Septa I

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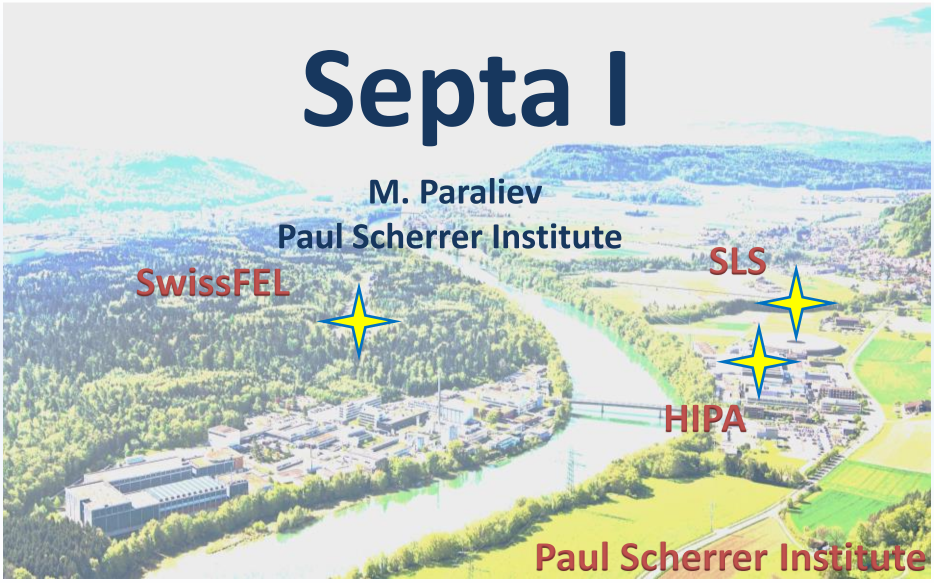


Presentation overview

- Why do we need another magnet type?

Septum

- Basic concept and terminology
- Key objectives
- Types of septa
- Electrostatic deflection
- Magnetic deflection
- Electrostatic vs Magnetic deflection

Electrostatic septa

- Types
- Examples, design specifics

What to remember

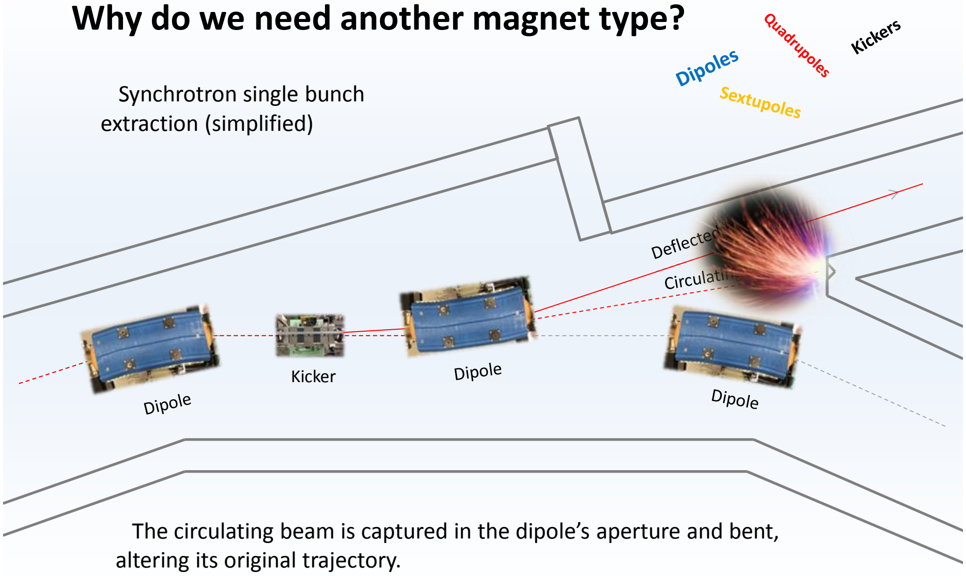
Literature

Additional material

- Electrostatic deflection derivation
- Magnetic deflection derivation
- Equivalence of electrostatic and magnetic deflection for relativistic particles

Why do we need another magnet type?

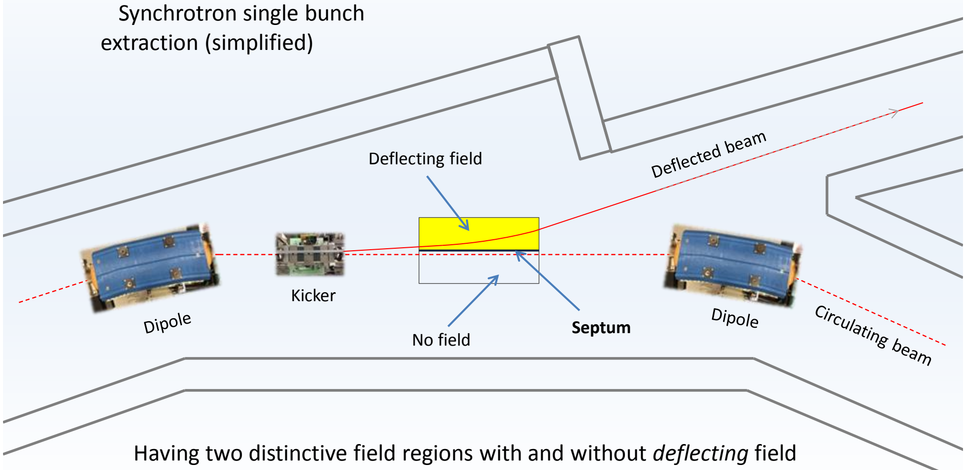
Synchrotron single bunch extraction (simplified)



The circulating beam is captured in the dipole's aperture and bent, altering its original trajectory.

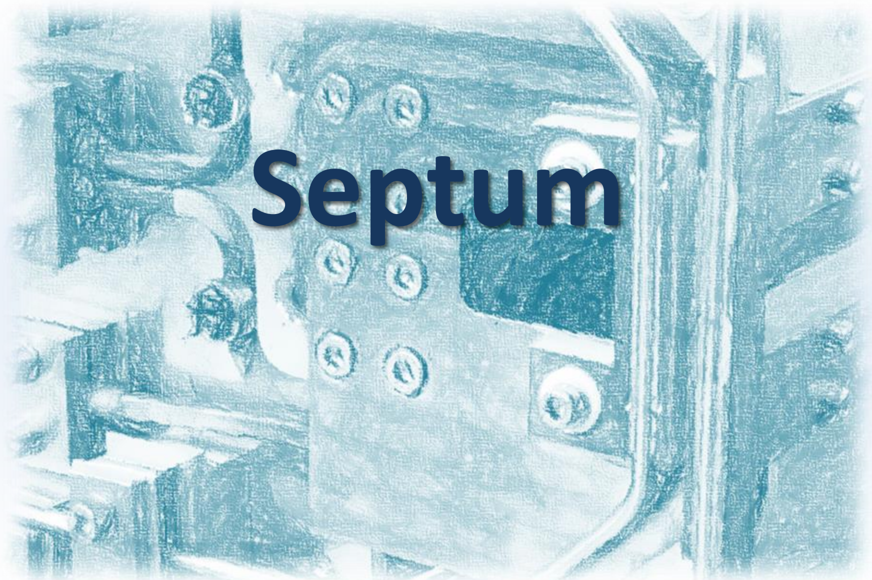
Why do we need another magnet type?

Synchrotron single bunch extraction (simplified)



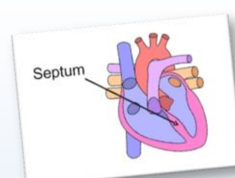
Having two distinctive field regions with and without *deflecting* field meets the desired requirements.

Septum

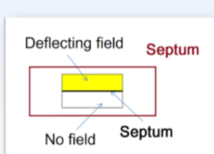


Septum

- A **septum** (plural **septa**) is a partition, a wall, a barrier that separates two cavities or two chambers (biology, mechanics, part. physics, etc.).
- Latin origin - *saepio* (*sēpiō*) - surround, enclose, fence in.
- In particle accelerators, a **septum** separates two distinctive field regions in order to selectively deflect particle beams.
- Used for **injection** and **extraction** of the beam



The septum separates the left and the right side of the heart



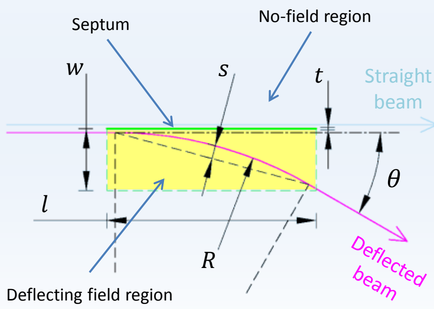
Schematic representation of a septum device

- Often the device that embodies the **septum** is called **septum** as well (*electrostatic septum*, *septum magnet*, etc.)

~~Septums~~ ✓ **Septa**

Basic concept and terminology

- A septum shares a lot with dipole (bending) magnets
- Has an abrupt field change between field and no-field region



Schematic representation of a septum

θ – Bending angle
R – Bending radius
s – Sagitta
w – Deflecting gap width
t – Septum thickness
l – Septum length

If deflecting gap does not follow the trajectory of the deflected beam (as shown here) the deflecting gap should be wide enough to accommodate the deflected beam trajectory.

Key objectives

- Field region (type, magnitude, direction, flatness)
- Field-free region (field leakage)
- Beam trajectory (sagitta), beam optics
- Beam impedance, wake fields
- Vacuum (UHV materials, conductance, bake-out)
- Positioning and mechanical stability
- Synchrotron radiation
- Radiation effects (beam loss region)
- Thermal management
- Machine / personnel protection
- Reliability / Serviceability / Reparability
- Economics (building costs vs exploitation costs)

Types of septa

How do we deflect charged particles beams?

Electric, magnetic and “exotic” (e.g. crystals - plane channeling*)

Lorentz force** – the force exerted on a point charge by electromagnetic fields.

$$\vec{F} = \underbrace{q\vec{E}}_{\text{Electric}} + \underbrace{q\vec{v} \times \vec{B}}_{\text{Magnetic}}$$

$\vec{F}$ – Force exerted on the point charge [N]

q – Electric charge [C]

$\vec{E}$ – Electric field [V/m]

$\vec{v}$ – Velocity of the point charge [m/s]

$\vec{B}$ – Magnetic flux density [T]

- Often charged particles in accelerators move with relativistic speeds – relativistic dynamics should be applied.

Hendrick Lorentz
1853 - 1928

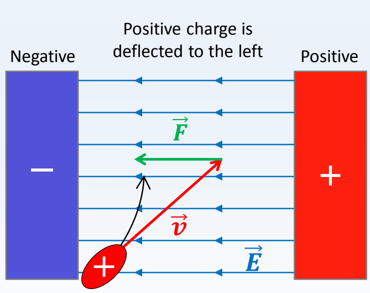
*Plane channeling of protons (> 10 GeV) in Si mono-crystals[21, 22]

**First derivation is often attributed to Oliver Heaviside or James Maxwell

Electric force

Deflecting force is collinear with the electric field – positive charges are deflected in the direction of the electric field lines, negative charges are deflected on the opposite direction.

$$\vec{F} = q\vec{E}$$



Positive charge moving into plane of page

Conventions:

- Force on a *positive* point charge.
- Electric field lines go from ***positive*** electrode to the ***negative*** one.
- *Opposite* electric charges attract each other and *like* electric charges repel.

Electrostatic deflection

Electrostatic bending angle θ_E of particles with single elementary charge [3, 20]

$$\theta_E \approx \frac{E \cdot l_{eff}}{10^9 \cdot \beta \cdot p} = \frac{U \cdot l_{eff}}{10^9 \cdot \beta \cdot p \cdot d}$$

Where:

θ_E – electrostatic bending angle* [rad]

E – deflecting electric field [V/m]

l_{eff} – effective length of the septum [m], usually different from the mechanical length due to fringe fields

β – relativistic coefficient that gives the fraction of the speed of light at which the particles travel [-]

p – particles momentum [GeV/c]

U – deflecting voltage [V]

d – distance between the deflecting electrodes [m]

*Small angle approximation

$\tan(\theta) \approx \theta$

up to ~0.17 rad (~10°) error is <1%

Don't get caught by the units!!!

$$p[\frac{\text{kg.m}}{\text{s}}] = \frac{q_e}{c} p[\frac{\text{eV}}{\text{c}}]$$

(Derivation in the additional material slides at the end of presentation)

Types of septa

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*Plane channeling of protons (> 10 GeV) in Si mono-crystals[21, 22]

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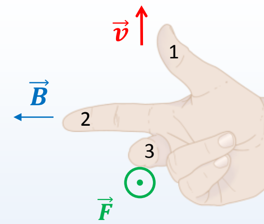
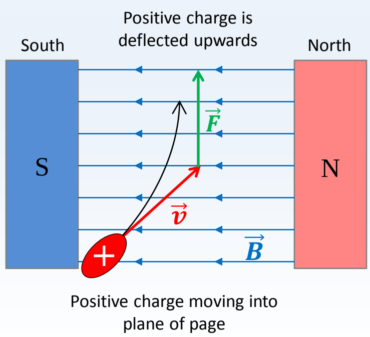
Magnetic force

Deflecting force F is cross product of v and B

$$\overset{3}{\vec{F}} = q\overset{1}{\vec{v}} \times \overset{2}{\vec{B}}$$





Conventions:

- Force on a positive point charge.
- Magnetic field lines go from ***North pole*** to the ***South pole*** of the magnet.
- Right hand rule: If the thumb points in the direction of motion and the index finger is in the direction of the magnetic field, the force goes in the direction of the middle finger.

Magnetic deflection

Magnetic bending angle θ_M of particles with elementary charge [1, 3, 20, 23]

$$\theta_M \approx \frac{0.3 \cdot B \cdot l_{eff}}{p} \approx \frac{3.76 \cdot n \cdot I \cdot l_{eff}}{10^7 \cdot p \cdot d}$$

Where:

θ_M – magnetic bending angle* [rad]

B – deflecting magnetic flux density [T]

l_{eff} – effective length of the septum [m], usually different from the mechanical length due to fringe fields

p – particles momentum [GeV/c]

n – number of turns [-]

I – current [V]

μ_0 – permeability of free space [H/m]

d – distance between magnetic poles [m]

*Small angle approximation

$\tan(\theta) \approx \theta$

up to ~0.17 rad (~10°) error is <1%

$$B \approx \frac{\mu_0 n I}{d}$$

Don't get caught by the units!!!

$$p[\frac{\text{kg. m}}{\text{s}}] = \frac{q_e}{c} p[\frac{\text{eV}}{\text{c}}]$$

(Derivation in the additional material slides at the end of presentation)

Electrostatic vs Magnetic deflection

Which one to use? How to compare?[24] - *Duality of electromagnetism*

Deflecting field	Stored energy per unit volume (free space)	Scalar form of Lorentz force
Electric	$W_E = \frac{\varepsilon_0 E^2}{2}$	$F_E = qE$
Magnetic	$W_M = \frac{B^2}{2\mu_0}$	$F_M = qvB$

For $W_E = W_M$ $(E = cB)$ and **relativistic** particles $(v = c)$

$$F_E = qcB = F_M$$

(Derivation in the additional material slides at the end of presentation)

Electrostatic vs Magnetic deflection

But something is different…

old greek dictum of wisdom:
if you are clever, you use magnetic fields in an accelerator wherever it is possible.

Dr. Bernhard Holzer [1]

- It is more practical to use ***magnetic field!***
- Too high electric field in vacuum could provoke electric breakdown. It is widely accepted that 10 MV/m is a practical limit[24].
- Electric deflection could be beneficial for **non-relativistic** particles (e.g. low energy beams, heavy ions etc.)

$$B[\mathrm{T}] = \frac{3.3}{\beta} E[\mathrm{GV/m}]$$

Comparison between electric and magnetic deflection for beams with different momentum p

β, -	γ, -	$p_{electrons}$, MeV/c	$p_{protons}$, GeV/c	Electric field, MV/m	Equivalent magnetic field, T
0.001	1.000	0.0005	0.0009	10.00	33.356
0.01	1.000	0.0051	0.0094	10.00	3.336
0.1	1.005	0.0514	0.0944	10.00	0.334
0.3	1.048	0.1607	0.2955	10.00	0.111
0.5	1.155	0.2950	0.5425	10.00	0.067
0.9	2.294	1.0552	1.9401	10.00	0.037
0.99	7.089	3.5864	6.5944	10.00	0.034
0.999	22.366	11.4185	20.9955	10.00	0.033
0.9999	70.712	36.1328	66.4386	10.00	0.033
0.99999	223.607	114.2698	210.1114	10.00	0.033
0.999999	707.107	361.3552	664.4349	10.00	0.033

Non-relativistic

Relativistic

1 T → 300 MV/m

Electrostatic vs Magnetic deflection

… and it goes further… There is no “universal” solution!

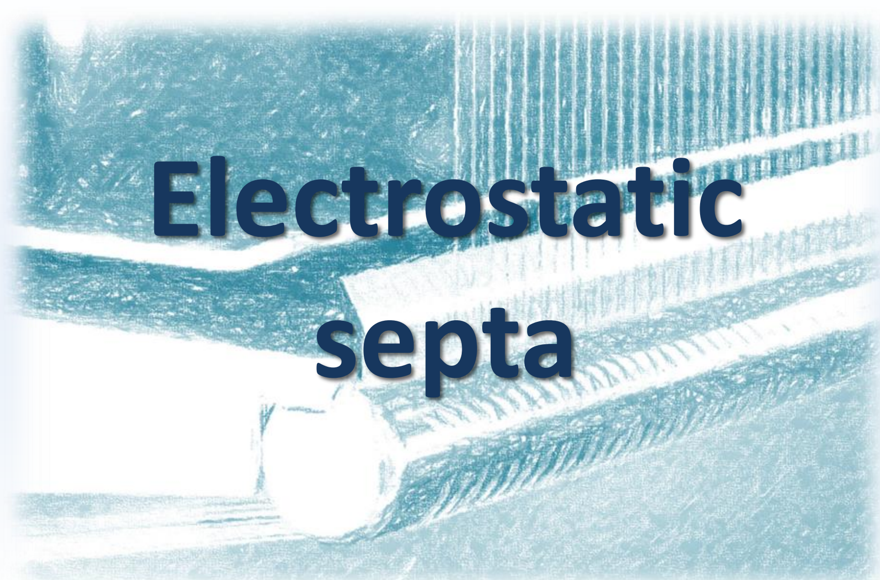
Advantages and disadvantages of the two schemes

Electric septum

- \+ Near perfect no-field region
- \+ Thin septum
- \+ Low mass density (low beam interaction)
- \+ Better for non-relativistic beams
- \- Difficult to have high fields
- \- Less effective for relativistic beams
- \- High voltages handling
- \- Strictly in-vacuum design

Magnetic septum

- \+ Strong deflection
- \+ More effective for relativistic beams
- \+ In-vacuum and in-air design is possible
- \- Thick septum
- \- Field leakage
- \- Non-uniform field region
- \- Interaction with other magnets
- \- High currents handling



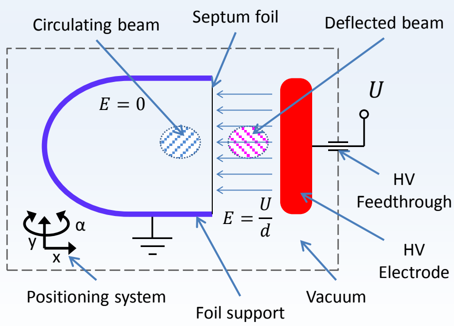
Basic scheme

Electric field is established between a HV electrode and a septum foil.

The extracted beam passes through the electric field region and it is deflected.

Using Faraday cage effect the foil and the foil support create a zero-filed region for the circulating beam that goes straight.

Faraday cage used to protect a person while demonstrating discharges from Tesla transformer

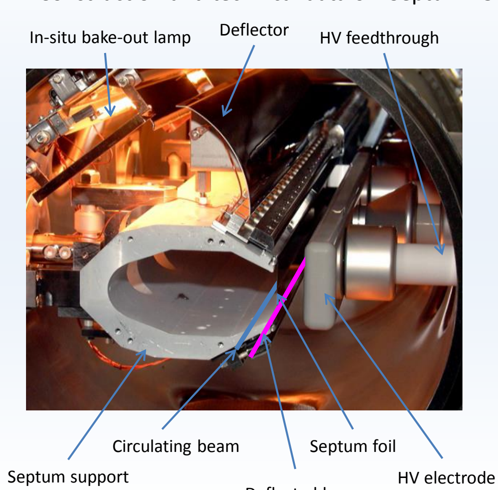


Schematic view of electrostatic septum

- Thin foil is used to minimize the interaction with beam (reduce beam losses and radiation levels)
- To utilize precise alignment with respect to the circulating beam often the septum is mounted on precision mover system.
- Care should be taken to ensure good vacuum conduction in order to maintain low background pressure

Example I – Foil septum

Construction and technical data of “Septum 23” (PS, CERN)[13, 15]

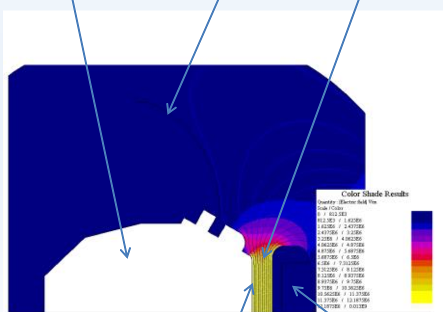


Courtesy of CERN Septa Section

- Electrode length : 778 mm
- Gap width: 17 mm
- Beam momentum 24 GeV/c
- Deflection angle: 0.28 mrad
- Septum thickness: 100 µm
- Vacuum: 10^{-9} mbar
- Voltage: 260 kV
- Electric field: up to 15 MV/m
- Septum foil material: Molybdenum
- Electrode material: Anodised Peraluman 300 aluminum alloy
- In-situ bake-able

Example I – Foil septum

Electric field considerations and HV aspects

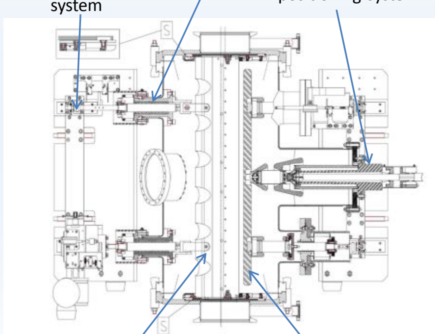


- Good field region 40 mm
- Polished molybdenum foil
- Polished st. steel deflectors to control field distribution
- Anodized HV electrode
- Good vacuum to avoid residual gas ionization triggered HV breakdown
- Every 3 months the insulating oil of the HV feedthroughs is changed (radiation deterioration) and septum is HV conditioned
- Annually the close HV cables are replaced
- Maintenance - radioactive activation

Courtesy of CERN Septa Section

Example I – Foil septum

Positioning and mechanical considerations

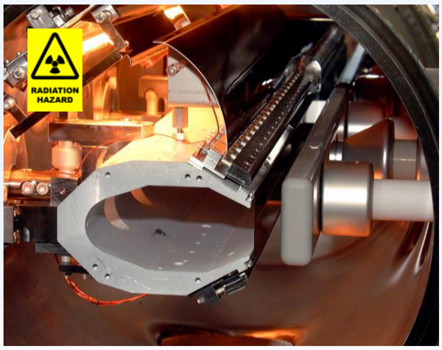


- Septum positioning in radial and angular direction
- HV electrode positioning in radial direction
- Angular resolution: 0.01 mrad
- Translation resolution: 100 µm
- Initial problems with bake-out above 90°C (different thermal expansion coefficients)[15]
- Later fixed by rebuilding the foil tensioner[13]

Courtesy of CERN, M. Hourican, A. Prost

Example I – Foil septum

General consideration

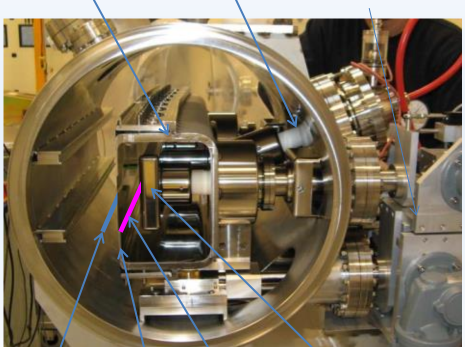


- Septum is activated – difficult to service
- In-situ bake-out system – mechanical stress due to different thermal expansion coefficients
- Thermal loading of septum (slow extraction systems)
- Beam impedance – reduce discontinuities using proper screening
- Machine protection system (vacuum interlock)

Courtesy of CERN Septa Section

Example II – Foil septum with inverted design

Construction and technical data of injection septum for CNAO (PIMMS)[12, 17]

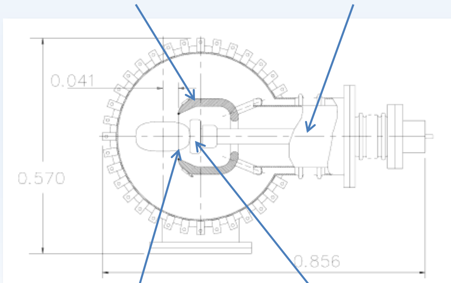


- Electrode length : 800 mm
- Gap width: 25 mm
- Beam momentum 20 MeV/c
- Deflection angle: 60 mrad
- Septum thickness: 100 μm
- Vacuum: 10^{-9} mbar
- Voltage: 69 kV
- Electric field: up to 2.8 MV/m
- Septum: Molybdenum foil
- Electrode material: St. steel

Courtesy of CERN Septa Section

Example II – Foil septum with inverted design

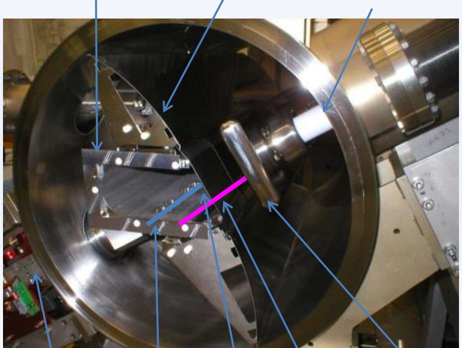
Design specifics[17]



- HV electrode inside the septum support
- Mechanical support and remote positioning systems are form the same side, freeing up space for other equipment on the other side of the beam
- Electric field is fully enclosed – field distribution control, well defined exposed HV surfaces
- Two sections to follow the beam trajectory
- Electrodes positioning range ±5 mm

Example III – Diagonal foil septum

Construction and technical data of ER.SEH10 septum (LEIR, CERN)[11]

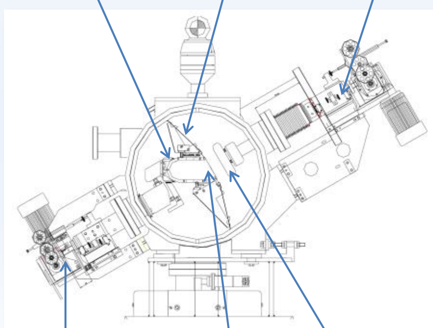


Courtesy of CERN, M. Hourican, A. Prost

- Electrode length : 720 mm
- Gap width: 40 mm
- Beam energy: 4.2 MeV/nucleon
- Deflection angle: 28.9 mrad
- Septum thickness: 100 µm
- Vacuum: 10^{-12} mbar
- Voltage: 51 kV
- Electric field: up to 1.12 MV/m
- Septum: Molybdenum foil
- Electrode material: Titanium
- Deflectors material: St. steel

Example III – Diagonal foil septum

Design specifics

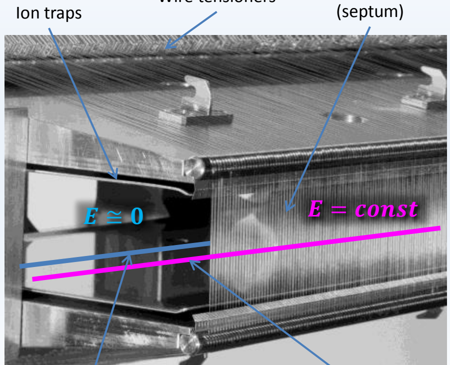


Courtesy of CERN, M. Hourican, A. Prost

- Polished molybdenum foil, HV electrode and deflectors
- Retractable septum and HV electrode
- Diagonal field
- Remote displacement at 30° from horizontal plane
- Good field region 40 mm (±0.5 % field uniformity)
- After bake out at 300°C the alignment of the foil remains within 100 µm, so that the effective septum foil thickness as seen by the beam is less than 200 µm.

Wire septum

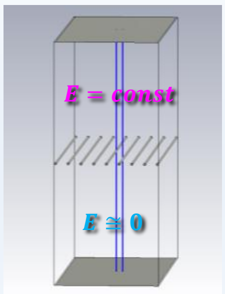
The thin foil septum is substituted with array of thin wires



- An array of wires reduces the effective density of the septum, decreasing beam loss and radiation
- High electric field possible
- Field leakage in no-field region
- Wires array increase the vacuum conductivity to the screened volume.
- Individual tensioner on each wire
- Ionization of residual gas in the field-free region can provoke a HV breakdown. Care should be taken to remove ions from the volume

Static field simulation - modeling

How dense should the wire array be to have sufficient field screening?



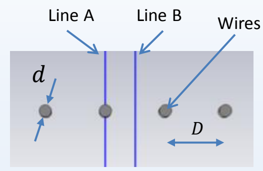
Wire density coefficient k

$$k = \frac{d}{D}$$

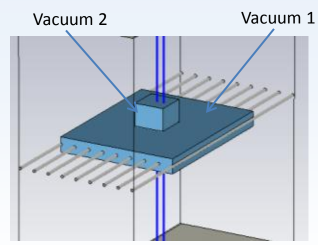
d – wire diameter

D – distance between wires (unit length)

Parametric electrostatic simulation – CST model



Problem definition

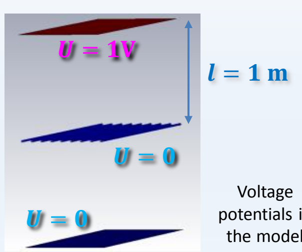


“Vacuum bodies” help to control mesh densities

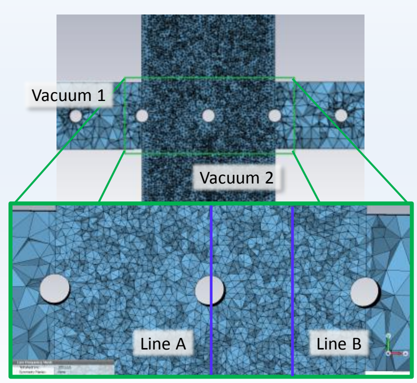
- Choose proper boundary conditions
- Use planes of symmetry
- Define areas of interest
- Use parameters when building the model in order to run parametric simulations easily

Static field simulation – potentials and meshing

Setting the field at nominal 1 V/m will make resulting field strength and field enhancement factor the same



Voltage potentials in the model

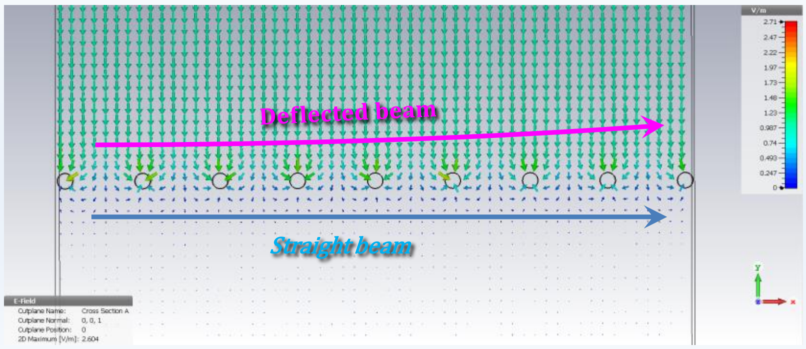


Mesh density is large in areas of interest

- Keep aspect ratios as low as possible (but be aware of proximity effects!)
- Use adequate meshing (adaptive mesh, vacuum bodies with different mesh parameters etc.)

Static field simulation – field results

Field is uniform above the wire array and vanishes below it.

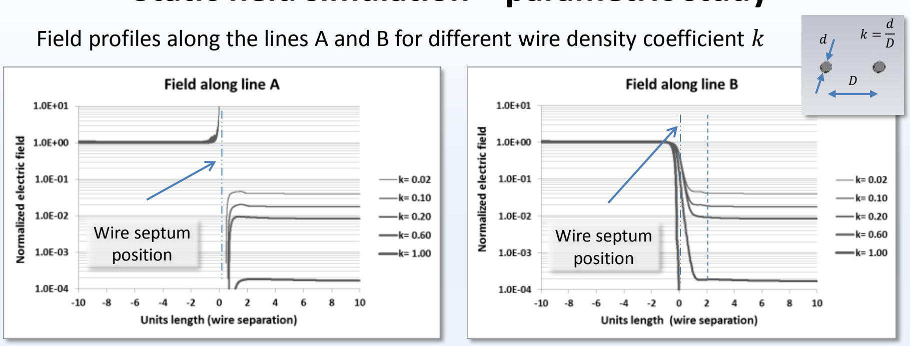


2D simulated electric field

- Explore carefully the field enhancement around the elements
- Use post-processing to have more detailed (tabulated) results

Static field simulation – parametric study

Field profiles along the lines A and B for different wire density coefficient k

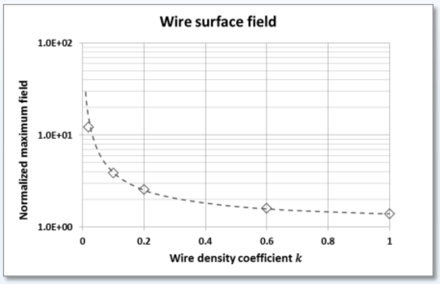


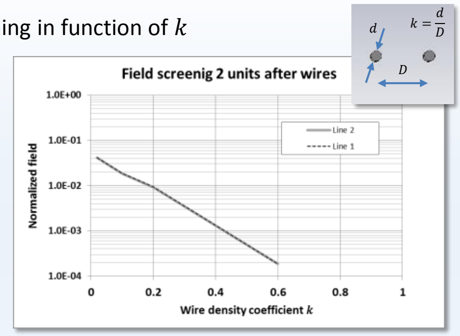
Wire septum is at 0 units

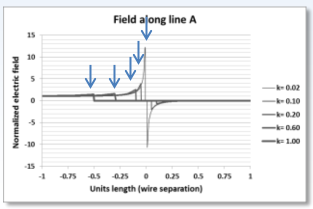
- Away from wire proximity the field profile is identical on both lines
- Even $k = 0.1$ gives significant screening (field reduction of ~53 at 2 units distance)
- At trajectories closer than 1 unit the field is not homogeneous

Static field simulation – parametric study

Field on the wires' surface and field screening in function of k



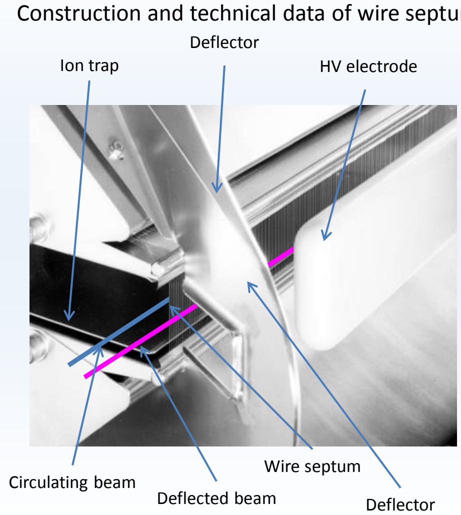




- The field on the wires' surface could be *several times* larger than the main field and it could provoke a HV breakdown
- Maximum wires' surface field follows nicely $\frac{1}{k}$ depedance

Example IV – Wire septum

Construction and technical data of wire septum ZS (5 units, SPS, CERN)[10]

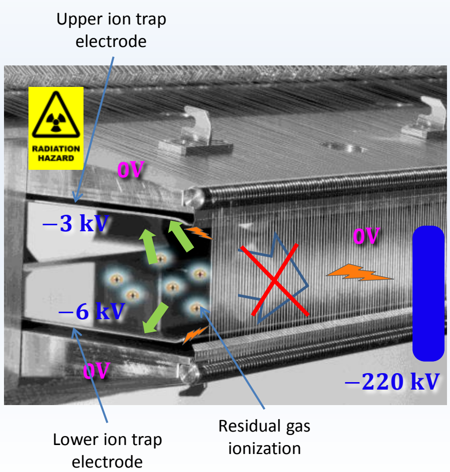


Courtesy of CERN Septa Section

- Electrode length : 2997 mm
- Wire material: W74Re26
- Wire diameter: 50 – 100 µm
- Wire spacing: 1.5 mm
- Gap width: 20 mm
- Beam momentum: 450 GeV/c
- Deflection angle: ~70 µrad
- Vacuum: 10^{-12} mbar
- Voltage: 220 kV
- Ion traps voltage: 3 to 6.5 kV
- Electric field: up to 11 MV/m
- Electrode material: Anodized aluminum

Example IV – Wire septum

Design specifics and general considerations

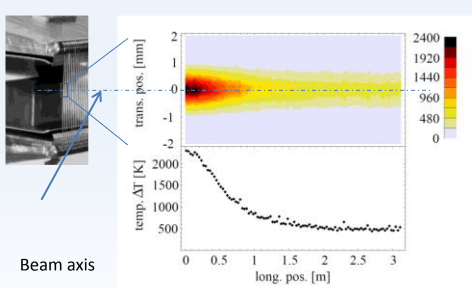


- Septa are activated – difficult to service
- Retractable design
- Powered by -300 kV (Cockcroft-Walton) HV generator
- The device is prone to breakdowns due to background gas ionization – ion traps (electrodes) clear the zero-field volume and partially neutralize the leakage field[10]
- Spark protection– deactivates the electrodes in case of too frequent sparks[5]

Example IV – Wire septum

Thermal loading - often the accelerators are used in different operation modes (different beam momentum, beam intensity and so on) In case of system failure or operator's error high intensity beam could land on the septum and damage it[18]

Short pulses – instantaneous heating – no heat conduction



Simulation of transverse and longitudinal temperature distribution on ZS septum wires[18].

$$\frac{dE}{dV} = \rho \int_{T_0}^{T_0+\Delta} c_p(T)dT$$

Where:

E - deposited energy

V - volume of the material where the energy is deposited

ρ - material density

$c_{p(T)}$ - specific heat of the material

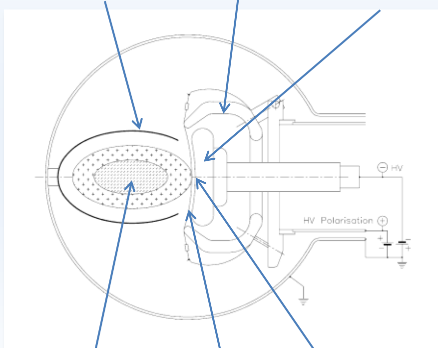
T_0 - initial temperature

ΔT - temperature change

Example V – curved wire septum

Construction and technical data of ES Mini-Wire-Septum collimator[14, 19]*

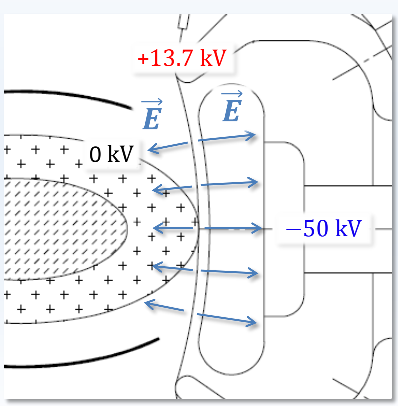
*Proposed design!



- Electrode length : 500 mm
- Wire material: Beryllium (Z = 4)
- Wire diameter: 50 µm
- Wire spacing: 5 mm
- Gap width: 10 mm
- Beam momentum: 0.13 to 1.6 GeV/c
- Deflection angle: 10.2 to 1.14 mrad
- Voltage: 50 kV
- Electric field: 5 MV/m
- Wire polarization: 13.7 kV

Example V – curved wire septum

Design specifics of ES Mini-Wire-Septum collimator



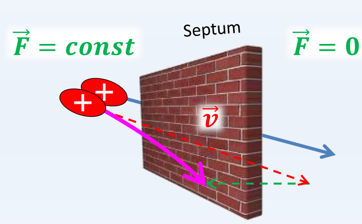
- Ultra thin wire and low atomic number ($N_{Be} = 4$) for low beam interaction
- Wire curved due to electrostatic force of 28 mN/m[19]
- Perfect alignment of the wires is difficult but even might not be necessary, reduces further beam interaction
- Wire septum is polarized (13.7 kV) – compensates the leakage field (no ion traps necessary, field on both sides of the septum)

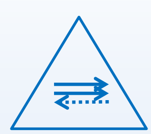
Practical considerations

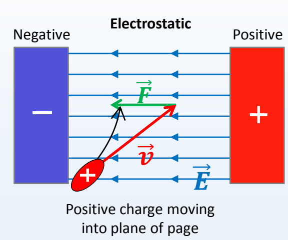
- ❑ Very thin septa – mechanical, thermal stress
- ❑ Low atomic number material – reduce beam interaction
- ❑ High electric field – surface finish and conditioning, spark protection
- ❑ Good field region – geometry of gap, deflectors
- ❑ Field leakage compensation (for wire septa only)
- ❑ Beam impedance – proper screening
- ❑ HV handling – creepage distances (feedthroughs, supports)
- ❑ Insulators degradation (insulation oil, cables)
- ❑ Alignment – remote positioning systems
- ❑ Good vacuum – bake-out capabilities, vacuum conductivity
- ❑ Background gas ionization – ion traps
- ❑ Machine protection (vacuum, system failure, operator mistakes)
- ❑ Activation – maintenance limitations

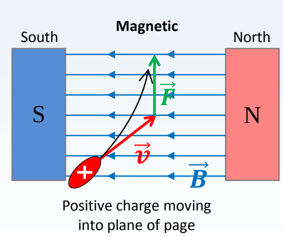
What to remember

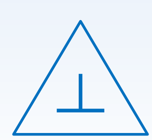
*Septum is a **wall**!*











old greek dictum of wisdom:
if you are clever, you use magnetic fields in an accelerator wherever it is possible.
Dr. Bernhard Holzer

When possible choose…
*… **magnetic**!*

Acknowledgments

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Thank you for your attention

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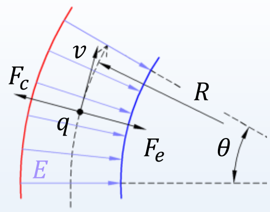
Additional material – Electrostatic deflection derivation



Derivation of electrostatic deflection of moving charged particle with velocity v, rest mass m_0 and charge q_0, in electric field E perpendicular to particle's direction of travel. The electric force F_e balances the centrifugal force F_c and the particle travels on an arc trajectory with radius R.

Bending angle θ is found using bending radius R and the length of the field l in the limits of small angle approximation $\tan(\theta) \approx \theta$

$$F_e = q_e E, \quad F_c = \frac{\gamma m_0 v^2}{R}, \quad p = \gamma m_0 \beta c, \quad v = \beta c$$

and

$$p[\frac{\mathrm{kg.m}}{\mathrm{s}}] = \frac{10^9 q_e}{c} p[\frac{\mathrm{GeV}}{\mathrm{c}}]$$

$\therefore$

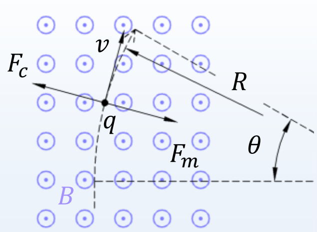
$$R = \frac{\gamma m_0 v^2}{q_e E} = \frac{p[\frac{\mathrm{kg.m}}{\mathrm{s}}]\beta c}{q_e E} = \frac{10^9 p[\frac{\mathrm{GeV}}{\mathrm{c}}]\beta}{E}$$

$\therefore$

$$\theta \approx \frac{l}{R} = \frac{El}{10^9 . \beta . p[\frac{\mathrm{GeV}}{\mathrm{c}}]}$$

Where:
- F_c - centrifugal force [N]
- F_e - electrostatic force [N]
- E - electric field [V/m]
- R - bending radius [m]
- q_e - particle charge (elementary charge) [C]
- m_0 - particle's rest mass [kg]
- p - beam momentum [kg.m/s] or [GeV/c]
- γ - relativistic gamma [-]
- β - relativistic beta [-]
- c - speed of light [m/s]
- l - length of the field [m]

Additional material – Magnetic deflection derivation



Derivation of magnetic deflection of moving charged particle with velocity v, rest mass m_0 and charge q_0, in magnetic field B going out of plane of paper. The magnetic force F_m balances the centrifugal force F_c and the particle travels on an arc trajectory with radius R.

Bending angle θ is found using bending radius R and the length of the field l in the limits of small angle approximation $\tan(\theta) \approx \theta$

$$F_m = q_e v B, \quad F_c = \frac{\gamma m_0 v^2}{R}, \quad p = \gamma m_0 \beta c, \quad v = \beta c$$

and

$$p[\frac{\mathrm{kg.m}}{\mathrm{s}}] = \frac{10^9 q_e}{c} p[\frac{\mathrm{GeV}}{\mathrm{c}}]$$

$\therefore$

$$R = \frac{\gamma m_0 v}{q_e B} = \frac{p[\frac{\mathrm{kg.m}}{\mathrm{s}}]c}{q_e B} = 3.3\frac{p[\frac{\mathrm{GeV}}{\mathrm{c}}]}{B}$$

$\therefore$

$$\theta \approx \frac{l}{R} = \frac{0.3Bl}{p[\frac{\mathrm{GeV}}{\mathrm{c}}]}$$

Where:
- F_c - centrifugal force [N]
- F_m - magnetic force [N]
- B - magnetic flux density [T]
- R - bending radius [m]
- q_e - particle charge (elementary charge) [C]
- m_0 - particle's rest mass [kg]
- p - beam momentum [kg.m/s] or [GeV/c]
- γ - relativistic gamma [-]
- β - relativistic beta [-]
- c - speed of light [m/s]
- l - length of the field [m]

Additional material – Equivalence of electrostatic and magnetic deflection

Comparing capabilities of electric and magnetic field with same volumetric energy density (in vacuum) to deflect relativistic charged particles

Where:
U_e - electric field energy density [J/m^3]
U_m - magnetic field energy density [J/m^3]
ε_0 - vacuum permittivity [F/m]
μ_0 - vacuum permeability [H/m]
q_e - particle charge (elementary charge) [C]
E - electric field [V/m]
B - magnetic flux density [T]
c - speed of light [m/s]
F_e - electric force [N]
F_m - magnetic force [N]

Electric field energy density $U_e = \frac{\varepsilon_0 E^2}{2}$

Magnetic field energy density $U_m = \frac{B^2}{2\mu_0}$

For $U_e = U_m$ and $c^2 = \frac{1}{\varepsilon_0 \mu_0}$ $\therefore$

$\frac{\varepsilon_0 E^2}{2} = \frac{B^2}{2\mu_0}$ or $E = cB$

Magnetic (Lorentz) deflection force $F_m = q_e v B$

Relativistic regime $v \cong c \therefore$

$F_m = q_e v B \cong q_e c B = q_e E$

Electrostatic (Lorentz) deflection force $F_e = q_e E$

or

$F_e \cong F_m$

Electric and magnetic field, <u>with same energy density</u>, have same effectiveness in deflecting <u>relativistic</u> charged particles.