Low Energy Beam Transport Design

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Examples

Particle	Magnetic rigidity $B\rho$	Electric rigidity $E\rho$	Photos
300 keV Electron	0.002 Tesla-metre	0.49 MV	ELinac LEBT
300 keV H$^-$	0.079 Tesla-metre	0.60 MV	Cyc. Inj. Quads, Periodic
50 MeV Electron	0.17 Tesla-metre	51 MV	ELinac HEBT
60 keV Heavy Ion	0.19 Tesla-metre	0.12 MV	Quads, Spherical Dipole
500 MeV Proton	3.6 Tesla-metre	826 MV	BL1A

Rigidities: Remember force law: $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$. So

$$\text{Electric: } qE = \frac{mv^2}{\rho} \Rightarrow E\rho = \frac{2T}{q} = 2V_\mathrm{B}\text{; Relativistic: } \left(1 + \frac{1}{\gamma}\right) V_\mathrm{B}$$

$$\text{Magnetic: } qvB = \frac{mv^2}{\rho} \Rightarrow B\rho = \frac{p}{q} = \frac{\sqrt{2V_\mathrm{M}V_\mathrm{B}}}{c}\text{; Relativistic: } \frac{\sqrt{(\gamma+1)V_\mathrm{M}V_\mathrm{B}}}{c}$$

Notation: $T \equiv \frac{1}{2}mv^2$ or relativistic $(\gamma - 1)mc^2$, $V_\mathrm{B} \equiv \frac{T}{q}$, $V_\mathrm{M} \equiv \frac{mc^2}{q}$. $V_\mathrm{B} = (\gamma - 1)V_\mathrm{M}$.

Electrostatic +/-:

- ($\pm$) Low energy, direct from source, optics scale with extraction voltage, don't depend on mass.
- ($+$) Does not affect polarization.
- ($+$) No hysteresis.
- ($+$) Cheap. Dipoles and quadrupoles are simple aluminum shapes; no coils.
- ($-$) Relatively larger vacuum chamber.
- ($-$) Open circuit: No way to tell if electrode has correct voltage.
- ($-$) Energy is limited because electric field is (Breakdown).

Magnetic +/-:

- $(\pm)$ Scales with momentum, not energy, so mass-dependent.
- $(-)$ Affects polarization. (E.g. Forces 245MeV pEDM exp. to be electrostatic)
- $(-)$ Hysteresis: worst at lowest energy. Somewhat mitigated by going shorter.
- $(+)$ Continuity: Easy to tell whether elements powered.
- $(+)$ No principle limit on magnetic field.

Focal Lengths

For a magnetic quad,

$$\frac{1}{f} = \frac{B_T L}{aB\rho} \propto \frac{1}{\beta\gamma}$$

For an electrostatic quad,

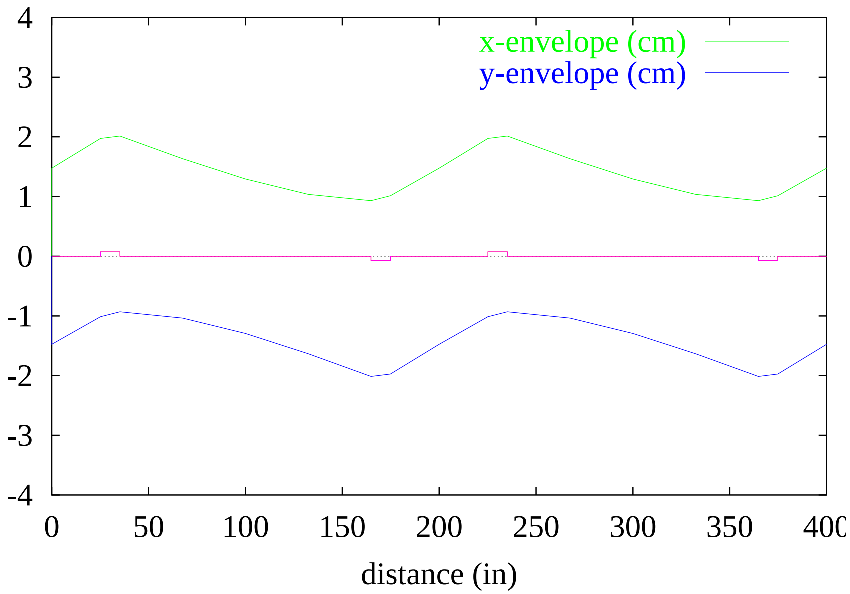
$$\frac{1}{f} = \frac{V_E L}{a^2 V_B}\left(\frac{2\gamma}{\gamma+1}\right) \propto \frac{1}{\beta^2\gamma}$$

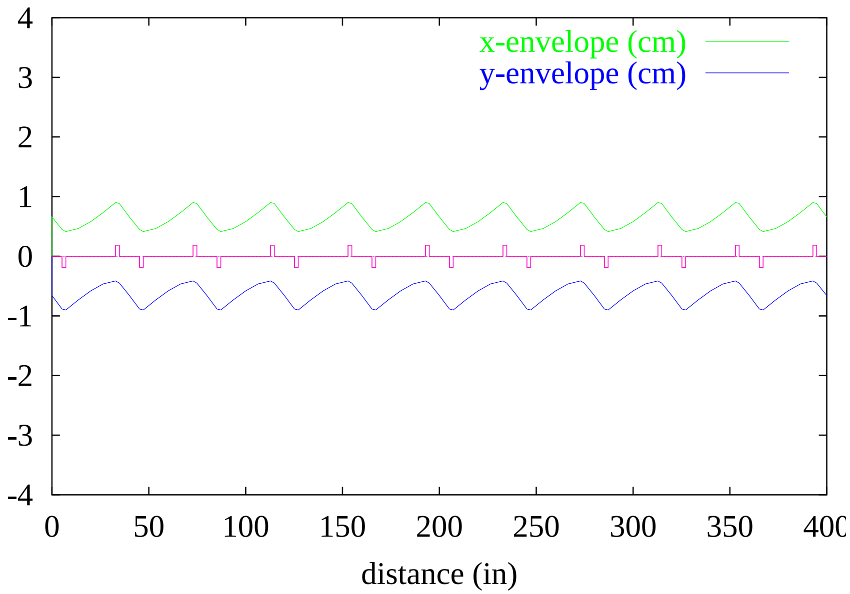
where L is the effective length.

E.g. for a 60 keV beam, electrodes at ± 1 kV with $L/a = 2$, $a = 25$ mm, $f = 60 \times 25\,\mathrm{mm}/2 = 0.75$ m.

Size Scale

Large scale or small scale...





Size Scale – Economics

One would like to transport a beam of given emittance. Say the aperture radius is a, and the distance between doublets (the cell length) is L_c. Then maximum angles are a/L_c and acceptance is on the order of $\hat{\epsilon} \sim a^2/L_\mathrm{c}$. (More precisely, $\hat{\epsilon} = a^2/\beta_\mathrm{T}$, where β_T is the Twiss β-function, but $\beta_\mathrm{T} \sim L_\mathrm{c}$).

For example, say we want $\hat{\epsilon} = 200$ mm-mrad. Then the following combinations will work:

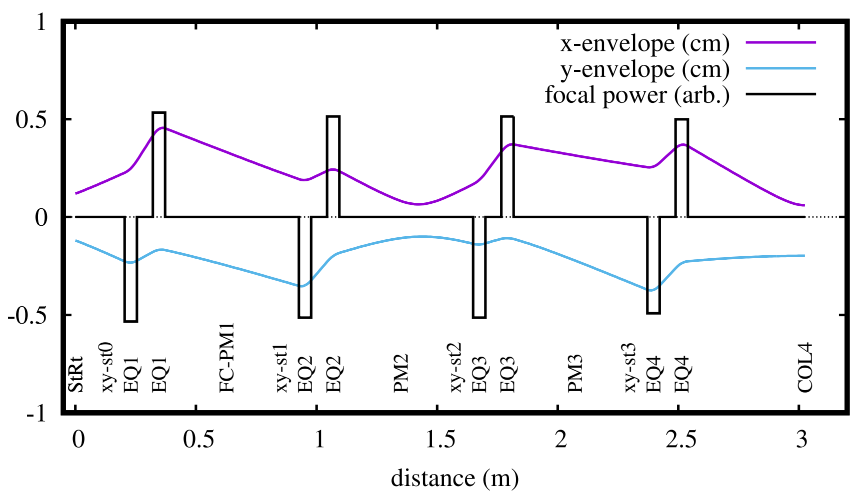
Length Scale	aperture	Looks like ...
5 m	50 mm	CERN-ISOLDE
1 m	22 mm	TRIUMF-ISAC
2 cm	3 mm	an RFQ

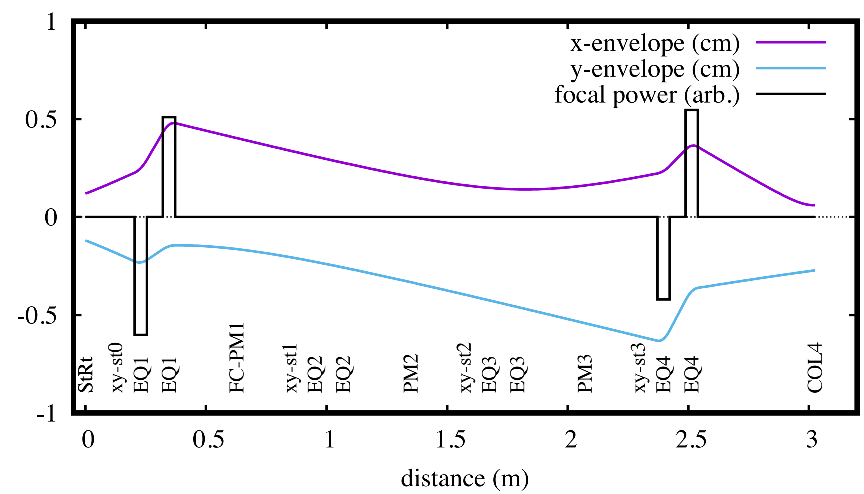
Also, cost is roughly $\propto a^2$ (cross-sectional area of vacuum chamber), and $\propto 1/L_\mathrm{c}$ (i.e. the number of optical elements assuming a fixed distance from source to destination).

Therefore, cost $\propto a^2/L_\mathrm{c} \sim \hat{\epsilon}$, so whether L_c is 5 or 1 metre, the cost is roughly the same. In other words, we have plenty of freedom to choose a scale for the optics.

Optics focal strength

Why high energy sparse, low energy dense? Why can't have just 4 quads? Only need to match 4 parameters: $\alpha_x, \beta_x, \alpha_y, \beta_y$. (OK, maybe need longitudinal as well: 1 buncher, 1 debuncher.)

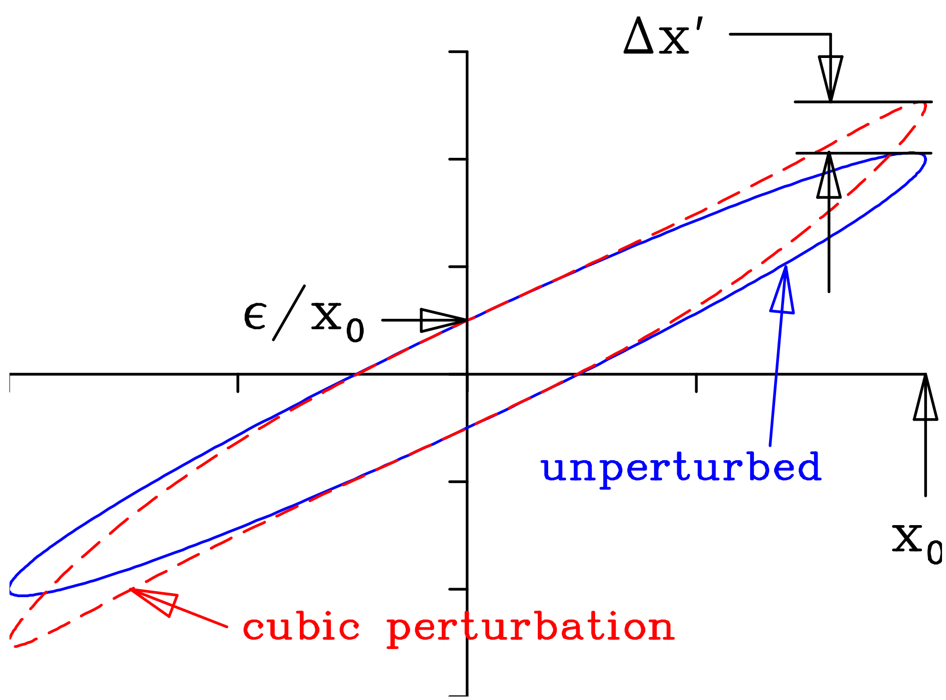




OK. Let's see how this works out. To "throw" the beam a large distance D requires divergence θ to be proportionally small. For fixed emittance, beam size x_0 is then large.

$$\theta \sim x_0/D,\ x_0\theta = \epsilon,\ x_0 \sim \sqrt{\epsilon D}$$

So in phase space, beam has unfortunate aspect ratio $x_0/\theta \sim D$.



Important: It is not the "divergence" $\sqrt{\gamma\epsilon}$ that matters, but $\epsilon/x_0 = \sqrt{\epsilon/\beta}$. Emittance will grow by

$$\frac{\Delta\epsilon}{\epsilon} \sim \frac{\Delta x'}{\epsilon/x_0} \propto x_0^n$$

Skinny ellipse means sensitivity to:

- Misalignments ($\Delta x/D \div \epsilon/x_0 \propto x_0$)
- Stray fields ($D/\rho \div \epsilon/x_0 \propto x_0$)
- Space charge (strong focus is better).
- Ripple ($x_0/f \div \epsilon/x_0 \propto x_0^2$)
- Nonlinearities (aberrations; $x_0^3/(f^2L) \div \epsilon/x_0 \propto x_0^4$)

So: Large beam bad, small beam good. I cover **RED** cases.

Stray Fields

Why do we care? Can always re-steer... but do we really want all those steering "knobs"?

The equation of motion for smooth focus approx. is

$$x'' + x/\beta_{\mathrm{T}}^2 = 1/\rho \tag{1}$$

where $\rho = (B\rho)/B_{\mathrm{stray}}$ is the radius of curvature of the beam in the stray field. So on average, the displacement Δx due to the stray field is $\beta_{\mathrm{T}}^2/\rho \sim L_{\mathrm{c}}^2/\rho$. We would like Δx to be small compared with the beam size $\sqrt{\epsilon\beta_{\mathrm{T}}} \sim \sqrt{\epsilon L_{\mathrm{c}}}$. In other words,

$$L_{\mathrm{c}} \ll (\rho^2\epsilon)^{1/3} \tag{2}$$

This clearly favours small optics.

Example:

In ISAC, the smallest $B\rho$ is 400 gauss-metres. This is for the lightest mass ($A = 6$) at the energy 2 keV/u required for the RFQ. In the earth's magnetic field, we find ρ=1 km. For acceptance ϵ=100 mm-mrad, we would like $L_\mathrm{c} \ll 5$ m. In fact, we have $L_\mathrm{c} = 1$ m. This makes us fairly tolerant, but CERN-ISOLDE at $L_\mathrm{c} \sim 3$ m is much more susceptible.

Space Charge

To estimate the effect, we can compare the space charge and emittance terms in the Kapchinsky-Vladimirsky envelope equation.

$$
\begin{aligned}
a'' + k^2(z)a + \epsilon^2/a^3 + 2A/(a+b) &= 0 \qquad (3)\\
b'' + k^2(z)b + \epsilon^2/b^3 + 2A/(a+b) &= 0, \qquad (4)
\end{aligned}
$$

where a and b are x- and y-envelopes, A is the space charge parameter.

$$
A = \frac{\text{space charge potential well depth}}{\text{beam voltage}} = \frac{I \times 30\,\Omega/\beta}{V_{\mathrm{B}}} \qquad (5)
$$

(Potential well depth does not depend on beam size!)

This gives the following condition for the negligibility of space charge.

$$A \ll \epsilon^2/a^2 \sim \epsilon/L_{\rm c} \qquad (6)$$

This means that in spite of smaller beam sizes and stronger space charge forces, stronger focusing allows higher current.

Example:

In ISAC, $L_{\rm c} \sim 1$ m, and a good beam has emittance 10 mm-mrad. This leads to $A \ll 10^{-5}$. As $V_{\rm B} \sim 30$ kV, and $\beta = 0.002$, we find that to avoid space charge effects, we need $I \ll 20\,\mu$A.

Quad Aberration Misconceptions

All of following statements are **FALSE**.

1. Electrostatic quads have worse aberrations than magnetic.
   No, they're about the same.
2. Fringe fields should be minimized.
   No, they're actually better if longer. Hard-edge quads have infinite fifth and higher order aberrations.
3. Large aperture to length ratio is bad.
   No, for same length, the large aperture quad is much better.
4. Aberrations are complicated/difficult to understand.
   No, you'll see...

Quad Aberrations

The quadrupole potential field

$$V(x,y) = \frac{k}{2}(x^2 - y^2) \tag{7}$$

is a solution to Laplace's equation, but only if the quadrupole is infinitely long (k=constant). For finite quads, we use the expansion

$$V(x,y,z) = \frac{k}{2}(x^2 - y^2) - \frac{k''}{24}(x^4 - y^4) + \frac{k''''}{720}(x^6 - y^6) - ... \tag{8}$$

The quartic term gives a cubic force term which leads to the following focusing error,

$$\Delta x' = \frac{-1}{f^2 L_Q}\left(\frac{7}{6}x^3 - \frac{1}{2}xy^2\right), \tag{9}$$

where L_Q is the quad length and f the focal length. It is important to note that this is **independent of aperture size** or fringe field hardness: indeed, the aberration is not affected by changing the fringe field shape. See misconceptions p.16.

For the Hamiltonian technique used to derive these, see R. Baartman, PAC97, Physics ArXiv (2014). In latter reference, I refine electrostatic for relativistic case:

$$\Delta x' = \frac{-1}{f^2 L_\mathrm{Q}}\left(\frac{7-3\beta^2}{6}x^3 - \frac{1-\beta^2}{2}xy^2\right).$$

Similar formula for magnetic quads:

$$\Delta x' = \frac{-1}{f^2 L_{\mathrm{Q}}}\left(\frac{1}{3}x^3 + xy^2\right), \tag{10}$$

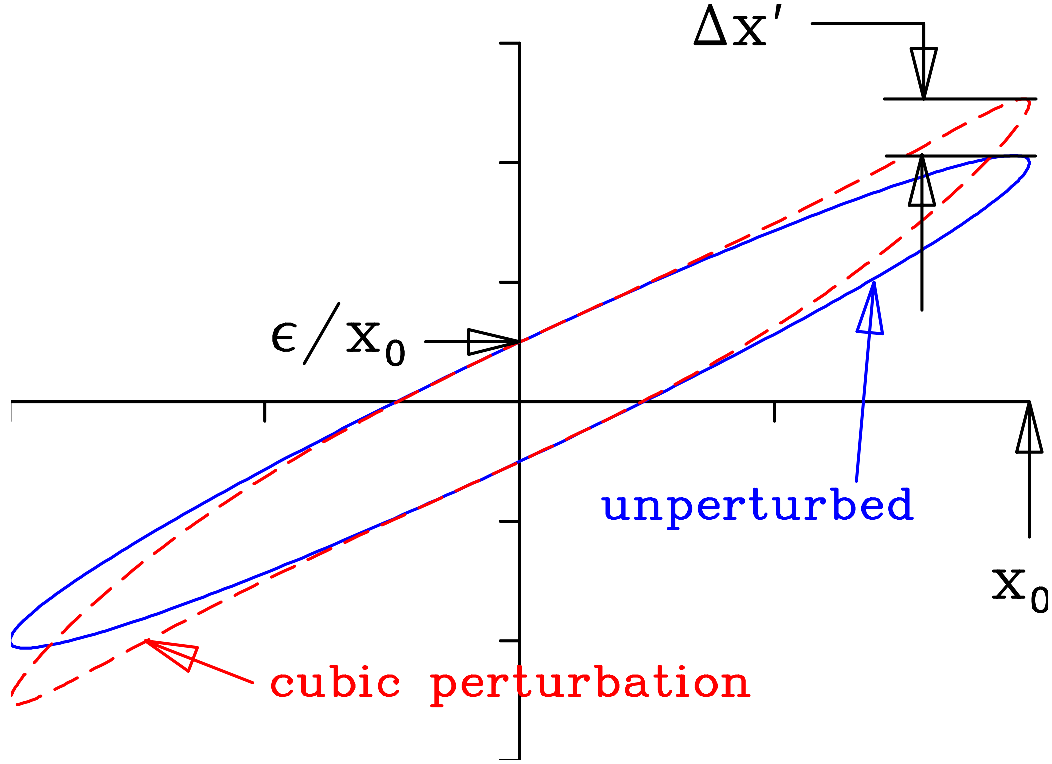
So electric not really worse than magnetic quads; might actually be better for e.g. ribbon beams.

Yes, these simple formulas agree with COSY-∞

See lines `300000, 102000, 201000, 003000.`

0.5784930	-5.297711	0.000000	0.000000	0.000000	100000
-0.1326415	2.943331	0.000000	0.000000	0.000000	010000
0.000000	0.000000	0.8984788E-01	-7.731608	0.000000	001000
0.000000	0.000000	0.1635628	-2.945021	0.000000	000100
-6948.877	-133240.7	0.000000	0.000000	0.000000	300000
7215.431	131640.4	0.000000	0.000000	0.000000	210000
-2510.038	-43908.78	0.000000	0.000000	0.000000	120000
297.0303	4892.746	0.000000	0.000000	0.000000	030000
0.000000	0.000000	-770.3541	-5935.619	0.000000	201000
0.000000	0.000000	756.8731	4562.864	0.000000	111000
0.000000	0.000000	-157.4319	-838.8645	0.000000	021000
0.000000	0.000000	-386.5590	-2791.754	0.000000	200100
0.000000	0.000000	359.9824	2137.825	0.000000	110100
0.000000	0.000000	-73.24476	-384.5105	0.000000	020100
-950.4457	-2513.761	0.000000	0.000000	0.000000	102000
332.4558	2367.618	0.000000	0.000000	0.000000	012000
-861.8040	-3307.773	0.000000	0.000000	0.000000	101100
300.5206	2391.129	0.000000	0.000000	0.000000	011100
-196.2168	-960.3432	0.000000	0.000000	0.000000	100200
67.66693	598.8534	0.000000	0.000000	0.000000	010200
0.000000	0.000000	-98320.05	-441758.0	0.000000	003000
0.000000	0.000000	-133404.3	-601011.7	0.000000	002100
0.000000	0.000000	-60362.23	-272510.6	0.000000	001200
0.000000	0.000000	-9107.138	-41188.80	0.000000	000300

Nonlinearity – cont'd



It is clear that for the higher order errors to have a negligible effect on the beam quality, the distortion in the emittance ellipse, $\Delta x'$, should be small compared with the local divergence ϵ/x.

Nonlinearity and Matching

Roughly speaking,

$$\frac{\Delta\epsilon}{\epsilon} = \frac{\Delta x'}{\epsilon/x} \sim \frac{x^4}{\epsilon f^2 L_{\mathrm{Q}}} \tag{11}$$

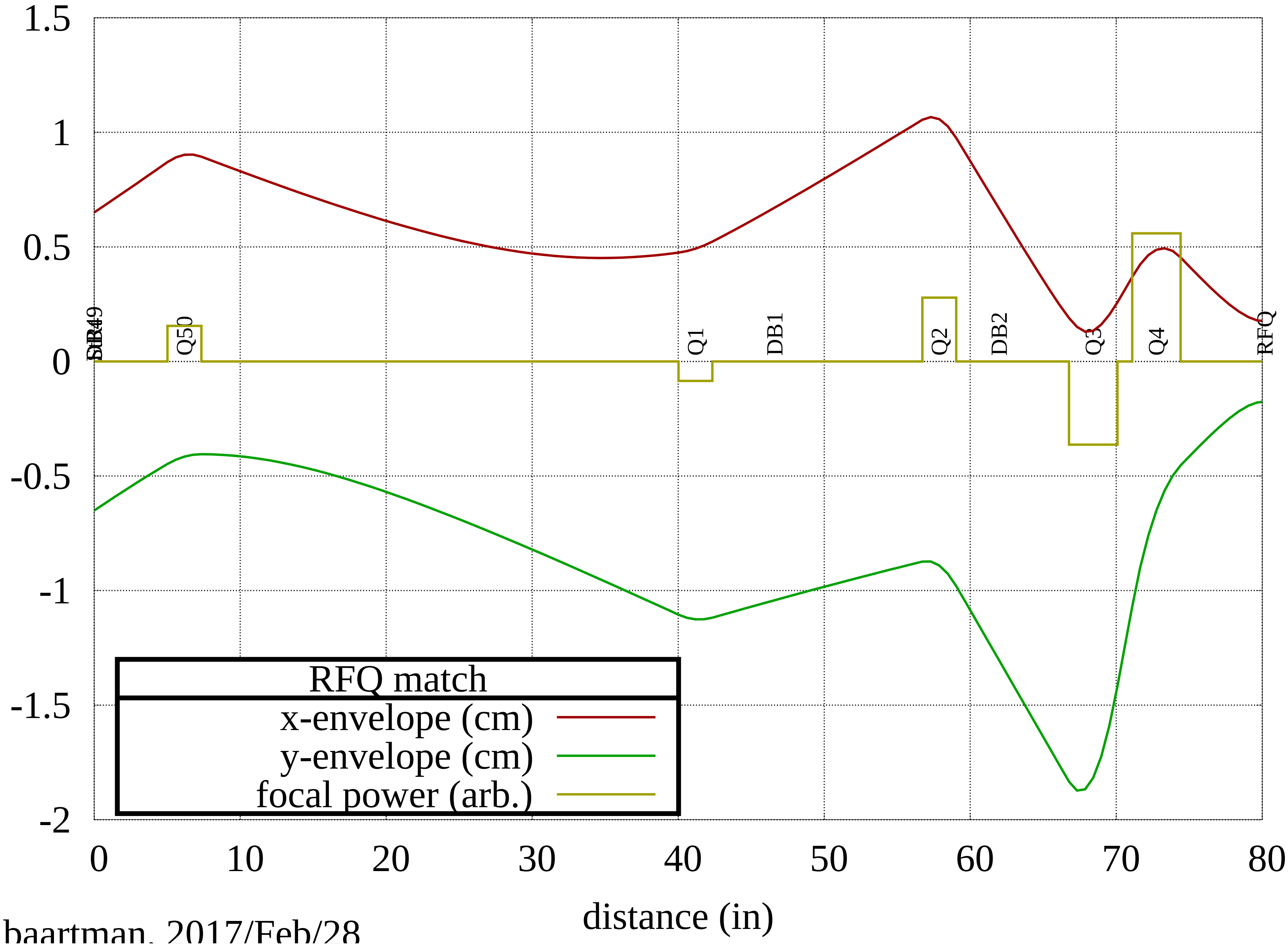
Example:

We apply this to the example below: $x \sim 2\,$cm, $f \sim 10\,$cm, $L_{\mathrm{Q}} \sim 10\,$cm, $\epsilon = 0.005\,$cm (commonly called "50π mm-mrad"). We get

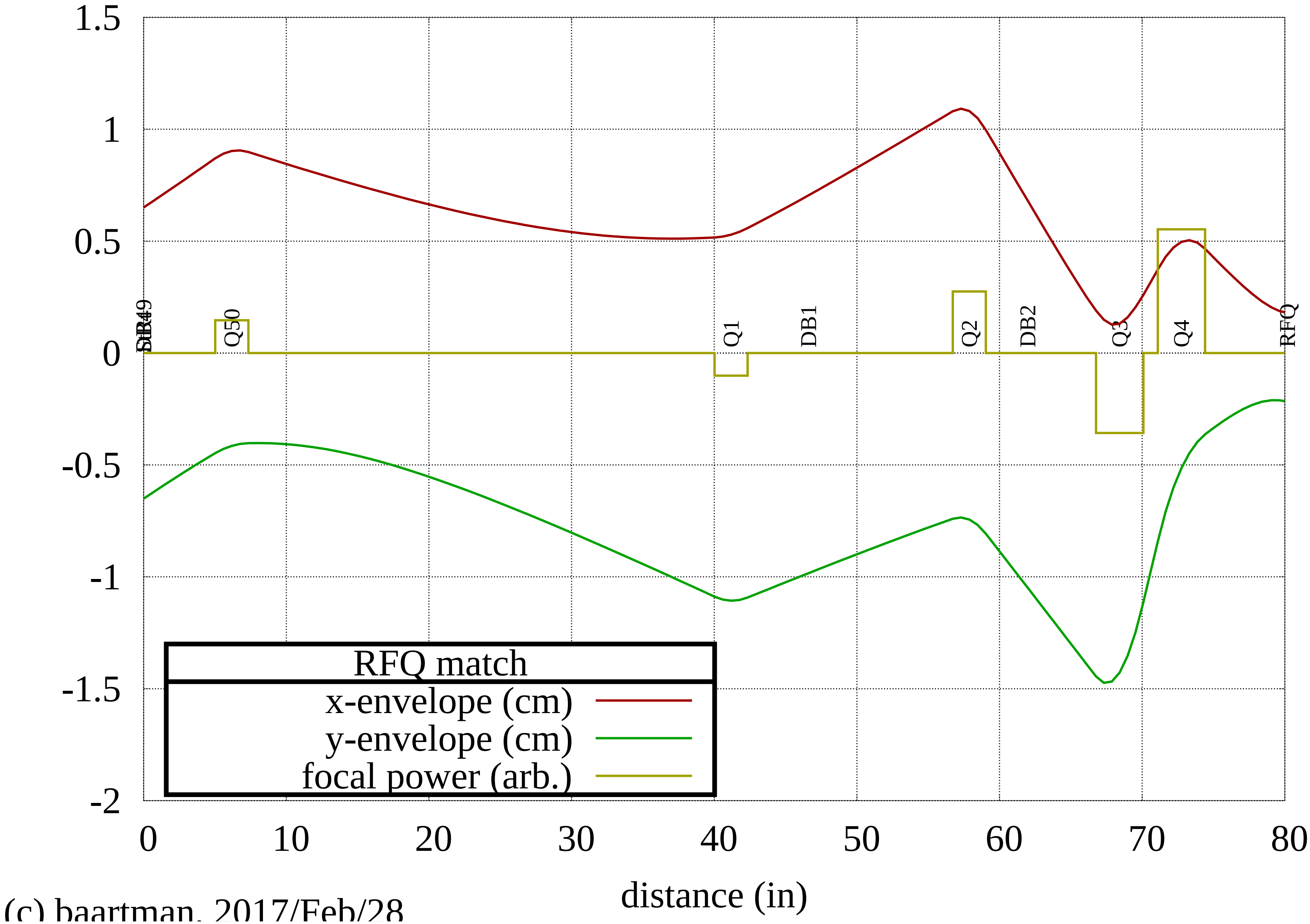
$$\frac{\Delta\epsilon}{\epsilon} \sim \frac{2^4}{0.005 \times 10^2 \times 10} = 3.2$$

In the case of the optimized example, beam size is one half, so result is $1/16$ or ~ 0.2. (COSY gives 0.1.)

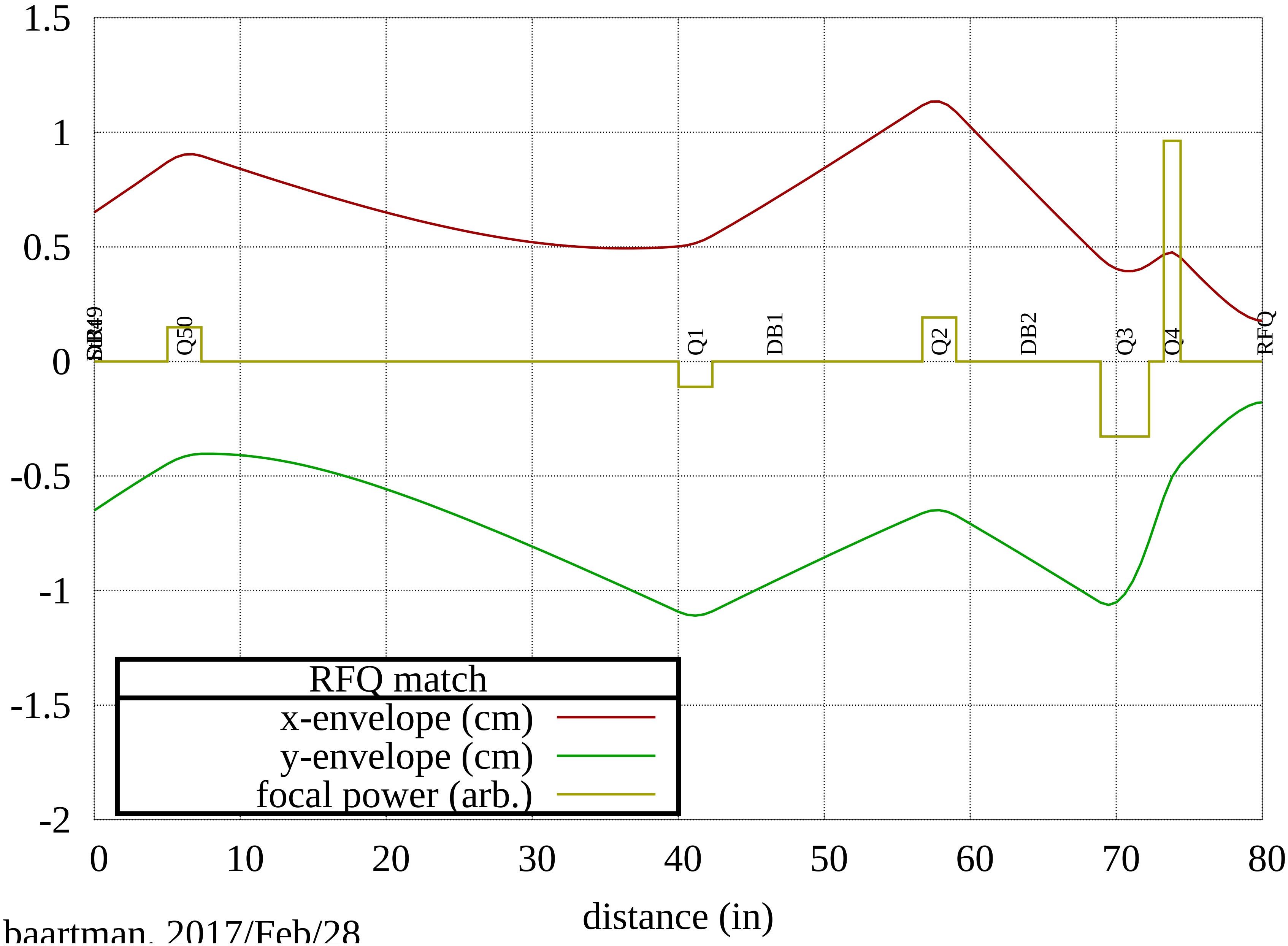
Long quads, exact match, $\epsilon_{\rm f}/\epsilon_{\rm i} = 4$



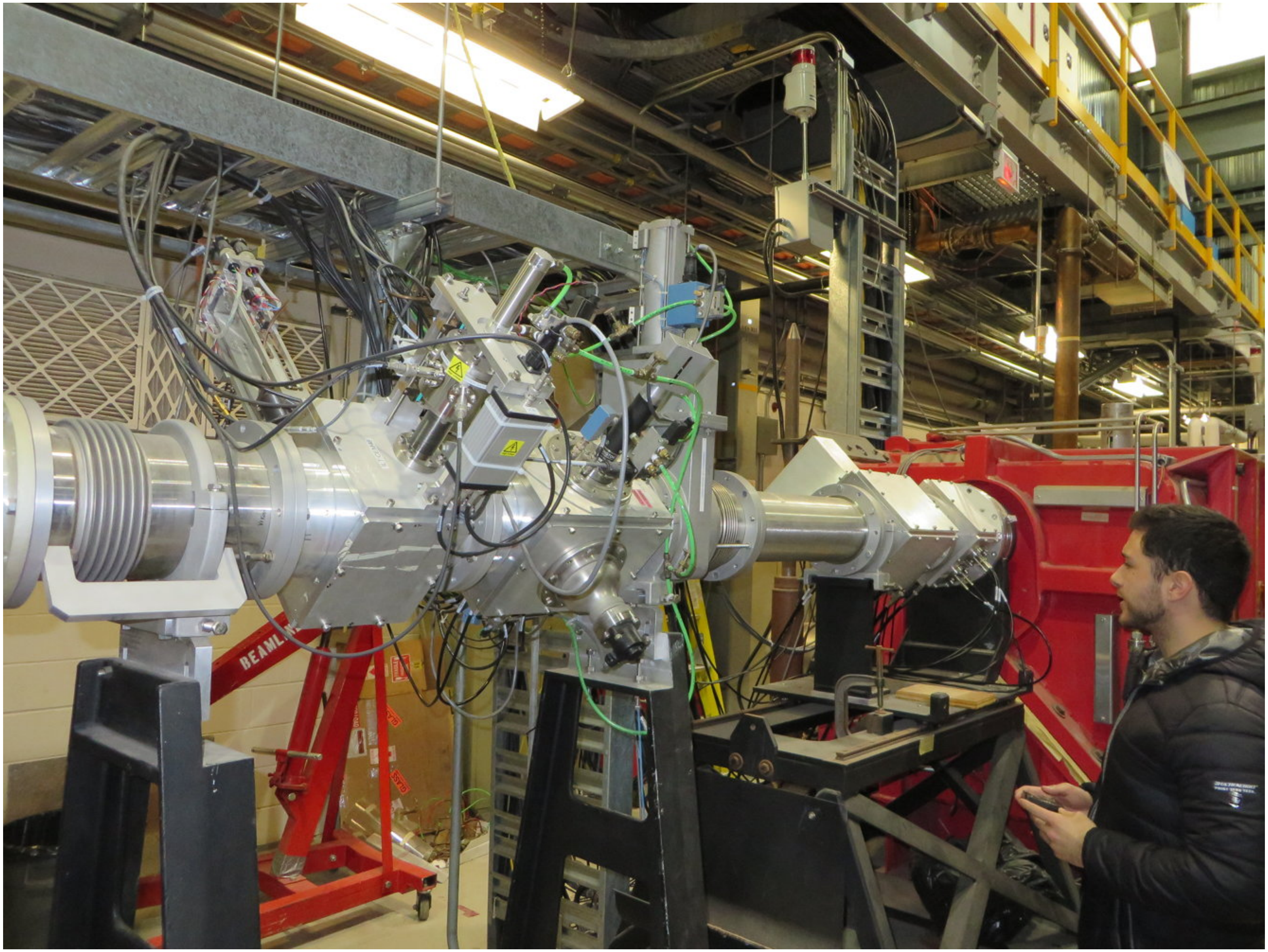
Long quads, compromised match, $\epsilon_{\mathrm{f}}/\epsilon_{\mathrm{i}} = 2$:



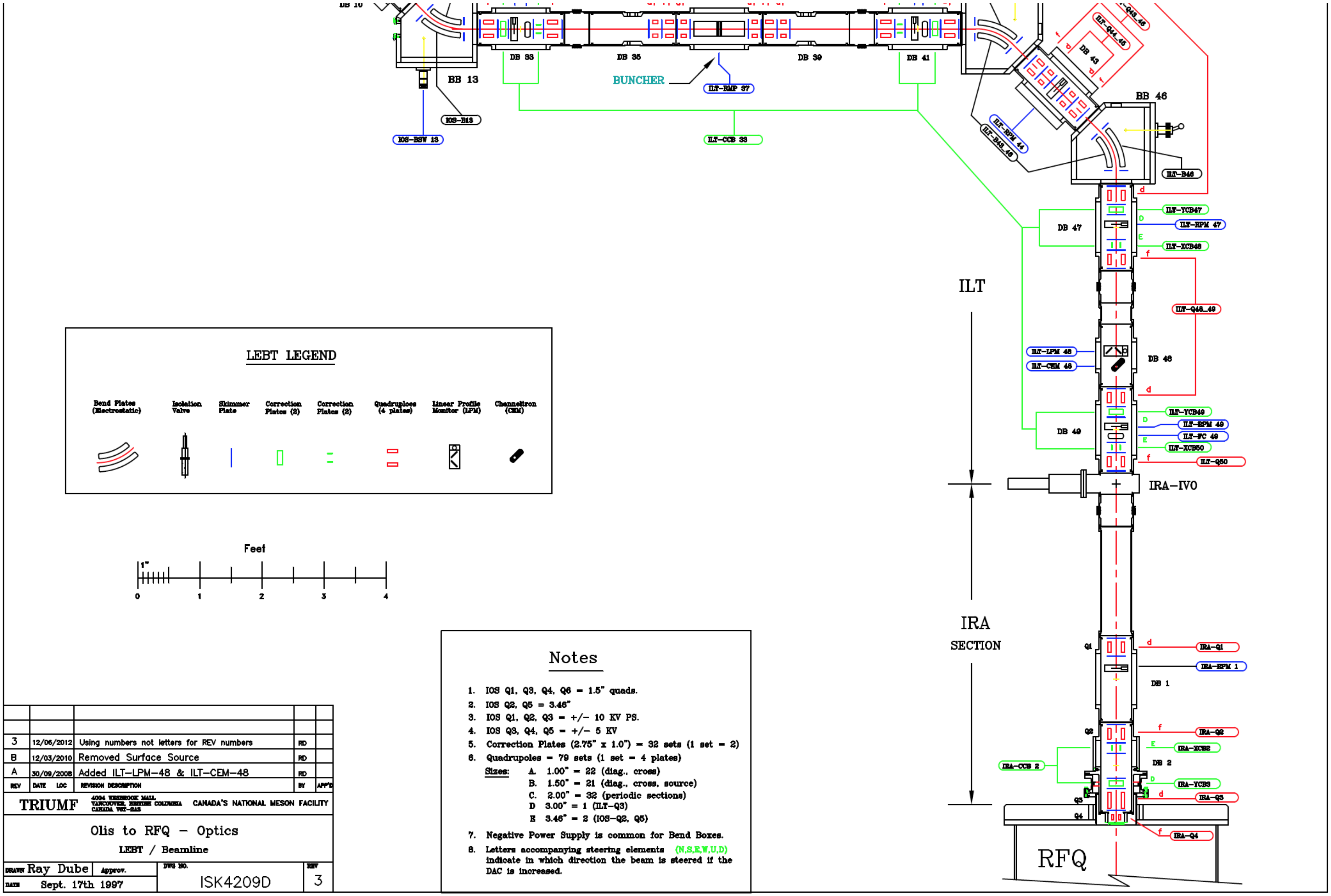
Optimized RFQ match, $\epsilon_f/\epsilon_i = 1.1$



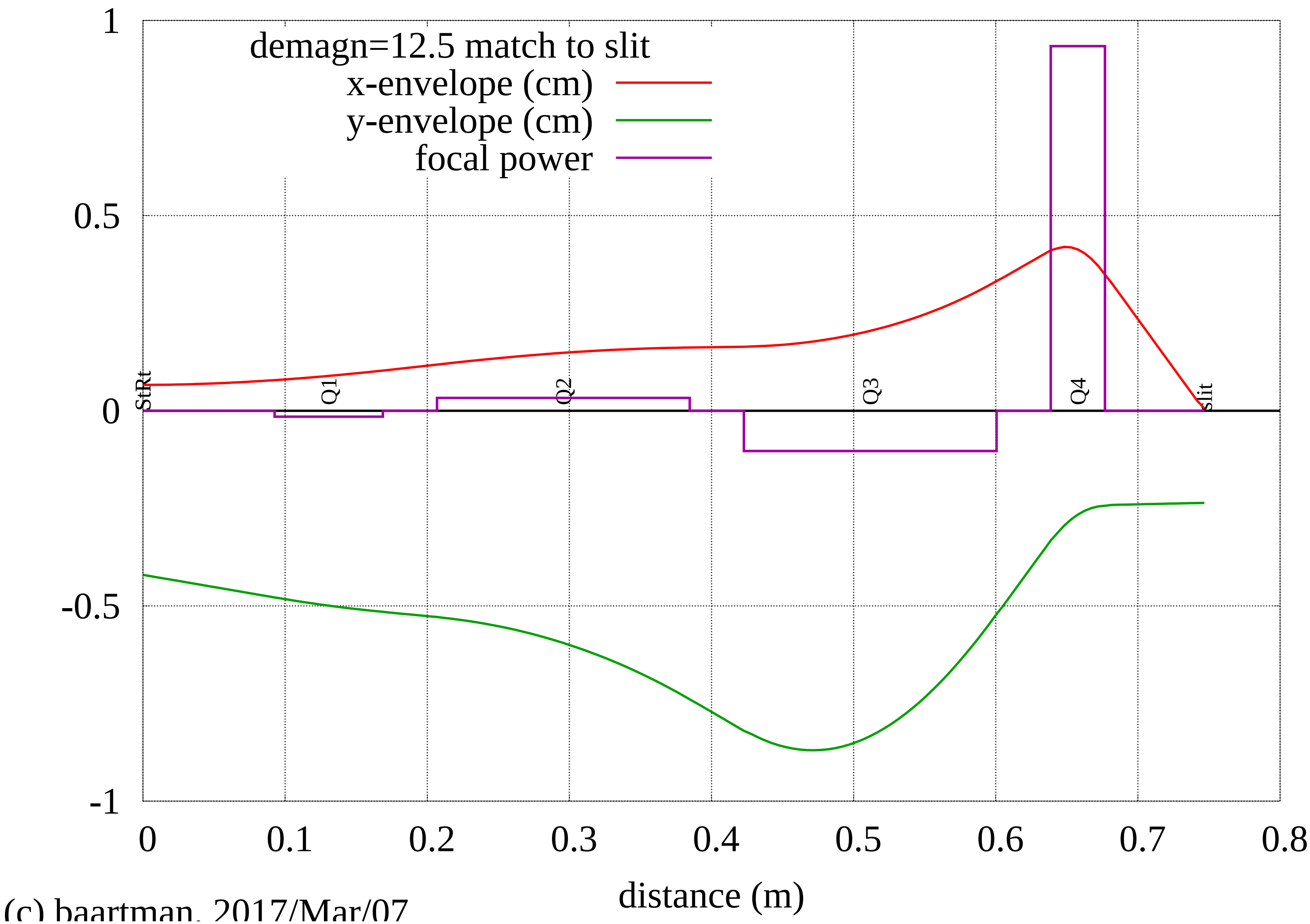
RFQ Match section photo



RFQ Match section drawing



Other example: Mass Separator Slit



Other example: EMMA

Beamline to an experiment that requires a 1 mm spot size.

Click here for Technical Note.

Other Element Types

Magnetic Sector Bend (angle θ, radius ρ)	$\Delta x' = -\frac{\theta y^2}{2\rho^2}$	$\Delta y' = -\frac{\theta xy}{\rho^2}$
Electric Cylindrical Bend (angle θ, radius ρ)	$\Delta x' = -\frac{4\theta x^2}{\rho^2}$	$\Delta y' = 0$
Electric Spherical Bend (angle θ, radius ρ)	$\Delta x' = \frac{\theta(-3x^2+y^2)}{2\rho^2}$	$\Delta y' = \frac{\theta xy}{\rho^2}$
Solenoid (aperture radius a, $\rho = \frac{B\rho}{B_{\mathrm{sol}}}$)	$\Delta r' = -\frac{r^3}{12a\rho^2}$	

Solenoid third order aberration depends on aperture. Quad aberration does not. Bend aberration does not in second order, but does in third order (not given here).

Optimization technique: Just add emittance growths from aberrations and from mismatch!

Once we know formulas for emittance growth, we simply add (in quadrature) together, and minimize this as we vary parameters such as quad position, length, strengths.

The mismatch factor is from Bovet et al.(1970).

CERN/MPS-SI/Int. DL/68-3 - Rev. 1
23 March, 1970

A SELECTION OF FORMULAE AND DATA USEFUL FOR THE DESIGN OF A.G. SYNCHROTRONS

C. Bovet, R. Gouiran, I. Gumowski, K.H. Reich

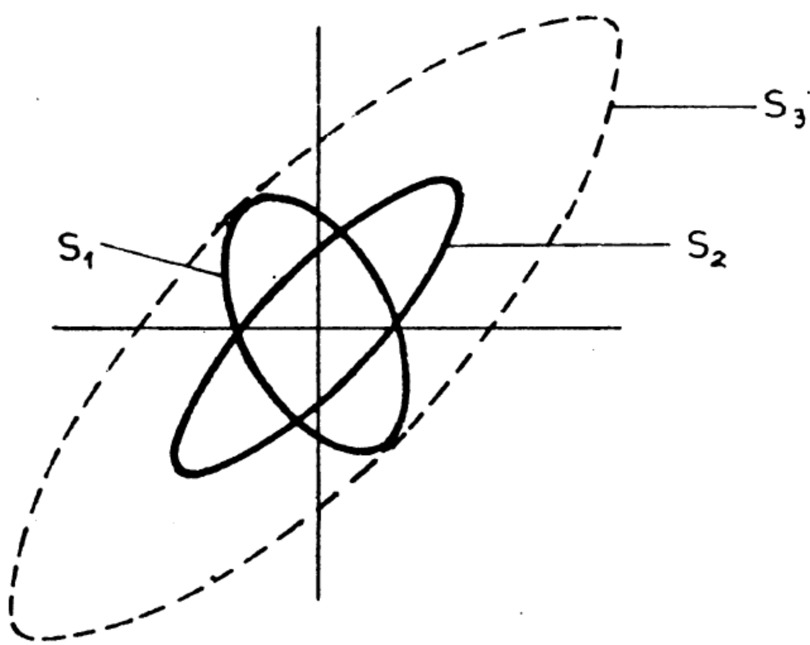
where $D = \frac{1}{2}(\beta_2\gamma_1 + \gamma_2\beta_1 - 2\alpha_1\alpha_2)$

$$= 1 + \frac{(L_2 - L_1)^2 + (S_2 - S_1)^2}{2L_1L_2} .$$

For meaning of $\alpha, \beta, \gamma, L, S$ see 3.2 .

3.4.5 Three ellipses

a) Area of ellipse S_3, similar to S_2, such that S_3 circumscribes S_1: (area S_1 = area $S_2 = S$)



$$\frac{S_3}{S} = D + \sqrt{D^2 - 1}$$

See 3.4.4 for meaning of D

Cute animation.

There are many optimization techniques for designing and tuning beam transport lines. Some are built into the transport codes themselves. Almost all of these work on the basis of reducing an error to zero by finding local derivatives of the error with respect to the parameters.

I don't do this.

Instead, use a downhill simplex method. It's fast and robust.

It is also easily modified to incorporate simulated annealing for more than 3 free parameters. I use routines from the book *Numerical Recipes* by Press, Flannery, et al.

More details regarding my code: `TRANSOPTR` (tech. note), or browse my entire work directory.

Conclusions

- Optics of any scale can be made to work: elements can be many metres apart, down to cm scale.
- Smaller scale means more focusing per unit length. This makes the beam less sensitive to perturbations (stray fields, misalignments, space charge, ...)
- Beam transport design can be optimized with simple tools like first order matrix codes, augmented by “back of the envelope” formulas for higher orders.

Copies of this lecture, software, etc. are available on my website `http://lin12.triumf.ca` or email me at `baartman@triumf.ca`