RF Systems

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Lecture I

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  - RF system anatomy
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  - Iris-loaded waveguides
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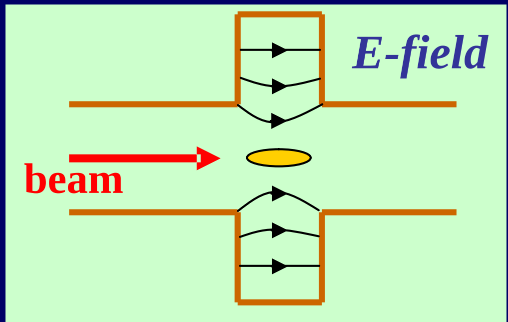
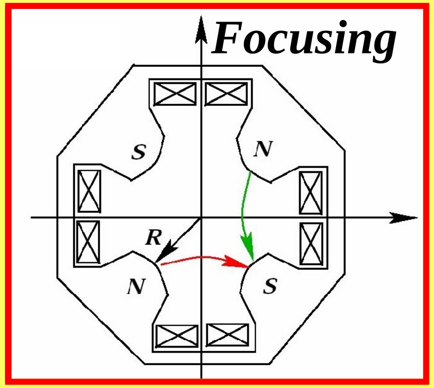
What are RF Systems for in Particle Accelerators?

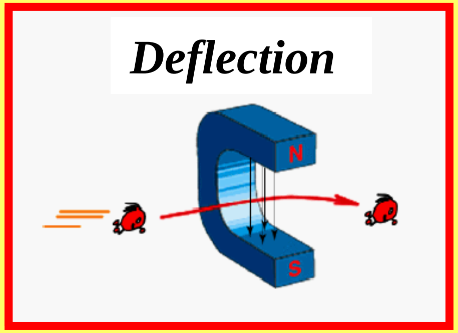
The motion of the charged particles in particle accelerators is governed by the Lorentz force:



$$\frac{d\vec{P}}{dt} = q\cdot\left(\boxed{\vec{E}}+\vec{v}\times\boxed{\vec{B}}\right)$$



B-field



Static magnetic field $\vec{B}(\vec{x})$ is used to model the beam trajectory and to focus the particle transverse motion (see Transverse Dynamics Lectures).

Longitudinal electric field $E_{//}$ is necessary to accelerate (≡ increase the energy) of charged particles. With the exception of pioneer electro-static accelerators, the longitudinal E-field component of variable electromagnetic fields $\vec{E}(\vec{r},t);\vec{B}(\vec{r},t)$ is always used for acceleration for practical reasons.

What are RF Systems for in Particle Accelerators? (cnt'd)

The energy gained by a charged particle interacting with a variable electric field along a portion of its trajectory is given by:

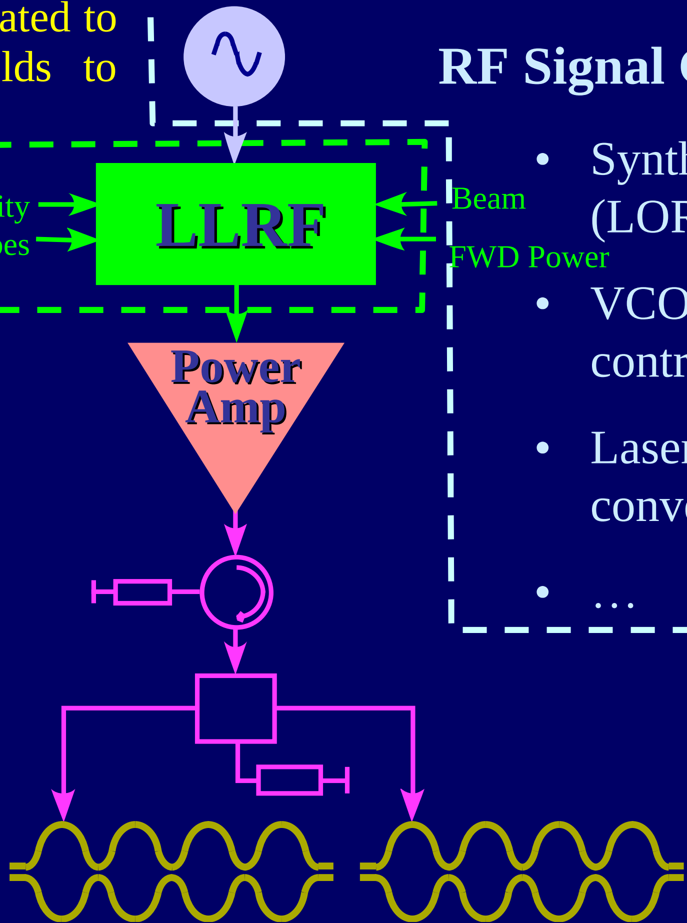
$$\Delta W_{21} = q\int_{\vec{r}_1}^{\vec{r}_2} \vec{E}\left(\vec{r}(t),t\right)\cdot d\vec{r} = q\int_{z_1}^{z_2} E_z\left(z(t),t\right)dz$$

where $\vec{r}(t)$ is the particle motion law.

The systems devoted to the generation of the accelerating E-fields have been mainly developed in the **radio-wave region** of the e.m. spectrum, because of both technical (availability of power sources) and physics (synchronization with the revolution frequencies of synchrotrons and cyclotrons) motivation.

RF Systems: Basic Definition and Anatomy

The RF systems in particle accelerators are the hardware complexes dedicated to the generation of the e.m. fields to accelerate charged particle beams.



RF Signal Generation

- Synthesized oscillators (LORAN stabilized)
- VCOs (driven by low-level controls)
- Laser-to-voltage reference converters
- …

Low-level RF control

- Amplitude and phase set of the accelerating fields
- Tuning control of the accelerating structures
- Beam loading compensation
- RF and beam feedback systems
- …

RF Systems:
Basic Definition and Anatomy

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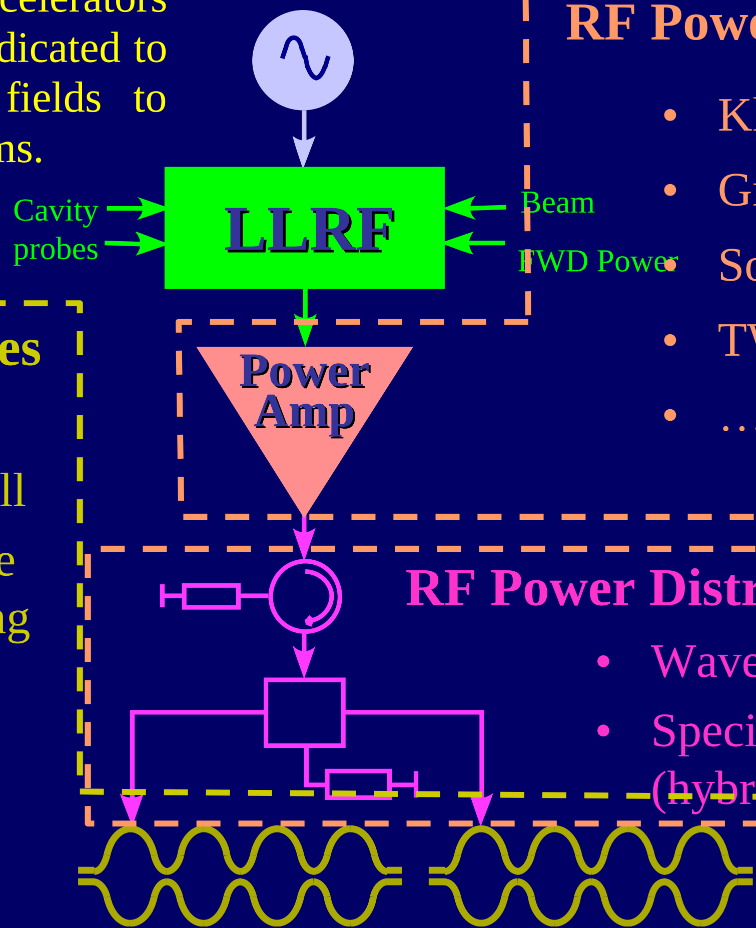


RF Power Generation

- Klystrons
- Grid Tubes
- Solid State Amps
- TWTs
- …

Accelerating Structures

- Resonant Cavities
  - Single or multi-cell
  - Room-temperature or Superconducting
- Travelling wave sections
- RF Deflectors (either SW or TW)

RF Power Distribution

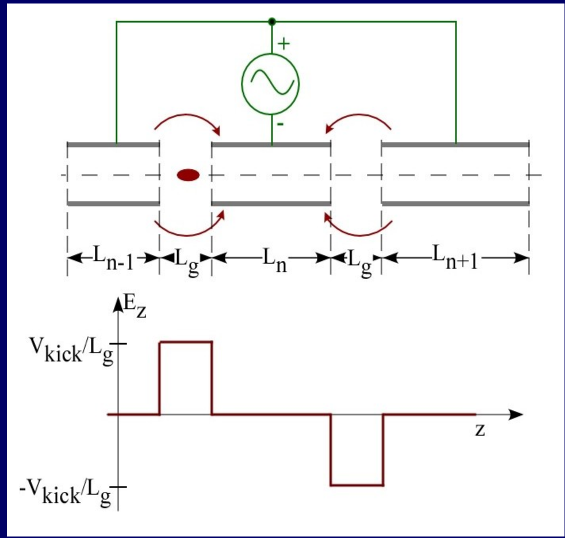
- Waveguide network
- Special Components (hybrids, circulators, …)

RF Systems: some more details

- Actual RF system frequencies in particle accelerators span from ≈10 MHz to 30 GHz and beyond (μ-wave spectrum);
- Depending on the characteristics of the beam RF systems can operate either continuously (CW) or in pulsed regime.
- RF power levels up to 100 MW rms and ≈10 GW pulsed;
- Accelerations for energy increase (toward final energy, as in linacs, synchrotrons, cyclotrons, …) or for energy restoring (around nominal energy, as in storage rings …)
- RF accelerating fields also provide particle capture and longitudinal focusing (see Longitudinal Dynamics lectures);
- In some cases, through different mechanisms, accelerating fields also influence the beam transverse equilibrium distributions (adiabatic damping in linacs, radiation damping in lepton synchrotrons and storage rings …)

RF Accelerating structures

The Wideröe Drift Tube Linac is an historical and didactical example of acceleration based on variable fields. The beam is accelerated while crossing the gap between equipotential drift tubes. The length of the tubes has to match the beam velocity in order to cross each gap in phase with the accelerating field:



$$\frac{L_n}{v_n} = \frac{T_{RF}}{2} \rightarrow \begin{cases} L_n = \frac{1}{2} v_n T_{RF} = \frac{1}{2} \beta_n \lambda_{RF} \\ < \Delta U >_{ave} = qV_{kick} / L_n = 2qV_{kick} / \beta_n \lambda_{RF} \end{cases}$$

There are two main consequences from the previous conditions:

1. The tube equipotentiality requires $\lambda_{RF} << L_n \rightarrow \beta_n << 1$, which only applies for non-relativistic beams. To efficiently accelerate relativistic beams distributed fields have to be used.
2. Average gradients are inversely proportional to the RF wavelengths. The use of high RF frequencies increases the acceleration efficiency.

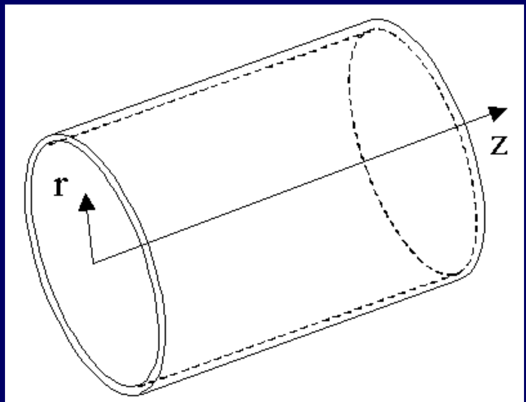
The natural evolution of the drift tubes was represented by high frequency, field distributed structures like <u>resonant RF cavities</u> and <u>disk-loaded waveguides</u>.

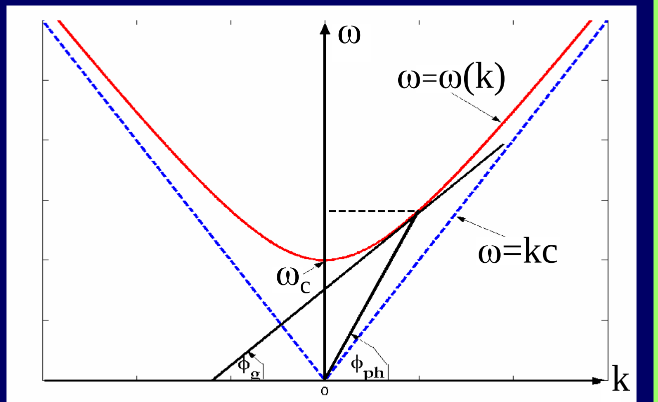
RF Accelerating structures: TW iris-loaded waveguides

According to Maxwell equations an e.m. wave travelling along a transverse uniform guide has always a phase $\boldsymbol{v_{ph}}$ velocity larger than the light speed $\boldsymbol{c}$ and can not be synchronous with a particle beam.

$$\vec{E}(x,y,z,t)=\sum_n \underbrace{\vec{E}_n(x,y)}_{\text{waveguide modes}}\, e^{j[\omega t-\underbrace{k_n(\omega)}_{\text{mode propagation constant}}z]}$$

$$k_n(\omega)=\sqrt{\left(\frac{\omega}{c}\right)^2\underbrace{-k_{c_n}^2}_{\text{mode wavenumber cut-off}}}<\frac{\omega}{c}\;\rightarrow\;\begin{cases} v_{ph}=\dfrac{\omega}{k}>c \\ v_g=\dfrac{d\omega}{dk}<c \end{cases}$$



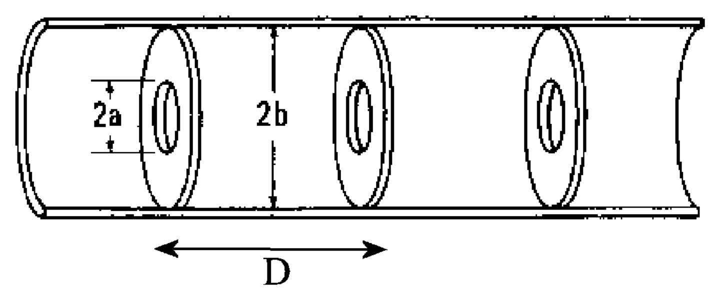


The ω(k) function is the so-called **dispersion curve** of the guide mode.

- The wave phase velocity v_{ph} at the operation frequency ω is equal to $tan(\phi_{ph})$;
- Modes propagate only if $\omega>\omega_c$ ($\omega_c=k_c\cdot c$ = ***mode cut-off*** frequency);
- The wave group velocity v_g at the operation frequency ω is equal to $tan(\phi_g)$;
- Negative k in the plot correspond to waves travelling toward negative-z.

RF Accelerating structures: TW iris-loaded waveguides

To "slow-down" the phase velocity of the guided wave periodic structures, also called "iris loaded", are used. According to the Floquet theorem, the field in the structure is that of a special wave travelling within a spatial periodic profile, with the same spatial period D of the structure. The periodic field profile can be Fourier expanded in a series of spatial harmonics with different phase velocity according to:



$$\vec{E}(x,y,z,t)=\sum_n \vec{E}_n(x,y,z)\,e^{j[\omega t-k_n(\omega)z]}$$

$$\vec{E}_n(x,y,z+D)=\vec{E}_n(x,y,z)$$

$$\vec{E}_n(x,y,z)=\sum_{i=-\infty}^{+\infty}\vec{E}_{n,i}(x,y)\,e^{-j\frac{2\pi i}{D}z}$$

$$\vec{E}(x,y,z,t)=\sum_{n,i}\vec{E}_{n,i}(x,y)\,e^{j\left[\omega t-\left(k_n+\frac{2\pi i}{D}\right)z\right]}$$

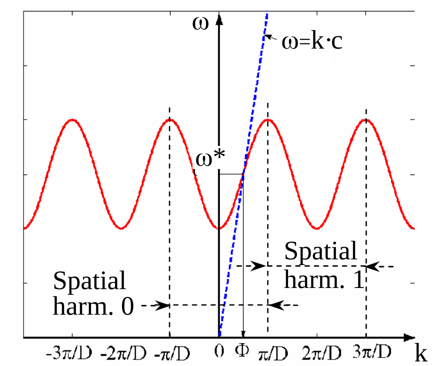
An iris loaded structure typical dispersion curve is plotted aside. In this case:

a) The plot is periodic in k, with a period of $2\pi/D$;

b) Every plot period is the dispersion curve of a particular spatial harmonic;

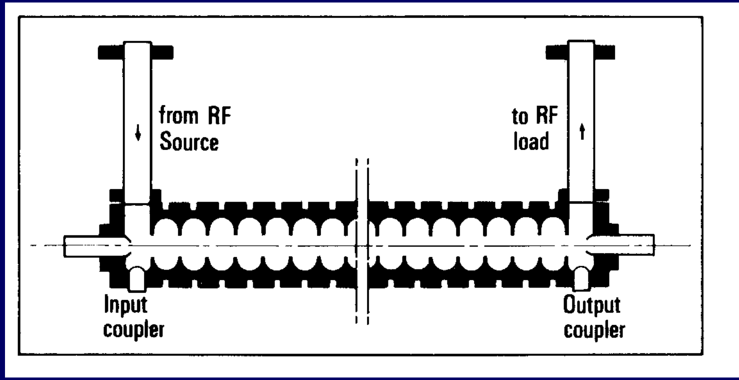
c) By design, at a certain excitation frequency ω* the 1st spatial harmonic can be made synchronous with the beam;

d) The high order spatial harmonics (i=1,2,3,...) are not synchronous and do not accelerate the beam over long distances.



RF Accelerating structures: TW iris-loaded waveguides

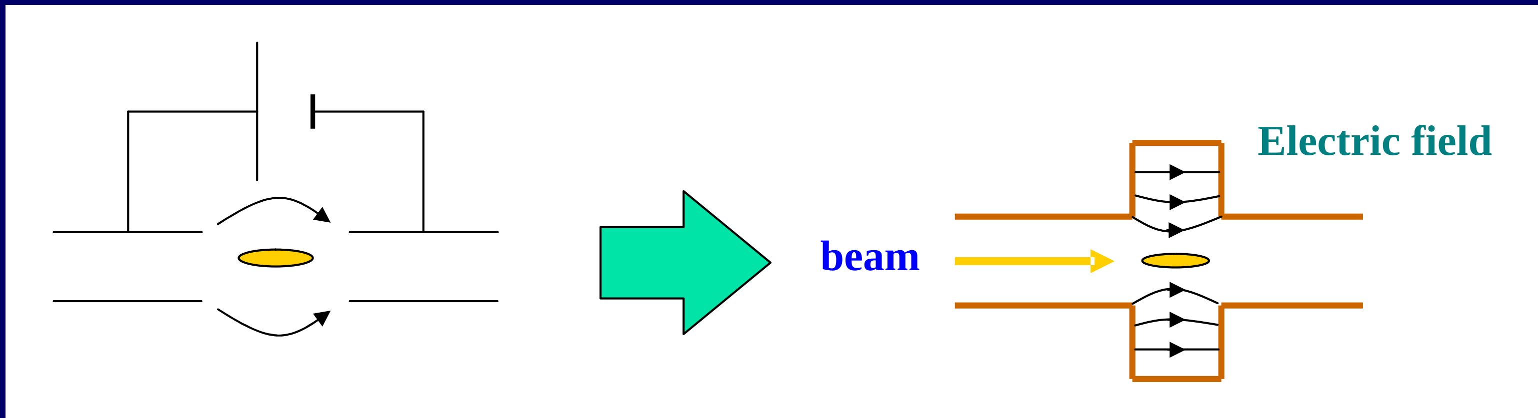
The most famous TW structure is the iris loaded accelerating waveguide developed at the "Stanford Linear Accelerator Center" (SLAC). It is an S-band structure (f = 2856 MHz) composed by 86 accelerating cells, with input/output couplers, capable of delivering accelerating fields up to 30MV/m.



More detailed presentation of TW structures in the "Introduction to Linacs" lectures by Maurizio Vretenar!

RF Accelerating structures: Resonant Cavities

High frequency accelerating fields synchronized with the beam motion are obtained by exciting metallic structures properly designed. In this case the structure physical dimensions are comparable with the e.m. field wavelength, and the exact spatial and temporal field profiles have to be computed (analytically or numerically) by solving the Maxwell equations with the proper boundary conditions.

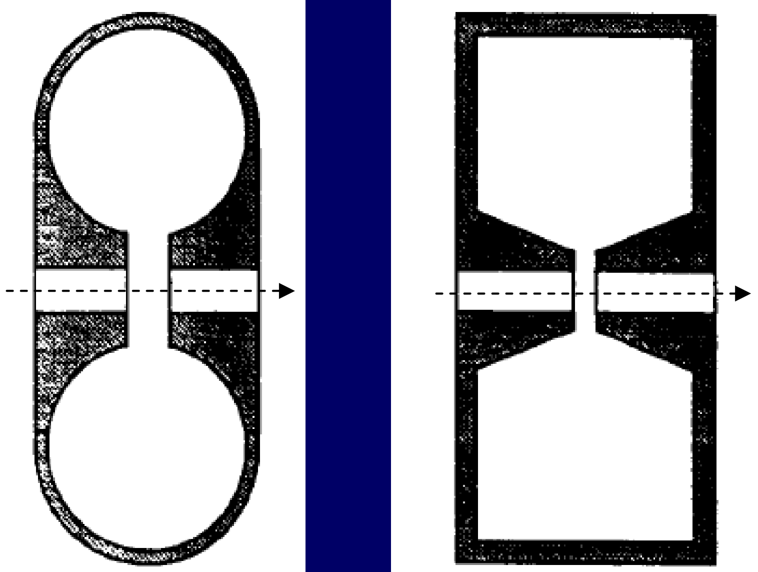


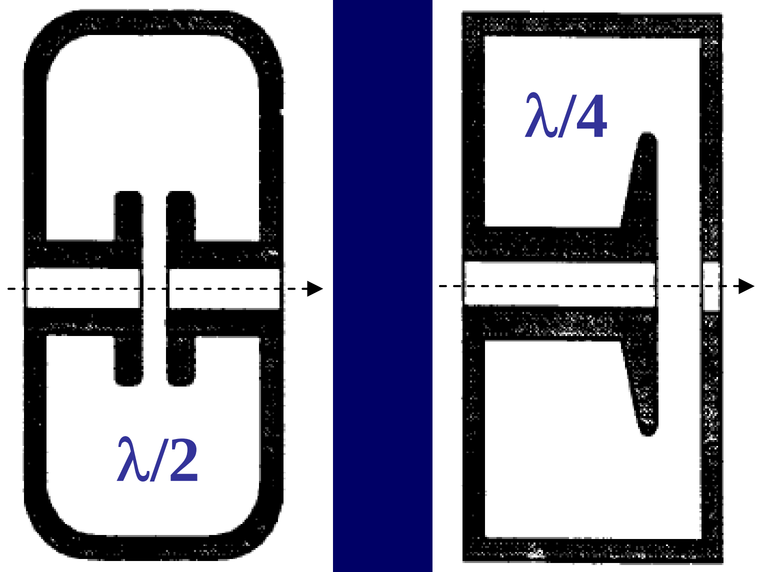
Resonant cavities are (almost) closed volumes were the e.m fields can only exists in the form of particular spatial conformations (resonant modes) rigidly oscillating at some characteristics frequencies (Standing Waves).

Cavity Profiles

Resonant cavities are (almost) closed volumes were the e.m fields can only exists in the form of particular spatial conformations (resonant modes) rigidly oscillating at some characteristics frequencies (Standing Waves).

Reentrant or nose-cone cavities





Disk-loaded or coaxial cavities

Modes of a Resonant Cavity: General Problem

The resonant cavity modes are solutions of the homogeneous Maxwell equations inside closed volumes surrounded by perfectly conducting walls. The mathematical problem has the following formal expression:

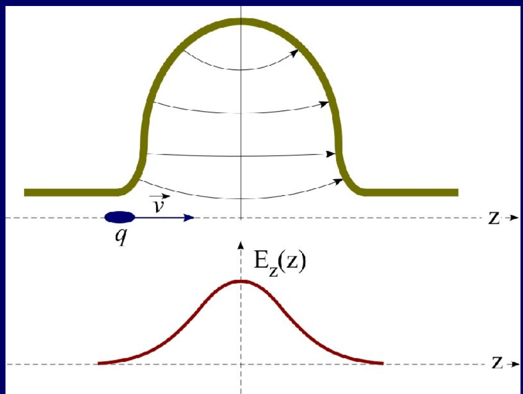
$$\begin{cases} \textit{Wave equation} \\ \textit{Field solenoidality} \\ \textit{Perfect boundary} \end{cases} \rightarrow \begin{cases} \nabla^2 \vec{E}(\vec{r},t) = \varepsilon\mu \dfrac{\partial^2}{\partial t^2}\vec{E}(\vec{r},t) \\ \vec{\nabla}\cdot\vec{E}(\vec{r},t) = 0 \\ \vec{n}\times\vec{E}(\vec{r},t) = 0 \end{cases} \xrightarrow[\textit{Phasors}]{} \begin{cases} \nabla^2 \vec{E}(\vec{r}) = -k^2\vec{E}(\vec{r}) \\ \vec{\nabla}\cdot\vec{E}(\vec{r}) = 0 \\ \vec{n}\times\vec{E}(\vec{r}) = 0 \end{cases}$$

According to the theory of linear operators, the solution is represented by a discrete set of eigen-functions $\vec{E}_n(\vec{r})$ and their associated eigenvalues $k_n = \omega_n / c$. The magnetic field eigenfunctions $\vec{B}_n(\vec{r})$ can be obtained from the Maxwell 3rd equation: $\vec{\nabla}\times\vec{E} = -\dfrac{\partial\vec{B}}{\partial t} \xrightarrow[\textit{phasors}]{} \vec{\nabla}\times\vec{E}_n(\vec{r}) = -j\omega_n\vec{B}_n(\vec{r})$

The $\vec{E}_n(\vec{r})$ functions are the cavity modes, each one resonating at a certain specific frequency ω_n. The eigen-functions are also a linear independent base, so that the actual fields $\vec{E}(\vec{r},t)$, $\vec{B}(\vec{r},t)$ can always be represented as a linear superposition of the cavity modes: $\vec{E}(\vec{r},t) = \sum_n a_n\vec{E}_n(\vec{r})e^{j\omega_n t}$; $\vec{B}(\vec{r},t) = \sum_n a_n\vec{B}_n(\vec{r})e^{j\omega_n t}$

Energy Gain

Cavities are normally designed to exploit the field of a particular resonant mode to accelerate the beam (accelerating mode). The lowest frequency mode (fundamental mode) is generally used for this task.



Let's consider a charge q travelling at constant speed v along a cavity gap on the reference frame z-axis. Assume that only the accelerating mode $\vec{E}_a(r,\varphi,z)$ is excited. The longitudinal E-field on the axis is:

$$E_z(z,t) = \Re\left[\vec{E}_a(r=0,z)\cdot \hat{z}\, e^{j\omega_a t}\right] = \Re\left[E_z(z)e^{j\omega_a t}\right]$$

Being the charge eq. of motion $z = v(t-t_0)$, the energy gain for a single gap transit is:

$$\Delta U = q\int_{-L/2}^{L/2}\Re\left[E_z(z)e^{j\omega_a(z/v-t_0)}\right]dz = q\,\Re\left[e^{-j\varphi_0}\int_{-L/2}^{L/2}E_z(z)e^{j\omega_a z/v}dz\right] \quad with \quad \varphi_0 = \omega_a t_0$$

The maximum energy gain $\Delta U_{\max}$ is obtained at the optimal value of the injection phase φ_0:

$$\Delta U_{\max} = qV_{\max} = q\left|\int_{-L/2}^{L/2}E_z(z)e^{j\omega_a z/v}dz\right| \xrightarrow[E_z(z)=even\ function]{} = q\int_{-L/2}^{L/2}E_z(z)\cos(\omega_a z/v)dz$$

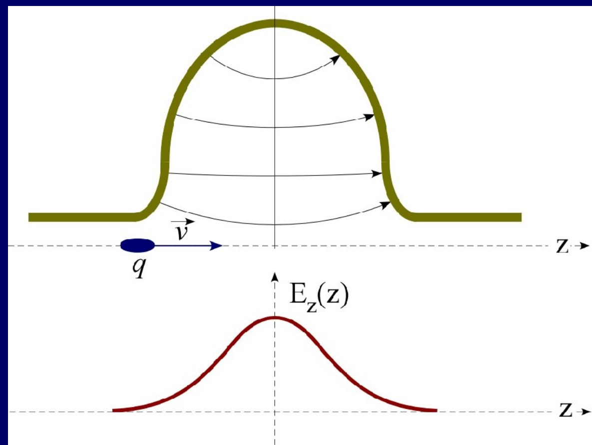
Transit Time Factor

The Transit Time Factor T of the cavity accelerating mode is defined according to

$$T = \left| \int_{-L/2}^{L/2} E_z(z) e^{j\omega_a z/v} dz \right| \Bigg/ \int_{-L/2}^{L/2} E_z(z) dz = V_{\max} / V_0$$

where $V_{\max}$ is the actual maximum integrated gradient and V_0 is the integral along the axis of the static field profile, calculated neglecting the effects of the field variation during the time of fly of the charge across the gap.

For field profiles $E_z(z)$ with odd symmetry the denominator in the T expression vanishes, and the definition can be misleading.



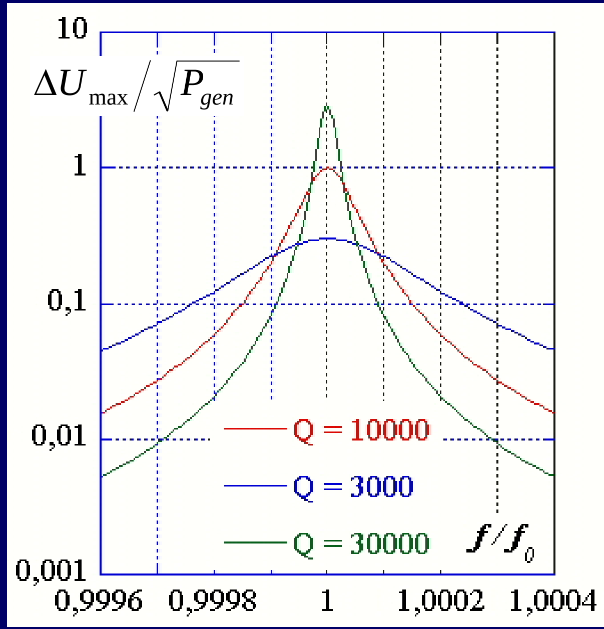
In the simplest case of a constant accelerating field E_0 along a gap of length L, the T factor is:

$$T = E_0 \int_{-L/2}^{L/2} \cos(\omega_a z / v) dz \Bigg/ E_0 L = \frac{\sin(\pi L / \beta \lambda)}{\pi L / \beta \lambda}$$

Cavity Loss – Q factor

Real cavities are lossy. Surface currents dissipate energy, so that a certain amount of RF power must be provided from the outside to keep the accelerating field at the desired level. If the external excitation is turned off, fields inside the cavity decay exponentially with a time constant τ_n characteristic of any given mode.

In frequency domain, the dissipation makes the modes resonating not only at the eigenvalue frequency ω_n but in a frequency band of width $\Delta\omega_n$ around ω_n. Both the bandwidth $\Delta\omega_n$ and the decay time τ_n are related to the quality factor Q of the mode defined as:



$$Q = \omega_n \frac{U}{P} \quad \to \begin{cases} \tau_n = 2Q/\omega_n \\ \Delta\omega_n\big|_{3dB} = \omega_n/Q \end{cases}$$

where U and P are the e.m. energy stored in the mode and the corresponding power dissipation on the walls.

General expressions for U and P are:

$$U = \int_{Vol} \left(\frac{1}{4}\varepsilon\left|\vec{E}\right|^2 + \frac{1}{4}\mu\left|\vec{H}\right|^2 \right) d\tau; \quad P = \frac{1}{2} R_s \int_{Surf.} H_{tan}^2 \, d\sigma \quad with \quad R_s = \frac{1}{\sigma\delta} = \sqrt{\frac{\omega\mu_0}{2\sigma}}$$

Cavity Loss – Superconductivity

Superconductivity is a very well-known physical phenomenon that is widely used in RF for particle accelerators.

Surface resistance is reduced by orders of magnitude (5 typically) in superconducting (SC) cavities, which are then very suitable when large gradients have to be sustained continuously or in a high duty-cycle regime.

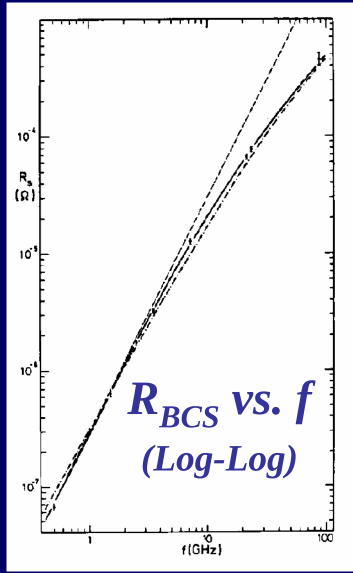
Surface resistance $R_S = R_{BCS} + R_{res}$

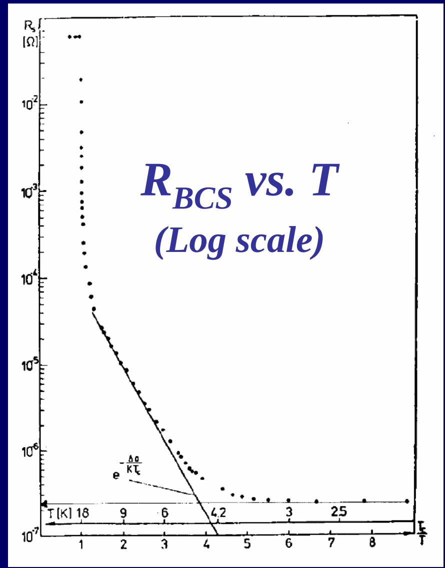
$$R_{BCS} = A\frac{1}{T}\omega^2 \exp\left(-\frac{\Delta(T)}{kT}\right) \qquad T \leqslant \frac{T_c}{2}$$

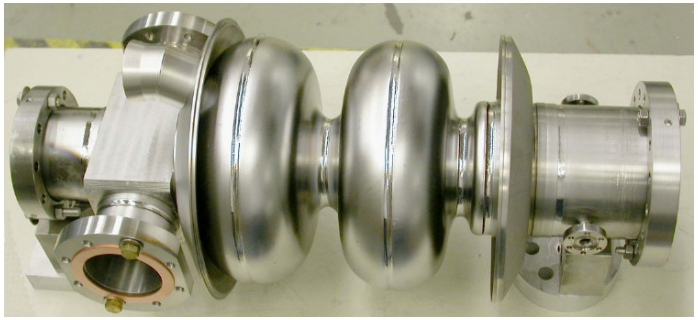
k the Boltzmann constant

A is a material parameter,

$\Delta(T)$ is the energy gap of the superconducting material







1.3 GHz, 2-cell cavity for Cornell ERL injector.

Shunt impedance

One of the most important parameter to characterize the cavity accelerating modes is the shunt impedance R defined as:

$$R = \frac{V^2}{2P} = \frac{1}{R_s} \frac{\left| \int E_z(z) e^{j\omega z/v} dz \right|^2}{\int_{Surf.} H_{tan}^2 d\sigma}$$

Frequency and material conductivity

Mode field configuration

The shunt impedance is the parameter that qualifies the efficiency of an accelerating mode. The highest the value of R, the larger the attainable accelerating voltage for a given power expenditure.

Another very useful parameter is the ratio between shunt impedance R and quality factor Q:

$$\left(\frac{R}{Q}\right) = \frac{V^2}{2P} \frac{P}{\omega U} = \frac{1}{2\omega} \frac{\left| \int_{traject.} E_z(z) e^{j\omega z/v} dz \right|^2}{\int_{Vol.} \left(\frac{1}{4}\varepsilon \left|\vec{E}\right|^2 + \frac{1}{4}\mu \left|\vec{H}\right|^2 \right) d\tau}$$

The (R/Q) is a pure geometric qualification factor. In fact, for a given mode it is straightforward that the (R/Q) does not depend on the cavity wall conductivity, and its value is preserved if homothetic expansions of a given geometry are considered.

The (R/Q) is a qualification parameter of the cavity geometrical design.

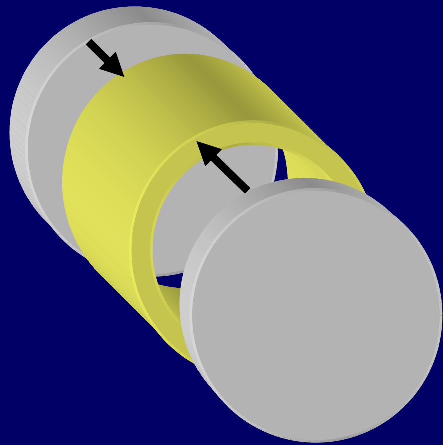
Merit Figures

PARAMETER	DEFINITION	FREQUENCY SCALING	TYPICAL VALUES
Quality factor Q Ratio between energy stored U and energy dissipated per cycle U_{loss}. Defines the resonance bandwidth and the field decay time.	$Q = \omega \dfrac{U}{P} = 2\pi \dfrac{U}{U_{loss}}$	$Q \div \dfrac{1}{R_s} \div \begin{cases} f^{-1/2} \ (NC) \\ f^{-2} \ (SC) \end{cases}$ $\left[Q \div e^{\Delta(T)/kT} \ \ (SC)\right]$	$\begin{cases} 10^4 \div 10^5 \ (NC) \\ 10^9 \div 10^{10} \ (SC) \end{cases}$
Shunt Impedance R Square of the accelerating voltage V normalized to the RF power P dissipated to sustain the field.	$R = \dfrac{V^2}{2P}$	$R \div \dfrac{1}{R_s} \div \begin{cases} f^{-1/2} \ (NC) \\ f^{-2} \ (SC) \end{cases}$	single cell: $\begin{cases} 1 \div 5 \, M\Omega \, (NC) \\ 100 \div 500 \, G\Omega \, (SC) \end{cases}$
Shunt Impedance per unit length Z Square of the accelerating voltage V normalized to the RF power P and cavity gap length L.	$Z = \dfrac{R}{L} = \dfrac{V^2}{2PL}$	$Z \div \dfrac{1}{R_s L} \div \begin{cases} f^{1/2} \ (NC) \\ f^{-3/2} \ (SC) \end{cases}$	$\begin{cases} 2 \div 20 \, M\Omega/m \, (NC) \\ 0.1 \div 5 \, T\Omega/m \, (SC) \end{cases}$
Geometric factor (R/Q) Geometric factor qualifying the accelerating cavity shape regardless to frequency and wall conductivity.	$(R/Q) = \dfrac{V^2}{2PQ} = \dfrac{V^2}{2\omega U}$	$(R/Q) \div f^0$!!!	single cell: $50 \div 100 \, \Omega$

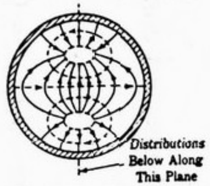
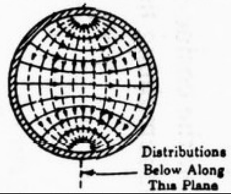
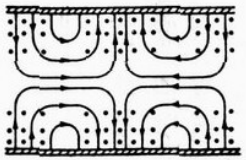
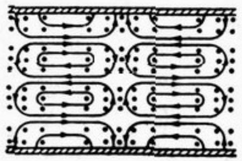
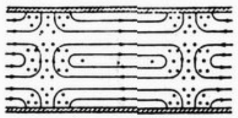
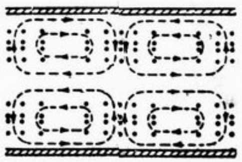
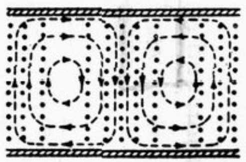
Analytical field solutions: the Pill-box cavity

In the simplest cases the mode field configuration can be calculated analytically, while in almost all practical cases the solutions are computed numerically by means of dedicated computer codes.

One of the most interesting didactical case is the cylindrical or “pill-box” cavity. The pill-box cavity can be seen as a piece of circular waveguide short-circuited at both ends by metallic plates.



Circular waveguide modes

Wave Type	TM_{01}	TM_{02}	TM_{11}	TE_{01}	TE_{11}
Field distributions in cross-sectional plane, at plane of maximum transverse fields					
Field distributions along guide					

The Pill-box fundamental mode TM_{010}

Considering the standing wave configurations generated by the superposition of forward and backward TM_{nm} and TE_{nm} waves, the pill-box resonates only at those frequencies corresponding to standing wave patterns compatibles with the short-circuit plate boundary condition (vanishing transverse E-field). The cavity modes can be enumerated adding a 3rd suffix "k" (indicating the longitudinal periodicity of the standing wave pattern) to the waveguide propagating modes.

$$TM_{nmk}$$

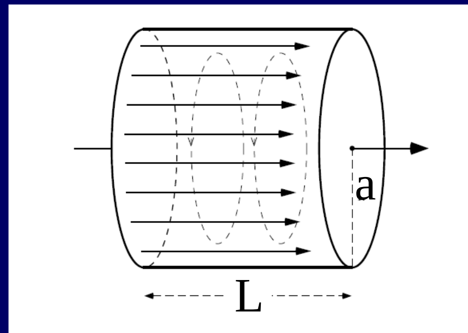
$$TE_{nmk}$$

n = azimuthal periodicity

m = radial periodicity

k = long. periodicity

Pill-box fund. mode TM_{010}



$$\lambda_{TM_{010}} = 2\pi a / p_{01}\,;\quad \omega_{TM_{010}} = p_{01}\, c/a$$

$$E_r = E_\theta = 0;\ H_r = H_z = 0$$

$$E_z(r) = E_0 J_0(p_{01} r/a)$$

$$H_\theta(r) = -j\sqrt{\varepsilon/\mu}\,E_0 J_1(p_{01} r/a)$$

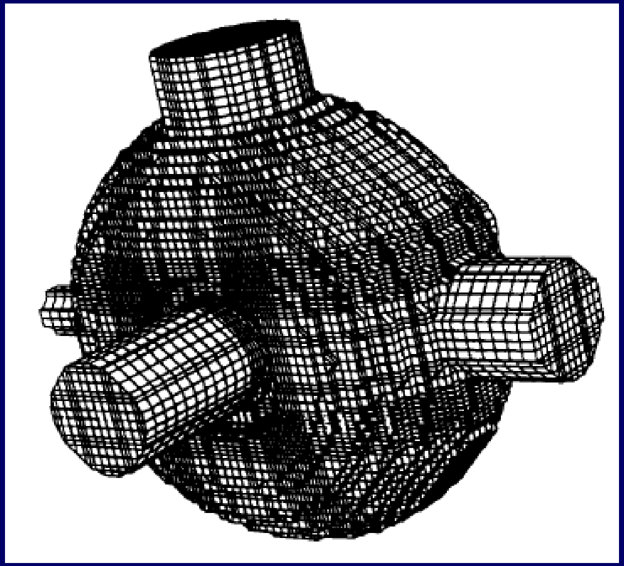
$$Q_{TM_{010}} = \sqrt{\frac{a\,\sigma\,p_{01}}{2\,\varepsilon\,c}}\ \frac{1}{1+a/L}$$

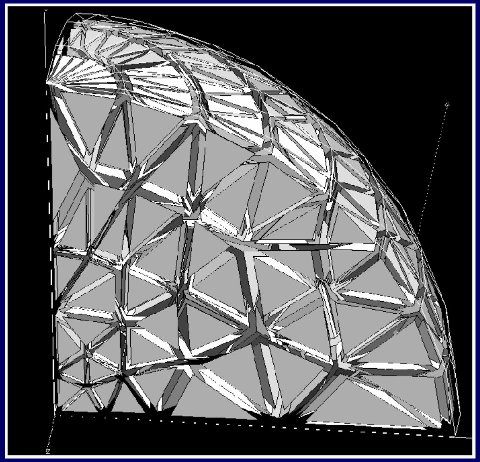
$$R_{TM_{010}} = \frac{1}{\pi\,\varepsilon}\sqrt{\frac{a\,\sigma\,\mu}{2\,p_{01}\,c}}\ \frac{\mathrm{sinc}^2(p_{01}L/2\beta a)}{J_1^2(p_{01})}\ \frac{L^2}{a(a+L)}$$

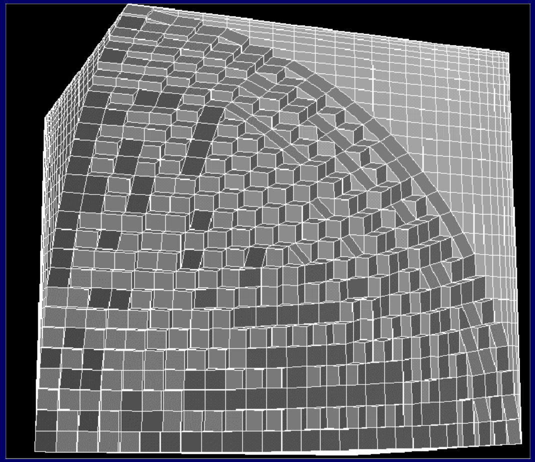
Numerical solutions

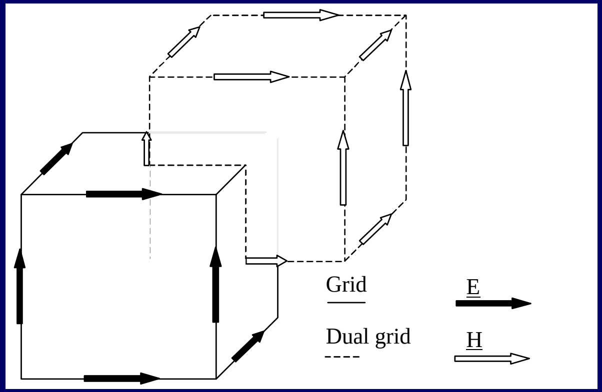
In the majority of cases analytical field solutions are not available and numerical methods are applied.
There are various codes dedicated to the solution of the Maxwell equations in closed and/or open volumes starting from a discretized model of the studied structure. Codes can be classified in various ways:

- 2D (2-dimensionals) and 3D (3-dimensionals) codes;
- Finite differences and finite elements codes;
- Time-domain and frequency domain codes.





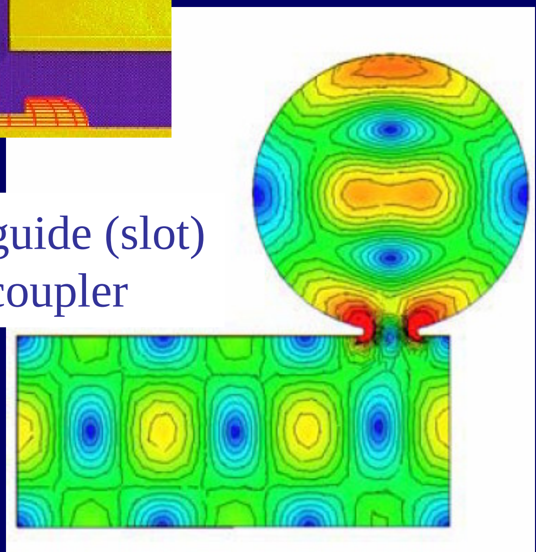




Q_0

Input-Output coupling

Magnetic (loop) input coupler



Waveguide (slot) input coupler

Real cavities are never completely closed volumes. At the least, apertures for beam transit (beam tubes) are required, as well as RF input couplers to feed the cavity, and RF output couplers to probe the field inside. The RF couplers can be of different types:

- Electric couplers (Antennas): the inner of a coaxial line connected from the outside couples to the cavity mode E-field;
- Magnetic couplers (Loops): the cavity mode B-field couples to a loop connecting inner and outer conductors of a coaxial line.
- Waveguide couplers: the cavity mode fields are coupled to an external waveguide of proper shape and cut-off through a hole or a slot in the cavity walls

Coupling and loading parameters of a cavity

The coupling strength of a port can be measured as the amount of power P_{out} extracted form the cavity through the port itself for a given level of the mode fields inside. This leads to the definitions of the ***external-Q*** (Q_{ext}) (in analogy with the definition of the resonance quality factor Q) and coupling coefficient β of a coupler according to :

$$Q_{ext} = \omega \frac{U}{P_{out}}; \qquad \beta = \frac{Q_0}{Q_{ext}} = \frac{P_{out}}{P_{walls}}$$

where Q_0 is the usual quality factor of the resonant mode, related only to the dissipation P_{walls} on the cavity walls.

The extra-power flow through the cavity couplers, in addition to the power loss in the walls, may significantly change the characteristics of the resonance. This effect is known as "cavity loading". The loaded cavity Q-factor Q_L is lowered by the power coupled out through the cavity ports and result to be:

$$Q_L = \omega \frac{U}{P_{Tot}} = \omega \frac{U}{P_{walls} + \sum P_{out_n}} \rightarrow \frac{1}{Q_L} = \frac{P_{walls}}{\omega U} + \sum \frac{P_{out_n}}{\omega U} \rightarrow \begin{cases} \dfrac{1}{Q_L} = \dfrac{1}{Q_0} + \sum \dfrac{1}{Q_{ext_n}} \\[2ex] \dfrac{1}{Q_L} = \dfrac{1 + \sum \beta_n}{Q_0} \end{cases}$$

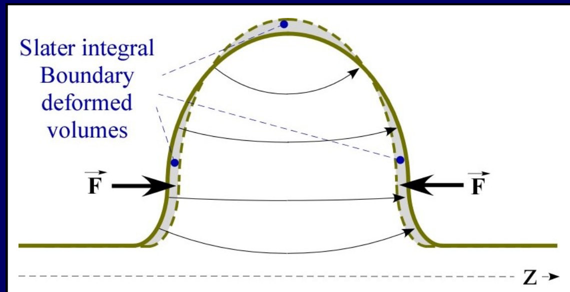
Tuning

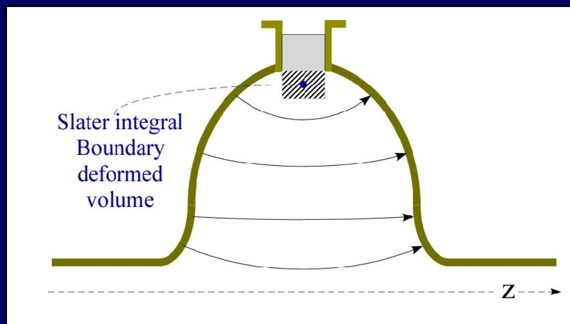
The cavity resonant frequencies need to be continuously controlled during operation. Actual frequencies are affected by thermal drifts and, in case of superconducting cavities, by pressure variations in the cryogenic bath.

Storage ring cavities have to be largely detuned during beam injection to compensate the beam loading. Synchrotron cavity frequencies have to follow beam velocity increase associated to the energy ramping.

The frequency control is normally obtained through small deformations of the cavity boundaries. The Slater theorem can be used to compute the resonant frequency change, according to:

$$\frac{\Delta\omega}{\omega_0} = \frac{\int_{\Delta V}\left(\mu H^2 - \varepsilon E^2\right)d\tau}{\int_V \left(\mu H^2 + \varepsilon E^2\right)d\tau} = \frac{\Delta U_H - \Delta U_E}{U}$$





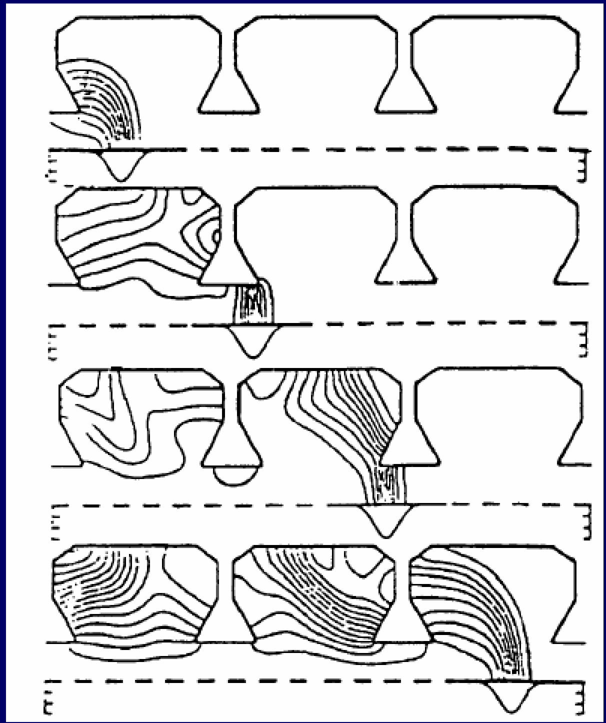
Cavity tuning is normally actuated through:

- Cooling fluid temperature control (linac TW or SW sections);
- Structure pushing/stretching by application of axial forces (SC and multi-cell cavities);
- Variable penetration of tuning plungers in the cavities volume (room-temperature, single cell cavities under heavy beam-loading).

Cavity High Order Modes (HOMs)

A resonant cavity has always an infinite number of resonant modes. The beam pipes connected to the cavity act as waveguides coupling out the fields of the modes with resonant frequencies higher than the pipe cut-off. The modes with frequencies lower than the beam pipe cut-off do not propagate out the cavity volume, and are called trapped modes.

In general, only the lowest frequency mode (fundamental) is useful for acceleration while all the other trapped modes, the so called High Order Modes (HOMs), can interact with the beam and are dangerous for the beam dynamics.



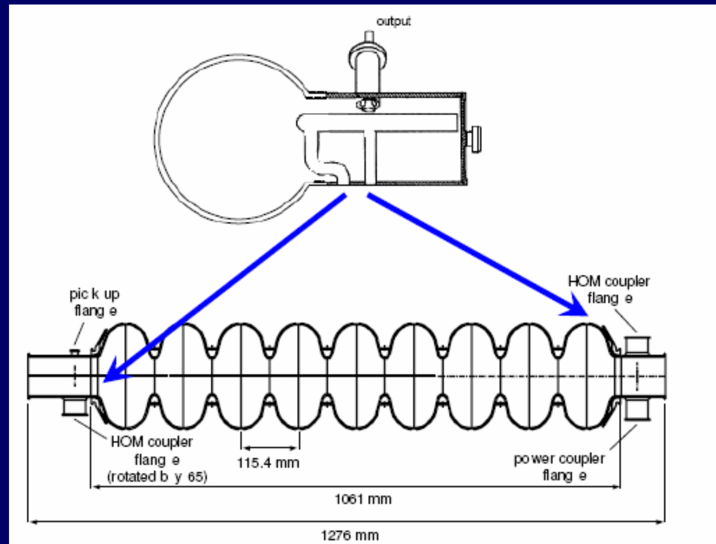
A bunch travelling across a cavity gap excites HOMs. If the HOMs Q values are high enough, the excited wakefields last for a long time and can possibly interact with following bunches.

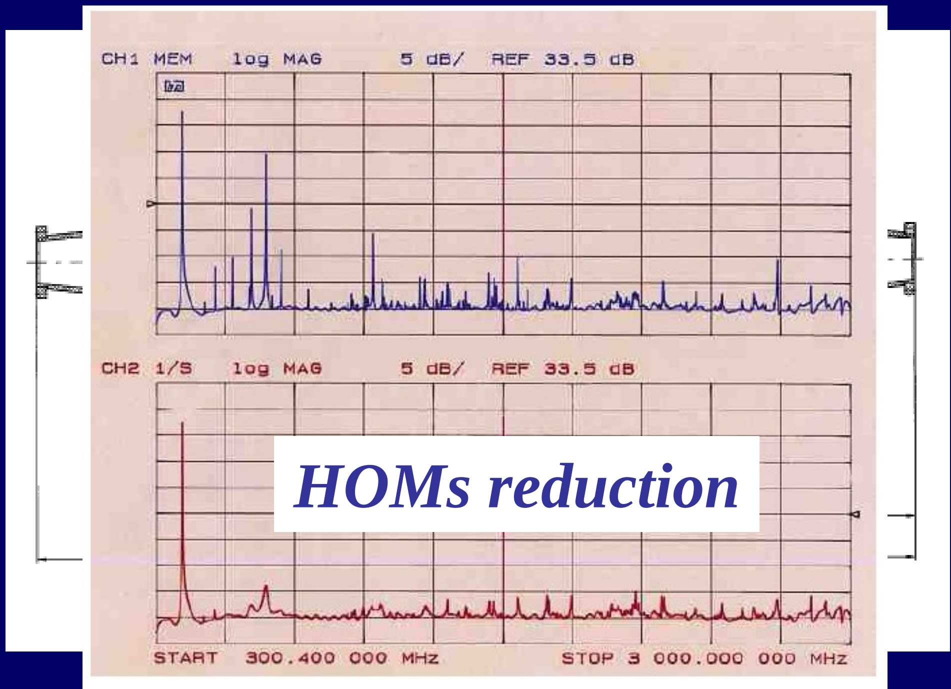
The HOMs fields may grow in this cumulative interaction leading to an overall beam instability (Coupled Bunch Instability), that may occurs both in longitudinal and in transverse beam motion.

To shorten the HOMs fields decay time, the modes can be loaded with dedicated couplers (dampers) to decrease their Qs.

HOM damping

In SC cavities HOM dampers are installed on the beam tubes since any additional device hosted in the cells would spoil the cavity performances. HOMs couplers are designed to reject the fundamental mode by means of embedded notch filters.

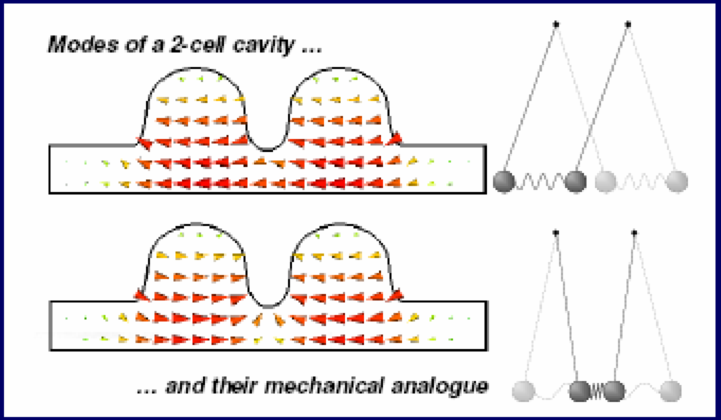


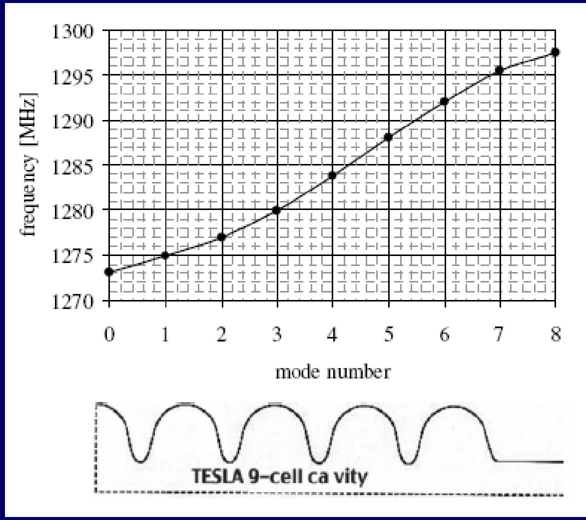


In room-temperature cavities for high beam currents the HOM damping can be obtained by means of waveguide couplers attached to the cavity bodies and/or the beam tubes. The natural high-pass behaviour of a single-conductor waveguide is exploited in this case to reject the fundamental mode.

Multi-cell cavities

Multi-cell resonant cavities are very effective in reducing the number of RF power sources and input couplers. The N-cell structure behaves like a system composed by N oscillators coupled together.





According to the theory of coupled oscillators, each single-cell mode degenerates in a set of N possible coupled oscillation modes of the whole system, characterized by a cell-to-cell phase advance given by:

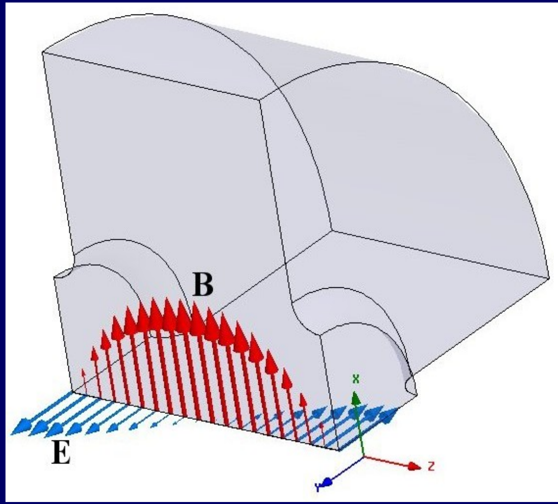
$$\Delta\phi_n = \frac{n\pi}{N-1} \qquad n = 0,1,\ldots,N-1.$$

SW multi-cell cavities are designed to use the last mode ($\Delta\phi_{N-1} = \pi$) for acceleration. Provided that $L_{cell} = \lambda_\pi / 2$ the beam and the RF are in-phase all along the structures, which brings to:

$$R_\pi \approx N\, R_{cell}; \qquad Q_\pi \approx Q_{cell}$$

RF Deflecting Structures

For some special applications RF fields are used to deflect charged beams more than to accelerate it. Structures called RF deflectors are designed for this task, mostly based on circular waveguide dipole modes TM_{1m} and TE_{1m} (mode showing an azimuthal periodicity of order 1) properly iris-loaded (for TW deflectors) or short-circuited (for SW deflecting cavities).



The figure of merit qualifying the efficiency of an RF deflecting structure is the transverse shunt impedance $R_\perp$ defined as:

$$R_\perp = \frac{V_\perp^2}{2P} \quad with\ V_\perp = \left| \int_{-L/2}^{L/2} \left[E_y(z) + v\,B_x(z) \right] e^{j\omega z/c}\, dz \right| = \frac{v}{q} \Delta p_\perp$$

where a deflection in the *y*-direction for a charge *q* moving along the *z*-direction with a velocity *v* has been considered, and *P* is the RF power absorbed by the structure.

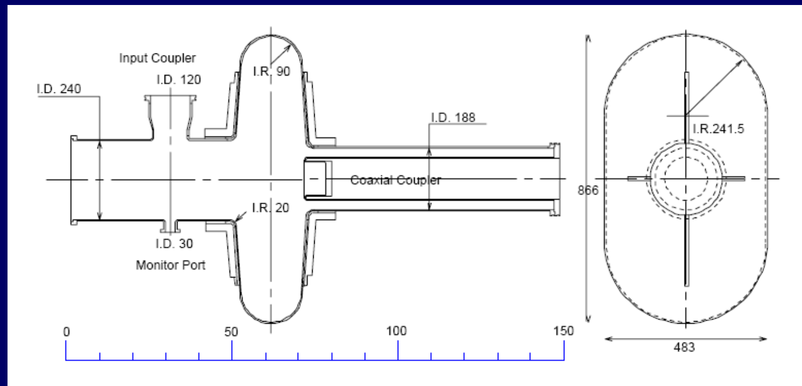
It turns out that the deflection angle of the charge is: $\phi_{def} \approx \frac{\Delta p_\perp}{p} = \frac{qV_\perp}{\beta^2 W}$

where $\Delta p_\perp$ is the transverse component of the momentum $\vec{p}$, and W is the particle energy.

RF Deflecting Structures: Examples

There are many different applications requiring deflecting RF structures:

- Cavities for crab crossing, to obtain head-on bunch collisions in colliders where beam trajectories cross with an angle.
- RF injection kickers, to create closed bump orbits rapidly varying with time in a ring for exotic stacking configuration.
- Diagnostics RF deflectors, for intra-bunch tomography of the longitudinal phase space.
- RF separators, to separate and collect different ion species produced by a source.
- …



KEK-B SC Crab Cavity

5-cells SW RF Deflector for beam diagnostics at SPARC

TW RF Deflector for CTF3 combiner ring