# Linac Driven Free Electron Lasers (III)

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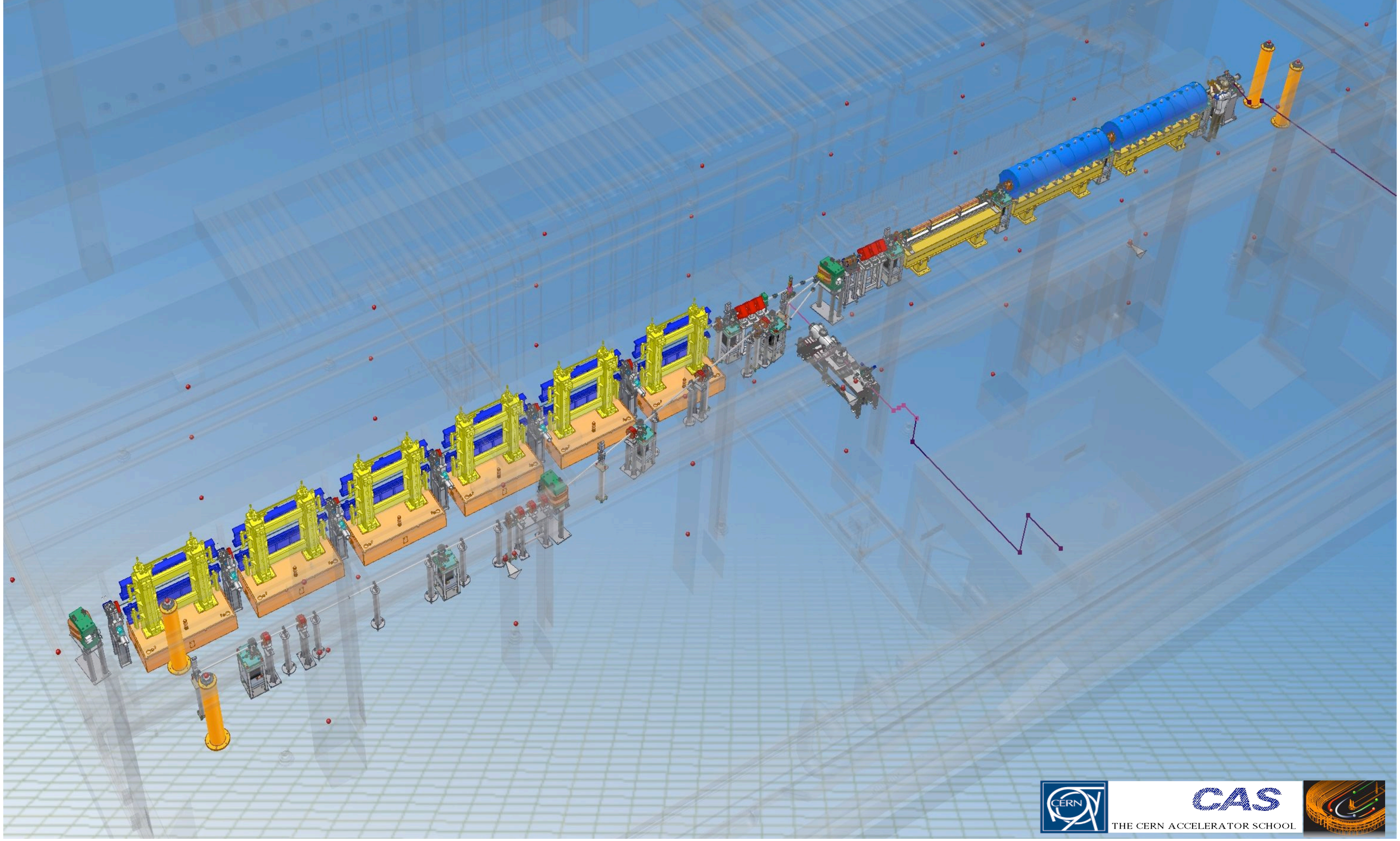


CERN CAS THE CERN ACCELERATOR SCHOOL

# SASE FEL Electron Beam Requirements: High Brightness $B_n$

$$\lambda_r^{MIN} \propto \sigma_\delta \sqrt{\frac{\left(1 + K^2/2\right)}{\gamma B_n K^2}}$$

minimum radiation wavelength

energy spread

undulator parameter

$$B_n = \frac{2I}{\varepsilon_n^2}$$

$$L_g \propto \frac{\gamma^{3/2}}{K\sqrt{B_n \left(1 + K^2/2\right)}}$$

gain length

R. Saldin et al. in *Conceptual Design of a 500 GeV e+e- Linear Collider with Integrated X-ray Laser Facility*, DESY-1997-048

# Short Wavelength SASE FEL Electron Beam Requirement: High Brightness $B_n > 10^{15}$ A/m$^2$

$$B_n \approx \frac{2I}{\varepsilon_n^2}$$

Bunch compressors

(RF & magnetic)

Laser Pulse shaping

Emittance compensation

Cathode emittance

# The paradox of relativistic bunch compression

Low energy electron bunch injected in a linac:

$$\gamma \approx 1$$
$$L_b = 3mm = L'_b$$
$$I = 100\,A$$

Length contraction?

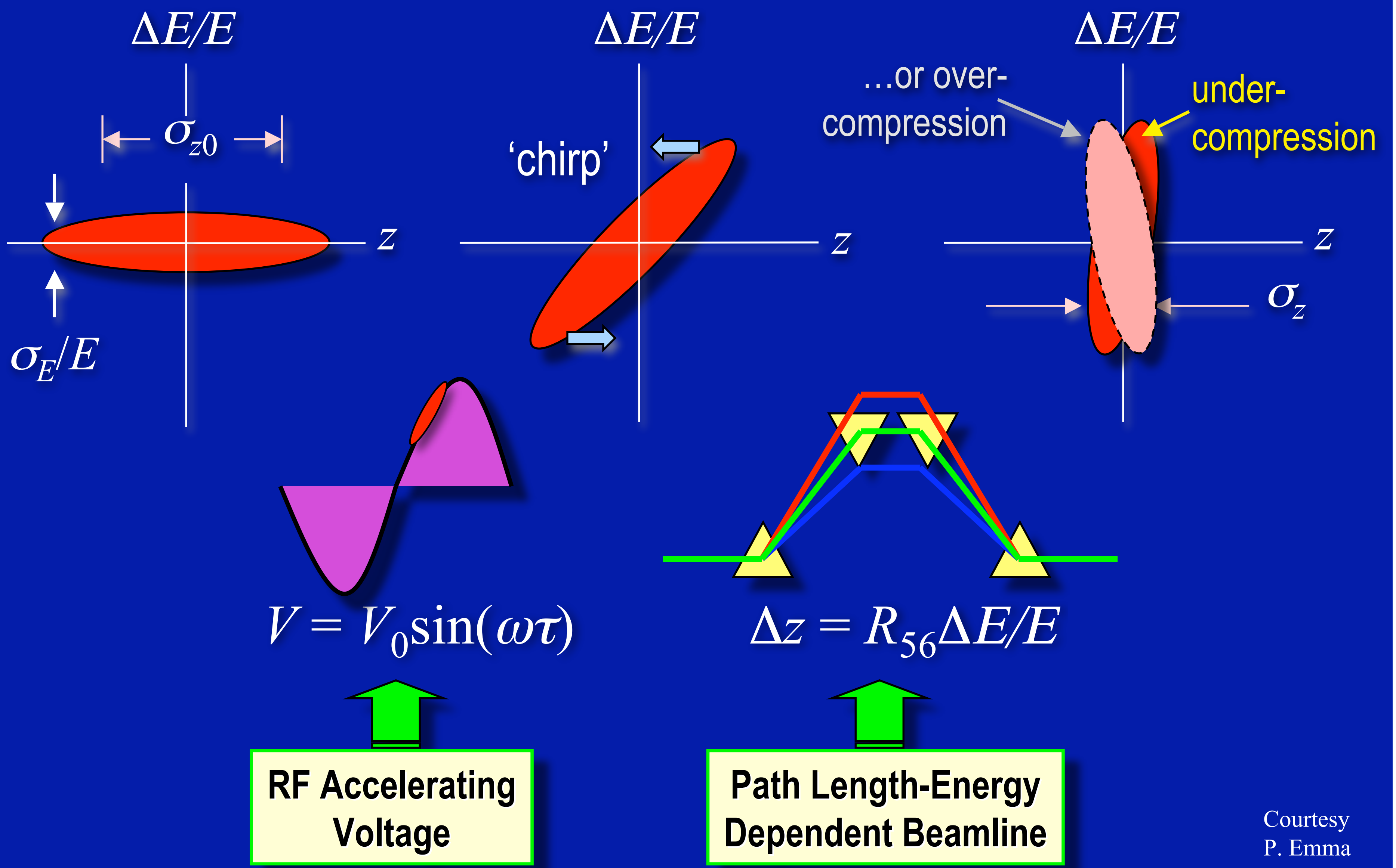
$$\gamma = 1000$$
$$L_b = \frac{L'_b}{\gamma} = 3\,\mu m$$
$$I = 100\,kA$$

Why do we need a bunch compressor?

$$L''_b = \gamma L_b = 30\,m$$
$$L_b = \frac{L''_b}{\gamma} = 3\,mm$$
$$I = 100\,A$$

# Magnetic compressor (Chicane)

# Magnetic compressor (Chicane)




Courtesy
P. Emma

# Transfer Matrix including longidunial phase space

$$\mathbf{R}_{\text{drift}} = \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}$$

$$\mathbf{R}_{\text{quad}} = \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix}$$

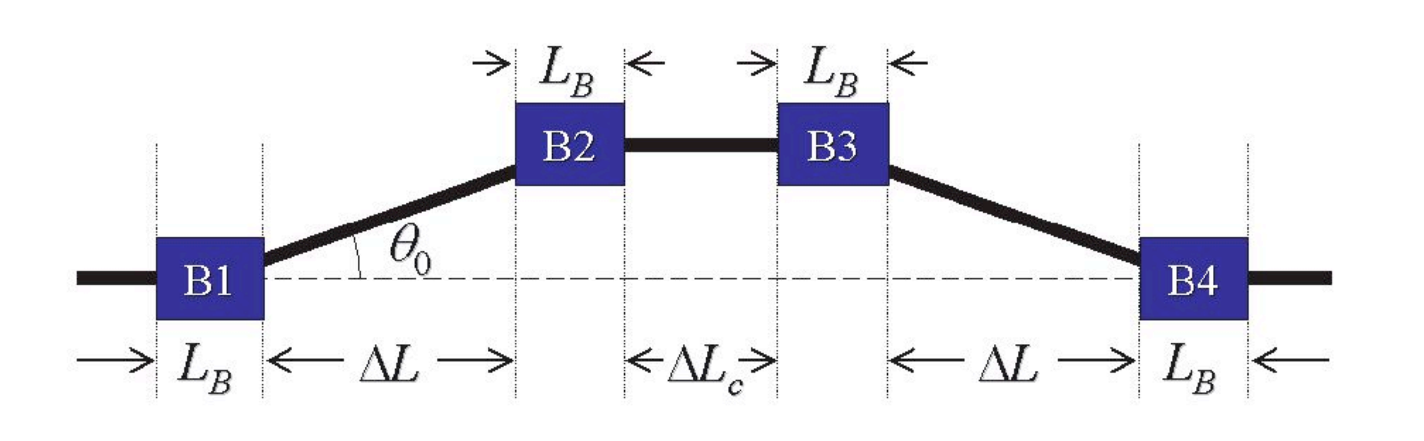
$$\mathbf{R}_{\text{dipole}} = \begin{pmatrix} \cos\theta & \rho\sin\theta \\ -\frac{1}{\rho}\sin\theta & \cos\theta \end{pmatrix}$$

$$\begin{pmatrix} x \\ x' \\ y \\ y' \\ l \\ \delta \end{pmatrix} = \begin{pmatrix} R_{11} & R_{12} & 0 & 0 & 0 & R_{16} \\ R_{21} & R_{22} & 0 & 0 & 0 & R_{26} \\ 0 & 0 & R_{33} & R_{34} & 0 & R_{36} \\ 0 & 0 & R_{43} & R_{44} & 0 & R_{46} \\ R_{51} & R_{52} & R_{53} & R_{54} & R_{55} & R_{56} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x_0 \\ x'_0 \\ y_0 \\ y'_0 \\ l_0 \\ \delta_0 \end{pmatrix}$$

$$l_f \approx l_i + R_{56}\delta_i$$

In general, any curved beam line section introduces a path length difference for particles with a relative energy (momentum) deviation δ: (with η being the longitudinal dispersion)

$$\Delta S = \eta(\delta)\delta = R_{56}\delta + T_{566}\delta^2 + \ldots$$




Path length:

$$S(\vartheta) = \frac{4L_B\vartheta}{\sin\vartheta} + \frac{2\Delta L}{\cos\vartheta} + \Delta L_c$$

Additional Path length:

$$\Delta S(\vartheta) = S(\vartheta) - (4L_B + 2\Delta L + \Delta L_c) \approx \vartheta^2\left(\frac{2}{3}L_B + \Delta L\right)$$

Expandig θ in terms of a small relative energy deviation δ :

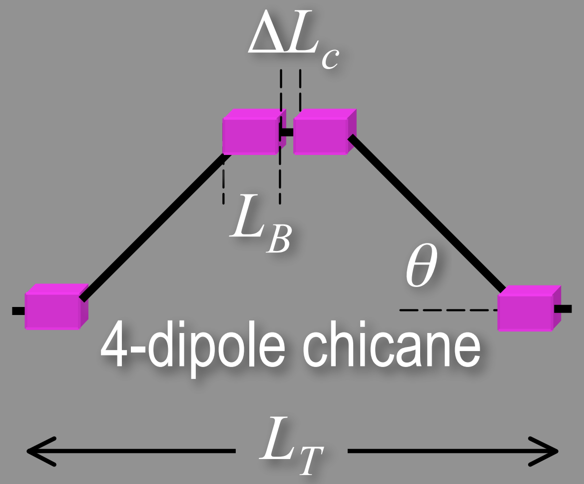
$$\vartheta^2 = \frac{\vartheta_o^2}{(1+\delta)^2} \approx \vartheta_o^2\left(1 - 2\delta + 3\delta^2 + \ldots\right)$$

we obtain:

$$R_{56} = -2\vartheta_o^2\left(\frac{2}{3}L_B + \Delta L\right)$$

$$\Delta S = -\frac{R_{56}}{2} \qquad T_{566} = -\frac{3}{2}R_{56}$$

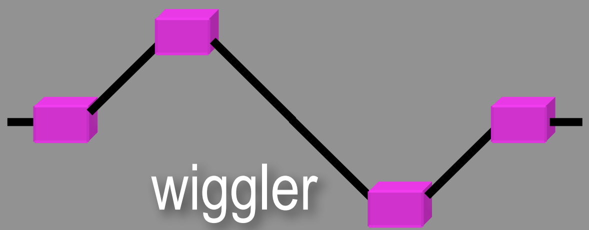
# Types of Compressor




LEUTL,…
LCLS,
TTF-BC1,2,
TESLA-BC1

$$R_{56} \approx -2\theta^2\left(\frac{L_T}{2} - \frac{4}{3}L_B - \frac{\Delta L_c}{2}\right) < 0$$

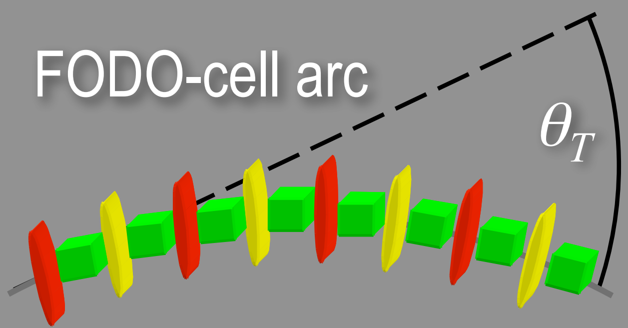
simple, achromatic




TESLA-BC2,3

$$R_{56} \approx -2\theta^2\left(\frac{L_T}{2} - \frac{4}{3}L_B\right) < 0$$

achromatic



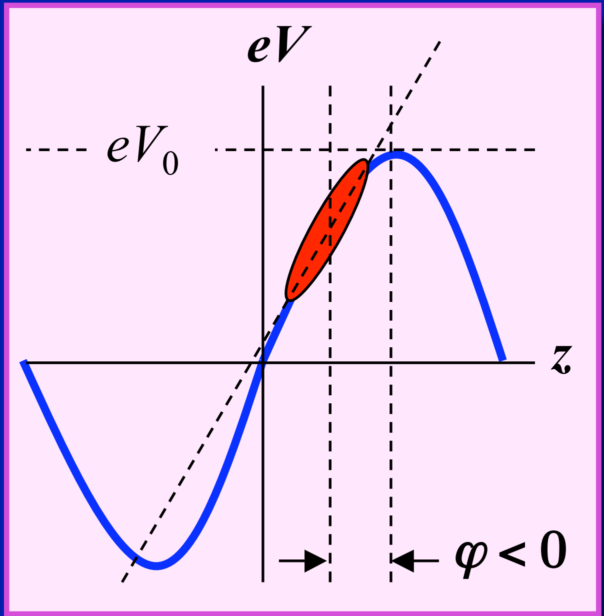

SLC RTL,
SLC arcs
NLC BC2

$$R_{56} \approx \frac{\theta_T^2 L_T}{4N_c^2 \sin^2(\mu_x/2)} > 0$$

reverse sign

But $T_{566} > 0$ in all cases…

# Single-Stage Bunch Compression




The bunch head is in the z<0 direction

Energy of a particle after acceleration in a RF linac where **z** is the longitudinal position in the bunch:

$$E(z_0) = eV\cos(\phi + kz_0),$$

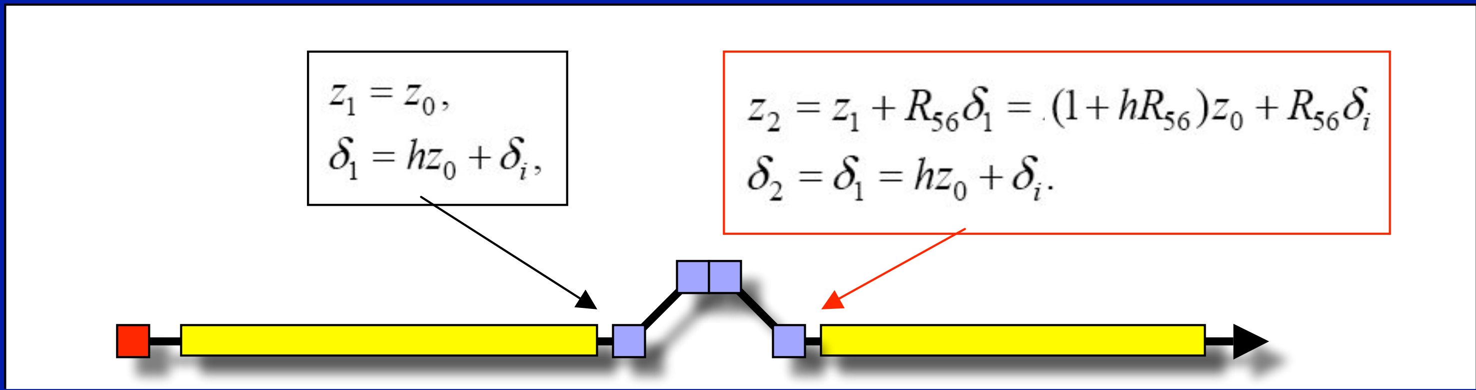
$$E(z_0) = E\left(1 + p'\cdot z_0 + \frac{1}{2}p''\cdot z_0^2 + \frac{1}{6}p'''\cdot z_0^3 + \mathcal{O}(z_0^4)\right),$$

Defining the Energy Chirp factor h:

$$h \equiv p' = -\frac{eV}{E}k\sin\phi,$$

The relative **correlated** energy deviation of a particle at a longitudinal position $z_o$ becomes:

$$\Delta E / E \equiv \delta = hz_0$$




Taking the average over all particles we obtain the final bunch length:

$$\sigma_{z_2} = \sqrt{\langle z_2^2 \rangle} = \sqrt{(1 + hR_{56})^2 \sigma_{z_o}^2 + R_{56}^2 \sigma_{\delta_i}^2}$$

Initial uncorrelated energy spread: $\sigma_{\delta_i} = \sqrt{\langle \delta_i^2 \rangle}$ Such that: $\langle z_o \delta_i \rangle = 0$

$\sigma_\delta$ in a FEL is extremely small and we can simplify

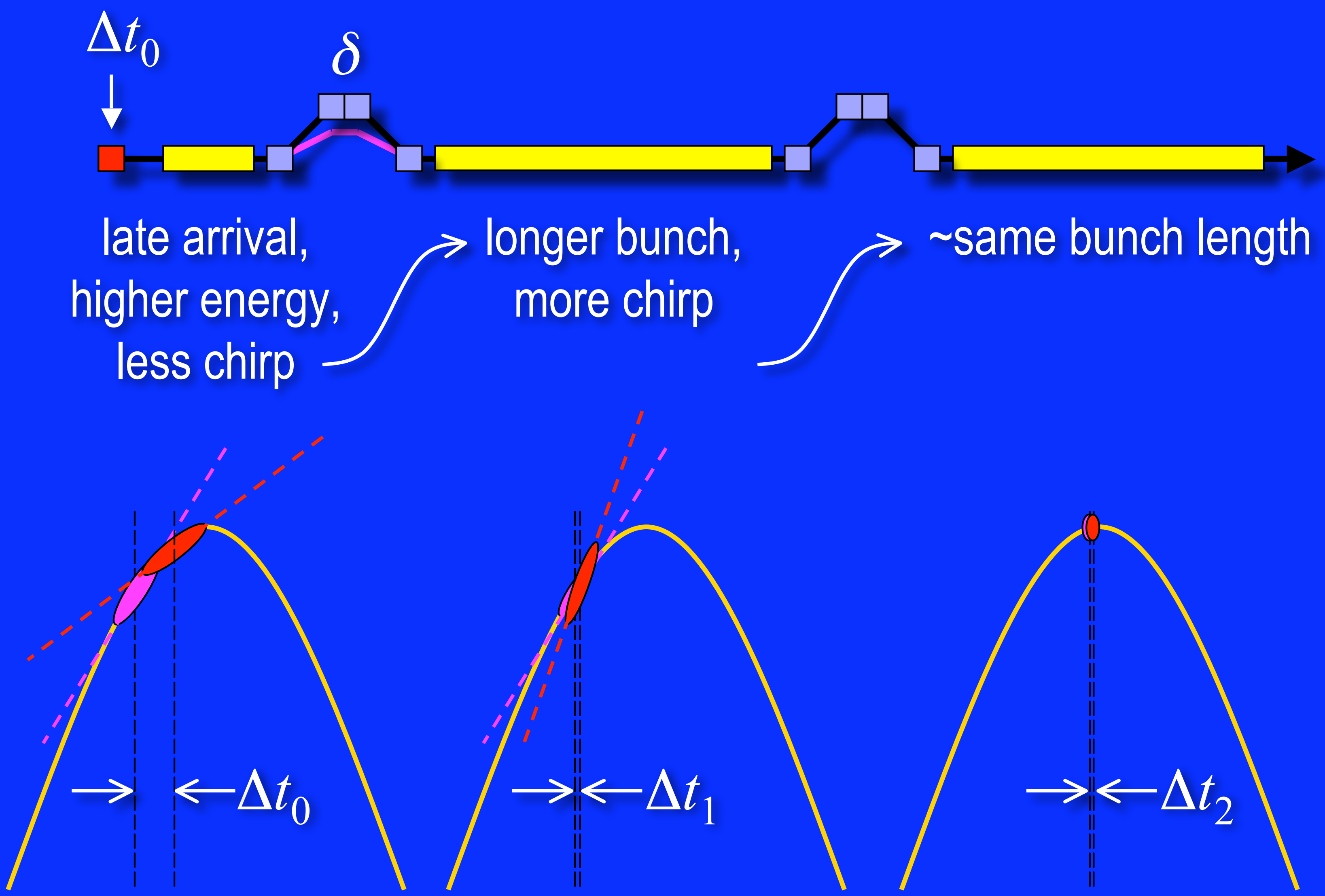
$$\sigma_{z_2} = |1 + hR_{56}|\sigma_{z_o} = \frac{\sigma_{z_o}}{C}$$

Compression factor $C = \frac{\sigma_{z_o}}{\sigma_{z_2}} >> 1$

bunch length stability with RF phase jitter…

$$\frac{\Delta\sigma_z}{\sigma_z} \approx -\left(\frac{\sigma_{z0}}{\sigma_z} \mp 1\right)\Delta\varphi \cot(\varphi) \quad \Rightarrow \quad \frac{\sigma_{z0}}{\sigma_z} = 40: \quad 25\% \text{ jitter} / 0.1 \text{ psec} \quad @ -15^\circ$$

# Two-Stage Compression Used for Stability




Courtesy
P. Emma

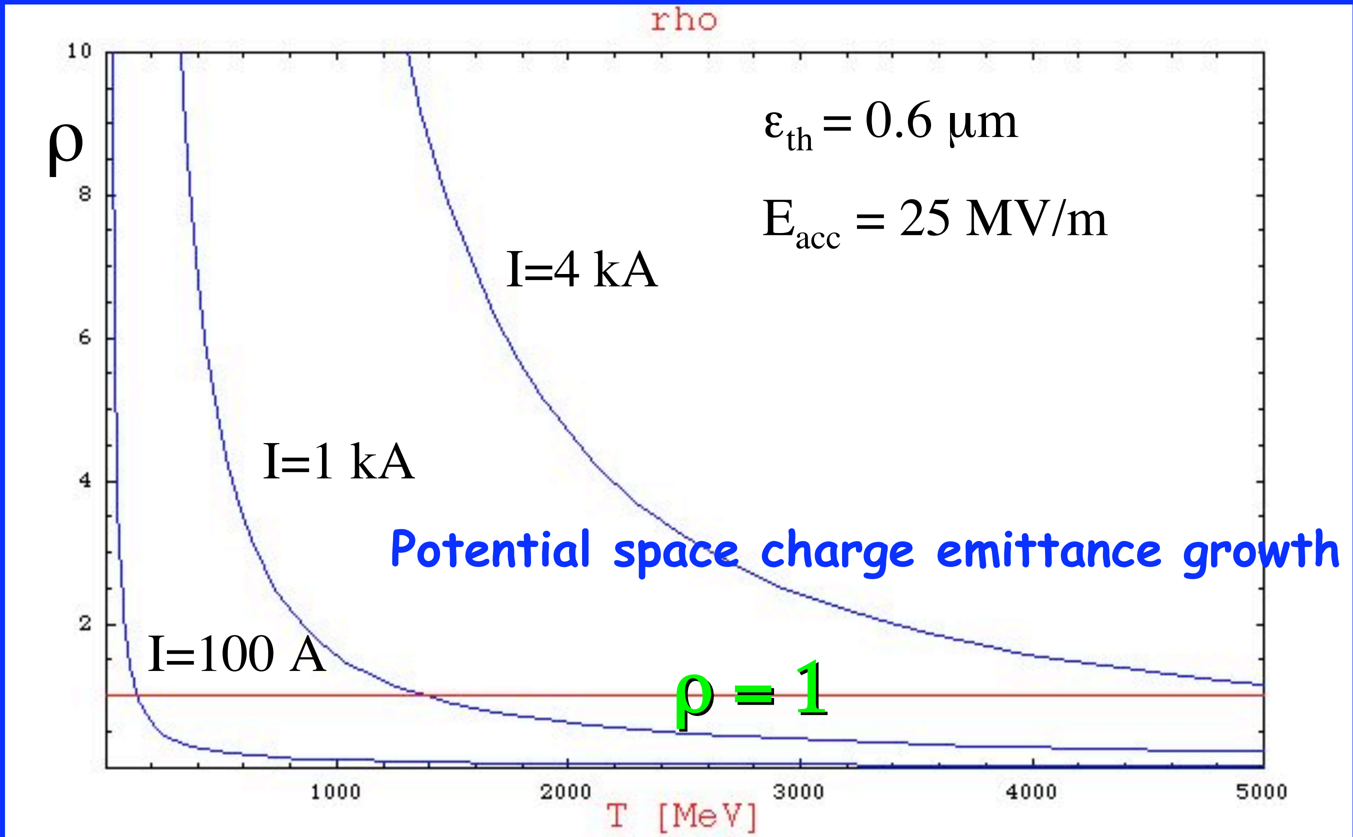
System can be optimized for stability against timing & charge jitter

## Laminarity parameter

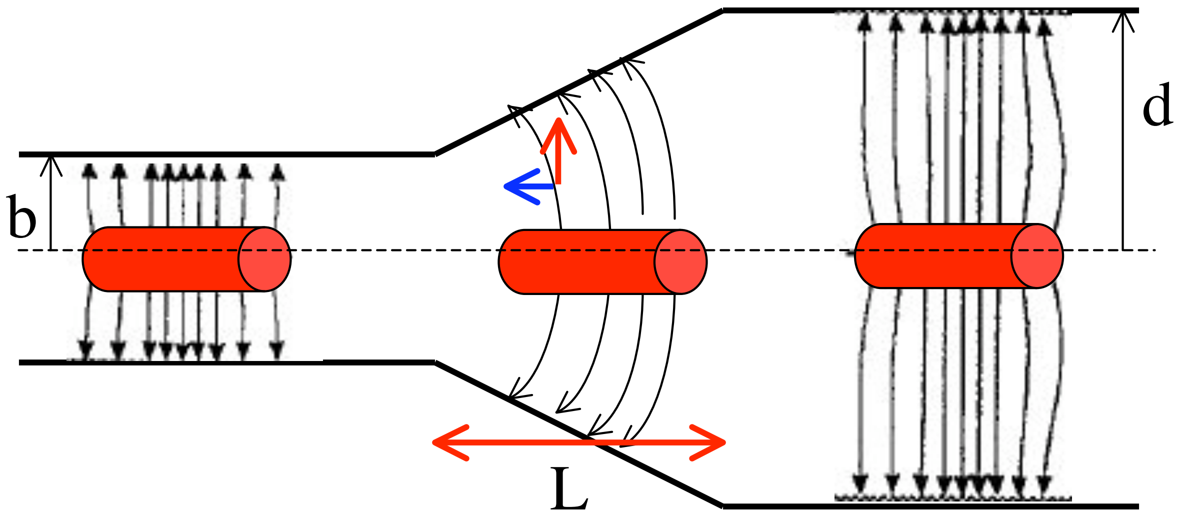
$$\rho = \frac{2I\sigma^2}{\gamma I_A \varepsilon_n^2} \equiv \frac{2I\sigma_q^2}{\gamma I_A \varepsilon_n^2} = \frac{4I^2}{\gamma'^2 I_A^2 \varepsilon_n^2 \gamma^2}$$

## Transition Energy (ρ=1)

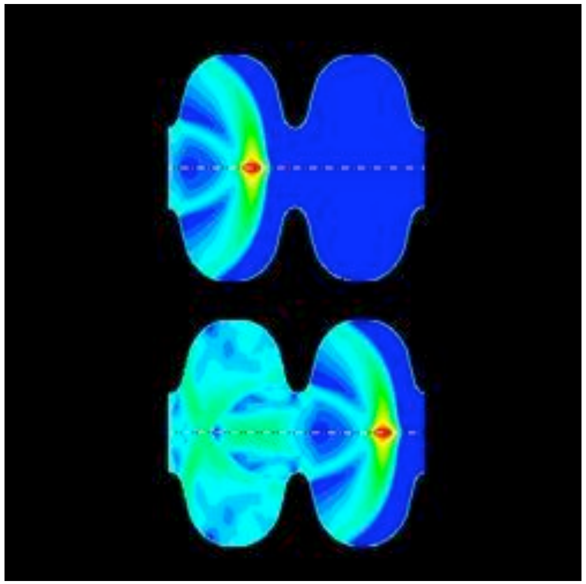
$$\gamma_{tr} = \frac{2I}{\gamma' I_A \varepsilon_n}$$



# Longitudinal Geometric Wakefields




There is a longitudinal $E_z(r,z)$ field in the transition

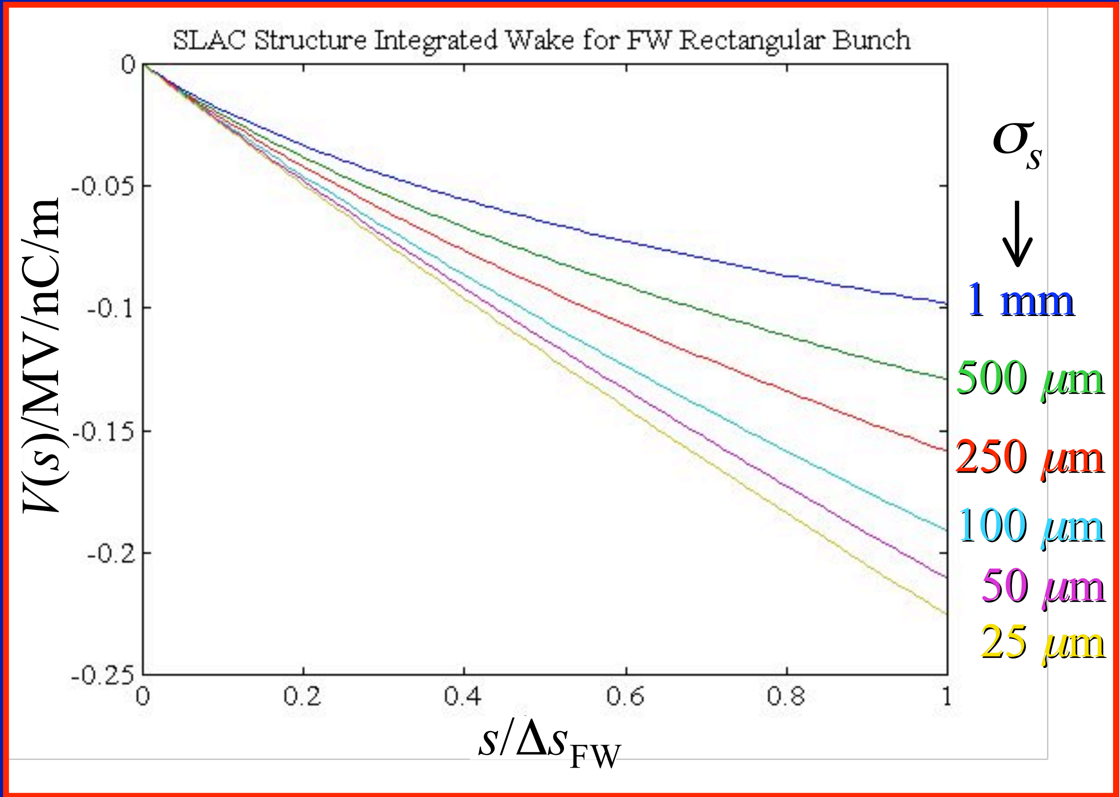


# Longitudinal Geometric Wakefields

Longitudinal point-wake:

$$W(s) \approx \frac{Z_0 c}{\pi a^2} e^{-\sqrt{s/s_0}}$$

K. Bane




***SLAC* S-Band:**

$s_0 \approx 1.32$ mm

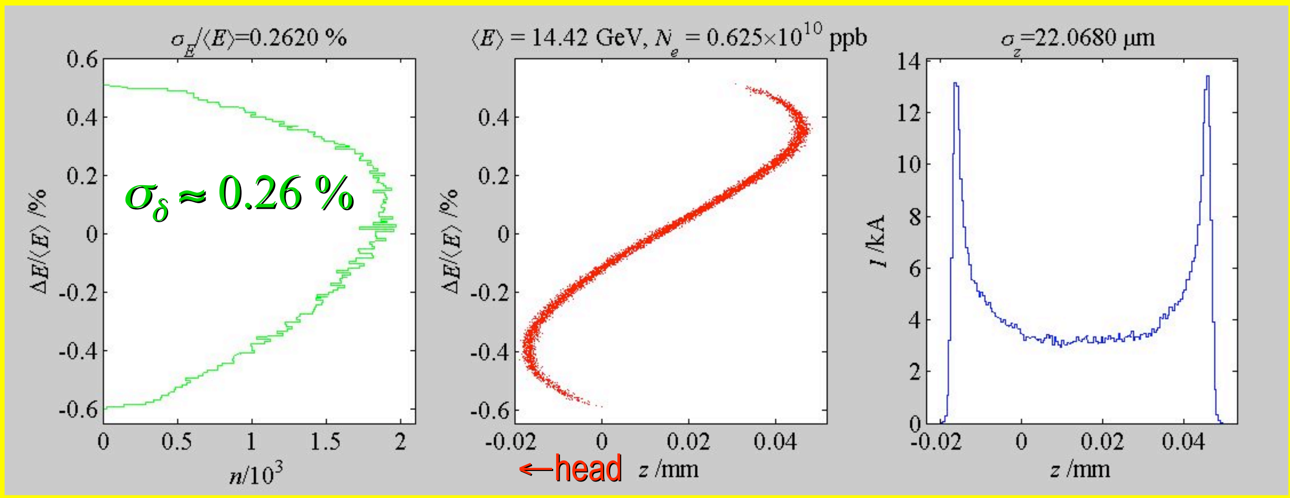
$a \approx 11.6$ mm

$s <$ ~6 mm

Induced voltage along bunch:

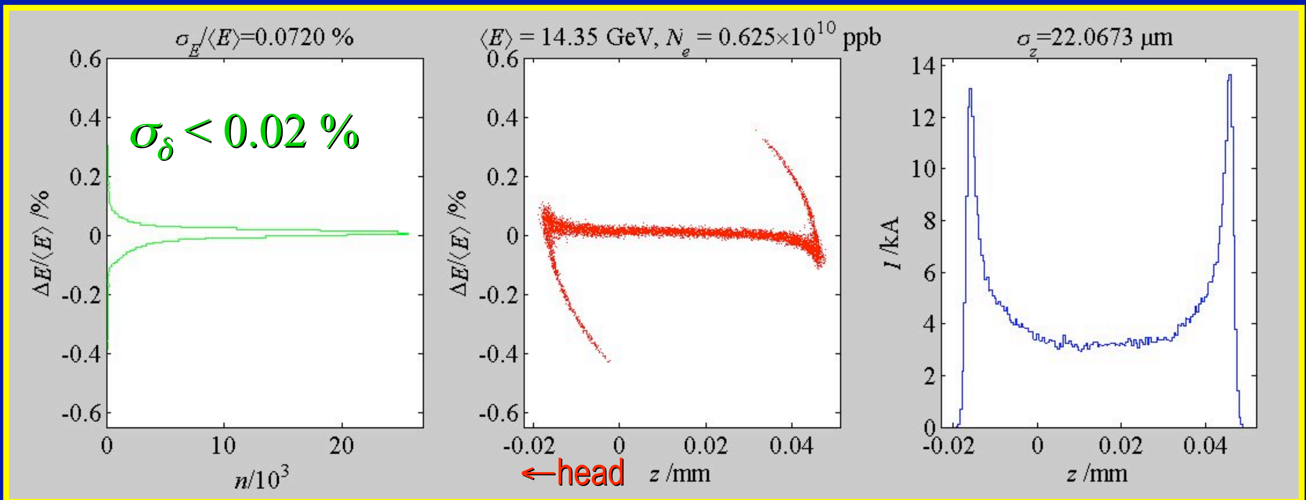
$$V(s) = -NeL \int_{-\infty}^{s} W(s - s') f(s') ds'$$

# *LCLS* Example of Wakefield Use




wakefield ‘OFF’

end of *LCLS* linac




wakefield ‘ON’

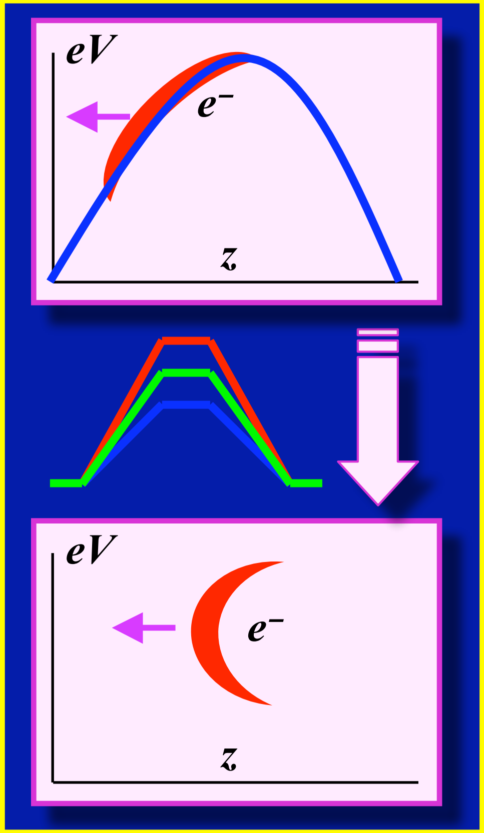
$L \approx 550$ m,
$N \approx 6.2\times10^9$,
$\Delta z \approx 75\ \mu$m,
$E = 14$ GeV

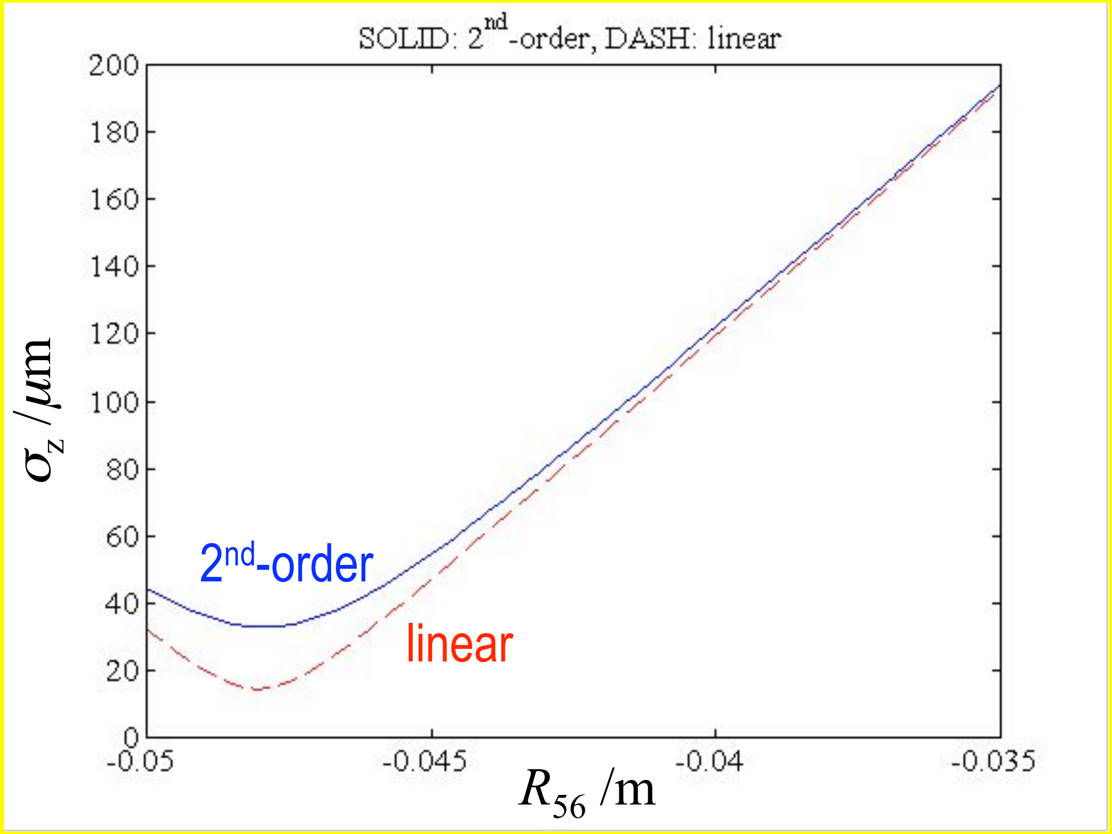
wake-induced energy spread

$$\sigma_\delta \approx \frac{2Ne^2cZ_0s_0L}{\pi\sqrt{12}a^2\Delta zE}\left[1-\left(1+\sqrt{\Delta z/s_0}\right)e^{-\sqrt{\Delta z/s_0}}\right]$$

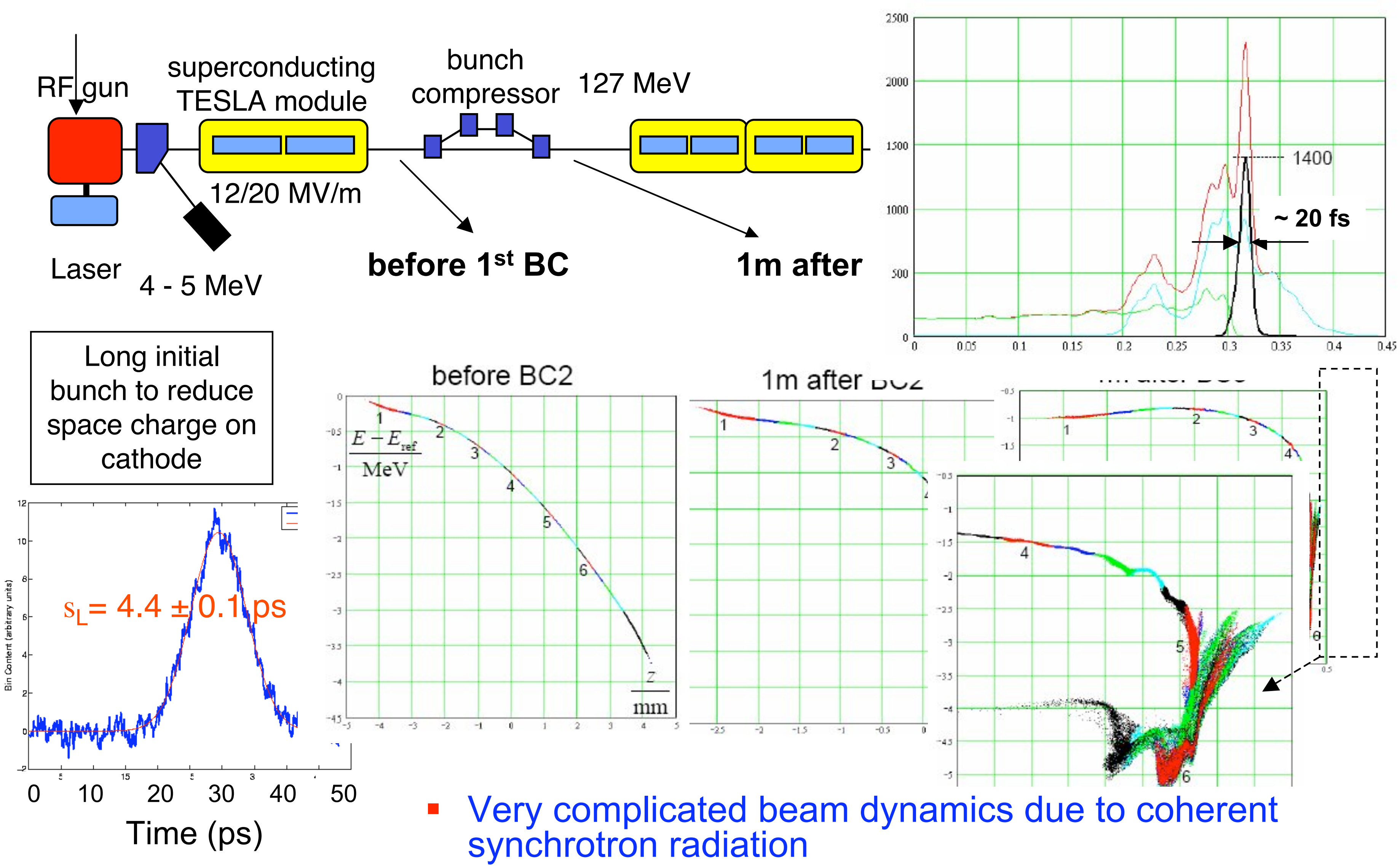
for uniform distribution

For chicane and accelerating phase, RF curvature and $T_{566}$ always add, limiting the minimum bunch length ...



# Courtesy Joerg Rossbach (@FLASH)




- Very complicated beam dynamics due to coherent synchrotron radiation
- Difficult access to relevant parameters
- Ultra-short photon pulses created ~20fs FWHM

## RF non linear distortion correction by an harmonic RF section

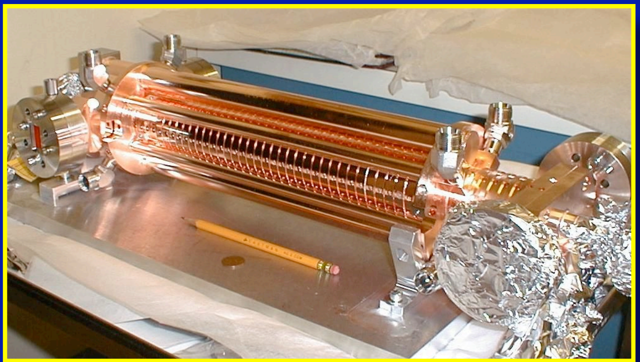
Due to sinusoidal nature of RF voltage

$$V = V_0 \sin(\phi_0) + \Delta\phi V_0 \cos(\phi_0) - \frac{1}{2}\Delta\phi^2 V_0 \sin(\phi_0) + \dots$$

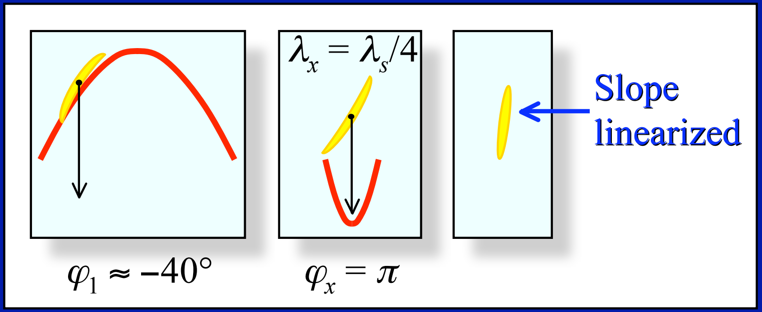
can be corrected by using a higher-harmonic cavity

$$V_h = V_h \sin(\phi_h) + h\Delta\phi V_h \cos(\phi_h) - \frac{1}{2}(h\Delta\phi)^2 V_h \sin(\phi_h) + \dots$$

- The second order term in *V* can be cancelled by using a decelerating phase $\varphi_h$
- Working on crest ($\sin(\varphi_0)=1$) the compensation occours for $V_h=V_0/h^2$ and $\sin(\varphi_h) = -1$

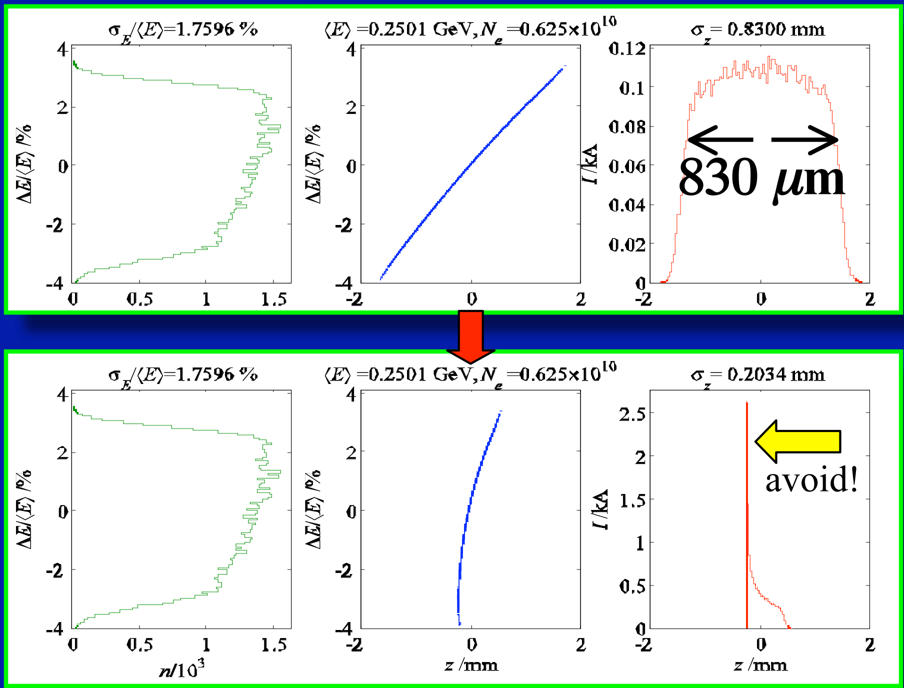


0.5-m X-band section for *LCLS* (22 MV, 11.4 GHz)

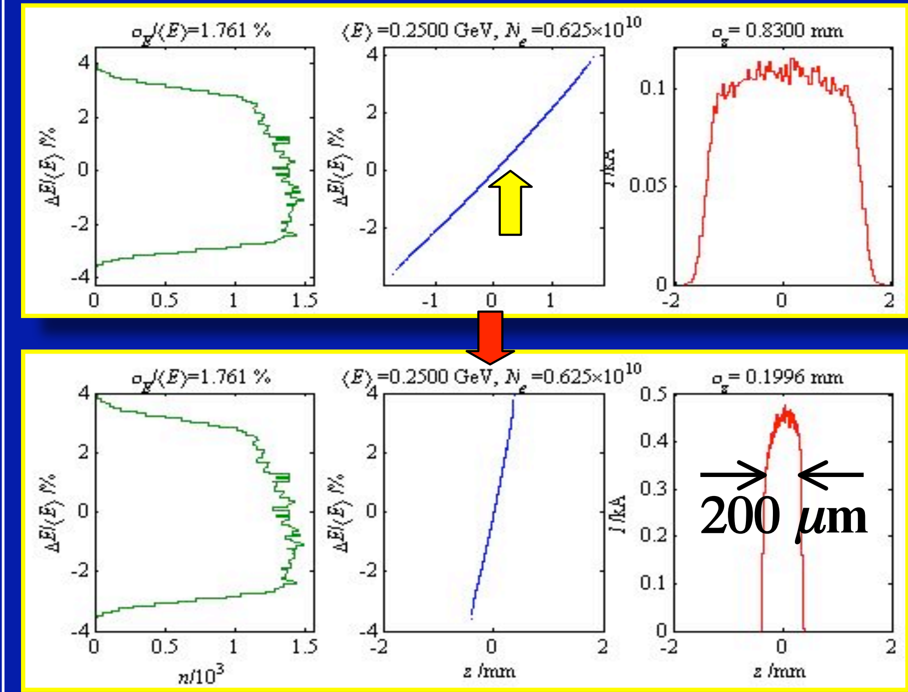


# Harmonic RF used to Linearize Compression

RF curvature and 2nd-order compression cause current spikes

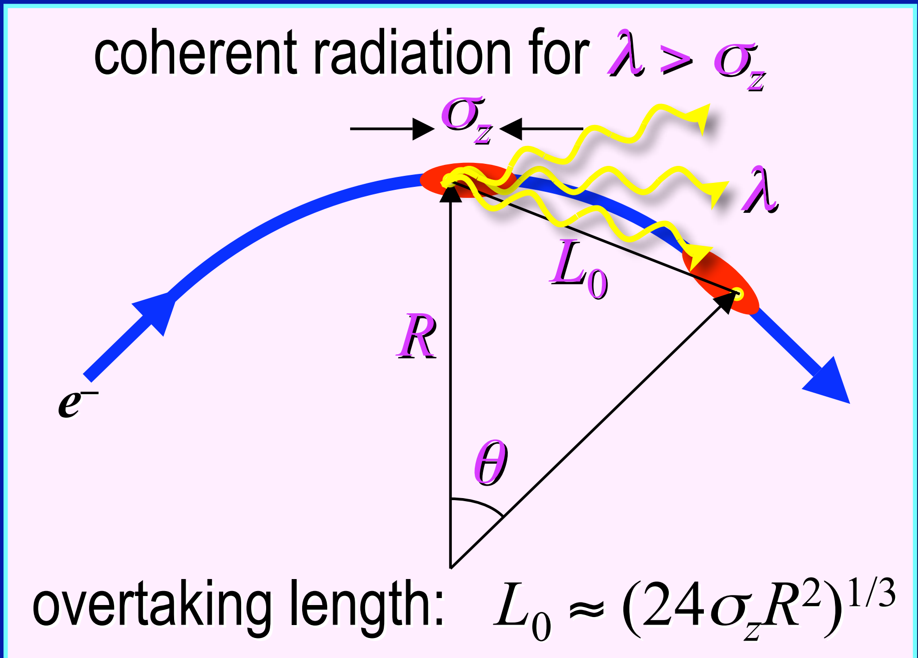


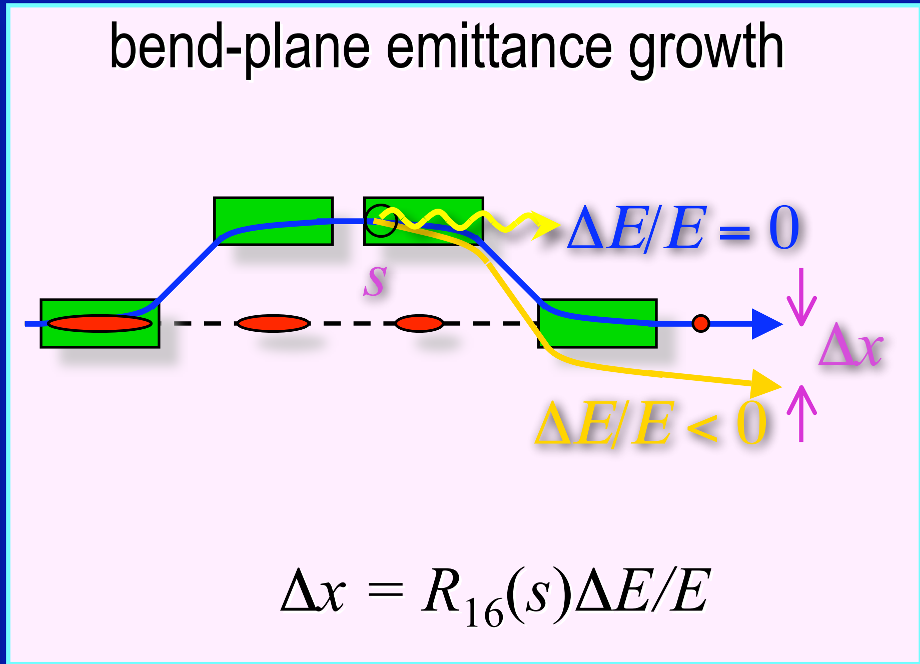

Harmonic RF at decelerating phase corrects 2nd-order and allows unchanged *z*-distribution



# Coherent Synchrotron Radiation (CSR)

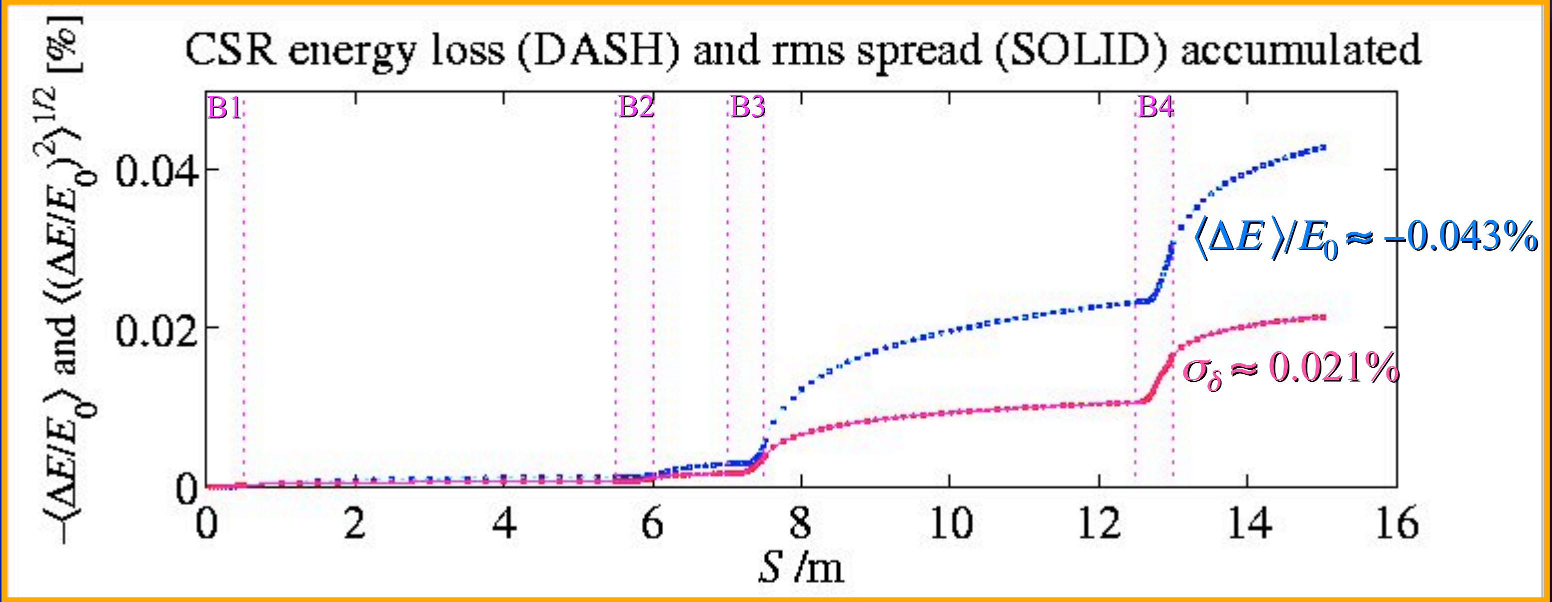
- Powerful radiation generates energy spread in bends
- Energy spread breaks achromatic system
- Causes bend-plane emittance growth (short bunch worse)

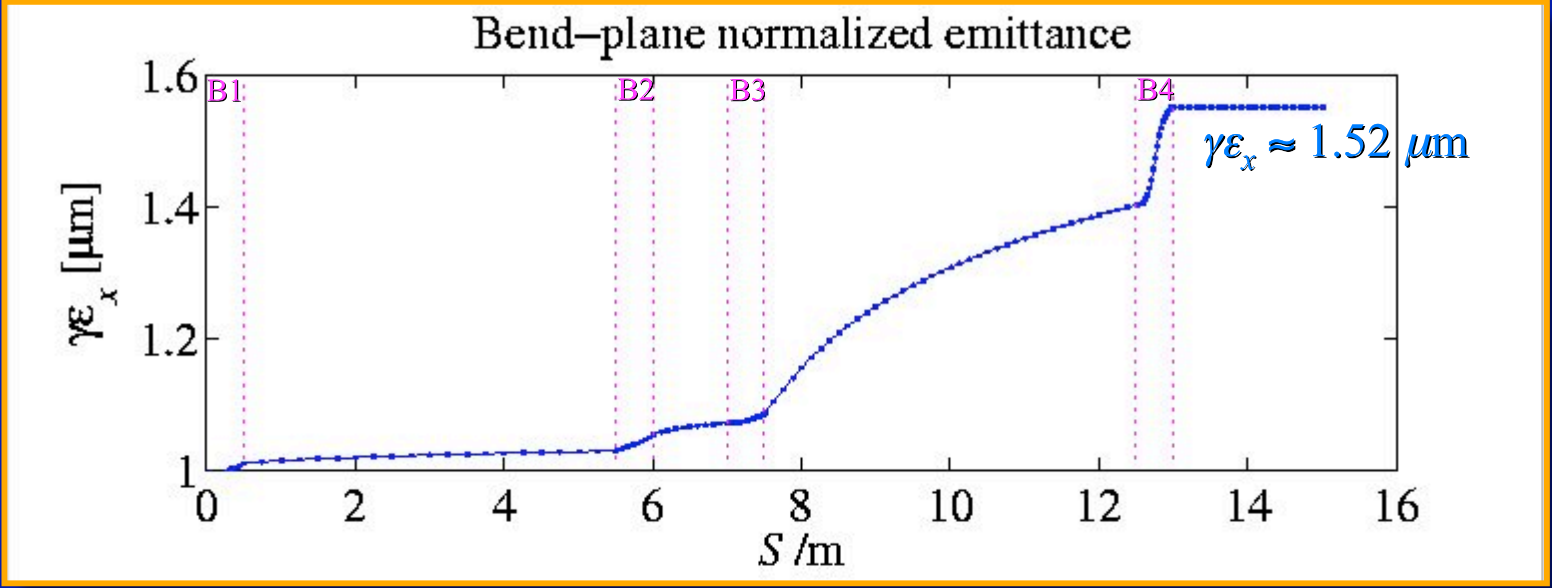



Courtesy
P. Emma

# Projected Emittance Growth



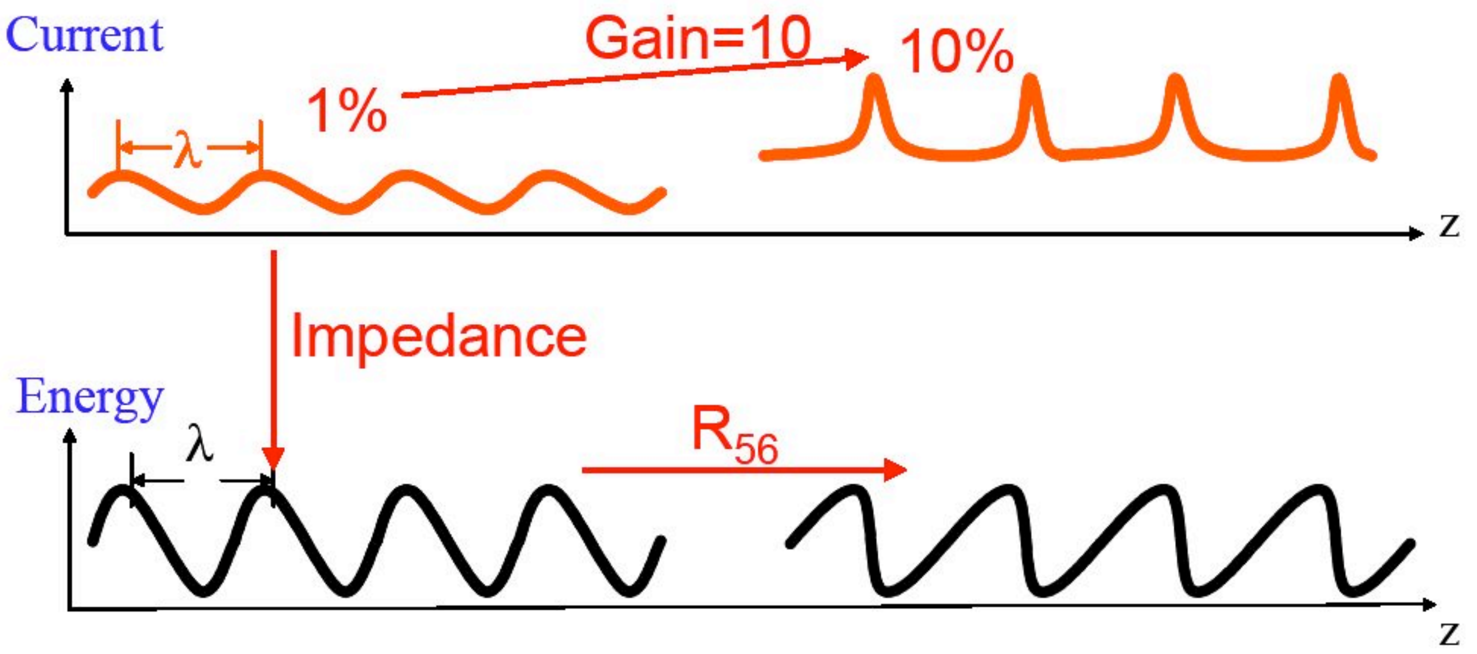
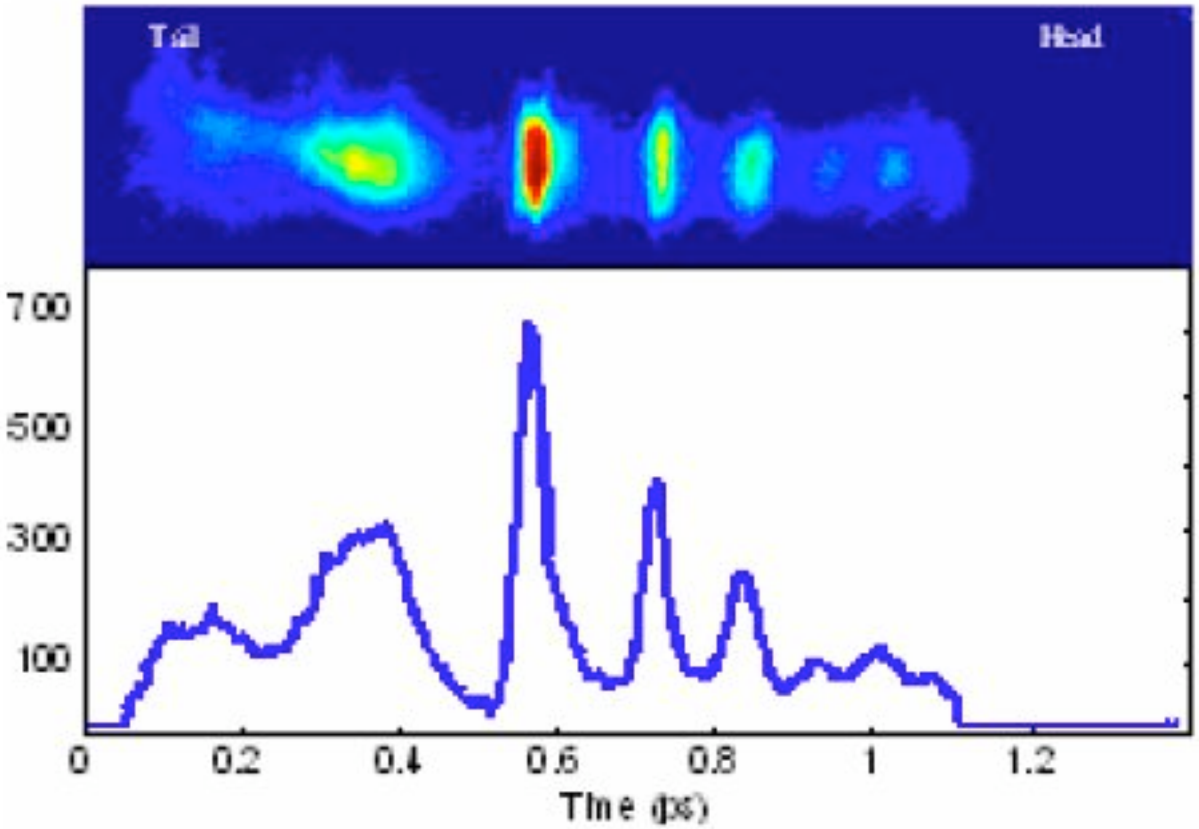


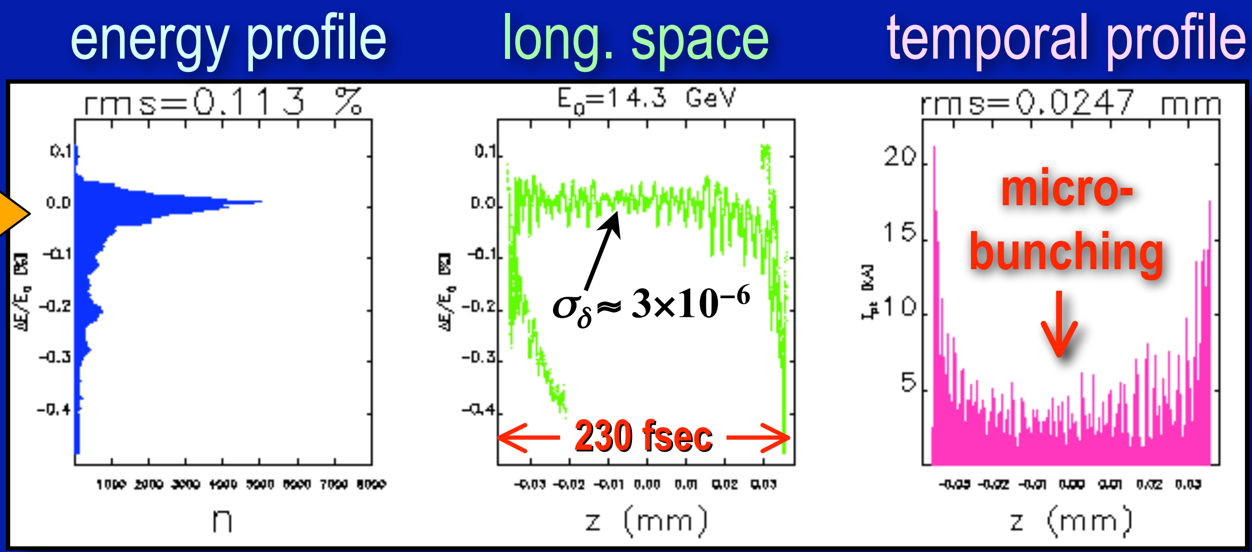

**Figure 1**: An illustration of microbunching instability in a bunch compressor.



Tail
Head
700
500
300
100
0
0.2
0.4
0.6
0.8
1
1.2
Time (ps)

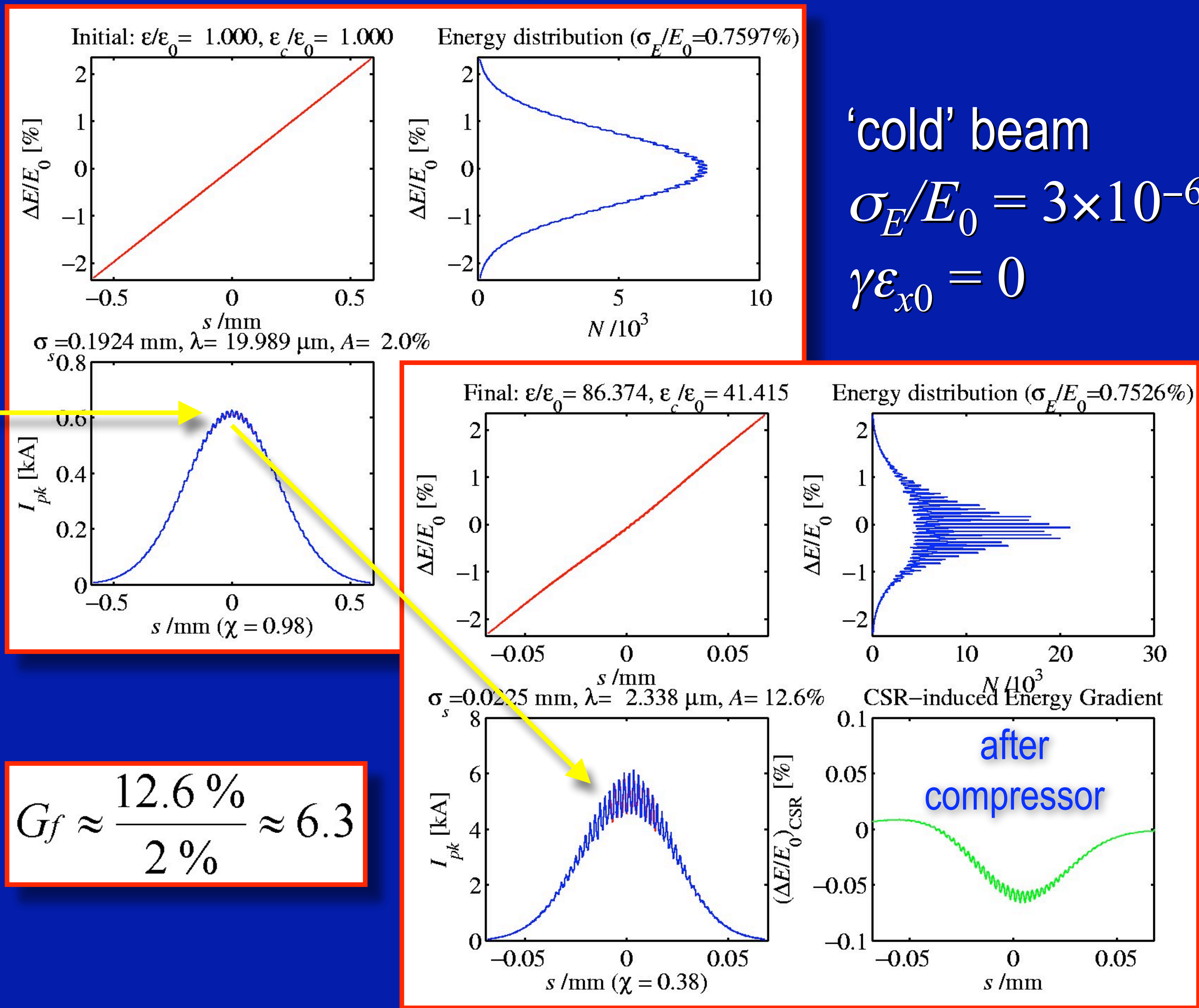
# CSR Microbunching in *LCLS*

CSR can amplify small current modulations:



# CSR Microbunching Gain in *LCLS* BC2

add 2% current & energy modulation




'cold' beam

$\sigma_E/E_0 = 3\times10^{-6}$

$\gamma\varepsilon_{x0} = 0$

$$G_f \approx \frac{12.6\,\%}{2\,\%} \approx 6.3$$

# Laser heater

**Motivation:**

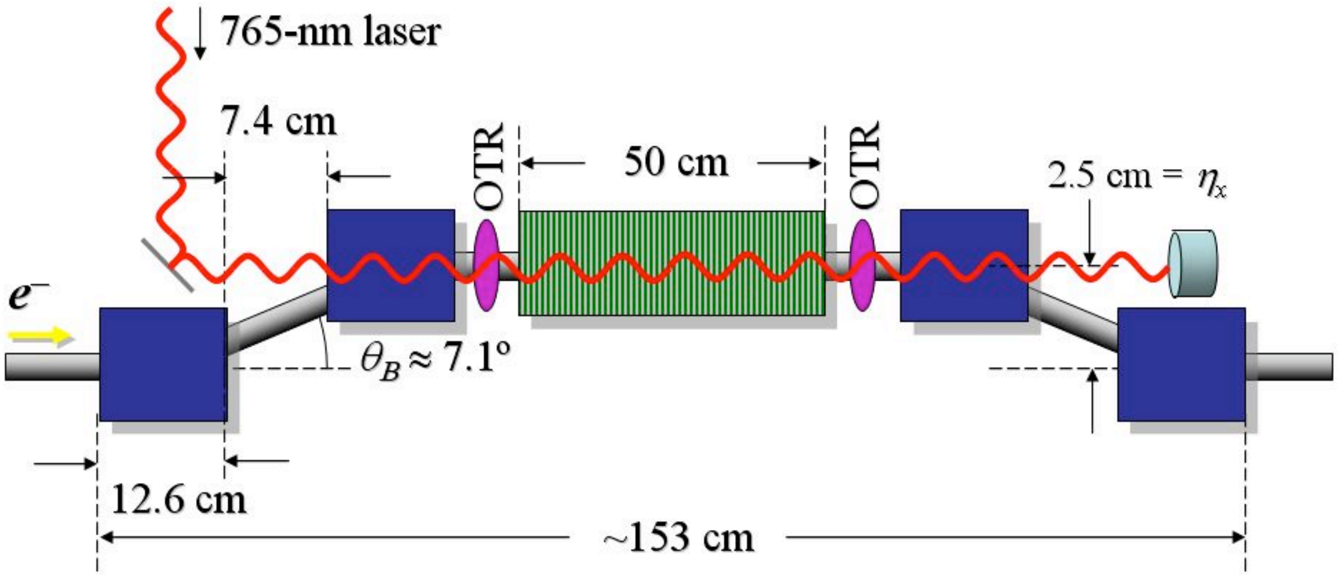
•Collective effect: SP/CSR drive micro-bunch instabilities

•Residual energy-spread ~ 1-3keV ⇒ No Landau damping

•Energy-spread can be larger for FELs ($\sigma_E/E < \rho \sim$ 5e-4)

**⇒ increase $\varepsilon_E$ →10-50 keV**

Example LCLS design



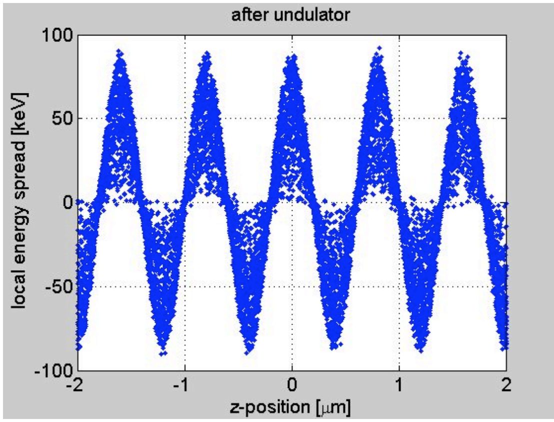

Courtesy H. Schlarb

Z. Huang et al., Phys. Rev. STAB 7, 074401 (2004)

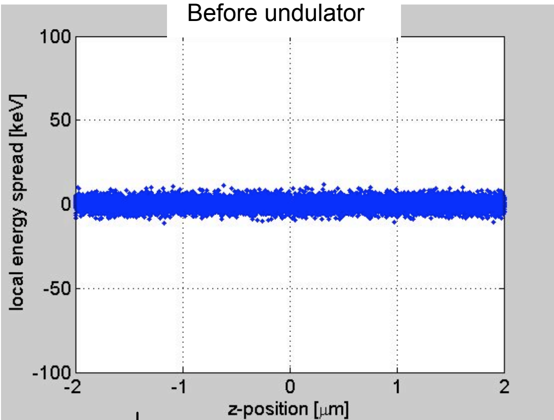
J. Wu et al., SLAC-PUB-10430

# Laser heater

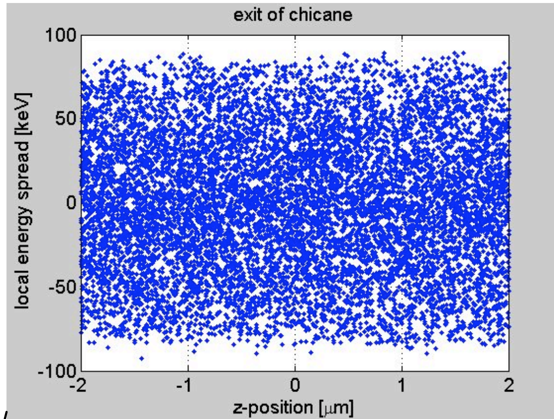
heating $\sigma_L \sim$ 40keV

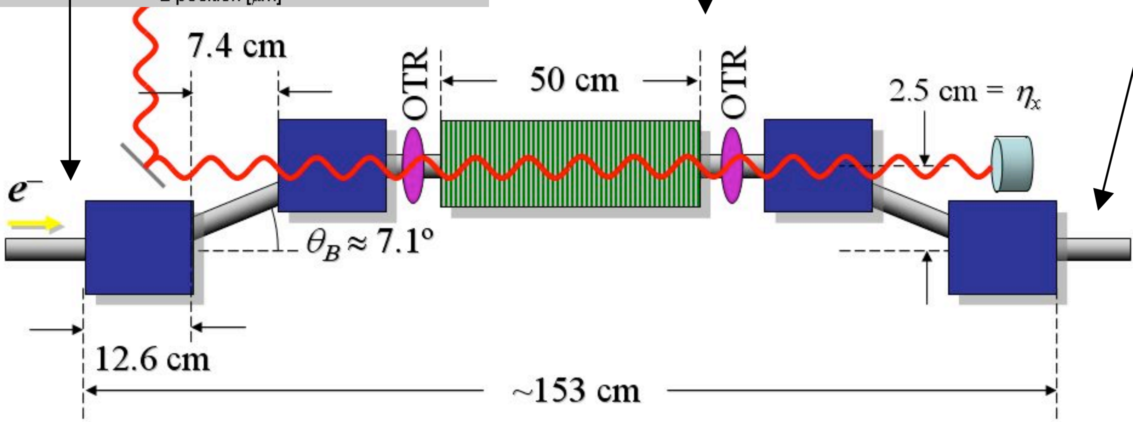



Residual $\sigma_E \sim$ 1-3keV

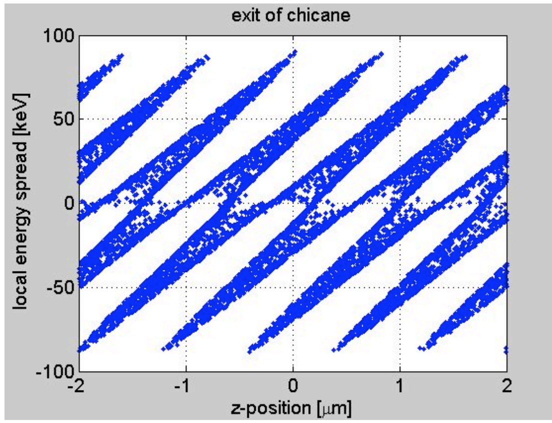



$R_{56}$ = -0.024




$R_{56}$=0



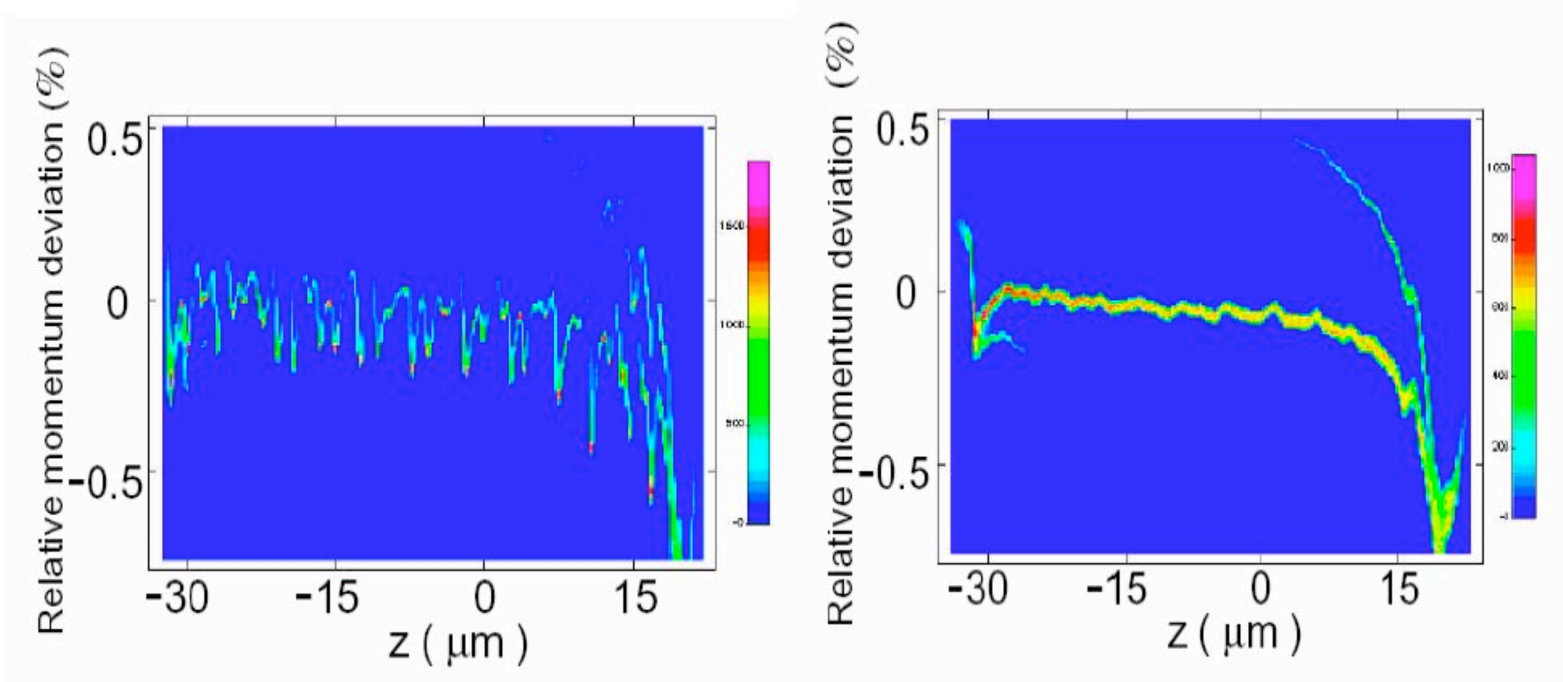
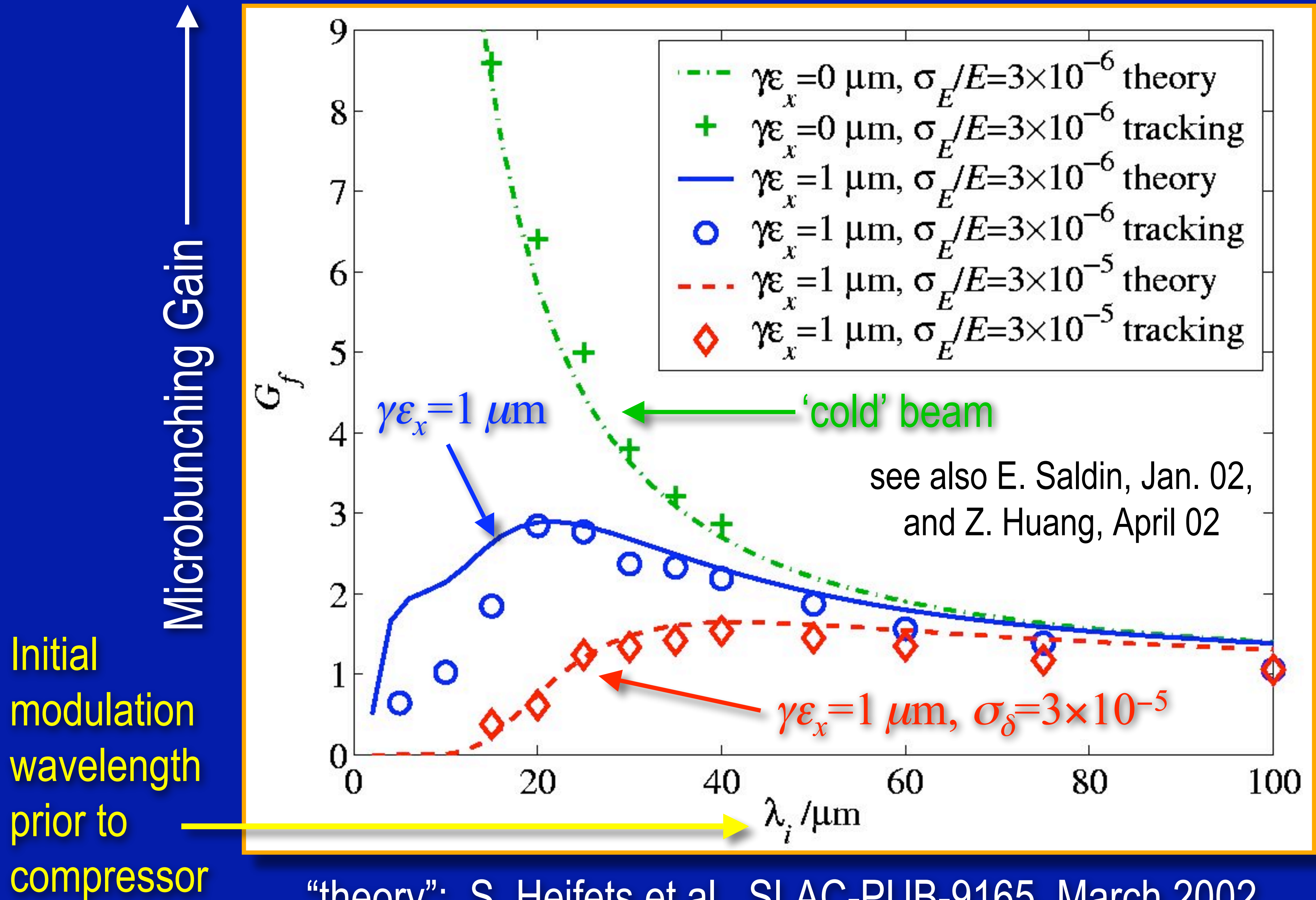


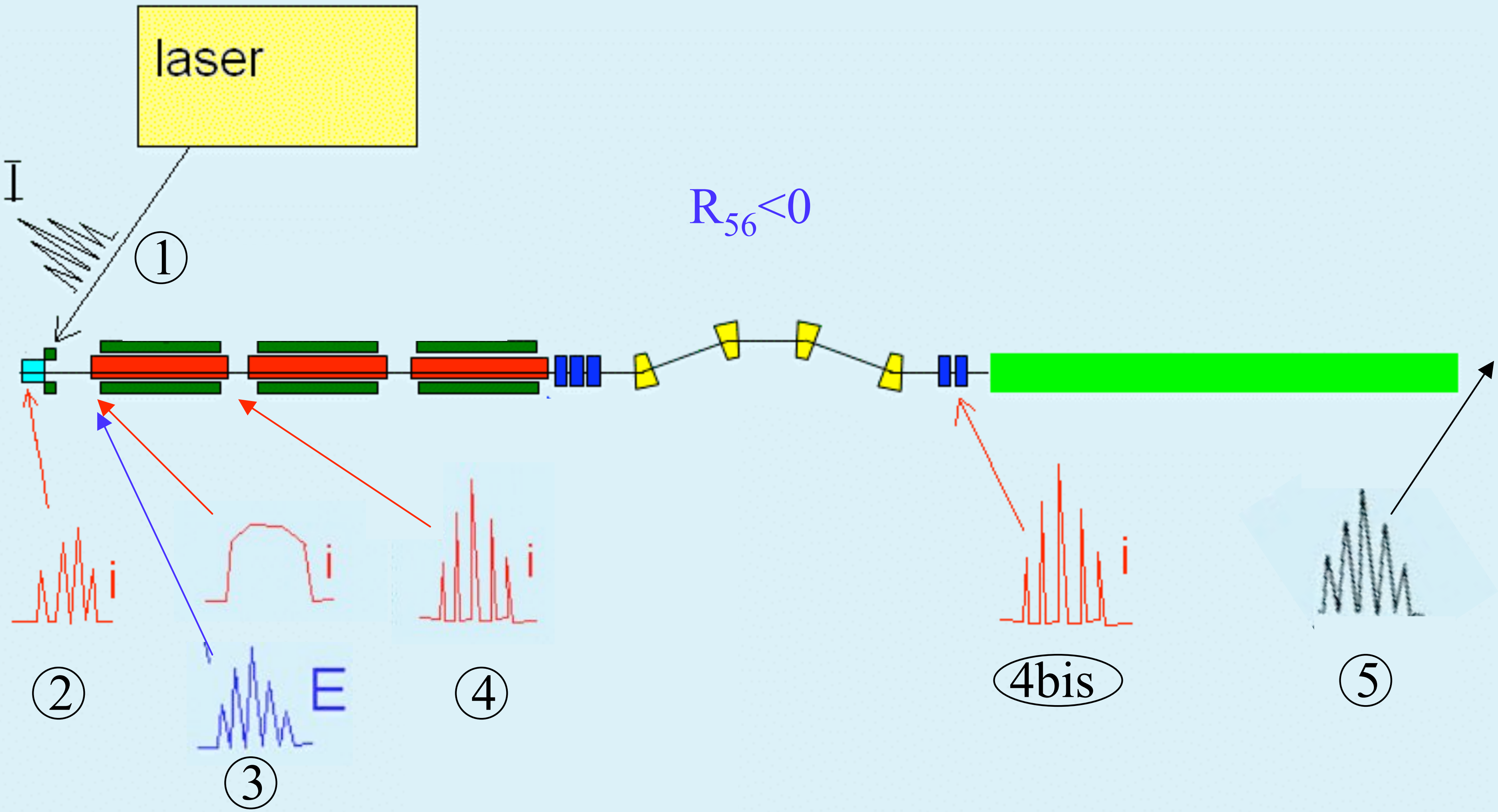

**Figure 5:** Longitudinal phase space distribution at the entrance of the LCLS undulator for an initial ±8% laser intensity modulation at $\lambda_0 = 150\ \mu m$ in the start-to-end simulation without the laser-heater (left plot) and with the laser heater (right plot).

# *LCLS* BC2 CSR Microbunching Gain vs. $\lambda$

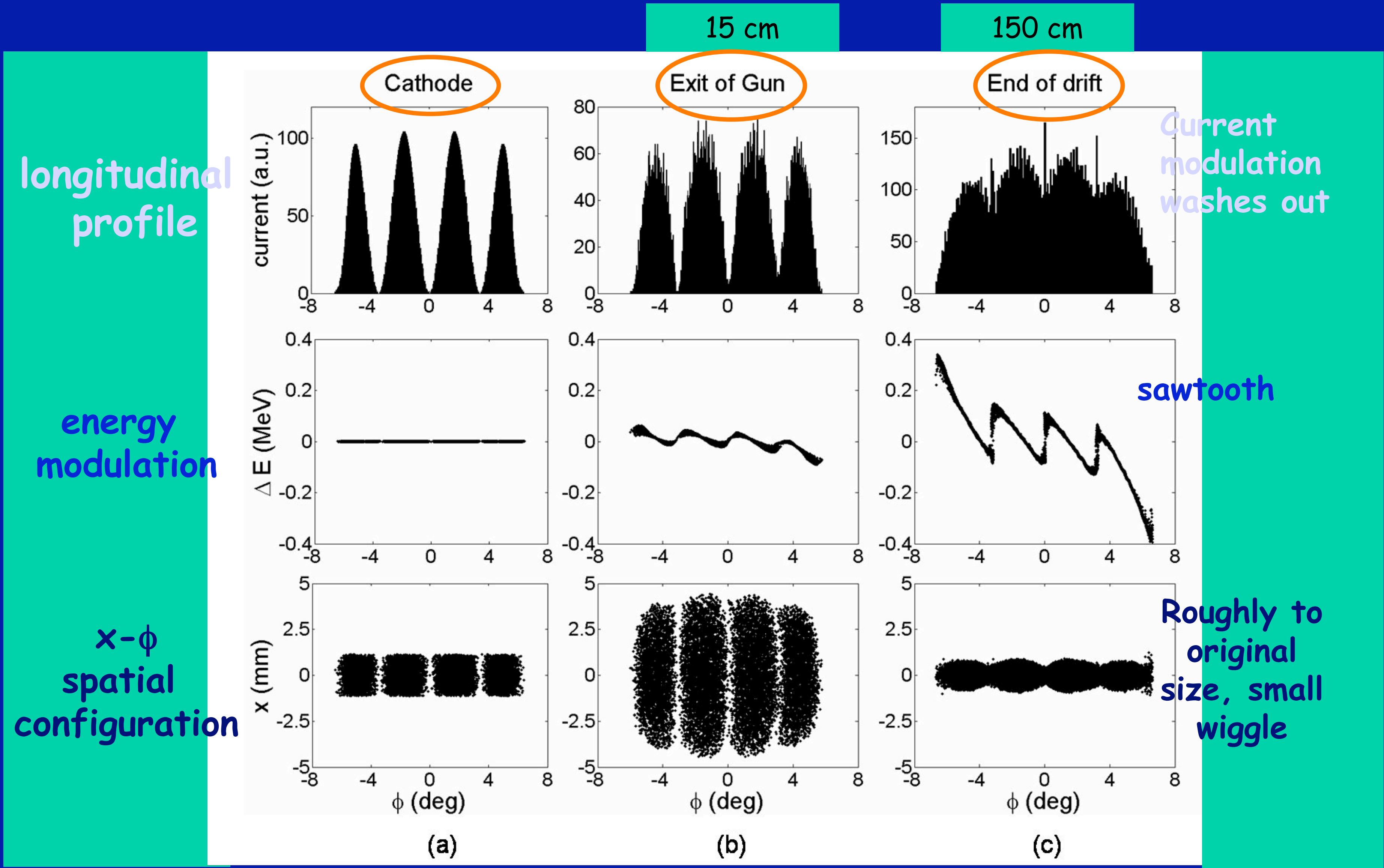



"theory": S. Heifets et al., SLAC-PUB-9165, March 2002

# Laser Comb: a giant microbunch instability



# Example of typical behavior: N = 4



# Laser Comb: a giant microbunch instability

M. Boscolo et al. / Nuclear Instruments and Methods in Physics Research A 593 (2008) 106–110

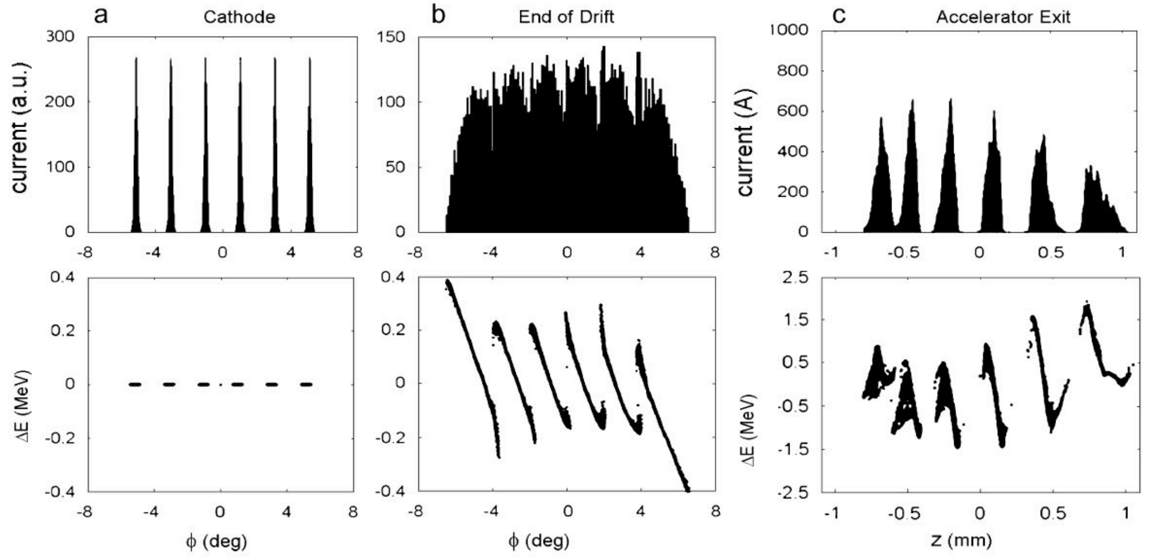
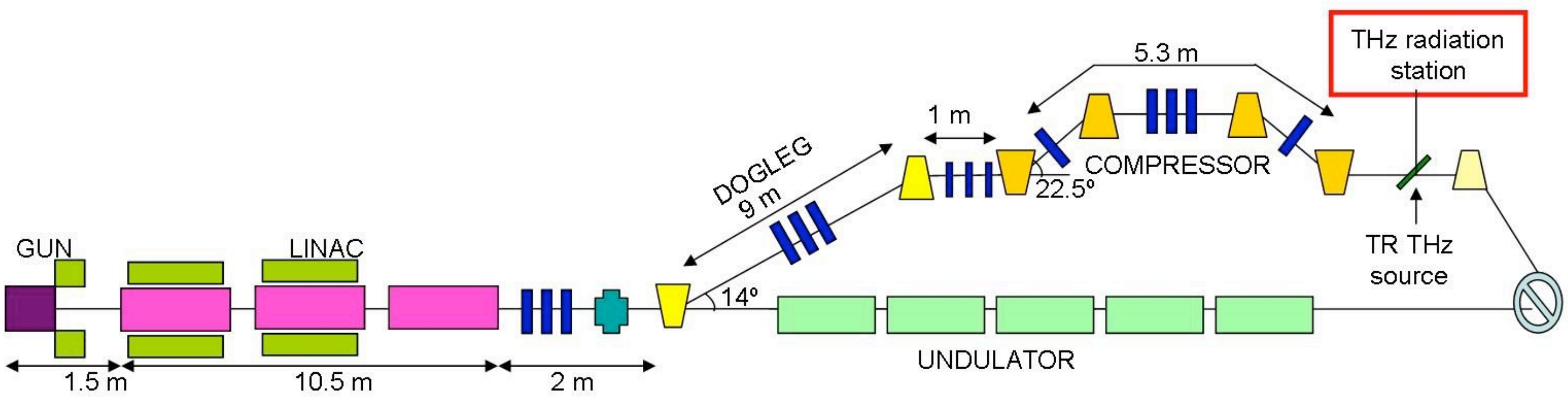
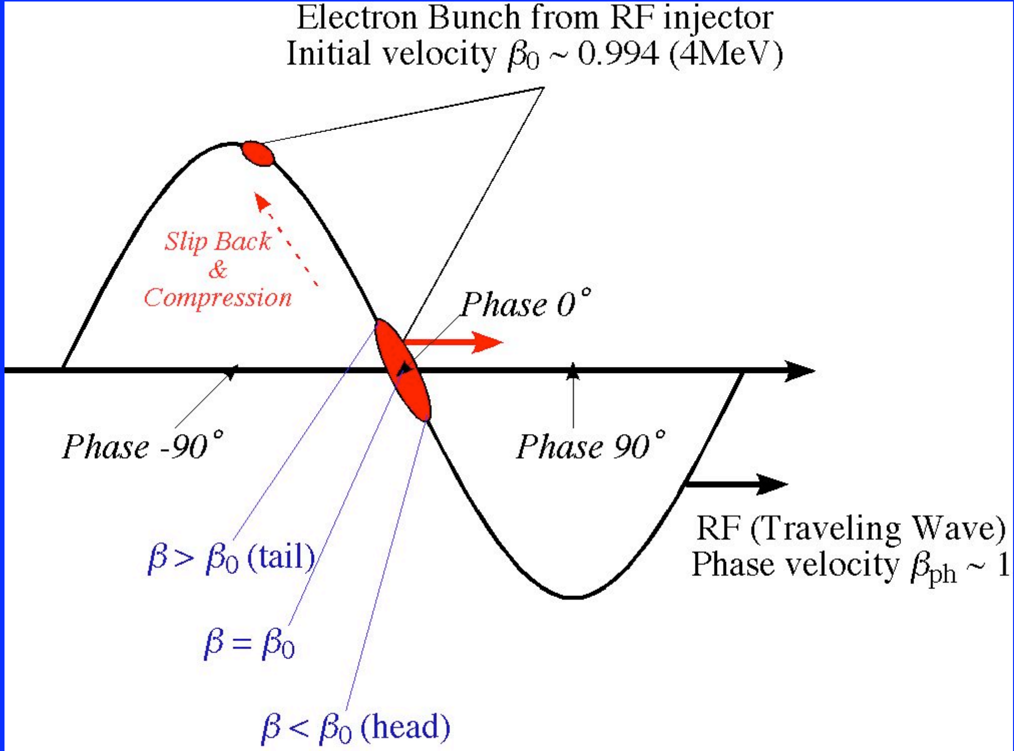




**Fig. 1.** Evolution of a six bunches electron beam train: the columns from left refer, respectively, to (a) the cathode, (b) the end of the drift at 150 cm and (c) the end of linac at 12 m far from cathode. The rows from top refer, respectively, to longitudinal profile and to energy modulation $\Delta E$ (MeV).

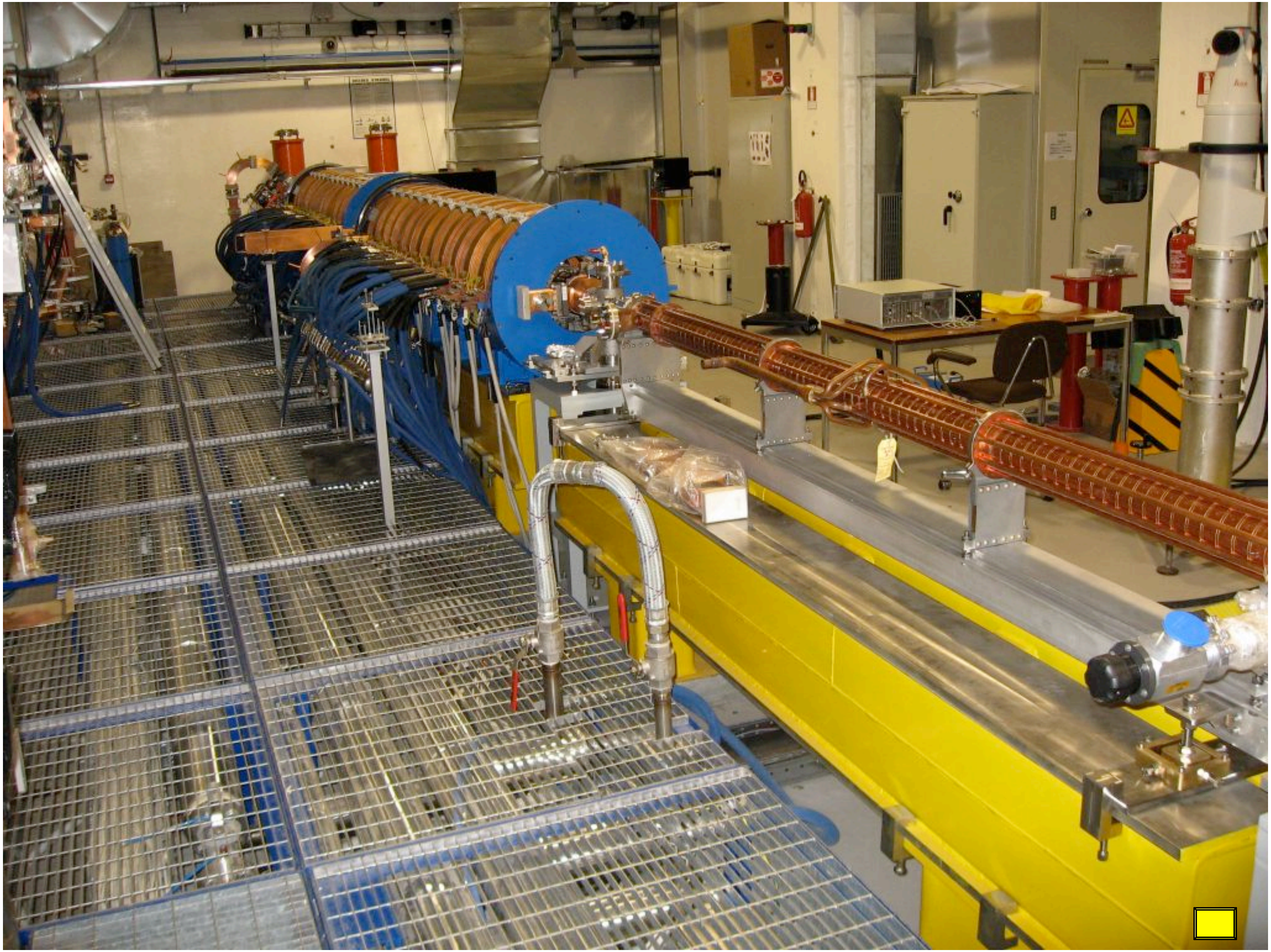


# Velocity bunching concept (RF Compressor)

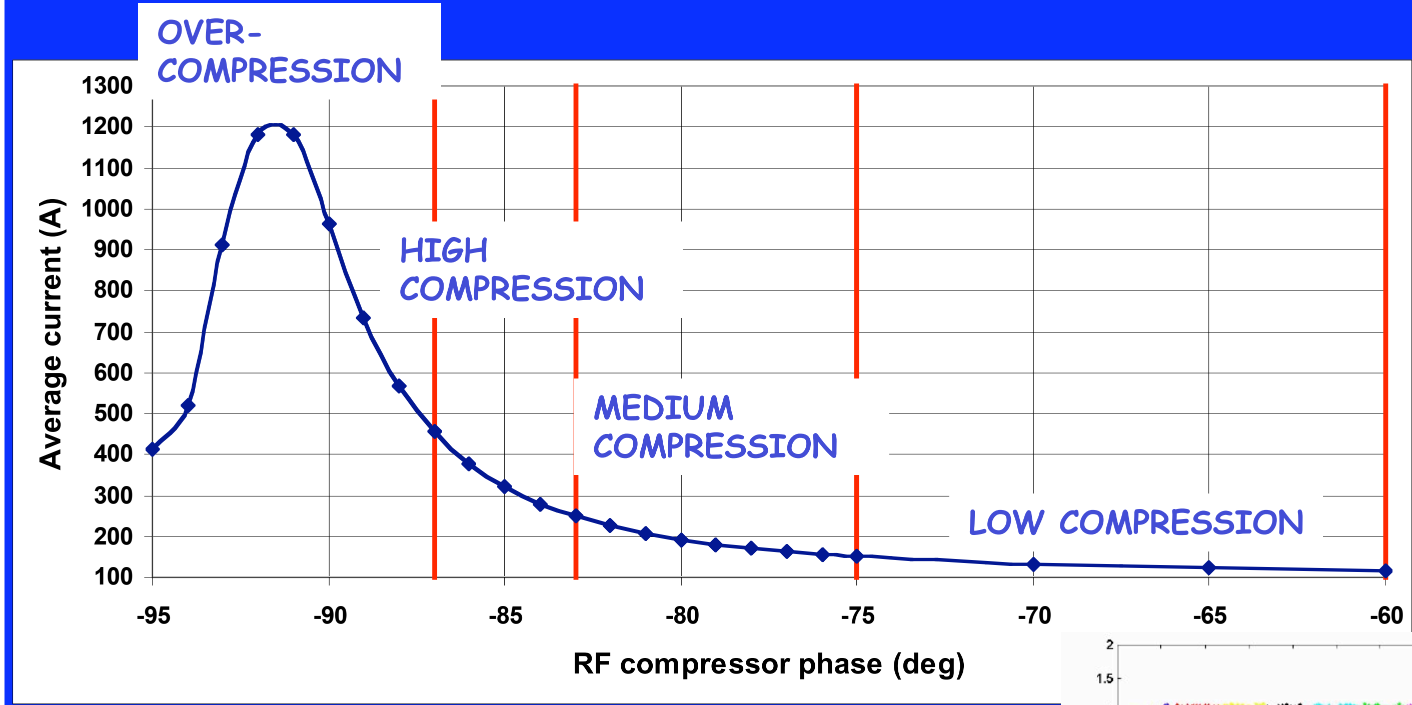
If the beam injected in a long accelerating structure at the crossing field phase and it is slightly slower than the phase velocity of the RF wave , it will slip back to phases where the field is accelerating, but at the same time it will be chirped and compressed.

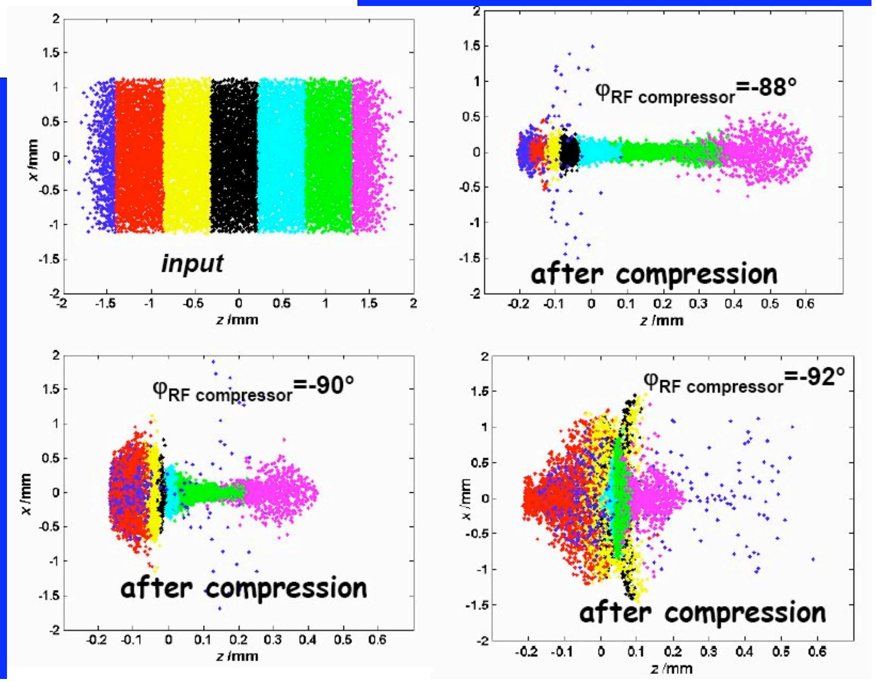


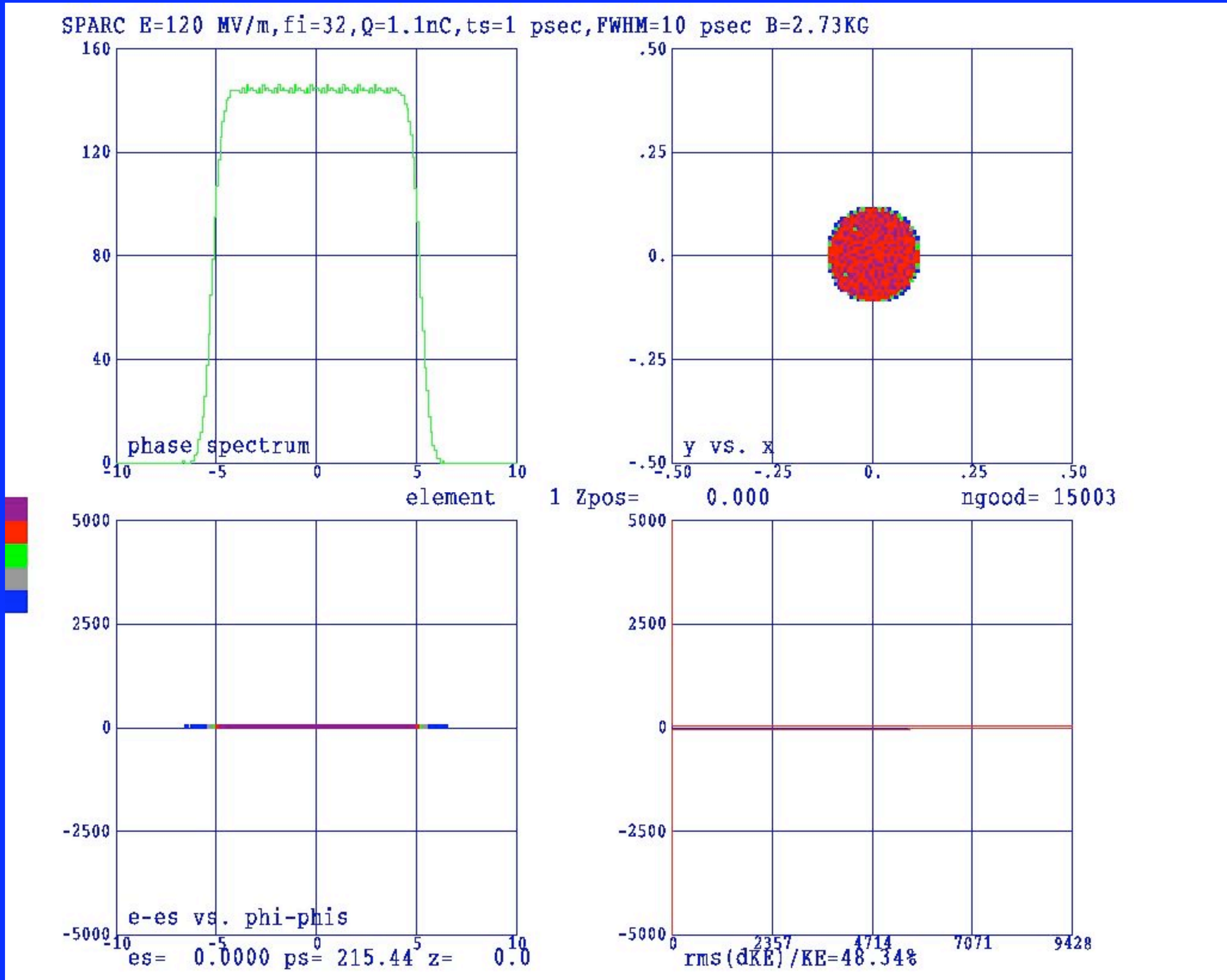

The key point is that compression and acceleration take place at the same time within the same linac section, actually the first section following the gun, that typically accelerates the beam, under these conditions, from a few MeV (> 4) up to 25-35 MeV.



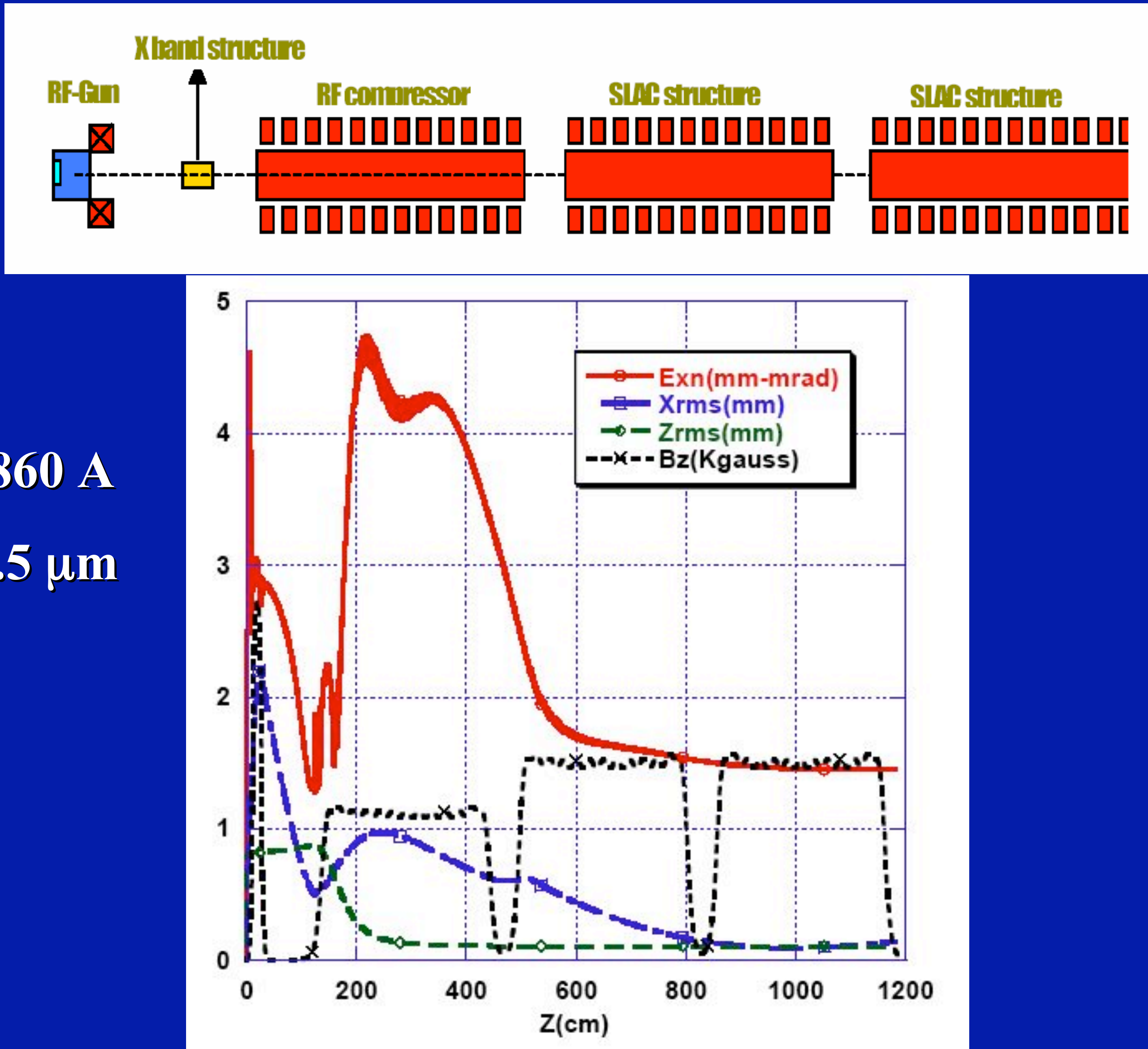
# Average current vs RF compressor phase



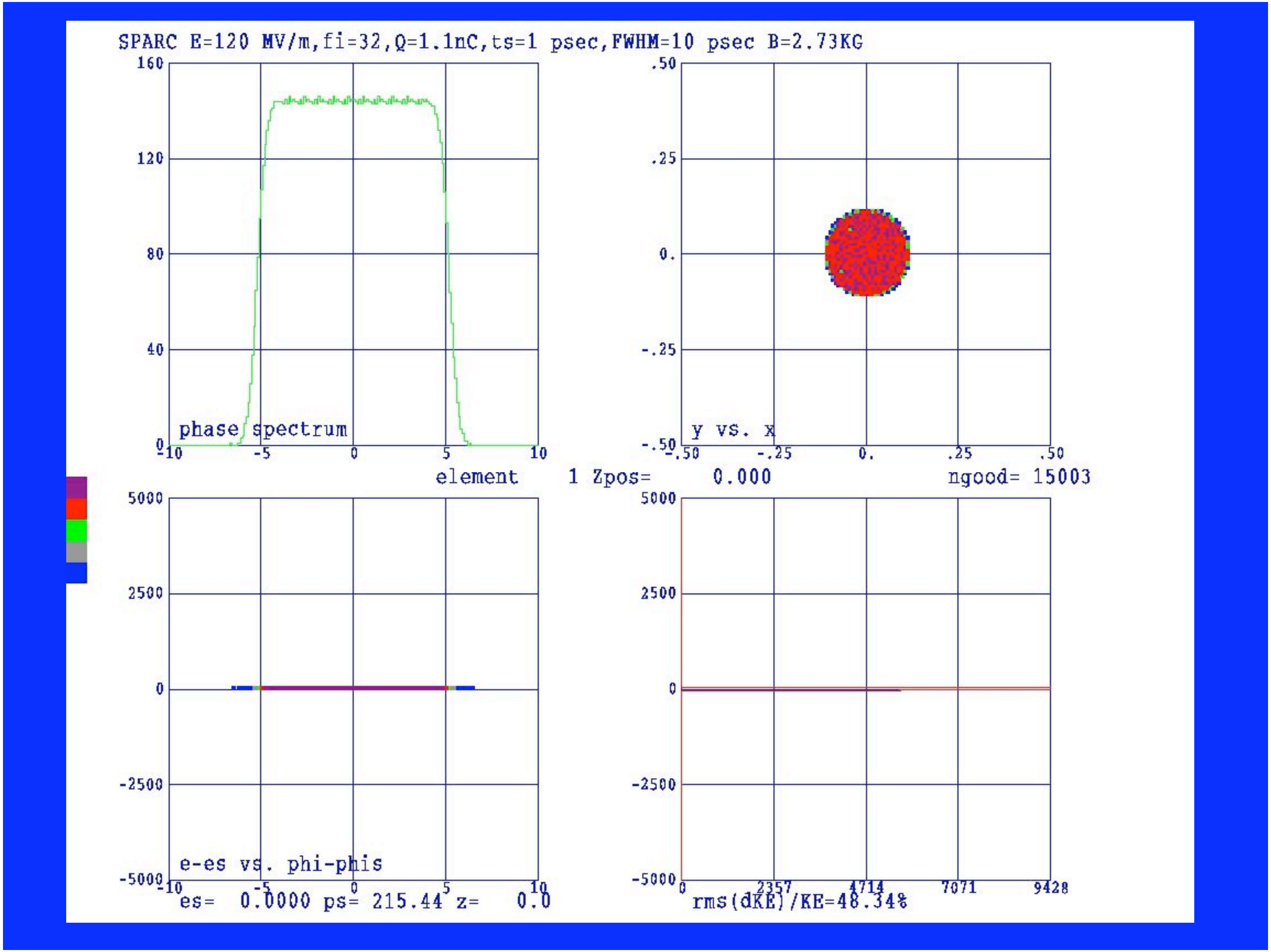


SPARC E=120 MV/m,fi=32,Q=1.1nC,ts=1 psec,FWHM=10 psec B=2.73KG
phase spectrum
y vs. x
element 1 Zpos= 0.000
ngood= 15003
e-es vs. phi-phis
es= 0.0000 ps= 215.44 z= 0.0
rms(dKE)/KE=48.34%



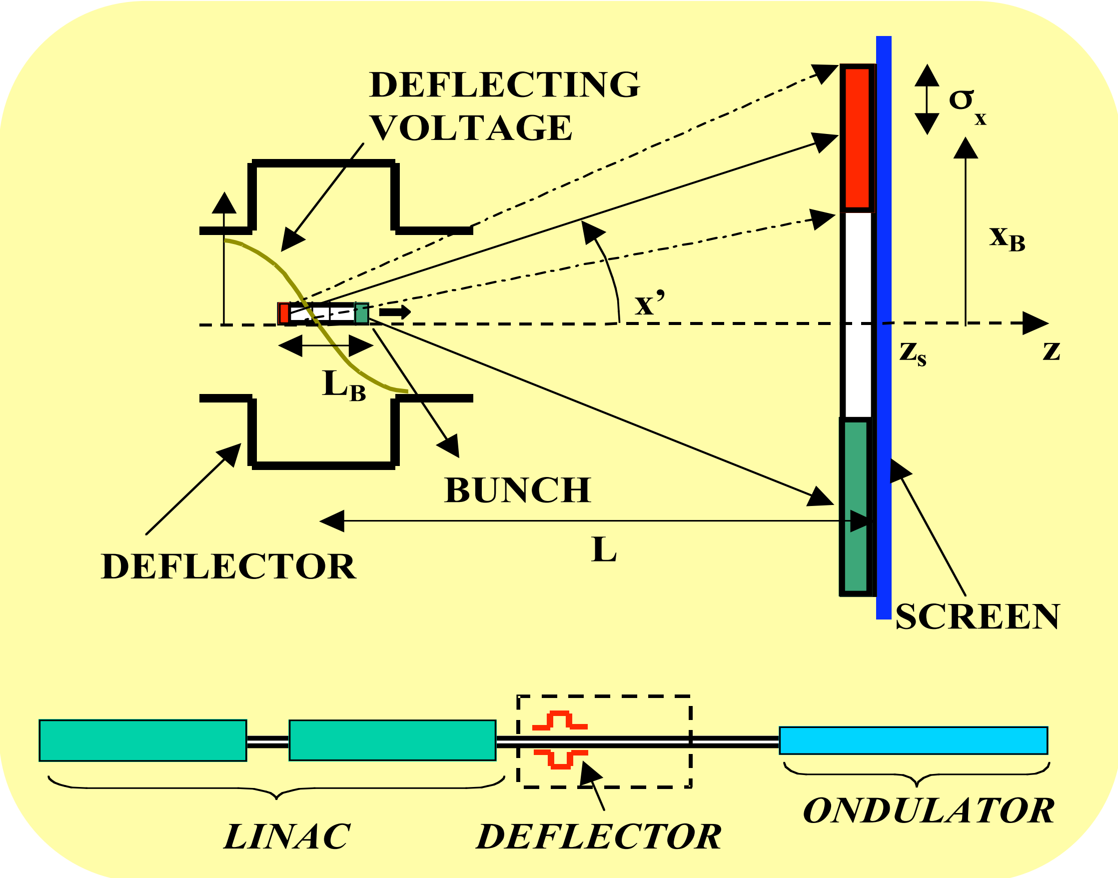

$\langle I \rangle = 860$ A

$\varepsilon_{nx} = 1.5\ \mu m$



SPARC E=120 MV/m,fi=32,Q=1.1nC,ts=1 psec,FWHM=10 psec B=2.73KG
phase spectrum
y vs. x
element 1 Zpos= 0.000
ngood= 15003
e-es vs. phi-phis
es= 0.0000 ps= 215.44 z= 0.0
rms(dKE)/KE=48.34%

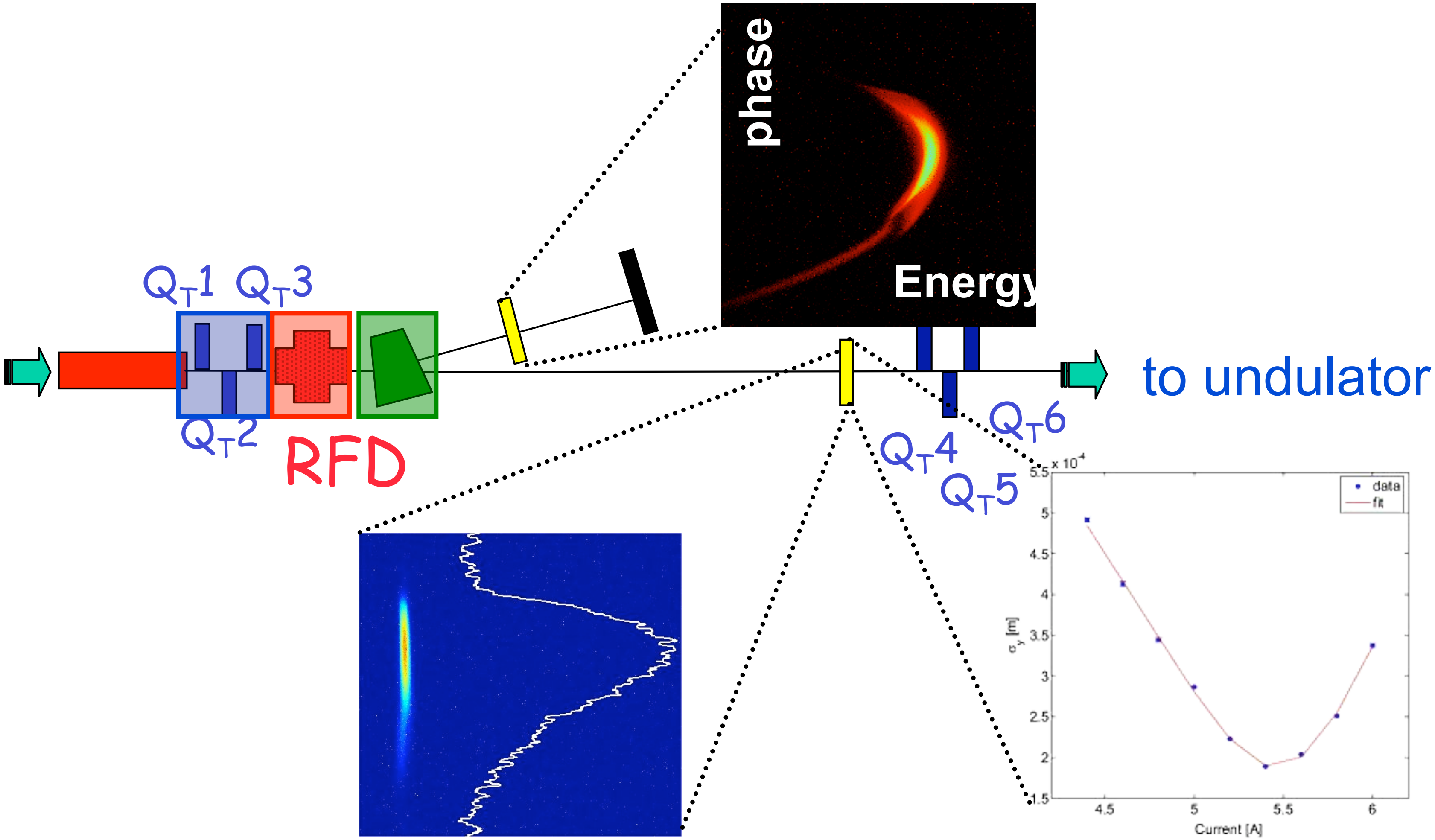
# RF deflector



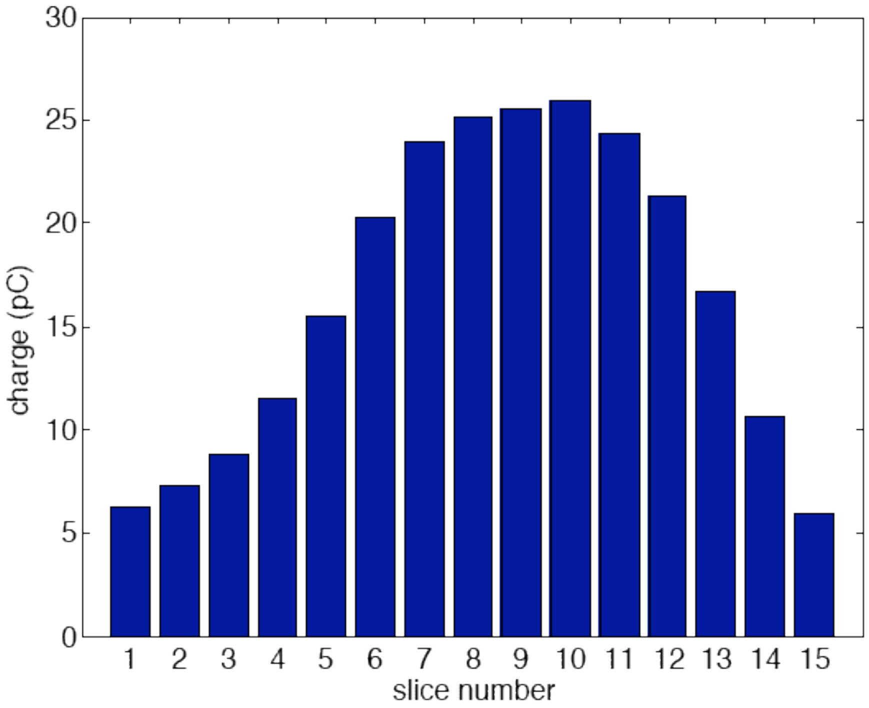

$$x_B = \frac{\pi f_{RF} L L_B V_\perp}{cE/e}$$

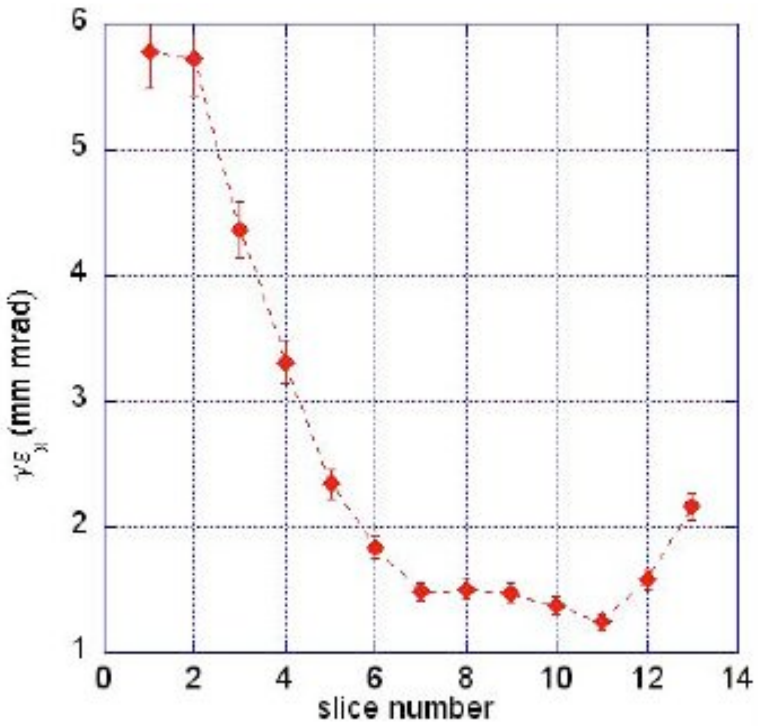
$$V_\perp = \frac{\sigma_x cE/e}{\pi f_{RF} L L_{res}}$$

# SPARC Diagnostic Section

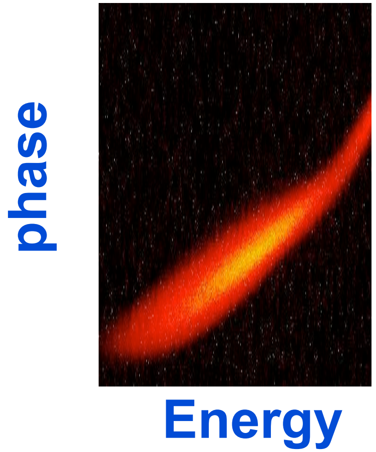


# Slice emittance measurements

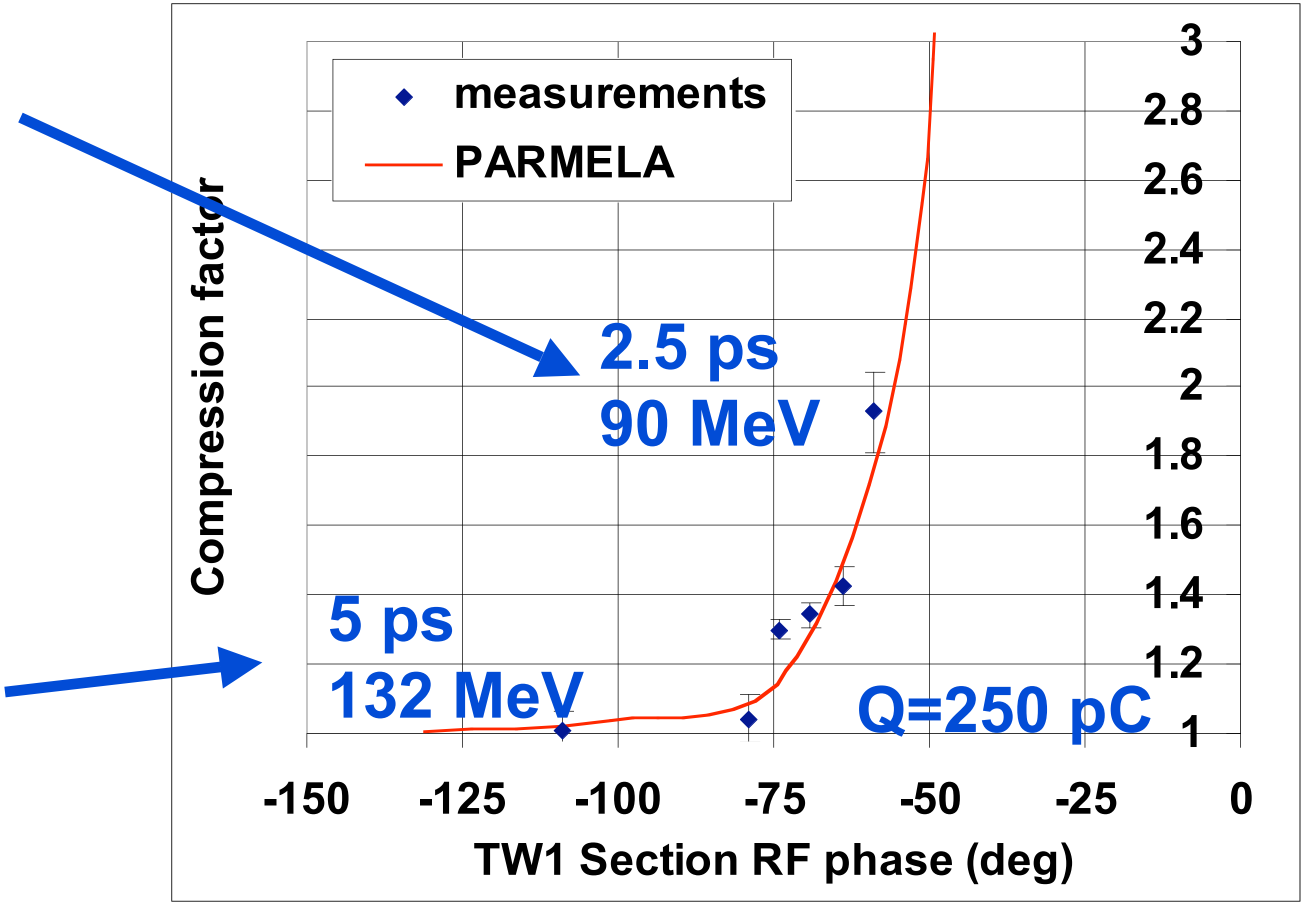


# Preliminary results for the velocity bunching at SPARC







# Velocity bunching equations

$$\frac{d}{dz}\left(\gamma mc^2\right) = eE\,sin\left(kz - \omega t + \varphi_o\right)$$
$$\frac{d\gamma}{dz} = \frac{eE}{mc^2}\,sin\left(kz - \omega t + \varphi_o\right)$$
$$\frac{d\gamma}{dz} = \alpha k\,sin\left(\varphi\right)$$

$$\frac{d\varphi}{dz} = \frac{d}{dz}\left(kz - \omega t + \varphi_o\right) = \left(k - \omega\frac{dt}{dz}\right) = \left(k - \frac{\omega}{\beta c}\right)$$
$$= k\left(1 - \frac{1}{\beta}\right) = k\left(1 - \frac{\gamma}{\sqrt{\gamma^2 - 1}}\right)$$

$$\begin{cases} \dfrac{d\gamma}{dz} = \alpha k\,sin\left(\varphi\right) \\ \dfrac{d\varphi}{dz} = k\left(1 - \dfrac{\gamma}{\sqrt{\gamma^2 - 1}}\right) \end{cases}$$

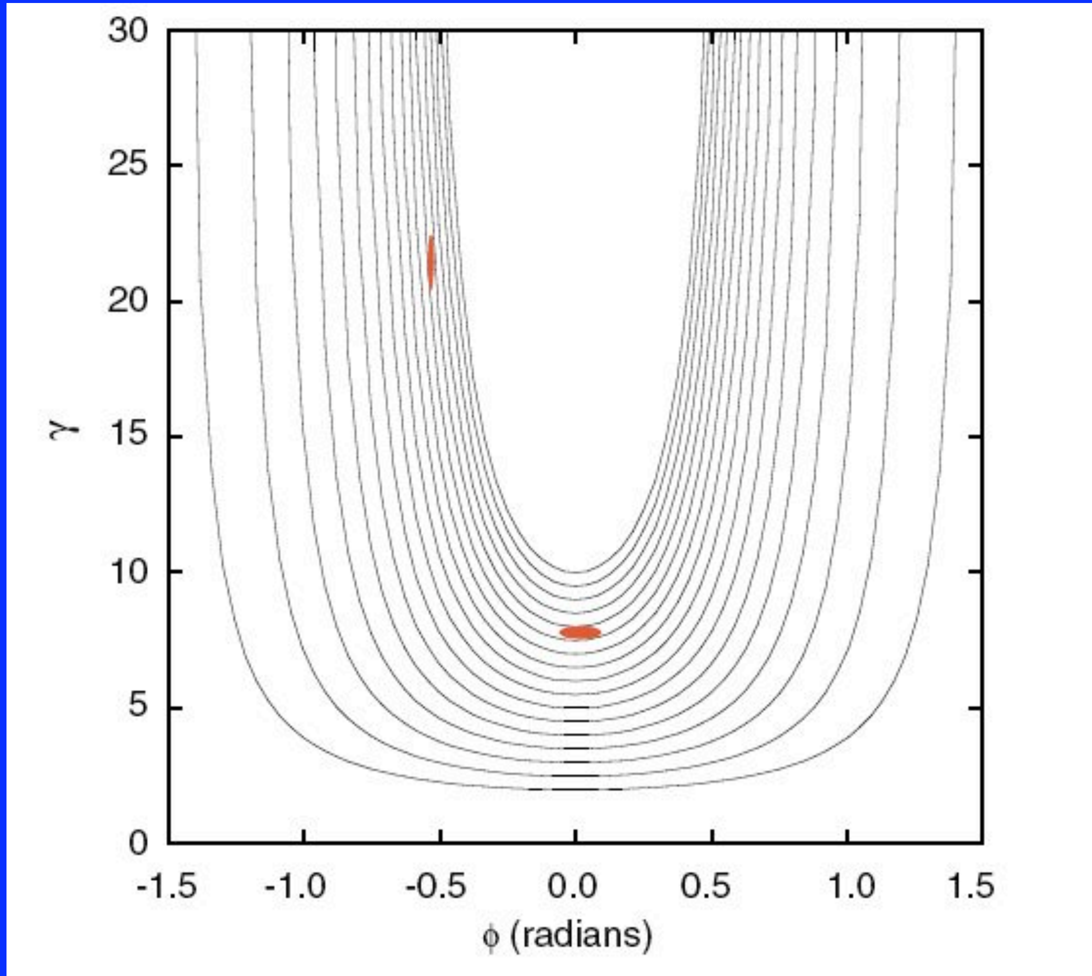
Such a system is solved using the variable separation technique to yield a constant of the motion (total energy):

$$H = \gamma - \sqrt{\gamma^2 - 1} - \alpha\cos(\phi)$$

$$H = \gamma - \sqrt{\gamma^2 - 1} - \alpha \cos(\phi)$$

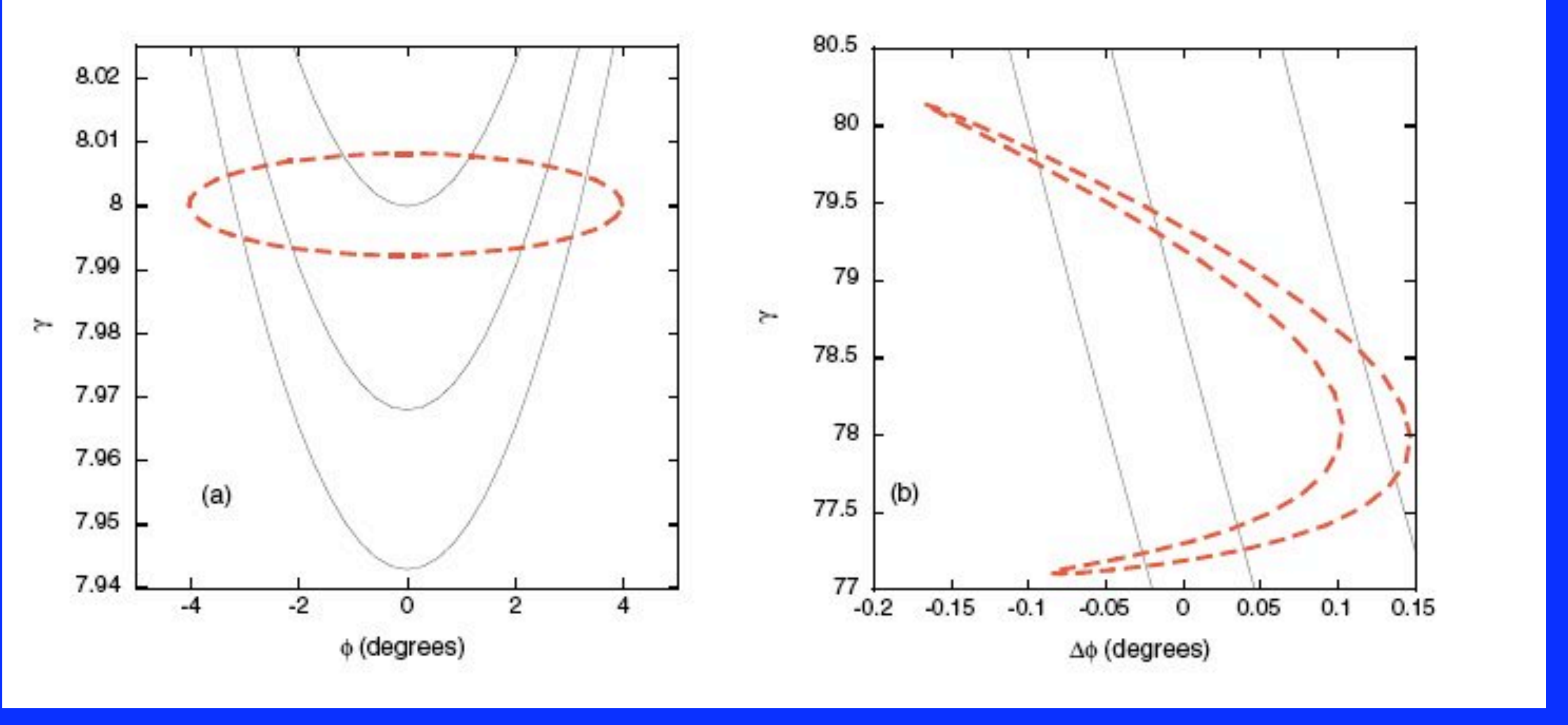
$$H = -\alpha \cos\phi_\infty = \gamma_0 - \sqrt{\gamma_0^2 - 1} - \alpha \cos\phi_0$$

$$\phi_\infty \cong \cos^{-1}\left[\cos\phi_0 - \frac{1}{2\alpha\gamma_0}\right]$$



# Second order effects

$$\Delta\phi_\infty = \frac{\sin\phi_0}{\sin\phi_\infty}\Delta\phi_0 + \frac{1}{2\alpha\gamma_0^2\sin\phi_\infty}\Delta\gamma_0 + \frac{1}{2}\left[\frac{\cos\phi_0}{\sin\phi_\infty} - \frac{\cos\phi_\infty\sin^2\phi_0}{\sin^3\phi_\infty}\right](\Delta\phi_0)^2.$$



# THE END

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http://icfa-usa.jlab.org/archive/newsletter.shtml

-Zhirong Huang, Juhao Wu (SLAC) Timur Shaftan (BNL), Microbunching instability due to bunch compression, ICFA Beam Dynamics Newsletters 38
http://icfa-usa.jlab.org/archive/newsletter.shtml

-L. Serafini and M. Ferrario, Velocity Bunching In Photo-injectors, AIP Conference Proceedings, 581, 87, (2001) also in LNF note LNF-00/036 (P)