Beam Diagnostic Requirements Overview

Gero Kube

DESY (Hamburg)

- Measurement Principles
- Specific Diagnostics Needs for Hadron Accelerators
- Specific Diagnostics Needs for Electron Accelerators

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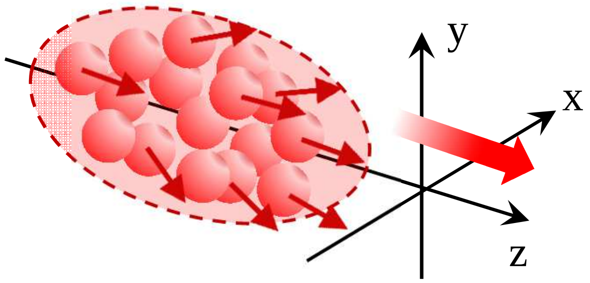
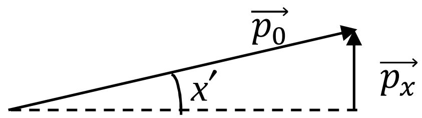
HELMHOLTZ RESEARCH FOR GRAND CHALLENGES

Beam of Particles

- particle beam (p, p̄, $e^{\pm}$, n, γ, $\mu^{\pm}$, heavy ions, …)
  - ensemble of ***N*** particles in 6-dimensional phase space
    → based on canonical coordinates ($x, y, z; p_x, p_y, p_z$)



- phase space in accelerator physics
  - use projection onto 3 orthogonal planes
    → instead of phase space in (x, p_x) use (x, $x' = p_x / p_0$)



- beam characterization → statistical ensemble
  - 1^{st} order: ***beam centroid***

    mean values $\langle r_i \rangle$

    - beam momenta p_x, p_y, p_z

      moving along "z"

      → $p_z \approx p_0 \gg p_x, p_y$
    - beam location $z(t)$
    - beam positions x, y
    - beam angles $x' = p_x/p_0, \quad y'$
  - 2^{nd} order: ***beam distribution***

    rms values $\langle r_i^2 \rangle$ and correlations $\langle r_i r_j \rangle$

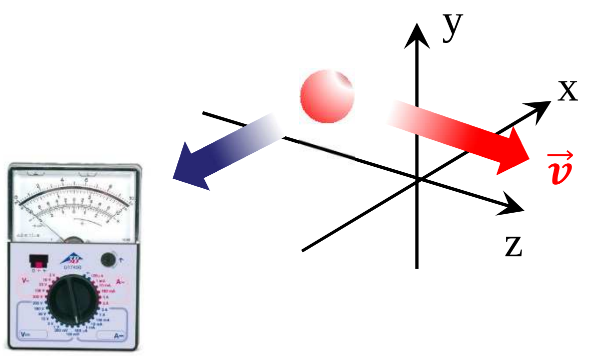
    - momentum spread $\sigma_{\Delta p/p}$
    - bunch length $\sigma_{\Delta z}$
    - beam sizes σ_x , σ_y
    - beam divergences $\sigma_{x'} , \sigma_{y'}$
    - … correlations …

courtesy:
A. Streun (PSI)

➡ how to get information about beam ?

Beam Information Transfer

- extraction of beam information
  - information transfer from ***beam particles*** to ***measuring device***
    - → information transfer characterized by ***interaction***
  - information transfer / interaction with beam preferably
    - → non-disturbing for beam
    - → strong (good signal quality)
    - → long-range (measuring device in certain distance from beam)



- fundamental particle interactions

Interaction	Gravitational	Weak	Electromagnetic	Strong
acting on	mass-energy	flavor	electric charge	colour charge
particles experiencing	all particles with mass	quarks, leptons	electrically charged particles	quarks, gluons
exchange particle	Graviton (?)	$W^{\pm}$, Z^0	γ (photon)	g (gluon)
relative strength	6×10^{-39}	10^{-5}	1/137	1
range [m]	∞	10^{-18}	∞	10^{-15}

→ restriction to charged particle beams

Electromagnetism

- described by ***Maxwell's equations*** → in SI units
  - Gauss' flux theorem

$$\vec{\nabla}\cdot\vec{E}(\vec{r},t)=\frac{\rho(\vec{r},\mathrm{t})}{\varepsilon_0} \qquad \oiint_S \vec{E}(\vec{r},t)\cdot \mathrm{d}\vec{S}=\frac{1}{\varepsilon_0}\iiint_V \rho(\vec{r},\mathrm{t})\,\mathrm{dV}$$

  - Gauss' law for magnetism

$$\vec{\nabla}\cdot\vec{B}(\vec{r},t)=0 \qquad \oiint_S \vec{B}(\vec{r},t)\cdot \mathrm{d}\vec{S}=0$$

  - Faraday's law of induction

$$\vec{\nabla}\times\vec{E}(\vec{r},t)=-\frac{\partial\vec{B}}{\partial t}(\vec{r},\mathrm{t}) \qquad \oint_C \vec{E}(\vec{r},t)\cdot \mathrm{d}\vec{l}=-\frac{\mathrm{d}}{\mathrm{dt}}\iint_S \vec{B}\,(\vec{r},\mathrm{t})\cdot \mathrm{d}\vec{S}$$

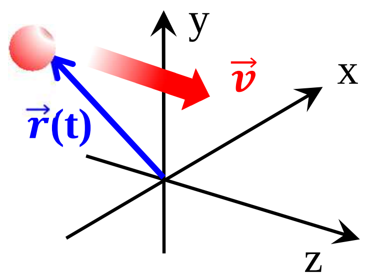
  - Ampère's law + displacement current

$$\vec{\nabla}\times\vec{B}(\vec{r},t)=\mu_0\vec{J}(\vec{r},\mathrm{t})+\frac{1}{c^2}\frac{\partial\vec{E}}{\partial t}(\vec{r},\mathrm{t}) \qquad \oint_C \vec{B}(\vec{r},t)\cdot \mathrm{d}\vec{l}=\mu_0\iint_S \vec{J}(\vec{r},\mathrm{t})\cdot \mathrm{d}\vec{S}+\frac{1}{c^2}\frac{\mathrm{d}}{\mathrm{dt}}\iint_S \vec{E}(\vec{r},\mathrm{t})\cdot \mathrm{d}\vec{S}$$

- application to beam particle in accelerator
  - consider point-like particle with charge ***Q***, moving with ***v = const.***
  - input → particle properties (kinematics)

$$\rho(\vec{r},\mathrm{t})=\mathrm{Q}\,\delta[\vec{r}(\mathrm{t})] \qquad \vec{J}(\vec{r},\mathrm{t})=\mathrm{Q}\,\vec{v}\,\delta[\vec{r}(\mathrm{t})]$$

  - output → electromagnetic fields → ***„information carrier"*** about beam
  - typical particle accelerator: ***v » 1 (→ c)***

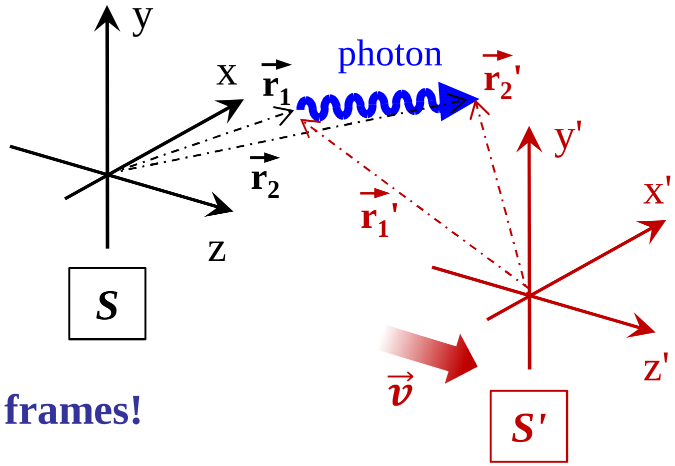


take into account relativistic motion

Special Relativity: a Glimpse

- postulates of special relativity
  - principle of relativity (relativistic or Lorentz invariance)
    → laws of physics are invariant under a transformation between two coordinate frames moving at constant velocity w.r.t. each other
  - invariance of c
    → velocity of light is the same for all observers

$$c = \frac{|\vec{r}_2 - \vec{r}_1|}{(t_2 - t_1)} = \frac{|\vec{r'}_2 - \vec{r'}_1|}{(t'_2 - t'_1)} = \left|\frac{d\vec{r}}{dt}\right| = \left|\frac{d\vec{r'}}{dt'}\right| = \text{const.}$$



⇒ $$\boxed{d(ct)^2 - dx^2 - dy^2 - dz^2 = 0}$$

is an invariant, i.e. same value (=0) in all frames!

- Lorentz transformation
  - primed frame ***S'*** moves with velocity v in z-direction w.r.t. fixed reference frame ***S***
  - reference frames coincide at t = t' = 0
  - point z' is moving with primed frame

⇒ Lorentz transformation (from ***S*** to ***S'***)

$$x' = x \qquad z' = \gamma\cdot(z - \beta\, ct)$$
$$y' = y \qquad ct' = \gamma\cdot(ct - \beta\, z)$$

Quantities used in Accelerator Calculations

- Lorentz transformation
  - reduced velocity: $\beta = \frac{|\vec{v}|}{c}$
  - Lorentz factor: $\gamma = \frac{1}{\sqrt{1-\beta^2}}$
- particle momentum

$$\vec{p} = m\,\vec{v} = \gamma m_0\,\vec{v} \;= \gamma m_0\,\vec{\beta}\,c$$

with m_0: rest mass

- total energy

$$E = m\,c^2 = \gamma m_0 c^2 \quad \Rightarrow \quad E^2 = (pc)^2 + (m_0 c^2)^2$$

with $E_0 = m_0 c^2$: rest mass energy

- kinetic energy

$$E = T_{kin} + m_0\,c^2 \quad \Rightarrow \quad T_{kin} = m_0 c^2\,(\gamma - 1)$$

- useful formulas

$$\gamma = \frac{E}{m_0 c^2} = 1 + \frac{T_{kin}}{m_0 c^2} \qquad \beta = \frac{pc}{E}$$

- example
  - proton with E = 1 TeV

  → value of β ? ⇒ $m_0 c^2$ for proton: 938 MeV

$$\gamma = \frac{E}{m_0 c^2} = \frac{1\ \mathrm{TeV}}{938\ \mathrm{MeV}} = 1066.1$$

$$\beta = \sqrt{1-\gamma^{-2}} = 0.99999956$$

Relativity and Electro-Magnetic Fields

- kinematics / dynamics
  - trajectory transformation: (x, y, z, ct) in rest frame ***S*** → (x', y', z', ct') in moving frame ***S'***
  - Lorentz transformation parameters: reduced velocity β Lorentz factor γ
- transformation of "information carrier"
  - electro-magnetic field transformation

→ as before:
- system ***S'*** moves with *v* = const. along z-axis of rest frame ***S***
- (x, y, z, ct) in rest frame ***S*** → (x', y', z', ct') in moving frame ***S'***

$$E'_x = \gamma\left[E_x - vB_y\right] \qquad B'_x = \gamma\left[B_x + \frac{v}{c^2}E_y\right]$$

$$E'_y = \gamma\left[E_y + vB_x\right] \qquad B'_y = \gamma\left[B_y - \frac{v}{c^2}E_x\right]$$

$$E'_z = E_z \qquad B'_z = B_z$$

- transformation from moving frame ***S'*** to rest frame ***S***: $\vec{v} \longrightarrow -\vec{v}$

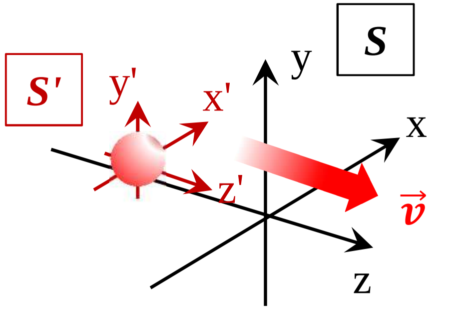
→ convention:
- rest frame ***S***: LAB frame
- moving frame ***S'***: rest frame of moving charge

- comment: different structure of transformation for space-time coordinates and fields

→ field vectors: cannot form 4-vectors (E-field: polar vector, B-field: axial vector)

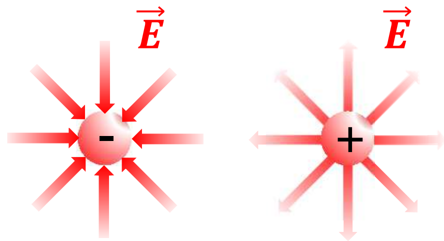
Electro-Magnetic Field of moving Charge

- example
  - point charge Q: moving with v = *const.* along z-axis
  - task: electro-magnetic fields in LAB frame



- rest frame S' of point charge
  - pure electro-static problem

    → radial symmetric *Coulomb field*



$$\vec{E}'(\vec{r}\,') = \frac{Q}{4\pi\varepsilon_0}\frac{\vec{r}\,'}{r'^3} = \frac{Q}{4\pi\varepsilon_0}\frac{1}{[x'^2+y'^2+z'^2]^{3/2}}\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix}$$

- electromagnetic fields in LAB frame S (rest frame)
  - apply Lorentz transformation equations using $\vec{v} \longrightarrow -\vec{v}$
  - 1st step: Lorentz transformation for fields $\Longrightarrow \vec{E}(\vec{r}\,')$
  - 2nd step: Lorentz transformation for space-coordinates $\Longrightarrow \vec{E}(\vec{r})$

$$\boxed{\vec{E}(\vec{r},t) = \frac{1}{4\pi\varepsilon_0}\frac{\gamma Q}{[x^2+y^2+\gamma^2(z-vt)^2]^{3/2}}\begin{pmatrix} x \\ y \\ z-vt \end{pmatrix}}$$

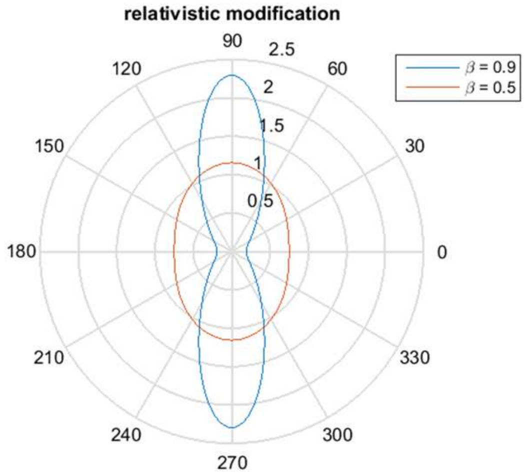
Electro-Magnetic Field of moving Charge (2)

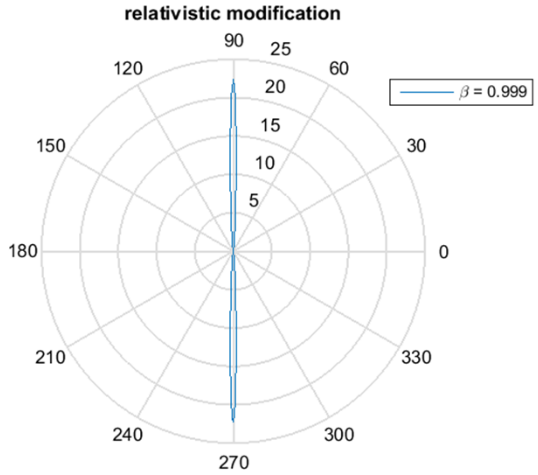
- snap-shot
  - point charge in origin of $\boldsymbol{S}$ and $\boldsymbol{S'}$: t = 0

$$\vec{E}(x,y,z) = \frac{1}{4\pi\varepsilon_0}\frac{\gamma Q}{[x^2+y^2+\gamma^2 z^2]^{3/2}}\begin{pmatrix}x\\y\\z\end{pmatrix} \Rightarrow \boxed{\vec{E}(\vec{r}) = \frac{1-\beta^2}{(1-\beta^2\sin^2\vartheta)^{3/2}}\cdot\frac{Q}{4\pi\varepsilon_0}\cdot\frac{\vec{r}}{r^3}}$$

ϑ: $\measuredangle(z,\vec{r})$

  - relativistic modification of Coulomb field: $\boxed{A_{rel} = \frac{1-\beta^2}{(1-\beta^2\sin^2\vartheta)^{3/2}}}$





- field components
  - longitudinal: $\vartheta = 0 \quad \Longrightarrow \quad E_{\parallel} = \frac{1}{\gamma^2}\cdot\frac{Q}{4\pi\varepsilon_0}\frac{1}{r^2}$
  - transverse: $\vartheta = \frac{\pi}{2} \quad \Longrightarrow \quad E_{\perp} = \gamma\cdot\frac{Q}{4\pi\varepsilon_0}\frac{1}{r^2}$

$\Rightarrow$ ultra-relativistic particles $\rightarrow$ pure transverse E-field

Electro-Magnetic Field of moving Charge (3)

- magnetic field
  - E-field in particle rest frame $\boldsymbol{S'}$ generates B-Field in LAB frame $\boldsymbol{S}$

    → consequence of transformation properties:

$$B_x = -\gamma \frac{v}{c^2} E'_y \qquad B_y = \gamma \frac{v}{c^2} E'_x \qquad B_z = 0$$

  - combined

$$\vec{B}(\vec{r},t) = \frac{\mu_0 Q}{4\pi} \frac{\gamma v}{[x^2+y^2+\gamma^2(z-vt)^2]^{3/2}} \begin{pmatrix} -y \\ x \\ 0 \end{pmatrix}$$

  - snapshot (t = 0) in non-relativistic limit: $\gamma \rightarrow 1$

$$\vec{B}(\vec{r}) = \frac{\mu_0 Q}{4\pi} \frac{v}{[x^2+y^2+z^2]^{3/2}} \begin{pmatrix} -y \\ x \\ 0 \end{pmatrix} = \frac{\mu_0}{4\pi} Q \frac{1}{r^3} \begin{pmatrix} -vy \\ vx \\ 0 \end{pmatrix}$$

  - re-writing

$$\boxed{\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} Q \frac{\vec{v} \times \vec{r}}{r^3}}$$

⇒ Biot Savart law for point charge

Interim Conclusion

- information transfer from / to particle beam
  - electro-magnetic interaction

    → restriction to charged particle beams
- electro-magnetic field of beam particles
  - acts as information carrier about beam properties
- description of particle field
  - basic knowledge of ***Maxwell equations*** and ***special relativity***
- electro-magnetic field of relativistic point charge
  - electric field almost transversal
  - magnetic field → generated due to particle motion

$$\boxed{E_{\parallel} \propto \frac{1}{\gamma^2} \ , \quad E_{\perp} \propto \gamma}$$

- monitor for charge particle beam diagnostics
  - **has to extract information from charged particle beam via electro-magnetic interaction**

    (i) coupling to ***particle*** electro-magnetic field ***carried*** by moving charge

    (ii) coupling to ***particle*** electro-magnetic field ***separated*** from moving charge (freely propagating)

    (iii) exploiting energy deposition due to ***particle*** electro-magnetic field ***interaction*** with matter

    (iv) exploiting ***interaction*** of ***external*** electro-magnetic field with charged particle

Coupling to Particle Electro-Magnetic Field carried by Moving Charge

- Beam Charge and Beam Current Measurements
- Beam Position Monitoring
- …

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Tuusula (Finland), 2-15 June 2018

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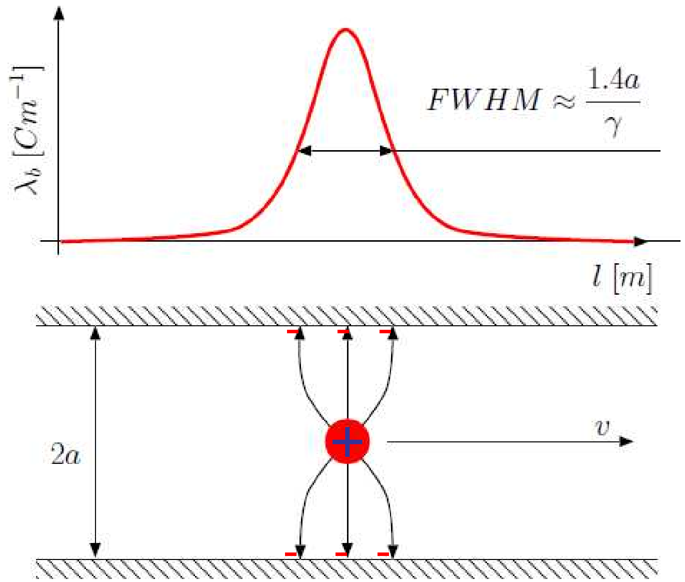
HELMHOLTZ RESEARCH FOR GRAND CHALLENGES

Non-propagating Particle Field

- concept of ***Wall Image Current***
  - charged particle travels through metallic beam pipe of accelerator
    - → beam pipe: evacuated tube, bounded by *electrically conducting material*
  - moving charged particle
    - → generates electro-magnetic field: *electric field* ↔ *charge,* *magnetic field* ↔ *charge movement*
    - → relativistic motion: *Lorentz boost* ↔ *electric field contracts in direction of motion*
  - E-field induces image charge
    - → generated at inner diameter of vacuum chamber
    - → opposite sign
  - moving charge
    - → induced image charge is dragged
    - → creation of ***Wall Image Current (WIC)***



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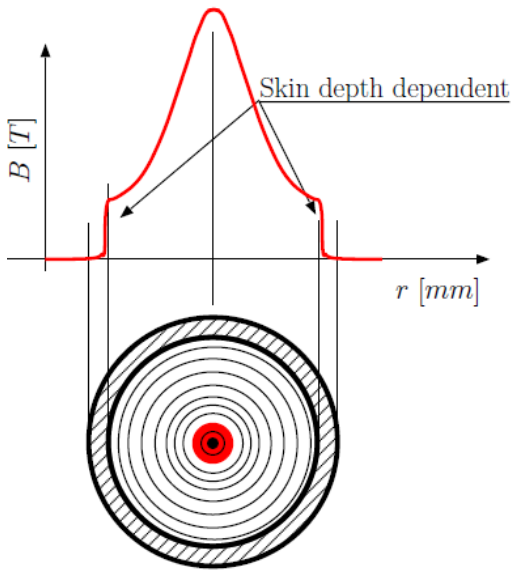
- no electrical field outside vacuum chamber
  - Gauss' flux theorem: $$\oiint_S \vec{E}(\vec{r},t)\cdot d\vec{S} = \frac{1}{\varepsilon_0}\iiint_V \rho(\vec{r},t)\ dV$$
    - → charge and image charge cancels outside beam pipe

➡ no coupling to E-field outside vacuum chamber

Non-propagating Particle Field (2)

- magnetic field
  - Ampère's law $$\oint_C \vec{B}(\vec{r},t)\cdot \mathrm{d}\vec{l} = \mu_0 \iint_S \vec{J}(\vec{r},t)\cdot \mathrm{d}\vec{S}$$
    → integration path: circle ***C*** around beam tube
  - WIC: equal magnitude but opposite sign to beam current (in 1st order)
    → sum of beam and image current cancels out
    → magnetic field outside the beam tube is cancelled

➡ no coupling to B-field outside vacuum chamber



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- field strength reduction
  - corresponds to attenuation of EM-wave propagating through conductor
    → characteristic length: ***skin depth*** (amplitude reduction e^{-1} → -8.69dB)
    non-magnetic, electrically good conductor:
    $$\delta[\mathrm{m}] = \frac{\sqrt{10^7}}{2\pi}\sqrt{\frac{\rho[\Omega/\mathrm{m}]}{f\,[\mathrm{Hz}]}}$$
- consequences for beam monitors
  - no access to particle electro-magnetic field outside metallic beam pipe

➡ coupling to beam field inside vacuum chamber

➡ allow beam field to extend outside

(ceramic gap)

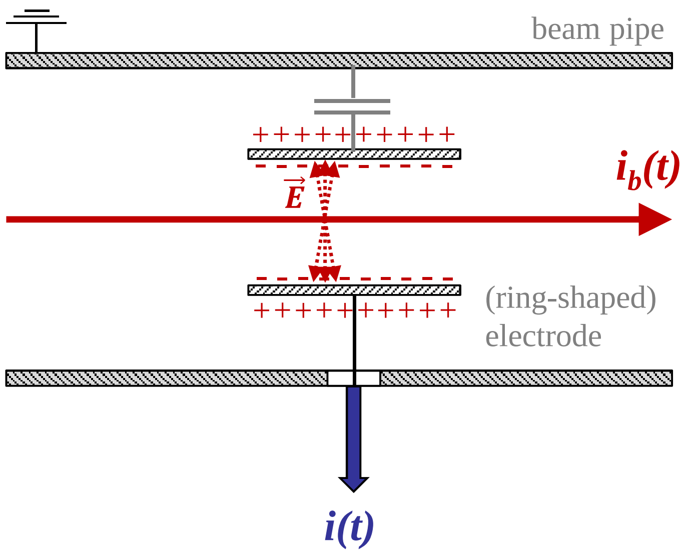
Principles of Signal Extraction

- no electro-magnetic field outside beam pipe
  - place coupling antenna inside vacuum chamber
- charged particle possesses electric / magnetic field
  - 2 different coupling schemata:

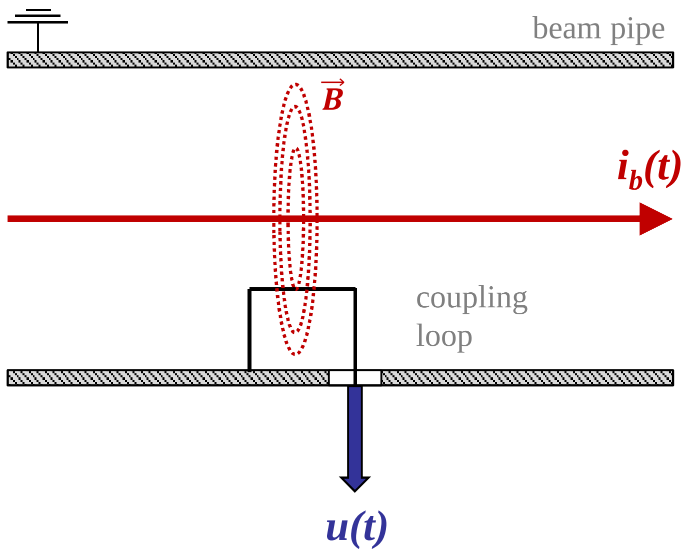
    → coupling to electric field: ***capacitive coupling***

    → coupling to magnetic field: ***inductive coupling***

- capacitive coupling



- inductive coupling



Capacitive versus Inductive Coupling

- capacitive coupling
  - output signal → displacement current

$$i_{cap}(t) = \varepsilon_0 \frac{\mathrm{d}}{\mathrm{d}t} \iint_S \vec{E}(\vec{r},t) \cdot \mathrm{d}\vec{S}$$

<u>comment:</u> consider plate capacity

$$E = \frac{Q}{\varepsilon_0 \cdot S} \quad \longleftrightarrow \quad Q = \varepsilon_0 \cdot S \cdot E$$

with $i(t) = \dot{Q} \quad \Longrightarrow \quad i\,(t) = \varepsilon_0 \cdot S \cdot \dot{E}$

- inductive coupling
  - output signal → Faraday's law of induction

$$u_{ind}(t) = -\frac{\mathrm{d}}{\mathrm{d}t} \iint_S \vec{B}(\vec{r},t) \cdot \mathrm{d}\vec{S}$$

- consider relation between E/B-field:
  - here: $\vec{v} = v\hat{e}_z$
  - relativistic case: $\vec{E} \approx E\hat{e}_r = E_r$

$$\vec{B} = \frac{1}{c^2}\vec{v} \times \vec{E}$$

➡ $$B_\vartheta = \frac{1}{c^2} v E_r = \frac{\beta}{c} E_r$$

- comparison

$$\left|\frac{i_{cap}(t)}{u_{ind}(t)}\right| = \frac{c}{\beta}\varepsilon_0 \frac{\frac{\mathrm{d}}{\mathrm{d}t}\iint_{electrode\,surface} E_r\,\mathrm{d}S}{\frac{\mathrm{d}}{\mathrm{d}t}\iint_{loop\,area} E_r\,\mathrm{d}S}$$

→ practical design: $$\frac{\frac{\mathrm{d}}{\mathrm{d}t}\iint_{electrode\,surface} E_r\,\mathrm{d}S}{\frac{\mathrm{d}}{\mathrm{d}t}\iint_{loop\,area} E_r\,\mathrm{d}S} \approx 1$$

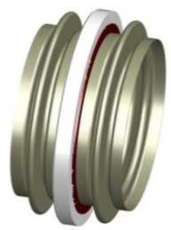
  - broadband signal processing → impedance ***R = 50 Ω***

$$\boxed{\left|\frac{R \cdot i_{cap}(t)}{u_{ind}(t)}\right| = \left|\frac{u_{cap}(t)}{u_{ind}(t)}\right| \approx \frac{Rc\varepsilon_0}{\beta} = \frac{0.133}{\beta}}$$

- practical reasons ➡ capacitive coupling → less prone to stray fields

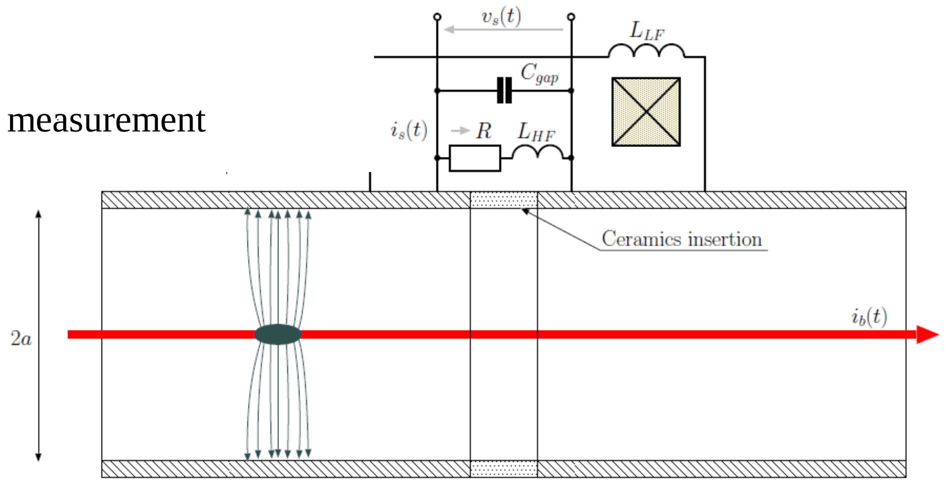
WIC alternative Path

- no electro-magnetic field outside beam pipe
  - provide alternative path for ***Wall Image Current (WIC)***
    - → conducting path in metallic vacuum chamber has to be broken
- technical realization
  - non-conducting material (usually ceramic) inserted electrically in series with metallic beam pipe
    - → interruption forces WIC to find new path
  - beam diagnostics
    - → alternative path under instrument designer's control, outside of vacuum chamber



(ceramic gap)

- example
  - ***Wall Current Monitor***
    - → broadband ($\geq$ 5 GHz) beam charge measurement



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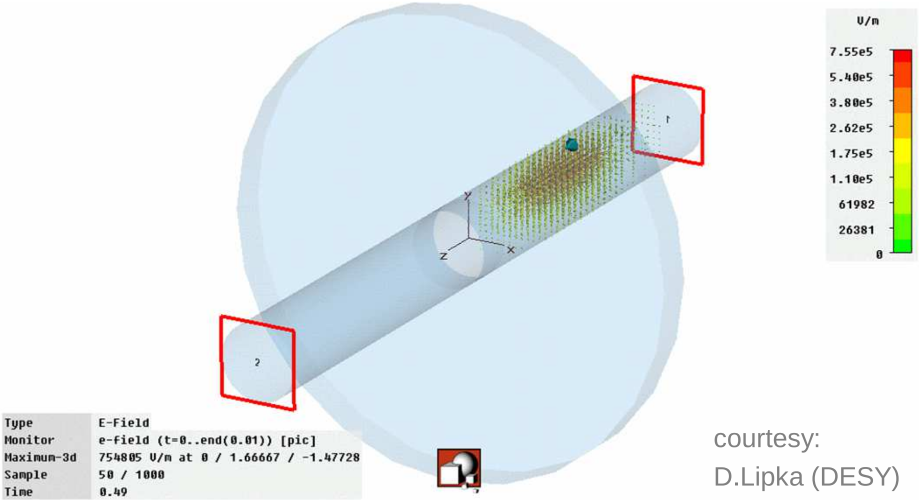
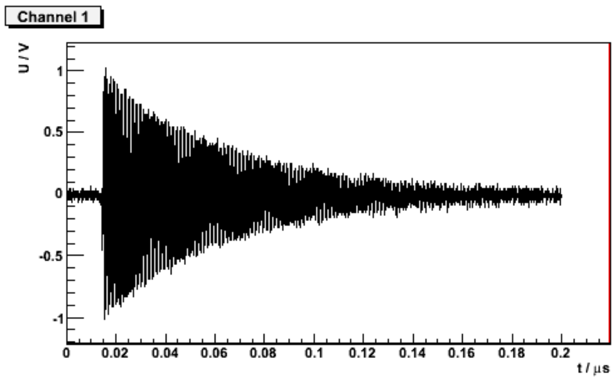
Cavity Resonators

- beam signal generation using ***passive cavity resonator***
  - passive (beam driven) cavity resonator
    - → electro-magnetic discontinuity in beam pipe
    - → charged particle passing resonator excites (several) resonator modes
  - example
    - → E-field excitation in pillbox cavity



courtesy:
D.Lipka (DESY)

- advantage of resonator
  - electro-magnetic energy dissipation for one period
    - → small compared to accumulated energy



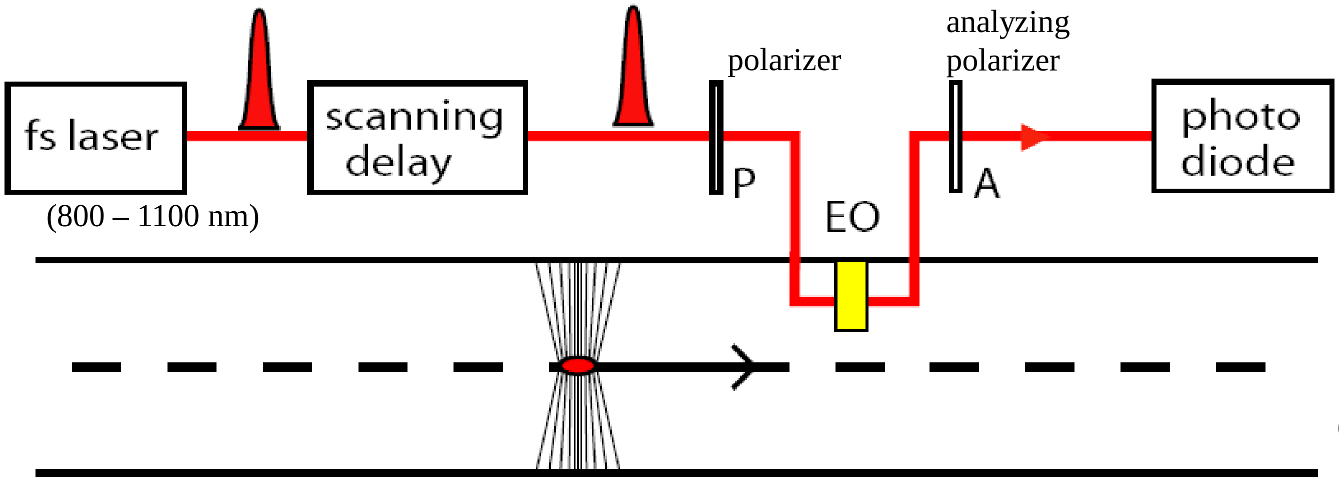
  - signal averaging over long time
    - → good signal quality, high accuracy

- task for beam diagnostics
  - design cavity for high signal level in resonator mode of interest
    - → suppress contribution from disturbing modes

Environment Modification

- application: ***Electro Optical (EO) techniques***
  - bunch length diagnostics
    → fsec electron bunches
  - placing EO crystal into beam pipe
    → direct measurement of Coulomb field from ultra-relativistic bunches in time-domain
    → Coulomb-field carried by sub-psec bunches reaches in THz region
  - Coulomb field induces ***refractive index change*** in ***birefringent crystal***
    → Pockels effect in optically active crystal (e.g. ZnTe, GaP)
  - probing of refractive index change by short-pulse (fsec), high bandwidth (some tens of nm) laser
    → detect linearly polarized light intensity variation

birefringence:
splitting ray into 2 parallel rays polarized perpendicular



courtesy V. Schlott (PSI), B. Steffen (DESY)

Coupling to freely propagating Particle Electro-Magnetic Field

- Bunch Length Measurements
- transverse Beam Profile Diagnostics
- …

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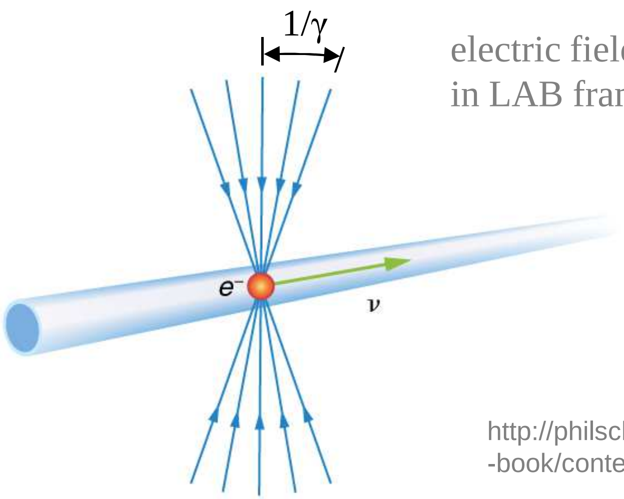
Propagating Particle Field

- freely propagating particle field
  - electro-magnetic field not bound to charged particle

    ⇒ emitted as radiation (preserving information from beam)

- radiation generation via particle electro-magnetic field
  - particle electro-magnetic field



electric field lines in LAB frame

http://philschatz.com/physics-book/contents/m42535.html

  - relativistic boost characterized by Lorentz factor

$$\boxed{\gamma = \frac{E}{m_0 c^2}}$$

E: total energy

m_0c^2: rest mass energy

proton: $m_p c^2 = 938.272$ MeV

electron: $m_e c^2 = 0.511$ MeV

- limiting case: $\gamma \rightarrow \infty$ ⇒ plane wave
  - $m_0c^2 = 0$ MeV : light → „real photon“
  - ultra relativistic energies : idealization → „virtual photon“ (basis of Weizsäcker-Williams method)

Separation of Particle Field

- electro-magnetic field bound to particle

  observation in far field (large distances)

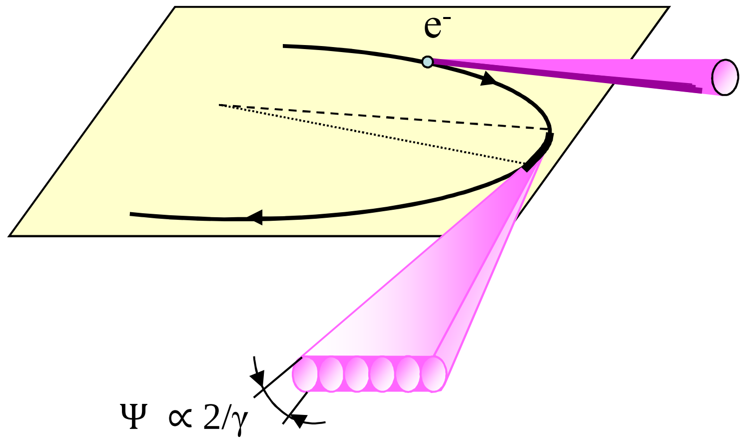
  } separate field from particle

- separation mechanisms
  - ***bending*** of ***particle*** via magnetic field

    synchrotron radiation

    ➡ circular accelerators

  linear accelerator → no particle bending…



- separation mechanisms at linear accelerators
  - ***diffraction/reflection*** of particle ***electro-magnetic field*** at material structures

    exploit analogy between real/virtual photons:

    - light reflection/refraction at surface ↔ backward/forward transition radiation (TR)
    - light diffraction at edges ↔ diffraction radiation (DR)
    - light diffraction at grating ↔ Smith-Purcell radiation
    - light (X-ray) diffraction in crystal ↔ parametric X-ray radiation (PXR) …

Radiation Generation and Mass Shell

- consider mass hyperboloid
  - hyperboloid in energy–momentum space describing the solutions to equation

$$\boxed{E^2 = (\vec{p}c)^2 + (m_0 c^2)^2}$$

  - charged particle behavior governed by this equation

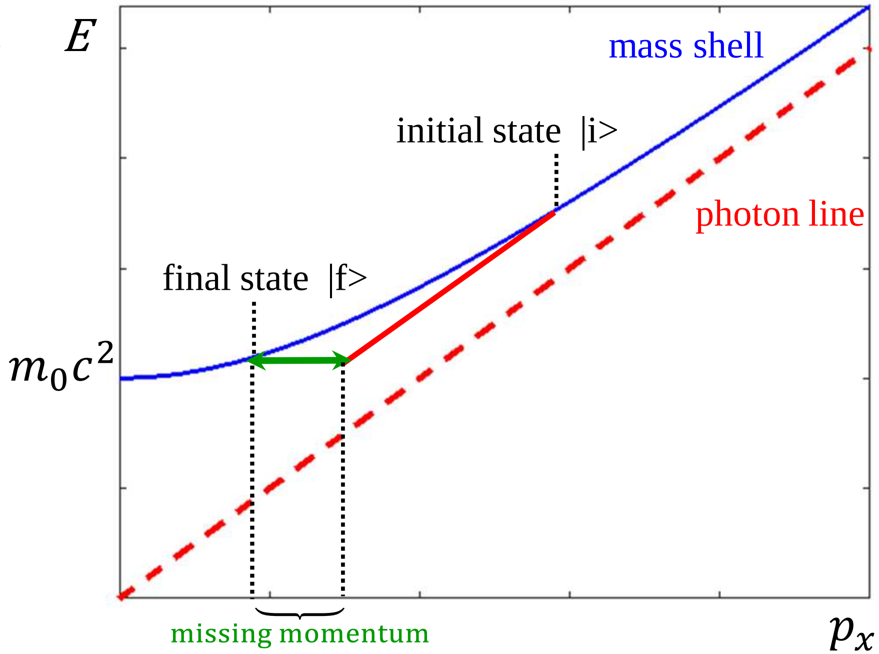
    → sitting on the ***mass shell***

- energy loss via radiation emission
  - transition from ***initial*** |*i*> to ***final*** |*f*> state
  - photon: massless particle → $E = pc$

- energy / momentum conservation has to be fulfilled
  - missing momentum remains

    ➡ externally provided

    (radial force, material structure, …)

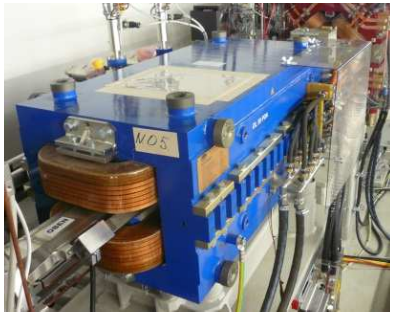


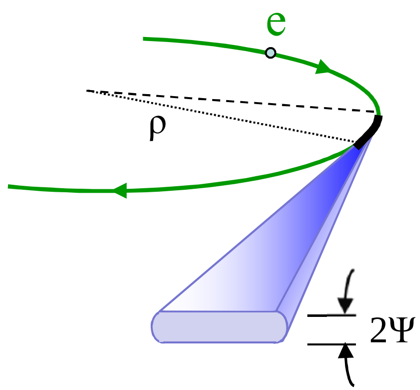
- Cherenkov radiation as special case
  - direct transition from ***initial*** |*i*> to ***final*** |*f*> state without external momentum

    → slope of photon line decreased: $c \longrightarrow {}^{c}/_{n}$ (*n*: index of refraction)

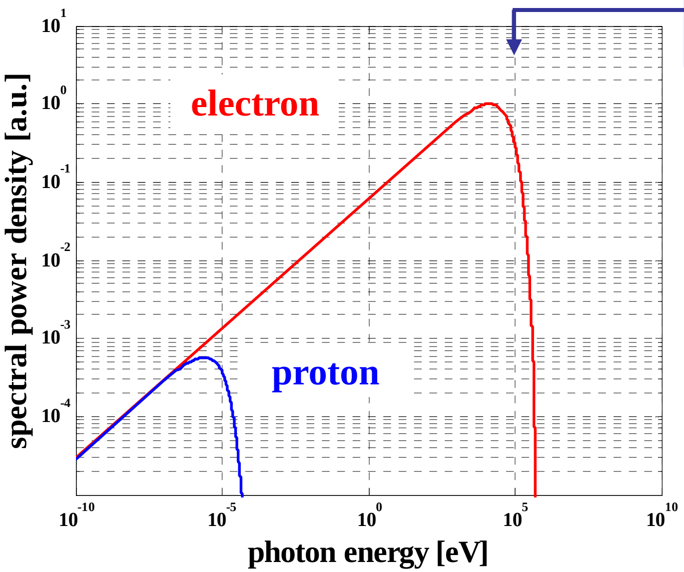
Synchrotron Radiation

- circular accelerator: radiation source available for free
  - bending magnet (wiggler, undulator)
- minimum-invasive
  - unavoidable losses
- strong collimation (vertical)
  - opening angle: $\boxed{\Psi \propto 1/\gamma}$





- emission over wide spectral range
  - choice of operational range



$$\hbar\omega_c = \frac{3}{2}\hbar c\frac{\gamma^3}{\rho}$$

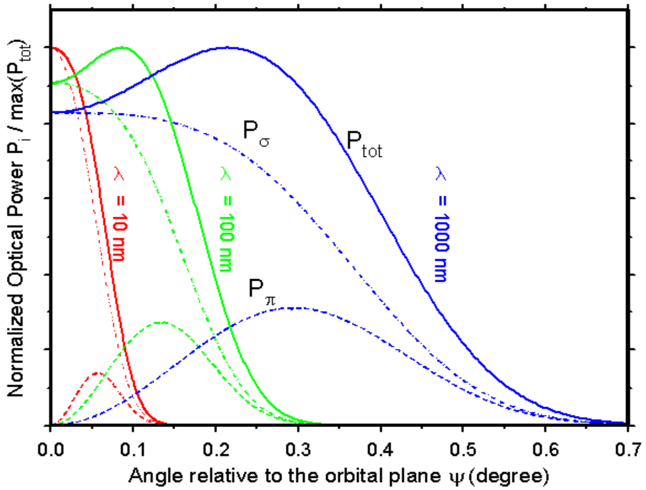
γ: Lorentz factor

ρ: bending radius

E_{kin} = 20 GeV

ρ= 370 m

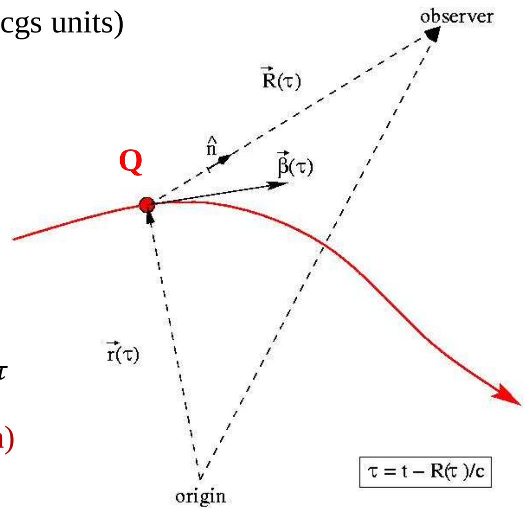
- polarized
  - define vertical angular divergence



SR Field: Standard Text Book

- source field: particle field described by **Liénard-Wiechert potentials:** (in cgs units)

$$\varphi(t) = \left(\frac{Q}{R\left(1-\hat{n}\cdot\vec{\beta}\right)}\right)_\tau , \qquad \vec{A}(t) = \left(\frac{Q\vec{\beta}}{R\left(1-\hat{n}\cdot\vec{\beta}\right)}\right)_\tau$$



  - field derivation: $\vec{E}(t) = -\vec{\nabla}\varphi(t) - \frac{1}{c}\dot{\vec{A}}(t)$, $\vec{H}(t) = \vec{\nabla}\times\vec{A}(t)$

$$\Rightarrow \quad \vec{E}(t) = Q\left(\cancel{\frac{\left(1-\beta^2\right)\left(\hat{n}-\vec{\beta}\right)}{R^2\left(1-\hat{n}\cdot\vec{\beta}\right)^3}} + \frac{\hat{n}\times\left[\left(\hat{n}-\vec{\beta}\right)\times\dot{\vec{\beta}}\right]}{cR\left(1-\hat{n}\cdot\vec{\beta}\right)^3}\right)_\tau , \qquad \vec{H}(t) = \left(\hat{n}\times\vec{E}\right)_\tau$$

neglect velocity term (far field **approximation**)

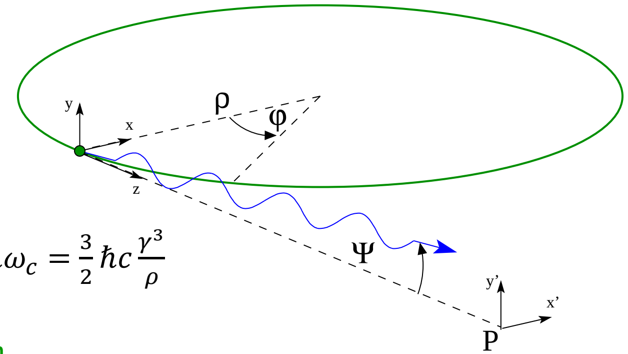
  - Fourier transform: $\vec{E}(\omega) \approx \frac{i\omega Q}{cR}\int_{-\infty}^{+\infty} \mathrm{d}\tau \left[\hat{n}\times\left[\hat{n}\times\vec{\beta}\right]\right] e^{i\omega(\tau+R(\tau)/c)}$

- special case: charged particle moving on circular orbit

$$E_x(\omega) = E_\sigma = A_\sigma \frac{\hbar\omega}{2\hbar\omega_c}\left(1+\gamma^2\Psi^2\right)\cdot \mathrm{K}_{2/3}\left[\frac{\hbar\omega}{2\hbar\omega_c}\left(1+\gamma^2\Psi^2\right)^{3/2}\right]$$

$$E_y(\omega) = E_\pi = A_\pi \frac{\hbar\omega}{2\hbar\omega_c}\gamma\Psi\sqrt{1+\gamma^2\Psi^2}\cdot \mathrm{K}_{1/3}\left[\frac{\hbar\omega}{2\hbar\omega_c}\left(1+\gamma^2\Psi^2\right)^{3/2}\right]$$

with $\hbar\omega_c = \frac{3}{2}\hbar c\frac{\gamma^3}{\rho}$



$\Rightarrow$ analytical field description

- comments: (i) approximative field description → far field approximation

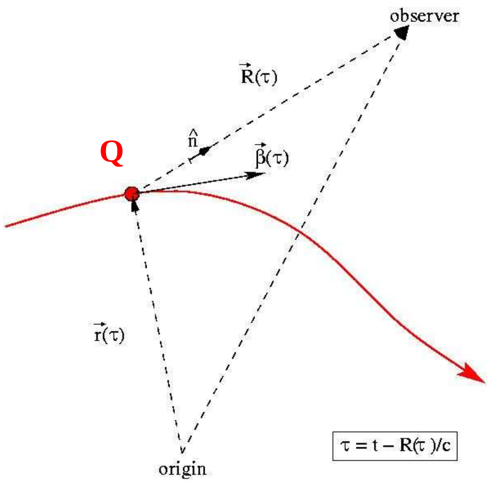
  (ii) emission from single point on orbit → additional contributions: depth of field, orbit curvature

Synchrotron Radiation Field

- second representation: starting point again **Liénard-Wiechert potentials**

O.Chubar and P.Elleaume, Proc. EPAC96, Stockholm (1996) 1177

$$\varphi(t) = \left(\frac{Q}{R\left(1-\hat{n}\cdot\vec{\beta}\right)}\right)_\tau, \qquad \vec{A}(t) = \left(\frac{Q\vec{\beta}}{R\left(1-\hat{n}\cdot\vec{\beta}\right)}\right)_\tau$$



- Fourier transform of potentials:

$$\varphi(\omega) = Q\int_{-\infty}^{+\infty} \mathrm{d}\tau \frac{1}{R(\tau)} e^{i\omega(\tau+R(\tau)/c)}, \quad \vec{A}(\omega) = Q\int_{-\infty}^{+\infty} \mathrm{d}\tau \frac{\vec{\beta}(\tau)}{R(\tau)} e^{i\omega(\tau+R(\tau)/c)}$$

- field derivation:

$$\boxed{\vec{E}(\omega) = \frac{i\omega Q}{c}\int_{-\infty}^{+\infty} \mathrm{d}\tau \left[\frac{(\vec{\beta}-\hat{n})}{R(\tau)} - \frac{ic}{\omega}\frac{\hat{n}}{R^2(\tau)}\right] e^{i\omega(\tau+R(\tau)/c)}}$$

$$\text{with} \quad \tau = \int_0^z \frac{\mathrm{d}z}{c\beta_z(z)} = \frac{1}{c}\int_0^z \mathrm{d}z\left[1 + \frac{1+(\gamma\beta_x)^2+\left(\gamma\beta_y\right)^2}{2\gamma^2}\right]$$

➡ knowledge of arbitrary particle orbit: $\vec{E}(\omega)$ determined

➡ arbitrary magnetic field configuration: determines orbit and $\vec{E}(\omega)$

- comments:

(i) exact field description → numerical near field calculation

(ii) includes depth of field & curvature → no additional contributions, only field propagation

(iii) free codes available → easy field calculation, even field propagation!

SRW: http://www.esrf.eu/Accelerators/Groups/InsertionDevices/Software/SRW (Chubar & Elleaume, ESRF)

Spectra: http://radiant.harima.riken.go.jp//spectra/index.html (Tanaka & Kitamura, SPring8)

SR for Heavy Particles

- synchrotron radiation spectrum
  - characterized by critical energy / wavelength

$$\hbar\omega_c = \frac{3}{2}\hbar c\frac{\gamma^3}{\rho} \quad \Leftrightarrow \quad \lambda_c = \frac{4\pi}{3}\frac{\rho}{\gamma^3}$$

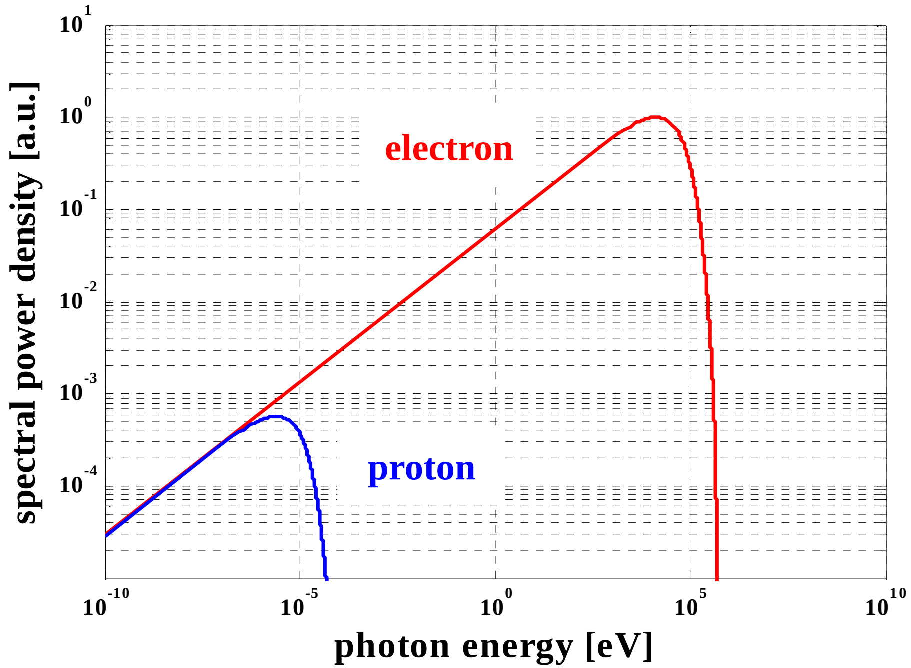
γ: Lorentz factor

ρ: bending radius

- heavy particles (protons)
  - large mass (protons: factor 1836 larger than for electrons)

→ small Lorentz factor $\gamma = E/m_0c^2$

- comparison of SR spectra



T_{kin} = 20 GeV, ρ= 370 m

- example
  - **HERA-p**: *E = 40…920 GeV*

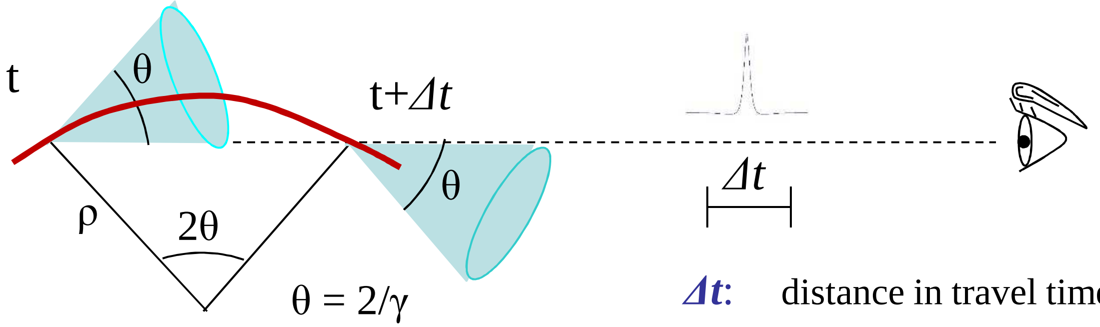
→ **λ_c = 55 mm … 4.5 µm**

→ large diffraction broadening, expensive optical elements,…

→ smaller λ achieveable ???

SR Single Particle Time Structure

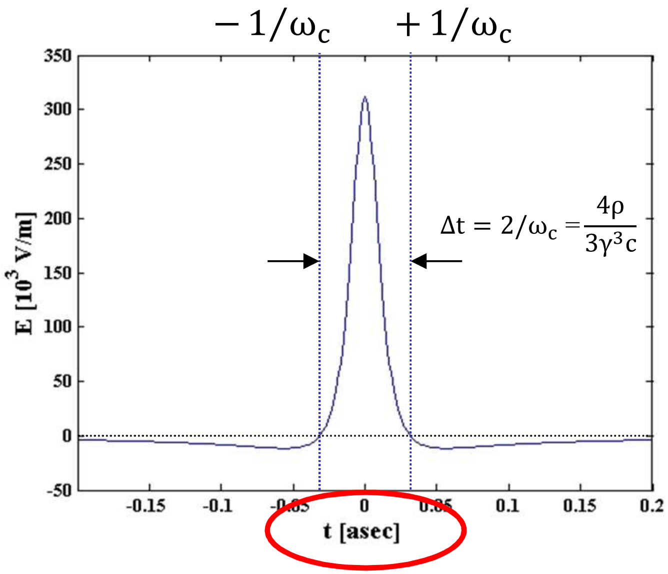
- geometrical interpretation



$$\Delta t = \frac{4\rho}{3\gamma^3 c}$$

Δt: distance in travel time between photon and particle

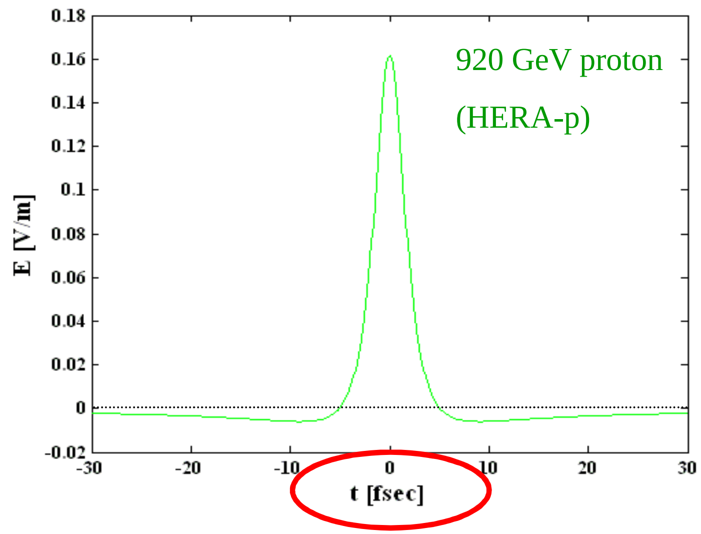
- radiation field in time domain



6 GeV electron,

field in orbit plane

- comparison with protons

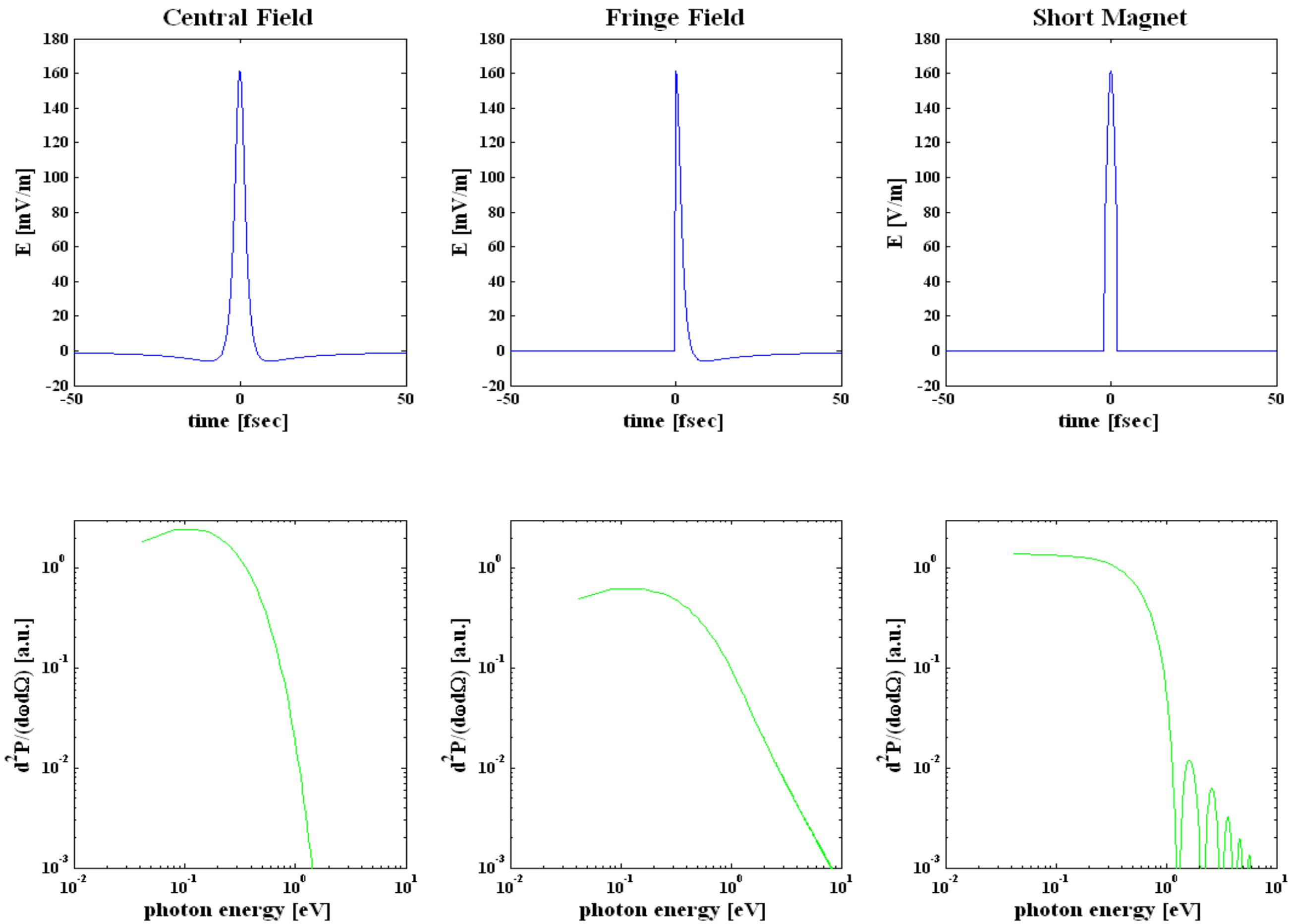


spectrum defined by time interval from maximum to zero crossing (ω_c)

"time squeezing" required

Time Squeezing

- introduce sharp "cut-off" in time domain



$$\frac{d^2N}{d\Omega d\omega/\omega} \propto \left|\vec{E}_\omega\right|^2 \qquad \text{with} \qquad \vec{E}_\omega = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} dt \vec{E}(t) e^{i\omega t}$$

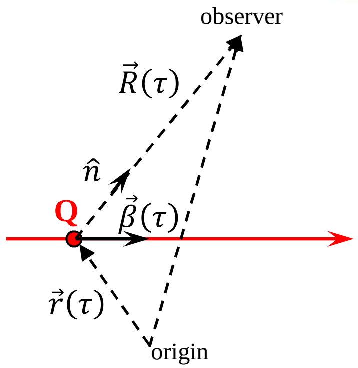
"frequency boost" possible

Constant Linear Motion

- source field
  - point charge with **constant** velocity v → Liénard-Wiechert fields

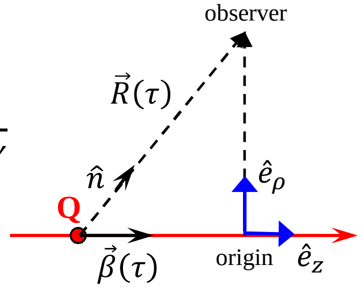
$$\Rightarrow \quad \vec{E}(t) = Q\left(\frac{(1-\beta^2)(\hat{n}-\vec{\beta})}{R^2(1-\hat{n}\cdot\vec{\beta})^3} + \cancel{\frac{\hat{n}\times[(\hat{n}-\vec{\beta})\times\dot{\vec{\beta}}]}{cR(1-\hat{n}\cdot\vec{\beta})^3}}\right)_\tau , \qquad \vec{H}(t) = (\hat{n}\times\vec{E})_\tau$$

no acceleration term



  - common representation → cylindrical coordinate system

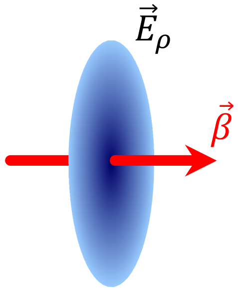
$$\Rightarrow \quad \vec{E}(\rho,\varphi,z,\omega) = \frac{Q\alpha}{\pi v} e^{i\frac{\omega}{v}z}\left(K_1(\alpha\rho)\hat{e}_\rho - \frac{i}{\gamma}K_0(\alpha\rho)\hat{e}_z\right) \quad \text{with} \quad \alpha = \frac{\omega}{\gamma v} = \frac{2\pi}{\lambda\beta\gamma}$$



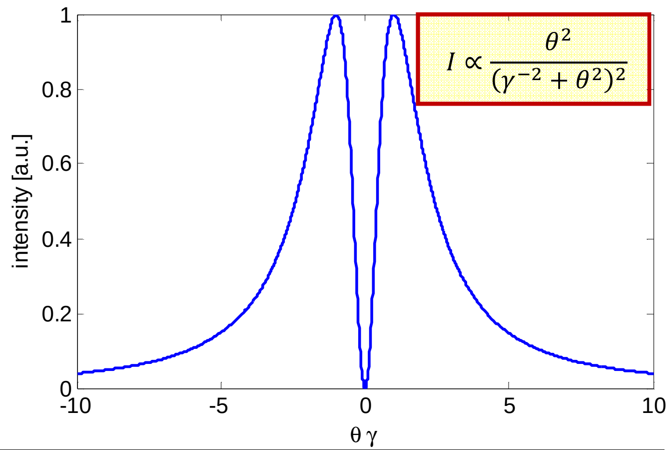
  - ultra-relativistic particles ($\gamma >> 1$)
    - → neglect longitudinal field component
    - → pure transverse „pancake“ structure
    - → radial extension: $\alpha\rho \approx 1$

$$\boxed{\rho = \frac{\lambda\beta\gamma}{2\pi} \approx \gamma\lambda}$$

virtual photon range



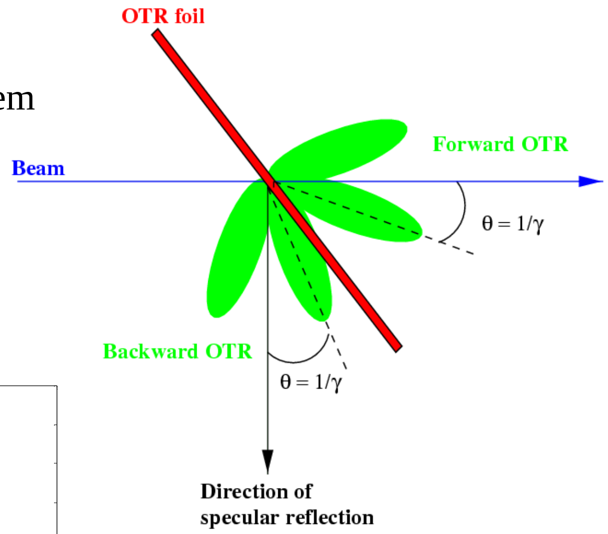
  - angular distribution



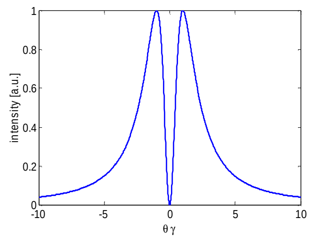
  - separation of field → different radiation sources

Transition Radiation

- transition radiation: electromagnetic radiation emitted when a charged particle crosses boundary between two media with different optical properties
- visible part: Optical Transition Radiation (OTR)
- beam diagnostics: backward OTR
  typical setup: image beam profile with optical system

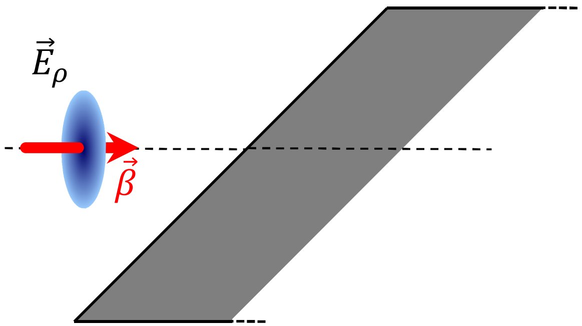


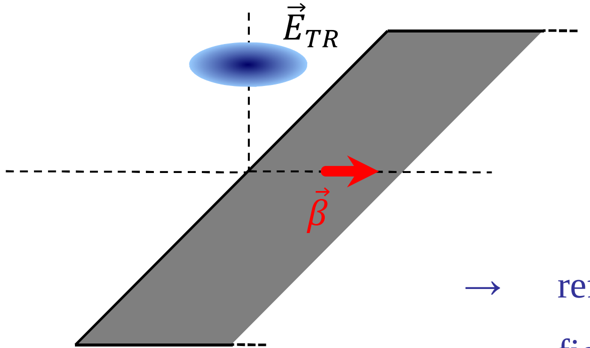
- advantage: fast single shot measurement
  linear response (neglect coherence !)
- disadvantage: high charge densities may destroy radiator
  → limitation on bunch number



- field separation mechanism

  → reflection at boundary (perfect conductivity)





→ reflected and incident field are the same

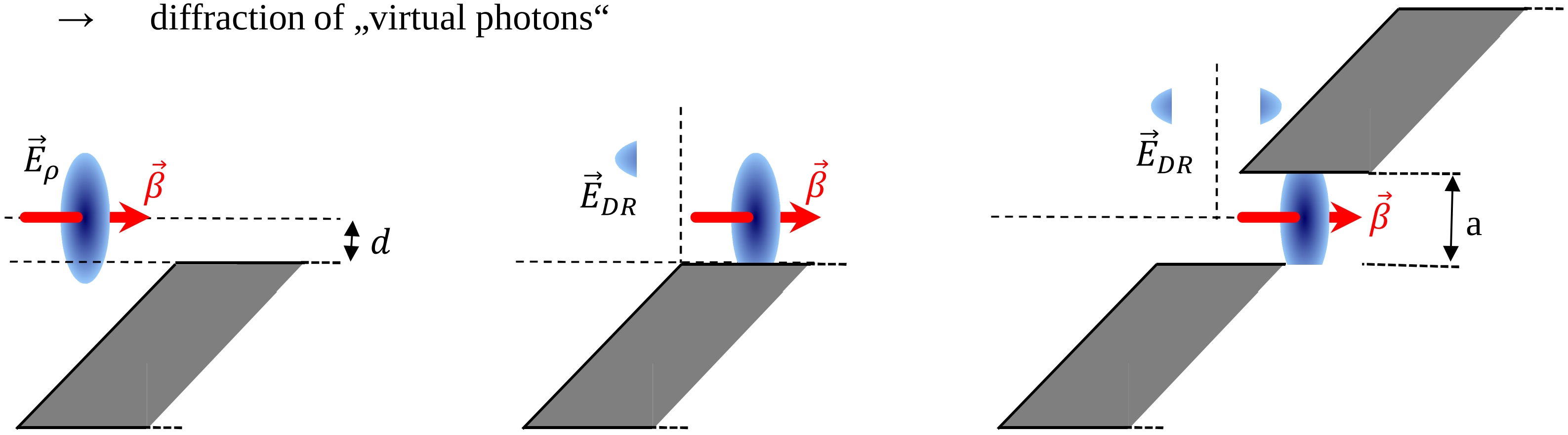
Diffraction Radiation

- problem OTR: screen degradation / damage

  → limited to only few bunch operation, no permanent observation

- Optical Diffraction Radiation (ODR): non-intercepting beam diagnostics
  - DR generation via interaction between particle EM field and conducting screen

    → diffraction of „virtual photons“



screen: half-plane

slit screen

- radial field extension

  → radius $\lambda\beta\gamma / 2\pi \approx \lambda\gamma$

- limiting cases

  $a >> \lambda\gamma$: no radiation

  $a \cong \lambda\gamma$: DR

  $a << \lambda\gamma$: TR

Parametric X-Ray Radiation (PXR)

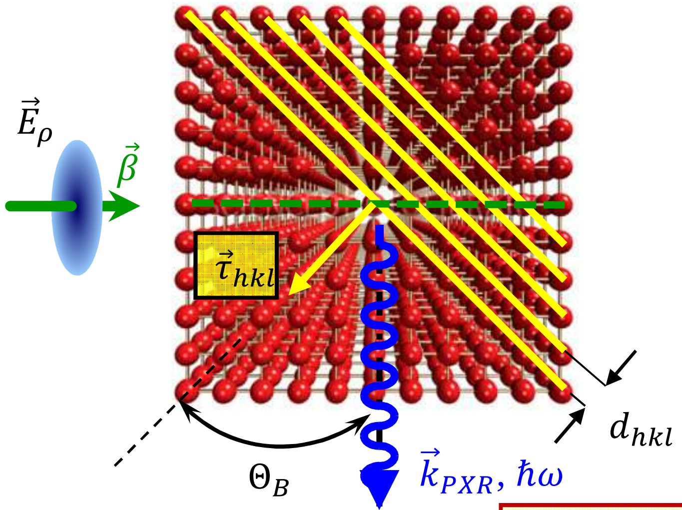
- idea: higher photon energies ħω
  - better resolution
  - insensitive on coherent effects
- real photons
  - X-rays ↔ Bragg reflection, crystals
- virtual photons
  - field separation by Bragg reflection at crystal lattice

    → radiation field: **Parametric X-Ray Radiation (PXR)**
- crystal periodicity (3D)
  - discrete momentum transfer (reciprocal lattice vector $\vec{\tau}_{hkl}$)

    → emission of line spectrum

courtesy: M.J. Winter

(Science Photo Library)



$$\sin\Theta_B = \frac{\lambda}{2d_{hkl}}$$

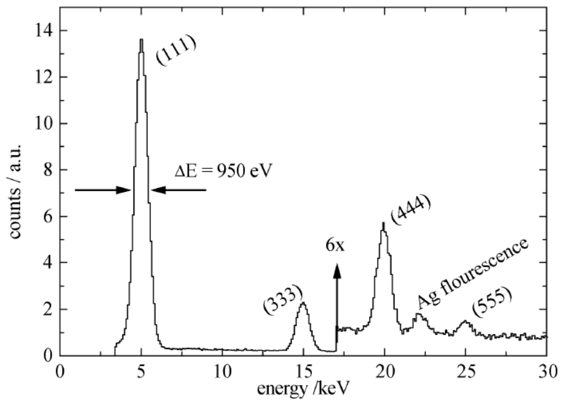
$$\vec{p}_i = \vec{p}_f + \hbar\vec{k} + \hbar\vec{\tau}_{hkl}$$

$$\delta E = \left(\vec{p}_i - \vec{p}_f\right)\cdot\vec{v} = \hbar\vec{k}\cdot\vec{v} + \hbar\vec{\tau}_{hkl} = \hbar\omega$$

$$\boxed{\hbar\omega_{hkl} = \hbar c\frac{|\vec{\beta}\cdot\vec{\tau}_{hkl}|}{1-\sqrt{\varepsilon}\vec{\beta}\cdot\hat{k}}}$$

$\varepsilon = 1 - |\chi_0|$

dielectric constant (≈ 1)



Si crystal

E = 855 MeV

Θ_B = 22.5 °

K.H. Brenzinger et al.,

Z. Phys. **A 358** (1997) 107

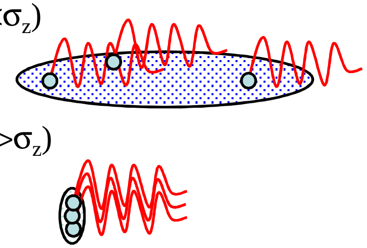
Smith-Purcell Radiation

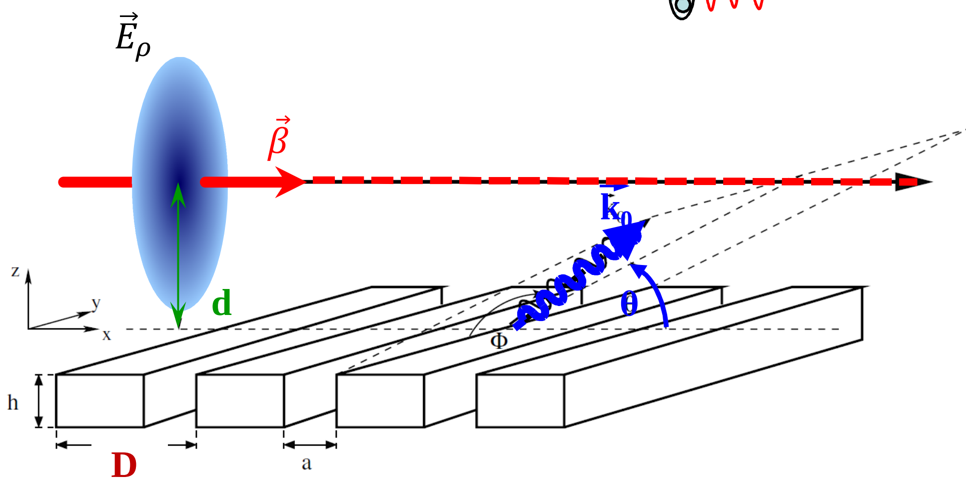
- idea: dispersive radiation generation for bunch length diagnostics
  - Coherent Radiation Diagnostics (CRD)
    - → compact setup (combined radiator / analysator)
- Smith-Purcell radiation (SPR)
  - field separation
    - → virtual photon diffraction at 1D *Bravais-structure* (grating)
    - → grating provides 1D discrete momentum

CRD: standard method for radiation based bunch length diagnostics

long bunch ($\lambda<\sigma_z$)

short bunch ($\lambda>\sigma_z$)





momentum conservation:

$$\vec{p}_i = \vec{p}_f + \hbar\vec{k} + \hbar n\frac{2\pi}{D}\hat{v}$$

$$(\vec{p}_i - \vec{p}_f)\cdot\vec{v} = \hbar\omega = \hbar\vec{k}\cdot\vec{v} + \hbar n\frac{2\pi}{D}\hat{v}\cdot\vec{v}$$

$$2\pi\frac{c}{\lambda} = \frac{2\pi}{\lambda}\mathrm{v}\cos\theta + n\frac{2\pi}{D}\mathrm{v}$$

$$\boxed{n\lambda = \mathrm{D}\left(\frac{1}{\beta} - \cos\theta\right)}$$

→ **SPR dispersion relation**

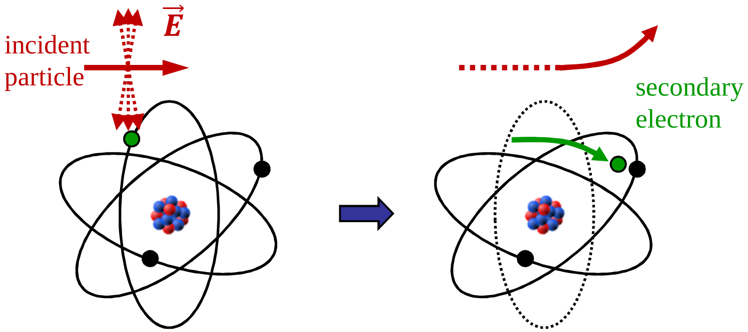
Particle Electro-Magnetic Field Interaction with Matter

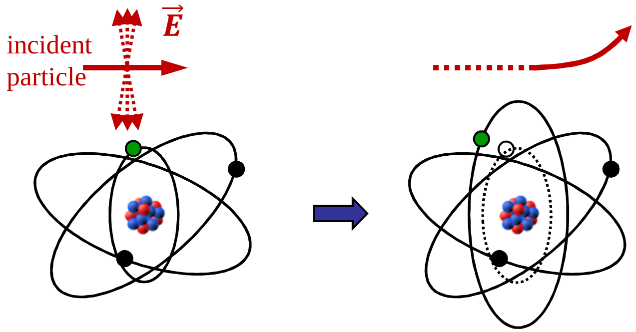
- Beam Loss Monitoring
- Beam Charge Measurements (Faraday Cup)
- Beam Profile Measurements (Wire Scanner, SEM, Scintillator)
- …

Charged Particle Interaction with Matter

- energy deposition of charged particles in matter
  - applied for beam monitoring → scintillating light generation, secondary electron emission, …
- types of particle interaction
  - charged particle transmits some of its energy to particles in medium → excitation of medium particles via:

→ ***ionization*** → ***excitation of optical states*** → …





- level of particle–particle interaction: important modes of interaction
  - elastic scattering → incident particle scatters off target particle, total T_{kin} of system remains constant
  - inelastic scattering → incident particle excites atom to higher electronic/nuclear state
  - annihilation
  - Bremsstrahlung emission
  - Cherenkov & Transition Radiation, …

Interaction of Heavy Charged Particles

- "heavy" particles: $A \geq 1$ (p, α, ions,…) ➡ 2 electro-magnetic interaction channels …
- interaction modes

  (1) *Rutherford (Coulomb) scattering* → elastic scattering

  › Coulomb force interaction between ***incident particle*** and ***target nucleus*** → not applied for beam diagnostics

  (2) *passage of particles through matter*

  › number of electronic/nuclear mechanisms, through which charged particle can interact with medium particles

  › net result of all interactions → ***reduction of particle energy***

  › underlying interaction mechanisms are complicated

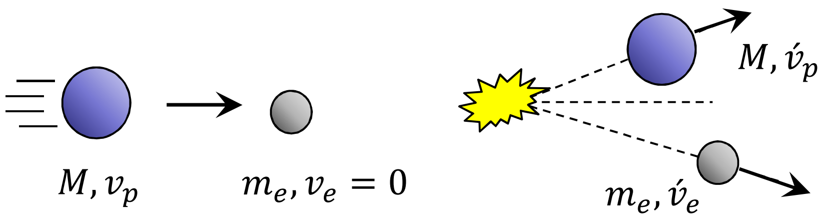
  → rate of energy loss fairly accurately predicted by semi-empirical relations

  ➡ relevant for beam diagnostics

- energy transfer from projectile to target → dominated by elastic collisions with shell electrons

  › projectile → ***beam particle***

  › target → ***atomic shell electron***



  › maximum energy transfer → head-on collision

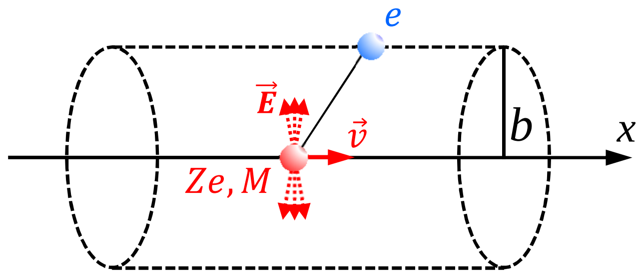
$$\frac{\Delta E_{max}}{T_{kin}} = 4\frac{m_e M}{(m_e + M)^2} \xrightarrow{M \gg m_e} 4\frac{m_e}{M}$$

proton beam: $\frac{\Delta E_{max}}{T_{kin}} = 4 \cdot \frac{1}{1836} \sim \frac{1}{500}$

➡ small energy transfer in single collision

Energy Loss by Ionization – Bohr

- classical derivation by Bohr (1913):
  - particle with ***charge Ze*** moves with ***velocity v*** through medium with ***electron density n***
  - electrons are conidered free and initially at rest   (assumption of elastic collisions $\rightarrow$ losses in fact inelastic)
- momentum transfer to single electron



$$\Delta\vec{p}_\perp = \int \mathrm{d}t\, \vec{F}_\perp = \int \mathrm{d}x\, \vec{F}_\perp \frac{\mathrm{d}t}{\mathrm{d}x} = \int \vec{F}_\perp \frac{\mathrm{d}x}{v} = e \int \vec{E}_\perp \frac{\mathrm{d}x}{v}$$

$\Delta\vec{p}_\parallel$: averages to ***zero*** $\rightarrow$ symmetry

  - apply Gauss' flux theorem (in cgs units): $\int \vec{E}\cdot \mathrm{d}\vec{S} = 4\pi Z e$

$$\int \vec{E}_\perp \cdot 2\pi b\, \mathrm{d}x = 4\pi Z e \quad \Longrightarrow \quad \int \vec{E}_\perp\, \mathrm{d}x = \frac{2Ze}{b}$$

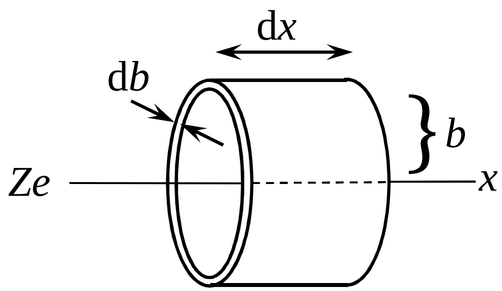
$$\boxed{\Longrightarrow \quad \Delta\vec{p}_\perp = \frac{2Ze^2}{bv}}$$

- energy transfer to ***single*** electron, located at transverse distance ***b***

$$\Delta E(b) = \frac{\Delta\vec{p}^2}{2m_e} \qquad \boxed{\Longrightarrow \quad \Delta E(b) = \frac{2Z^2e^4}{m_e v^2 b^2}}$$

- integration over ***all*** electrons in medium
  - consider cylindrical barrel with N_e electrons

$$N_e = n\, 2\pi b\, \mathrm{d}b\, \mathrm{d}x$$



Energy Loss by Ionization (2) – Bohr

- energy loss per path length **d*x*** for distance between ***b*** and ***b*+d*b*** in medium with electron density ***n***:

$$-\mathrm{d}E(b) = \frac{\Delta p^2}{2m_e} N_e = \frac{4\pi Z^2 e^4}{m_e v^2} n \frac{\mathrm{d}b}{b}\,\mathrm{d}x$$

$$\Rightarrow \quad -\frac{\mathrm{d}E}{\mathrm{d}x} = \frac{4\pi Z^2 e^4}{m_e v^2} n \int_{b_{min}}^{b_{max}} \frac{\mathrm{d}b}{b} = \frac{4\pi Z^2 e^4}{m_e v^2} n \ln \frac{b_{max}}{b_{min}}$$

- determination of relevant ***b*** range
  - ***b**$_{min}$*: for head-on collisions in which kinetic energy transfer is maximum $W_{max} = 2m_e c^2 \beta^2 \gamma^2$

$$\Delta E_{max}(b_{min}) = \frac{2Z^2 e^4}{m_e v^2 {b_{min}}^2} \stackrel{\text{def}}{=} W_{max} \quad \Rightarrow \quad b_{min} = \frac{Ze^2}{\gamma m_e v^2}$$

  - ***b**$_{max}$*: principle of adiabatic invarianc → e$^-$ bound to atom, circulating nucleus with mean orbital frequency $\bar{\nu}$

→ energy transfer: time interval of distortion ≤ period duration

$$\Delta t = \frac{b}{\gamma v} \le \tau = \frac{1}{\bar{\nu}} \quad \Rightarrow \quad b_{max} = \frac{\gamma v}{\bar{\nu}}$$

$$\Rightarrow \quad \boxed{-\frac{\mathrm{d}E}{\mathrm{d}x} = \frac{4\pi n Z^2 {r_e}^2 m_e c^2}{\beta^2} \ln\left(\frac{\gamma^2 m_e v^3}{Z e^2 \bar{\nu}}\right)}$$

with $r_e = \frac{e^2}{4\pi\varepsilon_0 m_e c^2}$ → classical electron radius, $n = N_A \rho \frac{Z_T}{A_T}$ → electron density

Bethe–Bloch (–Sternheimer) Formula

- quantum mechanical based calculation of ***collisional*** energy loss:

$$-\left\langle \frac{\mathrm{d}E}{\mathrm{d}x} \right\rangle_{coll} = 4\pi N_A {r_e}^2 m_e c^2 \cdot \rho \frac{Z_t}{A_t} \cdot \frac{{Z_p}^2}{\beta^2} \ln\left(\frac{2m_e c^2 \beta^2 \gamma^2}{I} - \beta^2 - \frac{\delta}{2} - \frac{C}{Z_t} \right)$$

- fundamental constants

r_e:	classical electron radius
m_e:	mass of electron
N_A:	Avogadro's number
c:	speed of light

- absorber medium

I:	mean ionization potential
Z_t:	atomic number of absorber
A:	atomic weight of absorber
ρ:	density of absorber
δ:	density correction
C:	shell correction

- incident particle

Z_p:	charge of incident particle
β:	reduced velocity
γ:	Avogadro's number

$W_{max} = 2m_e c^2 \beta^2 \gamma^2$

max. energy transfer in single collision

→ density correction δ:

shielding of distant electrons because of polarization ***(high energies)***

→ shell correction C:

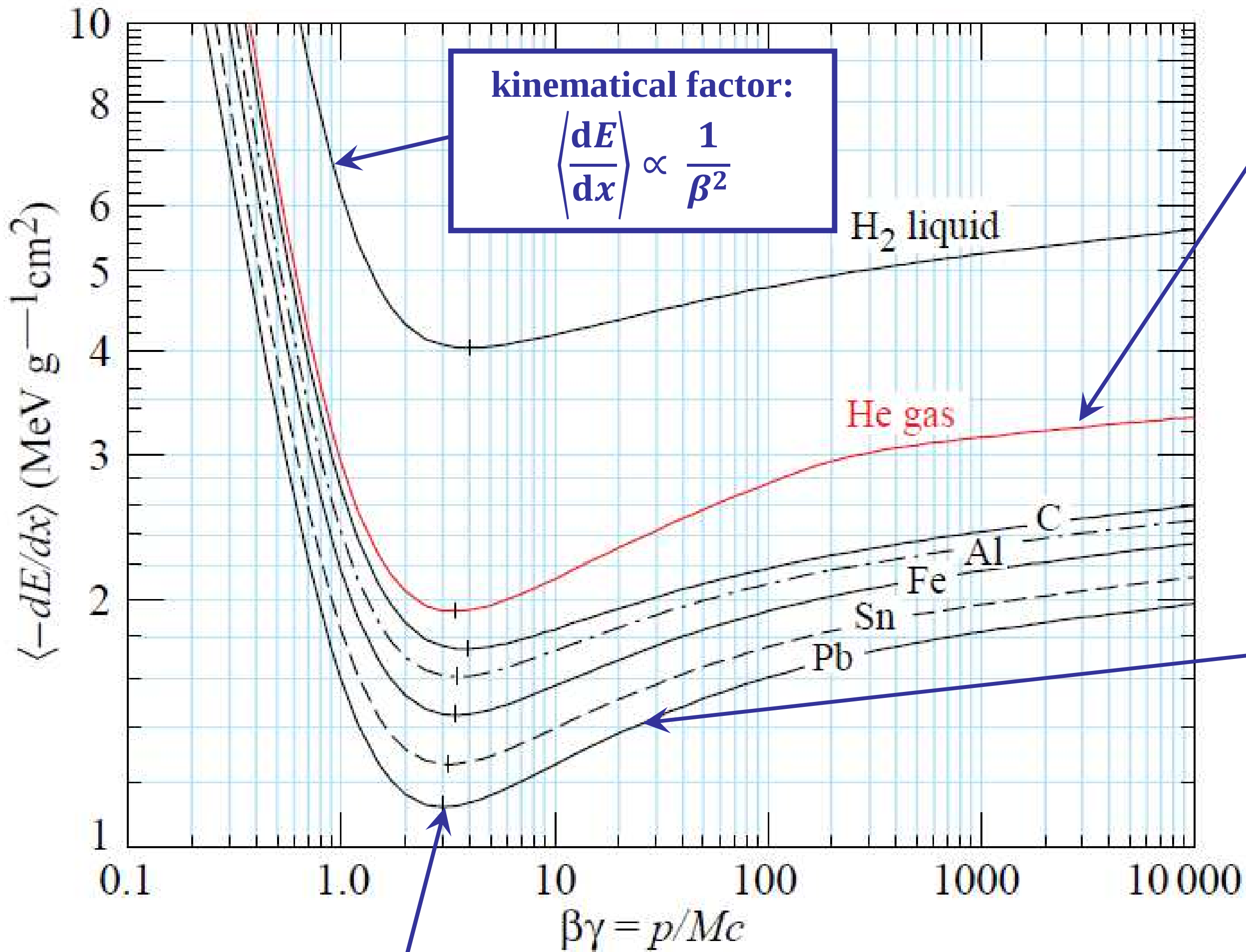
depends on electron orbital velocities ***(low energies)***

- general form

$$\frac{\mathrm{d}E}{\mathrm{d}x} \propto \frac{{Z_p}^2}{\beta^2} \ln(a\beta^2\gamma^2)$$

Bethe–Bloch Formula (2)

- collisional energy loss rates for different materials



kinematical factor:

$$\left\langle \frac{\mathbf{d}E}{\mathbf{d}x} \right\rangle \propto \frac{1}{\beta^2}$$

minimum ionizing particle, MIP:

$\beta\gamma \approx 3 - 4$

Fermi plateau:

medium polarization

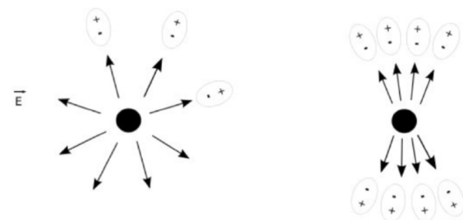
→ effective shielding of electric field far from particle path (density correction δ)

relativistic rise:

$$\left\langle \frac{\mathbf{d}E}{\mathbf{d}x} \right\rangle \propto ln(\beta^2\gamma^2)$$

Lorentz boost

→ increased range of particle field



C. Patrignani et al. (*Particle Data Group*), Chin. Phys. C, 40, 100001 (2016)

Bethe–Bloch and Particle Range

- comments
  - instead of energy loss per distance → frequently use of $\frac{1}{\rho}\frac{\mathrm{d}E}{\mathrm{d}x}$ with mass distribution d$x = \rho$ ds

    Mass Stopping Power S with ds in [cm], ρ in [g/cm^3]
  - $\frac{1}{\rho}\frac{\mathrm{d}E}{\mathrm{d}x}$ for MIP weakly depends on absorber matereial → typically ***~ 2 MeV g^{-1} cm^2***
  - description of mean energy loss due to ionization and excitation for all charged particles → exception: **e$^\pm$**

    for **e$^\pm$**: equal particle masses → different impact kinematics
- average distance heavy charged particle will travel → range
  - energy loss → statistical process
  - heavy charged particles loose only small fraction of their energy in collisions with atomic electrons

    → experience only slight deflection from scattering with electrons

    → travel in nearly straight lines through matter
  - small gradual amount of energy transferred from beam particle to absorber

    → particle passage treated as ***continuous slowing down*** process
- mean particle range
  - Continuous Slowing Down Approximation

    → ***CSDA***-range

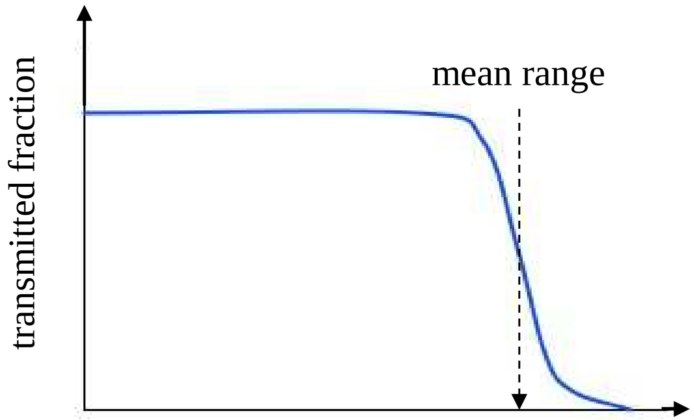
$$\boxed{R_{CSDA}(T) = \int_0^T \mathrm{d}T \left[-\frac{\mathrm{d}E}{\mathrm{d}x}\right]^{-1}}$$

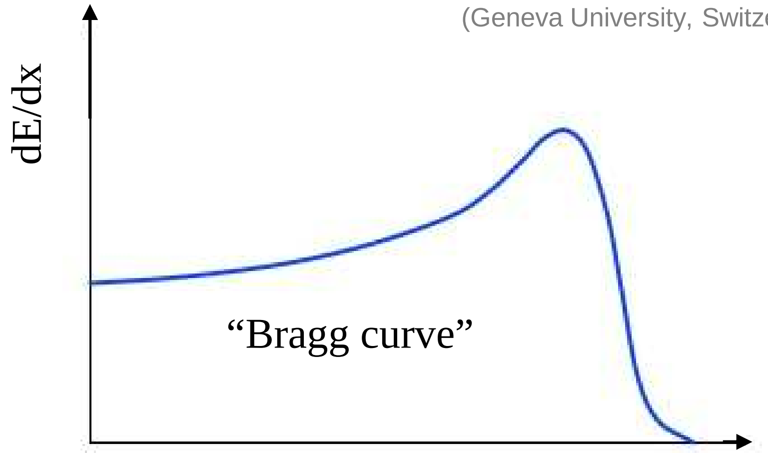
Particle Range of Heavy Particles

- transmitted fraction / energy loss as function of penetration depth

courtesy: D. Futyan

(Geneva University, Switzerland)

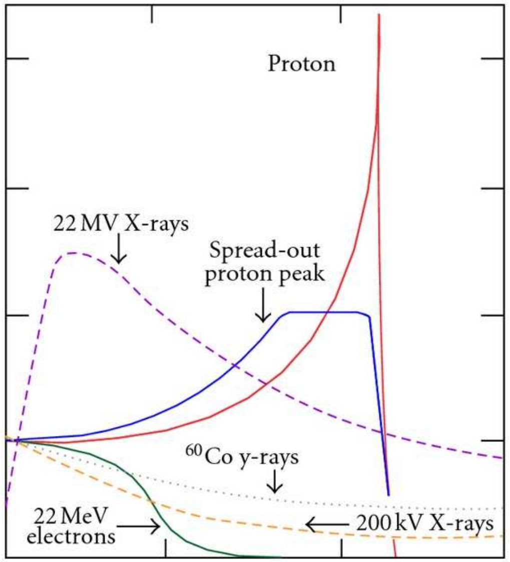




- application: tumor therapy
  - possibility to deposit rather precise dose at well defined depth (body) by variation of beam energy

    → initially with protons

    → later also with heavier ions (e.g. ^{12}C)



M. Cianchetti and M. Amichetti, International Journal of Otolaryngology, Vol. 2012, Article ID 325891

e^+ / e^- Interaction – Basic Considerations

- e^+ / e^- are "quickly" relativistic
  - small rest mass energy $E_0 = m_e c^2 = 511$ keV

    $\rightarrow$ relativistic effects have to be taken into account to deduce meaningful results

- large energy transfer possible
  - simple (non-relativistic) kinematical consideration:

    maximum energy transfer $\rightarrow$ head-on collision

$$\boxed{\frac{\Delta E_{max}}{T_{kin}} = 4\frac{m_e M}{(m_e + M)^2} \xrightarrow{M=m_e} 1}$$

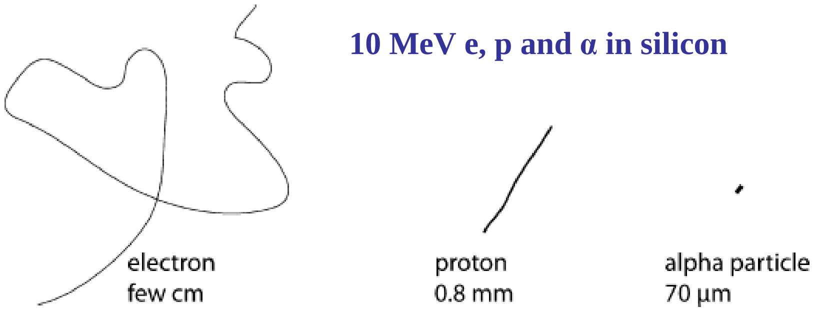
- incident electron and target electron are indistinguishable
  - convention:

    electron with higher energy $\rightarrow$ "beam particle"
  - maximum energy transfer $\rightarrow$ ***T/2***

different energy loss for electrons and positrons

- incident positron can transfer all energy to target electron in single collision
  - maximum energy transfer $\rightarrow$ ***T***

10 MeV e, p and α in silicon



- large angular deviations possible due to large energy transfer
  - curled electron / positron trajectories

- radiative losses
  - emission of Bremsstrahlung

Electron / Positron Interaction with Matter

- interaction modes

(1) ionization

→ distant collisions (small transferred energy), same procedure as for Bethe-Bloch equation

(2) Møller ($e^{\pm}$ - $e^{\pm}$) scattering

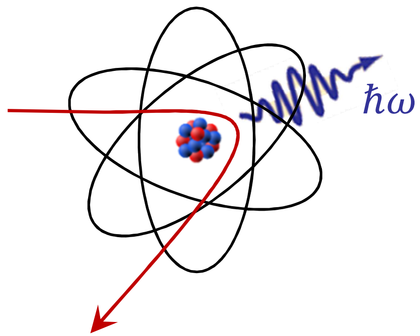
→ close collisions (large transferred energy), taking into account relativistic, spin and exchange effect

(3) Bhabha ($e^- + e^+ \rightarrow e^- + e^+$) scattering

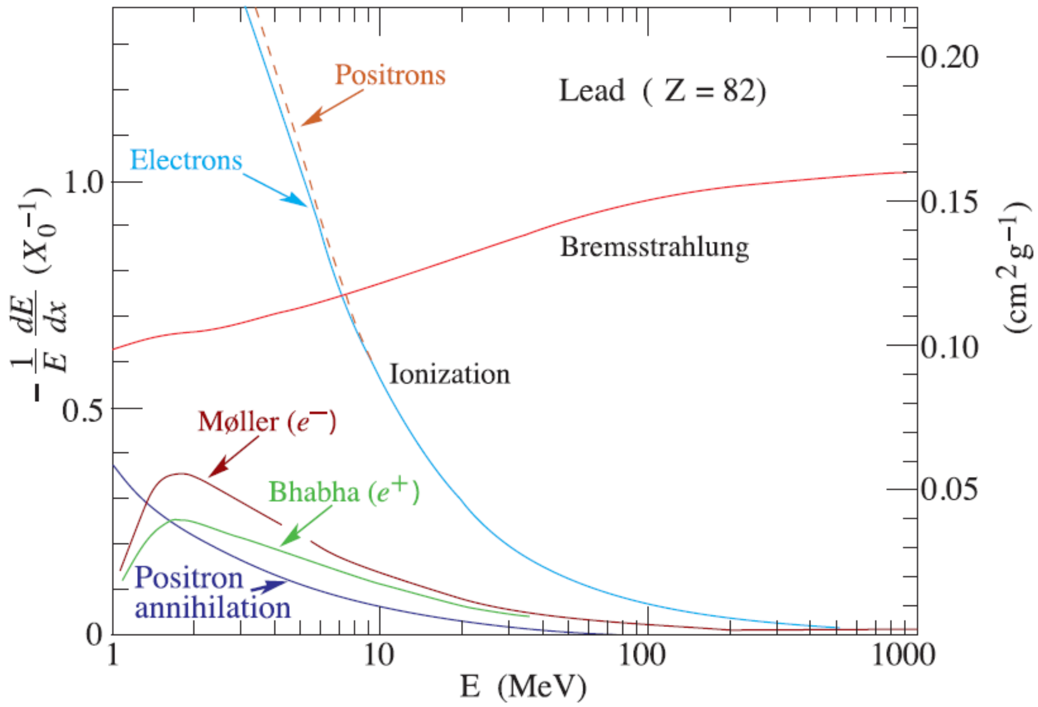
→ similar to Møller scattering

(4) electron-positron annihilation

(5) Bremsstrahlung



el.-magn. radiation emission by an electron in Coulomb field of nucleus



C. Patrignani et al. (*Particle Data Group*), Chin. Phys. C, 40, 100001 (2016)

Collisional Stopping Power

- modified Bethe-Bloch formula
  - not only includes inelastic impact ionization process
    - → also scattering mechanisms such as Møller or Bhabha scattering

$$S_{coll} = -\left\langle \frac{1}{\rho}\frac{\mathrm{d}E}{\mathrm{d}x} \right\rangle_{coll} = 4\pi N_A {r_e}^2 m_e c^2 \cdot \frac{Z_t}{A_t} \cdot \frac{1}{\beta^2} \left[\ln\left(\frac{T}{I}\right) + \frac{1}{2}\ln\left(1+\frac{\tau}{2}\right)^{1/2} + F^{\mp}(\tau) - \frac{\delta}{2} \right]$$

with T: kinetic energy of electron / positron

$\tau = \frac{T}{m_e c^2}$

  - electrons:

$$F^{-}(\tau) = \frac{1-\beta^2}{2}\left[1 + \frac{\tau^2}{8} - (2\tau+1)\ln 2\right]$$

  - positrons:

$$F^{+}(\tau) = \ln 2 - \frac{\beta^2}{24}\left[23 + \frac{14}{\tau+2} + \frac{10}{(\tau+2)^2} + \frac{4}{(\tau+2)^3}\right]$$

- free codes / tables available
  - collisional, radiative, nuclear stopping power and more for e, p, α particles
    - → https://physics.nist.gov/PhysRefData/Star/Text/intro.html

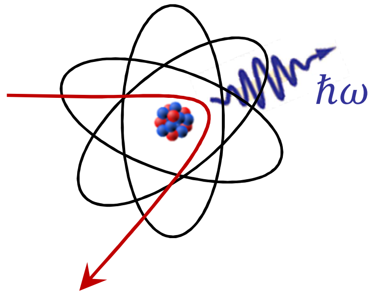
NIST National Institute of Standards and Technology Physical Meas. Laboratory

estar astar pstar

Radiative Stopping Power

- Bremsstrahlung
  - photon emission by charged particles, accelerated in Coulomb field of nucleus

    $\longrightarrow$ QED process (Fermi 1924, Weizsäcker-Williams 1938)



- energy loss / stopping power
  - screening of nucleus due to atomic electrons not taken into account

    $\longrightarrow$ only valid for large particle energies E

$$\boxed{S_{rad} = -\left\langle \frac{1}{\rho}\frac{\mathrm{d}E}{\mathrm{d}x}\right\rangle_{rad} = 4\alpha N_A \underbrace{\left(\frac{e^2}{mc^2}\right)^2}_{r_e} \cdot \frac{Z_t(Z_t+1)}{A_t} \cdot E \cdot \ln\left(\frac{183}{{Z_t}^{1/3}}\right)}$$

C. Patrignani et al. (*Particle Data Group*), Chin. Phys. C, 40, 100001 (2016)

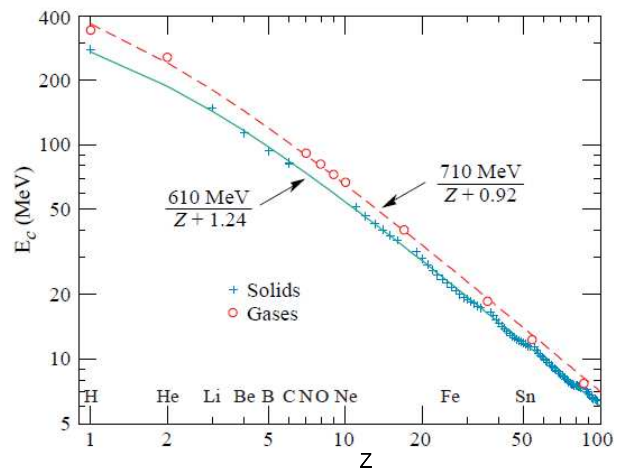
$$\Longrightarrow \quad S_{rad} \propto {Z_t}^2 \frac{E}{m^2} \quad \longrightarrow \text{ light particles } (e^\pm), \text{ high energies } E$$

- critical energy: $S_{rad}(E_c) \stackrel{\text{def}}{=} S_{coll}(E_c)$
  - different approximations

$$E_c = \frac{800\ \text{MeV}}{Z_t + 1.2}$$

B. Rossi, *High Energy Particles*, Prentice-Hall Inc., 1952

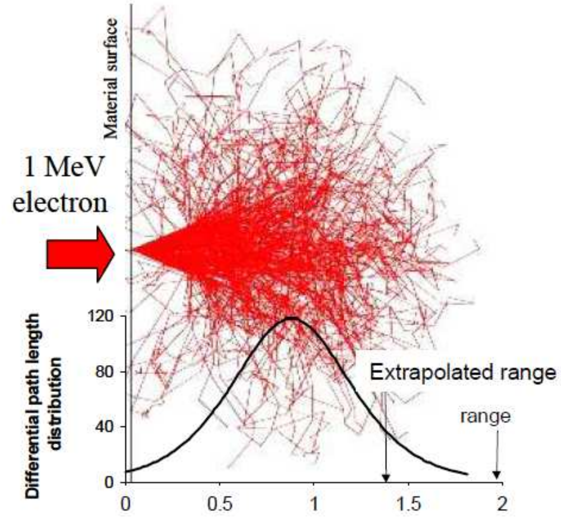
$$E_c = \frac{610\ \text{MeV}}{Z_t + 1.24} \quad \textit{for solids,} \qquad E_c = \frac{710\ \text{MeV}}{Z_t + 0.92} \quad \textit{for gas}$$

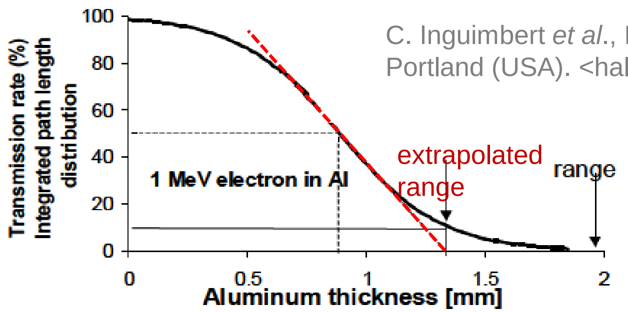


M.J. Berger, S.M. Seltzer, NASA-SP-3012, 1964

Total Stopping Power and Range

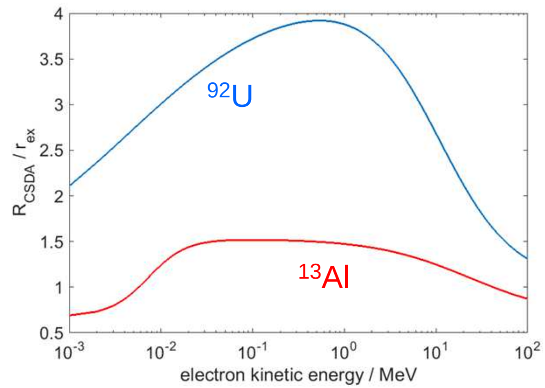
- total stopping power: sum of individual contributions $\Longrightarrow \quad S_{tot} = S_{coll} + S_{rad}$
- range
  - notion *„range of electrons“* not so clear than for heavy particles
    - $\rightarrow$ e-trajectory cannot be considered as straight line
    - $\rightarrow$ large angular deviations possible
    - $\rightarrow$ important fraction of energy may be lost in single collision
  - penetration depth / trajectory length
    - $\rightarrow$ random with large distributions $\rightarrow$ straggling
  - CSDA range: $R_{CSDA}(T) = \int_0^T \mathrm{d}T [S_{tot}]^{-1}$
    - $\rightarrow$ overestimates penetration depth
  - several alternative range definitions
    - $\rightarrow$ ***extrapolated range*** $\boldsymbol{r_{ex}}$ often in use





C. Inguimbert *et al.*, NRSEC 2016, Portland (USA). <hal-01372004>

- different parametrizations for $\boldsymbol{r_{ex}}$



e.g.: T. Tabata *et al.*, NIM **B119** (1996) 463

Quintessence

- particle interaction in matter difficult to treat analytically
  - approximative expressions and parametrizations exists

    → good for first insight → have a feeling what's going on…

- typical domain of simulation toolkits
  - depending on task / lab strategy / personal interest…

    → different codes with different pros and cons

  - Geant — http://geant4.web.cern.ch/
  - Fluka — http://www.fluka.org/fluka.php
  - EGS — EGS5 Web Page — http://rcwww.kek.jp/research/egs/egs5.html

  - …

Particle Interaction with external Electro-Magnetic Field

- Bunch Length Measurements
- transverse Beam Profile Diagnostics (Laser Wire)
- …

Tuusula (Finland), 2-15 June 2018

Interaction with external EM Fields

- external electromagnetic field acting as
  - signal source: photon scattered at beam particles

    → probing beam shape with external laser (laser wire)
  - beam manipulator

    → atomic excitations of ion beams

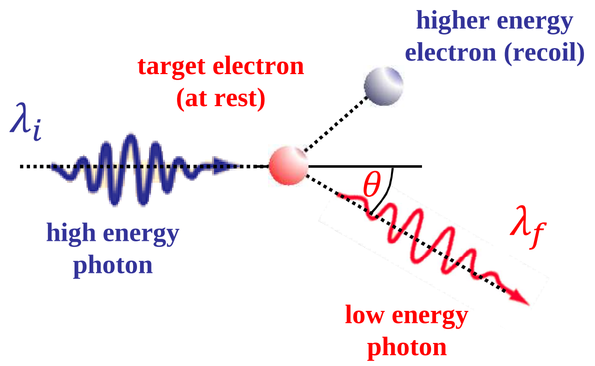
    → force acting on charged particle beam

- scattering of photons on charged particles
  - ***Compton effect:*** photon scattered on a „quasi free“ electron

    $\gamma + \text{Atom} \longrightarrow \gamma + e^{-} + \text{Ion}^{+}$

    → *photon energy large* compared to binding energy of electron

    → photon is deflected and wavelength λ changes due to energy transfer → ***photon loses energy***



- cross section: Klein-Nishina formula

$$\frac{d\sigma_c}{d\Omega} = \frac{1}{2}\underbrace{\left(\frac{e^2}{m_0c^2}\right)}_{r_e}^{2}\left\{\frac{1}{1+\varepsilon(1-\cos\theta)}\right\}^{2}\left[1+\cos^2\theta+\frac{\varepsilon^2(1-\cos\theta)^2}{1+\varepsilon(1-\cos\theta)}\right]$$

with $\varepsilon = \frac{\hbar\omega}{m_0c^2}$

$$\Longrightarrow \quad \frac{d\sigma_c}{d\Omega} \propto \frac{1}{(m_0c^2)^2}$$

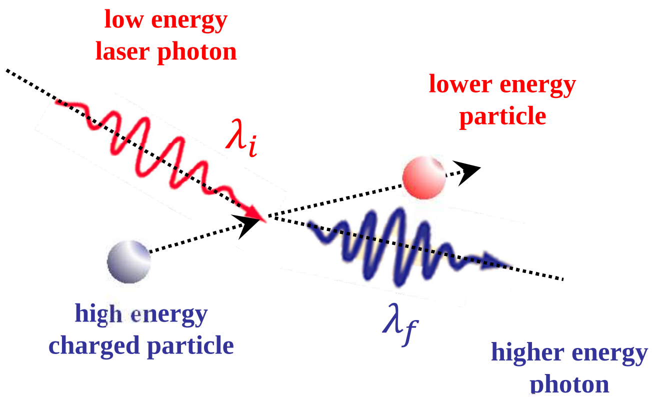
➡ only relevant for $e^{\pm}$

Inverse Compton Scattering

- electron / positron accelerator
  - target particles not at rest

    → application of Klein-Nishina only in particle rest frame → Lorentz boost to LAB frame

- inverse situation at accelerator ➡ ***photon gains energy in scattering process***
  - high energy $e^{\pm}$ (beam particles)
  - low energy photons (optical laser)



- **inverse Compton scattering**
  - cross section

$$\frac{d\sigma_{ic}}{d\varpi} = \frac{3}{8}\frac{\sigma_T}{\epsilon_1}\left[\frac{1}{1-\varpi} + 1 - \varpi + \left\{\frac{\varpi}{\epsilon_1(1-\varpi)}\right\}^2 - \frac{2\varpi}{\epsilon_1(1-\varpi)}\right]$$

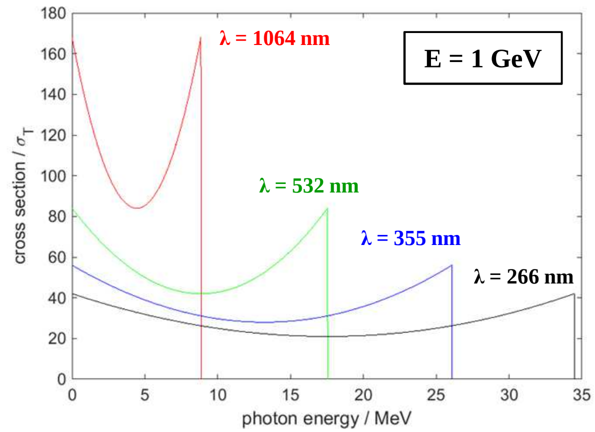
T. Shintake, Nucl. Instrum. Meth. **A311** (1992) 453

with

$\sigma_T = \frac{8\pi r_e{}^3}{3}$: Thomson cross section

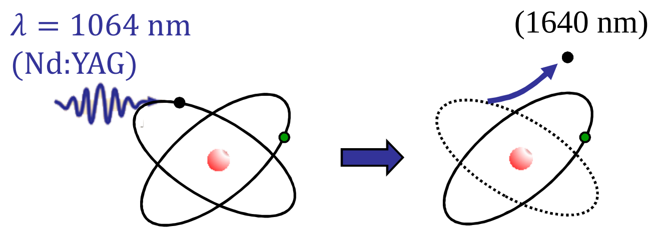
$\epsilon_1 = \frac{\gamma\hbar\omega_0}{m_e c^2}$: normalized energy of laser photons

$\varpi = \frac{\hbar\omega_\gamma}{E}$: normalized energy of emitted photons



Beam Manipulation with EM Fields

- no direct beam diagnostics
  - preparation for beam diagnostics measurement
    - → beam current (difference), beam profile, …
- laser based photoejection of H^- beams
  - proton accelerator → H^- gun
  - stripping for ***p*** generation → charge exchange via ***foil***
    → ***laser*** (2 electron photoejection)
  - laser photo neutralization for beam diagnostics
    - → e.g. difference in bunch charge before / after neutralization
- Transverse Deflecting Structure (TDS)
  - iris loaded RF waveguide structure
  - designed to provide hybrid deflecting modes ($HEM_{1,1}$)
    - → linear combination of $TM_{1,1}$ and $TE_{1,1}$ dipole modes
    - → resulting in transverse force that act on synchronously moving relativistic particle beam
  - used as RF deflector → ***intra-beam streak camera***
    (bunch length diagnostics)



$$H^- + \hbar\omega \longrightarrow H^0 + e^-$$

H^- photo neutralization cross section