Introduction to Particle Accelerators

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CAS – Vacuum for Particle Accelerators

Örenäs Slott – Glumslöv, Sweden June 2017

MAX IV

Introduction to Particle Accelerators

- Pre-requisites: classical mechanics & electromagnetism + matrix algebra at the undergraduate level.
- No specific knowledge of accelerators assumed.
- Objectives
  - Provide motivations for developing and building particle accelerators
  - Describe the basic building blocks of a particle accelerator
  - Describe the basic concepts and tools needed to understand how the vacuum system affects accelerator performance.

Caveat: I will focus the discussion/examples on one type of accelerator, but most of the discussion can be translated into other accelerator models.

Outlook

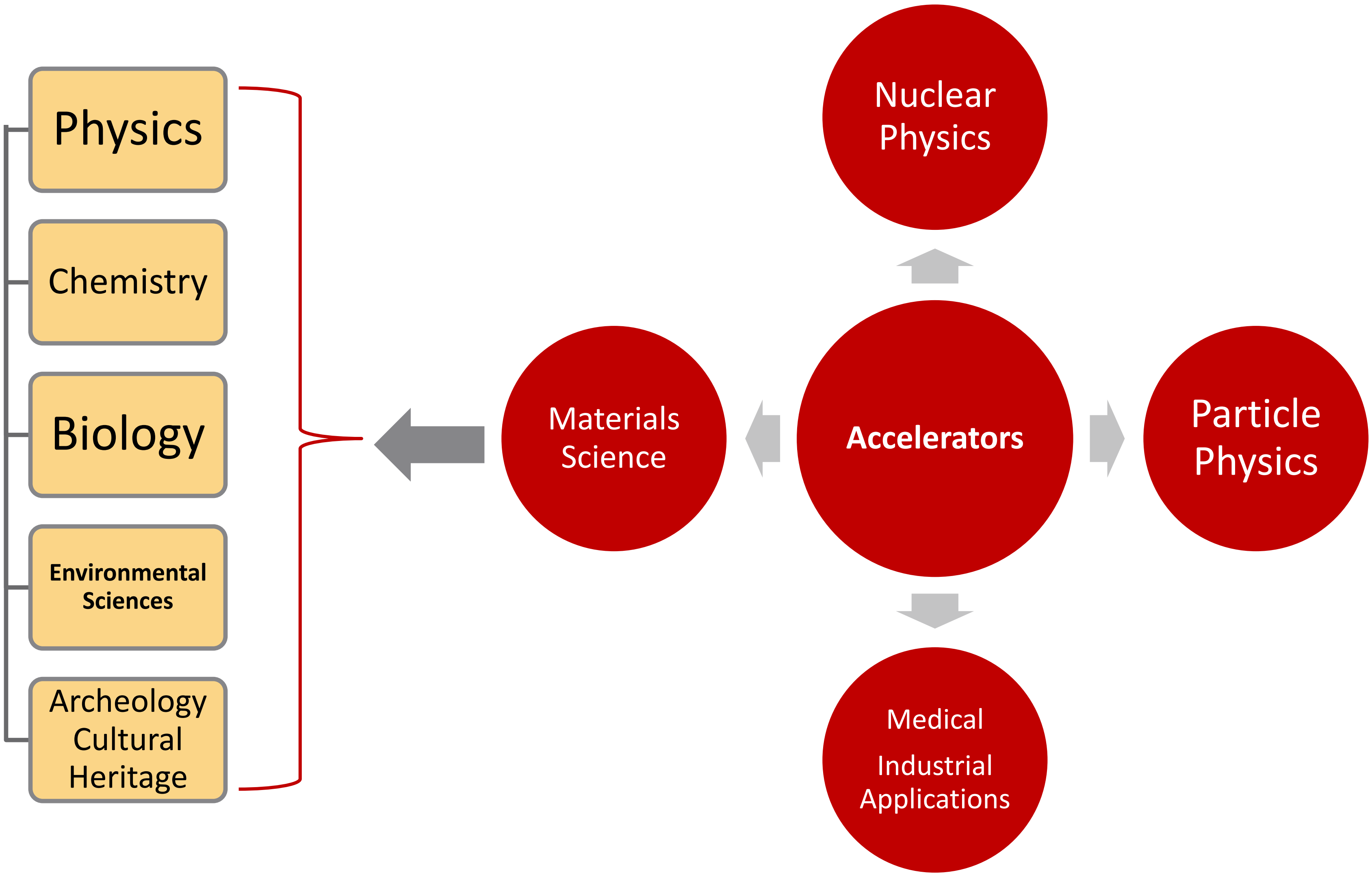
- Why Particle Accelerators ?
  - Why Synchrotron Light Sources ?

- Storage Ring Light Sources: *accelerator building blocks*

- Basic Beam Dynamics in Storage Rings.
  - Transverse dynamics: twiss parameters, betatron functions and tunes, chromaticity.
  - Longitudinal dynamics: RF acceleration, synchrotron tune
  - Synchrotron light emission, radiation damping and emittance

- How vacuum affects accelerator performance.

Why Particle Accelerators ?

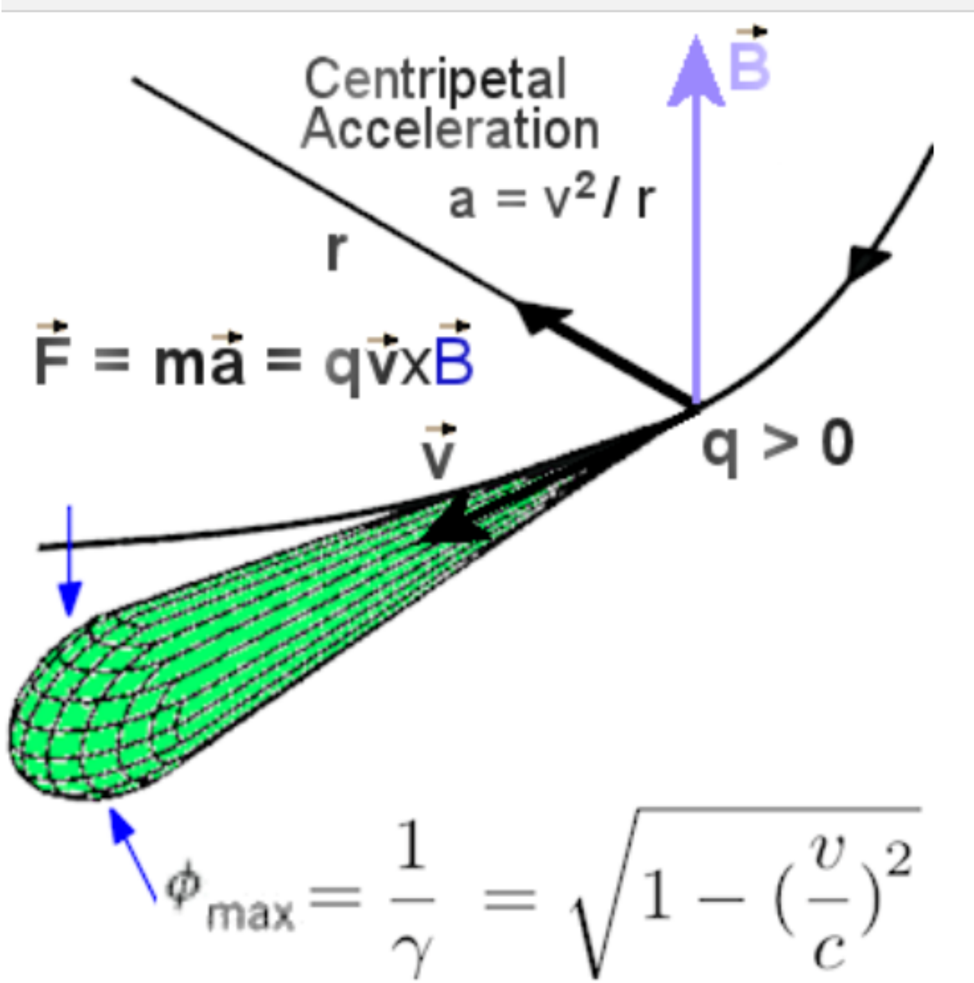


Beams for Materials Research

Photon Sources

Neutron Sources

What is Synchrotron Light ?



Properties:

Wide band

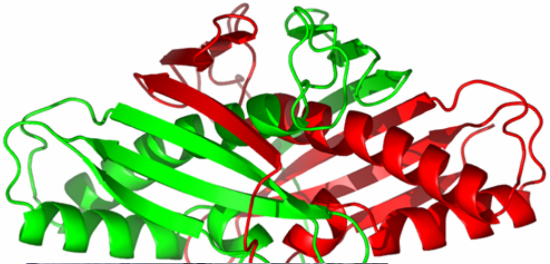
High intensity/Brightness

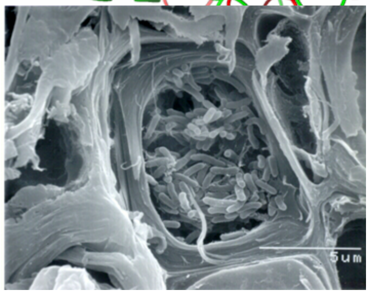
Polarization

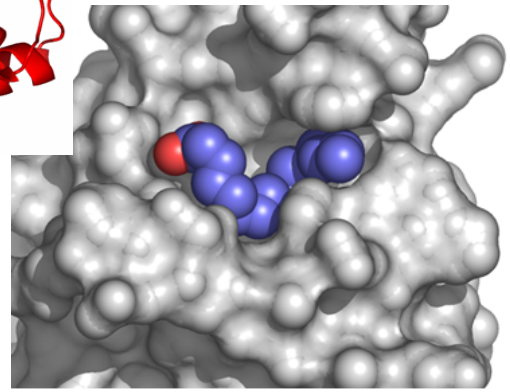
Time structure

Picture: https://universe-review.ca/I13-15-pattern.png

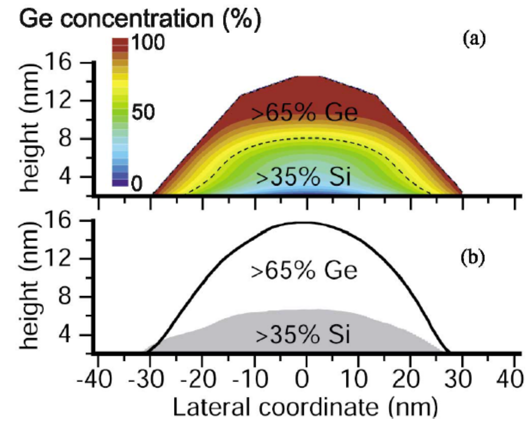
Why Synchrotron Light ?





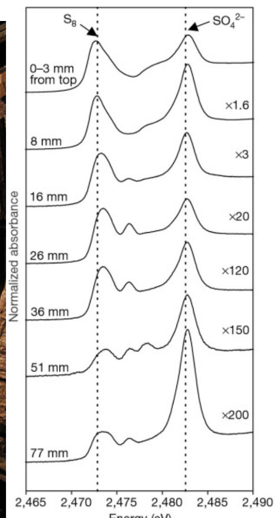


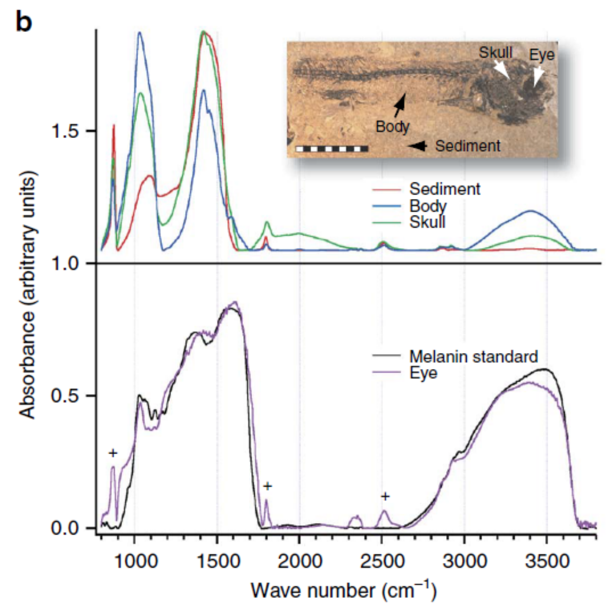
A.Malachias et ak, *3D Composition of Epitaxial Nanocrystals by Anomalous X-Ray Diffraction*, PRL **99,** 17 (2003)



OLIVEIRA, M. A. et al. *Crystallization and preliminary X-ray diffraction analysis of an oxidized state of Ohr from Xylella fastidiosa.* Acta Crystallographica. Section D, Biological Crystallography, v. D60, p. 337-339, 2004

Photo: Vasa Museum

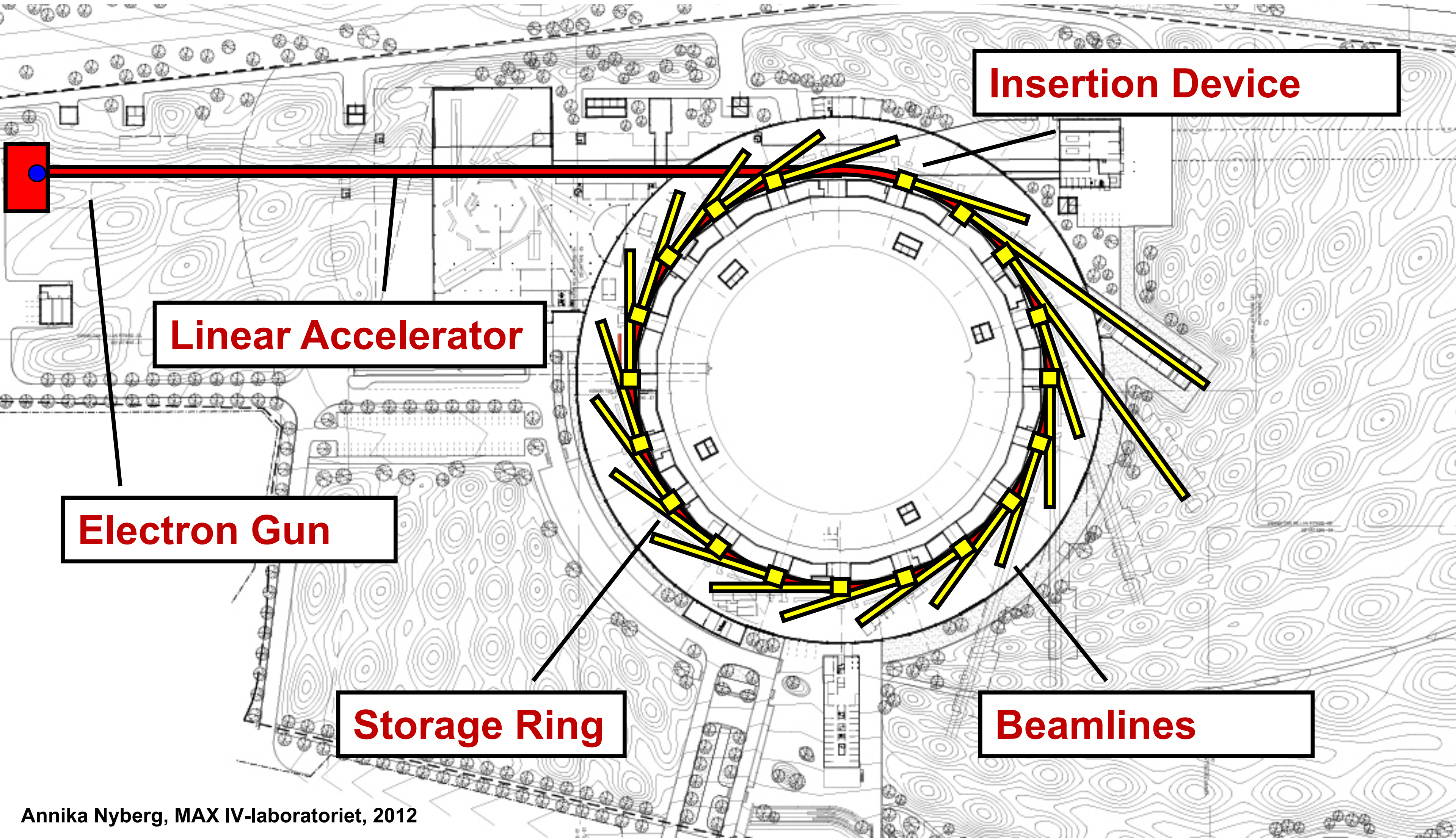




Sandstrom, M. *et al*. **Deterioration of the seventeenth-century warship vasa by internal formation of sulphuric acid**. *Nature* **415,** 893 - 897 (2002)

J.Lindgren et al, **Molecular preservation of the pigment melanin in fossil melanosomes**, Nature Communications DOI: 10.1038/ncomms1819 (2012)

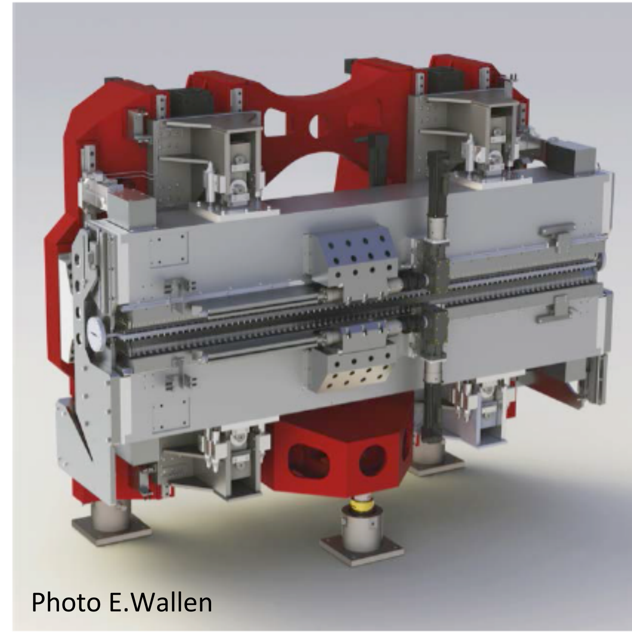
Building Blocks of a SR based Light Source



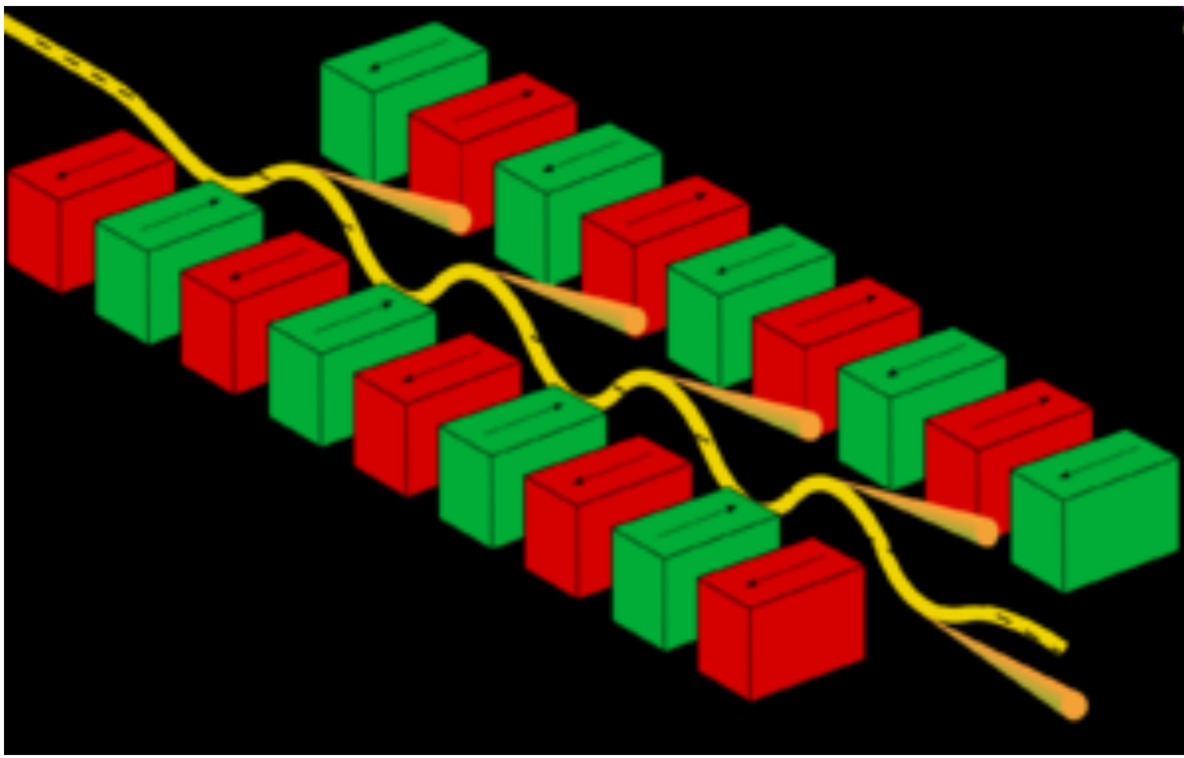
Annika Nyberg, MAX IV-laboratoriet, 2012

Insertion Devices

Undulator



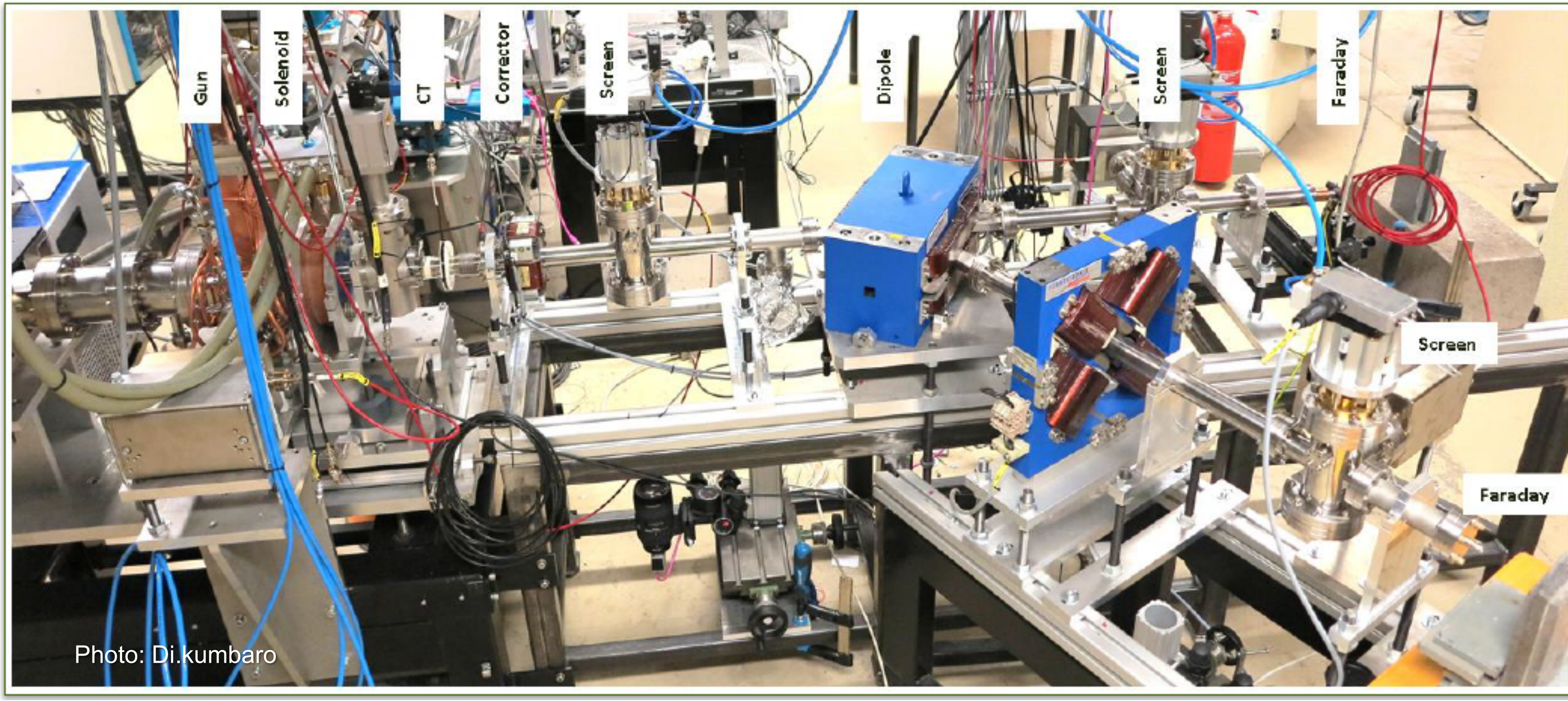
Periodic arrays of magnets cause the beam to “undulate”



Wiggler

www.lightsources.org

Electron Sources

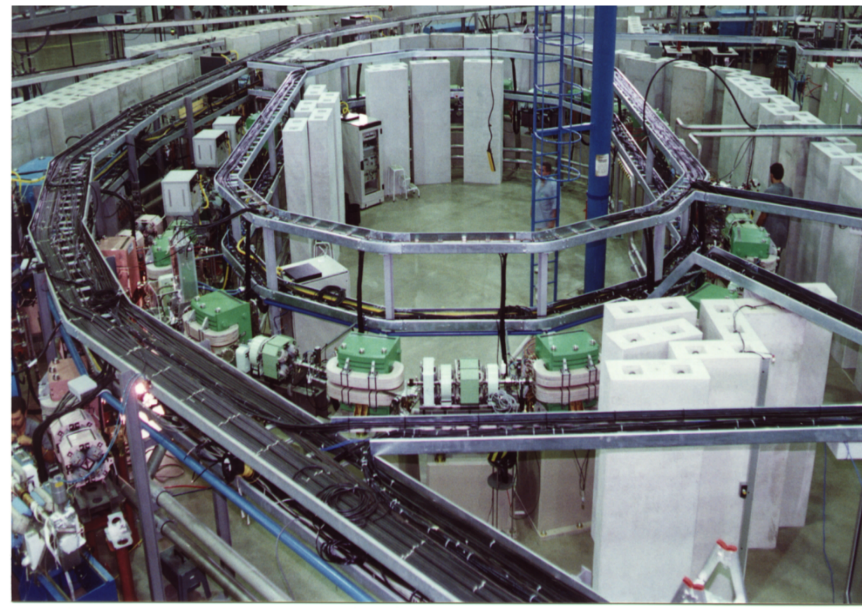


Injector Systems

Linear Accelerator

MAX IV Full Energy Injector LINAC

LINAC + Booster



LNLS - Brazil

Storage Ring Subsystems

Accumulate and maintain particles circulating stably for many turns

- Magnet System : *Guiding and Focussing*
  - DC
  - Pulsed

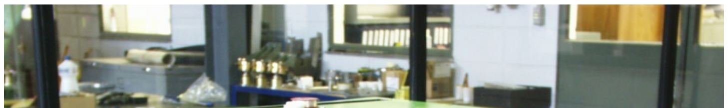
- Radio-Frequency System: *Replace lost energy*

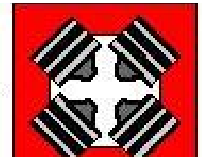
- Diagnostic and Control System: *Measure properties, feedback if necessary*

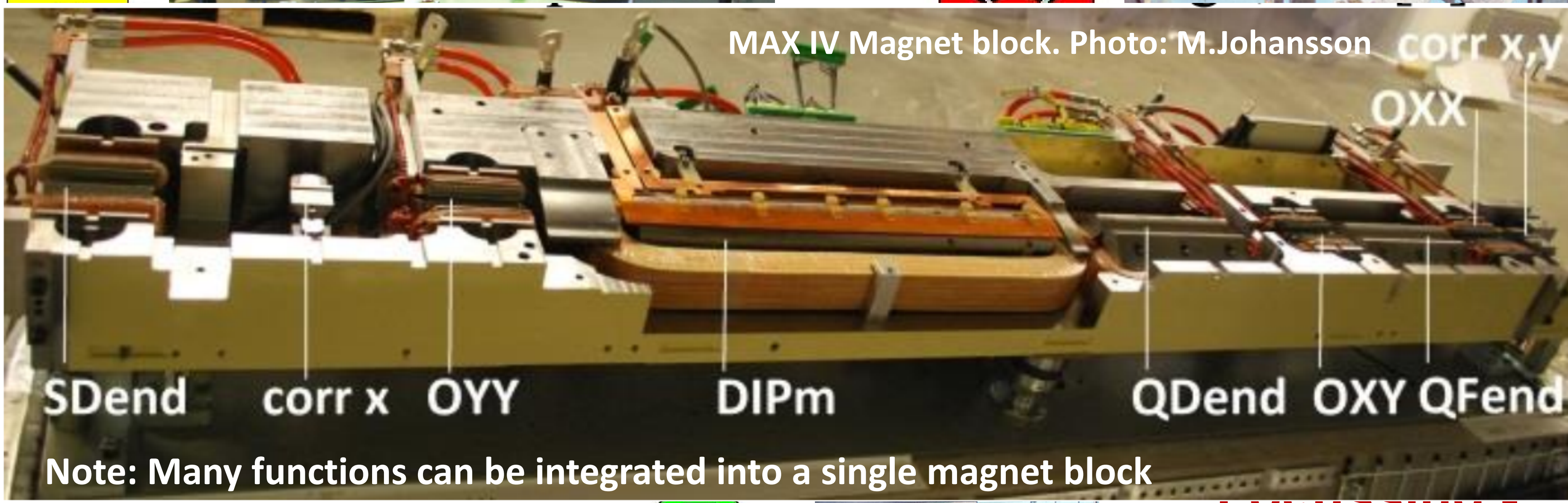
- Vacuum System: *Prevent losses and quality degradation*

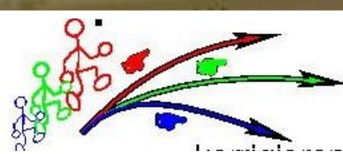
The magnet Lattice

Guiding - Dipoles

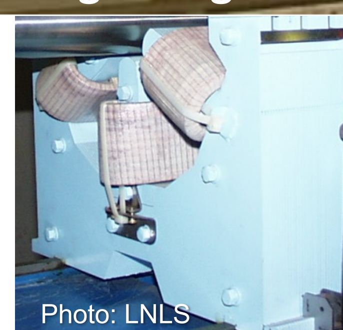








Correction of Chromatic aberrations Sextupoles



Quadrupoles

Icons adapted from BESSY
www.helmholz-berlin.de

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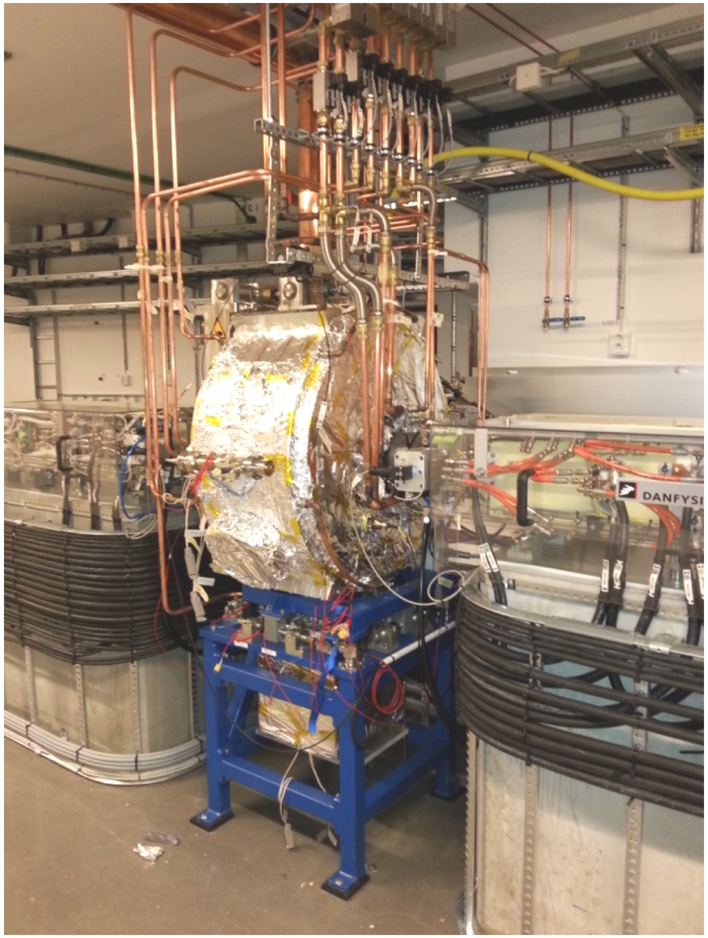
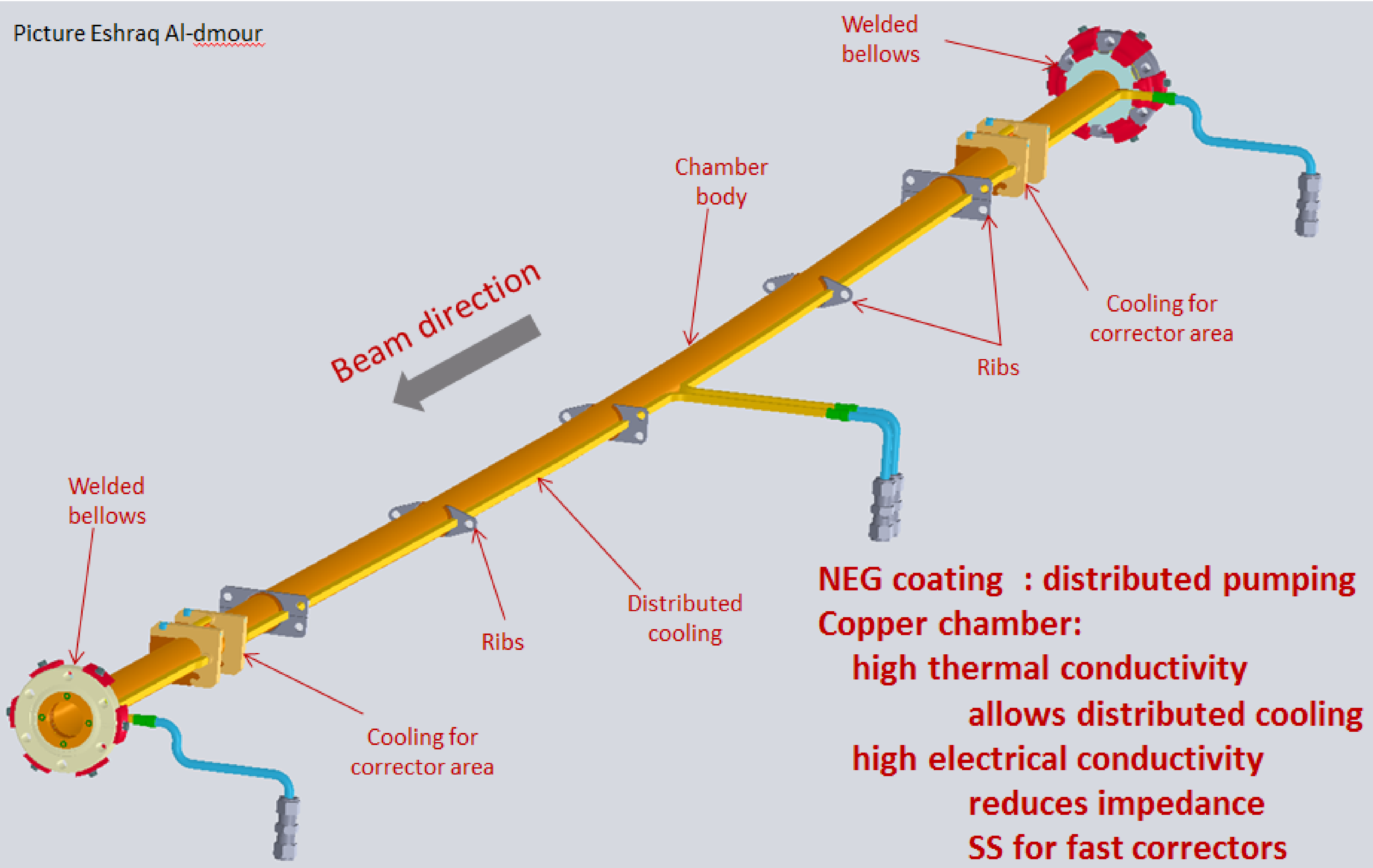
The Radio-Frequency System



100 MHz Copper Cavities

Solid State UHF Amplifiers

The Vacuum System



NEG coating : distributed pumping

Copper chamber:

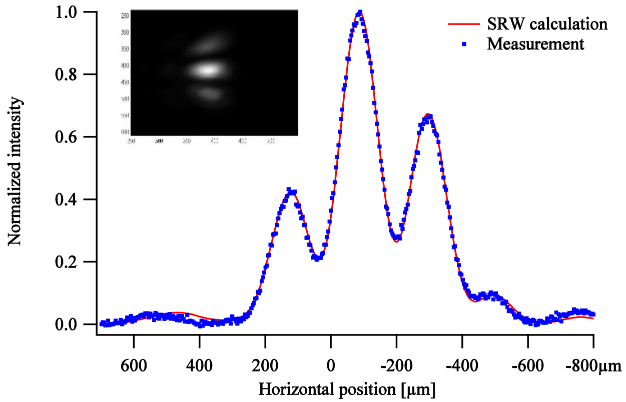
high thermal conductivity
allows distributed cooling

high electrical conductivity
reduces impedance

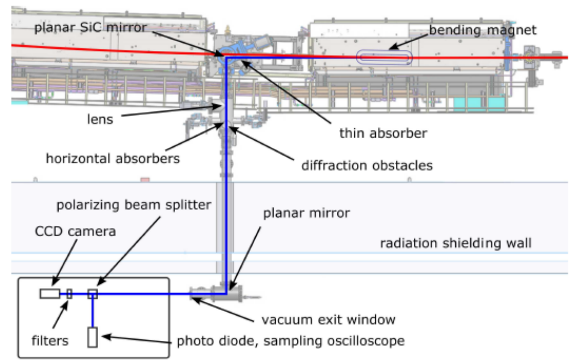
SS for fast correctors

Diagnostics and Controls

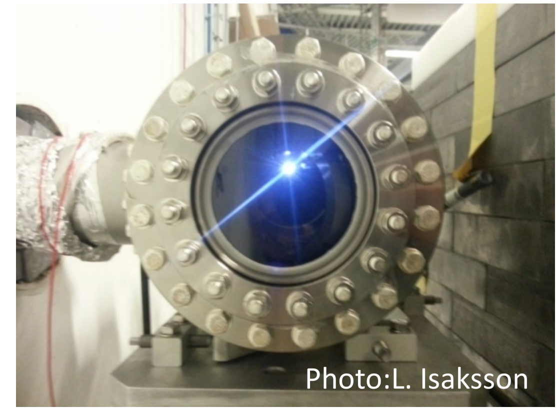
Optical Diagnostics



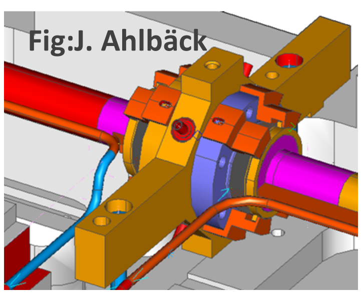
σ_x = 20.86 ± 0.14 µm (fit uncertainty)
σ_y = 15.70 ± 0.15 µm (fit uncertainty)



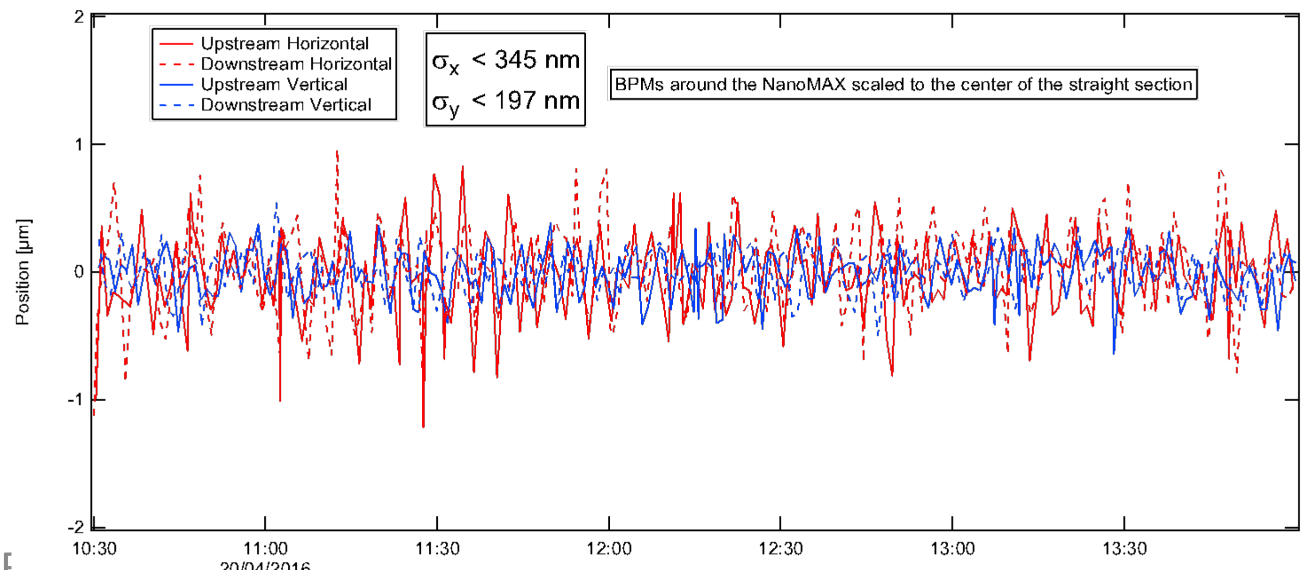
B320B diagnostic beamline
(visible SR radiation)



Electrical Diagnostics and Controls



RF BPMs



Outlook

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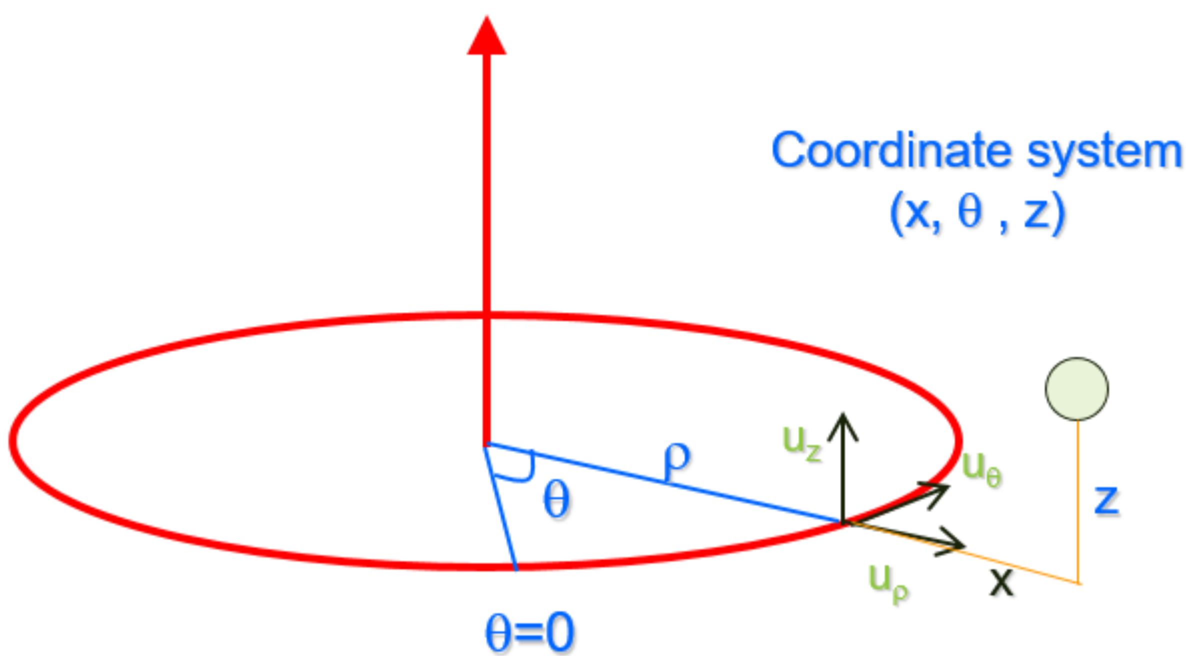
Storage Ring Beam Dynamics

- Goals:
  - To determine necessary conditions for the beam to circulate stably for many turns, while optimizing photon beam parameters – larger intensity and brilliance.
  - We want to study motion close to an *ideal* or *reference* orbit: Only small deviations w.r.t this reference are considered.
  - Understand the behaviour of a system composed of a large number (~ 10^{10} particles) of ***non-linear coupled oscillators governed by both classical and quantum effects***.



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Symmetry conditions for the Field

$$\begin{cases} B_z(x,\theta,z) = \quad B_z(x,\theta,-z) \\ B_x(x,\theta,z) = - B_x(x,\theta,-z) \\ B_\theta(x,\theta,z) = 0 \end{cases}$$

- Only transverse components (no edge effects)
- Only vertical component on the symmetry plane

$$\Downarrow$$

$$\boxed{\begin{gathered} B_z(x,\theta,z) = B_0 - g\,x \\ B_x(x,\theta,z) = - g\,z \end{gathered}}$$

First order expansion for the field close to the design orbit

Equations of Motion

$$\vec{F} = -e_0 \vec{v} \times \vec{B}$$

Lorentz Force

$$\gamma m \frac{d\vec{v}}{dt} = -e_0 \vec{v} \times \vec{B}$$

$$\vec{r}(t) = r\vec{u}_r + z\vec{u}_z$$

$$\vec{v}(t) = \dot{r}\vec{u}_r + r\dot{\theta}\vec{u}_\theta + \dot{z}\vec{u}_z$$

$$\frac{d\vec{v}}{dt} = (\ddot{r} - r\dot{\theta}^2)\vec{u}_r + (2\dot{r}\dot{\theta} + r\ddot{\theta})\vec{u}_\theta + \ddot{z}\vec{u}_z$$

$$\vec{v} \times \vec{B} = r\dot{\theta}B_z\vec{u}_r + (\dot{z}B_r - \dot{r}B_z)\vec{u}_\theta - r\dot{\theta}B_r\vec{u}_z$$

$$= r\dot{\theta}(B_0 - gx)\vec{u}_r + (-\dot{z}gz - \dot{r}(B_0 - gx))\vec{u}_\theta + r\dot{\theta}gz\vec{u}_z$$

Paraxial Approximation

- Azimuthal velocity >> transverse velocity
- Small deviations
- Independent variable t => s

$$r = \rho + x$$

$$x << \rho$$

$$p = p_0 + \Delta p$$

$$\Delta p << p_0$$

$$x''(s) + \left[1/\rho(s)^2 - K(s)\right]x(s) = \frac{1}{\rho}\frac{\Delta p}{p_0} \qquad K = \frac{e_0 g}{p_0}$$

$$z''(s) + K(s)z(s) = 0 \qquad \rho = \frac{p_0}{e_0 B_0}$$

K(s) periodic

Oscillatory (stable) solutions

$$s = \theta\rho$$

How to guarantee stability ?

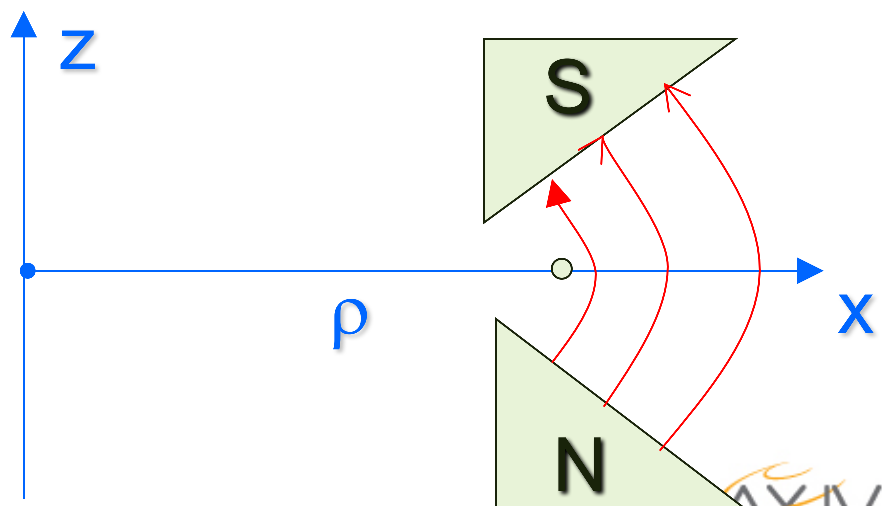
Weak focussing

Azimuthally symmetric machine: $y''(s) + Ky(s) = 0$ Oscillatory requires K>0

$$K_x = \frac{1}{\rho^2} - K > 0 \quad \Rightarrow \quad 0 < K < \frac{1}{\rho^2}$$

$$K_z = K > 0$$

Combined function magnets



Weak Focusing Limitations

- Magnet apertures scale with machine energy and become impractical

SOLUTION

Alternating Gradient

Courant/Snyder

Eliminate azimuthal symmetry and
alternate field gradients of opposite signs

On-Energy - General Solution

$$x(s) = x_0 C(s) + x_0' S(s)$$

On-energy particles $\dfrac{\Delta p}{p_0} = 0$

$$C(0) = 1 \qquad S(0) = 0$$

$$C'(0) = 0 \qquad S'(0) = 1$$

Particular solutions

$$\begin{pmatrix} x \\ x' \end{pmatrix}_s = \begin{pmatrix} C(s) & S(s) \\ C'(s) & S'(s) \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}_0$$

Matrix Solution

$$\begin{pmatrix} x \\ x' \end{pmatrix}_s = M(s) \begin{pmatrix} x \\ x' \end{pmatrix}_0$$

Combining elements means multiplying matrices

Transfer Matrices - Examples

Field-Free Straight section

$$x''(s) = 0$$

$$C(s) = 1$$

$$S(s) = s$$

$$M = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}$$

$$x(L) = x_0 + x_0' L$$

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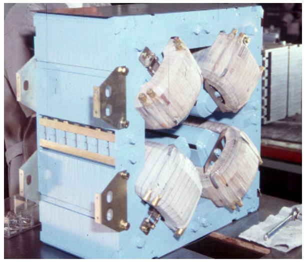
Transfer Matrices - Examples



Focussing Quad

$$x''(s) + Kx(s) = 0$$

$$C(s) = \cos(\sqrt{K}\, s)$$

$$S(s) = \frac{1}{\sqrt{K}} \sin(\sqrt{K}\, s)$$

$$M = \begin{pmatrix} \cos(\sqrt{K}\, L) & \frac{1}{\sqrt{K}} \sin(\sqrt{K}\, L) \\ -\sqrt{K} \sin(\sqrt{K}\, L) & \cos(\sqrt{K}\, L) \end{pmatrix}$$

Thin Lens Approximation

$$M = \begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix}$$

$$L \to 0$$

$$KL \to \frac{1}{f}$$

Stability Analysis – Periodic Systems

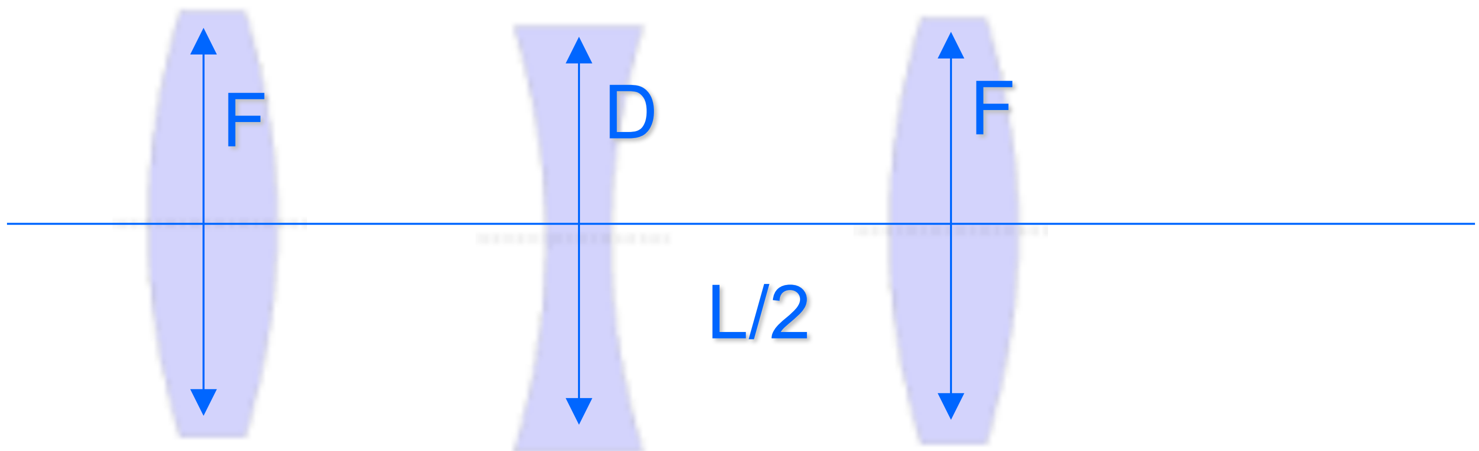
Transfer Matrix for a full period

$$M(s) = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

Stability ➔ matrix elements remain bounded

$$\begin{pmatrix} x \\ x' \end{pmatrix}_N = M^N \begin{pmatrix} x \\ x' \end{pmatrix}_0$$

Alternating Gradient: Stability



$$M = \begin{pmatrix} 1 & 0 \\ -\dfrac{1}{2f} & 1 \end{pmatrix} \begin{pmatrix} 1 & \dfrac{L}{2} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \dfrac{1}{f} & 1 \end{pmatrix} \begin{pmatrix} 1 & \dfrac{L}{2} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\dfrac{1}{2f} & 1 \end{pmatrix}$$

$$M = \begin{pmatrix} 1 - \dfrac{L^2}{8f^2} & L\left(1 + \dfrac{L}{4f}\right) \\ -\dfrac{L}{4f^2}\left(1 - \dfrac{L}{4f}\right) & 1 - \dfrac{L^2}{8f^2} \end{pmatrix}$$

Stability Analysis

$$M = \begin{pmatrix} 1-\frac{L^2}{8f^2} & L\left(1+\frac{L}{4f}\right) \\ -\frac{L}{4f^2}\left(1-\frac{L}{4f}\right) & 1-\frac{L^2}{8f^2} \end{pmatrix} = \cos(\mu)I + \sin(\mu)J$$

$$\cos(\mu) = 1-\frac{L^2}{8f^2}$$

$$\sin(\mu) = \frac{L}{2f}\sqrt{\left(1-\frac{L}{4f}\right)\left(1+\frac{L}{4f}\right)}$$

$$\beta = 2f\sqrt{\frac{\left(1+\frac{L}{4f}\right)}{\left(1-\frac{L}{4f}\right)}}$$

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$J = \begin{pmatrix} 0 & \beta \\ -\frac{1}{\beta} & 0 \end{pmatrix}$$

$$I^2 = I$$

$$J^2 = -I$$

$$M^2 = \cos(2\mu)I + \sin(2\mu)J$$

$$M^n = \cos(n\mu)I + \sin(n\mu)J$$

Stable if μ real

$$\left|1-\frac{L^2}{8f^2}\right| < 1 \Rightarrow f > \frac{L}{4}$$

Off-Energy Particles

Non-homogenuous term

$$D''(s) + \left[1/\rho(s)^2 - K(s)\right]D(s) = \frac{1}{\rho}\frac{\Delta p}{p_0}$$

D(s) can be obtained from the solution to the homogeneous eqs.

$$D(s) = S(s)\int_0^s \frac{ds'}{\rho(s')}C(s') - C(s)\int_0^s \frac{ds'}{\rho(s')}S(s')$$

Matrix Solution $\begin{pmatrix} x \\ x' \\ \frac{\Delta p}{p_0} \end{pmatrix}_s = M(s)\begin{pmatrix} x \\ x' \\ \frac{\Delta p}{p_0} \end{pmatrix}_{s=0}$ $M = \begin{pmatrix} C(s) & S(s) & D(s) \\ C'(s) & S'(s) & D'(s) \\ 0 & 0 & 1 \end{pmatrix}$

Example: Sector Dipole Magnet

$$K(s) = 0$$
$$\rho(s) = \rho_0$$

$$C(s) = \cos\left(\frac{s}{\rho_0}\right)$$

$$S(s) = \rho_0 \sin\left(\frac{s}{\rho_0}\right)$$

$$\rightarrow \quad D(s) = \rho_0 \left\{1 - \cos\left(\frac{s}{\rho_0}\right)\right\}$$

$$M = \begin{pmatrix} C(s) & S(s) & D(s) \\ C'(s) & S'(s) & D'(s) \\ 0 & 0 & 1 \end{pmatrix} \approx \begin{pmatrix} 1 & s & \dfrac{s^2}{2\rho_0} \\ 0 & 1 & \dfrac{s}{\rho_0} \\ 0 & 0 & 1 \end{pmatrix}$$

Small bending angle

General pseudo-harmonic solution

$$\begin{cases} x''(s) + \left[1/\rho(s)^2 - K(s)\right]x(s) = \dfrac{1}{\rho}\dfrac{\Delta p}{p_0} \\ z''(s) + K(s)z(s) = 0 \end{cases}$$

Pseudo-harmonic solution

$$x(s) = \sqrt{\varepsilon\beta(s)}\cos(\phi(s) - \phi_0) + \eta(s)\frac{\Delta p}{p}$$

Betatron Function

Dispersion Function

Periodic

Betatron Phase Advance

$$\phi(s) = \int_0^s \frac{ds'}{\beta(s')}$$

Betatron Tune

$$Q = \frac{\mu}{2\pi} = \frac{\phi(L)}{2\pi}$$

Equation for Betatron Function

$$\frac{1}{2}\beta(s)\beta''(s) - \frac{1}{4}\beta'^2(s) + \beta^2(s)K(s) = 1$$

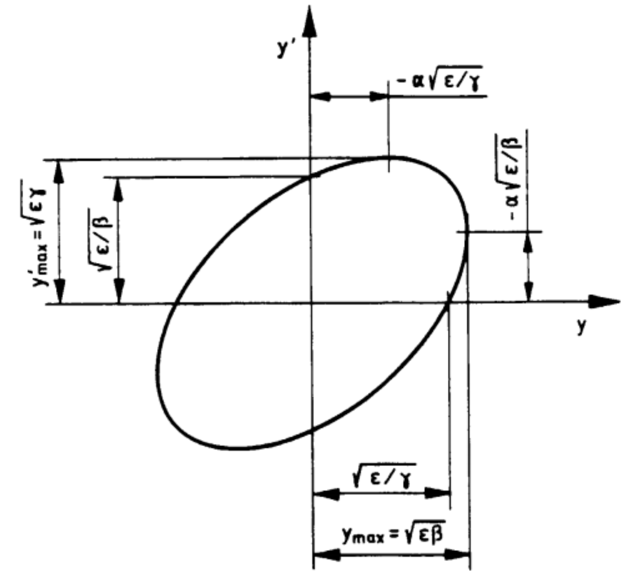
Twiss Parameters

$$\beta(s)$$

$$\alpha(s) = -\frac{1}{2}\beta'(s)$$

$$\gamma(s) = \frac{1+\alpha^2(s)}{\beta(s)}$$

Figure E.Willson, CAS 2003



Courant Snyder Invariant

$$\varepsilon = \gamma(s)x^2(s) + 2\alpha(s)x(s)x'(s) + \beta(s)x'^2(s)$$

These are properties of the ring, defined by how the focussing is distributed along the accelerator and give us a conveniente way to describe any trajectory (in linear approximation)

Twiss Parameters and Beam sizes

Equilibrium beam parameters: Emittance, Energy Spread $\epsilon_x, \epsilon_y, \sigma_\delta$

$$\sigma_x(s) = \sqrt{\epsilon_x \beta_x(s) + {\sigma_\delta}^2 \eta(s)^2}$$

$$\sigma_{x'}(s) = \sqrt{\epsilon_x \gamma_x(s) + {\sigma_\delta}^2 \eta'(s)^2}$$

$$\sigma_y(s) = \sqrt{\epsilon_y \beta_y(s)}$$

$$\sigma_{y'}(s) = \sqrt{\epsilon_y \gamma_y(s)}$$

Twiss Parameters and Perturbations

- Localized dipole error (***θ***) – perturbation of the *closed orbit* (periodic solution)

$$\Delta x_{c.o.}(s) = \frac{\sqrt{\beta(s)\beta_0}\theta_0 \cos(\phi(s) - \pi Q)}{2\sin(\pi Q)}$$

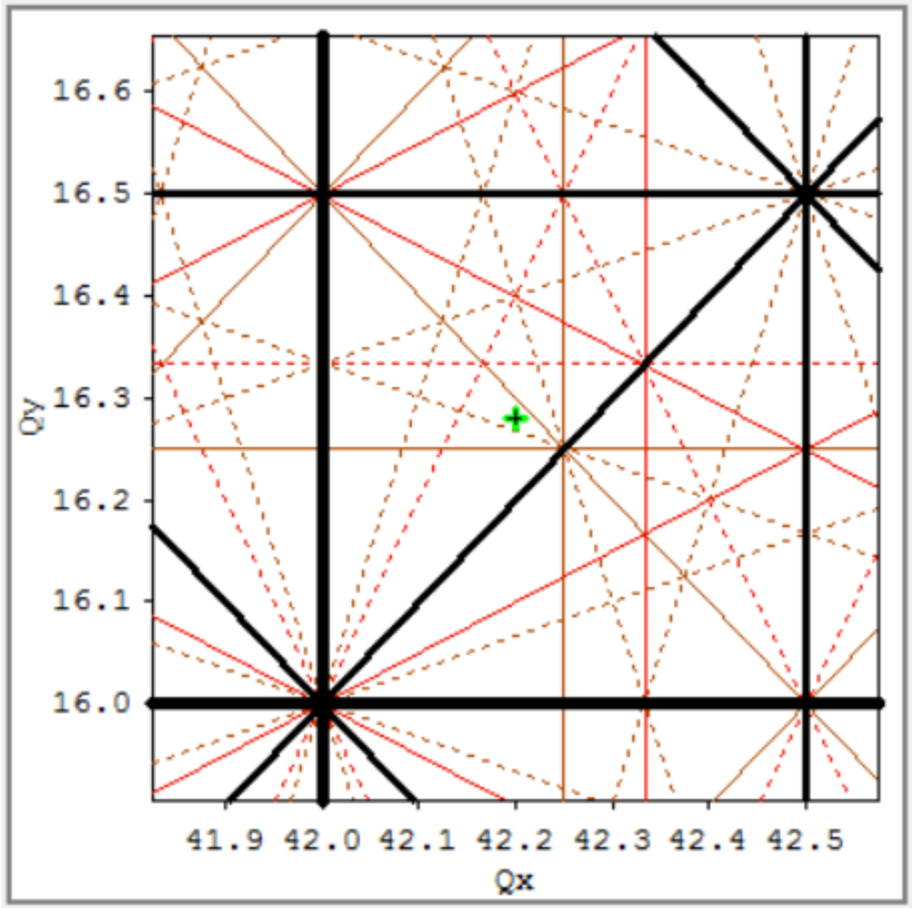
- Localized quadrupole error (ΔK) – perturbation of the tune and beta function

$$\frac{\Delta\beta}{\beta} = -\frac{\beta_0}{2\sin(2\pi Q)}\cos(2\pi Q)\Delta K$$

Perturbations

Some freqs. (tunes) must be avoided to prevent resonances.

$mQx+nQy=p$ m,n,p integer



Resonance Diagram for the MAX IV 3 GeV Ring

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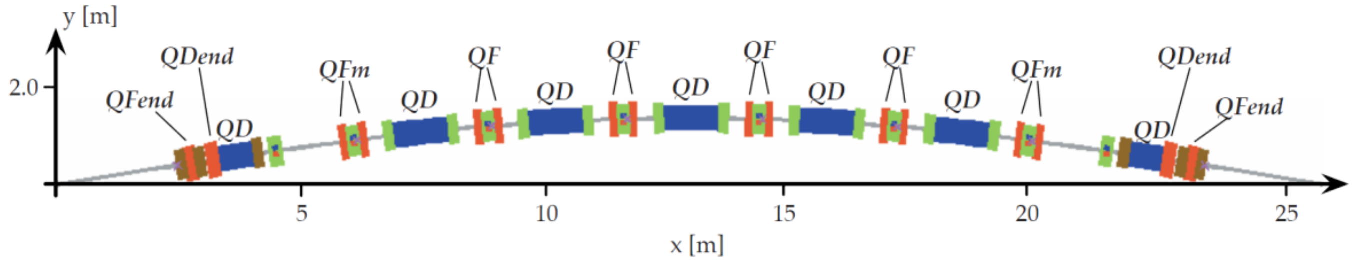
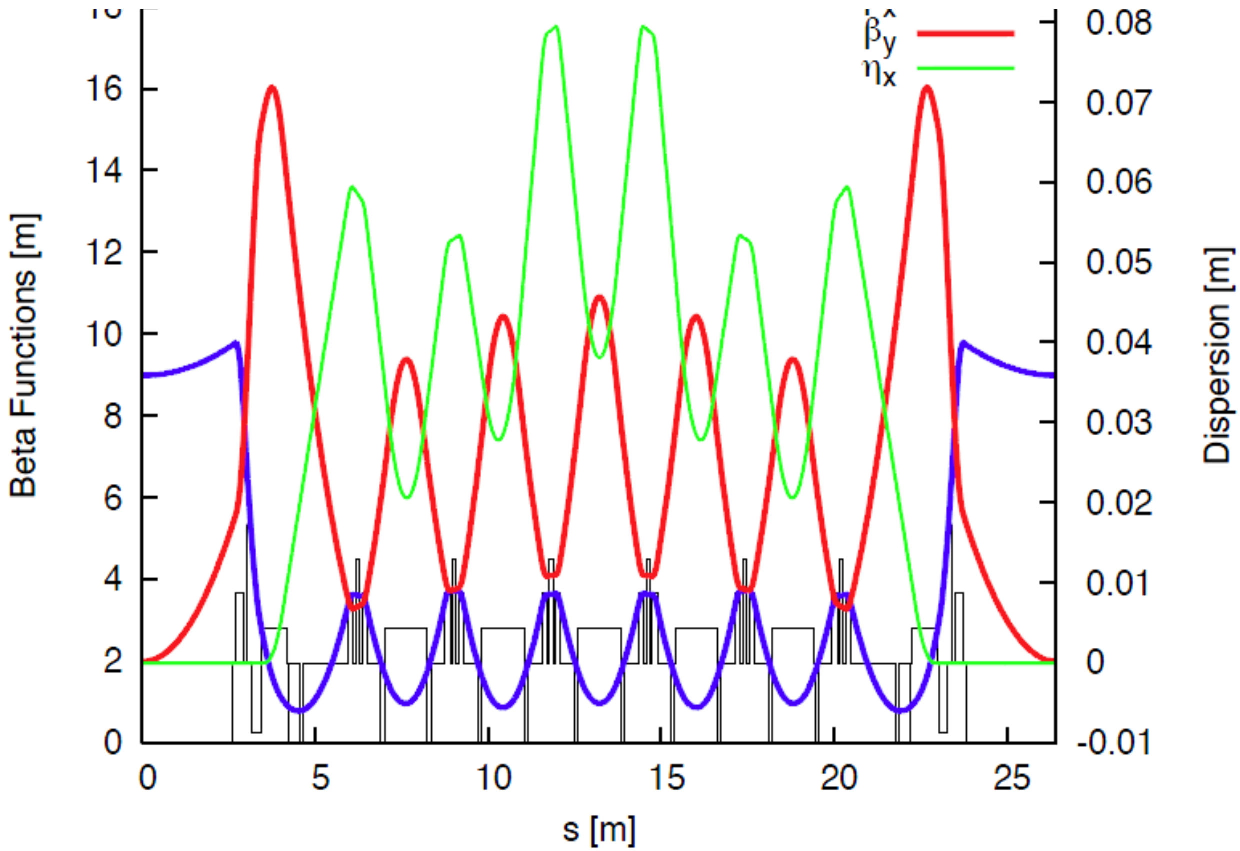
Twiss Parameters MAX IV 3 GeV Ring



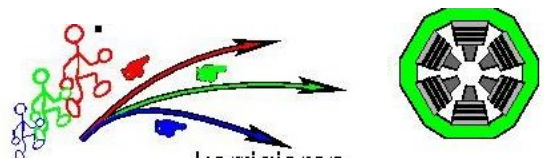


Non-linear perturbations

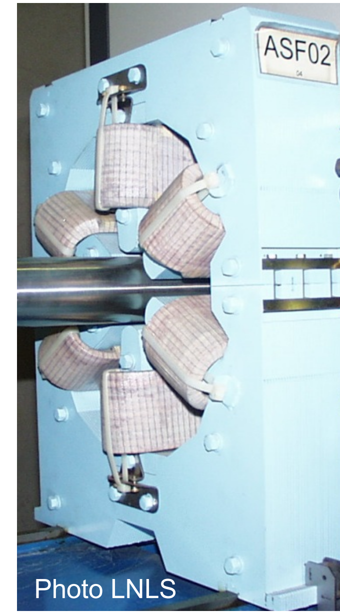
$$B_z(x) = Sx^2$$

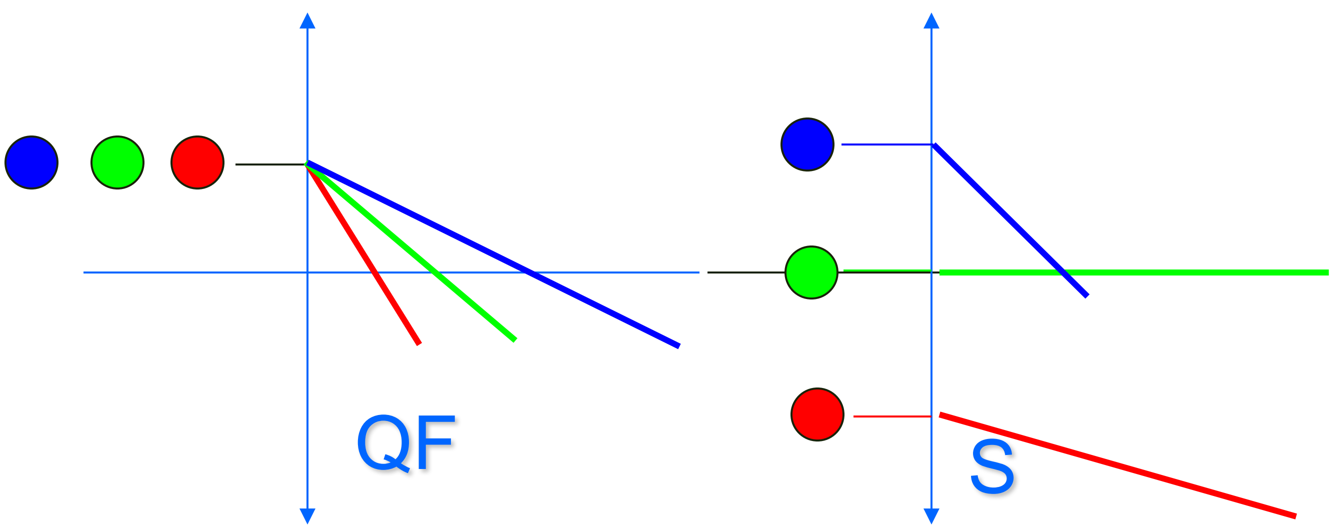
$$G(x) = 2Sx$$

Chromaticity: quad strength varies with energy.



Correction of chromatic aberration with sextupoles

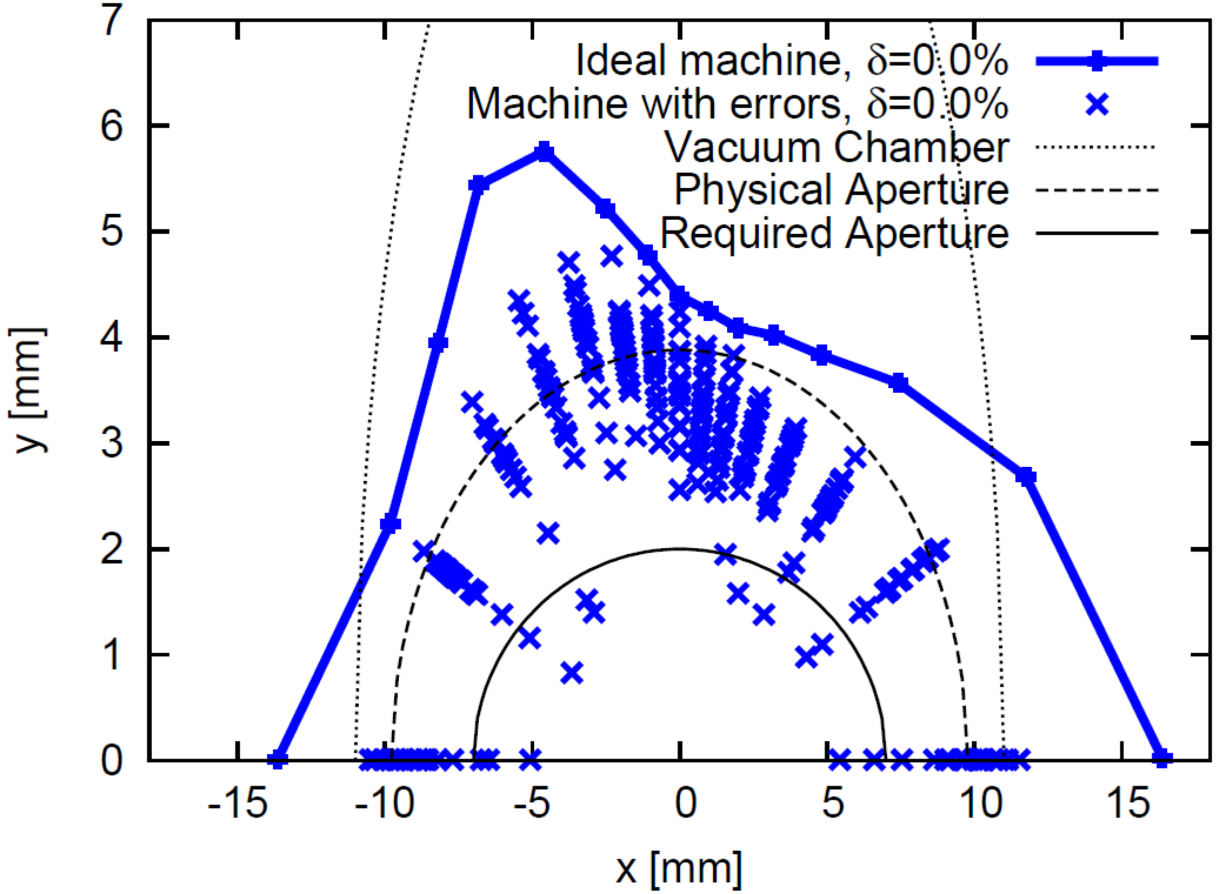




A sextupole produces a position dependent focussing

Sextupoles are non-liear elements and introduce perturbations

Non-Linear Perturbations and Dynamic Aperture

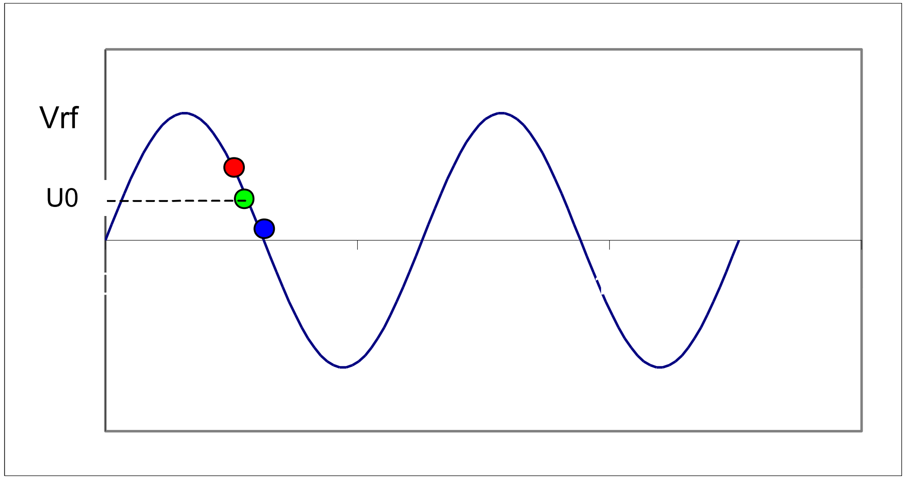


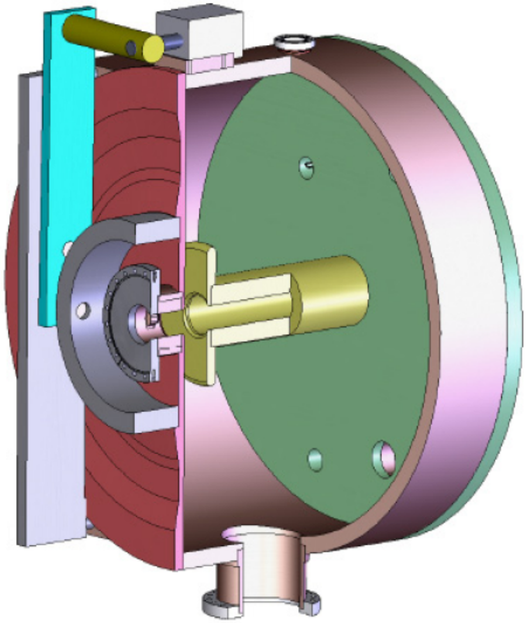
MAX IV DDR, 2010

Longitudinal Dynamics: Phase Stability

Synchrotron Oscillations

Particles with different energies have different revolution periods





MAX IV 100 MHz RF Cavity

For small amplitudes: simple harmonic motion
Larger amplitudes: non-linearities (like a pendulum)

$$\ddot{\tau} + \omega_s^2 \tau = 0$$

Brief Recap – Beam Dynamics

Transverse Plane:

$$x''(s) + \left[1/\rho(s)^2 - K(s)\right]x(s) = \frac{1}{\rho}\frac{\Delta p}{p_0}$$

$$z''(s) + K(s)z(s) = 0$$

Longitudinal Plane: $\ddot{\tau} + \omega_s^2 \tau = 0$

- The beam is a collection of many 3D – oscillators.
- If parameters are properly chosen (magnet lattice, RF system), *stable oscillations* are realized in all planes.
- Non-linearities cause distortions that may reduce the available stable area in phase space: reduction of the *dynamic aperture.*

Linear Oscillations – Twiss Parameters

- $Q, \beta(s), \alpha(s), \gamma(s)$
- Are a property of the lattice (the whole accelerator).
- Provide a convenient way to summarize all about the linear behaviour of the accelerator: trajectories, sizes, sensitivity to errors

Outlook

- Why Particle Accelerators ?
  - Why Synchrotron Light Sources ?

- Storage Ring Light Sources: *accelerator building blocks*

- Basic Beam Dynamics in Storage Rings.
  - Transverse dynamics: twiss parameters, betatron functions and tunes, chromaticity.
  - Longitudinal dynamics: RF acceleration, synchrotron tune
  - Synchrotron light emission, radiation damping and emittance

- How vacuum affects accelerator performance.

How Vacuum Systems affect SR Performance

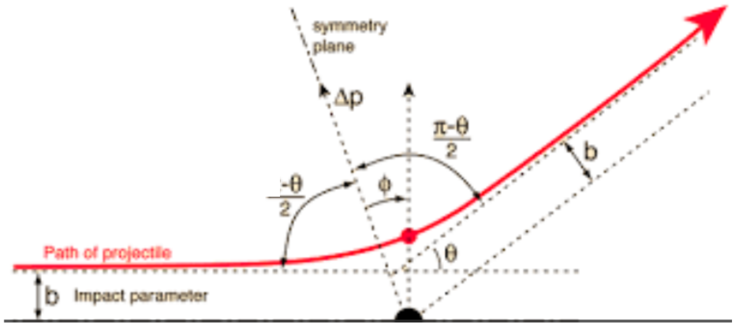
	Quantity	Quality	
Coherent	Wakefields		limits max. current bunch dimensions
	Ion Trapping		lifetime tune shifts emittance growth transverse stability
Incoherent	Scattering Elastic/Inelastic		lifetime : need to top-up more often

Elastic Scattering

Illustration from http://hyperphysics.phy-astr.gsu.edu

Rutherford Scattering cross-section

$$\frac{d\sigma_{el}}{d\Omega} = \left(\frac{Ze^2}{2pc}\right)^2 \frac{1}{sin\left(\frac{\theta}{2}\right)^4}$$



Particles are lost if scattered by angles larger than:

$$\theta_{max} = \frac{\left(A^2/\beta\right)_{min}}{\beta_0}$$

Aperture limitation around the whole ring

Beta function where the collision occured

electrons

Gas density

$$\frac{1}{\tau_{el}} = -\frac{1}{N}\frac{dN}{dt} = cn\int_{\theta_{max}}^{\pi} \frac{d\sigma_{el}}{d\Omega} d\Omega$$

Assuming Nitrogen

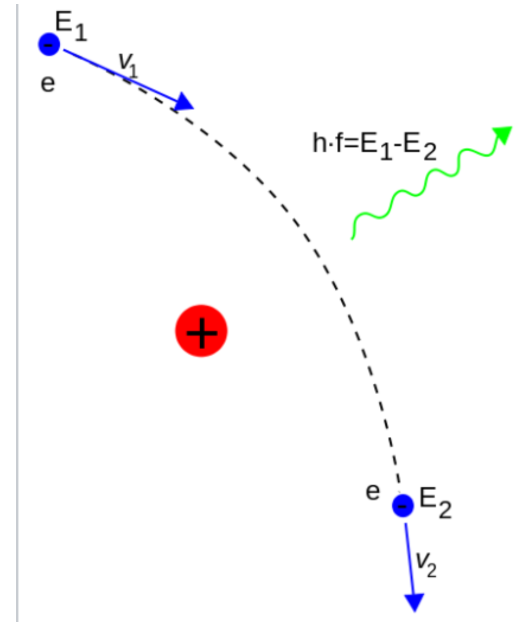
$$\tau_{el}[hr] = \frac{10.25E[GeV]^2\epsilon_A[mmmrad]}{\langle\beta\rangle(m)P[ntorr]}$$

Watch out for:

- **low energy**
- **small apertures**
- **high pressure at high beta locations**
- **High Z**

Inelastic Scattering (bremsstrahlung)

- Particle lose energy through radiation emission in collision with nuclei and electrons.
- If energy loss is larger than acceptance, particle is lost



Ilustration https://de.wikipedia.org

$$\frac{d\sigma_{BS}}{d\varepsilon}=\frac{\alpha 4Z^2r_e^2}{\varepsilon}\left\{\left[\frac{4}{3}\left(1-\frac{\varepsilon}{E}\right)+\left(\frac{\varepsilon}{E}\right)^2\right]\ln\left(\frac{183}{Z^{\frac{1}{3}}}\right)+\frac{1}{9}\left(1-\frac{\varepsilon}{E}\right)\right\}$$

Particles are lost if they lose energy larger than the acceptance: ε_{acc}

$$\frac{1}{\tau_{BS}}=-\frac{1}{N}\frac{dN}{dt}=cn\int_{\theta_{max}}^{\pi}\frac{d\sigma_{BS}}{d\Omega}d\Omega=cn4Z^2r_e^2\left\{\frac{4}{3}\left(\ln\left(\frac{E}{\varepsilon}\right)-\frac{5}{8}\right)\ln\left(\frac{183}{Z^{\frac{1}{3}}}\right)+\frac{1}{9}\left(\ln\left(\frac{E}{\varepsilon}\right)-1\right)\right\}$$

Watch out for high Z
Weak dependence on energy and energy acceptance

Ion Trapping

- circulating electrons collide with residual gas molecules producing positive ions that can be captured (trapped) by the beam

↓

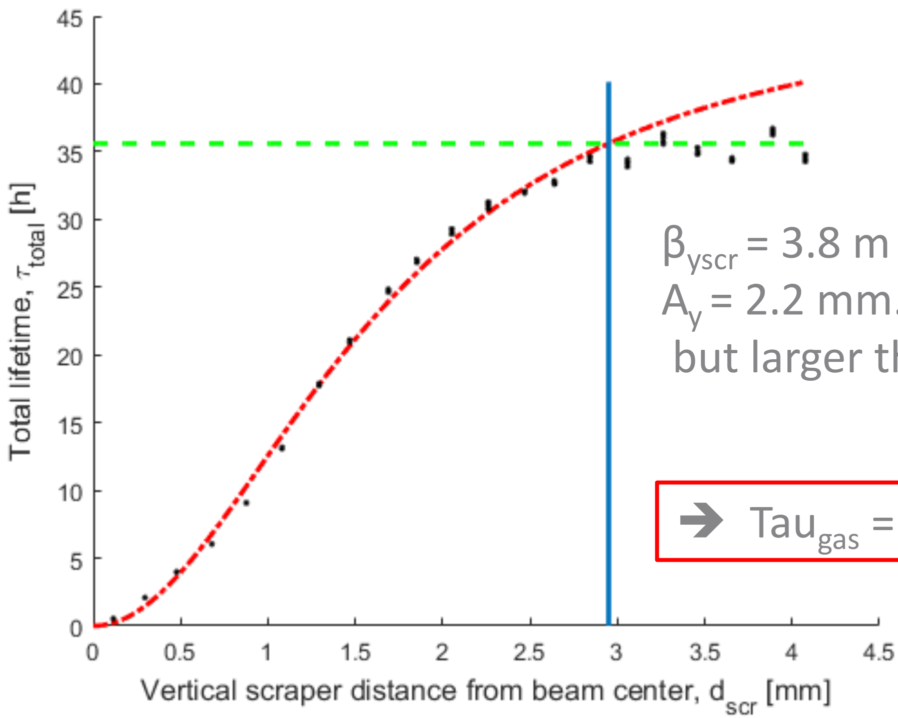
- Reduces beam lifetime : increased local pressure.
- Tune –shifts, Tune spreads
- Emittance Growth
- Coherent Collective instabilities (multi-bunch)

This had some nearly catastrophic effects on some early low energy injection machines

Lifetime contributions at the MAX IV 3 GeV Ring

Tau_{tot} = 36 h or I*Tau_{tot} = 2.5 Ah

Slide courtesy Åke Andersson



β_{yscr} = 3.8 m
A_y = 2.2 mm.mrad (less than physical: A_y ~ 4 mm.mrad, but larger than future ID-chambers: A_y ~ 1 mm.mrad)

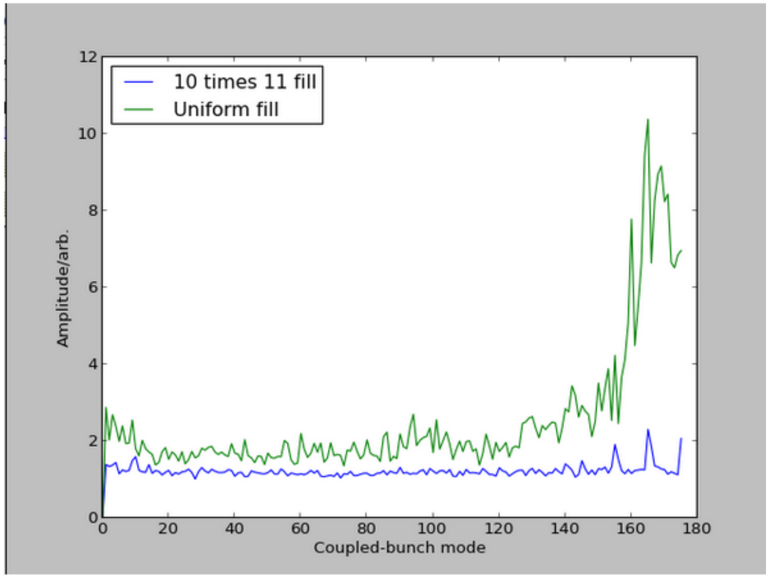
➔ Tau_{gas} = 60 h or I*Tau_{gas} = 4.2 Ah

Vertcal Scraper Measurements
by Jens Sundberg

➔ Tau_{Tou} = 90 h or I*Tau_{Tou} = 6.2 Ah

Transverse collective instabilities driven by ions

Increased stability by adding a gap to the bunch train



2016/07/06: MAX IV 3 GeV Ring
Early commissioning

Transverse beam blow up due to ion trapping

Ion Clearing ON

Ion Clearing OFF

LNLS 1.37 GeV electron storage rig

R.H.A.Farias et at at: *Optical Beam Diagnostics for the LNLS Synchrotron Light Source,*EPAC98, p.2238.

Thank you for your attention

- References
  - H. Wiedemann, *Particle Accelerator Physics I and II,* Springer Verlag.
  - M.Sands, *The Physics of Electron Storage Rings*
  - D.A.Edwards and M.J.Syphers, *An Introduction to the Physics of High Energy Accelerators*, Wiley
  - CAS – CERN Accelerator Schools (Basic and Advanced)

Back up slides

Why Synchrotron Light ?

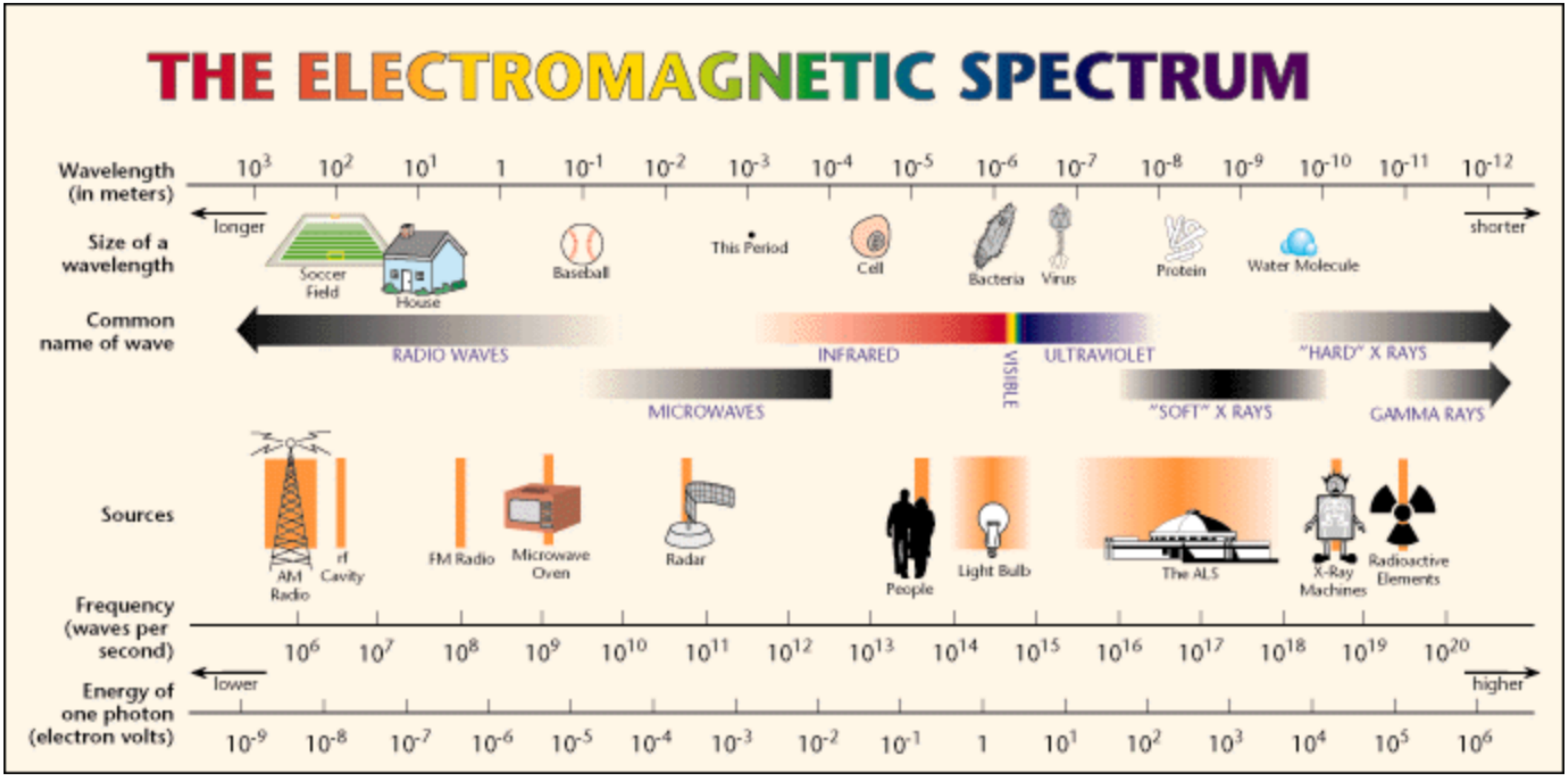


Image: Lawrence Berkely Lab

Lattice Design for *Low Emittance* Rings

General Problem Statement – Scaling Laws

$$\varepsilon_0 = C_q \frac{\gamma^2}{J_x} \frac{\left\langle \frac{H}{\rho^3} \right\rangle}{\left\langle \frac{1}{\rho^2} \right\rangle}$$

$$H(s) = \beta(s)\eta'^2(s) + 2\alpha(s)\beta(s) + \gamma(s)\eta^2(s)$$

$$J_x = 1 - \mathcal{D}$$

$$\mathcal{D} = \frac{\oint \frac{\eta(s)}{\rho^3(s)}\left(1 + 2\rho^2(s)k(s)\right)}{\oint \frac{ds}{\rho^2(s)}}$$

$$\varepsilon_0 = C_q \frac{\gamma^2}{J_x} \frac{\langle H \rangle_{dip}}{\rho}$$

isomagnetic

Defining the Basic Parameters of a SR based Light Source

$$E_1 = \frac{hc}{\lambda_p} \frac{2\gamma^2}{\left(1 + \frac{1}{2} K_u^2\right)}$$

Energy → Photon energy range + Insertion device Technology + Top-up Injection

$$B_0[Tesla] = 3.694 \exp\left(-5.068 \frac{g}{\lambda_p} + 1.52 \left(\frac{g}{\lambda_p}\right)^2\right)$$

Diameter → Emittance (brightness) requirements

$$\varepsilon_0 = C_q \frac{\gamma^2 \theta^3}{12\sqrt{15} J_x} F$$

Electrostatic SR

Stores 25 keV ions.

S.Moller, EPAC98

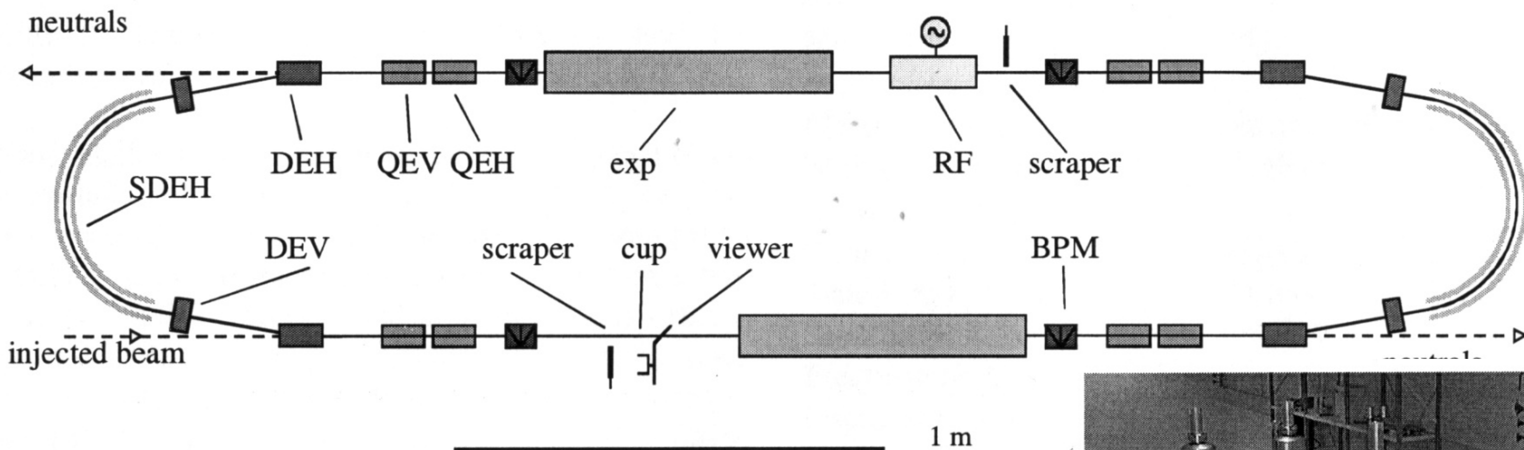


Figure 1: Layout of the ELISA storage ring. The abbreviations are explained i

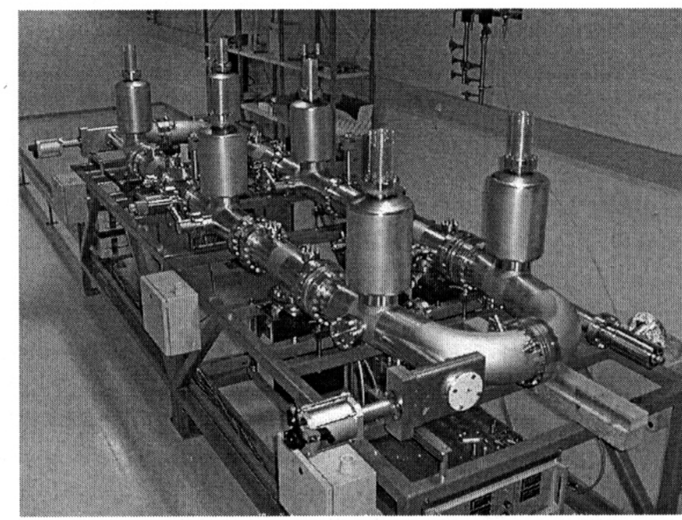


Figure 3: Picture of the ELISA storage ring.

Beam Guiding

Why magnetic fields ?

Lorentz Force $$\vec{F} = -e_0(\vec{E} + \vec{v} \times \vec{B})$$

at 3.0 GeV,

B= 1.0 T

E = 500 MV/m !!

Transverse Beam Dynamics

- Zeroth order: guide fields (dipoles)
- First order : Focusing – linear oscillations (quadrupoles). ***Alternating Gradient*.**
- Second order: Chromatic Aberrations and corrections (sextupoles)
- Effects of perturbations, non-linearities ***Dynamic Aperture*.**

Damping/Excitation of Longitudinal Oscillations

- Photon emission depends on particle energy (larger energy, more emission). This adds a **dissipative** term to the eqs. of motion.
- However, emission happens in the form of **discrete** events (photons). At each emission, there is a **sudden** change in particle energy (but no sudden change in particle position.
- Both effects together lead to an **equilibrium** state that defines the bunch dimensions in longitudinal phase space

Energy spread

$$\sigma_p^2 = C_q \frac{\gamma^2}{J_s} \frac{\left\langle 1/\rho^3 \right\rangle_s}{\left\langle 1/\rho^2 \right\rangle_s}$$

Bunch length

$$\sigma_l = \frac{c\alpha}{\Omega_s}\sigma_p$$

Depends on lattice

Damping/Excitation of Transverse Oscillations Oscillations

- Discrete photon emission changes momentum along the direction of propagation If this happens in a **dispersive** region of the magnet lattice, a transverse (betatron) oscillation will be **excited**.
- Momentum is regained at the RF cavity only along the longitudinal direction. This causes a reduction of the particle angles (**damping**).
- Both effects together lead to an **equilibrium** state that define the transverse beam dimension and angular spread, i.e., the **emittance**.

$$\varepsilon_0 = C_q \frac{\gamma^2}{J_x} \langle 1/ \ldots$$

Isomagnetic

$$\varepsilon_0 = C_q \frac{\gamma^2 \theta^3}{12\sqrt{15} J_x} F$$

Number of dipoles

$$H(s) = \beta(\ldots + \gamma(s)\eta(s)^2$$

Lattice

The Challenge of High Brightness Source Source Design: a beam dynamics perspective