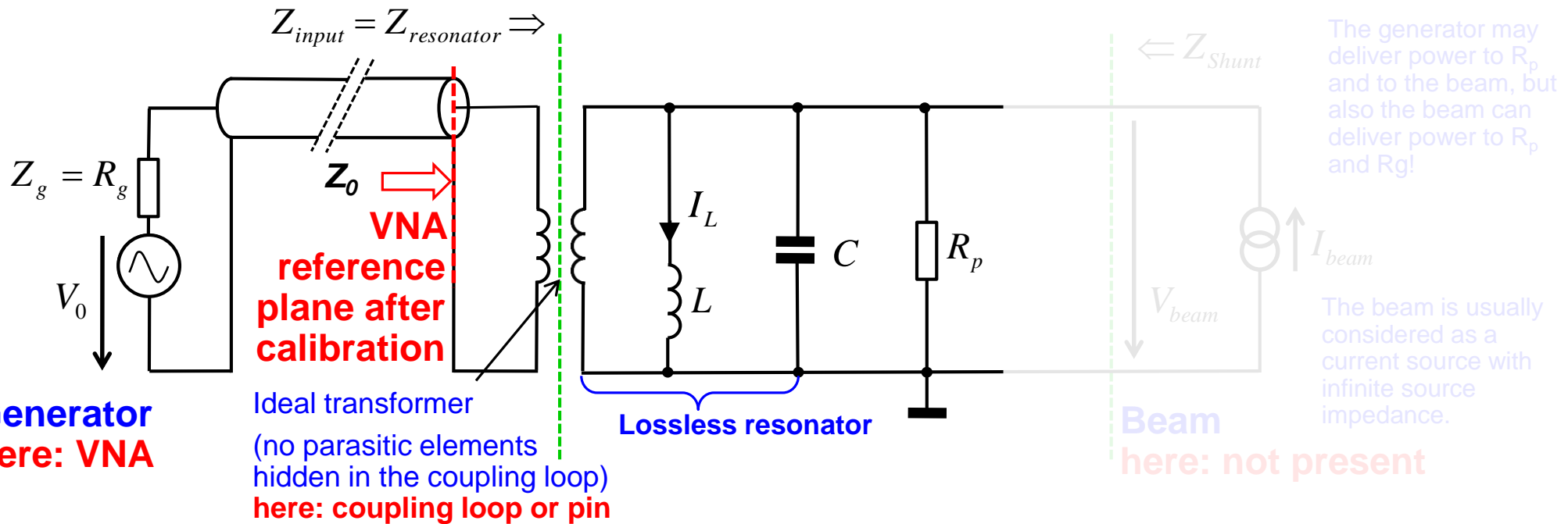


classical mistake!



- ◆ **Identify the correct (TM010) mode frequency**
 - Introduce a small perturbation, e.g. metallic rod or wire on the z-axis and observe the shift of the mode frequencies
- ◆ **Calibrate the VNA and measure S11**
 - Tune the coupling loop for critical coupling
 - Display the resonant circle in the Smith chart using a sufficient number of points

Remember: The Equivalent Circuit

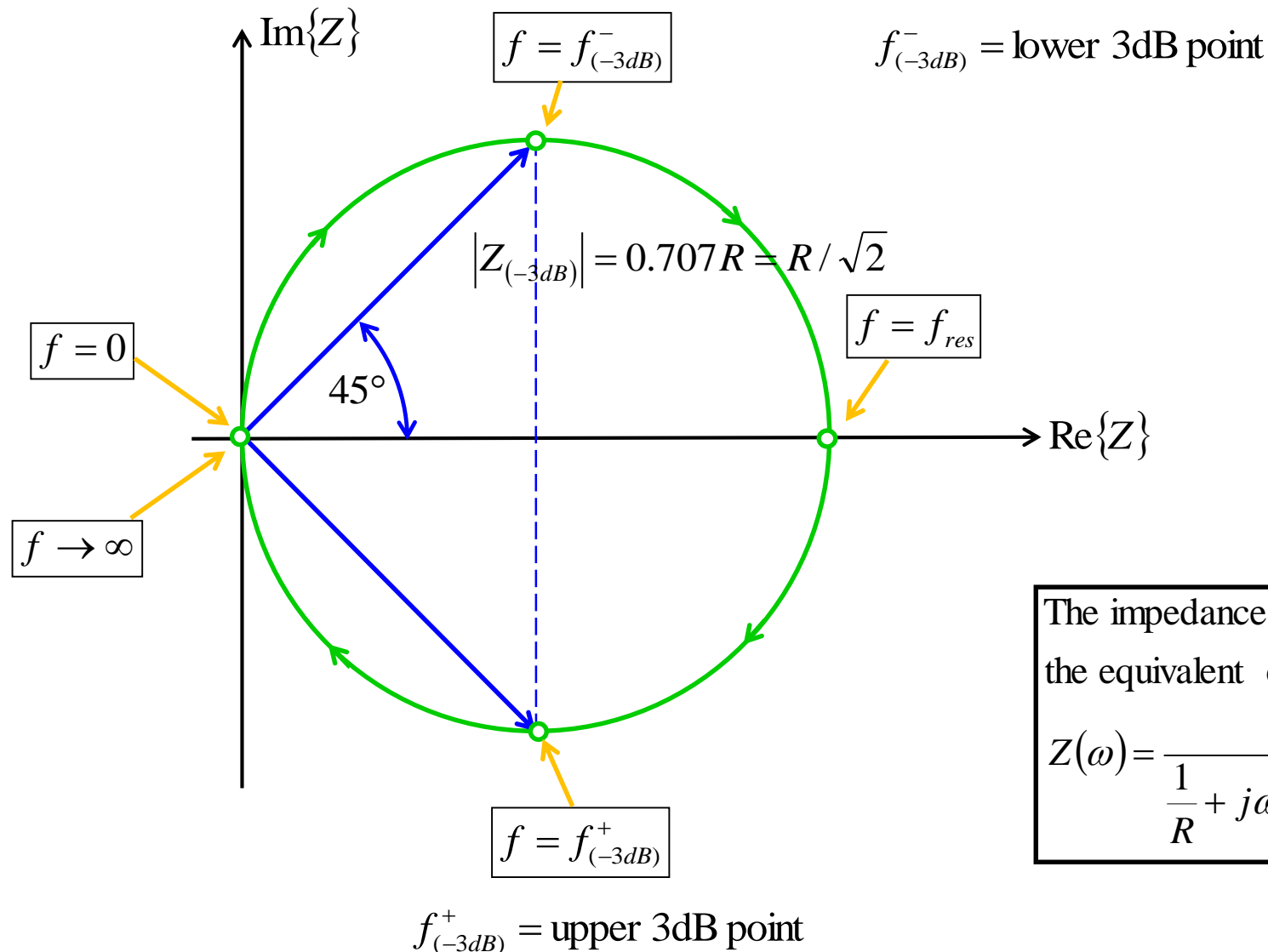


R_p = resistor representing the losses of the parallel RLC equivalent circuit (resonator losses)

We have resonance condition, when $\omega L = \frac{1}{\omega C}$

→ Resonance frequency: $\omega_{res} = 2\pi f_{res} = \frac{1}{LC} \Rightarrow f_{res} = \frac{1}{2\pi\sqrt{LC}}$

Input Impedance in the Complex Z-Plane



The impedance Z for the equivalent circuit is :

$$Z(\omega) = \frac{1}{\frac{1}{R} + j\omega C + \frac{1}{j\omega L}}$$

Useful Formulas of the Equivalent Circuit

◆ Characteristic impedance "R upon Q"

$$X = \frac{R}{Q} = \omega_{res} L = \frac{1}{\omega_{res} C} = \sqrt{L/C}$$

◆ Stored energy at resonance

$$U = \frac{C V_C^2}{2} = \frac{L I_L^2}{2}$$

V_C ... Voltage at the capacitor
 I_L ... Current in the inductor

◆ Dissipated power

$$P = \frac{V^2}{2 R}$$

◆ Q-factor

$$Q = \frac{R}{X} = \frac{\omega_{res} U}{P}$$

U ... stored energy
 P ... dissipated power over 1 period

◆ Shunt impedance (circuit definition)

$$R = \frac{V^2}{2 P}$$

◆ Tuning sensitivity

$$\frac{\Delta f}{f} = \frac{1}{2} \frac{\Delta C}{C} = -\frac{1}{2} \frac{\Delta L}{L}$$

◆ Coupling parameter (shunt impedance over generator or feeder impedance)

$$k^2 = \frac{R}{R_{input}}$$

tune for critical coupling

The Quality Factor (Q-Value)

- ◆ The quality (Q) factor of a resonant circuit is defined as ratio of the stored energy U over the energy dissipated P in one oscillation cycle:

$$Q = 2\pi \frac{\text{energy stored}}{\text{energy dissipated in 1 cycle}} = \frac{\omega_{res} U}{P}$$

- ◆ The Q-factor of an impedance loaded resonator:

- Q_0 : unloaded Q-value of the unperturbed system
- Q_L : loaded Q-value, e.g. measured with the impedance of the connected generator
- Q_{ext} : external Q-factor, representing the effects of the external circuit (generator and coupling circuit)

$$\frac{1}{Q_L} = \frac{1}{Q_0} + \frac{1}{Q_{ext}}$$

- ◆ Q-factor and bandwidth

- This is how we actually "measure" the Q-factor!

$$Q = \frac{f_{res}}{f_{BW}} \quad \text{with: } f_{BW} = f_{+3dB} - f_{-3dB}$$

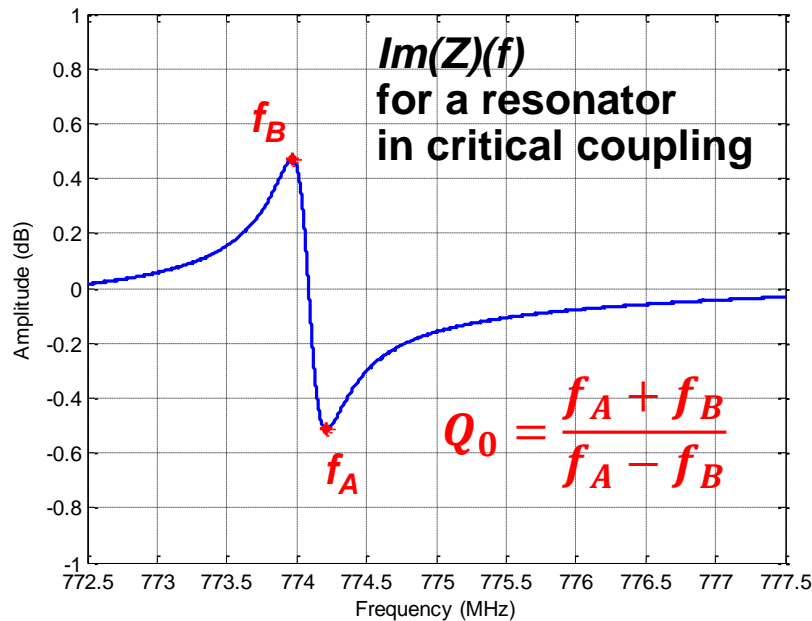
tune k for
critical coupling:

$$Q_0 = Q_{ext}$$

$$\Rightarrow Q_0 = 2 Q_L$$

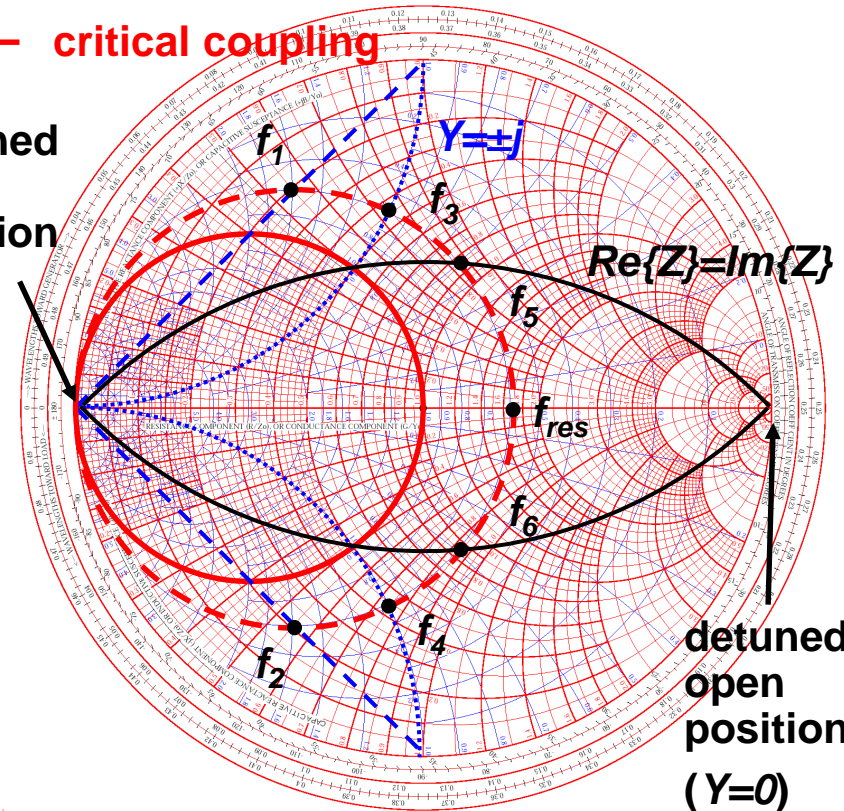
With Q_L being our
measured Q-value

Q-factor from S11 Measurement



----- arbitrary coupling (here: over-critical)
 ——— critical coupling

detuned
short
position
($Z=0$)



- ◆ **Correct for the uncompensated effects of the coupling loop**
 - Electrical length adjustment: "straight" $Im\{Z\}(f)$
- ◆ **Adjust the locus circle to the detuned short location**
 - Phase offset
- ◆ **Verify no evanescent fields penetrating outside the beam ports**
 - i.e. no frequency shifts if the boundaries at the beam ports are altered

Frequency marker points in the Smith chart:

- ◆ $f_{1,2}$: $Im\{S_{11}\} = max.$ to calculate Q_L
- ◆ $f_{3,4}$: $Y = Re \pm j1$ to calculate Q_{ext}
- ◆ $f_{5,6}$: $Re\{Z\} = Im\{Z\}$ to calculate Q_0

R/Q Measurement – Slater's Theorem

◆ Remember from the equivalent circuit:

$$\frac{R}{Q} = \frac{V_{acc}^2 / 2P_d}{\omega_{res} U / P_d} = \frac{V_{acc}^2}{2\omega_{res} U} \quad \text{with:} \quad V_{acc} = \left| \int E_z(z) \cos\left(\frac{\omega z}{\beta c_0}\right) dz \right|$$

transit time related

- V_{acc} is based on the integrated longitudinal E-field component E_z along the z-axis ($x=y=0$)

◆ Based on Slater's perturbation theorem:

$$\frac{\Delta f}{f_{res}} = \frac{1}{U} \left[\mu \left(k_{\parallel}^H |H_{\parallel}|^2 + k_{\perp}^H |H_{\perp}|^2 \right) - \varepsilon \left(k_{\parallel}^E |E_{\parallel}|^2 + k_{\perp}^E |E_{\perp}|^2 \right) \right]$$

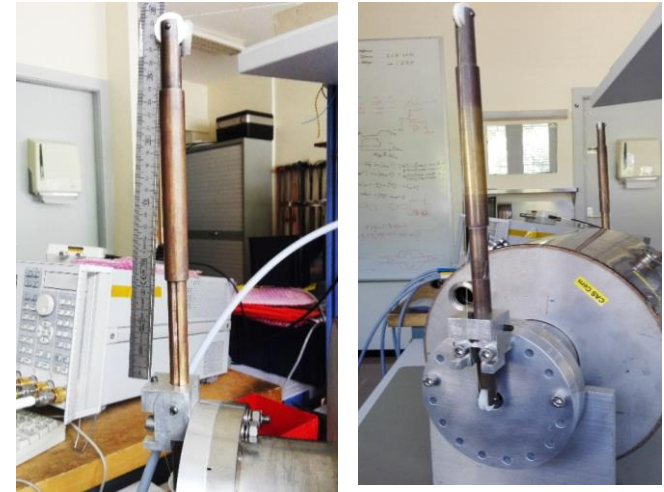
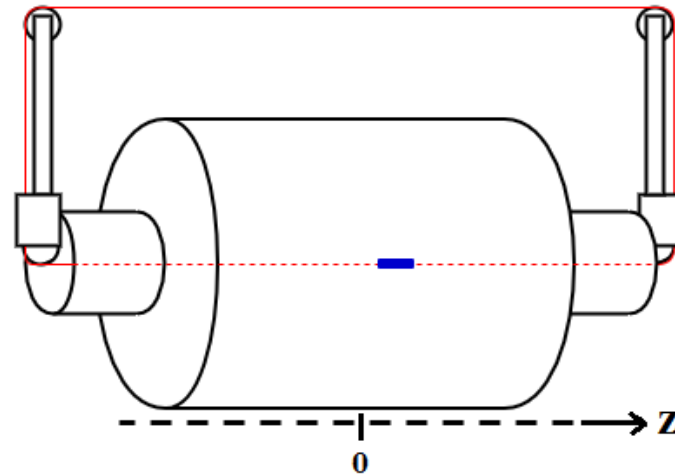
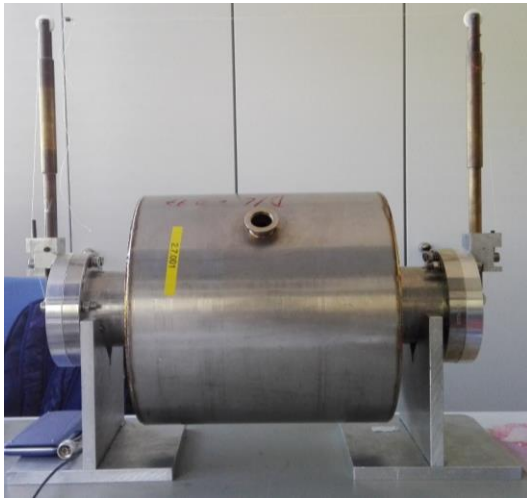
- Resonance frequency shift due to a small perturbation object, expressed in longitudinal and transverse E and H field components
- k : coefficients proportional to the electric or magnetic polarizability of the perturbation object (here: only k_{\parallel}^E for a longitudinal metallic object)

◆ E-field characterization along the z-axis

$$E(z) = \sqrt{\frac{\Delta f(z)}{f_{res}} \cdot \frac{-1}{k_{\parallel}^E \varepsilon_0}}$$

with: $k_{\parallel}^E = \frac{\pi}{3} l^3 \left[\sinh^{-1} \left(\frac{2l}{3\pi a} \right) \right]^{-1}$
 (metallic ellipsoid, e.g. syringe needle of half length l and radius a)

R/Q Measurement - Bead Pull Method



◆ E-field characterization by evaluating

- The frequency shift (S11 reflection measurement with a single probe) or
- The phase shift ϕ at f_{res} (S21 transmission measurement with 2 probes)

$$\frac{\Delta f}{f_{res}} = \frac{1}{2 Q_0} \tan \phi$$

◆ Exercise with a manual bead-pull through a known cavity

- requires: fishing wire, syringe needle, ruler and VNA
- Compare the measured E_z at the maximum f or ϕ shift (in the center of the cavity) with the theoretical estimation (e.g. numerical computed value)

Where to go from here...?!

◆ Setup your own mode characterization experiment

- Start with a simple analytically solvable structure, e.g. brick-style or cylindrical ("pillbox") resonator
- Unfortunately you need to go through the math of the modal expansion of the vector potential Ψ ... 😊

Laplace equation:

$$k_0^2 = \omega^2 \epsilon_0 \mu_0$$

k_0 : free space wave number

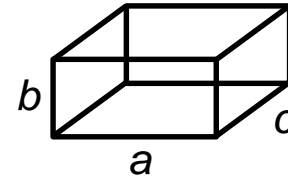
$$\Delta \Psi + k_0^2 \epsilon_r \mu_r \Psi = 0$$

$$k_0 = 2\pi / \lambda_0$$

λ_0 : free space wave length

Product ansatz (*Cartesian* coordinates):

$$\Psi = X(x)Y(y)Z(z)$$



standing waves

General solution (field components):

$$\Psi = \left\{ \begin{array}{l} A \cos(k_x x) + B \sin(k_x x) \\ \hat{A} e^{-jk_x x} + \hat{B} e^{-jk_x x} \end{array} \right\} \left\{ \begin{array}{l} C \cos(k_y y) + D \sin(k_y y) \\ \hat{C} e^{-jk_y y} + \hat{D} e^{-jk_y y} \end{array} \right\} \left\{ \begin{array}{l} E \cos(k_z z) + F \sin(k_z z) \\ \hat{E} e^{-jk_z z} + \hat{F} e^{-jk_z z} \end{array} \right\}$$

↙ (pointing to the first brace)
↙ (pointing to the second brace)

separation condition:

$$k_x^2 + k_y^2 + k_z^2 = k_0^2 \epsilon_r \mu_r$$

$$k_x = \frac{m\pi}{a} \quad k_y = \frac{n\pi}{b} \quad k_z = \frac{p\pi}{c}$$

travelling waves

Eigen frequencies:

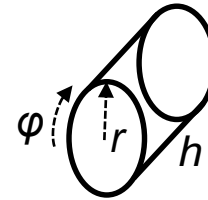
$$f_{mnp} = \frac{c_0}{2\pi \epsilon_r \mu_r} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{c}\right)^2}$$

Pillbox Eigenmodes

- ◆ Same procedure, but now with cylindrical functions ☹️☹️

Product ansatz (cylindrical coordinates):

$$\Psi = R(\rho)F(\varphi)Z(z)$$



standing waves

General solution (field components):

$$\Psi = \begin{cases} A J_m(k_r r) + B N_m(k_r r) \\ \hat{A} H_m^{(2)}(k_r r) + \hat{B} H_m^{(2)}(k_r r) \end{cases} \begin{cases} C \cos(m\varphi) + D \sin(m\varphi) \\ \hat{C} e^{-jm\varphi} + \hat{D} e^{-jm\varphi} \end{cases} \begin{cases} E \cos(k_z z) + F \sin(k_z z) \\ \hat{E} e^{-jk_z z} + \hat{F} e^{-jk_z z} \end{cases}$$

$J_m, N_m, H_m^{(1,2)}$: cylindrical functions (Bessel, Hankel, Neumann)

travelling waves

separation condition:

see Abramowitz and Stegun

$$k_r^2 + k_z^2 = k_0^2 \epsilon_r \mu_r$$

Eigen frequencies:

$$f_{TMmnp} = \frac{c_0}{2\pi\epsilon_r\mu_r} \sqrt{\left(\frac{j_{mn}}{r}\right)^2 + \left(\frac{p\pi}{h}\right)^2}$$

j_{mn} being the n^{th} root of $J_m(x)$

j'_{mn} being the n^{th} root of $J'_m(x)$

$$f_{TEmnp} = \frac{c_0}{2\pi\epsilon_r\mu_r} \sqrt{\left(\frac{j'_{mn}}{r}\right)^2 + \left(\frac{p\pi}{h}\right)^2}$$

Compare your Results

◆ Analytical – Numerical – Measurement

- There is also an analytical expression for the R/Q of the TM₀₁₀ mode of a cylindrical resonator:

$$\frac{R}{Q} = \frac{4 \eta}{j_{01}^3 \pi J_1^2(j_{01})} \frac{\sin^2\left(\frac{j_{01} h}{2 r}\right)}{h/r} \approx 128 \frac{\sin^2\left(1.2024 \frac{h}{r}\right)}{h/r} \quad \eta = \sqrt{\frac{\mu_0}{\epsilon_0}} \approx 377 \Omega$$

- ...and for the Q_0 as well: $Q_0 = \frac{r}{\delta} \left[1 + \frac{r}{h}\right]^{-1}$ $\delta = \sqrt{\frac{2}{\omega_{res} \sigma \mu}}$ (skin depth)
- There are student versions for 3D numerical EM software available
 - Visualize E and H fields to ensure you got the expected results!
- Remember: f_{res} and R/Q are only determined by the geometry of the cavity, the Q-factor however depends on the material properties (conductivity!)
 - The results of the Q-factor may vary between measurement and computation
- Don't forget to compute the values of the equivalent circuit!
- Make use of mode charts to understand the order of the eigen-frequencies
- If the phase-shift method is used to measure the R/Q with the bead-pull, set the VNA to "Zero Span" at f_{TM010} and use a very weak coupling to stay precisely on the eigen-frequency
 - Compare the measured phase before and after the bead-pull! If the value has changed the phase drifted because of temperature effects!
Perform the bead-pull at a faster pace!