BEAM INSTRUMENTATION

Transverse Emittance Measurement

Enrico Bravin – CERN BE-BI

Cern Accelerator School on Beam Instrumentation
2-15 June 2018, Tuusula, Finland

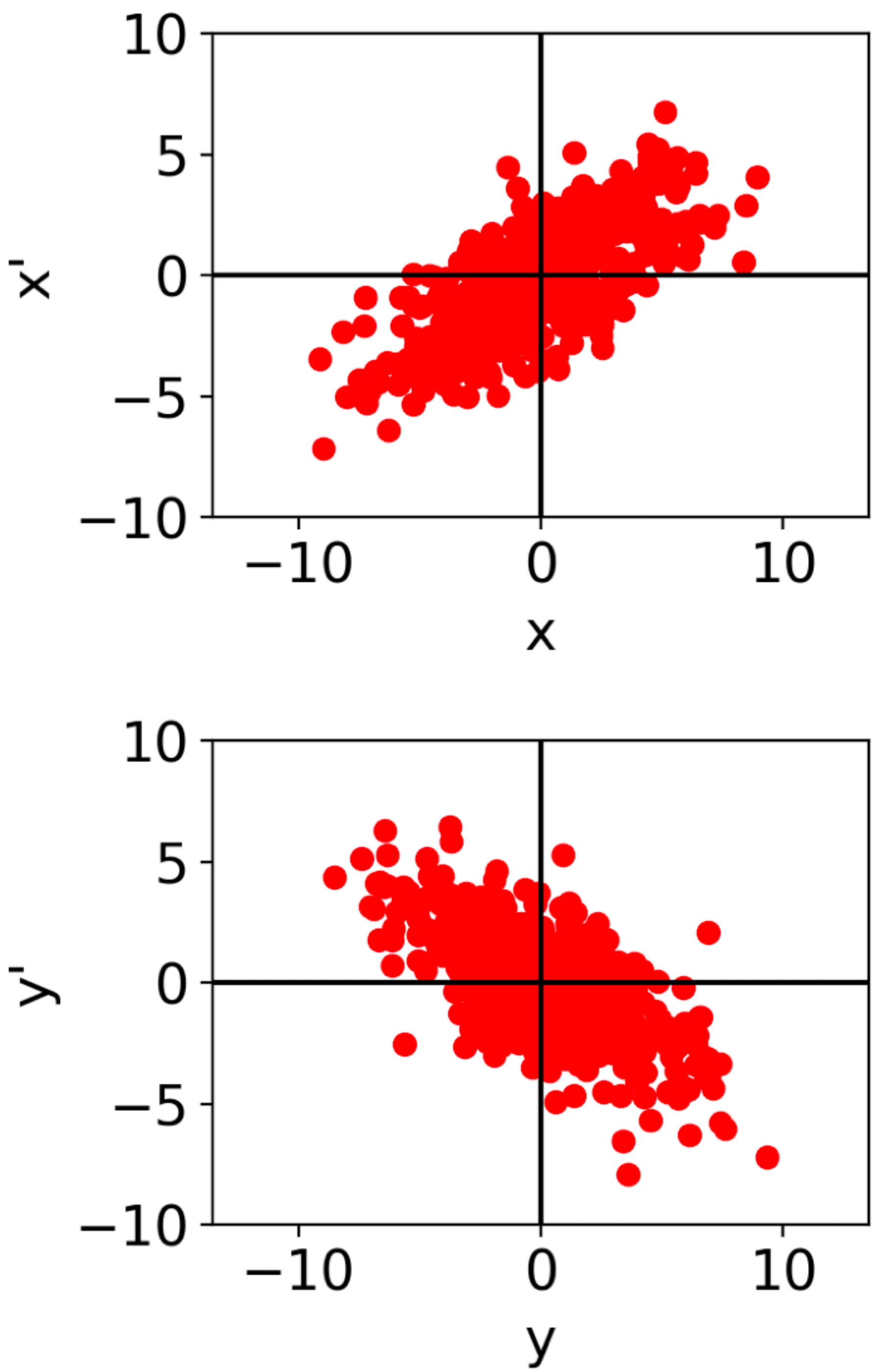
Content

- Definition of beam matrix and emittance
- Adiabatic damping and dispersion
- Why is emittance important
- Phase space sampling (direct)
- Phase space sampling (indirect)
- Emittance measurement in rings
- Emittance in e^- LINACs

Transverse spaces

- The REAL (x,y) space and the PHASE space are different things
- Their projections along x or y are however the same thing
- Phase spaces contain the information needed for beam dynamic calculations
- x,y space is easier to sample
- Perform measurement in x,y and use optics parameters and beam dynamic theories to calculate the phase space

Phase space



Assume centre of distribution is (0, 0) for simplicity $x_i = \bar{x}_i - \langle \bar{x} \rangle$ $\quad x'_i = \bar{x'}_i - \langle \bar{x'} \rangle$

$$x^2_{rms} = \langle x^2 \rangle = \frac{1}{N} \sum_{i=1}^{N} x_i^2$$

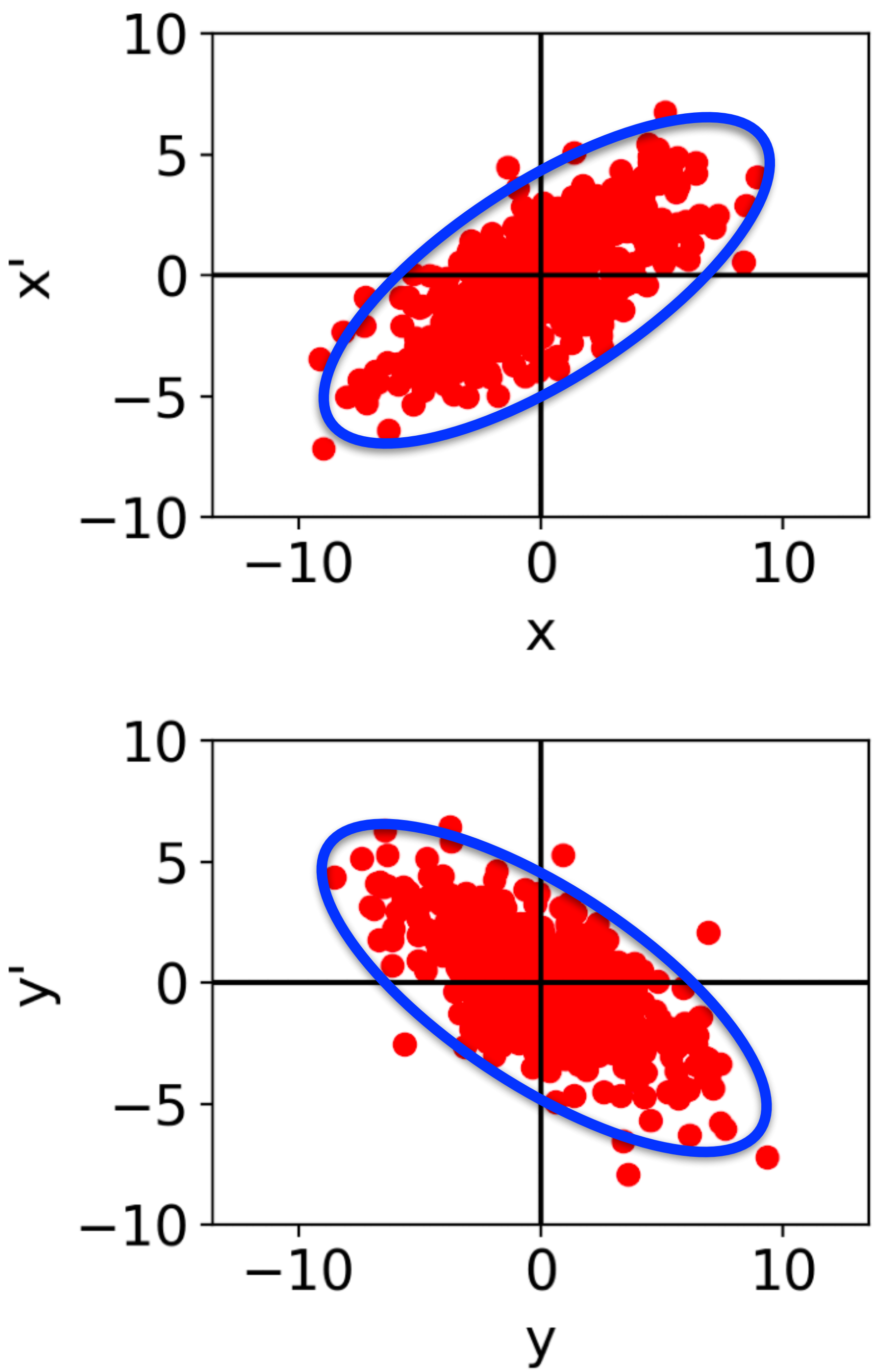
$${x'}^2_{rms} = \langle {x'}^2 \rangle = \frac{1}{N} \sum_{i=1}^{N} {x'}_i^2$$

$$\langle x x' \rangle = \frac{1}{N} \sum_{i=1}^{N} x_i {x'}_i$$

$$\Sigma = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{12} & \Sigma_{22} \end{bmatrix} = \begin{bmatrix} \langle x^2 \rangle & \langle x x' \rangle \\ \langle x x' \rangle & \langle {x'}^2 \rangle \end{bmatrix}$$ Beam matrix

$$\boxed{\varepsilon_{rms} = \sqrt{\Sigma_{11}\Sigma_{22} - \Sigma_{12}^2} = \sqrt{\det \Sigma}}$$ $\text{Area} = \pi \varepsilon$

Phase space



Assume centre of distribution is (0, 0) for simplicity $x_i = \bar{x}_i - \langle \bar{x} \rangle \qquad {x'}_i = \bar{x'}_i - \langle \bar{x'} \rangle$

$$x^2_{rms} = \langle x^2 \rangle = \frac{1}{N}\sum_{i=1}^{N} x_i^2$$

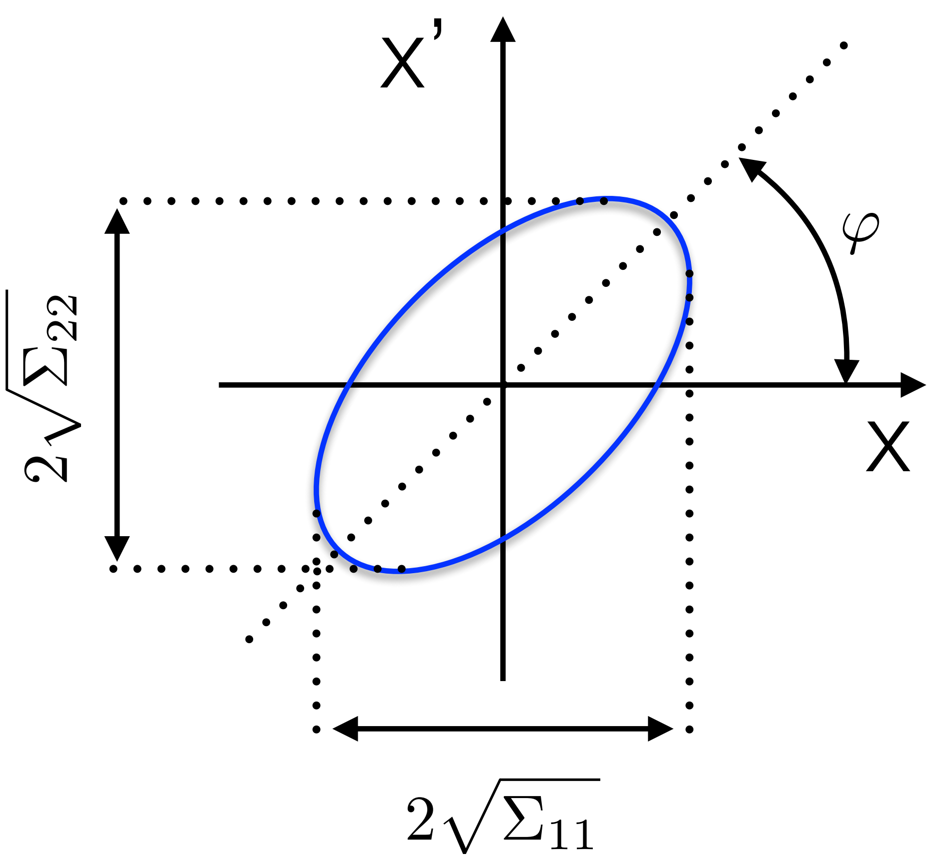
$${x'}^2_{rms} = \langle {x'}^2 \rangle = \frac{1}{N}\sum_{i=1}^{N} {x'}_i^2$$

$$\langle xx' \rangle = \frac{1}{N}\sum_{i=1}^{N} x_i {x'}_i$$

$$\Sigma = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{12} & \Sigma_{22} \end{bmatrix} = \begin{bmatrix} \langle x^2 \rangle & \langle xx' \rangle \\ \langle xx' \rangle & \langle {x'}^2 \rangle \end{bmatrix}$$ Beam matrix

$$\boxed{\varepsilon_{rms} = \sqrt{\Sigma_{11}\Sigma_{22} - \Sigma_{12}^2} = \sqrt{\det \Sigma}} \qquad \text{Area} = \pi\varepsilon$$

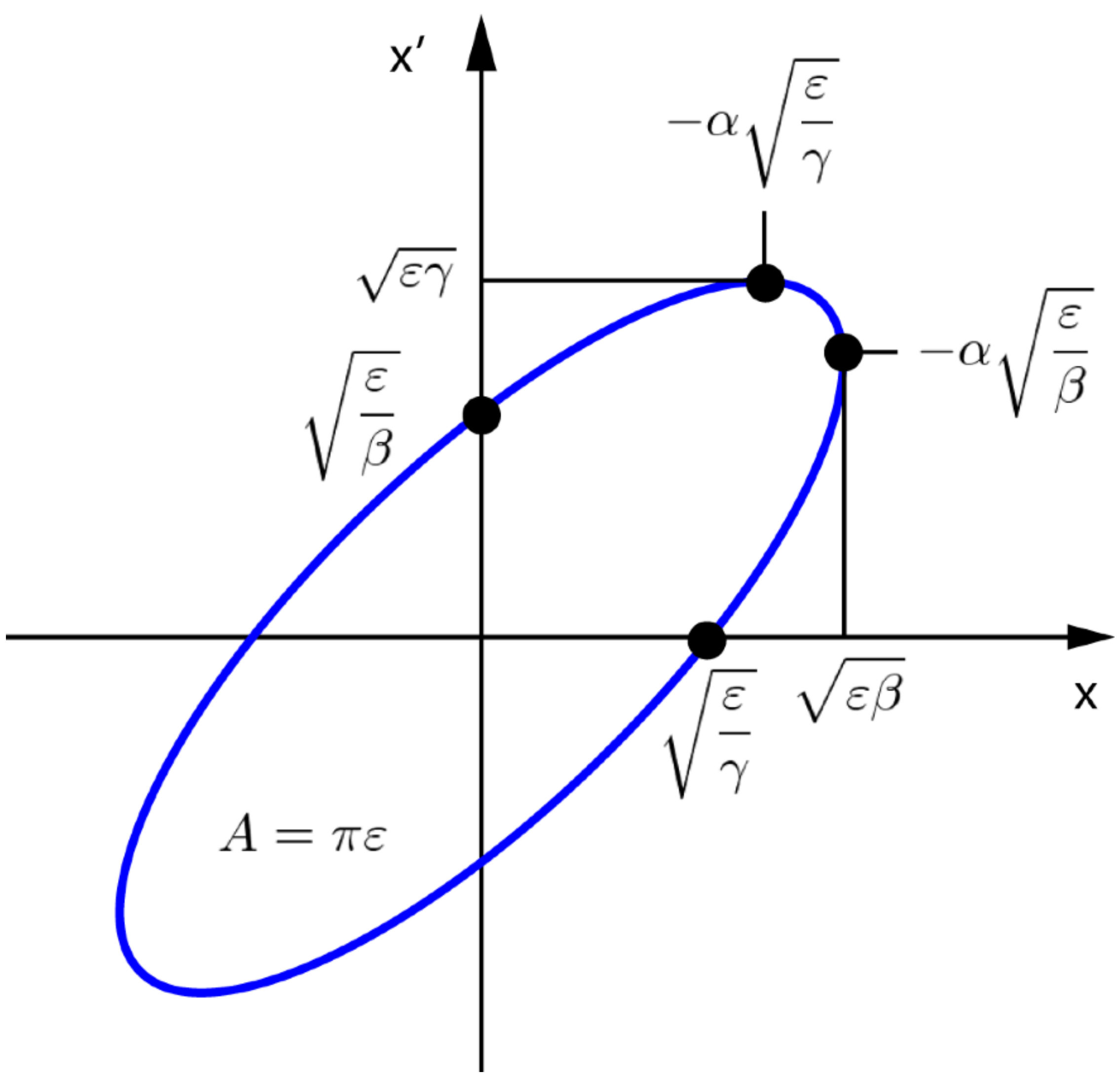
Beam matrix



$$\Sigma = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{12} & \Sigma_{22} \end{bmatrix} = \begin{bmatrix} \langle x^2 \rangle & \langle xx' \rangle \\ \langle xx' \rangle & \langle x'^2 \rangle \end{bmatrix}$$

$$\tan 2\varphi = -\frac{2\Sigma_{12}}{\Sigma_{22} - \Sigma_{11}}$$

Courant-Snyder parameters



The phase space ellipse can be defined by 4 parameters:

$$(\varepsilon, \beta, \alpha, \gamma)$$

$$\gamma = \frac{1+\alpha^2}{\beta}$$

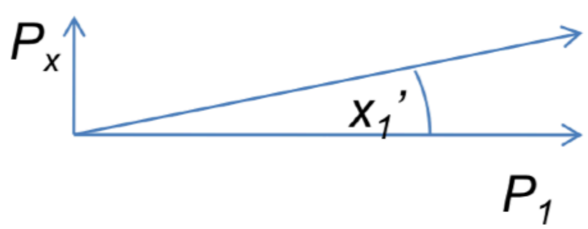
And the equation of the ellipse is:

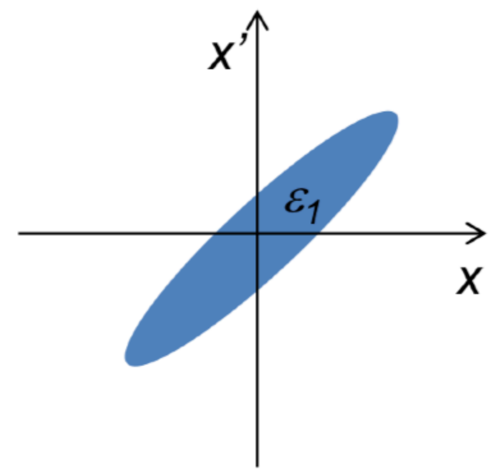
$$\varepsilon = \gamma x^2 + 2\alpha x x' + \beta x'^2$$

$$\Sigma = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{12} & \Sigma_{22} \end{bmatrix} = \varepsilon \begin{bmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{bmatrix}$$

Adiabatic damping

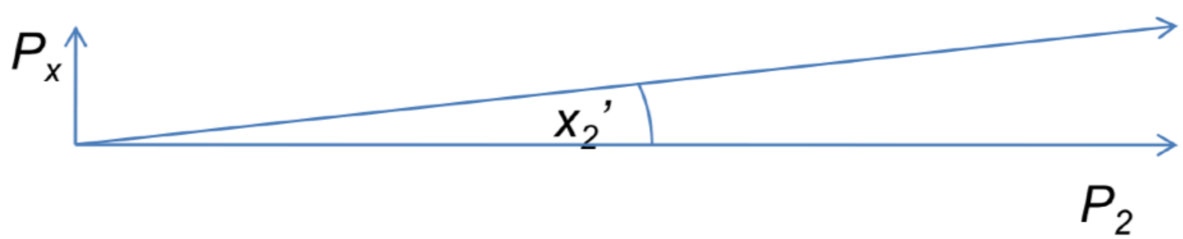
Emittance is only constant in beamlines without acceleration

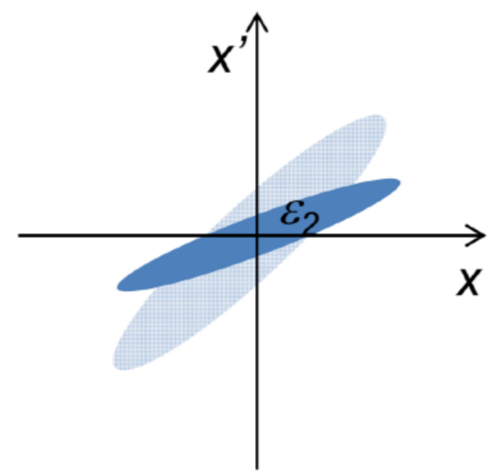




acceleration

$$\varepsilon_2 = \varepsilon_1 \frac{P_1}{P_2} \quad \text{with} \quad P = m\beta_{rel}\gamma_{rel}\, c$$



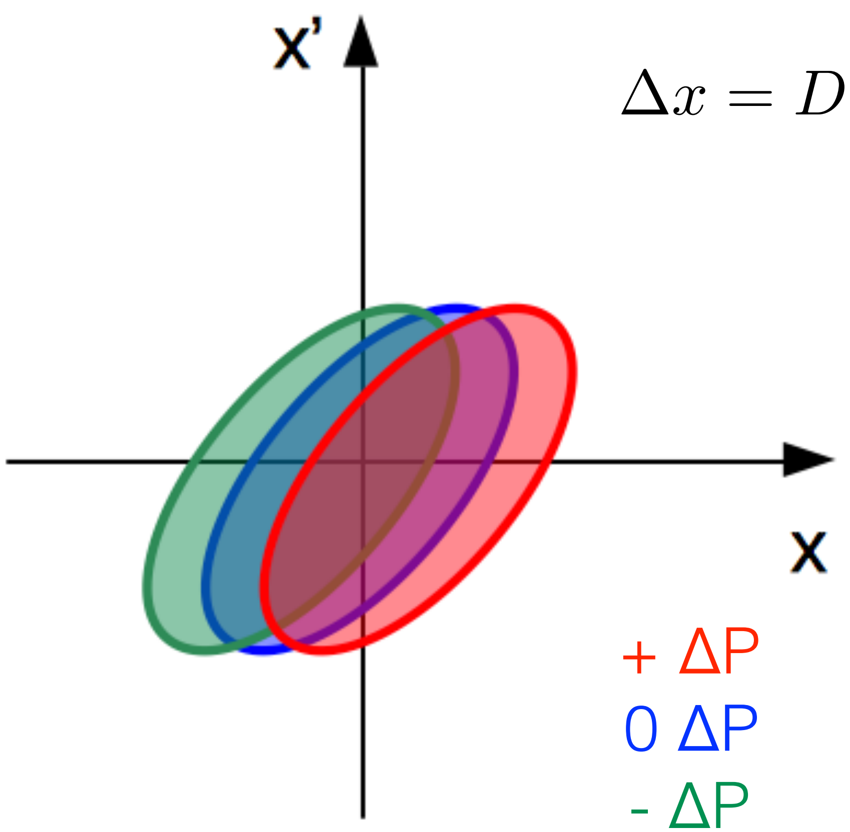


Normalised emittance
vs.
Geometric emittance

$$\varepsilon_N = \beta_{\mathrm{rel}}\gamma_{\mathrm{rel}}\varepsilon_{\mathrm{geo}}$$

- We measure the geometric emittance!
- The normalised emittance is constant during acceleration

Effect of dispersion



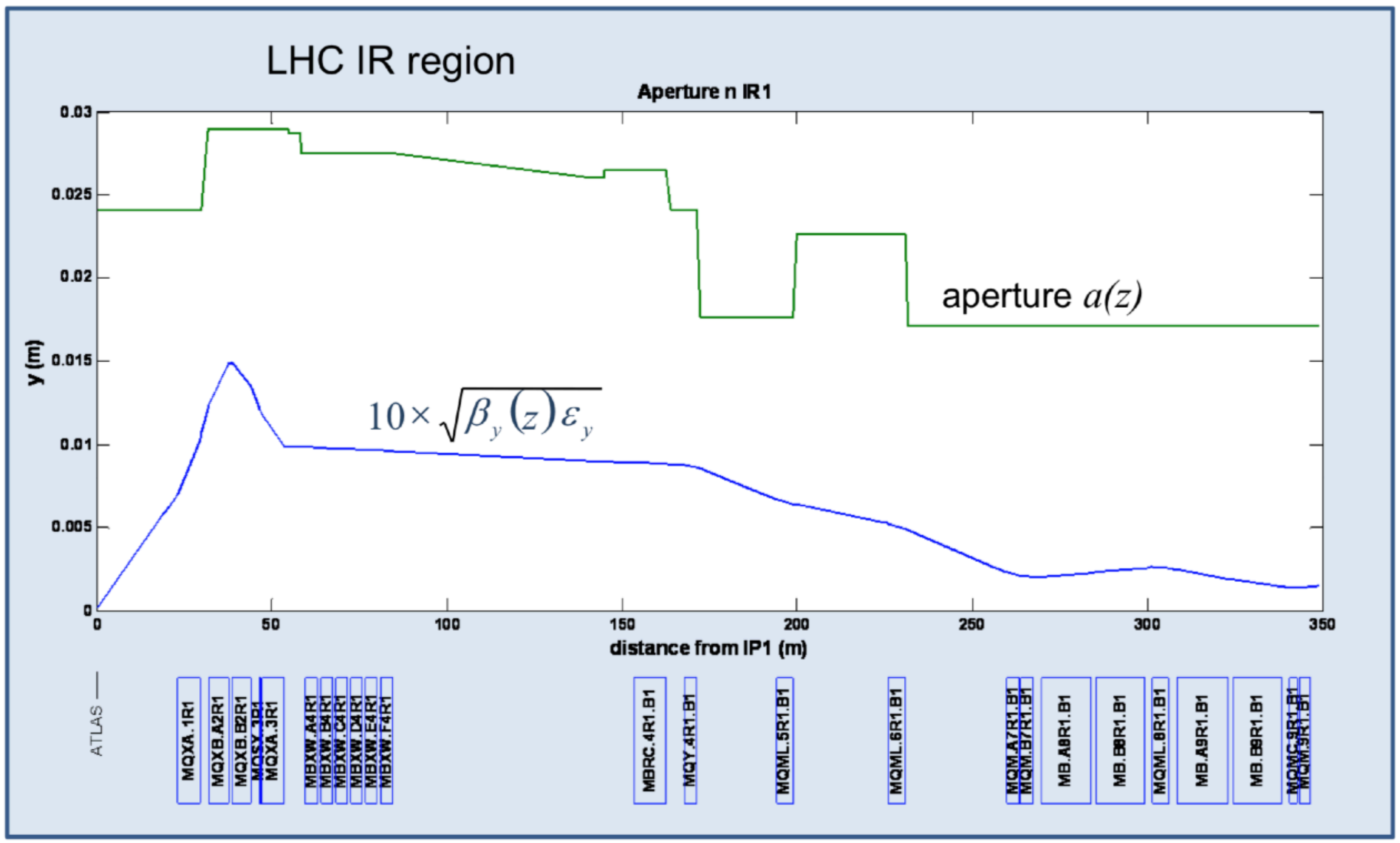
$$\Delta x = D\frac{\Delta P}{P}$$

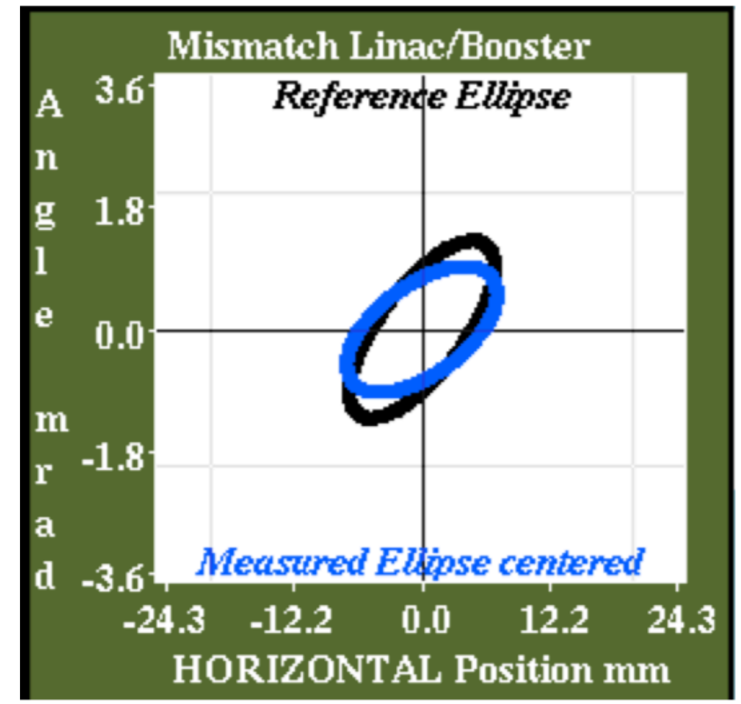
- The phase space area covered by the particles depends also on dispersion and ΔP
- Emittance is still conserved: The volume of the 6 dimension phase space is invariant
- Better measure the emittance where D is 0 (can decouple individual planes)

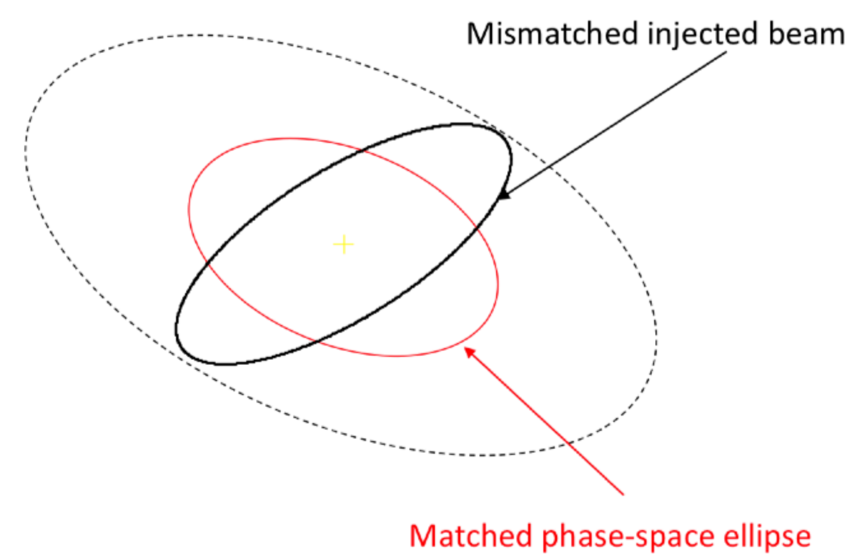
Why measure the emittance?

- Emittance has a fundamental role in the size of the beams!
  - Will the beam hit the accelerator aperture limitations?
  - Is the phase-space of the beam matched to the Courant-Snyder ellipse of the ring I am injecting into?
  - Small dense beams is often what you want to produce
    - Luminosity in colliders
    - Brightness in light sources

Why measure the emittance







Collider luminosity

- Luminosity determines the rate at which collisions take place in a collider.

- Colliders are tuned to maximise the luminosity

$$L = \frac{N_{b1} N_{b2} f_{rev} k_b}{2\pi \sqrt{(\sigma_{x1}^2 + \sigma_{x2}^2)(\sigma_{y1}^2 + \sigma_{y2}^2)}}$$

$$L = \frac{N_{b1} N_{b2} f_{rev} k_b}{4\pi \bar{\sigma}^2}$$

Equal, round beams

$$\sigma_i = \sqrt{\varepsilon_i \beta_i} \quad i \in [x_1, y_1, x_2, y_2]$$

Synchrotron light sources

- Experiment rely on diffraction of short, intense, spatial coherent synchrotron radiation photon pulses
- This applies to both storage rings and FEL

Beam brightness

$$\bar{B} = \frac{2I}{\pi^2 \varepsilon_x \varepsilon_y}$$

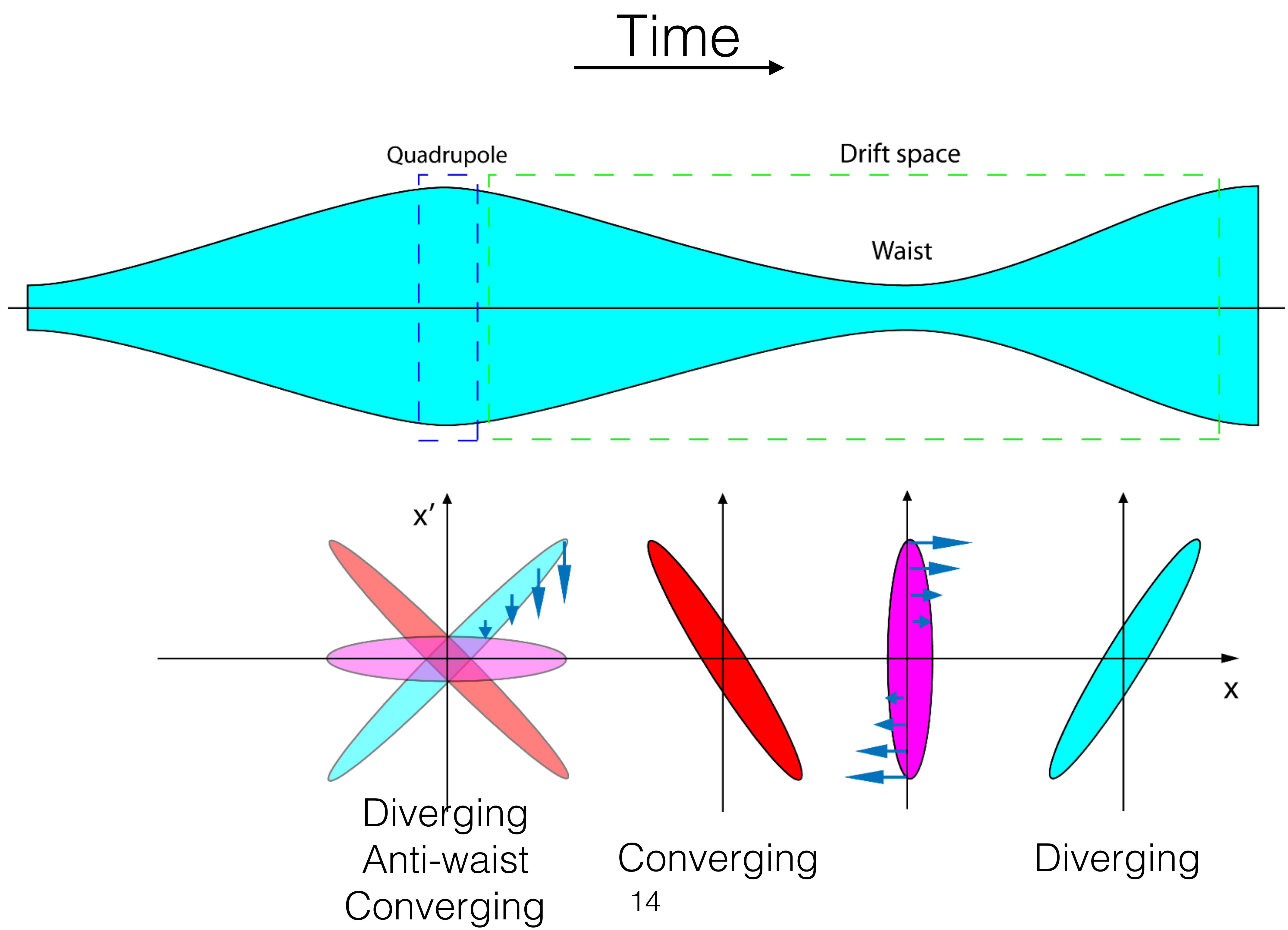
SR Spectral brightness

$$B = \frac{d^4 N}{dt\, d\Omega\, dS\, d\lambda/\lambda}$$

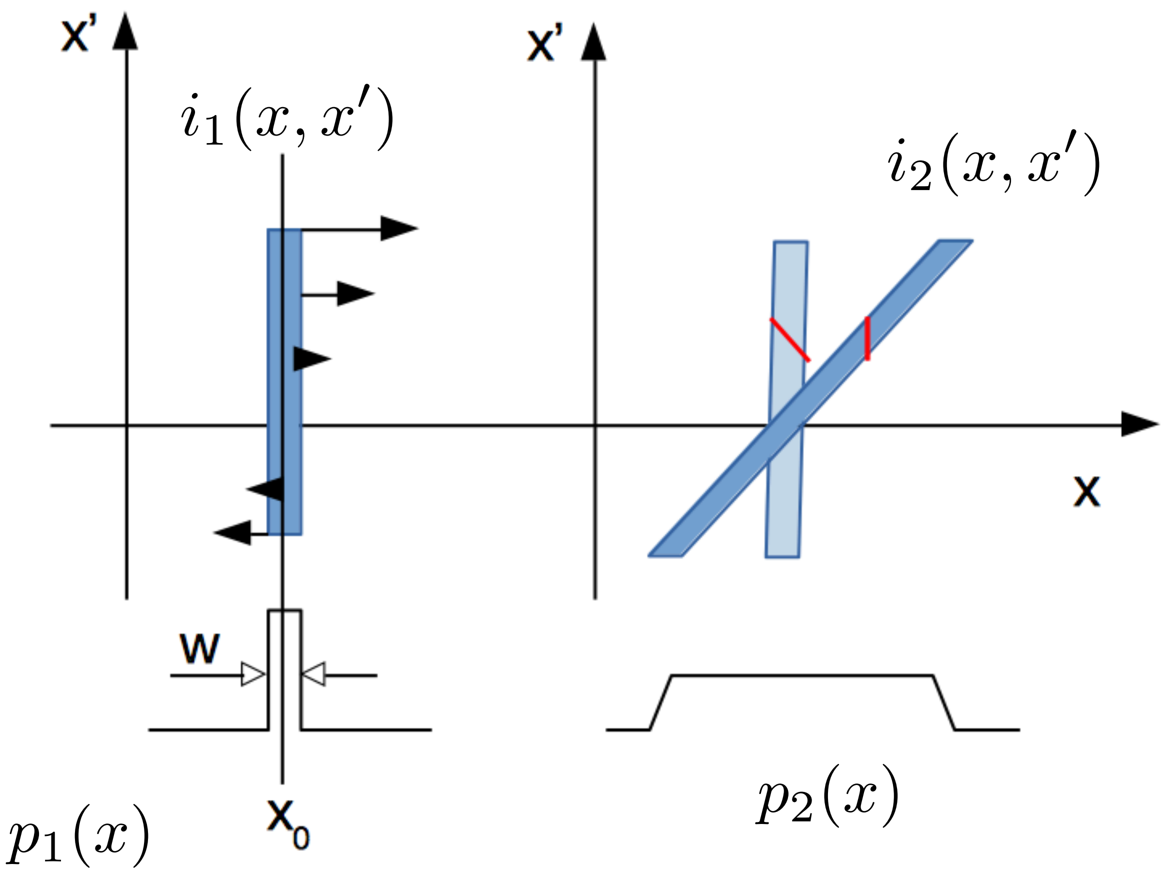
Measuring the emittance

- Two options
  - Sample the phase space directly
    - This is the preferred method for low energy beams (sources, LEBT)
  - Measure the transverse beam distributions in real space and use beam dynamics relations (Twiss parameters) to infer the emittance
    - Single profile measurement (rings)
    - Multiple profiles measurement (transfer lines)
    - Quadrupolar scans (transfer lines, LINACs)

Phase space dynamics



Drift space



$$x_2 = x_1 + x_1' L$$

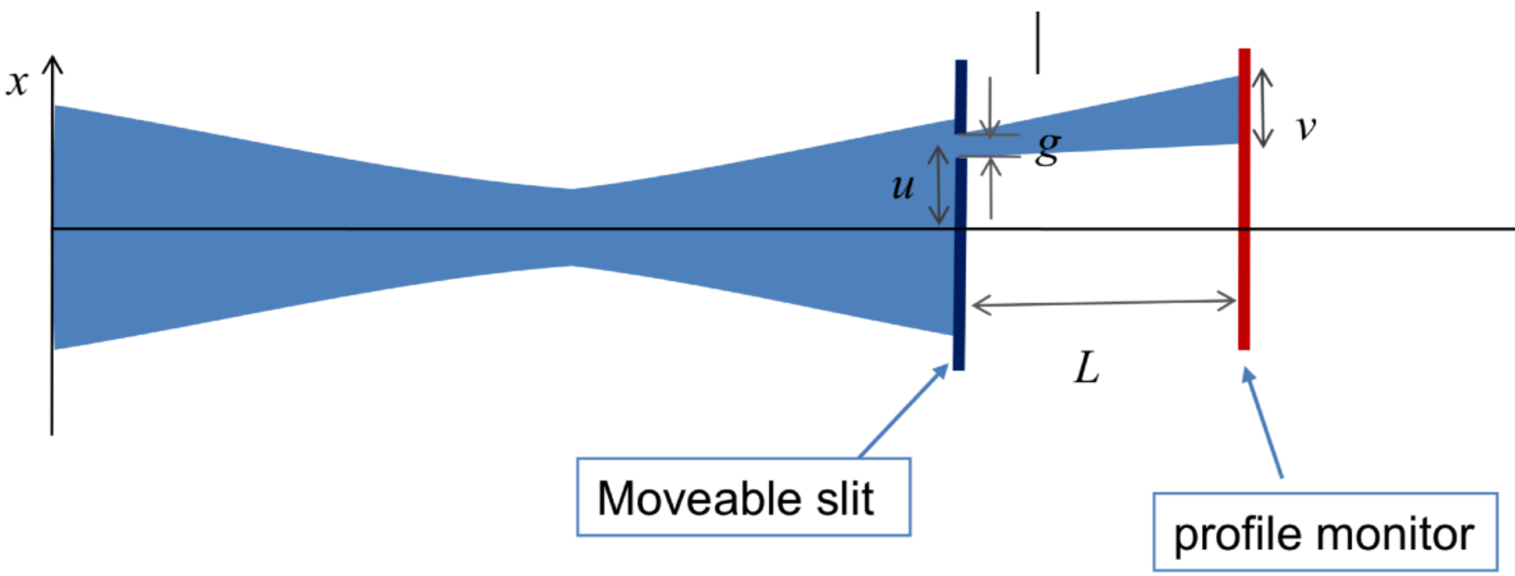
$$x_2' = x_1'$$

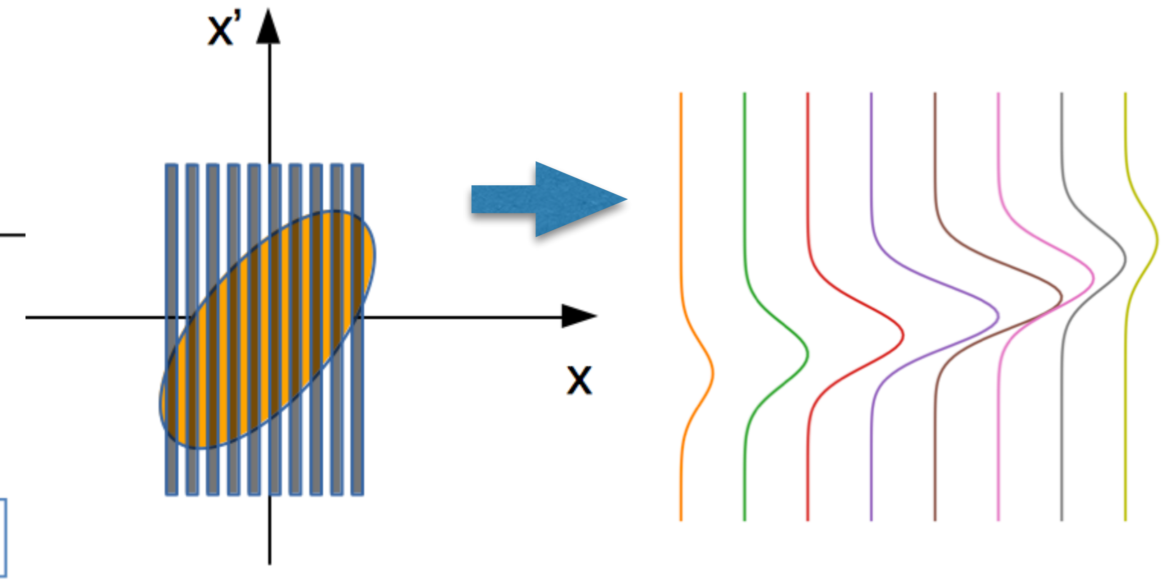
$$p_1(x) = \int i_1(x,x')dx'$$

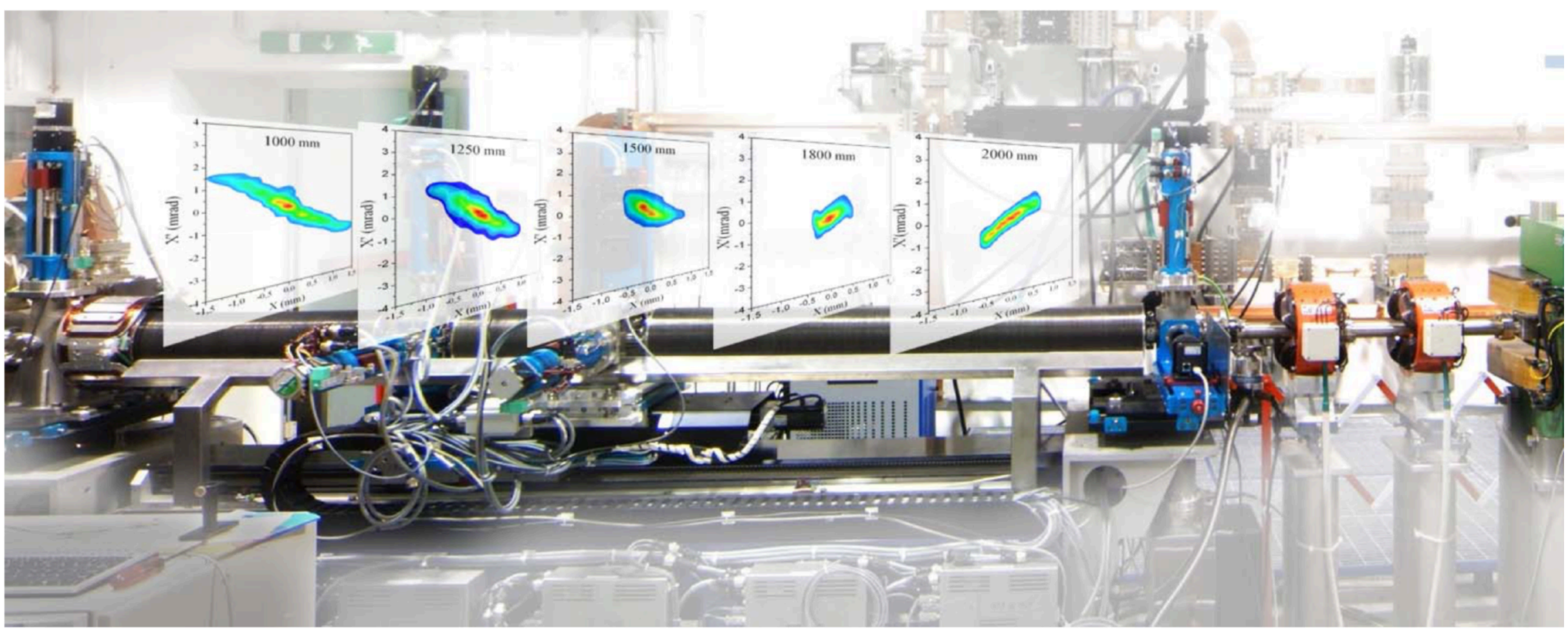
$$p_2(x) = \int i_2(x,x')dx' = \int i_1(x - x'L, x')dx'$$

$$\lim_{w \to 0} \begin{cases} i_1(x,x') & = \delta(x - x_0)\xi(x') \\ p_2(x) & = i_1(x_0, \frac{x - x_0}{L}) = \xi(\frac{x - x_0}{L}) \end{cases}$$

Slit and Grid

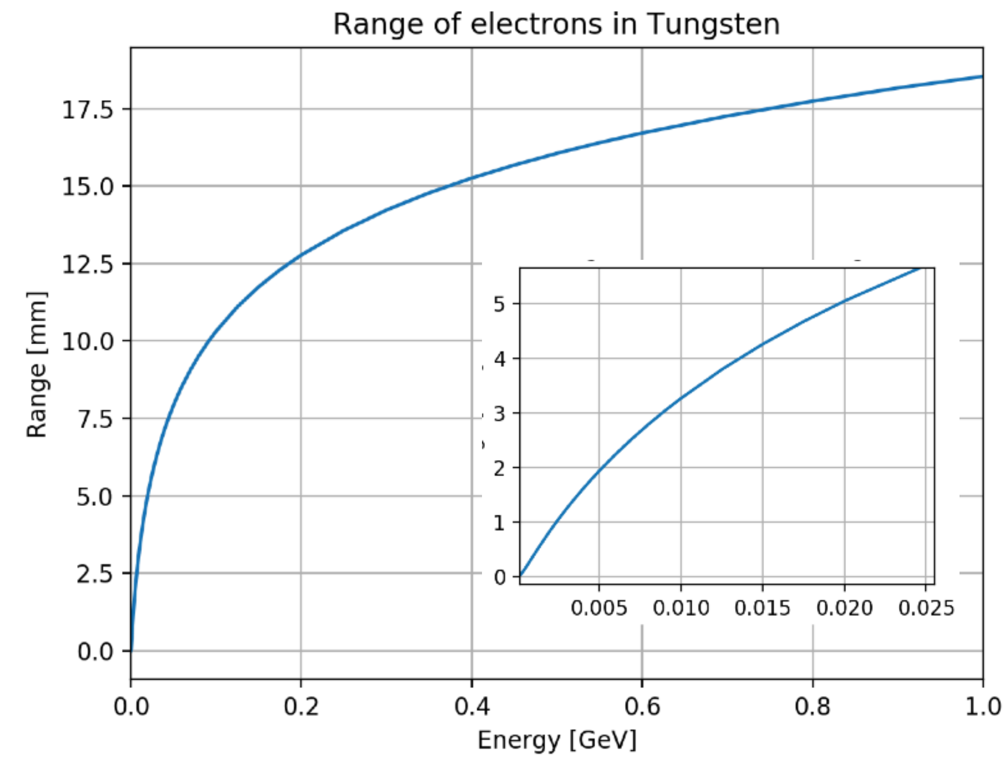


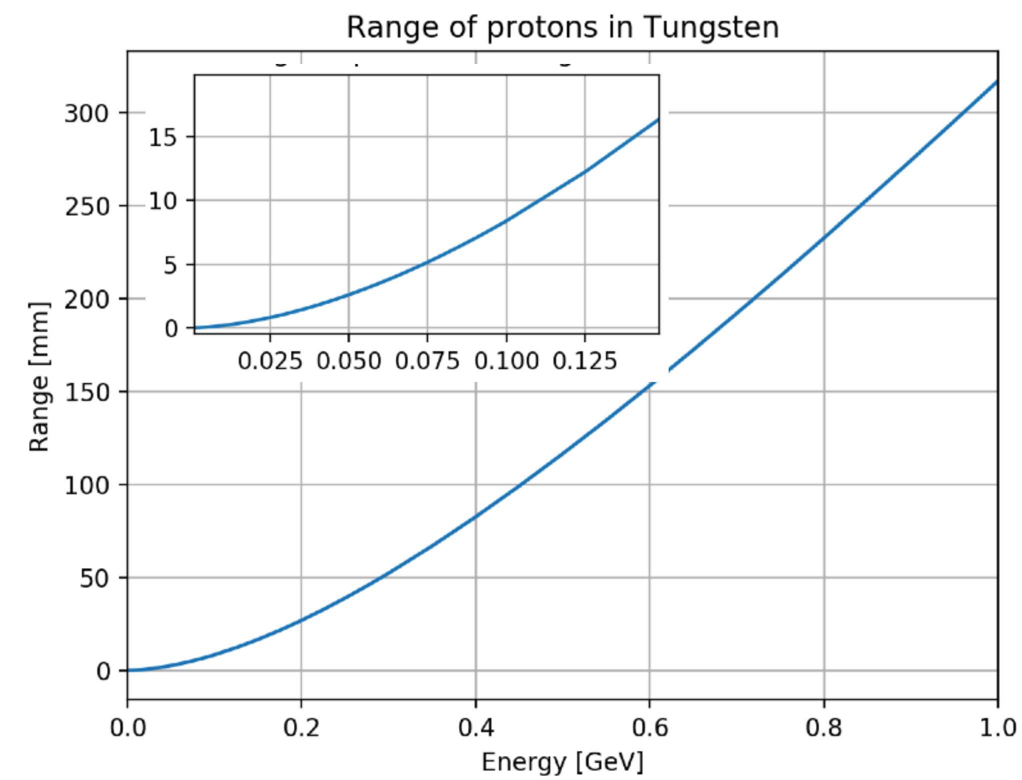




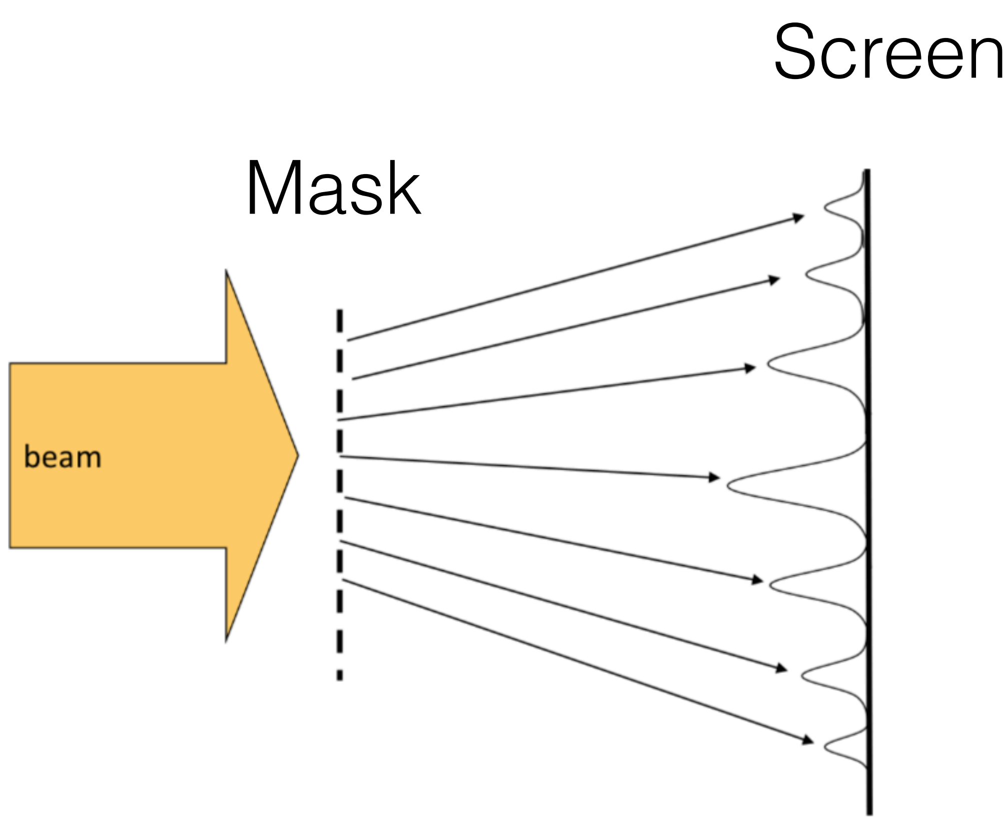
Slit and Grid

- Slit must be narrow
    - Few particles go trough
    - Possible scattering on the sides of the slit
- Slit must be tick enough to stop particles
- Distance between slit and grid must be optimised
    - Large to increase the sensitivity
    - Beamlets should however fit in the profile monitor
- Grid is often moved with the slit to reduce the number of channels required



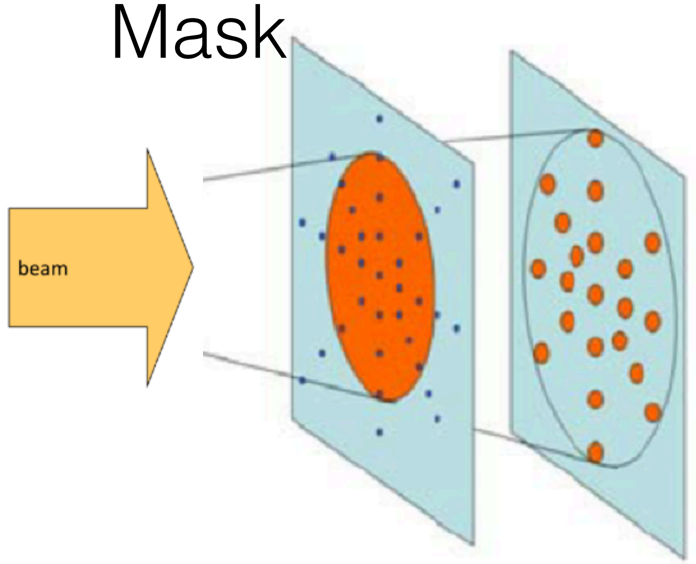


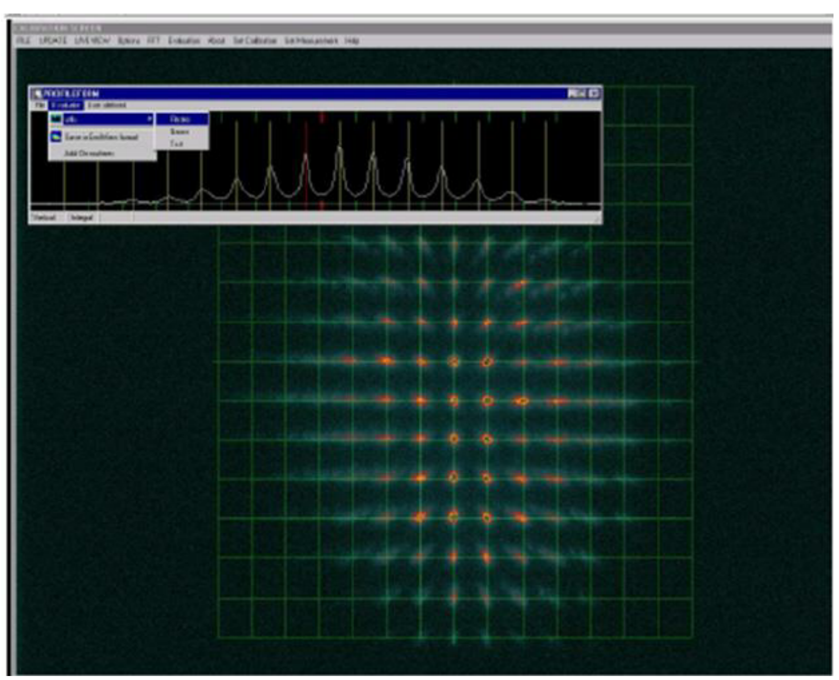
Multi Slit



- Extend the concept of the slit and grid
  - Why not adding many slits on the same blade?
    - No need to scan the slit
      - Single shot measurement
- Grid replaced by screen

Pepper Pot





- Extend the concept of the multi slit
    - Why not replacing the slits with holes?
        - Both planes (x, y) at the same time
        - Data analysis more complicated
- High resolution on the screen required

Particles transport

In a linear system, like a system composed of drift space and quadrupoles, the coordinates of a particle in phase space can be transported using a simple matrix notation

$$\begin{bmatrix} x_1 \\ x'_1 \end{bmatrix} = M_1 \begin{bmatrix} x_0 \\ x'_0 \end{bmatrix} \qquad \begin{bmatrix} x_2 \\ x'_2 \end{bmatrix} = M_2 \begin{bmatrix} x_1 \\ x'_1 \end{bmatrix} \qquad \begin{bmatrix} x_3 \\ x'_3 \end{bmatrix} = M_3 \begin{bmatrix} x_2 \\ x'_2 \end{bmatrix}$$

$$\begin{bmatrix} x_3 \\ x'_3 \end{bmatrix} = M_3 M_2 M_1 \begin{bmatrix} x_0 \\ x'_0 \end{bmatrix} = M_{0 \Rightarrow 3} \begin{bmatrix} x_0 \\ x'_0 \end{bmatrix}$$

$$M_{\mathrm{Drift}} = \begin{bmatrix} 1 & L \\ 0 & 1 \end{bmatrix} \qquad M_{\mathrm{Quad}} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix} = \begin{bmatrix} \cos(\sqrt{k} L_Q) & \frac{1}{\sqrt{k}} \sin(\sqrt{k} L_Q) \\ -\sqrt{k} \sin(\sqrt{k} L_Q) & \cos(\sqrt{k} L_Q) \end{bmatrix}$$

$L_Q \rightarrow 0$ (QF, for QD it is different)

Twiss parameters transport

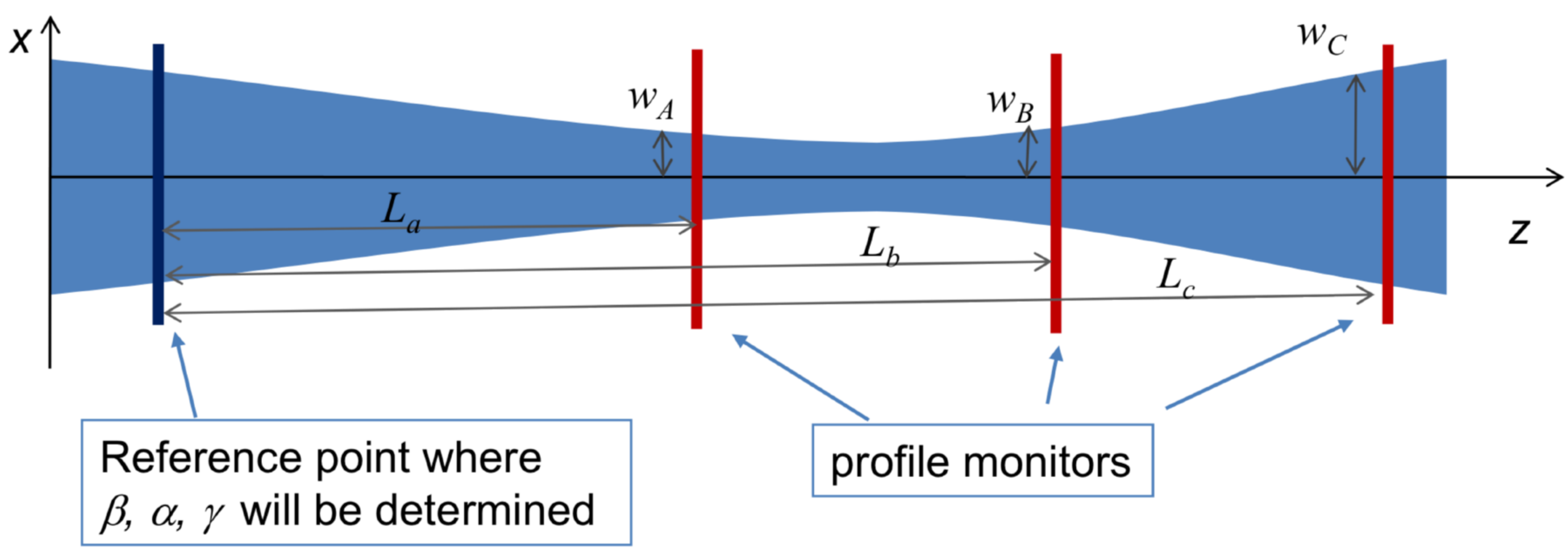
If one can transport each point of the phase space one can also transport the ellipse and thus the Courant-Snyder, a.k.a. Twiss, parameters

$$\begin{bmatrix} x_1 \\ x'_1 \end{bmatrix} = \begin{bmatrix} c & s \\ c' & s' \end{bmatrix} \begin{bmatrix} x_0 \\ x'_0 \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} \beta_1 \\ \alpha_1 \\ \gamma_1 \end{bmatrix} = \begin{bmatrix} c^2 & 2cs & s^2 \\ cc' & cs' + c's & -ss' \\ c'^2 & -2c's' & s'^2 \end{bmatrix} \begin{bmatrix} \beta_0 \\ \alpha_0 \\ \gamma_0 \end{bmatrix}$$

3 profiles emittance measurement

- 3 Unknown (ε, α, β) (γ is calculated from α, β)

- If we can make three measurement and write three linear independent equations we can solve the system



3 profiles emittance

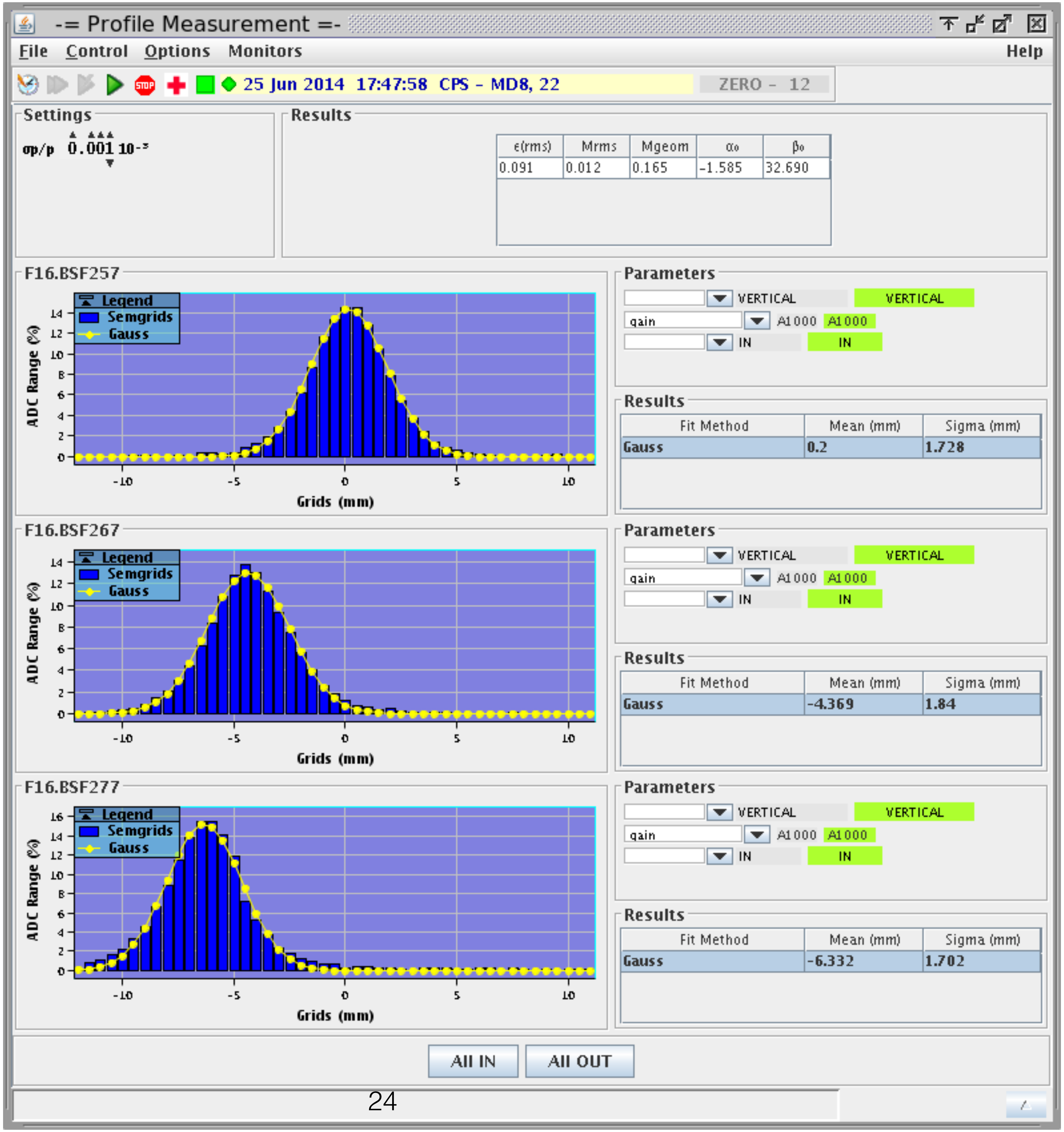
$$\beta_1 = \begin{bmatrix} c_1^2 & 2c_1 s_1 & s_1^2 \end{bmatrix} \begin{bmatrix} \beta_0 \\ \alpha_0 \\ \gamma_0 \end{bmatrix} \qquad \begin{bmatrix} \varepsilon\beta_1 \\ \varepsilon\beta_2 \\ \varepsilon\beta_3 \end{bmatrix} = \varepsilon \begin{bmatrix} c_1^2 & 2c_1 s_1 & s_1^2 \\ c_2^2 & 2c_2 s_2 & s_2^2 \\ c_3^2 & 2c_3 s_3 & s_3^2 \end{bmatrix} \begin{bmatrix} \beta_0 \\ \alpha_0 \\ \gamma_0 \end{bmatrix}$$

$$\boxed{\sqrt{\varepsilon\beta} = \text{Beam Size}}$$

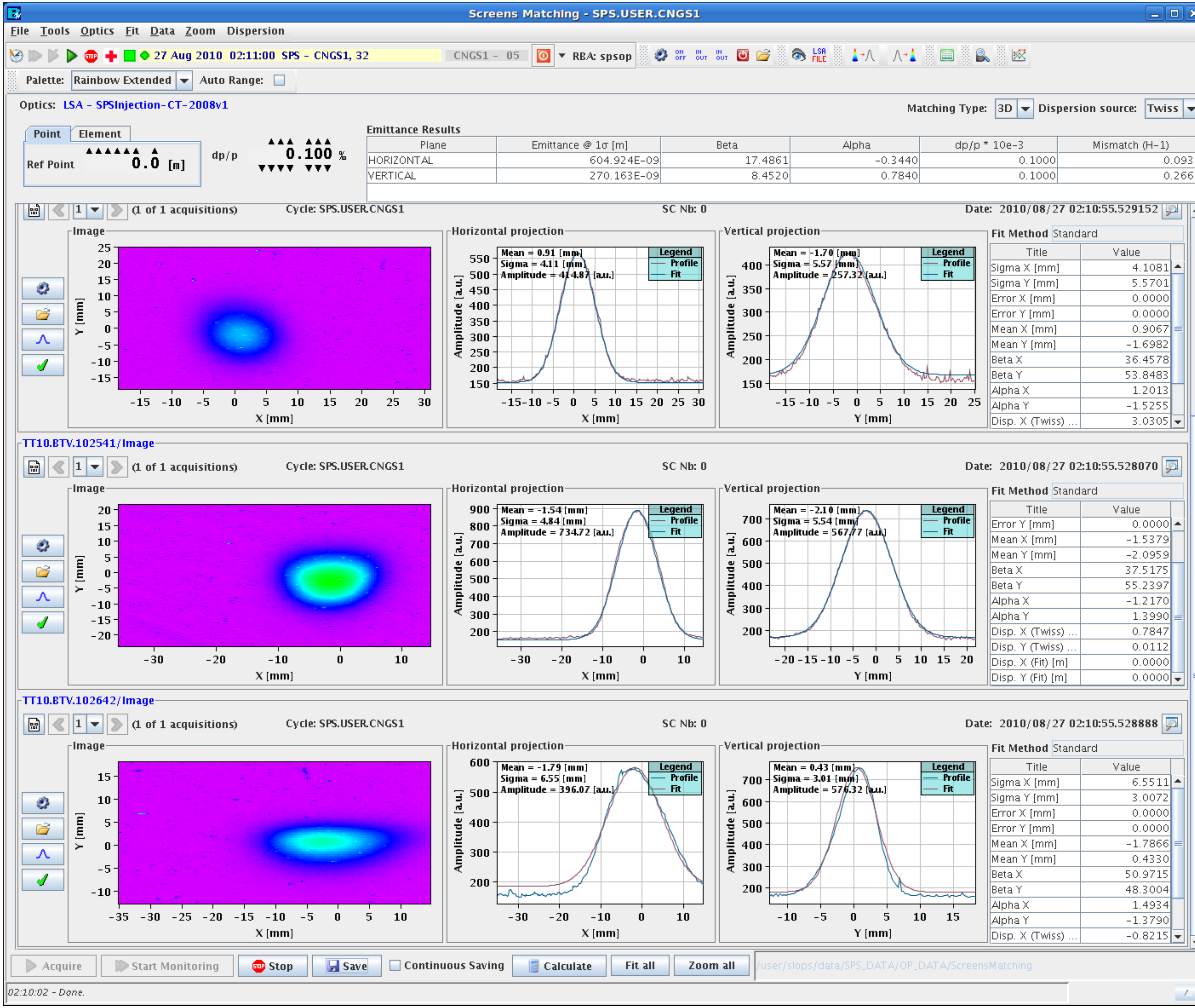
$$\begin{bmatrix} \varepsilon\beta_1 \\ \varepsilon\beta_2 \\ \varepsilon\beta_3 \end{bmatrix} = \varepsilon M \begin{bmatrix} \beta_0 \\ \alpha_0 \\ \gamma_0 \end{bmatrix} \quad \Rightarrow \quad M^{-1} \begin{bmatrix} \varepsilon\beta_1 \\ \varepsilon\beta_2 \\ \varepsilon\beta_3 \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \varepsilon \begin{bmatrix} \beta_0 \\ \alpha_0 \\ \gamma_0 \end{bmatrix}$$

$$\begin{cases} a = \varepsilon\beta_0 \\ b = \varepsilon\alpha_0 \\ c = \varepsilon\gamma_0 \\ \gamma_0 = \dfrac{1+\alpha_0^2}{\beta_0} \end{cases} \quad \Rightarrow \quad \begin{cases} \beta_0 = \dfrac{a}{\sqrt{ac-b^2}} \\ \alpha_0 = \dfrac{b}{\sqrt{ac-b^2}} \\ \gamma_0 = \dfrac{c}{\sqrt{ac-b^2}} \\ \varepsilon = \sqrt{ac-b^2} \end{cases}$$

3 profiles emittance



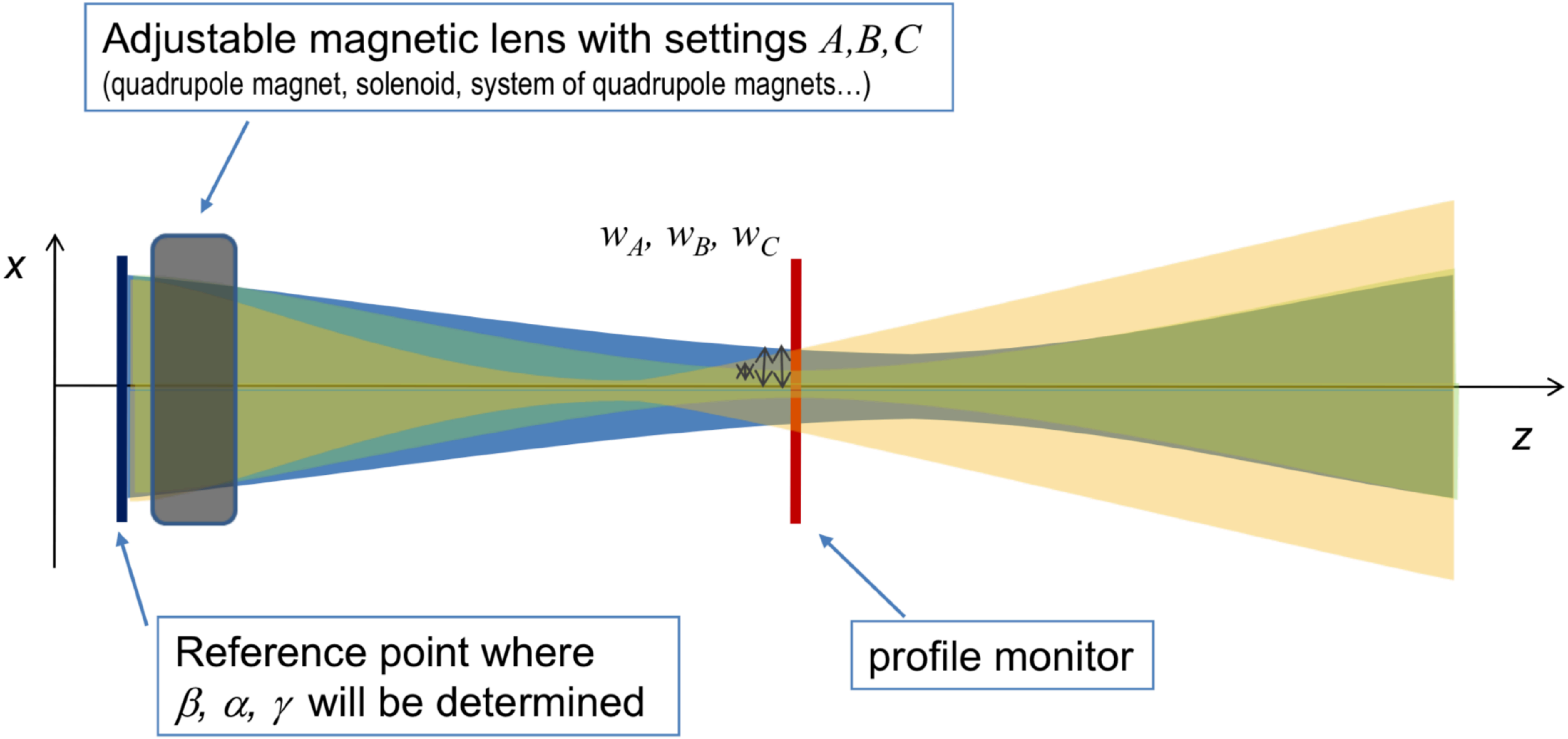
3 profiles emittance



Quadrupole strength scan

- Instead of measuring the beam size at different position measure multiple times at the same position changing the optics upstream in a known way
  - Simplest solution is to change the strength of a focusing quadrupole upstream the profile monitor
  - The beam cannot be transported after the quadrupole (we change the optics substantially!)
  - Method cannot be used with “dangerous” beams

Quad scan



$$\begin{pmatrix} c & s \\ c' & s' \end{pmatrix} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} m_{11}(I_{mag}) & m_{12}(I_{mag}) \\ m_{21}(I_{mag}) & m_{22}(I_{mag}) \end{pmatrix}$$

Quad scan

- In principle three different values of the quadrupole strength are sufficient (same equations as for the 3 profiles method)

- In practice it is very easy to acquire many measurement points with the quad scan (while it is not easy to add more than 3 profile monitors in a line)

- If you have more than 3 measurements the problem is over constrained → use a minimisation routine (fit)

$$\{$$

Minimisation (fit)

Residuals

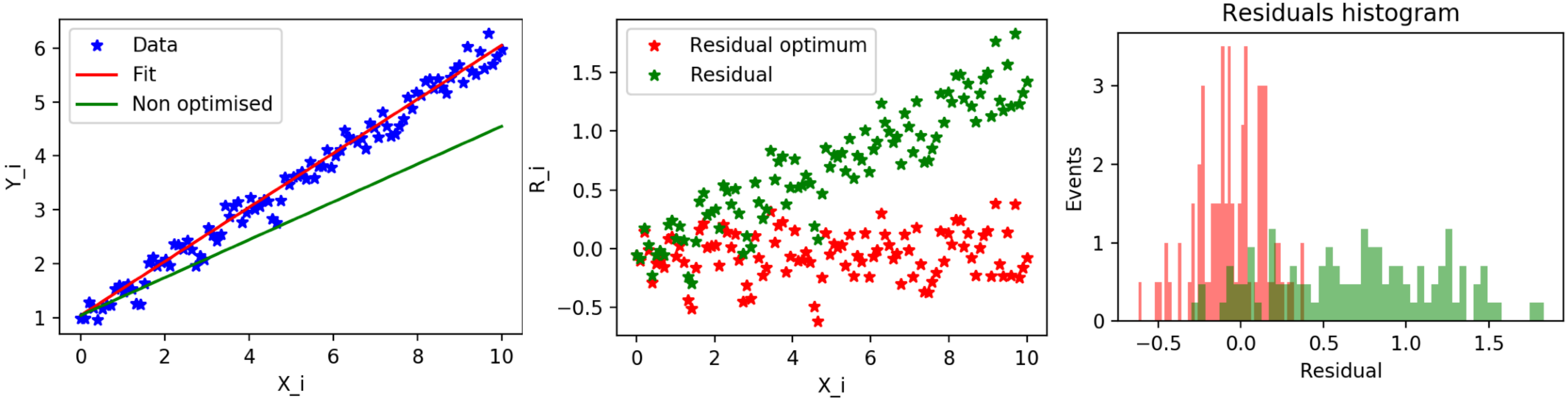
$$r_i = \text{measurement}_i - \text{model}(p_1, p_2, \cdots)_i$$

$$\text{Error} = \sqrt{\sum_i r_i^2}$$

$$\text{Min}(Error) \Rightarrow (\bar{p}_0, \bar{p}_1, \cdots)$$

- Input
  - Model (parameters)
  - Measurements
- Free parameters (unknowns)
- Procedure
  - Vary the free parameters until you find a minimum of the Error

Fit example



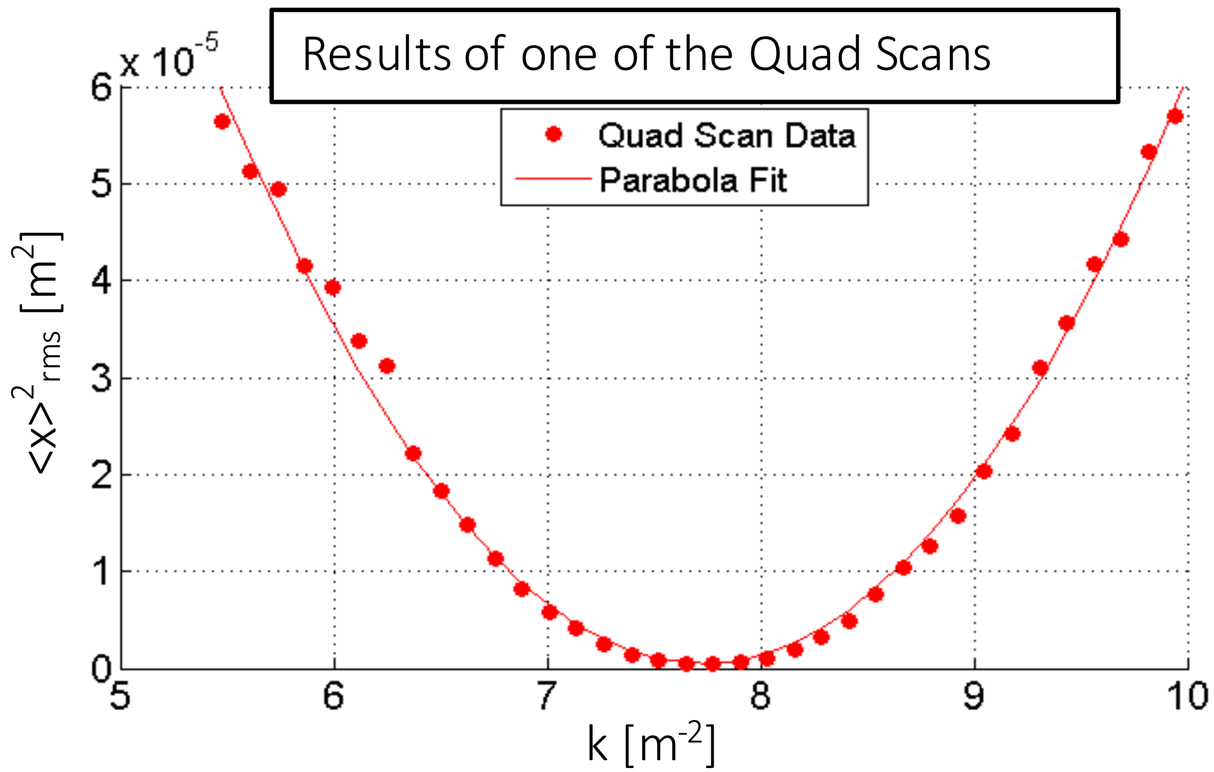
Model: $y(x) = a + b \cdot x$

Measurements: $(x, y)_i$

Parameters: (a, b)

}

Quadrupole scan



- Plot the measured beam sizes (Σ_{11}) agains the strength of the quadrupole field (k)
- Fit a parabola
- Extract ε, α, β, γ from the parameters of the parabola

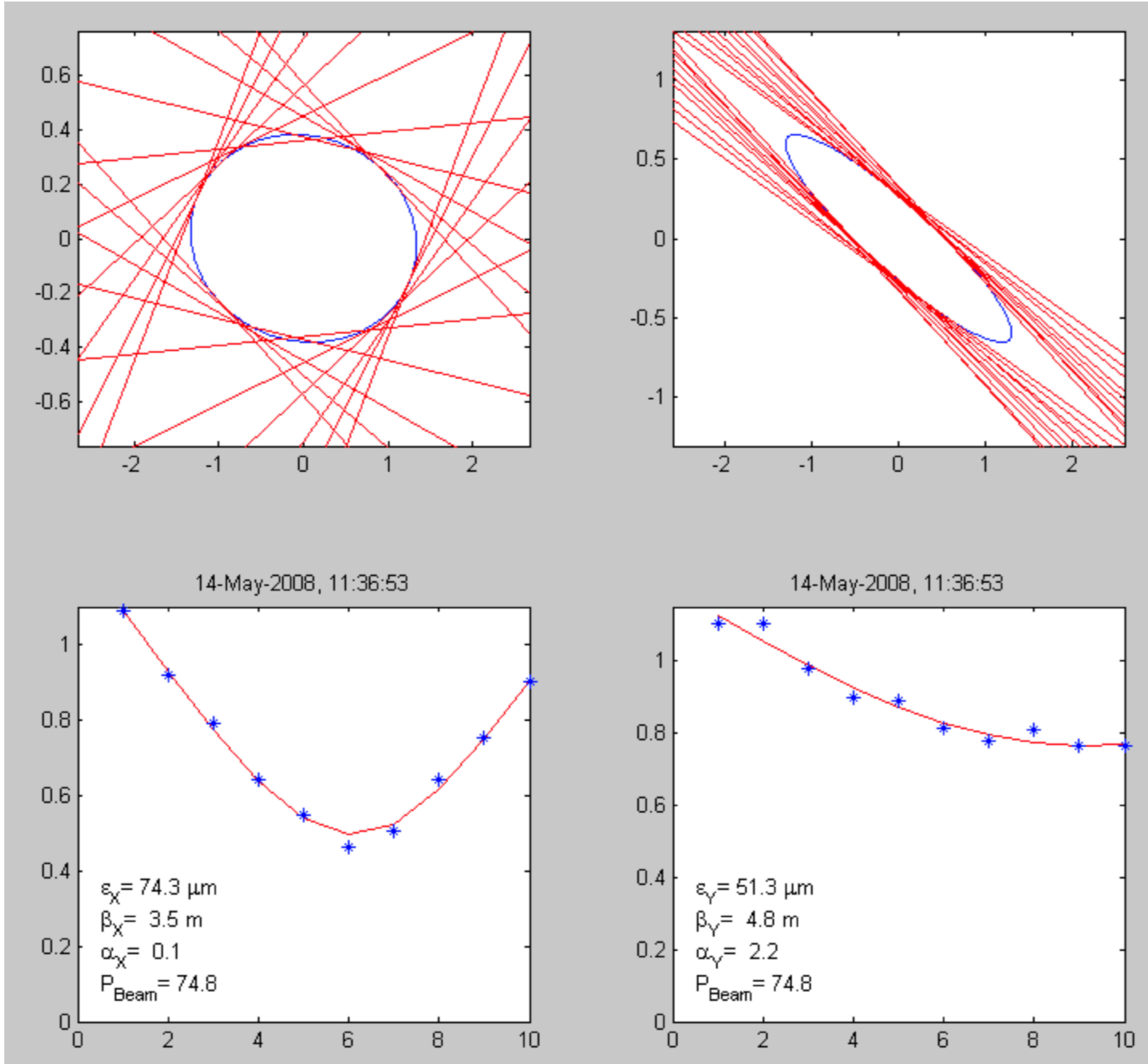
Fit: $w^2 = ak^2 - 2abk + ab^2 + c$

$$\Sigma = \begin{bmatrix} \varepsilon\beta & -\varepsilon\alpha \\ -\varepsilon\alpha & \varepsilon\gamma \end{bmatrix} = \begin{bmatrix} \frac{a}{d^2l^2} & \frac{a}{d^2l^2}(bl - \frac{1}{d}) \\ \frac{a}{d^2l^2}(bl - \frac{1}{d}) & \frac{c}{d^2} + \frac{a}{d^2l^2}(bl - \frac{1}{d})^2 \end{bmatrix}$$

$$\varepsilon = \frac{\sqrt{ac}}{d^2l^2}$$

d= drift space (L), l= quadrupole length (L_Q)

Quad scan at CTF



Emittance measurement in synchrotrons

$$\begin{bmatrix}\beta(s)\\ \alpha(s)\\ \gamma(s)\end{bmatrix} = \begin{bmatrix}\beta(s+C)\\ \alpha(s+C)\\ \gamma(s+C)\end{bmatrix} = \begin{bmatrix} c^2 & 2cs & s^2 \\ cc' & cs' + c's & -ss' \\ c'^2 & -2c's' & s'^2 \end{bmatrix} \begin{bmatrix}\beta(s)\\ \alpha(s)\\ \gamma(s)\end{bmatrix}$$

$$\beta(s) = \frac{2s}{\sqrt{(2-c-s')(2+c+s')}}$$

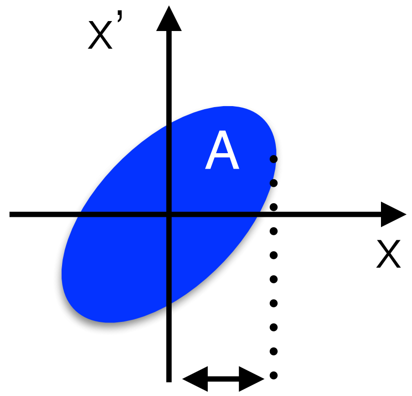
$$\sigma = \sqrt{\beta\varepsilon}$$

$$\sigma = \sqrt{\beta\varepsilon + \left(D\frac{\Delta p}{p}\right)^2}$$

Measured Optics Measured?

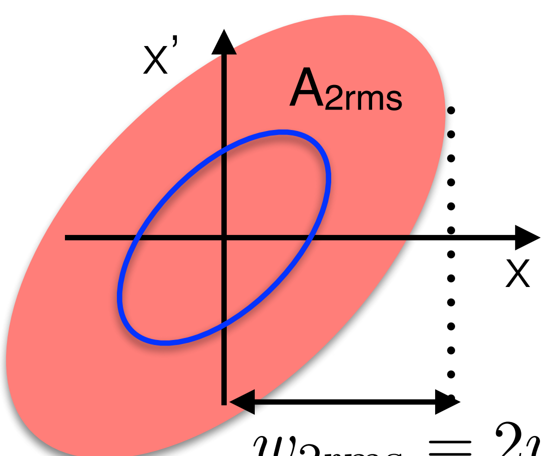
- One can calculate the value of β directly from this constraint (using the transport matrix)
- It is possible to measure the β function around the ring using BPMs, k-modulation etc.
- Measure the beam size and derive the emittance from the optics functions
- Measure in a dispersion free if possible

Various emittance definitions



$$w_{\rm rms} = \sqrt{\Sigma_{11}}$$

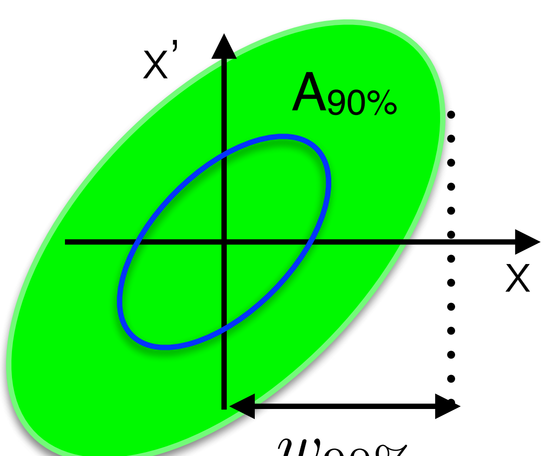
$$\varepsilon_{\rm rms} = \frac{A}{\pi}$$



$$w_{\rm 2rms} = 2 w_{\rm rms}$$

$$\varepsilon_{\rm 2rms} = \frac{A_{\rm 2rms}}{\pi}$$

$$\varepsilon_{\rm 2rms} = 4 \varepsilon_{\rm rms}$$



$$w_{90\%}$$

$$A_{90\%} \supset 90\% \text{ of particles}$$

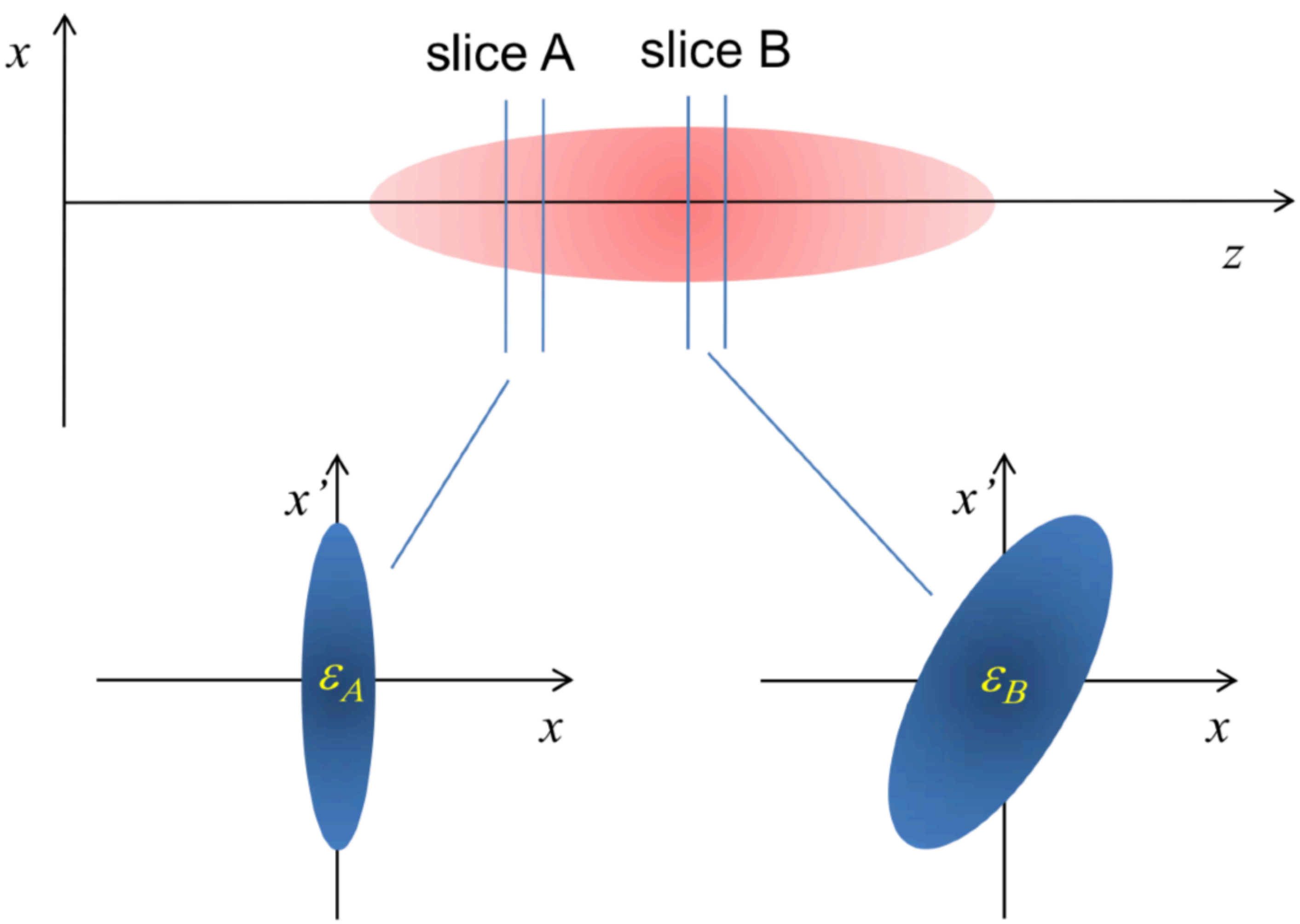
$$\varepsilon_{90\%} = \frac{A_{90\%}}{\pi}$$

- We have seen the RMS emittance (from phase space moments)
- Different people use different definitions
- $\varepsilon_{90\%}$ the ellipse that contains 90% of particles
- $\varepsilon_{95\%}$ the ellipse that contains 95% of particles
- ε_{2rms} the ellipse at twice the rms

Gaussian phase space

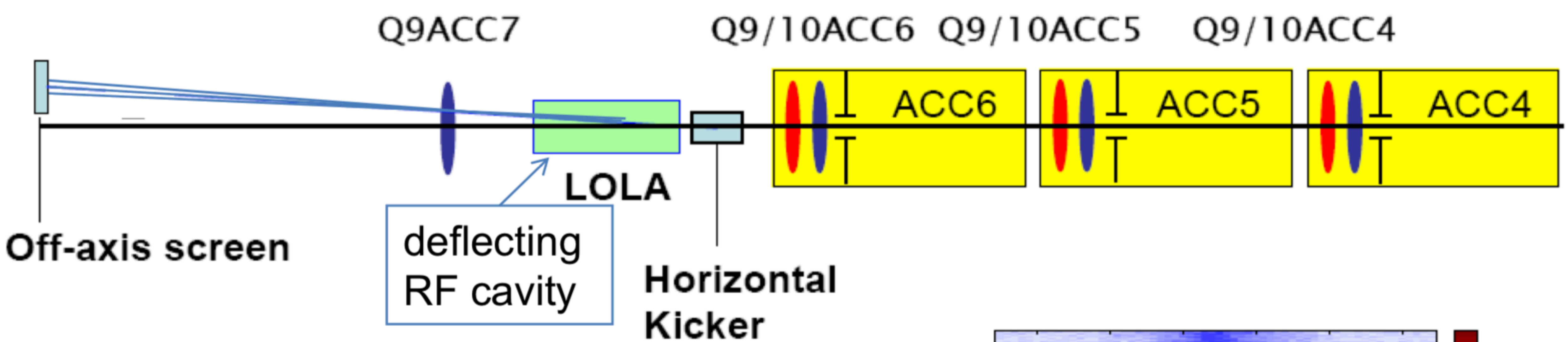
- $A = \pi\varepsilon_{rms}$ contains ~40% of particles w= σ
- $A = \pi\varepsilon_{2rms}$ contains ~86% of particles w= 2σ
- $A = \pi\varepsilon_{90\%}$ contains 90% of particles w= 2.15σ
- $A = \pi\varepsilon_{95\%}$ contains 95% of particles w= 2.45σ

Slice emittance

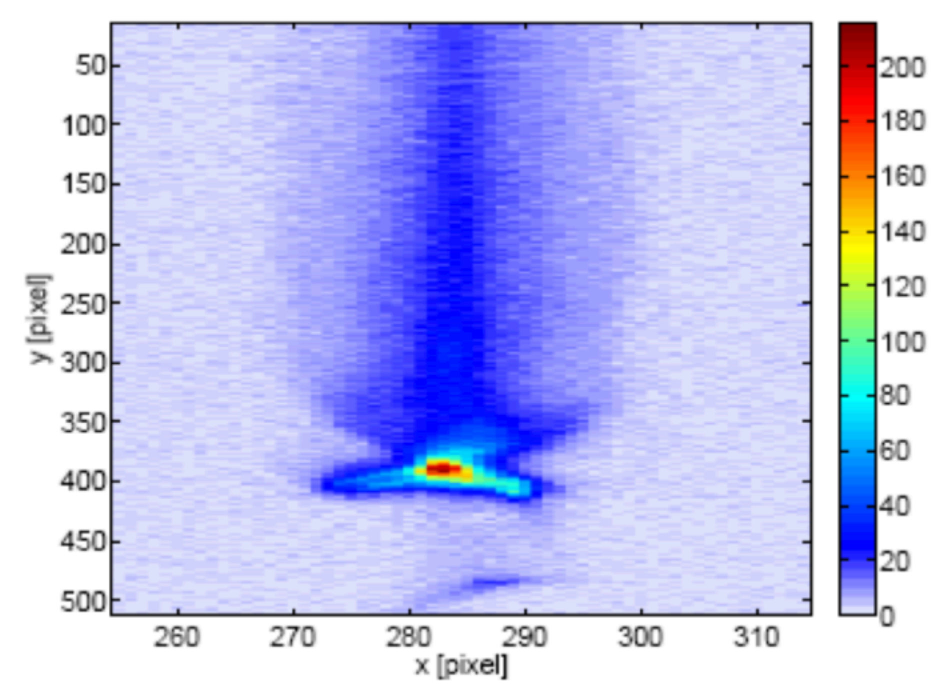


- In electron LINACs emittance may change along the bunch
- Need for a time resolved profile measurement
  - Streak camera
  - Deflecting cavity

Slice emittance



Example of set-up at FLASH in DESY



That’s all folks!

Thank you for your attention