



# Introduction to Accelerators

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# Why an Introduction?

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- The time where each accelerator sector was working alone in its corner is definitively **over!**
- Modern machines are at the **limit of technology**, so that individual (ideal) requests often cannot be fulfilled.
- The **only solution** is to find a **reasonable compromise**, which implies for each sector to understand the (sometimes extreme) requests from the other sectors, and to be able to explain its (sometimes extreme) requests to the other sectors:

(e.g. beam dynamics  $\leftrightarrow$  magnets  $\leftrightarrow$  Power Converters )

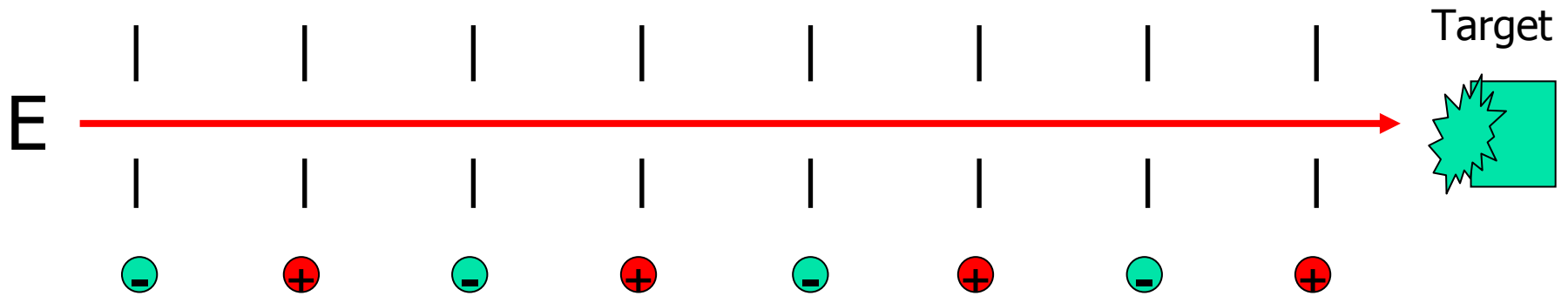
- This lecture is an attempt to introduce a few basic concepts of transverse beam dynamics, which might be useful for vacuum considerations.

# Accelerators in the world (2002)

Basic and Applied Research		Medicine	
High-energy phys.	120	Radiotherapy	7500
S.R. sources	50	Isotope Product.	200
Non-nuclear Res.	1000	Hadron Therapy	20
Industry			
Ion Implanters	7000		
Industrial e- Accel.	1500	<b>Total:</b>	<b>17390</b>

Courtesy: W. Mondelaers JUAS 2004

# Ideal linear machines (linacs)



$$\text{Available Energy : } E_{c.m.} = m \cdot (2+2\gamma)^{1/2} = (2m \cdot (m+E))^{1/2}$$

$$\text{with } \gamma = E/E_0$$

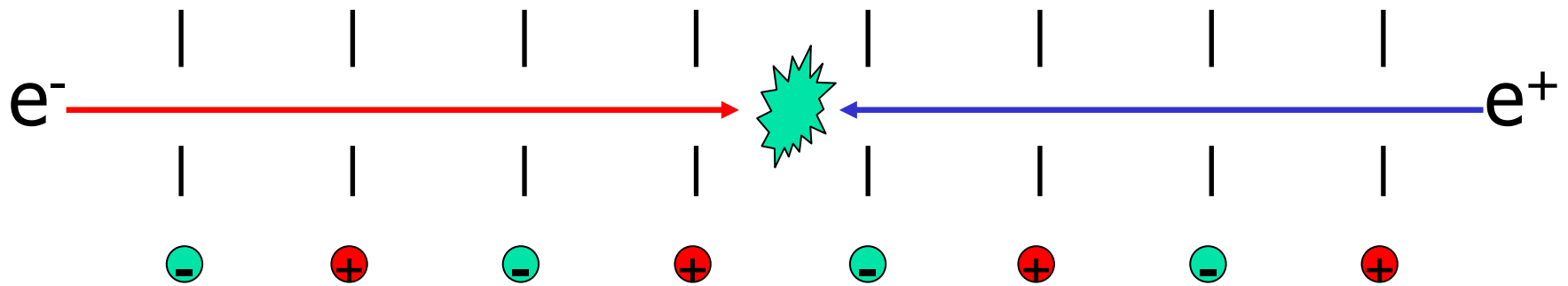
Advantages: Single pass

High intensity

Drawbacks: Single pass

Available Energy

# Improved solution for $E_{c.m.}$



Available Energy :  $E_{c.m.} = 2m\gamma = 2E$

with  $\gamma = E/E_0$

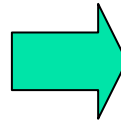
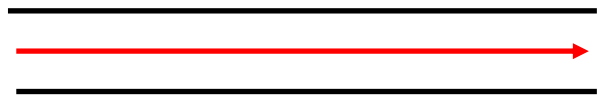
Advantages: High intensity

Drawbacks: Single pass

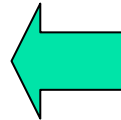
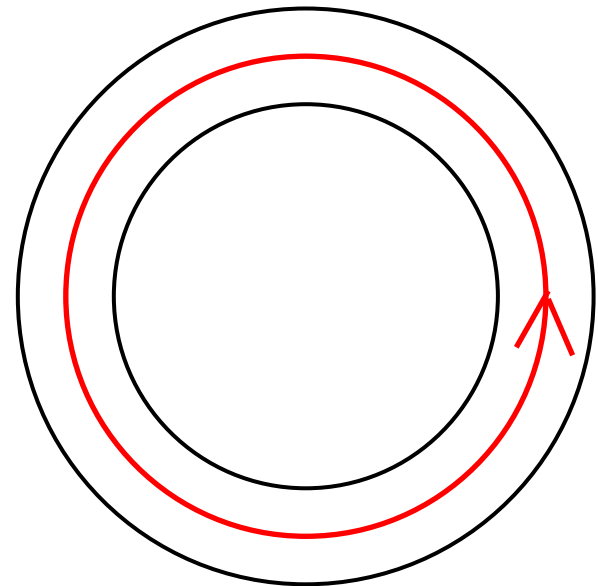
Space required

# Keep particles: circular machines

Basic idea is to re-use the particles or keep them in the machine.  
Move from the linear design

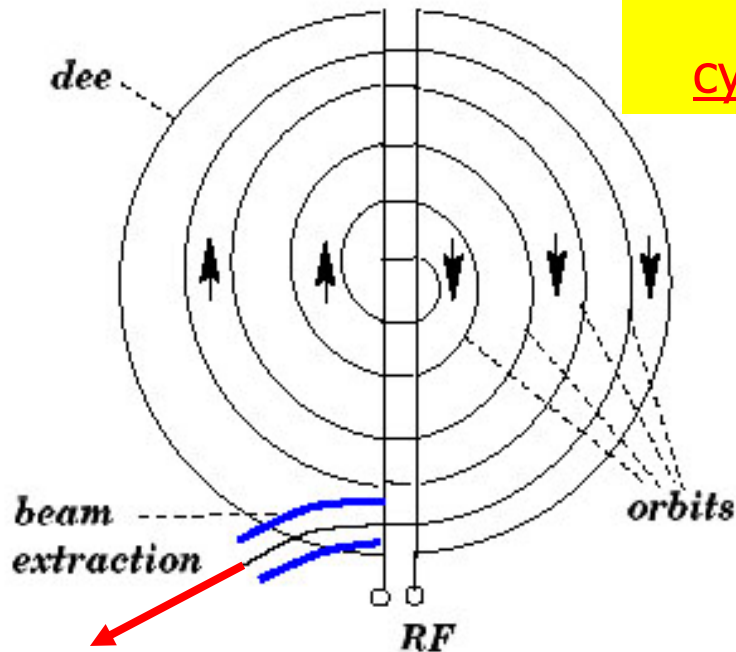


To a circular one:



- Need Bending
- Need **Dipoles!**

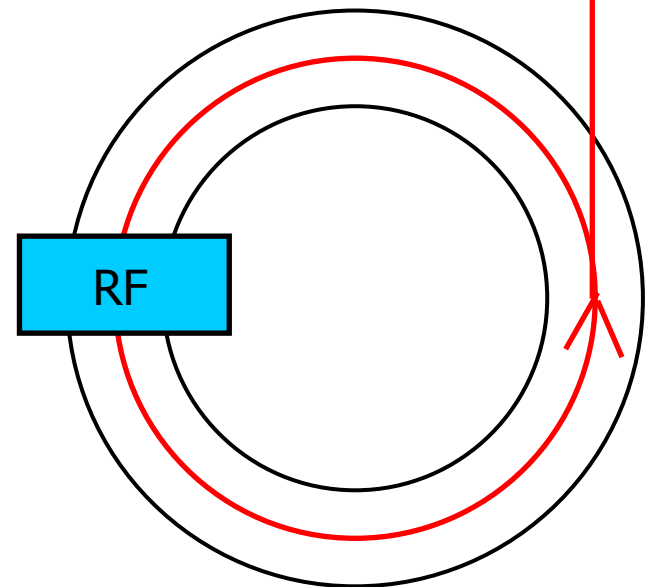
# Circular machines ( $E_{c.m.} \sim (mE)^{1/2}$ )



fixed target:  
cyclotron

huge dipole, compact design,  
B = constant  
low energy, single pass.

fixed target:  
synchrotron



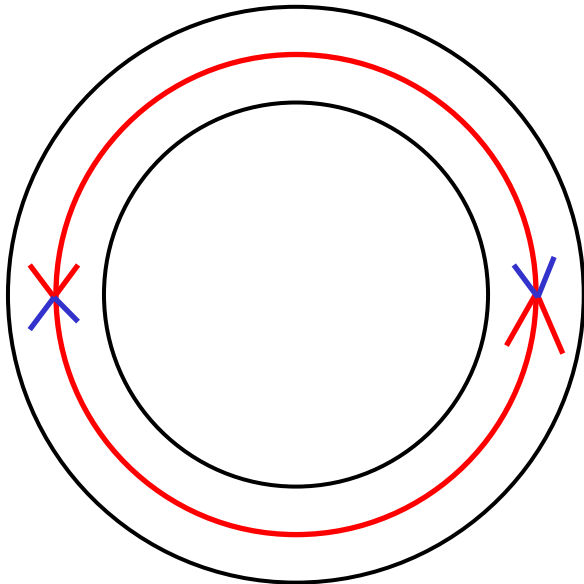
varying B, small magnets, high energy

# Colliders ( $E_{c.m.} = 2E$ )

## Colliders:

electron – positron  
proton - antiproton

Colliders with the same type of particles (e.g. p-p) require two separate chambers. The beam are brought into a common chamber around the interaction regions



Ex: LHC

8 possible interaction regions  
4 experiments collecting data

Question: What about LHC physics with fixed target?





# Beam Dynamics

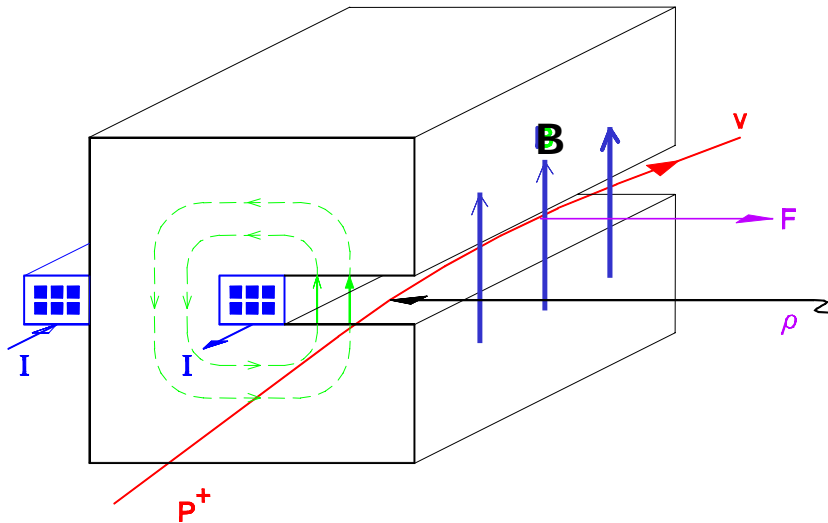
A particle is described by:

- Its azimuthal position along the machine:  $s$
- Its momentum:  $p$
- Its horizontal position:  $x$
- Its horizontal slope:  $x'$
- Its vertical position:  $y$
- Its vertical slope:  $y'$

i.e. a sixth dimensional phase space

$$(s, p, x, x', y, y')$$

# Circular machines: Dipoles



Classical mechanics:

Equilibrium between two forces

Lorentz force

Centripetal force

$$F = e.(\underline{v} \times \underline{B})$$

$$F = mv^2/\rho$$

$$evB = mv^2/\rho$$

$$p = m_0.c.(\beta\gamma)$$



Magnetic rigidity:

$$B\rho = mv/e = p/e$$

Relation also holds for relativistic case provided the classical momentum  $mv$  is replaced by the relativistic momentum  $p$



# Simple but fundamental relation!

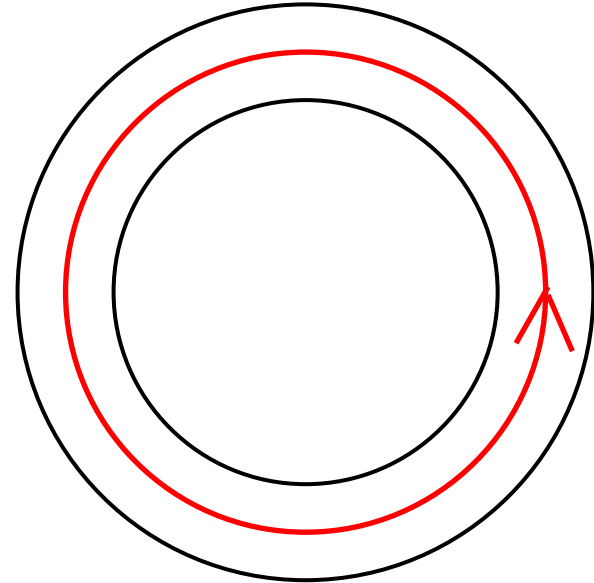
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- If you already have a site and you know the physics (E) to be achieved -> **you know the magnets required (SC or warm).**
- If you have the site and do not want to go for SC magnets -> **you know the energy range you can cover (fixed target or collider).**
- Now, check LHC physics with fixed target....!

# Ideal circular machine:

- Neglecting radiation losses in the dipoles
- Neglecting gravitation

ideal particle would happily circulate on axis in the machine for ever!



Unfortunately: real life is different!

Gravitation:  $\Delta y = 20$  mm in 64 msec!

Alignment of the machine

Limited physical aperture

Ground motion

Field imperfections

Energy error of particles **and/or**  $(x, x')_{inj} \neq (x, x')_{nominal}$

Error in magnet strength (power supplies and calibration)



# From Ideal to Real machines...

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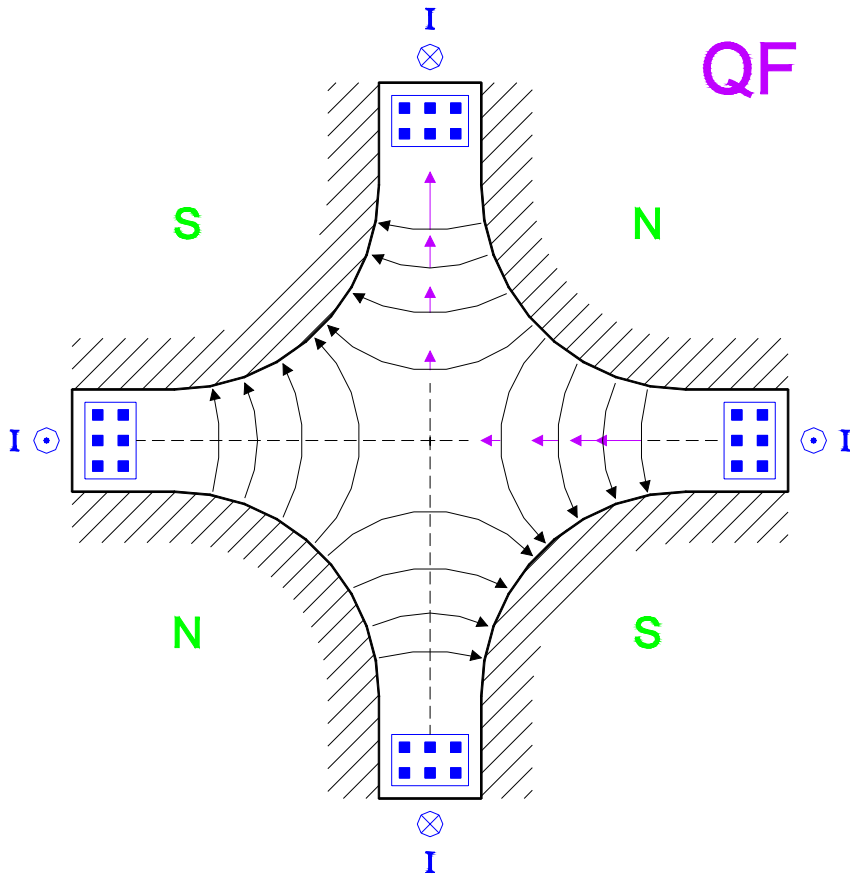
- Most of the particles are NOT ideal particles

$$(x, x', y, y', p) \neq (x, x', y, y', p)_{\text{ideal}}$$

- Sooner or later these particles will hit the walls and will be lost!

- How can we keep these particles within the vacuum chamber?

# Focusing with quadrupoles



$$F_x = -g \cdot x$$

$$F_y = g \cdot y$$

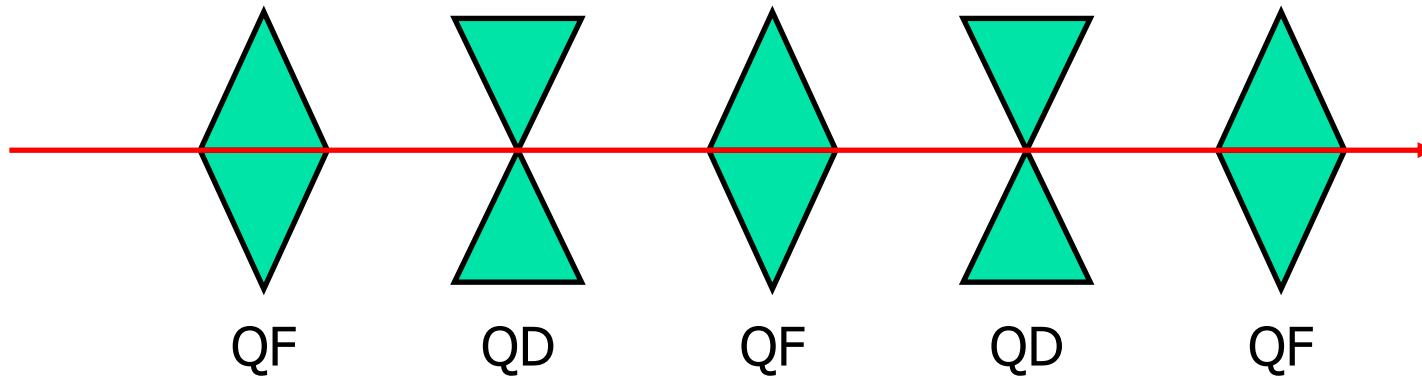
Force increases **linearly** with displacement.

Unfortunately, effect is **opposite** in the two planes (H and V).

Remember: **this** quadrupole is **focusing** in the **horizontal** plane but **defocusing** in the **vertical** plane!

# Alternating gradient focusing

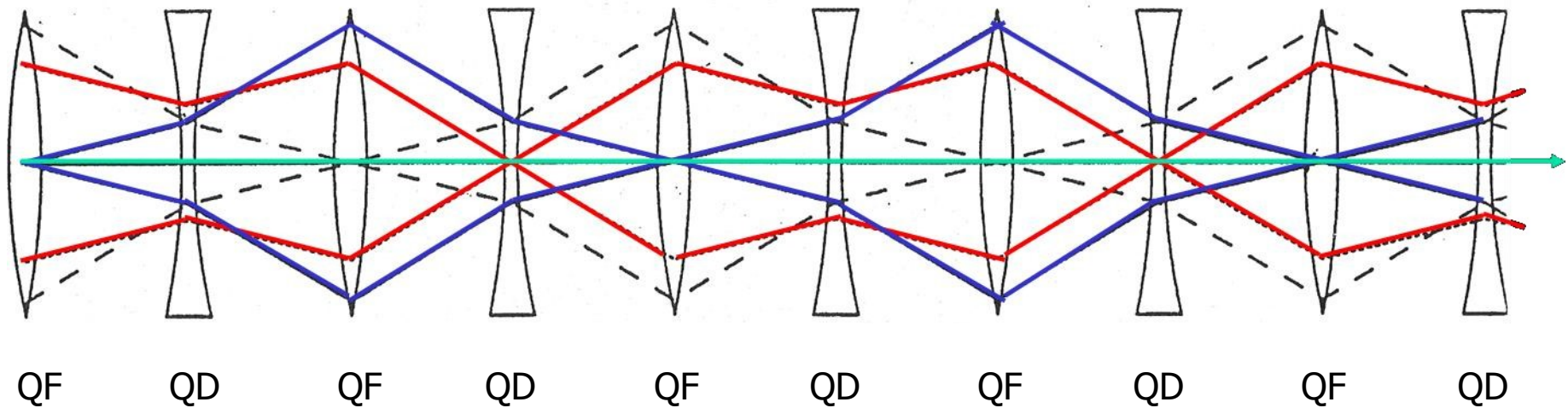
Basic new idea:  
Alternate QF and QD



Let us first consider a « non-ideal » injection in **position and slope**  
( $x, x'$ ) or ( $y, y'$ )

# Alternating gradient focusing

It can be shown that a section composed of alternating **focusing** and **defocusing** elements has a **net focusing effect**, provided the quadrupoles are correctly placed. **What happens to « non-ideal » particles?**



The « non-ideal » particles perform an **oscillation** around the « ideal » trajectory.



# Thin lens analogy of AG focusing

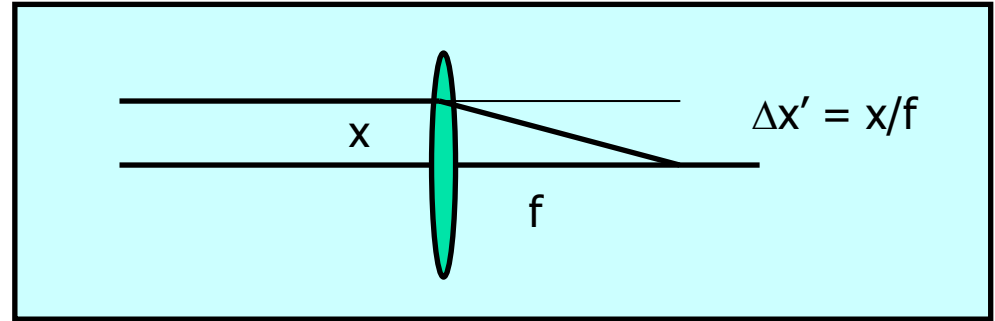
$$\begin{pmatrix} x \\ x' \end{pmatrix}_{\text{out}} = \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}_{\text{in}}$$

$$\begin{aligned} X_{\text{out}} &= X_{\text{in}} + 0 \cdot X'_{\text{in}} \\ x'_{\text{out}} &= (-1/f) \cdot x_{\text{in}} + x'_{\text{in}} \end{aligned}$$

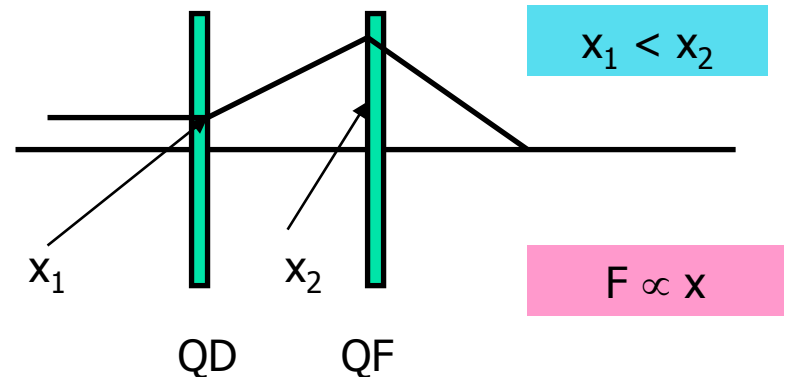
$$\text{Drift} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}$$

$$\text{QF-Drift-QD} = \begin{pmatrix} 1-L/f & L \\ -L/f^2 & 1+L/f \end{pmatrix}$$

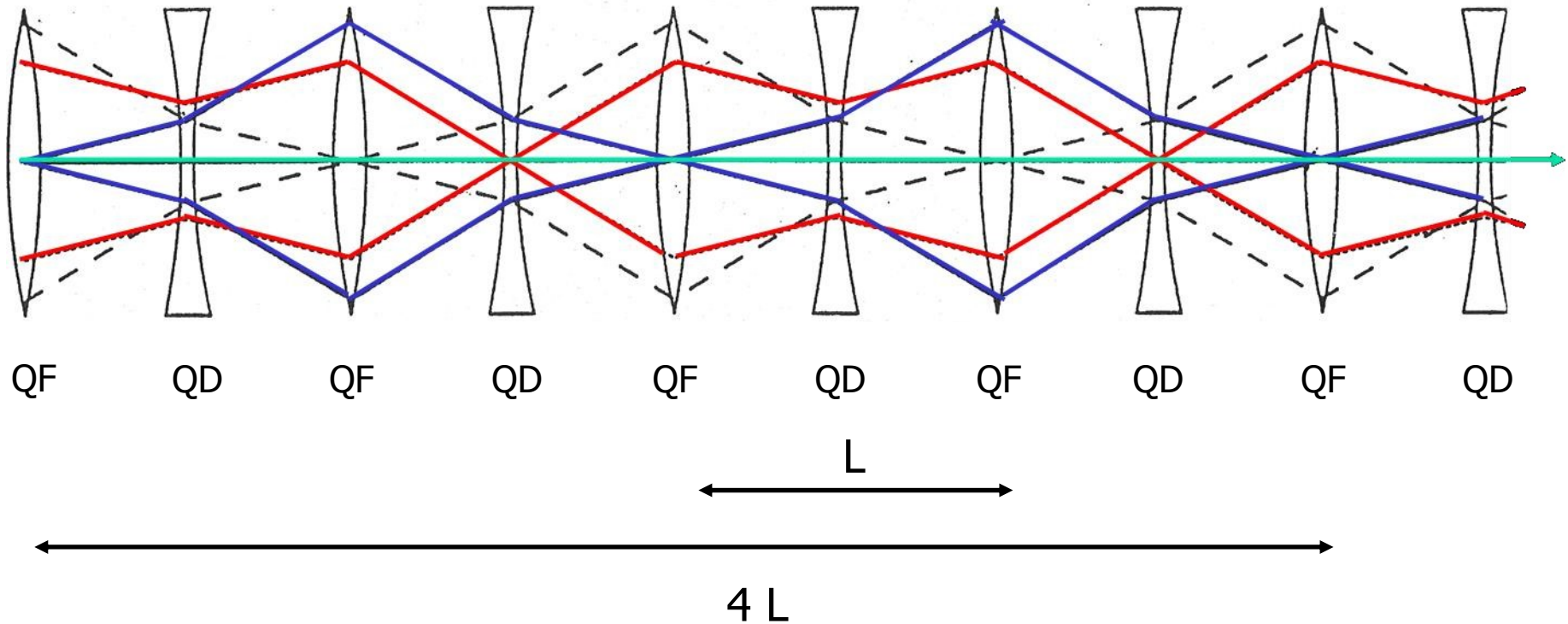
Initial:  $x = x$  and  $L < f$   
 $x' = 0$



More intuitively:



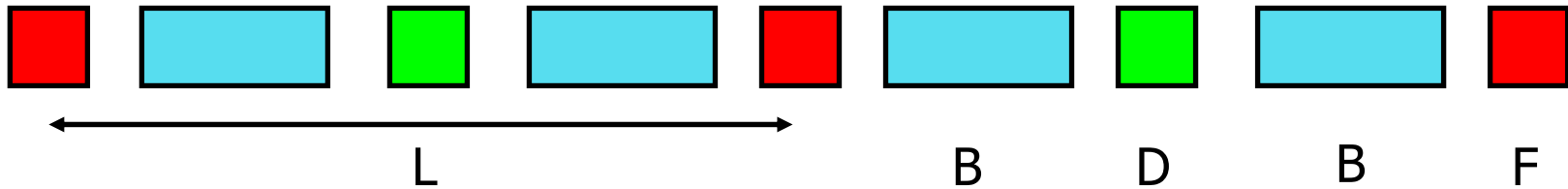
# The FODO cell:



One complete oscillation in 4 cells  $\Rightarrow 90^\circ / \text{cell} \Rightarrow \mu = 90^\circ$

# Real circular machines (no errors!)

The accelerator is composed of a **periodic** repetition of **FODO** cells:



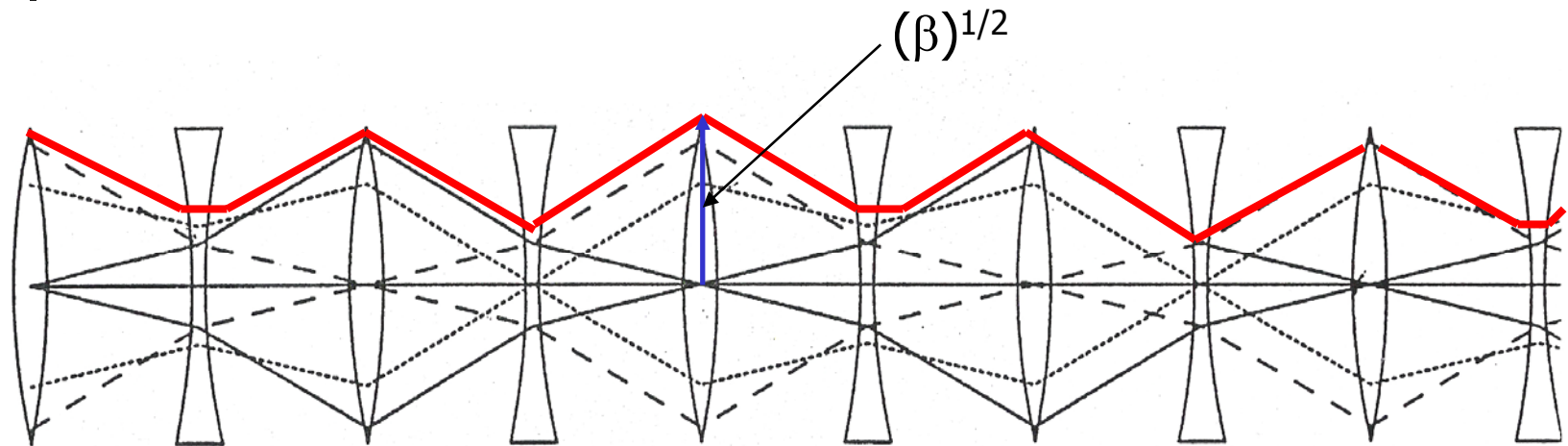
➤ The phase advance per cell  $\mu$  can be modified, in each plane, by varying the strength of the quadrupoles.

➤ The ideal particle will follow a **particular** trajectory, which closes on itself after one revolution: **the closed orbit**.

➤ The real particles will perform oscillations **around the closed orbit**.

➤ The number of **oscillations for a complete revolution** is called the **Tune Q** of the machine ( $Q_x$  and  $Q_y$ ).

# The beta function $\beta(s)$



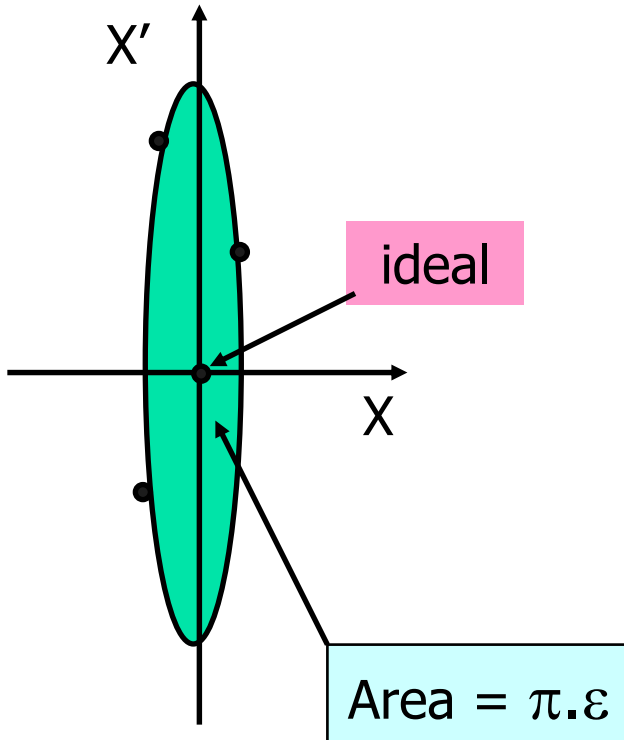
The  $\beta$ -function is the **envelope** around all the trajectories of the particles circulating in the FODO lattice.

The  $\beta$ -function has a **minimum at the QD** and a **maximum at the QF**, ensuring the net focusing effect of the lattice.

It is a periodic function in the FODO lattice. The oscillations of the particles are called **betatron motion or betatron oscillations**.

# Phase space at some position (s)

- Select the particle in the beam with the **largest betatron motion** and plot its **position vs. its phase** (x vs. x') at some location in the machine for many turns.

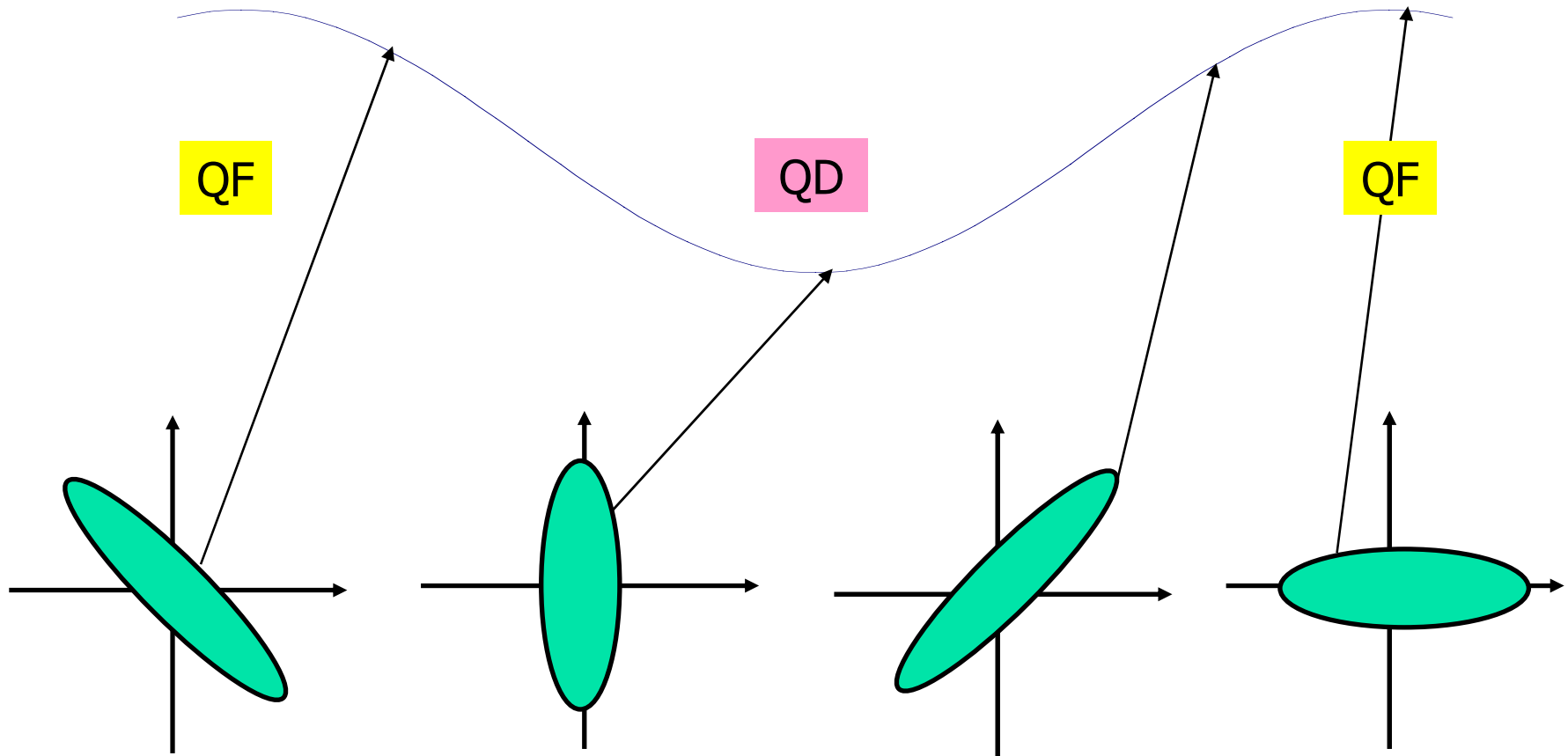


- $\epsilon$  Is the emittance of the beam [ $\pi$  mm mrad]
- $\epsilon$  is a **property of the beam** (quality)
- Measure of how much particle depart from ideal trajectory.
- $\beta$  is a **property of the machine** (quadrupoles).

Beam size [m]

$$\sigma(s) = (\epsilon \cdot \beta(s))^{1/2}$$

# Emittance conservation



The shape of the ellipse varies along the machine, but its area (the emittance  $\varepsilon$ ) remains constant for a given energy.

# Recapitulation

- The fraction of the oscillation performed in a FODO cell is called the **phase advance  $\mu$  per cell** (x or y).
- The total number of oscillations over one full turn of the machine is called the **betatron tune  $Q$**  (x or y).
- The envelope of the betatron oscillations is characterised by the **beta function  $\beta(s)$** . This is a property of the quadrupole settings.
- The quality of the (injected) beam is characterised by the **emittance  $\varepsilon$** . This is a property of the beam and is invariant around the machine.
- The r.m.s. beam size (measurable quantity) is  $\sigma = (\beta \cdot \varepsilon)^{1/2}$ .

What about a « non-ideal » injection energy  $\Delta p/p \neq 0$  ?



# Off momentum particles:

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- For particles with  $\Delta p/p \neq 0$ , the magnets induce other important effects like:

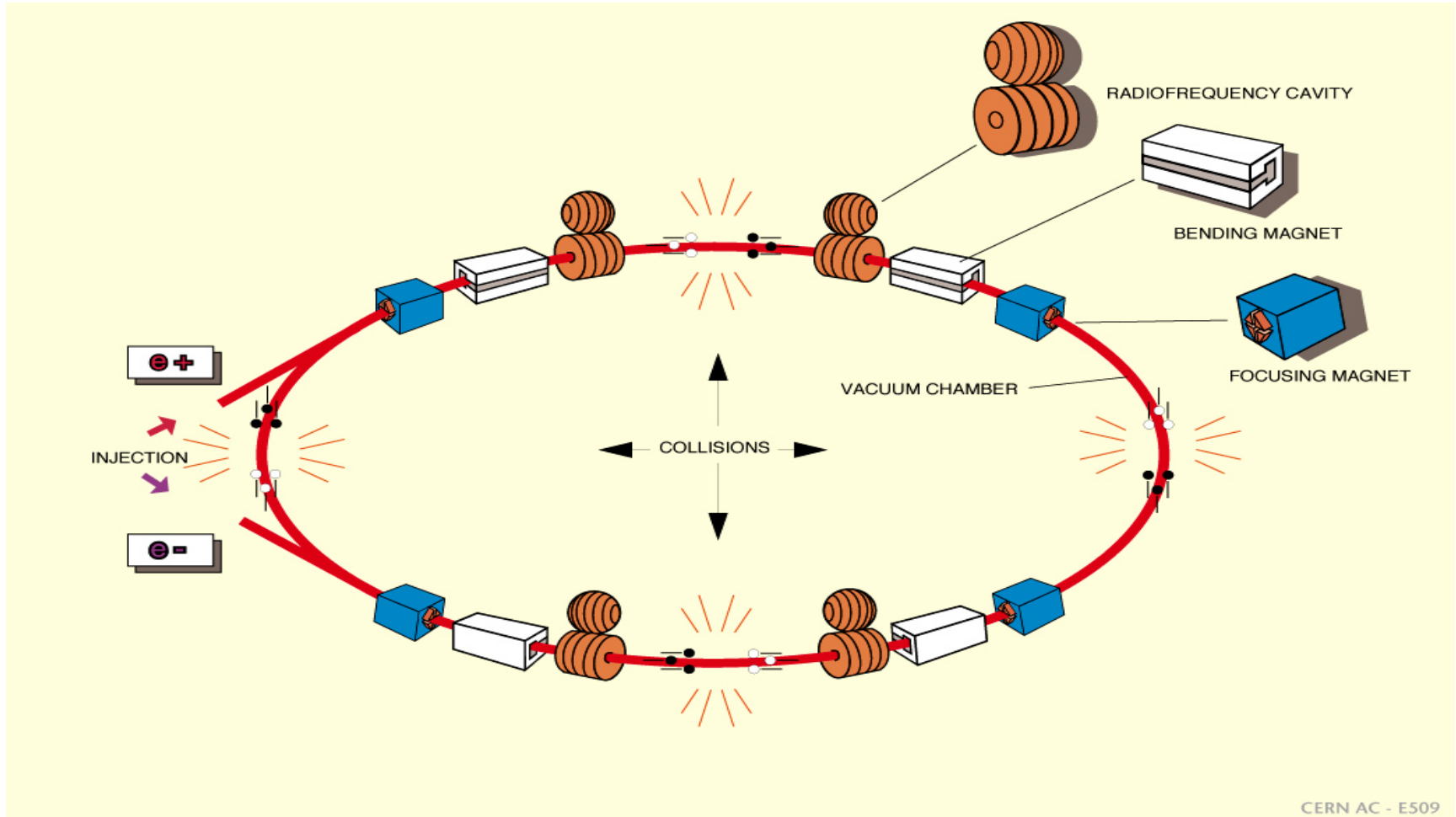
The dispersion (dipoles)

The chromaticity (quadrupoles)

These effects will not be treated in this lecture!



# Real high energy collider





# Vacuum and Accelerators

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- In HEP, accelerators are used to produce:
  - **Collisions** (fixed target or colliders)
  - **Synchrotron light**

This requires:

- **High intensities**
- **As small as possible beams**
- **As long as possible runs**

# e.g. for colliders: the Luminosity

$$dN/dt = L \times \sigma$$

$$[1/s] = [1/(cm^2.s)] \times [cm^2]$$

$$L = N_1 \cdot N_2 \cdot f \cdot k / (4 \cdot \pi \cdot \sigma_x \cdot \sigma_y)$$

Vacuum

with:

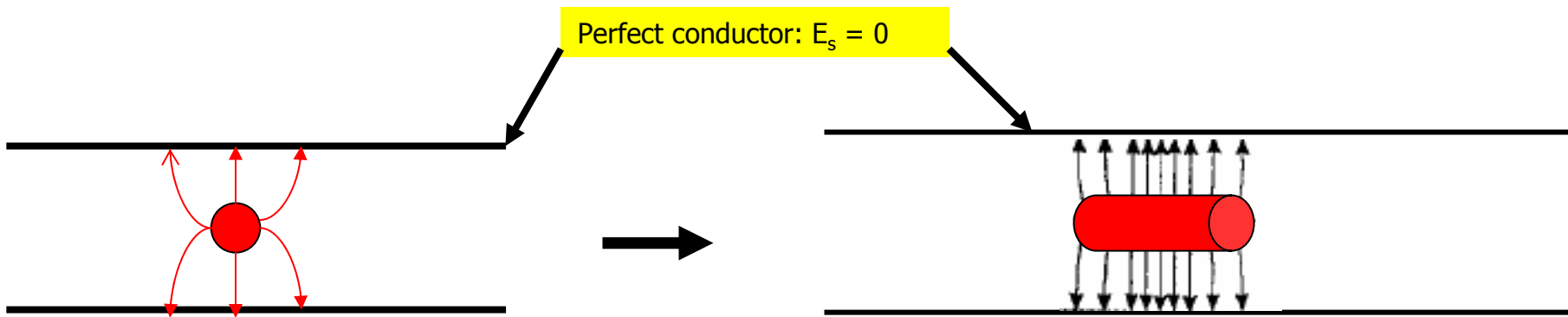
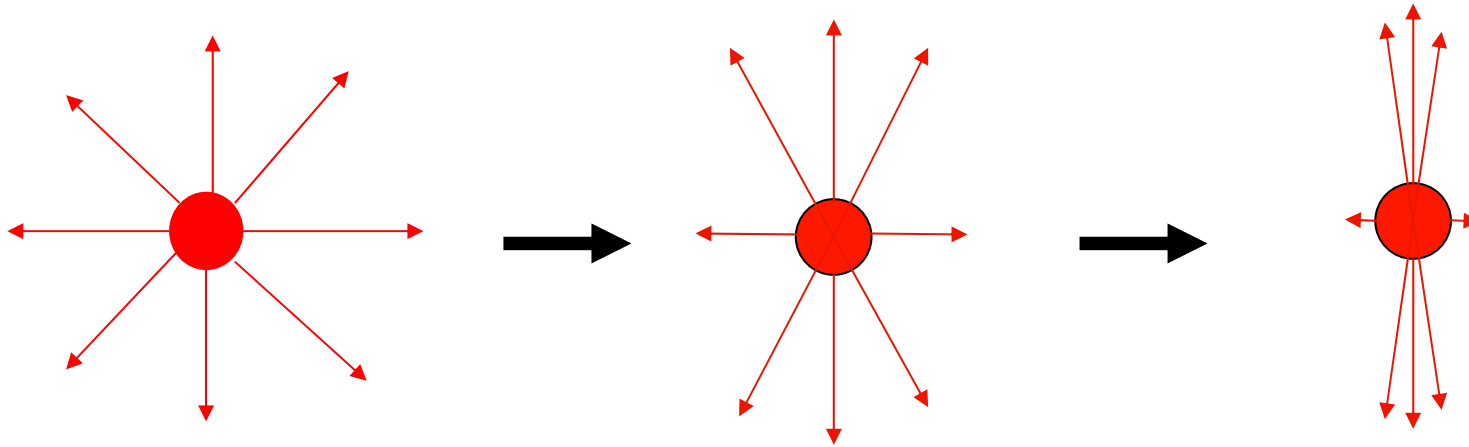
$N_{1,2}$  = Number of particles per bunch

$f$  = revolution frequency

$k$  = number of bunches

$\sigma_{x,y}$  = horizontal and vertical beam size

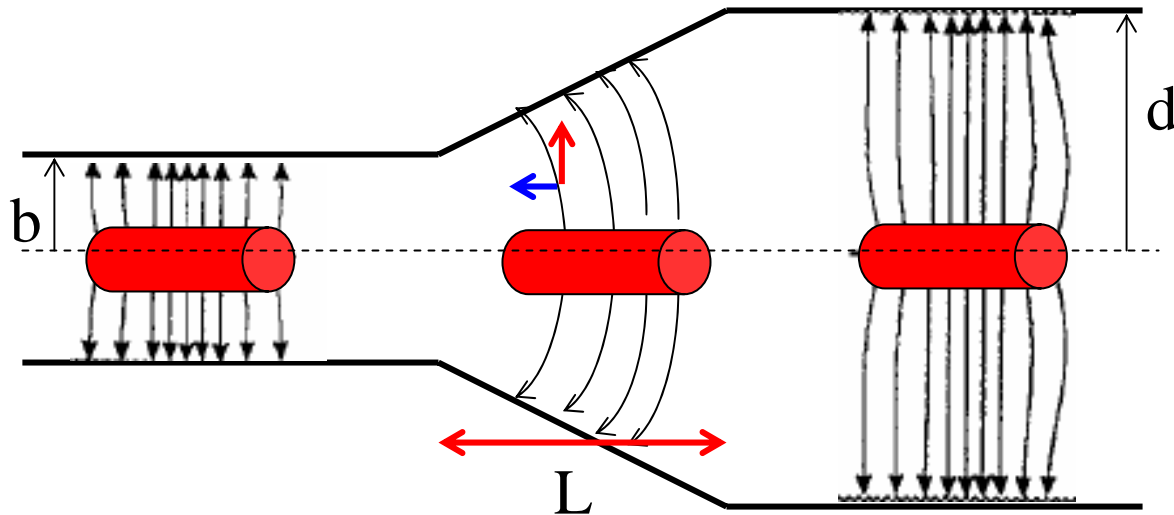
# Intensity: Impedance $Z_L(\omega)$ (1)



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# Impedance $Z_L(\omega)$ (2)

- If conductor is **not perfect**, or, even worse, if  **$b \neq \text{const.}$**

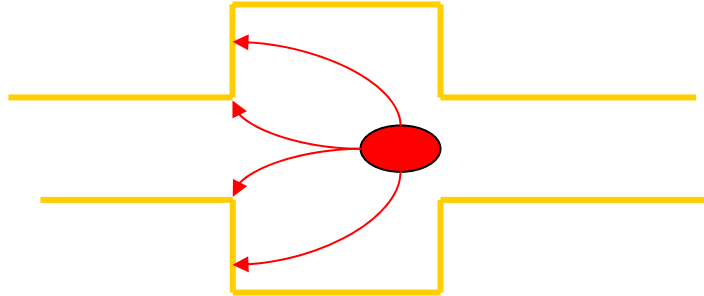


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**$E_s \neq 0$**  => there is an interaction between the beam and the wall!

# Impedance $Z_L(\omega)$ (3)

**Worst case:** abrupt changes in the cross-section of the pipe:



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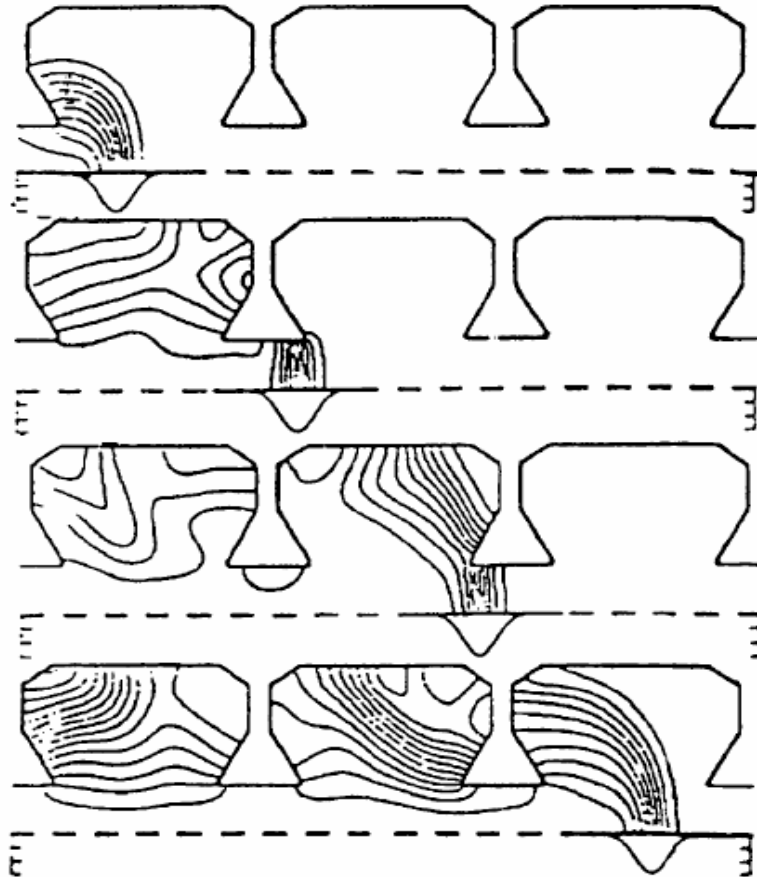
The beam loses energy (heating problems), but the induced fields can act back on the bunch or on the following bunches:

**=> Instabilities!**

# Induced fields

e.m. fields induced in the RF cavities during the passage of a bunch.

The fields can act back either on the bunch itself or, on the following bunches



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# Impedance $Z_L(\omega)$ (4)

Not surprisingly:  $I_{\max} \propto 1/Z_L(\omega)$

- Select carefully the **materials** you are using.
- Avoid any (unnecessary) **change** in the **cross-section**.
- When variations of the cross-section are unavoidable, use **smooth tapers** ( $\alpha \leq 15^\circ$ ).

$Z_L(\omega)$  is a complex function. The quality of the design is characterised by the value of  $|Z/n|$  with  $n=\omega/\omega_0$ .



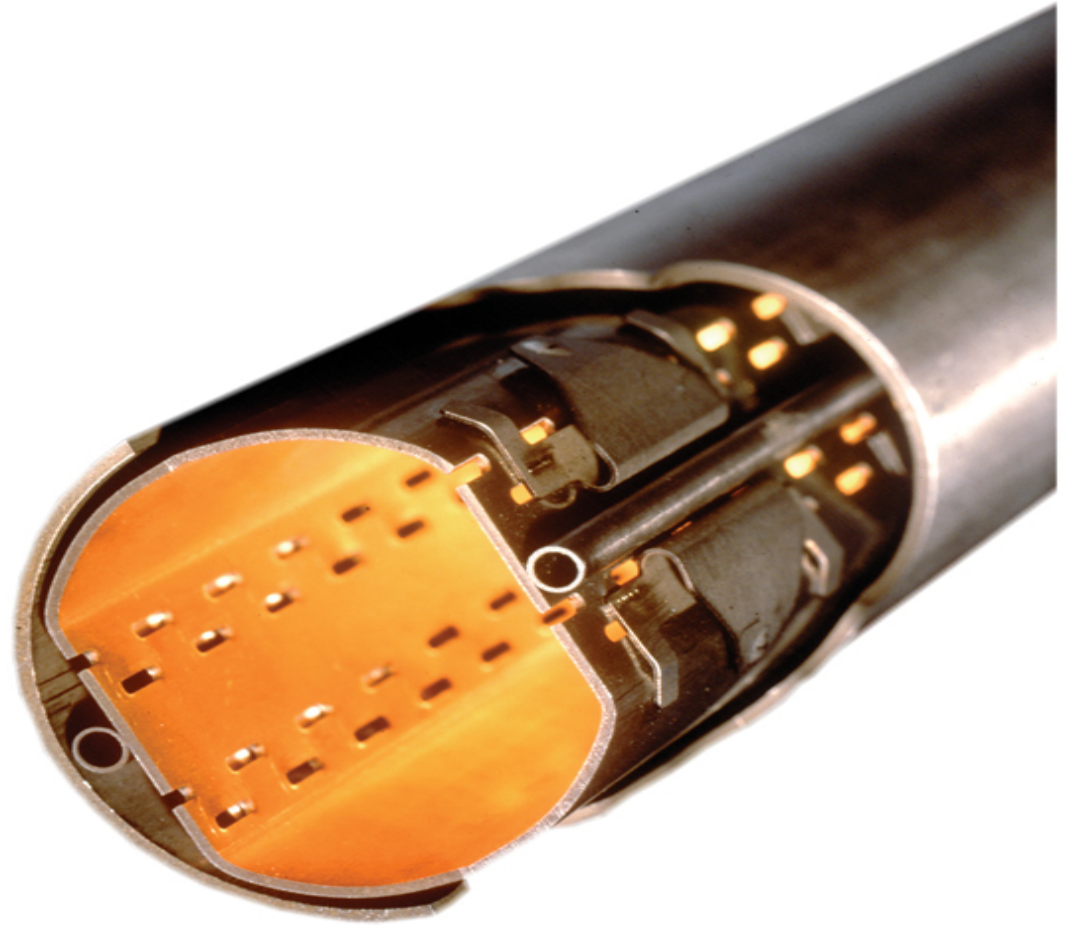


# $|Z/n|$ as a function of time:

Machine	$ Z/n $ [ $\Omega$ ]
PS ( $\sim$ 1960)	$> 50$
SPS ( $\sim$ 1970)	$\sim 20$
LEP ( $\sim$ 1990)	$\sim 0.25$ ( <b>1.0</b> )
LHC ( $\sim$ 2007)	$\sim 0.10$ ( <b>0.25</b> )

# LHC Beam-Screen (material)

- Without proper Cu-coating of the beam-screen, nominal intensity foreseen for the LHC could not circulate in the machine!



# Transverse Impedance $Z_T(\omega)$

- In case of a broad-band impedance, there is a very convenient relation between the longitudinal and the transverse impedances, namely:

$$Z_T(\omega) = (2R/b^2) \cdot |Z_L(\omega)/n| \quad [\Omega/m]$$

This relation clearly shows that **magnets designers**, **vacuum experts** and **financial considerations** might favour solutions which are opposite to those of accelerator physicists!

The only solution is to understand each other's constraints and to find the best possible compromise.



# Vacuum: beam sizes and Lifetime

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- A “poor” vacuum can affect the beam in different ways:
  - Blow-up of the beam sizes (performance).
  - Induce losses by deflecting particles outside the aperture (lifetime):
    - Physical aperture
    - Dynamic aperture
    - RF acceptance



# Gas scattering

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- Particles scatter with residual gas molecules in the vacuum chamber:
  - Two main SINGLE particle effects:
    - Elastic collisions (particles deflected resulting in increase of betatron oscillation => apertures!).
    - Inelastic collisions (particles loose energy either by energy transfer to the residual gas molecule or by photon emission – Bremsstrahlung effect => RF acceptance)



# Elastic scattering

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- Apertures are very important for elastic scattering.
- The lifetime at a given position (s) in the machine for elastic scattering behaves like:

$$\tau \propto 1/\langle \beta_s P_s \rangle$$

i.e. inversely proportional to the product of **gas partial pressure** and the **beta function** => pumping at places with large beta functions is appropriate!

# Synchrotron radiation

- Charged particles bent in a magnetic field emit synchrotron radiation!

Energy loss:

$$eU_0 = A \cdot \gamma^4 / \rho$$

with  $\gamma = E/E_0 = m/m_0$  and  $m_0$  is the rest mass

$$m_0 \text{ proton} = 0.938 \text{ GeV}/c^2$$

$$m_0 \text{ electron} = 0.511 \text{ MeV}/c^2$$

$$(m_{0-p}/m_{0-e})^4 = (1836)^4 \cong 10^{13}$$

Collider	B (T)	E/beam (GeV)	$\gamma$	$eU_0$ (GeV)
LEP ( $e^+ e^-$ )	0.12	100	196000	2.92
LHC (p-p)	8.3	7000	7500	0.00001

# The power is all too real!

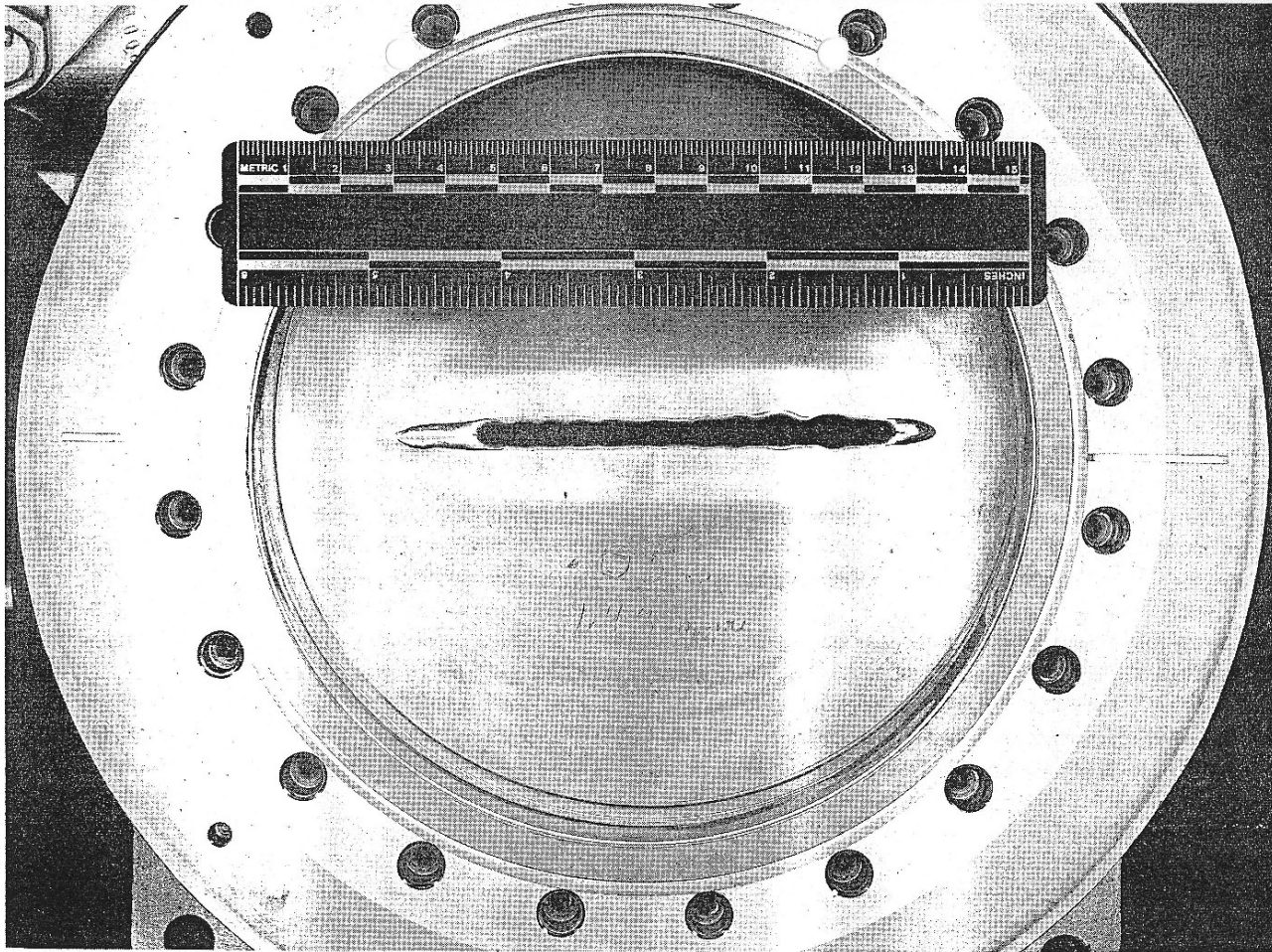


Fig. 12. Damaged X-ray ring front end gate valve. The power incident on the valve was approximately 1 kW for a duration estimated to 2–10 min and drilled a hole through the valve plate.

L. Rivkin

CAS-Trieste2005





# Synchrotron radiation (2)

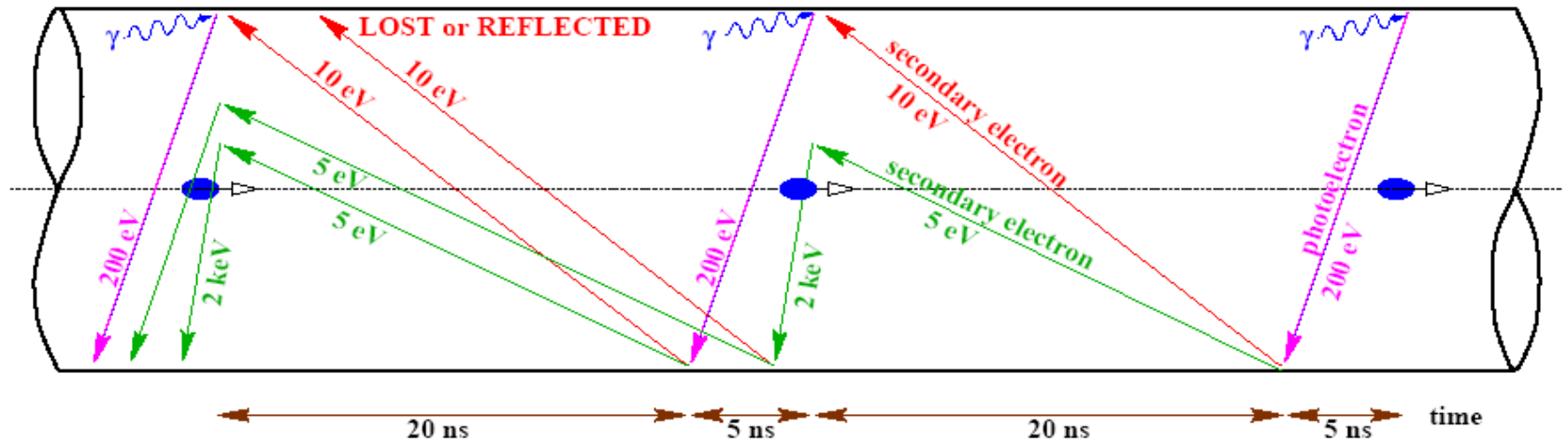
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Desorption of gas molecules by synchrotron radiation is the main source of residual gas in synchrotron light sources!

$$P_{\gamma} \propto E^4/\rho^2$$

# E-cloud effect

- Accumulation of electrons which can perturb the circulating beam.



➤ Strongly depends on intensity, bunch spacing and surface conditioning...



# Conclusions

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- Vacuum design has to take into account the accelerator physics constraints (and vice-versa).
- On top of considering beam-gas interactions, the choice of the correct **materials** and of the most appropriate **geometry** is **essential** in the design phase of the vacuum system.
- A poor vacuum design will definitively affect the optimal operation of the accelerator (**apertures**, **instabilities**).

➤ I hope to have convinced you that the time where experts from individual fields were working alone is definitively over !