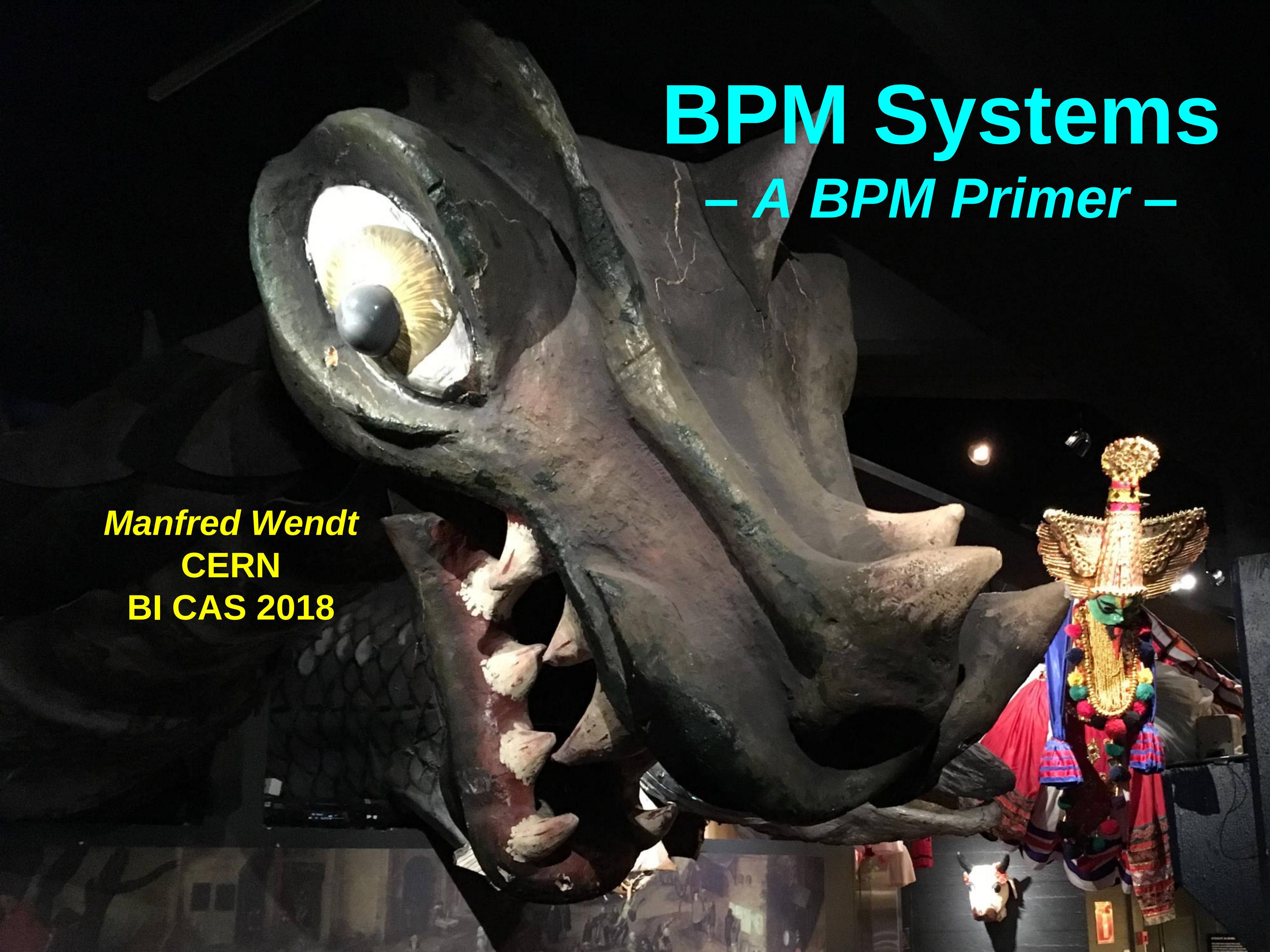
BPM Systems

– A BPM Primer –

Manfred Wendt
CERN
BI CAS 2018



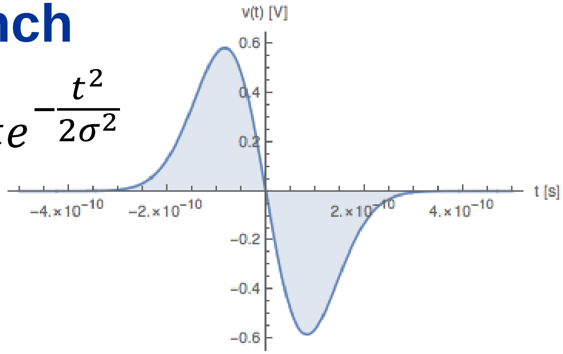
BPM Systems Part 2

- **Leftover Part 1**
  - **Bunch signals from broadband BPMs**
  - **Cavity & other BPMs**
- **Low-β beams**
- **Beam coupling impedance**
- **BPM read-out electronics**
  - **Analog & digital systems**
  - **RF signal conditioning and impedance matching**
  - **Digital signal processing**
  - **Long-term drift calibration**
  - **Signal-to-noise and resolution limit**
  - **Performance check applying SVD**
- **Summary & final remarks**

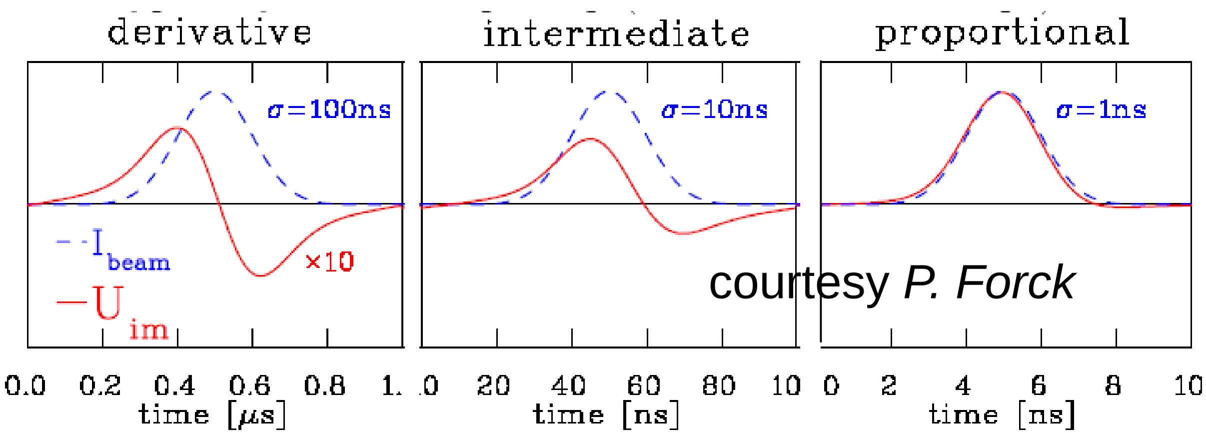
Bunch Signals from broadband BPMs

- **Button BPM output signal to a Gaussian bunch**

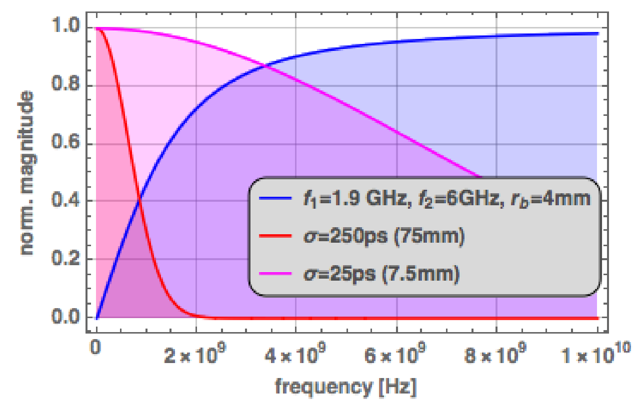
$$v_{button}(t) \approx \frac{A_{button} R_{load}}{\pi r_{pipe} \beta c} \frac{d i_{beam}(t)}{dt} = \frac{r_{button}^2 R_{load}}{2 r_{pipe} \beta c} \frac{eN}{\sqrt{2\pi}\sigma^3} t e^{-\frac{t^2}{2\sigma^2}}$$



  – **Only valid for $\omega \ll \omega_1$ (low frequency range)!**



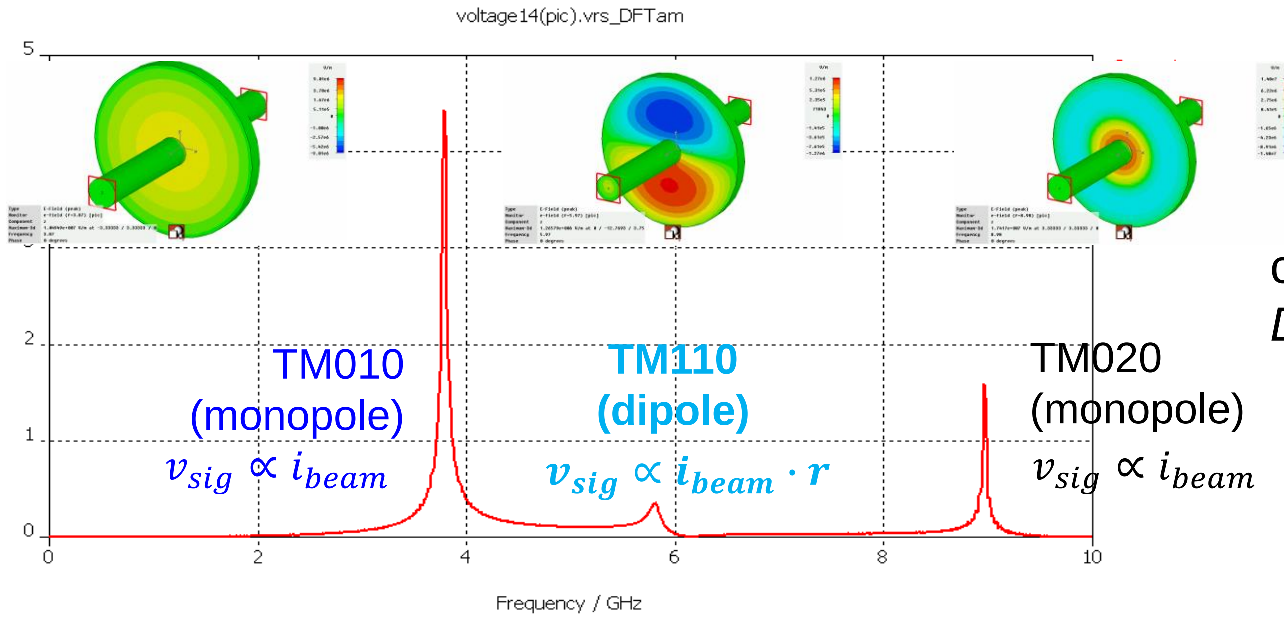
courtesy *P. Forck*



- **Stripline BPM output signal**
  – **For $\ell \gg \sigma\beta c$ the bunch shape can be well reproduced**
    - **Separation: $2\,{}^{\ell}/_{c}$**
    - **enables e.g. head-tail mode detection**

Resonant Cavity BPM

- **Based on a beam-excited, passive resonator**
  - **Often a cylindrical "pillbox" cavity is used**
  - **Operating on the TM110 dipole-eigenmode offers a higher resolution potential than comparable broadband BPMs (button, stripline).**
    - **No common-mode Σ signal, only a difference Δ signal**
    - **High transfer impedance, typically in the kΩ/mm range**



courtesy *D. Lipka*

Towards a Cavity BPM…

- **Eigenmodes in a brick-style resonator**
  - **1st step towards a cavity BPM**
  - **Unfortunately you need to go through the math of the modal expansion of the vector potential Ψ…😊**

Laplace equation:

$$\boldsymbol{\Delta\Psi + k_0^2 \varepsilon_r \mu_r \Psi = 0}$$

$$k_0^2 = \omega^2 \varepsilon_0 \mu_0$$

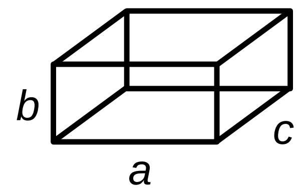
$$k_0 = {}^{2\pi}/_{\lambda_0}$$

k_0: free space wave number

λ_0: free space wave length

Product ansatz (*Cartesian* coordinates):

$$\boldsymbol{\Psi = X(x)Y(y)Z(z)}$$



General solution (field components):

standing waves

$$\Psi = \begin{Bmatrix} A\cos(k_x x) + B\sin(k_x x) \\ \acute{A}e^{-jk_x x} + \acute{B}e^{-jk_x x} \end{Bmatrix} \begin{Bmatrix} C\cos(k_y y) + D\sin(k_y y) \\ \acute{C}e^{-jk_y y} + \acute{D}e^{-jk_y y} \end{Bmatrix} \begin{Bmatrix} E\cos(k_z z) + F\sin(k_z z) \\ \acute{E}e^{-jk_z z} + \acute{F}e^{-jk_z z} \end{Bmatrix}$$

travelling waves

separation condition:

$$\boldsymbol{k_x^2 + k_y^2 + k_z^2 = k_0^2 \varepsilon_r \mu_r}$$

$$k_x = \frac{m\pi}{a} \quad k_y = \frac{n\pi}{b} \quad k_z = \frac{p\pi}{c}$$

Eigen frequencies:

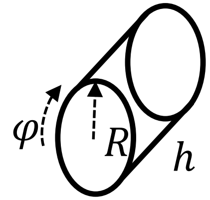
$$\boxed{\boldsymbol{f_{mnp} = \frac{c_0}{2\pi\varepsilon_r\mu_r}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{c}\right)^2}}}$$

Cylindrical "Pillbox" Cavity Resonator

- **Same procedure, but now with cylindrical functions 😢😢**

Product ansatz (cylindrical coordinates):

$$\boldsymbol{\Psi = R(\rho)F(\varphi)Z(z)}$$



General solution (field components):

$$\Psi = \begin{Bmatrix} A\,J_m(k_r\rho) + B\,N_m(k_r\rho) \\ \acute{A}H_m^{(2)}(k_r\rho) + \acute{B}H_m^{(2)}(k_r\rho) \end{Bmatrix} \begin{Bmatrix} C\,cos(m\varphi) + D\,sin(m\varphi) \\ \acute{C}e^{-jm\varphi} + \acute{D}e^{-jm\varphi} \end{Bmatrix} \begin{Bmatrix} E\,cos(k_z z) + F\,sin(k_z z) \\ \acute{E}e^{-jk_z z} + \acute{F}e^{-jk_z z} \end{Bmatrix}$$

standing waves

travelling waves

$J_m, N_m, H_m^{(1,2)}$: cylindical functions $(Bessel, Hankel, Neumann)$

(1,2) see *Abramowitz* and *Stegun*

separation condition:

$$\boldsymbol{k_r^2 + k_z^2 = k_0^2 \varepsilon_r \mu_r}$$

Eigen frequencies:

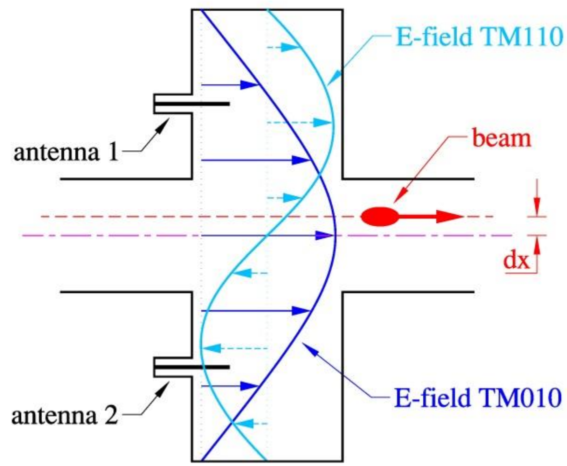
$$\boxed{\boldsymbol{f_{TMmnp} = \frac{c_0}{2\pi\varepsilon_r\mu_r}\sqrt{\left(\frac{j_{mn}}{R}\right)^2 + \left(\frac{p\pi}{h}\right)^2}}}$$

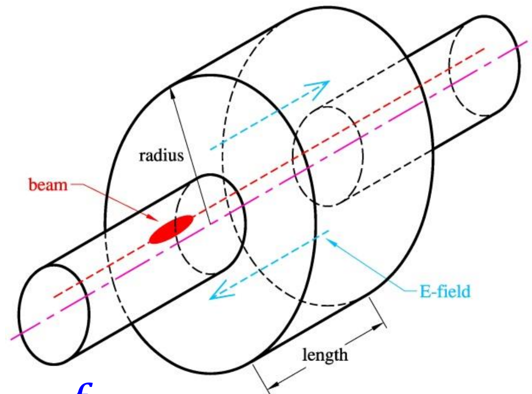
$$\boxed{\boldsymbol{f_{TEmnp} = \frac{c_0}{2\pi\varepsilon_r\mu_r}\sqrt{\left(\frac{j'_{mn}}{R}\right)^2 + \left(\frac{p\pi}{h}\right)^2}}}$$

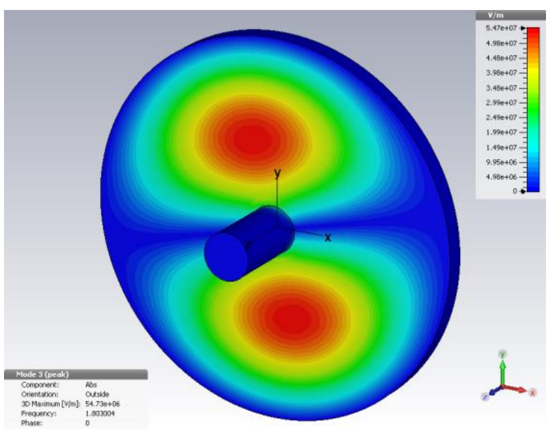
j_{mn} being the n^{th} root of $J_m(x)$

j'_{mn} being the n^{th} root of $J'_m(x)$

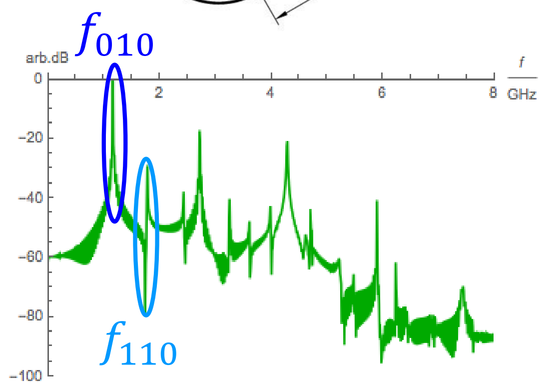
Cavity BPM

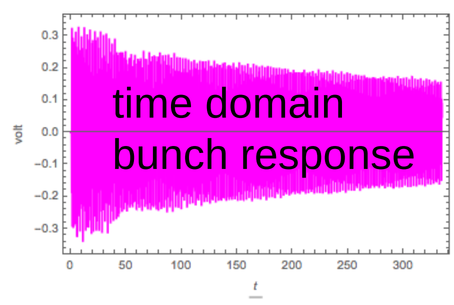






$r_{pipe} = 12.5mm$
$r_{cav} = 100mm$
$\ell_{cav} = 10mm$





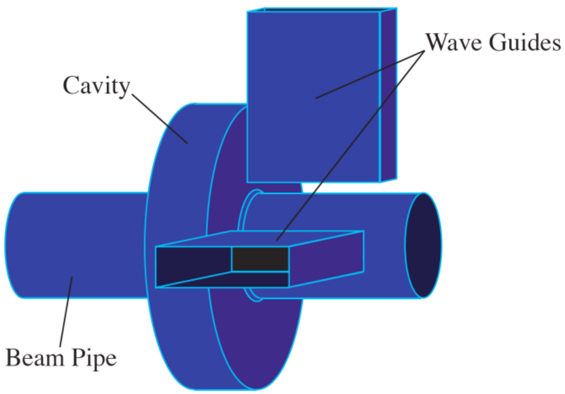
- **Beam couples to:**

$$E_z = C\, J_1\left(\frac{j_{11} r}{R}\right) e^{i\omega t} \cos\varphi$$

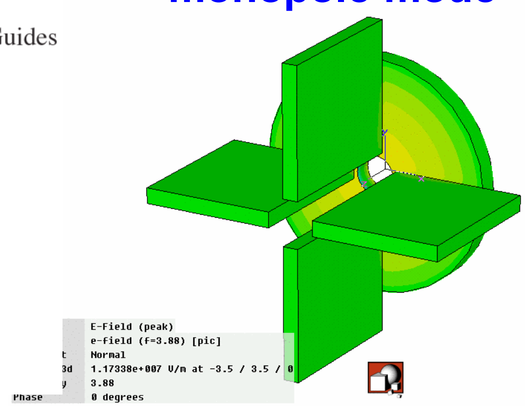
dipole (TM_{110}) and monopole (TM_{010}) & other modes

- **Common mode (TM_{010}) frequency discrimination**
- **Decaying RF signal response**
  - **Position signal: TM_{110}**
    - **Requires normalization and a phase reference**
  - **Intensity signal: TM_{010}**

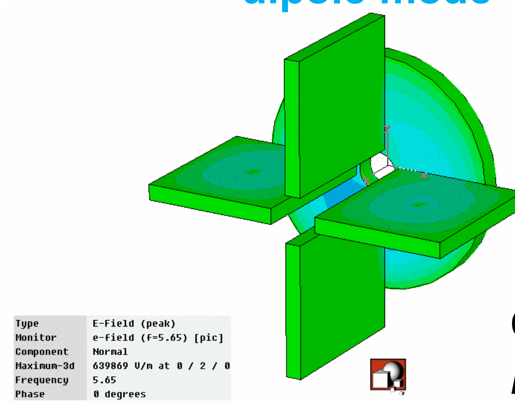
Common-mode free Cavity BPMs

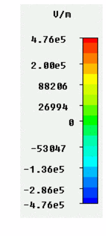


monopole mode

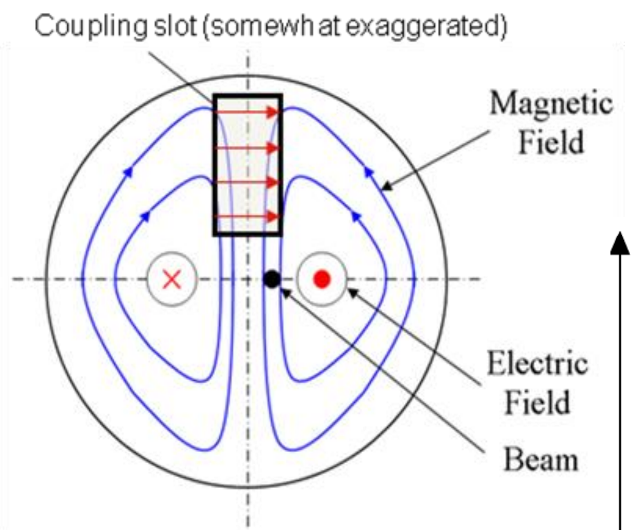


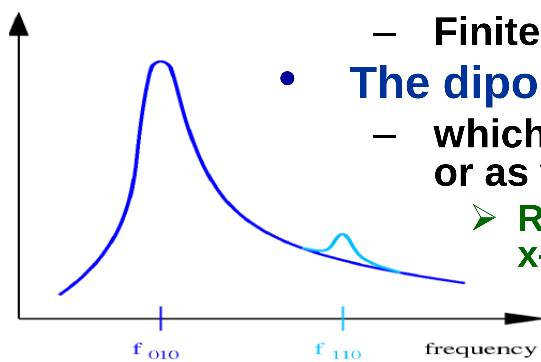
dipole mode





courtesy *D. Lipka*



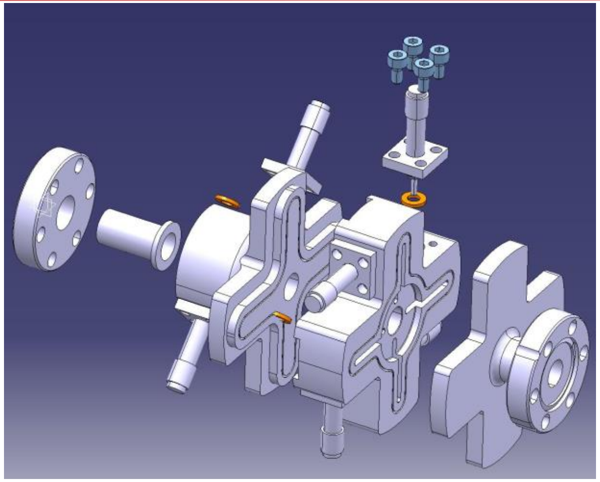


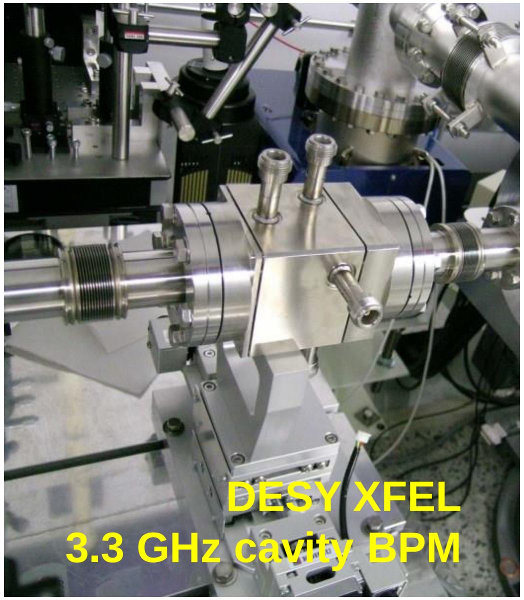
- **Add slot-coupled waveguide TE_{01}-mode high-pass filter**

$$f_{010} < f_{10} = \frac{1}{2a\sqrt{\varepsilon\mu}} < f_{110}$$

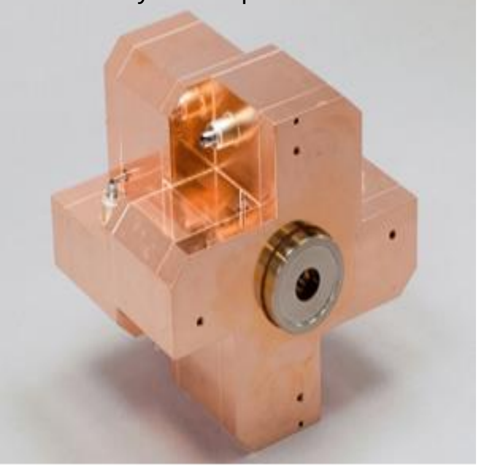
  between cavity and coaxial output port.
  - **Finite Q of TM_{010} still leaks into TM_{110}!**
- **The dipole mode has two polarizations**
  - **which will orientate along imperfections, or as wanted along the coupling slots**
    - ➢ **Requires tight tolerances to minimize x-y cross talk**

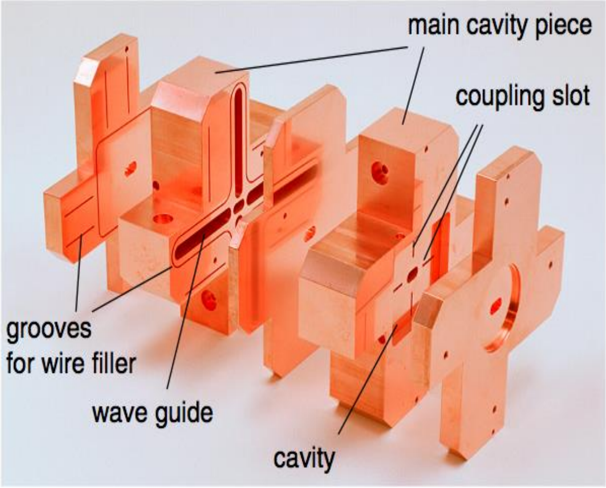
Examples of Cavity BPMs

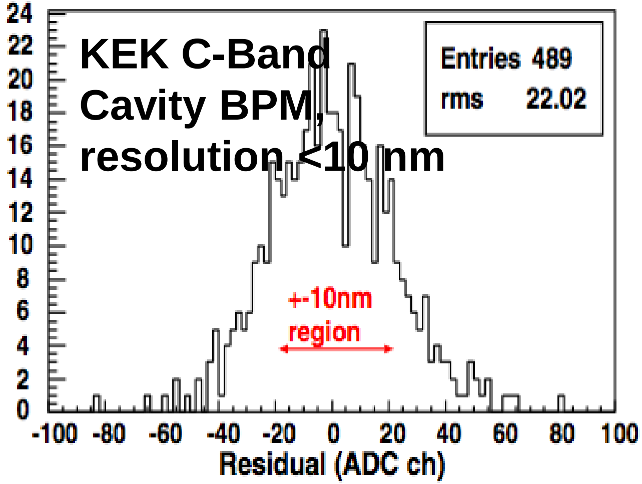




Courtesy of D. Lipka & Y. Honda







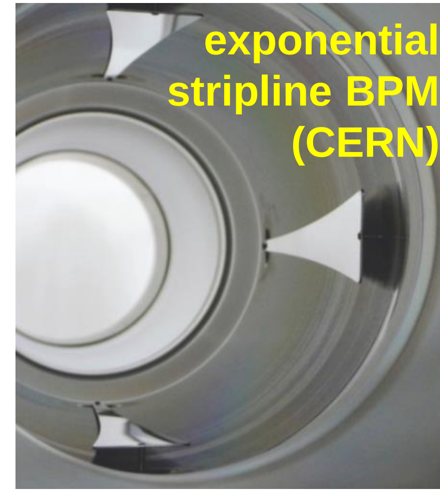
Cavity BPM

+ **Pros**
  - **No or minimum common mode signal contribution in the Δ-signal**
    - **Frequency discrimination of dipole (TM110) and monopole (TM010) modes**
  - **High resolution potential**
    - **High shunt (transfer) impedance of the TM110 mode**
      - Even for lower Q tuning of the TM110 mode
    - **Sub-μm signal pass resolution potential**

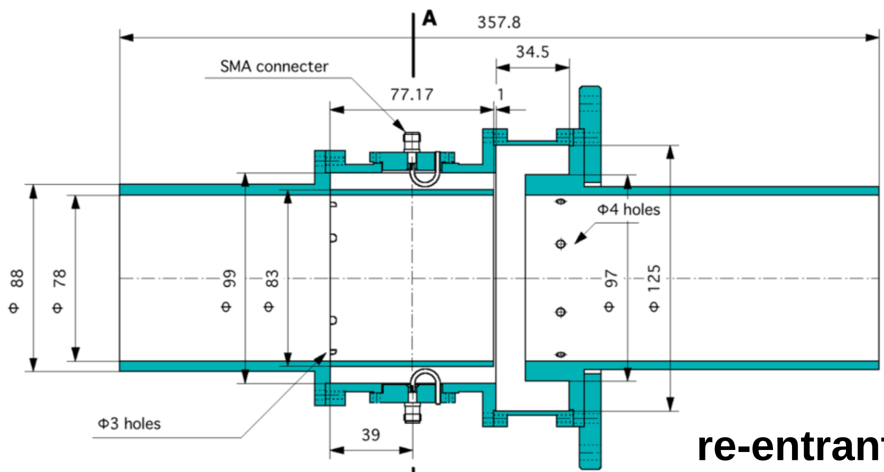
- **Cons**
  - **High beam coupling impedance**
    - **No free lunch: high impedance may cause beam break-up and/or instabilities**
    - **No or very limited use in ring accelerators**
  - **Requires a reference monopole mode (TM010) resonator**
    - **Beam phase and intensity**
  - **Limited position range**
    - **~half aperture**
  - **Requires advanced RF read-out electronics**
  - **High-Q resonator may not be suitable for single bunch position measurements**

Other Types of BPM Pickups

- **Less popular, but sometimes better suited for a specific application**
  - **Stripline BPM with shorted downstream ports**
  - **Exponentially tapered stripline BPM**
  - **Re-entrant cavity BPM**
  - **Resonant stripline of button BPM**
  - **Inductive BPM, …**
    - ➢ **In common: based on symmetry**

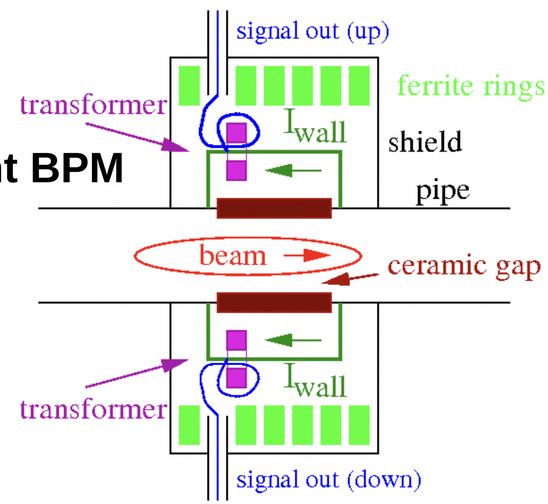


exponential stripline BPM (CERN)



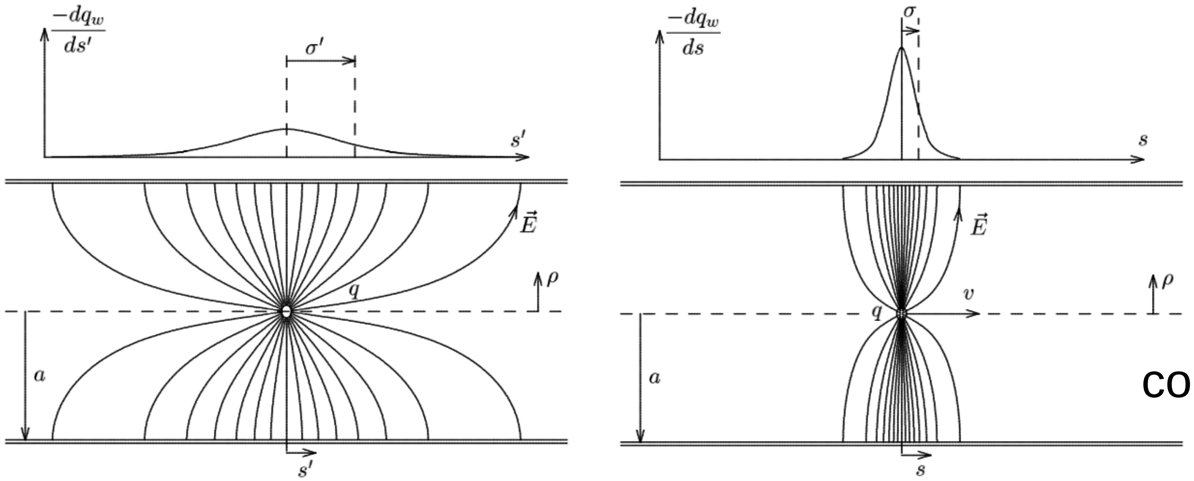
re-entrant cavity BPM (KEK)

inductive wall-current BPM (CERN)



Effects of Low-β Beams

- **At $\beta \ll 1$ the EM-field of a point charge develops longitudinal field components (non-TEM field)**
  - **Point charge in a cylindrical beam pipe of radius $r_{pipe} = a$ at rest and at $\gamma = 4$ $(\beta \approx 0.97)$**



$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

$$\beta = \frac{v}{c}$$

courtesy *A. Hofmann*

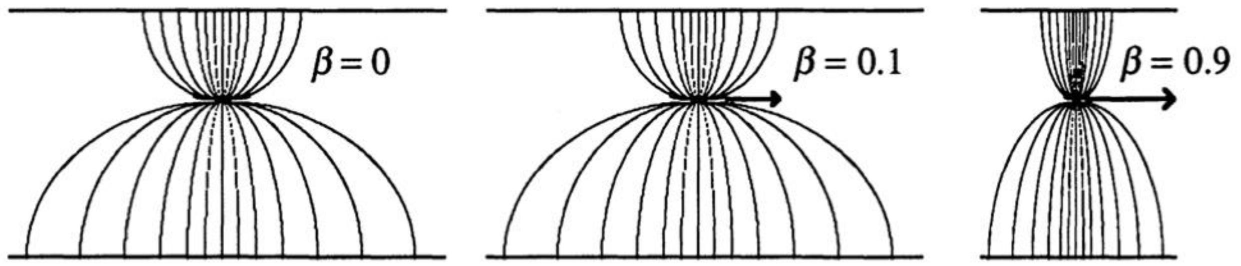
  - **The longitudinal image charge distribution $-dq_w/ds$ follows a complicated expression from a Bessel-Fourier series expansion**
    - ➢ **Fortunately the RMS value is simply:**

$$\sigma_s = \frac{r_{pipe}}{\sqrt{2}\gamma}$$

Position Monitoring of Low-β Beams

E-field for an off-center beam moving at:

courtesy *R. Shafer*



- **For an off-center beam in a cylindrical beam pipe:**
  - **Image charges integrated on the right, horizontal electrode A**
    - **Some simplifications could be applied for $gr < gR \ll 1$**

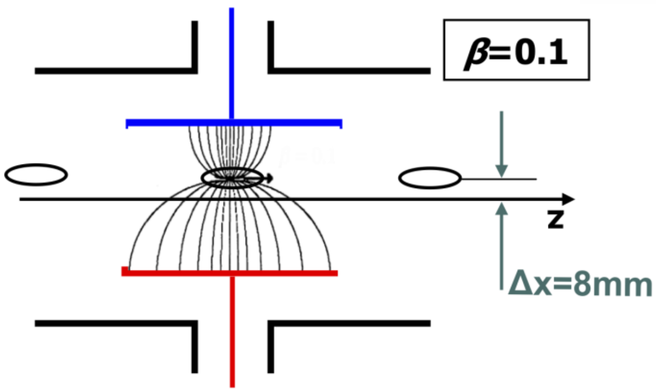
$$I_A = -\frac{I_{beam}}{2\pi} s_A[r, \varphi, \alpha, g(\omega)]$$

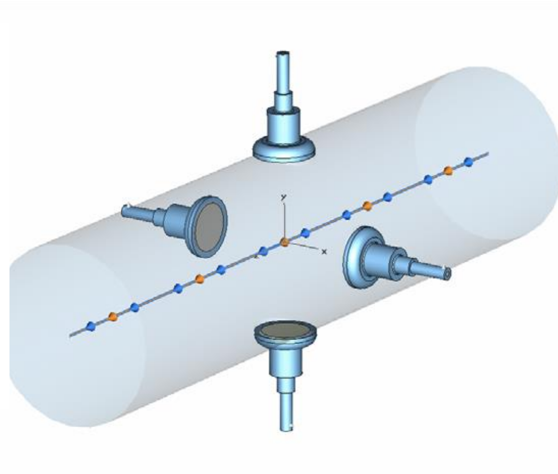
$$s_A[r, \varphi, \alpha, g(\omega)] = \alpha \frac{J_0(gr)}{J_0(gR)} + 4 \sum_{m=1}^{\infty} \frac{1}{m} \frac{J_m(gr)}{J_m(gR)} \sin\left[m\left(\frac{\alpha}{2} - \varphi\right)\right]$$

with: $g(\omega) = \dfrac{\omega}{\beta\gamma c}$, $\quad J_m(arg)$: mod. Bessel function of m^{th} order

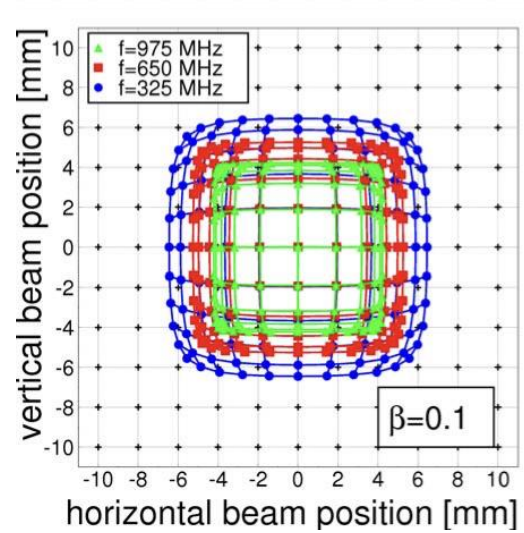
- **Result:**
  - **The position characteristic of a broadband BPM for low-β beams is frequency depending!**

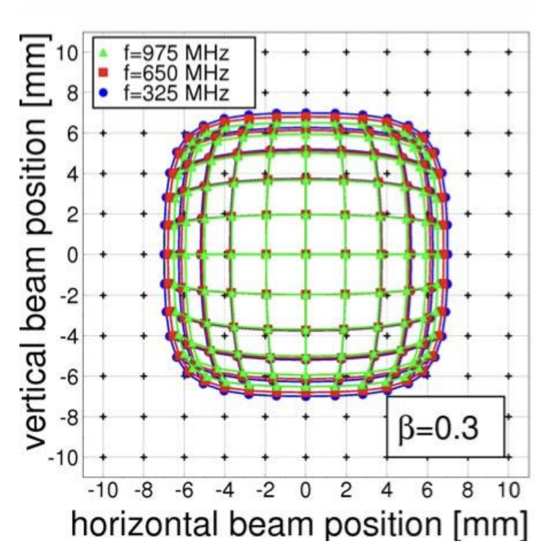
Numerical Analysis of Low-β Beam Effects





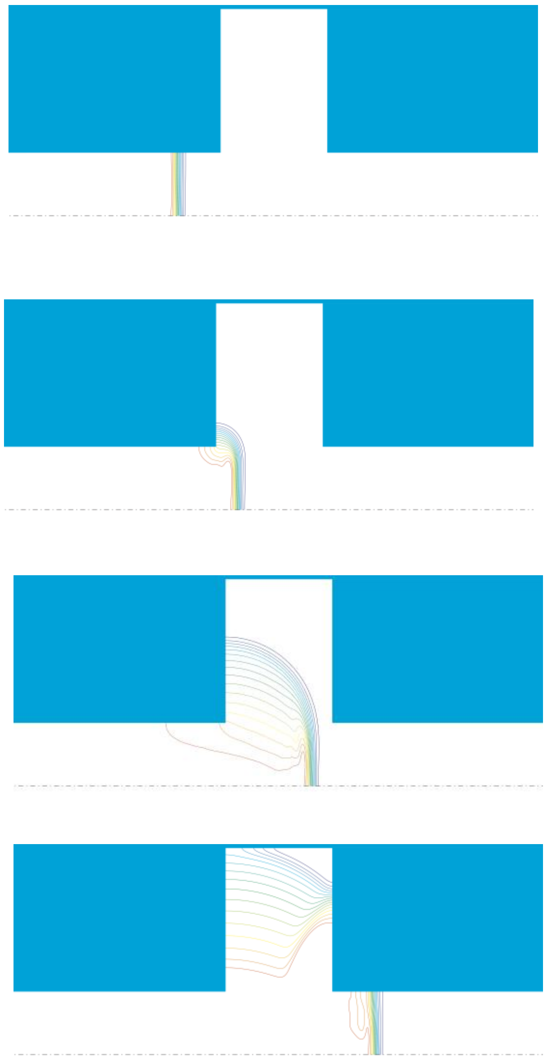
courtesy *P. Kowina*





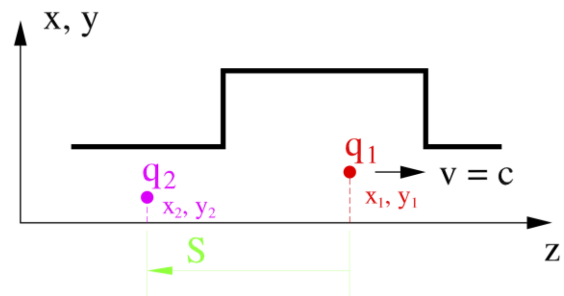
- **Button BPM analysis**
  - **Beam pipe $R = 30\, mm$**
  - **Gaussian bunch $\sigma = 0.15\, ns$**
  - **Beam velocity $0.1 < \beta < 0.3$**
  - **Operating frequencies $f = 325, 650, 975\, MHz$**
- **Discussion of the results**
  - **BPM electrode signals, i.e. the waveform and frequency spectrum are position dependent**
    - **Therefore the BPM position characteristic is frequency dependent for low β beams**
    - **The position sensitivity is reduced at low β, particular when operating at high frequencies**

Beam Coupling Impedance



- **The wake potential**
  - **Lorenz force on q_2 by the wake field of q_1:**

$$\vec{F} = \frac{d\vec{p}}{dt} = q_2\left(\vec{E} + c\vec{e}_z \times \vec{B}\right)$$



  - **Wake potential of a structure, e.g. a discontinuity driven by q_1**

$$\vec{W}(x_2, y_2, x_1, y_1, s) = \frac{1}{q_1}\int_0^L dz\left(\vec{E} + c\vec{e}_z \times \vec{B}\right)_{t=(s+z)/c}$$

  - ➢ **Longitudinal and transverse components of the wake potential are related (*Panofski-Wenzel* theorem)**
  - ➢ **The beam coupling impedance is the frequency domain representation of the wake potential**
    - For resonant structures the wake potential can be described by a multipole expansion of the eigenmodes (HOMs), e.g.:

$$W_\perp^{(n)}(s) = c\sum_i \left(\frac{R^{(n)}}{Q}\right)_i \sin\left(\frac{\omega_i s}{c}\right) exp\left[-\frac{\omega_i s}{2(Q_{ext})_i c}\right]$$

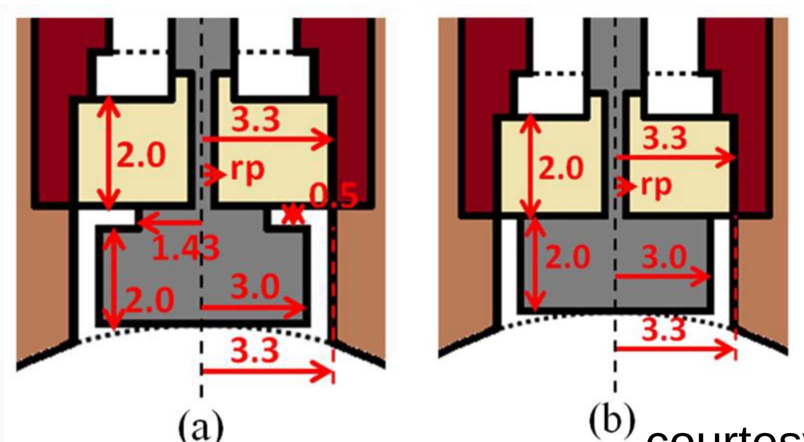
Button BPM Beam Coupling Impedance

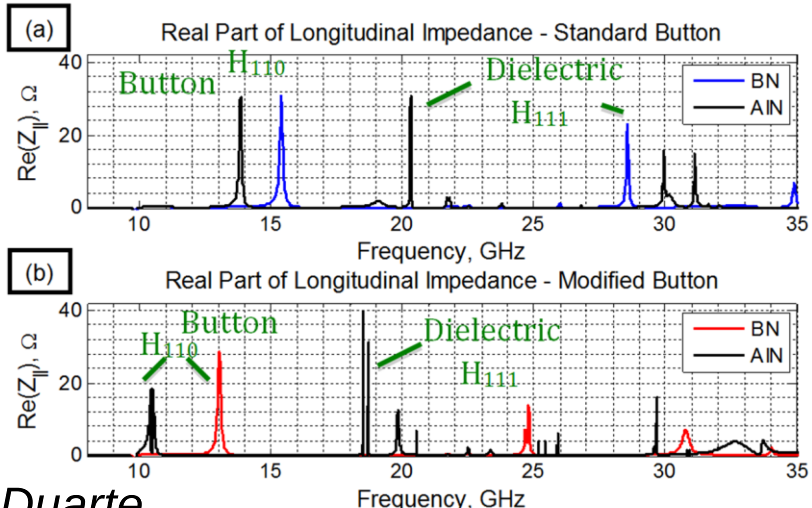
- **Longitudinal coupling impedance of a button BPM electrode**
  - **Related to the transfer impedance $Z_{button}(\omega)$ and scales with r_{button}^4**

$$Z_{\|button}(\omega) = \phi\left(\frac{\omega_1}{\omega_2}\right) Z_{button}(\omega)$$

  - **The gap between button and pipe acts as slot resonator:**
  - **Thickness and shape of the button have significant influence on the coupling impedance**

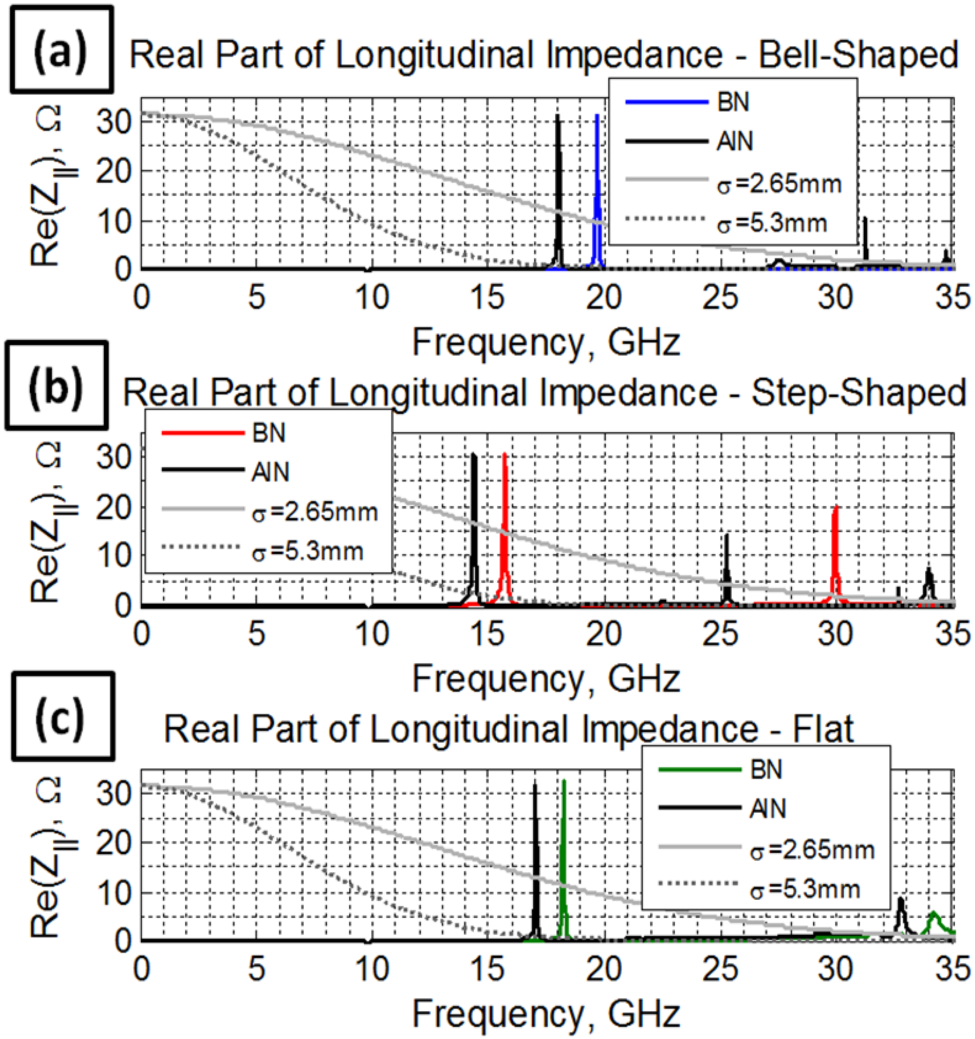
$$Z_{\|gap}(\omega) \approx j\left(\frac{Z_0\omega\left(r_{button}+w_{gap}\right)^3}{8cr_{pipe}^2\{\ln[32(r_{button}+w_{gap})/w_{gap}]-2\}}\right)$$

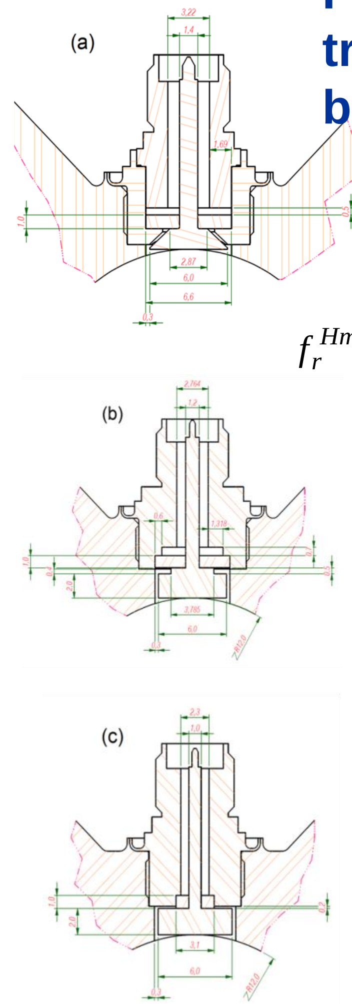




courtesy *H. Duarte*

Coupling Impedance Studies for Sirius





- **Frequencies of trapped modes in the button electrode:**

Trapped H-modes in the insulator dielectric

$$f_r^{Hm1p} = \frac{1}{\sqrt{\varepsilon_r}} \frac{c}{2\pi} \sqrt{\left(\frac{2m}{r_p + r_h}\right)^2 + \left(\frac{\pi p}{t_c}\right)^2}$$

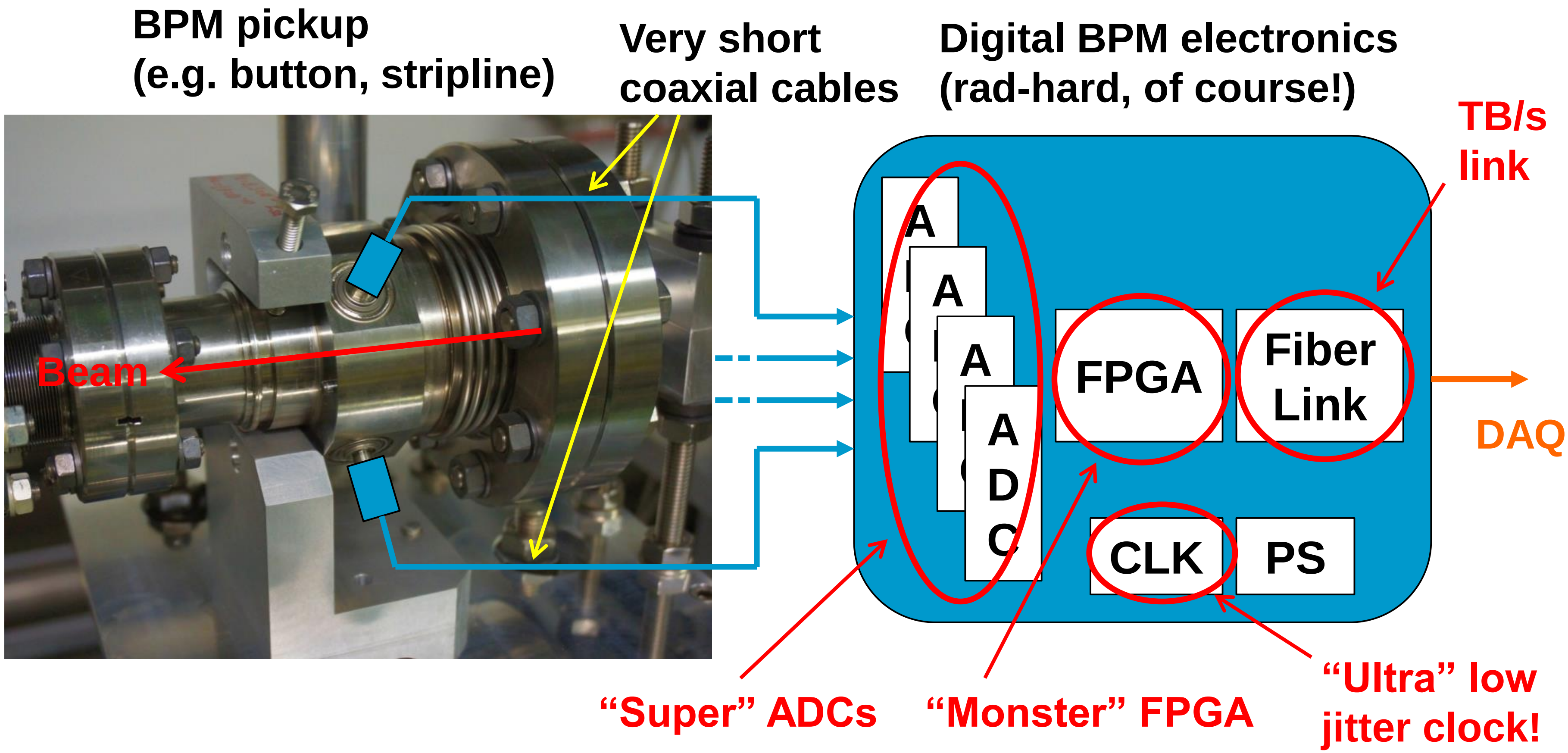
Trapped H-modes in the button

$$f_c^{Hm1} = \frac{c}{\pi} \frac{m}{r_b + r_h}$$

- ε_r: dielectric permittivity
- m: azimuthal index and
- p: longitudinal mode number
- r_p: insulator pin radius
- r_h: housing radius
- r_b: button radius
- t_c: ceramics thickness

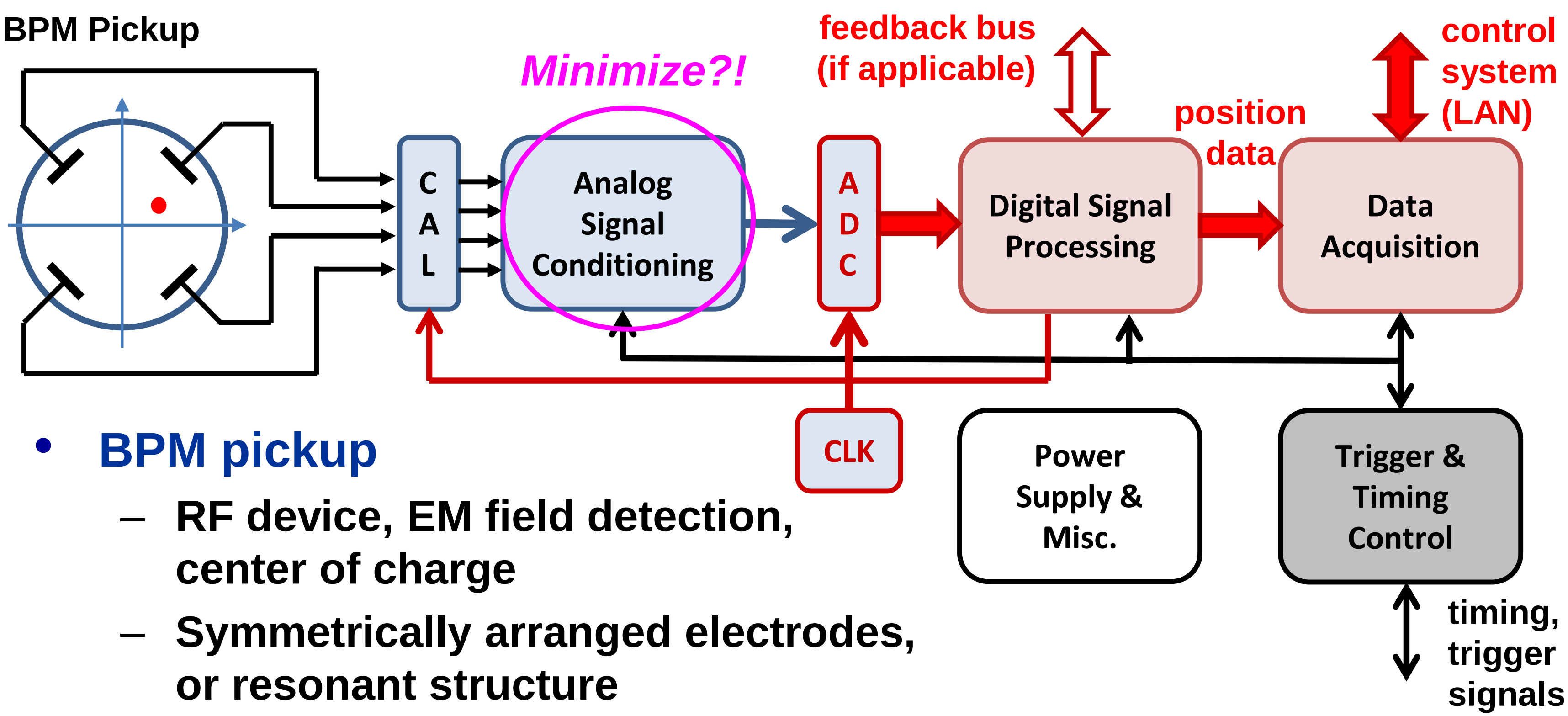
courtesy *H. Duarte*

The Ideal BPM Read-out Electronics!?



- **Time multiplexing of the BPM electrode signals:**
  - **Interleaving BPM electrode signals by different cable delays**
  - **Requires only a single read-out channel!**

BPM Building Blocks



- **BPM pickup**
  - RF device, EM field detection, center of charge
  - Symmetrically arranged electrodes, or resonant structure
- **Read-out electronics**
  - Analog signal conditioning
  - Signal sampling (ADC)
  - Digital signal processing
  - Data acquisition and control system interface
  - Trigger, CLK & timing signals
  - Provides calibration signals or other drift compensation methods

Bunched Beam BPM Signals

- **Bunched beam signals from a broadband BPM are short in time**
  - **Single bunch responses convert to *nsec* or sub-*nsec* pulse signals**
  - **The beam position information is amplitude modulated (AM) on a large (common mode) beam intensity signal!**
- **In ring accelerators, the beam position varies turn-by-turn**
  - **The position signal spectrum is related to some machine parameters**
  - **Dipole moment spectrum of a single Gaussian bunch (simplistic case):**

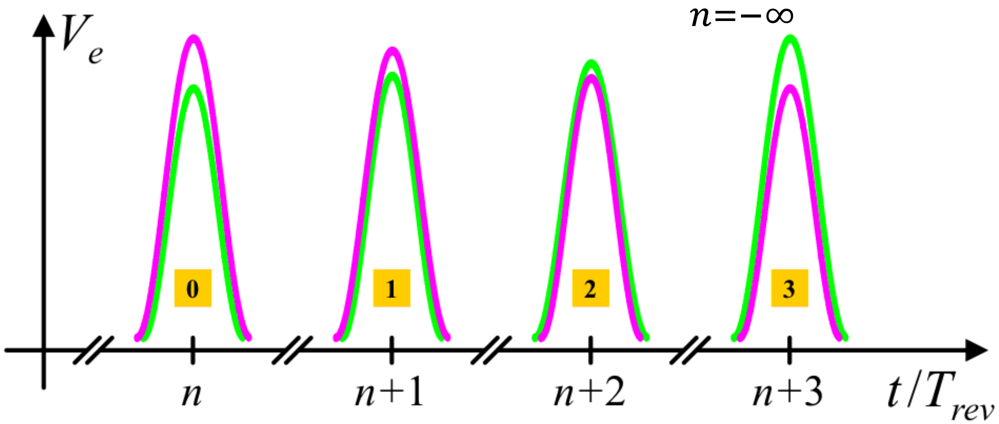
courtesy *R. Siemann*

$$Z_{bpm}(\omega) I_{beam}(\omega) =$$

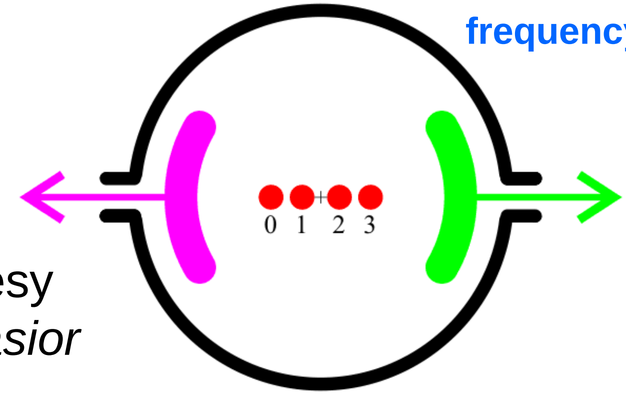
betatron frequency

$$D(\omega) = \omega_{rev} A_0 Q \sum_{n=-\infty}^{+\infty} \delta\left[\omega - \left(n\omega_{rev} + \boldsymbol{\omega_\beta}\right)\right] exp\left[-\frac{\left(\omega - \boldsymbol{\omega_\beta} - \boldsymbol{\omega_\xi}\right)^2 \sigma^2}{2}\right]$$

chromatic frequency

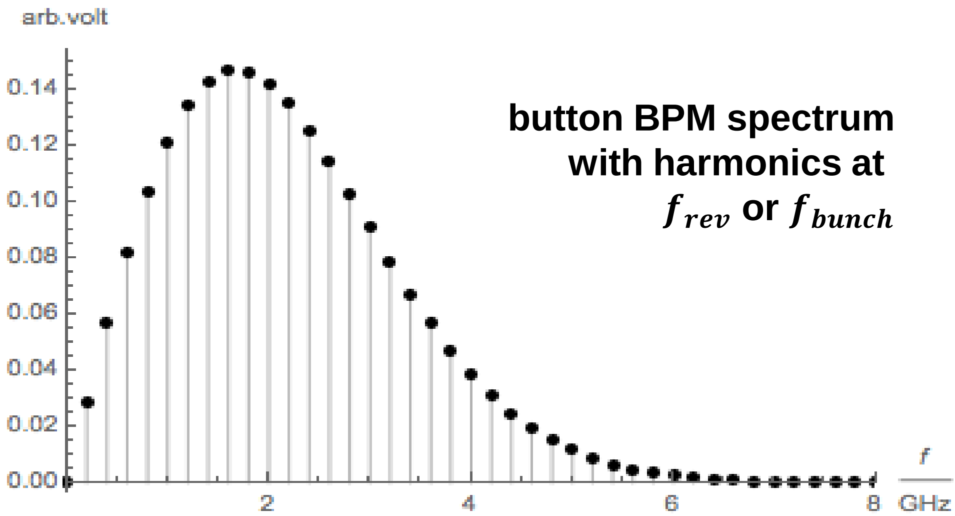


courtesy
M. Gasior



Bunched Beam BPM Signals (cont.)

- **Bunch length and beam formatting define the signal spectrum**
  - E.g. f_{rev}, f_{bunch} in circular or linear accelerators



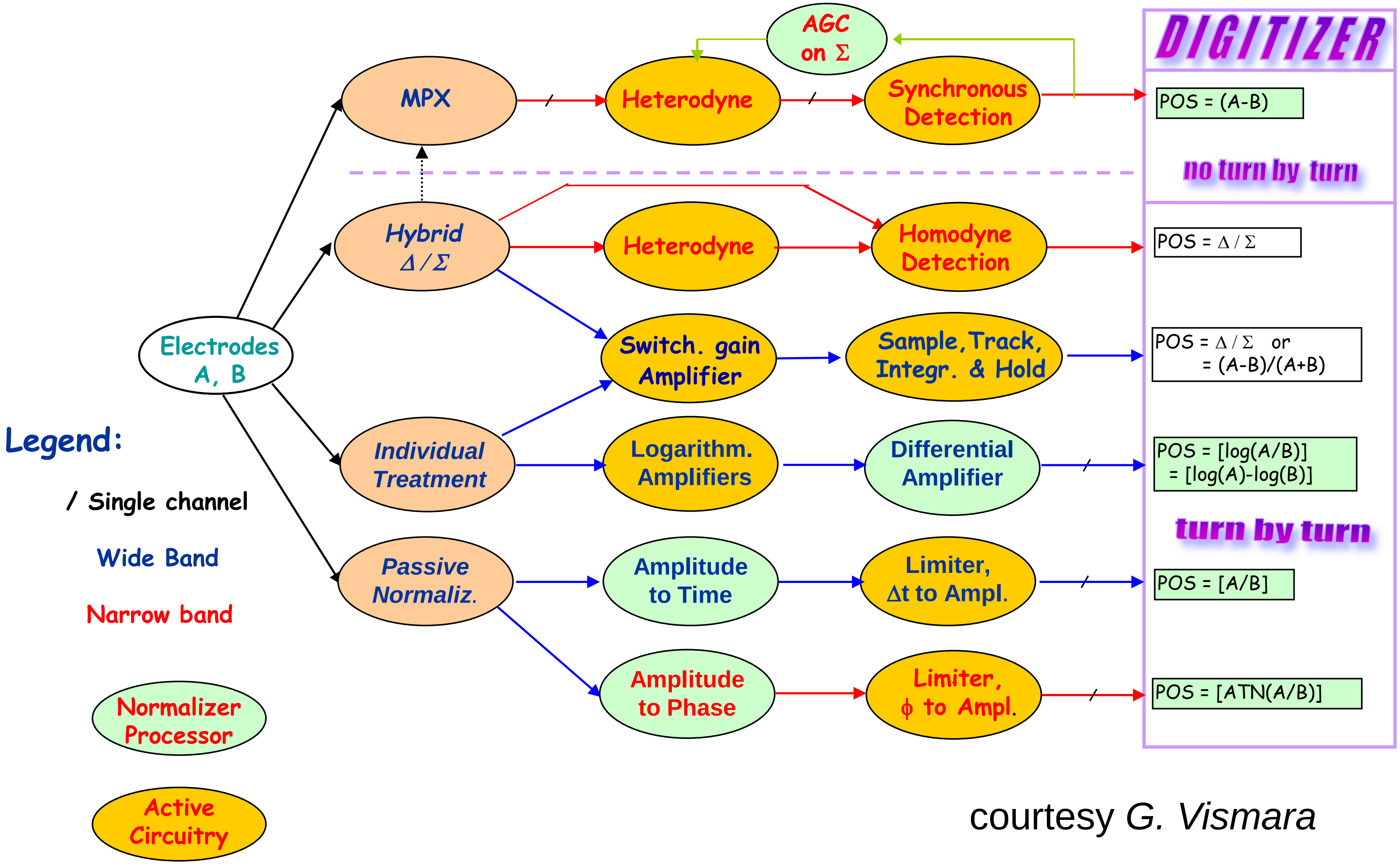
button BPM spectrum
with harmonics at
f_{rev} or f_{bunch}

- **Basically, the position information of a broadband BPM is available at any frequency**
  - and is independent of the frequency for relativistic beams $v \approx c$
  - the broad spectral response of the BPM can be band limited without compromising the position detection:
    Apply appropriate analog signal conditioning!

Signal Processing & Normalization

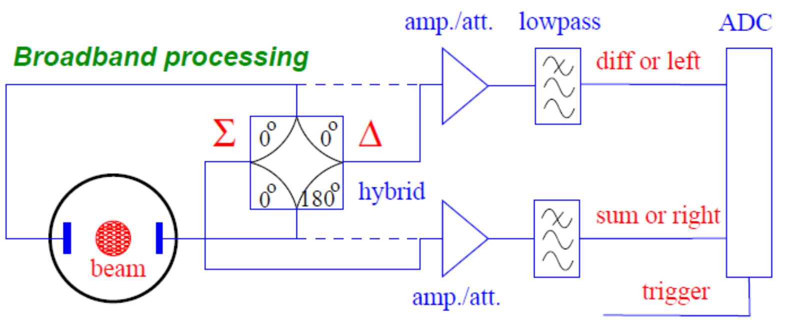
- Extract the beam position information from the electrode signals: Normalization
  - Analog using Δ-Σ or 90^0-hybrids, followed by filters, amplifiers mixers and other elements, or logarithmic amplifiers.
  - Digital, performing the math on individual digitized electrode signals.
- Decimation / processing of broadband signals
  - BPM data often is not required on a bunch-by-bunch basis
    - Exception: Fast feedback processors
    - Default: Turn-by-turn and “narrowband” beam positions
  - Filters, amplifiers, mixers and demodulators in analog and digital to decimate broadband signals to the necessary level.
- Other aspects
  - Generate calibration / test signals
  - Correct for non-linearities of the beam position response of the BPM
  - Synchronization of turn-by-turn and /or bunch-by-bunch data
  - Optimization on the BPM system level to minimize cable expenses.
  - BPM signals keep other very useful information other than that based on the beam displacement, e.g.
    - Beam intensity, beam phase (timing)

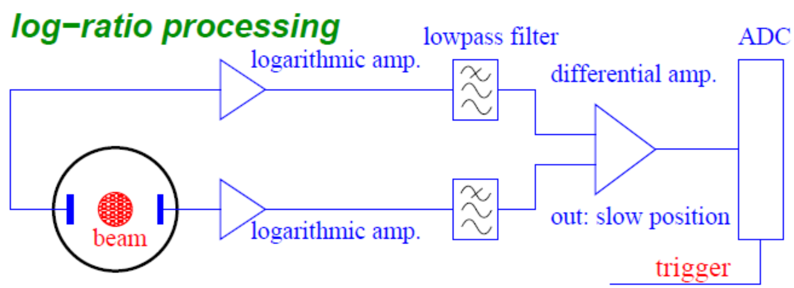
Analog Signal Processing Options



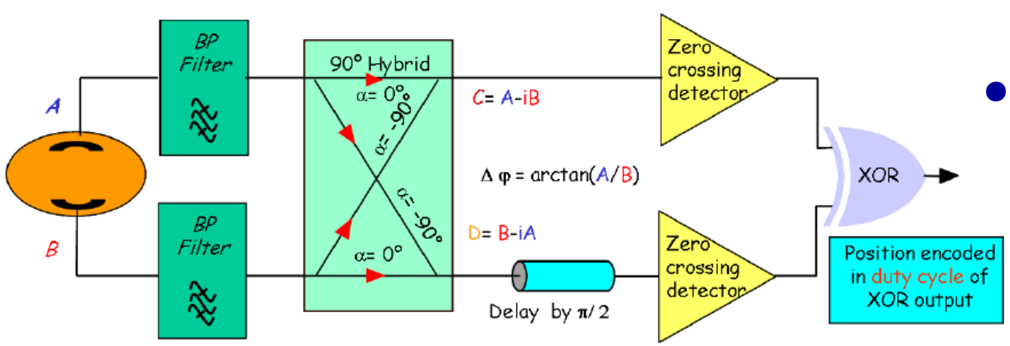
courtesy *G. Vismara*

Examples: RF Analog BPM Processors





courtesy *M. Bozzolan*

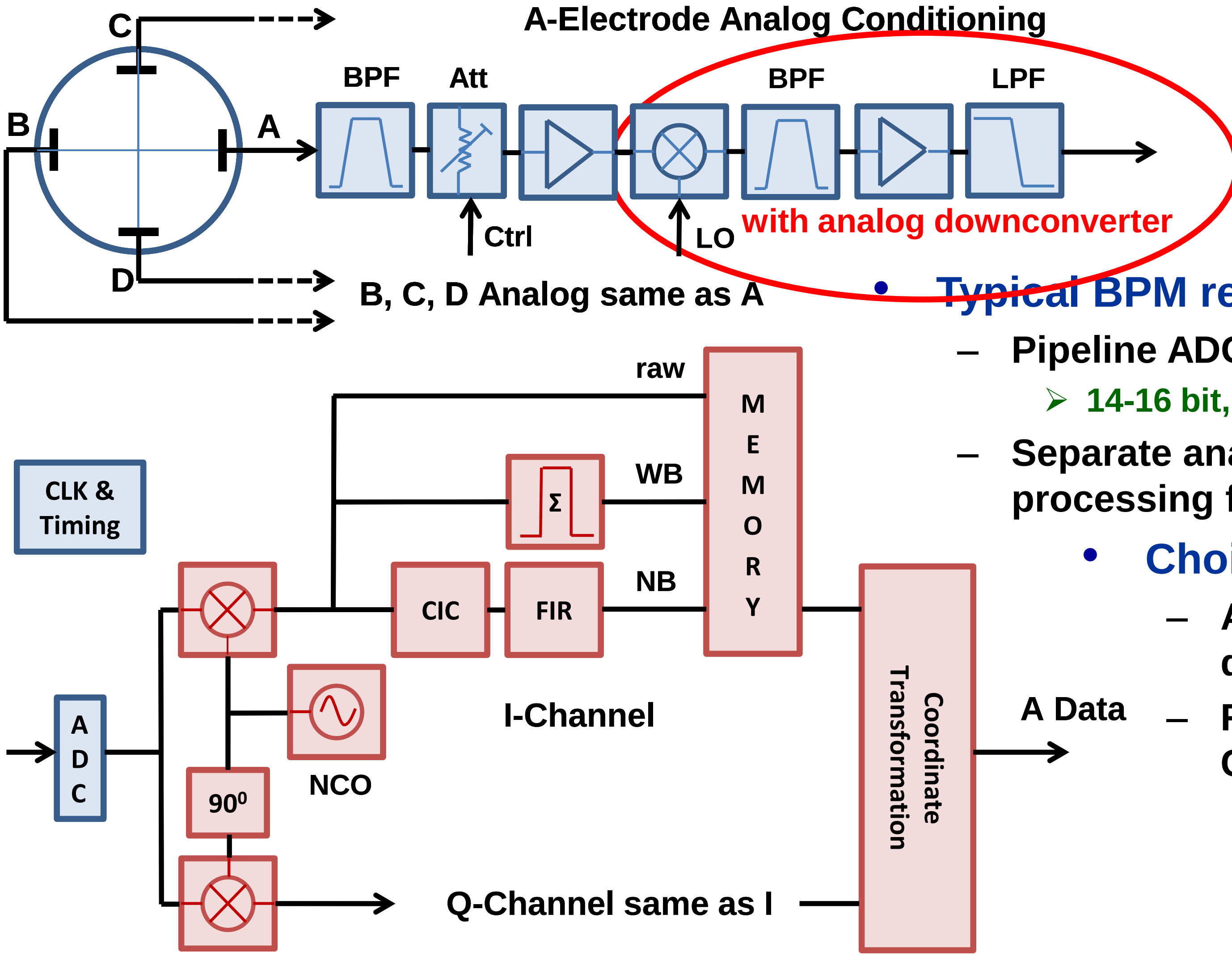


- **Δ/Σ broadband**
  - **Hybrid performance**
  - **Phase-matched cables**
  - **Gain switching**
- **LogAmps: $\log_{10}(A/B)$**
  - **Dynamic compression**
    - **Reduced position sensitivity**
  - **Limited bandwidth**
    - **TbT: yes, BbB: maybe?!**
- **$\pi/2$-hybrid: $\arctan(A/B)$**
  - **Broadband: BbB**
  - **Phase-matching**
  - **~40 dB dynamic range**

Digital BPM Signal Processing

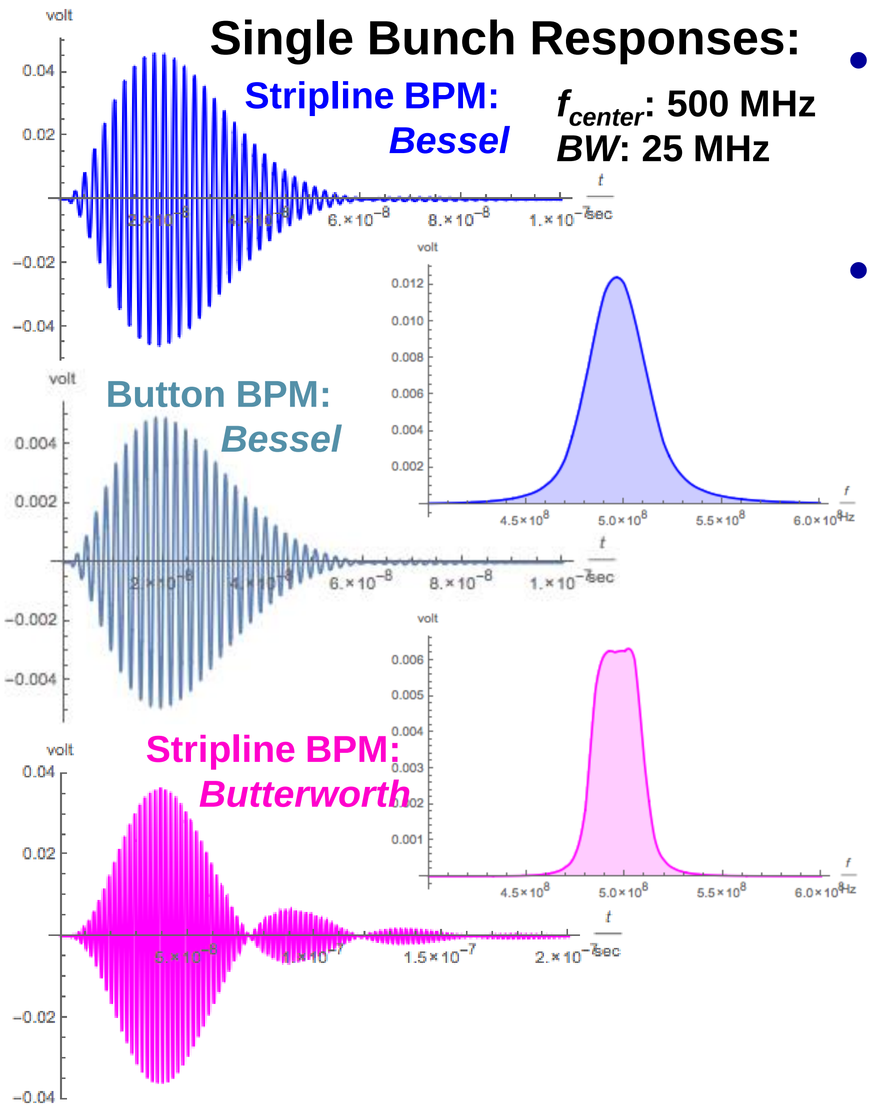
- **Why digital signal processing?**
  - **Better reproducibility of the beam position measurement**
    - **Robust to environmental conditions, e.g. temperature, humidity, (radiation?)**
    - **No slow aging and/or drift effects of components**
    - **Deterministic, no noise or statistical effects on the position information**
  - **Flexibility**
    - **Modification of FPGA firmware, control registers or DAQ software to adapt to different beam conditions or operation requirements**
  - **Performance**
    - **Often better performance, e.g. higher resolution and stability compared to analog solutions**
    - **No analog equivalent of digital filters and signal processing elements.**
- **BUT: Digital is not automatically better than analog!**
  - **Latency of pipeline ADCs (FB applications)**
  - **Quantization and CLK jitter effects, dynamic range & bandwidth limits**
  - **Digital BPM solutions tend to be much more complex than some analog signal processing BPM systems**
    - **Manpower, costs, development time, firmware / software maintenance**

Typical BPM Read-out Electronics



- **Typical BPM read-out scheme**
  - Pipeline ADC & FPGA
    - ➢ 14-16 bit, >300 MSPS, >60 dB S/N
  - Separate analog signal processing for the channels
    - **Choices:**
      - Analog downconverter?!
      - RF locked (sync) CLK & LO signals?!
        - ➢ No I-Q required

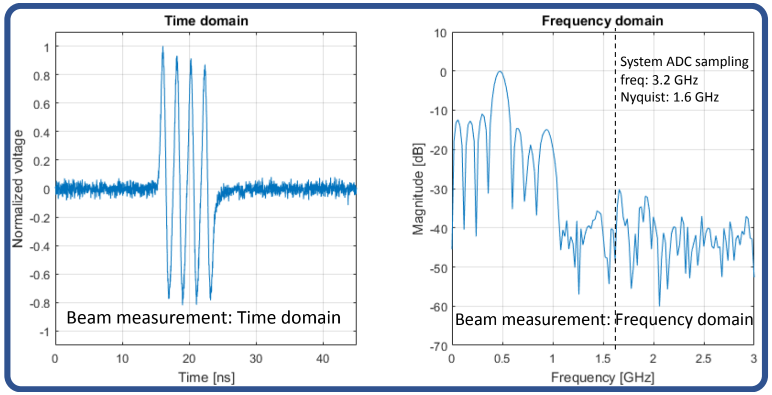
"Ringing" Bandpass-Filter (BPF)

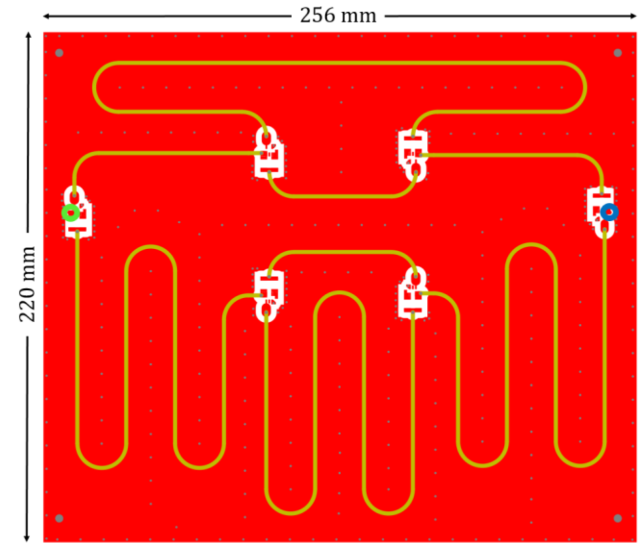


- **BPM electrode signal energy is highly time compressed**
  - **Most of the time: "0 volt"!**
- **A "ringing" bandpass filter "stretches" the signal**
  - **Passive RF BPF**
    - ➢ **Matched pairs!**
  - **f_{center} matched to f_{rev} or f_{bunch}**
    - ➢ **Quasi sinusoidal waveform**
  - **Reduces output signal level**
    - ➢ **Narrow BW: longer ringing, lower signal level**
  - **Linear group delay designs**
    - ➢ **Minimize envelope ringing**
    - ➢ ***Bessel*, *Gaussian*, time domain designs**

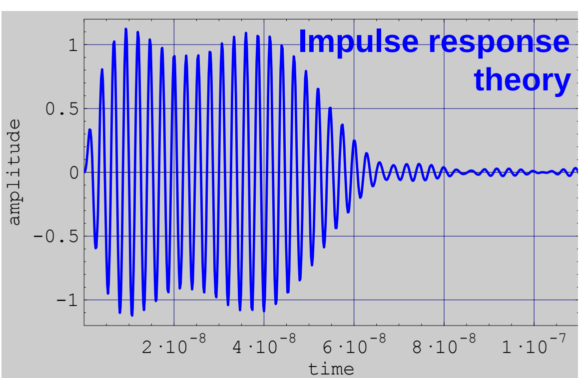
Time Domain BPF Optimization

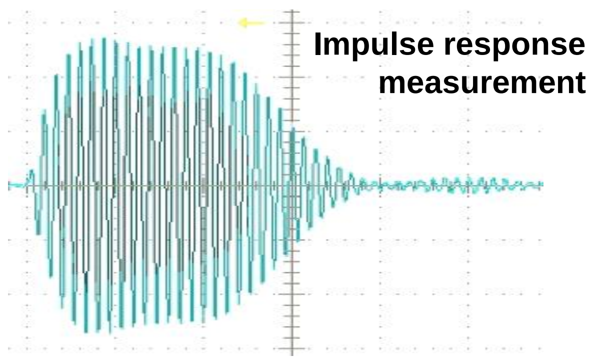
- **Delay-line "comb" or analog FIR BPF**
  - **500 MHz BPF R&D for LHC iBPMs**

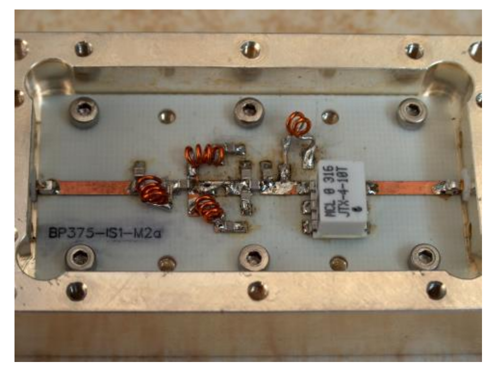




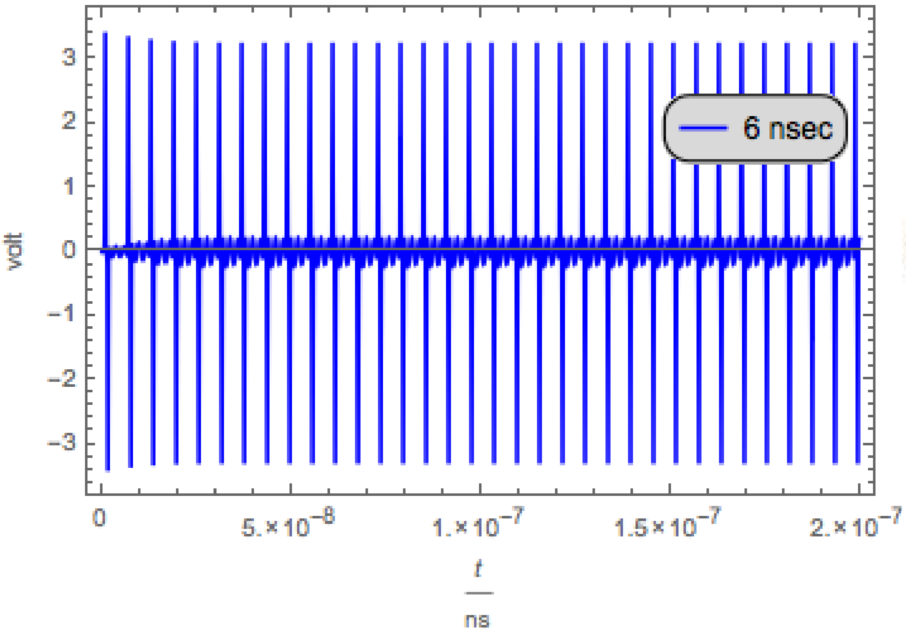
- **Rectangular impulse response approximation**
  - **375 MHz lumped element BPF, BW ~10 MHz**

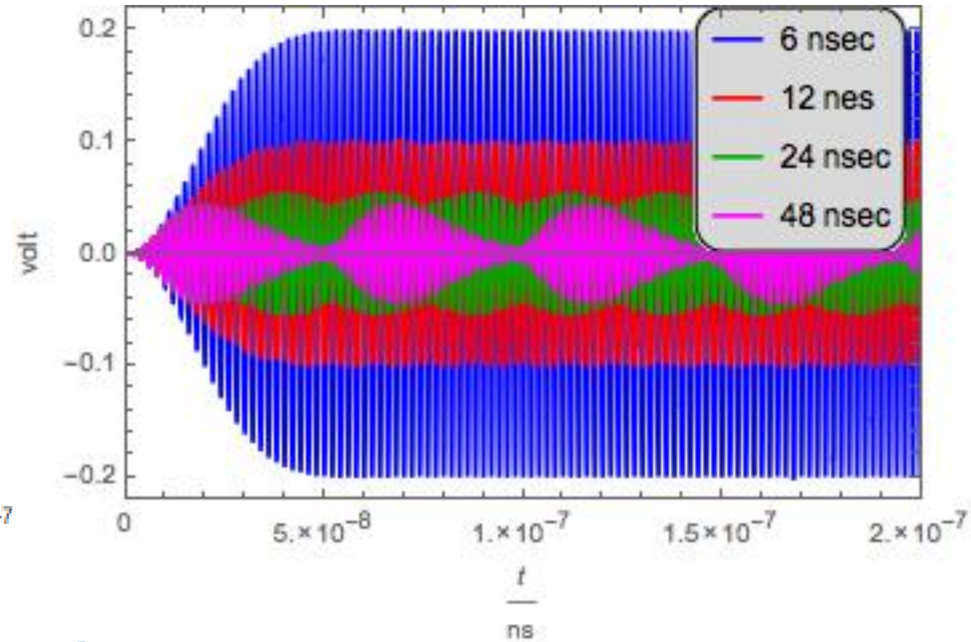






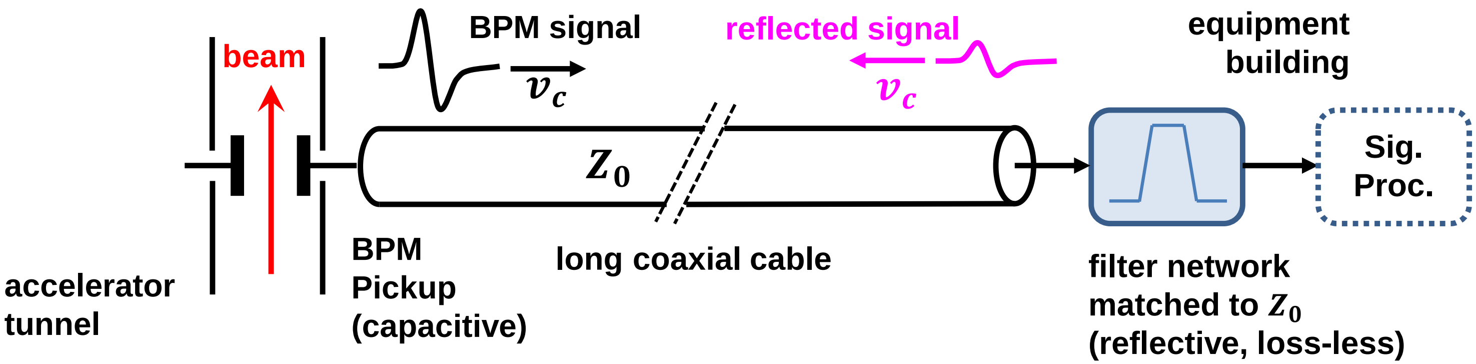
“Ringing” BPF & Multi-Bunches

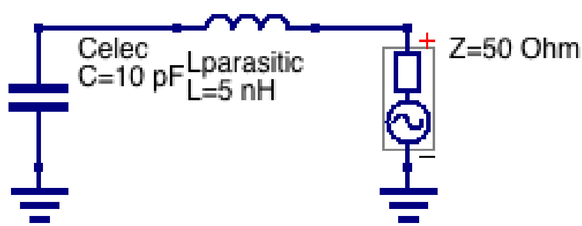


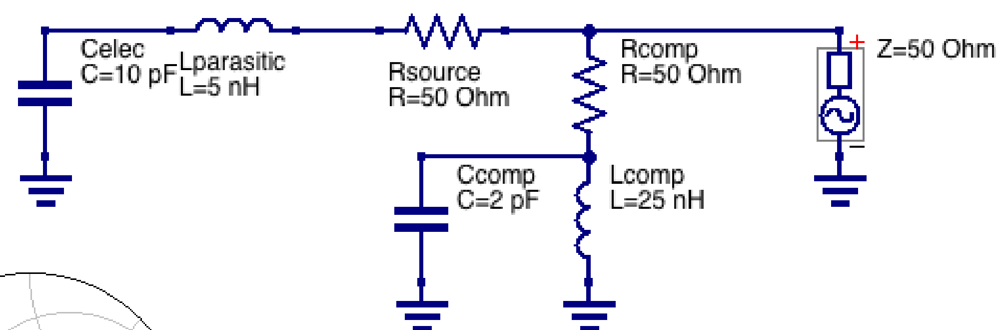


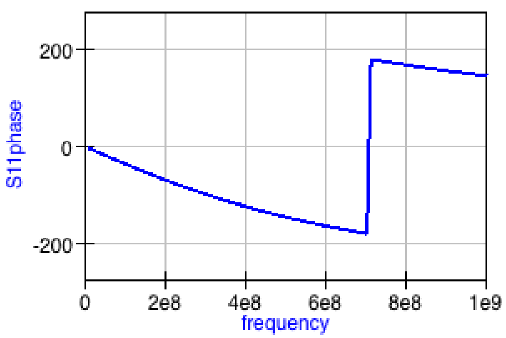
- **Bunch spacing < BPF ringing time:**
  - **Superposition of single bunch BPF responses**
  - **More continuous “ringing”, smearing of SB responses**
- **Bunch spacing < BPF rise time**
  - **Constructive signal pile-up effect**
    - ➢ **Output signal level increases linear with decreasing bunch spacing**

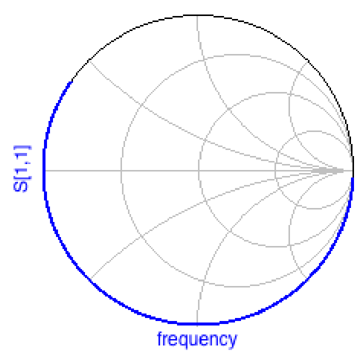
Fighting Reflections!





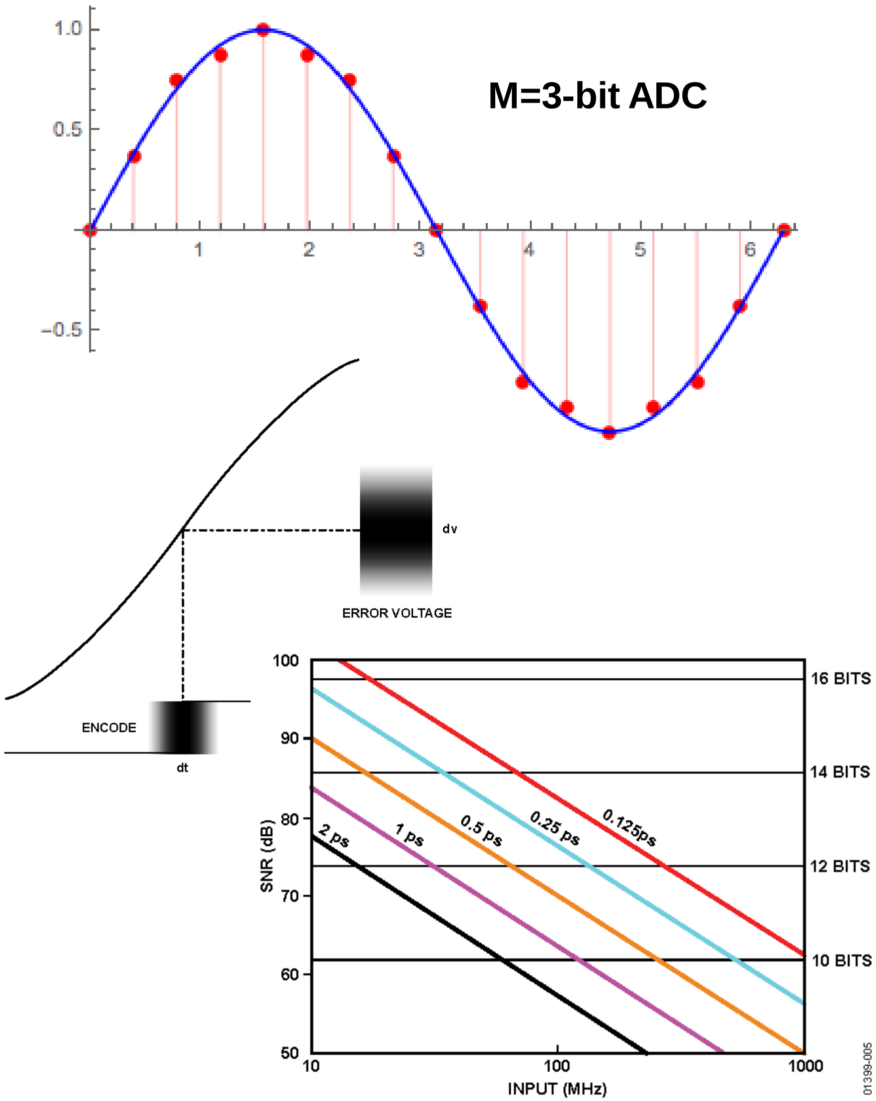






- **Impedance matching of a button BPM**
  - **Broadband with dual network!**

Analog Digital Converter



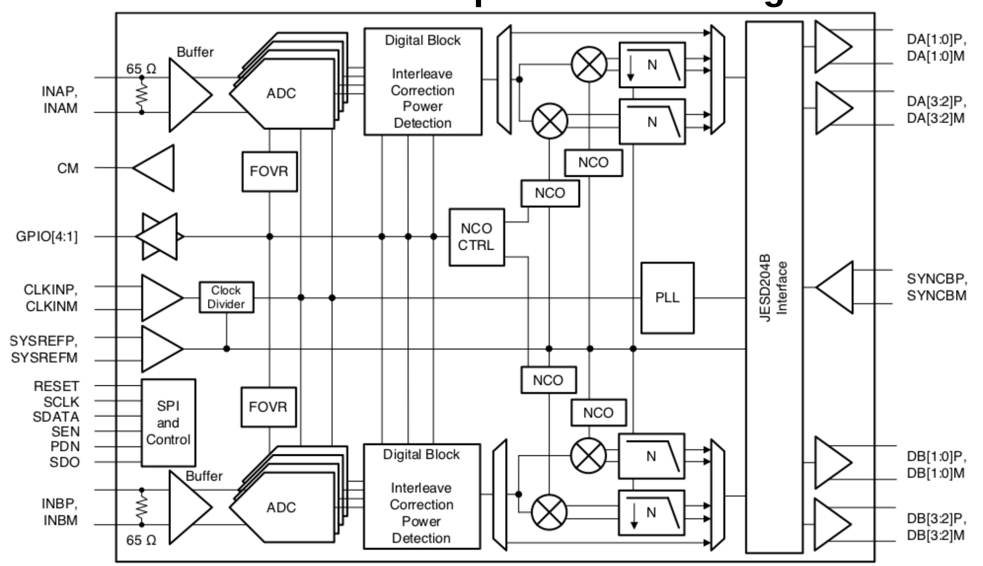
- **Quantization of the continuous input waveform at equidistant spaced time samples**
  - **Digital data is discrete in amplitude and time**
- **LSB voltage (resolution)** $Q = \dfrac{V_{FSR}}{2^M}$
  - **E.g. 61 µV (14-bit),**
    15 µV (16-bit) @ 1 volt V_{FSR}
- **Quantization error (dynamic range)** $SQNR = 20\log_{10}(2^M)$
  - **E.g. 84 dB (14-bit), 96 dB (16 bit)**
- **SNR limit due to aperture jitter** $SNR = -20\log_{10}(2\pi f t_a)$
  - **E.g. 62 dB@500 MHz, 0.25 psec (equivalent to EOB=10.3)**

14-16 bit ADC Technology (2018)

	Type	Res. [bit]	Ch.	Power [W]	f_s (max) [GSPS]	BW [GHz]	SNR @ f_{in} [dB @ GHz]
AD	AD9208	14	2	3.3	3	9	59.5 @ 2.6
TI	ADC32RF45	14	2	6.4	3	3.2	56.8 @ 2.6
TI	ADS54J60	16	2	2.7	1	1.2	67.5 @ 0.35

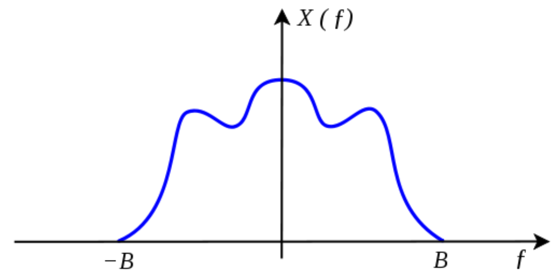
- **Dual Channel**
  - **I-Q sampling with separate ADCs**
- **Pipeline architecture**
  - **Continuous CLK**
  - **Data latency**
- **Signal post-processing**
  - **Mixers, NCO, CIC, etc.**

TI AD9208 Simplified Block Diagram

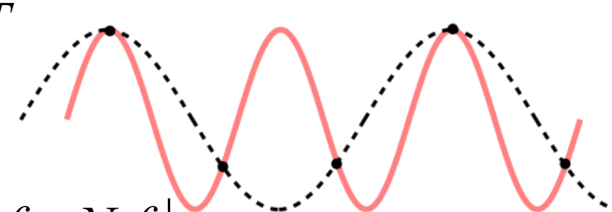


Sampling Theory

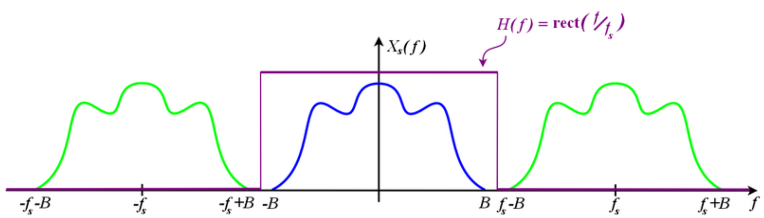
- **A band limited signal *x(t)* with $B=f_{max}$ can be reconstructed if** $f_s \geq 2 f_{\max}$
  - **Nyquist-Shannon theorem**
  - **The exact reconstruction of *x(t)* by $x_n=x(nT)$:**

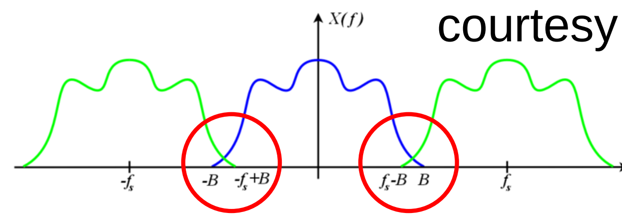


$$x(t) = \sum_{n=-\infty}^{+\infty} x_n \frac{\sin \pi(2 f_{\max} t - n)}{\pi(2 f_{\max} t - n)} = \sum_{n=-\infty}^{+\infty} x_n \,\mathrm{sinc}\, \frac{t - nT}{T}$$

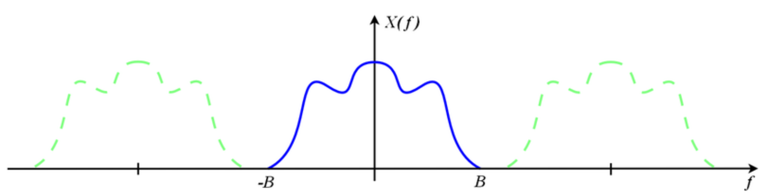


- **Aliasing of a sampled sin-function**
  - **Samples can be interpreted by** $f_{alias}(N) = \left| f - N f_s \right|$

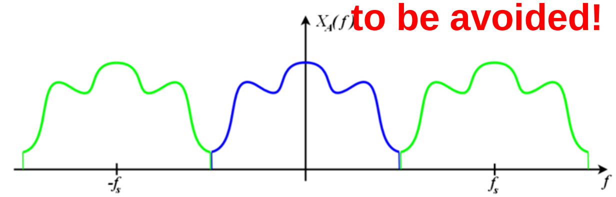




courtesy *Wikipedia*



to be avoided!



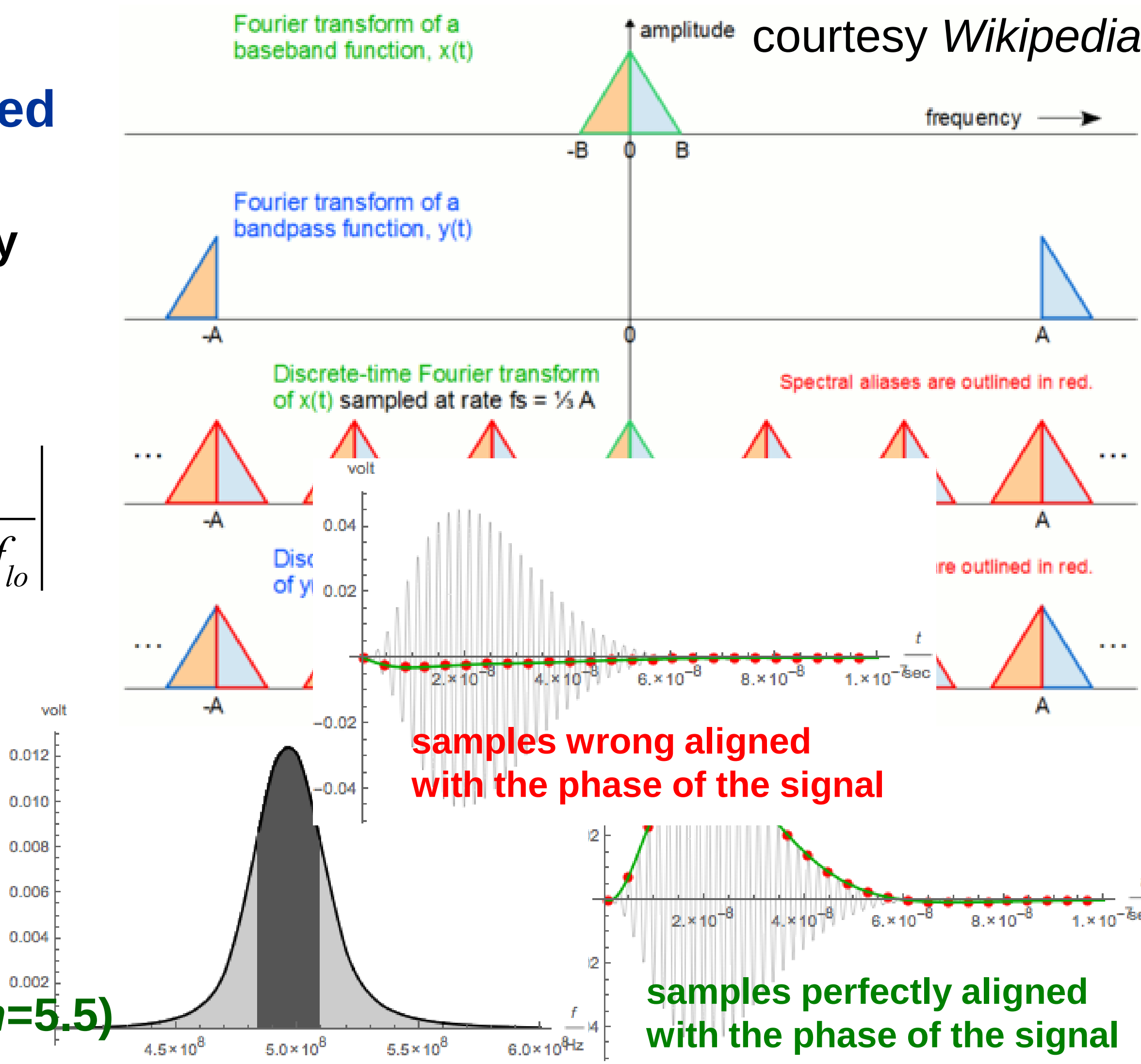
Bandpass or Undersampling

- **A bandpass signal $f_{lo}=A$, $f_{hi}=A+B$ is down-converted to baseband**
  - **The sampling frequency has to satisfy:**

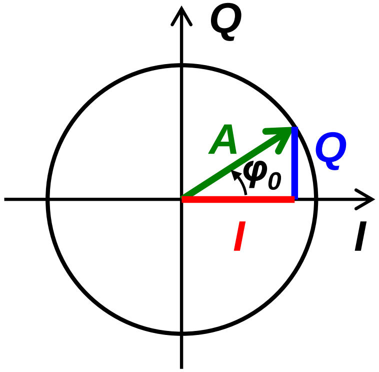
$$\frac{2f_{hi}}{n} \le f_s \le \frac{2f_{lo}}{n-1}$$

$$\text{with: } 1 \le n \le \left|\frac{f_{hi}}{f_{hi}-f_{lo}}\right|$$

- **Digital down-conversion (DDC) of BPM signals**
  - **BPM -> BPF (Bessel)**
    - ➢ ***f*$_{center}$: ~500 MHz**
      ***BW* (3 dB): 25 MHz**
    - ➢ ***T*=4 ns, *f*$_s$=200 MHz**
      (f_{hi}/f_{lo}=550/450 MHz, *n*=5.5)



I-Q Sampling



$$y(t) = I \sin \omega t + Q \cos \omega t$$

$$I = A \cos \varphi_0 \qquad A = \sqrt{I^2 + Q^2}$$

$$Q = A \sin \varphi_0 \qquad \varphi_0 = \arctan\left(\frac{Q}{I}\right)$$

- **Vector representation of sinusoidal signals:**
  - **Phasor rotating counter-clockwise (pos. freq.)**

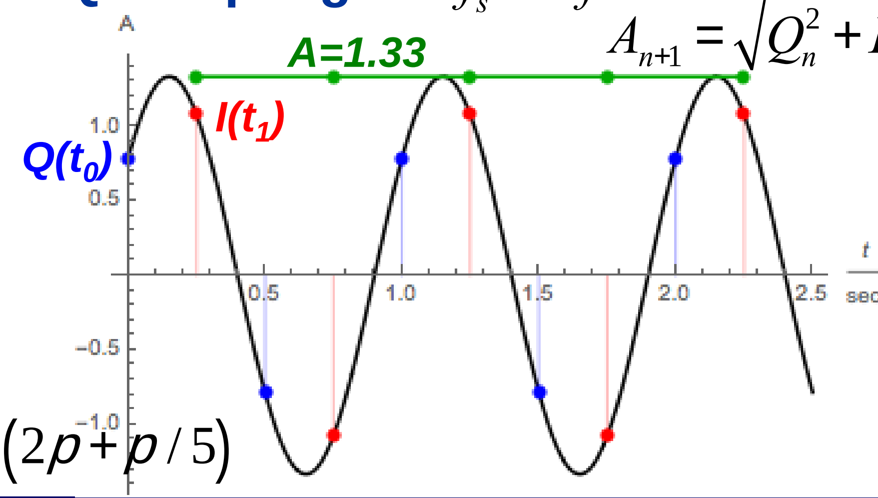
$$y(t) = A \sin\left(\omega t + \varphi_0\right)$$

$$y(t) = \underbrace{A \cos \varphi_0}_{=: I} \sin \omega t + \underbrace{A \sin \varphi_0}_{=: Q} \cos \omega t$$

I: in-phase component **Q: quadrature-phase component**

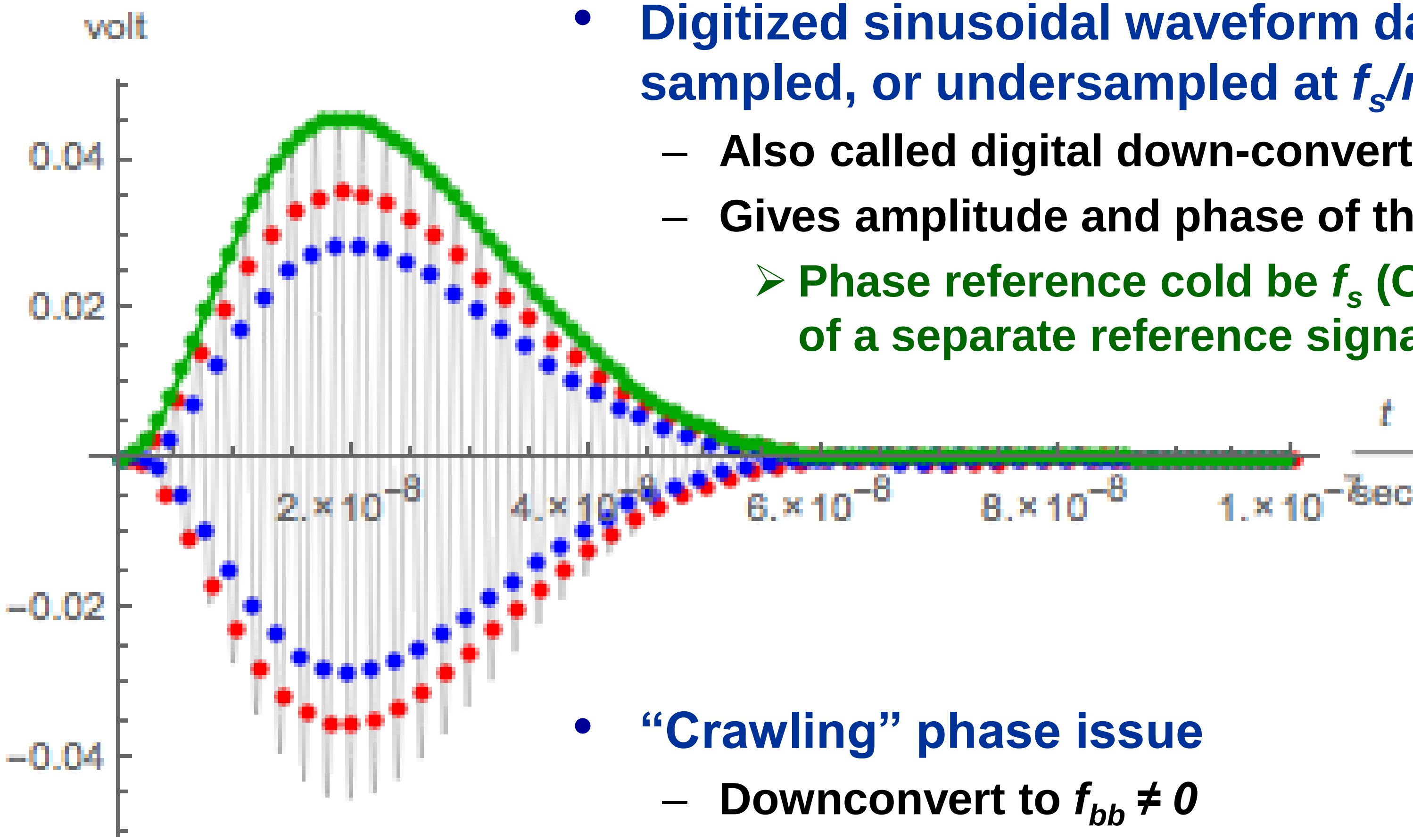
- **I-Q sampling at:** $f_s = 4f$

$$A_{n+1} = \sqrt{Q_n^2 + I_{n+1}^2}$$



$$y(t) = 1.33 \sin\left(2\pi + \pi / 5\right)$$

I-Q Demodulation of BPM Signals



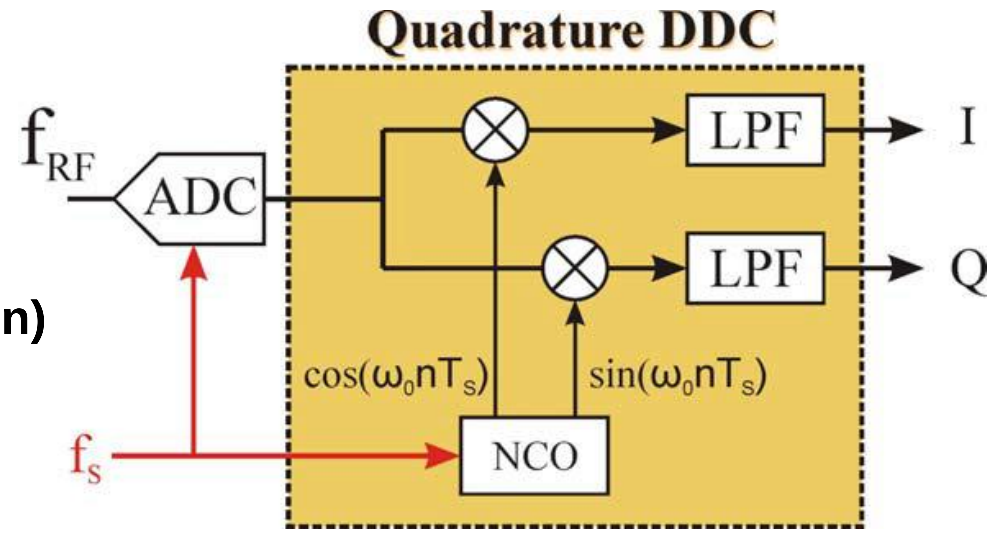
***n*=2**

- **Digitized sinusoidal waveform data is sampled, or undersampled at $f_s/n = 4xf_{in}$**
  - **Also called digital down-converter (DDC)**
  - **Gives amplitude and phase of the input signal**
    - **Phase reference cold be f_s (CLK), of a separate reference signal**

- **"Crawling" phase issue**
  - **Downconvert to $f_{bb} \neq 0$**
  - **For TbT BPM measurements ensure $f_{bb}=i\ f_{rev}$**

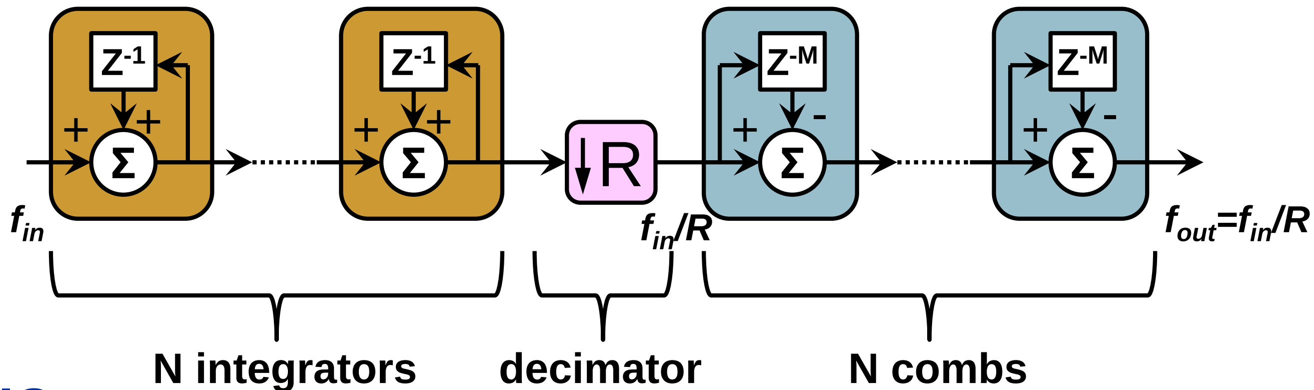
Digital Down-Converter

- **Goals**
  - **Convert the band limited RF-signal to baseband (demodulation)**
  - **Data reduction (decimation)**
- **DDC Building blocks:**
  - **ADC**
    - ➢ **Single fast ADC (oversampling)**
  - **Local oscillator**
    - ➢ **Numerically controlled oscillator (NCO) based on a direct digital frequency synthesizer (DDS)**
  - **Digital mixers ("ideal" multipliers)**
  - **Decimating low pass (anti alias) filters**
    - ➢ **Filtering and data decimation.**
    - ➢ **Implemented as CIC and/or FIR filters**



courtesy *T. Schilcher*

Cascaded Integrator Comb Filter (CIC)



- **Decimating CIC**
  - **Boxcar filter (anti aliasing)**
    - ➢ **non-recursive moving average filter**
  - **Decimator**
    - ➢ **Data rate reduction**
  - **Comb filter**
    - ➢ **Recursive running-sum**
- **Economical implementation**
  - **No multiplier, minimum storage requirements**

$$H(z) = H_I^N(z)\,H_C^N(z)$$

$$= \frac{1}{\left(1 - z^{-1}\right)^N}\left(1 - z^{-RM}\right)^N$$

$$= \left(\sum_{k=0}^{RM-1} z^{-k}\right)^N$$

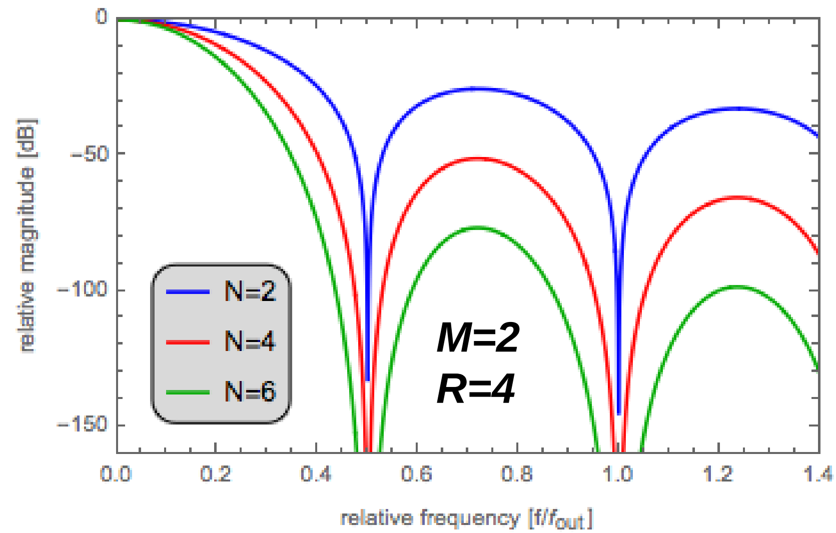
FIR filter (stable)

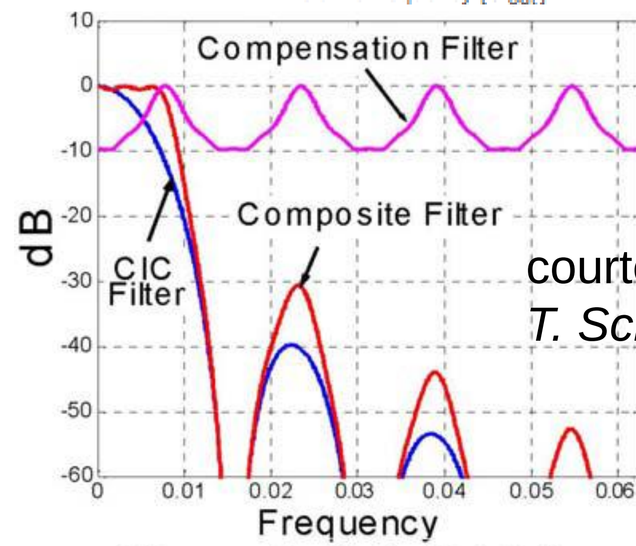
CIC Filter (cont.)

- **CIC frequency response**

$$H(f)=\left(\frac{\sin \pi M \frac{f}{f_{out}}}{\sin \frac{\pi}{R}\frac{f}{f_{out}}}\right)^{N}$$

  - **With respect to the output frequency:** $f_{out}=\frac{f_{in}}{R}$
  - ***M:* differential delay, determines the location of the zeros:** $f_0=k\frac{f_{out}}{M}$
  - ***N*: number of CIC stages**
  - ***R*: decimation ratio**
    - ➢ **Has little influence on the filter response**
- **CIC plus FIR compensation filter**
  - **Compensate CIC passband drop**

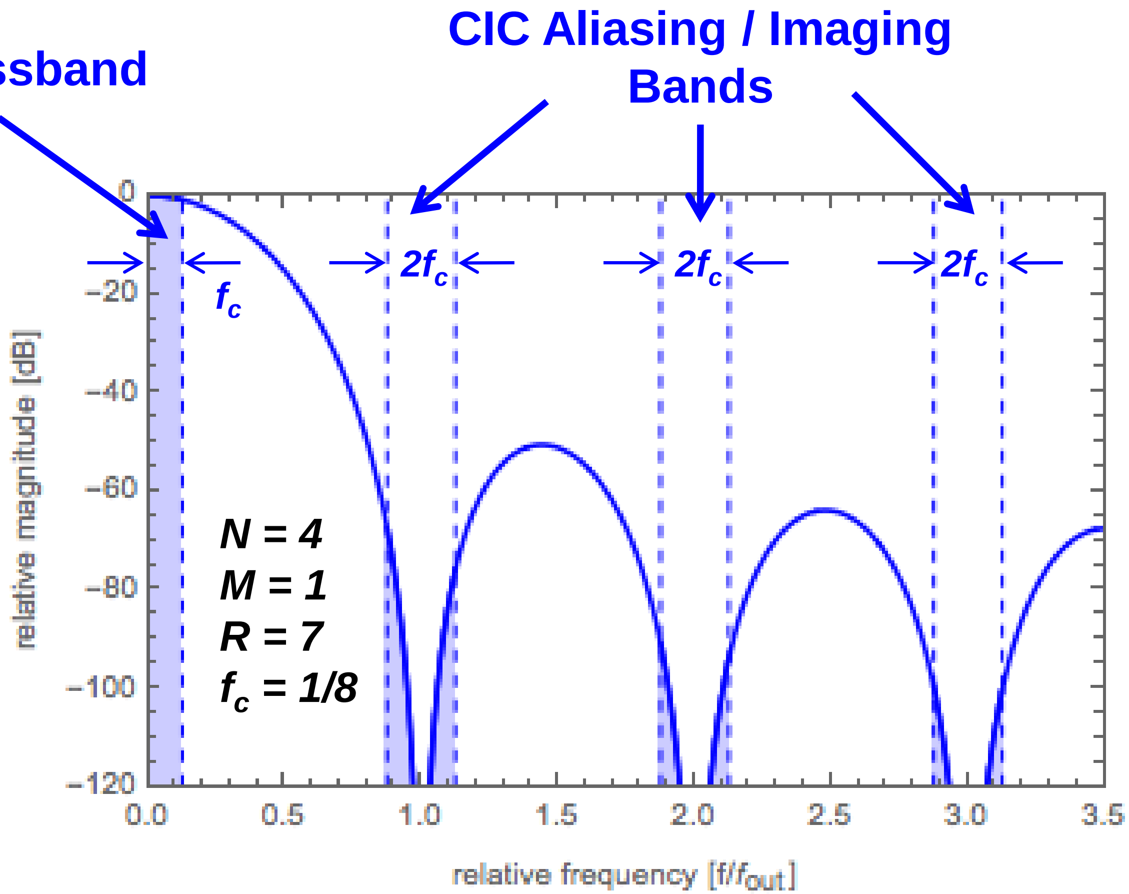




CIC compensation filter for M = 3, R = 64.

courtesy
T. Schilcher

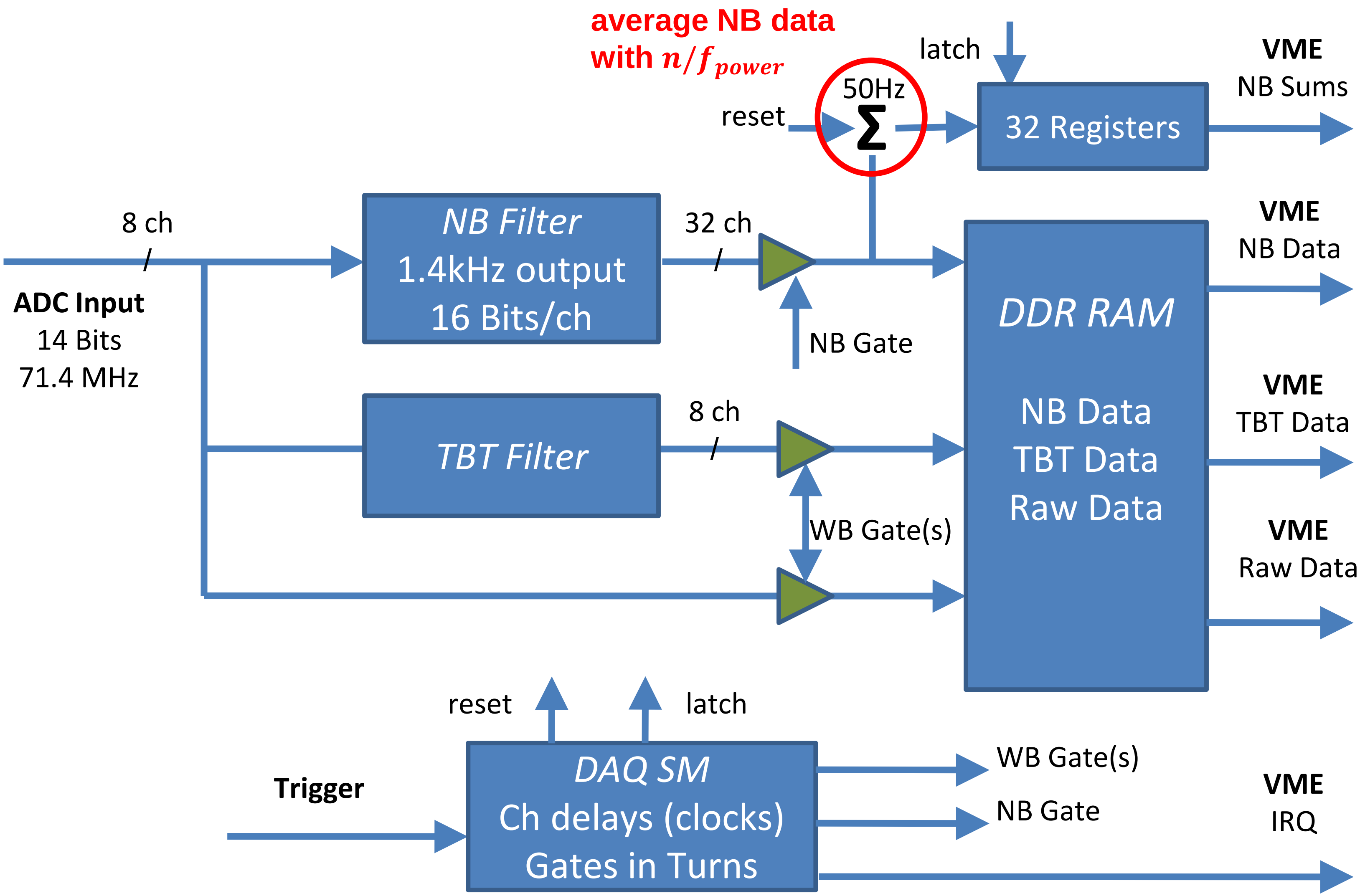
CIC Aliasing – Imaging



courtesy
E. Hogenauer

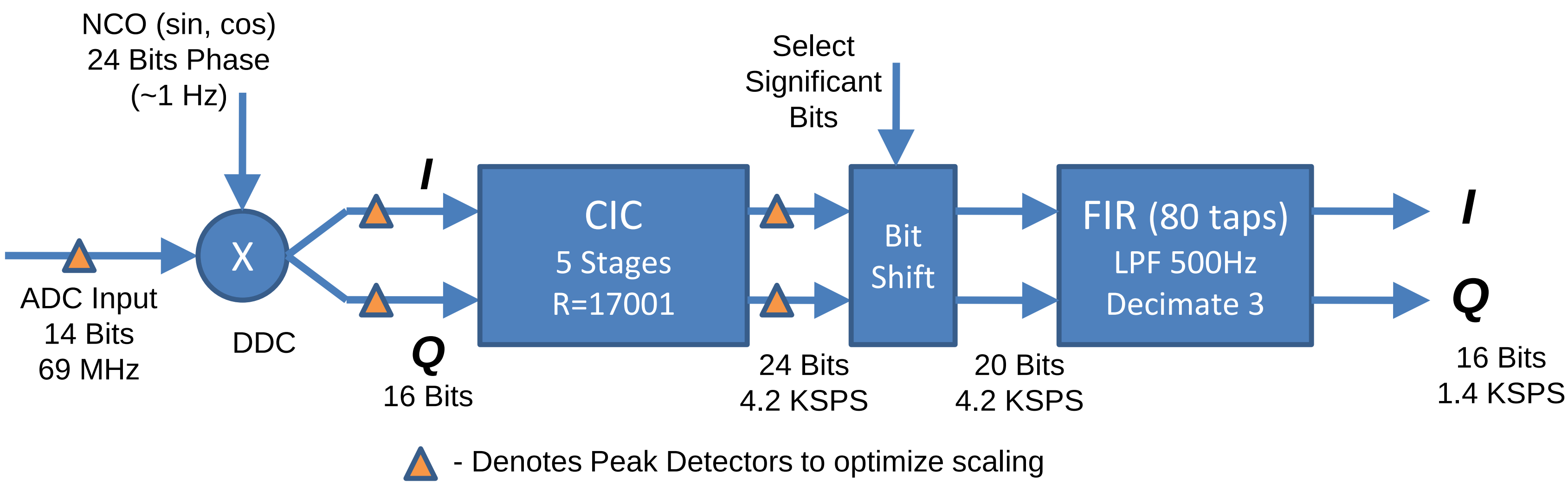
- **CIC aliasing / imaging bands are around:** $(i - f_c) \le f \le (i + f_c)$

Example: ATF DR BPM Signal Processing

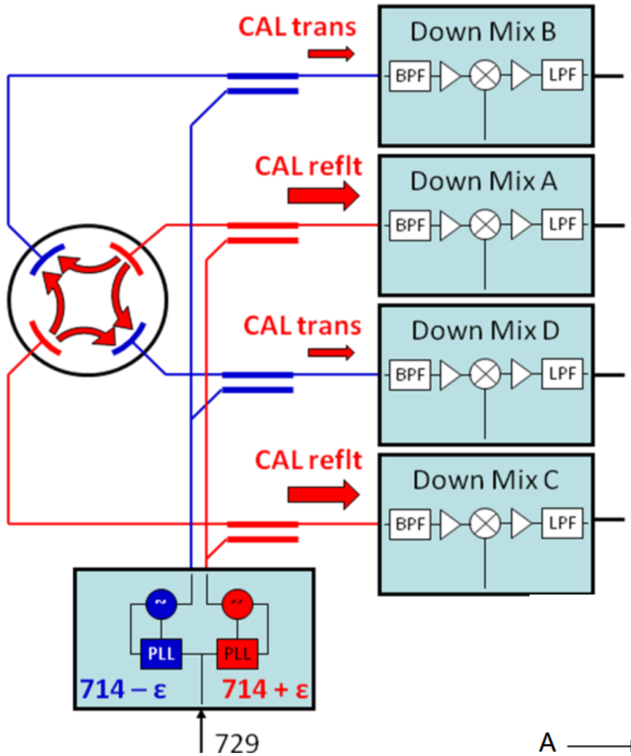


ATF BPM Narrowband Signal Processing

- **Process 8 ADC channels in parallel up to FIR filter**
  - **Digitally downconvert each channel into *I,Q* then filter I,Q independently**
  - **CIC Filters operating in parallel at 71.4MHz**
    - **Decimate by 17KSPS to 4.2KSPS output rate**
  - **1 Serial FIR Filter processes all 32 CIC Filter outputs**
    - **80 tap FIR (400 Hz BW, 500 Hz Stop, -100 db stopband) -> 1KHz effective BW**
    - **Decimate by 3 to 1.4 KSPS output rate -> ability to easily filter 50Hz**
  - **Calculate Magnitude from *I,Q* at 1.4KHz**
    - **Both Magnitude and I,Q are written to RAM**
    - **Also able to write I,Q output from CIC to RAM upon request**



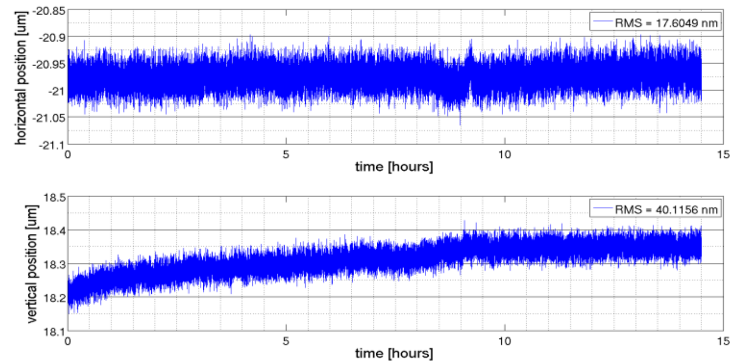
Long-Term Drift Compensation

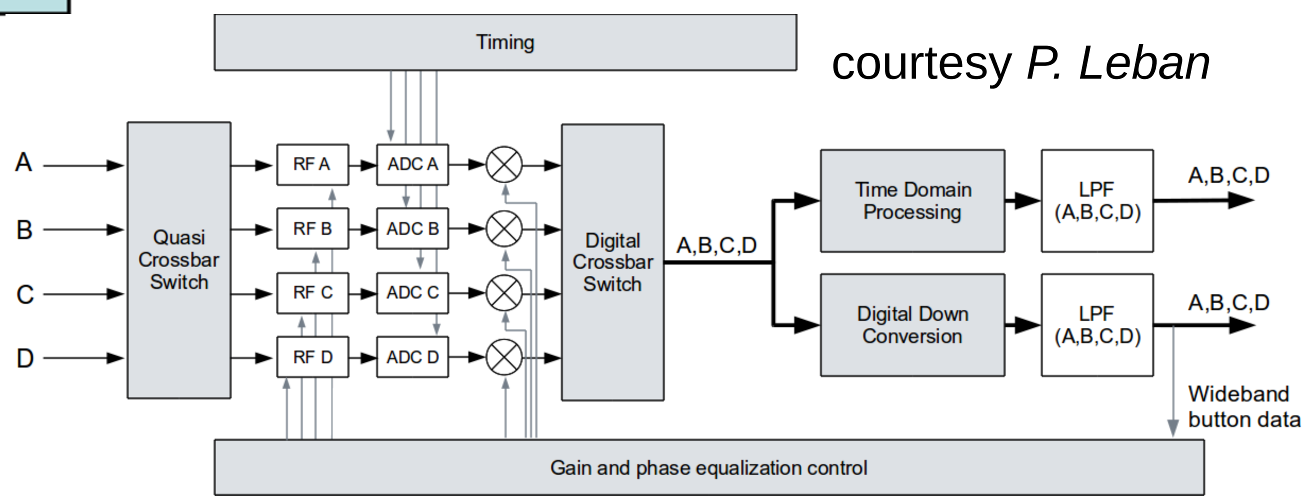


courtesy *N. Eddy*

- **Calibration tone technique (only in narrowband operation)**
  ATF (KEK)

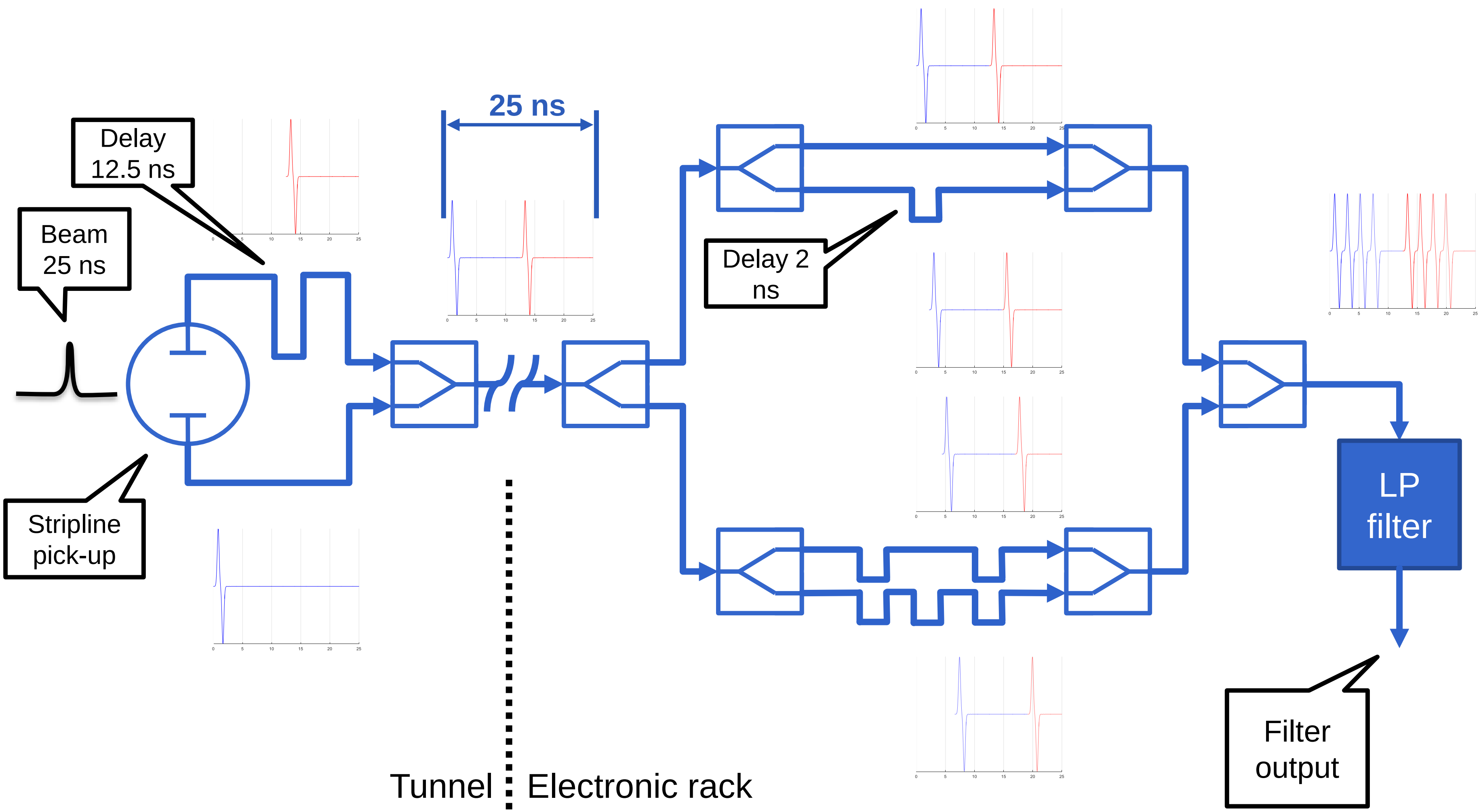
- ***Libera* crossbar switching technique**
  - **<100 nm stability over 14 hours**





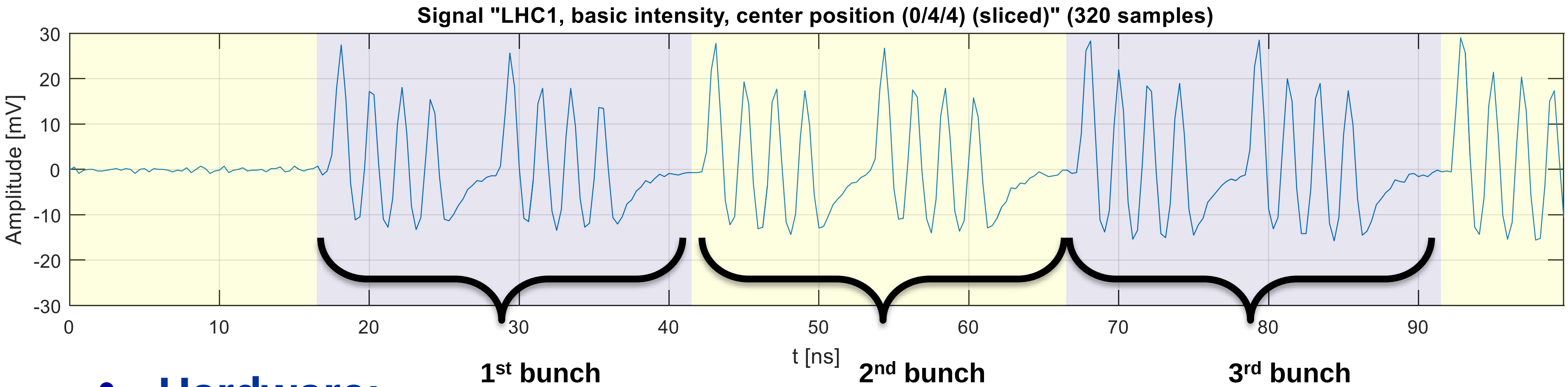
courtesy *P. Leban*

Time-multiplexed BPM Read-out



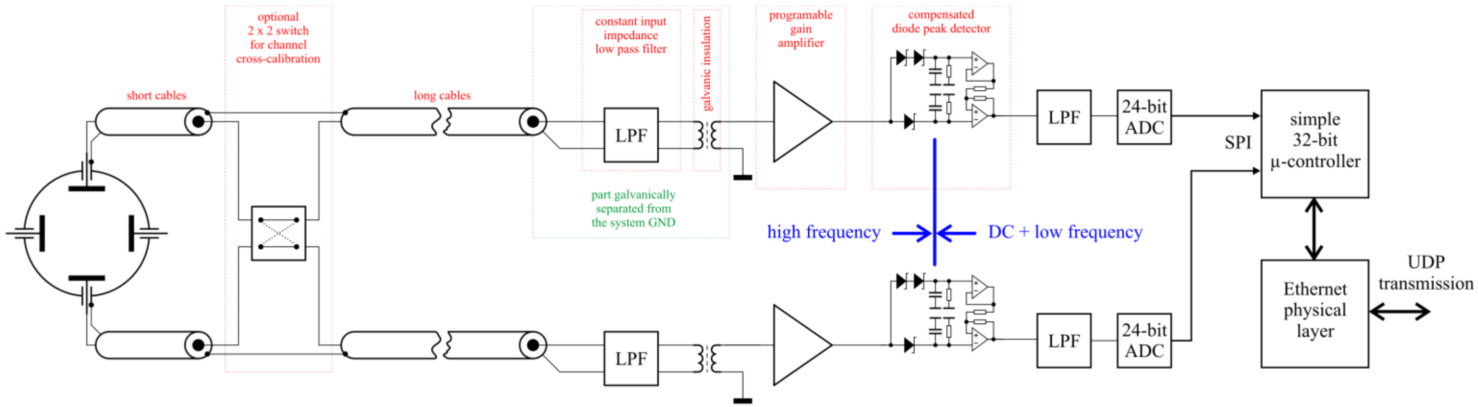
Prototyping Time-multiplexed BPM

- **Target: LHC interlock BPMs**
  - **Typical one-turn acquisition (first 100 ns):**



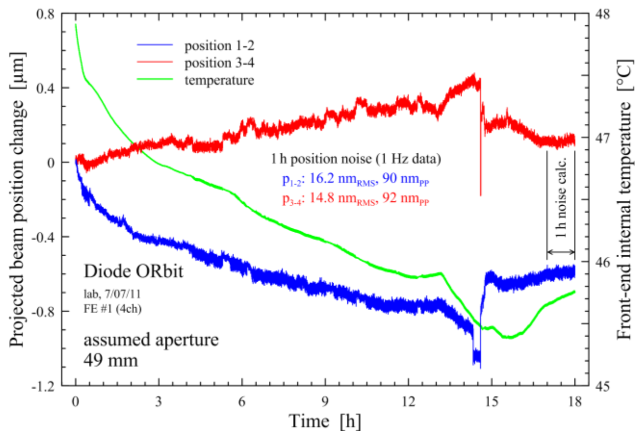
- **Hardware:**
  - **LHC stripline BPM with delay-lines and in-house comb BPF**
  - **Commercial FMC digitizer Vadatech FMC225 (12-bit, 4 GSPS)**
  - **CERN VME FMC carrier**
- **Raw data analysis**
  - **Python scripts, bunch-by-bunch RMS algorithm**

Compensated Diode Detector for BOM

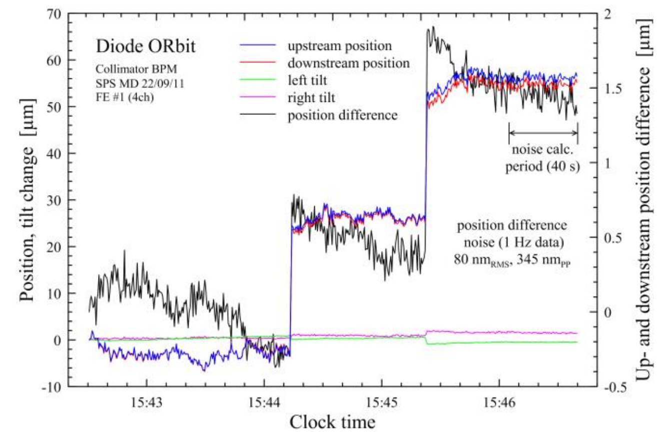


courtesy *M. Gasior*

Diode ORbit (DOR) Measurement



2 channels shown for one pick-up plane,
one 19" 1U unit accommodates 8 channels



- Sub-micrometre resolution can be achieved with relatively simple hardware and signals from any position pick-up.
- To be used for the future LHC collimators with embedded BPMs.

Signal/Noise & Theoretical Resolution Limit

- **Minimum noise voltage at the 1st gain stage:** $v_{noise} = \sqrt{4 k_B T R \Delta f}$
  - **With the stripline BPM and Bessel BPF example:** ***R*** **= 50 Ω,** ***Δf*** **= 25 MHz ➔** v_{noise} **= 4.55 μV (-93.83 dBm)**
- **Signal-to-noise ratio:** $S/N = \frac{\Delta v}{v_{noise}}$
  - **Where** ***Δv*** **is the change of the voltage signal at the 1st gain stage due to the change of the beam position (*Δx, Δy*).**
  - **Consider a signal level v ≈ 22.3 mV (-20 dBm)**
    - **Bessel BPF output signal of the stripline BPM example**
  - **22.3 mV / 4.55 μV ≈ 4900 (73.8 dB) would be the required dynamic range to resolve the theoretical resolution limit of the BPM**
    - **Under the given beam conditions, e.g. n=1e10, σ=25mm, single bunch, etc.**
    - **The equivalent BPM resolution limit would be: Δx=Δy=0.66μm (assuming a sensitivity of ~2.7dB/mm)**

S/N & BPM Resolution (cont.)

- **Factors which reduce the S/N**
  - **Insertion losses of cables, connectors, filters, couplers, etc.**
    - **Typically sum to 3…6 dB**
  - **Noise figure of the 1st amplifier, typically 1…2 dB**
  - **The usable S/N needs to be >0 dB, e.g. 2.3 dB is sometimes used as lowest limit. (*HP* SA definition)**
  - **For the given example the single bunch / single turn resolution limit reduces by ~10 dB (~3x): 2…3 µm**
- **Factors to improve the BPM resolution**
  - **Increase the signal level**
    - **Increase BPM electrode-to-beam coupling, e.g. larger electrodes**
    - **Higher beam intensity**
  - **Increase the measurement time, apply statistics**
    - **Reduce the filter bandwidth (S/N improves with $1/\sqrt{BW}$)**
    - **Increase the number of samples (S/N improves with $\sqrt{n}$)**

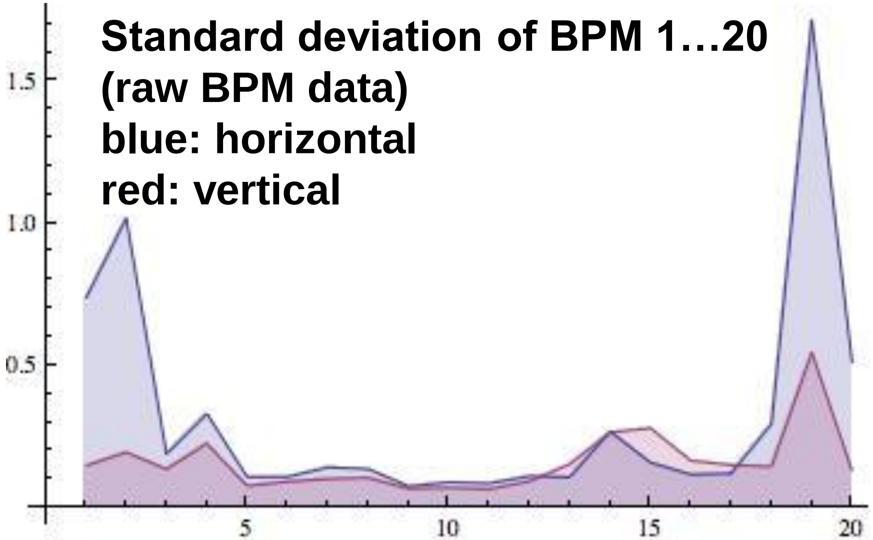
Singular Value Decomposition (SVD)

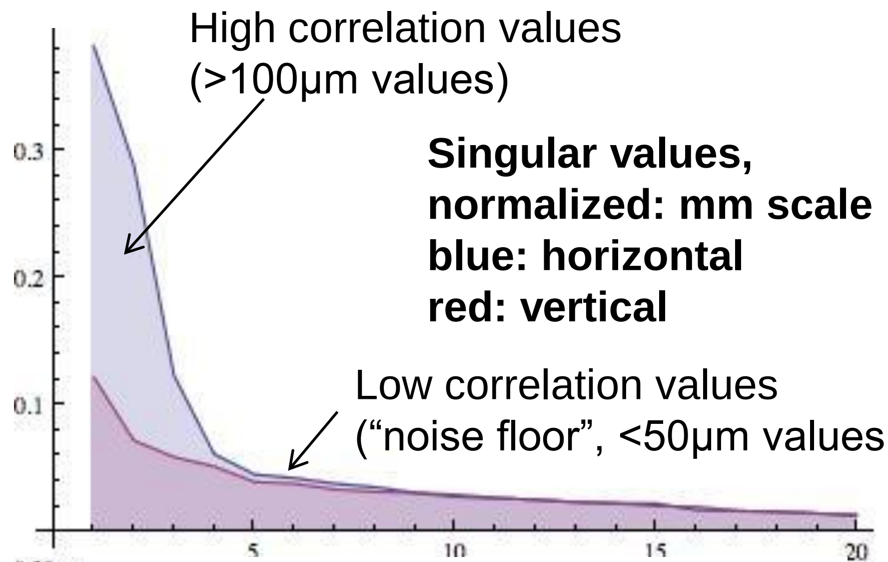
- **SVD:** ***B = U S V^T***

$$\begin{pmatrix} B_1s_1 & B_2s_2 & \cdots & B_Ms_1 \\ B_1s_2 & B_2s_2 & \cdots & B_Ms_2 \\ B_1s_3 & B_2s_3 & \cdots & B_Ms_3 \\ B_1s_4 & B_2s_4 & \cdots & B_Ms_4 \\ \cdots & \cdots & \cdots & \cdots \\ B_1s_P & B_2s_P & \cdots & B_Ms_P \end{pmatrix} = \begin{pmatrix} u_{11} & u_{12} & u_{13} & u_{14} & \cdots & u_{1P} \\ u_{21} & u_{22} & u_{23} & u_{24} & \cdots & u_{2P} \\ u_{31} & u_{32} & u_{33} & u_{34} & \cdots & u_{3P} \\ u_{41} & u_{42} & u_{43} & u_{44} & \cdots & u_{4P} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ u_{P1} & u_{P2} & u_{P3} & u_{P4} & \cdots & u_{PP} \end{pmatrix} \begin{pmatrix} s_{11} & 0 & \cdots & 0 \\ 0 & s_{22} & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & s_{PP} \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 0 \end{pmatrix} \begin{pmatrix} v_{11} & v_{12} & \cdots & v_{1M} \\ v_{21} & v_{22} & \cdots & v_{2M} \\ \cdots & \cdots & \cdots & \cdots \\ v_{M1} & v_{M2} & \cdots & v_{MM} \end{pmatrix}$$

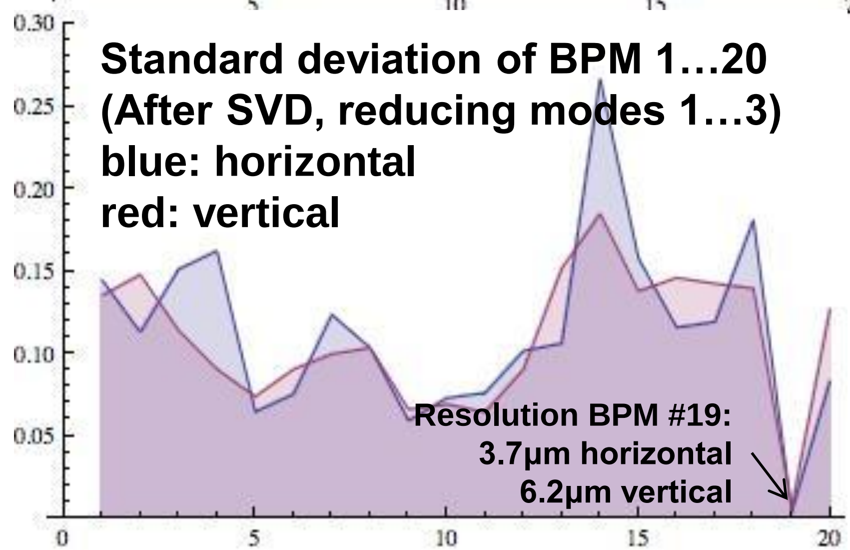
- **The BPM matrix *B* is decomposed into 3 matrices, *U, S, V*.**
  - **BPM numbers $B_1 \ldots B_M$, shot numbers $s_1 \ldots s_P$**
- **The values of the diagonal of the S matrix expresses the level of correlation between U (temporal) and V (spatial) orthogonal matrices**
  - **Correlation appears, e.g. due to beam motion effects (x, x', phase, energy,…) or common systematics (CLK jitter,…) in all BPMs.**
  - **The SVD algorithm assumes an over constrained system**
    - # of BPMs >> degrees of freedom of correlated data, e.g. beam motion
  - **We can set some high value $S_{nn} = 0$ (with great care!) to estimate the uncorrelated noise of the individual BPMs (resolution).**

CERN Linac 2: BPM Analysis





- **SVD analysis of a new BPM read-out electronics**
  - **Could identify new electronics**
  - **Removed beam motion using SVD, modes 1…3**
    - ➢ **Alternative: Split BPM electrode signal to both inputs**



Summary & Final Remarks

- An introduction in the technology of BPMs was presented
  - Basics on BPM pickups and beam signals
  - Some technical aspects on read-out electronics
- Many interesting details cold not be covered
  - BPM pickup design and optimization
    - Including the minimization of the beam coupling impedance
  - Details on RF feedthroughs
  - BPM system aspects
    - Infrastructure, trigger and timing signals, commissioning
  - In-house design vs. industry solutions
  - Testing and calibration
- BPMs are complex instrumentation systems
  - Teamwork, teamwork, teamwork!!!
- Refinements, improvements, corrections, and a few additional aspects on BPMs in the BI CAS proceedings

THANK YOU!