Field Solvers

TECHNISCHE UNIVERSITÄT DARMSTADT

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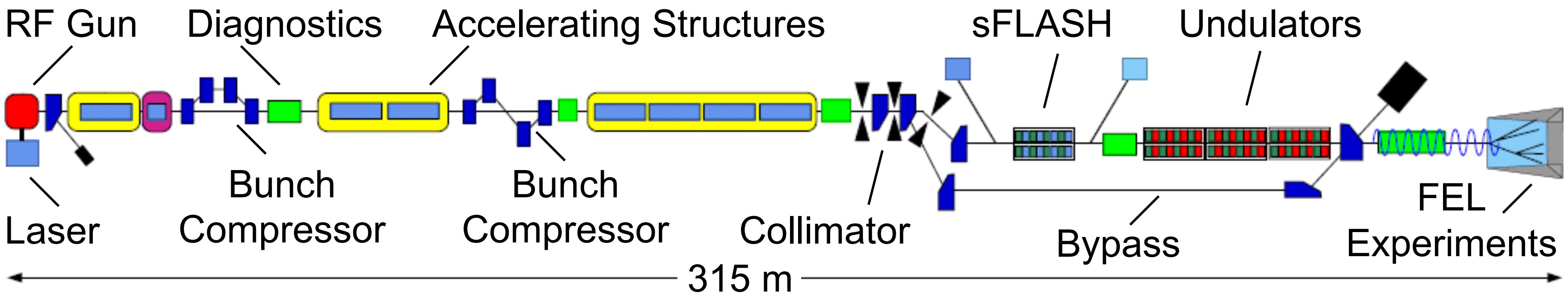
CERN Accelerator School 2018
Thessaloniki, Greece, 11-23 November 2018

Lecture 3 : Simulating Accelerating Cavities

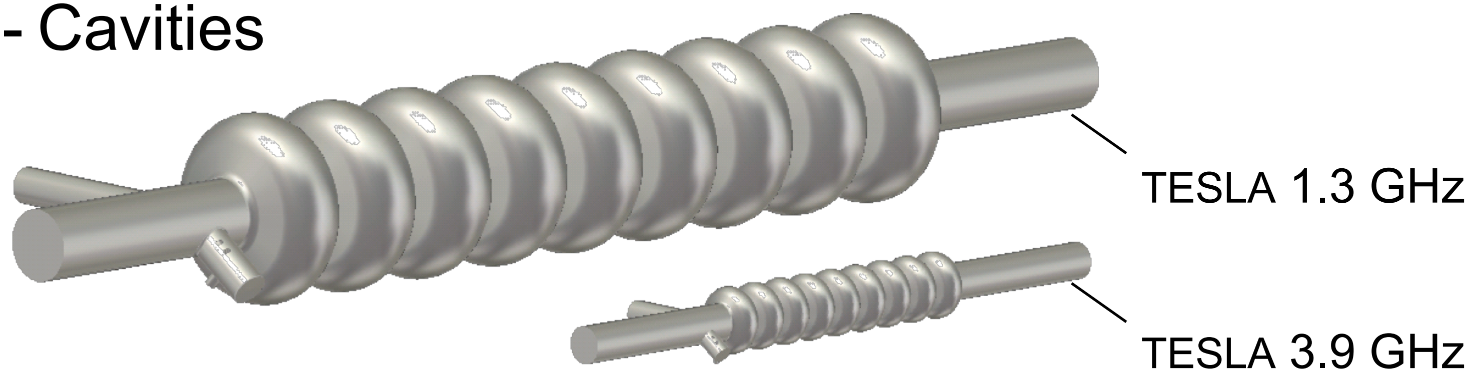
Motivation

- Particle accelerators
  - FLASH at DESY, Hamburg

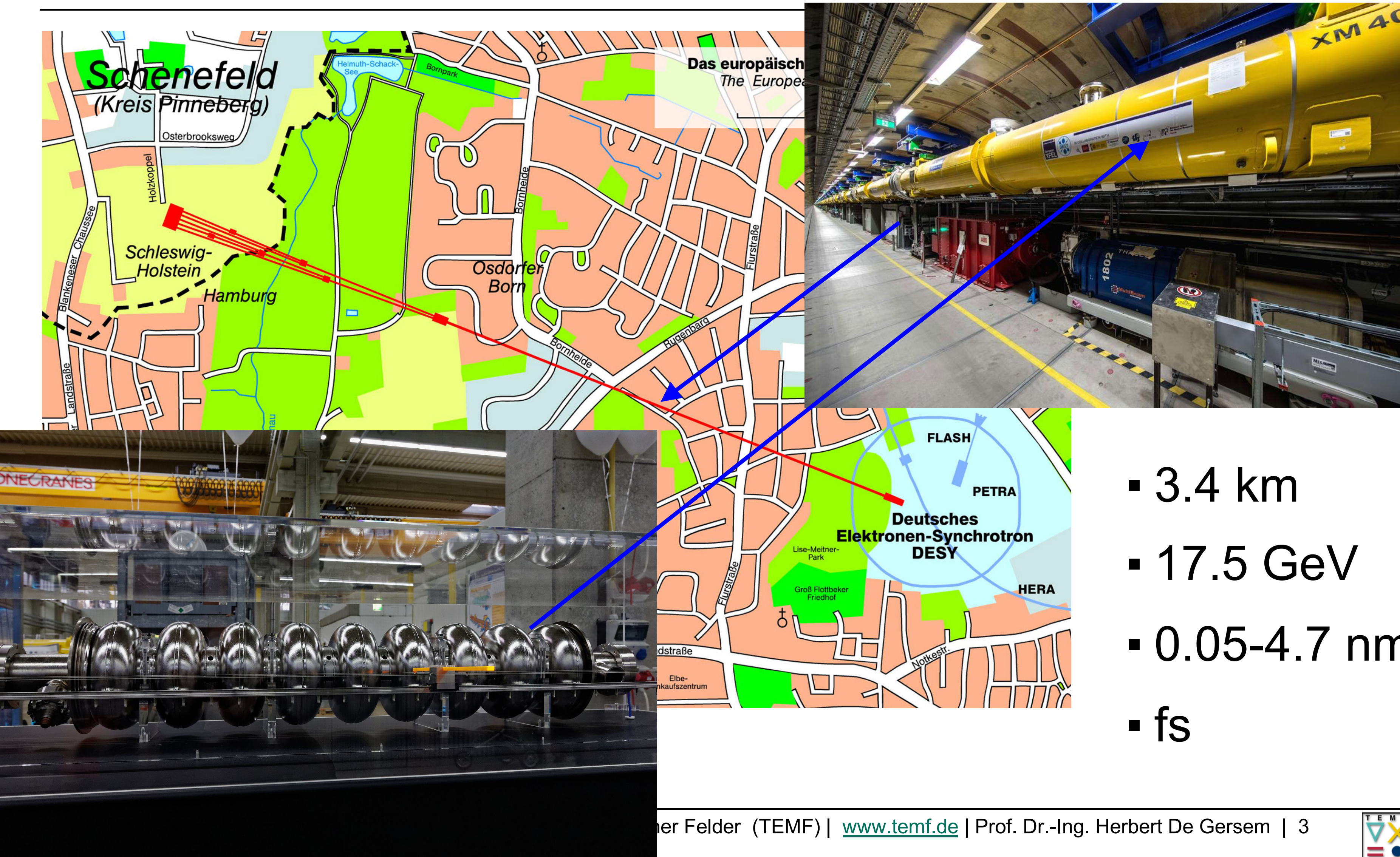


http://www.desy.de

  - Cavities

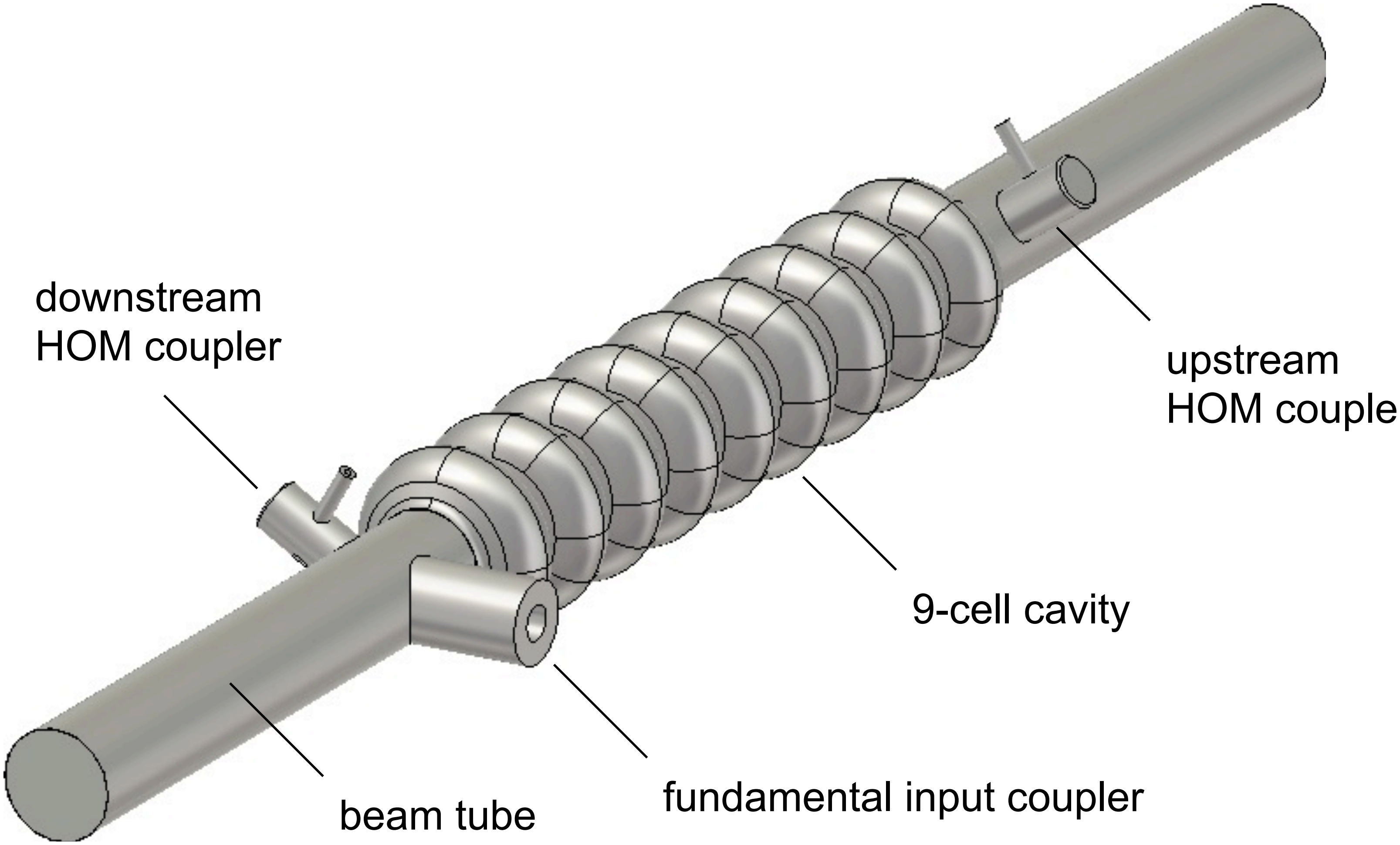


XFEL @DESY, Hamburg

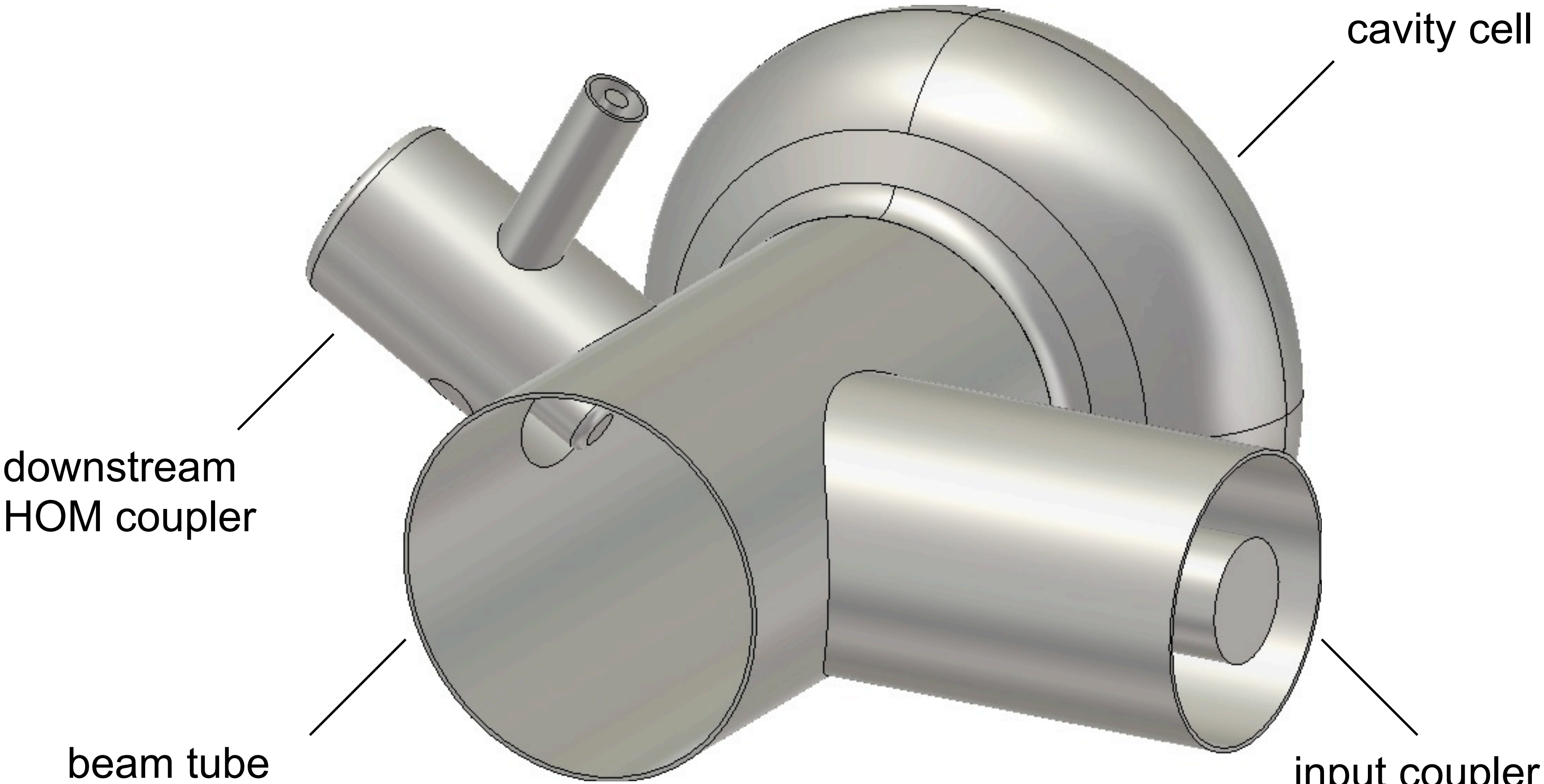


- 3.4 km
- 17.5 GeV
- 0.05-4.7 nm
- fs

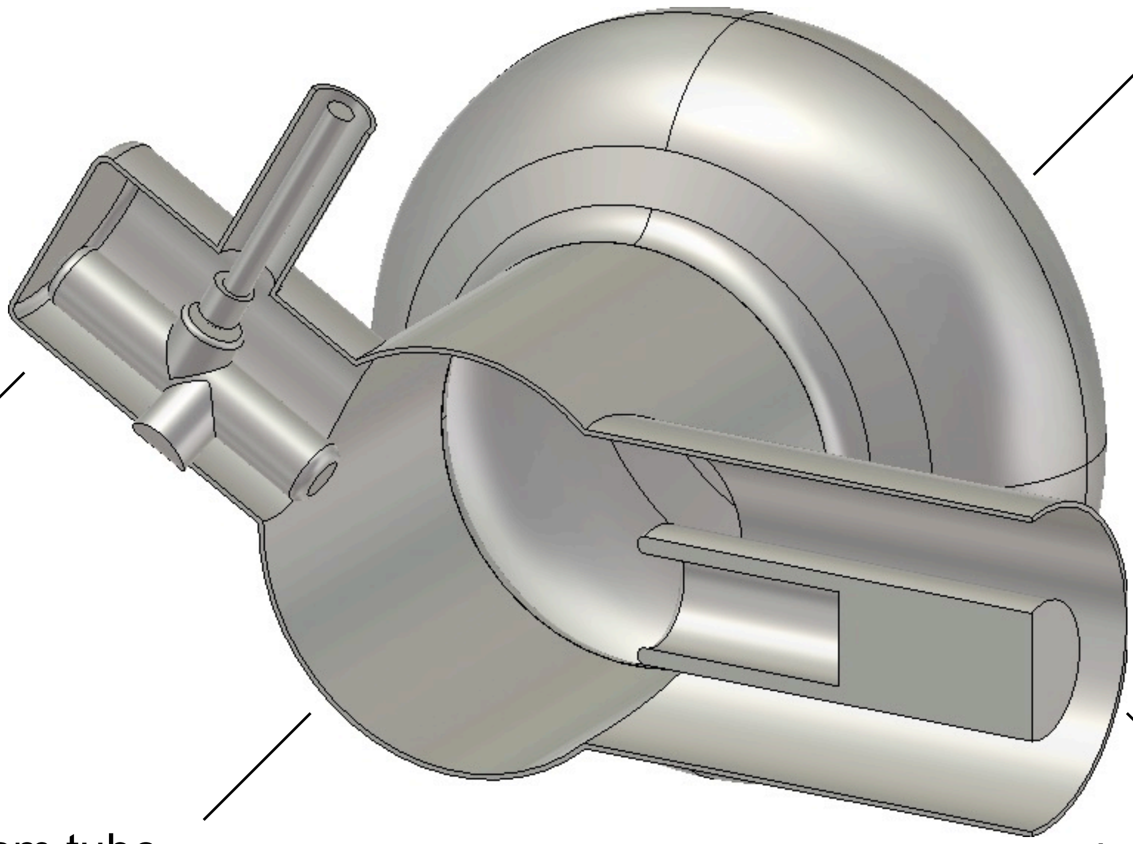
TESLA 3.9 GHz Cavity



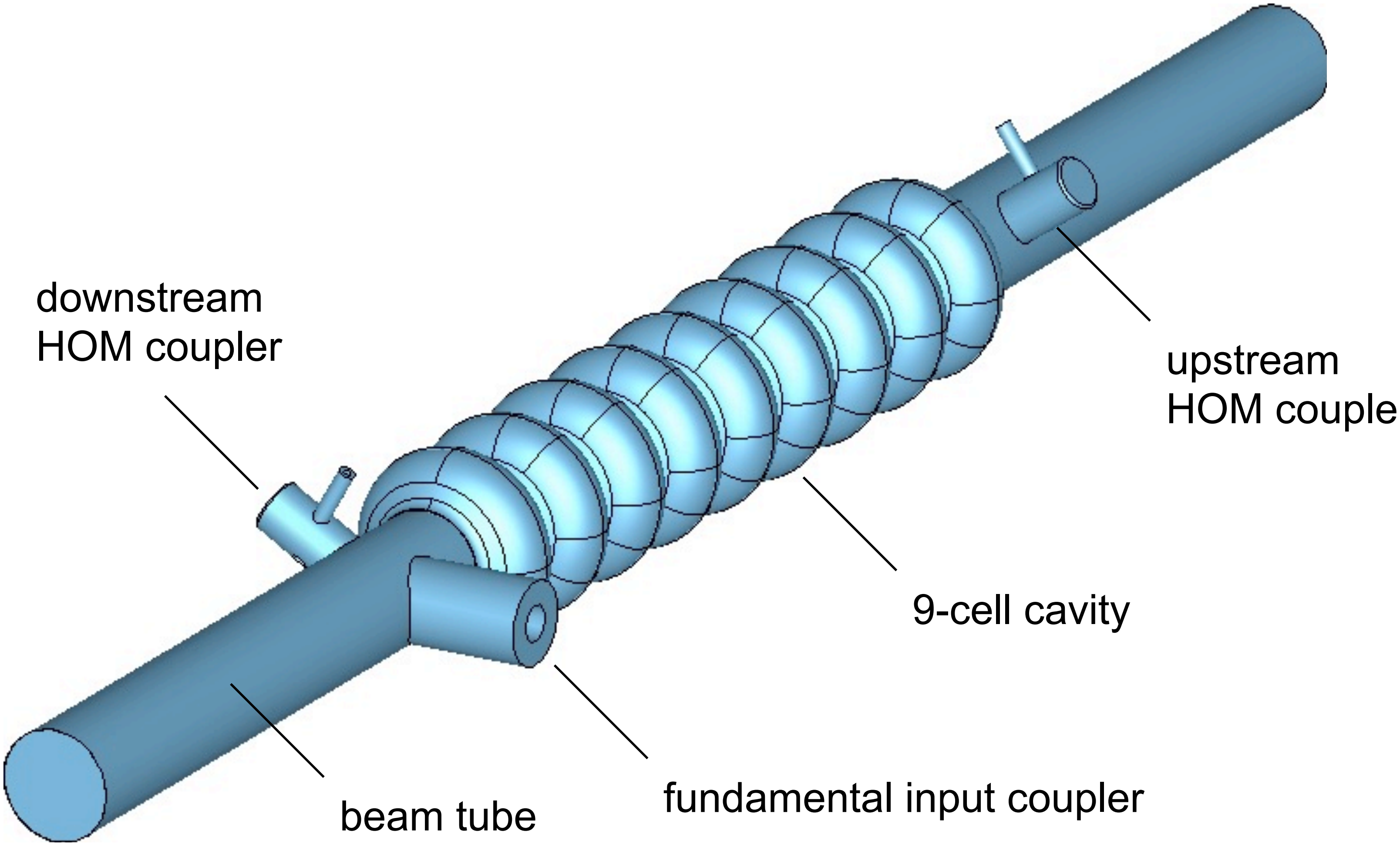
TESLA 3.9 GHz Cavity



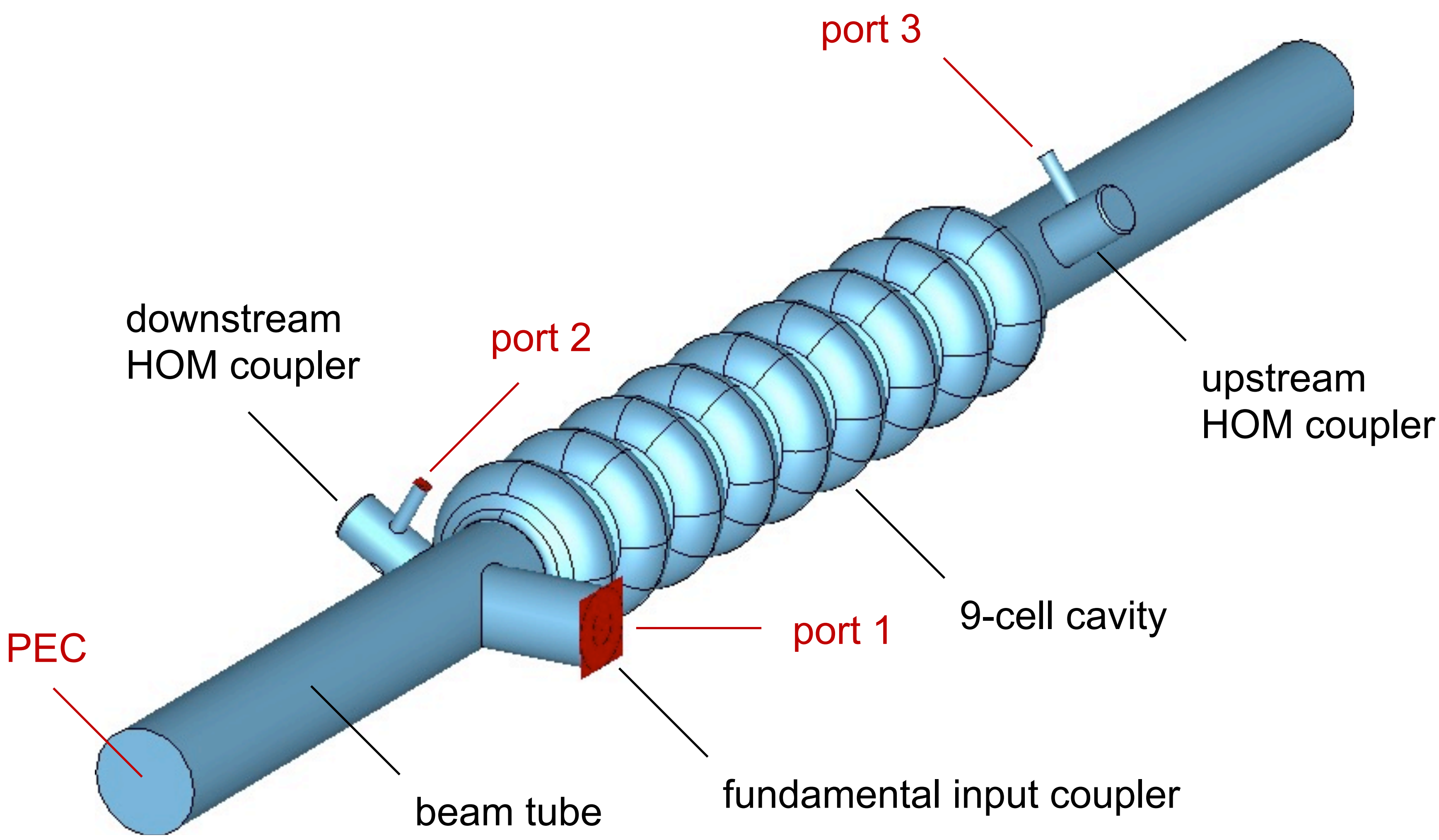
TESLA 3.9 GHz Cavity



TESLA 3.9 GHz Cavity (Model)



TESLA 3.9 GHz Cavity (Model)



TESLA 3.9 GHz Cavity (Model)

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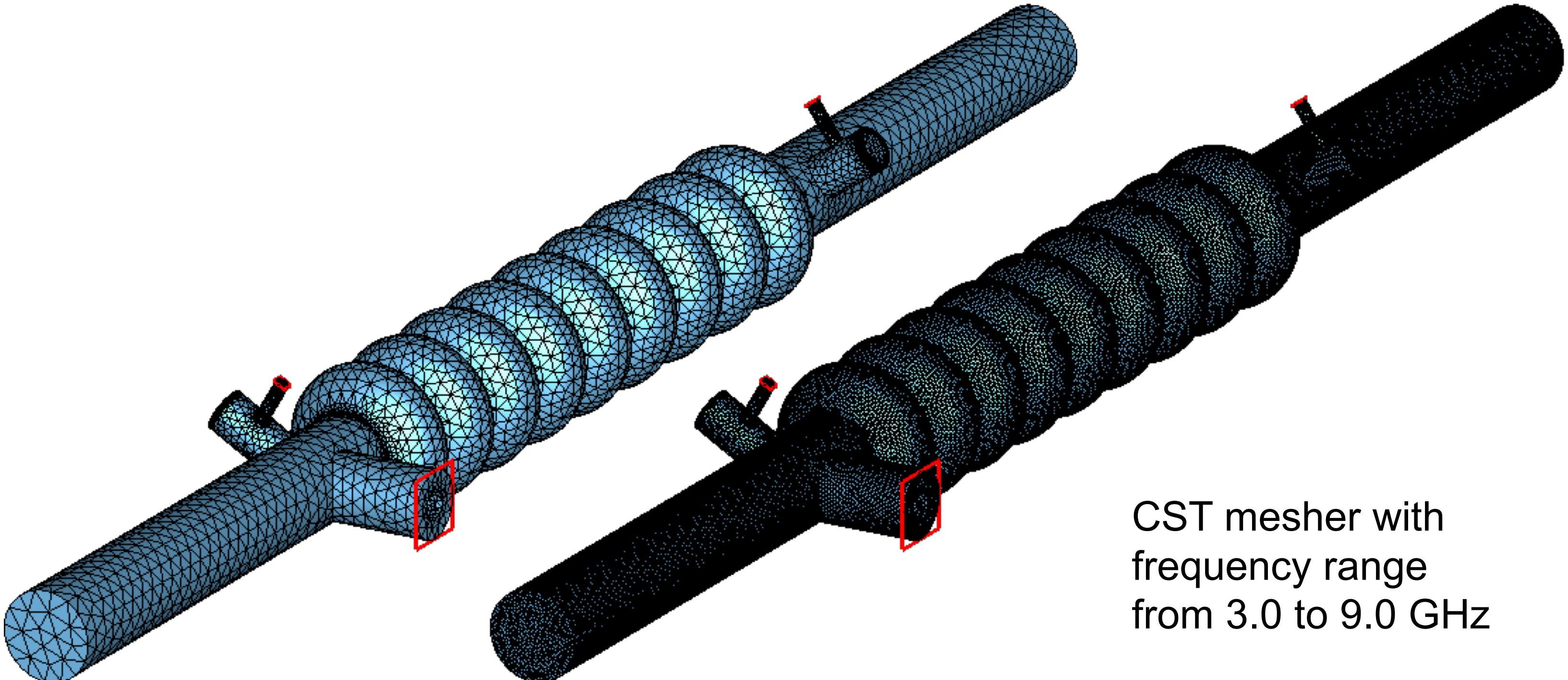


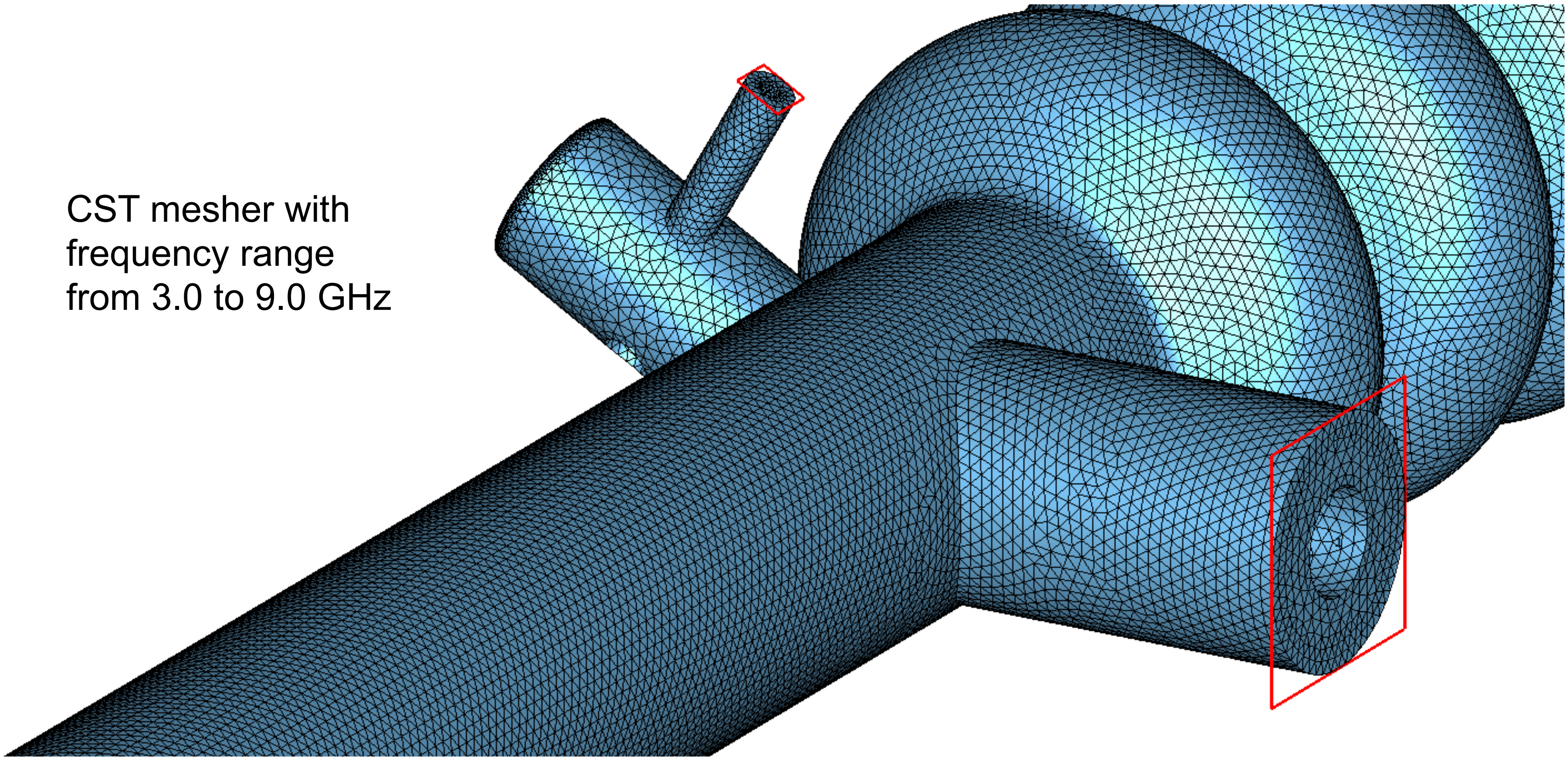
CST mesher with frequency range from 3.0 to 9.0 GHz

LPW	4	6	8	10	12	14	16	18	20
tetrahedra	136.443	187.435	304.833	480.376	767.271	1.177.883	1.704.528	2.432.978	3.337.736
complex DOF	761.820	1.079.488	1.802.314	2.885.154	4.668.072	7.227.096	10.509.404	15.064.232	20.721.334

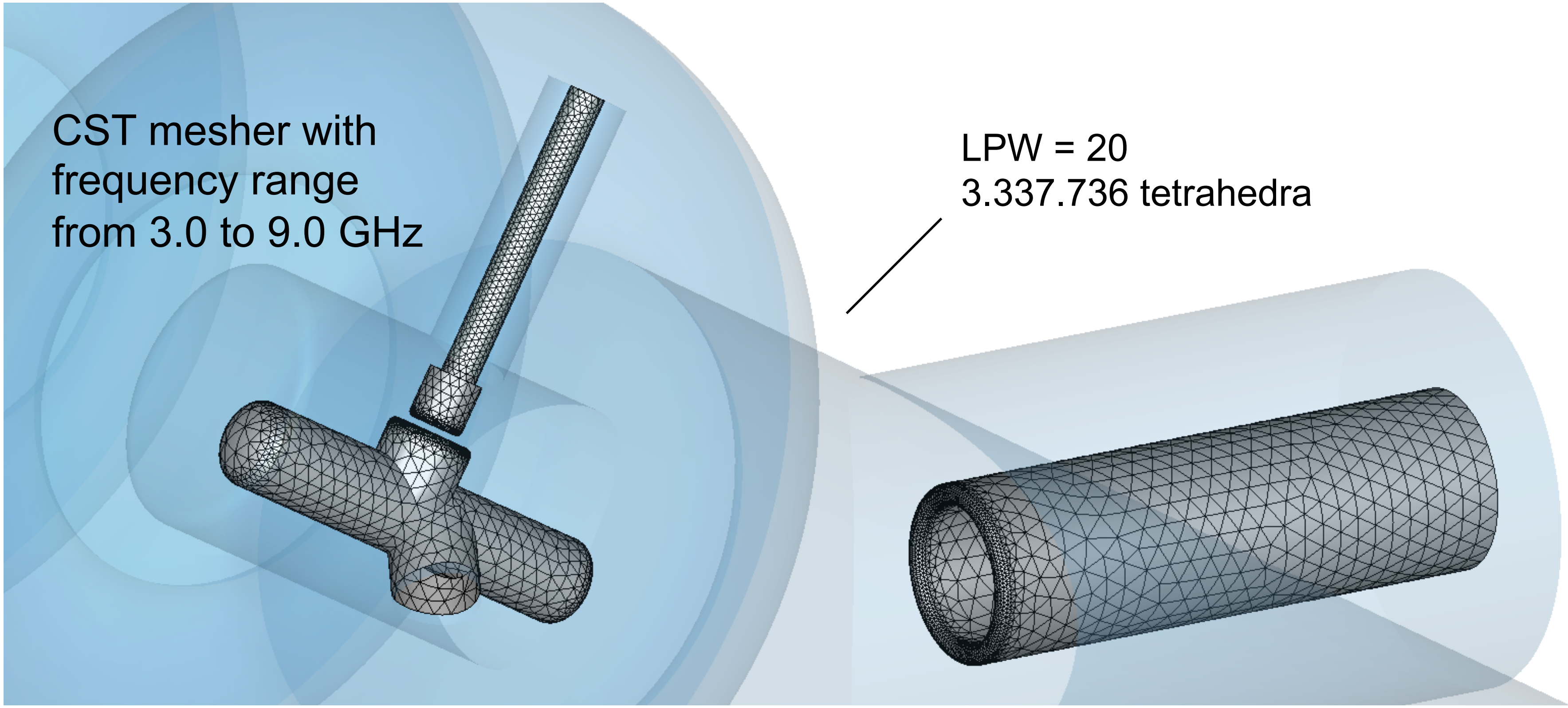
TESLA 3.9 GHz Cavity (Model)

LPW = 20
3.337.736 tetrahedra

CST mesher with frequency range from 3.0 to 9.0 GHz



TESLA 3.9 GHz Cavity (Model)



Outline

- XFEL + Tesla 3.9 GHz cavities
- FE eigenmode solver + on-axis fields
- Kirchhoff integrals + symmetric meshes

Finite-Element Eigenmode Solver

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FE discretisation

- local Ritz approach

$$\vec{E} = \vec{E}(\vec{r}) = \sum_{i=1}^{n} \alpha_i \, \vec{w}_i(\vec{r})$$

$\vec{w}$ vectorial function

α_i scalar coefficient

i global index

n number of DOFs

$$\mathrm{curl}\ 1/\mu_\mathrm{r}\ \mathrm{curl}\vec{E} = \left(\frac{\omega}{c_0}\right)^2 \varepsilon_\mathrm{r}\, \vec{E}\,\Big|_{\vec{r}\in\Omega}$$

$$\mathrm{div}(\varepsilon\vec{E})\Big|_{\vec{r}\in\Omega} = 0 \qquad \text{+ boundary conditions}$$

continuous eigenvalue problem

Galerkin

$$A_{ij} = \iiint_\Omega 1/\mu_\mathrm{r}\, \mathrm{curl}\vec{w}_i \cdot \mathrm{curl}\vec{w}_j \,\mathrm{d}\Omega$$

$$B_{ij} = \iiint_\Omega \varepsilon_\mathrm{r}\, \vec{w}_i \cdot \vec{w}_j \,\mathrm{d}\Omega$$

$$C_{ij} = \iiint_\Omega Z_0\, \sigma\, \vec{w}_i \cdot \vec{w}_j \,\mathrm{d}\Omega$$

$$A\vec{\alpha} + j\,\frac{\omega}{c_0} C\vec{\alpha} + \left(j\,\frac{\omega}{c_0}\right)^2 B\vec{\alpha} = 0$$

discrete eigenvalue problem

Finite-Element Eigenmode Solver

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Jacobi-Davidson method

- important properties
  - direct solution difficult because of dense matrix in correction equation.
  - iterative solution not immediately applicable because vectors $\Delta\vec{x}$ with $\Delta\vec{x} \in R\{(V_B)_\perp\}$ are not mapped back onto $R\{(V_B)_\perp\}$ again.
- preconditioning
  - JD - preconditioner

$$PC = \{I - M^{-1}V_B[(M^{-1}V_B)^T V_B]^{-1} V_B^T\} M^{-1}$$
$$= M^{-1} - M^{-1}V_B[(M^{-1}V_B)^T V_B]^{-1} V_B^T M^{-1}$$

retains the property $\Delta\vec{x} \in R\{(V_B)_\perp\}$ for any preconditioner M^{-1}.

➡ simplest case: $M^{-1} = I \quad \hookrightarrow \quad PC = I - VV_B^T = P$

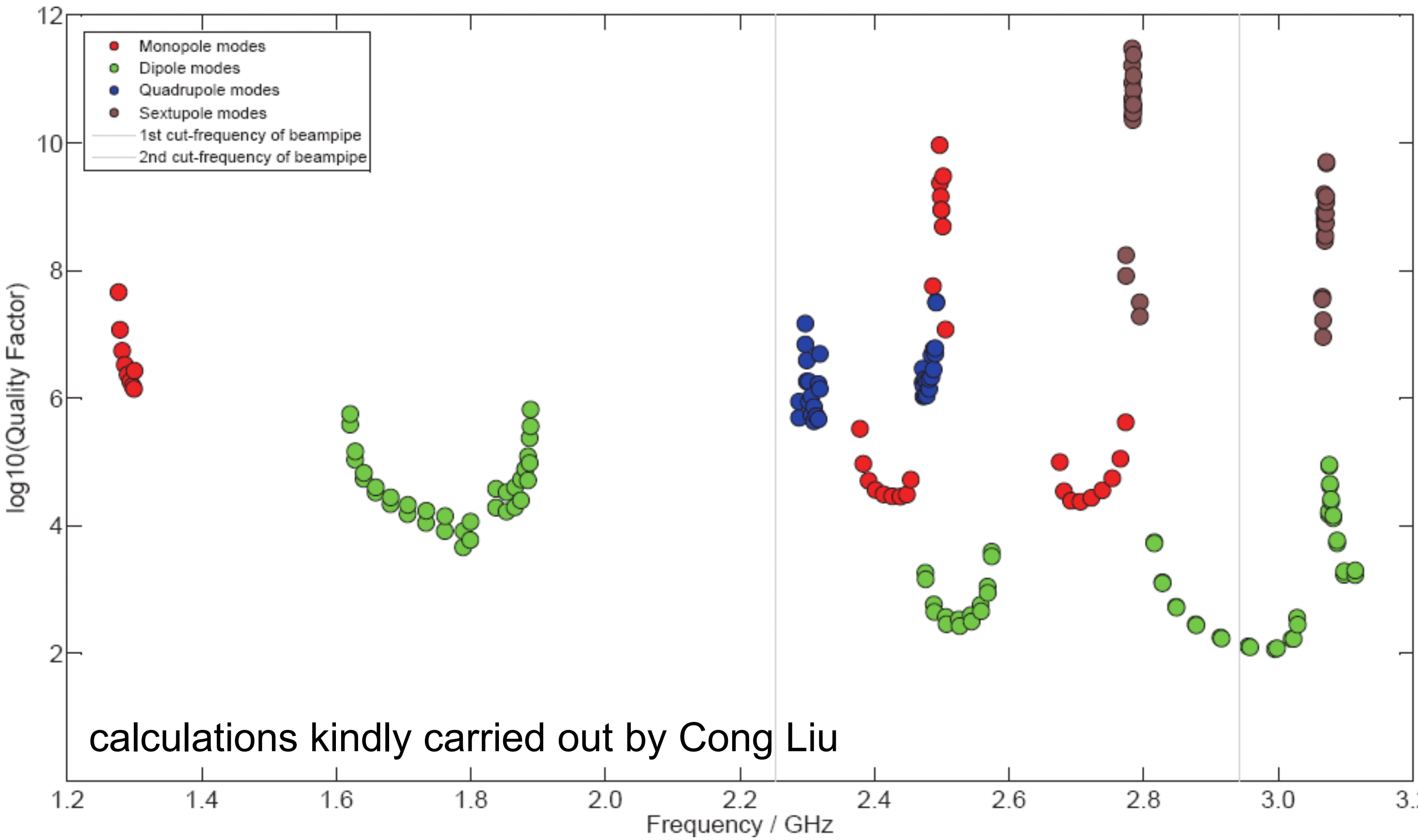
Finite-Element Eigenmode Solver

parallelisation on a compute cluster

200 nodes
400 CPUs
2400 cores
90kW cooling power
80kW power
3200 GB memory
~7t weight
~1M€ investment cost
40 GBIT connection

Numerical Examples

quality factor versus frequency



calculations kindly carried out by Cong Liu

Mode Atlas

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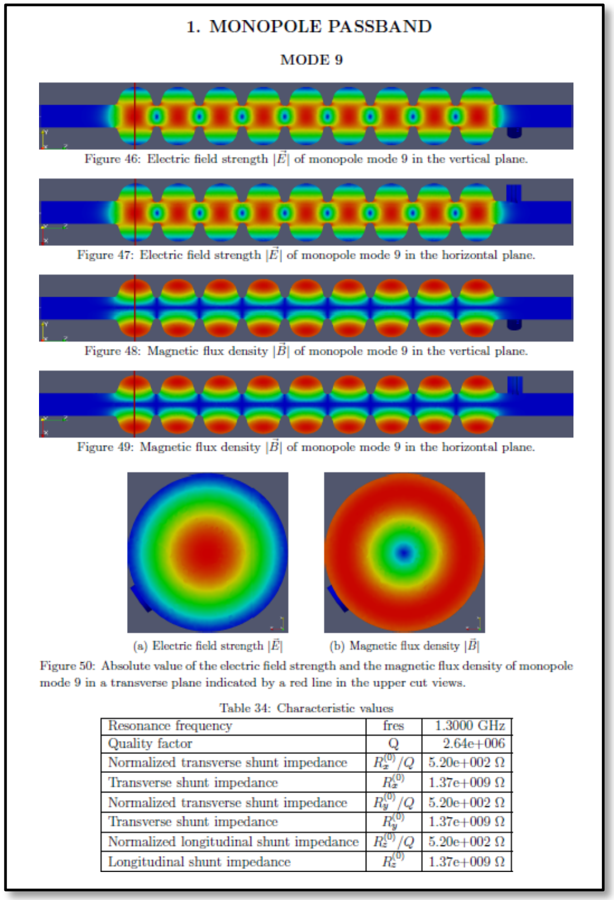
collection of the first 194 modes (selected page)



1. MONOPOLE PASSBAND

MODE 9

Figure 46: Electric field strength $|\vec{E}|$ of monopole mode 9 in the vertical plane.

Figure 47: Electric field strength $|\vec{E}|$ of monopole mode 9 in the horizontal plane.

Figure 48: Magnetic flux density $|\vec{B}|$ of monopole mode 9 in the vertical plane.

Figure 49: Magnetic flux density $|\vec{B}|$ of monopole mode 9 in the horizontal plane.

(a) Electric field strength $|\vec{E}|$ (b) Magnetic flux density $|\vec{B}|$

Figure 50: Absolute value of the electric field strength and the magnetic flux density of monopole mode 9 in a transverse plane indicated by a red line in the upper cut views.

Table 34: Characteristic values

Resonance frequency	fres	1.3000 GHz
Quality factor	Q	2.64e+006
Normalized transverse shunt impedance	$R_x^{(0)}/Q$	5.20e+002 Ω
Transverse shunt impedance	$R_x^{(0)}$	1.37e+009 Ω
Normalized transverse shunt impedance	$R_y^{(0)}/Q$	5.20e+002 Ω
Transverse shunt impedance	$R_y^{(0)}$	1.37e+009 Ω
Normalized longitudinal shunt impedance	$R_z^{(0)}/Q$	5.20e+002 Ω
Longitudinal shunt impedance	$R_z^{(0)}$	1.37e+009 Ω

magnitude of the electric field strength (longitudinal cut)

magnitude of the magnetic flux density (longitudinal cut)

magnitude of the electric field and the magnetic flux density (transverse cut)

resonance frequency, quality factor and shunt impedances

TESLA 3.9 GHz Cavity (Results)

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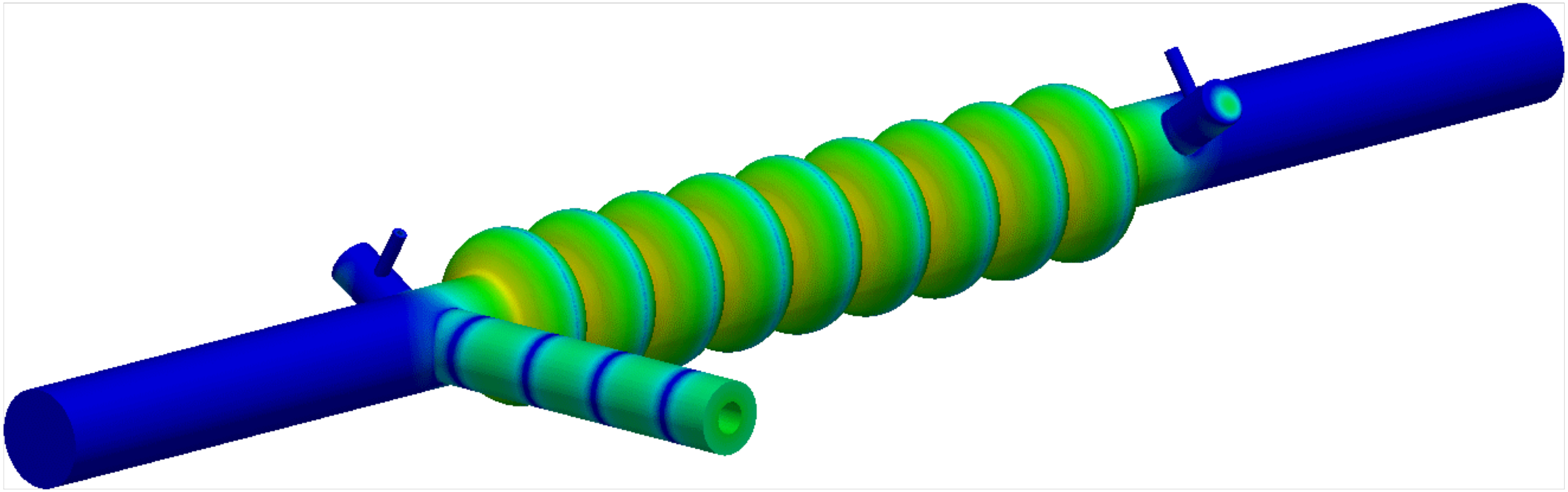
fundamental mode

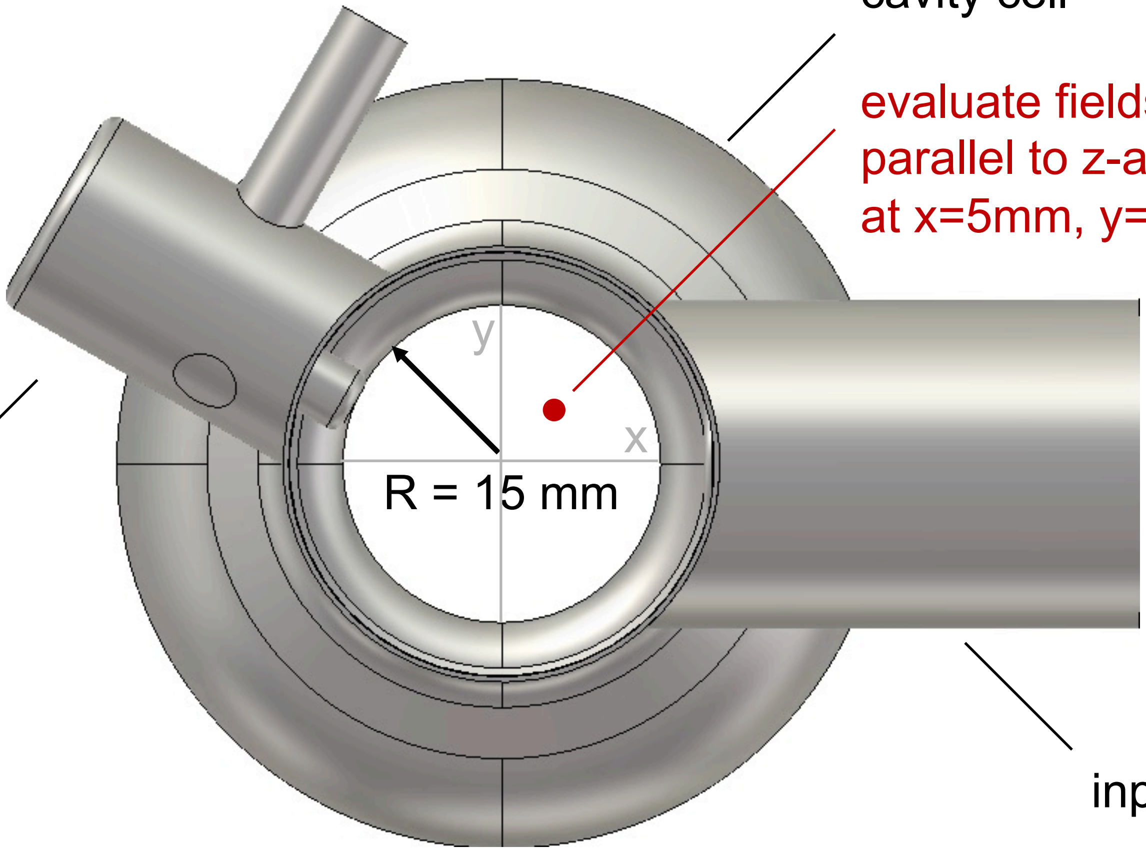
absolute value of the electric field strength $|\vec{E}|$



logarithmic scale from 10^4 to 10^7 V/m

LPW = 20
3.337.736 tetrahedra

TESLA 3.9 GHz Cavity (Results)



TESLA 3.9 GHz Cavity (Results)

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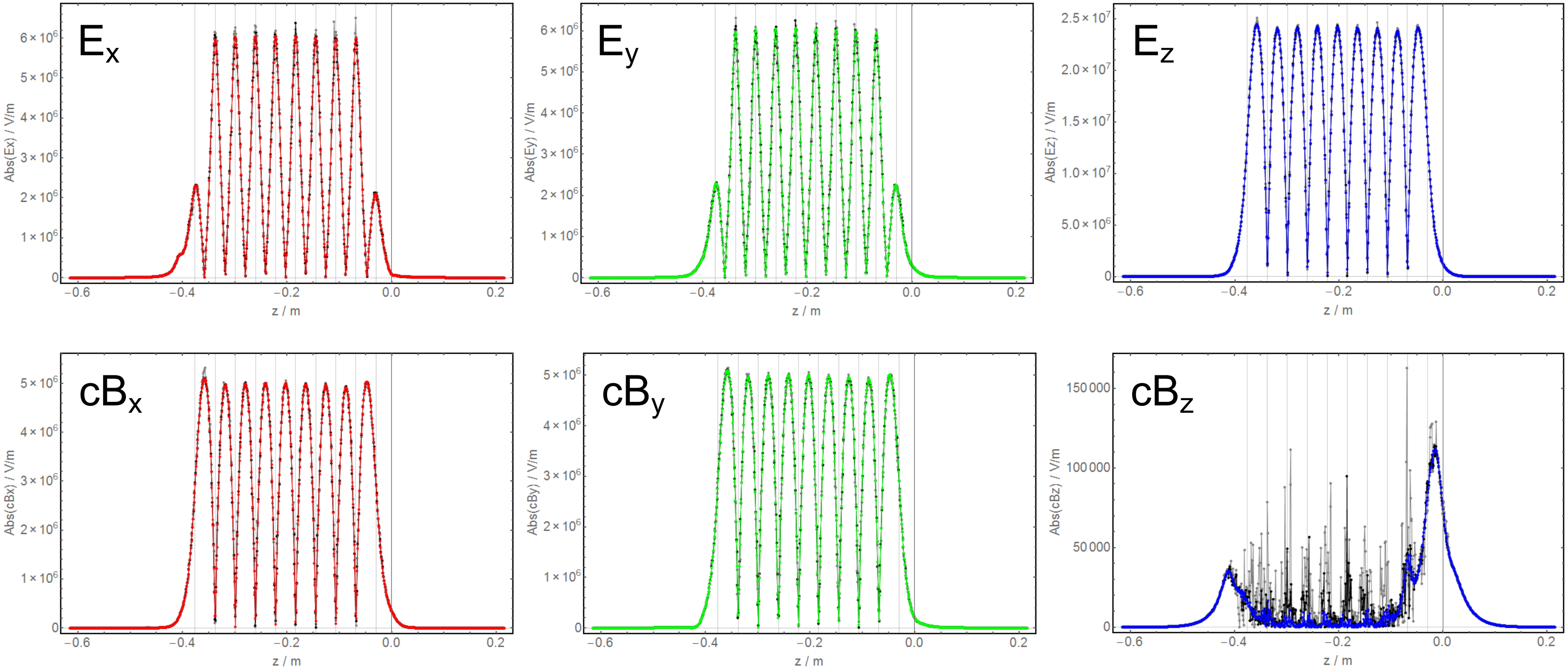
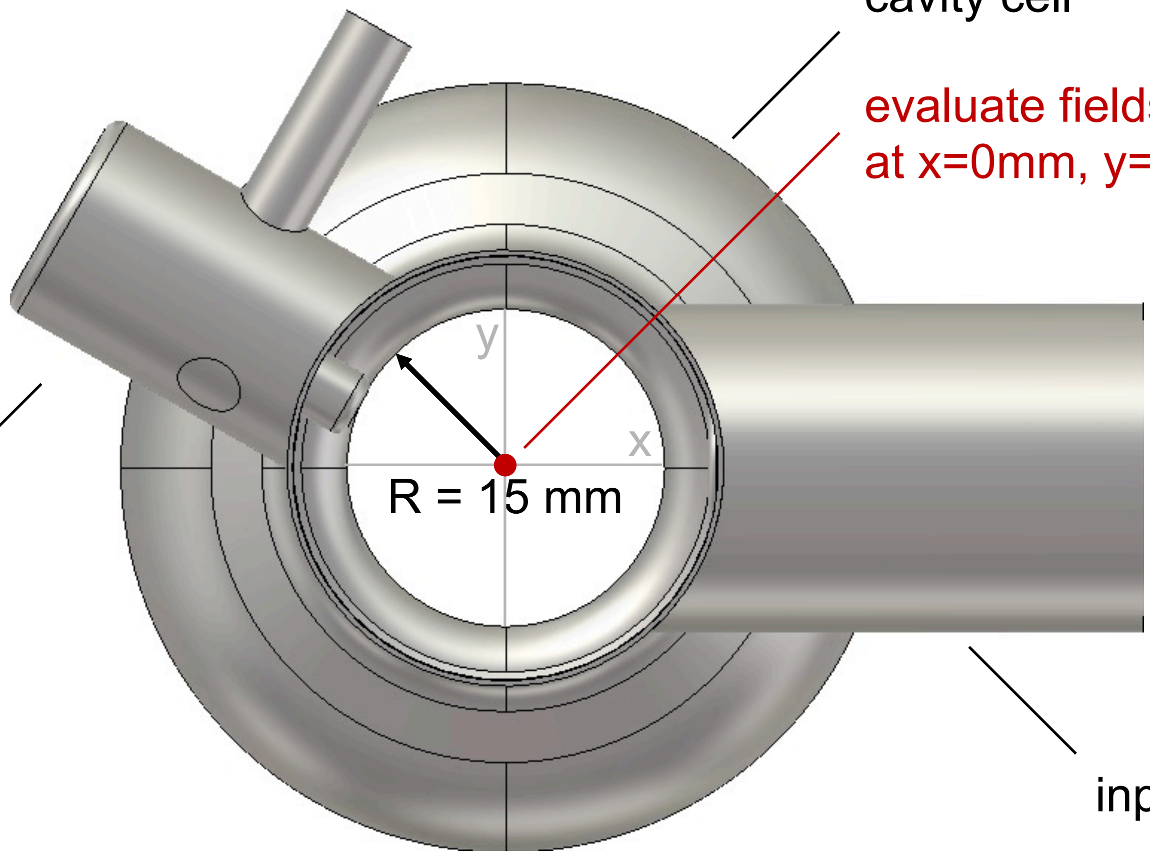
- Field components parallel to the cavity axis (LPW 4,8,16)
  - Transversal offset at x_0 = 5 mm, y_0 = 5 mm

off-axis



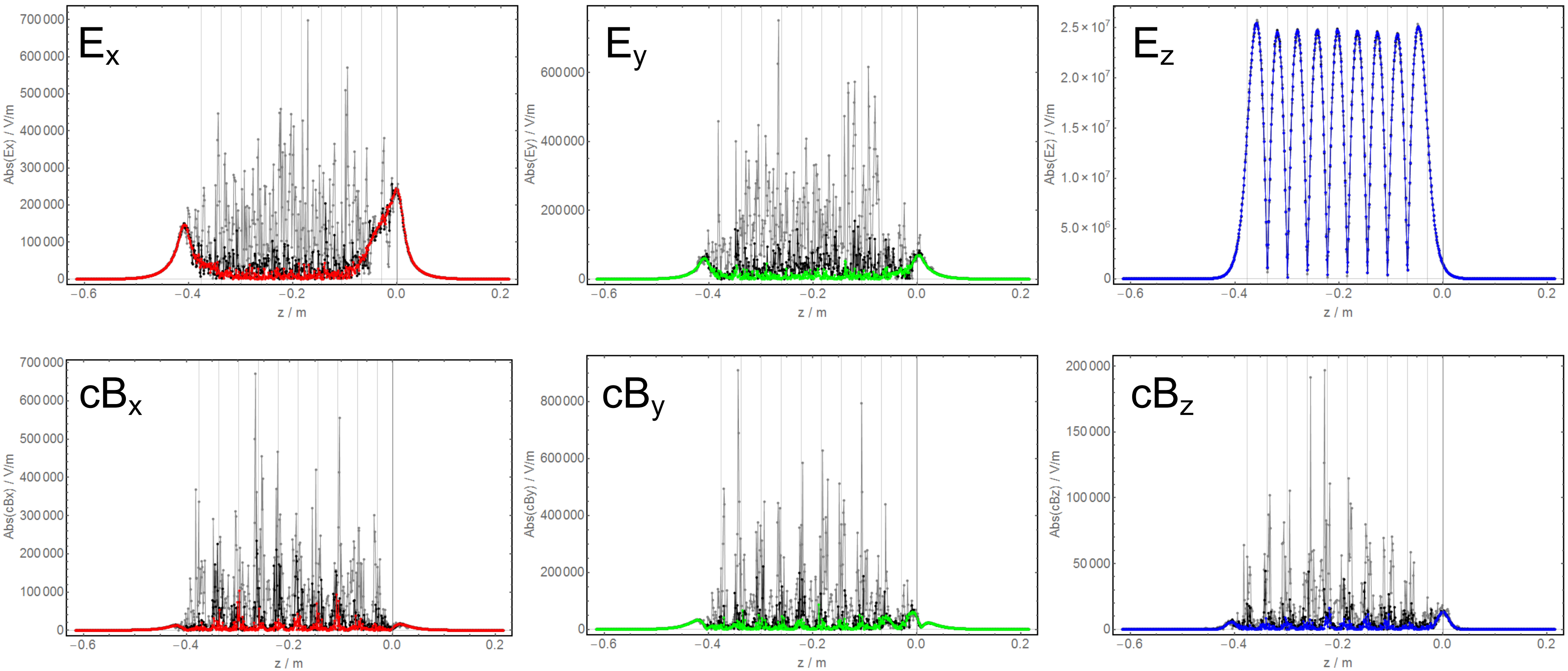
TESLA 3.9 GHz Cavity (Results)



TESLA 3.9 GHz Cavity (Results)

- Field components parallel to the cavity axis (LPW 4,8,16)
  - Transversal offset at x_0 = 0 mm, y_0 = 0 mm

on-axis (standard)



TESLA 3.9 GHz Cavity (Results)

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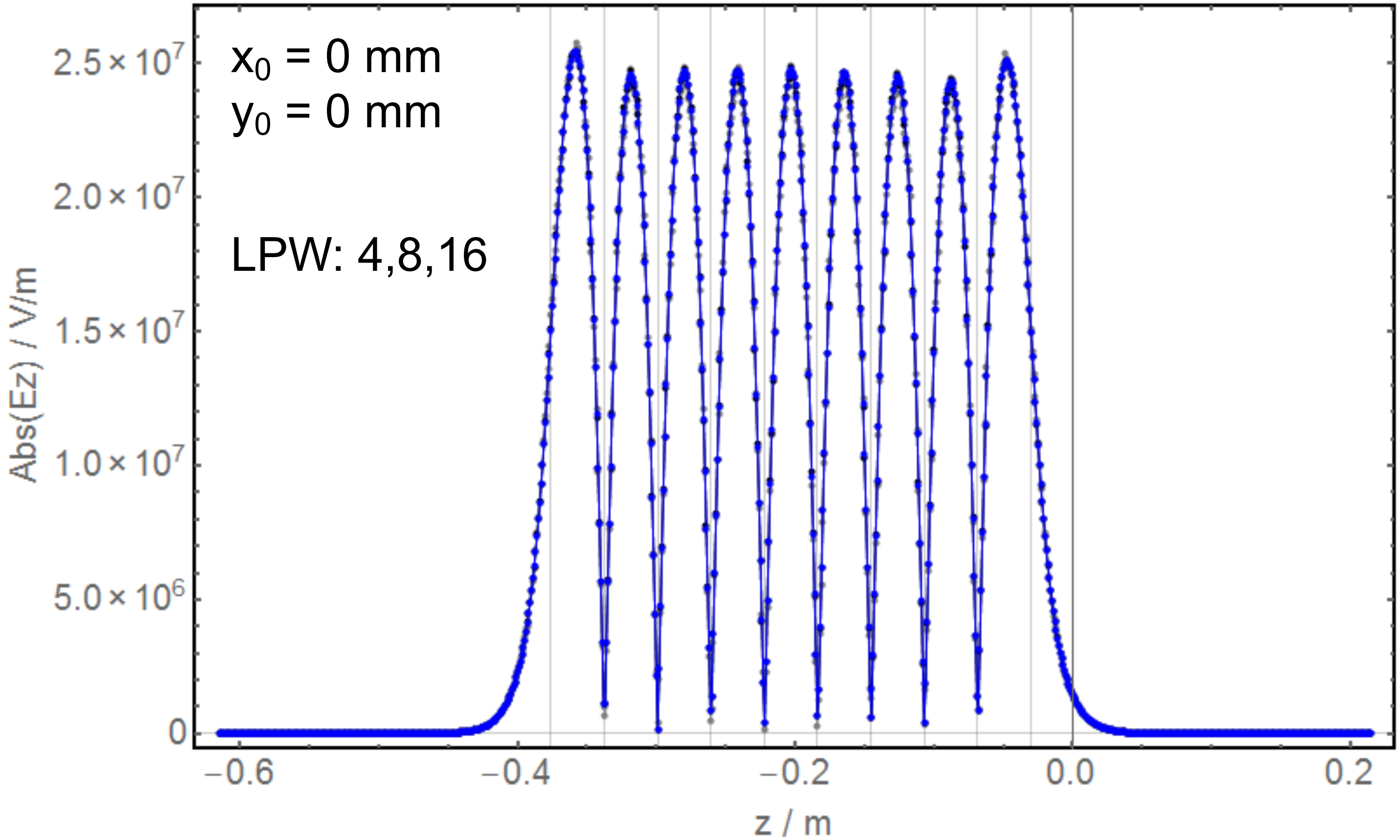
- Field component E_z parallel to the cavity axis

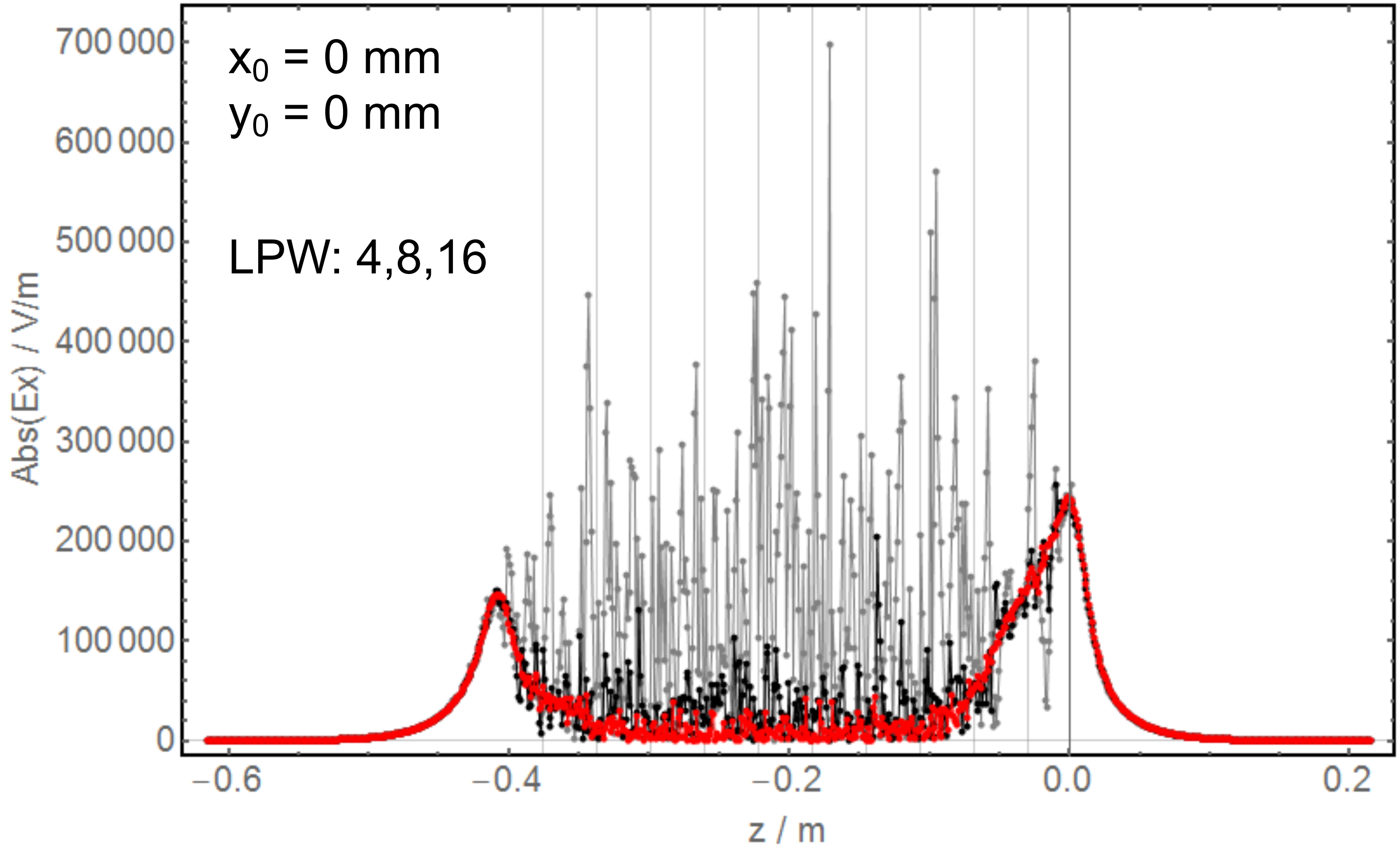
on-axis (standard)



TESLA 3.9 GHz Cavity (Results)

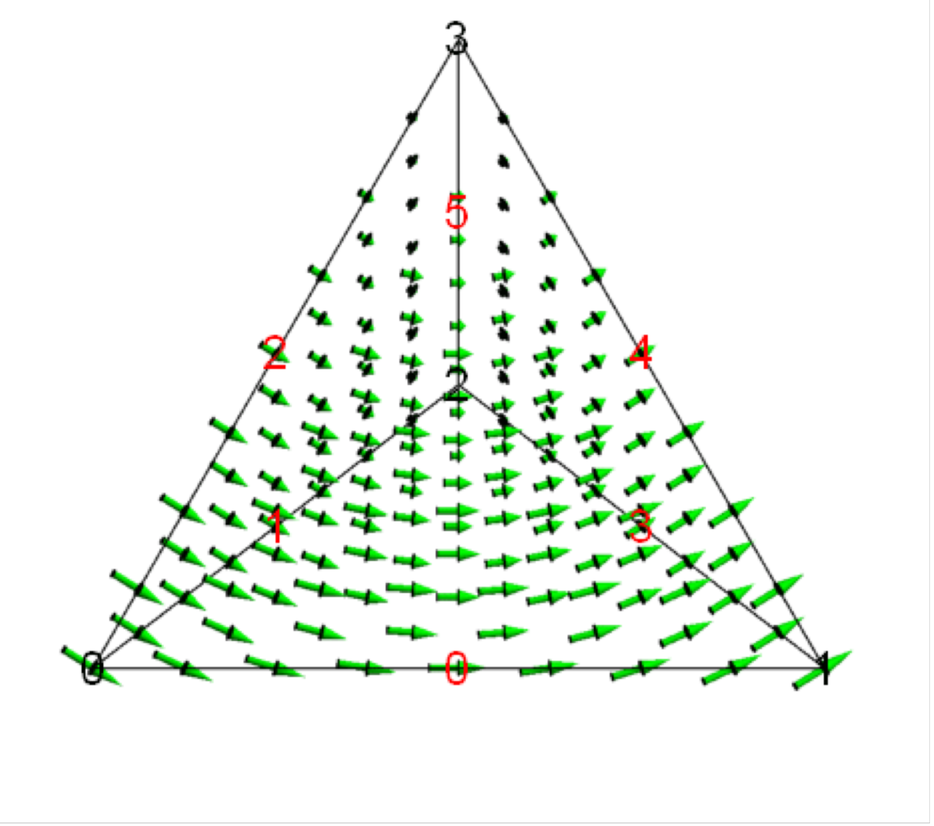
- Field component E_x parallel to the cavity axis

on-axis (standard)



Finite-Element Method

- Field representation in the finite-element method
  - Edge shape funktion $\vec{w}_0(\vec{r})$



example:
equilateral tetrahedron

point	x	y	z
0	0	0	0
1	1	0	0
2	$\frac{1}{2}$	$\frac{1}{2}\sqrt{3}$	0
3	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\sqrt{2/3}$

Finite-Element Method

- Field representation in the finite-element method
  - Representation of the electric field strength

$$\vec{f}(\vec{r}) = \sum_{i=0}^{N-1} a_i \; \vec{w}_i(\vec{r})$$

  - Projection of an arbitrary electric field strength $\vec{f}$ on the basis $\vec{w}_i$

$$\sum_{i=0}^{N-1} a_i \underbrace{\iiint_{\Omega} \vec{w}_i \cdot \vec{w}_j \, d\Omega}_{\text{mat}} = \underbrace{\iiint_{\Omega} \vec{f} \cdot \vec{w}_j \, d\Omega}_{\text{vec}}$$

solve linear system to obtain the weighting coefficients a_i

Finite-Element Method

- Field representation in the finite-element method
  - Residuals of vector fields

$$\vec{R}(\vec{r}) = \sum_{i=0}^{N-1} a_i\ \vec{w}_i(\vec{r}) - \vec{f}(\vec{r})$$

  - Fundamental field components

$$\vec{f}(\vec{r}) = \begin{pmatrix}1\\0\\0\end{pmatrix} \Rightarrow \vec{R}(\vec{r}) = \begin{pmatrix}0\\0\\0\end{pmatrix}$$

$$\vec{f}(\vec{r}) = \begin{pmatrix}x\\0\\0\end{pmatrix} \Rightarrow \vec{R}(\vec{r}) = \begin{pmatrix}0\\0\\0\end{pmatrix}$$

$$\vec{f}(\vec{r}) = \begin{pmatrix}x^2\\0\\0\end{pmatrix} \Rightarrow \vec{R}(\vec{r}) = \begin{pmatrix}(x-1)x + (33 + 10\sqrt{3}y + 5\sqrt{6}z)/180\\0\\0\end{pmatrix}$$

example:
FE method with full linear basis

order	DOFs per cell
0.5	6
1	12
1.5	20
2	30
2.5	45
3	60
3.5	84
4	105

Finite-Element Method

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- Field representation in the finite-element method
  - Residuals of vector fields

$$\vec{R}(\vec{r}) = \sum_{i=0}^{N-1} a_i \, \vec{w}_i(\vec{r}) - \vec{f}(\vec{r})$$

  - Fundamental field components

$$\vec{f}(\vec{r}) = \begin{pmatrix} x^2 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \vec{R}(\vec{r}) = \begin{pmatrix} (x-1)x + (33 + 10\sqrt{3}y + 5\sqrt{6}z)/180 \\ 0 \\ 0 \end{pmatrix}$$

$$\vec{f}(\vec{r}) = \begin{pmatrix} y^2 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \vec{R}(\vec{r}) = \begin{pmatrix} -7y/(6\sqrt{3}) + y^2 + (13 + 5\sqrt{6}z)/180 \\ 0 \\ 0 \end{pmatrix}$$

$$\vec{f}(\vec{r}) = \begin{pmatrix} z^2 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \vec{R}(\vec{r}) = \begin{pmatrix} 2/45 - 2/3\sqrt{2/3}z + z^2 \\ 0 \\ 0 \end{pmatrix}$$

Outline

- XFEL + Tesla 3.9 GHz cavities
- FE eigenmode solver + on-axis fields
- Kirchhoff integrals + symmetric meshes

Post-Processing: Kirchhoff Integral

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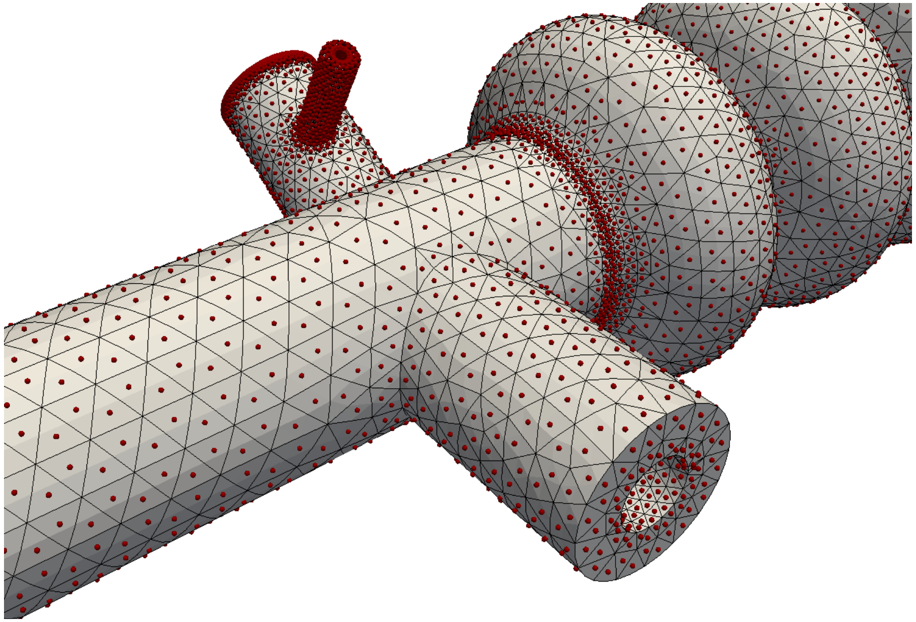
- Field reconstruction using the Kirchhoff integral
  - Field values inside a closed surface can be determined once the surface field components are available
  - Kirchhoff integral



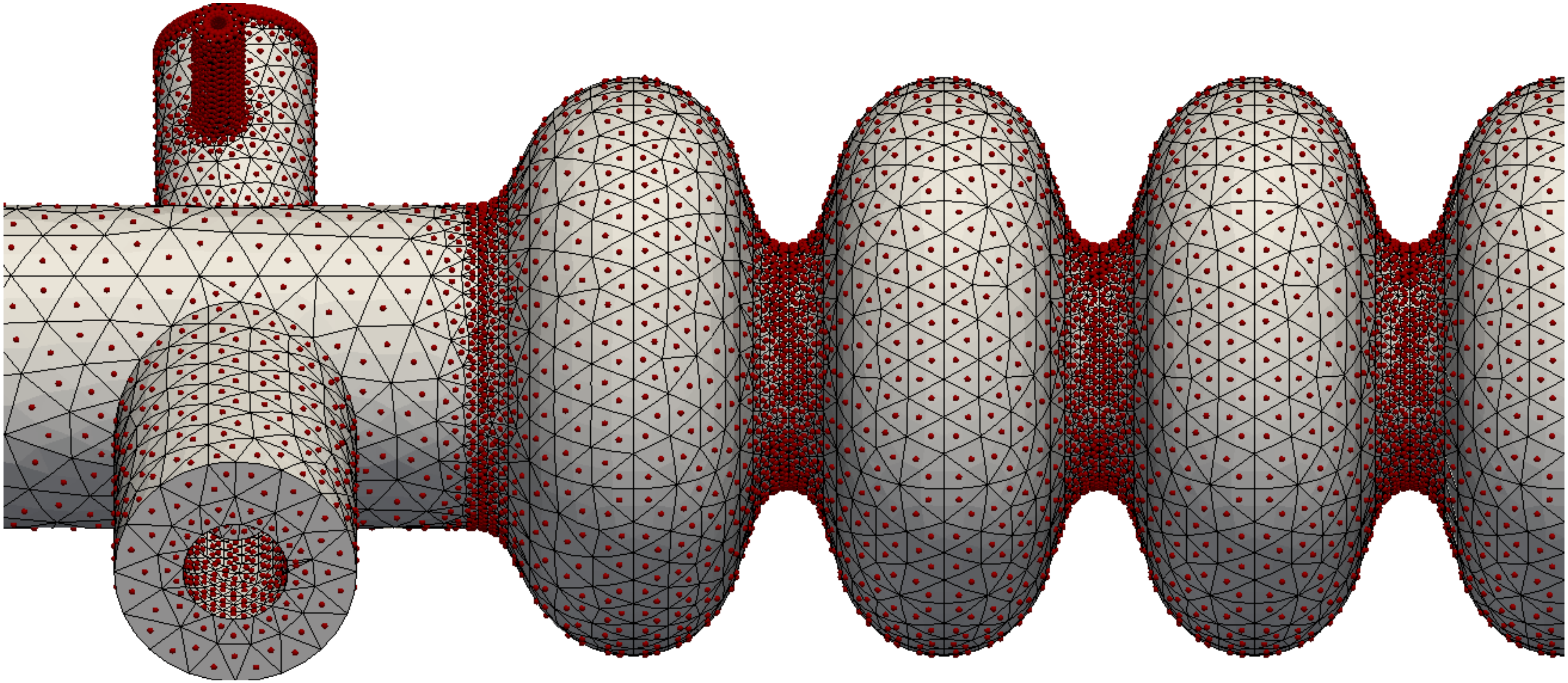
$$G = \frac{e^{-ik|\vec{r}-\vec{r}\,'|}}{4\pi|\vec{r}-\vec{r}\,'|} \qquad k = \frac{2\pi f}{c_0}$$

$$\vec{E}(\vec{r}) = \int \Big(k(\vec{n}' \times ic_0\vec{B}')\, G - (\vec{n}' \times \vec{E}') \times \nabla G - (\vec{n}' \cdot \vec{E}')\, \nabla G \Big)\, \mathrm{d}A'$$

$$ic_0\vec{B}(\vec{r}) = \int \Big(k(\vec{n}' \times \vec{E}')\, G - (\vec{n}' \times ic_0\vec{B}') \times \nabla G - (\vec{n}' \cdot ic_0\vec{B}')\, \nabla G \Big)\, \mathrm{d}A'$$

Post-Processing: Kirchhoff Integral

- Field reconstruction using the Kirchhoff integral
  - Surface selection



Post-Processing: Kirchhoff Integral

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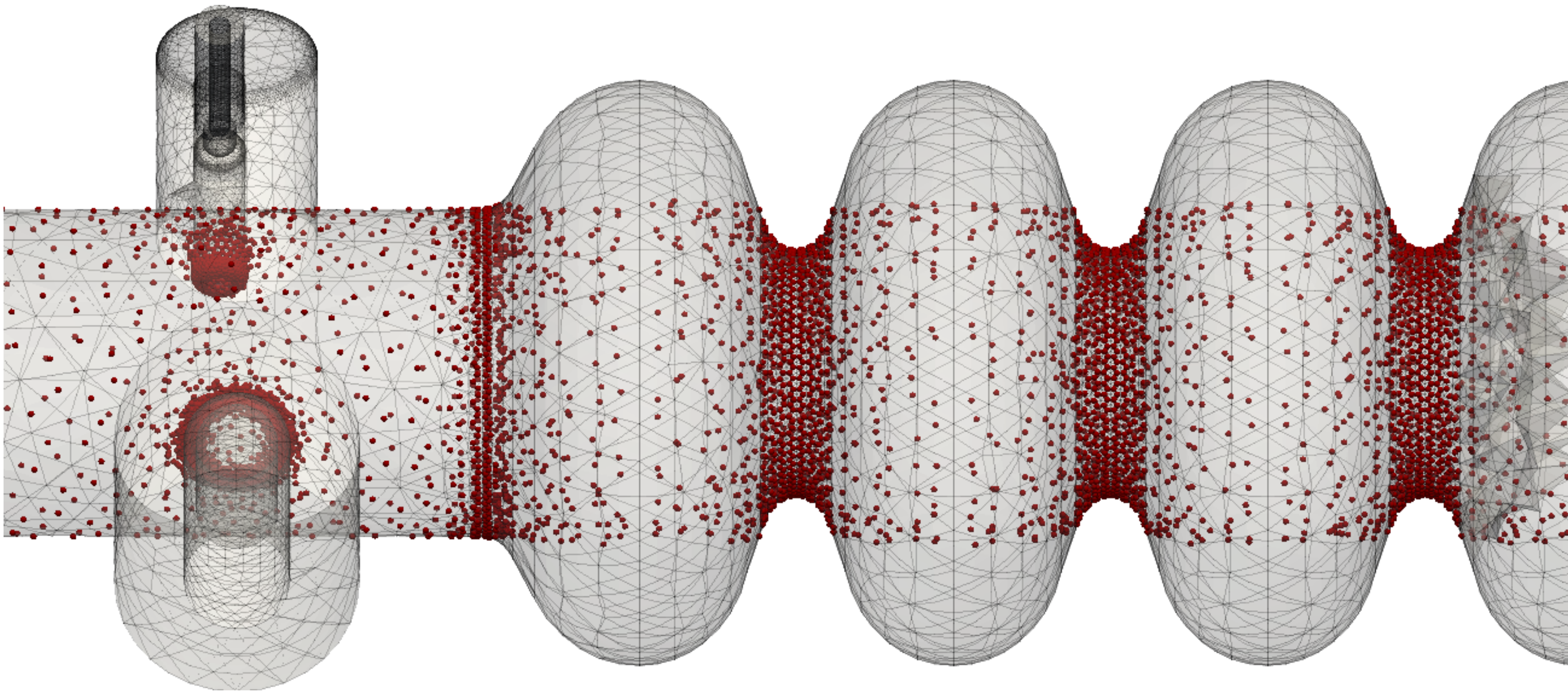
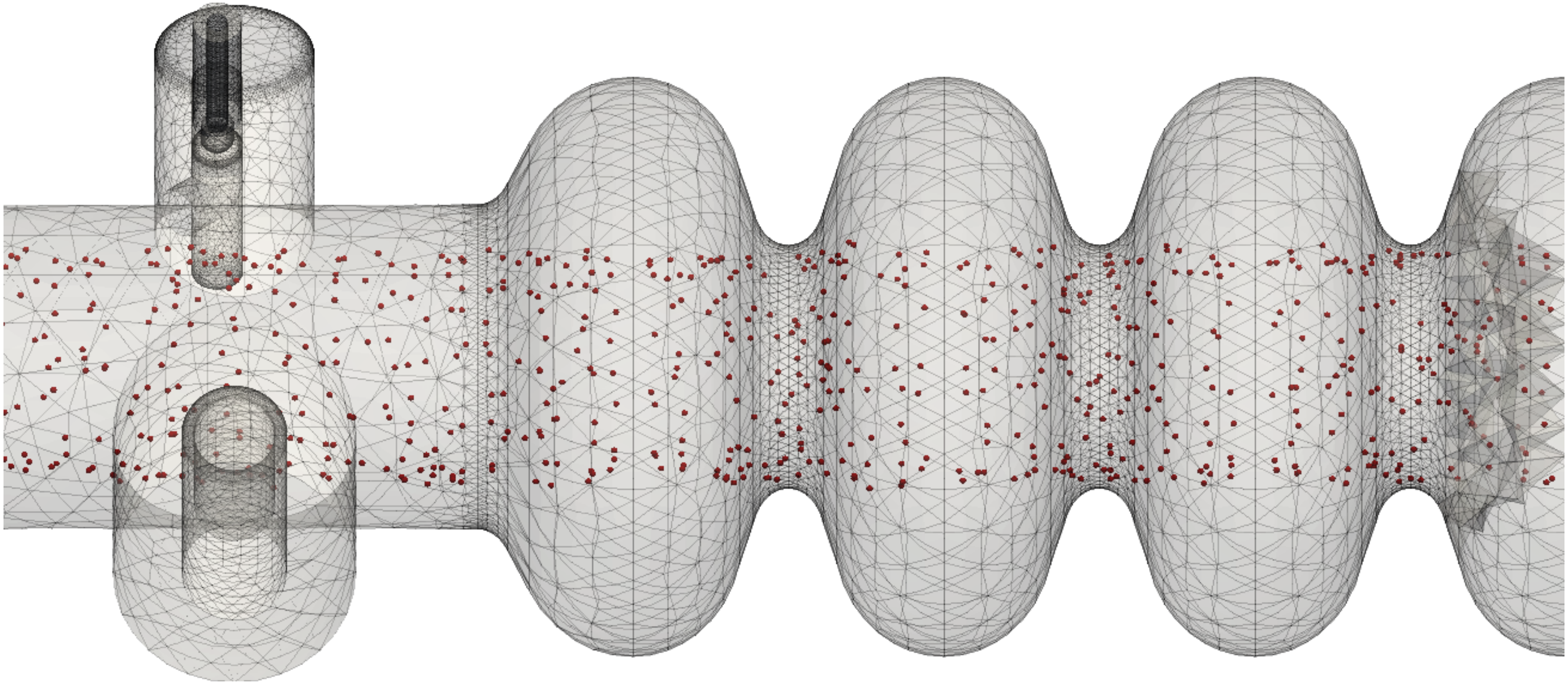
- Field reconstruction using the Kirchhoff integral
  - Surface selection



Post-Processing: Kirchhoff Integral

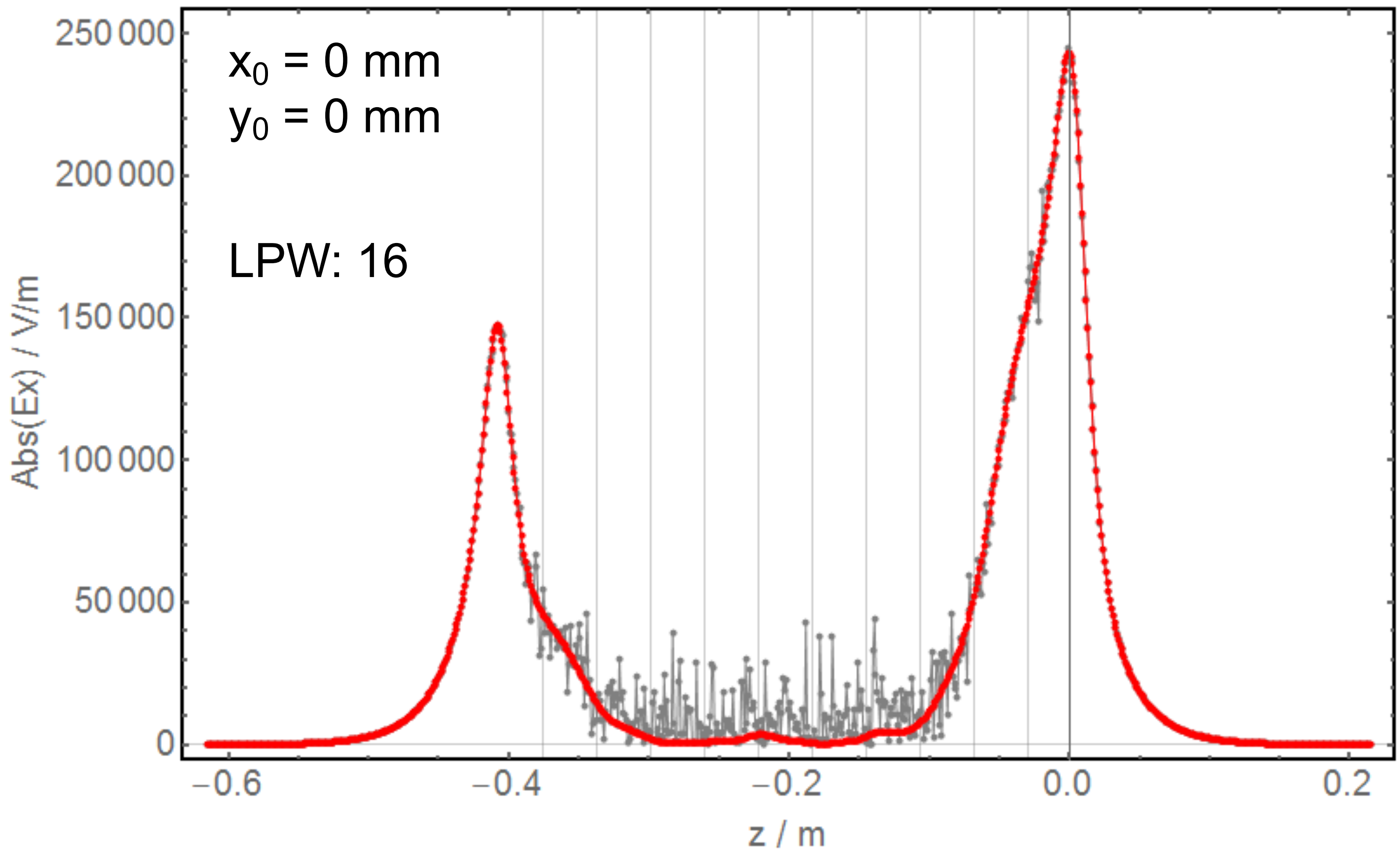
- Field reconstruction using the Kirchhoff integral
  - Surface selection



TESLA 3.9 GHz Cavity (Results)

- Field component E_x parallel to the cavity axis

on-axis (Kirchhoff)



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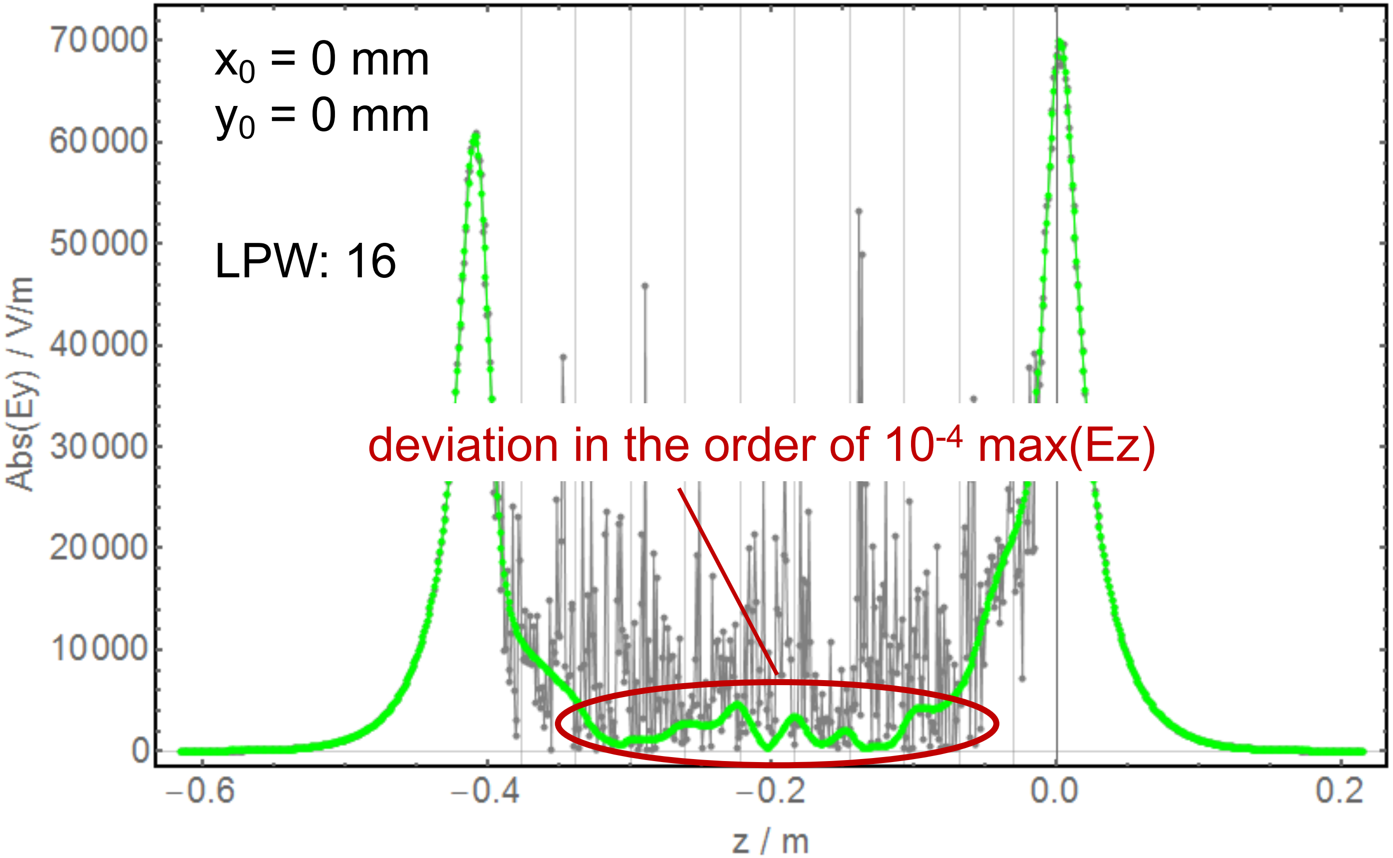
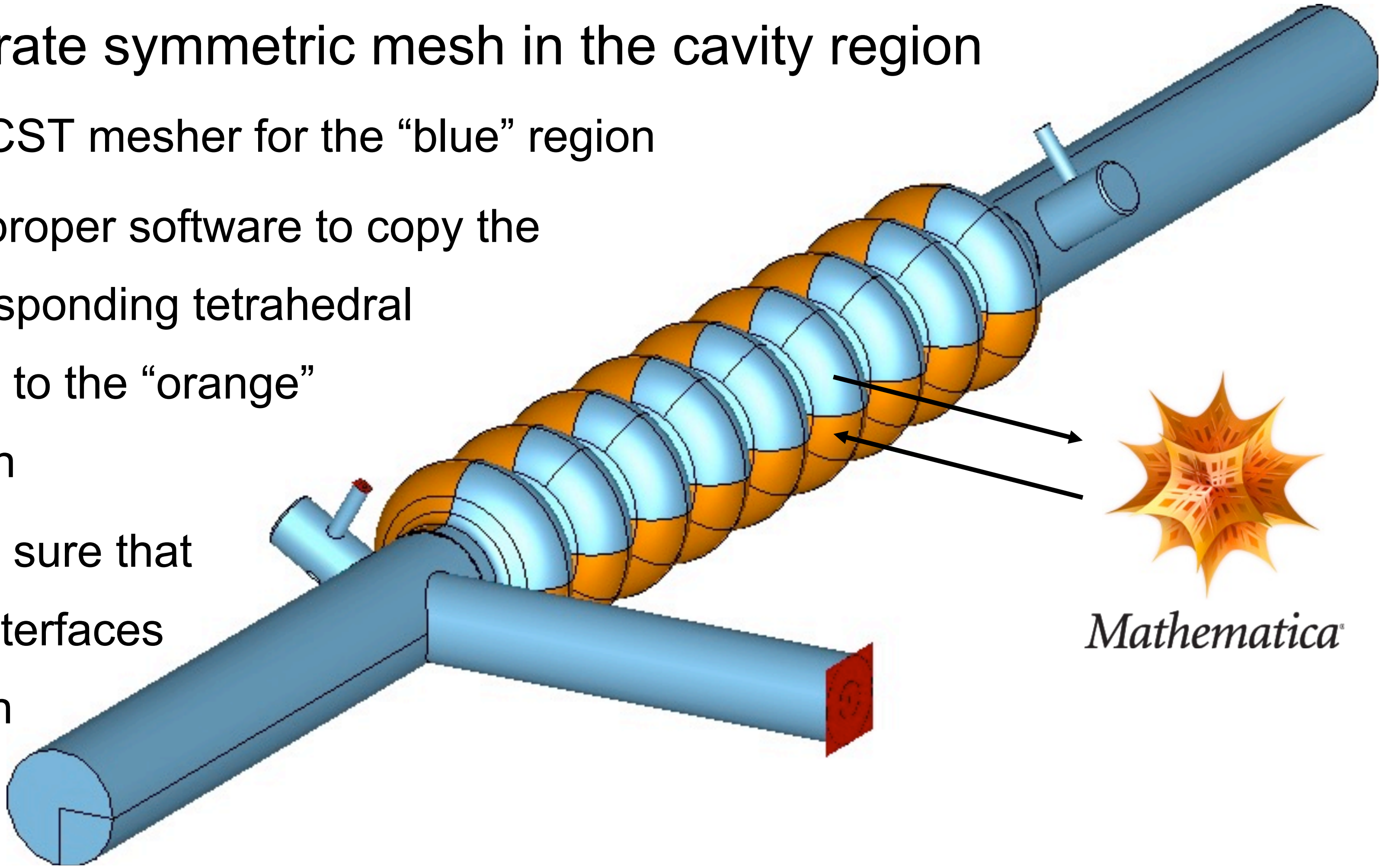


Outline

- XFEL + Tesla 3.9 GHz cavities
- FE eigenmode solver + on-axis fields
- Kirchhoff integrals + symmetric meshes

TESLA 3.9 GHz Cavity (Meshing)

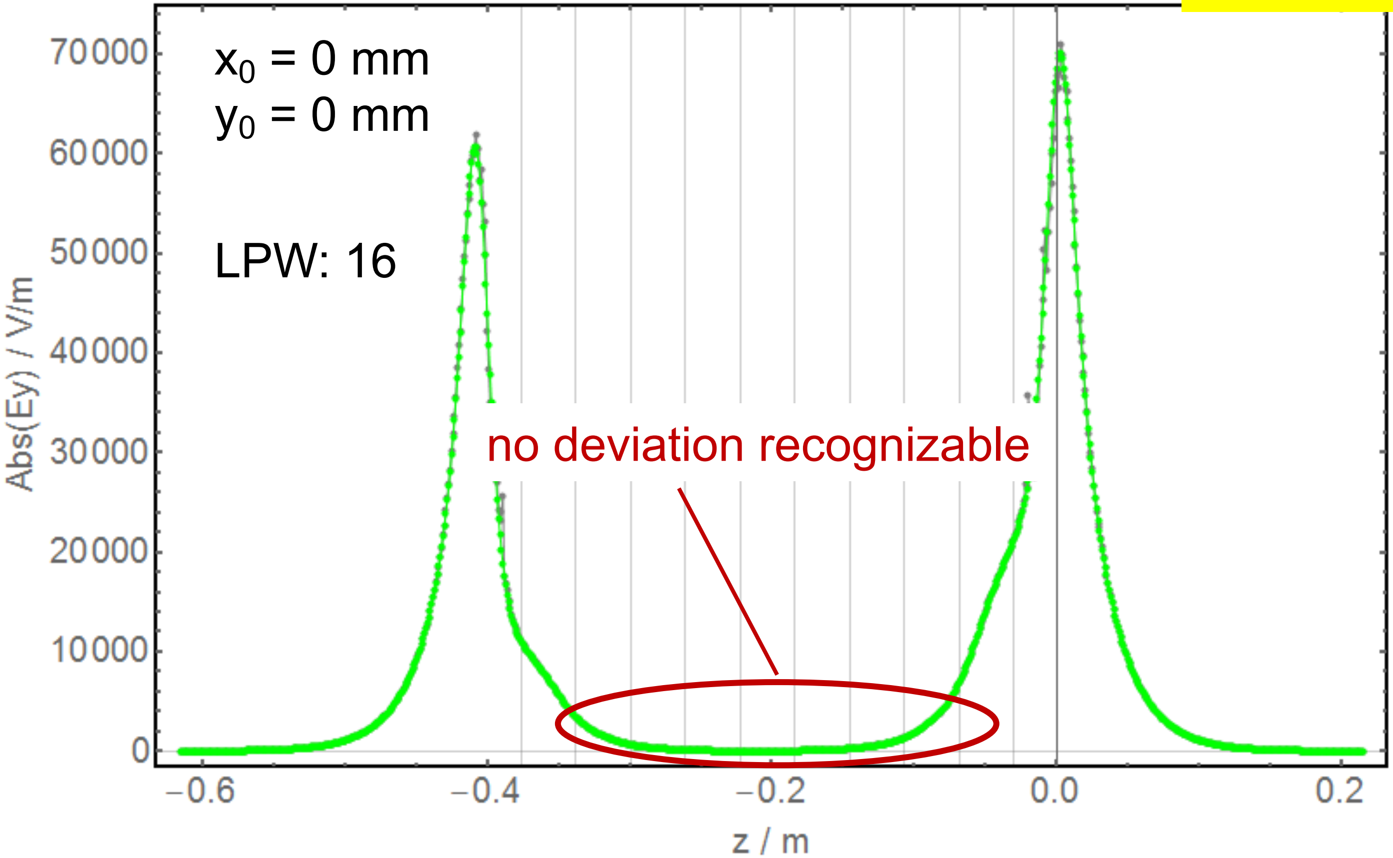
- Generate symmetric mesh in the cavity region
  - Use CST mesher for the “blue” region
  - Use proper software to copy the corresponding tetrahedral mesh to the “orange” region
  - Make sure that the interfaces match



TESLA 3.9 GHz Cavity (Results)

- Field component E_y parallel to the cavity axis

on-axis (Kirchhoff+mesh)



TESLA 3.9 GHz Cavity (Results)

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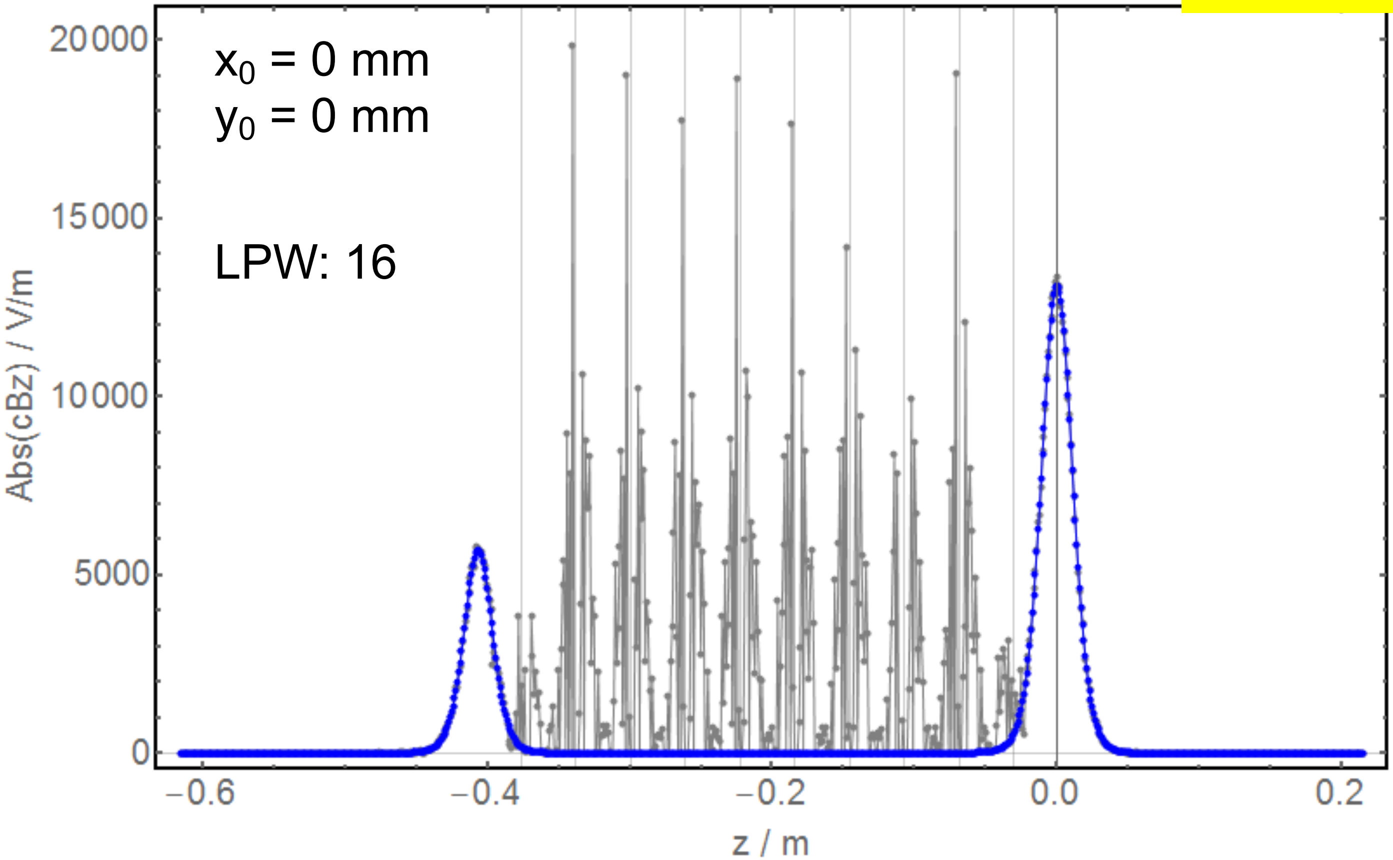
- Field component cB_z parallel to the cavity axis

on-axis (Kirchhoff+mesh)



Conclusions

- in accelerator cavities, (transversal) field maps are challenging
- noise because of unstructured tetrahedral meshes
- a-posteriori improvement by Kirchhoff integrals
- a-priori caution: use symmetric meshes