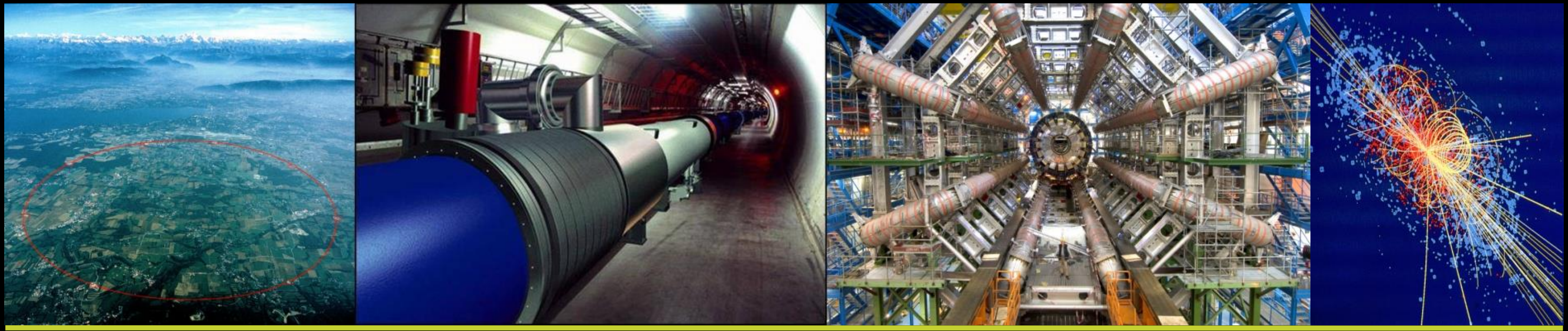


Tune, Chromaticity and Coupling Measurement

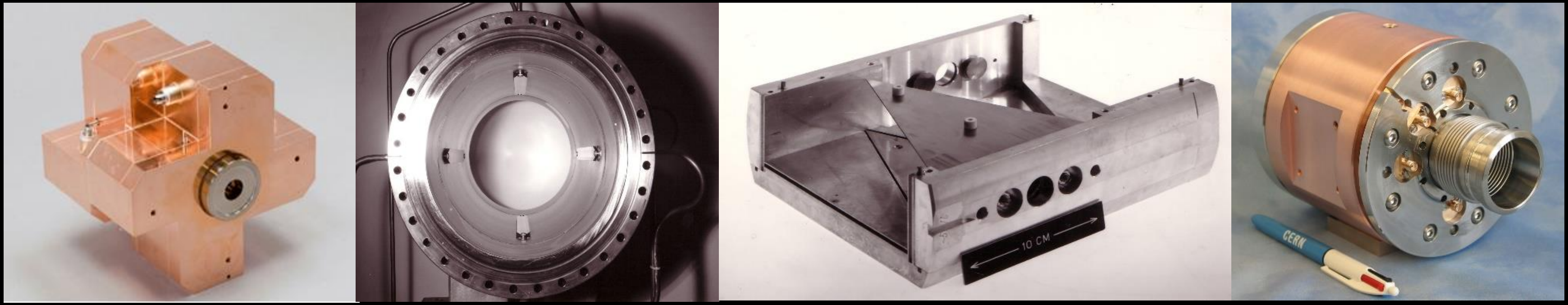
BI CAS 2018

Tuusula, Finland

2nd – 15th June, 2018

Rhodri Jones

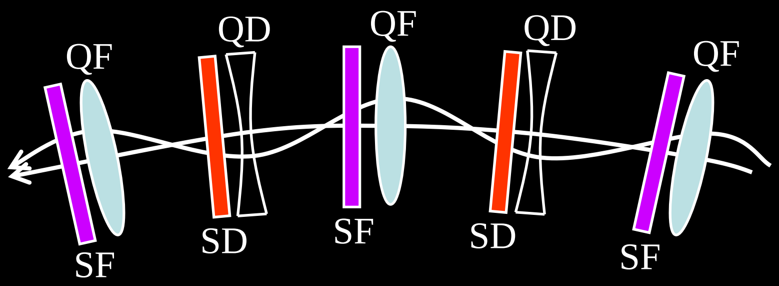
Head of the CERN Beam Instrumentation Group



Tune Measurement

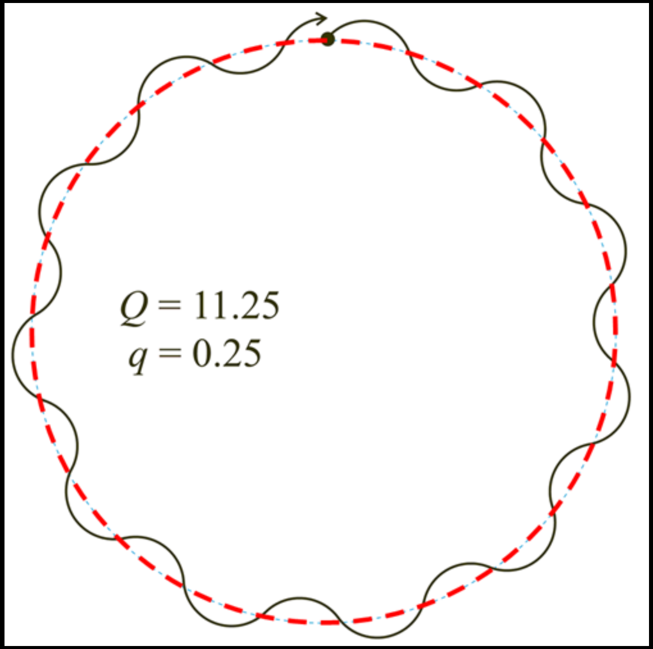
Machine Tune

- Machine Tune



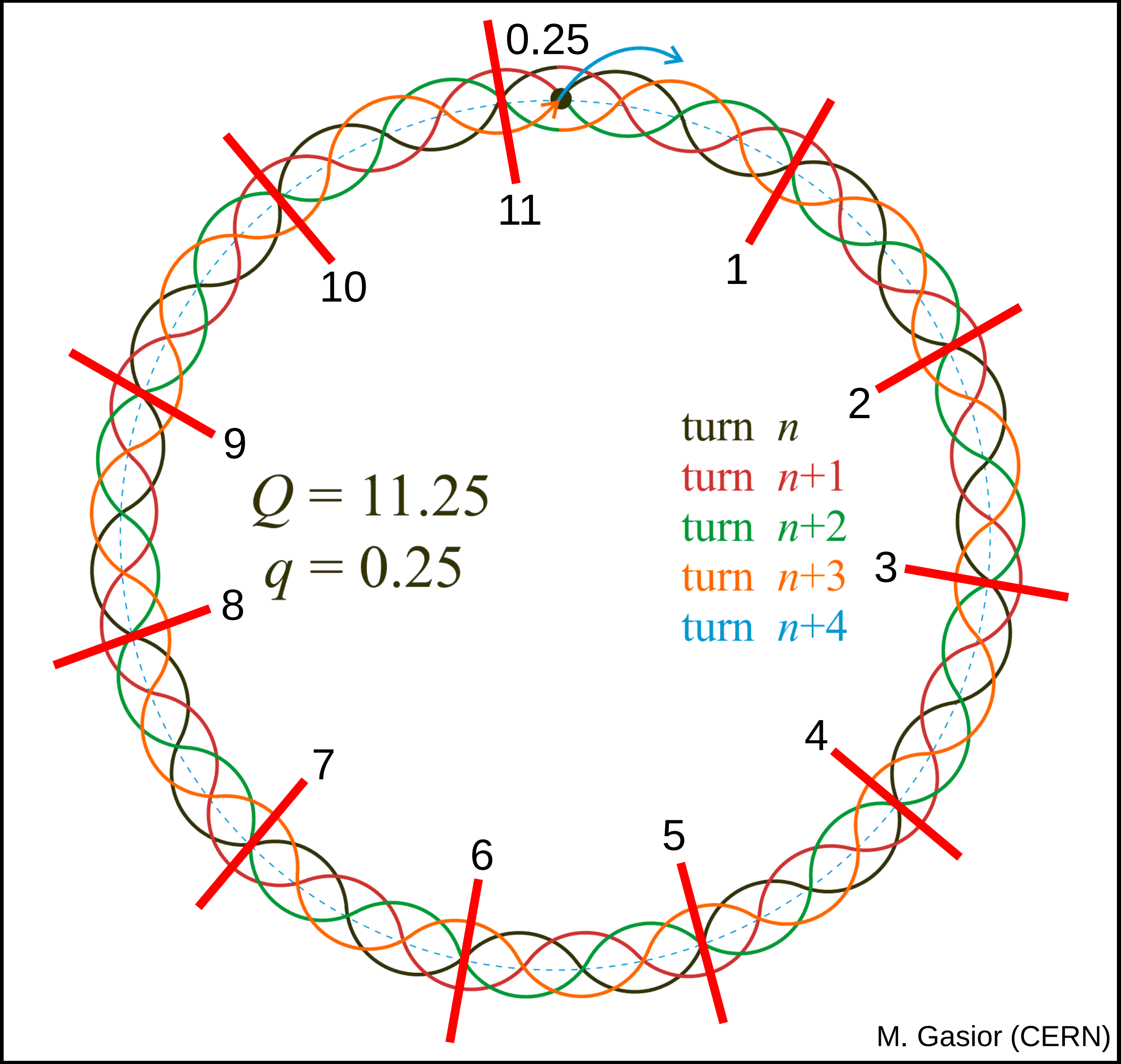
Characteristic Frequency of the Magnetic Lattice

Given by the strength of the Quadrupole magnets



- Parameters per plane
  - Q : Full betatron tune
  - q : Fractional tune (operating point)
- Real life more complex
  - horizontal & vertical oscillations couple
  - betatron motion at large amplitudes non-linear

Betatron motion and the Tune



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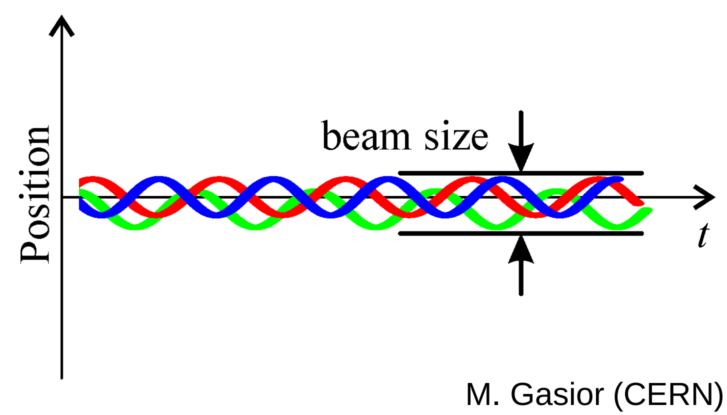
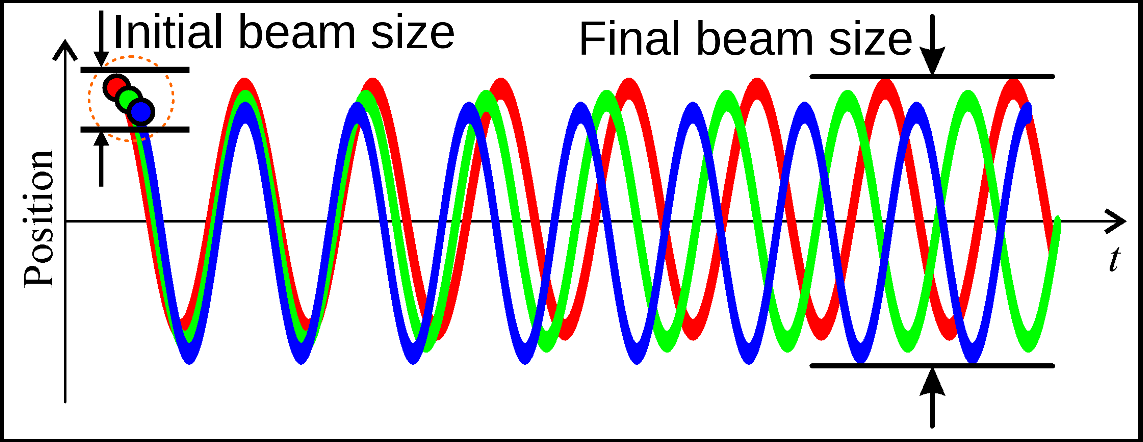
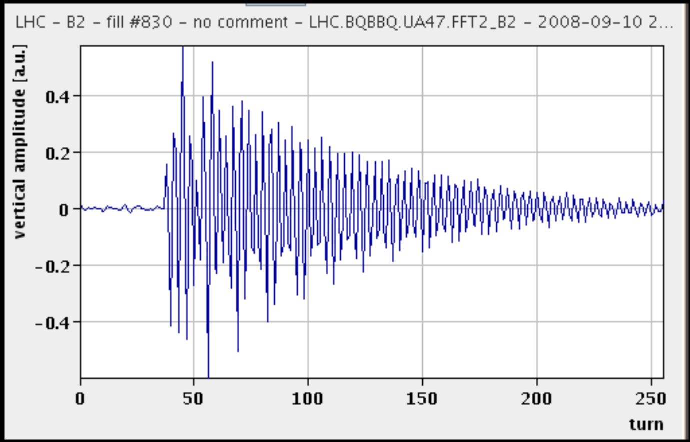
Betatron motion and the Tune



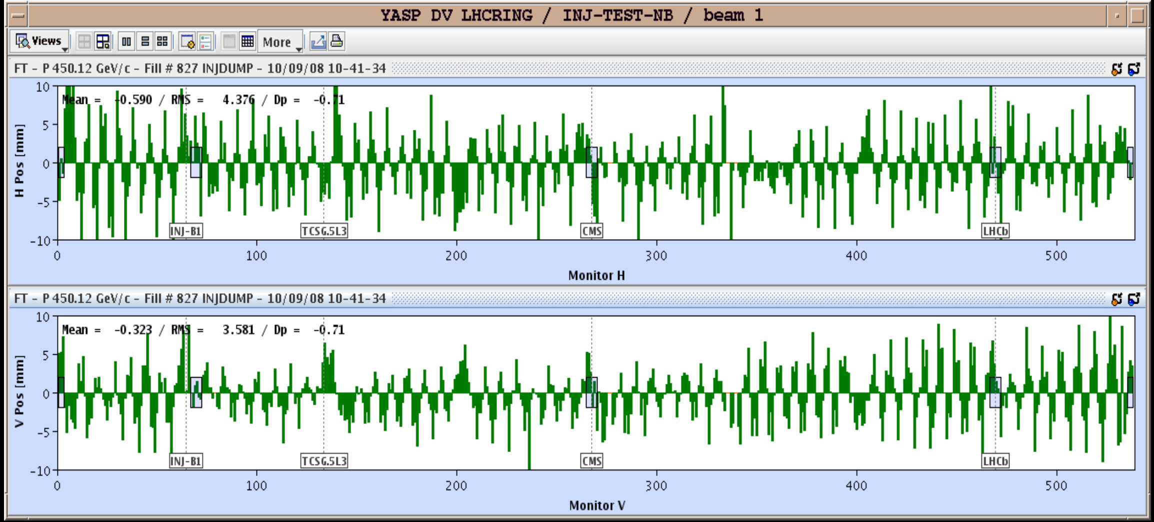
- Beam size
  - defined by incoherent betatron motion of all particles
- Particles have momentum spread
  - gives spread in focussing by quadrupoles
  - gives rise to spread in the frequency of the betatron oscillations (chromaticity)
- Coherent oscillations will de-cohere
  - Hadrons do not forget!
  - once hit they oscillate (practically) forever
  - any excitation must be kept very small





Tune Measurement

- Integer tune
  - seen in orbit response
  - ~550 dual plane BPMs
  - H: 59, V: 64 for LHC



YASP DV LHCRING / INJ-TEST-NB / beam 1

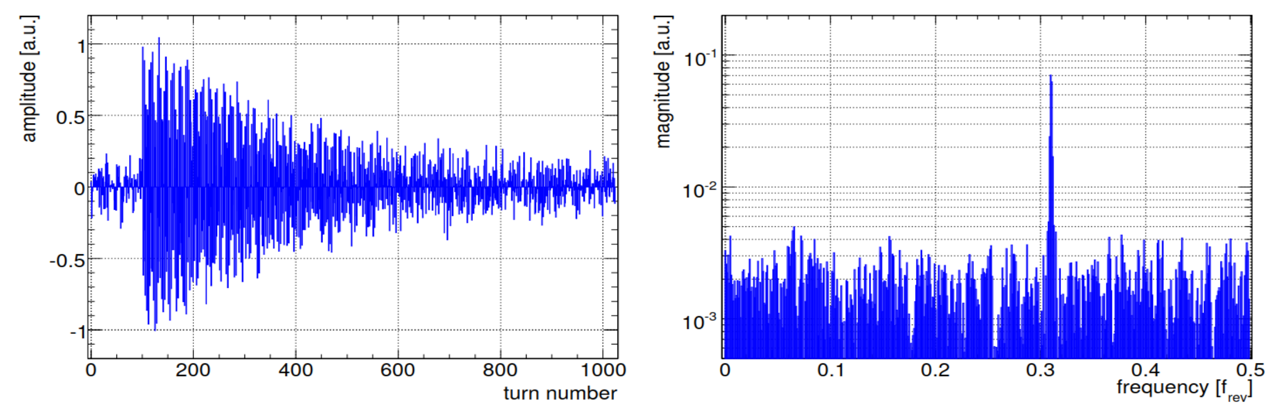
FT - P 450.12 GeV/c - Fill # 827 INJDUMP - 10/09/08 10-41-34

Mean = -0.590 / RMS = 4.376 / Dp = -0.71

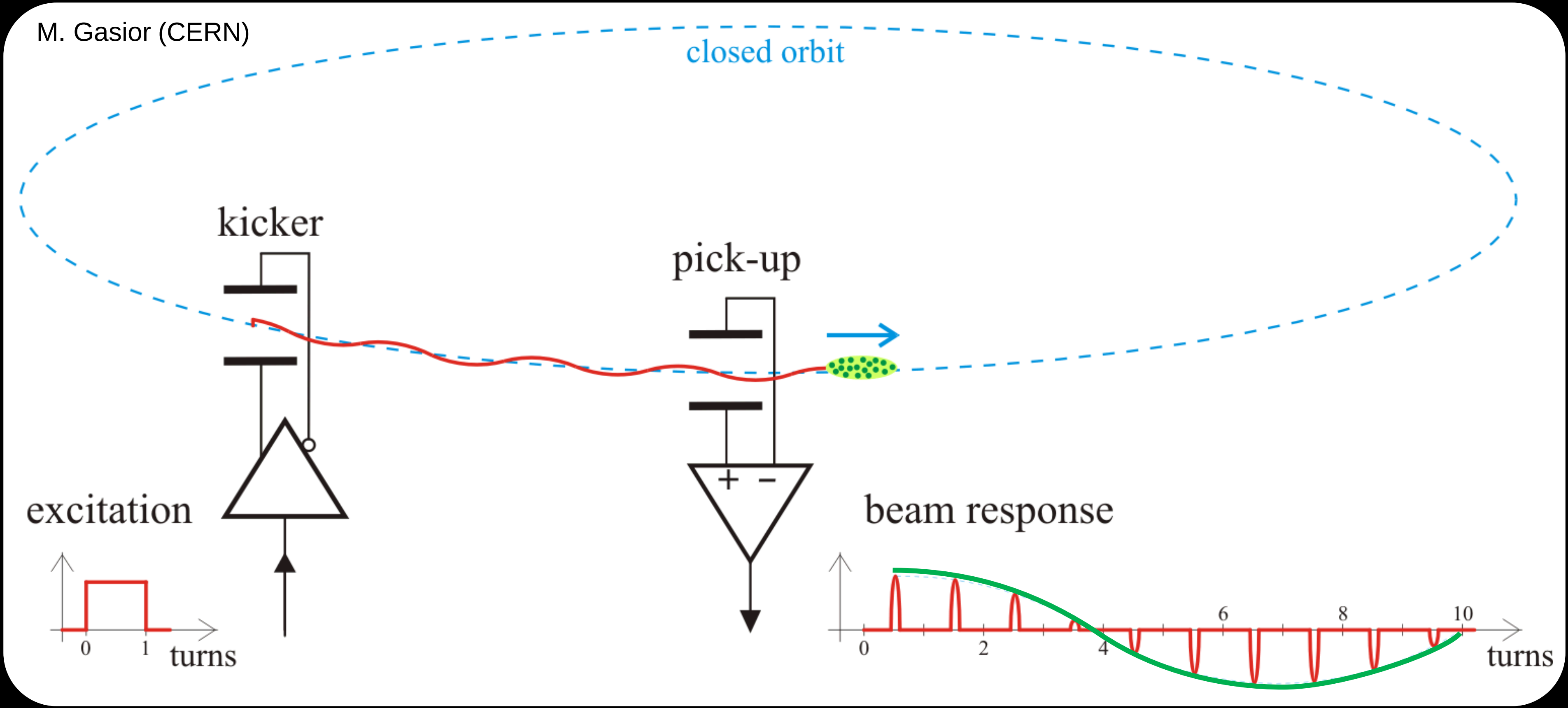
FT - P 450.12 GeV/c - Fill # 827 INJDUMP - 10/09/08 10-41-34

Mean = -0.323 / RMS = 3.581 / Dp = -0.71

- Fractional tune (q)
  - Seen from turn-by-turn signal of single BPM if beam is given a kick
  - Fast Fourier Transform (FFT) of oscillation data gives resonant frequency (q)



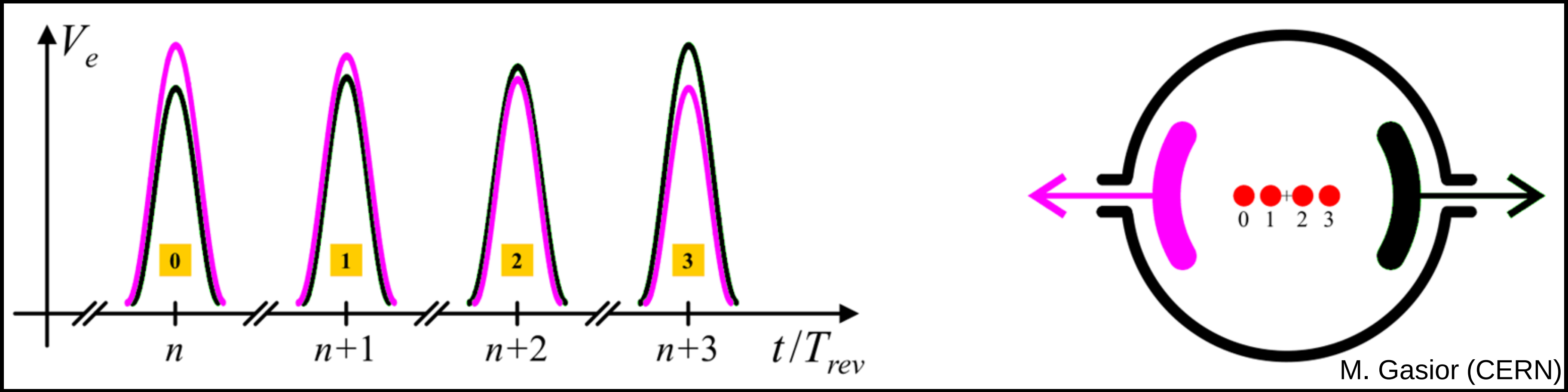
Tune Measurement – the principle


M. Gasior (CERN)

- A stimulus is needed to globally excite the beam
  - Resulting betatron oscillations observed on a position pick-up
  - Time domain signals usually converted to frequency domain
    - Displays which frequencies are present in the oscillations

Tune Measurement – the principle

- Observable is the turn-by-turn position from a BPM
- BPM electrode signal has temporal shape related to the temporal structure (intensity profile) of the passing beam
  - Most of the signal produced is linked to intensity
- On top we look for very small variations linked to position
  - Such signals are very difficult to simulate in the lab



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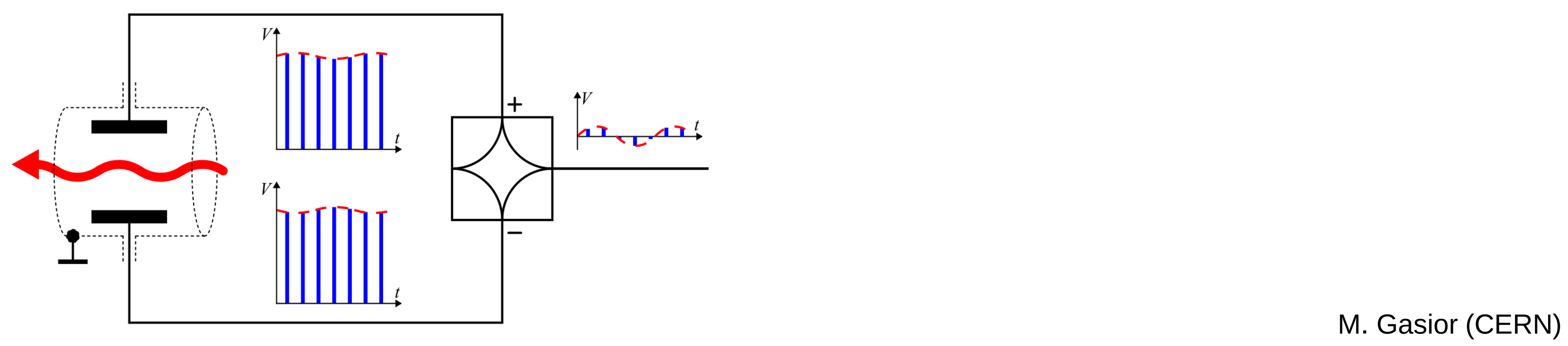
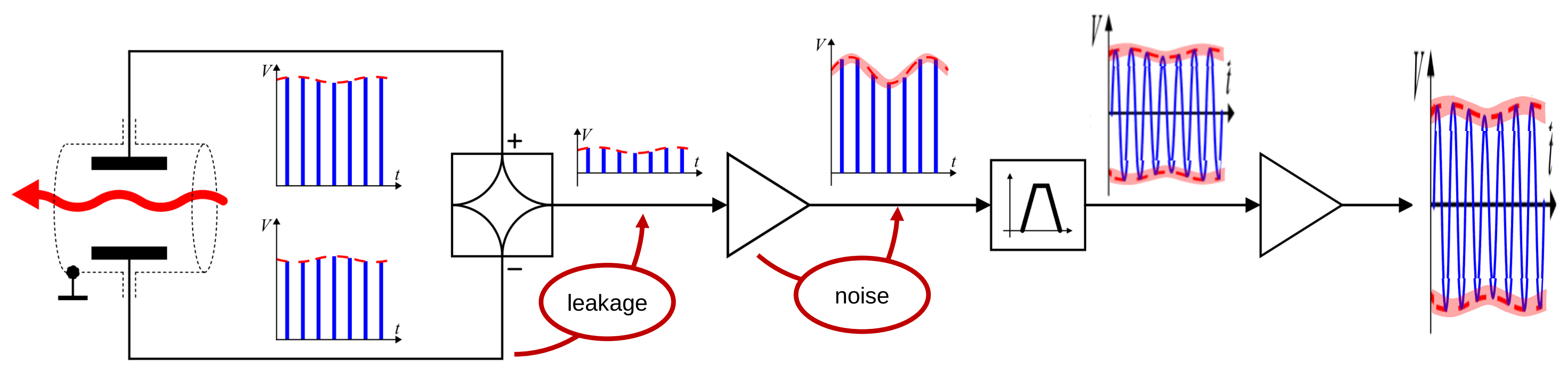
Tune Measurement – the principle

- A typical perfect detection scheme



M. Gasior (CERN)

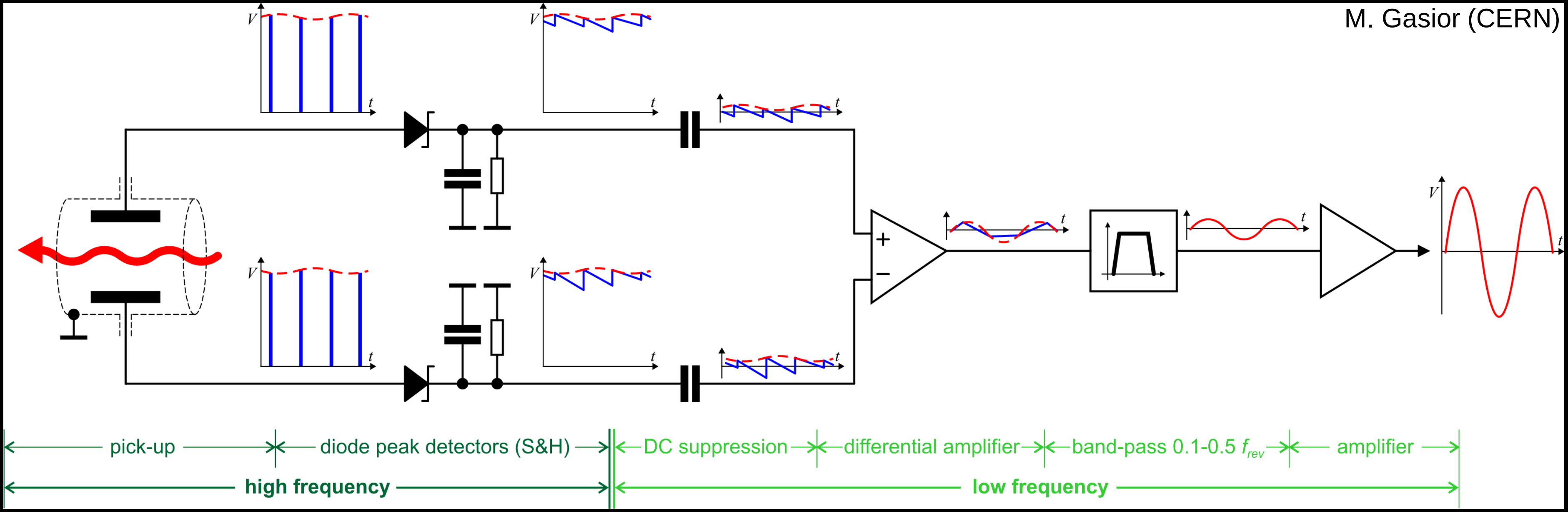
- Reality



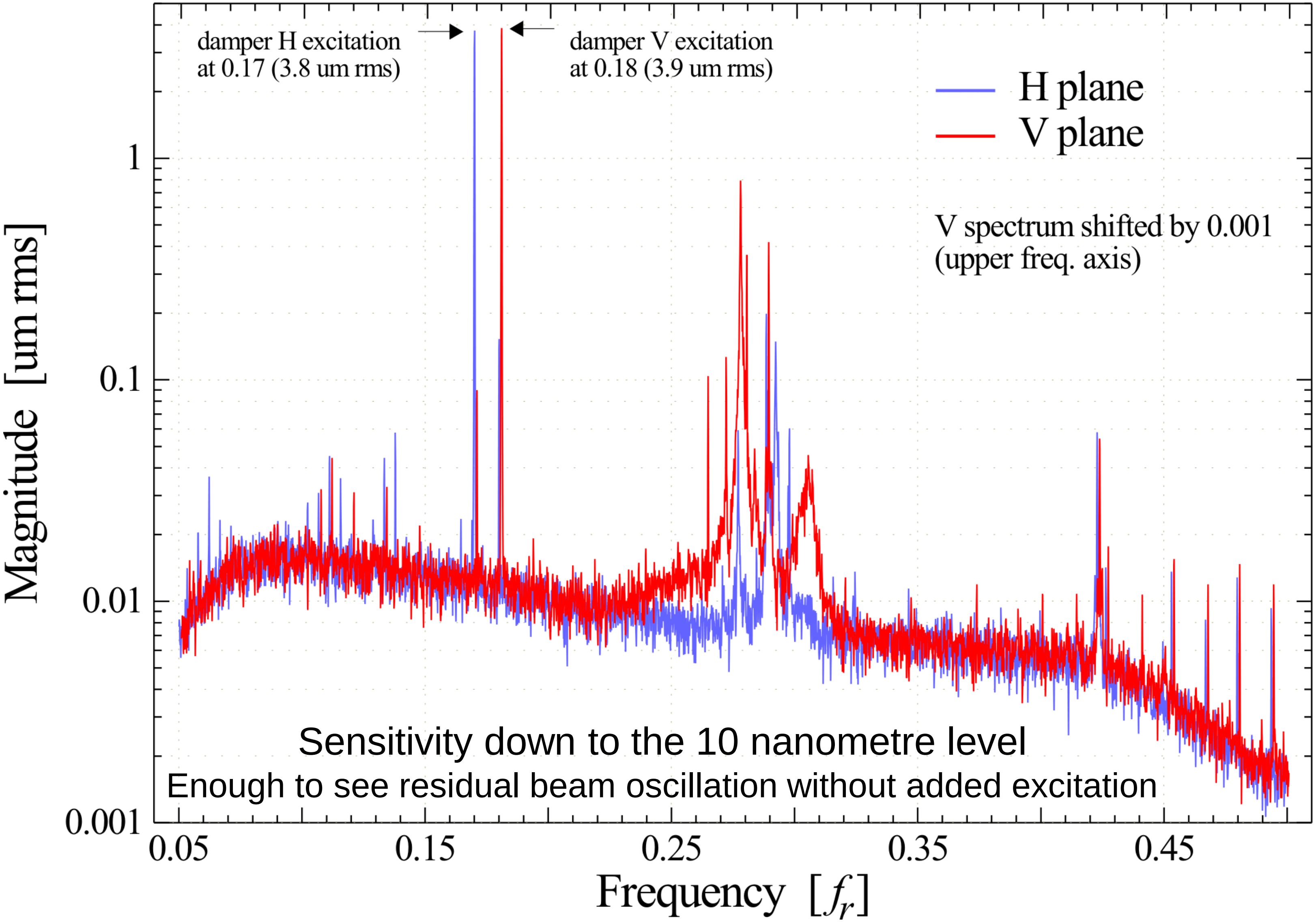
- Dynamic range issues
  - Signals related to betatron oscillations are small with respect to beam offset signals
  - Even for centred beam leakage is of order 1-10 % (of 100V!) for ns beam pulses

BaseBand Tune (BBQ) Measurement System

- Direct Diode Detection – the advantages
  - Single RF Schottky diode can handle up to 50 V pulses
    - Higher with a few diodes in series (LHC detector has 6 diodes)
  - Betatron modulation downmixed to below the revolution frequency
    - Allows efficient signal processing with inexpensive, high resolution ADCs
  - Just AM radio receiver – so what's new?
    - Slow discharge & use of low noise, high impedance amplifiers
    - Brutal filtering of revolution line & everything outside band of interest



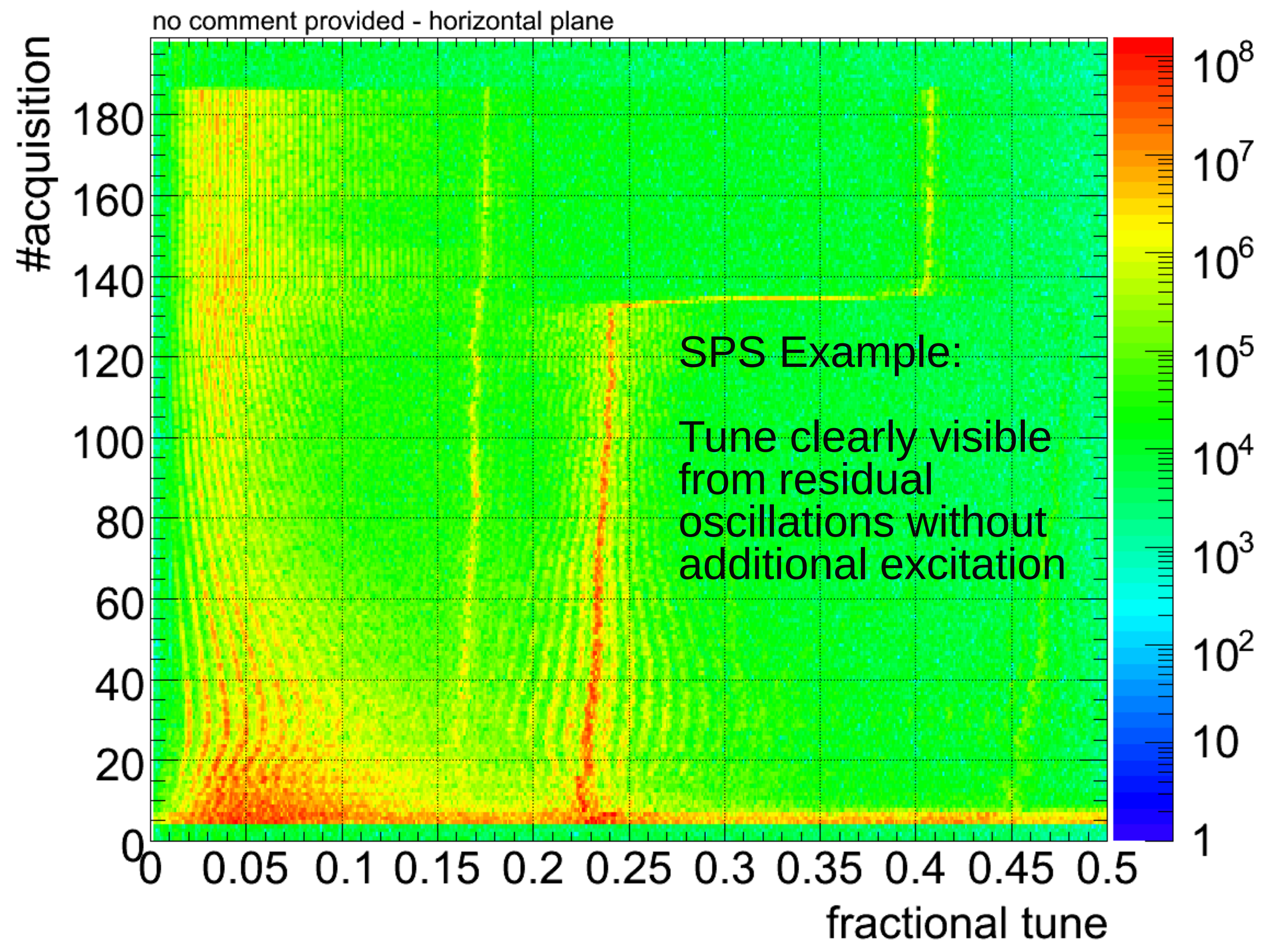
LHC BBQ System Performance



Sensitivity down to the 10 nanometre level

Enough to see residual beam oscillation without added excitation

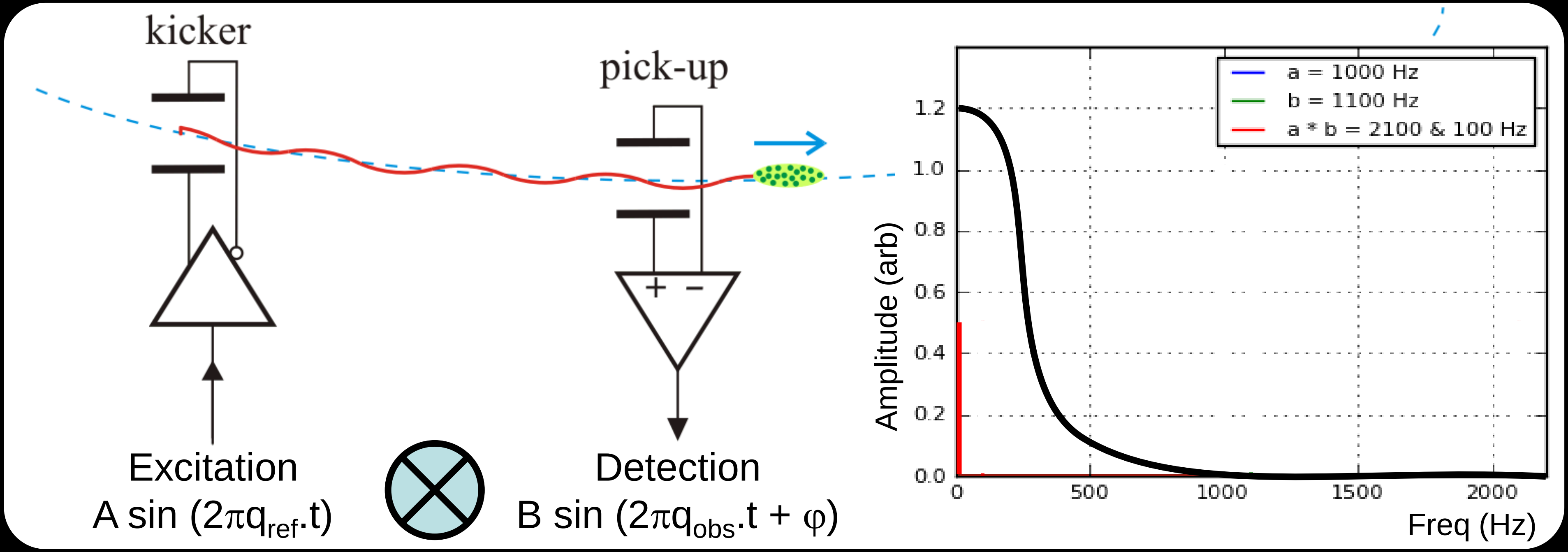
Real-Time Tune Display



SPS Example:

Tune clearly visible from residual oscillations without additional excitation

PLL Tune Measurement



Excitation
A sin $(2\pi q_{ref}.t)$

$\otimes$

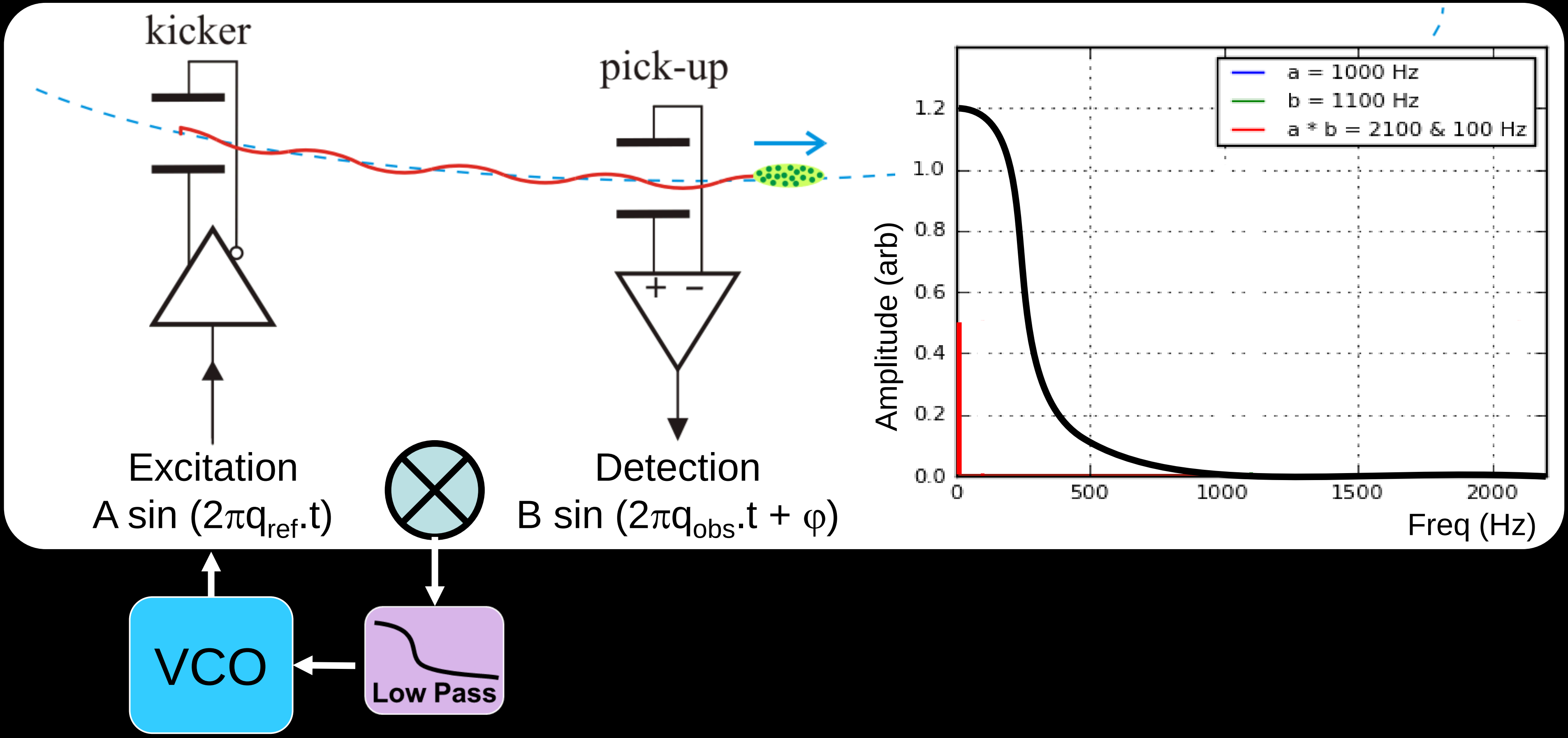
Detection
B sin $(2\pi q_{obs}.t + \varphi)$

$$= \frac{1}{2}AB[\underbrace{\cos(2\pi(q_{ref} - q_{obs})t - \varphi)}_{\text{Low frequency}} - \cancel{\underbrace{\cos(2\pi(q_{ref} + q_{obs})t + \varphi)}_{\text{High frequency}}}]$$

Low Pass Filter : $\frac{1}{2}AB[\cos(2\pi(\cancel{q_{ref} - q_{obs}})t - \varphi)]$

But $q_{obs} = q_{ref}$ for forced harmonic oscillator (our beam) $\Rightarrow$ **½AB [cos (- φ)]** (DC value)

PLL Tune Measurement

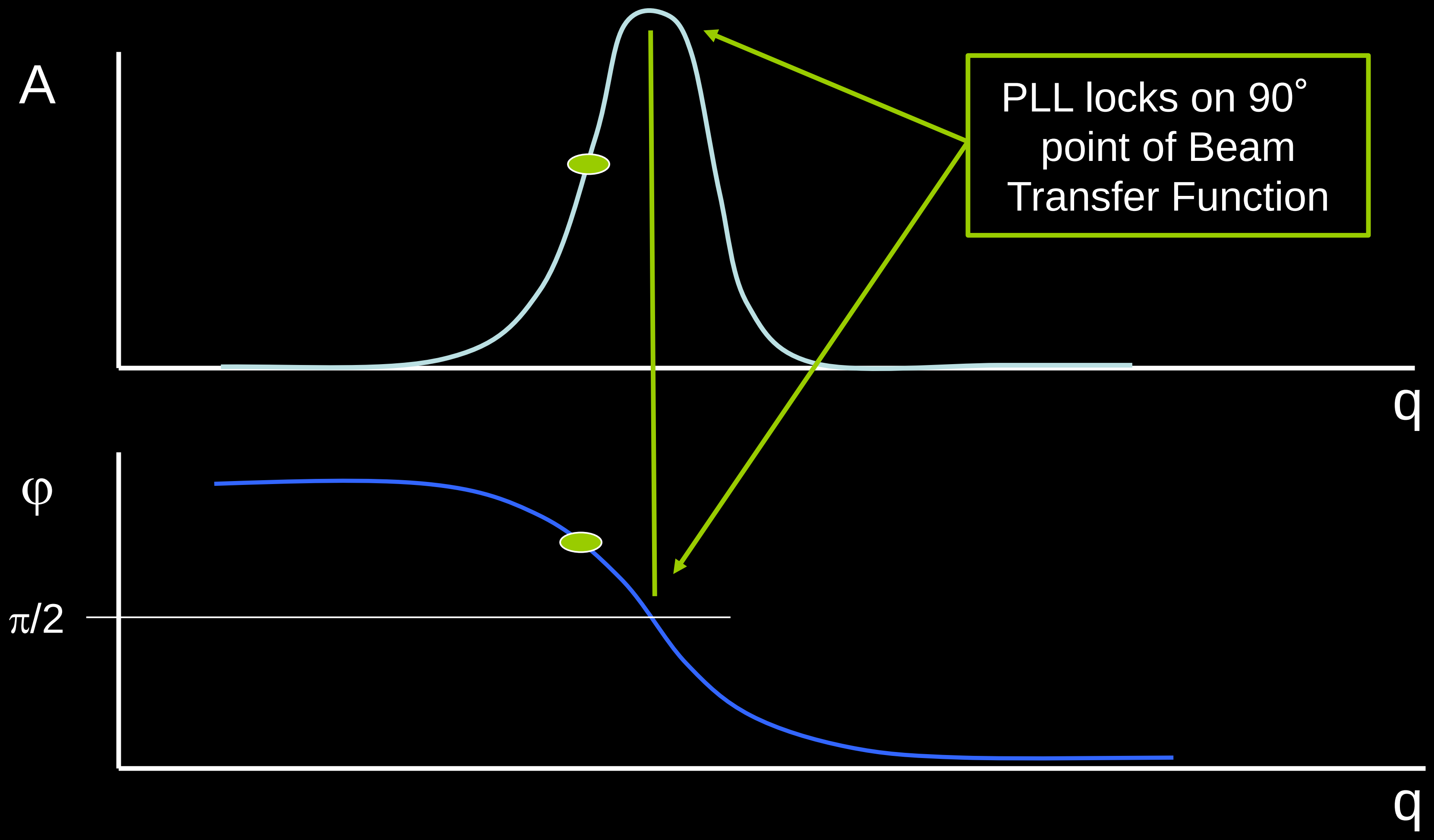


But $q_{obs} = q_{ref}$ for forced harmonic oscillator (our beam) ⇒ **½AB [cos (- φ)]** (DC value)

Feed back this value to change exciter frequency (voltage controlled oscillator)

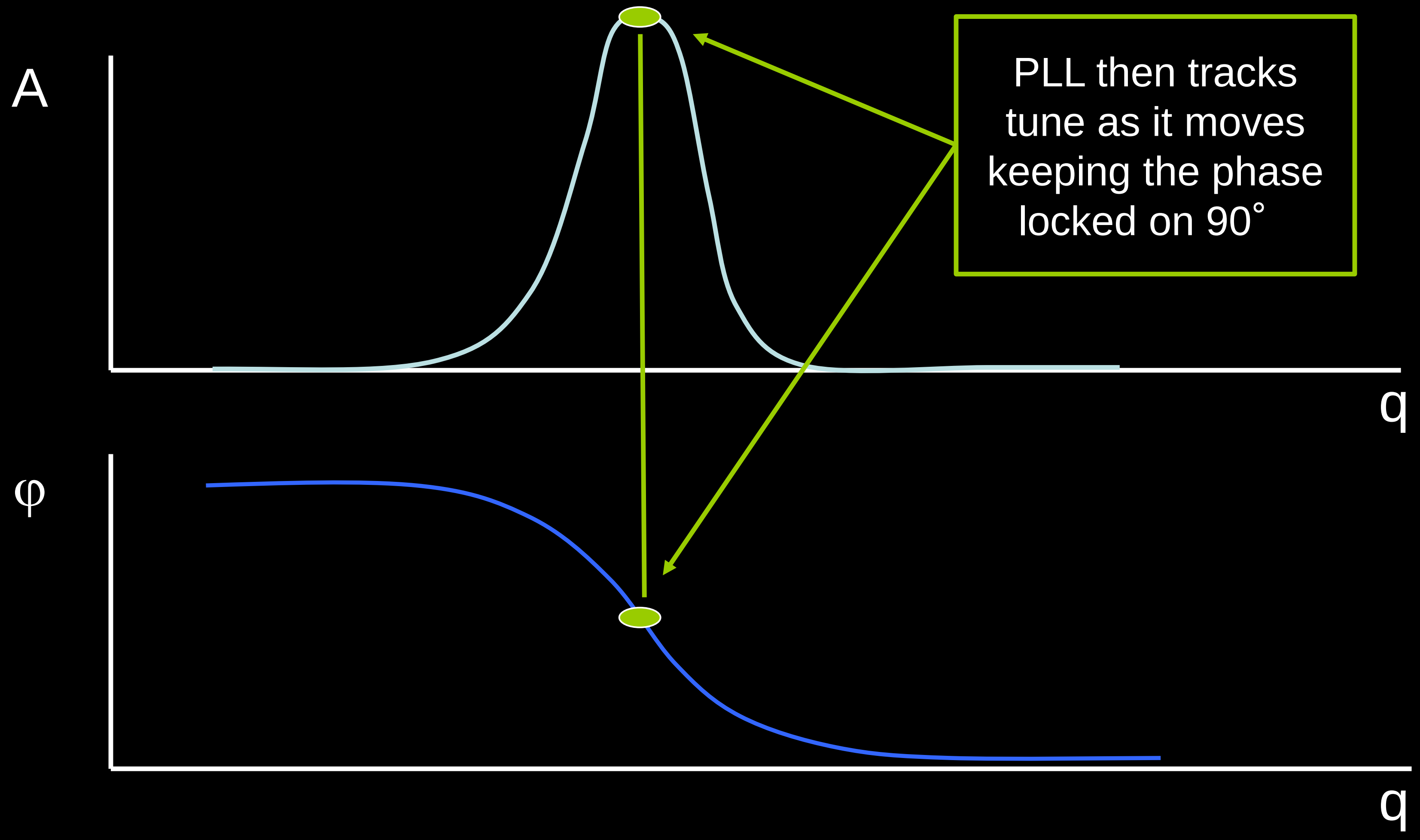
when $q_{ref} = q_{tune}$ then $\varphi = \pi/2$ for harmonic oscillator : ½AB [cos (-π/2)] = 0

Illustration of PLL tune locking



PLL locks on 90˚ point of Beam Transfer Function

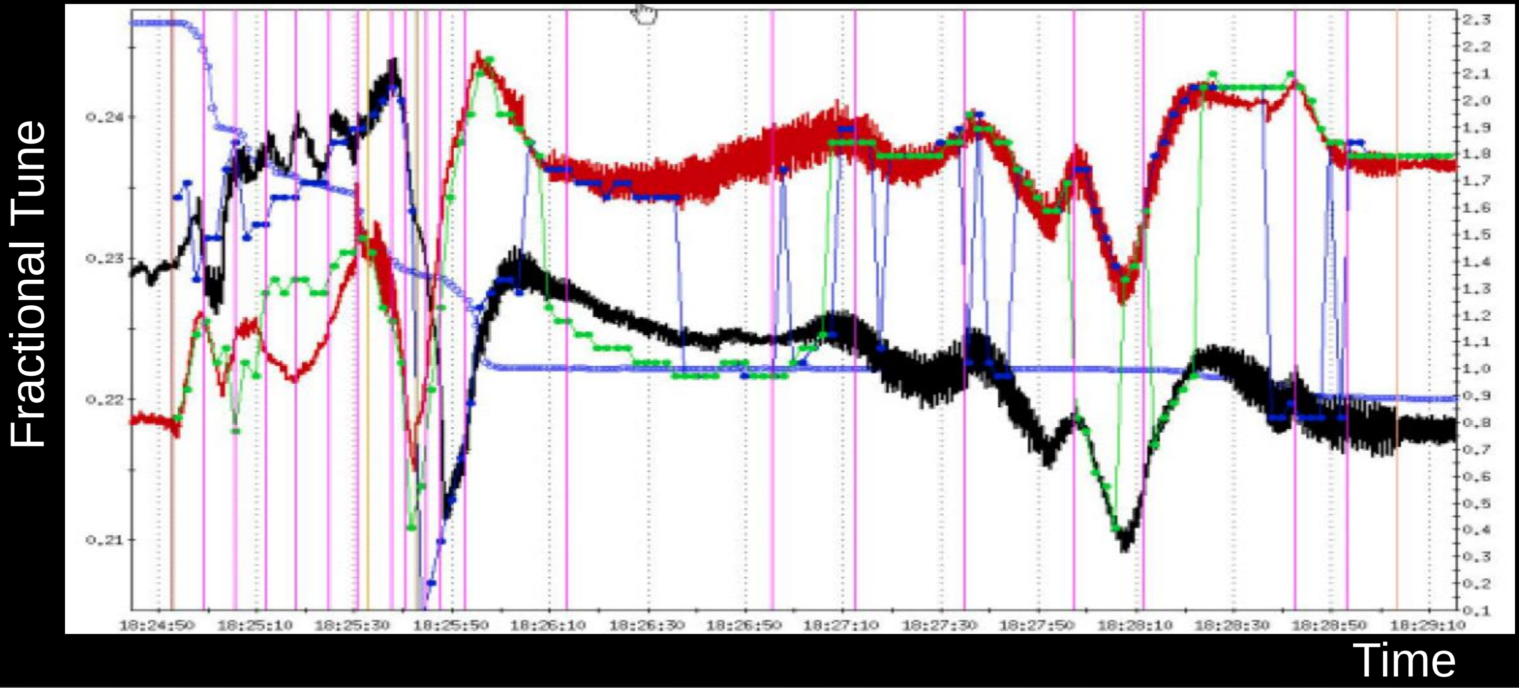
Illustration of PLL tune tracking

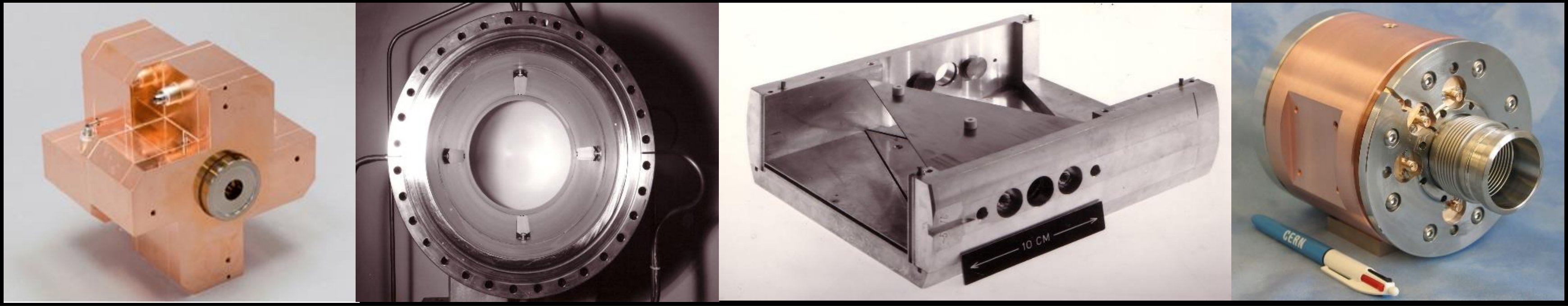


PLL then tracks tune as it moves keeping the phase locked on 90°

PLL Tune Tracking Example

- Early example of PLL Tune tracking at RHIC during a ramp
  - Comparison to kicked tune measurement (green & blue solid dots)
- Advantage
  - Much lower excitation frequency possible due to known detection frequency
  - Allows continual tracking without significant emittance blow-up
- Disadvantage
  - Can lock to synchrotron sidebands or spurious peaks
    - Same is also true of peak fitter for standard FFT measurement

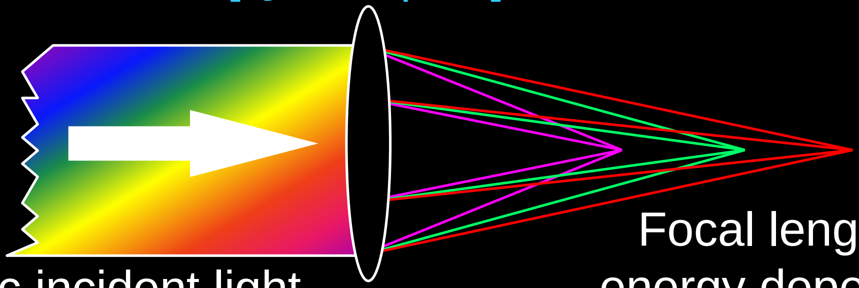




Chromaticity Measurement

Introduction

- Machine Chromaticity



Optics Analogy:

Lens

[Quadrupole]

Achromatic incident light

[Spread in particle energy]

Focal length is energy dependent

Spread in the Machine Tune due to Particle Energy Spread

Controlled by Sextupole magnets

First Order

$$\Delta Q = Q'\frac{\Delta p}{p} = \left(\frac{1}{\gamma^2} - \alpha\right)^{-1} Q'\frac{\Delta f}{f}$$

$$\xi = \frac{Q'}{Q}$$

Generalised

$$Q^{(n)} = \frac{\partial^{(n)} Q}{\partial \delta^{(n)}} \qquad \delta := \frac{\Delta p}{p}$$

Measurement Techniques

Tune change for different beam momenta	⇔	Standard method used on all machines. Can be combined with PLL tune tracking to give on-line measurement
Width of tune peak or damping time	⇔	Model dependent, non-linear effects, not compatible with active transverse damping
Amplitude ratio of synchrotron sidebands	⇔	Difficult to exploit in hadron machines with low synchrotron tune, Influence of collective effects
Width ratio of Schottky sidebands	⇔	Used on many machines & ideally suited to unbunched or ion beams. Measurement is typically very slow
Bunch spectrum variations during betatron oscillations	⇔	Difficult to disentangle effects from all other sources – e.g. bunch filling patterns, pick-up & electronics response
Head-tail phase advance (same as above, but in time domain)	⇔	Good results on several machines but requires kick stimulus ⇒ emittance growth!

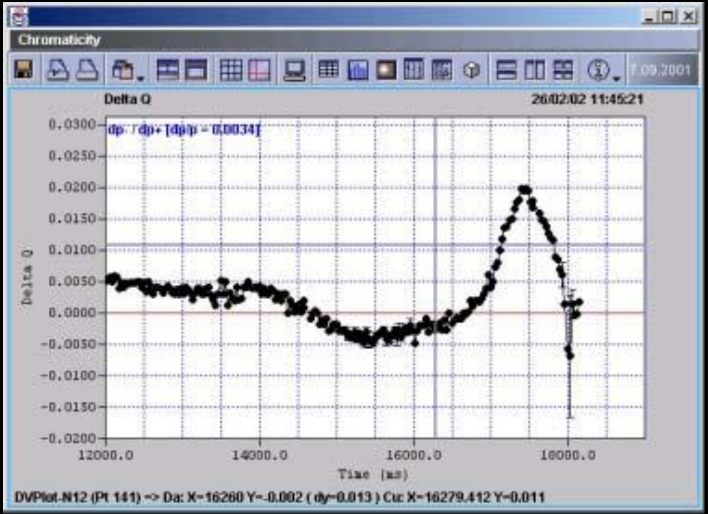
RF Momentum Modulation Techniques

- DC Change in Momentum
  - – Typically used on fast cycling machines
  - – Measure the tune throughout cycle
    - Requires continuous tune measurement capability
  - – Compare results for 2 or 3 cycles with slight energy offset to calculate chromaticity

$$Q' = \frac{\Delta Q}{\Delta p / p}$$

measured tune change

RF induced momentum change (known)



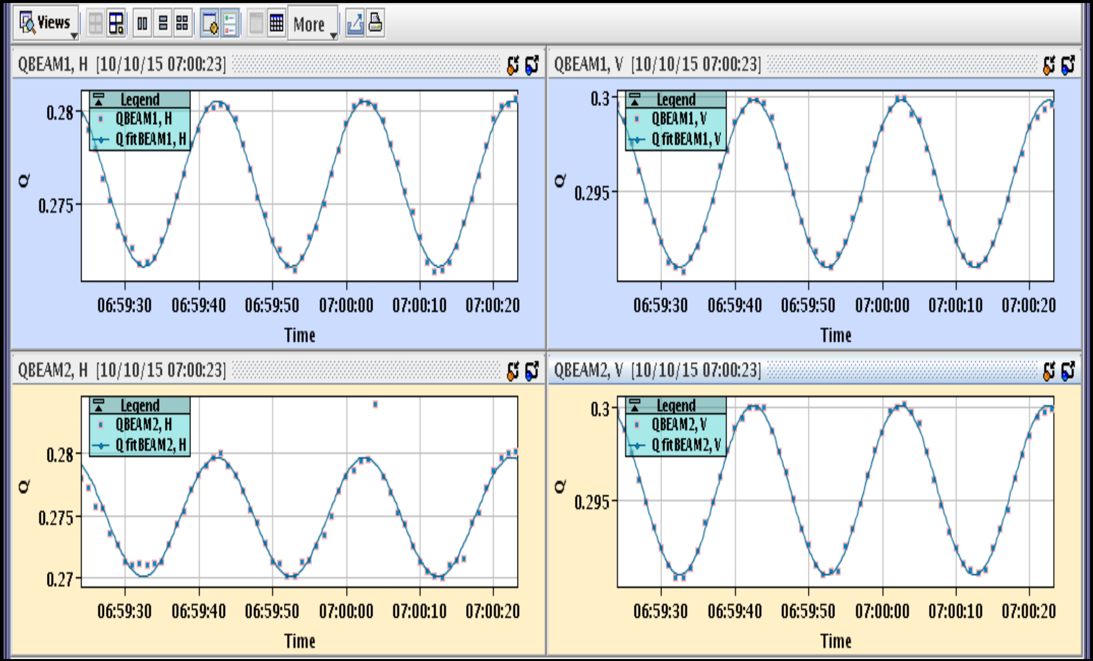
DVPlot-N12 (Pt 141) -> Da: X=16260 Y=-0.002 (dy=0.013) Cu: X=16279.412 Y=0.011

Old example from CERN-SPS

- – Q difference during the ramp for 2 radial steering (Δp/p) settings
- – Complete picture of chromaticity throughout the cycle

RF Momentum Modulation Techniques

- Slow RF Variation
  - Apply time varying RF modulation
  - Continuously measure the tune
    - Amplitude of tune variation proportional to chromaticity

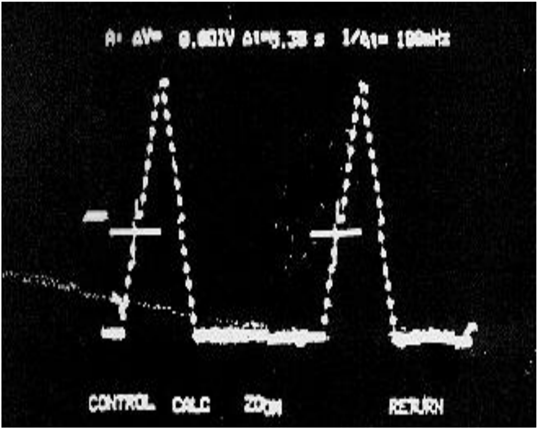


Example from the LHC

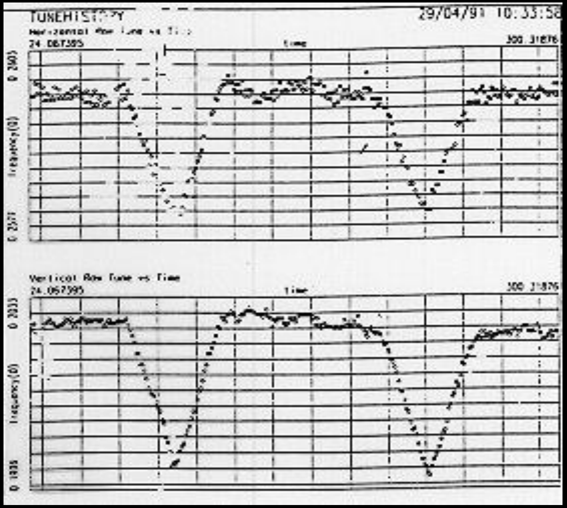
- Sinusoidal RF modulation at 0.05Hz
- Tune continuously tracked in all planes of both beams
- Chromaticity calculated once acquisition complete

RF Momentum Modulation Techniques

- Slow RF Variation
  - Apply time varying RF modulation
  - Continuously measure the tune
    - Amplitude of tune variation proportional to chromaticity



Applied Frequency Shift



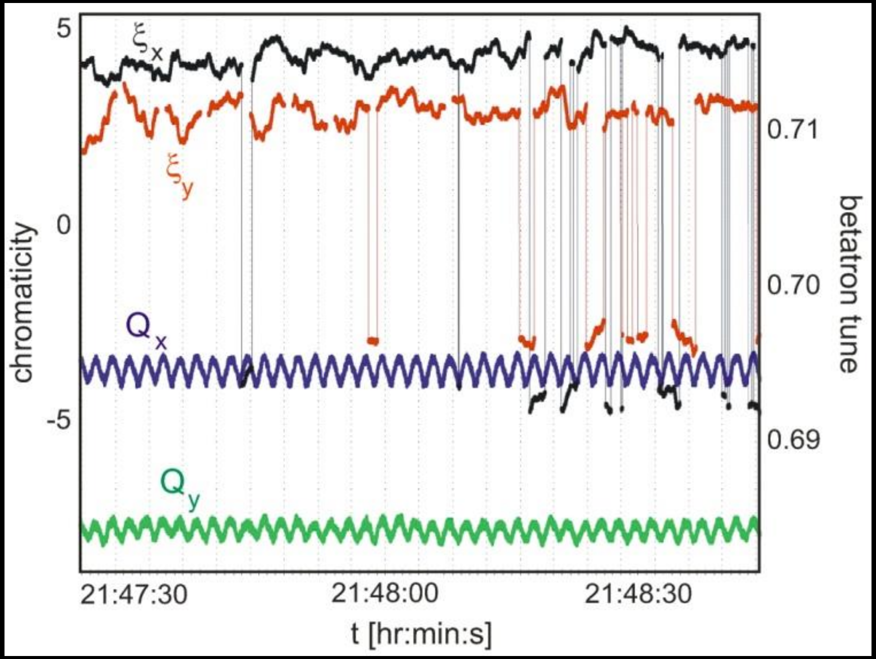
Q_h & Q_v Variation

Example from CERN-LEP

- Triangular RF modulation
- Allows sign to be easily determined

RF Momentum Modulation Techniques

- Slow RF Variation
  - Apply time varying RF modulation
  - Continuously measure the tune
    - Amplitude of tune variation proportional to chromaticity
  - Need to make sure of delays in RF modulation & acquisition chains to obtain correct sign when using symmetrical modulation function

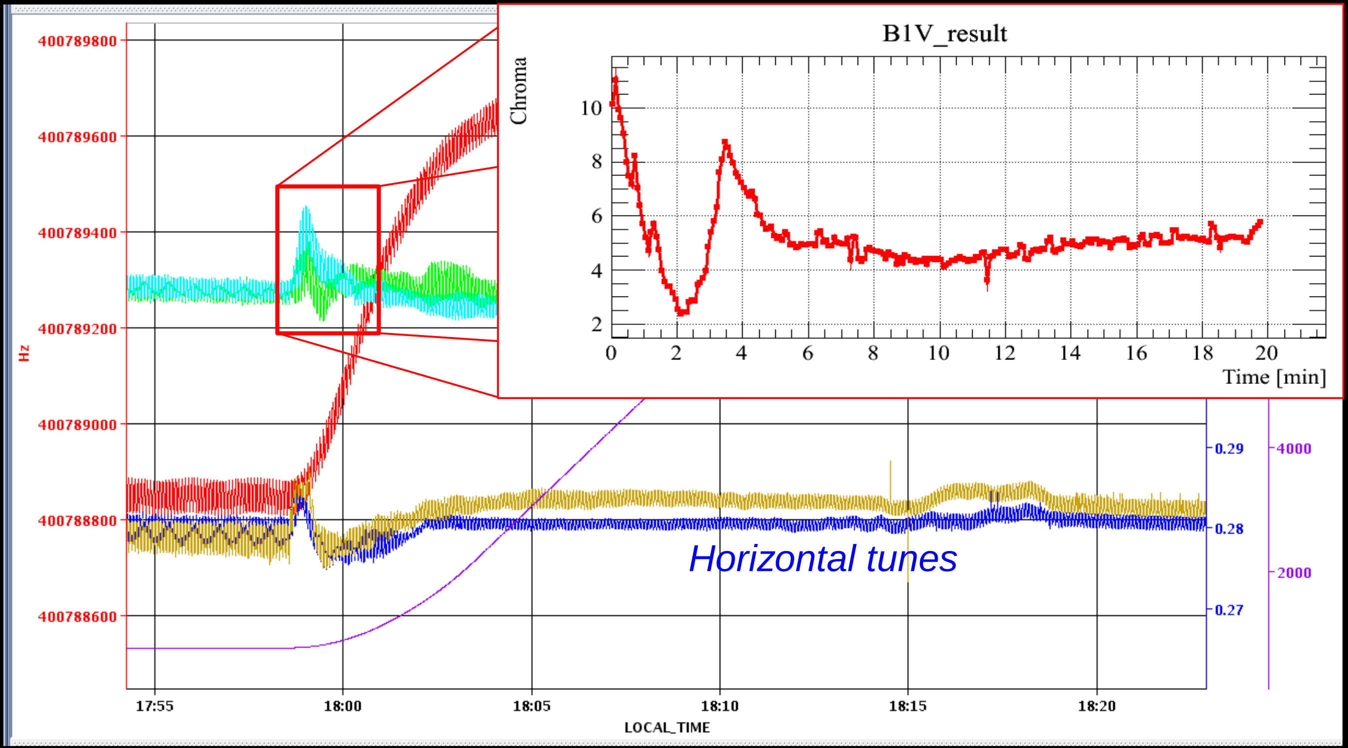


Early example from RHIC

- Phase slip between excitation & acquisition sometimes leads to incorrect sign of chromaticity

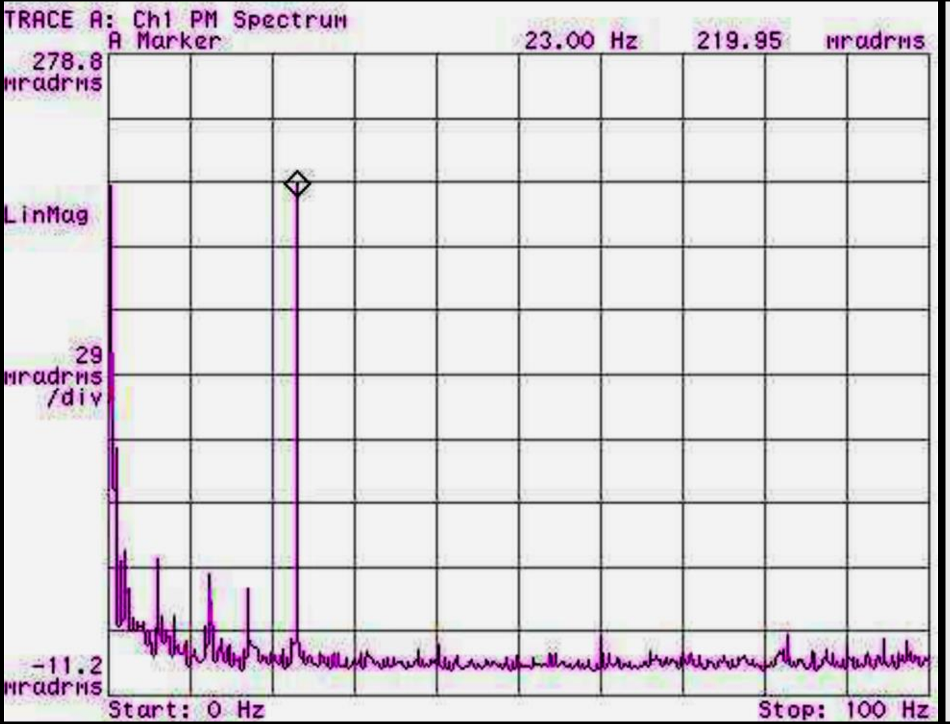
RF Momentum Modulation Techniques

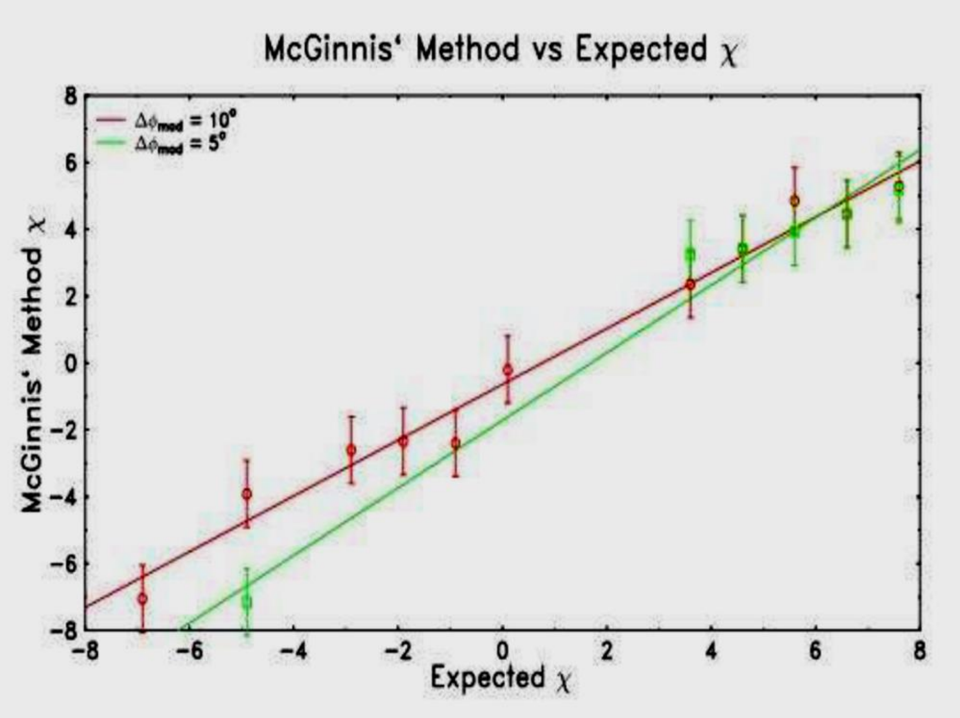
- Dynamic Measurement Examples
  - LHC Ramp
    - RF continuously modulated
    - Tune measured using the sensitive Base Band Tune (BBQ) system
    - Tune calculated from peak fitting of resulting frequency spectrum
    - Chromaticity calculated from amplitude of tune modulation



RF Momentum Modulation Techniques

- RF Phase Modulation
  - Proposed by D. McGinnis (FNAL)
  - Phase modulate the RF instead of frequency modulation
    - Possible to achieve faster modulation
  - Measure demodulated tune signal
    - Tune tracker supplies carrier frequency but does not need to track modulation
  - Tevatron Results for 5° modulation @ 23Hz





Summary of RF Momentum Modulation

- Frequency modulation
  - Standard chromaticity measurement technique for all machines
    - Usually using DC frequency offset or slow frequency modulation
  - On-line calculation requires continuous tune measurement
  - Typical parameters
    - RF momentum modulation of 10^{-4} to 10^{-3} @ < 1Hz
      - Generally acceptable orbit changes of 0.1 to 1mm for 1m dispersion
    - Chromaticity resolution of 1 unit requires tune resolution of 10^{-5} to 10^{-4}
    - Can be achieved by
      - Demodulating at correct frequency
      - Averaging tune measurements or adapting PLL bandwidth
    - Trade-off between RF modulation amplitude and frequency

- Phase modulation
  - Promising initial tests but not applied operationally
  - May be limited by RF power required at higher frequencies

Measurement Techniques

Tune change for different beam momenta	⇔	Standard method used on all machines. Can be combined with PLL tune tracking to give on-line measurement
Width of tune peak or damping time	⇔	Model dependent, non-linear effects, not compatible with active transverse damping
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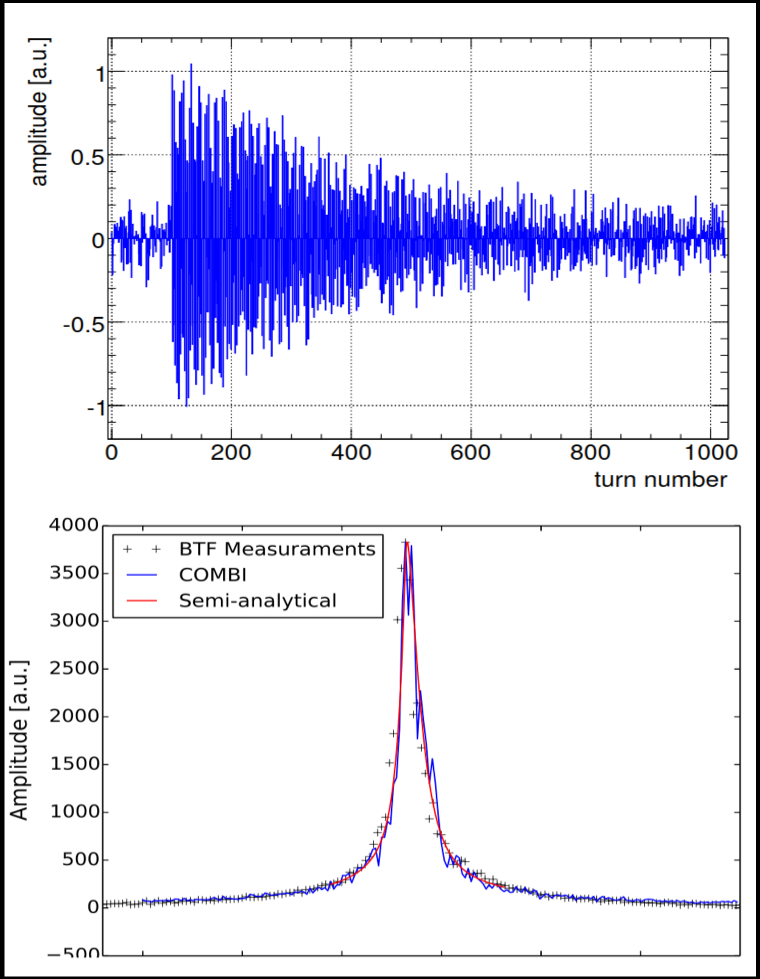
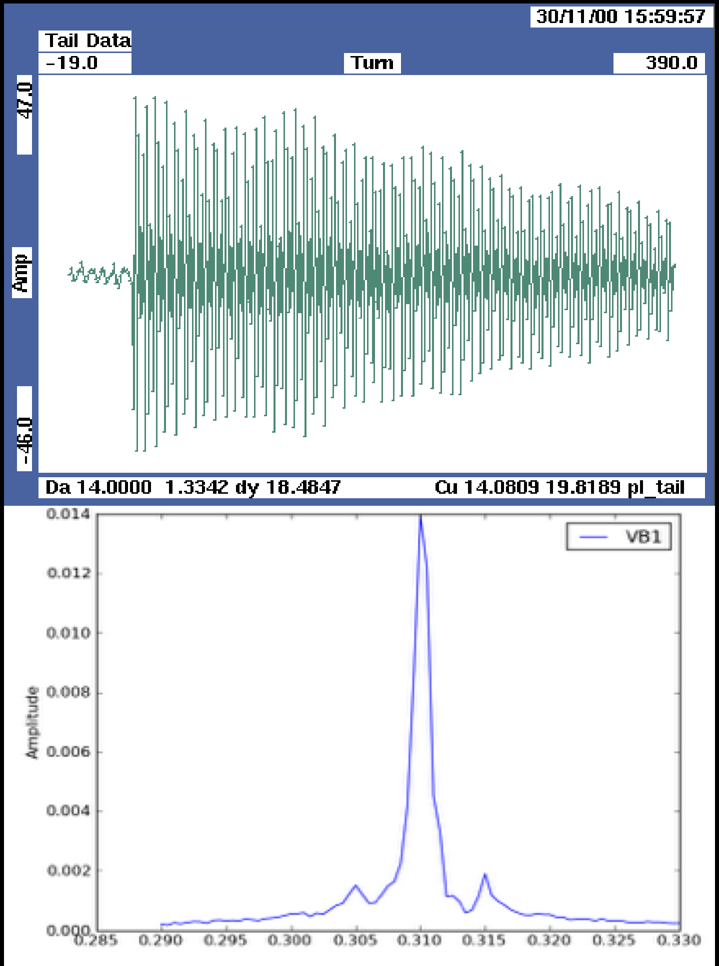
Head-tail phase advance (same as above, but in time domain)	⇔	Good results on several machines but requires kick stimulus ⇒ emittance growth!

Amplitude of Synchrotron Sidebands

- Chromaticity gives rise to synchrotron sidebands around tune
  - Tune variation with longitudinal motion at the synchrotron frequency



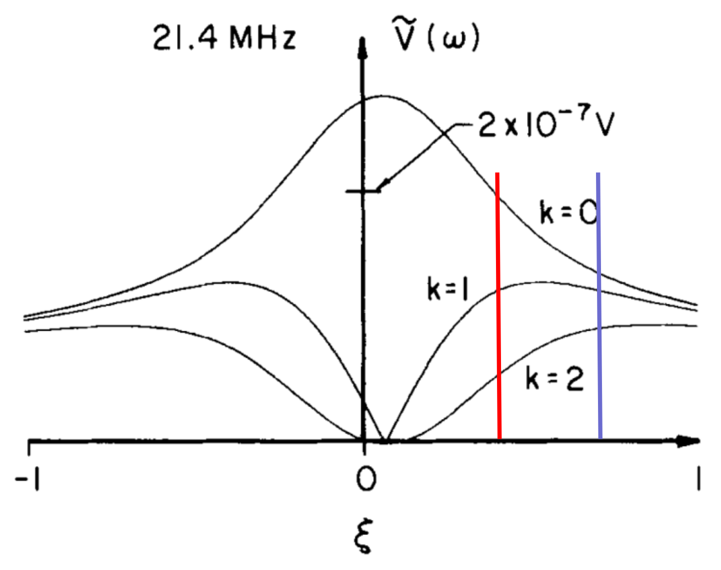
Zero Chromaticity



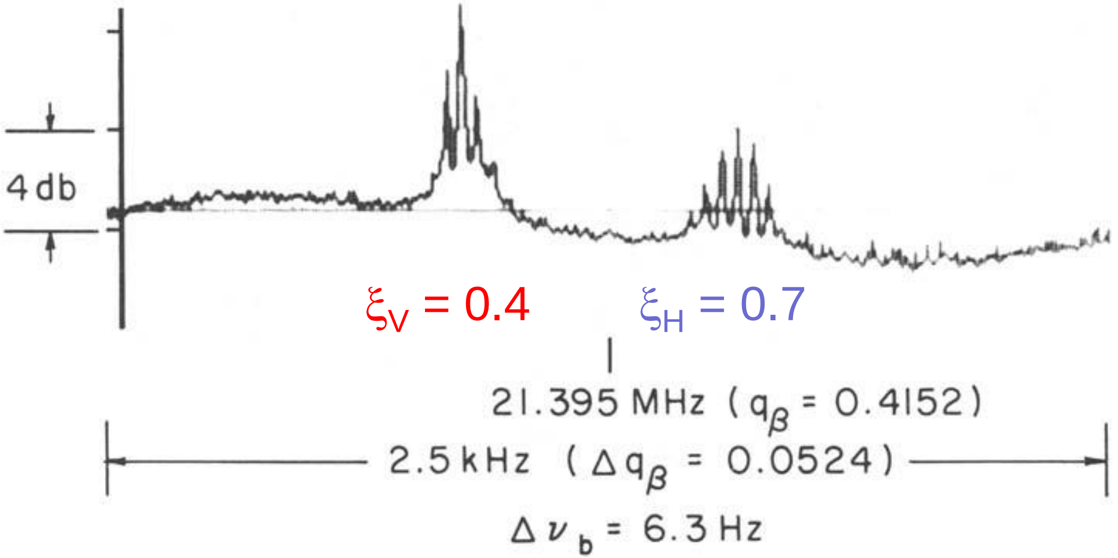
Non-zero Chromaticity

Amplitude of Synchrotron Sidebands

- Background
  - Presented by R.H. Siemann
    - Physics of Particle Accelerators (1989)
  - Demonstrated in the Tevatron
    - G. Jackson (1989)
- Chromaticity calculated from ratio of synchrotron sidebands
  - Sidebands follow Bessel functions with chromaticity term
  - Relies on absence of collective effects



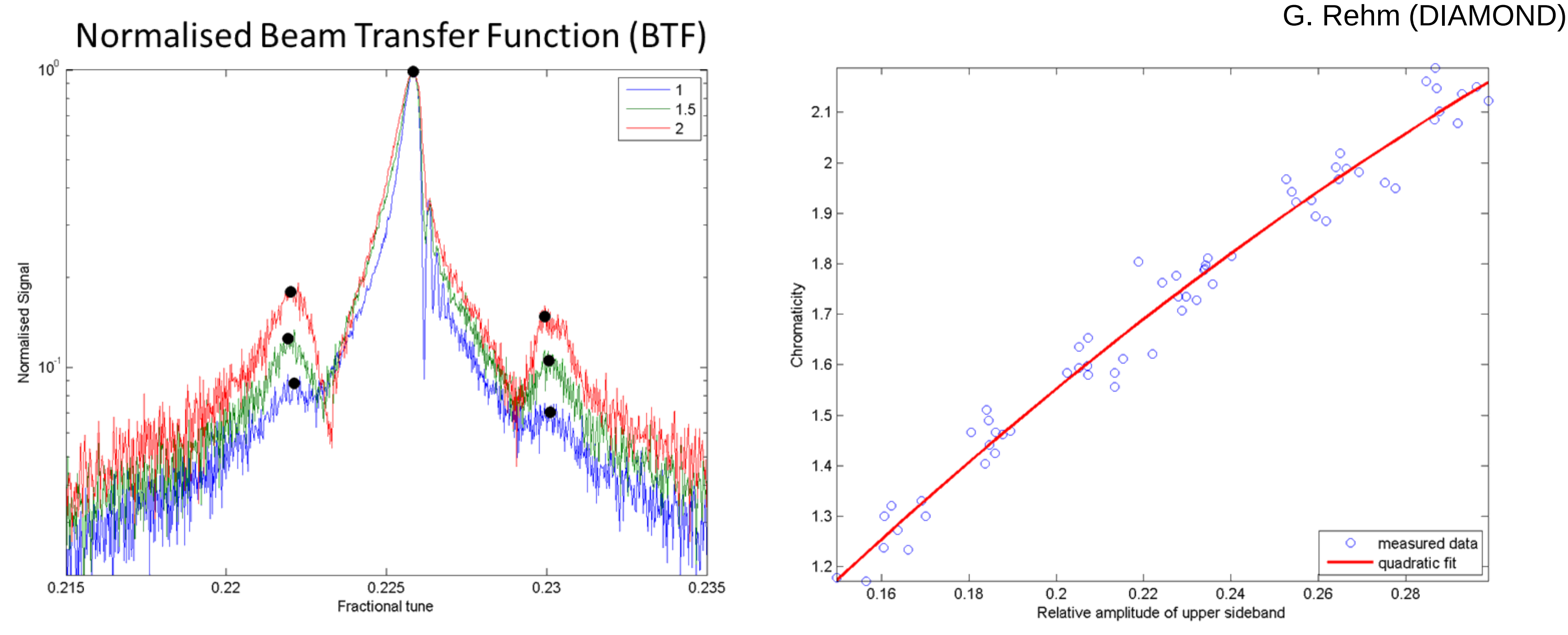
Tevatron Results



$\xi_V = 0.4$ $\xi_H = 0.7$

Amplitude of Synchrotron Sidebands

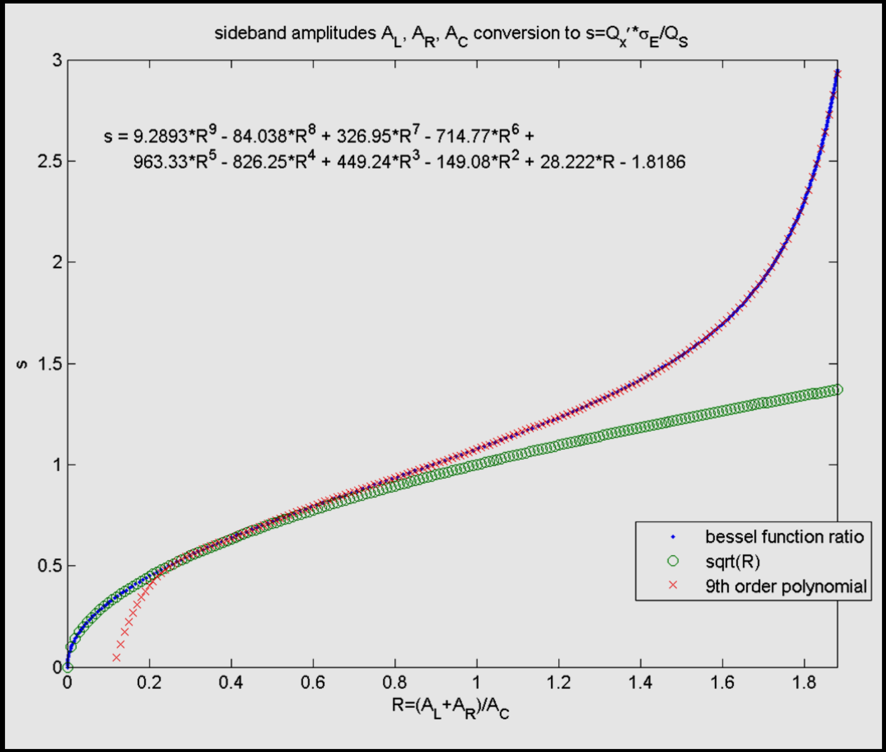
- Recent measurement - DIAMOND Light Source (UK)
  - RF modulation changes orbit - not compatible with user operation
  - Looking for technique to measure chromaticity on-line
    - Measure Beam Transfer Function (BTF) on single bunch
      - Using transverse bunch by bunch feedback system
      - Emittance blow-up of single bunch irrelevant

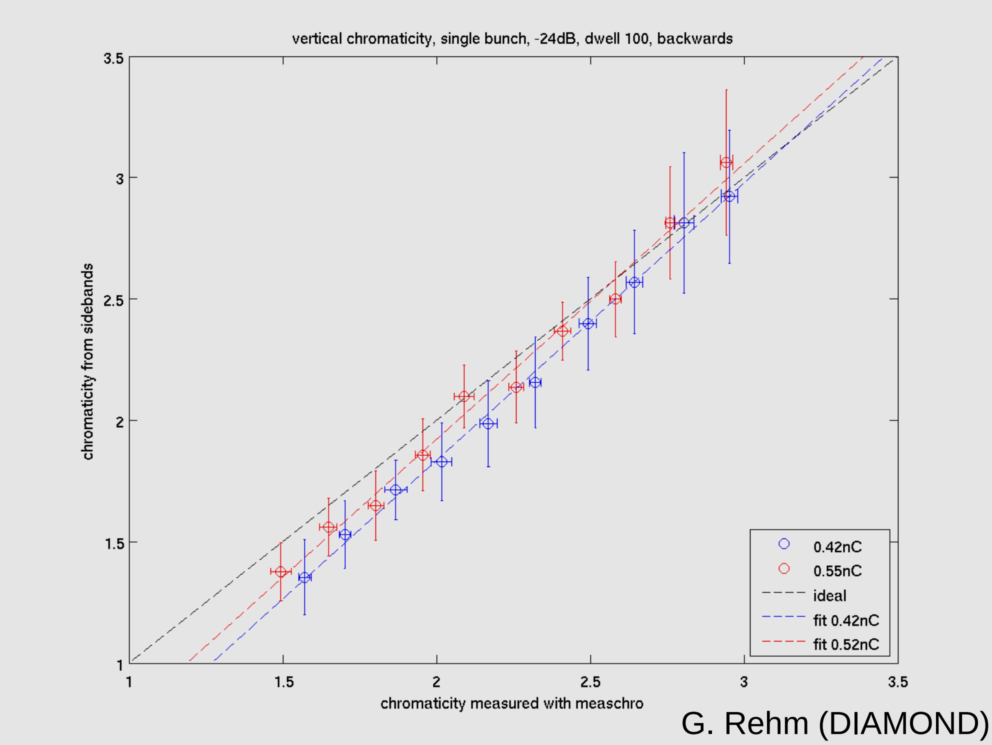


G. Rehm (DIAMOND)

Amplitude of Synchrotron Sidebands

- From Empirical Fit to Theoretical Approach @ DIAMOND
  - Use expression for sideband amplitude that is ratio of Bessel functions
  - As relationship cannot be inverted analytically, use a piecewise fit with a square root and a 9th order polynomial
  - The only other knowledge required is energy spread
    - Measured from beam size in two locations

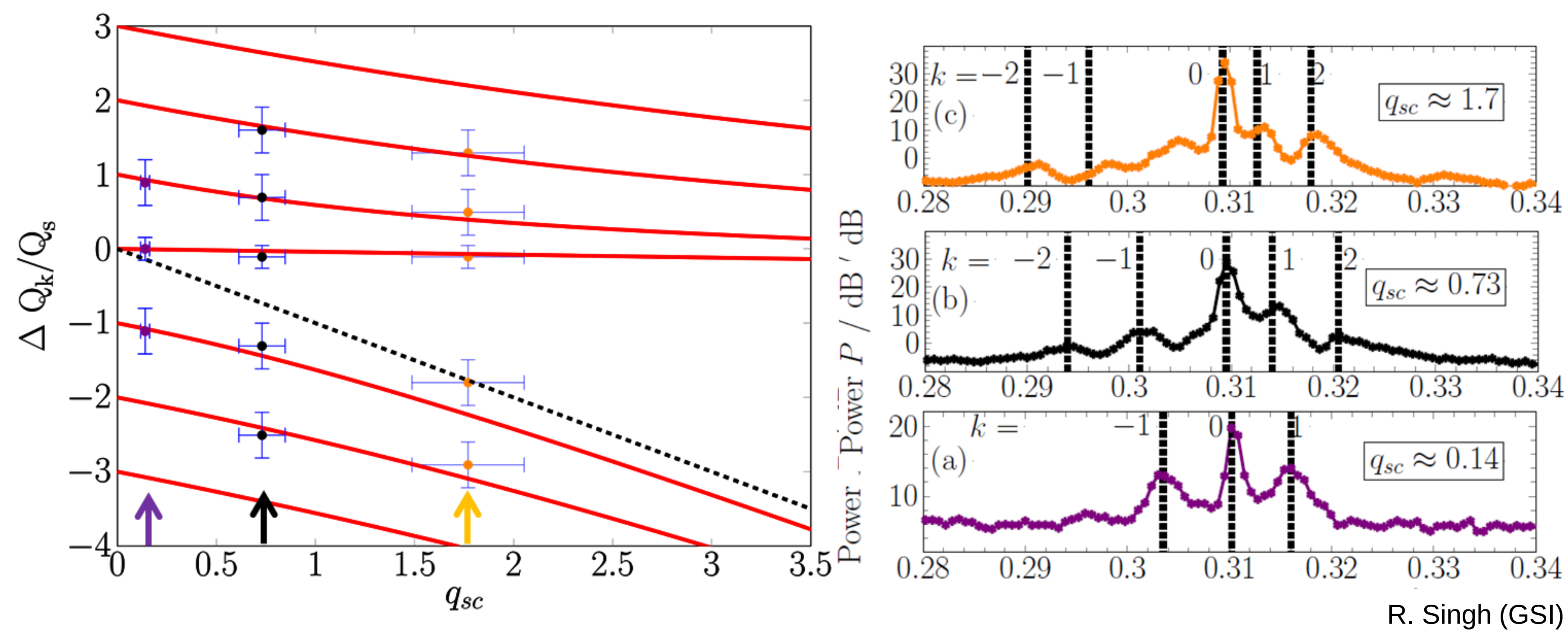




G. Rehm (DIAMOND)

Amplitude of Synchrotron Sidebands

- Dealing with High Intensity Effects @ GSI (DE)
  - Modification of tune spectra by space charge & impedance
  - Measured using Base Band Tune system
  - Relative heights & mode structure given by chromaticity
    - Can be calculated with simplified analytical models



R. Singh (GSI)

Measurement Techniques

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Chromaticity from Schottky Spectra

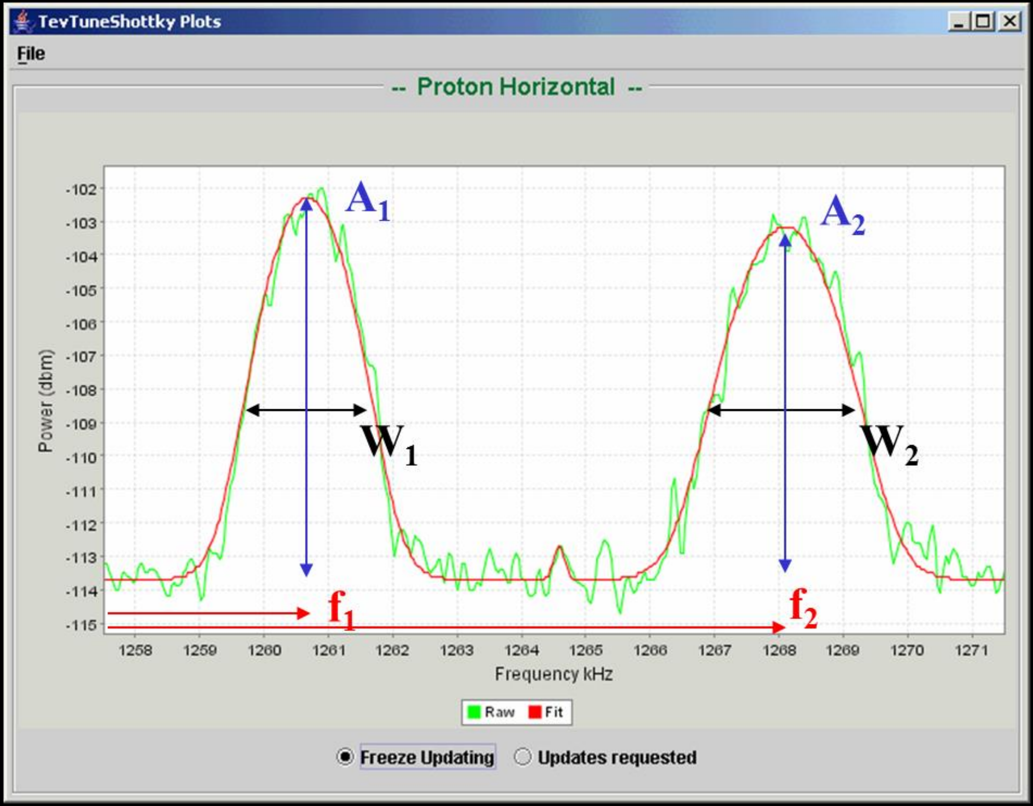
- Schottky a powerful tool for non-invasive measurements
  - Ideally suited to coasting (unbunched) beams & heavy ions (Z^2 relationship)
  - Rely on detecting statistical fluctuations in position of finite number of particles
    - Acquisition times are therefore typically long
  - Bunched beam Schottky challenge
    - Measurement of small signals in presence of revolution lines up to 100000 times higher

$$q = \frac{1}{2} + \frac{(f_2 - f_1)}{2 f_{\text{rev}}}$$

$$\frac{dp}{p} \propto W_1 + W_2$$

$$C \propto \frac{W_1 - W_2}{W_1 + W_2}$$

$$\varepsilon \propto A_1 W_1 + A_2 W_2$$



Chromaticity from Schottky Spectra

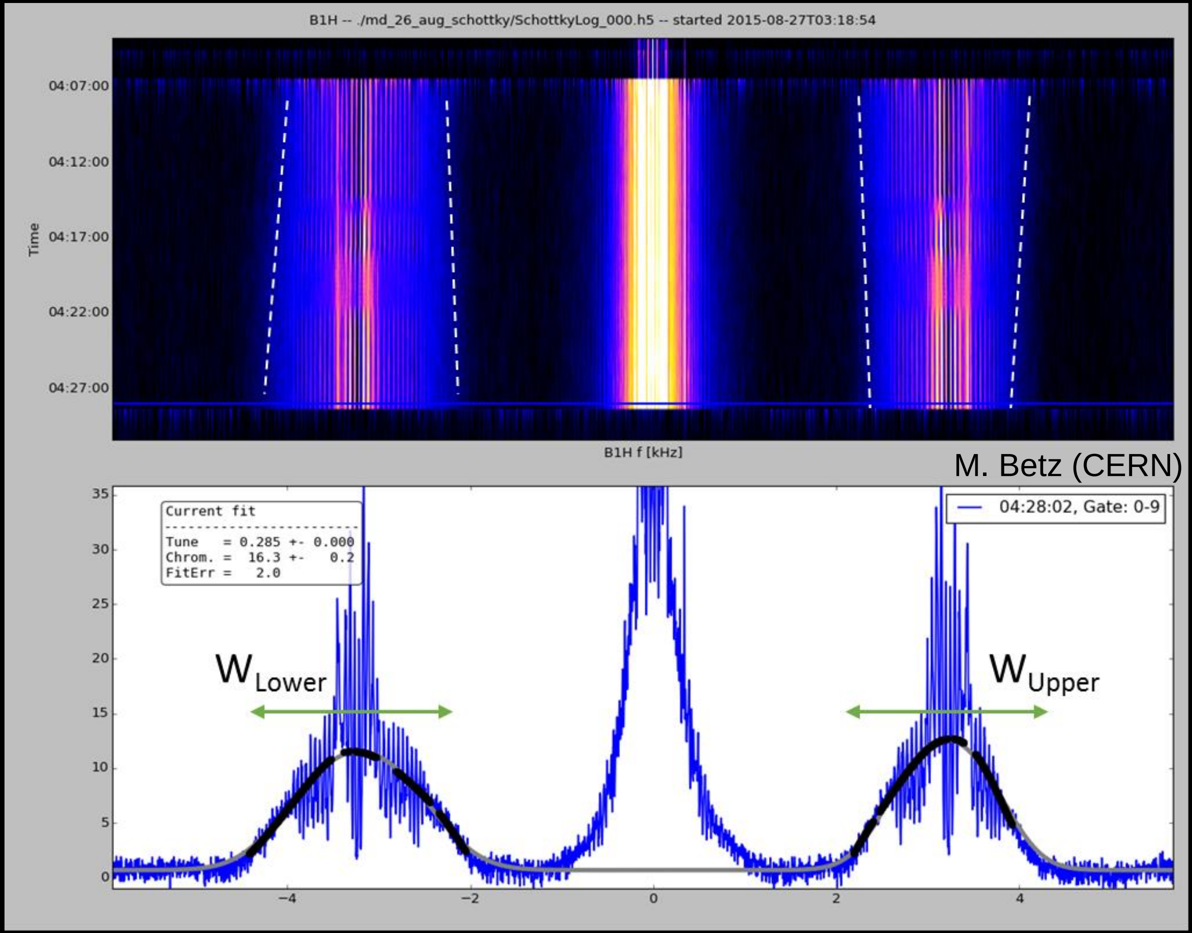
- Chromaticity from Schottky spectra
  - From difference in widths of lower & upper sidebands on given revolution (f_0) harmonic (n)

$$\Delta f_{\pm} = f_0 \frac{\Delta p}{p}[(n \pm q)\eta \pm Q\xi] \approx f_0 \frac{\Delta p}{p}[n \times \eta \pm Q']$$

  - Ideally detect at n where Q’ of same order as n x η
    - Width variations are then dominated by chromaticity
- Constraints
  - Need to avoid band overlap where sidebands merge
    - Keep n low
  - Need to be outside coherent bunch spectrum for bunched beams
    - Keep n high to minimise revolution components
  - Trade-off may not be optimal for chromaticity measurements
    - E.g. LHC where n ~ 430000 (4.8 GHz & f_0 = 11 kHz) gives n x η ~ 140
    - One unit of chromaticity represents < 2% variation in width difference

Chromaticity from Schottky Spectra

- Bunched beam Schottky example from the LHC
  - Variation in sideband widths as chromaticity is changed from 2 to 15



```
Current fit
-----------------------
Tune   = 0.285 +- 0.000
Chrom. =  16.3 +-   0.2
FitErr =   2.0
```

M. Betz (CERN)

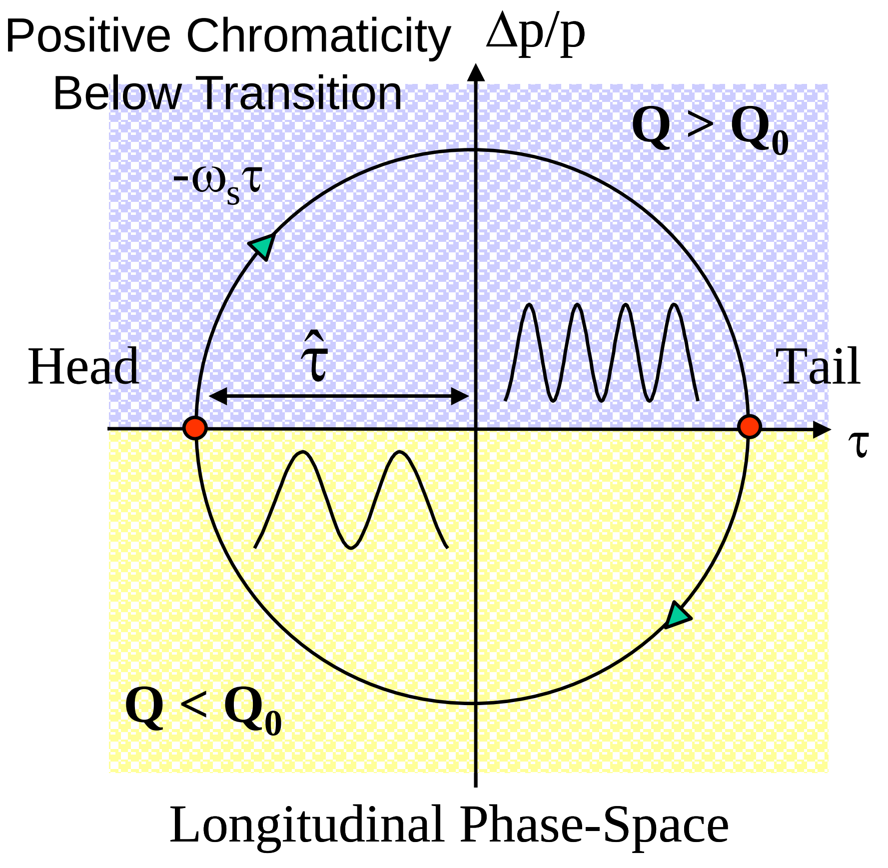
Frequency offset from revolution harmonic at 4.81 GHz [kHz]

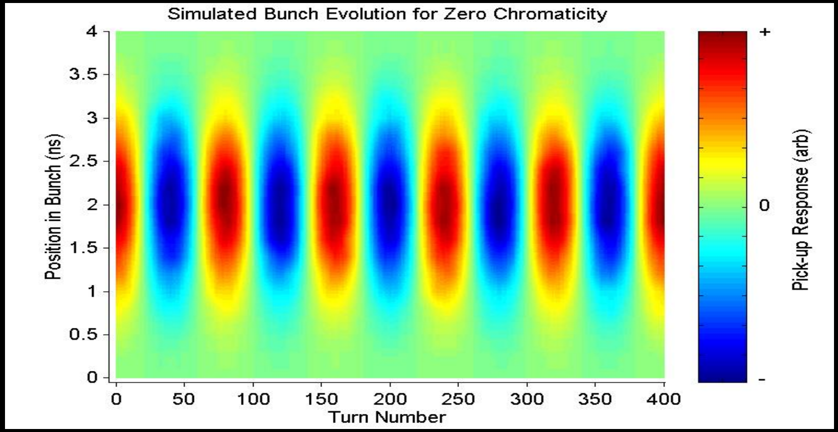
Measurement Techniques

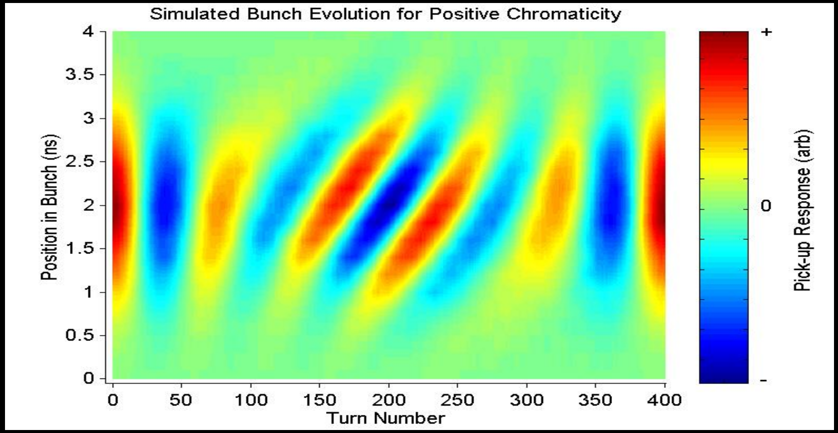
Tune change for different beam momenta	⇔	Standard method used on all machines. Can be combined with PLL tune tracking to give on-line measurement
Width of tune peak or damping time	⇔	Model dependent, non-linear effects, not compatible with active transverse damping
Amplitude ratio of synchrotron sidebands	⇔	Difficult to exploit in hadron machines with low synchrotron tune, Influence of collective effects
Width ratio of Schottky sidebands	⇔	Used on many machines & ideally suited to unbunched or ion beams. Measurement is typically very slow
Bunch spectrum variations during betatron oscillations	⇔	Difficult to disentangle effects from all other sources – e.g. bunch filling patterns, pick-up & electronics response
Head-tail phase advance (same as above, but in time domain)	⇔	Good results on several machines but requires kick stimulus ⇒ emittance growth!

Head-Tail Chromaticity Measurement

- Developed at CERN in late 1990’s for fast chromaticity measurement & possible alternative to RF modulation in LHC
  - Kick all particles in bunch to same initial phase
  - Measure subsequent phase difference of head & tail over synchrotron period

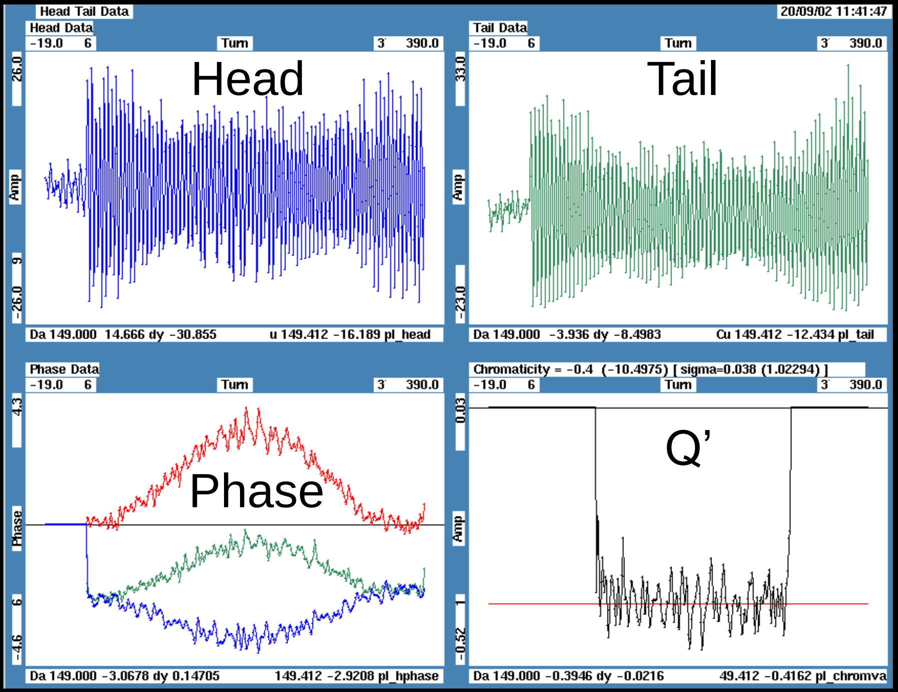



Simulated Bunch Evolution for Zero Chromaticity


Simulated Bunch Evolution for Positive Chromaticity

Head-Tail Chromaticity Measurement

- Demonstrated in CERN-SPS & Tevatron but had limitations
  - Needed strong kick to overcome static orbit offsets
    - Various electronic means attempted to overcome this with little success
  - Affected by space charge at low energy
  - Can suffer from short decoherence times (< synchrotron tune)
- Should also work with continuous excitation (S. Fartoukh)
  - Requires sensitive tune measurement gated on head & tail of bunch
  - Attempts to adapt base band tune system for this not successful to date

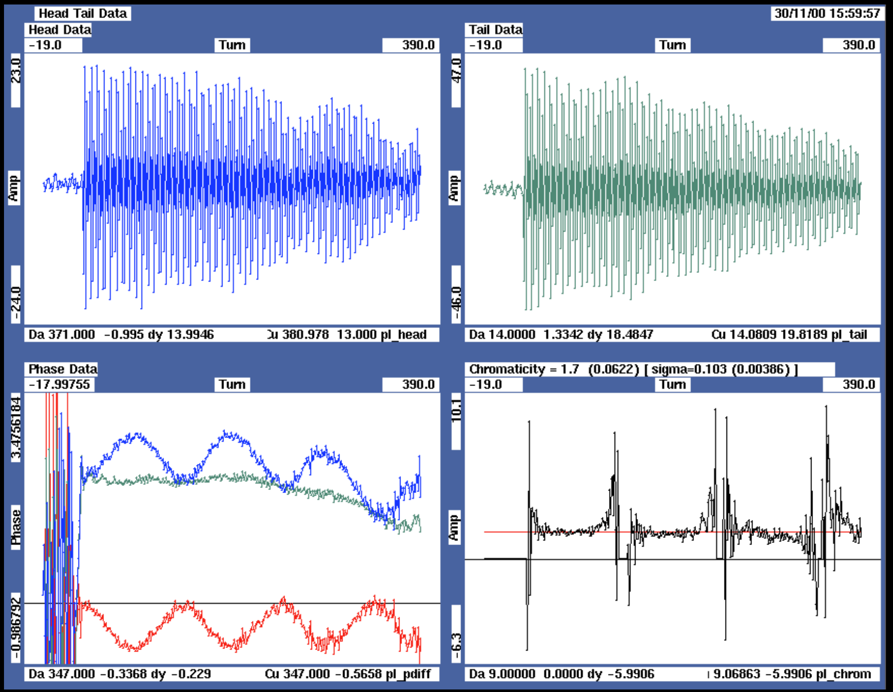

Chromaticity = -0.4 (-10.4975) [sigma=0.038 (1.02294)]

Da 149.000 14.666 dy -30.855 u 149.412 -16.189 pl_head

Da 149.000 -3.936 dy -8.4983 Cu 149.412 -12.434 pl_tail

Da 149.000 -3.0678 dy 0.14705 149.412 -2.9208 pl_hphase

Da 149.000 -0.3946 dy -0.0216 49.412 -0.4162 pl_chromva


Chromaticity = 1.7 (0.0622) [sigma=0.103 (0.00386)]

Da 371.000 -0.995 dy 13.9946 Cu 380.978 13.000 pl_head

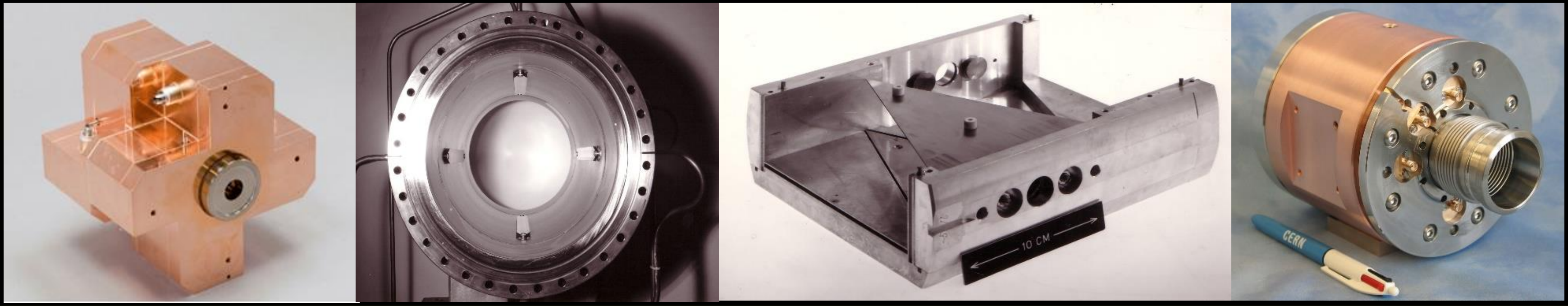
Da 14.0000 1.3342 dy 18.4847 Cu 14.0809 19.8189 pl_tail

Da 347.000 -0.3368 dy -0.229 Cu 347.000 -0.5658 pl_pdiff

Da 9.00000 0.0000 dy -5.9906 9.06863 -5.9906 pl_chrom

Chromaticity Summary

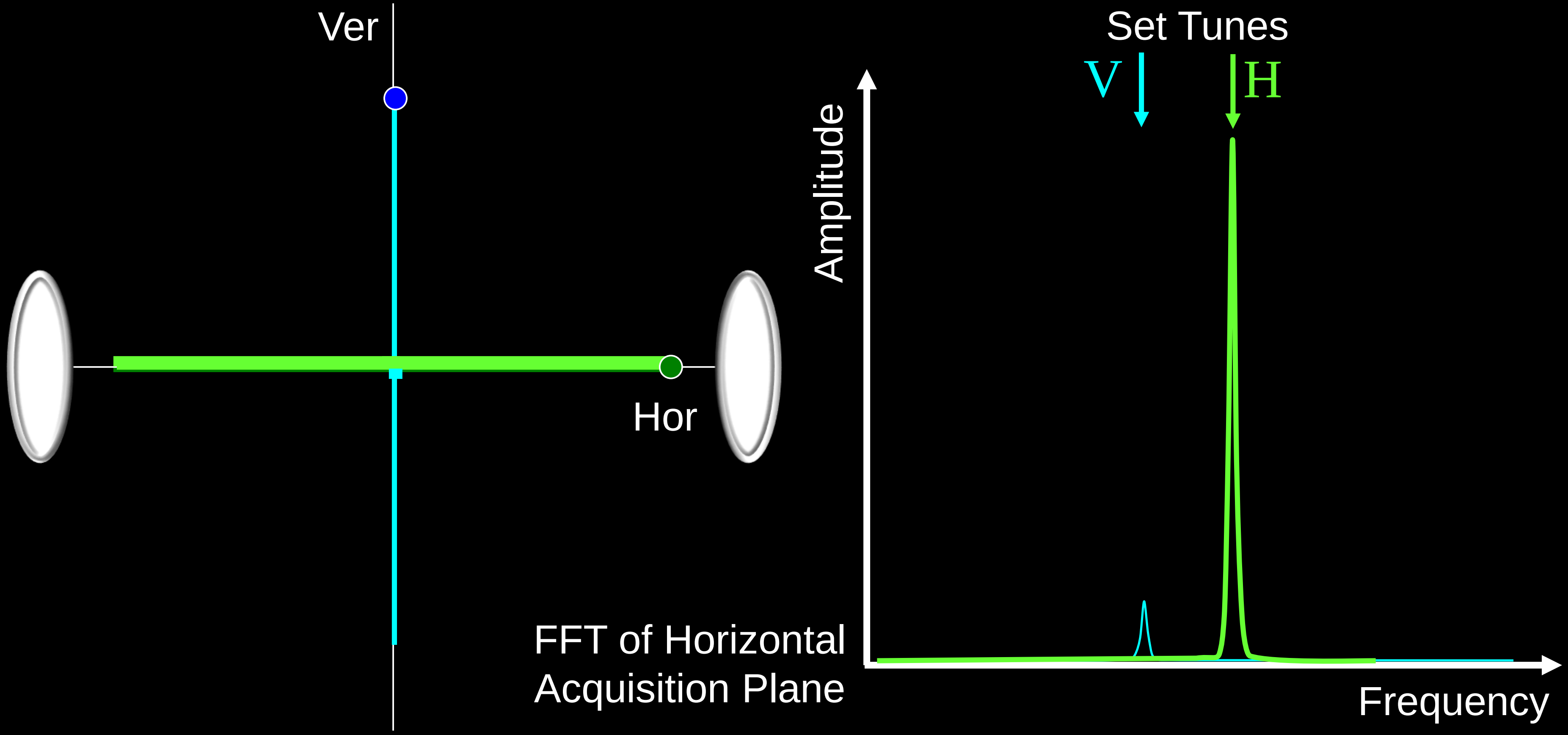
- Many techniques available to measure chromaticity
- RF frequency modulation most widely used
  - Sufficient for majority of machines
  - Possibility for on-line measurement
    - Requires sensitive, continuous tune measurement system
  - Main limitations
    - Induced orbit change prohibits on-line measurement at synchrotron light sources & high intensity colliders
    - Relatively slow measurement rate
- Schottky diagnostics
  - Ideal for unbunched beams & heavy ion machines
  - Challenging for bunched beams
- Several techniques could do with another look
  - Fast RF phase modulation with tune spectrum demodulation
  - Low excitation strength Head-Tail measurements



Coupling Measurement

Coupling

- Start with decoupled machine
  - Only horizontal tune shows up in horizontal FFT (& vertical in vertical FFT)
- Gradually increase coupling (skew quadrupole field)
  - Vertical mode shows up in horizontal FFT & frequencies shift



Coupling

$$\overbrace{Q_{I,II}}^{\text{Measured Tunes}} = \frac{1}{2}\left(\overbrace{Q_x + Q_y}^{\text{Set Tunes}} \pm \sqrt{\left(Q_x - Q_y\right)^2 + |C^-|^2}\right)$$

- Measured tunes - the physical observables seen in FFT
  - Often called the ‘normal modes’ or ‘eigenvalues’

- Set tunes
  - What the tunes would be in absence of coupling
  - Tune split $\Delta = (Q_x - Q_y)$
    - Difference between the set horizontal & vertical tunes

- Magnitude of the coupling coefficient $|C^-|$
  - The closest Q_I & Q_{II} can approach each other - ‘closest tune approach’
  - Any closer is a ‘forbidden zone’ in a system of coupled oscillators

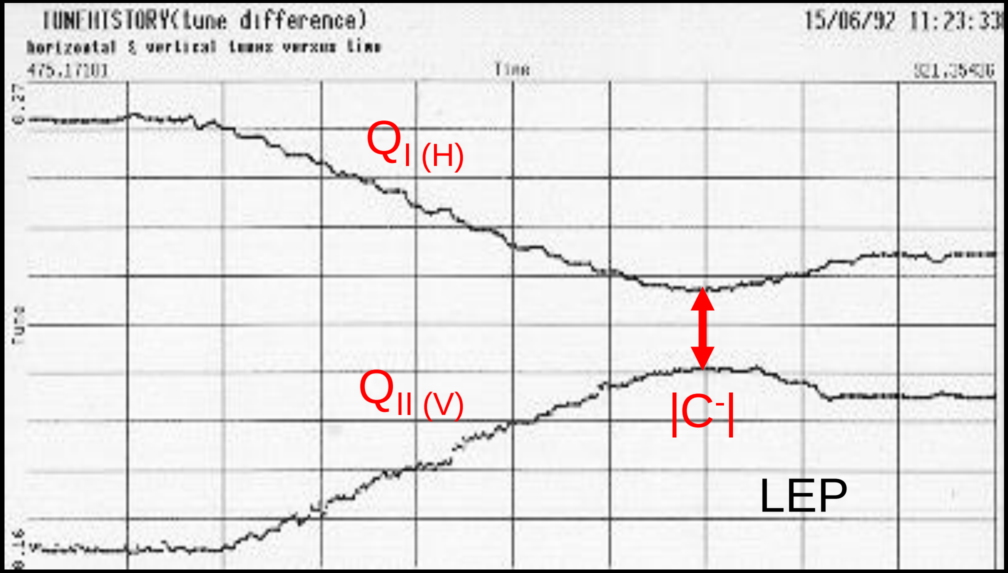
Measuring Coupling

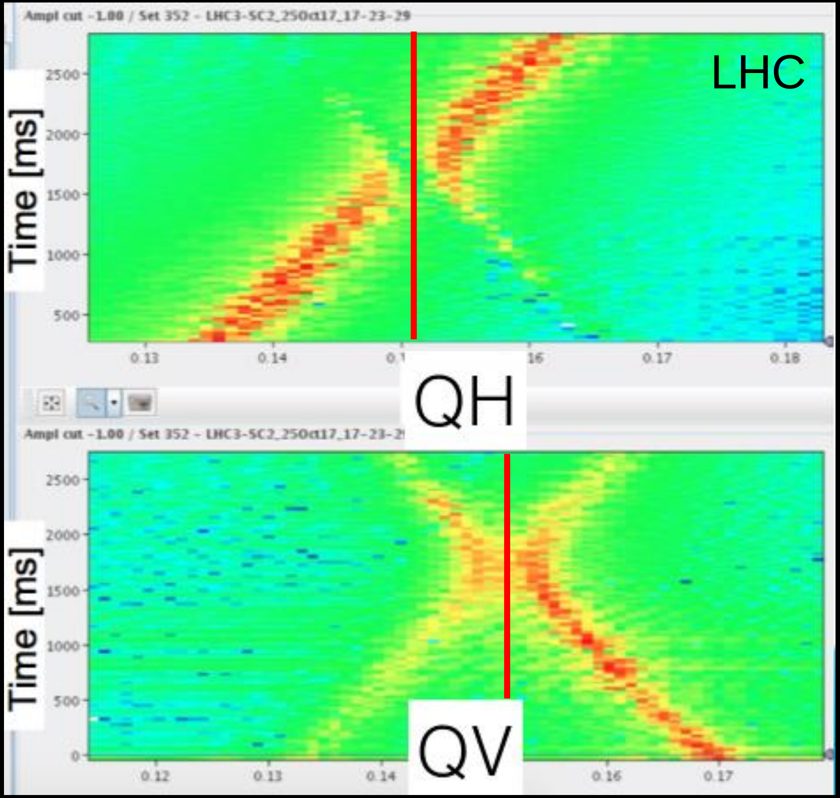
- 3 Main Methods
  - Orbit changes
    - Change orbit in one plane by exciting steering correctors or by changing injection conditions & measure effect in other plane
    - Large coupling sources identified as locations where horizontal orbit change generates a vertical kick & vice versa
    - Acquire large numbers of orbits for excitation of different correctors to determine skew quadrupole component of each magnet
  - Closest tune approach
    - Approach horizontal & vertical tunes until they cross
    - Coupling derived from how close tunes can approach
  - Kick response
    - Kick in one plane & measure in other using
      - Tune FFT or Phase Locked Loop
      - Pairs of BPMs to derive Resonance Driving Terms

Measuring Coupling – Closest Tune Approach

$$\overbrace{Q_{I,II}}^{\text{Measured Tunes}} = \frac{1}{2}\left(\overbrace{Q_x + Q_y}^{\text{Set Tunes}} \pm \sqrt{(Q_x - Q_y)^2 + |C^-|^2}\right)$$

- Measure tunes while changing the quadrupole strength
  - Coupling Measurement in LEP using Phase Locked Loop tune measurement
  - Coupling measurement in LHC using base band tune measurement

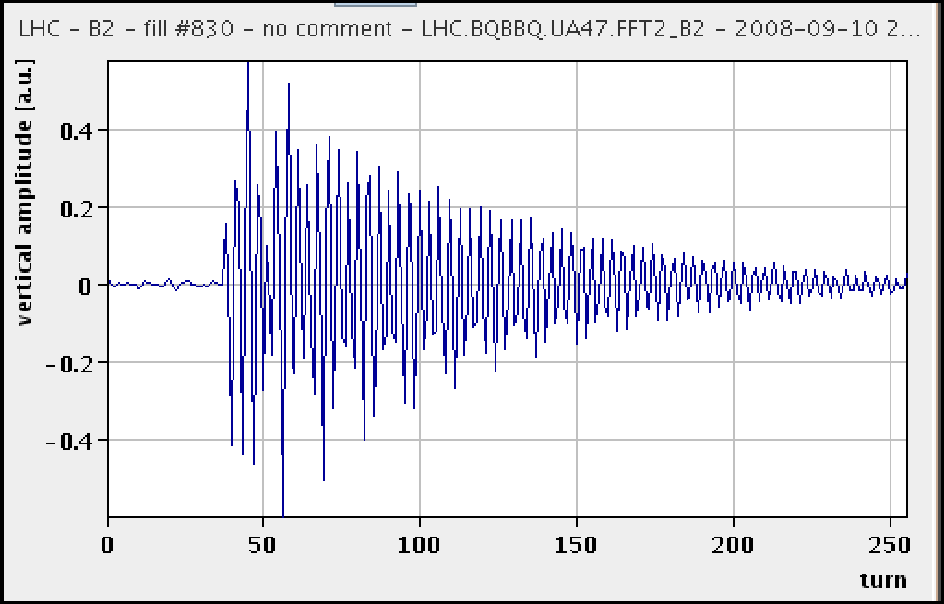


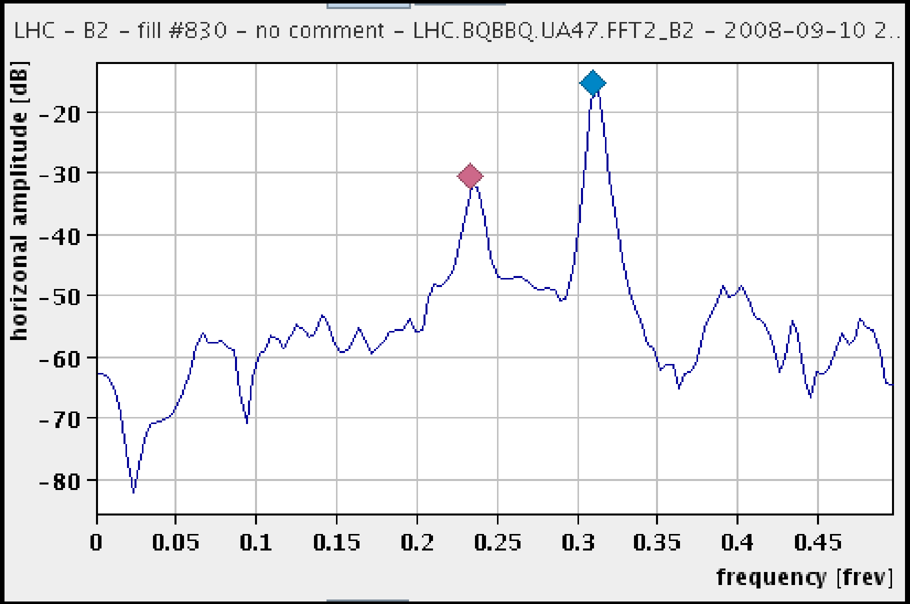


Measuring Coupling – Kick Response

- Kick Beam in one plane and measure oscillations in other
  - Observe with tune measurement system
  - Magnitude of local coupling can be derived from amplitude ratios of tune peaks

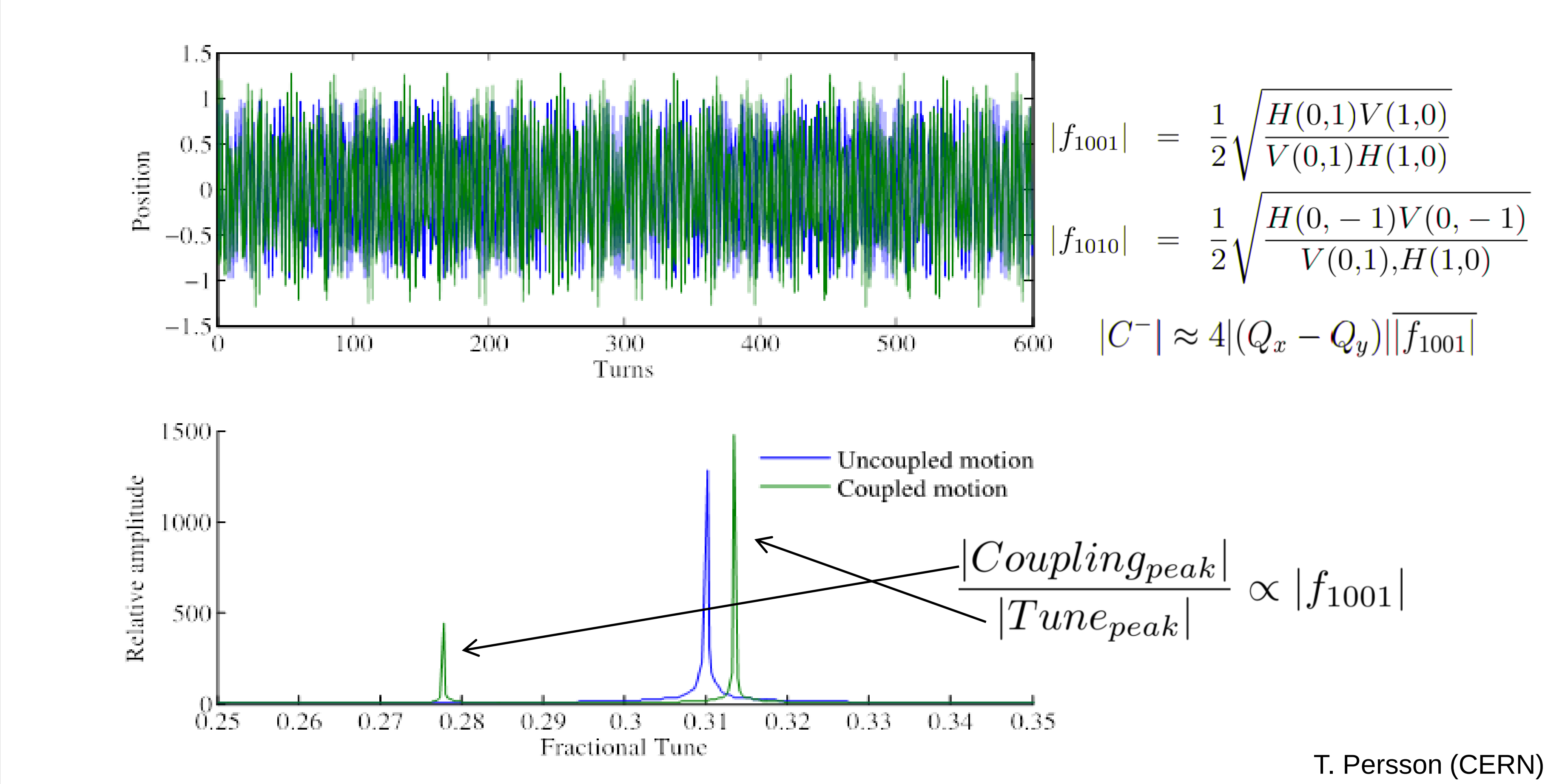
$$|C^-| \propto \frac{\sqrt{r_1 r_2}}{1 + r_1 r_2} \qquad r_1 = \frac{A_{1,y}}{A_{1,x}} \qquad r_2 = \frac{A_{2,x}}{A_{2,y}}$$





Measuring Coupling – Kick Response

- Resonant Driving Terms
  - Using the amplitude & phase of FFT spectrum RDTs proportional to the coupling strength can be calculated



$$|f_{1001}| = \frac{1}{2}\sqrt{\frac{H(0,1)V(1,0)}{V(0,1)H(1,0)}}$$

$$|f_{1010}| = \frac{1}{2}\sqrt{\frac{H(0,-1)V(0,-1)}{V(0,1),H(1,0)}}$$

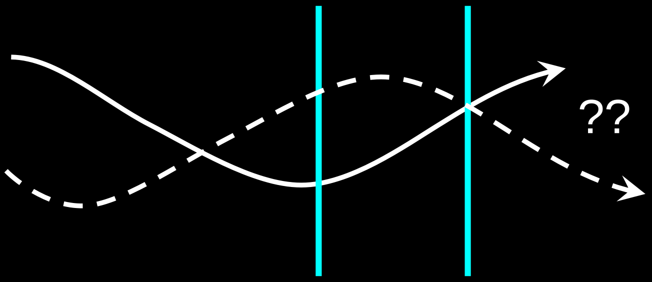
$$|C^-| \approx 4|(Q_x - Q_y)|\overline{|f_{1001}|}$$

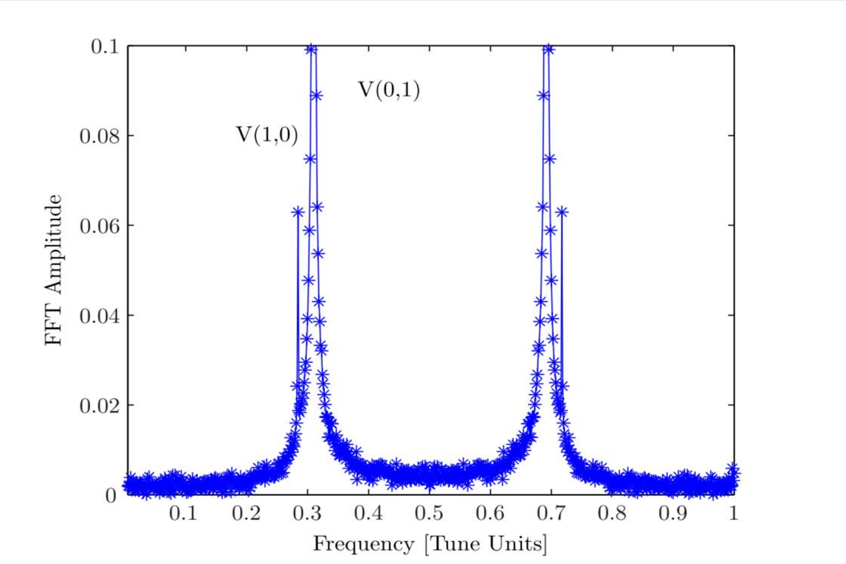
$$\frac{|Coupling_{peak}|}{|Tune_{peak}|} \propto |f_{1001}|$$

T. Persson (CERN)

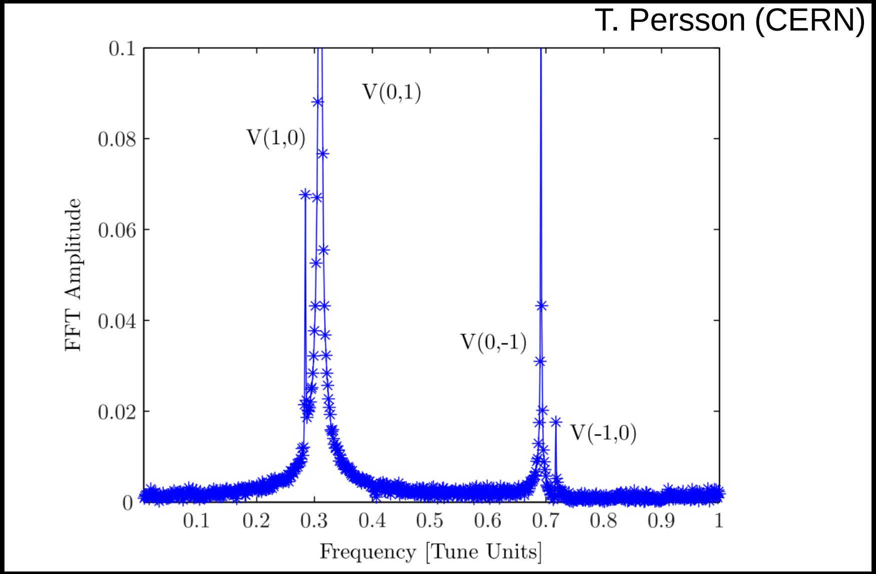
Coupling via Resonant Driving Terms

- Using a single BPM
  - – The Normal FFT (tune measurement)
  - – Spectrum is mirrored around half revolution
    - Cannot distinguish phase of oscillation
- Using a pair of BPMs
  - – Produce a complex variable
  - – Can reconstruct both amplitude & phase



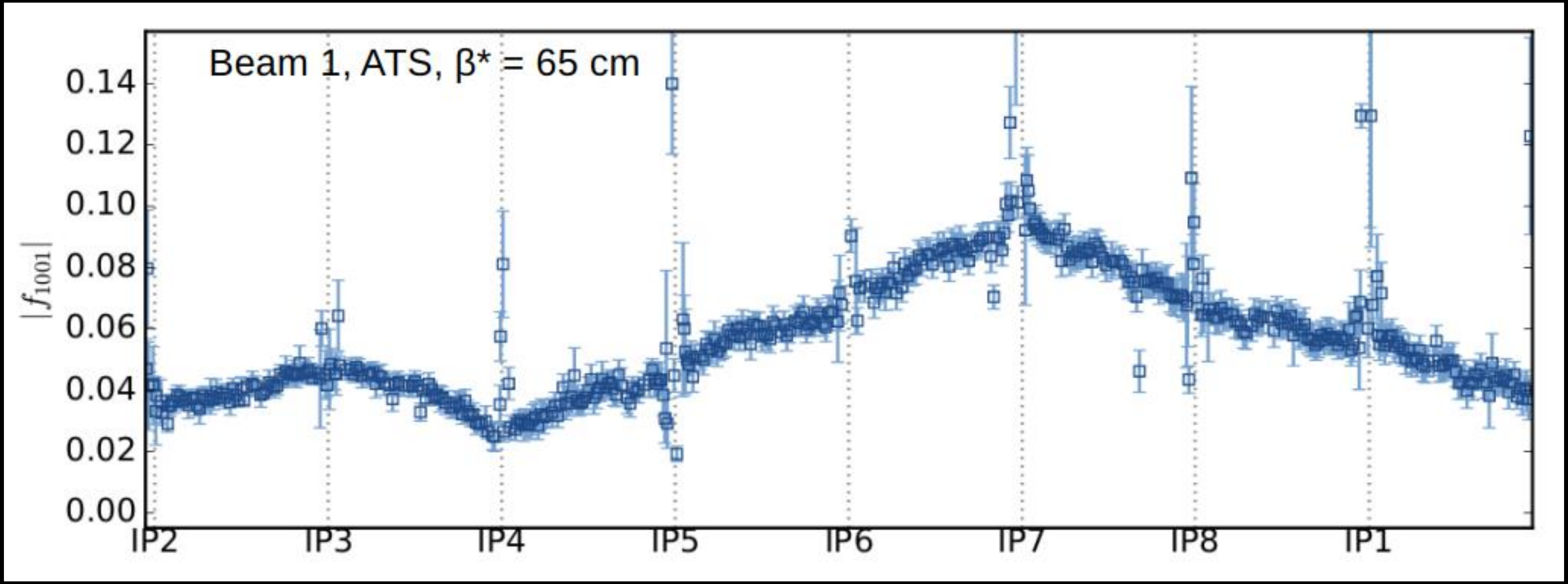


The Normal FFT



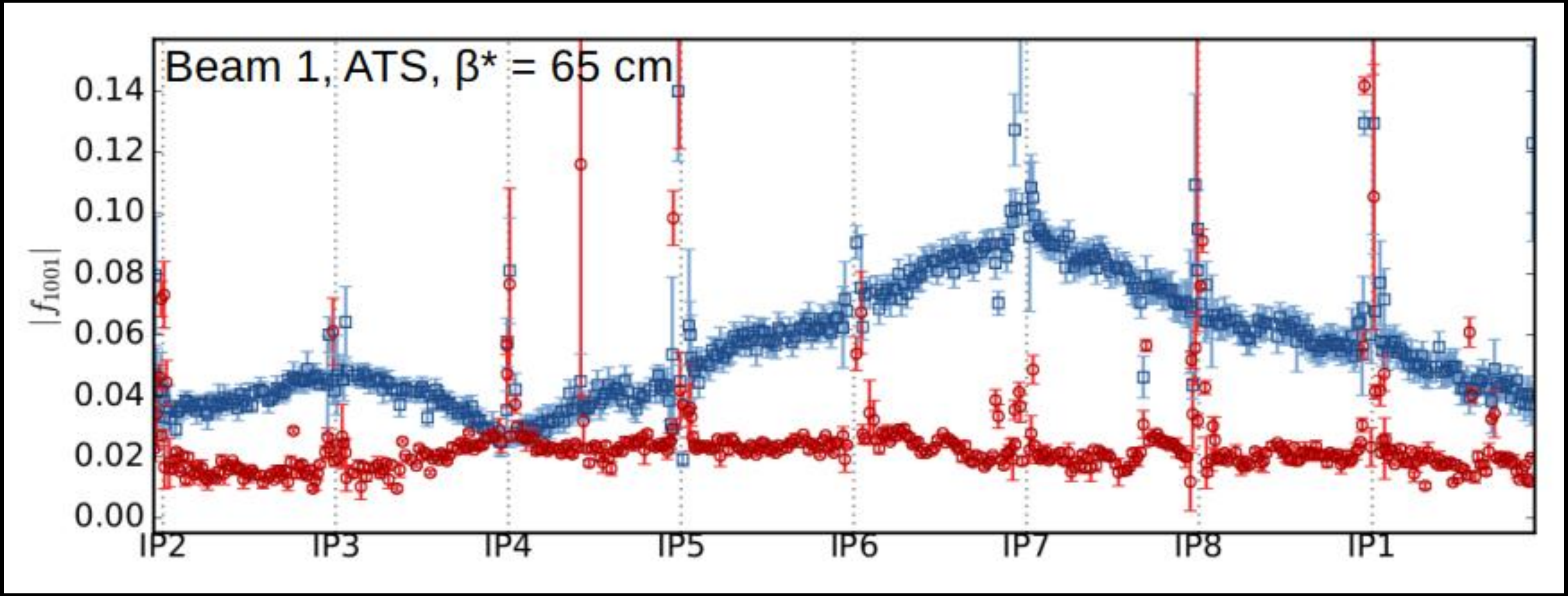
Using the Complex Variable

Coupling via Resonant Driving Terms



- Example from the LHC
  - Kick Beam & measure oscillations using BPMs throughout the ring
  - Reconstruct local coupling at each BPM location
  - Clear difference between local & average “global” coupling
    - FFT based on tune measurement system (single location) would not have detected such a structure

Coupling via Resonant Driving Terms



- Example from the LHC
  - Kick Beam & measure oscillations using BPMs throughout the ring
  - Reconstruct local coupling at each BPM location
  - Clear difference between local & average “global” coupling
    - FFT based on tune measurement system (single location) would not have detected such a structure
  - Correct on an arc by arc basis using skew quadrupoles

Summary

- Tune Measurement
  - A basic “must have” diagnostic system
  - Emittance preservation an important issue for hadron machines implying low amplitude excitation
  - Single kick, single measurement systems now replaced by continuous measurement systems
    - High sensitivity systems making use of μm level residual oscillations
    - Phase Locked Loops
- Chromaticity Measurement
  - Workhorse is tune measurement during RF modulation
  - Large array of other techniques available for specific situations
- Coupling Measurement
  - Important to decouple machine for beam stability & feedback stability
  - Kick response the main technique exploited
    - Tune FFT or Phase Locked Loop
    - Pairs of BPMs to derive Resonance Driving Terms