Fundamentals of Microwave Engineering
&
RF coupling issues

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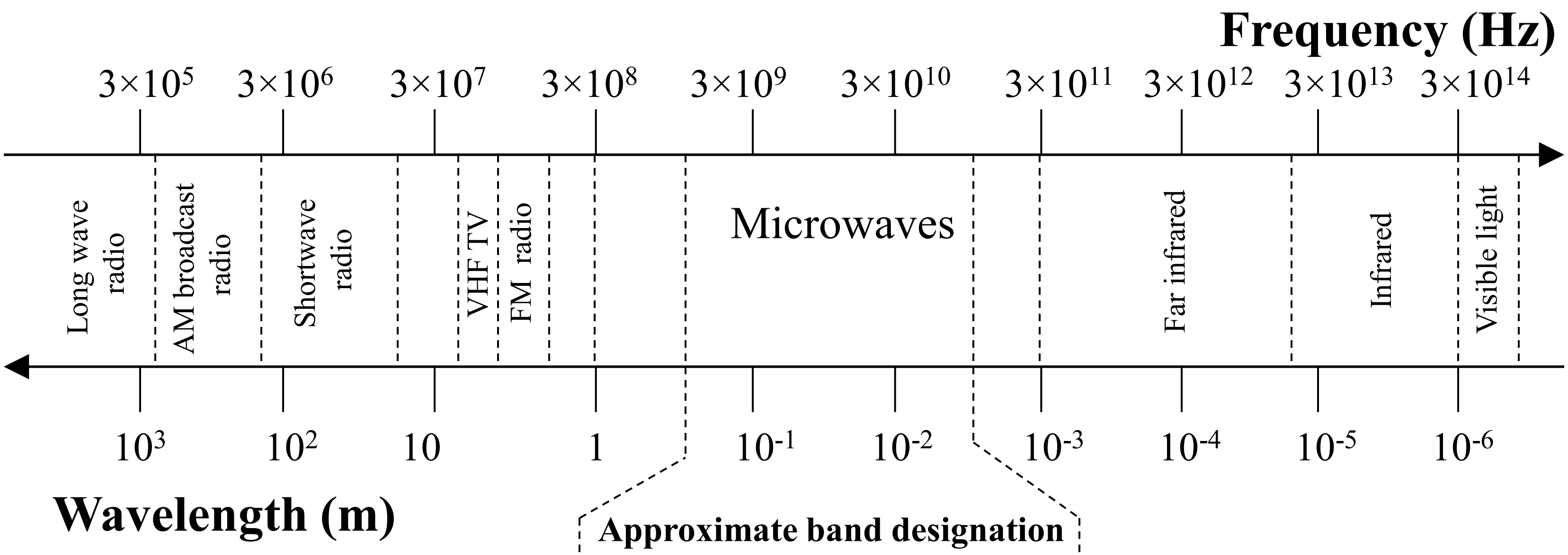
Frequency spectrum



Approximate band designation

L-band	1-2 GHz
S-band	2-4 GHz
C-band	4-8 GHz
X-band	8-12 GHz
Ku-band	12-18 GHz
K-band	18-26 GHz
Ka-band	26-40 GHz
U-band	40-60 GHz

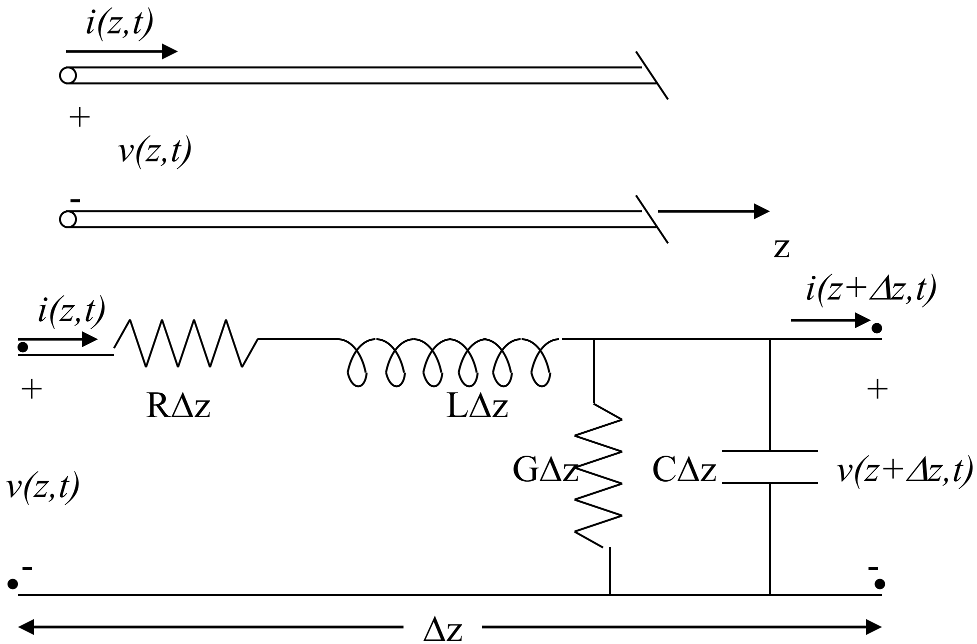
Low frequencies: λ so large that the phase variation across the dimensions is little.

Optical frequencies: λ much shorter than the component dimensions.

Geometrical optics regime

Phase of the voltage and currents changes significantly over the physical lenght of the device, microwave components are distributed, the lumped circuit element approximation are not valid.

Electrical Model of a Transmission Line



$$I_z = I_{z+\Delta z} + G\Delta z V_z + C\Delta z \frac{\partial V_z}{\partial t}$$

$$V_z = V_{z+\Delta z} + R\Delta z I_{z+\Delta z} + L\Delta z \frac{\partial I_{z+\Delta z}}{\partial t}$$

R= series resistance per unit lenght, for both conductors, in Ω/m.
L= series inductance per unit lenght, for both conductors, in H/m.
G= shunt conductance per unit lenght in S/m.
C= shunt capacitance per unit lenght in F/m.

Wave equation - Propagation constant

$$\frac{\partial^2 V_z}{\partial z^2} - \gamma^2 V_z = 0 \qquad \frac{\partial^2 I_z}{\partial z^2} - \gamma^2 I_z = 0$$

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$ **propagation constant**

$$V(z,t) = (V_0^+ \exp(-\gamma z) + V_0^- \exp(+\gamma z)) \times \exp(j\omega t)$$

$$I(z,t) = (I_0^+ \exp(-\gamma z) + I_0^- \exp(+\gamma z)) \times \exp(j\omega t)$$

wave travelling along +z direction — *wave travelling along -z direction*

$$\gamma = \sqrt{(R + j\omega L)\cdot(G + j\omega C)} = \alpha + j\beta$$

α **is called attenuation constant** [dB/m]

β **is called phase constant** [radians/m]

$$V(z) = V_0^+ e^{-\alpha z} \cdot e^{j(\omega t - \beta z)} + V_0^- e^{\alpha z} \cdot e^{j(\omega t + \beta z)}$$

$$I(z) = I_0^+ e^{-\alpha z} \cdot e^{j(\omega t - \beta z)} + I_0^- e^{\alpha z} \cdot e^{j(\omega t + \beta z)}$$

$$\lambda\beta = 2\pi \quad \Rightarrow \quad \lambda = \frac{2\pi}{\beta}$$

Characteristic Impedance

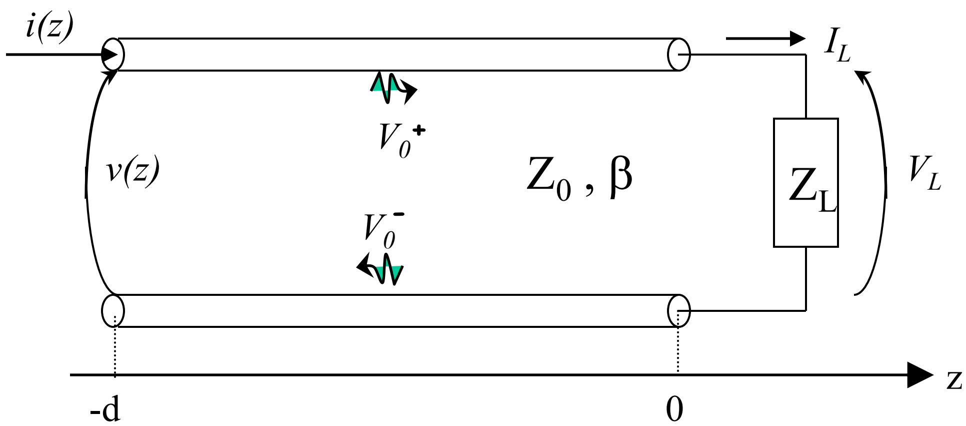
$$Z_0 = \frac{V_0^+}{I_0^+} \qquad Z_0 = -\frac{V_0^-}{I_0^-}, \quad I_0^-$$ *has negative sign*

$$V(z) = V_0^+ e^{-\alpha z} \cdot e^{j(\omega t - \beta z)} + V_0^- e^{\alpha z} \cdot e^{j(\omega t + \beta z)}$$

$$I(z) = (V_0^+ / Z_0) \cdot e^{-\alpha z} \cdot e^{j(\omega t - \beta z)} + (V_0^- / Z_0) \cdot e^{\alpha z} \cdot e^{j(\omega t + \beta z)}$$

$$Z_0 = \frac{V_0^+}{I_0^+} = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \qquad Z_0 = -\frac{V_0^-}{I_0^-} = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

Terminated lossless transmission line



$$V(z) = V_0^+ \cdot e^{-j\beta z} + V_0^- \cdot e^{j\beta z}$$

$$I(z) = (V_0^+ / Z_0) \cdot e^{-j\beta z} + (V_0^- / Z_0) \cdot e^{j\beta z}$$

$$@\, z = 0 \qquad Z_L = \frac{V(0)}{I(0)} = \frac{V_0^+ + V_0^-}{V_0^+ - V_0^-} Z_0 \quad \Rightarrow \quad V_0^- = \frac{Z_L - Z_0}{Z_L + Z_0} V_0^+$$

$$\Gamma = \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$V(z) = V_0^+ \cdot [e^{-j\beta z} + \Gamma \cdot e^{j\beta z}]$$

$$I(z) = \frac{V_0^+}{Z_0} \cdot [e^{-j\beta z} - \Gamma \cdot e^{j\beta z}]$$

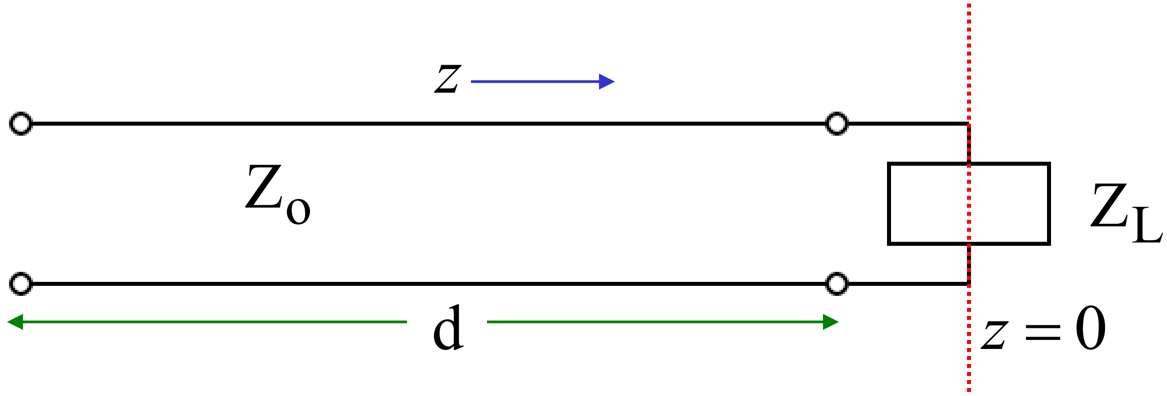
The amplitude of the reflected wave normalized to the amplitude of the incident wave is known as reflection coefficient

$$Z_L = Z_o \implies \Gamma = 0$$

NO REFLECTIONS (maximum power delivered to the load)
MATCHED LINE

Standing Waves

If $Z_L \neq Z_o$



$$V(z) = V_0^+ \cdot e^{-j\beta z} + V_0^- \cdot e^{j\beta z}$$

$$I(z) = (V_0^+ / Z_0) \cdot e^{-j\beta z} + (V_0^- / Z_0) \cdot e^{j\beta z}$$

At z=0

$$V_0^- = \Gamma V_0^+ = \frac{Z_L - Z_o}{Z_L + Z_o}$$

Along the transmission line:

$$V = V_0^+ e^{-j\beta z} + \Gamma V_0^+ e^{+j\beta z}$$

(by adding and substracting: $\Gamma V_0^+ e^{-j\beta z}$)

$$V = V_0^+ (1 - \Gamma) e^{-j\beta z} + 2 V_0^+ \Gamma \cos(\beta z)$$

traveling wave

standing wave

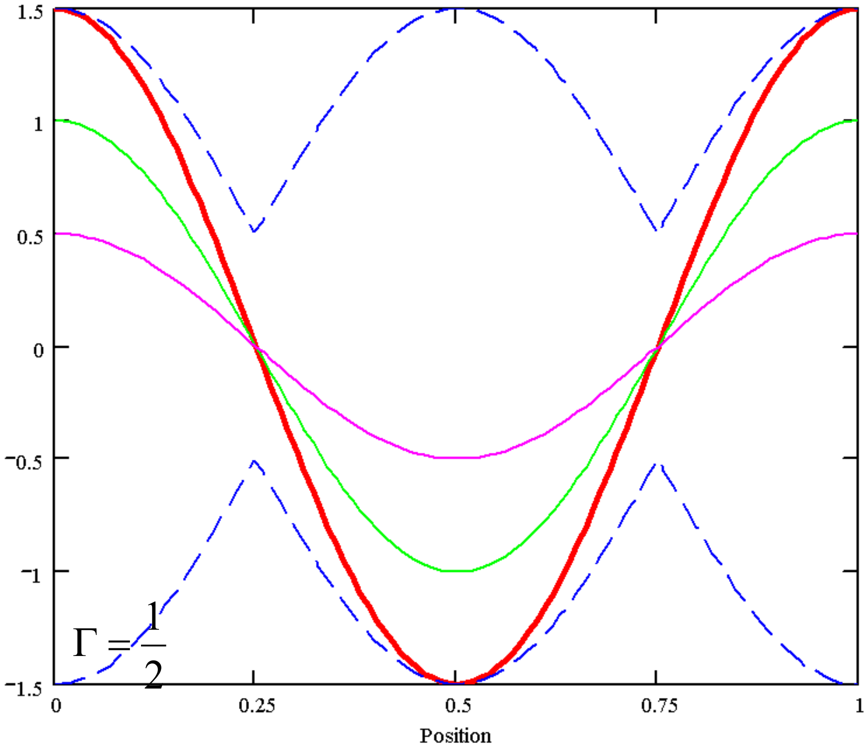
Voltage Standing Wave Ratio (VSWR)

$$V = V_0^+ e^{-j\beta z} + \Gamma V_0^+ e^{+j\beta z}$$

$$VSWR = \frac{V_{max}}{V_{min}} = \frac{V_0^+ \left(1 + |\Gamma|\right)}{V_0^+ \left(1 - |\Gamma|\right)} = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

The VSWR is always greater than 1

$$RL = -20 \log|\Gamma| \qquad [dB]$$

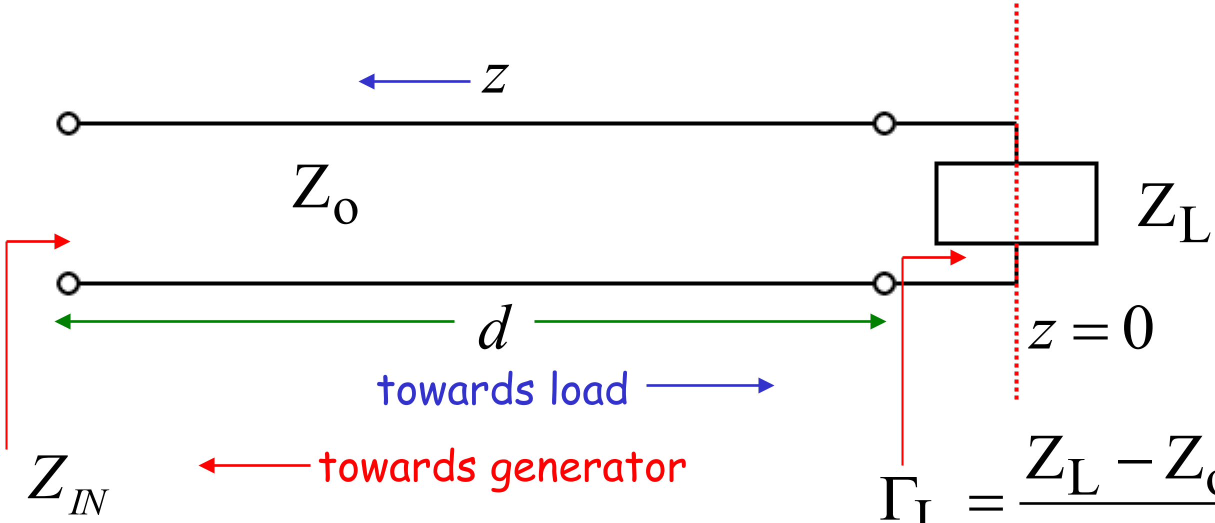


Incident wave

Reflected wave

Standing wave

Input Impedance of a Transmission Line



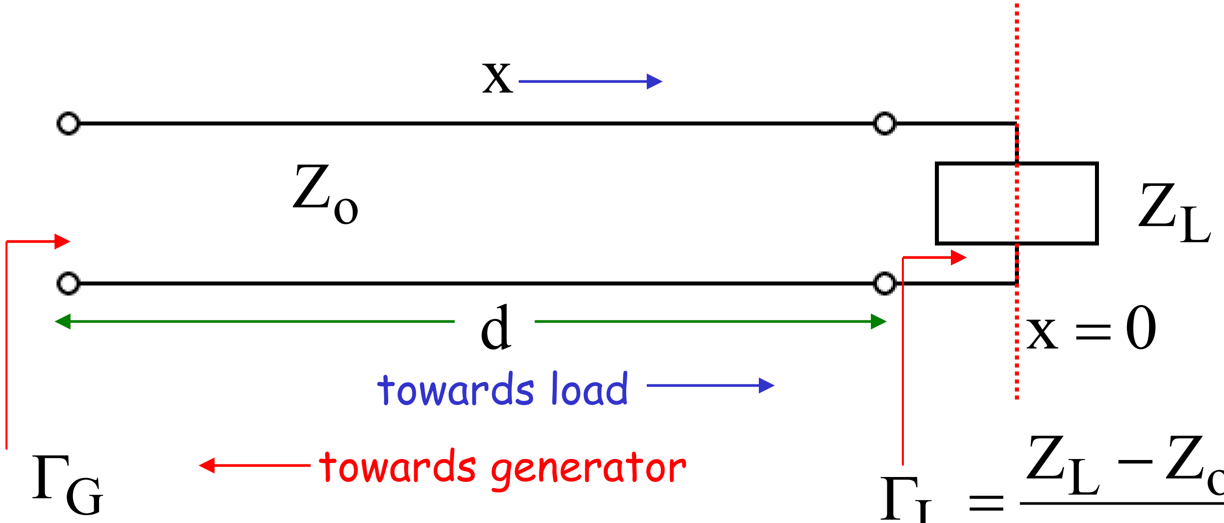
$$\Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o}$$

$$V(z) = V_0^+ \cdot [e^{-j\beta z} + \Gamma \cdot e^{j\beta z}]$$

$$I(z) = \frac{V_0^+}{Z_0} \cdot [e^{-j\beta z} - \Gamma \cdot e^{j\beta z}]$$

$$Z_{IN} = \frac{V(-d)}{I(-d)} = Z_0 \frac{1 + \Gamma \cdot e^{-2j\beta d}}{1 - \Gamma \cdot e^{-2j\beta d}} = Z_0 \frac{Z_L + jZ_0 \tan \beta d}{Z_0 + jZ_L \tan \beta d}$$

Reflection Coefficient Along a Transmission Line

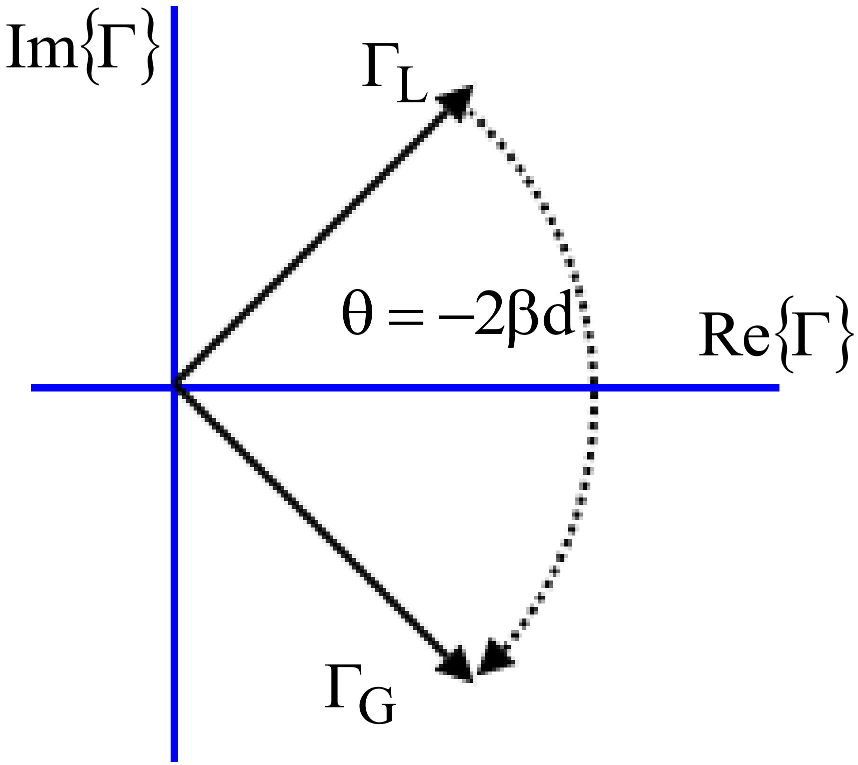


$$\Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o}$$

$$V(z) = V_0^+ \cdot [e^{-j\beta z} + \Gamma \cdot e^{j\beta z}]$$

Wave has to travel down and back

$$\Gamma_G = \left.\frac{V_{reverse}}{V_{forward}}\right|_{gen} = \Gamma_L \frac{V_0^+ e^{+j\beta(-d)}}{V_0^+ e^{-j\beta(-d)}} = \Gamma_L e^{-j2\beta d}$$



There is a one-to-one correspondence between Γ_G and Z_L

$$\Gamma_G = \frac{Z_G - Z_o}{Z_G + Z_o}$$

$$Z_G = Z_o \frac{1+\Gamma_G}{1-\Gamma_G}$$

$$Z_G = Z_o \frac{1+\Gamma_L e^{-j2\beta d}}{1-\Gamma_L e^{-j2\beta d}}$$

Focus on "special lenghts"

For a general transmission line with lenght:

•l=nλ/2

$$Z_{IN} = Z_L$$

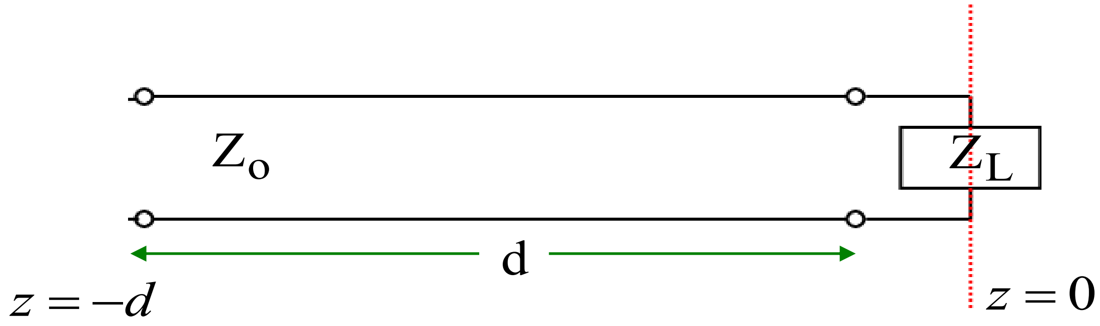
meaning that a half-wavelenght line does not alter or transform the load impedance

•l= λ/4 + nλ/2

$$Z_{IN} = \frac{Z_0^2}{Z_L}$$

•Such lines are called quarter wave transformers since they have the effect of trasforming the load impedance in an inverse manner depending on the characteristic impedance of the line.

Incident and Reflected Power



The rate of energy flowing through the plane at z=-d

$$P=\frac{1}{2}\mathrm{Re}\left\{V(z)I(z)^*\right\}=\frac{1}{2}\cdot\frac{\left|V_0^+\right|^2}{Z_0}\mathrm{Re}\left\{1-\Gamma^* e^{-2j\beta z}+\Gamma e^{2j\beta z}V(z)-\left|\Gamma\right|^2\right\}$$

$$P=\frac{1}{2}\cdot\frac{\left|V_0^+\right|^2}{Z_0}\cdot(1-\left|\Gamma\right|^2)=\frac{1}{2}\frac{\left|V_0^+\right|^2}{Z_o}-\frac{1}{2}\left|\Gamma_L\right|^2\frac{\left|V_0^+\right|^2}{Z_o}$$

forward power

reflected power

Incident and Reflected Power

- Power does not flow! Energy flows.
  - The net rate of energy transfer is equal to the difference in power of the individual waves
- To maximize the power transferred to the load we want:

$$\Gamma_L = 0$$

which implies:

$$Z_L = Z_o$$

When $Z_L = Z_o$, the load is matched to the transmission line

dB and dBm

A dB is defined as a **POWER** ratio. For example:

$$\Gamma_{dB} = 10\log\left(\frac{P_{rev}}{P_{for}}\right)$$

$$= 10\log\left(|\Gamma|^2\right)$$

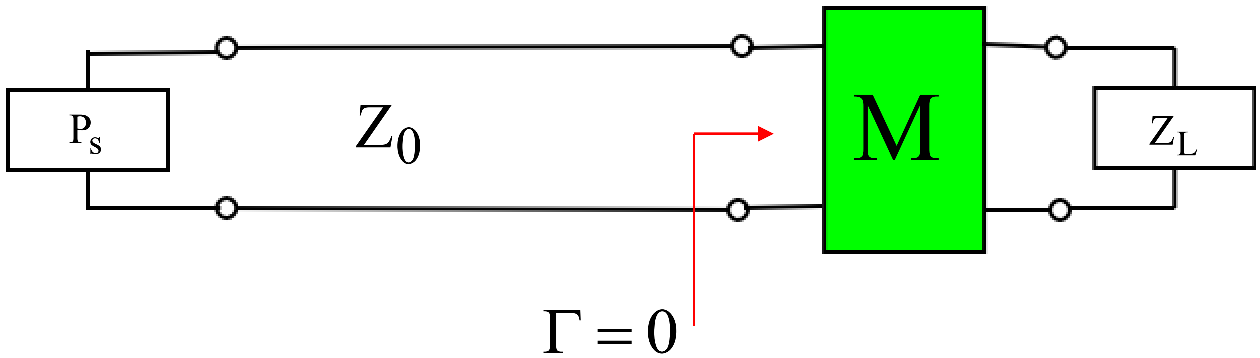
$$= 20\log\left(|\Gamma|\right)$$

A dBm is defined as log unit of power referenced to 1mW:

$$P_{dBm} = 10\log\left(\frac{P}{1mW}\right)$$

Load Matching

What if the load cannot be made equal to Z_o for some other reasons? Then, we need to build a matching network so that the source effectively sees a match load.



Typically we only want to use lossless devices such as capacitors, inductors, transmission lines, in our matching network so that we do not dissipate any power in the network and deliver all the available power to the load.

Normalized Impedance

It will be easier if we normalize the load impedance to the characteristic impedance of the transmission line attached to the load.

$$z_L = \frac{Z_L}{Z_o} = r_L + jx_L$$

$$z_L = \frac{1+\Gamma}{1-\Gamma}$$

Since the impedance is a complex number, the reflection coefficient will be a complex number

$$\Gamma = \Gamma_r + j\Gamma_i$$

$$r_L = \frac{1-\Gamma_r^2-\Gamma_i^2}{(1-\Gamma_r)^2+\Gamma_i^2} \qquad x_L = \frac{2\Gamma_i}{(1-\Gamma_r)^2+\Gamma_i^2}$$

The impedance as a function of reflection coefficient can be re-written in the form:

$$r_L = \frac{1-\Gamma_r^2-\Gamma_i^2}{(1-\Gamma_r)^2+\Gamma_i^2} \quad \Rightarrow \quad \left(\Gamma_r - \frac{r_L}{1+r_L}\right)^2 + \Gamma_i^2 = \frac{1}{(1+r_L)^2}$$

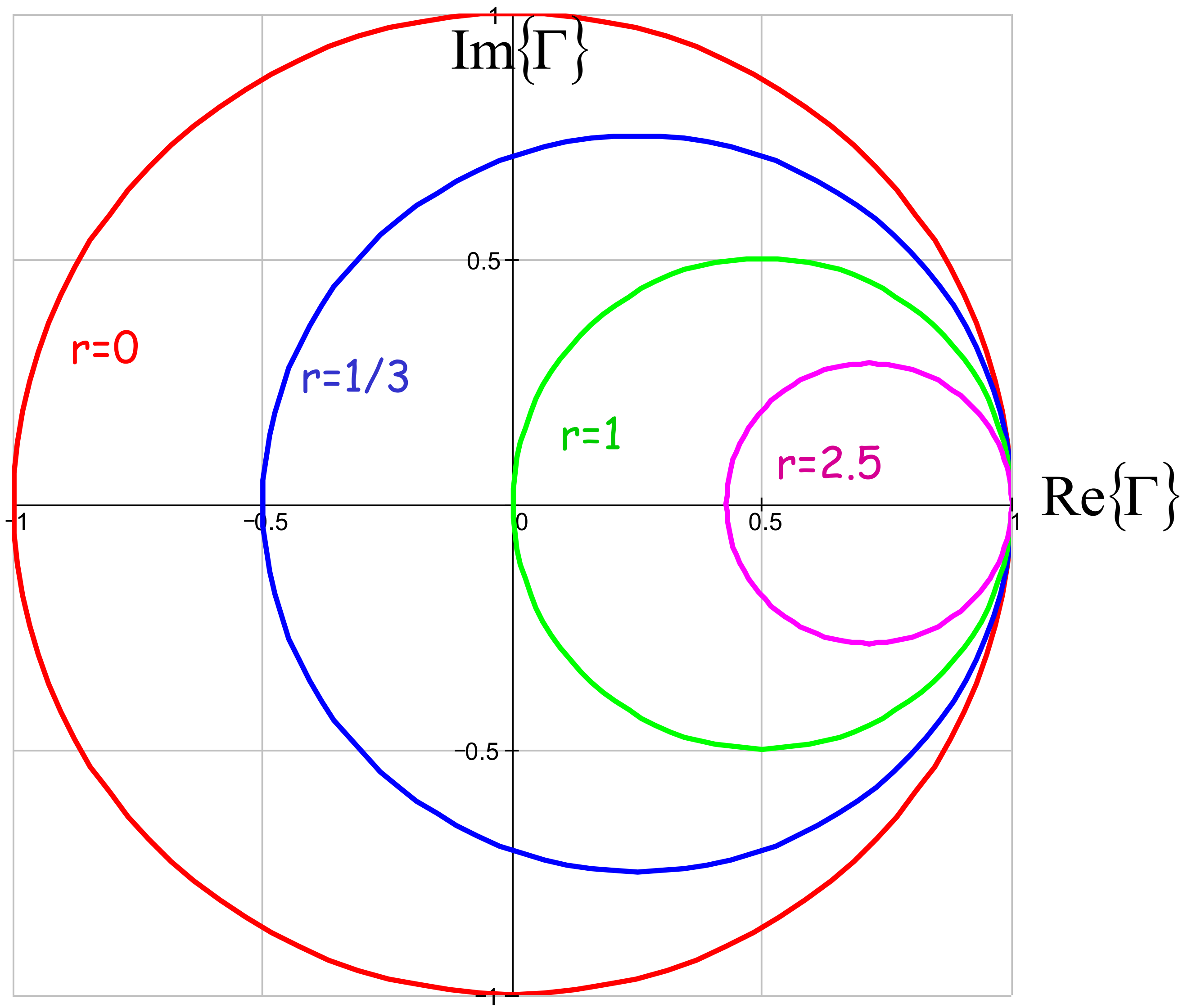
$$x_L = \frac{2\Gamma_i}{(1-\Gamma_r)^2+\Gamma_i^2} \quad \Rightarrow \quad (\Gamma_r - 1)^2 + \left(\Gamma_i - \frac{1}{x_L}\right)^2 = \frac{1}{x_L^2}$$

These are equations for circles on the (Γr,Γi) plane

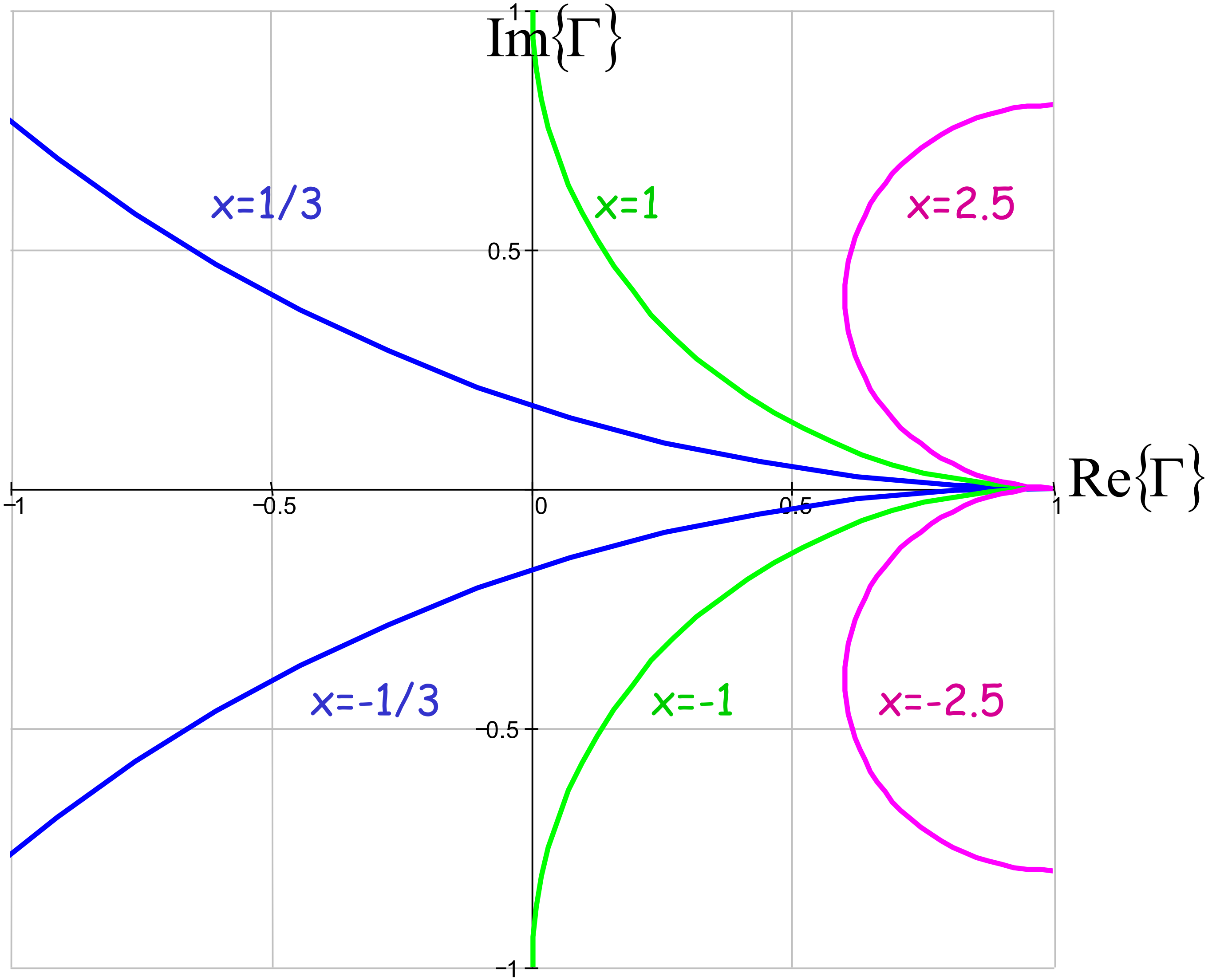
$$C \equiv \left(\frac{r_L}{1+r_L}, 0\right) \qquad C \equiv \left(1, \frac{1}{x_L}\right)$$

$$radius = \frac{1}{1+r_L} \qquad radius = \frac{1}{x_L}$$

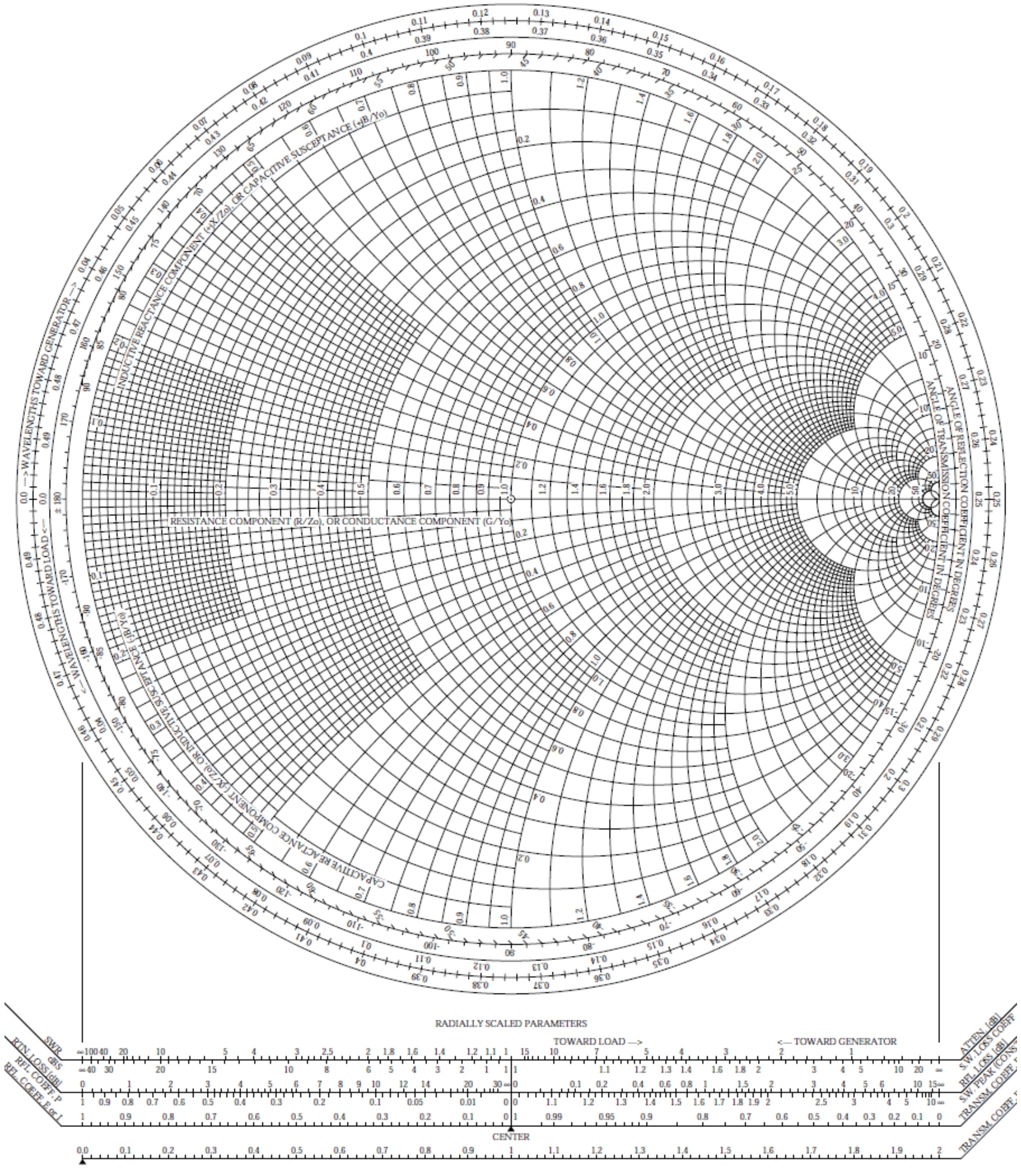
Smith Chart - Real Circles



Smith Chart - Imaginary Circles



Smith Chart



RESISTANCE COMPONENT (R/Zo), OR CONDUCTANCE COMPONENT (G/Yo)
INDUCTIVE REACTANCE COMPONENT (+jX/Zo) OR CAPACITIVE SUSCEPTANCE (+jB/Yo)
CAPACITIVE REACTANCE COMPONENT (-jX/Zo) OR INDUCTIVE SUSCEPTANCE (-jB/Yo)
ANGLE OF REFLECTION COEFFICIENT IN DEGREES
ANGLE OF TRANSMISSION COEFFICIENT IN DEGREES
WAVELENGTHS TOWARD GENERATOR
WAVELENGTHS TOWARD LOAD
RADIALLY SCALED PARAMETERS
TOWARD LOAD
TOWARD GENERATOR
CENTER
SWR
RTN. LOSS [dB]
RFL. COEFF, P
RFL. COEFF, E or I
ATTEN. [dB]
S.W. LOSS COEFF
RFL. LOSS [dB]
S.W. PEAK (CONST. P)
TRANSM. COEFF, P
TRANSM. COEFF, E or I

Smith Chart Example 1

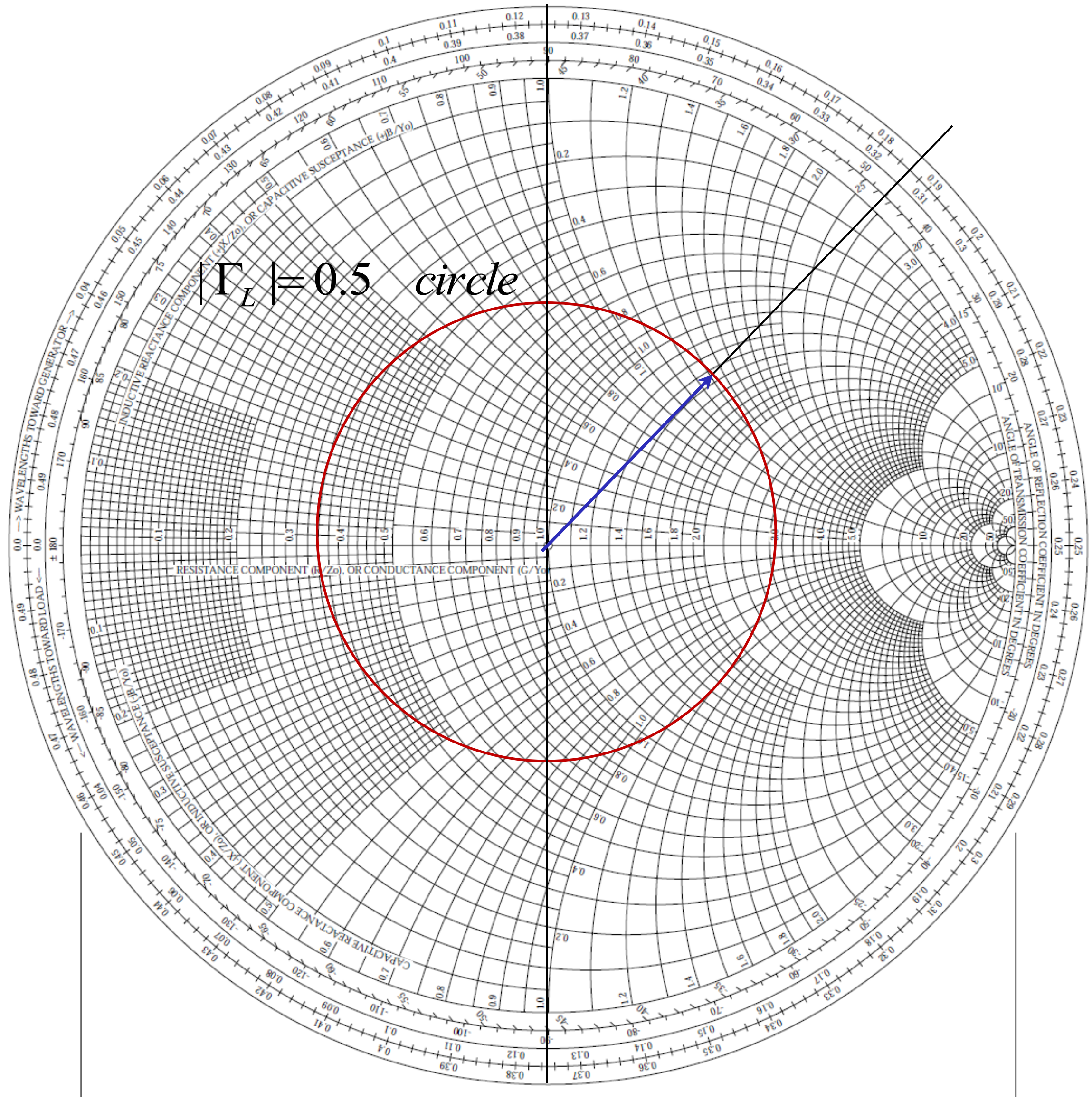
Given:

$$\Gamma_L = 0.5\angle 45°$$

$$Z_o = 50\Omega$$

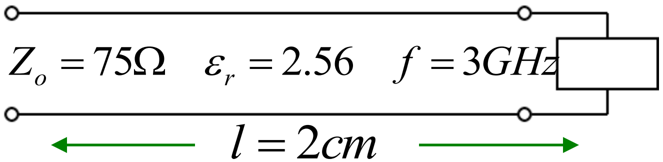
What is Z_L?

$$Z_L = 50\Omega(1.35 + j1.35)$$
$$= 67.5\Omega + j67.5\Omega$$



Smith Chart Example 2

Given: $Z_L = 37.5\Omega + j75\Omega$



$Z_o = 75\Omega \quad \varepsilon_r = 2.56 \quad f = 3GHz$

$l = 2cm$

$Z_{in} = ? \qquad SWR?$

$$z_L = 0.5 + j1.0$$

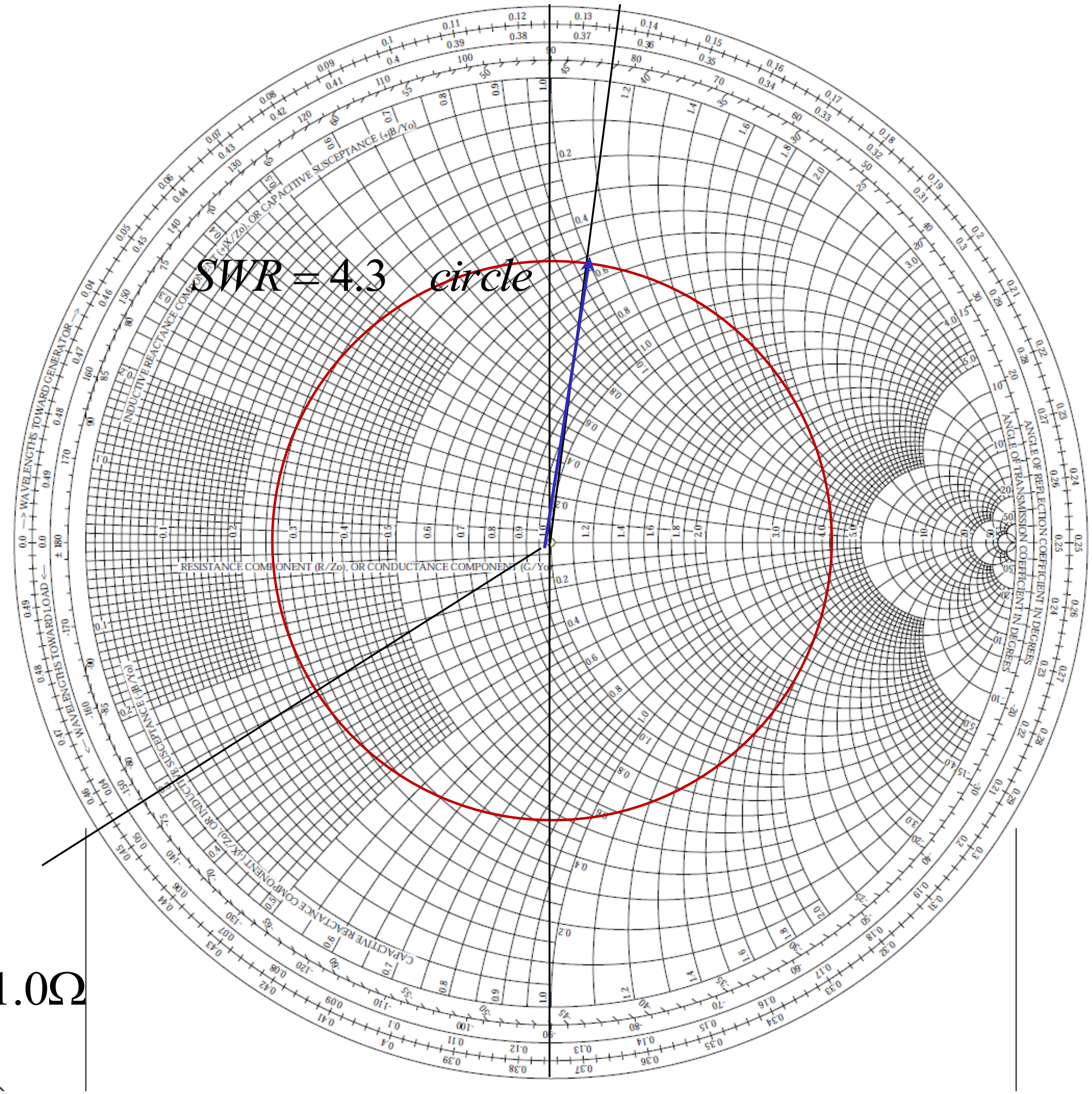
Ref. position of the load=0.135λ
We need to move toward to the gen. an electrical distance equal to the line length

$$\lambda = \frac{3 \cdot 10^8}{3 \cdot 10^9 \cdot \sqrt{2.56}} = 6.25cm$$

$$l = \frac{2.0}{6.25} = 0.32\lambda$$

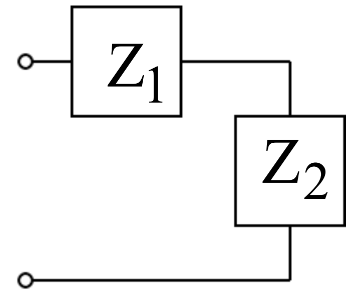
$$0.32\lambda + 0.135\lambda = 0.455\lambda$$

$$Z_{in} = 75\Omega(0.25 - j0.28) = 18.75 - j21.0\Omega$$



Admittance

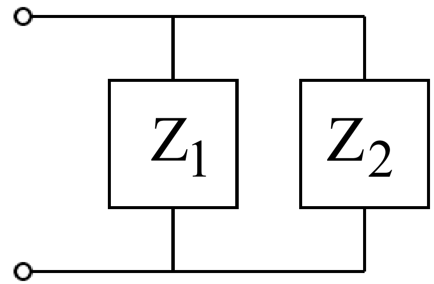
A matching network is going to be a combination of elements connected in series AND parallel.



Impedance is well suited when working with series configurations. For example:

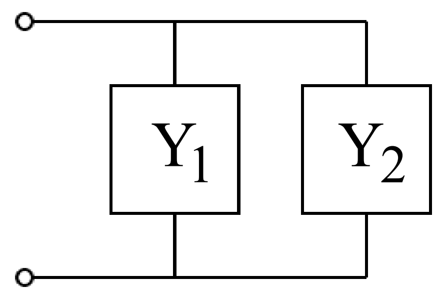
$$V = ZI$$

$$Z_L = Z_1 + Z_2$$



Impedance is NOT well suited when working with parallel configurations.

$$Z_L = \frac{Z_1 Z_2}{Z_1 + Z_2}$$



For parallel loads it is better to work with admittance.

$$I = YV$$

$$Y_1 = \frac{1}{Z_1}$$

$$Y_L = Y_1 + Y_2$$

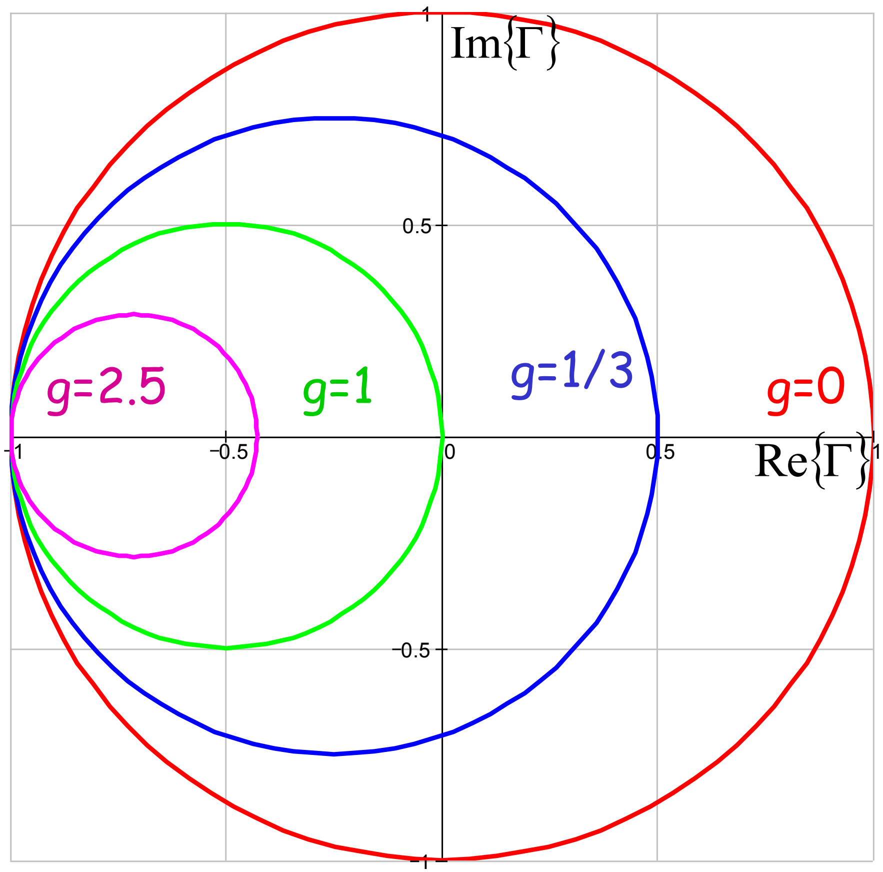
Impedance and Admittance Smith Charts

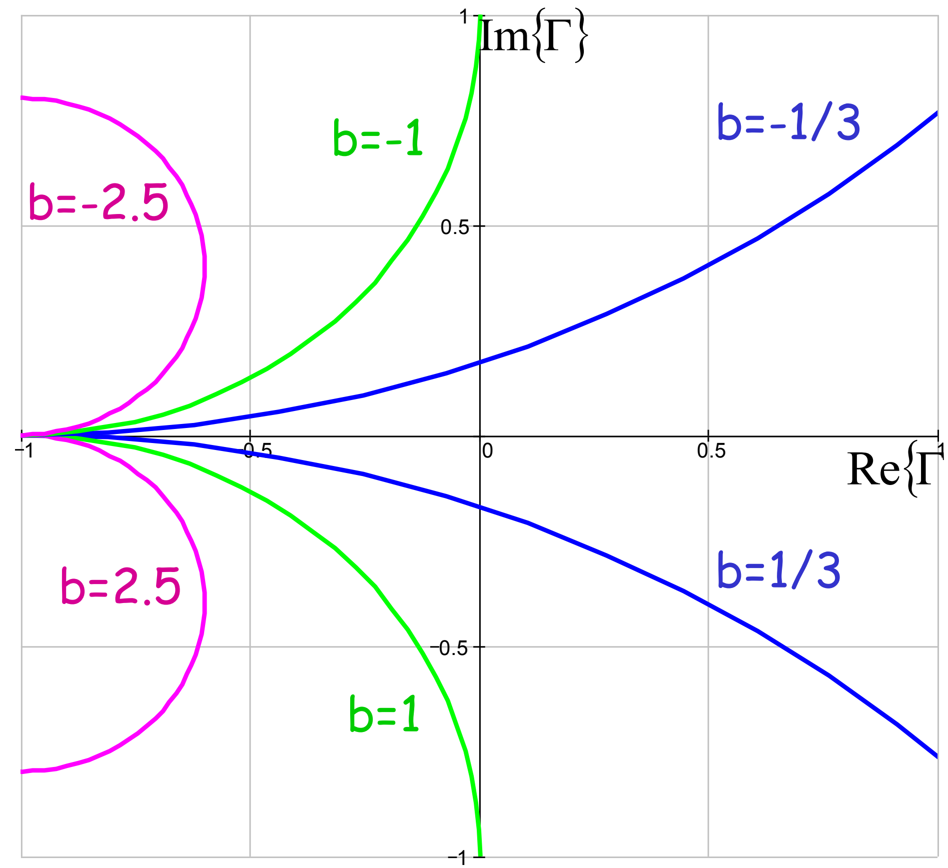
- For a matching network that contains elements connected in series and parallel, we will need two types of Smith charts
  - impedance Smith chart
  - admittance Smith Chart
- The admittance Smith chart is the impedance Smith chart rotated 180 degrees.
  - We could use one Smith chart and flip the reflection coefficient vector 180 degrees when switching between a series configuration to a parallel configuration.

Admittance Smith Chart

$$y = \frac{Y}{Y_o} = YZ_o = g + jb$$

$$y = \frac{1-\Gamma}{1+\Gamma}$$





Admittance Smith Chart Example 1

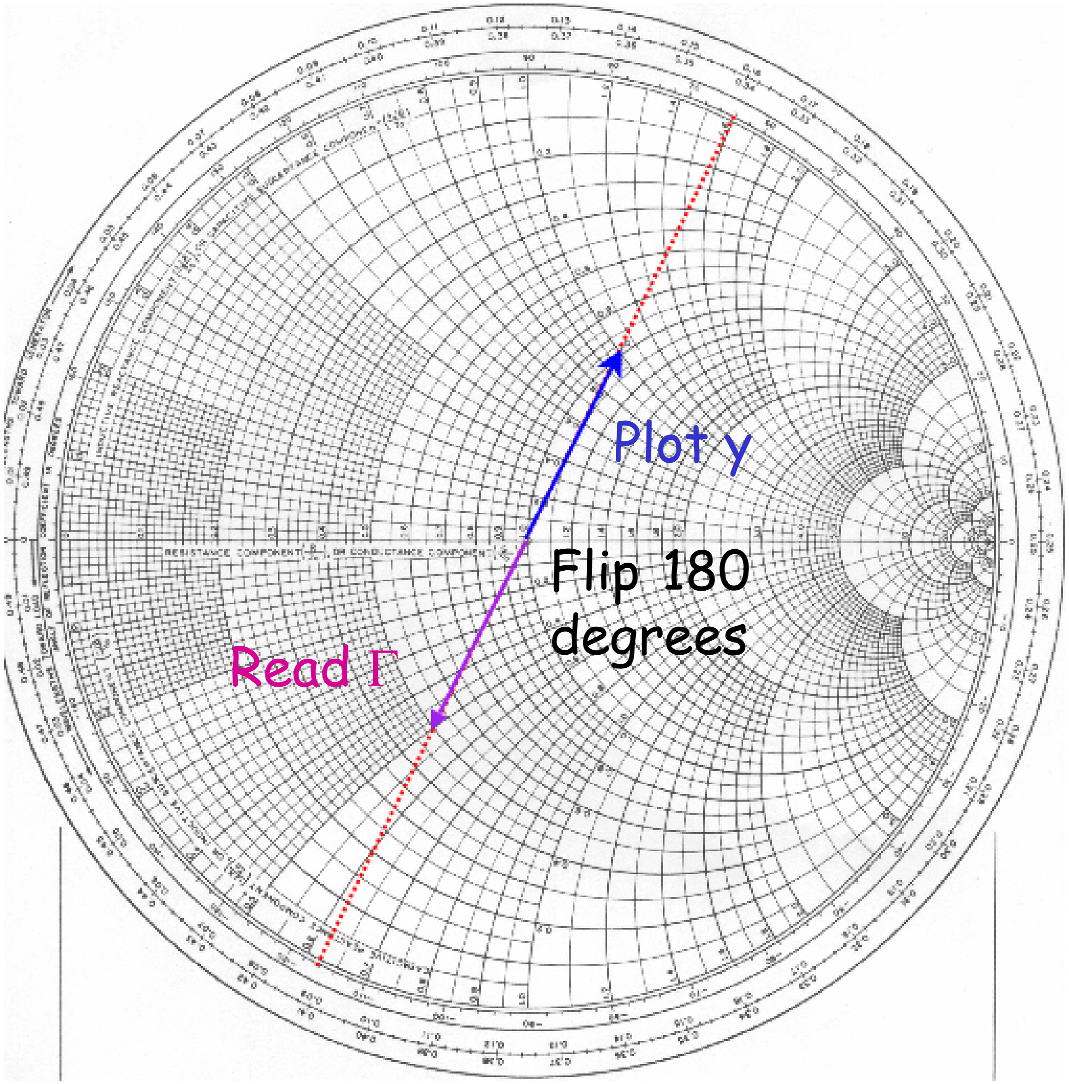
Given:

$$y = 1 + j1$$

What is Γ?

- Procedure:
  - Plot 1+j1 on chart
    - vector = $0.445\angle 64°$
  - Flip vector 180 degrees

$$\Gamma = 0.445\angle -116°$$



Admittance Smith Chart Example 2

Given:

$$\Gamma = 0.5\angle +45^\circ \quad Z_o = 50\Omega$$

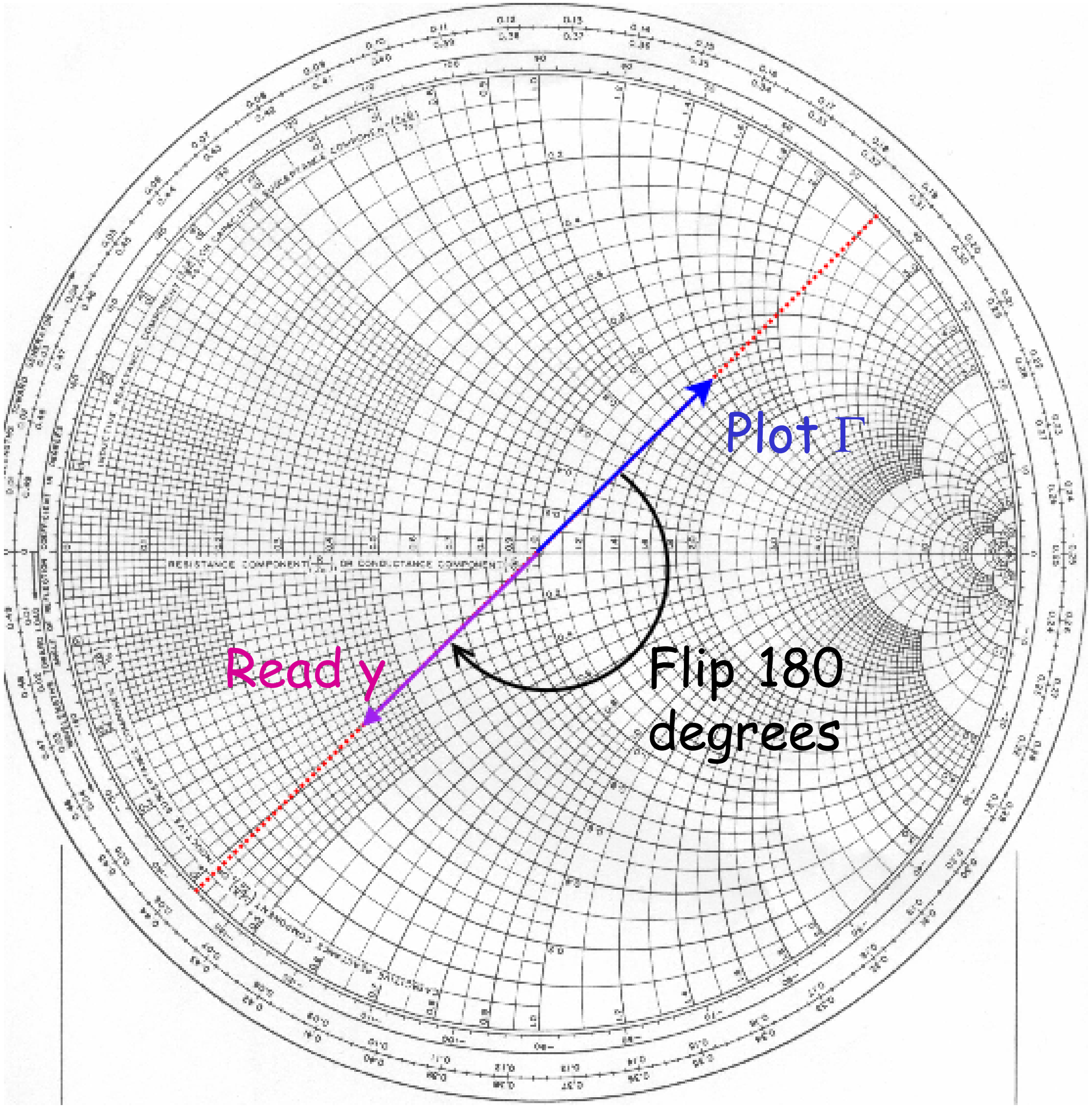
What is Y?

- Procedure:
  - Plot Γ
  - Flip vector by 180 degrees
  - Read coordinate

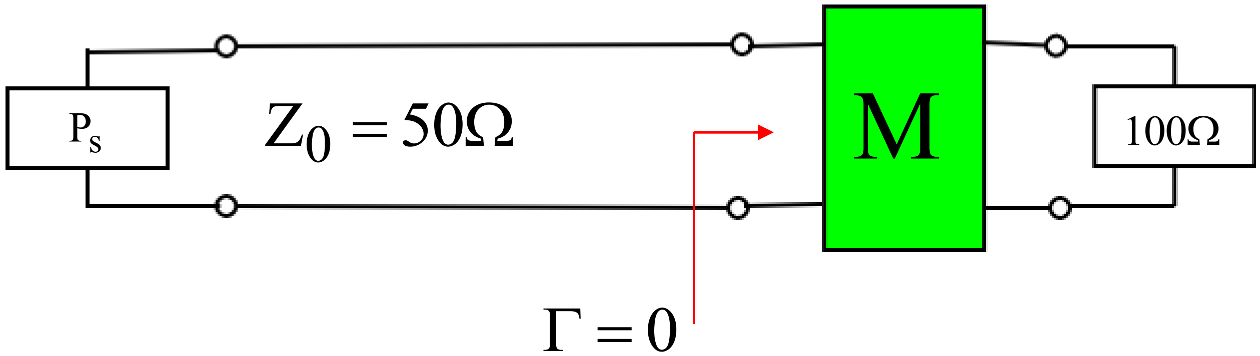
$$y = 0.38 - j0.36$$

$$Y = \frac{1}{50\Omega}(0.38 - j0.36)$$

$$Y = (7.6 - j7.2)x10^{-3}\,\text{mhos}$$



Matching Example

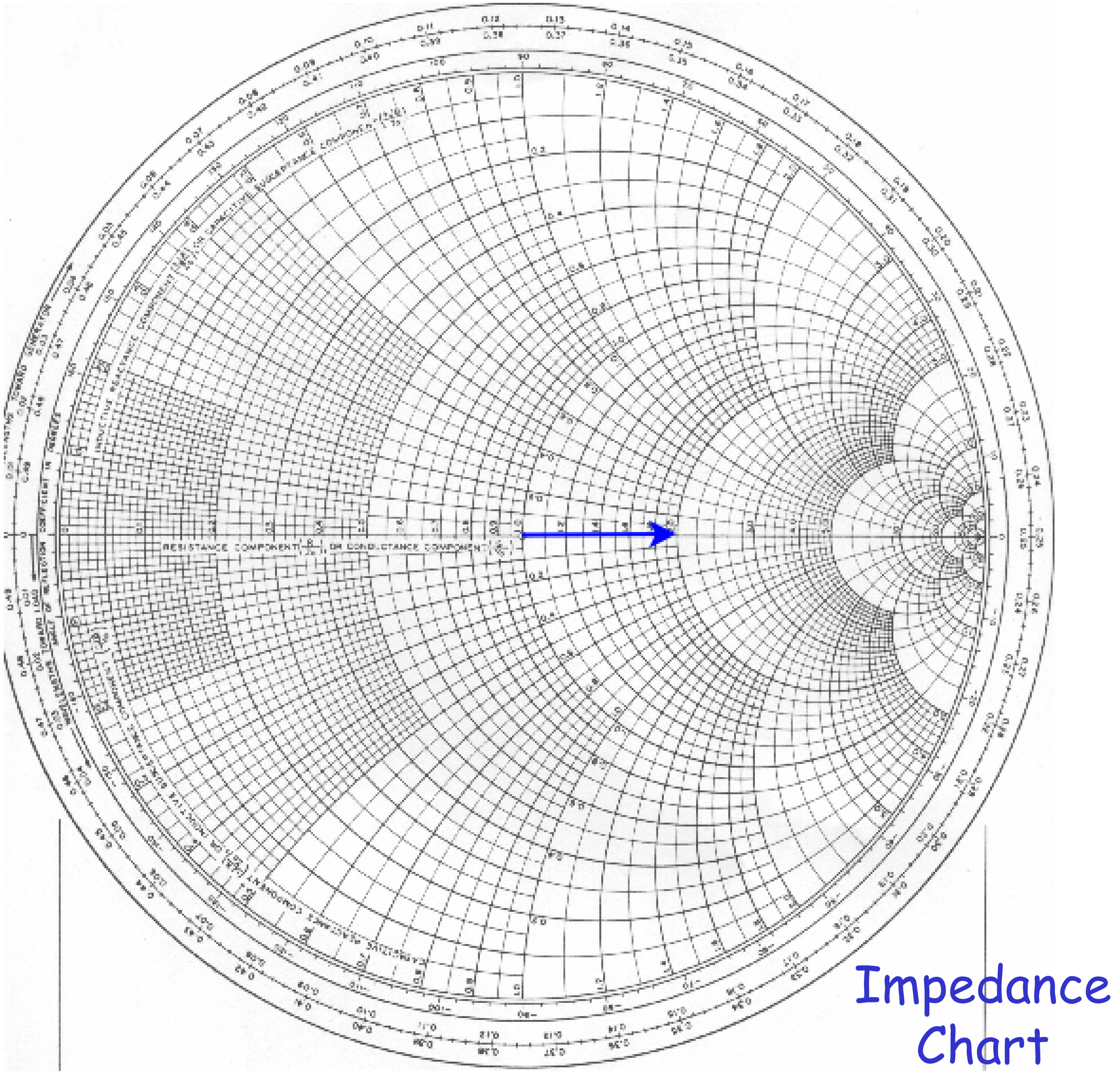


Match 100Ω load to a 50Ω system at 100MHz

A 100Ω resistor in parallel would do the trick but ½ of the power would be dissipated in the matching network. We want to use only lossless elements such as inductors and capacitors so we don't dissipate any power in the matching network

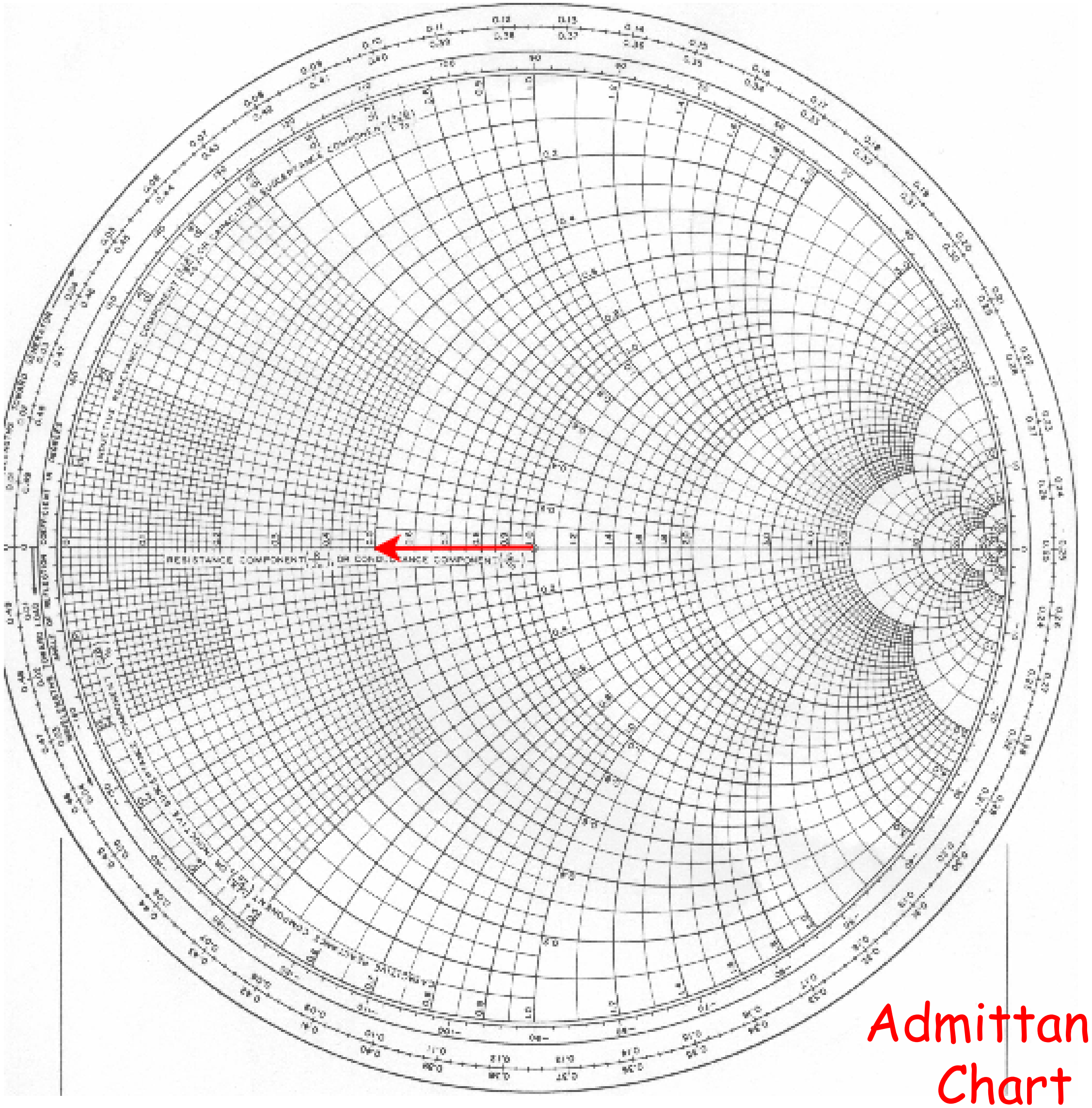
Matching Example

- We need to go from z=2+j0 to z=1+j0 on the Smith chart
- We won't get any closer by adding series impedance so we will need to add something in parallel.
- We need to flip over to the admittance chart



Matching Example

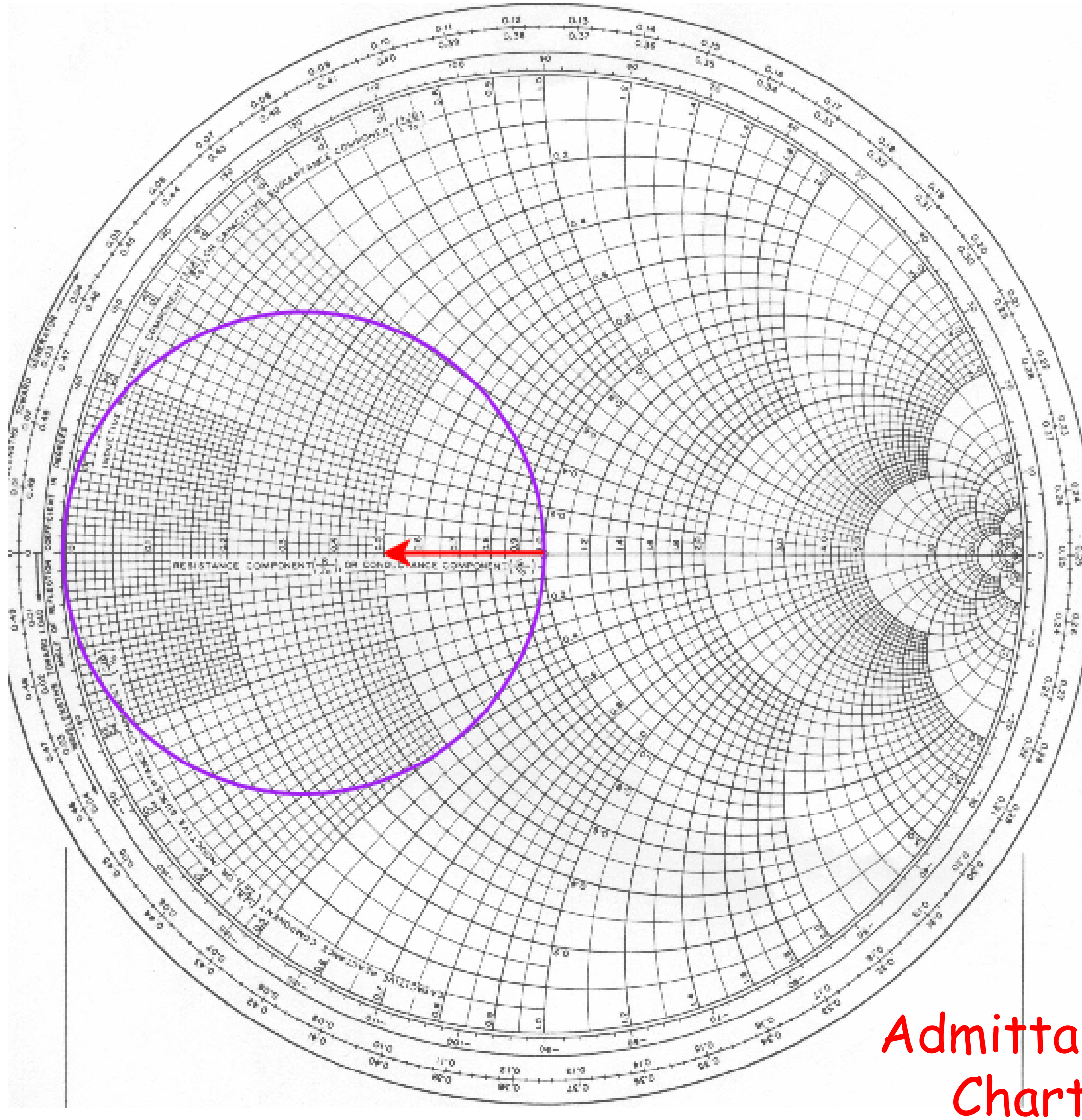
- y=0.5+j0
- Before we add the admittance, add a mirror of the r=1 circle as a guide.



Admittance Chart

Matching Example

- y=0.5+j0
- Before we add the admittance, add a mirror of the r=1 circle as a guide
- Now add positive imaginary admittance.



Admittance Chart

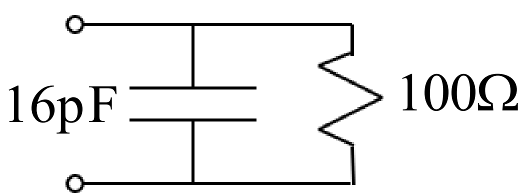
Matching Example

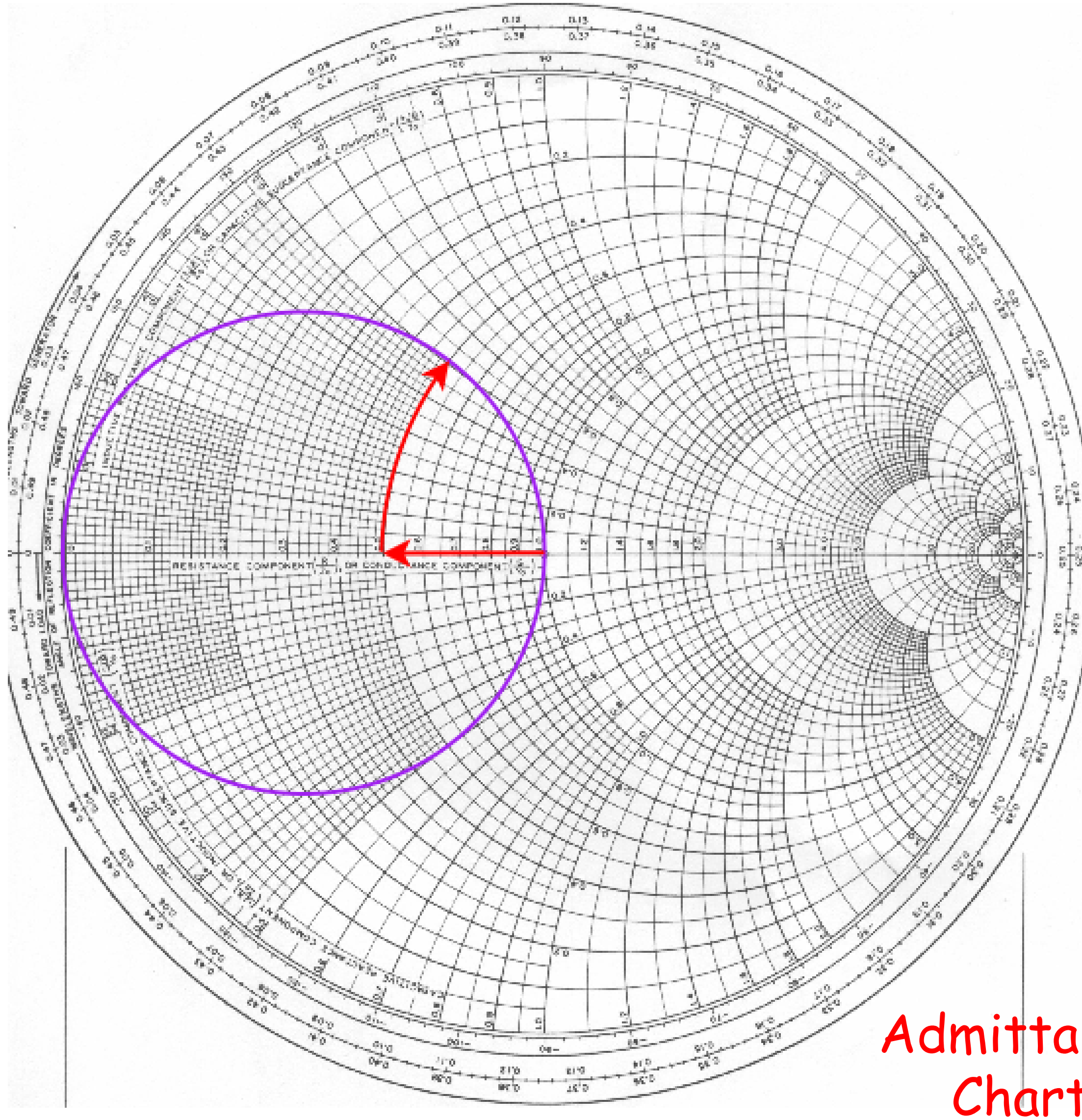
- y=0.5+j0
- Before we add the admittance, add a mirror of the r=1 circle as a guide
- Now add positive imaginary admittance jb = j0.5

$$jb = j0.5$$

$$\frac{j0.5}{50\Omega} = j2\pi(100\,\mathrm{MHz})C$$

$$C = 16\,\mathrm{pF}$$

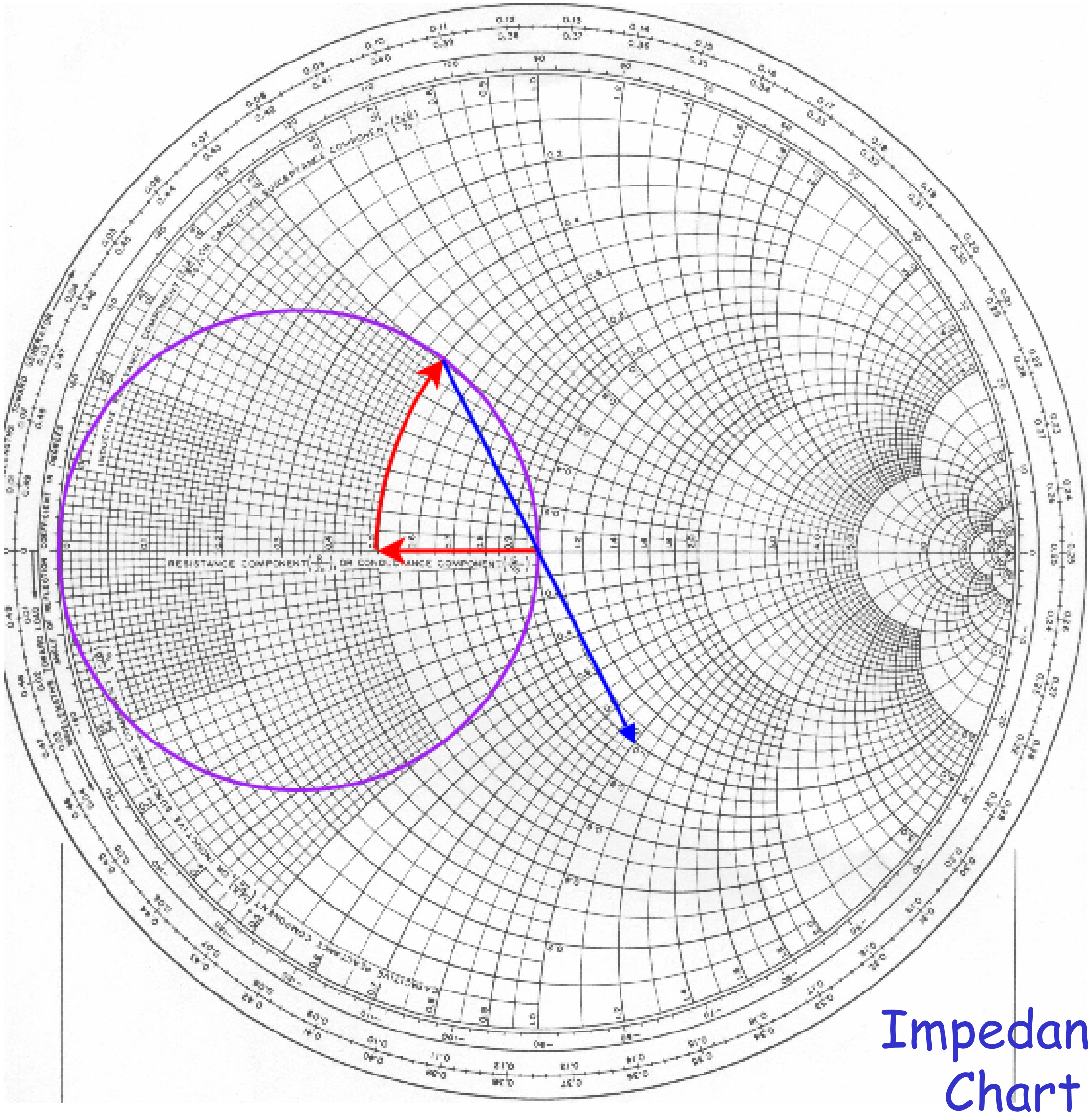




Admittance Chart

Matching Example

- We will now add series impedance
- Flip to the impedance Smith Chart
- We land at on the r=1 circle at x=-1



Impedance Chart

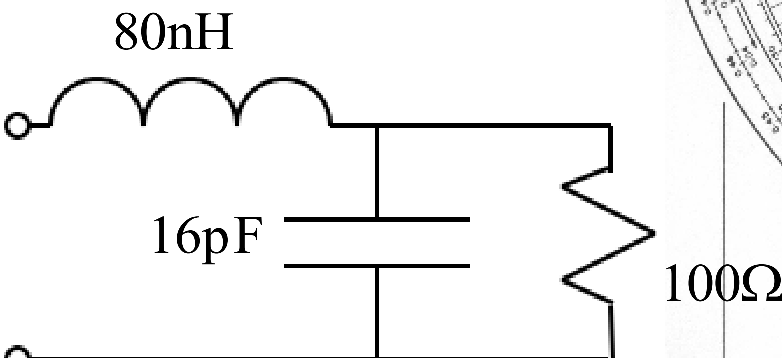
Matching Example

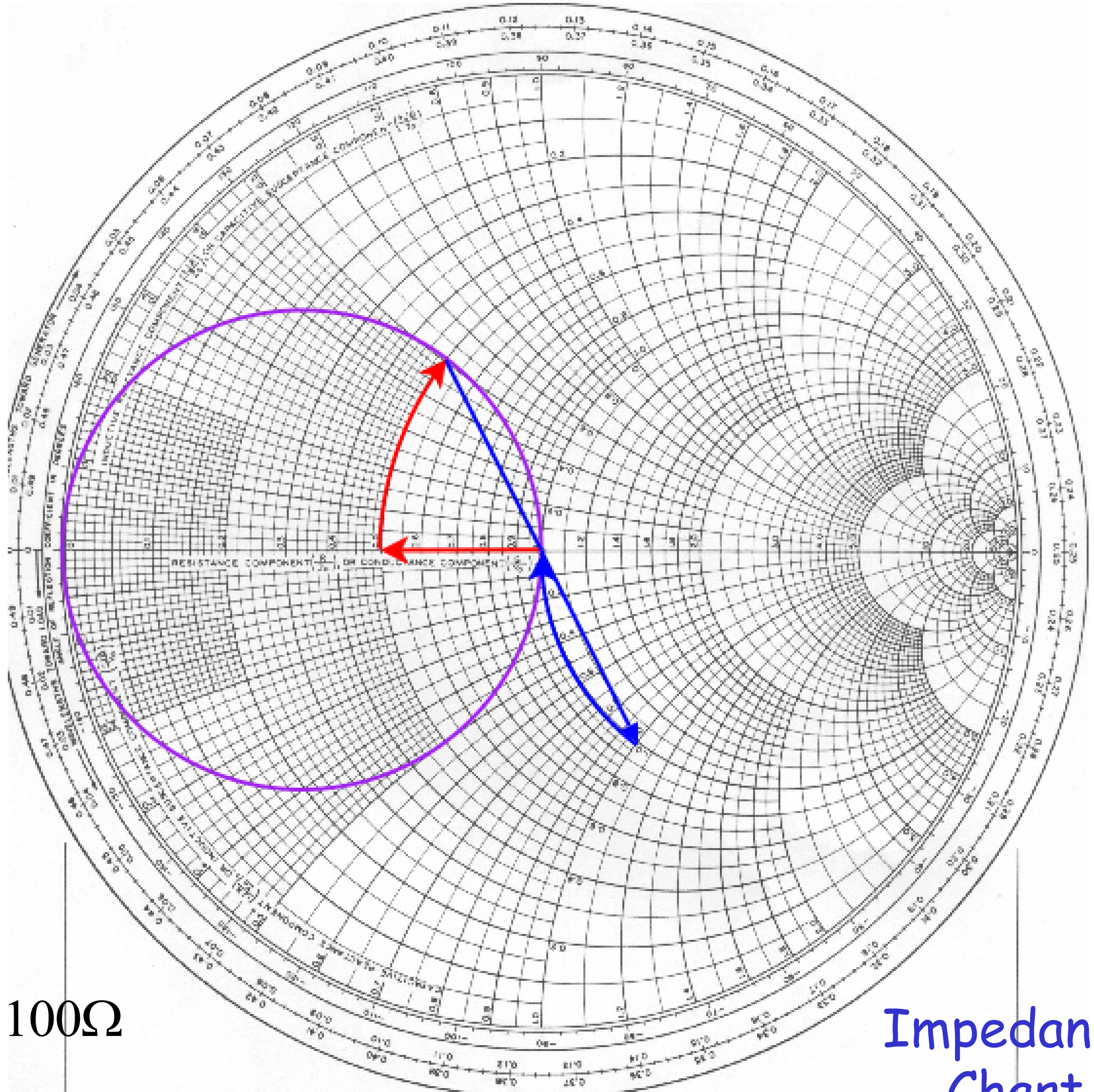
- Add positive imaginary admittance to get to z=1+j0

$$jx = j1.0$$

$$(j1.0)50\Omega = j2\pi(100\,\mathrm{MHz})L$$

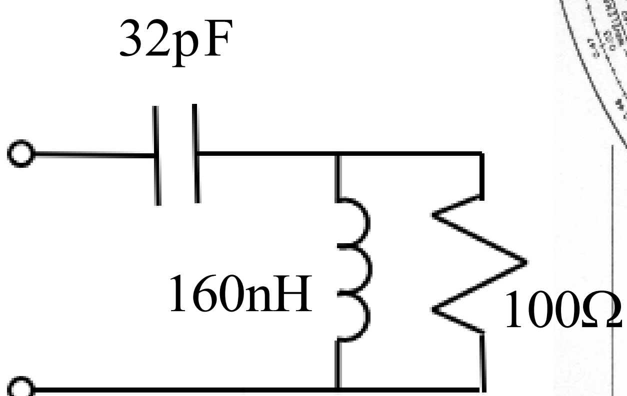
$$L = 80\,\mathrm{nH}$$

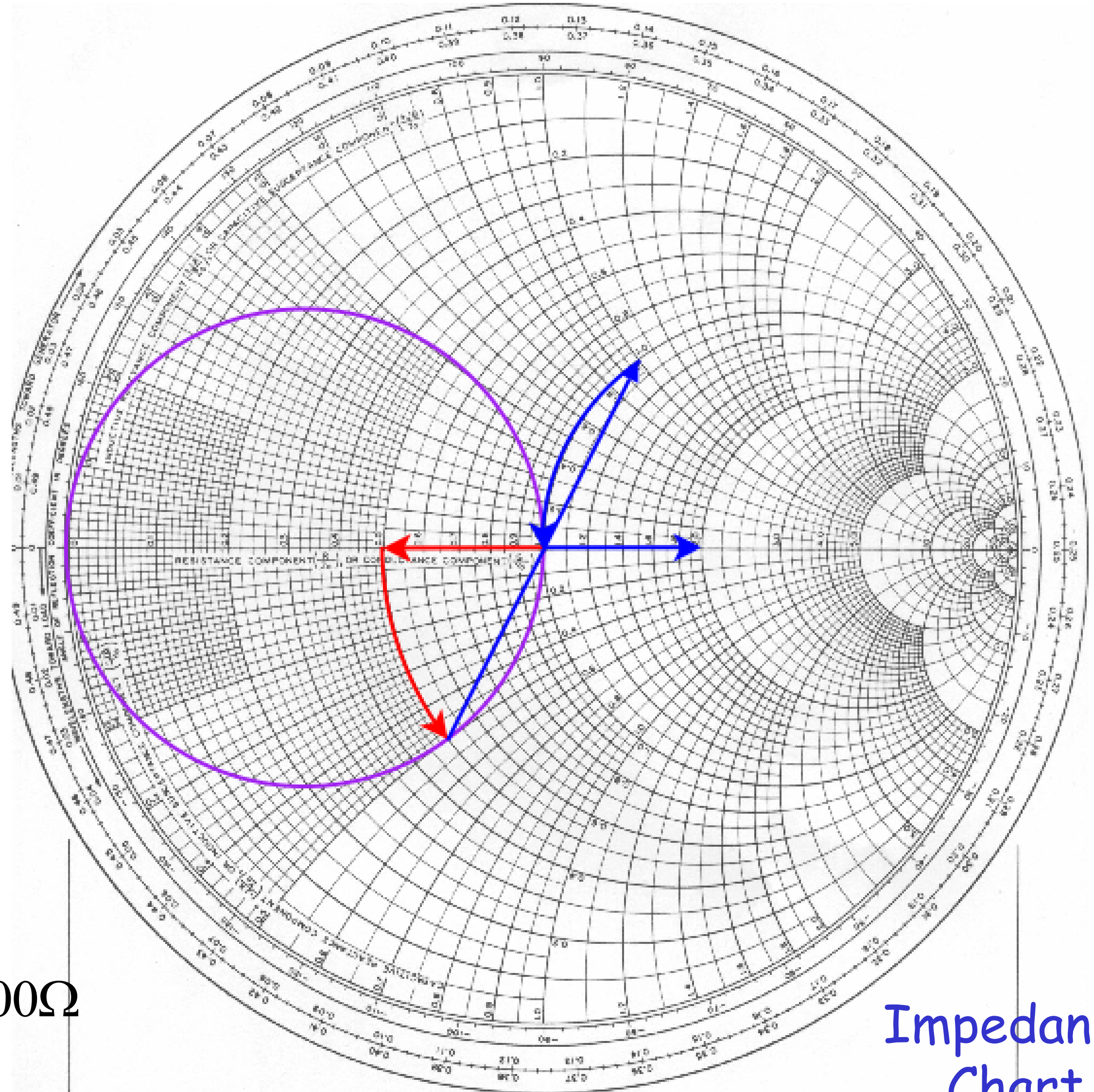




Impedance Chart

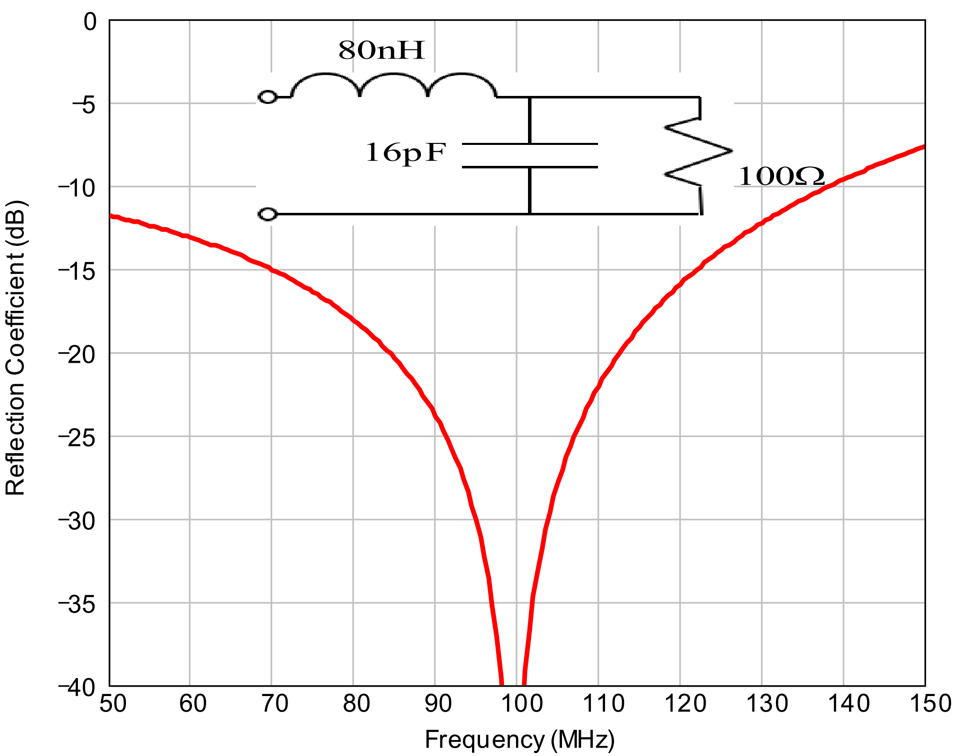
- This solution would have also worked

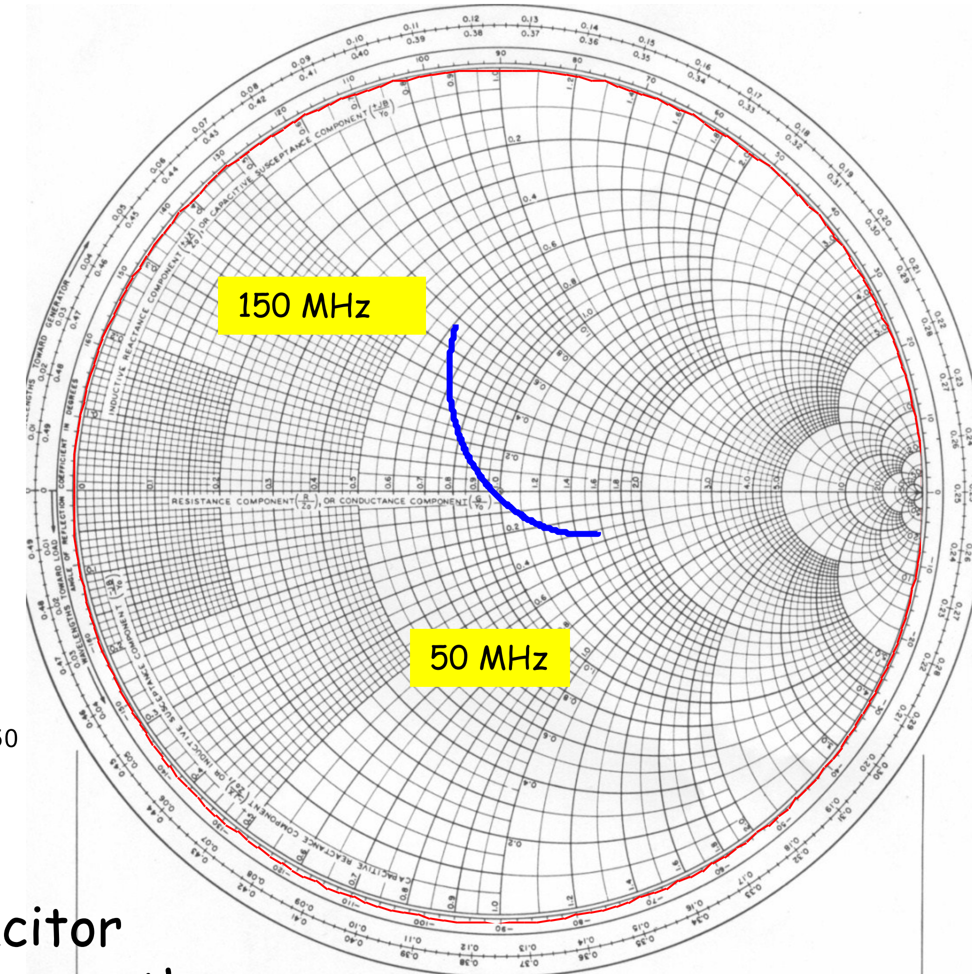




Impedance Chart

Matching Bandwidth



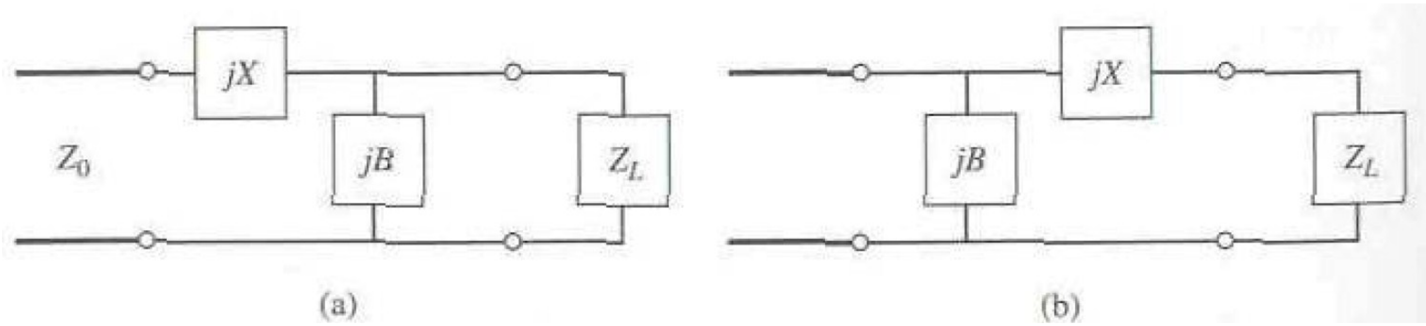


Because the inductor and capacitor impedances change with frequency, the match works over a narrow frequency range

Impedance Chart

Matching with lumped elements

The simplest type of matching network is therefore the L section which uses two reactive elements to match an arbitrary load impedance to a transmission line.



If $z_L=Z_L/Z_0$ is ***inside*** the 1+jx circle on the Smith chart the solution (a) is used.

If $z_L=Z_L/Z_0$ is ***outside*** the 1+jx circle on the Smith chart the solution (b) is used.

If the frequency is low enough (frequencies up to 1 GHz) lumped capacitance and inductance can be used.
There is, however, a large range of frequencies where this solution may be not realizable. This is the ***main limitation*** of the L matching section technique.

Analytical solution

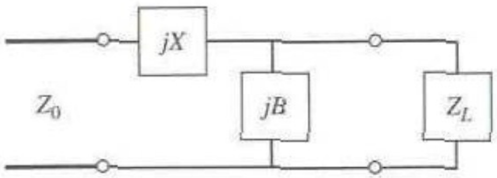
If $Z_L > Z_0$ (implies that z_L is inside the 1+jx circle on the Smith chart)

$$B = \frac{X_L \pm \sqrt{R_L / Z_0}\sqrt{R_L^2 + X_L^2 - R_L Z_0}}{R_L^2 + X_L^2}$$

$$X = \frac{1}{B} + \frac{X_L Z_0}{R_L} - \frac{Z_0}{R_L B}$$

Positive B=capacitor
Negative B=inductor

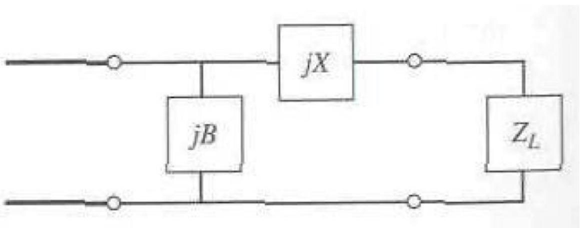
Positive X=inductor
Negative X=capacitor



If $Z_L < Z_0$ (implies that z_L is outside the 1+jx circle on the Smith chart)

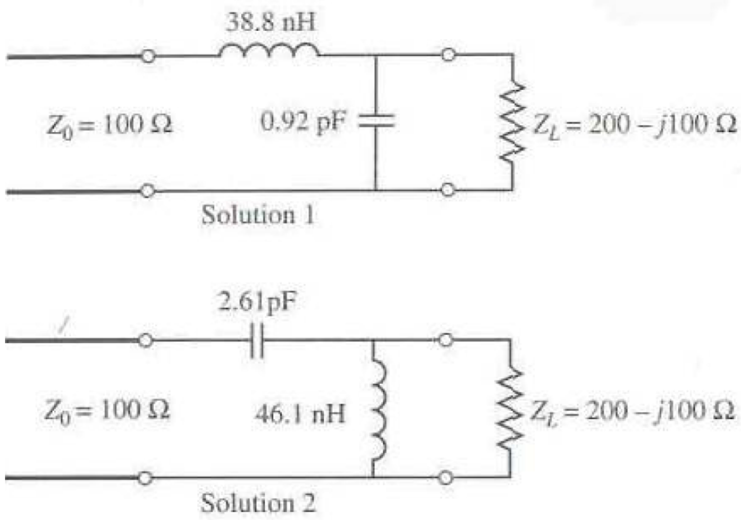
$$X = \pm\sqrt{R_L (Z_0 - R_L)} - X_L$$

$$B = \pm \frac{\sqrt{(Z_0 - R_L)/R_L}}{Z_0}$$



Two solutions, but…one may result in a significant smaller value for the reactive components and may be preferred due to the matching bandwidth or VSWR on the line.

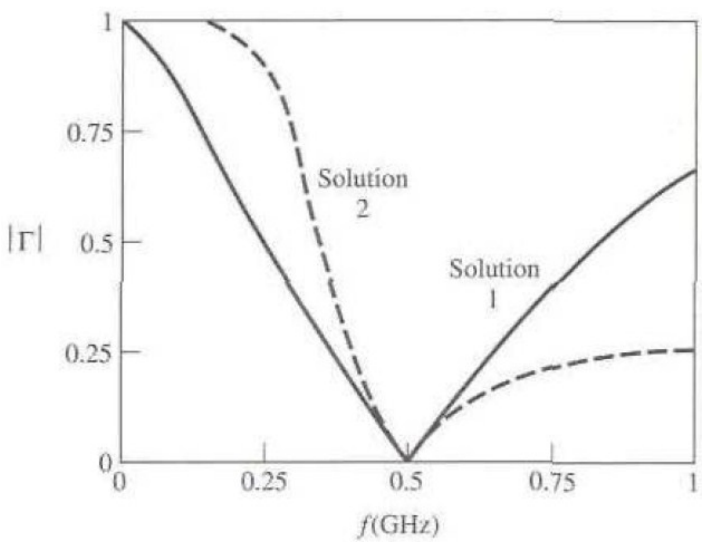
Example



Match a series RC load with impedance 200+j100Ω to a 100Ω line at 500 MHz

C_1=0.92 pF L_1=38.8μH

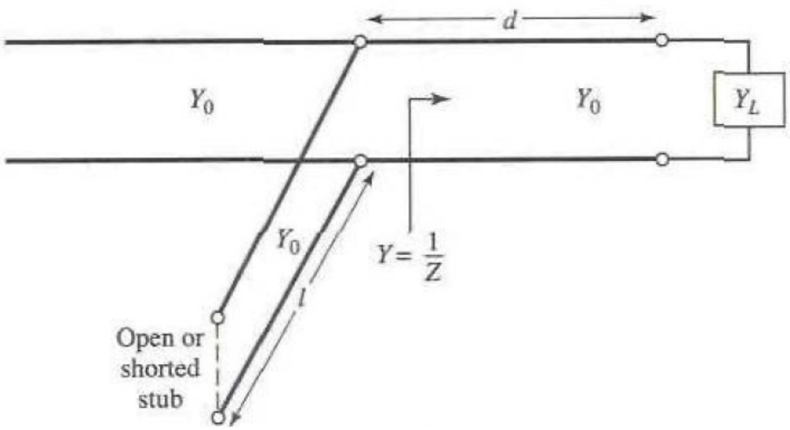
C_2=2.61 pF L_2=46.1μH

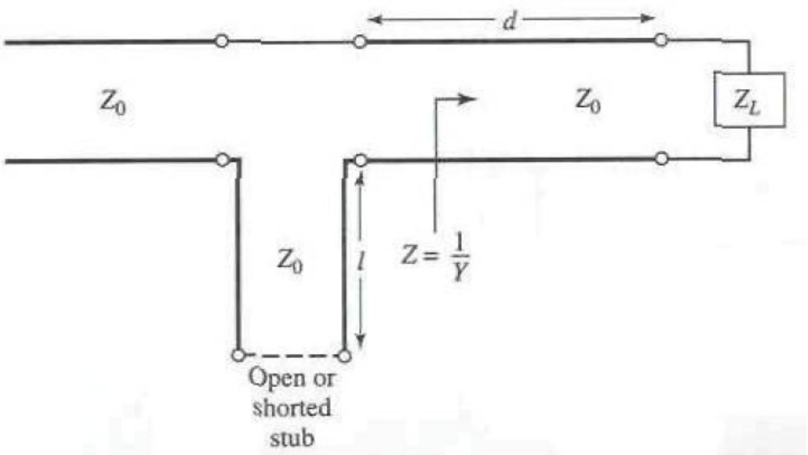


Solution 1 is slightly better than solution 2.

Single stub tuning

An important matching technique uses an open-circuited or a short-circuited transmission line (a "stub") either in parallel or in series with the transmission line at a certain distance from the load.





Two adjustable parameters: distance (d) from the load, susceptance or reactance provided by the shunt or series stub.

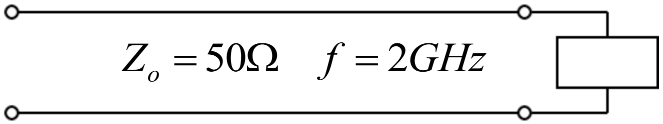
Shunt: select d so that the susceptance seen is Y_0+jB and add the stub with susceptance –jB.

Series: select d so that the impedance seen is Z_0+jX and add the stub with impedance –jX.

Tuning circuit very easy to implement

All together!

Given: $Z_L = 15\Omega + j10\Omega$



Assuming load consisting in a RL series matched at 2 Ghz, calculate the shunt stubs to have the match.

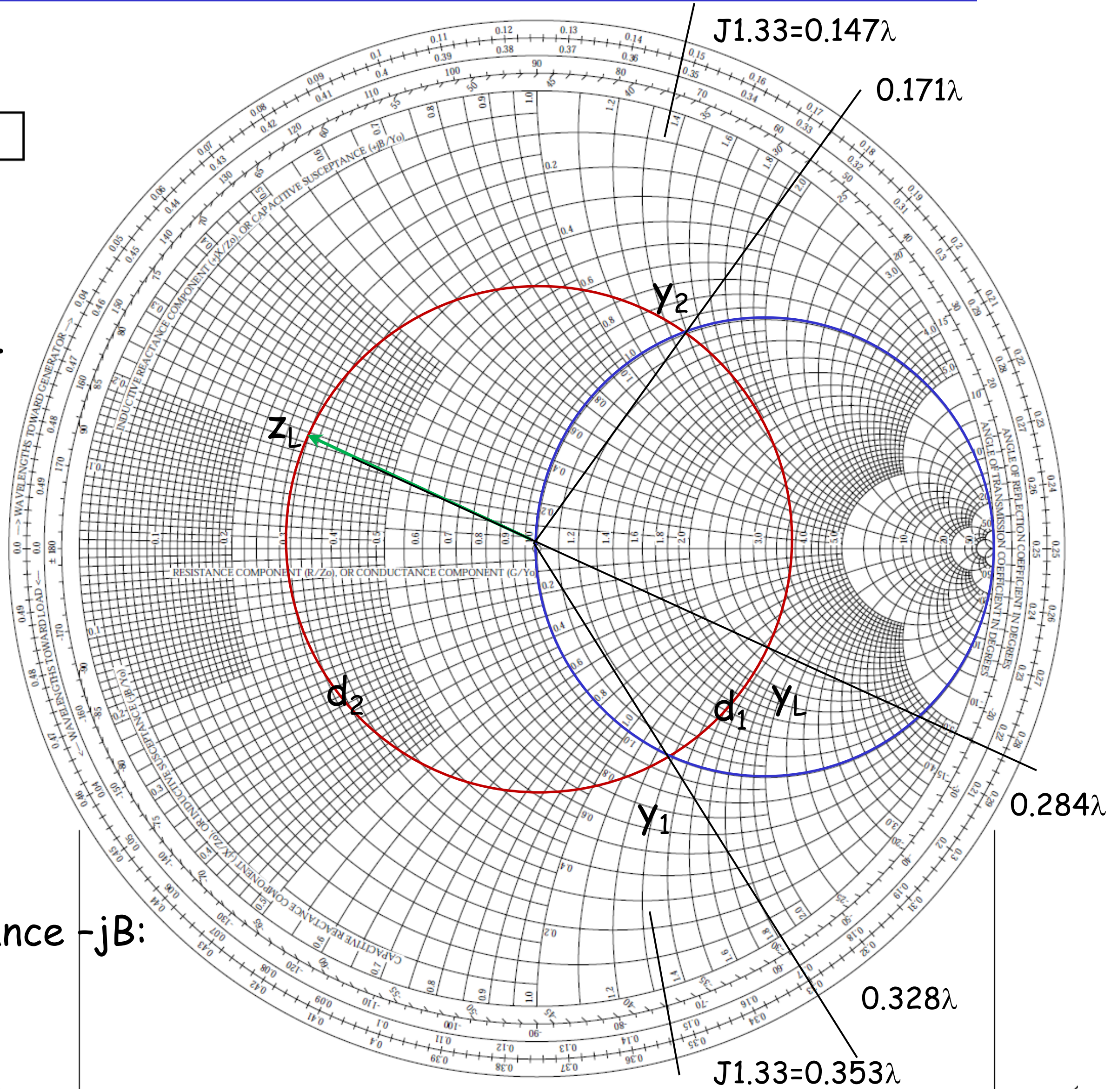
$$z_L = 0.3 + j0.2$$

$$y_1 = 1 - j1.33$$

$$y_2 = 1 + j1.33$$

$$d_1 = 0.328 - 0.284 = 0.044\lambda$$

$$d_2 = (0.5 - 0.284) + 0.171 = 0.387\lambda$$

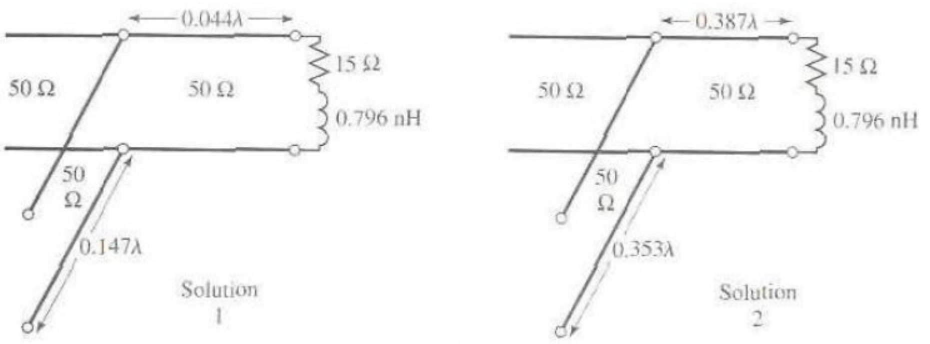


Add a lenght of stub with susceptance –jB:
Solution1) l=0.147λ
Solution2) l=0.353λ

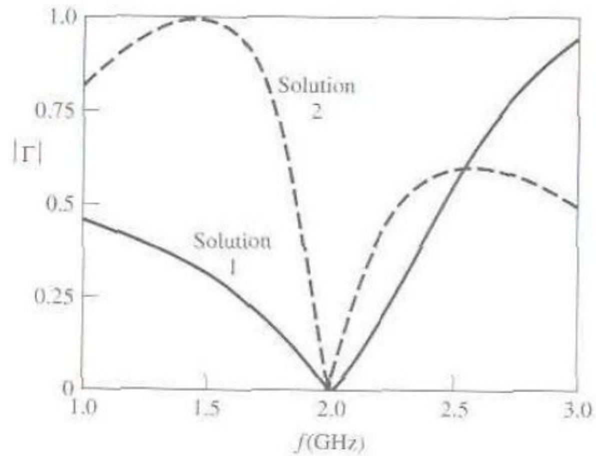
Frequency response

Two possibilities: it is normally desired to keep the matching stub as close as possible to the load to reduce the losses caused by a possibly large standing wave ratio on the line between the stub and the load.

Solution 1 has a significantly better bandwidht than solution 2.

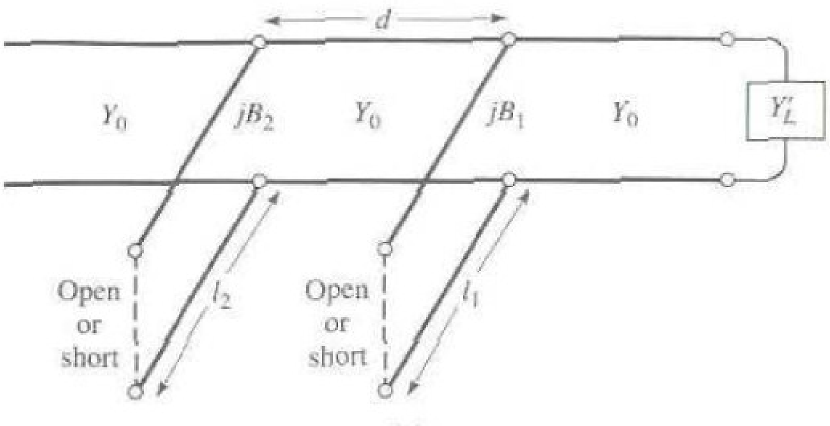


(b)



Double stub tuning

The single stub tuning is able to match any load impedance to a transmission line, but suffer of the disadvantage of requiring a varianle length of line between the load and the stub.



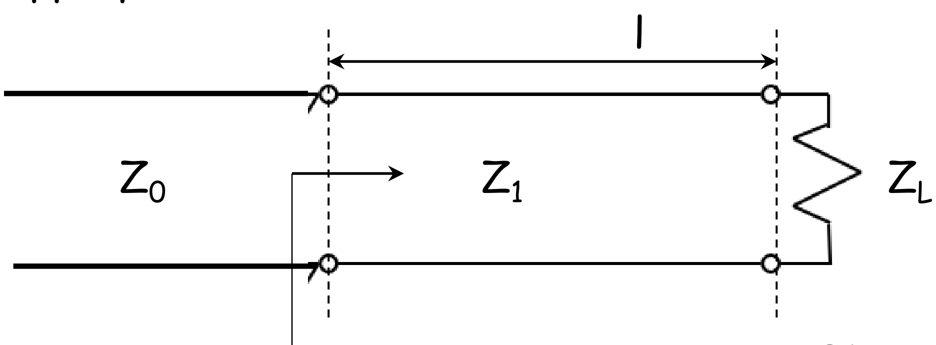
Design of matching with double stub tuner, or multiple stub tuner is far away from the scope of this course.

The Quarter wave transformer

Simple and useful method for matching a real load impedance to a transmission line. It can be extended to multisection design for a broader bandwidth.

Drawback: QWT can only match a real load impedance.

A complex load impedance can always be transformed to a real impedance by using an appropriate series or shunt reactive stub.



At f_0 $l=\lambda\backslash 4$ $\longrightarrow$ $\beta l = \frac{2\pi}{\lambda}\frac{\lambda}{4} = \frac{\pi}{2}$

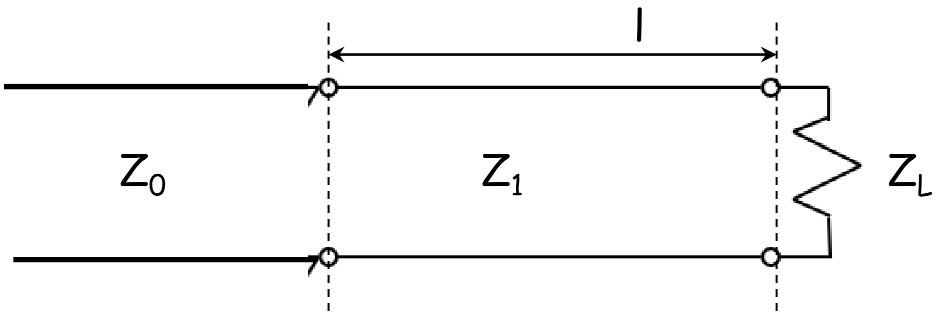
At $f_0+\Delta f$ $l\neq \lambda\backslash 4$

$$Z_{IN} = Z_1 \frac{Z_L + jZ_1 \tan \beta l}{Z_1 + jZ_L \tan \beta l} \longrightarrow Z_{IN} = \frac{Z_1^2}{Z_L}$$

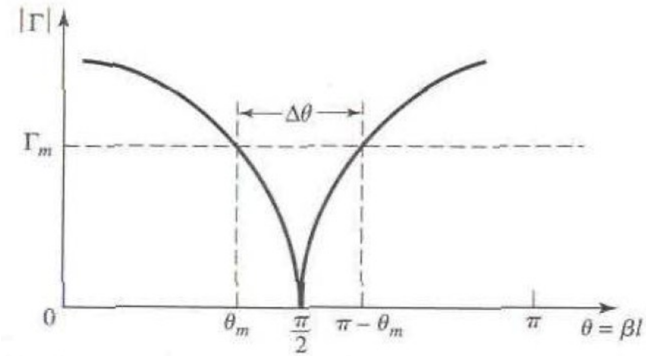
To get Γ=0 $\longrightarrow$ $Z_{IN} = Z_0$ $\longrightarrow$ $Z_1 = \sqrt{Z_0 \cdot Z_L}$

Geometric mean of load and source imp.

Mismatch vs. frequency



$$Z_1 = \sqrt{Z_0 \cdot Z_L}$$



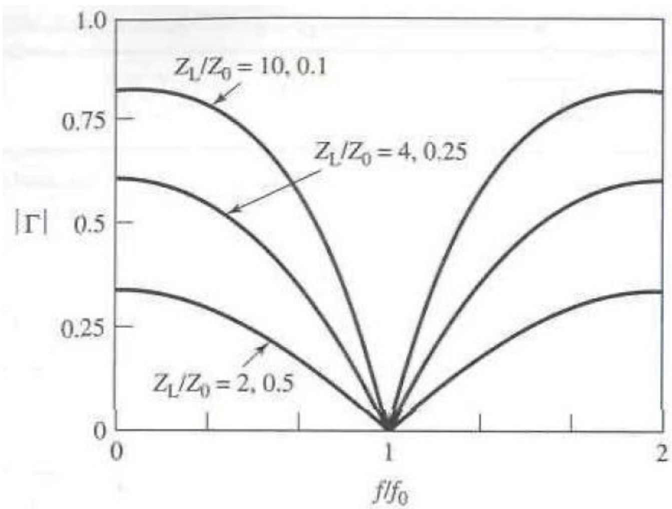
$$|\Gamma| \cong \frac{|Z_L - Z_0|}{2\sqrt{Z_0 Z_L}} \cdot |\cos\theta| \qquad with \qquad \theta = \frac{\pi}{2} \cdot \frac{f}{f_0}$$

For Θ near $\pi/2$

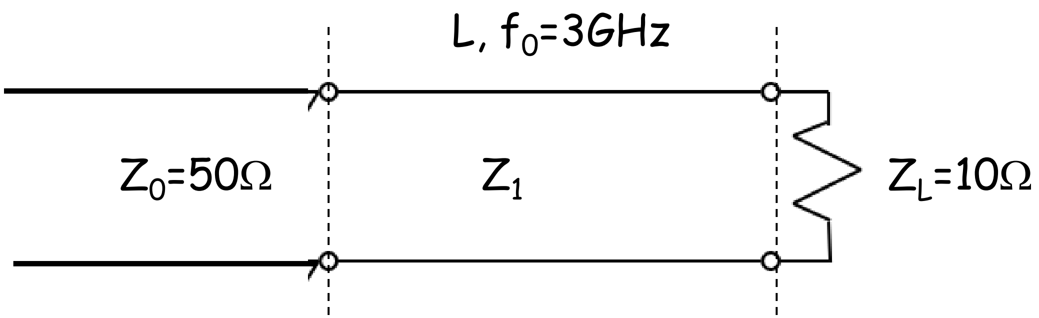
If we set the max value of Reflection Coefficient Γ_m that can be tolerated, the bandwidth of the matching transformer is:

$$f_m = \frac{2\theta_m f_0}{\pi}$$

$$\frac{\Delta f}{f_0} = \frac{2(f_0 - f_m)}{f_0} = 2 - \frac{4\theta_m}{\pi} = 2 - \frac{4}{\pi}\cos\left[\frac{\Gamma_m}{\sqrt{1-\Gamma_m^2}} \frac{2\sqrt{Z_0 Z_L}}{|Z_L - Z_0|}\right]$$



Example



Determine the percent bandwith for which the SWR<1.5

$$Z_1 = \sqrt{Z_0 \cdot Z_L} = 22.36\Omega$$

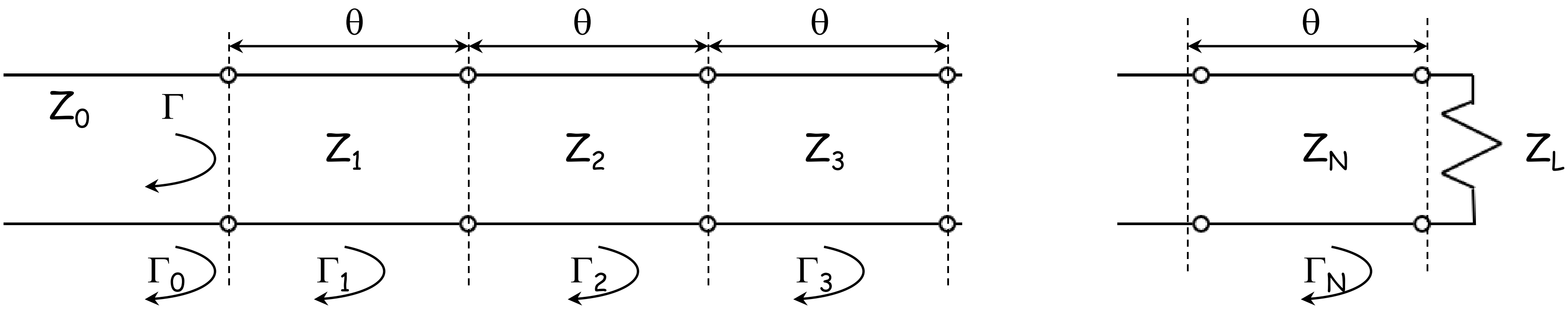
For this value of Z_1 L=λ/4 at f_0=3GHz. A SWR=1.5 correspond to:

$$\Gamma_m = \frac{SWR-1}{SWR+1} = 0.2$$

$$\frac{\Delta f}{f_0} = 2 - \frac{4}{\pi}\cos\left[\frac{\Gamma_m}{\sqrt{1-\Gamma_m^2}} \frac{2\sqrt{Z_0 Z_L}}{|Z_L - Z_0|}\right] = 0.29 = 29\%$$

$$\frac{\Delta f}{f_0} = 870 \quad MHz$$

Multisection transformer



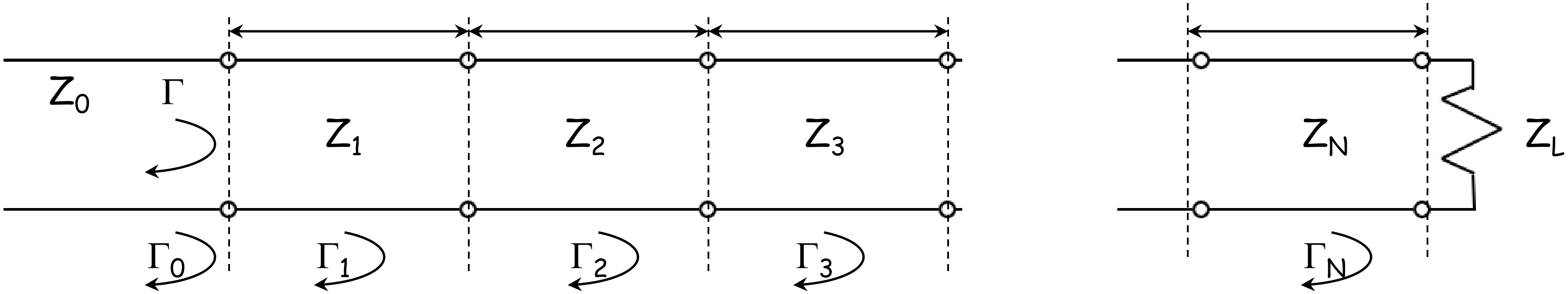
N equal sections of transmission lines
Z_L real
Z monotically decreasing: $Z_0 > Z_1 > Z_2 > Z_3 > \ldots > Z_N$

$$\Gamma(\theta) = \Gamma_0 + \Gamma_1 e^{-2j\theta} + \Gamma_2 e^{-4j\theta} \ldots + \Gamma_N e^{-2jN\theta}$$

We can synthesize any desired reflection coefficient response as a function of frequency (θ), by properly choosing the Γ_N and using enough sections.
Commonly used:

Binomial transformer (maximally flat design)
Chebyshev (equal ripple) transformer

Binomial Transformers



For a given number of sections, the passband response is as flat as possible near the design frequency (maximally flat design).

This kind of response is designed, for a N sections transformer, by setting the first N-1 derivatives of the reflection coefficient to zero.

$$\Gamma(\theta) = A(1+e^{-2j\theta})^N = A\sum_{n=0}^{N} C_n^N e^{-2jn\theta}$$

where:

$$A = 2^{-N}\left|\frac{Z_L - Z_0}{Z_L + Z_0}\right| \qquad C_n^N = \frac{N!}{(N-n)!n!}$$

Binomial Transformers Bandwidth

If we set the max value of Reflection Coefficient Γ_m that can be tolerated, the bandwidth of the matching transformer is:

$$\Gamma_m = 2^N \cdot |A| \cdot \cos^N \theta_m \quad \Rightarrow \quad \theta_m = a\cos\left[\frac{1}{2}\left(\frac{\Gamma_m}{A}\right)^{1/N}\right]$$

$$\frac{\Delta f}{f_0} = \frac{2(f_0 - f_m)}{f_0} = 2 - \frac{4\theta_m}{\pi} = 2 - \frac{4}{\pi}\left[\frac{1}{2}\left(\frac{\Gamma_m}{A}\right)^{1/N}\right]$$

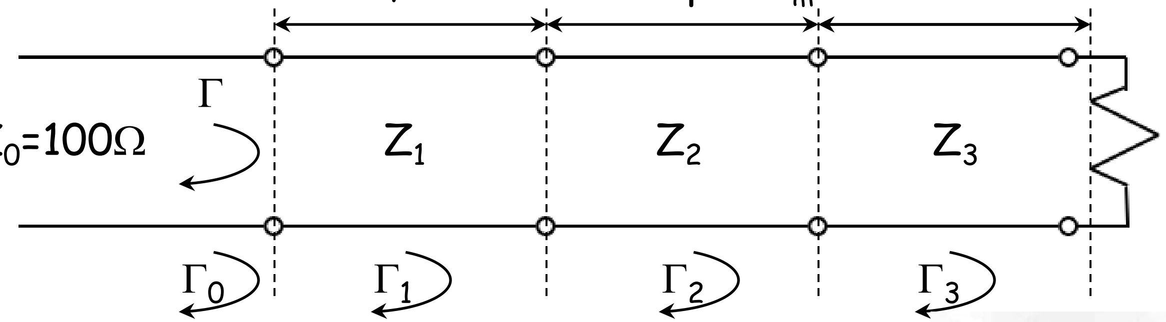
Tables

Z_L/Z_0	N = 2		N = 3			N = 4			
	Z_1/Z_0	Z_2/Z_0	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0	Z_4/Z_0
1.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1.5	1.1067	1.3554	1.0520	1.2247	1.4259	1.0257	1.1351	1.3215	1.4624
2.0	1.1892	1.6818	1.0907	1.4142	1.8337	1.0444	1.2421	1.6102	1.9150
3.0	1.3161	2.2795	1.1479	1.7321	2.6135	1.0718	1.4105	2.1269	2.7990
4.0	1.4142	2.8285	1.1907	2.0000	3.3594	1.0919	1.5442	2.5903	3.6633
6.0	1.5651	3.8336	1.2544	2.4495	4.7832	1.1215	1.7553	3.4182	5.3500
8.0	1.6818	4.7568	1.3022	2.8284	6.1434	1.1436	1.9232	4.1597	6.9955
10.0	1.7783	5.6233	1.3409	3.1623	7.4577	1.1613	2.0651	4.8424	8.6110

Z_L/Z_0	N = 5					N = 6					
	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0	Z_4/Z_0	Z_5/Z_0	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0	Z_4/Z_0	Z_5/Z_0	Z_6/Z_0
1.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1.5	1.0128	1.0790	1.2247	1.3902	1.4810	1.0064	1.0454	1.1496	1.3048	1.4349	1.4905
2.0	1.0220	1.1391	1.4142	1.7558	1.9569	1.0110	1.0790	1.2693	1.5757	1.8536	1.9782
3.0	1.0354	1.2300	1.7321	2.4390	2.8974	1.0176	1.1288	1.4599	2.0549	2.6577	2.9481
4.0	1.0452	1.2995	2.0000	3.0781	3.8270	1.0225	1.1661	1.6129	2.4800	3.4302	3.9120
6.0	1.0596	1.4055	2.4495	4.2689	5.6625	1.0296	1.2219	1.8573	3.2305	4.9104	5.8275
8.0	1.0703	1.4870	2.8284	5.3800	7.4745	1.0349	1.2640	2.0539	3.8950	6.3291	7.7302
10.0	1.0789	1.5541	3.1623	6.4346	9.2687	1.0392	1.2982	2.2215	4.5015	7.7030	9.6228

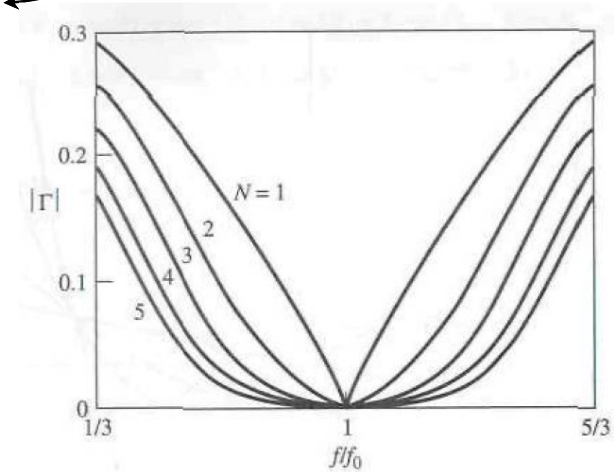
Example (analytical solution)

Design the 3 sections transformer, calculate BW per Γ_m=0.05



$$A = 2^{-N}\left|\frac{Z_L - Z_0}{Z_L + Z_0}\right| = 0.0417 \qquad \frac{\Delta f}{f_0} = 2 - \frac{4}{\pi} a\cos\left[\frac{1}{2}\left(\frac{\Gamma_m}{A}\right)^{1/N}\right] = 0.71$$

$$C_o^3 = \frac{3!}{2!0!} = 1 \qquad C_1^3 = \frac{3!}{2!1!} = 3 \qquad C_2^3 = \frac{3!}{2!1!} = 3$$



$$n=0; \quad \ln Z_1 = \ln Z_0 + 2^{-N} C_0^3 \ln\frac{Z_L}{Z_o} = \ln 100 + 2^{-3}(1)\ln\frac{50}{100} = 4.518; \qquad Z_1 = 91.7\Omega$$

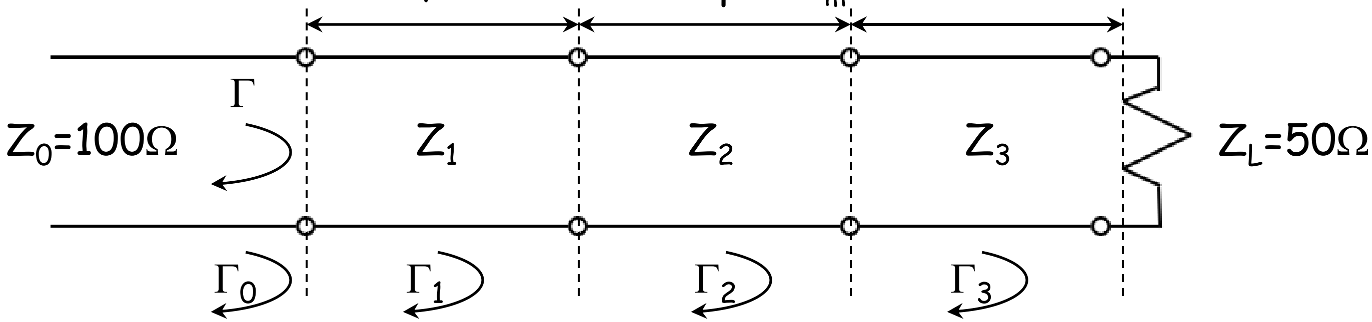
$$n=1; \quad \ln Z_2 = \ln Z_1 + 2^{-N} C_1^3 \ln\frac{Z_L}{Z_o} = \ln 01.7 + 2^{-3}(3)\ln\frac{50}{100} = 4.26; \qquad Z_2 = 70.7\Omega$$

$$n=2; \quad \ln Z_3 = \ln Z_2 + 2^{-N} C_0^3 \ln\frac{Z_L}{Z_o} = \ln 70.7 + 2^{-3}(1)\ln\frac{50}{100} = 4.00; \qquad Z_1 = 54.5\Omega$$

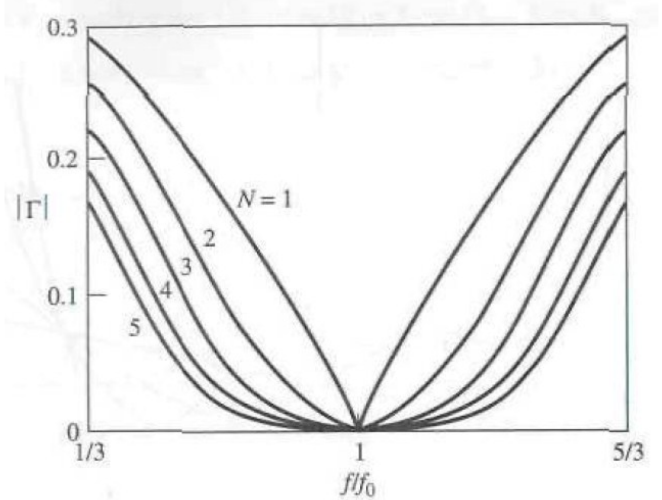
Greater BW!!!

Example with tables

Design the 3 sections transformer, calculate BW per Γ_m=0.05



	N = 2		N = 3		
Z_L/Z_0	Z_1/Z_0	Z_2/Z_0	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0
1.0	1.0000	1.0000	1.0000	1.0000	1.0000
1.5	1.1067	1.3554	1.0520	1.2247	1.4259
2.0	1.1892	1.6818	1.0907	1.4142	1.8337



$$Z_1 = \frac{Z_0}{1.0907} = 91.68\Omega$$

$$Z_2 = \frac{Z_0}{1.4142} = 70.71\Omega$$

$$Z_3 = \frac{Z_0}{1.8337} = 54.53\Omega$$

Greater BW!!!

Chebyschev Multisection matching transformers

In contrast with binomial transformers, the chebyschev ones optimize bandwidth at expenses of passband ripple.

Design procedure based on Chebyshev polynomials:

$$T_1(x) = x$$

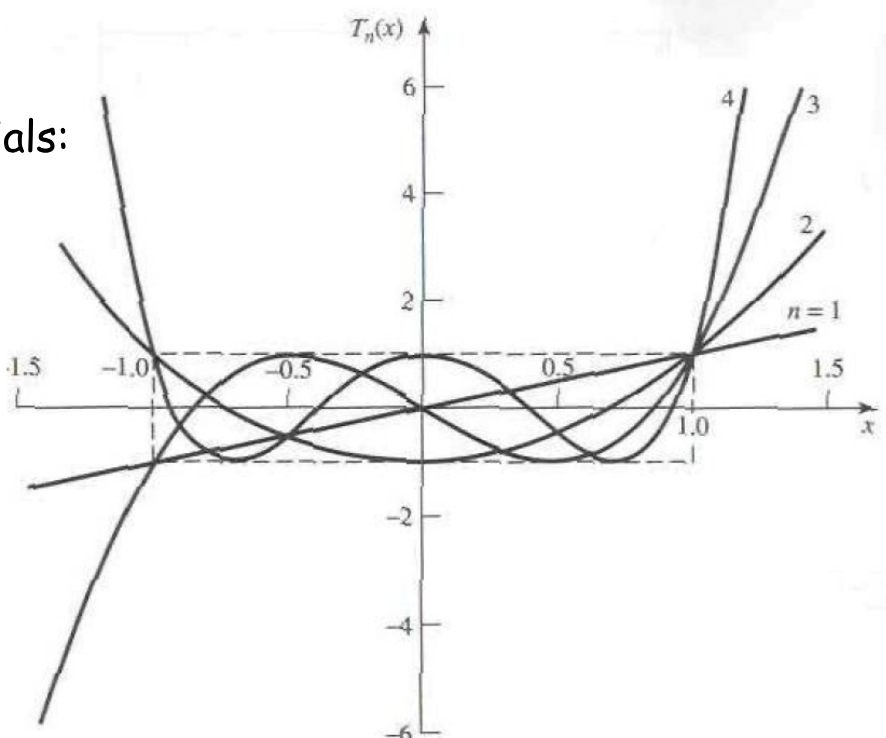
$$T_2(x) = 2x^2 - 1$$

$$T_3(x) = 4x^3 - 3x$$

$$T_4(x) = 8x^4 - 8x^2 + 1$$

....

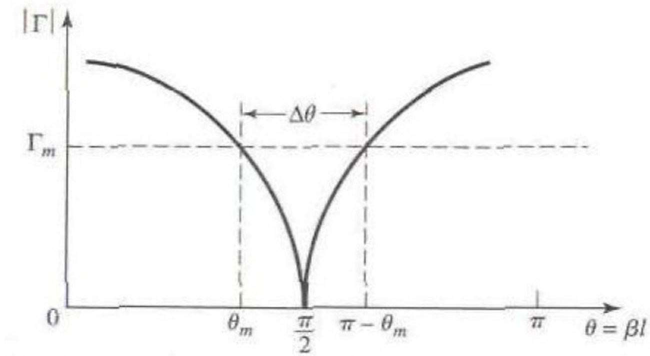
$$T_n(x) = 2xT_{n-1}(x) - T_{n-2}(x)$$

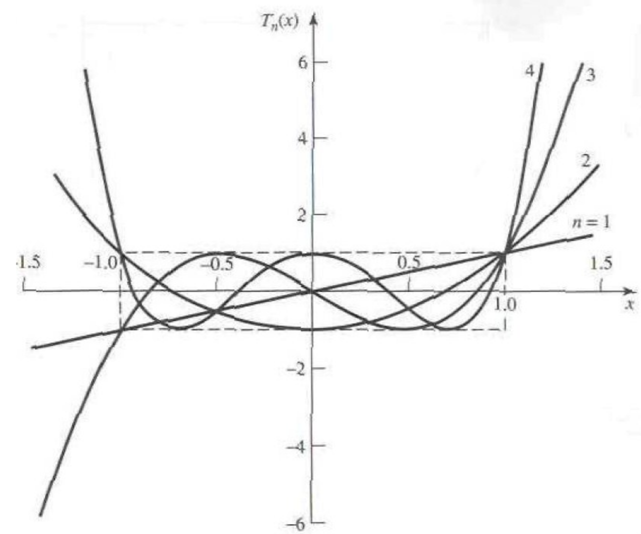


For -1<x<1, |Tn(x)|<1. In this range the polynomials oscillate between ±1. This is the equal ripple property and this region will be mapped to the passband of the matching transformer.

For |x|>1, |Tn(x)|>1. |Tn(x)| increases faster as n increases. This region will map to the frequency range outside the passband.

Chebyschev Multisection matching transformers





We need to map: θ_m to x=-1 and $\pi - \theta_m$ to x=1

$$T_1(\sec\theta_m \cos\theta) = \sec\theta_m \cos\theta$$

$$T_2(\sec\theta_m \cos\theta) = \sec^2\theta_m (1+\cos 2\theta) - 1$$

$$T_3(\sec\theta_m \cos\theta) = \sec^3\theta_m (\cos 3\theta + 3\cos\theta) - 3\sec\theta_m \cos\theta$$

$$T_4(\sec\theta_m \cos\theta) = \sec^4\theta_m (\cos 4\theta + 4\cos 2\theta + 3) - 4\sec^2\theta(\cos 2\theta + 1) + 1$$

....

$$T_n(x) = 2xT_{n-1}(x) - T_{n-2}(x)$$

Often used for D.C., filters…

Chebyschev Multisection matching transformers

$$\Gamma(\theta) = A e^{-Nj\theta} T_N(\sec\theta_m \cos\theta) \qquad where \qquad A = \frac{Z_L - Z_0}{Z_L + Z_0} \cdot \frac{1}{T_N(\sec\theta_m)}$$

Now if Γ_m is tha maximum allowable reflection coefficient in the passband then A=Γ_m, since the maximum value in the passband of $T_N(\sec\theta_m \cos\theta)$ is the unity.
Then θ_m and fractional bandwidth are given by:

$$\sec\theta_m = \cosh\left[\frac{1}{N} a\cosh\left(\frac{1}{\Gamma_m}\left|\frac{Z_L - Z_0}{Z_L + Z_0}\right|\right)\right]$$

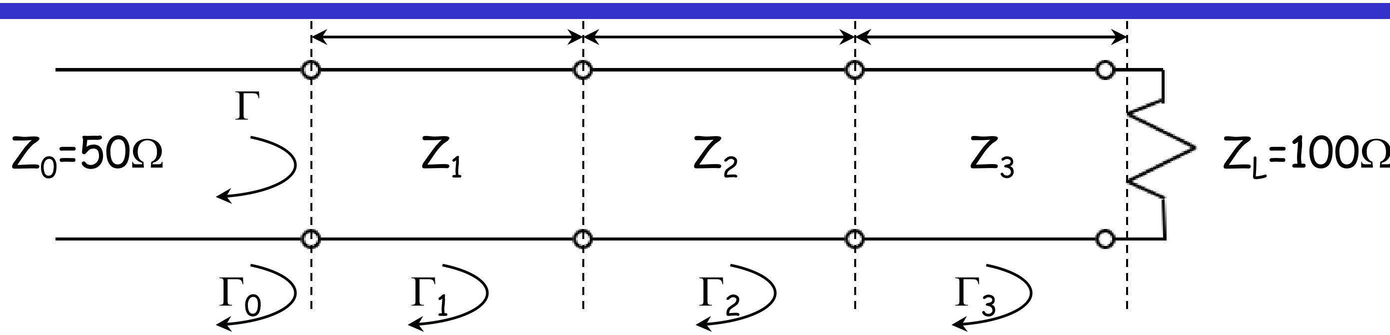
$$\frac{\Delta f}{f_0} = 2 - \frac{4\theta_m}{\pi}$$

Tables

Z_L/Z_0	$N=2$, $\Gamma_m=0.05$		$N=2$, $\Gamma_m=0.20$		$N=3$, $\Gamma_m=0.05$			$N=3$, $\Gamma_m=0.20$		
	Z_1/Z_0	Z_2/Z_0	Z_1/Z_0	Z_2/Z_0	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0
1.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1.5	1.1347	1.3219	1.2247	1.2247	1.1029	1.2247	1.3601	1.2247	1.2247	1.2247
2.0	1.2193	1.6402	1.3161	1.5197	1.1475	1.4142	1.7429	1.2855	1.4142	1.5558
3.0	1.3494	2.2232	1.4565	2.0598	1.2171	1.7321	2.4649	1.3743	1.7321	2.1829
4.0	1.4500	2.7585	1.5651	2.5558	1.2662	2.0000	3.1591	1.4333	2.0000	2.7908
6.0	1.6047	3.7389	1.7321	3.4641	1.3383	2.4495	4.4833	1.5193	2.4495	3.9492
8.0	1.7244	4.6393	1.8612	4.2983	1.3944	2.8284	5.7372	1.5766	2.8284	5.0742
10.0	1.8233	5.4845	1.9680	5.0813	1.4385	3.1623	6.9517	1.6415	3.1623	6.0920

Z_L/Z_0	$N=4$, $\Gamma_m=0.05$				$N=4$, $\Gamma_m=0.20$			
	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0	Z_4/Z_0	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0	Z_4/Z_0
1.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1.5	1.0892	1.1742	1.2775	1.3772	1.2247	1.2247	1.2247	1.2247
2.0	1.1201	1.2979	1.5409	1.7855	1.2727	1.3634	1.4669	1.5715
3.0	1.1586	1.4876	2.0167	2.5893	1.4879	1.5819	1.8965	2.0163
4.0	1.1906	1.6414	2.4369	3.3597	1.3692	1.7490	2.2870	2.9214
6.0	1.2290	1.8773	3.1961	4.8820	1.4415	2.0231	2.9657	4.1623
8.0	1.2583	2.0657	3.8728	6.3578	1.4914	2.2428	3.5670	5.3641
10.0	1.2832	2.2268	4.4907	7.7930	1.5163	2.4210	4.1305	6.5950

Example with tables



Design a transformer with maximum reflection coefficient equal to 0.05 ($A=\Gamma_m=0.05$).

	$N = 2$				$N = 3$			
	$\Gamma_m = 0.05$		$\Gamma_m = 0.20$		$\Gamma_m = 0.05$			Γ_m
Z_L/Z_0	Z_1/Z_0	Z_2/Z_0	Z_1/Z_0	Z_2/Z_0	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0	Z_1/Z_0
1.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1.5	1.1347	1.3219	1.2247	1.2247	1.1029	1.2247	1.3601	[illegible]
2.0	1.2193	1.6402	1.3161	1.5197	1.1475	1.4142	1.7429	[illegible]

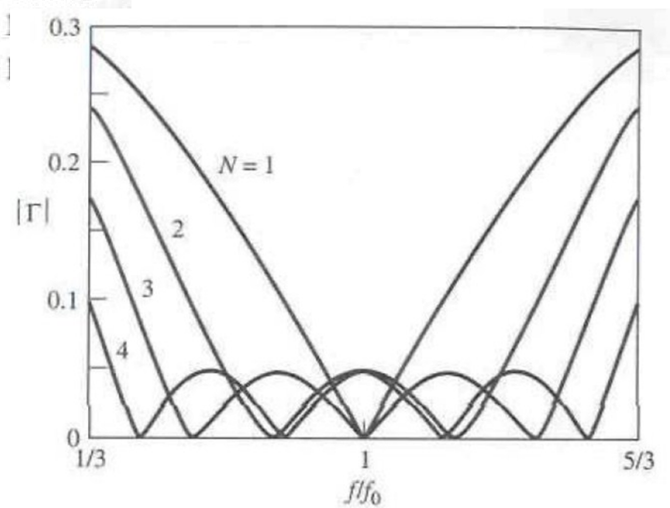
$$Z_1 = Z_0 \cdot 1.1475 = 57.37\Omega$$

$$Z_2 = Z_0 \cdot 1.4142 = 70.71\Omega$$

$$Z_3 = Z_0 \cdot 1.7429 = 87.15\Omega$$

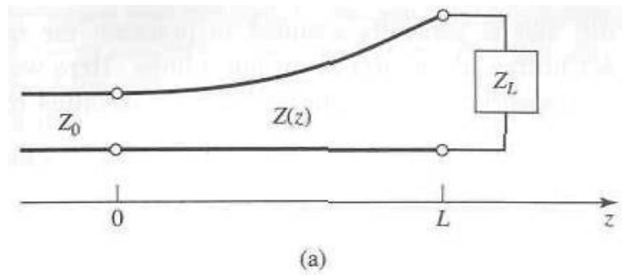
$$\sec\theta_m = \cosh\left[\frac{1}{N} a\cosh\left(\frac{1}{\Gamma_m}\left|\frac{Z_L - Z_0}{Z_L + Z_0}\right|\right)\right] = 1.395 \Rightarrow \theta_m = 44.2°$$

$$\frac{\Delta f}{f_0} = 2 - \frac{4\theta_m}{\pi} = 1.02 = 102\%$$

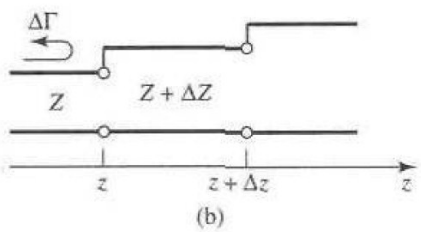


Tapered Lines

As the number of the discrete sections increases, the step changes in the characteristic impedance become smaller. Thus, in the limit of an infinite number of sections, we approach a continuosly tapered line.



By changing the type of taper, we can obtain different passband characteristics.



$$\Delta z \quad \rightarrow \quad \Delta Z(z)$$

$$\Gamma(\theta) = \frac{1}{2}\int_0^L e^{-j2\beta z}\frac{d}{dz}\ln\left(\frac{Z}{Z_0}\right)dz$$

Three special cases analyzed in the following:

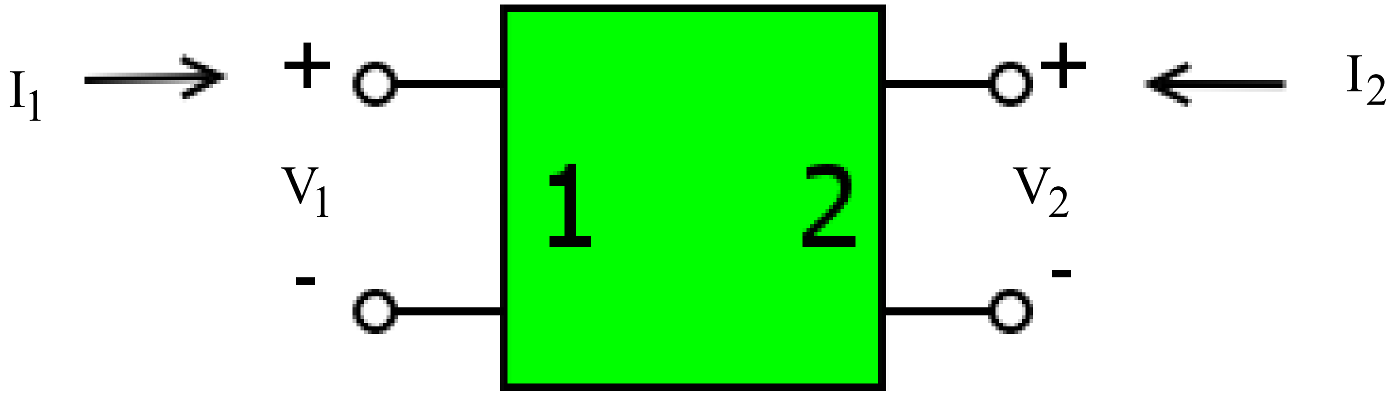
- Exponential taper
- Triangular taper
- Klopfenstein taper

For a given taper lenght the klpofstein impedance taper is the optimum in the sense that the reflection coefficient is minimum over the passband.

It is derived from a stepped Chebyshev transformer as the number of sections increas to infinity.

Impedance (Z) and Admittance (Y) Matrices

This linear two port "black box" can be fully described if we know how the voltage and the current at port 1 relate to the voltage and the current at port 2.



$$V_1 = z_{11}I_1 + z_{12}I_2$$

$$V_2 = z_{21}I_1 + z_{22}I_2$$

$$[V] = [Z][I]$$

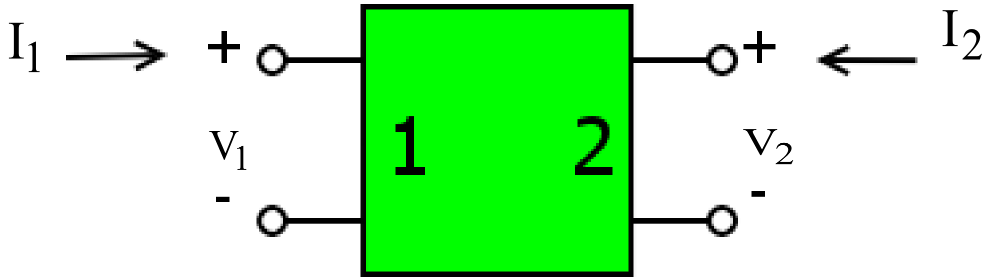
$$I_1 = y_{11}V_1 + y_{12}V_2$$

$$I_2 = y_{21}V_1 + z_{22}V_2$$

$$[I] = [Y][V]$$

zij and yij characterize the properties of the black box.

Impedance (Z) and Admittance (Y) Matrices



We need 4 measurements to determine [Y] or [Z]:

$$V_1 = z_{11}I_1 + z_{12}I_2 \qquad I_1 = y_{11}V_1 + y_{12}V_2$$

$$V_2 = z_{21}I_1 + z_{22}I_2 \qquad I_2 = y_{21}V_1 + z_{22}V_2$$

Open output (I_2=0)

$$z_{11} = \frac{V_1}{I_1} \qquad z_{21} = \frac{V_2}{I_1}$$

Short output (V_2=0)

$$y_{11} = \frac{I_1}{V_1} \qquad y_{21} = \frac{I_2}{V_1}$$

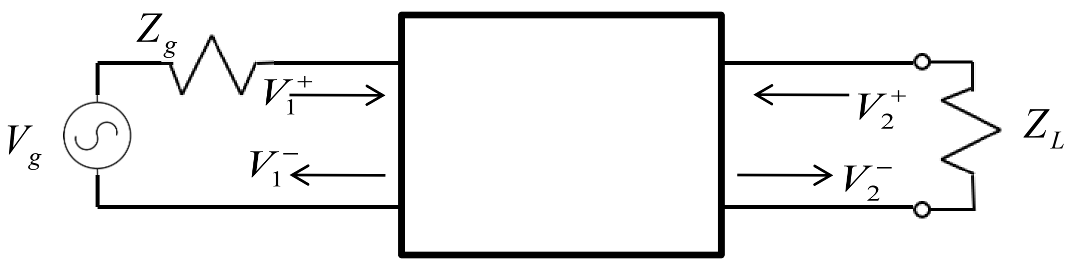
Open input (I_1=0)

$$z_{12} = \frac{V_1}{I_2} \qquad z_{22} = \frac{V_2}{I_2}$$

Short input (V_1=0)

$$y_{12} = \frac{I_1}{V_2} \qquad z_{22} = \frac{I_2}{V_2}$$

S matrix



$$V_1^- = s_{11}V_1^+ + s_{12}V_2^+$$
$$V_2^- = s_{21}V_1^+ + s_{22}V_2^+$$

It is helpful to think about travelling waves along a TRL!

$$s_{ij} = \frac{V_i^-}{V_j^+}$$

Means we are sitting and measuring the receding voltage wave at port I, while the driving source is at port j.

$$s_{11} = \frac{V_1^-}{V_1^+} \quad when \quad V_2^+ = 0$$

the input reflection coefficient

$$s_{21} = \frac{V_2^-}{V_1^+} \quad when \quad V_2^+ = 0$$

the input transmission coefficient

$$s_{12} = \frac{V_1^-}{V_2^+} \quad when \quad V_1^+ = 0$$

the reverse transmission coefficient

$$s_{22} = \frac{V_2^-}{V_2^+} \quad when \quad V_1^+ = 0$$

the reverse reflection coefficient

Normalized Scattering (S) Parameters

Since measuring voltages and currents at microwave frequencies becomes unpratical, the S- parameters are related to the incident and reflected power.

The S matrix defined previously is called the un-normalized scattering matrix. For convenience, define normalized waves:

$$a_i = \frac{V_i^+}{\sqrt{2Z_{o_i}}} \qquad b_i = \frac{V_i^-}{\sqrt{2Z_{o_i}}}$$

Where Z_{oi} is the characteristic impedance of the transmission line connecting port (i)

$|a_i|^2$ is the forward power into port (i)

$|b_i|^2$ is the reverse power from port (i)

Normalized Scattering (S) Parameters

The normalized scattering matrix is:

$$b_1 = s_{11}a_1 + s_{12}a_2$$
$$b_2 = s_{21}a_1 + s_{22}a_2$$

Where:

$$s_{i,j} = \sqrt{\frac{Z_{o_j}}{Z_{o_i}}} S_{i,j}$$

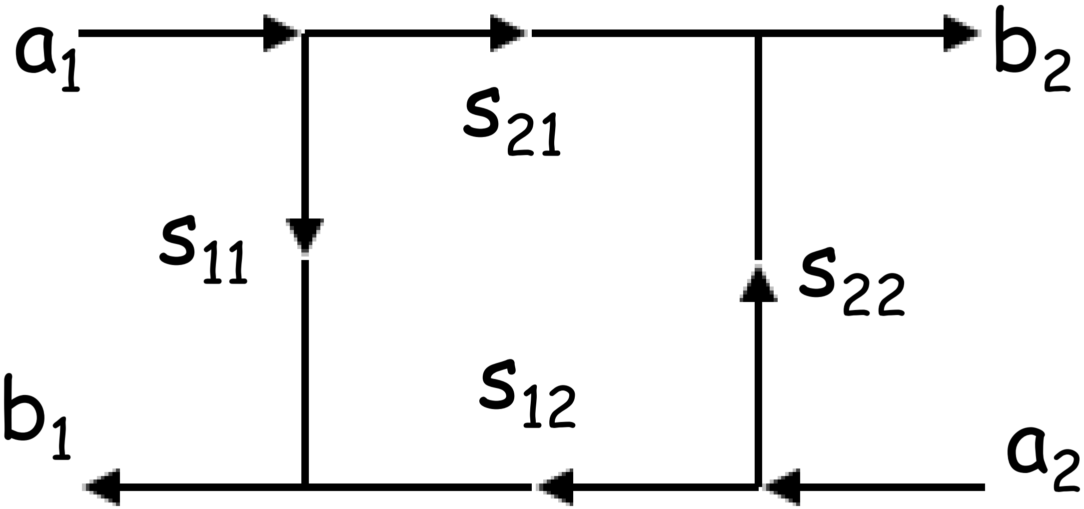
Energy conservation:
for a loss less network
SUM(Sij)=1

If the characteristic impedance on both ports is the same then the normalized and un-normalized S parameters are the same.

Normalized S parameters are the most commonly used.

Normalized S Parameters

The **s** parameters can be drawn pictorially



s_{11} and s_{22} can be thought of as reflection coefficients

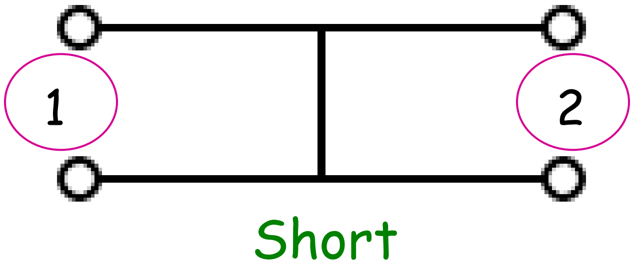
s_{21} and s_{12} can be thought of as transmission coefficients

s parameters are complex numbers where the angle corresponds to a phase shift between the forward and reverse waves

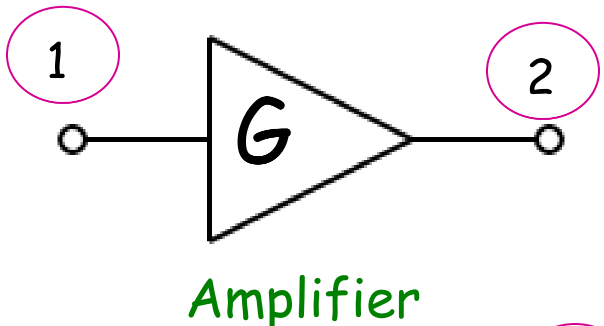
Why S-parameters?

- S-parameters are a powerful way to describe an electrical network
- The ease with which scattering parameters can be measured makes them especially well suited to describe transistor or other active devices.
- S-parameters change with freqency / load impedance / source impedance / network
- The most important advantage of S-parameters stems from the fact that travelling waves, unlike terminal voltage and currents, ***do not vary in magnitude*** along a transmission line.
- S_{11} is the *reflection coefficient*
- S_{21} describes the *forward transmission coefficient* (responding port 1st!)
- S-parameters have both magnitude and phase information
- Sometimes the gain (or loss) is more important than the phase shift and the phase information may be ignored
- S-parameters may describe large and complex networks

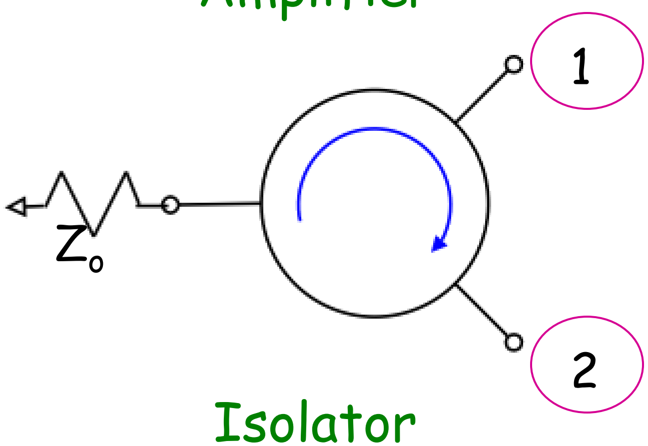
Examples of S parameters



$$[s]=\begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}$$



$$[s]=\begin{bmatrix}0 & 0\\ G & 0\end{bmatrix}$$



$$[s]=\begin{bmatrix}0 & 0\\ 1 & 0\end{bmatrix}$$

Lorentz Reciprocity

If the device is made out of linear isotropic materials (resistors, capacitors, inductors, metal, etc..) then:

$$[s]^T = [s]$$

or

$$s_{j,i} = s_{i,j} \qquad \text{for} \quad i \neq j$$

This is equivalent to saying that the transmitting pattern of an antenna is the same as the receiving pattern

reciprocal devices:	transmission line short directional coupler
non-reciprocal devices:	amplifier isolator circulator

Lossless Devices

The s matrix of a lossless device is unitary:

$$\left[s^*\right]^T [s] = [1]$$

$$1 = \sum_i \left|s_{i,j}\right|^2 \qquad \text{for all } j$$

$$1 = \sum_j \left|s_{i,j}\right|^2 \qquad \text{for all } i$$

Lossless devices:
- transmission line
- short
- circulator

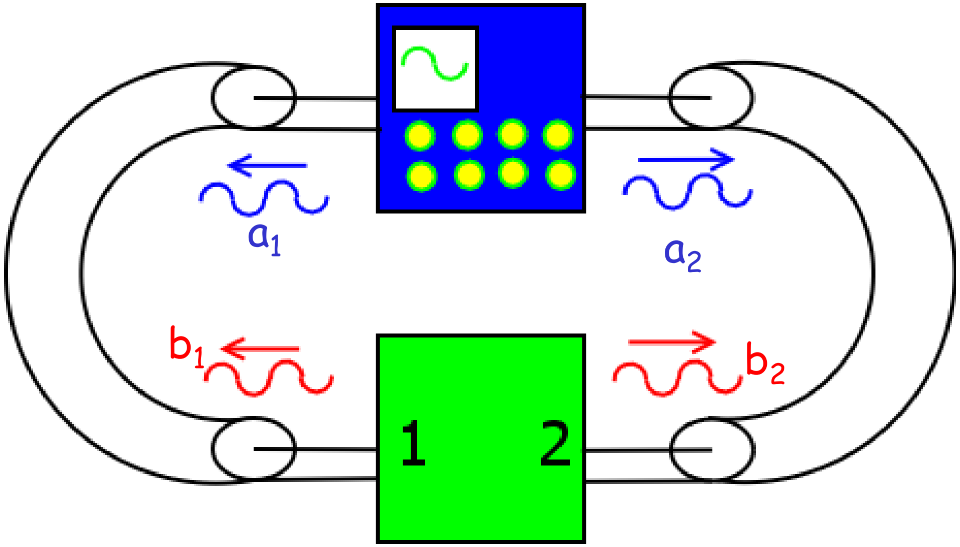
Non-lossless devices:
- amplifier
- isolator

Network Analyzers

- Network analyzers measure S parameters as a function of frequency
- At a **single** frequency, network analyzers send out forward waves a_1 and a_2 and measure the phase and amplitude of the reflected waves b_1 and b_2 with respect to the forward waves.

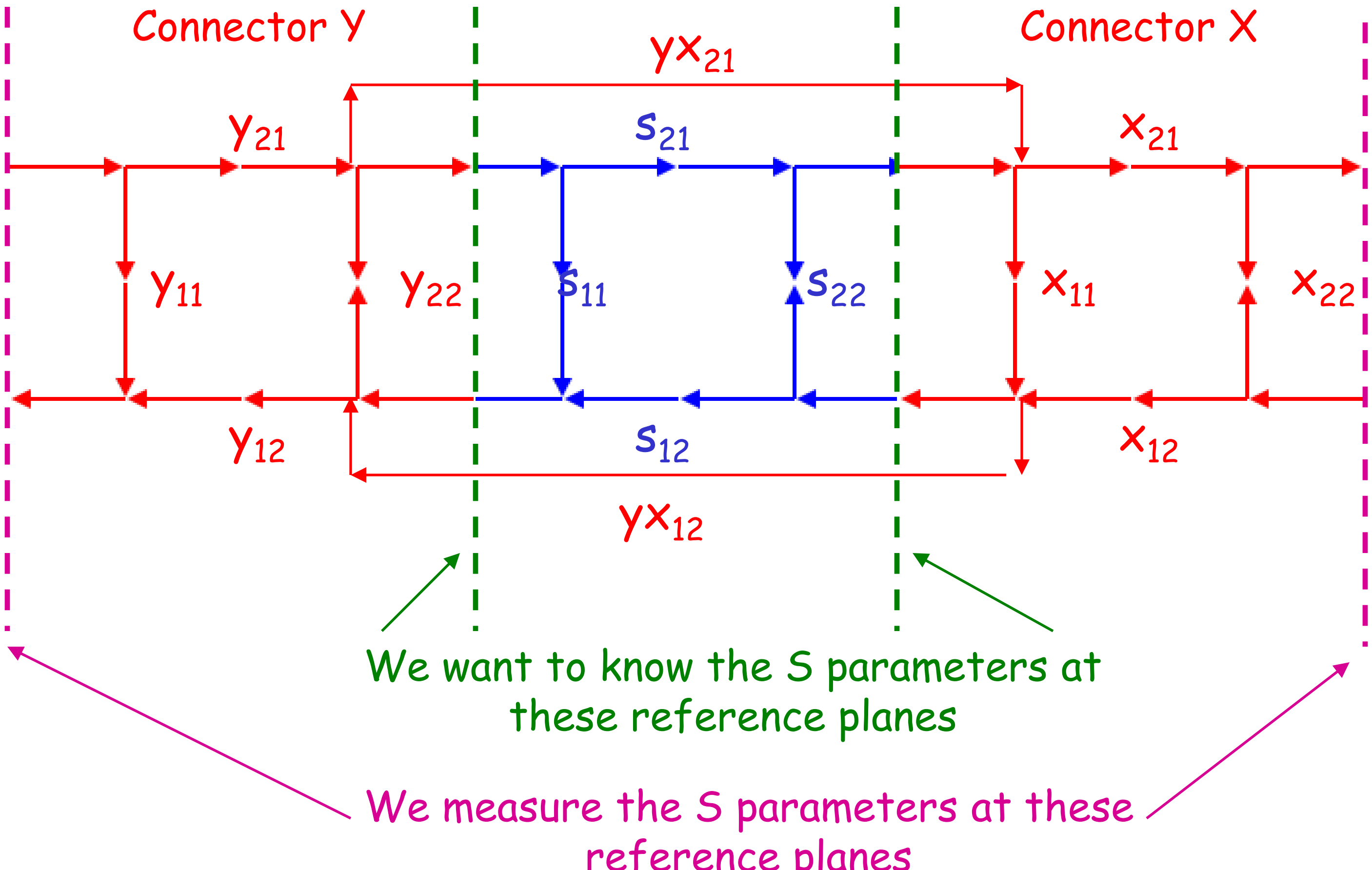
$$s_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0} \qquad s_{21} = \left.\frac{b_2}{a_1}\right|_{a_2=0}$$

$$s_{12} = \left.\frac{b_1}{a_2}\right|_{a_1=0} \qquad s_{22} = \left.\frac{b_2}{a_2}\right|_{a_1=0}$$



Network Analyzer Calibration

To measure the pure S parameters of a device, we need to eliminate the effects of cables, connectors, etc. attaching the device to the network analyzer



Network Analyzer Calibration

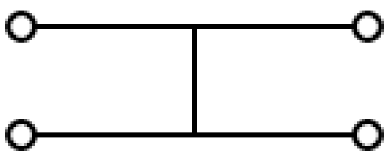
- There are 10 unknowns in the connectors
- We need 10 independent measurements to eliminate these unknowns
  - Develop calibration standards
  - Place the standards in place of the Device Under Test (DUT) and measure the S- parameters of the standards and the connectors
  - Because the S parameters of the calibration standards are known (theoretically), the S parameters of the connectors can be determined and can be mathematically eliminated once the DUT is placed back in the measuring fixtures.

Network Analyzer Calibration

- Since we measure four S parameters for each calibration standard, we need at least three independent standards.
- One possible set is:

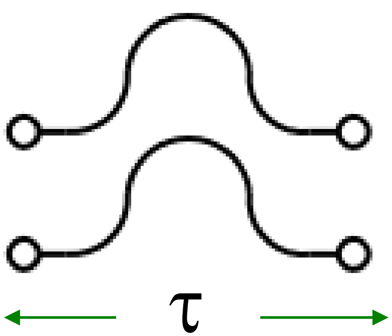
Thru

$$[s]=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$$



Short

$$[s]=\begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}$$



Delay*

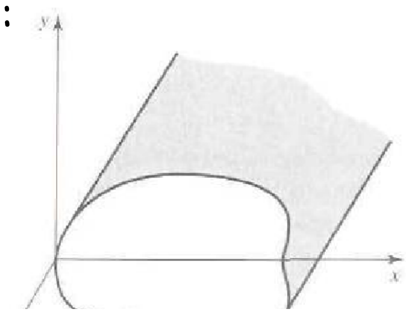
*$\omega\tau$~90degrees

$$[s]=\begin{bmatrix}0 & e^{-j\omega\tau}\\ e^{-j\omega\tau} & 0\end{bmatrix}$$

Waveguides

The mathematical solution to the waveguide problem is tedious. In order to find the various field components in a rectangular or circular guide six differential equations have to be solved.

Assuming that: a)waveguide region is source-free, b)z direction propagation (inifitely long structure), c) time harmonic fields with exp(jwt) dependence, d) perfectly conducting wall, the Maxwell equation can be rewritten as:

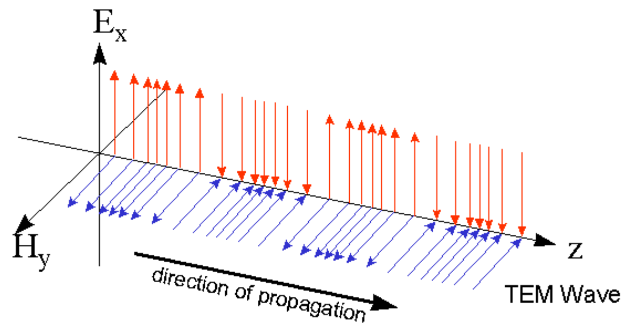


$$\nabla \times E = -j\omega\mu H \qquad E(x,y,z) = [e(x,y) + \hat{z}\, e_z(x,y)] \cdot e^{-j\beta z}$$

$$\nabla \times H = j\omega\varepsilon E \qquad H(x,y,z) = [h(x,y) + \hat{z}\, h_z(x,y)] \cdot e^{-j\beta z}$$

TEM waves **Ez=Hz=0**

TEM wave has no cutoff and can exist when two or more conducors are present.

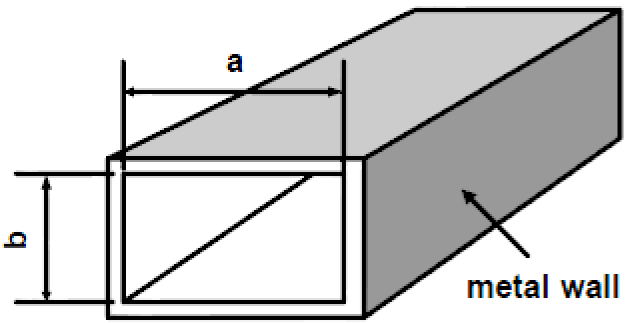


TE waves **Ez=0 Hz≠0**

TM waves **Ez≠0 Hz=0**

Cutoff!

Rectangular Waveguide



No TEM waves can propagate!

$$\nabla \times E = -j\omega\mu H$$
$$\nabla \times H = j\omega\varepsilon E$$

Boundary conditions

$$x=0\,,\ x=a \qquad E_y = E_z = 0$$
$$y=0\,,\ y=b \qquad E_y = E_z = 0$$

TE_{mn} waves **Ez=0 Hz≠0**

$$E_x = \frac{j\omega\mu n\pi}{k_c^2 b} A_{mn} \cos\frac{m\pi x}{a} \sin\frac{n\pi y}{b} e^{-j\beta z} \qquad H_x = \frac{j\beta m\pi}{k_c^2 a} A_{mn} \sin\frac{m\pi x}{a} \cos\frac{n\pi y}{b} e^{-j\beta z}$$

$$E_y = \frac{-j\omega\mu m\pi}{k_c^2 a} A_{mn} \sin\frac{m\pi x}{a} \cos\frac{n\pi y}{b} e^{-j\beta z} \qquad H_y = \frac{j\beta n\pi}{k_c^2 b} A_{mn} \cos\frac{m\pi x}{a} \sin\frac{n\pi y}{b} e^{-j\beta z}$$

$$E_z = 0 \qquad H_z = A_{mn} \cos\frac{m\pi x}{a} \cos\frac{n\pi y}{b} e^{-j\beta z}$$

$$\beta = \sqrt{k^2 - k_c^2} = \sqrt{k^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}$$

$\beta>0$ for propagation !!!

$$k > k_c = \sqrt{\left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}$$

$$k = \frac{2\pi}{\lambda} \quad wave \quad number$$

Propagation constant and cutoff frequency

Each mode (combination of m and n) has a cutoff frequency given by:

$$f_{Cmn} = \frac{k_c}{2\pi\sqrt{\varepsilon\mu}} = \frac{1}{2\pi\sqrt{\varepsilon\mu}}\sqrt{\left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}$$

At a given frequency f only those modes having f>fc will propagate! The contrary will lead to an imaginary β meaning that all field components will decay exponentially from the source excitation (evanescent mode)!

The mode with lowest cutoff is called dominant mode, if a>b the lowest fc occur for the TE_{10} mode:

$$f_{C10} = \frac{1}{2a\sqrt{\varepsilon\mu}}$$

$$E_x = E_z = H_y = 0$$

$$H_z = A_{10}\cos\frac{\pi x}{a}e^{-j\beta z}$$

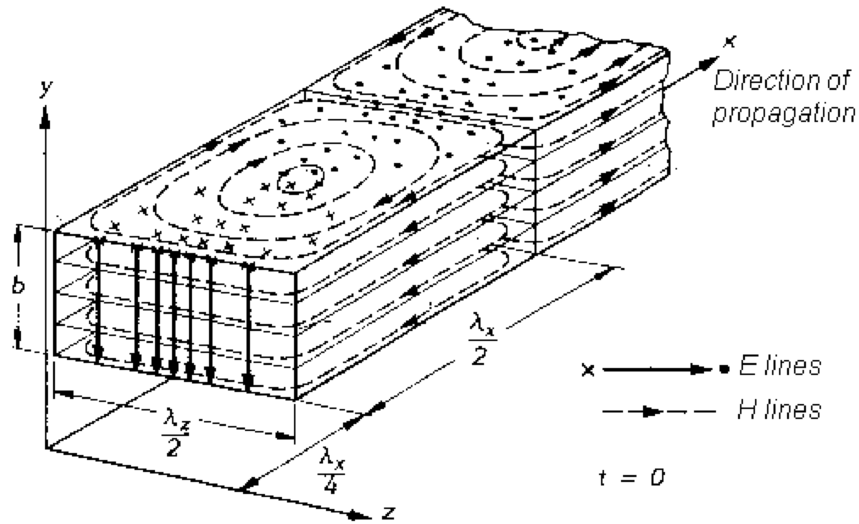
$$H_x = \frac{-j\beta a}{\pi}A_{10}\sin\frac{\pi x}{a}e^{-j\beta z}$$

$$E_y = \frac{-j\omega\mu a}{\pi}A_{10}\sin\frac{\pi x}{a}e^{-j\beta z}$$

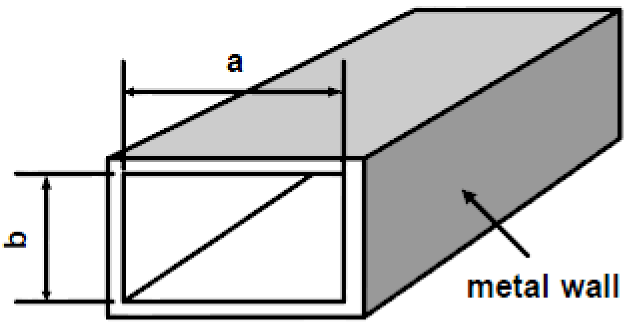
$$P_{10} = \frac{\omega\mu a^3|A_{10}|^2 b}{4\pi^2}\mathrm{Re}[\beta]$$

$$P_l = R_s|A_{10}|^2\left(b + \frac{a}{2} + \frac{\beta^2 a^3}{2\pi^2}\right)$$

$$\alpha_c = \frac{R_S}{a^3 b\beta k\eta}\cdot\left(2b\pi^2 + a^3k^2\right) \quad Np/m$$



Rectangular Waveguide



$$\nabla \times E = -j\omega\mu H$$
$$\nabla \times H = j\omega\varepsilon E$$

Boundary conditions

$x = 0\,,\, x = a \qquad E_y = E_z = 0$

$y = 0\,,\, y = b \qquad E_y = E_z = 0$

TM_{mn} waves **Ez ≠ 0 Hz=0**

$$E_x = \frac{-j\beta m\pi}{k_c^2 a} B_{mn} \cos\frac{m\pi x}{a} \sin\frac{n\pi y}{b} e^{-j\beta z} \qquad H_x = \frac{j\omega\varepsilon n\pi}{k_c^2 b} B_{mn} \sin\frac{m\pi x}{a} \cos\frac{n\pi y}{b} e^{-j\beta z}$$

$$E_y = \frac{-j\beta n\pi}{k_c^2 b} B_{mn} \sin\frac{m\pi x}{a} \cos\frac{n\pi y}{b} e^{-j\beta z} \qquad H_y = \frac{-j\omega\varepsilon m\pi}{k_c^2 a} A_{mn} \cos\frac{m\pi x}{a} \sin\frac{n\pi y}{b} e^{-j\beta z}$$

$$E_z = B_{mn} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} e^{-j\beta z} \qquad H_z = 0$$

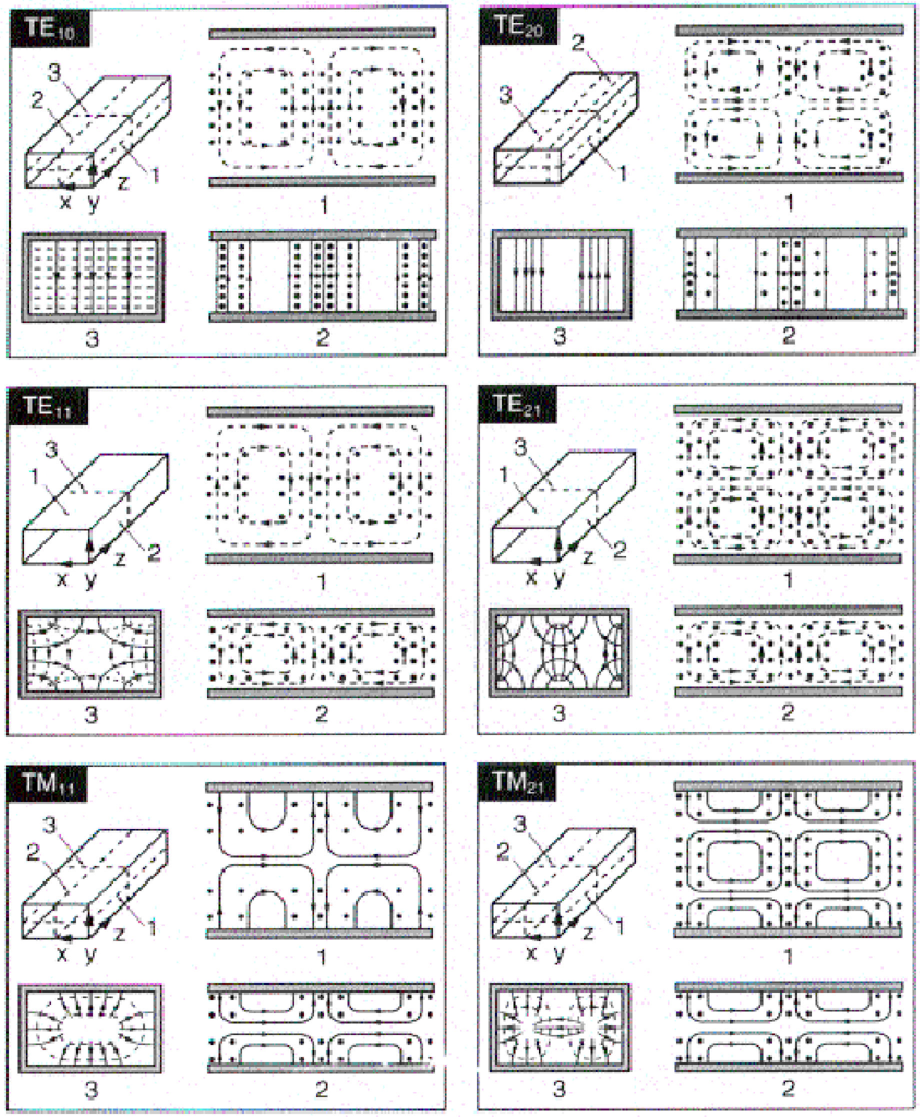
$$\beta = \sqrt{k^2 - k_c^2} = \sqrt{k^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}$$

β>0 for propagation !!!

$$f_{Cmn} = \frac{1}{2\pi\sqrt{\varepsilon\mu}} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

Fields are zero if the indexes m or n are zero!
The lowes TM mode that can propagate into the waveguide is the TM_{11}.

Field lines for some low order modes



Other parameters

$$Z_{TE} = \frac{-E_y}{H_x} = \frac{E_x}{H_y} = \frac{k\eta}{\beta}$$

$$\eta = \sqrt{\varepsilon / \mu}$$

wave impedance

$$Z_{TM} = \frac{-E_y}{H_x} = \frac{E_x}{H_y} = \frac{\beta\eta}{k}$$

medium intrinsic impedance

$$\lambda_g = \frac{2\pi}{\beta} > \frac{2\pi}{k} = \lambda$$

The guided wavelenght is the distance between two equal phase planes inside the waveguide

$$v_p = \frac{\omega}{\beta} > \frac{\omega}{k} = \frac{1}{\sqrt{\varepsilon\mu}}$$

Phase velocity

Table of rectangular waveguide

WG	EIA-WR	IEC-R	Fc1, GHz	Flo, GHz	Fhi, GHz	Fc2, GHz	A, in	B, in	A, mm	B, mm
00	2300	3	.257	0.32	0.49	.513	23.000	11.500	584.200	292.100
0	2100	4	.281	0.35	0.53	.562	21.000	10.500	533.400	266.700
1	1800	5	.328	0.41	0.625	.656	18.000	9.000	457.200	228.600
2	1500	6	.393	0.49	0.75	.787	15.000	7.500	381.000	190.500
3	1150	8	.513	0.64	0.96	1.026	11.500	5.750	292.100	146.050
4	975	9	.605	0.75	1.12	1.211	9.750	4.875	247.650	123.825
5	770	12	.766	0.96	1.45	1.533	7.700	3.850	195.580	97.790
6	650	14	.908	1.12	1.70	1.816	6.500	3.250	165.100	82.550
7	510	18	1.157	1.45	2.20	2.314	5.100	2.550	129.540	64.770
8	430	22	1.372	1.70	2.60	2.745	4.300	2.150	109.220	54.610
9A	340	26	1.736	2.20	3.30	3.471	3.400	1.700	86.360	43.180
10	284	32	2.078	2.60	3.95	4.156	2.840	1.340	72.136	34.036
11A	229	40	2.577	3.30	4.90	5.154	2.290	1.145	58.166	29.083
12	187	48	3.152	3.95	5.85	6.305	1.872	.872	47.549	22.149
13	159	58	3.712	4.90	7.05	7.423	1.590	.795	40.386	20.193
14	137	70	4.301	5.85	8.20	8.603	1.372	.622	34.849	15.799
15	112	84	5.260	7.05	10.0	10.519	1.122	.497	28.499	12.624
16	90	100	6.557	8.20	12.4	13.114	.900	.400	22.860	10.160
17	75	120	7.869	10.0	15.0	15.737	.750	.375	19.050	9.525
18	62	140	9.488	12.4	18.0	18.976	.622	.311	15.799	7.899
19	51	180	11.571	15.0	22.0	23.143	.510	.255	12.954	6.477
20	42	220	14.051	18.0	26.5	28.102	.420	.170	10.668	4.318
21	34	260	17.357	22.0	33.0	34.714	.340	.170	8.636	4.318
22	28	320	21.077	26.5	40.0	42.153	.280	.140	7.112	3.556
23	22	400	26.346	33.0	50.0	52.691	.224	.112	5.690	2.845
24	19	500	31.391	40.0	60.0	62.781	.188	.094	4.775	2.388
25	15	620	39.875	50.0	75.0	79.749	.148	.074	3.759	1.880
26	12	740	48.372	60.0	90.0	96.745	.122	.061	3.099	1.549
27	10	900	59.014	75.0	110	118.029	.100	.050	2.540	1.270
28	8	1200	73.768	90.0	140	147.536	.080	.040	2.032	1.016
29	7	1400	90.791	110	170	181.582	.065	.033	1.651	.826
30	5	1800	115.714	140	220	231.428	.051	.026	1.295	.648
31	4	2200	137.242	170	260	274.485	.043	.022	1.092	.546
32	3	2600	173.571	220	325	347.143	.034	.017	.864	.432
-	2	-	295.071	325	500	590.143	.020	.010	.508	.254

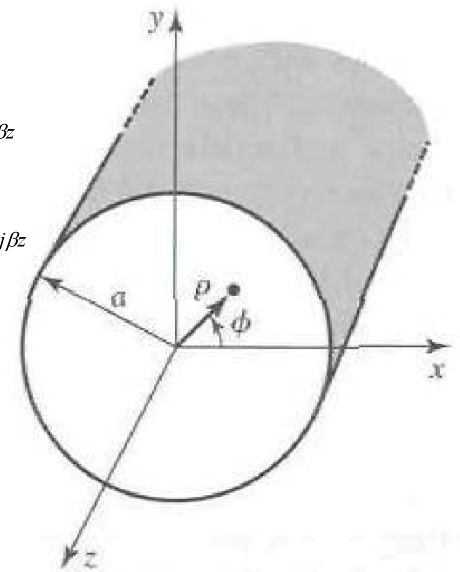
Circular waveguide

TE_{nm} modes

$$E_\rho = \frac{-j\omega\mu n}{k_c^2\rho}(A\cos n\phi - B\sin n\phi)J_n(k_c\rho)e^{-j\beta z} \qquad H_\rho = \frac{-j\beta}{k_c}(A\sin n\phi + B\cos n\phi)J_n'(k_c\rho)e^{-j\beta z}$$

$$E_\phi = \frac{j\omega\mu}{k_c}(A\sin n\phi - B\cos n\phi)J_n'(k_c\rho)e^{-j\beta z} \qquad H_\phi = \frac{-j\beta n}{k_c^2\rho}(A\cos n\phi - B\sin n\phi)J_n(k_c\rho)e^{-j\beta z}$$

$$E_z = 0 \qquad H_z = (A\sin n\phi + B\cos n\phi)J_n(k_c\rho)e^{-j\beta z}$$



$J_n(x)$ is the n-order Bessel function of first kind

$J'_n(x)$ is the n-order Bessel function derivative of first kind

$$\beta = \sqrt{k^2 - k_c^2} = \sqrt{k^2 - \left(\frac{p'_{nm}}{a}\right)^2}$$

$\beta>0$ for propagation !!!

where p'_{nm} is the mth root of $J'_n(x)$

$$f_{Cmn} = \frac{k_c}{2\pi\sqrt{\varepsilon\mu}} = \frac{p'_{nm}}{2\pi a\sqrt{\varepsilon\mu}}$$

Cutoff frequencies

	n=0	n=1	n=2	n=3	n=4	n=5	n=6	...
m=1	3.8318	1.8412	3.0542	4.2012	5.3175	6.4155	7.5013	...
m=2	7.0156	5.3315	6.7062	8.0153	9.2824	10.5199	11.7349	...
m=3	10.1735	8.5363	9.9695	11.3459	12.6819	13.9872	15.2682	...

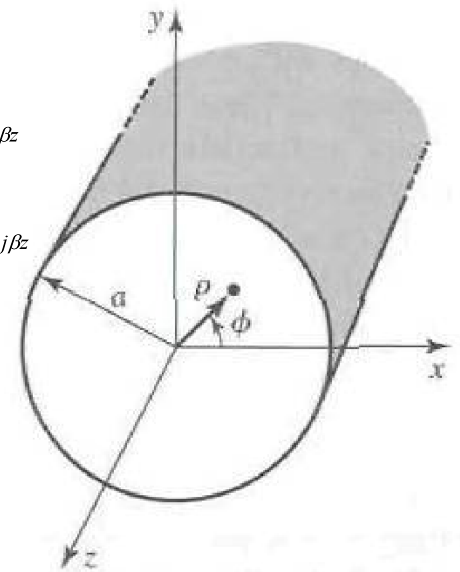
Circular waveguide

TM_{nm} modes

$$E_\rho = \frac{-j\beta}{k_c}(A\sin n\phi + B\cos n\phi)J_n'(k_c\rho)e^{-j\beta z} \qquad H_\rho = \frac{j\omega\varepsilon n}{k_c^2\rho}(A\cos n\phi - B\sin n\phi)J_n(k_c\rho)e^{-j\beta z}$$

$$E_\phi = \frac{-j\beta n}{k_c^2\rho}(A\cos n\phi - B\sin n\phi)J_n(k_c\rho)e^{-j\beta z} \qquad H_\phi = \frac{-j\omega\varepsilon}{k_c}(A\sin n\phi + B\cos n\phi)J_n'(k_c\rho)e^{-j\beta z}$$

$$E_z = (A\sin n\phi + B\cos n\phi)J_n(k_c\rho)e^{-j\beta z} \qquad H_z = 0$$



$J_n(x)$ is the n-order Bessel function of first kind

$J'_n(x)$ is the n-order Bessel function derivative of first kind

$$\beta = \sqrt{k^2 - k_c^2} = \sqrt{k^2 - \left(\frac{p_{nm}}{a}\right)^2}$$

$\beta>0$ for propagation !!!

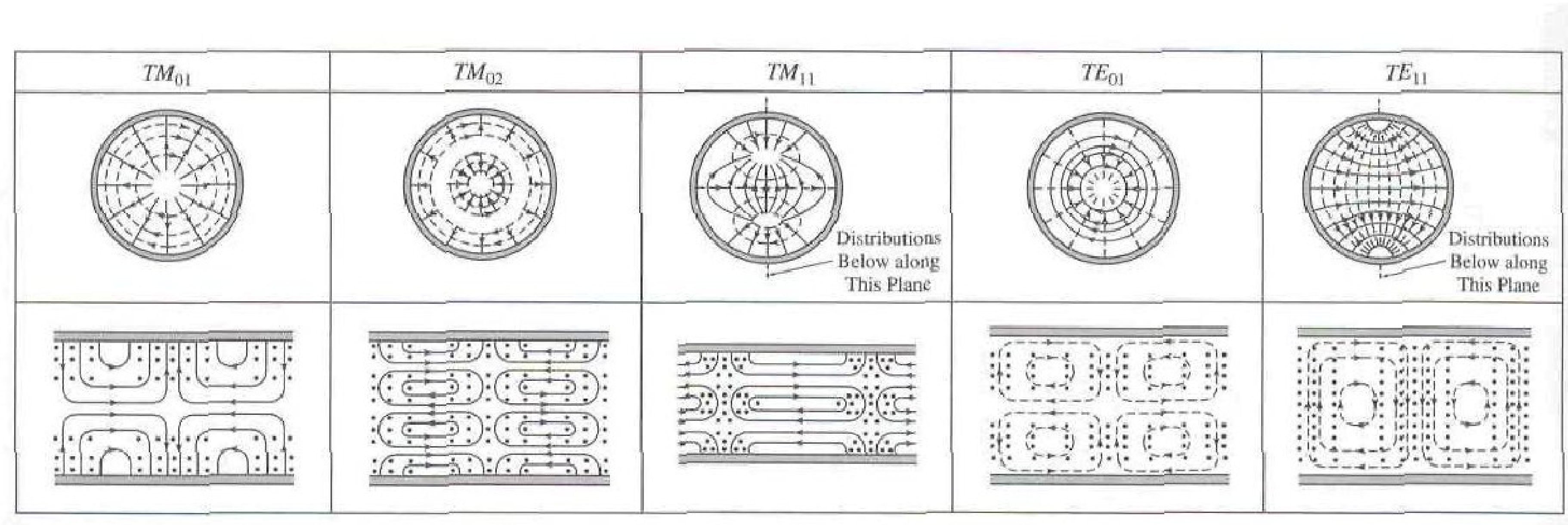
where p_{nm} is the mth root of $J_n(x)$

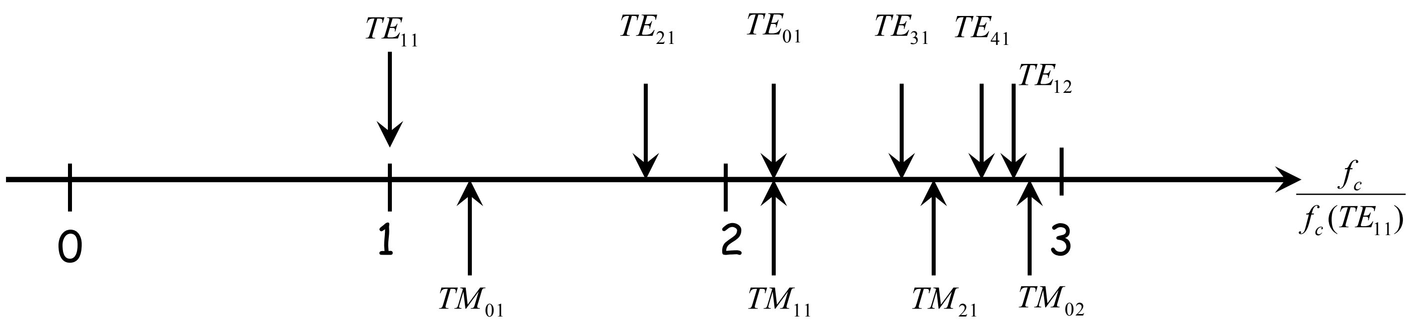
$$f_{Cmn} = \frac{k_c}{2\pi\sqrt{\varepsilon\mu}} = \frac{p_{nm}}{2\pi a\sqrt{\varepsilon\mu}}$$

Cutoff frequencies

	n=0	n=1	n=2	n=3	n=4	n=5	n=6	...
m=1	2.4049	3.8318	5.1357	6.3802	7.5884	8.7715	9.9361	...
m=2	5.5201	7.1056	8.4173	9.7610	11.0647	12.3386	13.5893	...
m=3	8.6537	10.1735	11.6199	13.0152	14.3726	15.7002	17.0038	...

Field lines for some low order modes

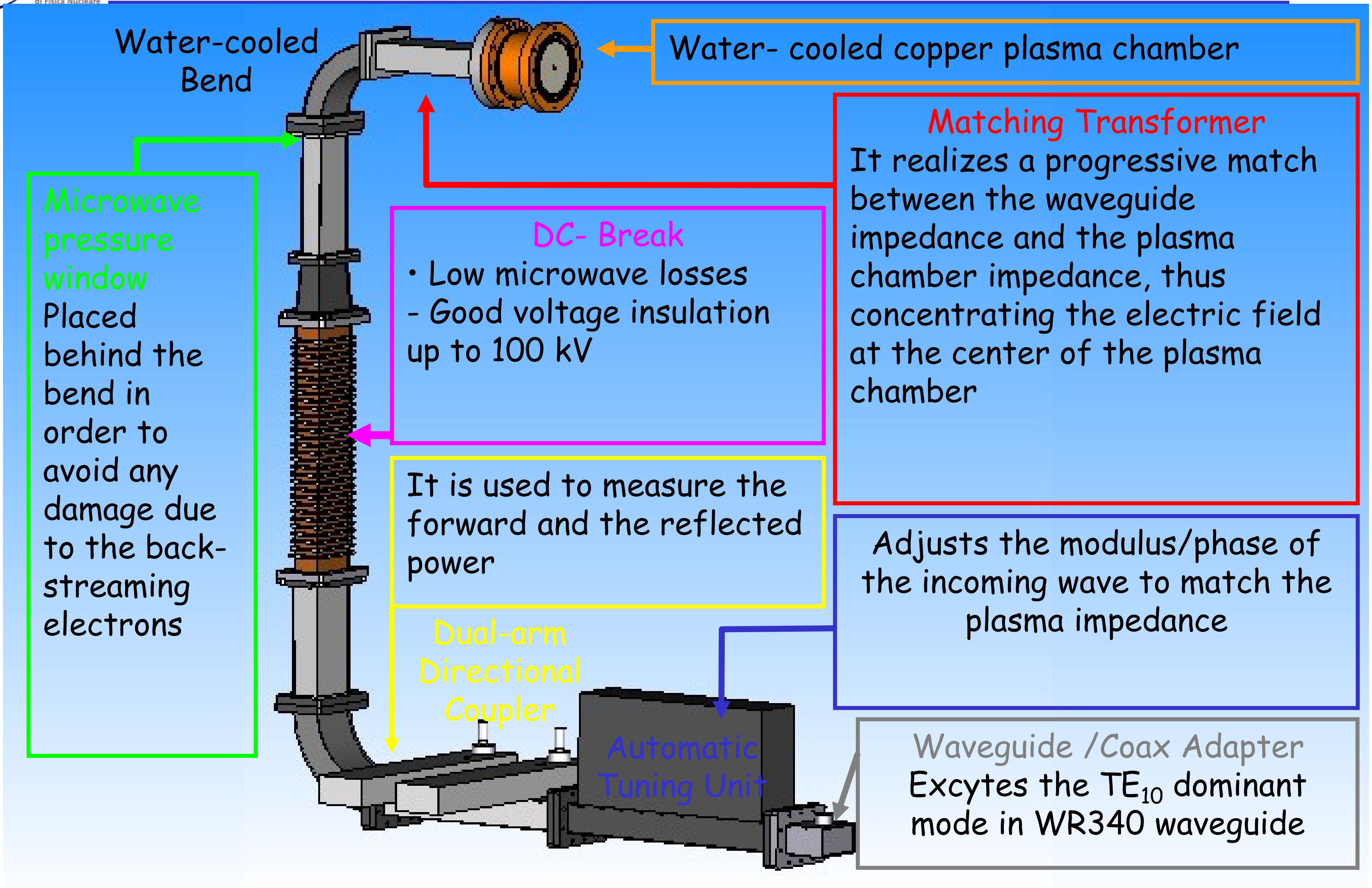




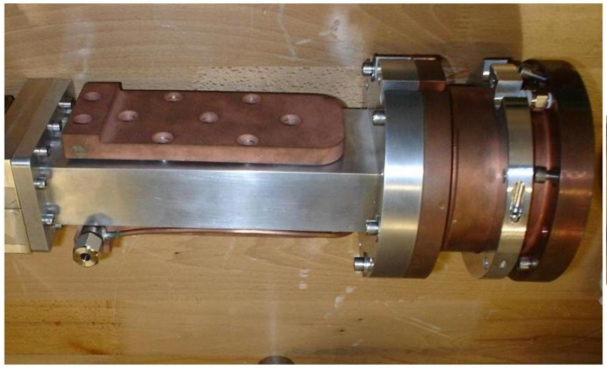
Circular waveguides table

Frequency Band	Frequency Range		Circular Waveguide Diameter, Inches (mm)	Cover Flange (Brass) MIL-F- 3922 UG	Flange Type
X	LOW	8.2-9.97	1.094 (27.79)	53-001 UG-39/U	Square
	MID	8.5-11.6	0.938 (23.83)		
	HIGH	9.97-12.4	0.797 (20.24)		
Ku	LOW	12.4-15.9	0.688 (17.48)	53-005 UG-1666/U	Square
	MID	13.4-18.0	0.594 (15.08)		
	HIGH	15.9-18.0	0.500 (12.70)		
K	LOW	17.5-20.5	0.455 (11.56)	54-001 UG-595/U	Square
	MID	20-24.5	0.396 (10.06)		
	HIGH	24-26.5	0.328 (8.33)		
Ka	LOW	26.5-33	0.315 (8.00)	54-003 UG-595/U	Square
	MID	33-38.5	0.250 (6.35)		
	HIGH	38.5-40	0.219 (5.56)		
Q	LOW	33-38.5	0.250 (6.35)	67B-006 UG-383/U	Round
	MID	38.5-43	0.219 (5.56)		
	HIGH	43-50	0.188 (4.78)		
U	LOW	40-43	0.210 (5.33)	67B-007 UG-383/U-M	Round
	MID	43-50	0.188 (4.78)		
	HIGH	50-60	0.165 (4.19)		
V	LOW	50-58	0.165 (4.19)	67B-008 UG-385/U	Round
	MID	58-68	0.141 (3.58)		
	HIGH	68-75	0.125 (3.18)		
E	LOW	60-66	0.136 (3.45)	67B-009 UG-387/U	Round
	MID	66-88	0.125 (3.18)		
	HIGH	88-90	0.094 (2.39)		
W	LOW	75-88	0.112 (2.84)	67B-010 UG-387/U-M	Round
	HIGH	88-110	0.094 (2.39)		
F	LOW	90-115	0.089 (2.26)	- UG-387/U-M	Round
	HIGH	115-140	0.075 (1.91)		
D	LOW	110-140	0.073 (1.85)	- UG-387/U-M	Round
	HIGH	140-160	0.059 (1.50)		
G	LOW	140-220	0.058 (1.47)	- UG-387/U-M	Round
	HIGH	170-260	0.049 (1.25)		

Microwave coupling to plasma source

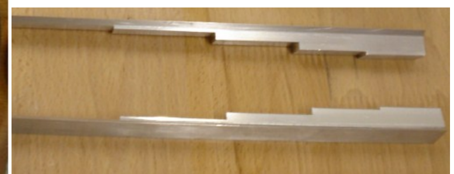


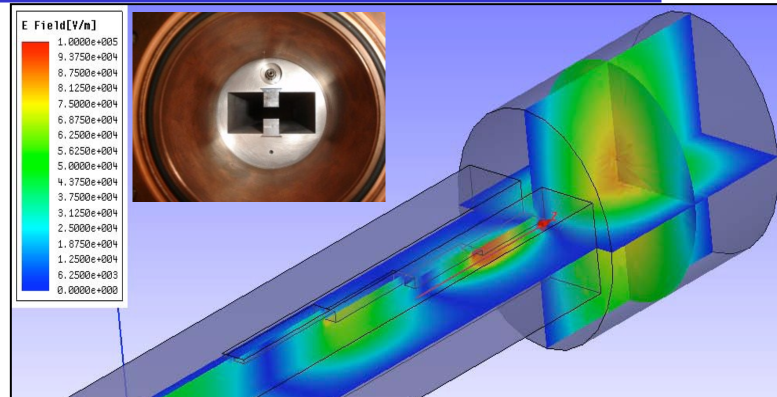
Multisection matching transformers in WG



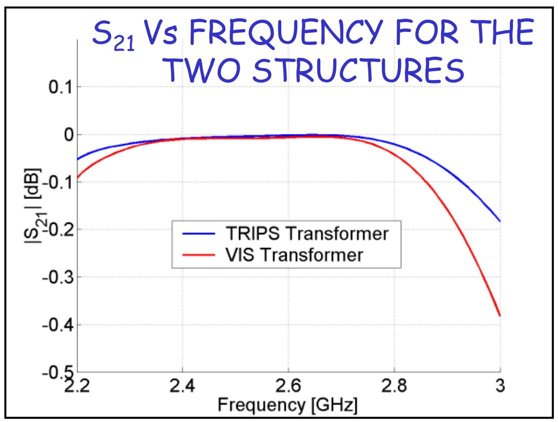
Matching transformer coupled to the plasma chamber

Four step double ridges

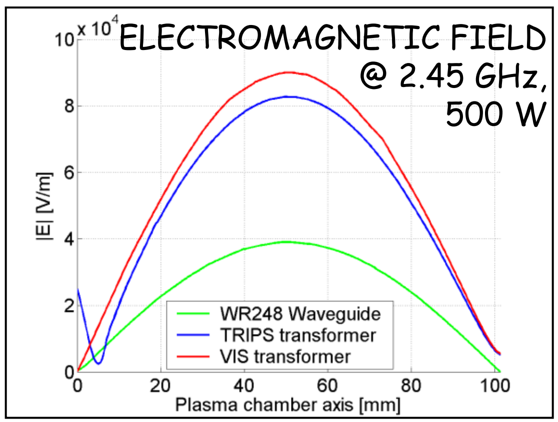




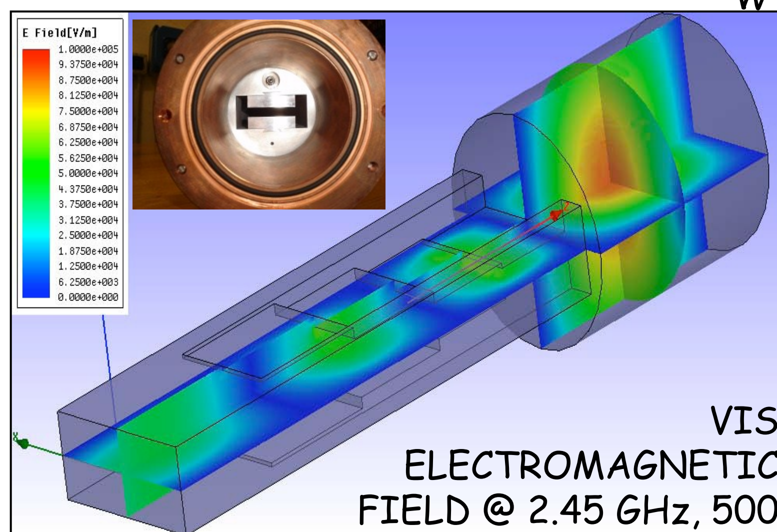
TRIPS ELECTROMAGNETIC FIELD @ 2.45 GHz, 500 W



VIS TRANSFORMER INSERTION LOSS 0.0085 dB @ 2.45 GHz



10 % ENHANCEMENT WITH VIS TRANSFORMER



VIS ELECTROMAGNETIC FIELD @ 2.45 GHz, 500 W

Microwave resonators

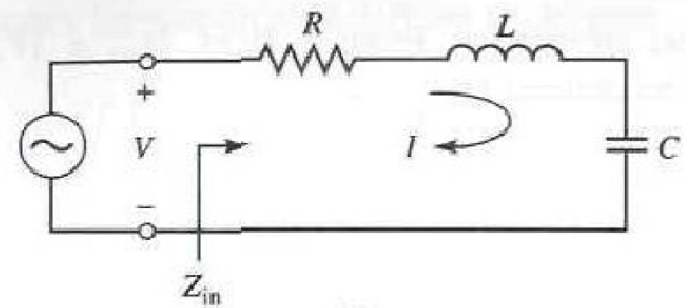
Used in a wide variety of applications: filters, oscillators, frequency meters, tuned amplifiers, etc…

The behaviour near resonance is very similar to the lumped element resonators.

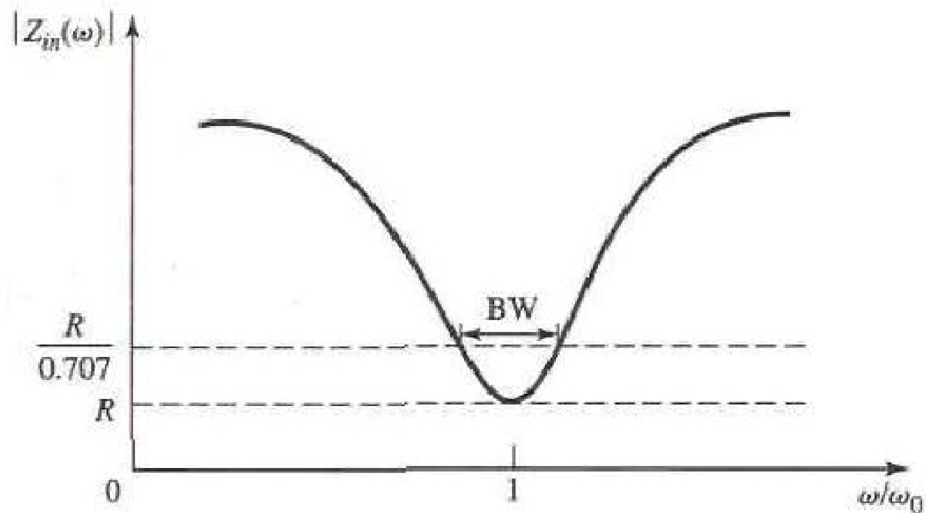
$$Z_{IN} = R + j\omega L - \frac{1}{j\omega C}$$

$$P_{IN} = \frac{1}{2}VI = \frac{1}{2}Z_{IN}|I|^2 = \frac{1}{2}|I|^2\left(R + j\omega L - \frac{1}{j\omega C}\right)$$

$$P_{IN} = P_{loss} + 2j\omega(W_m - W_E)$$



(a)



When resonance occurs the average storage electric and magnetic energies are equal

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$Q = \omega\frac{average\ \ energy\ \ stored}{energy\ \ loss/second} = \omega(W_m + W_E)/P_L$$

At resonance: $$Q = \frac{1}{\omega_0 RC} = \frac{\omega_0 L}{R}$$

Fractional BW: $$BW = \frac{1}{Q}$$

Rectangular waveguide cavities

Usually short circuited at both ends forming a closed box or a cavity.
Power dissipated on metallic walls as well as in the dielctric filling the cavity.

Cutoff wavenumber and resonant frequencies

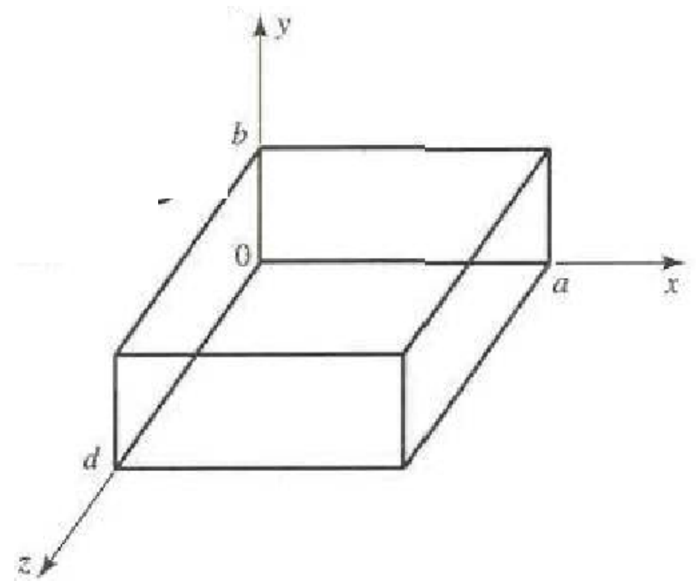
$$k_{mnl} = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{l\pi}{d}\right)^2}$$

$$f_{mnl} = \frac{ck_{mnl}}{2\pi\sqrt{\varepsilon_r\mu_r}} = \frac{c}{2\pi\sqrt{\varepsilon_r\mu_r}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{l\pi}{d}\right)^2}$$

Q for TE_{10l} mode

$$Q = \frac{(kad)^3 b\eta}{2\pi^2 R_S} \cdot \frac{1}{2l^2a^3b + 2bd^3 + l^2a^3d + ad^3}$$

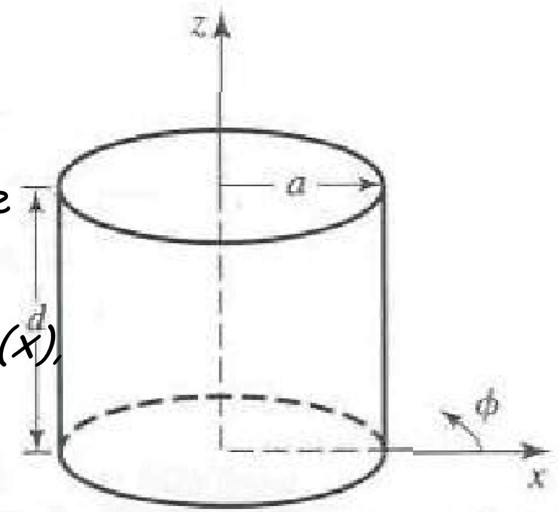
$$P_d = \frac{abd\omega\varepsilon|E_0|^2}{8}$$



Circular waveguide cavities

$$f_{nml} = \frac{c}{2\pi\sqrt{\varepsilon_r \mu_r}} \sqrt{\left(\frac{p'_{nm}}{a}\right)^2 + \left(\frac{l\pi}{d}\right)^2}$$ Resonant frequencies TE_{nml} mode

$$f_{nml} = \frac{c}{2\pi\sqrt{\varepsilon_r \mu_r}} \sqrt{\left(\frac{p_{nm}}{a}\right)^2 + \left(\frac{l\pi}{d}\right)^2}$$ Resonant frequencies TM_{nml} mode



where p'_{nm} is the mth root of $J'_n(x)$ and p_{nm} is the mth root of $J_n(x)$,

$J_n(x)$ is the n-order Bessel function of first kind

$J'_n(x)$ is the n-order Bessel function derivative of first kind

Q evaluation

$$Q_c = \frac{\omega_0 W}{P_c} = \frac{(ka)^3 \eta a d}{4(p'_{nm})^2 R_s} \frac{1 - \left(\frac{n}{p'_{nm}}\right)^2}{\left\{\frac{ad}{2}\left[1 + \left(\frac{\beta a n}{(p'_{nm})^2}\right)^2\right] + \left(\frac{\beta a^2}{p'_{nm}}\right)^2 \left(1 - \frac{n^2}{(p'_{nm})^2}\right)\right\}}.$$