Bunch Length Compressors

S. Di Mitri, *Elettra Sincrotrone Trieste*

Outline

- *Why do we need bunch length compressors ?*
  - FELs
- *What kind of compressors ?*
  - Magnetic insertions
- *Longitudinal beam dynamics in a LINAC*
  - Energy chirp
- *Longitudinal beam dynamics in a CHICANE*
  - Transport matrix

Credits and References

- ***Basics:***

  CAS Yellow Report 94-01, Vol.I (1995)

- ***Lectures:***

  S. Di Mitri & M. Venturini, *USPAS Course (2013, 2015)*

- ***Technical Notes:***

  Beam Dynamics Newletter No. 38 (2005)

  P. Emma, *LCLS-TN-01-1 (2001)*

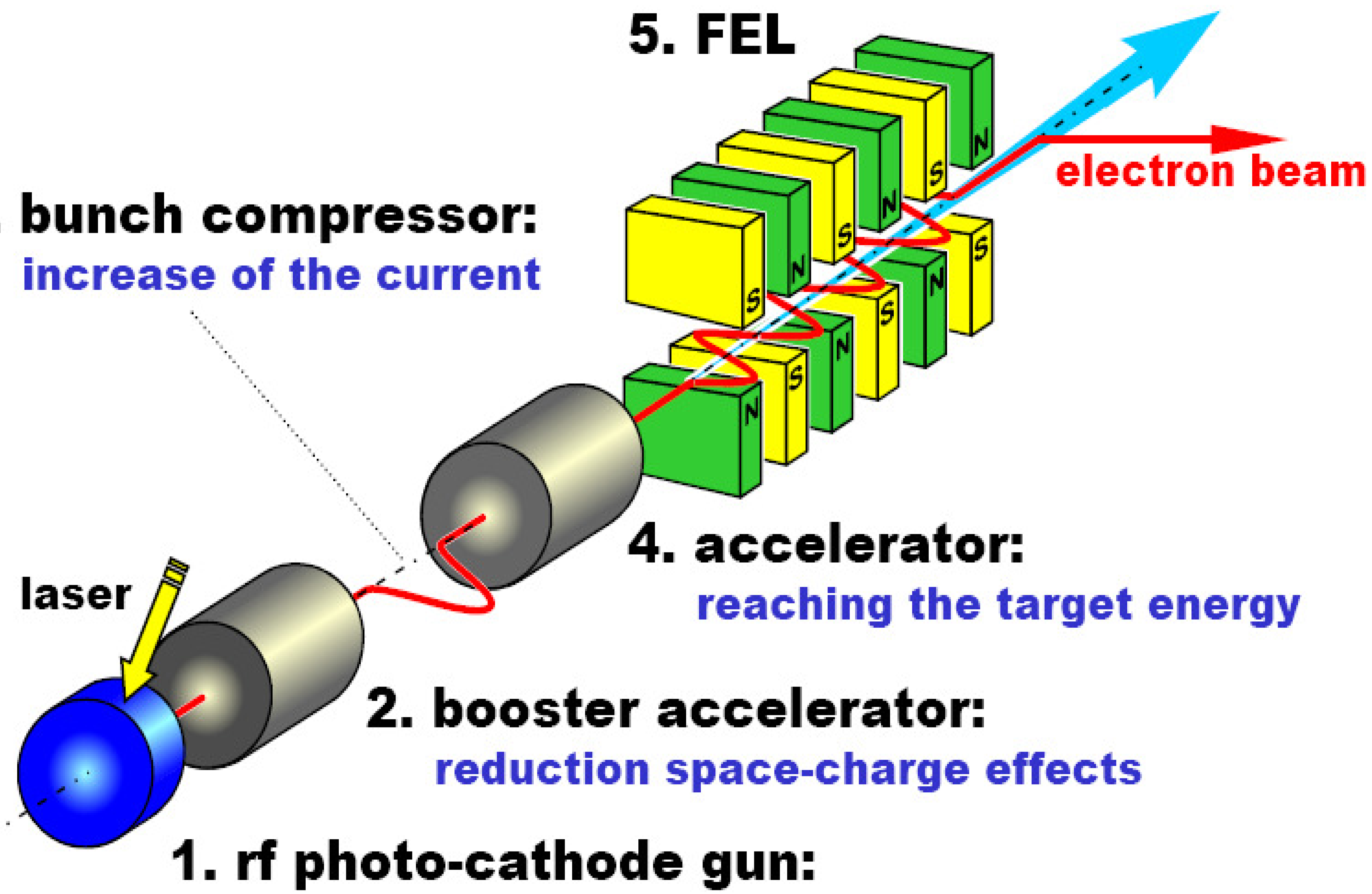
  S. Di Mitri & M. Cornacchia, *Physics Reports 539 (2014)*

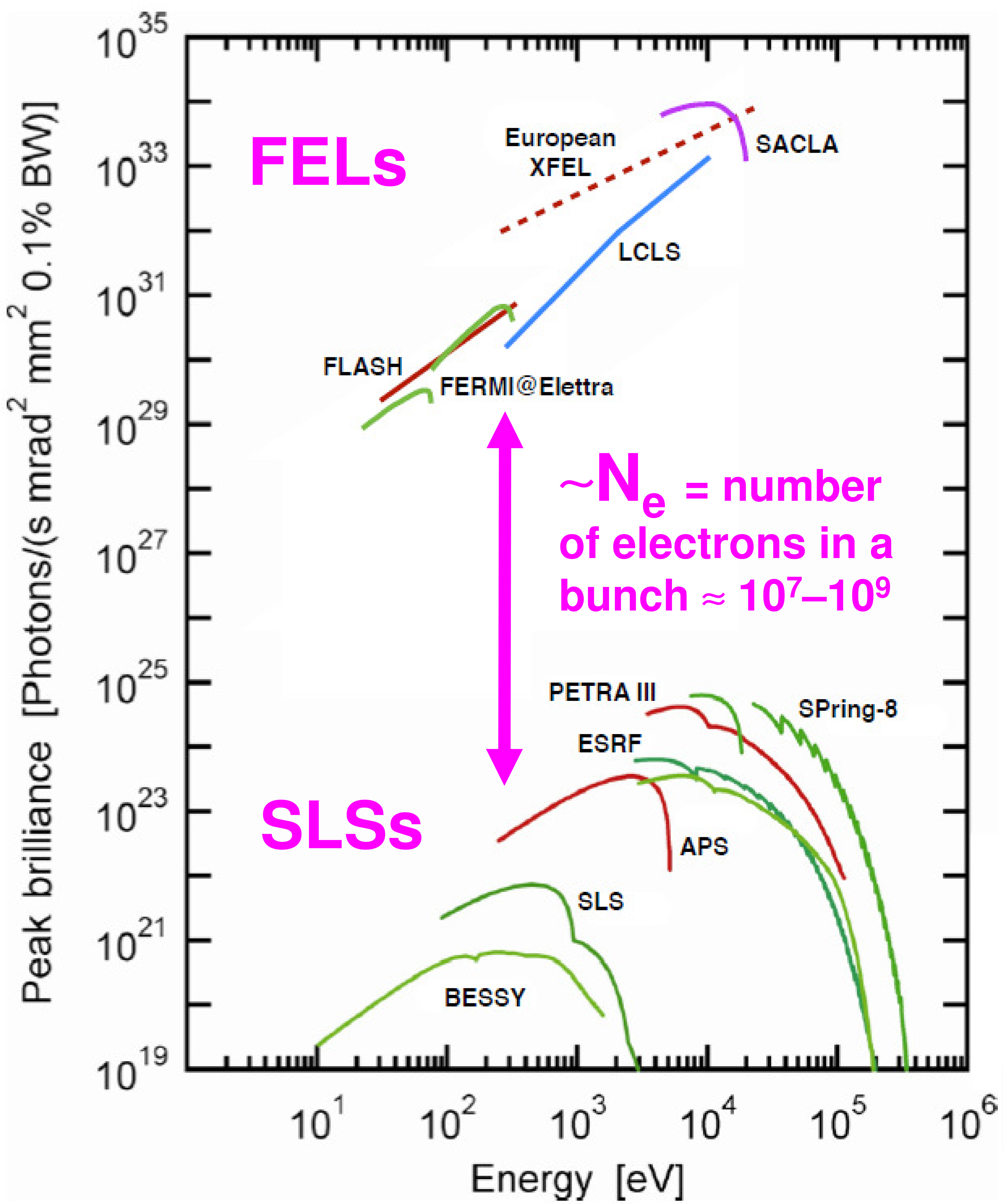
- ***Acknowledgment:***

  M. Venturini, for valuable support and figures

FEL Brilliance

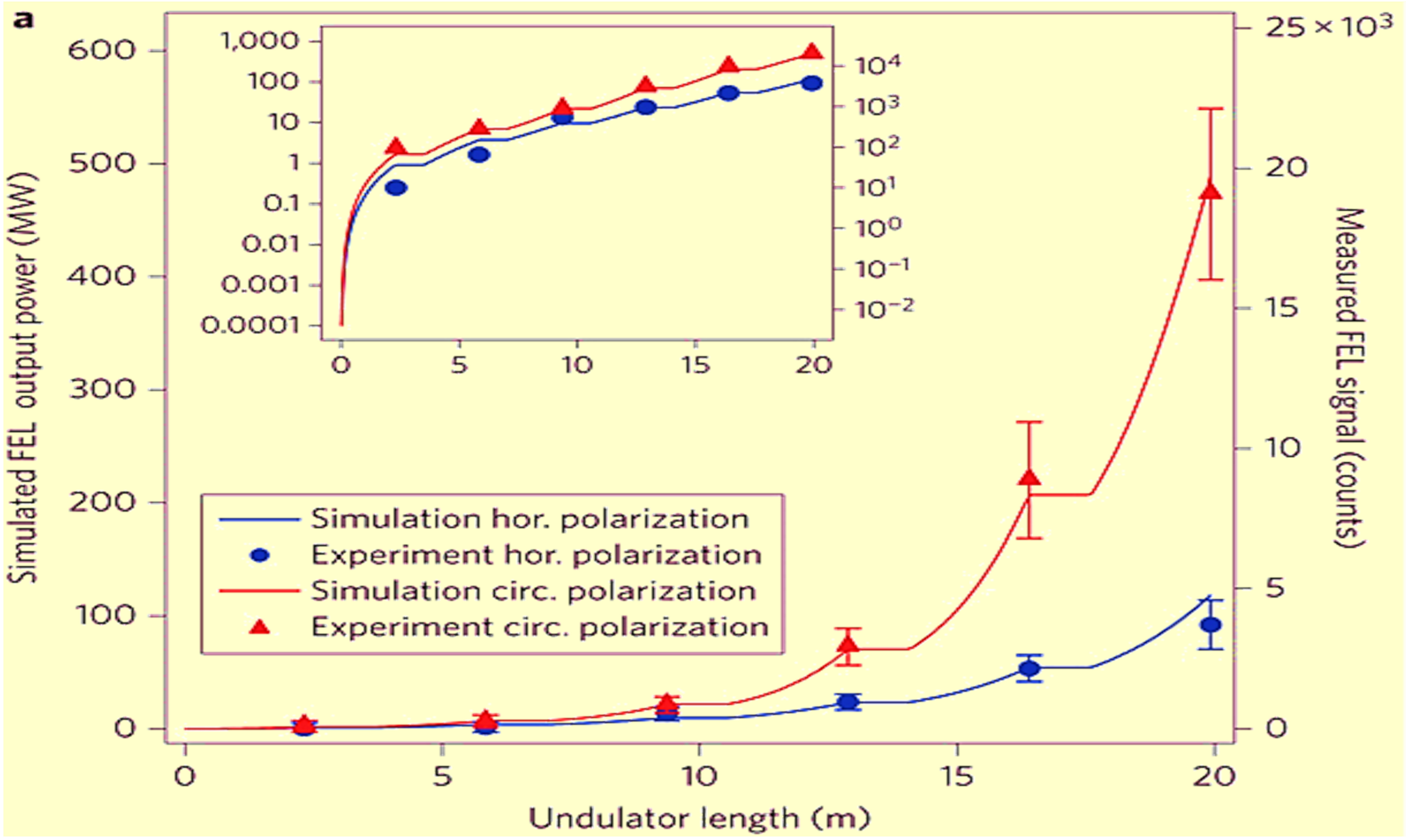
- FEL radiation is generated in undulators.
- Far higher degree of coherence and *peak* intensity than synchrotrons, at same wavelength.





Simone Di Mitri – simone.dimitri@elettra.eu

FEL Gain



Radiation power grows exponentially along the undulator, until *saturation*:

$$\boxed{P(s) = P_0 e^{\frac{4\pi\sqrt{3}}{\lambda_u}\rho s}}$$

Radiation power at saturation is proportional to the e-beam power $P_b = E_b I/e$ (SASE):

$$\boxed{P_{sat} \sim \rho\, P_b}$$

FEL power saturation length: this sets the scale for the undulator length (SASE):

$$\boxed{L_{sat} \sim \frac{\lambda_u}{\rho}}$$

Pierce or FEL parameter, ρ: the jack of all trades of ***1D FEL theory***. Typically $\rho \lesssim 10^{-3}$ for UV and X-rays.

e-beam peak current ***Undulator parameter***

$$\boxed{\rho = \frac{1}{4}\left[\frac{1}{\pi^2}\ \frac{I}{I_A}\ \frac{\lambda_u^2}{\gamma^3\sigma^2}(K \times JJ[K])^2\right]^{1/3}}$$

e-beam energy ***e-beam transverse size***

A **high peak current** makes ρ **large**:
⇒ large FEL gain, ⇒ high saturation power,
⇒ short saturation length.

Elettra Sincrotrone Trieste

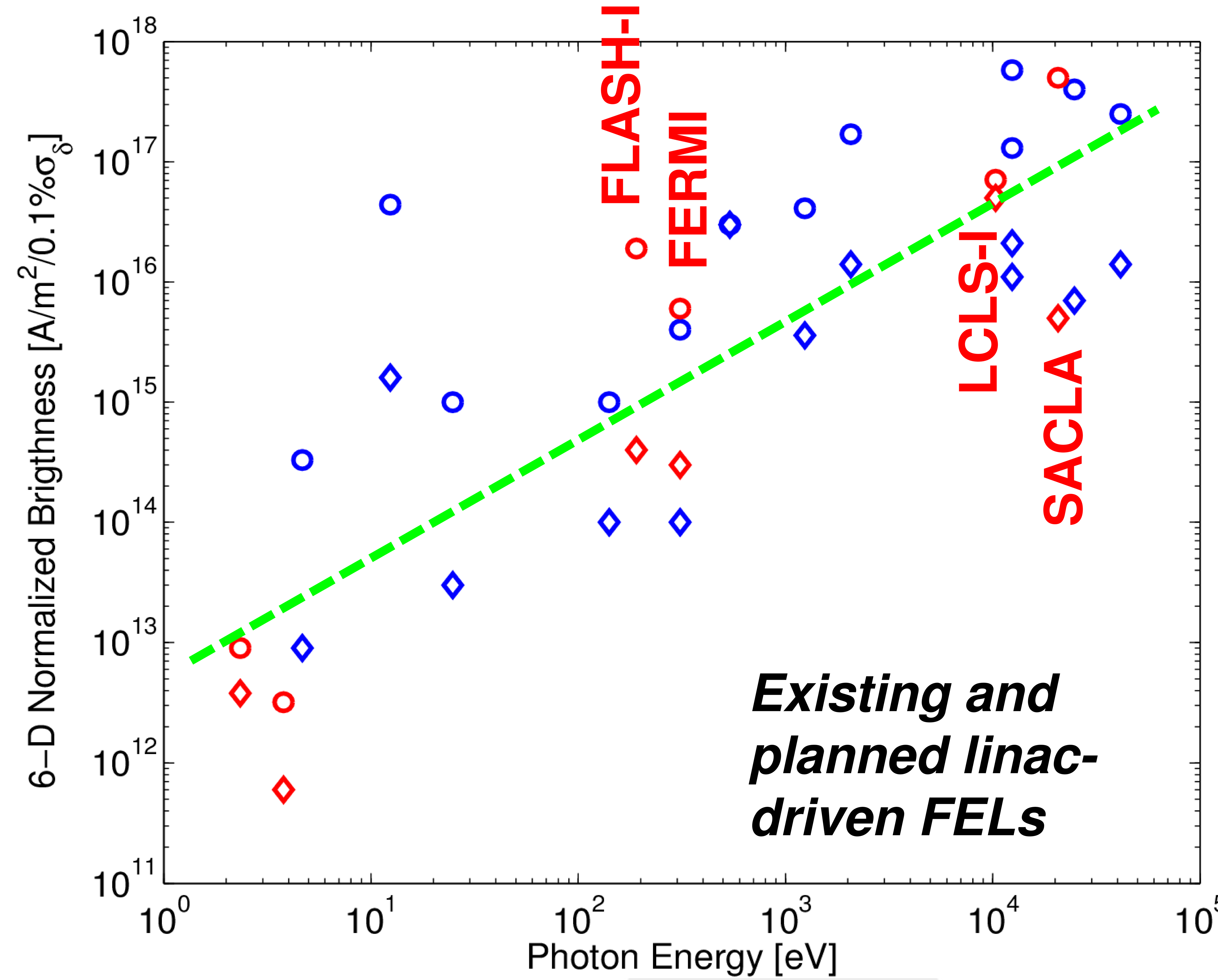
e-Beam Brightness

The **6D e-beam brightness** measures the charge density in the 6D phase space (energy-normalized). For a "diffraction limited" [$4\pi\varepsilon_{x,y}=\lambda$], "cold" [$\sigma_\delta<\rho$] e-beam, the **higher the brightness**, the **shorter the FEL wavelength** achievable with a decent power….

Bunch charge

$$B_{n,0}(\lambda) \equiv \frac{Q}{\underbrace{\varepsilon_{n,x}\varepsilon_{n,y}\varepsilon_{n,z}}_{\text{6-D energy-normalized emittance}}} = \frac{I}{c\sigma_E\gamma_0^2\varepsilon_0^2} \approx \frac{32\pi^2}{c}\frac{1}{\sigma_E\lambda_u\left(1+K^2/2\right)}\frac{I}{\lambda}$$

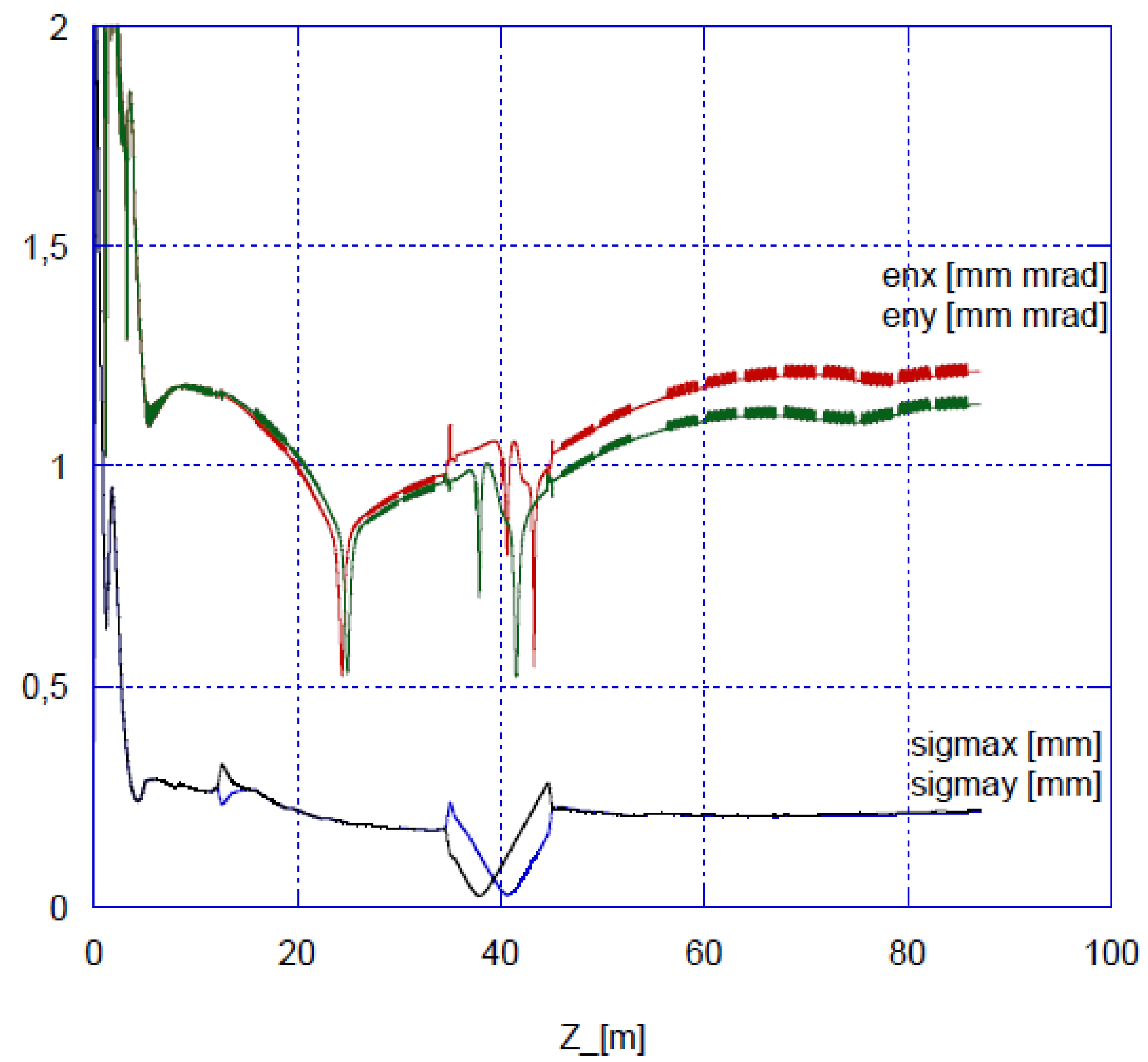
A **high peak current** makes the e-beam **6D brightness large** $\Rightarrow$ short FEL wavelengths with reasonable amount of power.



Why not Short Bunches from Injectors ?

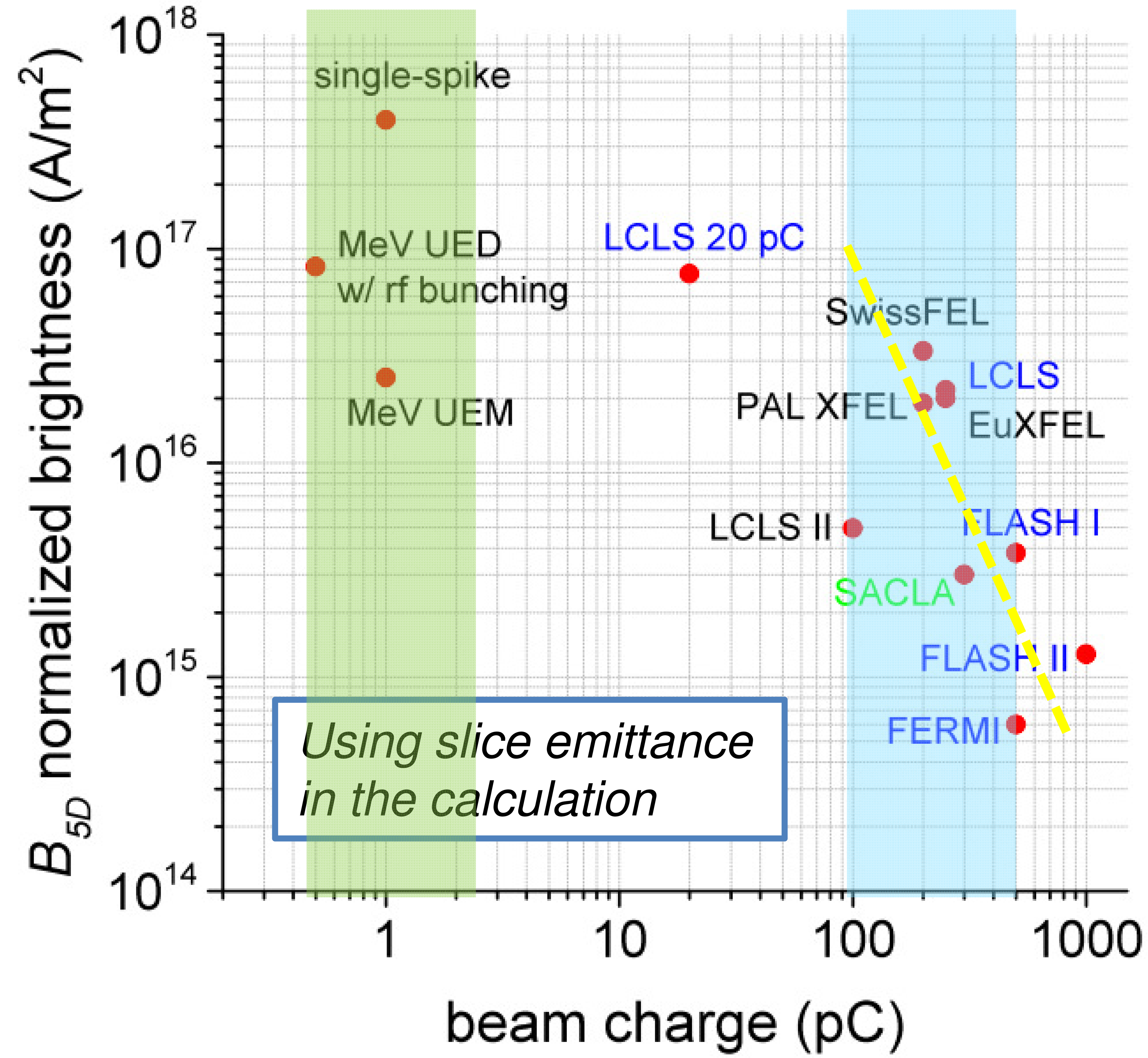
Very short bunches at low energy ($K \approx m_e c^2$) are diluted in the 6-D phase space by **"space charge" forces** (inter-particle Coulomb interaction).

- The intra-bunch repulsive force is stronger at lower beam energies ($\sim 1/\gamma^2$), and at higher charge density ($\sim I$).



$$\varepsilon_{tot} = \sqrt{(\varepsilon_{cathode}\,\sigma_r)^2 + (F\frac{Q}{\sigma_r^2\sigma_z})^2}$$

Transverse Emittance ~ $1/\sigma_z$

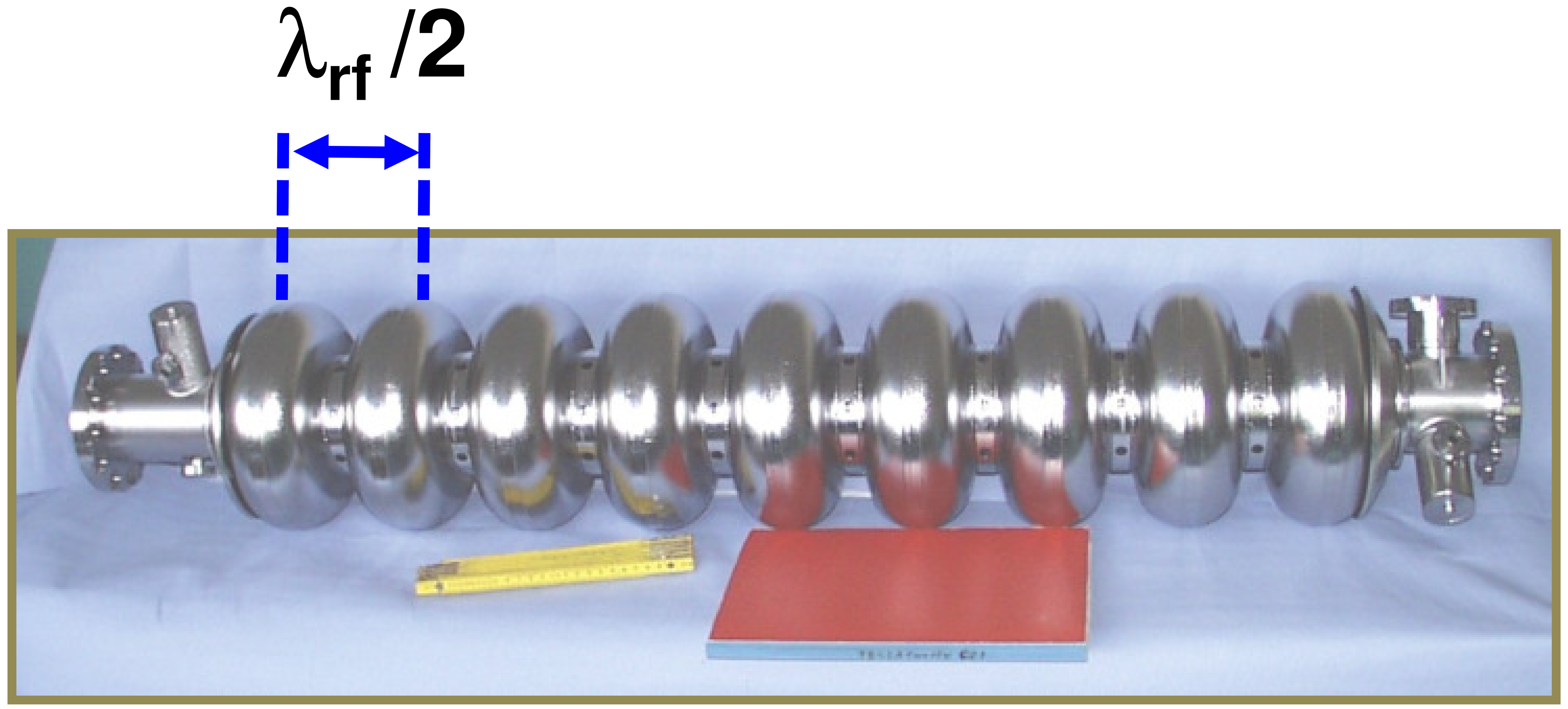


Bunch length compressor(s) are needed at beam energies higher than 100s of MeV to reach **100s A to kA peak current level**

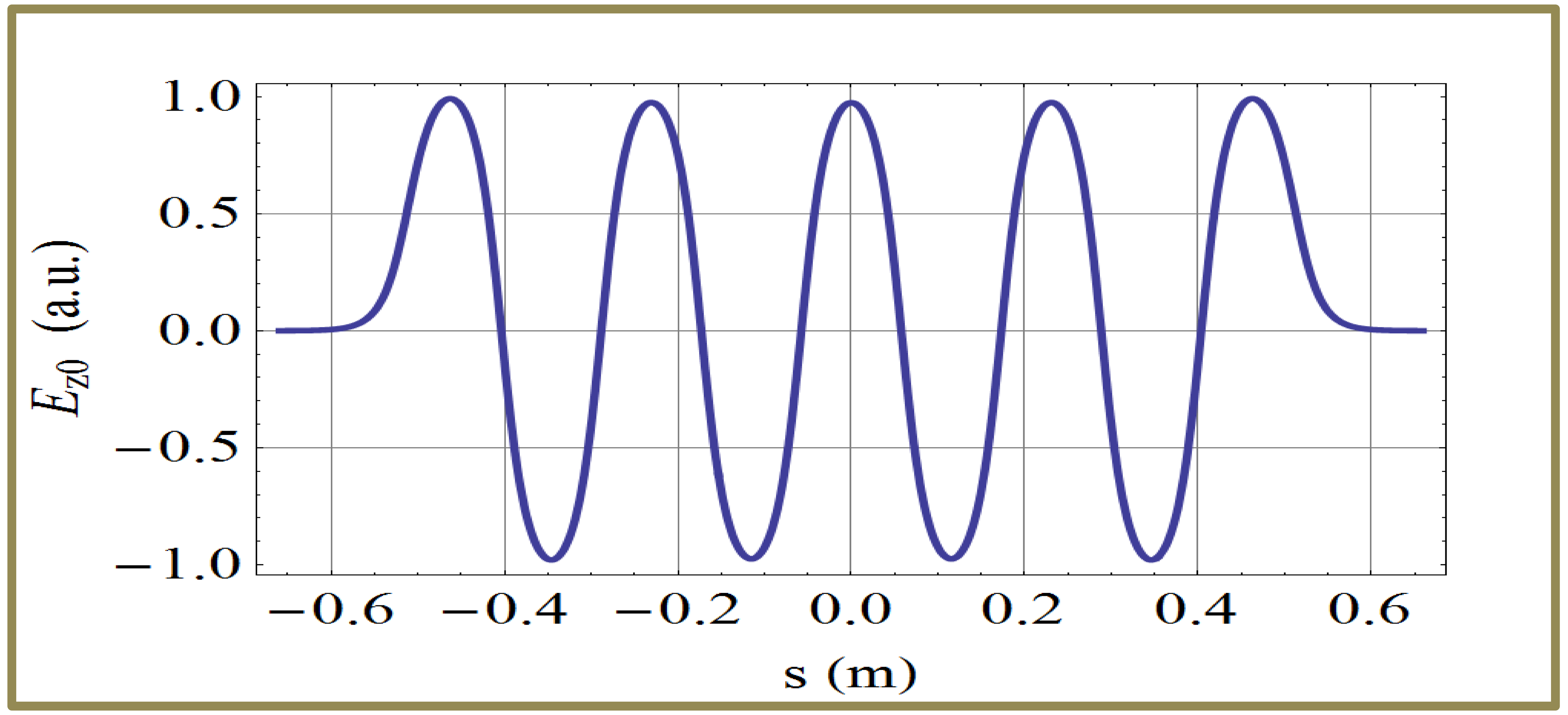
RF Structure (Standing-Wave)

- Dynamics is driven by the longitudinal component of electric field, E_z [MV/m].
- Consider **standing-wave** structures (traveling-wave structures have similar treatment).

1.3GHz , Super Conducting 9-Cell Tesla RF cavities are operated as Standing-Wave structures



On-axis Longitudinal E-field for TESLA Cavity



- Design structures so that, as the electron moves from cell to cell, it sees the same E_z:
  - the electron travels through one cell in half rf period,
  - cell length is half the rf wavelength: $\lambda_{rf} = \frac{c}{f_{rf}}$ ("π-mode").

Longitudinal Dynamics

❑ **On axis (x=y=0) electric field in a cell [-g/2, g/2]:**

$$E_z(s) = E_0(s)\cos(\omega_{rf}t(s)+\varphi_{rf}) \cong E_{z,0}\cos(k_{rf}s)\cos(\omega_{rf}t_{syn}+\omega_{rf}\Delta t+\varphi_{rf}) =$$
$$= E_{z,0}\cos(k_{rf}s)\cos(k_{rf}s+k_{rf}z+\varphi_{rf}) =$$
$$= E_{z,0}\cos(k_{rf}s)\left[\cos(k_{rf}s)\cos(k_{rf}z+\varphi_{rf})-\sin(k_{rf}s)\sin(k_{rf}z+\varphi_{rf})\right];$$

❑ **Energy change by an electron with coordinate z:**

$$\boxed{\Delta E(g,z)} = -e\int_{-g/2}^{g/2}E_z(s)ds \cong \left[-eE_{z,0}\int_{-g/2}^{g/2}ds\cos^2(k_{rf}s)\right]\cos(k_{rf}z+\varphi_{rf}) \equiv \boxed{e\Delta V(g)\cos(k_{rf}z+\varphi_{rf})}$$

Acceleration Peak Gradient

"Transit Time Factor"

Acceleration Peak Voltage

RF Phase (synchronous for z=0)

Approximations and Notes.

Fundamental mode of E-field.

Time of arrival of any particle relatively to the **reference ("synchronous") particle**: $\Delta t(s) = t(s) - t_r(s)$

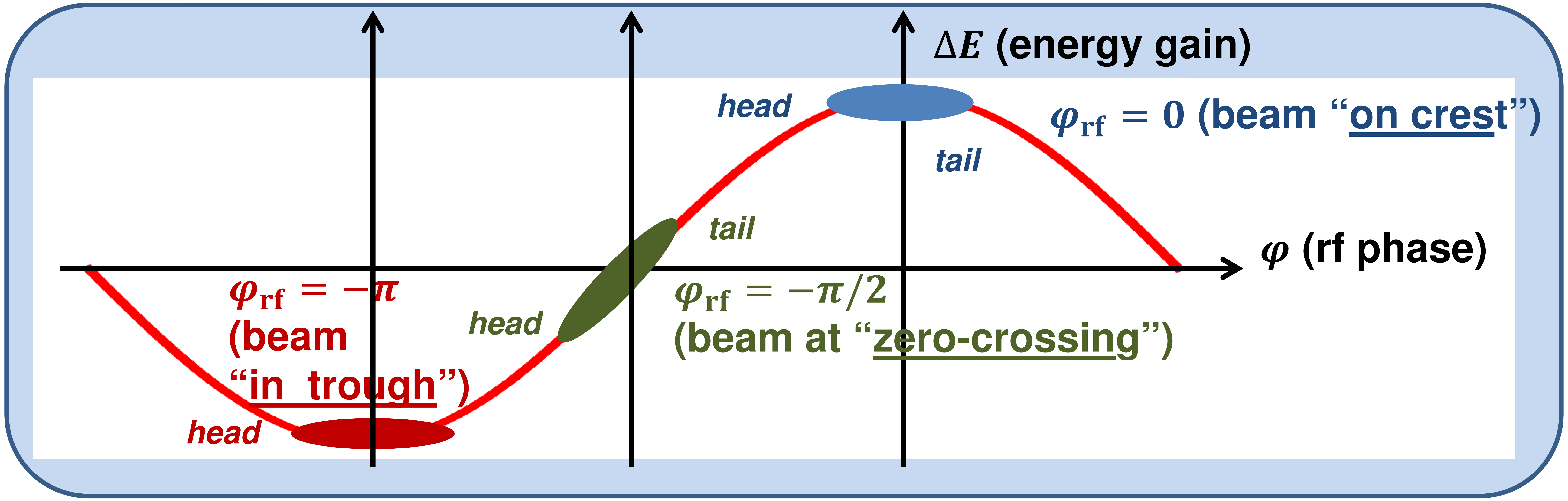
$z = \Delta t/c$

What's the meaning? $t(s)$ is the arrival time of the electron measured by an observer at longitudinal position s.

Δt(s)<0 or z<0, means particle is ahead of reference particle (it arrives earlier at s)

Ultra-relativistic particles: $\frac{ds}{dt} \simeq c$

How to Choose the RF Phase



"zero-phase is on crest"
rf-phase convention:

$$\boxed{\Delta E(z) = eV\cos(k_{\mathrm{rf}}z + \varphi_{\mathrm{rf}})}$$

- For maximum acceleration, the cavities should be operated on crest…

Q: Why do we ever want to operate the cavities off-crest?

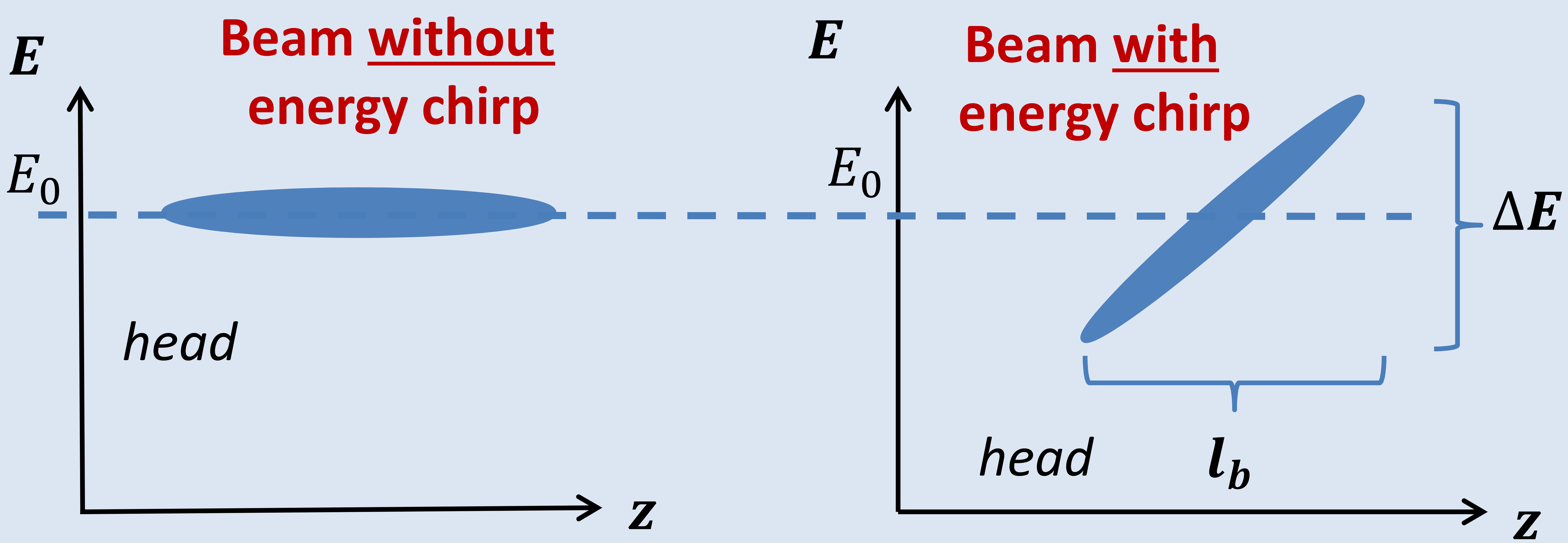
A: *To control the beam "**energy chirp**", i.e. the correlation between a particle position z within the bunch and its energy E*

- The ability to put an energy chirp on a beam is needed to do bunch compression through a magnetic insertion.

Elettra Sincrotrone Trieste

Energy Chirp

Electron beam longitudinal phase space



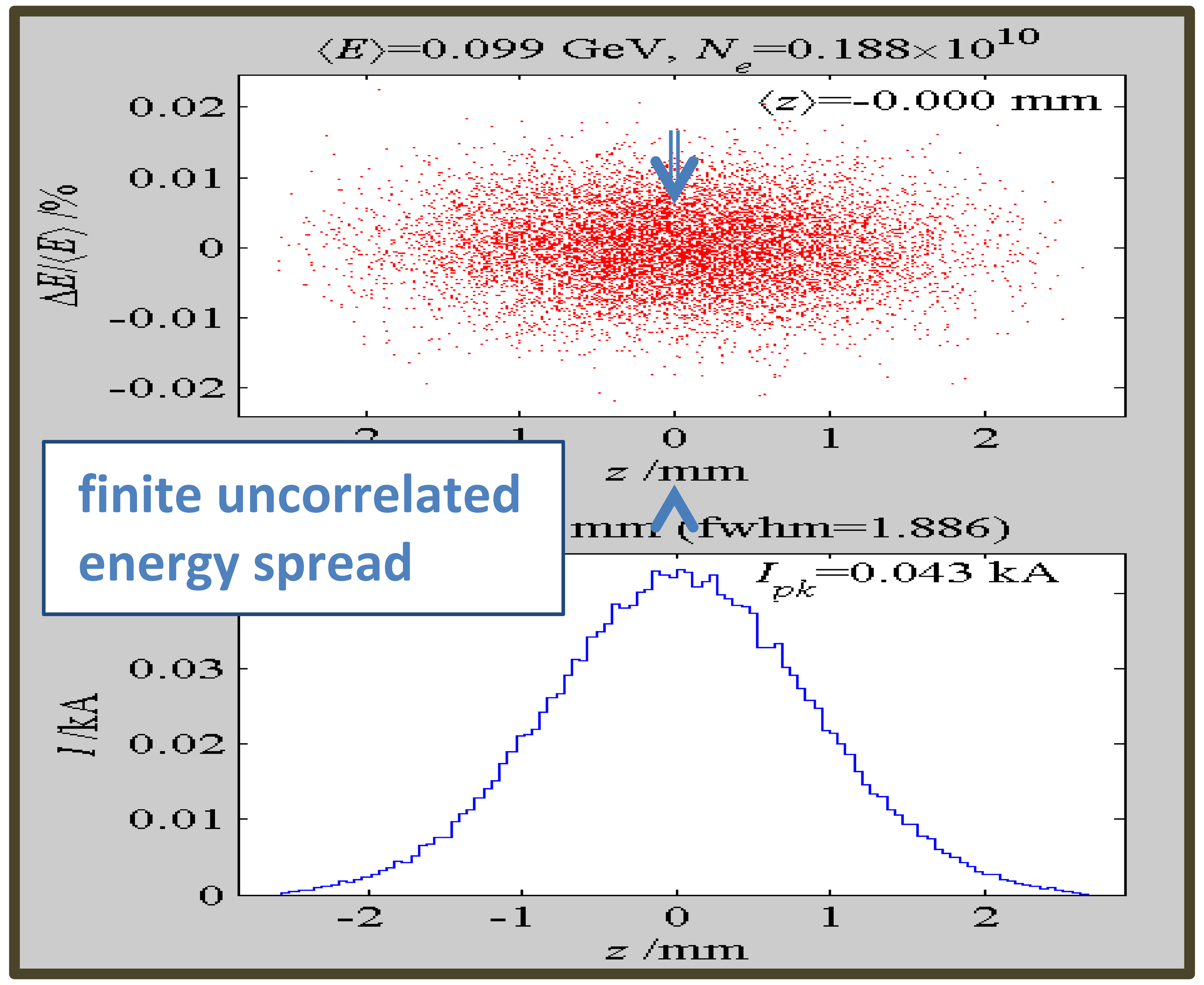
Taylor expand the energy through first order in z:

$$E(z) = E_i + eV\cos(k_{rf}z + \varphi_{rf}) \approx$$
$$\approx E_i + eV\cos\varphi_{rf} - eVk_{rf}\,z\sin\varphi_{rf} + O(z)^2$$

Linear chirp *at exit of structure*:

$$h_1 = \frac{1}{E(0)}\frac{dE(0)}{dz} = -\frac{eVk_{rf}\sin\varphi_{rf}}{(E_i + eV\cos\varphi_{rf})} \cong \frac{\sigma_{\delta corr}}{\sigma_z}$$

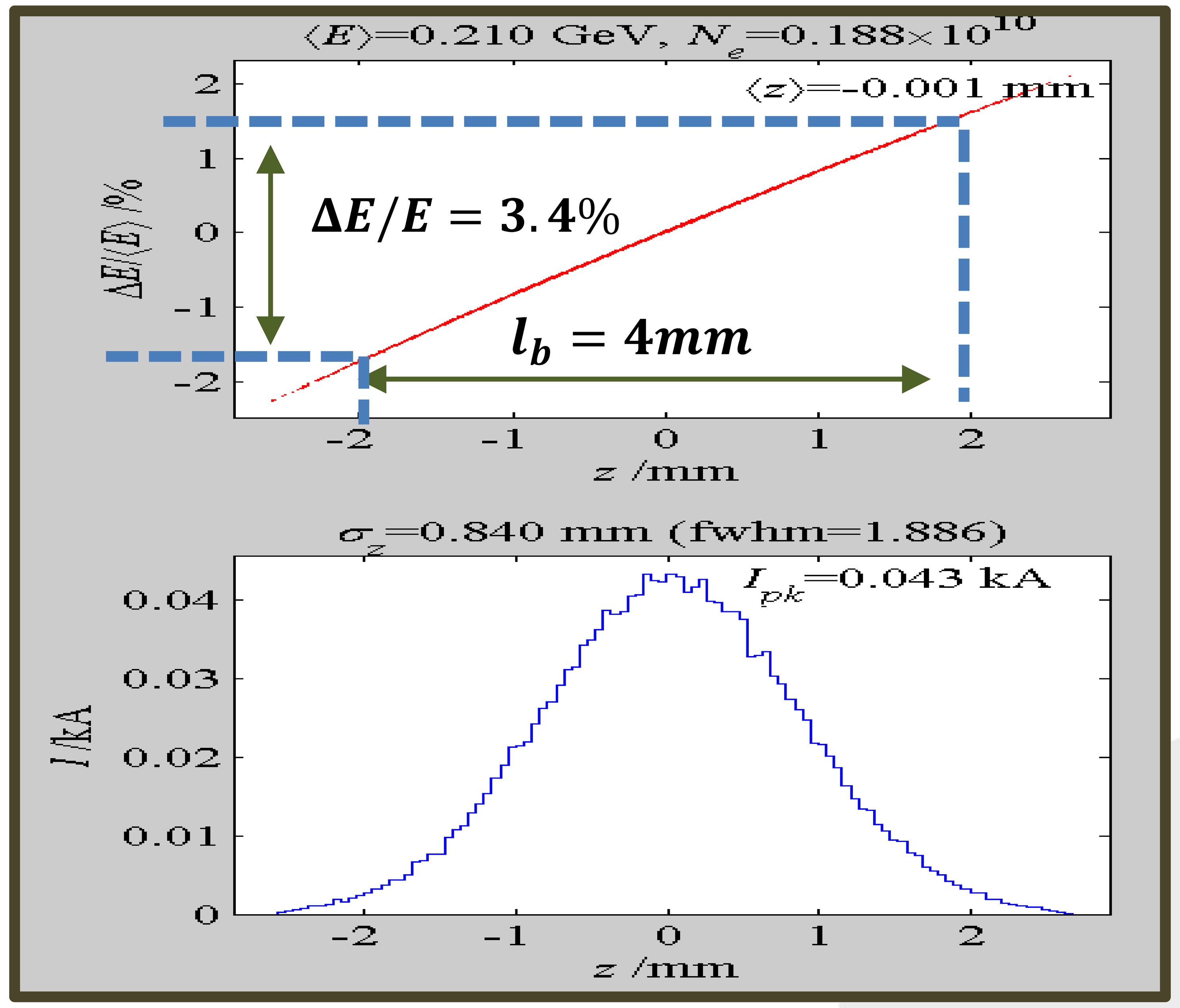
Beam @ entrance of structure



Off-crest acceleration:

f_{rf} =1.3 GHz
$\lambda_{rf} = 23\ cm$
$V_0 = 129\ MV$
$\varphi_{rf} = -30.3^o$

$$h_1 \sim \frac{0.034}{0.004m} = 8.5m^{-1}$$



Beam @ exit of structure

Simone Di Mitri – simone.dimitri@elettra.eu

Slippage of Ultra-Relativistic Particles

- Compress the bunch length: we need to change the electrons' longitudinal coordinate z (inside the bunch).
- We have problem: equation of motion of ultra-relativistic electron (trough an accelerating structure or transport line) gives:

$$\boxed{\frac{dz}{ds} \simeq 0}$$

Relative longitudinal position of particles in the bunch does not change (the beam is 'frozen').

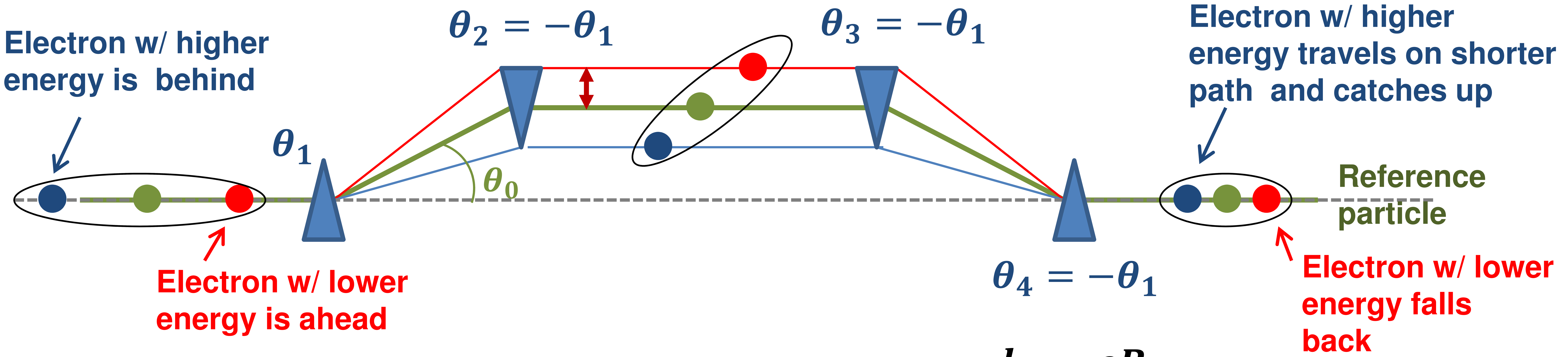
- However, $\boxed{\frac{dz}{dE} \neq 0}$ $\quad E(z) \approx E_i + eV\cos\varphi_{\mathrm{rf}} - eVk_{\mathrm{rf}}\,z\sin\varphi_{\mathrm{rf}} \equiv \left[(m_e c^2)^2 + (p_z(z)c)^2\right]^{1/2}$

- And the Lorentz force depends on the particle momentum: $p_z(z)c = eB_y R(z)$ $\quad$ $p_z[\mathrm{GeV}/c] = 0.2998 \cdot B_y[\mathrm{T}] \cdot R(\mathrm{m})$

R is radius of curvature

1. Establish a ***(z,E) correlation*** [energy chirp]
2. Pass through a ***magnetic field*** [magnetic insertion]
3. Particles with different energy will follow ***different paths***.
4. For same velocity (v≈c), different path lengths will lead to ***different arrival time***.
5. The bunch is '***time-compressed' !***

4-Dipoles C-shape Chicane



- Bend angle for **on-momentum** (reference) particle: $\theta_0 \simeq \frac{l_B}{R} = \frac{eB}{p_{z,0}} l_B$
- Bend angle for a particle **off-momentum**: $\theta = \frac{\theta_0}{1+\delta}$ $\quad \delta \equiv \frac{\Delta p}{p_0} \simeq \frac{\Delta E}{E_0}$ (ultra-relativistic approx.)
- The system is an **achromat** by design (barring magnet errors/imperfections): $\theta_1 + \theta_2 + \theta_3 + \theta_4 = 0$
- **Energy-dispersion function:**

$$\eta_x(s) = \frac{\Delta x(s)}{\Delta p_z / p_{z,0}} \to R(1-\cos\theta) \text{ for a single dipole}$$

$$\eta'_x(s) = \frac{\Delta x'(s)}{\Delta p_z / p_{z,0}} \to \sin\theta \quad \text{for a single dipole}$$

DEFs.

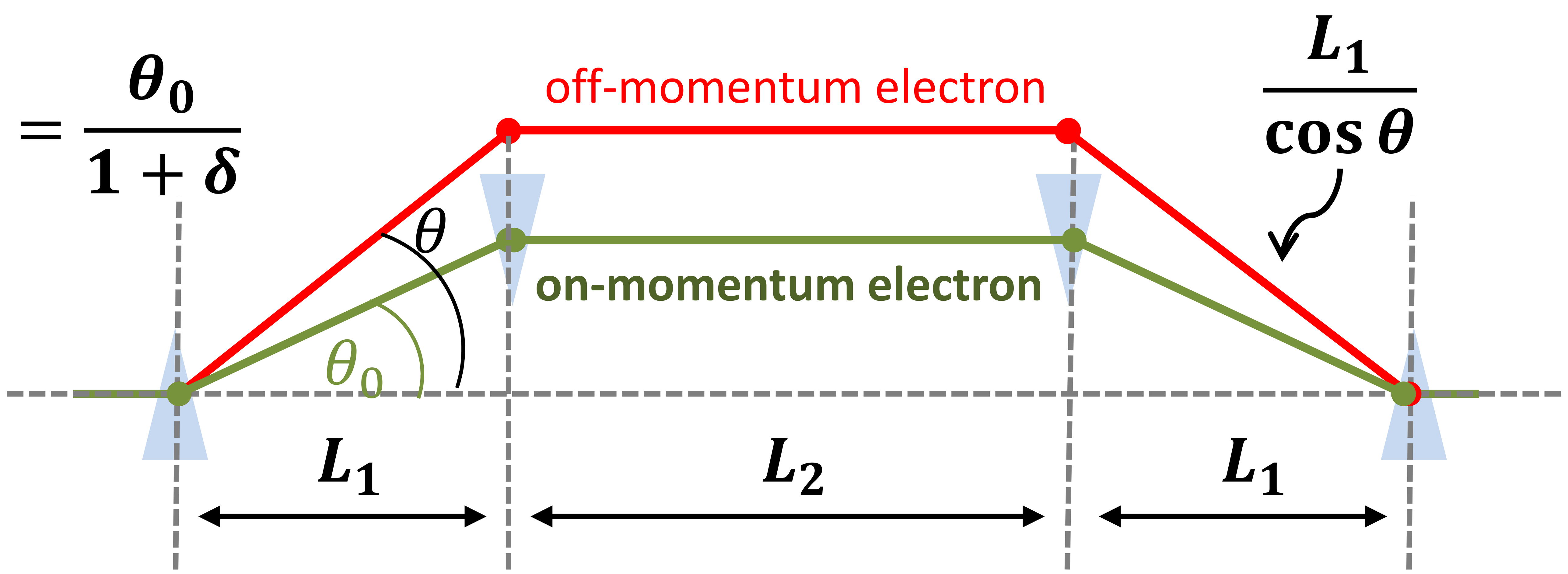
$x' := dx/ds$

$\Delta x(s) := x(s) - x_{ref}(s)$

$\Delta p_z := p_z - p_{z,0}$

Momentum Compaction

$$\theta = \frac{\theta_0}{1+\delta}$$



- **Longitudinal slippage?** $z_f = z_i + R_{51}x_i R_{52}x_i' + R_{56}\delta_i$
- **What is R_{56} for a chicane?** ($R_{51} = R_{52} = 0$ by design)
  - Since $z_f - zi = \Delta z = \Delta s \cong -2L_1\,\theta_0^2\delta_i$, we find:

$$R_{56} \cong -2L_1\theta_0^2$$

For finite dipoles' length L_b:

$$\boxed{R_{56} = -2\theta_0^2\left(L_1 + \frac{2}{3}L_b\right)}$$

- Thin lens approximation for the dipoles (finite bend angle resulting from infinitesimally short dipole and infinitely large magnetic field): $\theta = \frac{L_{B\to 0}}{R_{B\to 0}} = finite$
- **Path-length of off-momentum electron:** $s = \frac{2L_1}{\cos\theta} + L_2$
- **Path-length of on-momentum (reference-particle) electron:** $s_0 = \frac{2L_1}{\cos\theta_0} + L_2$
- **Path-length difference:**

General expression:

$$R_{56}(0\to s) = \int_0^s ds'\frac{\eta_x(s')}{R(s')}$$

"Momentum Compaction":

$$\alpha_c := \frac{R_{56}}{L_{tot}}$$

Simone Di Mitri – simone.dimitri@elettra.eu

Compression Factor

- Longitudinal action through the chicane:

$$z_f = z_i + R_{56}\delta_i$$

energy spread *correlated* with z

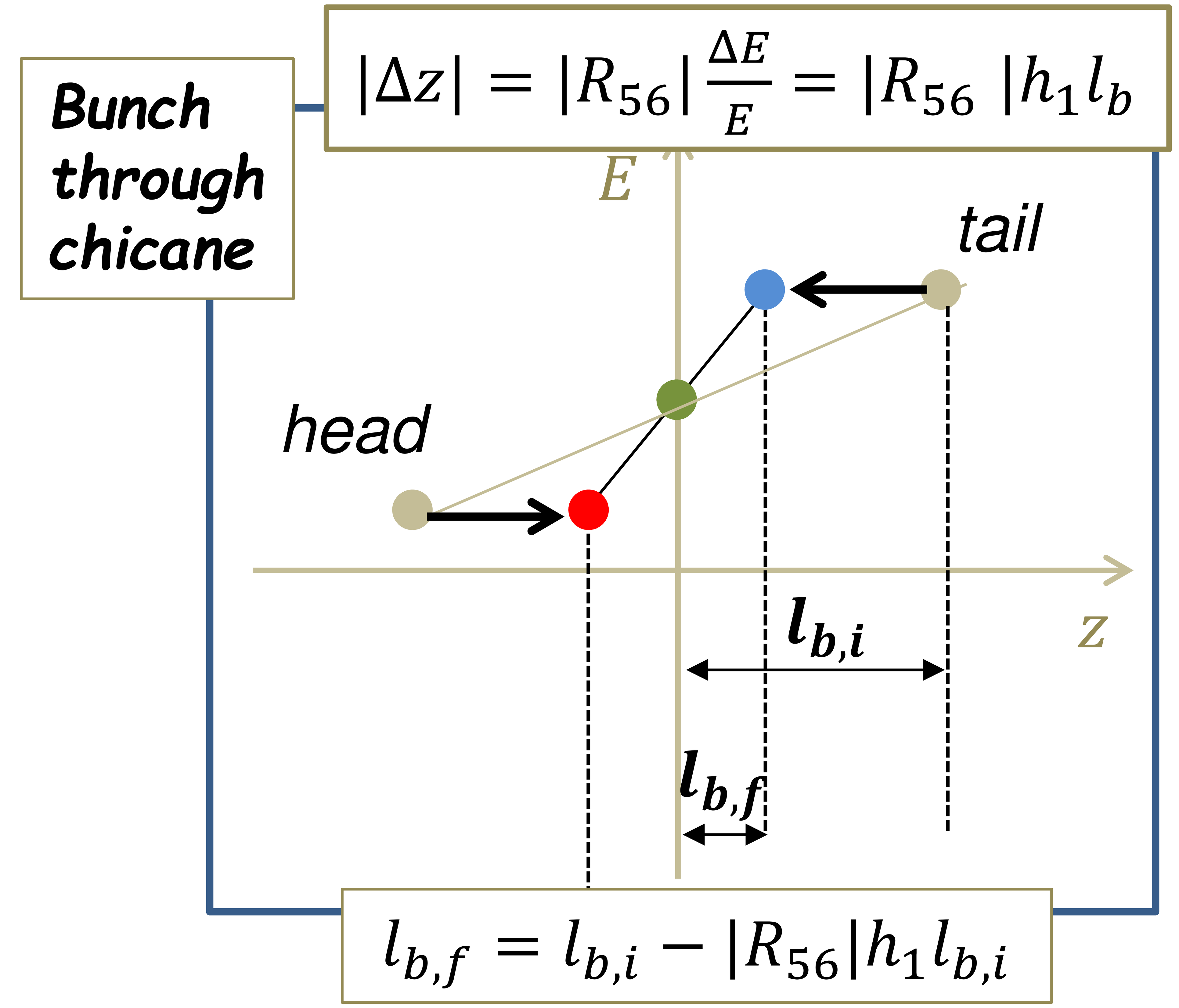
***uncorrelated* energy spread**

- Differentiate (pass from local position to bunch length):

$$dz_f = dz_i + R_{56}d\delta = dz_i + R_{56}\frac{dE}{E_0} = dz_i\left(1 + R_{56}\frac{1}{E_0}\frac{dE(z)}{dz_i}\right) + R_{56}\frac{dE_{unc}}{E_0} =$$

$$= dz_i\left(1 + hR_{56}\right) + R_{56}\delta_{unc} \equiv dz_i / C + R_{56}\delta_{unc}$$

$$\boldsymbol{C \equiv \left|\frac{dz_0}{dz_1(z_0)}\right| = \frac{l_{b,i}}{l_{b,f}} = \frac{1}{|1 + R_{56}h_1|}}$$



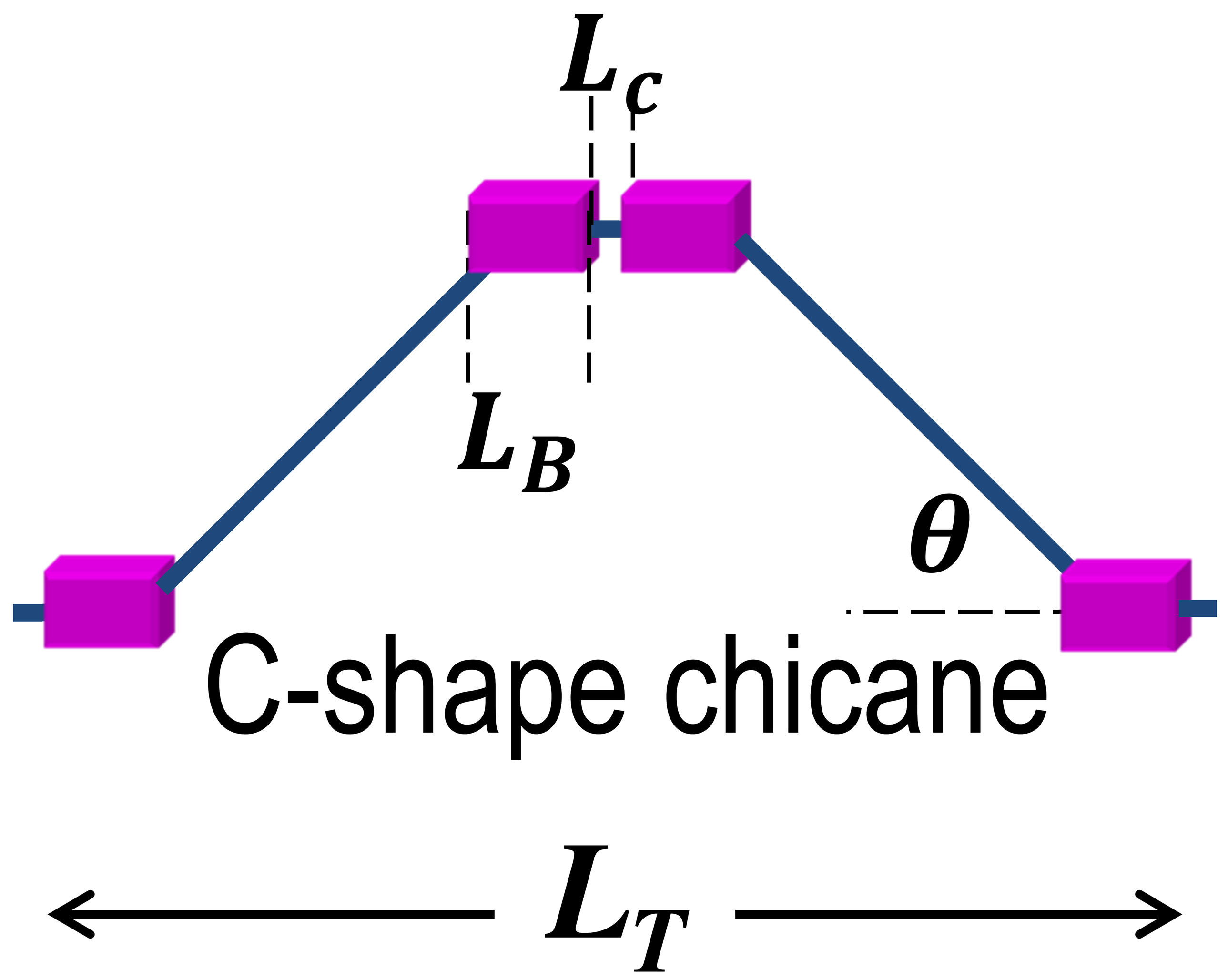
- *If $E(z)$ - the energy chirp - is nonlinear, then C depends on z (compression will vary along bunch). Generally, we refer to $C(z = 0)$ as the nominal (linear) compression factor.*

- *When $C \to \infty$, the minimum bunch length is set by the "**uncorrelated" energy spread**:*

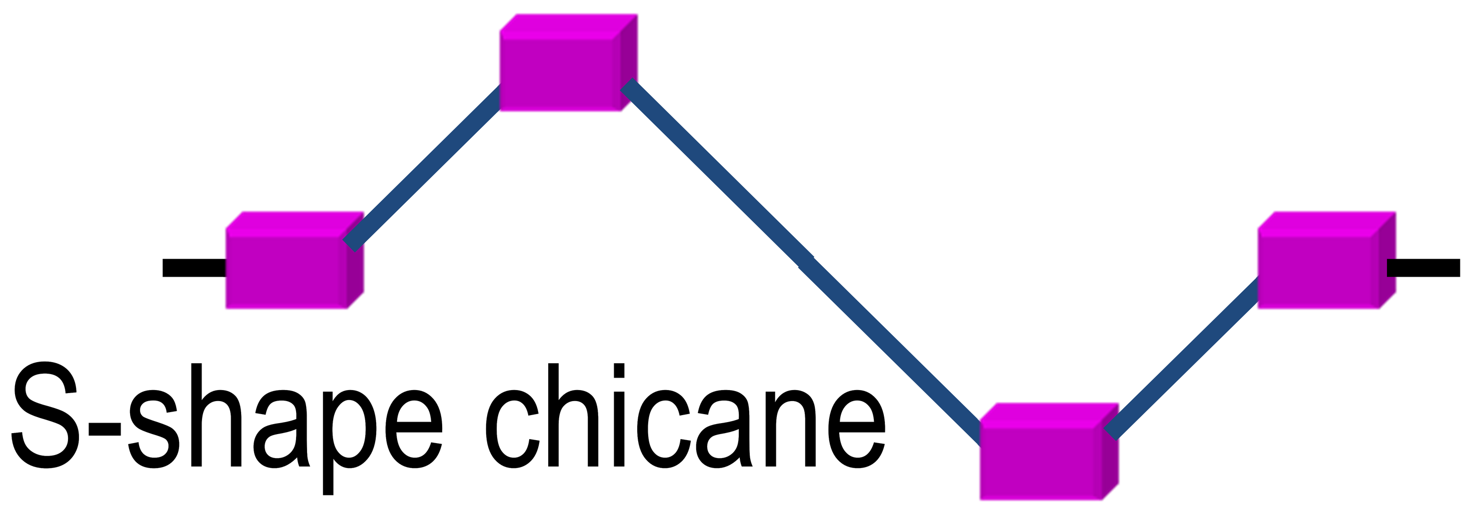
$$\sigma_{z,min} = R_{56}\sigma_{\delta,unc}$$

Elettra Sincrotrone Trieste

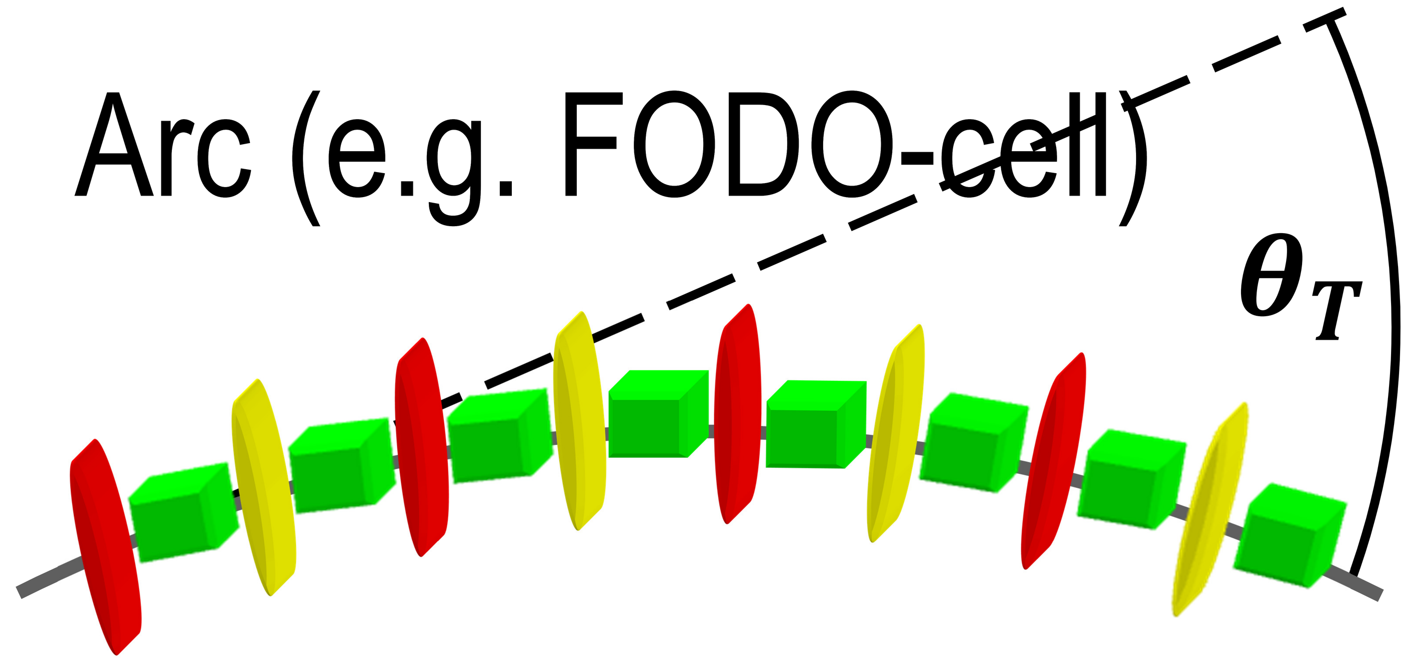
Various Types of Compressors…



FLASH LCLS FERMI X-FEL SACLA	R56 < 0



FLASH X-FEL	R56 < 0

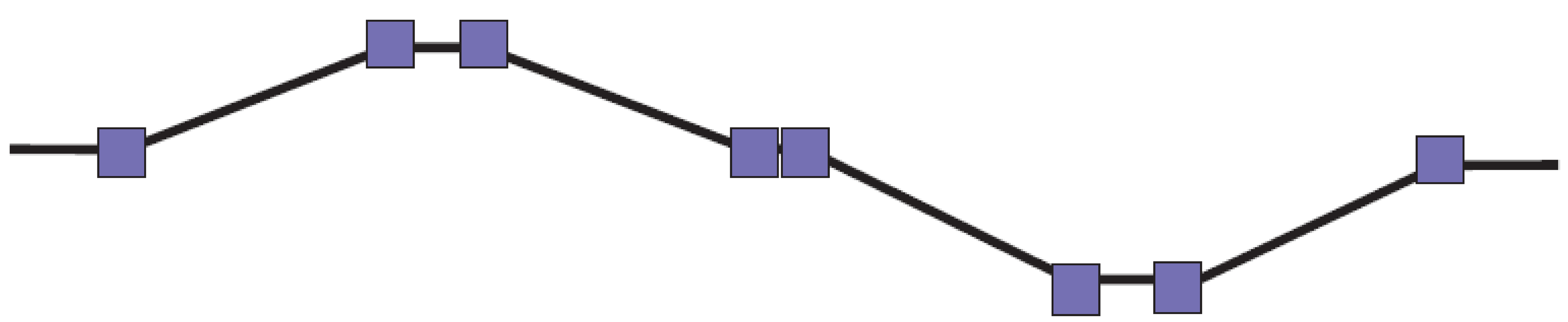


SLC arcs NLC BC2 ERLs ?	R56 > 0

- Sign of R_{56} sets the sign of the incoming energy chirp (thus RF phase in upstream linac).
- Compactness is usually important.
- In single-pass linacs, usually preferred a net zero-deflection from straight path.
- Arcs are a natural choice for ERLs.

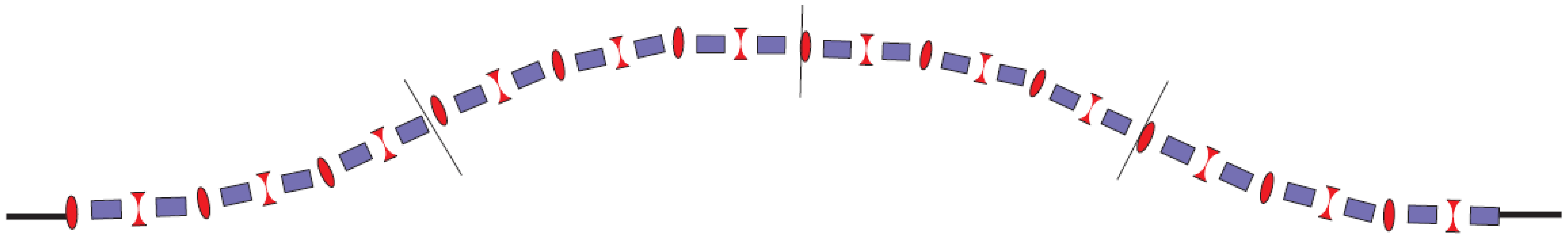
…Few More Ones

Double S-Chicane:



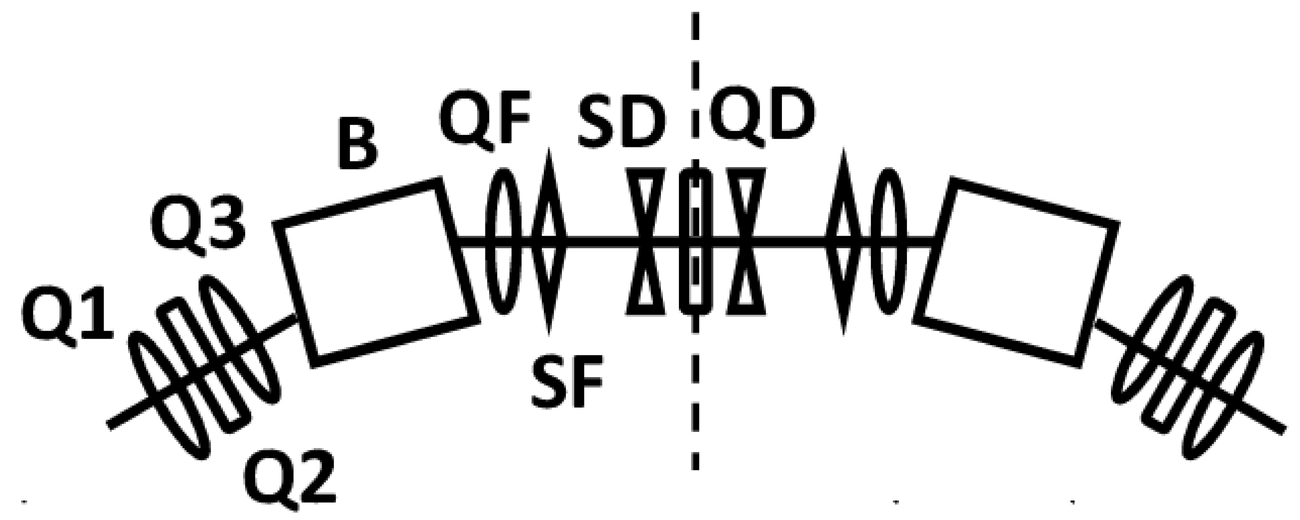
Proposed to counteract disrupting interaction of electron beam with its own emitted synchrotron radiation.

Four arcs form a "FODO"-Compressor:



MAX-IV SPF has two compressors, each one is half of this.

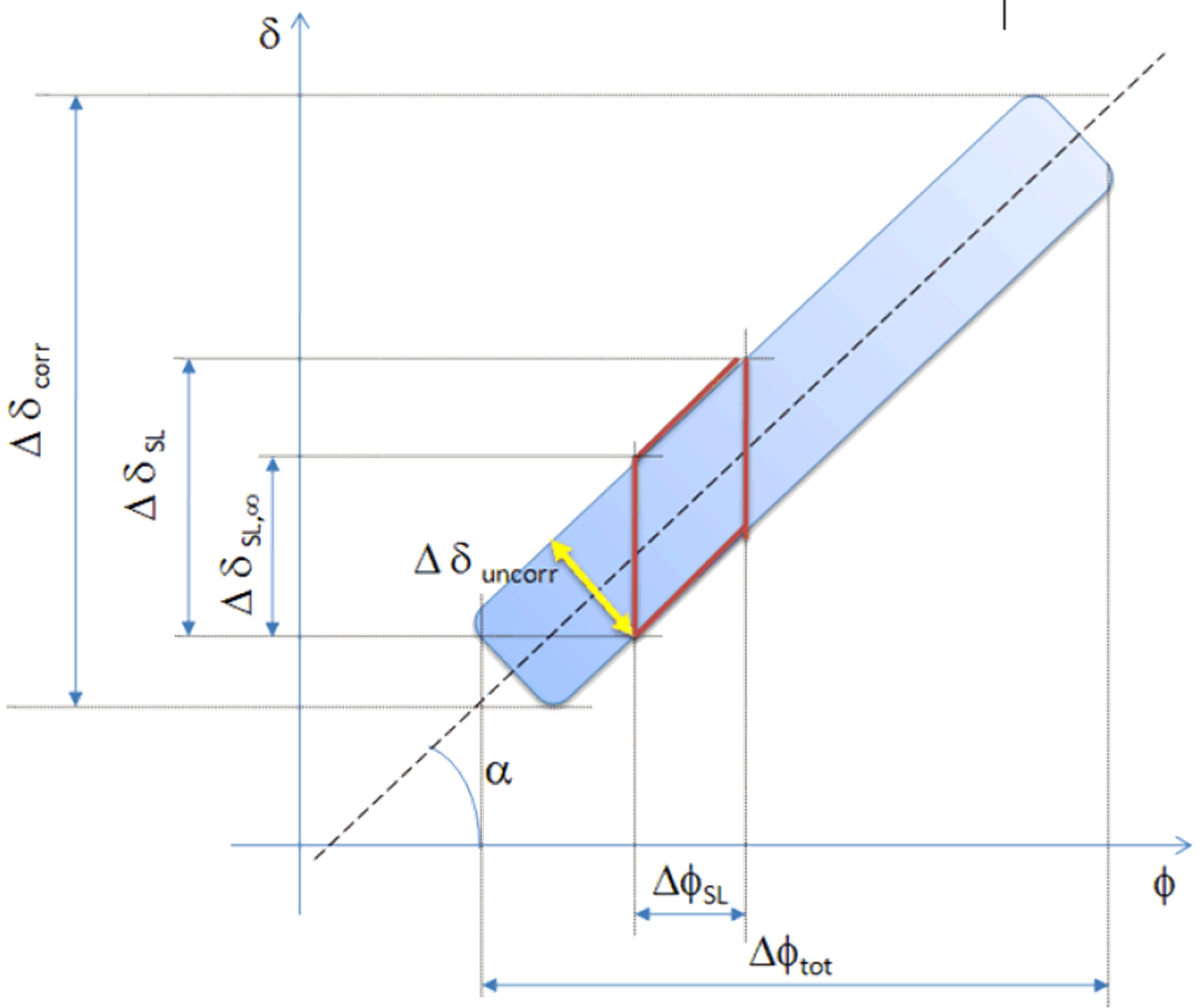
Arc based on "double-bend achromat" cell:



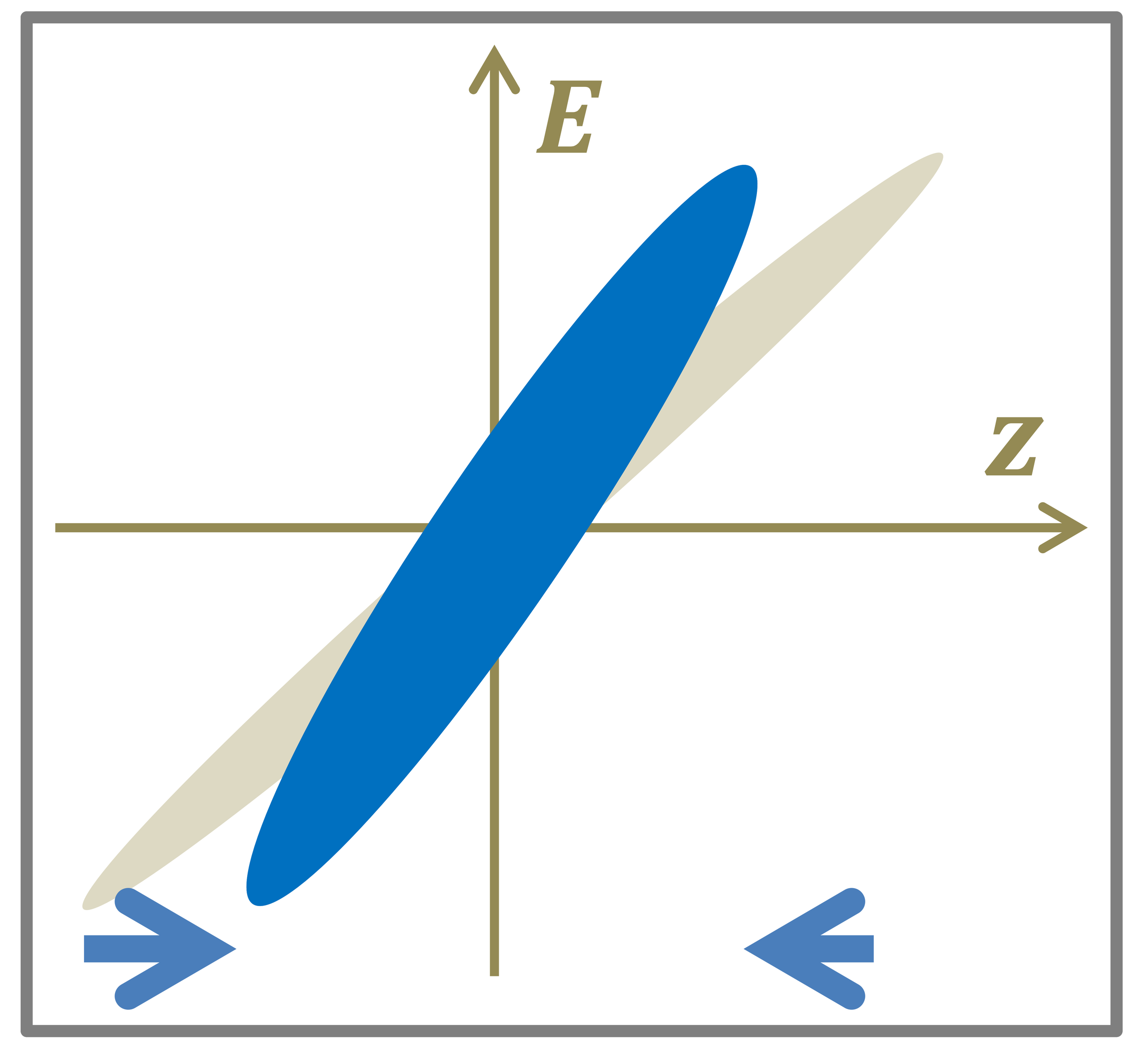
Proposed for recirculating accelerators.

Elettra
Sincrotrone
Trieste

Correlated, Uncorrelated, Slice Energy Spread

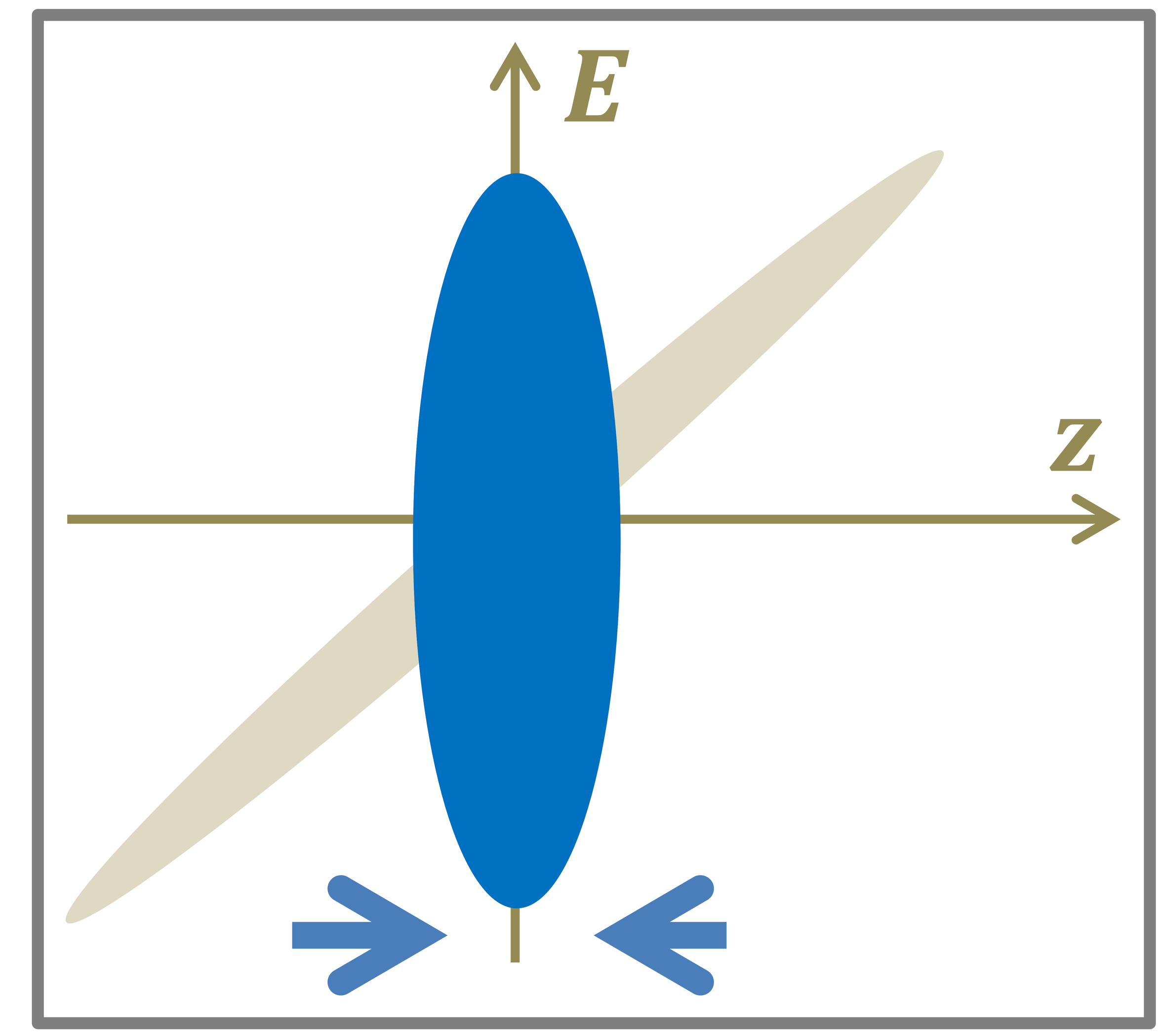


$1 + R_{56}h_1 > 0$



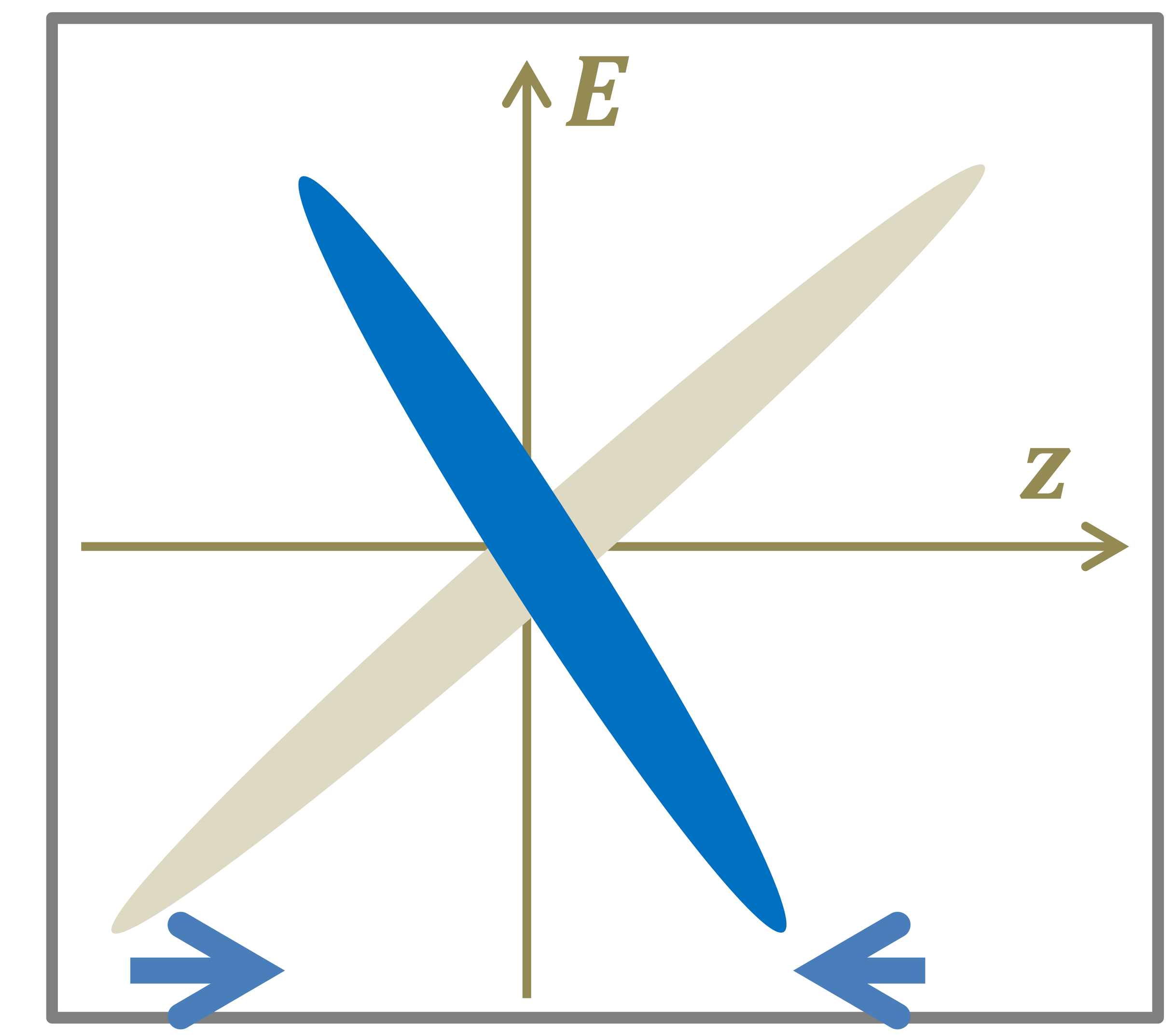
"Under-compression" (prevalent mode)

$1 + R_{56}h_1 = 0$



"Full-compression" (min. bunch length set by uncorrelated energy spread

$1 + R_{56}h_1 < 0$



"Over-compression" (sign of energy chirp is reversed)

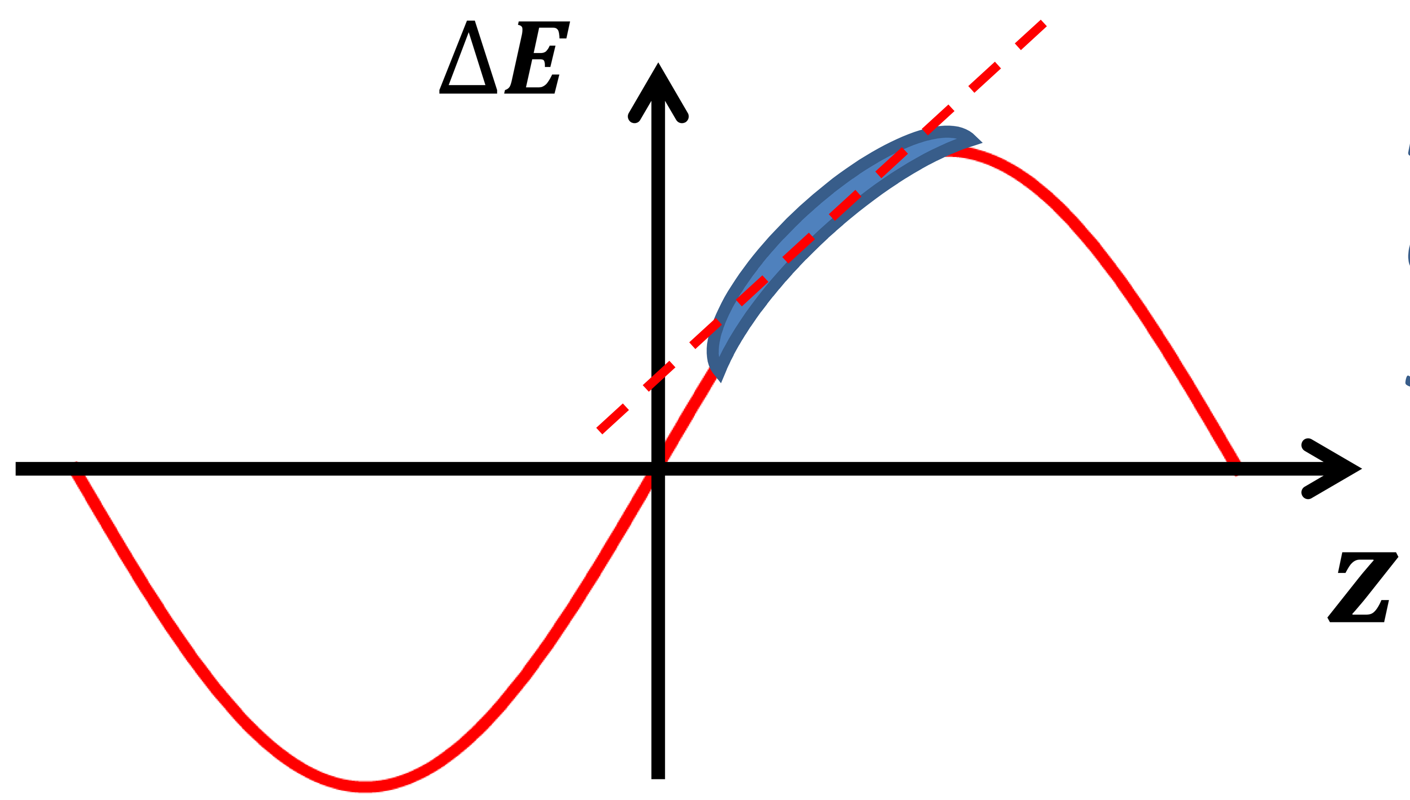
Uncorrelated δ **Correlated δ**

$$\sigma^2_{\delta,tot} = \sigma^2_{u,i} + \sigma^2_{c,i} = \boxed{\sigma^2_{u,i} + h_i^2\sigma^2_{z,i}} \equiv \sigma^2_{u,f} + h_f^2\sigma^2_{z,f} = \ldots = \boxed{\sigma^2_{u,f} + h_i^2\sigma^2_{z,i} + C^2R^2_{56}\sigma^2_{u,i}} + o\left(\sigma^4_{z,i},\sigma^4_{u,i}\right)$$

$$\begin{cases} \sigma^2_{z,f} = \sigma^2_{z,i}/C^2 + R^2_{56}\sigma^2_{u,i} + 9T^2_{566}h_i^4\sigma^4_{z,i} \\ h_f = Ch_i \end{cases}$$

$$\sigma^2_{u,f} = \sigma^2_{u,i}\left(1 - C^2R^2_{56}h_i^2\right) = C^2\sigma^2_{u,i}$$

RF Curvature



Energy of particle at exit of accelerating structure

$$E_I = E_i + eV\cos(kz+\varphi) \simeq E_i + \boxed{eV\cos\varphi} - kzeV_0\sin\varphi \boxed{- e\frac{Vk^2}{2}z^2\cos\varphi} + O(z^3)$$

0-order term >0 (acceleration)

Quadratic term <0

- How can we compensate the quadratic term?
  - Idea: pass beam through a second rf section (with different rf wavenumber)

0-order term <0

$$E_{II} = E_I + eV_H\cos(k_H z+\varphi_H) \simeq E_I + \boxed{eV_H\cos\varphi_H} - k_H z\, eV_H\sin\varphi_H \boxed{- e\frac{V_H k_H^2}{2}z^2\cos\varphi_H} + O(z^3)$$

$$-\frac{Vk^2}{2}\cos\varphi + \frac{V_H k_H^2}{2} = 0 \quad \Rightarrow \quad V_H = \frac{k^2}{k_H^2}V\cos\varphi$$

To cancel quadratic curvature from accelerating structure this term should be >0; i.e. $\cos\phi_H < 0$, say ($\cos\varphi = -1$). This structure is decelerating !

Q: *How can we win? (i.e. compensate 2nd order term and still have overall acceleration?)*

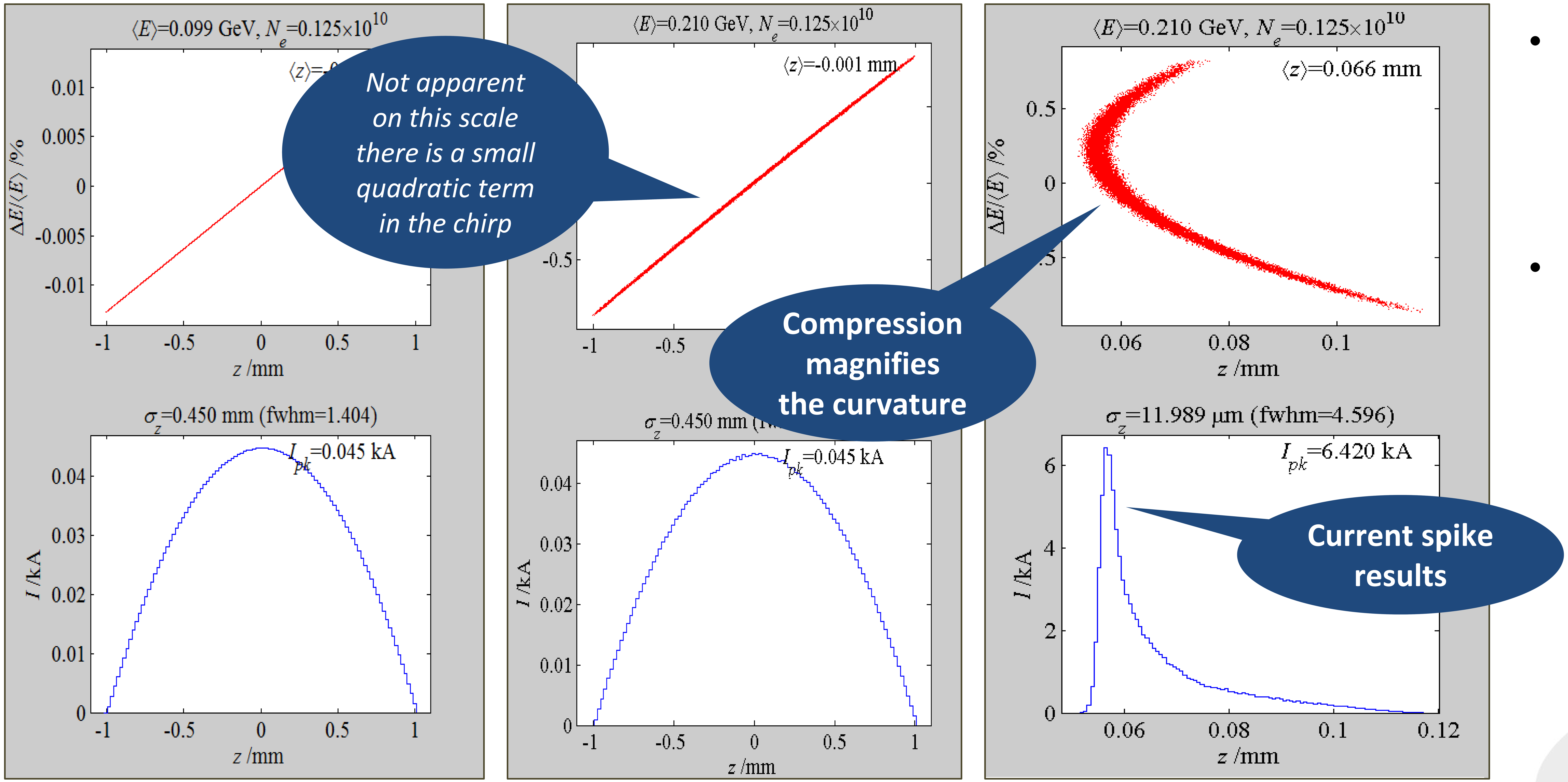
A: *Choose* $k_H > k$

Nonlinear Compression…

(Idealized) beam out of the injector E=100MeV

Beam accelerated off-crest to E=210MeV

Beam @ exit of compressor



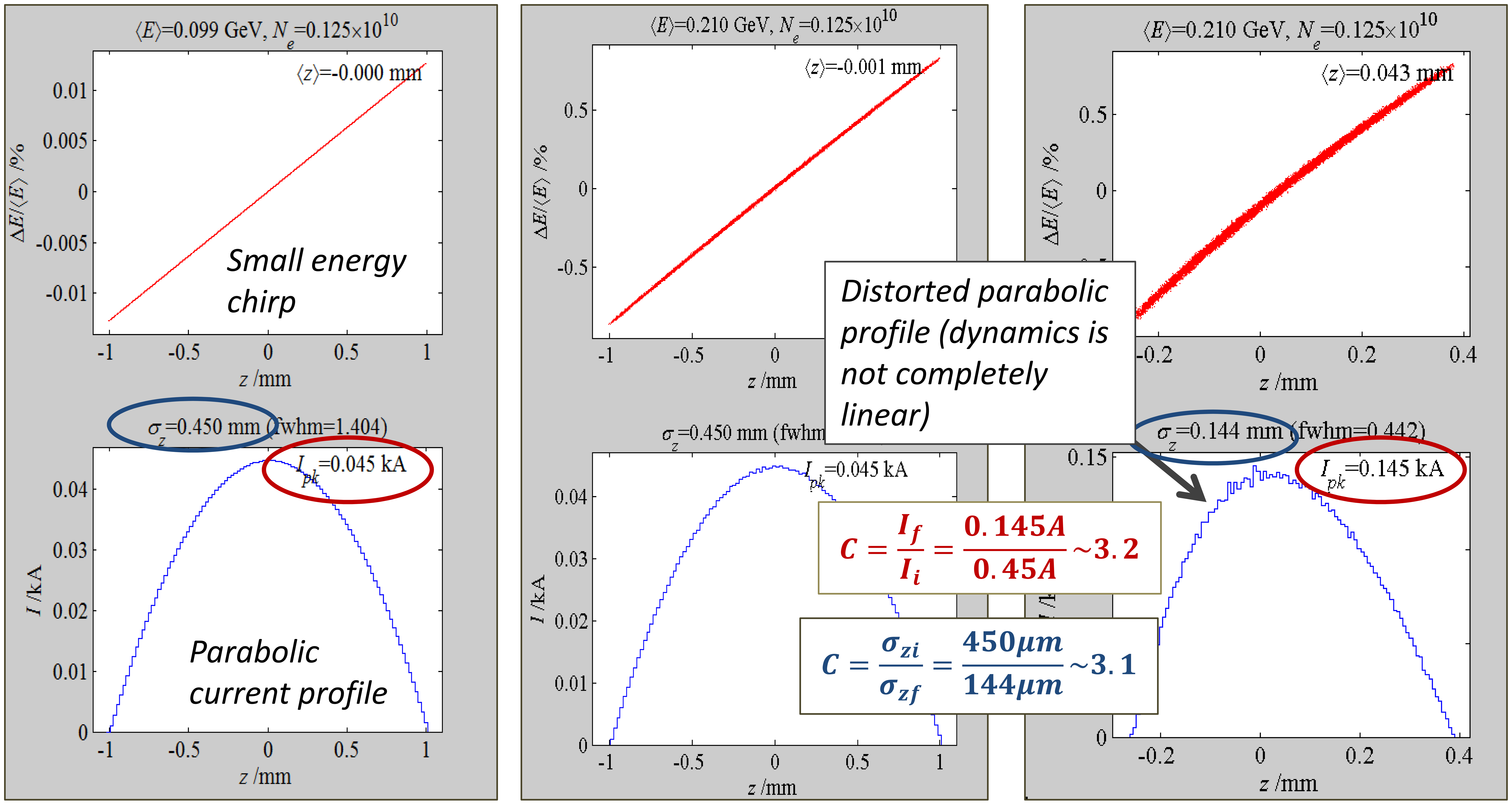
- Current spike and/or nonlinear energy chirp might not be good for FELs.
- In practice, it really depends on scientific target and application.

…Compression Linearized

(Idealized) beam out of the injector E=100MeV

Beam accelerated off-crest to E=210MeV

Beam @ exit of compressor

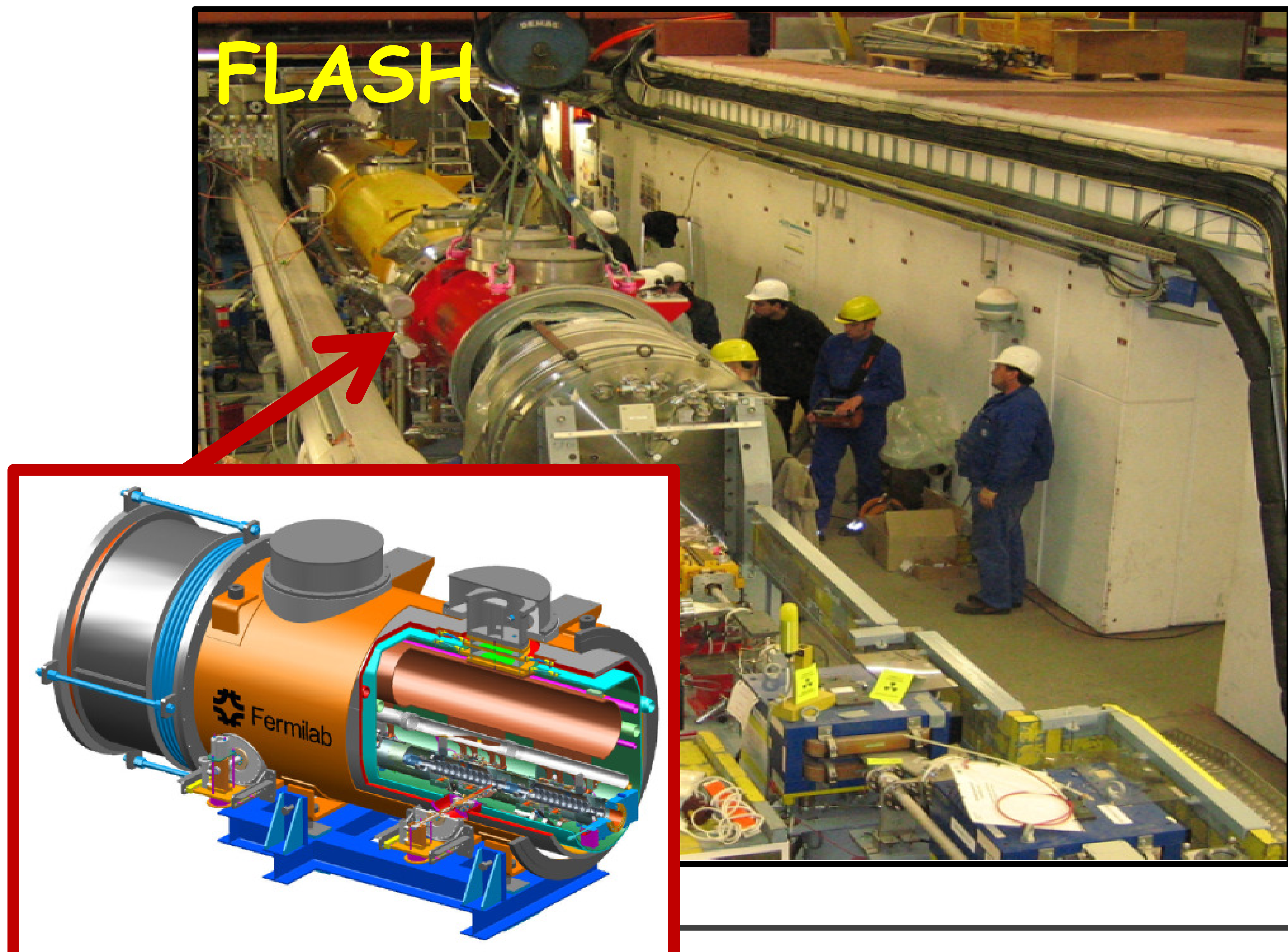


- *"Linear compression"* = linear transformation applied to the longitudinal phase space.
- Current shape is "preserved" through the compression process.

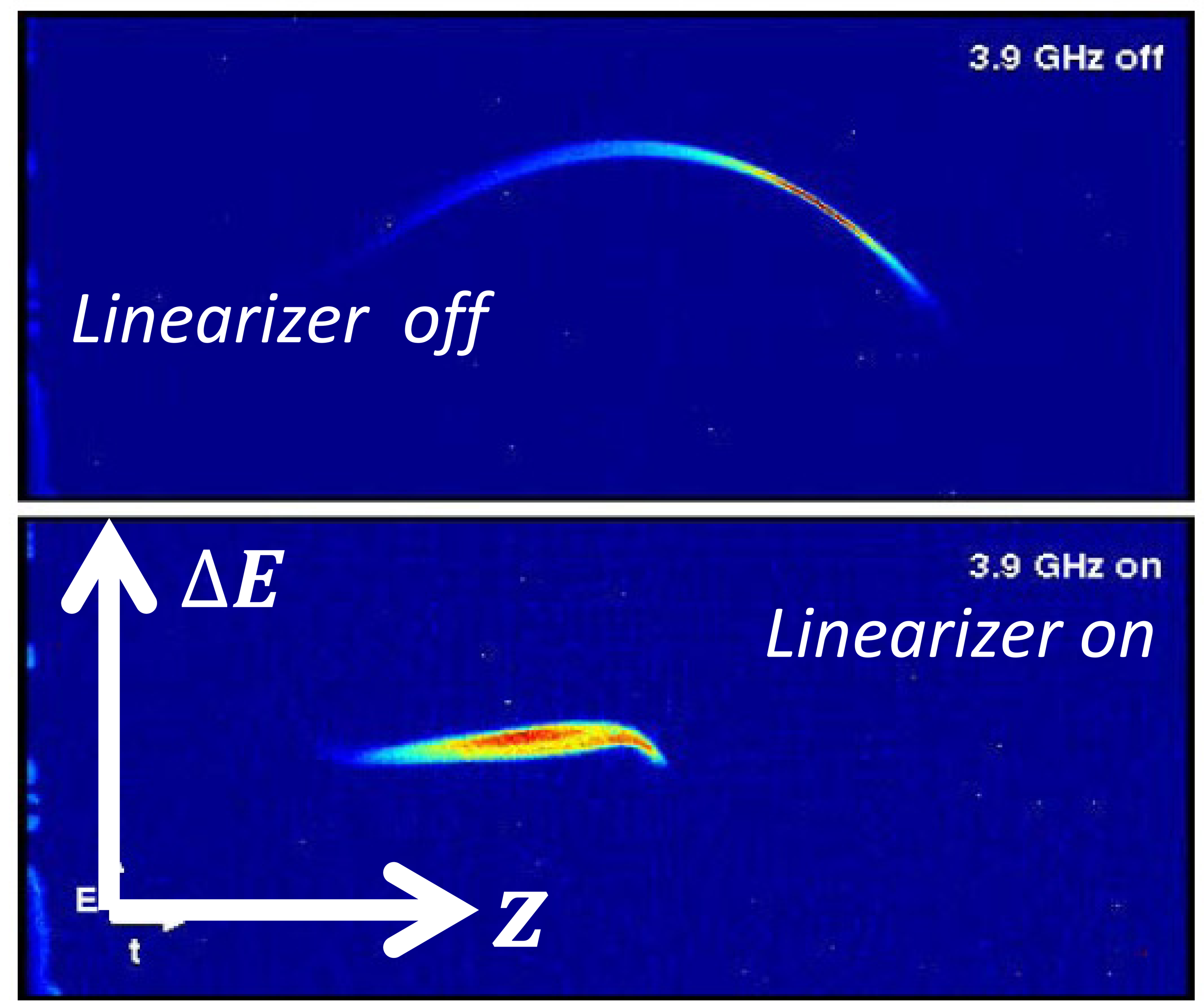
Harmonic Cavity in Action

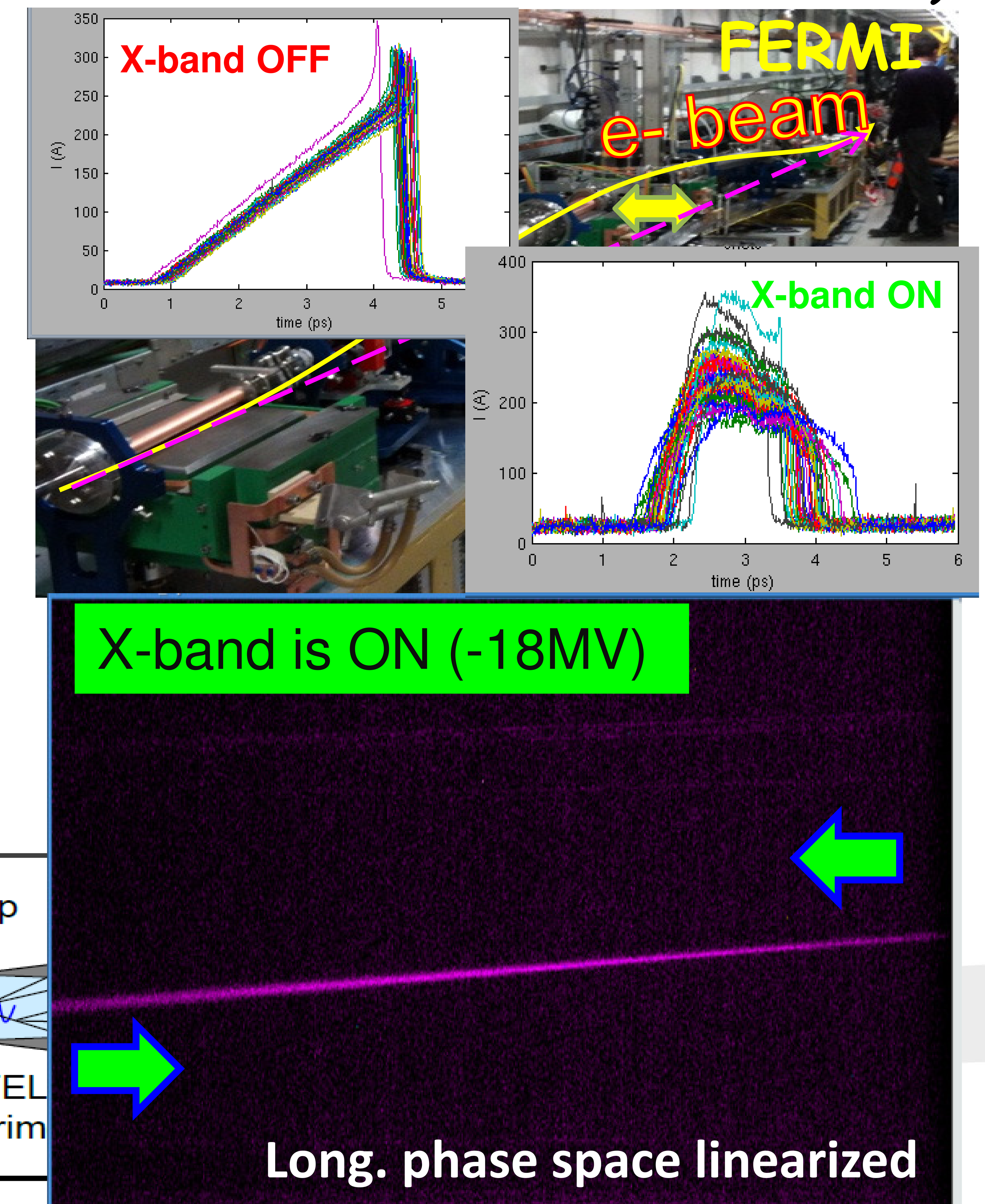
Operationally, linearizer rf frequency is best chosen to be a harmonic number of rf frequency of main linac (FLASH uses 3.9 GHz vs. 1.3 GHz SC linac; FERMI uses 11.4 GHz vs. 3.0 GHz NC linac; ….)

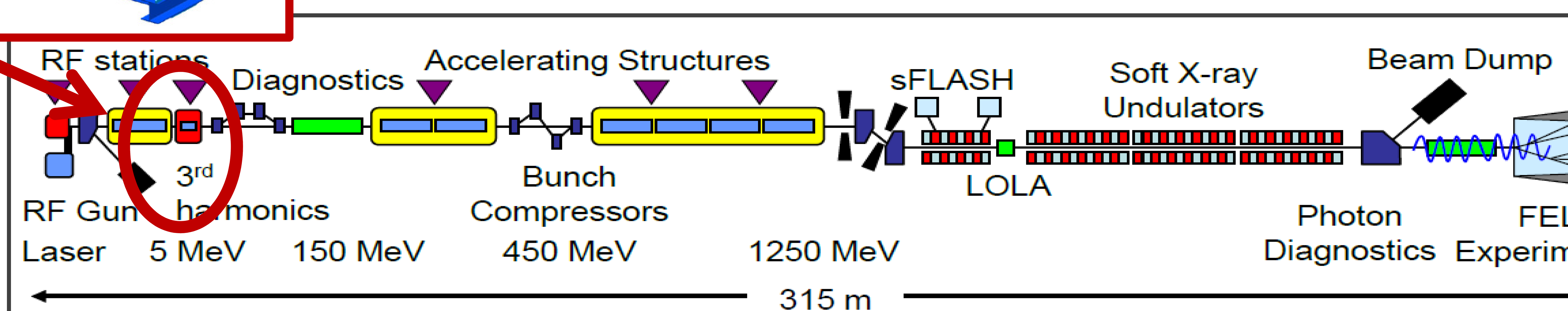
Installation of cryomodule w/ linearizer



Time-resolved measurements of longitudinal phase space







Simone Di Mitri – simone.dimitri@elettra.eu

Compression at Second Order in δ

- Nonlinear momentum compaction in chicane is usually non-negligible and has to be compensated:

$$z_1 = z_0 + R_{56}\delta_0 + \boldsymbol{T_{566}\delta_0^2}$$

For C-type chicanes: $\boldsymbol{T_{566} \simeq -\frac{3}{2}R_{56} > 0}$

- Modified setting of harmonic cavity when accounting for the 2nd order term $\boldsymbol{T_{566}}$ in momentum compaction:

$$\boldsymbol{eV_H = \frac{1}{(k_H^2/k^2 - 1)}\left\{E_{BC}\left[1 + \frac{2}{k^2}\frac{T_{566}}{|R_{56}|^3}\left(1 - \frac{1}{C}\right)^3\right] - E_i\right\}}$$

Energy of beam entering Linac section

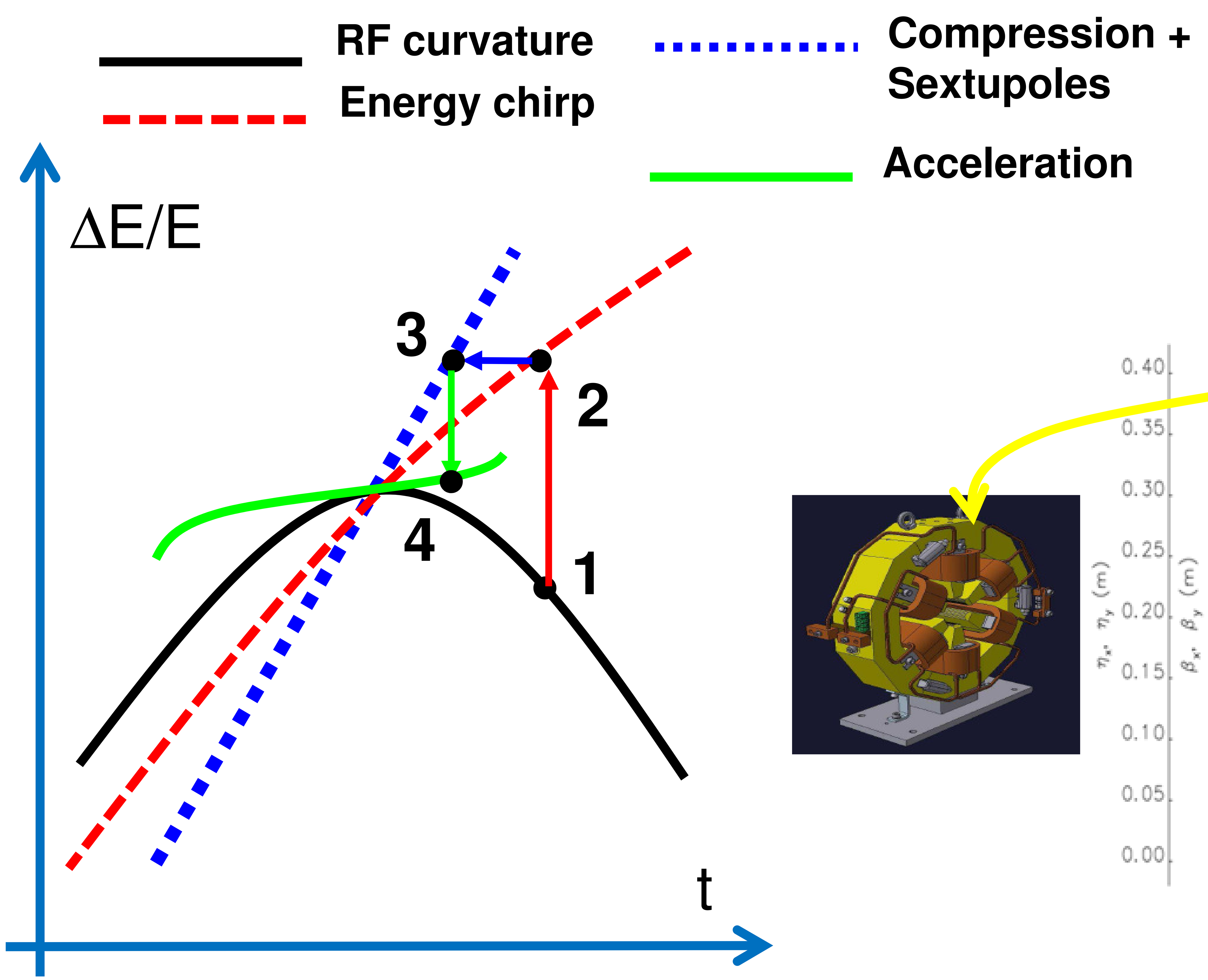
Beam energy at compressor
(minimizing V_H may imply compression at lower energy)

Linear compression factor $\boldsymbol{C = \frac{1}{|1 + R_{56}h_1|}}$

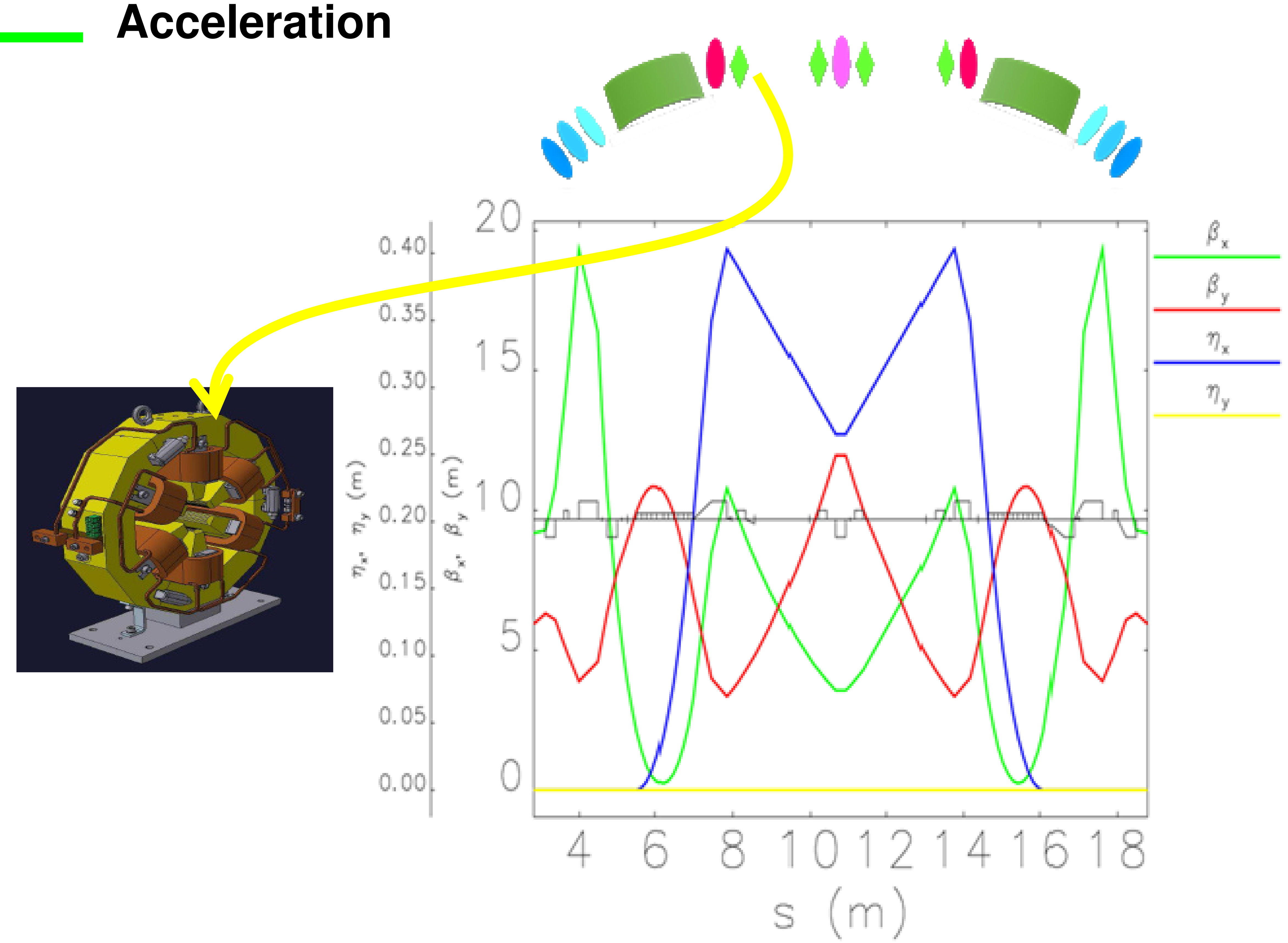
- Formula valid for $\boldsymbol{\phi_H = -180^o}$ and one-stage (single chicane) compression.
- If **multiple compressors** are present, V_H setting varies somewhat but typically not too much (after first BC, the bunch is shorter and less vulnerable to rf nonlinearities)
- Alternate method to linearize: sextupole magnets within magnetic compressor (works well in arc-like compressors, where large dispersion and separation between magnets is allowed).

Linearization with Sextupole Magnets

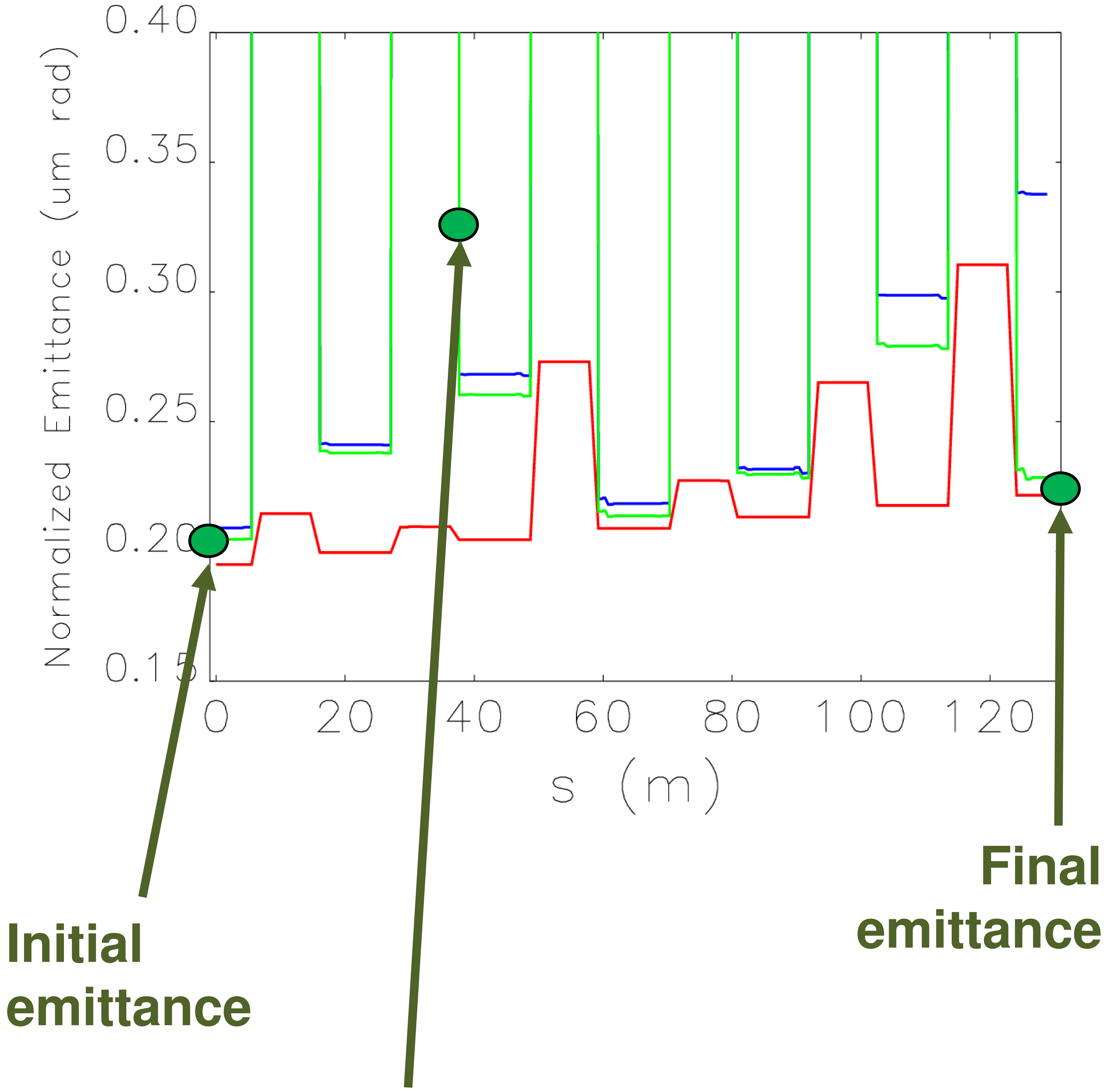
- **Sextupole magnets** can be used as an alternative to a harmonic cavity.
  - Usually included in "long" compressors such as **dog-legs** and **arcs**, in order to cumulate betatron phase advance to cancel out 2nd order optical aberrations, and eventually avoid emittance growth.



Double-Bend Achromat Cell



Horizontal Emittance through Arc



Green line depicts emittance value oscillation due to aberrations induced by sextupoles. Those eventually (almost) cancel.

Simone Di Mitri – simone.dimitri@elettra.eu

Jitter: Peak Current

- Consider the short-term **variation of RF phase** in a linac **upstream** of a **magnetic compressor**.
  - Evaluate how C varies in the presence of jitter on φ_{rf}:

$$\Delta\left(\frac{1}{C}\right) = \Delta(1 + hR_{56}); \quad \textit{from the definition of C}$$

$$\frac{\Delta C}{C^2} = -R_{56}\frac{\Delta h}{h}h = -R_{56}\frac{\Delta(\sin\varphi_{rf})}{\sin\varphi_{rf}}h = -hR_{56}\frac{\Delta\varphi_{rf}}{\tan\varphi_{rf}}; \quad \textit{from the definition of h, for } E_z \sim \cos\varphi_{rf}$$

$$\boxed{\frac{\Delta C}{C}} = -ChR_{56}\frac{\Delta\varphi_{rf}}{\tan\varphi_{rf}} = \boxed{(C-1)\frac{\Delta\varphi_{rf}}{\tan\varphi_{rf}}}. \quad \textit{at 1}^{st} \textit{ order in } \Delta\varphi_{rf}$$

- Fluctuations of rf structure parameters (voltage, phase) around set values are unavoidable.
  - They cause undesirable "jitters" in beam ***energy***, ***arrival time***, peak ***current***.

- Aggressive but not unreasonable targets for max. RF fluctuations are:
  - rf phase: 0.1 deg (NC) – 0.01 deg (SC)
  - rf voltage: 0.1% (NC) – 0.01% (SC)

Jitter: Arrival Time

- Consider the short-term **variation** of: **RF phase, RF voltage, dipole field, and arrival time at the linac entrance.**
  - Evaluate how t_{syn} at the exit of the chicane varies in the presence of the aforementioned jitters (not derived here):

$$\sigma_{t,f}^2 \cong \left(\frac{\sigma_{t,i}}{C}\right)^2 + \left(\frac{R_{56}}{c}\right)^2\left(\frac{\sigma_B}{B}\right)^2 + \left(\frac{R_{56}}{c}\frac{\Delta E_{linac}}{E_{BC}}\right)^2\left(\frac{\sigma_V}{V}\right)^2 + \left(\frac{C-1}{C}\right)^2\left(\frac{\sigma_\varphi}{ck}\right)^2$$

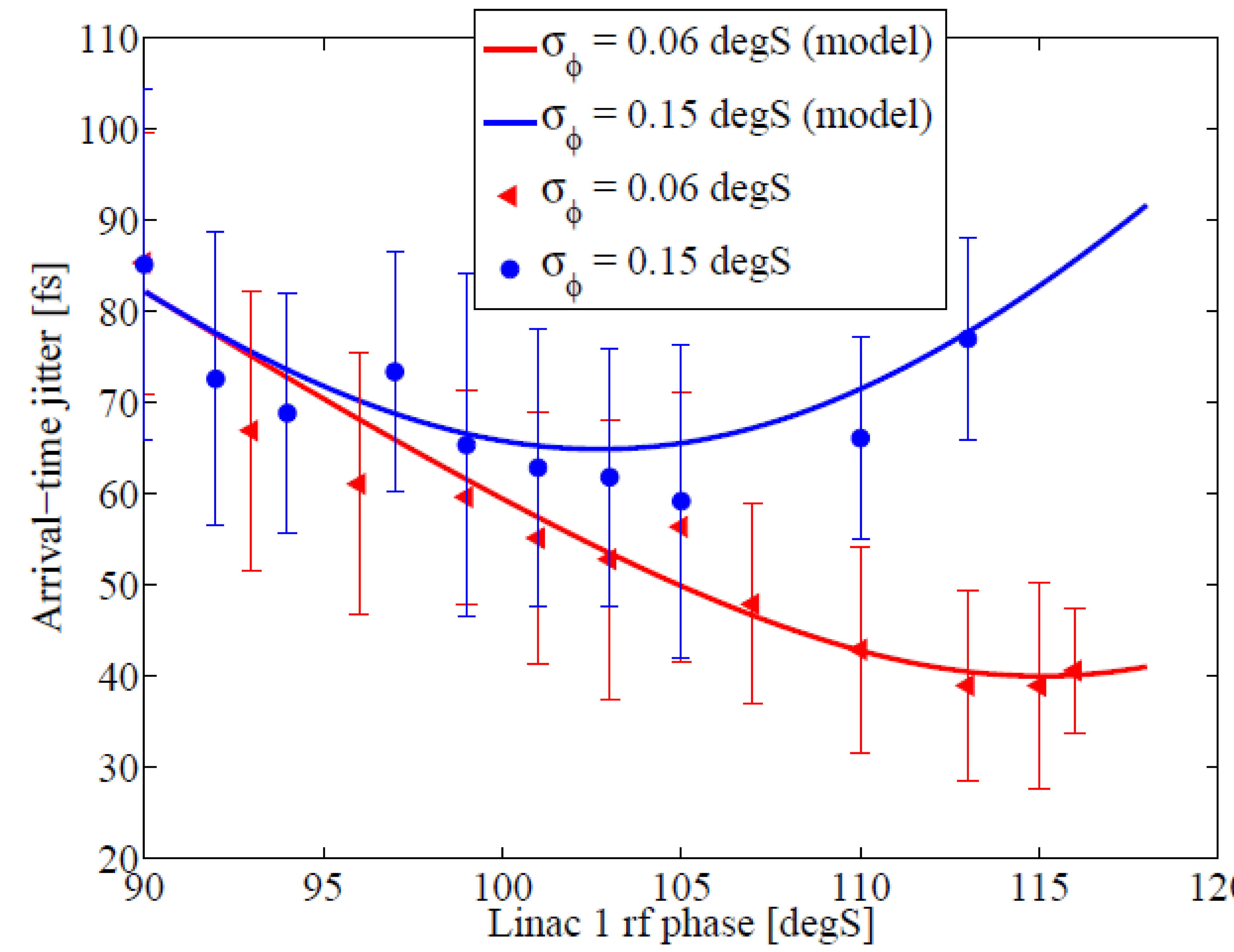
Initial arrival time is "compressed". Typical $\sigma_{t,i}$ ~ 150 fs

Power supply stability. Typical σ_B/B ~ 0.01%

RF peak voltage. Typical σ_V/V ~ 0.1% - 0.01%

RF phase. Typical σ_φ ~ 0.1 - 0.01 deg S-band

EXPERIMENTAL (FERMI linac)



- For given R_{56} in single-stage compression, the final arrival time jitter may show up a local minimum as a function of the linac RF phase (in that case, C is varying).

- *A multi-stage compression scheme has potentiality for reducing final beam jitters. In practice, tracking runs are used to determine a jitter tolerance budget and perform optimization.*

Magnetic Specs and Tolerances

Multipolar field expansion (normal mode):

$$\boxed{B_y(x) = \sum_0^n b_n \left(\frac{x}{R}\right)^n_{y=0} \qquad b_n = \frac{1}{n!}\left(\frac{\partial^n B_y}{\partial x^n}\right)_{y=0} R^n}$$

"x" is the particle's distance from the magnetic axis

"R" is the arbitrary distance at which the multipole field is sampled

"n" is the multipole order, e.g., n=0 'dipole', n=1 'quad', n=2 'sext',…

"skew" components (rotated magnets) have similar expressions.

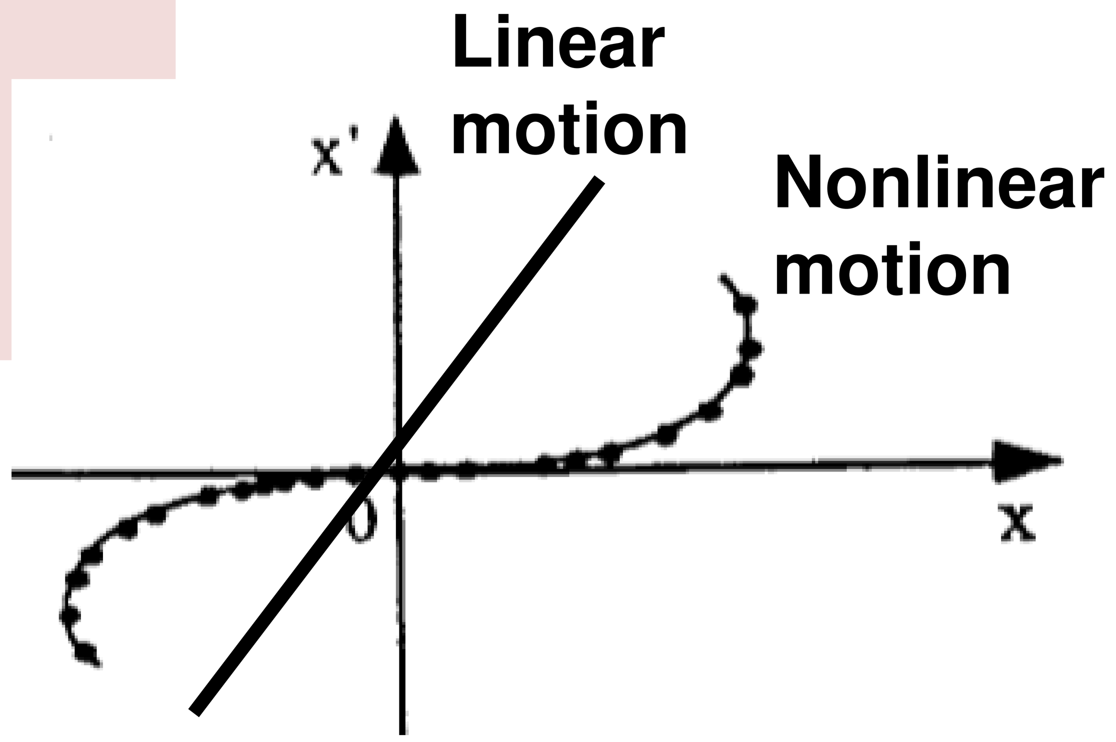
e.g., sextupole component in a dipole magnet.

$$\varepsilon_x = \sqrt{\det \varepsilon_x \begin{pmatrix} \beta_x & -\alpha_x \\ -\alpha_x & \gamma_x \end{pmatrix}} \equiv \sqrt{\det \begin{pmatrix} \langle x^2 \rangle & \langle xx' \rangle \\ \langle xx' \rangle & \langle x'^2 \rangle \end{pmatrix}}$$

transforms like $\Sigma_1 = M_{01}\Sigma_0 M_{01}^T$

Beam emittance ε in terms of **2nd order momenta** of the particle distribution in (x,x'). α, β, γ are **'Twiss parameters'**.

The "rms" emittance is NOT invariant under nonlinear motion (field).



1. Consider a nonlinear transport matrix with $M_{21} \sim b_n \neq 0$ (nonlinear field component).
2. Beam matrix transforms through M so that $\langle x_1'^2 \rangle = \langle x_0'^2 \rangle + Q_x^2(b_n, x_0) = \gamma_x \varepsilon_{x,0} + Q_x^2$

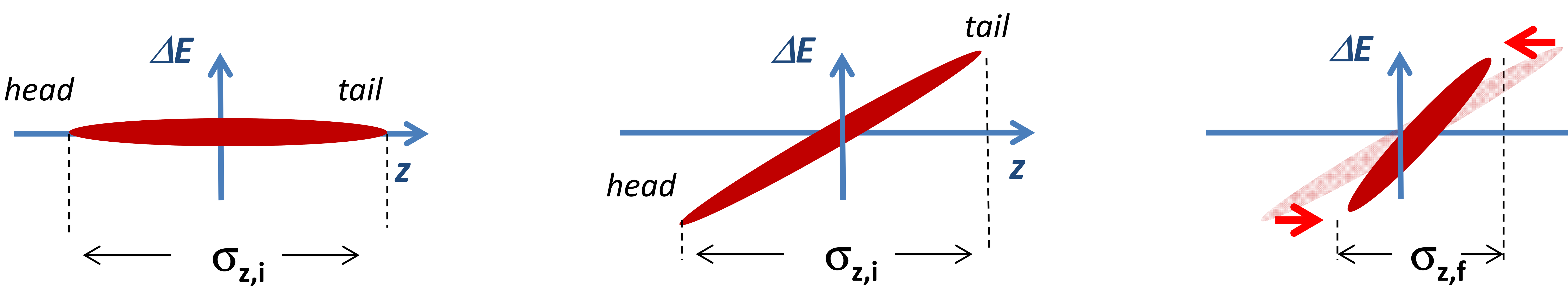
From the determinant of the perturbed beam matrix we find:

$$\left(\frac{\Delta\varepsilon_x}{\varepsilon_{x,0}}\right) \cong \frac{1}{2}\frac{\beta_x}{\varepsilon_{x,0}} Q_x^2(b_n)$$

This relationship sets a spec. on b_n vs. the maximum tolerated $\Delta\varepsilon_x$.

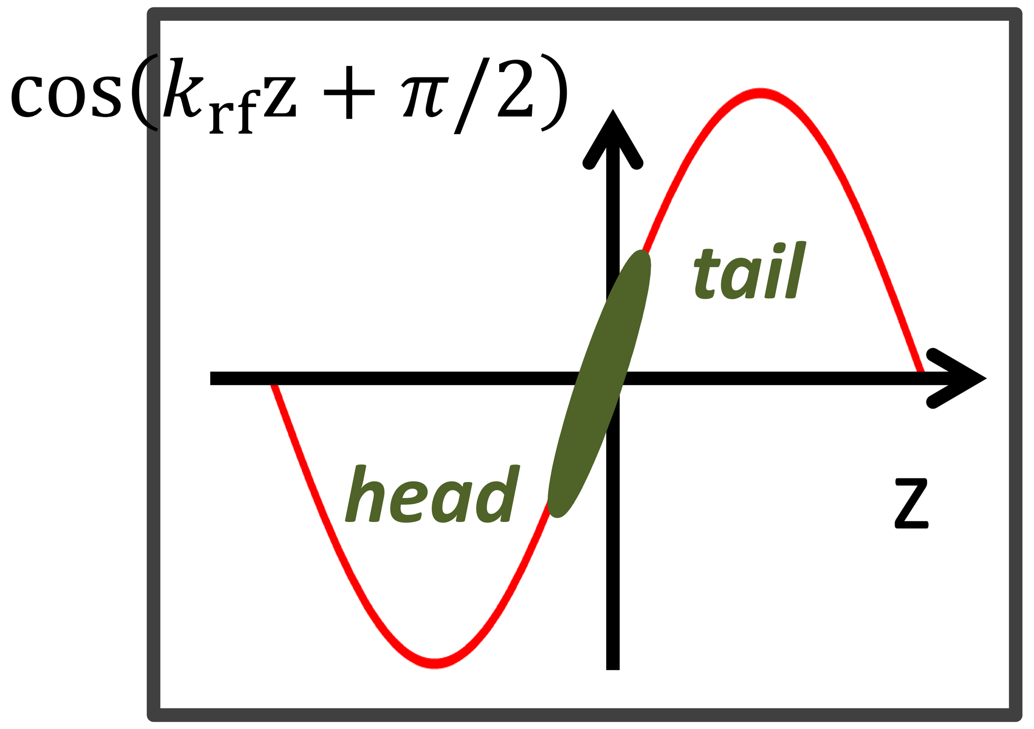
Compression in RF Structures

- At low beam energy, different particles energy means significant difference in velocity.
  - Particles travel different distances over the same time lap → compression in straight non-dispersive channels.
  - Trailing particles should have larger energy (velocity) than leading ones (same for compression in a chicane).
  - "Ballistic compression" is often referred to compression *without* acceleration. "Velocity bunching" is more generic.



$$\Delta z(s) = \Delta\beta_z \cdot s$$

RF-cavity operated at "zero-crossing"



Standing wave RF cavity

Relativistic equation for longitudinal motion

$$\begin{cases} \dfrac{dz}{ds} = \dfrac{\gamma}{\sqrt{\gamma^2 - 1}} \\ \dfrac{d\gamma}{ds} = -\dfrac{eE_{z,0}(s)}{m_e c^2}\cos(k_{\mathrm{rf}} z + \varphi_{\mathrm{rf}}) \end{cases}$$

Compression Factor at 'zero-crossing'

$$C\left(\varphi_{\mathrm{rf}} = \frac{\pi}{2}\right) \equiv \frac{\Delta z(s)}{\Delta z(s=0)} = \left[1 - \frac{eV_0 k_{\mathrm{rf}}}{m_e c^2}\frac{s}{(\gamma^2 - 1)^{\frac{3}{2}}}\right]^{-1}$$

Which, and How Many Compressors ?

- **Velocity Bunching (VB): max. C is limited by "space charge" (particles repulsive Coulomb interaction), in order to preserve beam quality.**
  - Presently operating Normal Conducting Photo-Injectors (LCLS, FERMI) usually do not employ VB at all.

- **Magnetic Compression (MC): commonly with chicanes, rare dog-legs (MAX-IV SPF), proposed arcs.**
  - usually done at energies high enough to limit adverse impact of "space charge" and emitted radiation…
  - … but too high energy is bad too (energy at first compression sets requirement for linearizer voltage) .

- **Favoring Multi-Stage magnetic compression:**
  - First gentle compression can be done at relatively low energies (100-300 MeV)
  - Further compression at higher energy minimizes synchrotron radiation effects on transverse emittance
  - Potential for larger overall compression
  - Reduced sensitivity to RF jitter.

- **Favoring Single-Stage magnetic compression:**
  - Some collective effects (microbunching instability) are alleviated by single-stage compression
  - Shorter and simpler machine layout (usually not a decisive factor)

Summary

- Bunch length compressors are fundamental tools for increasing the bunch **peak current**, e.g. for **FELs**.
- **Magnetic compressors** are made of a **linac** (properly rf-phased) + **magnetic insertion** (proper sign of R_{56}).
- Control of **current profile** requires linear compression, thus **linearization** of the compression process.

- Bunch length compression changes the **uncorrelated energy spread**.
- Bunch length compression implies peak current **jitter** as a function of RF parameters.
- Magnetic compressors require **magnetic specifications** also for the beam **transverse emittance**.

- **Velocity bunching** (VB) is complementary to magnetic compression (MC).
  - The choice of VB + MC, one- or multi-stage MC, depends on many (inter-dependent) parameters such as: emittance, collective effects, stability, infrastructure, final application…

Homework (1/2)

You should be able to work out all of the following ones, looking to the presented slides. You are encouraged to work together, use books, and ask for help if needed (I'll be around all night).

CAS "policy" adopted: homework are not mandatory, but your efforts in facing them will be appreciated !

1. Show that E_z in a standing-wave structure (assume for simplicity $E_{z,0}$ in the fundamental accelerating mode) can be written as the superposition of two counter-propagating e.m. waves [Note: in fact, the forward-traveling component depicts the accelerating field in a real traveling-wave structure; in a standing-wave structure, the counter-propagating wave does not contribute to acceleration on average].
   - *Hint: slide 9*
2. Demonstrate that in a standing-wave structure (assume for simplicity E_z in the fundamental accelerating mode) the effective accelerating voltage over a cell of length [-g/2, g/2] is always < $E_0 \cdot g$, even if E_z were ideally uniform along the gap.
   - *Hint: slide 9*
3. Derive the relationship $p_z = eB_yR$.
   - *Hint: slide 12 + Lorentz force*
4. Estimate the value of η_x' right at the exit of a dipole magnet ($\eta_x = \eta_x' = 0$ at its entrance). What is η_x' in the middle of a 4-dipoles C-shape chicane?
   - *Hint: slide 13*

Simone Di Mitri – simone.dimitri@elettra.eu

Homework (2/2)

5. Consider a Linac made of S-band structures, an X-band harmonic cavity, and followed by a magnetic chicane; the parameters are (refer to slides for notation): λ_{rf} = 3 GHz, λ_H = 11.4 GHz, R_{56} = -41 mm, E_{BC} = 280 MeV, E_i = 100 MeV, C = 10. What is the peak voltage of the harmonic cavity required to linearize the compression process (assume φ_H at the decelerating crest)? What is the peak voltage and the rf phase of the S-band linac ?
   - *Hint: slide 11, 22*

6. Consider a beam entering a 4-dipoles C-shape chicane; the parameters are (refer to slides for notation): R_{56} = -41 mm, $\sigma_{u,i}$ = 3 keV/280MeV, $\sigma_{c,i}$ = 5.6MeV/280MeV. What is the minimum achievable bunch length? Consider 2nd order terms for the beam transport through the chicane, but 1st order energy chirp only.
   - *Hint: slide 24*

7. Consider a 3.0 ps rms long Gaussian bunch with 0.1% correlated energy spread at the entrance of a symmetric double-bend achromatic (DBA) cell. Evaluate the dipole length of the DBA for achieving a total linear compression factor of 10, at the beam energy of 1 GeV, for a dipole magnetic field of 0.5 T. What is the minimum bunch length achievable, and therefore the maximum effective compression factor, if the beam initial uncorrelated energy spread is 20 keV rms ?
   - *Hint: slide 11-15, 24*