



Digital Signal Processing in RF Applications

Part II

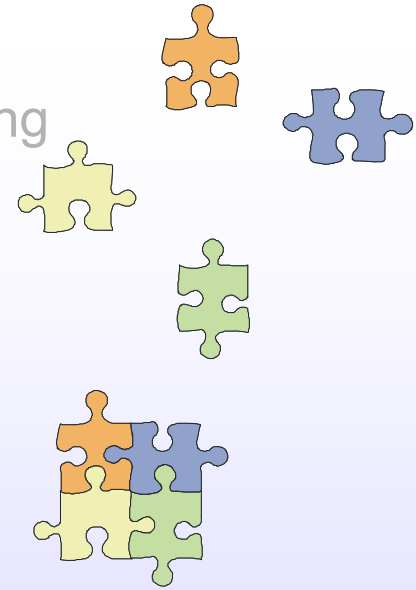
Thomas Schilcher

PAUL SCHERRER INSTITUT



Outline

1. signal conditioning / down conversion
2. detection of amp./phase by digital I/Q sampling
 - I/Q sampling
 - non I/Q sampling
 - digital down conversion (DDC)
3. upconversion
4. **algorithms in RF applications**
 - feedback systems**
 - cavity amplitude and phase**
 - radial and phase loops
 - adaptive feedforward
 - system identification



RF cavity: amplitude and phase feedback

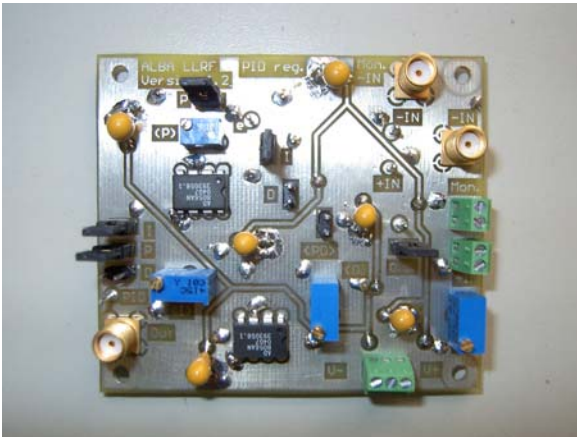
task: maintain phase and amplitude of the accelerating field within given tolerances to accelerate a charged particle beam

- **operating frequency:**
 - few MHz / ~50 MHz (cyclotrons)
 - 30 GHz (CLIC)
- **required stability:**
 - 10^{-2} – 10^{-4} in amplitude
 - (1% - 0.01%),
 - 1° - 0.01° (10^{-2} – 10^{-4} rad) in phase
 - (0.01° @ 1.3 GHz corresponds to 21 fs)
- **often: additional tasks required** like exception handling, built-in diagnostics, automated calibration, ...
- **design choices:**
 - analog / digital / combined
 - amplitude/phase versus IQ control
- **control of**
 - single cell/multicell cavity with one RF amplifier (klystron, IOT,...)
 - string of several cavities with single klystron (vector sum control)
 - pulsed / CW operation
 - normal / superconducting cavities

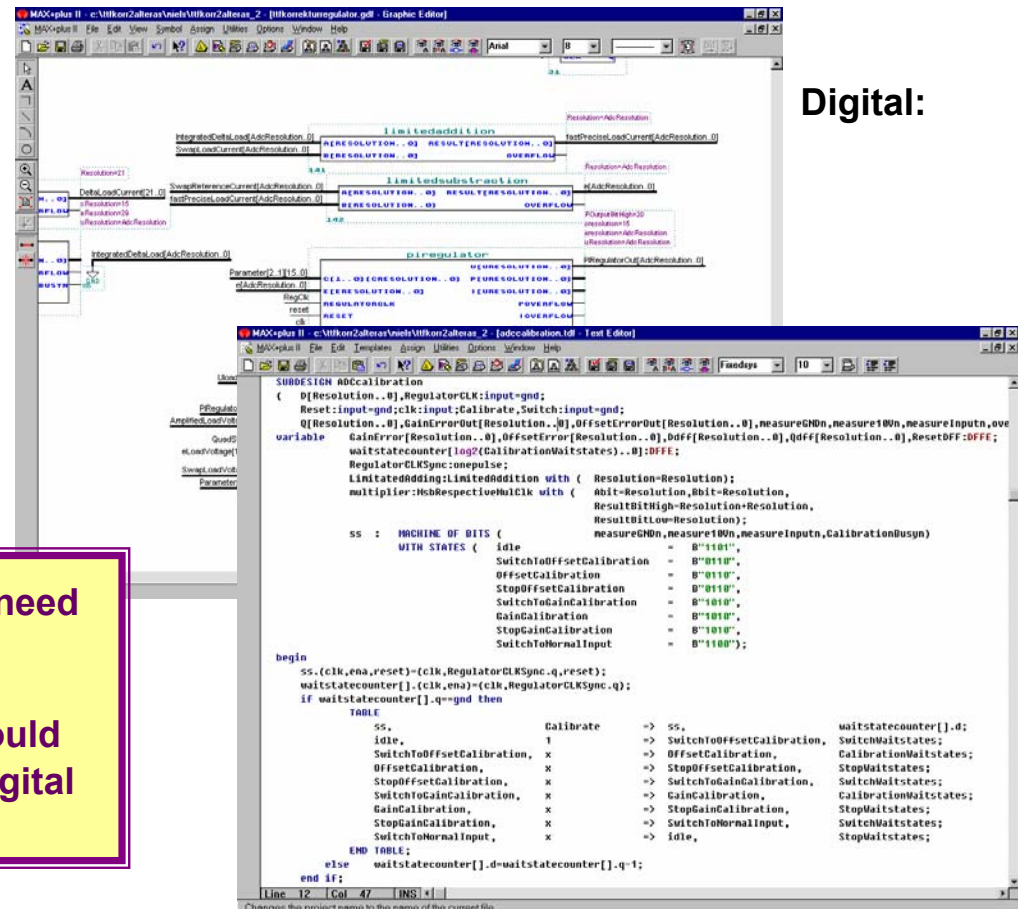
RF cavity: amplitude and phase feedback (2)

Analog/Digital LLRF comparison – Flexibility (ALBA)

Analog:



Digital:

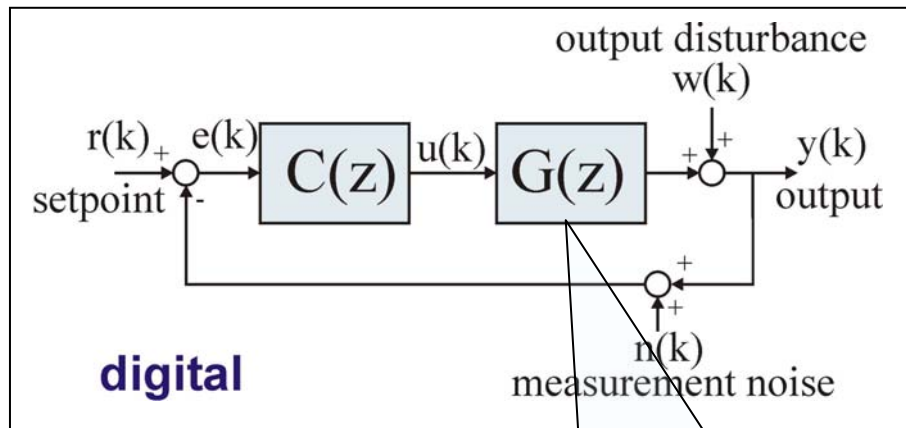


- **Analog:** any major change would need a new PCB design.
- **Digital:** most of future changes would be a matter of reprogramming the digital processor.

H. Hassanzadegan (CELLS)

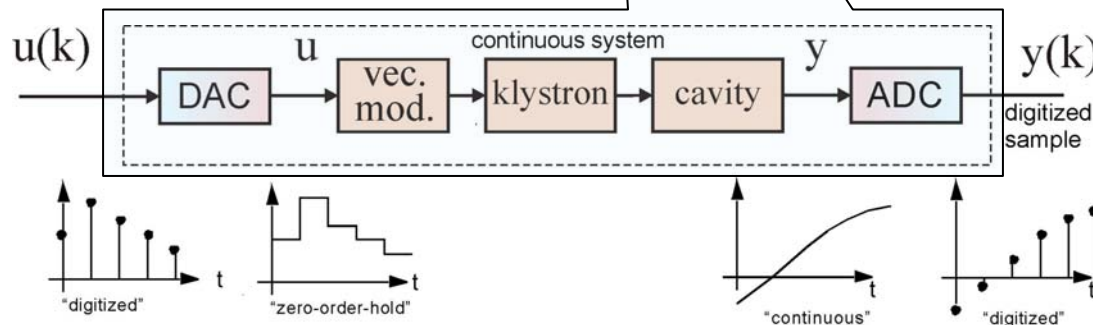
RF cavity: amplitude and phase feedback (3)

basic feedback loop:



r: setpoint
 y: plant output
 n: measurement noise
 w: output disturbance
 e: control error
 u: plant input
 C: controller
 G: plant
 (klystron, cavity, ...)

analog \rightarrow digital: $G(z) = \frac{z-1}{z} \mathcal{Z} \left\{ \mathcal{L}^{-1} \left\{ \frac{G(s)}{s} \right\} \Big|_{t=kT_S} \right\}$ (due to zero order hold function of DAC)



task:

model the plant
 to find $G(s)$ and
 transform it into Z -space

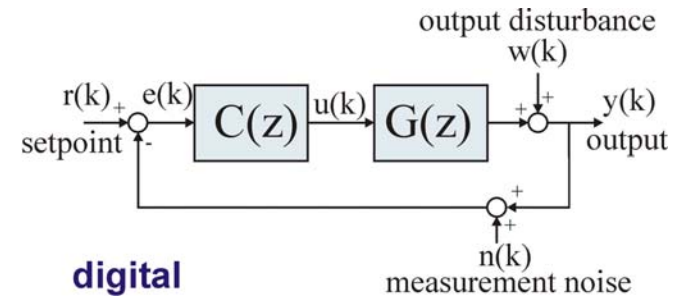
RF cavity: amplitude and phase feedback (4)

$$y = \underbrace{\frac{GC}{1+GC}}_T [r - n] + \underbrace{\frac{1}{1+GC}}_S w$$

**complementary
sensitivity function**

**sensitivity
function**

$$T + S = 1$$



$$e_T = \frac{GC}{1+GC} n + \frac{1}{1+GC} [r - w]$$

$$e_T = r - y \quad (\text{tracking error})$$

➤ **GC: open loop transfer function**

➤ for output y :

measurement error n behaves like a change in the setpoint r
(e.g. I/Q sampling error...)

➤ output y should be insensitive for low frequencies output disturbances w
(\rightarrow high gain with the controller to get $GC \gg 1$)

➔ T should be small
(robustness)

➔ S should be small
(performance)

**trade-off
between
performance
and
robustness**

RF cavity: amplitude and phase feedback (5)

LTI feedback: Bode integral theorem - waterbed effect

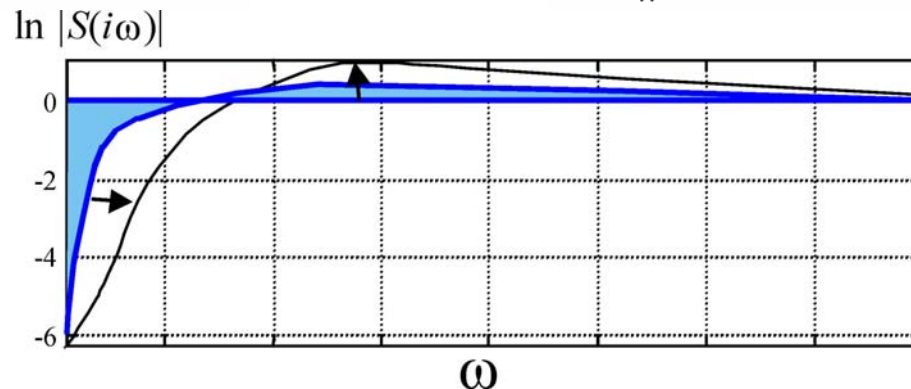
- if GC has no unstable poles and there are two or more poles than zeros:
(continuous: no poles in the right hand plane; discrete: no poles outside unity circle)

continuous:

$$\int_0^{\infty} \ln |S(i\omega)| d\omega = 0$$

discrete:

$$\int_{-\pi}^{\pi} \ln |S(e^{i\omega})| d\omega = 0$$



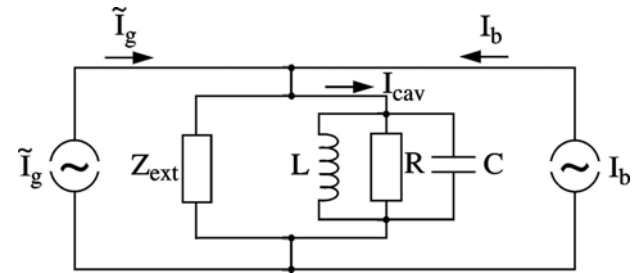
- Small sensitivity at low frequencies must be “paid” by a larger than 1 sensitivity at some higher frequencies **“waterbed effect”**

RF cavity: amplitude and phase feedback (6)

representation of RF cavity (transfer function / state space)

simplified model: LCR circuit

- $V(t)$: cavity voltage
- $I(t)$: driving current (from generator and beam)
- ω_0 : resonance frequency of undamped cavity
- Q_L : loaded quality factor of cavity
- R_L : cavity resistance incl. external load



differential equation for driven LCR circuit:

$$\ddot{\mathbf{V}}(t) + \frac{\omega_0}{Q_L} \dot{\mathbf{V}}(t) + \omega_0^2 \mathbf{V}(t) = \frac{\omega_0 R_L}{Q_L} \dot{\mathbf{i}}(t)$$

stationary solution for a harmonic driven cavity:

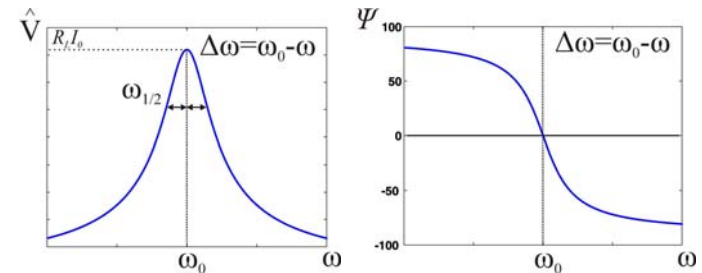
$$V(t) = \hat{V} \cdot \sin(\omega t + \psi)$$

ampl.: $\hat{V} \approx \frac{R_L I_0}{\sqrt{1 + (2Q_L \frac{\Delta\omega}{\omega})^2}}$

detuning angle: $\tan \psi \approx 2Q_L \frac{\Delta\omega}{\omega}$

bandwidth: $\omega_{1/2} = \frac{\omega_0}{2Q_L}$

detuning: $\Delta\omega = \omega_0 - \omega$



RF cavity: amplitude and phase feedback (7)

separate fast RF oscillations from the **slowly** changing amplitude/phases:

(slowly: compared to time period of RF oscillations)

$$\mathbf{V}(t) = \begin{pmatrix} V_r(t) \\ V_i(t) \end{pmatrix} \cdot e^{i\omega t}$$

$$\mathbf{I}(t) = \begin{pmatrix} I_r(t) \\ I_i(t) \end{pmatrix} \cdot e^{i\omega t}$$

(notation:
real and imaginary parts
instead of I/Q values)

$$\frac{d}{dt} \begin{pmatrix} V_r \\ V_i \end{pmatrix} = \begin{pmatrix} -\omega_{1/2} & -\Delta\omega \\ \Delta\omega & -\omega_{1/2} \end{pmatrix} \cdot \begin{pmatrix} V_r \\ V_i \end{pmatrix} + \begin{pmatrix} R_L\omega_{1/2} & 0 \\ 0 & R_L\omega_{1/2} \end{pmatrix} \cdot \begin{pmatrix} I_r \\ I_i \end{pmatrix}$$

Laplace transformation:

$$\underbrace{\begin{pmatrix} V_r(s) \\ V_i(s) \end{pmatrix}}_{\mathbf{V}(s)} = \underbrace{\frac{\omega_{1/2}}{\Delta\omega^2 + (s + \omega_{1/2})^2} \begin{pmatrix} s + \omega_{1/2} & -\Delta\omega \\ \Delta\omega & s + \omega_{1/2} \end{pmatrix}}_{H_{cav}(s)} \cdot \underbrace{\begin{pmatrix} R_L \cdot I_r(s) \\ R_L \cdot I_i(s) \end{pmatrix}}_{\mathbf{U}(s)}$$

cavity transfer matrix (continuous)

z transformation (continuous \rightarrow discrete with zero order hold):

$$H(z) = \frac{\omega_{12}}{\Delta\omega^2 + \omega_{12}^2} \begin{bmatrix} \omega_{12} & -\Delta\omega \\ \Delta\omega & \omega_{12} \end{bmatrix} - \left(\frac{\omega_{12}}{\Delta\omega^2 + \omega_{12}^2} \cdot \frac{z-1}{z^2 - 2ze^{i\omega_2 T_s} \cos(\Delta\omega T_s) + e^{2i\omega_2 T_s}} \right)$$

$$\cdot \left\{ \left((z - e^{i\omega_2 T_s} \cos(\Delta\omega T_s)) \begin{bmatrix} \omega_{12} & -\Delta\omega \\ \Delta\omega & \omega_{12} \end{bmatrix} \right) - e^{i\omega_2 T_s} \sin(\Delta\omega T_s) \begin{bmatrix} \Delta\omega & \omega_{12} \\ -\omega_{12} & \Delta\omega \end{bmatrix} \right\}$$

**let matlab
do the job
for you!**

state space:

$$\dot{\mathbf{x}}(t) = \mathbf{A} \cdot \mathbf{x}(t) + \mathbf{B} \cdot \mathbf{u}(t)$$

$$\mathbf{y}(t) = \mathbf{C} \cdot \mathbf{x}(t)$$

$$\mathbf{x}(t) = \begin{pmatrix} V_r(t) \\ V_i(t) \end{pmatrix} \quad \mathbf{u}(t) = \begin{pmatrix} I_r(t) \\ I_i(t) \end{pmatrix}$$

$$\mathbf{A} = \begin{pmatrix} -\omega_{1/2} & -\Delta\omega \\ \Delta\omega & -\omega_{1/2} \end{pmatrix} \quad \mathbf{C} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\mathbf{B} = \begin{pmatrix} R_L\omega_{1/2} & 0 \\ 0 & R_L\omega_{1/2} \end{pmatrix} \quad \mathbf{y}(t) = \mathbf{x}(t)$$

RF cavity: amplitude and phase feedback (8)

properties of cavity transfer functions:

$$H_{cav}(s) = \frac{\omega_{1/2}}{\Delta\omega^2 + (s + \omega_{1/2})^2} \begin{pmatrix} s + \omega_{1/2} & -\Delta\omega \\ \Delta\omega & s + \omega_{1/2} \end{pmatrix} = \begin{pmatrix} H_{11}(s) & H_{12}(s) \\ H_{21}(s) & H_{22}(s) \end{pmatrix}$$

➤ $\Delta\omega=0$ (cavity on resonance)

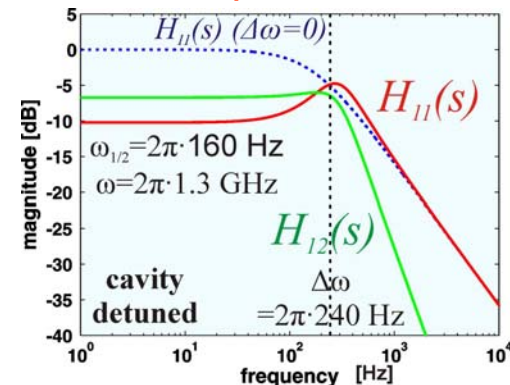
- cavity behaves like a **first order low pass filter** (20 dB roll off per decade)
- I/Q (or amplitude and phase) are **decoupled**

$$H_{11}(s) = H_{22}(s) = \frac{\omega_{1/2}}{s + \omega_{1/2}}$$

$$H_{12}(s) = H_{21}(s) = 0$$

➤ $\Delta\omega \neq 0$ (cavity off resonance)

- I/Q are **coupled**



- if $\Delta\omega = \Delta\omega(t)$

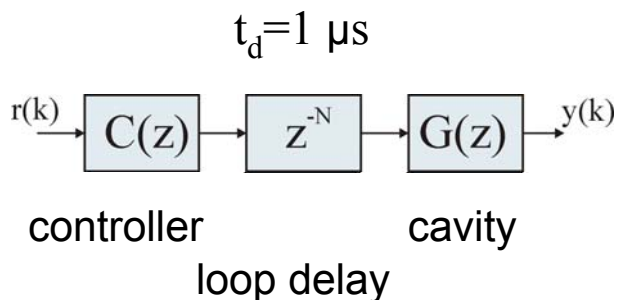
➔ **cavity models are time variant, control is more complex**

RF cavity: amplitude and phase feedback (9)

example: loop analysis in frequency domain (**simplified model !**)

superconducting cavity: $Q_L=3 \cdot 10^6$
 $f_0=1.3 \text{ GHz}$ $\omega_{1/2}=216 \text{ Hz}$

$\Delta\omega=0 \text{ Hz}$ $f_s=10 \text{ MHz}$



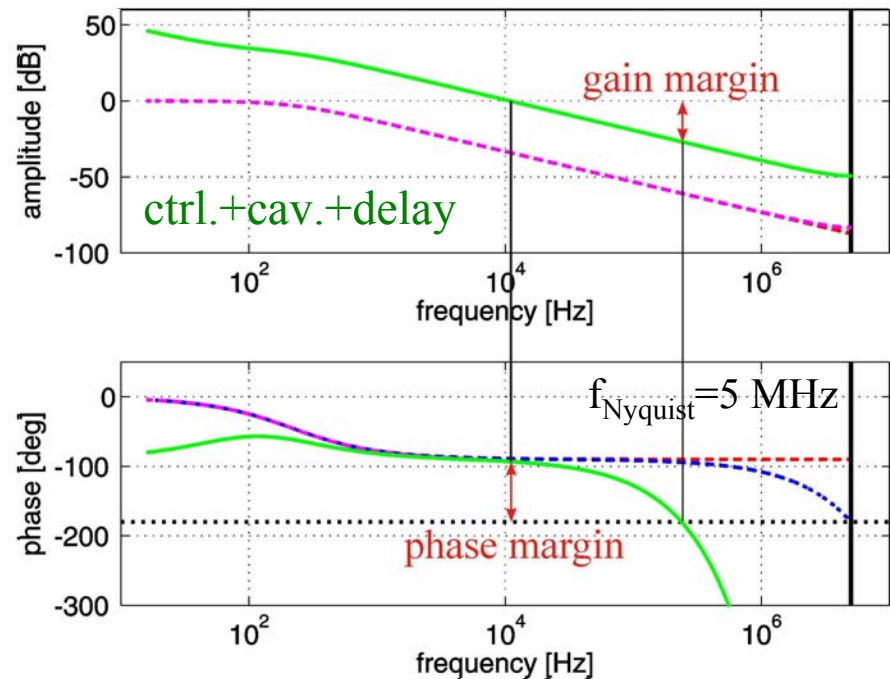
PID:
$$u(k) = u(k-1) + (K_p + K_i + K_d) \cdot e(k) - (K_p + 2K_d) \cdot e(k-1) + K_d \cdot e(k-2)$$

rule of thumb:

Gain margin at least between 6 and 8dB

Phase margin between 40° and 60°

open loop transfer function



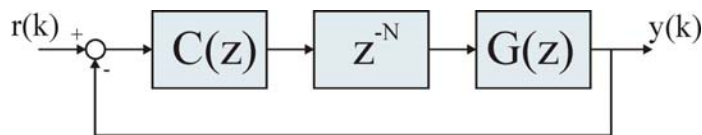
RF cavity: amplitude and phase feedback (10)

example: loop analysis in frequency domain

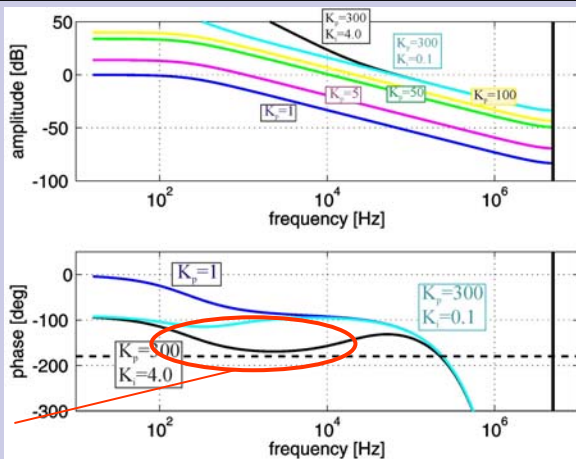
superconducting cavity: $Q_L = 3 \cdot 10^6$
 $f_0 = 1.3 \text{ GHz}$ $\omega_{1/2} = 216 \text{ Hz}$

$t_D = 1 \text{ } \mu\text{s}$ $f_S = 10 \text{ MHz}$

closed loop:

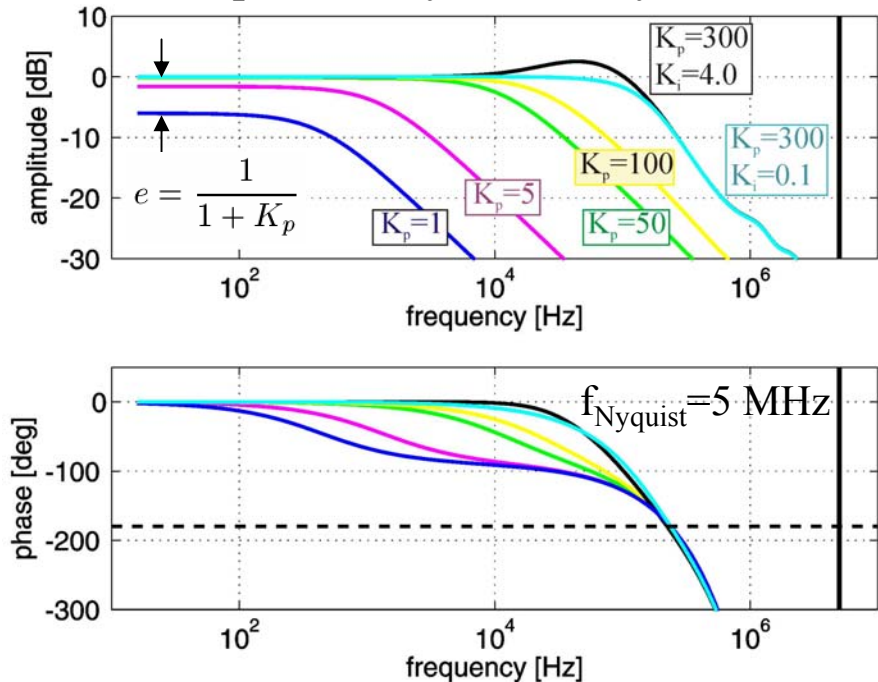


open loop:



phase margin!

complementary sensitivity function



RF cavity: amplitude and phase feedback (11)

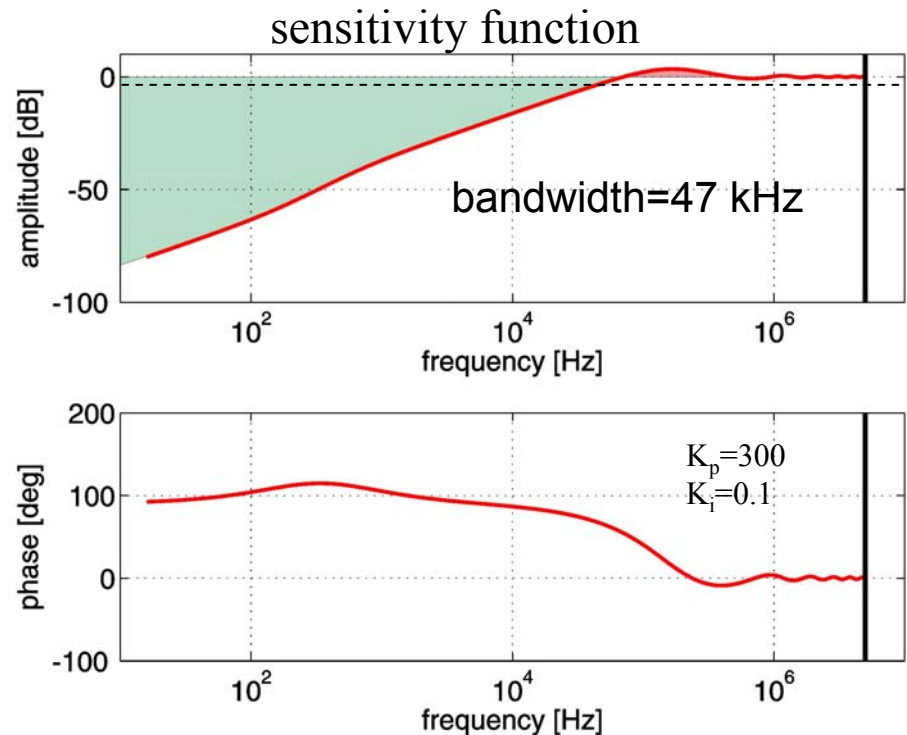
example: loop analysis in frequency domain

superconducting cavity: $Q_L=3 \cdot 10^6$
 $f_0=1.3 \text{ GHz}$ $\omega_{1/2}=216 \text{ Hz}$

$t_D=1 \mu\text{s}$ $f_S=10 \text{ MHz}$

choose parameter such that

- dominant disturbance frequencies are suppressed
- no dangerous lines show up in the range where the feedback can excite
- system performance will not be spoiled by sensor noise due to increasing loop gain



RF cavity: amplitude and phase feedback (12)

example: loop analysis in frequency domain

superconducting cavity: $Q_L=3 \cdot 10^6$
 $\omega_{1/2}=216 \text{ Hz}$

$\Delta\omega=0 \text{ Hz}$

$f_s=10 \text{ MHz}$

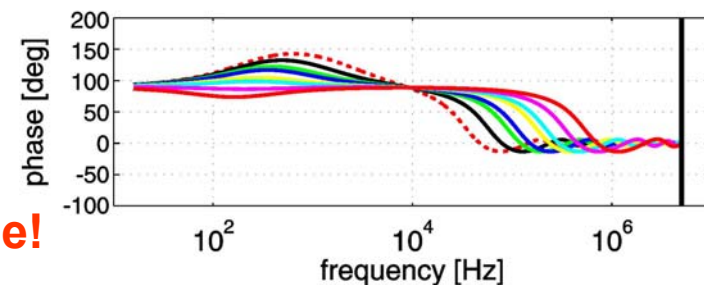
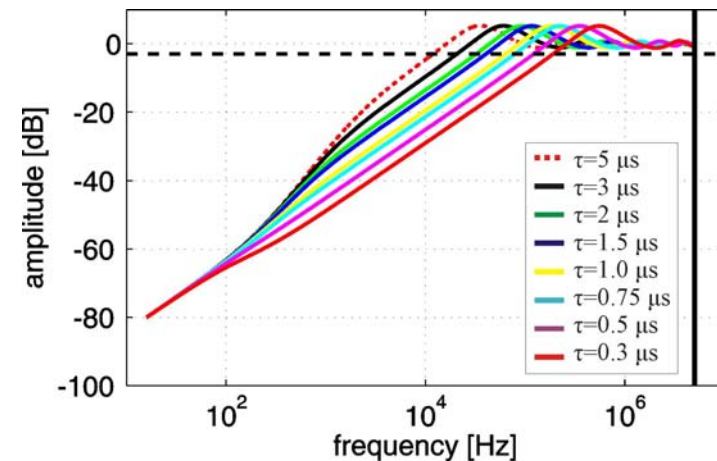
sensitivity function

variation of the loop delay

(boundary condition:

keep gain margin constant at 8 dB; $K_i=0.1$)

t_D	K_p	loop bandwidth (-3 dB)
5 μs	87	11.9 kHz
3 μs	145	20.6 kHz
2 μs	223	32.2 kHz
1.5 μs	278	40.3 kHz
1.0 μs	436	63.6 kHz
0.75 μs	539	78.6 kHz
0.5 μs	832	121 kHz
0.3 μs	1303	190 kHz



→ total loop delay is an important parameter; keep it as small as possible!

RF cavity: amplitude and phase feedback (13)

so far: loop analysis done for **fixed** detuning ($\Delta\omega=0$) only!

things get much more complicated if $\Delta\omega = \Delta\omega(t)$

→ **time varying control**

example: pulsed superconducting cavity
with high gradient

→ strong Lorentz forces

→ cavity detunes during the pulse

remember:
$$H_{cav}(s) = \frac{\omega_{1/2}}{\Delta\omega^2 + (s + \omega_{1/2})^2} \left(\frac{s + \omega_{1/2}}{\Delta\omega} \right)$$

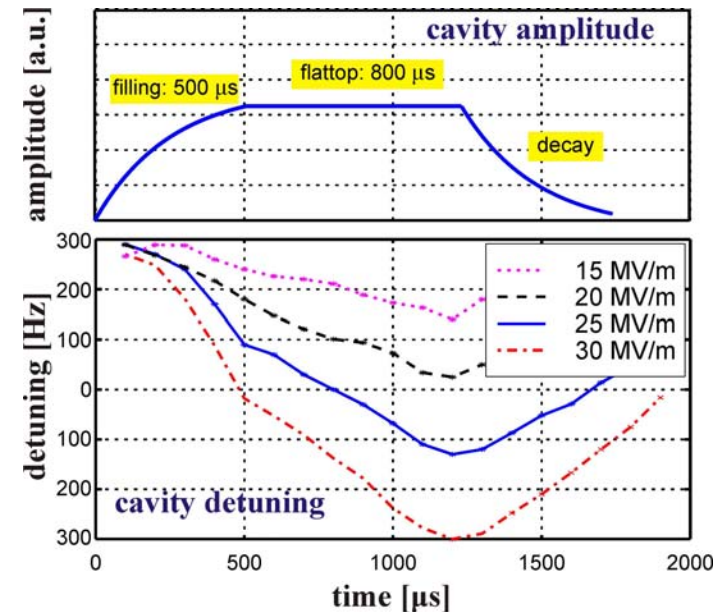
$\Delta\omega$: function of the gradient (time varying)

→ stability of the feedback loop has to be guaranteed under these parameter changes!

→ this might limit the feedback gain in contrast to the simple analysis !

→ design of “optimal” controller under study at many labs...

pulsed operation of superconducting cavity



RF cavity: amplitude and phase feedback (14)

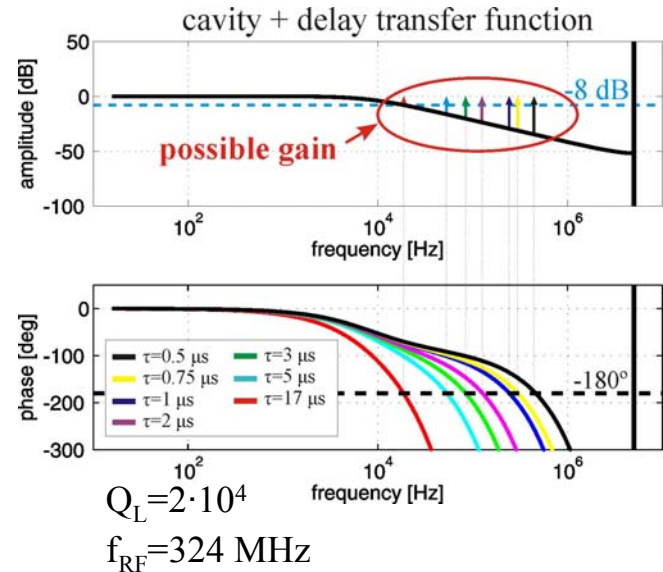
cavities

superconducting	normal conducting
<ul style="list-style-type: none"> Q_L: ~few $10^5 - 10^7$ cavity time constants $\tau_{cav} = Q_L / (\pi f_{RF})$: ~few 100 μs bandwidth $f_{1/2} = f_{RF} / (2Q_L)$: ~few 100 Hz feedback loop delay small compared to τ_{cav} 	<ul style="list-style-type: none"> Q_L: ~10 – 10⁵ cavity time constants τ_{cav}: ~few μs bandwidth $f_{1/2}$: ~100 kHz feedback loop delay in the order of τ_{cav}

loop latency limits high feedback gain
for high bandwidth cavities!

if the gains/bandwidths achieved by digital feedback
systems are not sufficient

➔ analog/digital hybrid system might be an alternative !?



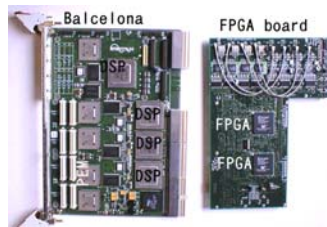
amplitude and phase feedback: example

LLRF: J-PARC linac (RFQ, DTL, SDTL)

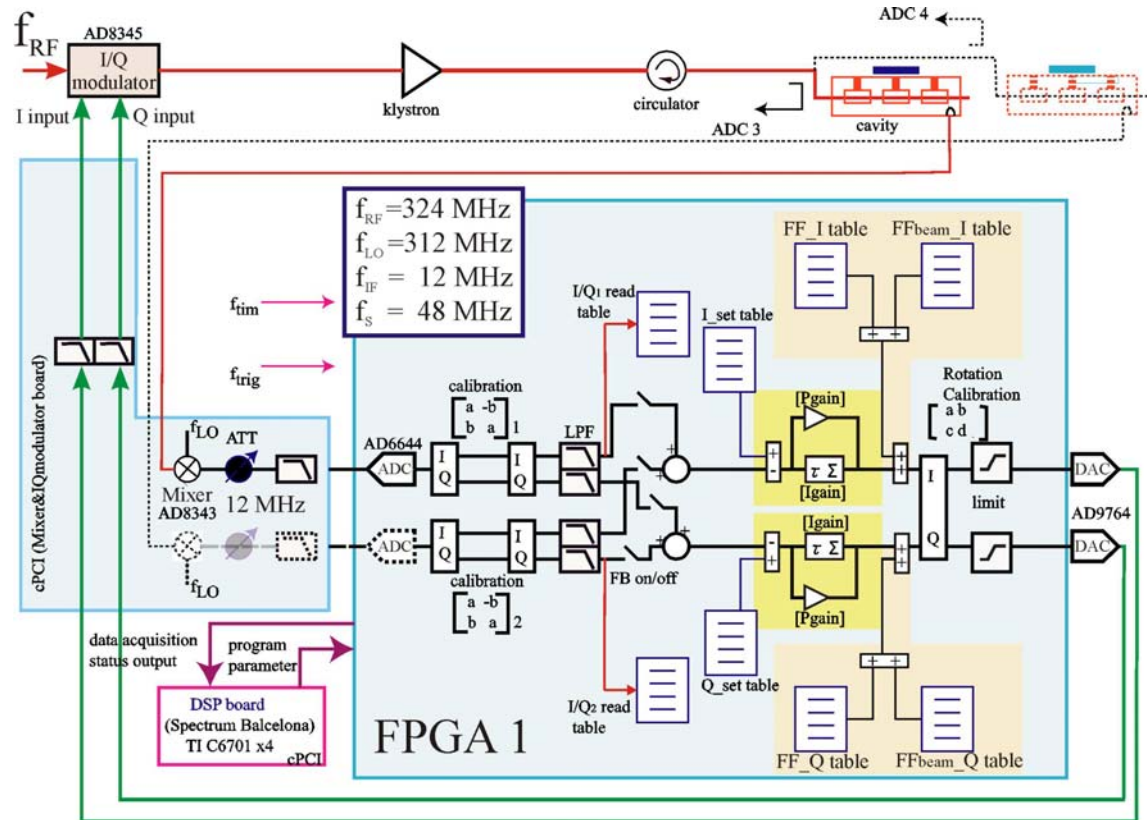
- 400 MeV proton linac
- pulsed operation; rep. rate: 12.5/25 Hz;
- pulse length: $\sim 500\text{-}650 \mu\text{s}$
- vector sum control
- normal conducting cavities; $Q_L \sim 8'000\text{-}300'000$
- $\tau_{cav} \sim 100 \mu\text{s}$

requirements / achieved:

- **amplitude:** $< +1\%$ / $< +0.15\%$
- **phase:** $< +1^\circ$ / $< +0.15^\circ$



combined
DSP/FPGA
board



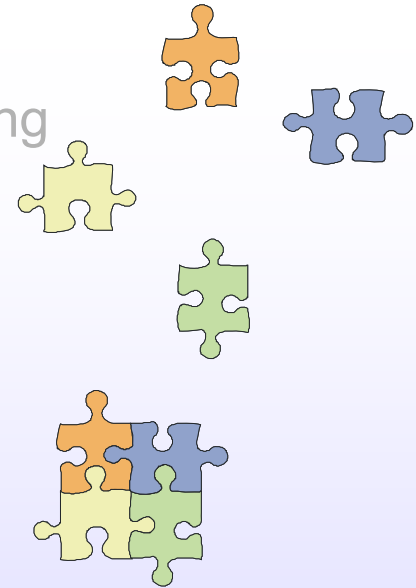
S. Michizono (J-PARC)

performance:

- loop delay: 500 ns
- bandwidth: $\sim 100 \text{ kHz}$
- gains: proportional ~ 10 , integral ~ 0.01

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Feedbacks in hadron/ion synchrotrons

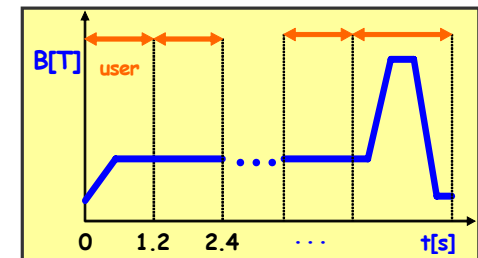
booster synchrotrons:

capture and adiabatically rebunch the beam and accelerate to the desired extraction energy.

Beam Control System

task: control of

- ❑ RF frequency during the ramp
(large frequency swings of up to a factor of ten, usually from several 100 kHz to several 10 MHz)
- ❑ cavity amplitude and phase
(ampl. can follow a pattern during acceleration)
- ❑ mean radial position of the beam
- ❑ phase between beam and cavity RF \longrightarrow deviations from Φ_S will lead to synchrotron oscillations
(synchronous phase Φ_S)
- ❑ synchronization to master RF phase
(to synchronize the beam transport to other accelerator rings)



Typical LEIR commissioning cycle.

$$\frac{dB}{B} = \gamma^2 \frac{df}{f} + (\gamma^2 - \gamma_{tr}^2) \frac{dR}{R}$$

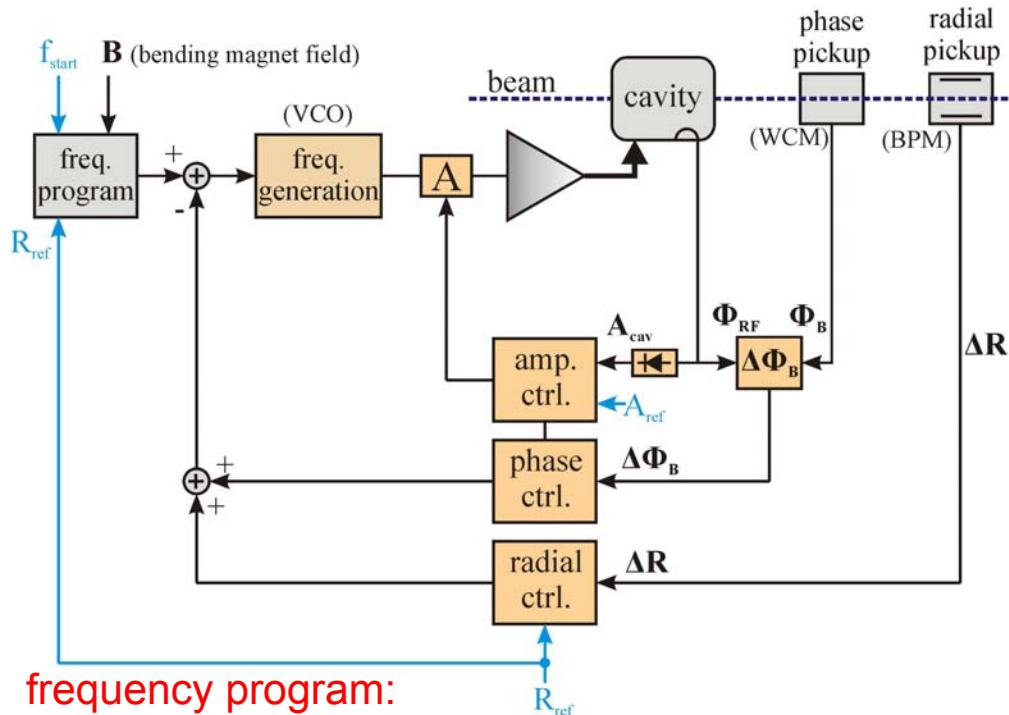
deviations from Φ_S will lead to synchrotron oscillations

in reality: errors due to phase noise, B field errors, power supply ripples, ...



feedbacks are required

Beam Control System



frequency program:

- 1) calculate frequency based on the B field, desired radial position
- 2) optimize the freq. ramp to improve injection efficiency
- 3) generate dual harmonic RF signals for cavities (bunch shaping)

beam phase loop

damps coherent synchrotron oscillations from

- 1) injection errors (energy, phase)
- 2) bending magnet noise
- 3) frequency synthesizer phase noise

radial loop

keeps the beam to its design radial position during acceleration

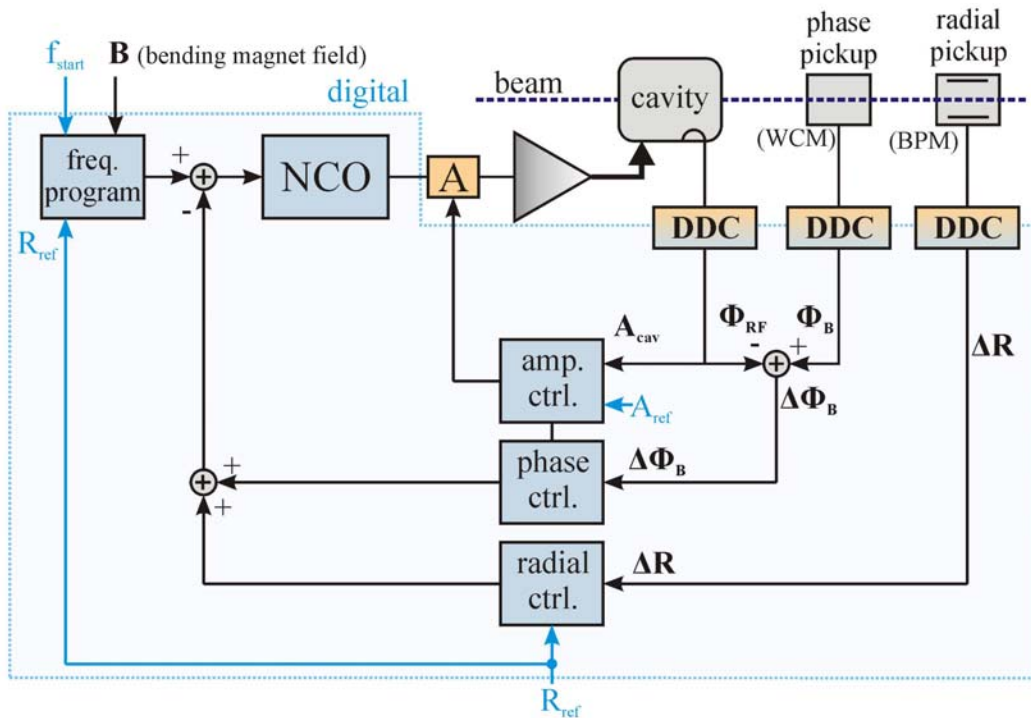
cavity amplitude loop

- 1) compensates imperfections in the cavity amplifier chain
- 2) amplitude has to follow a ramping function

synchronization loop (not shown)

locks the phase to a master RF

Beam Control System: from analog to digital



How do we setup the control loops?

➔ model required

in '80s: **DDS/NCO** replace **VCO**

(VCO:

lack of absolute accuracy,
stability limitations if
freq. tuning is required
over a broad range)

in recent years (LEIR, AGS, RHIC):

fully digital

beam control system

- digitize RF signals (I/Q, DDC)
- all control loops are purely digital
- feedback gains: function of the beam parameters (keep the same loop performances through the acceleration cycle)

Radial and phase loops

beam dynamics delivers the differential equations \longrightarrow transfer functions

transfer functions without derivation:

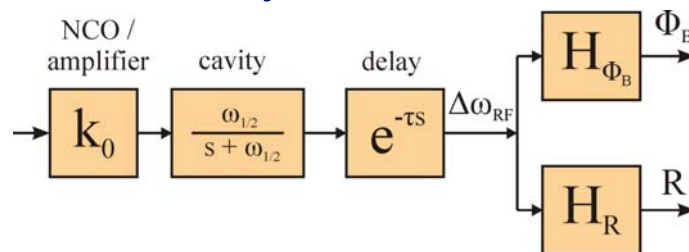
RF freq. (NCO output) to phase deviation of the beam from the synchronous phase

$$H_{\phi_B}(s) = \frac{\Delta\phi_B}{\Delta\omega_{RF}} = \frac{s}{s^2 + \omega_S^2}$$

RF freq. (NCO output) to radial position R

$$H_R(s) = \frac{R}{\Delta\omega_{RF}} = \frac{b}{s^2 + \omega_S^2}$$

model of the system:



$\omega_S = \omega_S(E)$: synchrotron frequency,
depend on the beam energy
 $b = b(E, \Phi_S)$: function of energy,
synchronous phase

since energy varies along the ramp

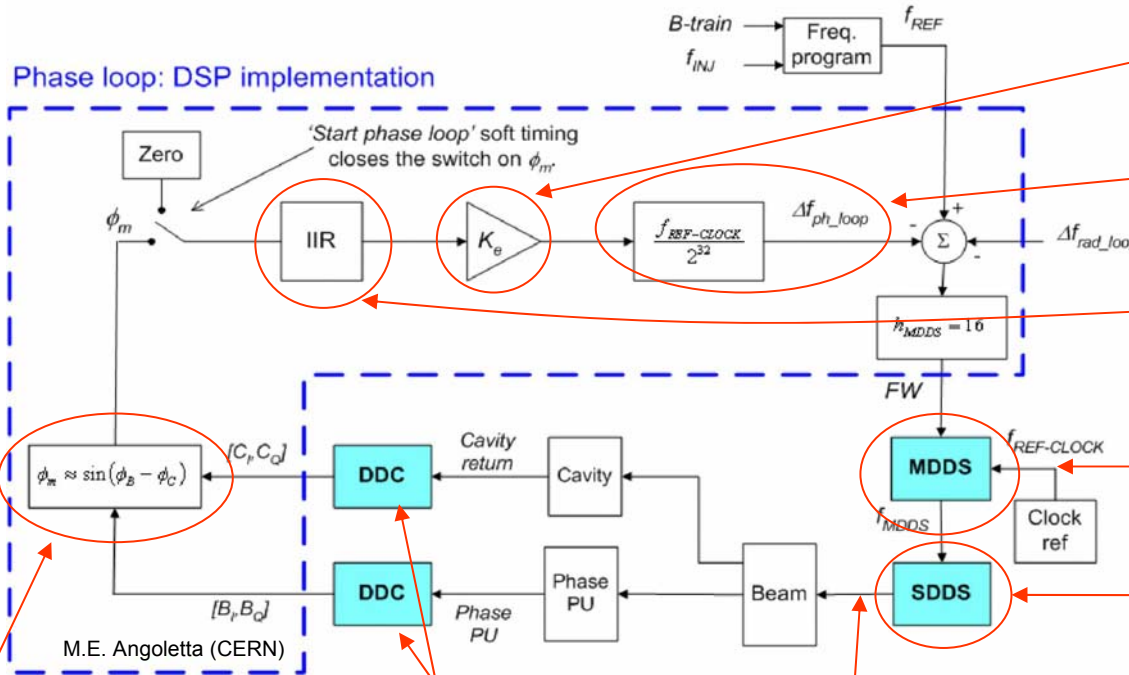
\longrightarrow time varying model !

LPV: linear parameter varying model

design of the controller: parameters have to be adjusted over time to meet the changing plant dynamics (guarantee constant loop performance and stability)

Phase loop: example

implementation example (test system for LEIR): **PS Booster @ CERN**



- programmable proportional gain
- phase-to-freq. conversion
- IIR: 1st order high pass cut-off: 10 Hz AC coupling
- 1 master DDS (MDDS)
- several slave DDS (SDDS)
 - cavity control (2 harmonics)
 - RF signal for diagnostics

$$\Delta\phi_B \approx \sin(\phi_B - \phi_{RF}) = \frac{I_c Q_b - I_b Q_c}{\sqrt{I_c^2 + Q_c^2} \sqrt{I_b^2 + Q_b^2}}$$

$$|\sin \varphi| = \frac{|\vec{u} \times \vec{v}|}{|\vec{u}||\vec{v}|}$$

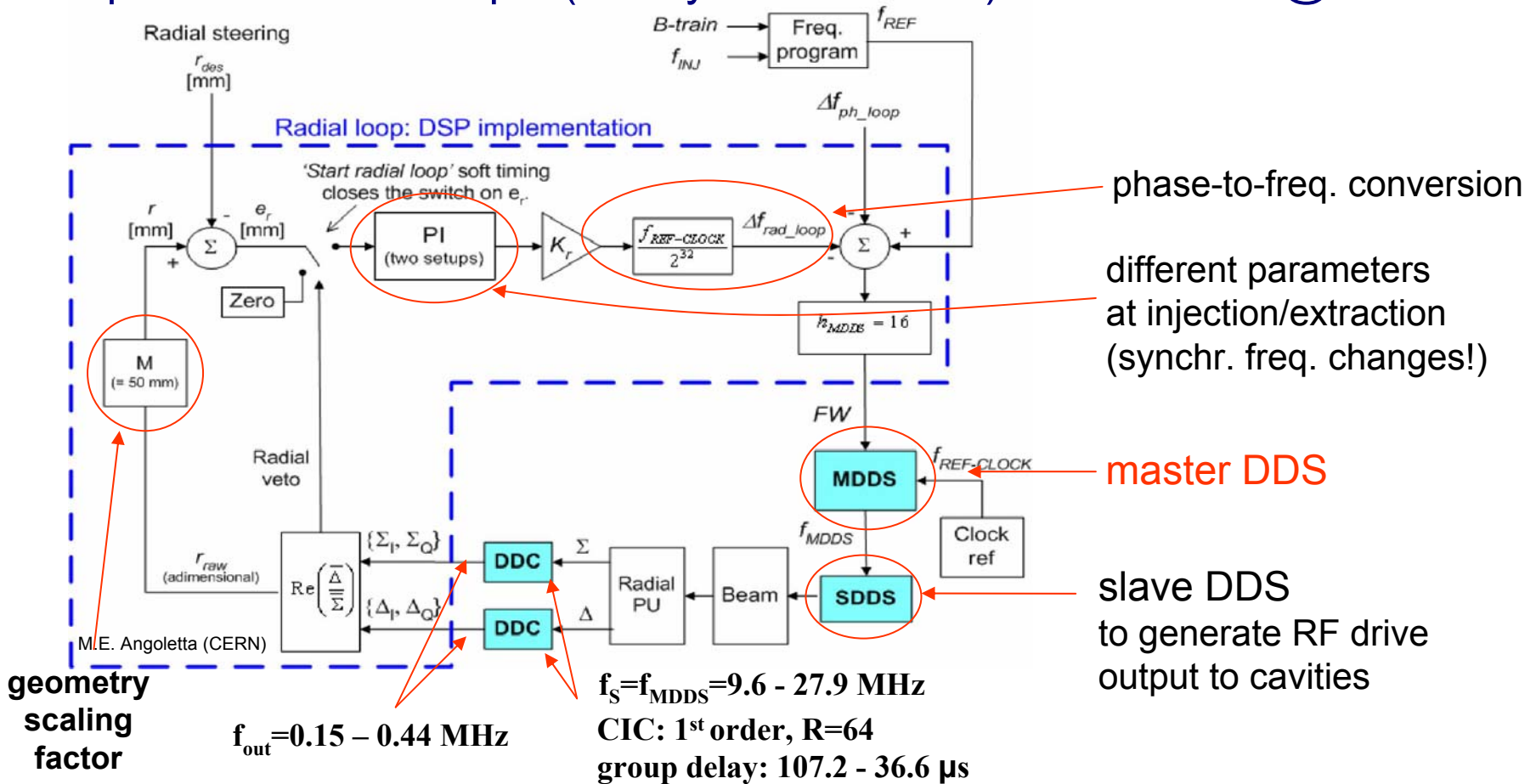
$f_s = f_{MDDS} = 9.6 - 27.9 \text{ MHz}$
CIC: 1st order, R=1
group delay: 1.67 - 0.57 μ s

$f_{rev} = 0.599 - 1.746 \text{ MHz}$
 (inj. / ext.)

loop bandwidth: 7 kHz

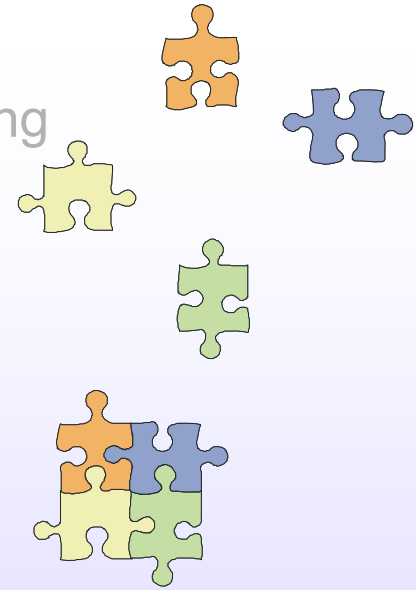
Radial loop: example

implementation example (test system for LEIR): **PS Booster @ CERN**



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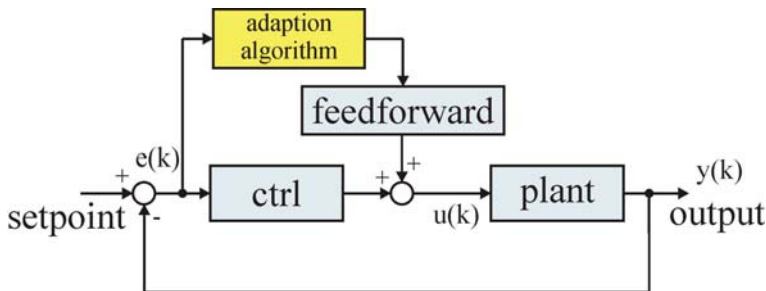
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Adaptive Feedforward

goal:

- suppress repetitive errors by feedforward in order to disburden the feedback
- cancel well known disturbances where feedback is not able to (loop delay!)
- adapt feedforward tables continuously to compensate changing conditions



warning:

adding the error (loop delay corrected) to system input **does not work!**
(dynamics of plant is not taken into account)

How to obtain feedforward correction?

we need to calculate the proper input which generates output signal $-e(k)$

➔ **inverse system model needed!**

Adaptive Feedforward (2)

in reality: model for plant not well known enough

- system identification \longrightarrow model
- measure system response (e.g. by step response measurements)

linear system (SISO):



$$\underbrace{\begin{pmatrix} \Delta y(\tau_1) \\ \vdots \\ \Delta y(\tau_p) \end{pmatrix}}_{\text{system output}} = \underbrace{\begin{pmatrix} R_{11} & & 0 \\ \vdots & \ddots & \\ R_{p1} & \dots & R_{pp} \end{pmatrix}}_{\text{system response matrix}} \cdot \underbrace{\begin{pmatrix} \Delta u(t_1) \\ \vdots \\ \Delta u(t_p) \end{pmatrix}}_{\text{test inputs}}$$

$$\tau_k = t_k + \tau_d$$

↑
loop delay

in successive measurements: apply $\Delta u(t_k)$ and measure response $\Delta \vec{y}$

\longrightarrow results in \mathbf{R} (with some math depending on the test input)

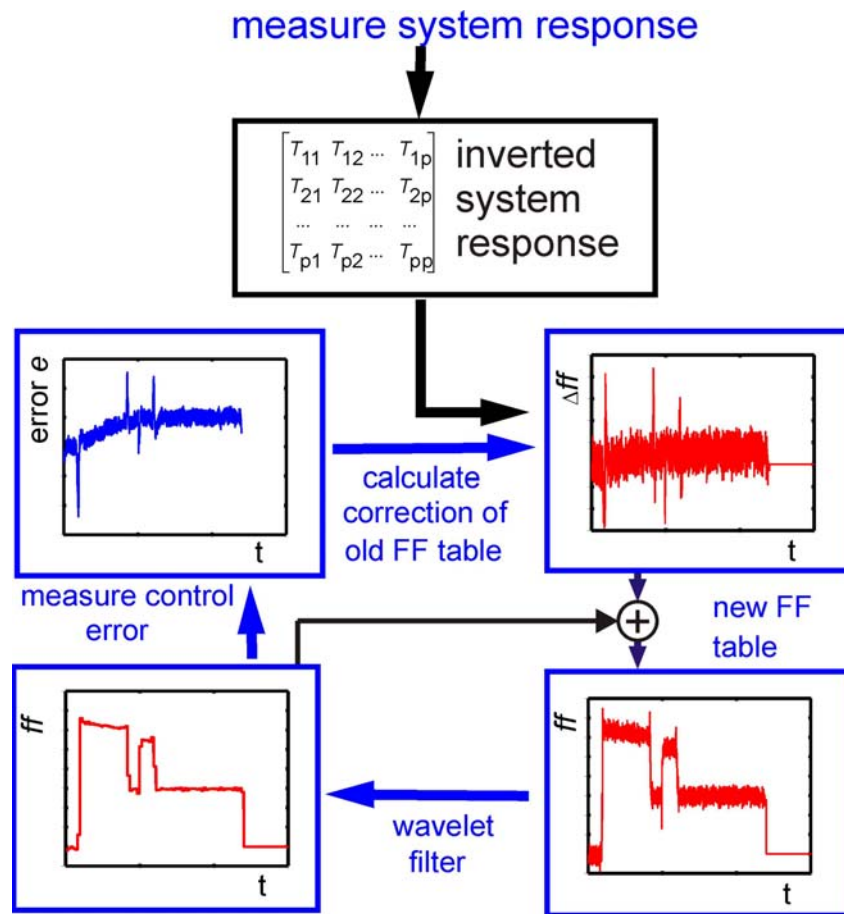
\longrightarrow invert response matrix $\mathbf{T} = \mathbf{R}^{-1}$ (possible due to definition of sampling time $\tau_k = t_k + \tau_d$)

$$\Delta \vec{u} = \mathbf{T} \cdot \Delta \vec{y}$$

feedforward for error correction:

$$\Delta \vec{ff} = \mathbf{T} \cdot \vec{e} = \mathbf{T} \cdot (\vec{r} - \vec{y})$$

Adaptive Feedforward (3)



pulsed superconducting 1.3 GHz cavity:
works fine in principle

but:

- remeasure T when operating point changes (amplitude/phase) (non-linearities in the loop)
- response measurement could not be fast enough

➔ need for a fast and robust adaptive feedforward algorithm!

Adaptive Feedforward (4)

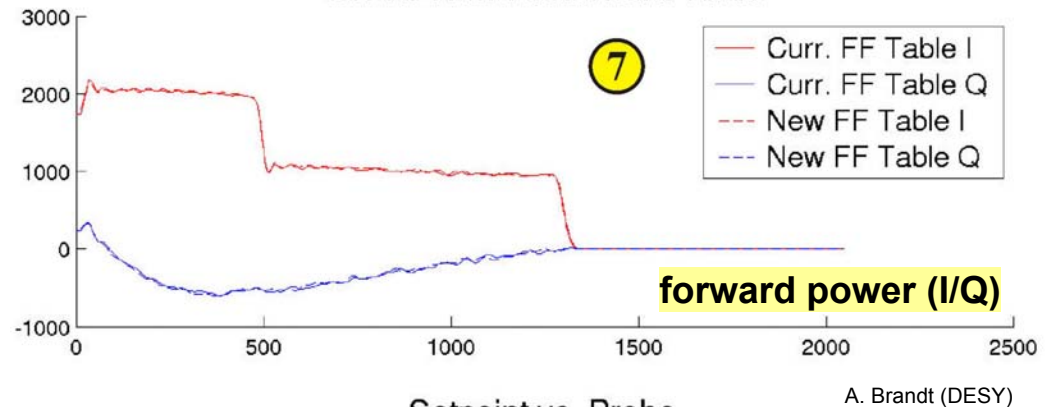
“time reversed” filtering:

- developed for FLASH, in use at FLASH/tested at SNS
- works only for pulsed systems
- not really understood but it works within a few iterations!

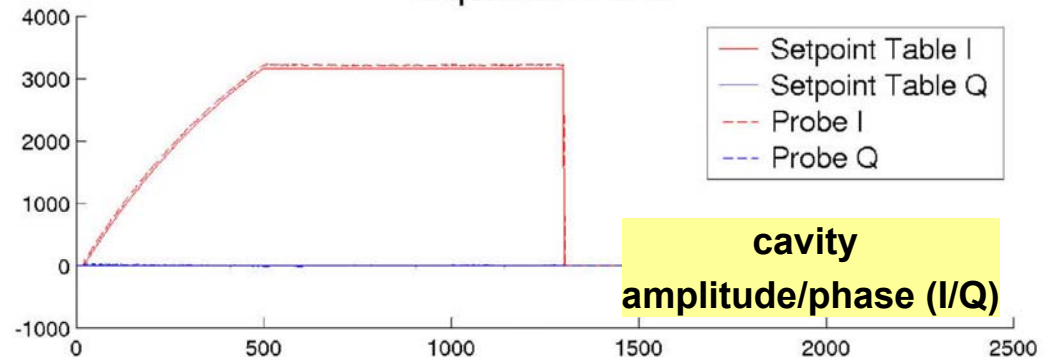
recipe:

- record feedback error signal $e(t)$
- time reverse $e(t) \rightarrow e(-t)$
- lowpass filter $e(-t)$ with ω_{LP}
- reverse filtered signal in time again
- shift signal in time (Δt_{AFF}) to compensate loop delay
- add result to the previous FF table

Old vs. New Feedforward Table



Setpoint vs. Probe

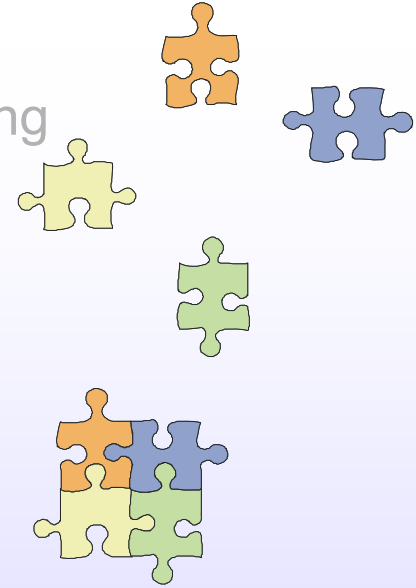


best results: $\omega_{LP} \approx$ closed loop bandwidth

$\Delta t_{AFF} \approx$ loop delay

Outline

1. signal conditioning / down conversion
2. detection of amp./phase by digital I/Q sampling
 - I/Q sampling
 - non I/Q sampling
 - digital down conversion (DDC)
3. upconversion
- 4. algorithms in RF applications**
 - feedback systems
 - cavity amplitude and phase
 - radial and phase loops
 - adaptive feed forward
 - system identification**

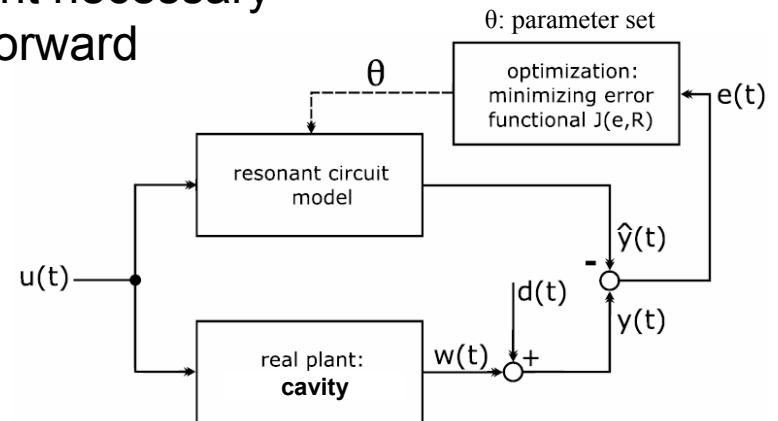


System Identification in RF plants

- goal:**
- design (synthesis) of high performance cavity field controllers is model based;
 - ➔ mathematical model of plant necessary
 - model required for efficient feedforward

➔ system identification

output error (OE) model structure:



system identification steps

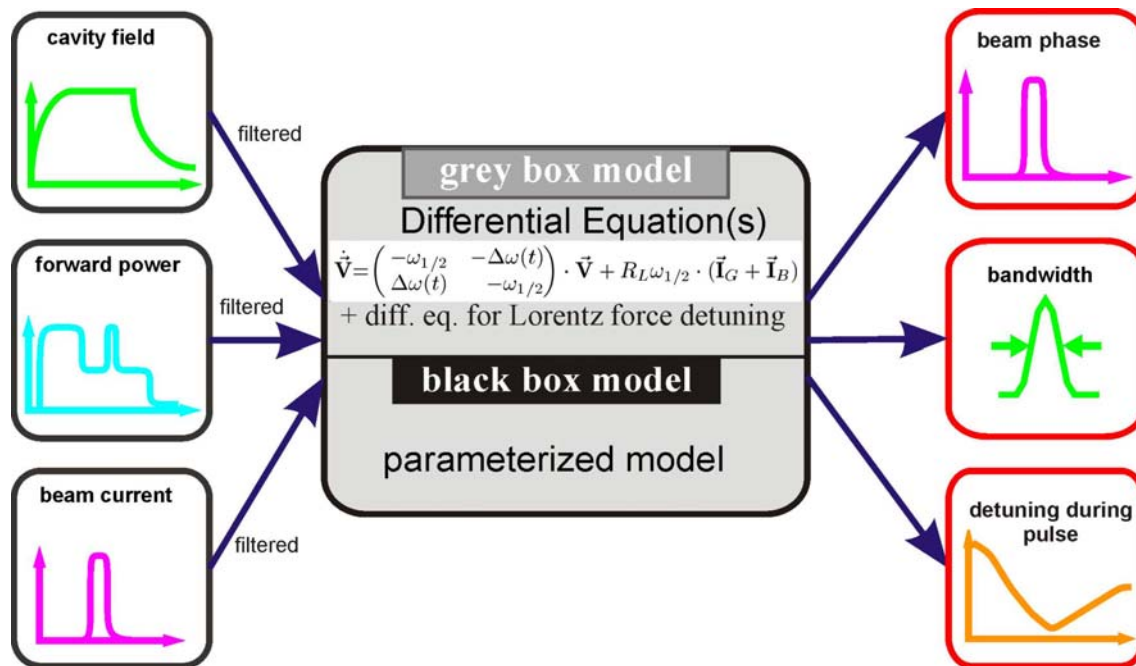
- record output data with proper input signal (step, impulse, white noise)
- choose model structure
 - ▣ grey box (preserves known physical structures with a number of unknown free parameters)
 - ▣ black box (no physical structure, parameters have no direct physical meaning)
- estimate model parameter (minimize $e(t)$)
- validate model with a set of data not included in the identification process

System Identification in RF plants (2)

example:

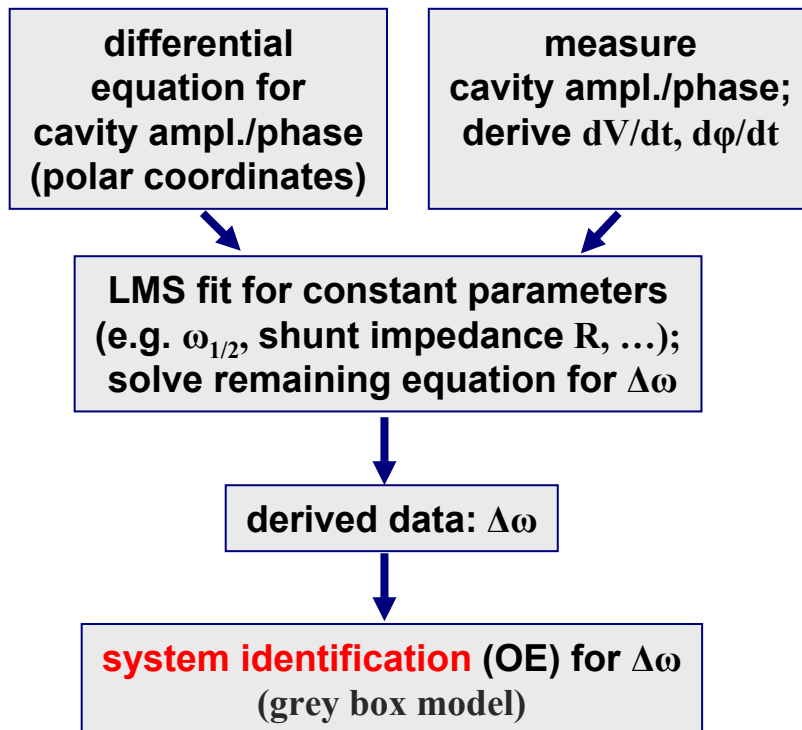
pulsed high gradient superconducting cavities with Lorentz force detuning

LPV: linear parameter varying model



System Identification in RF plants (3)

example: identification of Lorentz force detuning in high gradient cavity

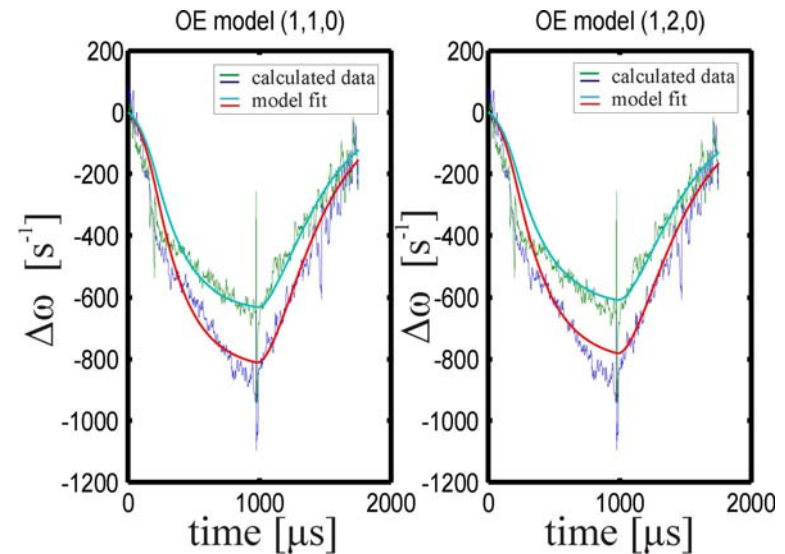


different parametric models:

$$\text{OE}(1,1,0): \Delta\dot{\omega}(t) = -\frac{1}{\tau_m}(\Delta\omega(t) - \Delta\omega_T) - 2\pi\frac{K}{\tau_m}E_{acc}^2$$

$$\text{OE}(1,2,0): \Delta\ddot{\omega}(t) = -\frac{1}{\tau_m}\Delta\dot{\omega}(t) - \omega_m^2(\Delta\omega(t) - \Delta\omega_T) - 2\pi K\omega_m^2 E_{acc}^2$$

↑ loop delay
 ↑ order of denominator polynomial in transfer function
 ↑ order of numerator polynomial in transfer function



M. Hüning (DESY)

Conclusion/ Outlook

- performance is very often dominated by systematic errors and nonlinearities of sensors and analog components
- digital LLRF does not look very different from other RF applications (beam diagnostics...) → common platforms?
- extensive diagnostics in digital RF systems allow automated procedures and calibration for complex systems (finite state machines...)
- digital platforms for RF applications provide playground for sophisticated algorithms



Now it's your turn
to contribute to this
exciting field!