Introduction to Insertion Devices

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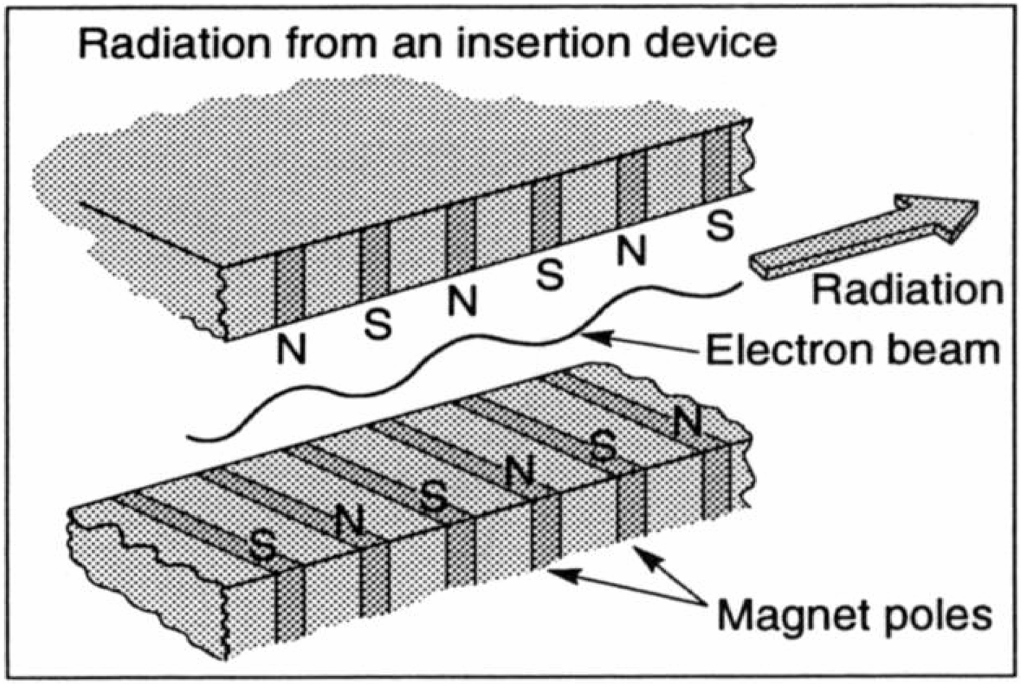
What is an Insertion Device ?



- Insertion Devices are also called Undulators and Wigglers
- Can be 1 to 20 m long, (typical 5 m) with a small magnetic gap (5-15 mm)
- **Intense Source of Synchrotron Radiation in e- Storage Ring Sources**
- Control of damping times in Electron Colliders (LEP, CESR,…)

Table of Content

Beam Dynamics

Electron Trajectory in an Insertion Device

Consider Ortogonal Frame Oxzs

Electron velocity : $\vec{v} = (v_x, v_z, v_s)$

Electron position : $\vec{R} = (x, z, s)$

Magnetic field : $\vec{B} = (B_x, B_z, B_s)$

Define : $\gamma = \dfrac{1}{\sqrt{1 - \dfrac{v^2}{c^2}}}$

Lorentz Force

$$\gamma m \frac{d\vec{v}}{dt} = e\vec{v} \times \vec{B}$$

$$=> \gamma m \frac{dv_x}{dt} = -e(v_s B_z - v_z B_s)$$

$$\boxed{Assume : v_x , v_z << v_s \approx c}$$

$$\frac{v_x(s)}{c} = -\frac{e}{\gamma mc} \int_{-\infty}^{s} B_z(s')ds'$$

$$x(s) = -\frac{e}{\gamma mc} \int_{-\infty}^{s} \int_{-\infty}^{s'} B_z(s'')ds''ds'$$

and similar expression for $v_z(s)$ and $z(s)$

Electron Trajectory in a Planar Sinusoidal Undulator

$$Consider\ \vec{B} = (\,0,\, B_0 \sin(2\pi \frac{s}{\lambda_0}),\, 0\,)$$

$$\boxed{\begin{aligned}
&\frac{v_x}{c} = \frac{K}{\gamma}\cos(2\pi\frac{s}{\lambda_0})\\
&\frac{v_z}{c} = 0\\
&\frac{v_s}{c} = 1 - \frac{1}{2\gamma^2}(1 + K^2\cos^2(2\pi\frac{s}{\lambda_0}))\\
&x \simeq -\frac{\lambda_0}{2\pi}\frac{K}{\gamma}\sin(2\pi\frac{s}{\lambda_0})
\end{aligned}}$$

with $$\boxed{K = \frac{eB_0\lambda_0}{2\pi mc} = 0.0934\, B_0[T]\,\lambda_0[mm]}$$

K is a fundamental parameter called : **Deflection Parameter**

$$Example: ESRF,\ Energy = 6GeV,\ Undulator\ \ \lambda_0 = 35\ \ mm,\ B_0 = 0.7\ \ T$$

$$=>\quad K = 2.3, \qquad \frac{K}{\gamma} = 200\ \mu rad, \qquad \frac{\lambda_0}{2\pi}\frac{K}{\gamma} = 1.1\ \mu m\ !!$$

$$B_x = 0$$

$$B_z = B_0 \cosh(2\pi \frac{z}{\lambda_0}) \cos(2\pi \frac{s}{\lambda_0})$$

$$B_s = -B_0 \sinh(2\pi \frac{z}{\lambda_0}) \sin(2\pi \frac{s}{\lambda_0})$$

Undulator Field Satisfying
Maxwell Equation

$$\gamma m \frac{d\vec{v}}{dt} = e\vec{v} \times \vec{B}$$

Lorentz Force Equation

2nd Order in γ^{-1}

$$\frac{d^2x}{ds^2} = 0$$

$$\frac{d^2z}{ds^2} = -\frac{1}{2}\left(\frac{eB_0}{\gamma mc}\right)^2 \frac{\lambda_0}{4\pi} \sinh(4\pi \frac{z}{\lambda_0}) \simeq -\left(K_z\right)^2 z$$

$$K_z = \frac{1}{2}\left(\frac{eB_0}{\gamma mc}\right)^2$$

A vertical Field Undulator is Vertically Focusing !

$$\frac{1}{F_z} = \int_{ID} K_z ds = \frac{1}{2}\left(\frac{eB_0}{\gamma mc}\right)^2 L$$

Interference with the beam dynamics in the ring lattice

- An Insertion Device is the first component of a photon beamline. Its field setting is fully controlled by the Users of the beamline . The beam dynamics in the whole ring may be altered if the field of an ID is changed => **crosstalk** of the source parameters in each beamline **must be avoided** .

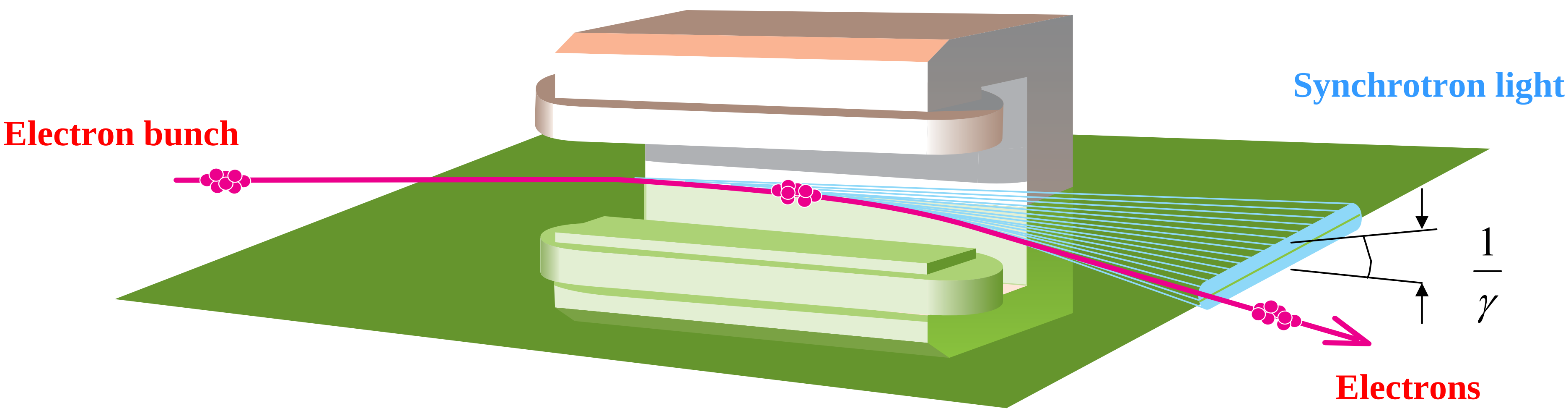
- As far as the lattices are concerned, **Insertion devices** should ideally behave like **drift space** but the reality is somewhat different :
  - Closed Orbit distortion (non zero field integrals generated by design and field errors)
  - Tune shift (induced by nominal field and by field errors)
  - Reduction of dynamic aperture (=>Lifetime reduction & reduced injection efficiency) induced by varying focusing properties inside the aperture for the beam => critical for modern sources operating in topping-up mode.
  - Very high field IDs may change the damping time, emittance , energy spread …

- By combining field shimming and local steering corrections, most of the perturbations are able to be solved.

- The problem of the reduction of dynamic aperture is severest on low energy rings with many insertion devices.

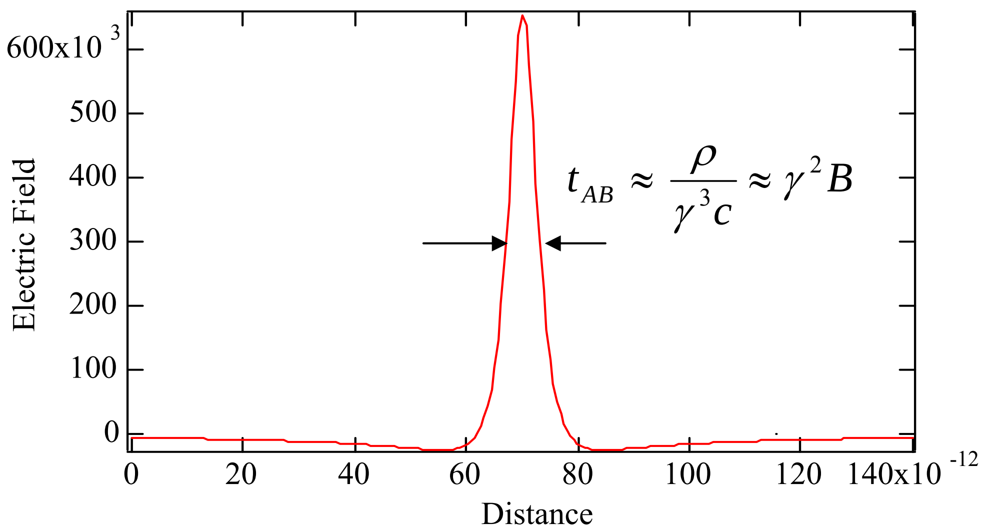
Radiation from IDs

Bending Magnet

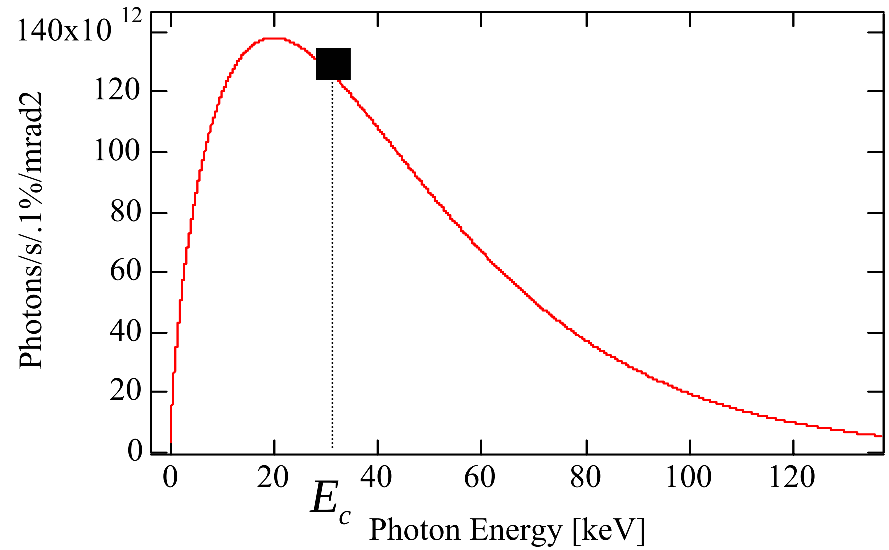


Critical Energy of Bending Magnet Radiation

Electric Field in the Time Domain



Angular Flux in Frequency Domain

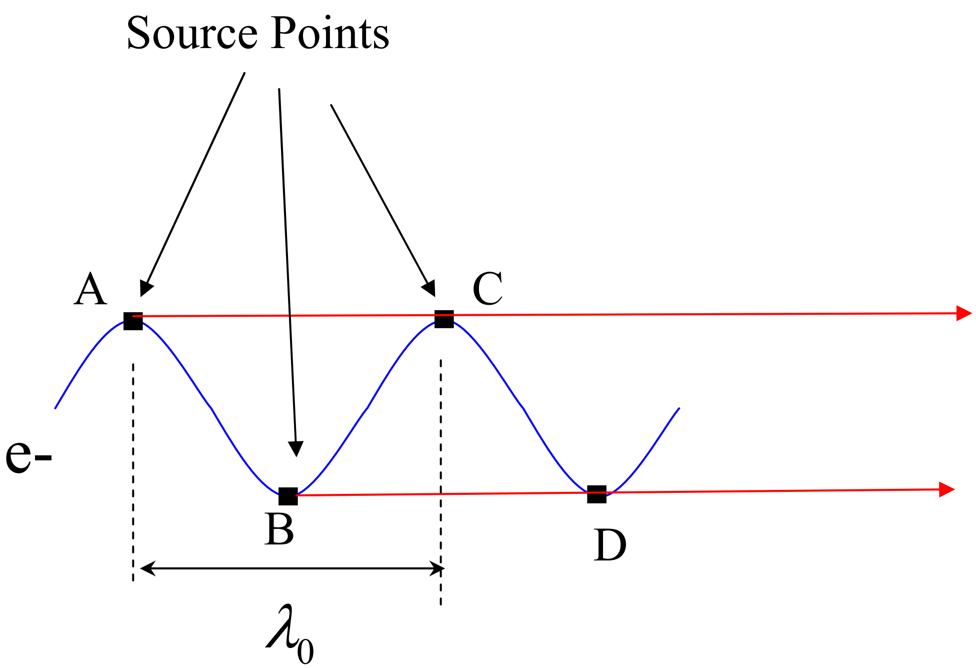


Computed for 6 GeV, I = 200 mA, B =1 tesla

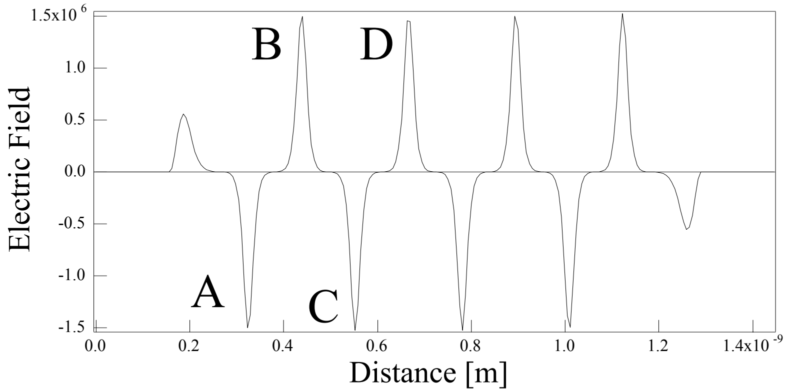
$$E_c = \frac{3hc}{4\pi}\frac{\gamma^3}{\rho} = \frac{3he}{4\pi m}\gamma^2 B$$

$$\boxed{E_c[keV] = 0.665\, E^2[GeV]\, B[T]}$$

Undulator

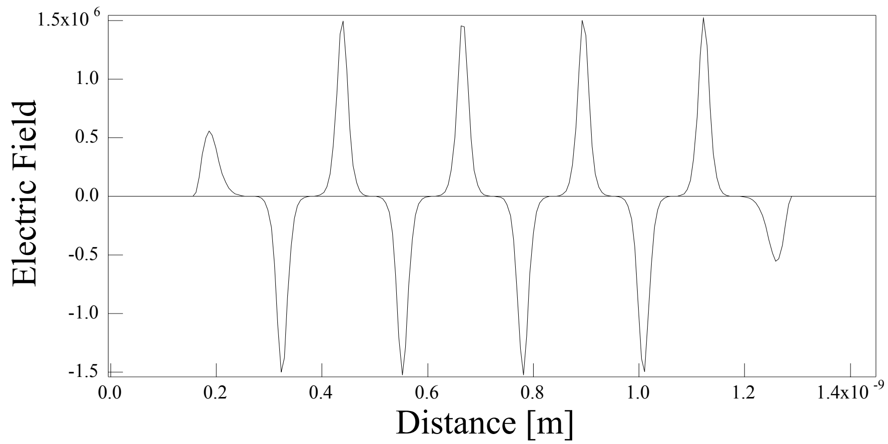


Electron trajectory

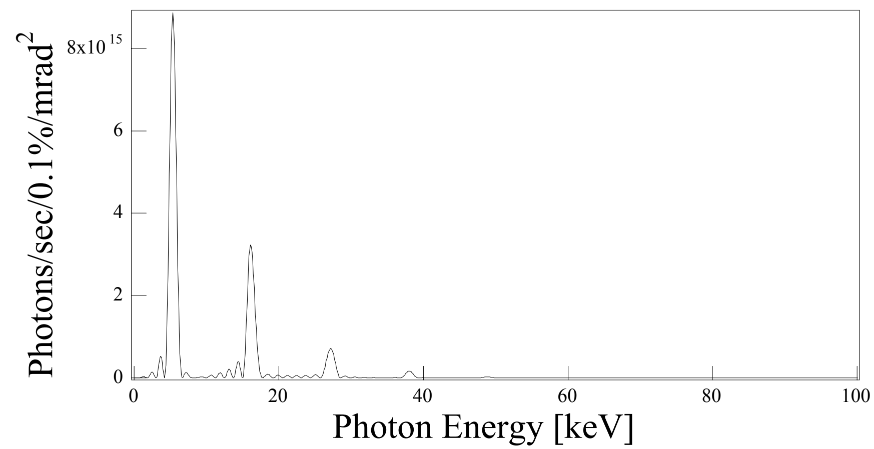


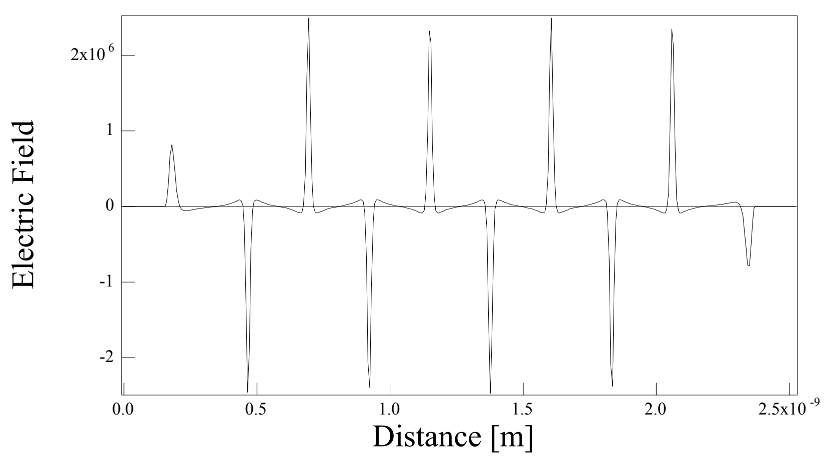
Electric Field in Time Domain

Electric Field and Spectrum vs K

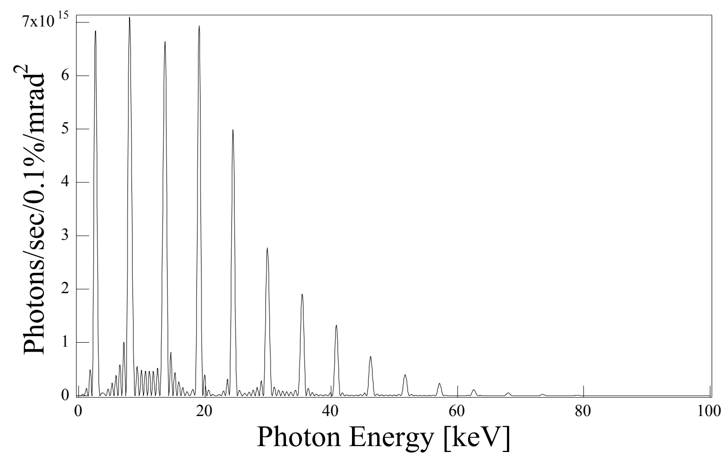


K=1



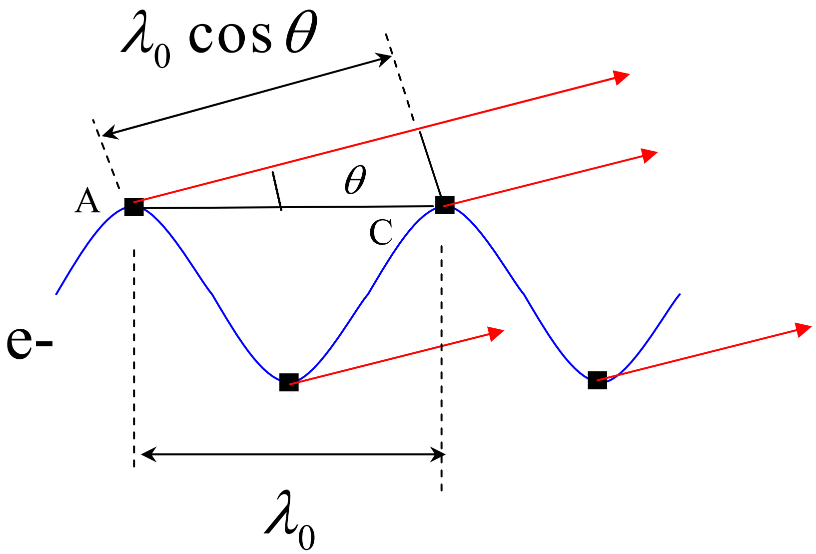


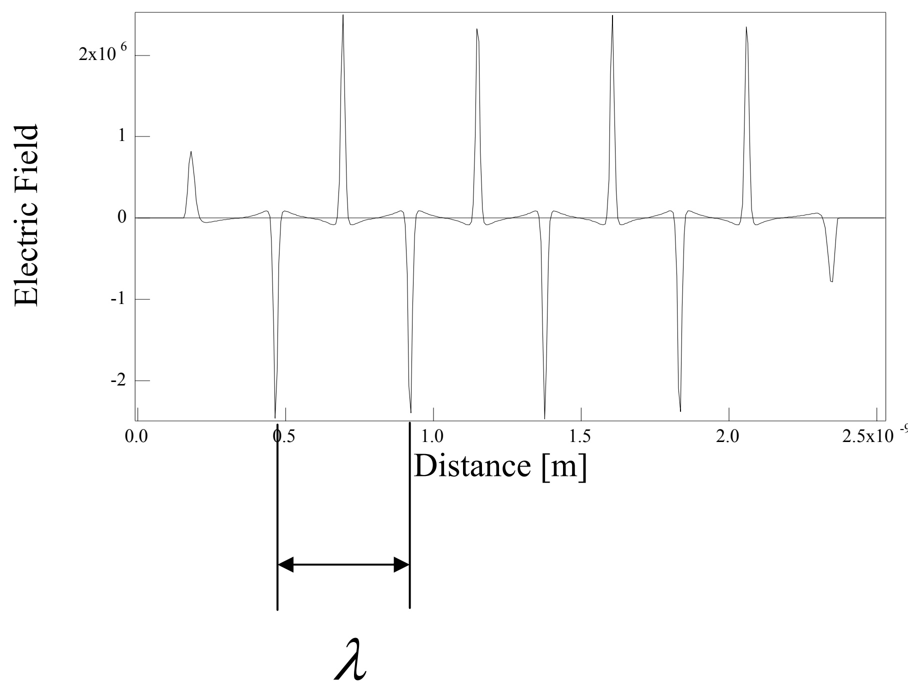
K=2



Computed for 6 GeV, I=200 mA, 35 mm period

Fundamental Wavelength of the Radiation Field





$$t_{AC} = \frac{\lambda_0}{<v_s>} = \frac{\lambda_0}{c(1 - \frac{1}{2\gamma^2}(1 + \frac{K^2}{2}))}$$

$$\lambda = \lambda_0 \cos\theta - ct_{AC} \cong \lambda_0 (1 - \frac{\theta^2}{2}) - \frac{\lambda_0}{(1 - \frac{1}{2\gamma^2}(1 + \frac{K^2}{2}))} \cong \frac{\lambda_0}{2\gamma^2}(1 + \frac{K^2}{2} + \gamma^2\theta^2)$$

Wavelength of the Harmonics

$$\boxed{\lambda_n = \frac{\lambda_0}{2n\gamma^2}\left(1+\frac{K^2}{2}+\gamma^2\theta^2\right)}$$

In an equivalent manner , the energy E_n of the harmonics are given by

$$\boxed{E_n[keV] = \frac{9.5\, n\, E^2[GeV]}{\lambda_0[mm]\left(1+\frac{K^2}{2}+\gamma^2\theta^2\right)}}$$

λ_n, E_n : *Wavelength, Energy of the* n^{th} *harmonic*

$n = 1, 2, 3, ..$: *Harmonic number*

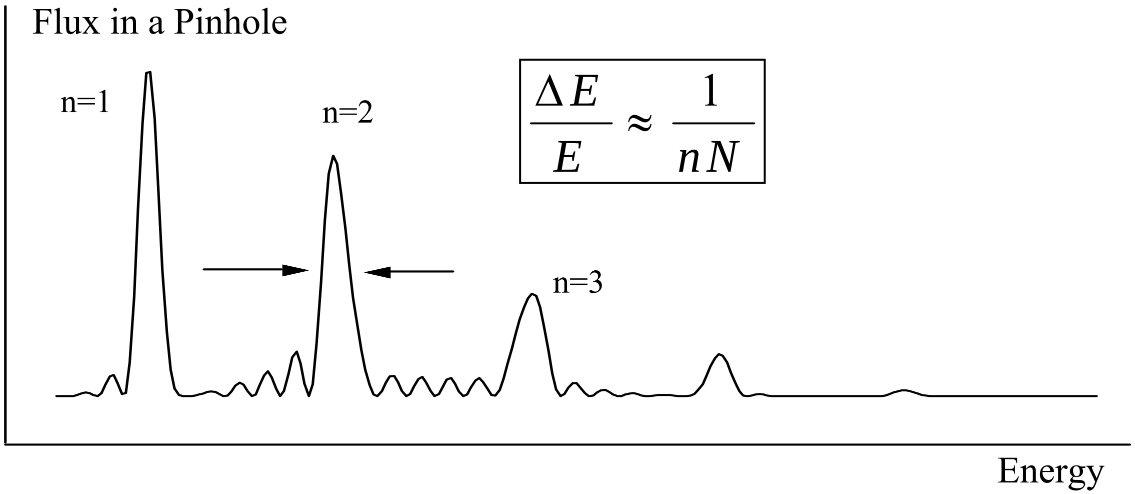
λ_0 : *Undulator period*

$E = \gamma mc^2$: *Electron Energy*

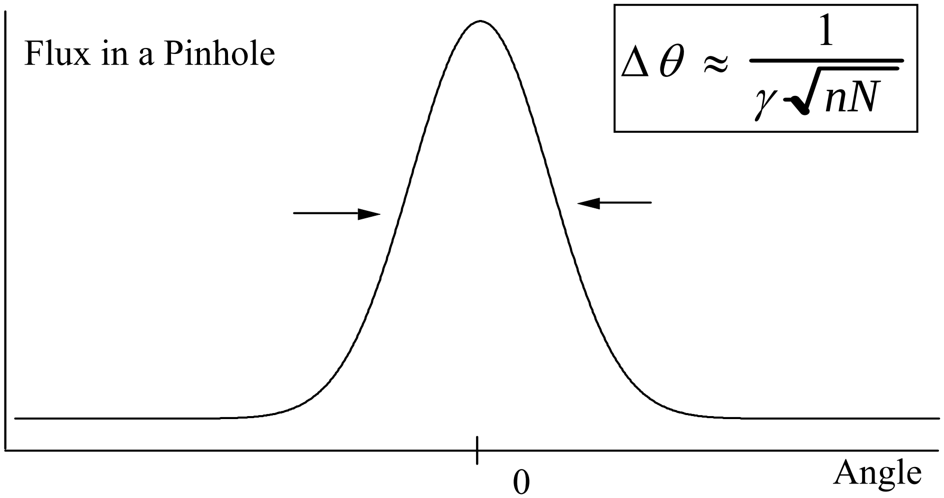
K : *Deflection Parameter* $= 0.0934\, B_0[T]\, \lambda_0[mm]$

θ : *Angle between observer direction and e – beam*

Undulator Emission by a Filament Electron Beam



$$\frac{\Delta E}{E} \approx \frac{1}{nN}$$



$$\Delta\theta \approx \frac{1}{\gamma\sqrt{nN}}$$

n : Harmonic number

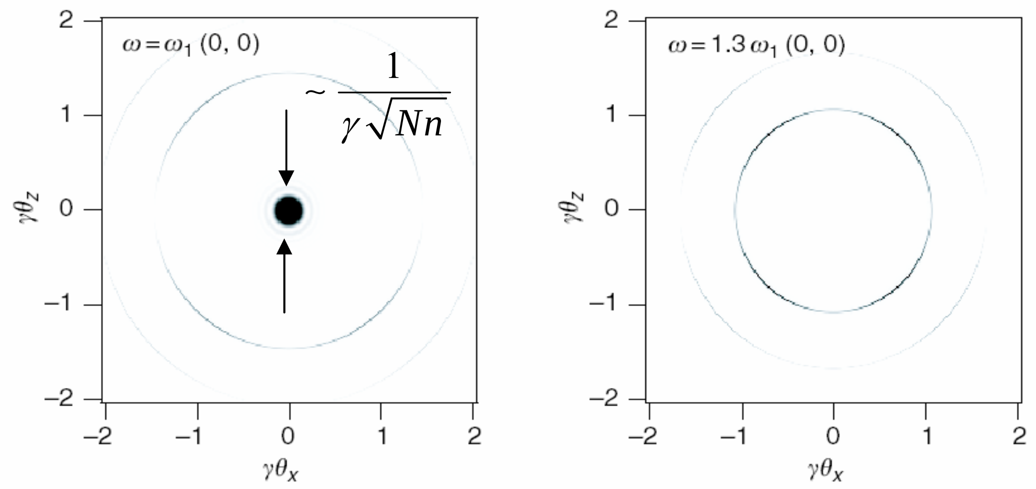
N : Number of Periods

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{E}{mc^2}$$

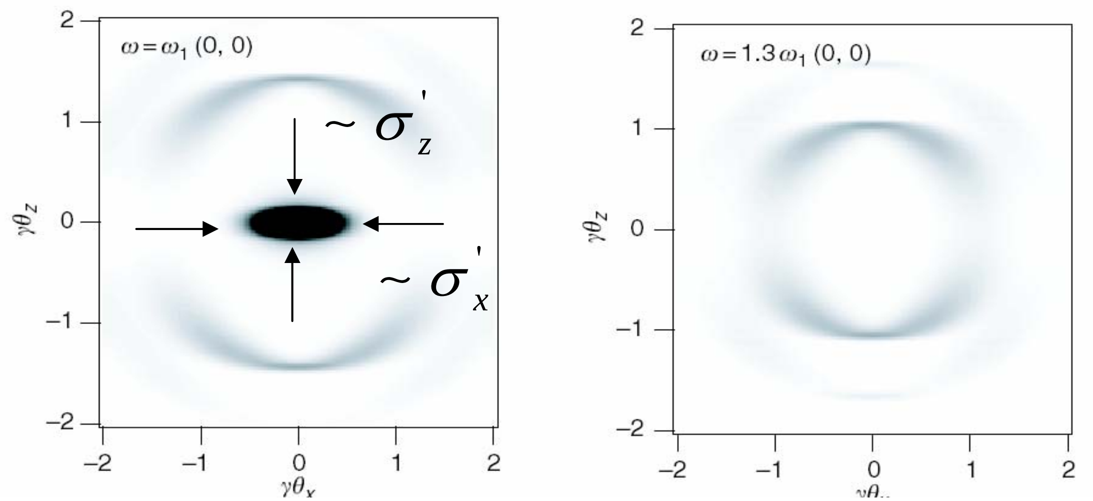
- Until now a Filament mono-energetic electron beam has been assumed.

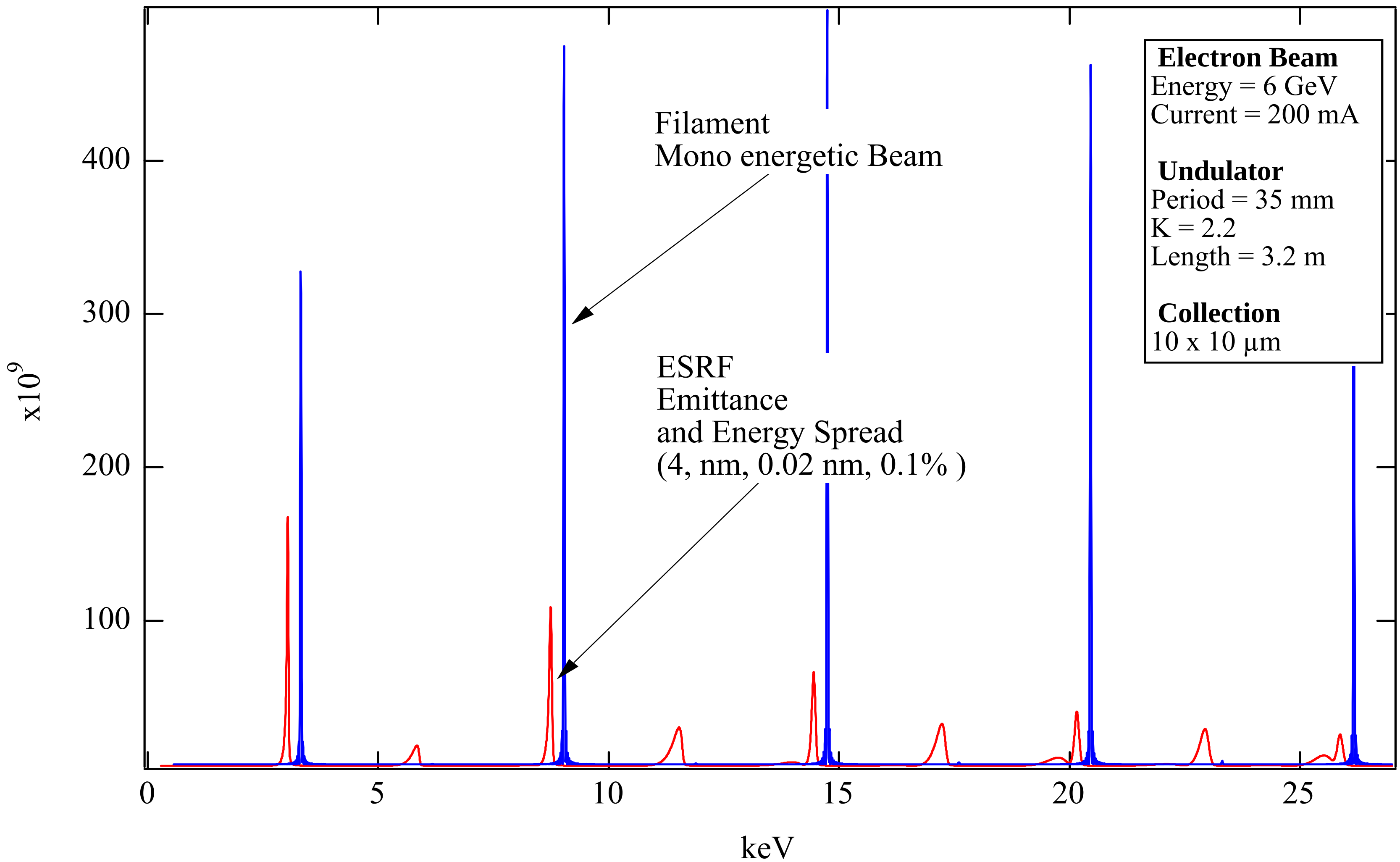
- What happens if the beam presents a finite emittance (size and divergence) and finite energy spread ?

Filament
e- Beam

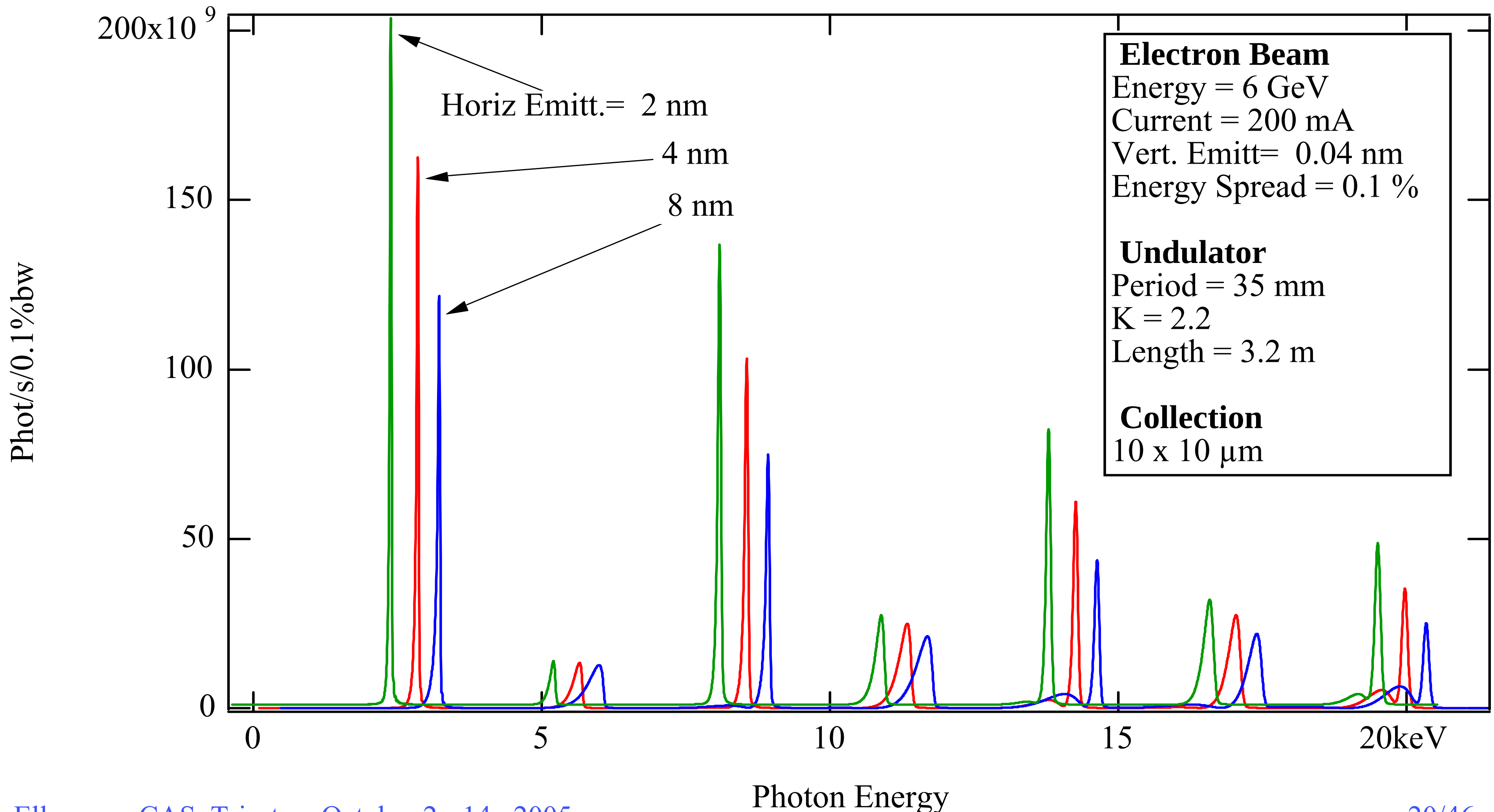


e- Beam with
Finite Divergence

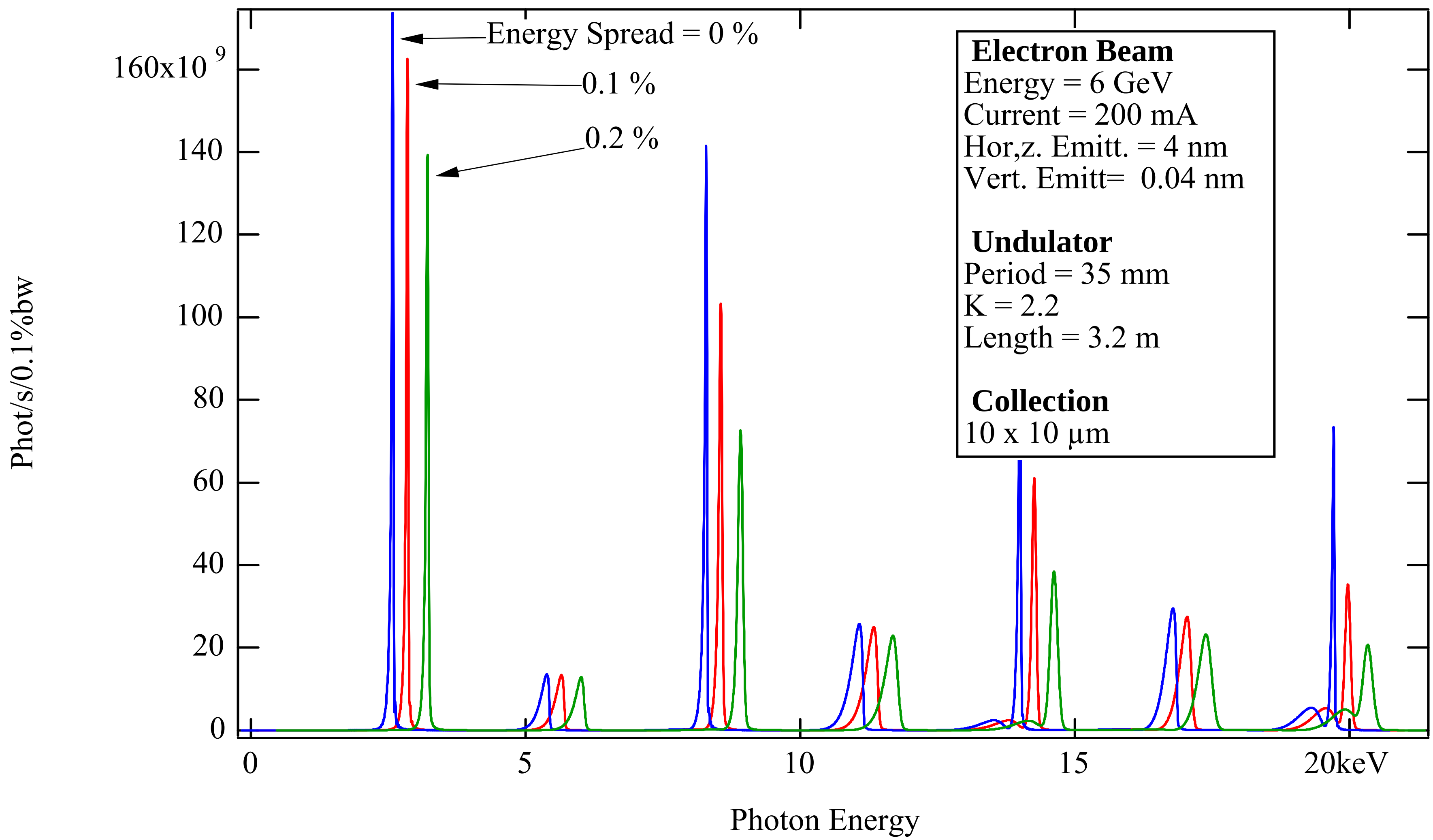



Electron Beam
Energy = 6 GeV
Current = 200 mA
Undulator
Period = 35 mm
K = 2.2
Length = 3.2 m
Collection
10 x 10 µm
Filament
Mono energetic Beam
ESRF
Emittance
and Energy Spread
(4, nm, 0.02 nm, 0.1%)
x10^9
400
300
200
100
0
5
10
15
20
25
keV

Broadening of the Harmonics by the Electron Emittance

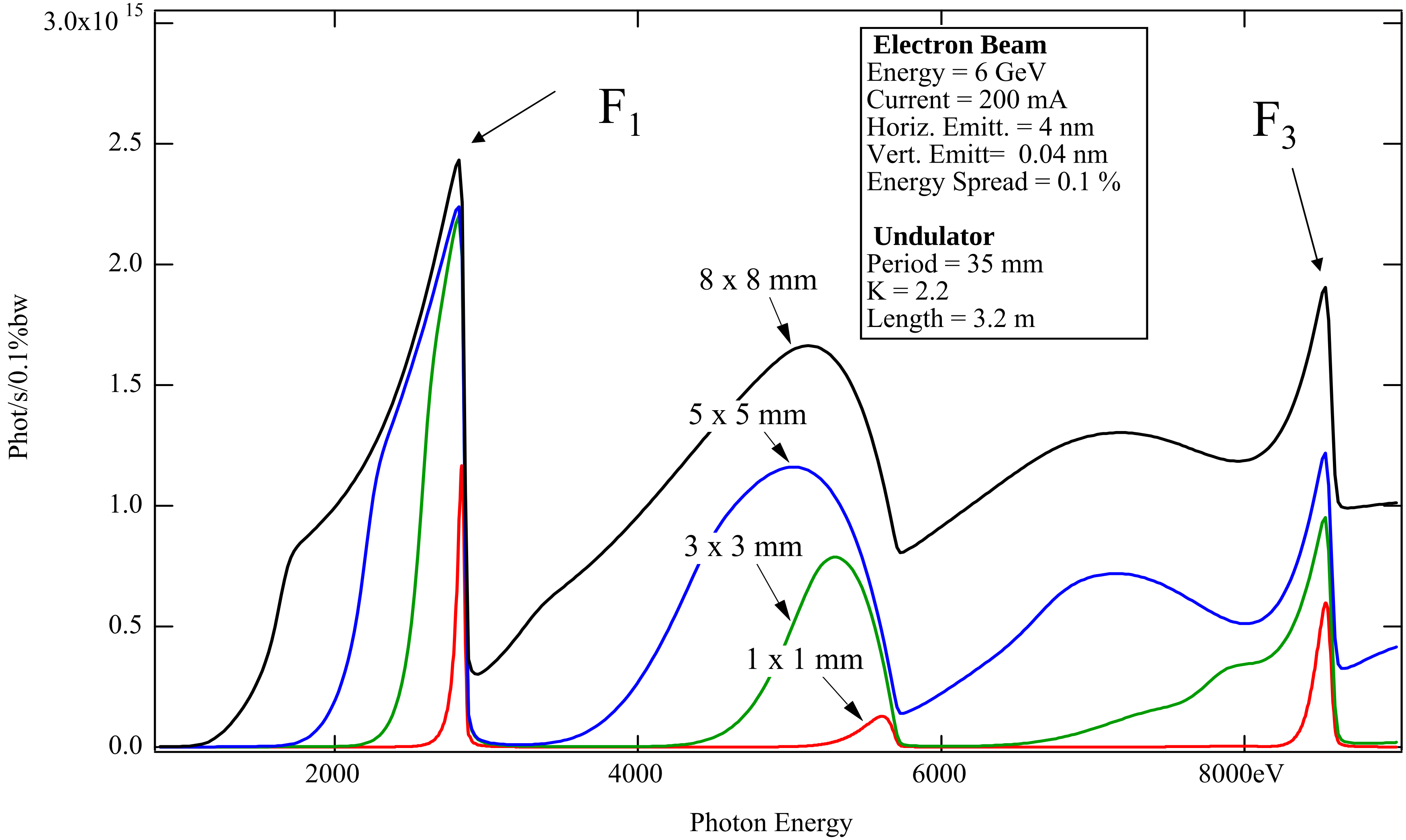


Broadening of the Harmonics by Electron Energy Spread



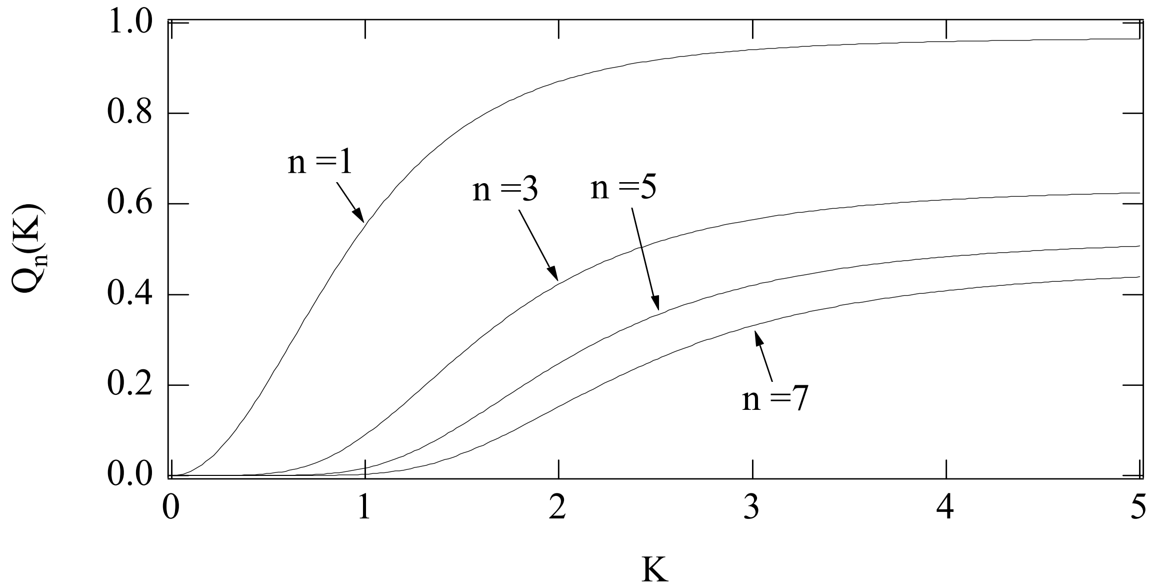
- To Make Optimum use of the **Undulators, The magnet Lattices of** synchrotron light sources are optimized to produce the **smallest emittance** and **smallest energy spread** of the electron beam possible.

Collecting Undulator Radiation in a variable Aperture



Maximum Spectral Flux On-axis on odd harmonics

$$F_n \ \ [Ph/\sec/ \ 0.1\%] \ = \ 1.431 \ \ 10^{14} \ \ N \ \ I[A] \ \ Q_n(K)$$



Brilliance (or Brightness)

$$B_n = \frac{F_n}{(2\pi)^2 \Sigma_x \Sigma'_x \Sigma_z \Sigma'_z}$$

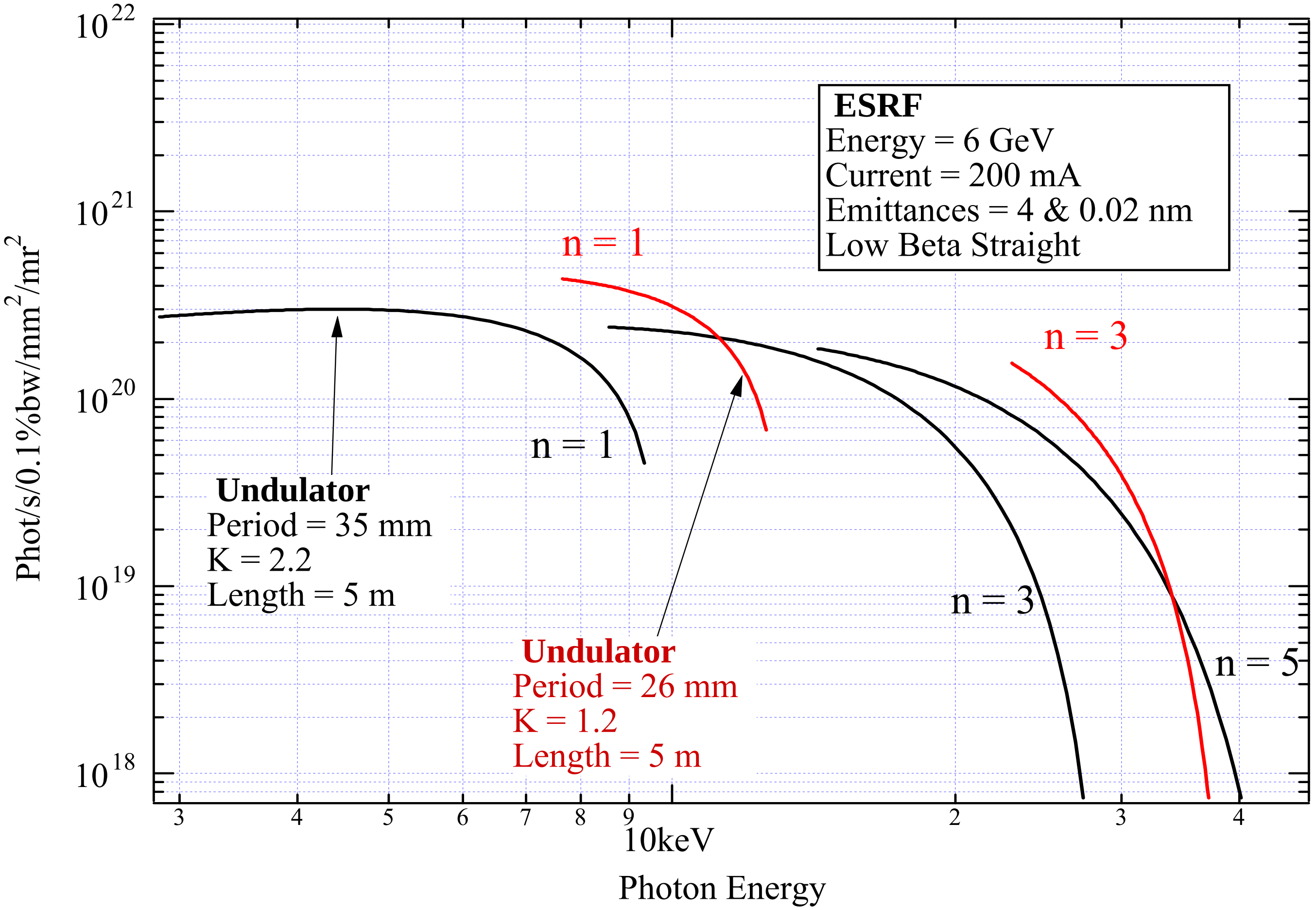
$$\Sigma_x = \sqrt{\sigma_x^2 + \frac{\lambda_n L}{(2\pi)^2}}$$

$$\Sigma'_x = \sqrt{\sigma'^2_x + \frac{\lambda_n}{2L}}$$

Electron beam

Single electron emission

Brilliance vs Photon Energy



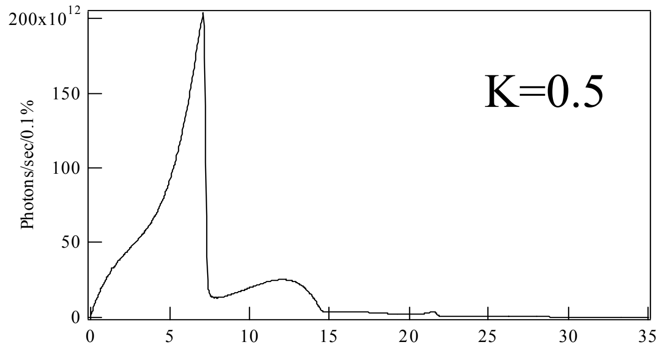
$$B_n = \frac{F_n}{(2\pi)^2 \Sigma_x \Sigma'_x \Sigma_z \Sigma'_z}$$

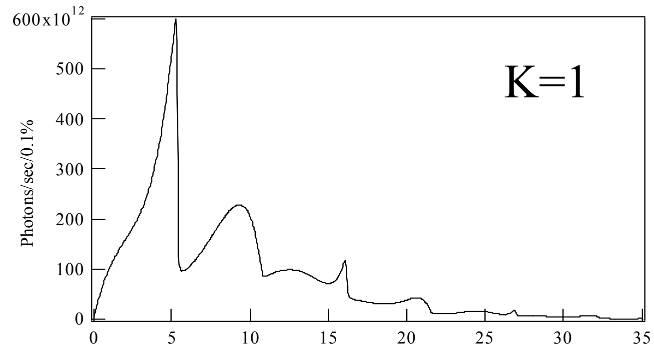
with

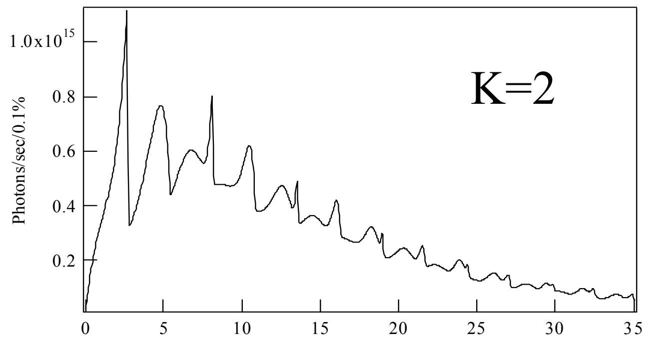
$$\Sigma_x = \sqrt{\sigma_x^2 + \frac{\lambda_n L}{(2\pi)^2}}$$

$$\Sigma'_x = \sqrt{\sigma'^2_x + \frac{\lambda_n}{2L}}$$

Angle Integrated Flux



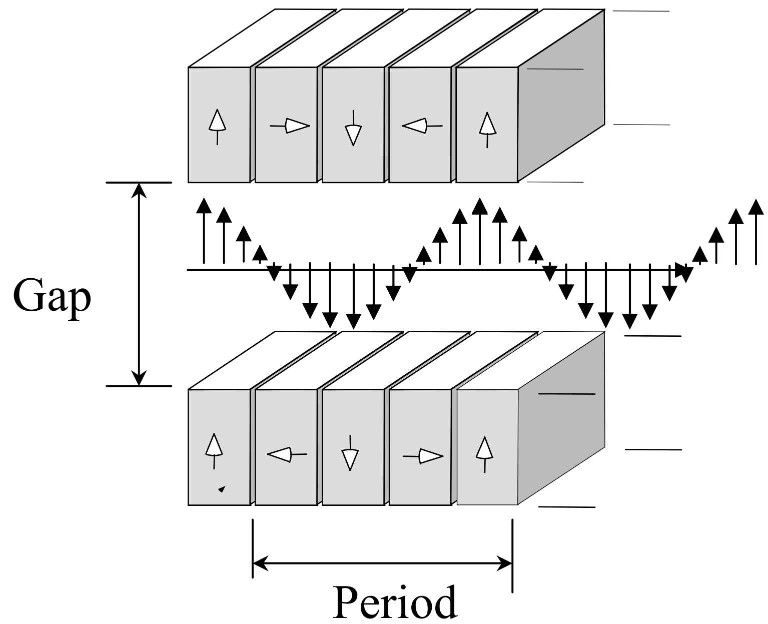




For Large K, the angle integrated spectrum from an Undulator tends toward that of a bending magnet x 2N
=> Such Devices are called **Wigglers**

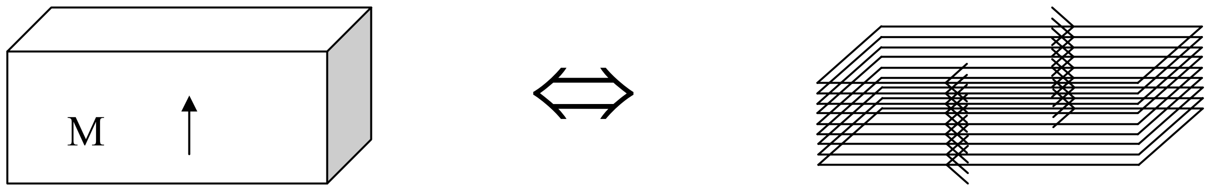
Technology

Technology of Undulators and Wigglers



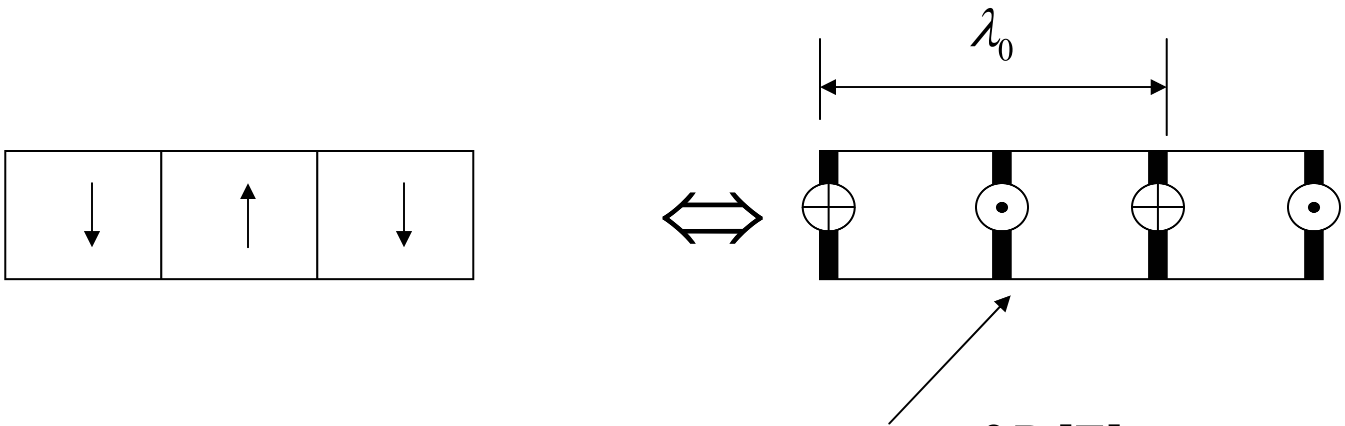
- The fundamental issue in the magnetic design of a planar undulator or wiggler is to produce a periodic field with a high peak field B and the shortest period λ_0 within a given aperture (gap).
- Three type of technologies can be used :
  - Permanent magnets (NdFeB , Sm_2Co_{17})
  - Room temperature electromagnets
  - Superconducting electromagnets

Current Equivalent of a Magnetized Material



$$\text{Air coil with Surface Current Density[A/m]} \cong \frac{B_r[T]}{\mu_0}$$

Periodic Array of Magnets



$$\text{Surface Current Density[A/m]} \cong \frac{2B_r[T]}{\mu_0}$$

$$\text{or Current Density[A/m}^2\text{]} \cong \frac{4B_r[T]}{\mu_0 \lambda_0}$$

Examples:

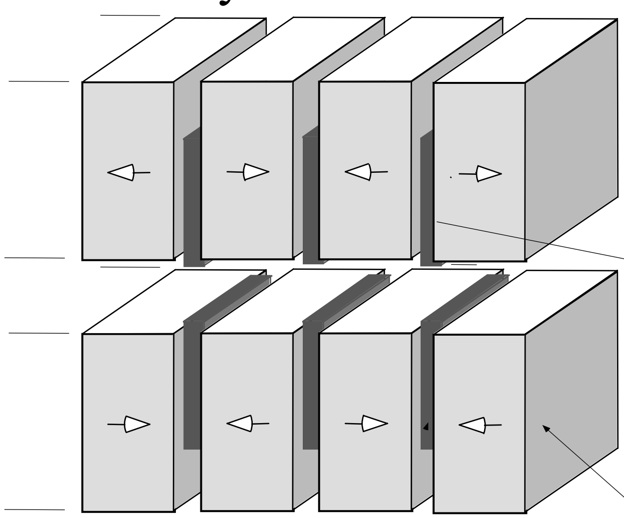
$B_r = 1$ T , $\lambda_0 = 20$ mm $\Rightarrow$ Equiv. Current Density=160A/mm^2 !!

$B_r = 1$ T , $\lambda_0 = 400$ mm $\Rightarrow$ Equiv. Current Density=8 A/mm^2

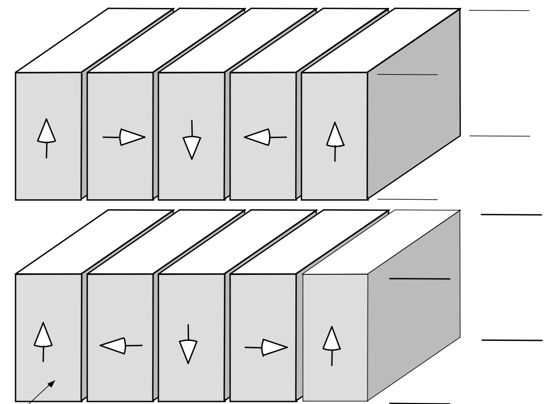
~ 95 % of Insertion Devices are made of Permanent Magnets !!

Permanent Magnet Undulator

Hybrid

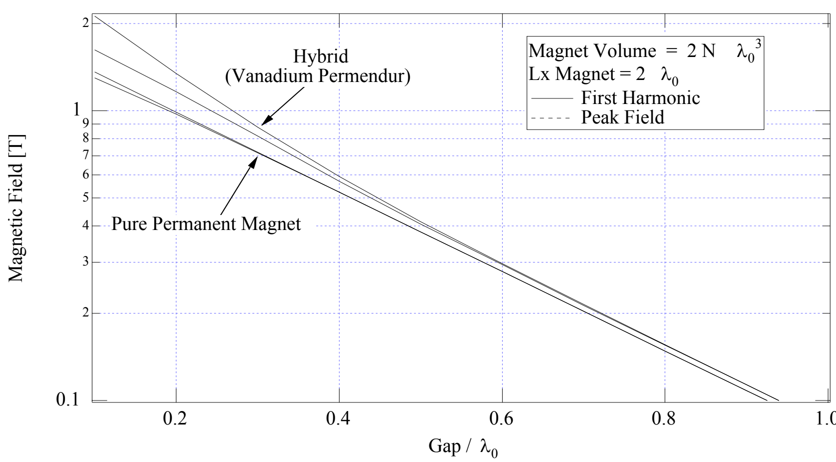


Pure Permanent Magnet



Pole (Steel)

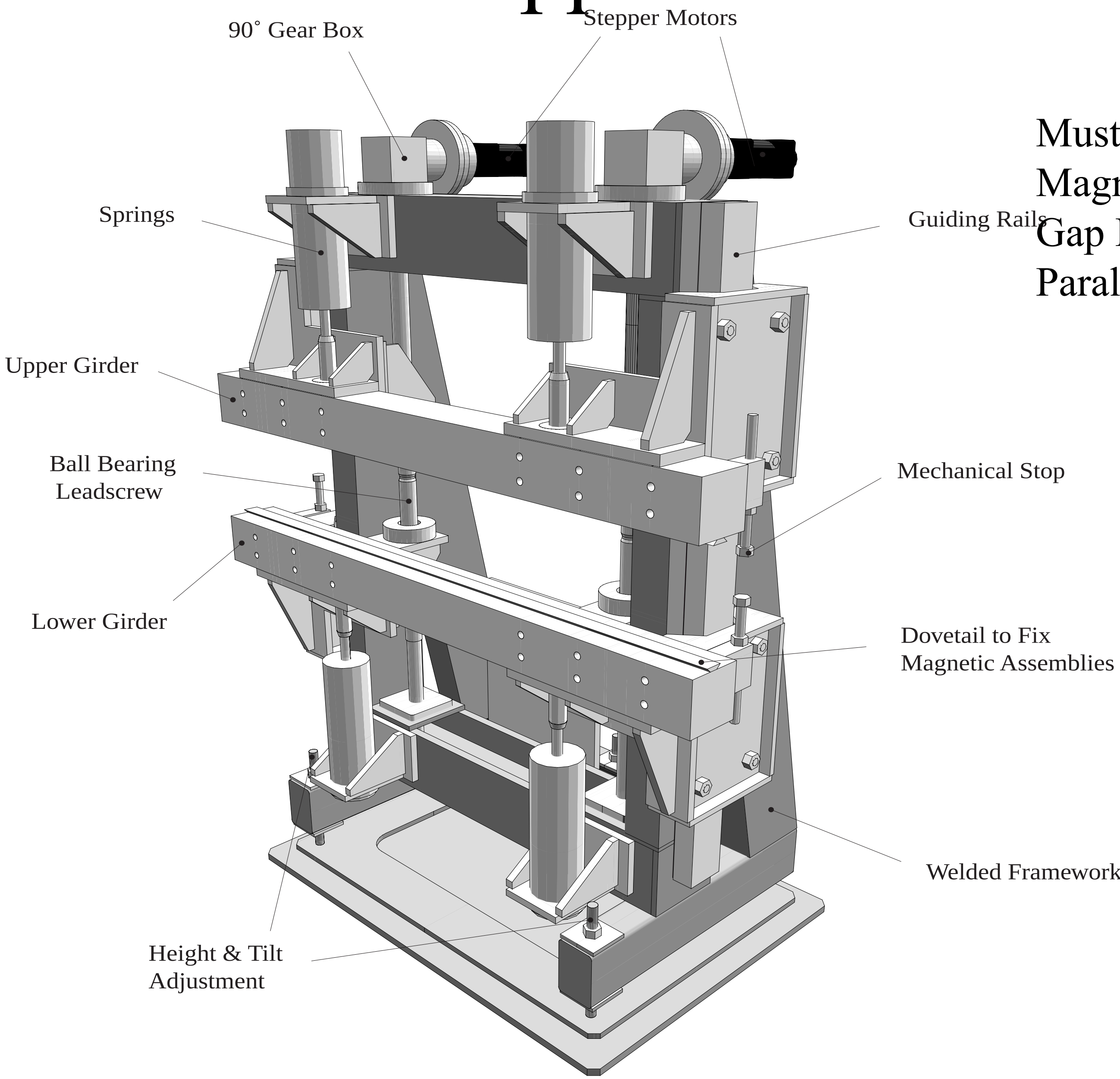
Magnet (NdFeB, Sm_2Co_{17},...)



magnet materials

Material	B_r [T]	$\mu_{r,\parallel}$	$\mu_{r,\perp}$	H_{cJ} [kA/m]	$10^{-2}/°C$
$SmCO_5$	0.9–1.01	1.05		1500–2400	−0.04
Sm_2CO_{17}	1.04–1.12	1.05–1.08		800–2000	−0.03
NdFeB	1.0–1.4	1.04–1.06	1.15–1.17	1000–3000	−0.10

Support Structure



Must Handle :

Magnetic Force : 1-20 Tons

Gap Resolution : $< 1\ \mu m$

Parallellism $< 20\ \mu m$

Undulators are Fundamentally Small Gap Devices

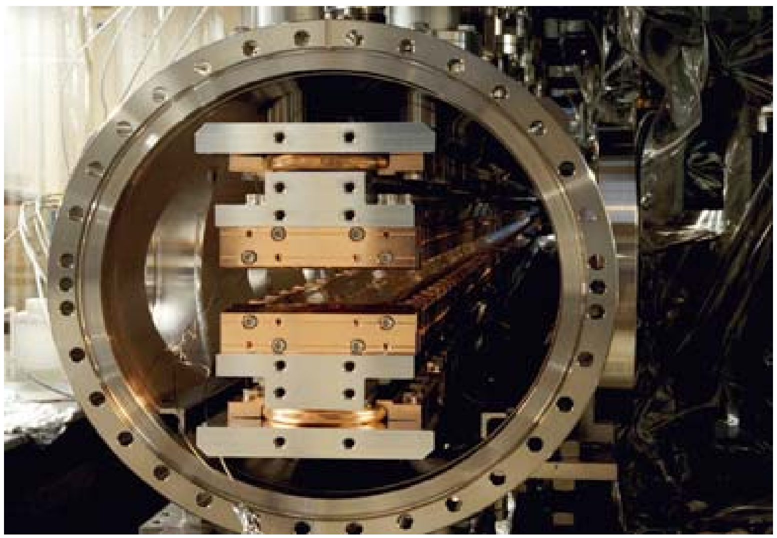
- For a permanent magnet undulator , shrinking all dimensions maintains the field unchanged. The peak field $B_p \sim B_r$ F(gap/period)

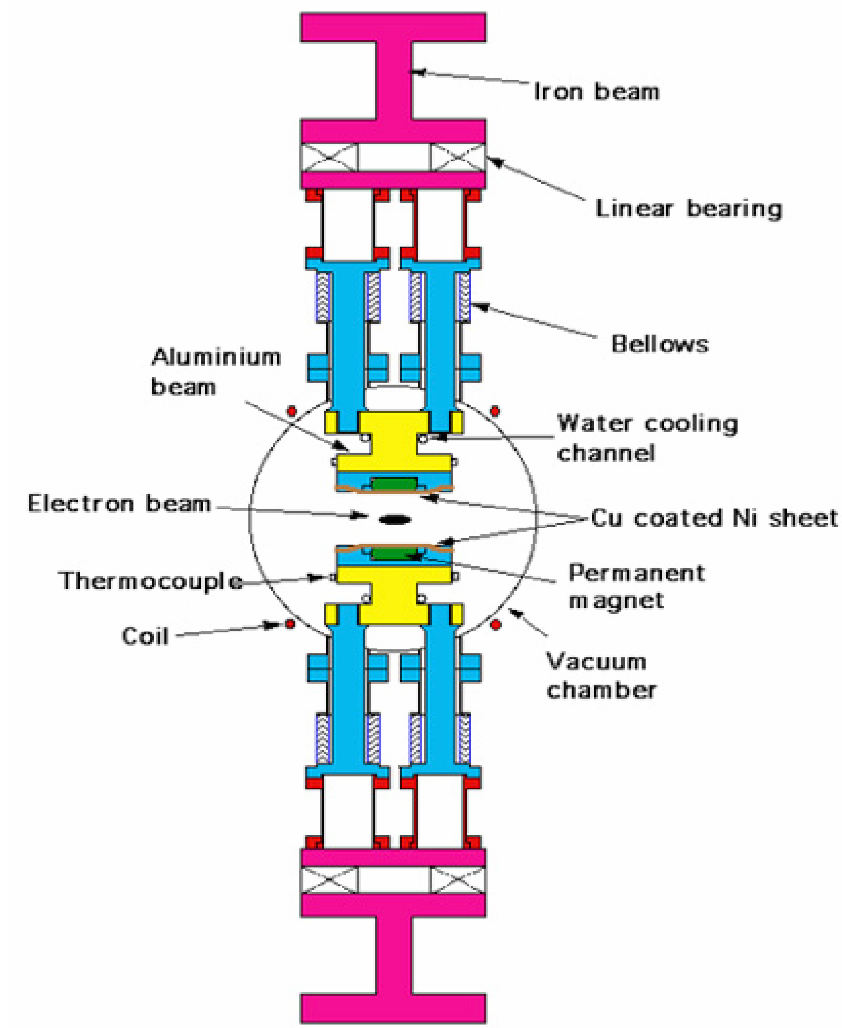
- Benefits of using small gaps Insertion Devices :
  - Decrease the volume of material (cost driving) ~ gap^3
  - The lower the gap, the higher the energy of the harmonics of the undulator emission => the lower the electron energy required to reach the same photon energy

$$\lambda = \frac{\lambda_0}{2\gamma^2}(1 + \frac{K^2}{2}) \qquad with \;\; K = \frac{eB_0\lambda_0}{2\pi mc}$$

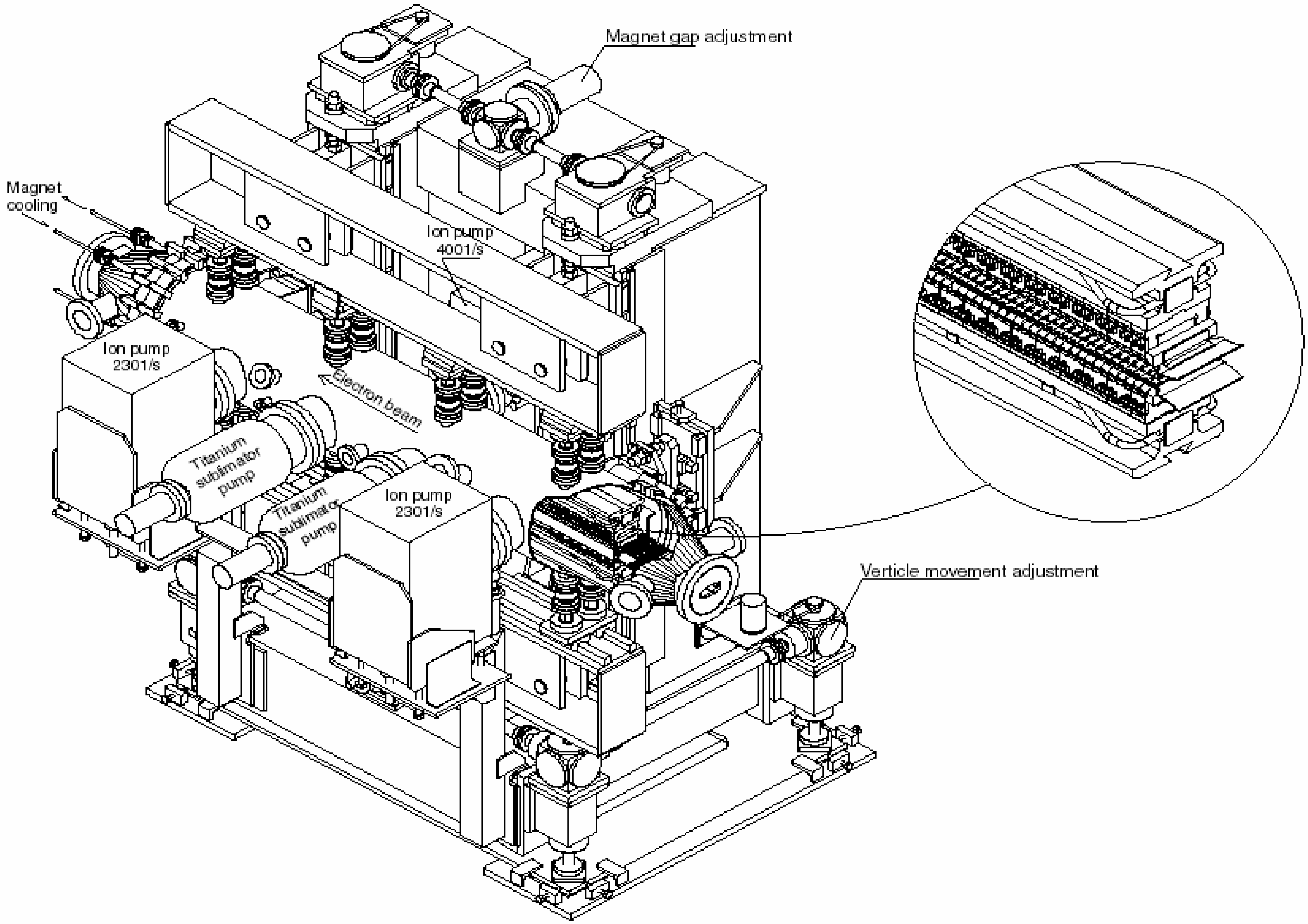
- The most advanced undulators have magnet blocks in the vacuum with an operating magnetic gap of 4-6 mm

In Vacuum Permanent Magnet Undulators





SPring-8 In-Vacuum Undulator

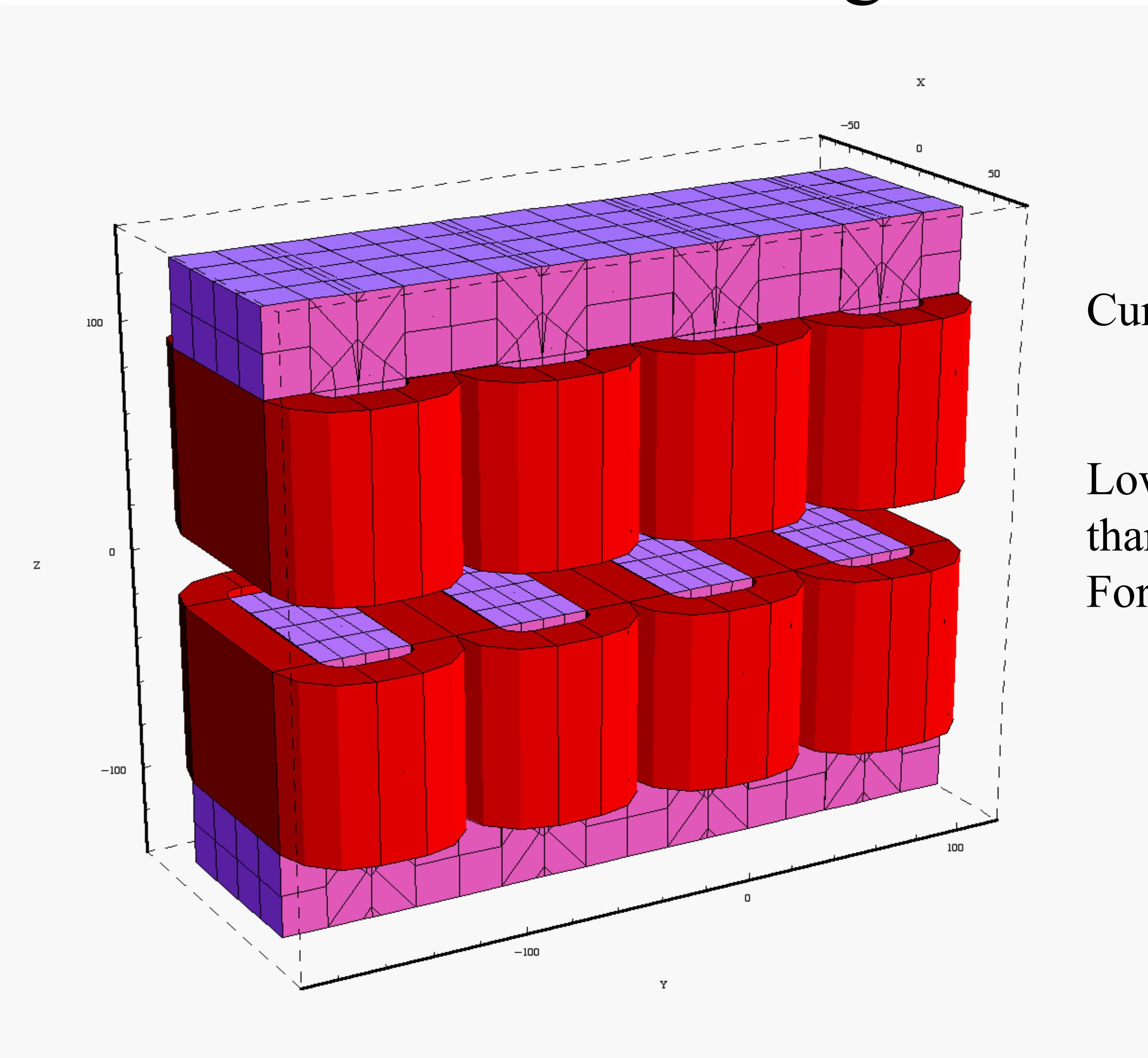

Magnet gap adjustment
Magnet
cooling
Ion pump
4001/s
Ion pump
2301/s
Electron beam
Titanium
sublimator
pump
Titanium
sublimator
pump
Ion pump
2301/s
Verticle movement adjustment

Application : Build a pure permanent magnet undulator with NdFeB Magnets (B_r = 1.2 T)

Undulator with K=1

Gap [mm]	B [T]	Period [mm]	Fundamental [keV] @ 6 GeV	Electron Energy [GeV] Fund = 15.2 keV
5	0.72	15	15.2	6.0
10	0.49	22	10.3	7.3
15	0.38	28	8.2	8.2

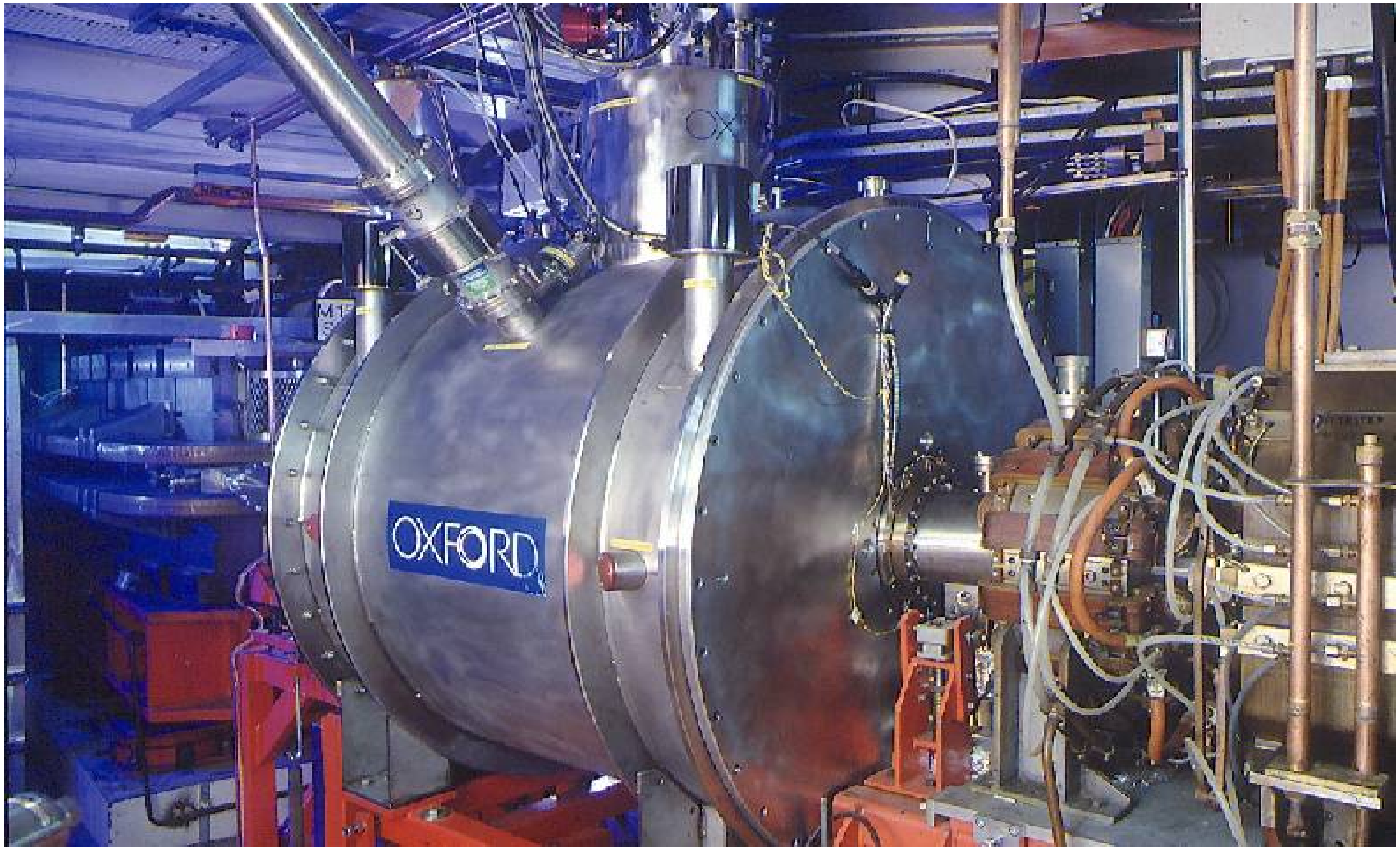
Electro-Magnet Undulator



Current Densities < 5-20 A/mm^2

Lower field
than permanent magnet
For small period / gap

Superconducting Wigglers



- High field : up to 10 T => Shift the spectrum to higher energies
- Complicated engineering & High costs

Magnetic Field Errors in Permanent Magnet Insertion Devices :

- Field errors originate from :
  - Non uniform magnetization of the magnet blocks (poles).
  - Dimensional and Positional errors of the poles and magnet blocks.
  - Interaction with environmental magnetic field (iron frame, earth field,…)

- Important to use highly uniform magnetized blocks
  - perform a systematic characterization of the magnetization
  - Perform a pairing of the blocks to cancel errors
  - Still insufficient …

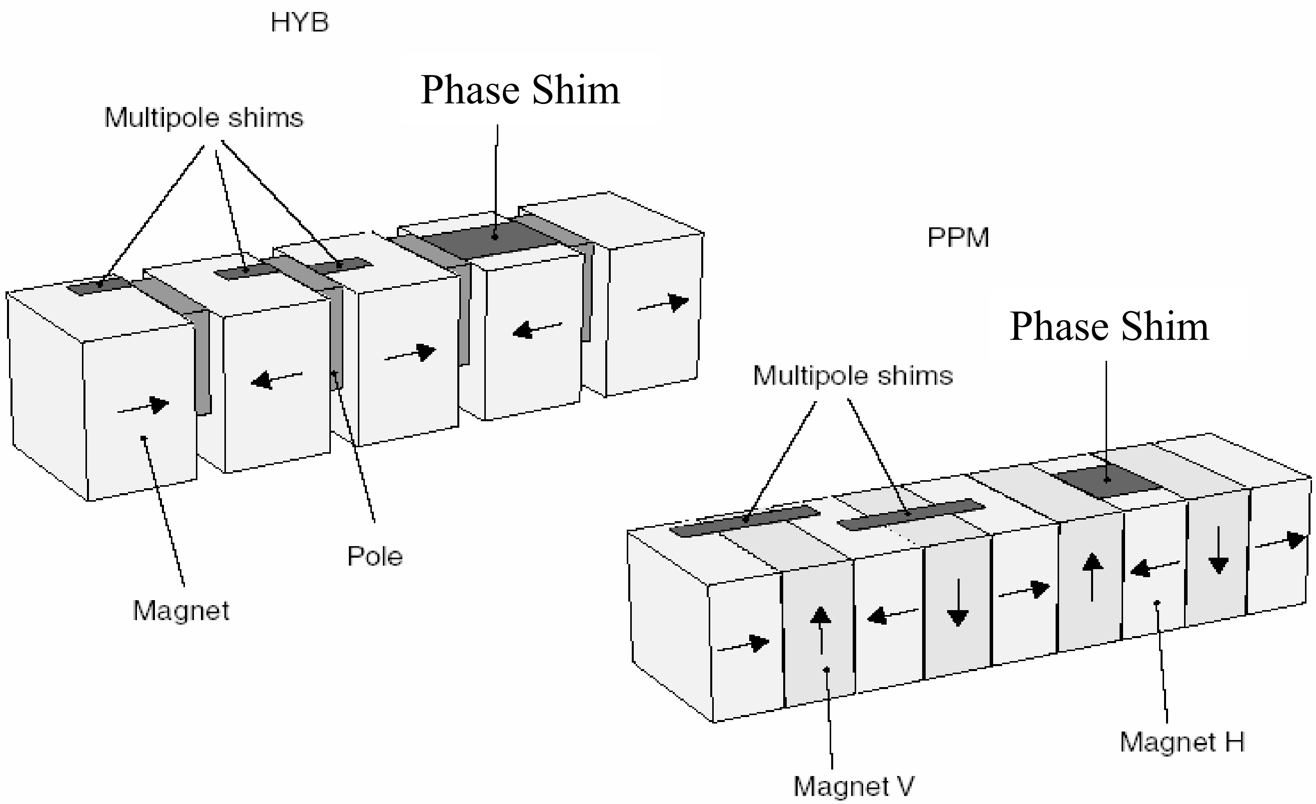
- Two main type of field errors remain
  - Multipole Field Errors (Normal and skew dipole, quadrupole, sextupole,…).
  - Phase errors which reduce the emission on the high harmonic numbers
  - Further corrections :
    - Active steerers
    - Shimming

Shimming

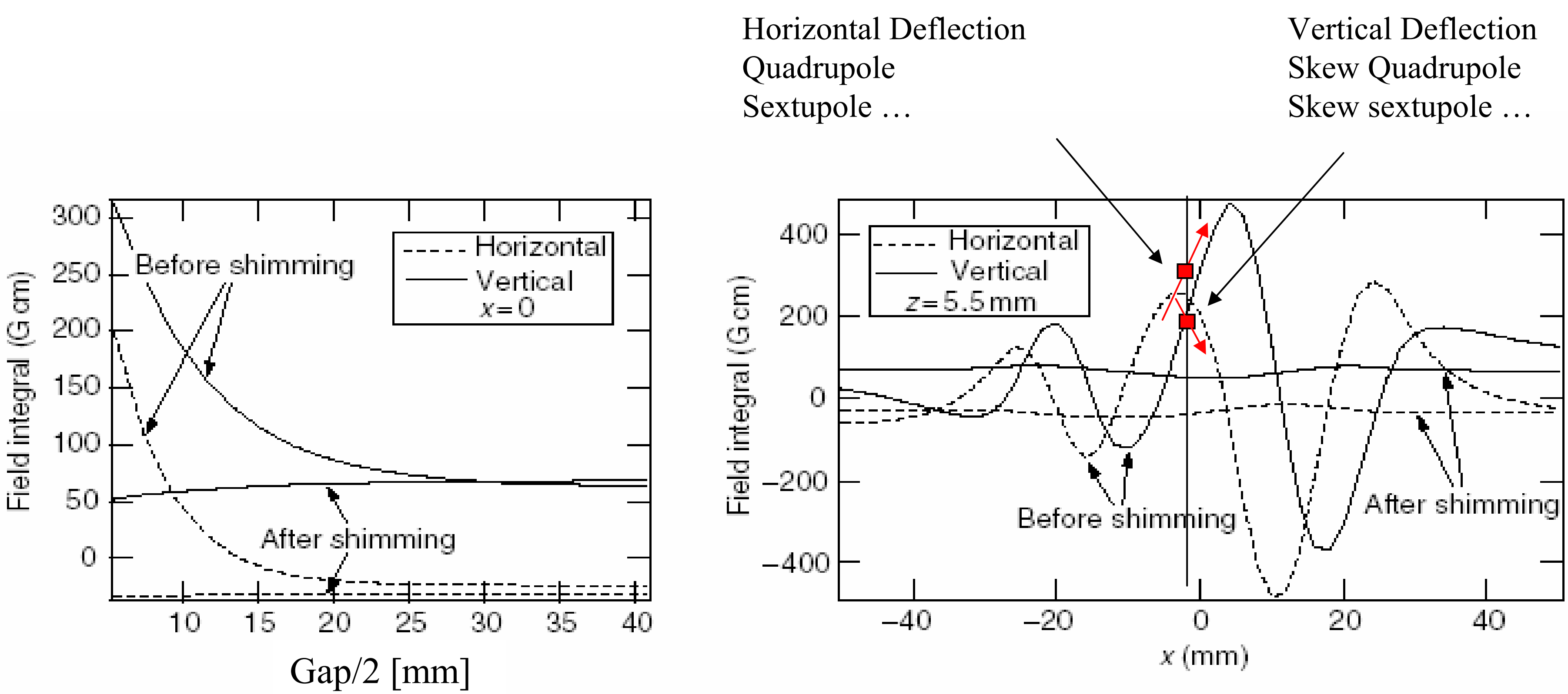
- Mechanical Shimming :
  - Moving permanent magnet or iron pole vertically or horizontally
  - Best when free space and mechanical fixation make it possible.

- Magnetic Shimming :
  - Add thin iron piece at the surface of the blocks
  - Reduce minimum gap and reduce the peak field

Magnetic shims



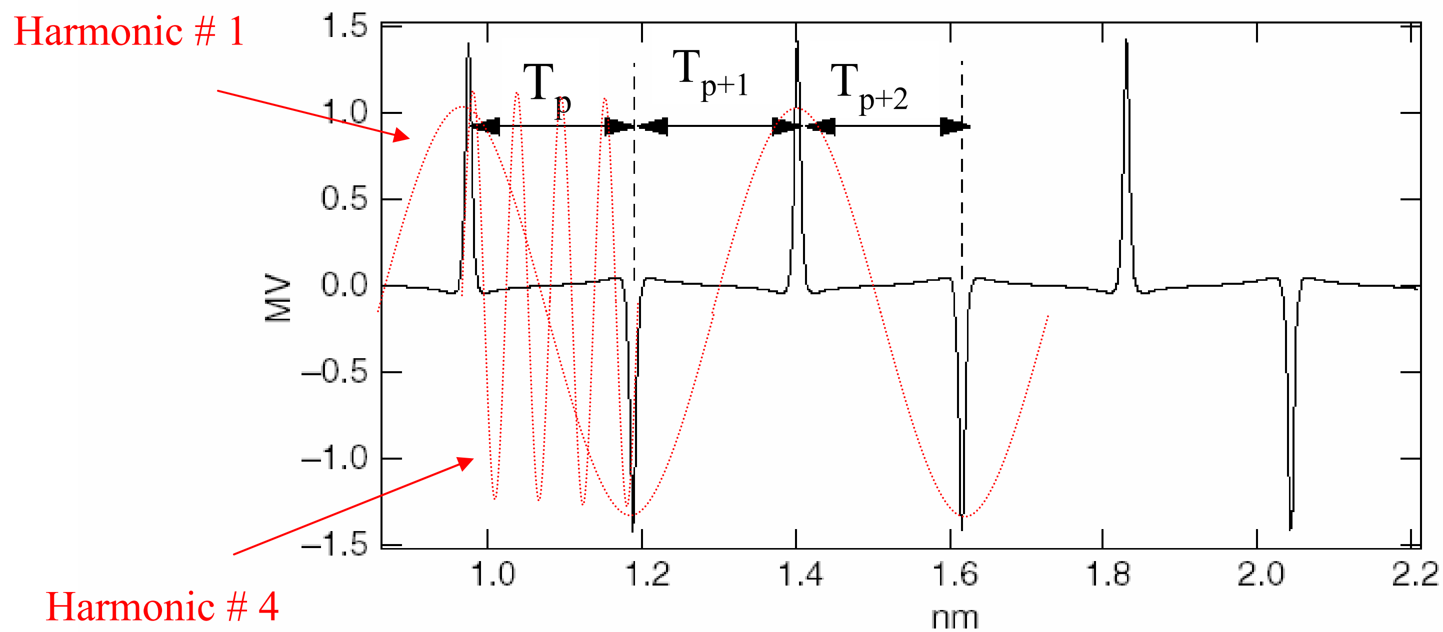
Field Integral and Multipole Shimming



T_p : time distance between successive peaks

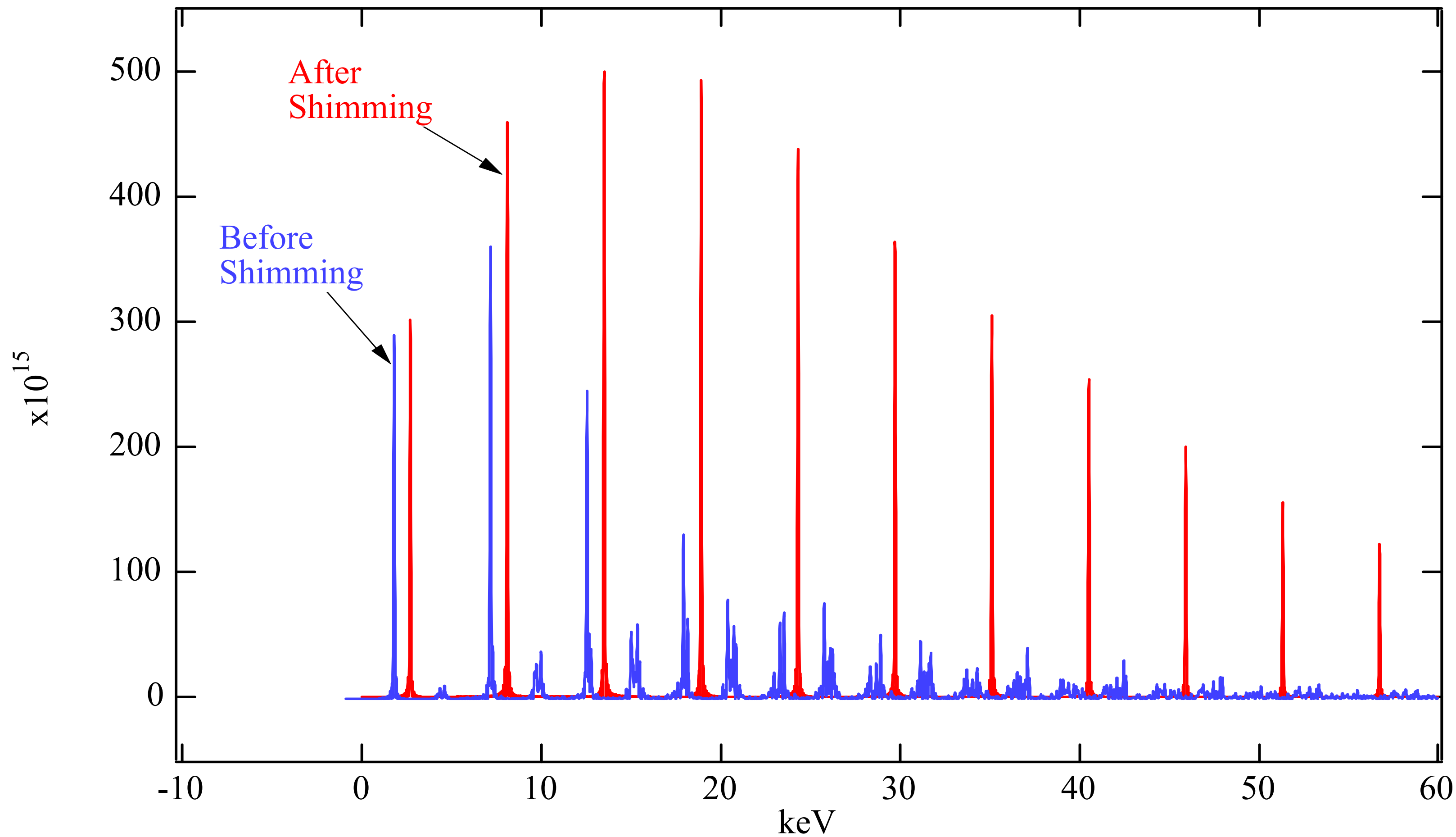
$$T_p = \frac{c\gamma^2}{\lambda_0 (1 + \frac{K^2}{2})}$$

T_p varies from one pole to the next due to period and peak field fluctuations



The Phase shimming consists of a set of local magnetic field corrections, which make T_p always identical.

Phase Shimming and the single electron spectrum



- I hope that this short introduction has incited your curiosity in the broad and exciting field that is Insertion Devices.

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