



**CAS RF**

**Ebeltoft – Denmark**  
8-17 June, 2010



**RF GYMNASTICS**  
**IN**  
**SYNCHROTRONS**  
**- Part 1 -**

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# OUTLINE



Part 1  
Part 2

1. Introduction
2. Longitudinal beam dynamics
3. Single bunch gymnastics
4. Multi-bunch gymnastics
5. Beam gymnastics with broadband RF systems
6. Practical implementation
7. Conclusions

# 1. Introduction

*Frequent need for changing the longitudinal beam characteristics  
In an accelerator complex. Example:*

**Conflict between the requirements of high energy hadron colliders and the ones of the lower energy Synchrotrons:**

	<b>HIGH ENERGY HADRON COLLIDER</b>	<b>INJECTORS' SYNCHROTRONS</b>
<b><i>RF frequency</i></b>	Large because of need for: <ul style="list-style-type: none"><li>- high voltage for acceleration</li><li>- high focusing for luminosity</li></ul>	Small because of need for: <ul style="list-style-type: none"><li>- acceptance</li><li>- large frequency swing</li><li>- gap without beam for kicker</li></ul>
<b><i>Bunch frequency</i></b>	$\ll$ RF frequency to maximize luminosity <ul style="list-style-type: none"><li>- limited by detector saturation</li></ul>	= RF frequency to maximise bunching factor and minimise Laslett tune-shift

# 1. Introduction (cont.)

COLLIDER			INJECTOR COMPLEX
Name (Lab.)	RF (MHz)	Bunch freq. (MHz)	Injector / Collider bunches
<i>RHIC</i> (BNL)	197	4.7	8/1
<i>LHC</i> (CERN)	400	40	~ 1/12
<i>HERA</i> (DESY)	208	10.4	1/1
<i>Tevatron</i> (FNAL)	53	2.5	~ 11/1

# 1. Introduction (cont.)

⇒ ***Need for beam gymnastics in the chain of injectors to change the number of bunches while satisfying the following constraints:***

- minimum blow-up of longitudinal emittance,
- minimum (no ?) losses,
- high reliability / high stability of performance,
- low cost (minimum modifications to existing injectors hardware)



**Modulation of RF parameters (amplitude, phase, frequency)  
Non-trivial manipulations are called “RF gymnastics”**

## **Preliminary comments:**

- Synchrotron radiation is not considered.
- Large numbers of gymnastics have been invented. Only the most typical and common ones are described.

## 2. Longitudinal beam dynamics [ref. 1 + 2]

### ◆ Particle parameters:

- charge, energy (rest energy):  $q, E (E_0)$
- speed, momentum, relativistic parameters:  $v, p, \beta, \gamma$
- revolution period (frequency), orbit length:  $T (\omega_R/2\pi), 2\pi R$

### ◆ Synchrotron parameters:

- $\alpha_P, \gamma_T, \eta$

$$\frac{\Delta R}{R} = \alpha_P \frac{\Delta p}{p}, \quad \alpha_P = \frac{1}{\gamma_T^2}$$

$$\frac{\Delta \omega}{\omega} = -\eta \frac{\Delta p}{p}, \quad \eta = \frac{1}{\gamma_T^2} - \frac{1}{\gamma^2}$$

- nominal orbit length:  $2\pi R_0$
- energy etc. on nominal orbit:  $E_0, v_0, p_0, \beta_0, \gamma_0$

### ◆ RF parameters

- frequency, harmonic number:  $h, \omega_R$        $\omega = h\omega_R$
- voltage (peak voltage), phase:  $\hat{V}, \varphi$        $V = \hat{V} \sin(\omega t + \varphi)$

## 2. Longitudinal beam dynamics

### ◆ Equations of motion

- Synchronous particle = “the particle whose energy  $E_S$  and phase  $\varphi_S$  are such that it sees the same voltage at the next revolution”
- Phase slip:

$$\text{per turn : } d\Delta\varphi = h\omega_{RS}(T - T_S)$$

$$\text{per second : } \frac{d\Delta\varphi}{dt} = h\omega_{RS} \frac{\Delta T}{T_S}$$



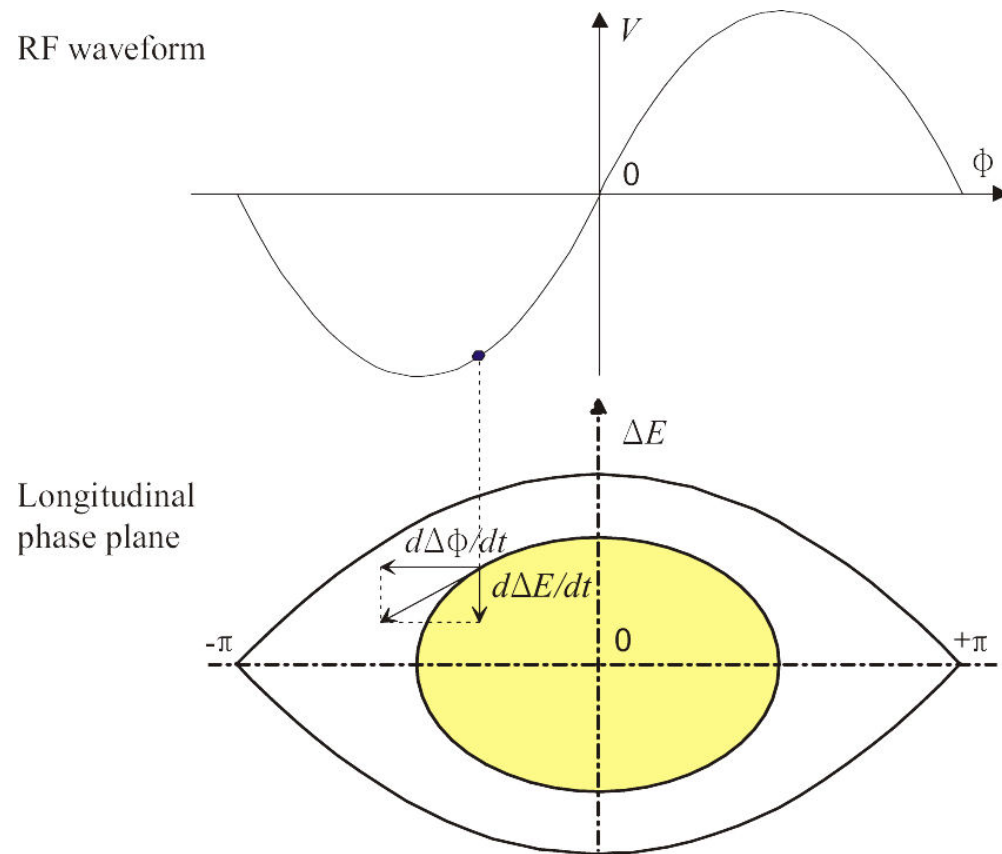
$$\frac{d\Delta\varphi}{dt} = \frac{h\eta\omega_{RS}}{\beta^2} \frac{\Delta E}{E_S}$$

- Energy variation:

$$\frac{d\Delta E}{dt} = \frac{q\omega_{RS}}{2\pi} [V(\Delta\varphi + \varphi_S) - V(\varphi_S)]$$

## 2. Longitudinal beam dynamics

### ◆ Phase stability (sinewave & stationary bucket)



#### Comment

On the unstable  
fixed points  $(\pm\pi, 0)$ :

$$\frac{d\Delta\phi}{dt} = 0, \quad \frac{d\Delta E}{dt} = 0$$



## 2. Longitudinal beam dynamics

### ◆ Basic relations and effects of parameters (case of a stationary bucket)

- Synchrotron frequency (for small amplitude oscillations around the synchronous particle)

$$\omega_S = \sqrt{\frac{h|\eta|\omega_{RS}^2 q \hat{V}}{2\pi\beta^2 E_S}}$$

- Bucket height

$$\Delta E_{MAX} = \sqrt{\frac{2E_S\beta^2}{\pi h|\eta|}} q \hat{V}$$

- Bunch dimensions at constant emittance

$$\Delta\hat{\phi} \propto \left[ \frac{|\eta|}{hE_S q \hat{V}} \right]^{\frac{1}{4}} \quad \Delta\hat{E} \propto \left[ \frac{hE_S q \hat{V}}{|\eta|} \right]^{\frac{1}{4}}$$

## 2. Longitudinal beam dynamics

### ◆ Adiabaticity

- Adiabatic = “Parameters are changed at a slow enough rate so that the distribution of particles is always at equilibrium.”
- Adiabaticity parameters:  $\varepsilon$

$$\varepsilon = \frac{1}{\omega_s^2} \left| \frac{d\omega_s}{dt} \right|$$

### ◆ Emittance preservation (Liouville's theorem)

Longitudinal motion is conservative (no energy dissipation effect like synchrotron radiation)

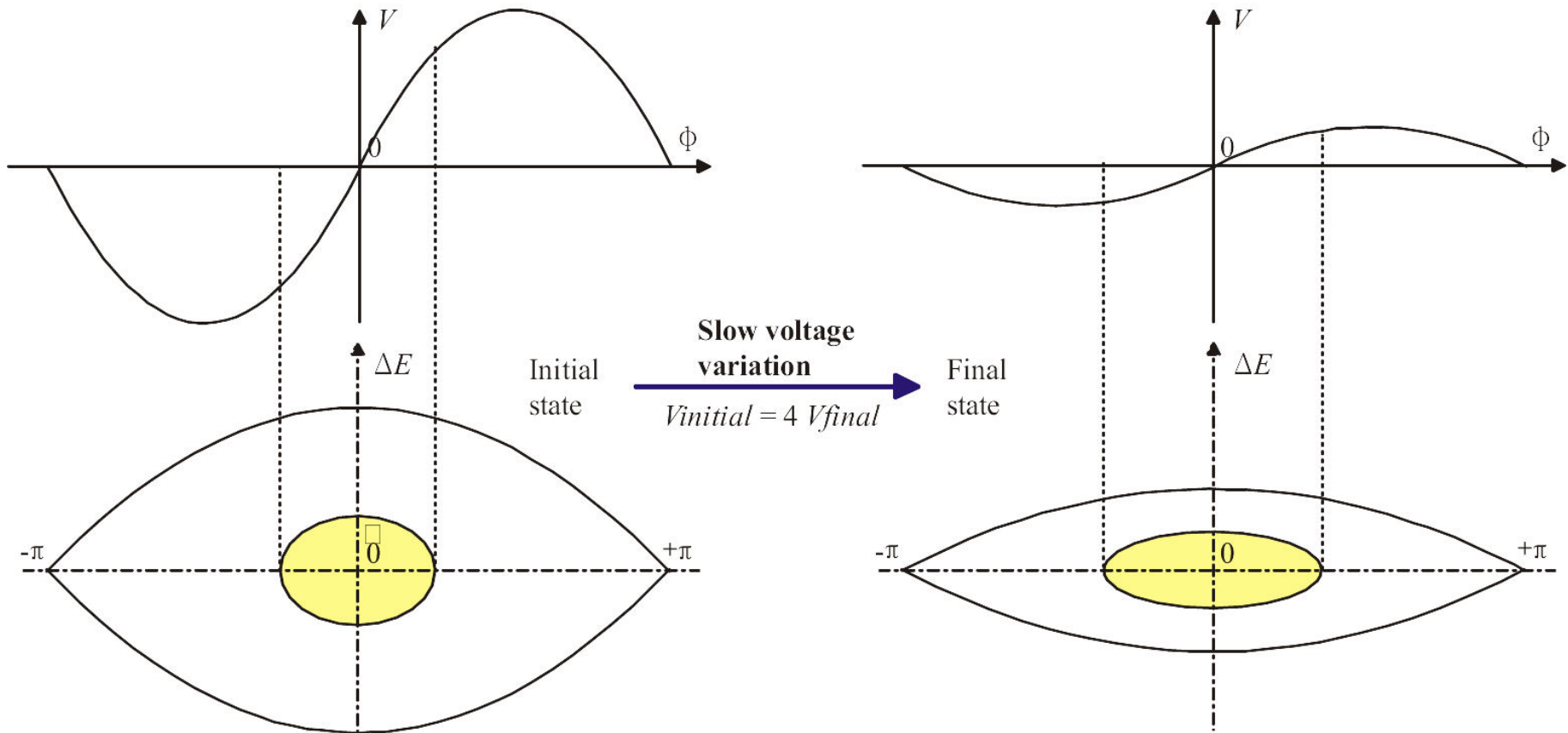


Constant local density of particles in the longitudinal phase plane

## 2. Longitudinal beam dynamics

### “Micro-” and “Macro-scopic” emittances

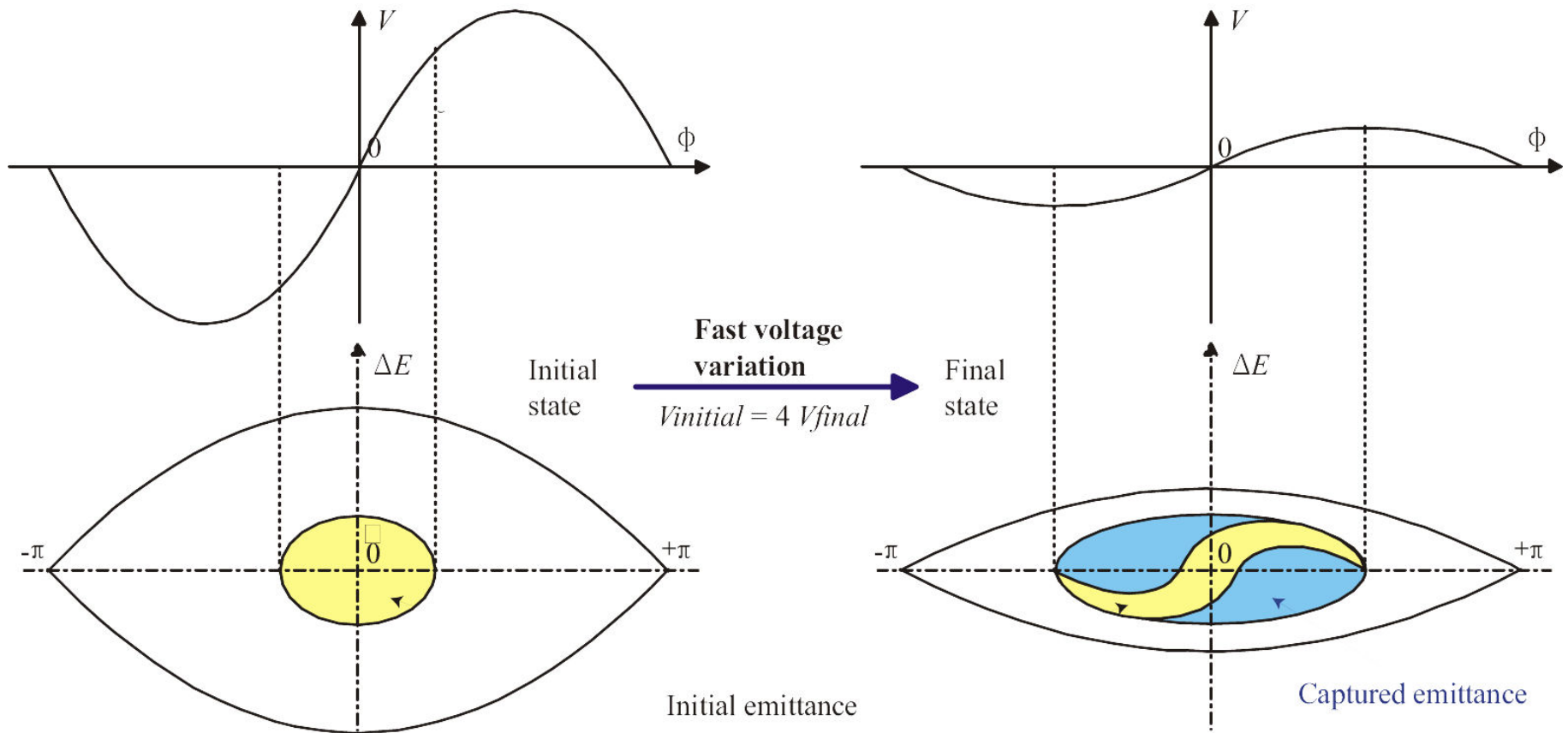
- Adiabatic RF voltage reduction



## 2. Longitudinal beam dynamics

### “Micro-” and “Macro-scopic” emittances

- Non-adiabatic RF voltage reduction



# 3. Single bunch gymnastics

## Bunch compression [ref. 3, 4, 5]

### ◆ Required to get:

short bunches and/or large energy spread (Liouville !)

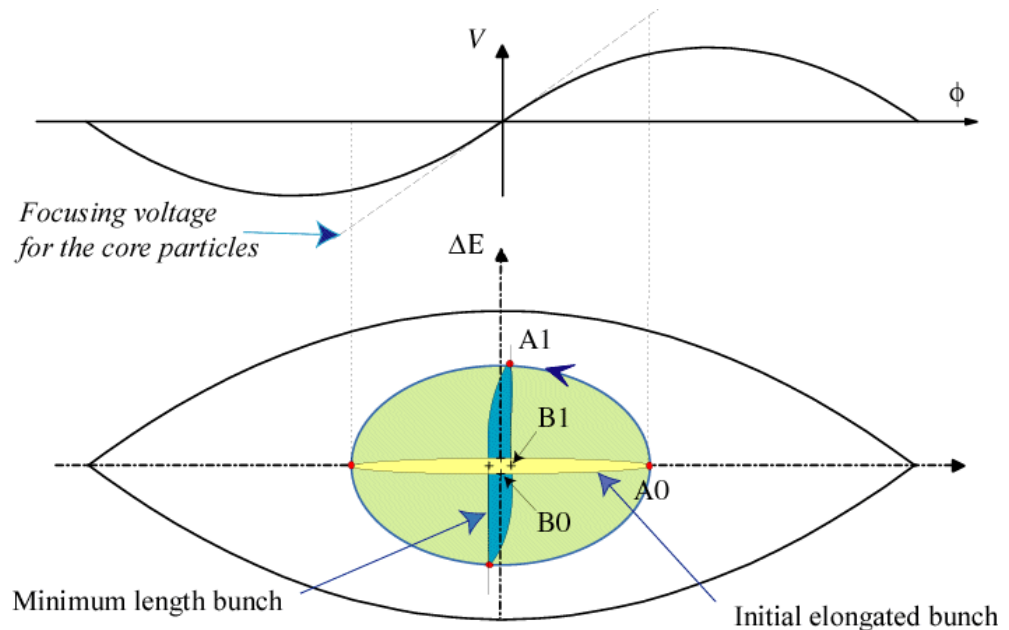
### ◆ Principle:

rotate a mismatched and long bunch in a large bucket.

Required characteristics are only momentarily achieved



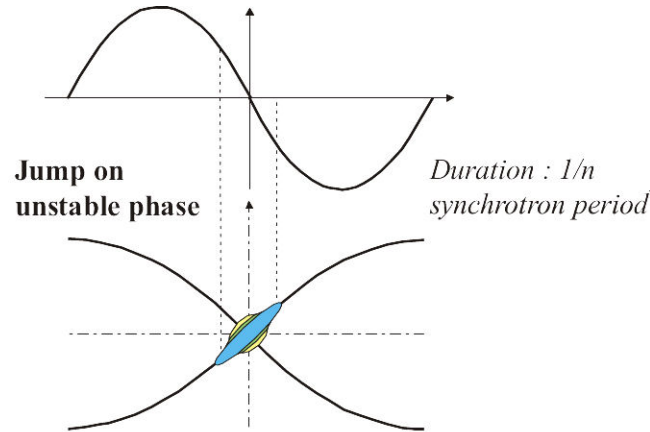
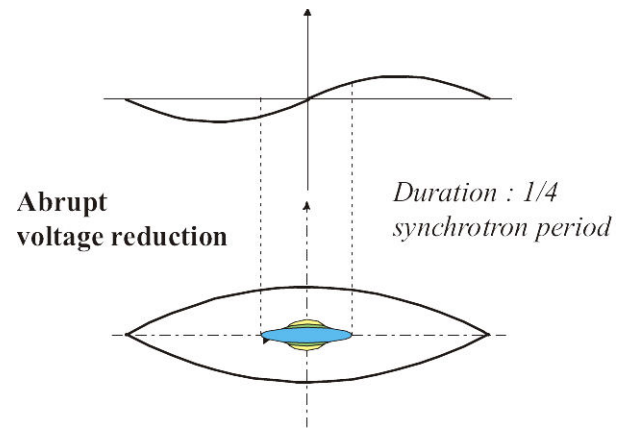
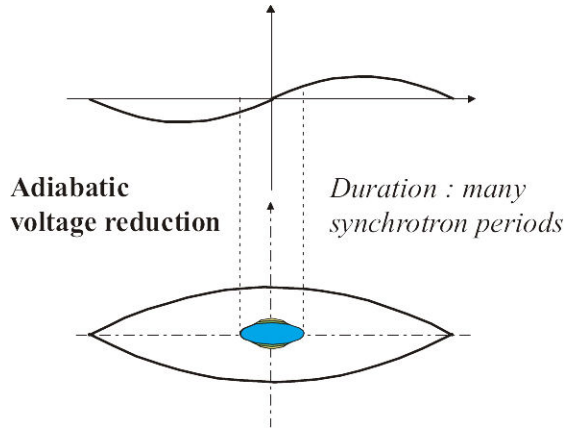
precise timing for ejection or RF turn-off



# 3. Single bunch gymnastics

## Bunch compression

### ◆ Bunch stretching techniques

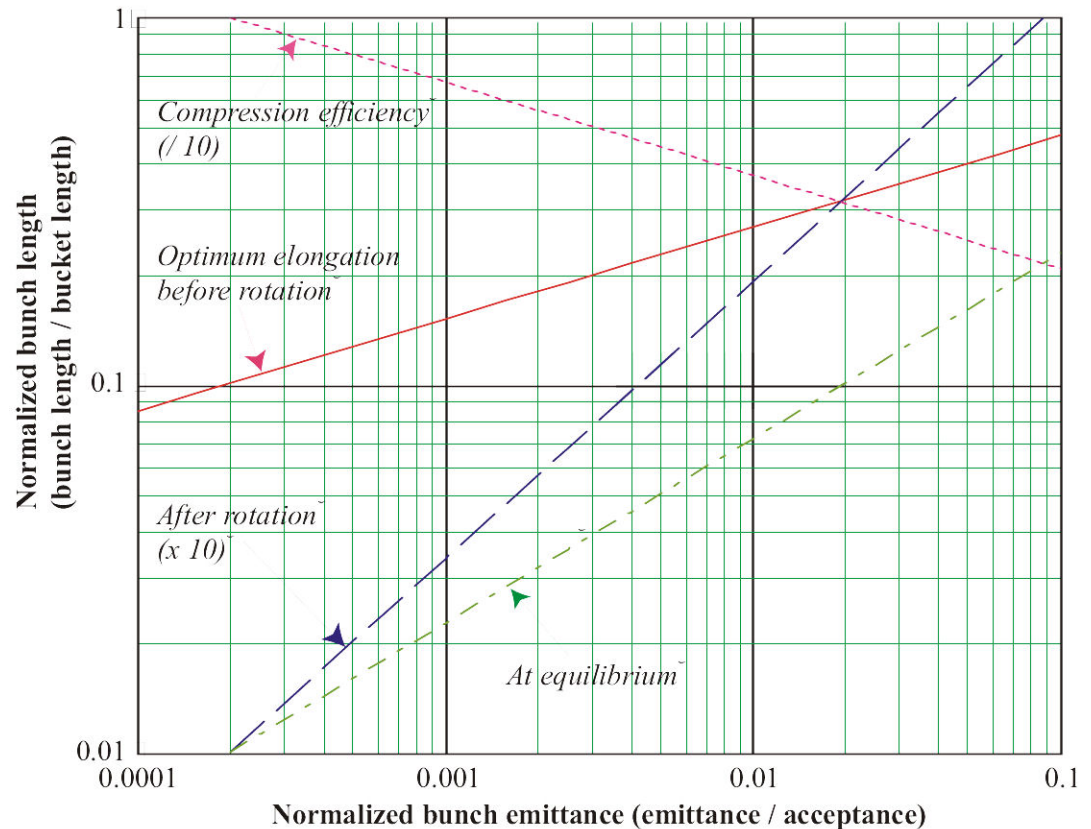
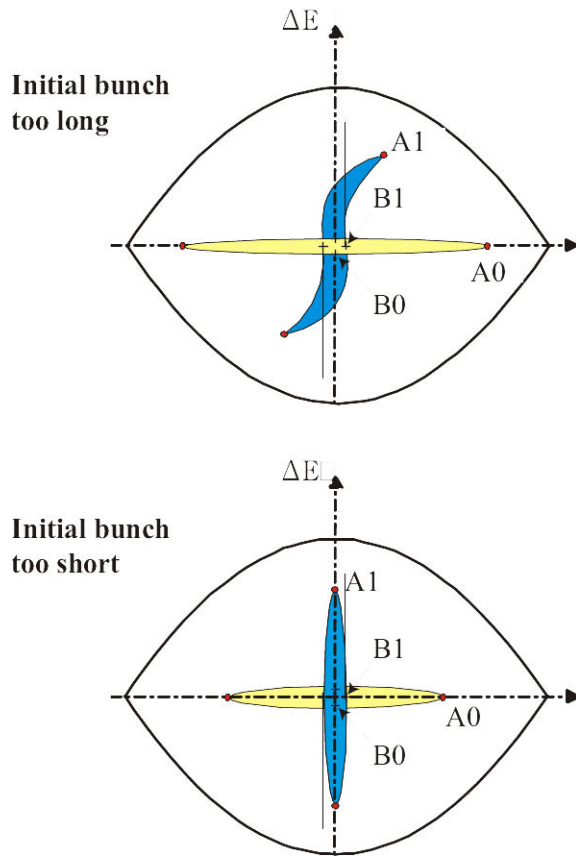


# 3. Single bunch gymnastics

## Bunch compression

### ◆ Performance:

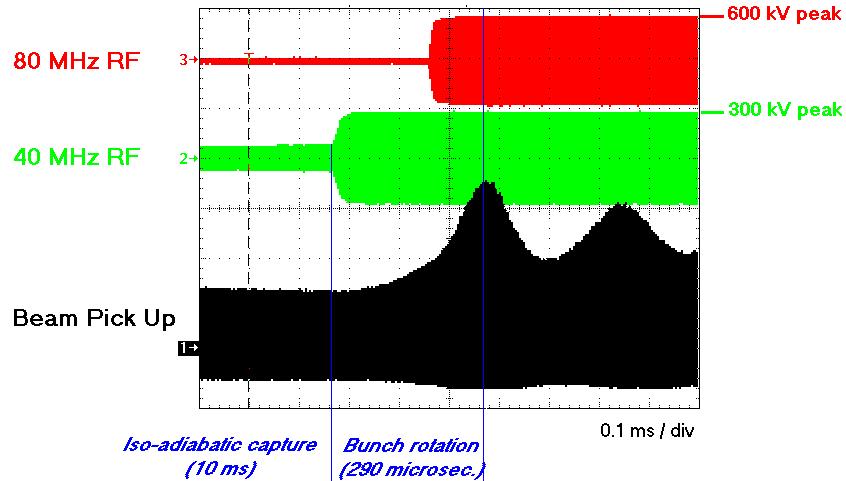
depends upon bunch length  
and normalized emittance



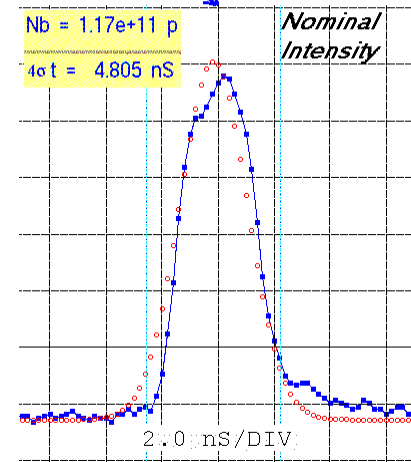
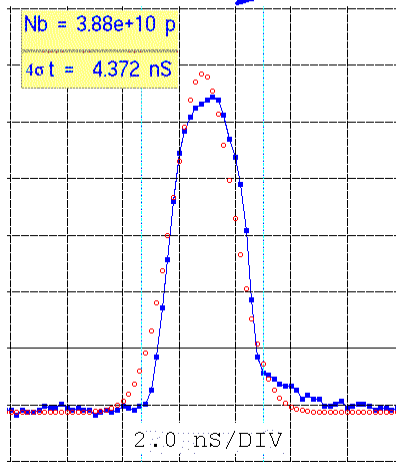
# 3. Single bunch gymnastics

## Bunch compression (experiment)

LHC bunch  
 Compression in the  
 CERN-PS before  
 ejection to the SPS  
 $p = 26 \text{ GeV}/c$



**SHORTEST BUNCHES  
 (EJECTION)**



### Comment

Phase & energy errors  
 are swapped

⇒ phase accuracy  
 depends upon  
 initial energy

⇒ energy accuracy  
 depends upon initial  
 phase

31/08/98



### 3. Single bunch gymnastics

#### Controlled blow-up [ref. 6, 7, 8]

#### ◆ Required for:

stabilizing the beam at high intensity

#### ◆ Principle:

increase the longitudinal “macroscopic” emittance creating filaments deliberately and accelerating their dilution

#### ◆ Technique:

superpose to the RF holding the beam a phase modulated voltage at a much higher frequency

$$V_H = \hat{V}_H \sin(h_H \omega_R t + \alpha \sin \omega_M t + \theta_H)$$

Resonances are induced which redistribute density inside the bunch. High frequency increases non-linearities and helps smooth the distribution (filamentation).

### 3. Single bunch gymnastics

#### Controlled blow-up

◆ Typical parameters

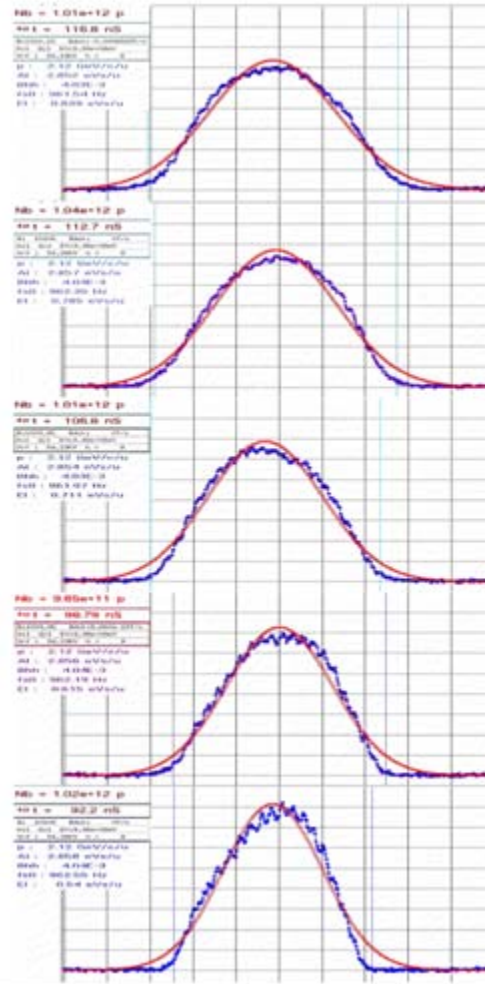
$\frac{\hat{V}_H}{\hat{V}}$	$\frac{h_H}{h}$	$\alpha$ (rad)	$\frac{\omega_M}{\omega_S}$	Duration
0.05 to 0.2	> 10 for fast blow-up	0.8 $\pi$ to 1.2 $\pi$	2 to 7	>10 synchrotron periods

# 3. Single bunch gymnastics

## Controlled blow-up (experiment)

Longitudinal controlled  
Blow-up in the CERN-PS  
T=1.4 GeV

$V(h=8) = 55 \text{ kV}$   
 $V(h=458) = 3 \text{ kV}$   
 $\alpha = \pi \text{ rad}$   
 $f_{\text{Synch}} = 0.95 \text{ kHz}$   
 $f_{\text{Mod}} = 7 \text{ kHz}$   
 Duration = 20 ms



$t = 20 \text{ ms}$   
 (final bunch)  
 $\epsilon = 0.84 \text{ eVs}$

$t = 15 \text{ ms}$   
 $\epsilon = 0.78 \text{ eVs}$

$t = 10 \text{ ms}$   
 $\epsilon = 0.71 \text{ eVs}$

$t = 5 \text{ ms}$   
 $\epsilon = 0.62 \text{ eVs}$

$t = 0 \text{ ms}$   
 (initial bunch)  
 $\epsilon = 0.54 \text{ eVs}$

# 4. Multi-bunch gymnastics

## Iso-adiabatic debunching-rebunching [ref. 9, 10]

◆ Used for changing the number of bunches

◆ Principle:

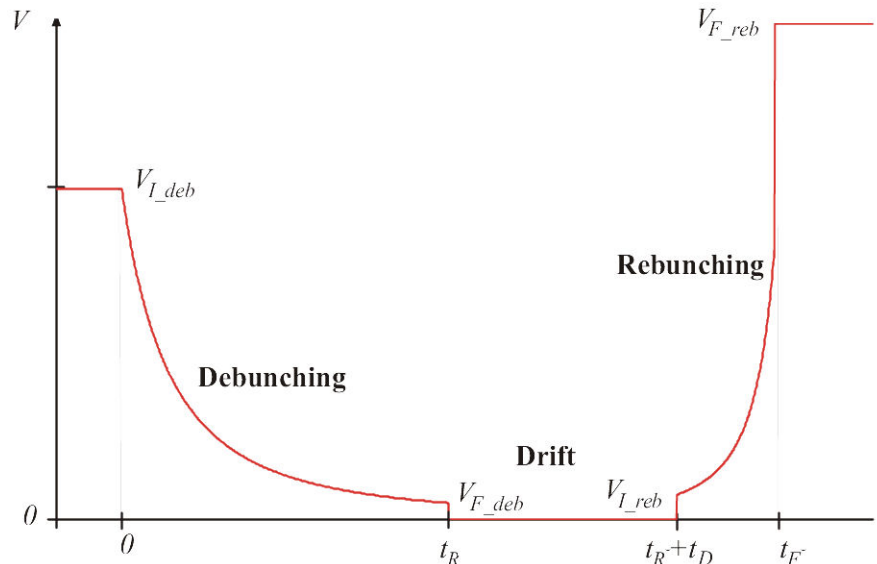
- Slow (adiabatic) voltage reduction with fast final  $h_1$  RF turn-off
- Drift period without RF
- Fast  $h_2$  RF turn-on followed by slow (adiabatic) voltage increase

Voltage variation

at constant  
adiabaticity  $\varepsilon = \frac{1}{\omega_S^2} \left| \frac{d\omega_S}{dt} \right|$

$$V_{deb}(t) = \frac{V_{I\_deb}}{\left[ 1 - \left( 1 - \sqrt{\frac{V_{I\_deb}}{V_{F\_deb}}} \right) \frac{t}{t_R} \right]^2}$$

$$V_{reb}(t) = \frac{V_{I\_reb}}{\left[ 1 - \left( 1 + \sqrt{\frac{V_{I\_reb}}{V_{F\_reb}}} \right) \frac{t}{t_F} \right]^2}$$



# 4. Multi-bunch gymnastics

## Iso-adiabatic debunching-rebunching

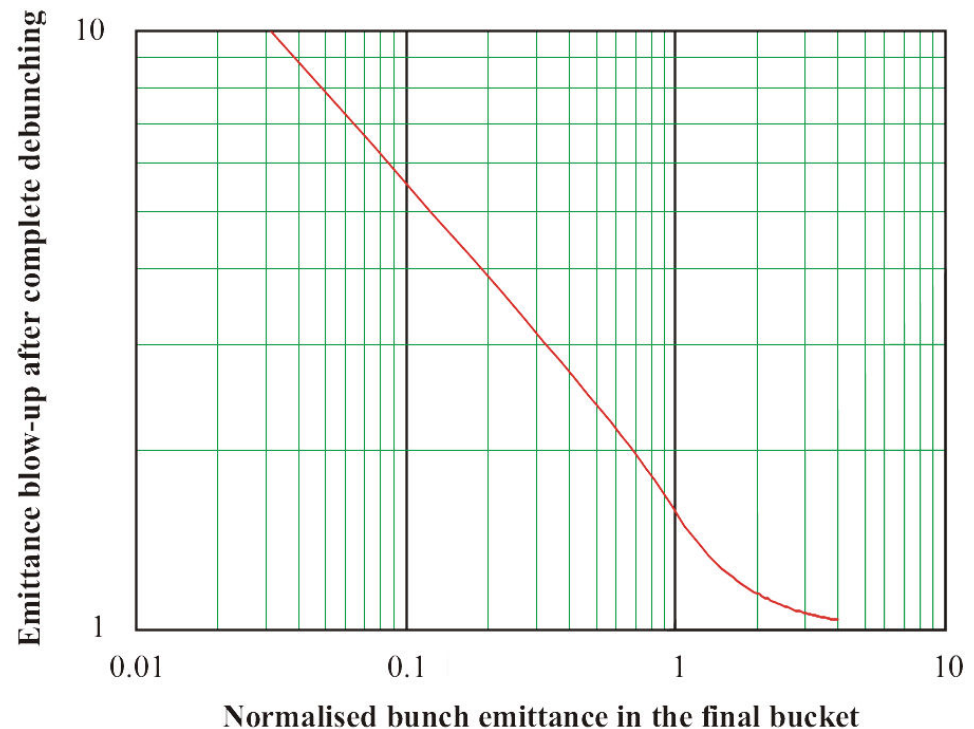
### ◆ Performance:

- depends upon adiabaticity and voltages at RF turn-off and -on
- high sensitivity to disturbance during drift without RF
- effect of beam induced voltage in cavities



- unavoidable blow-up

+ no gap without beam



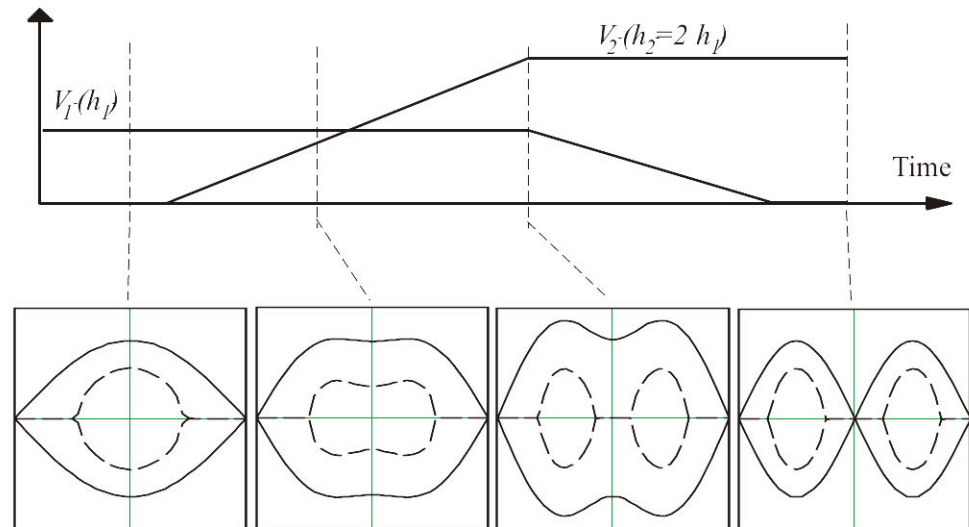
# 4. Multi-bunch gymnastics

## Splitting (Merging) [ref. 11, 12]

- ◆ **Changes the numbers of bunches** (in multiples of 2 and 3)
- ◆ **Principle:** adiabatically change the focusing voltage for a continuous evolution from the original to the final state
- ◆ **Technique:** apply simultaneously 2 RF voltages on  $h$  and  $2h$  (Example of splitting in 2)

$$V_1 = \hat{V}_1 \sin(h\omega_R t)$$

$$V_2 = \hat{V}_2 \sin(2h\omega_R t + \pi)$$

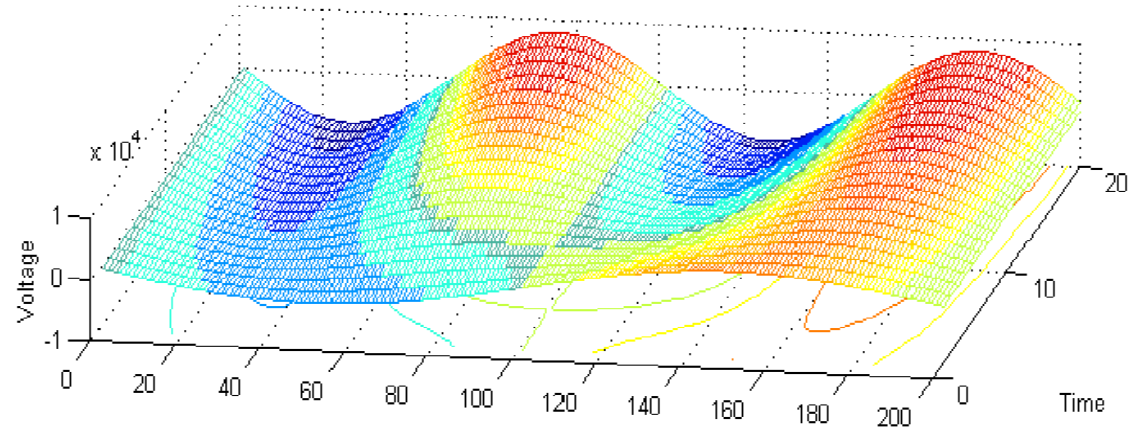


# 4. Multi-bunch gymnastics

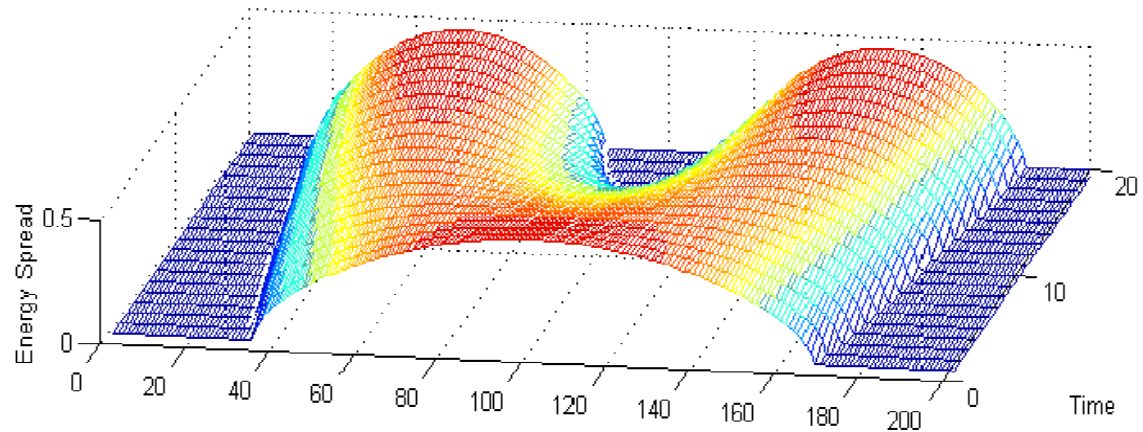
## Splitting in two (simulation)

◆ **Technique:**

**Time evolution of the RF voltage**



**Time evolution of the bunch(es)**

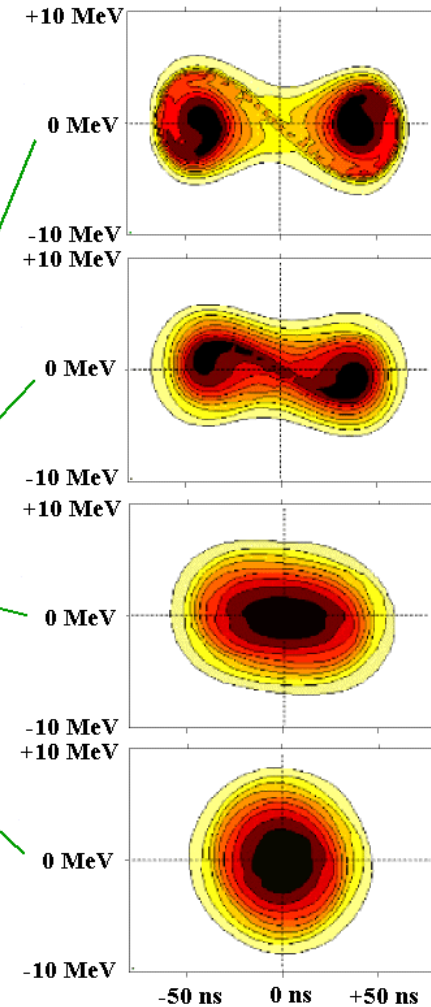
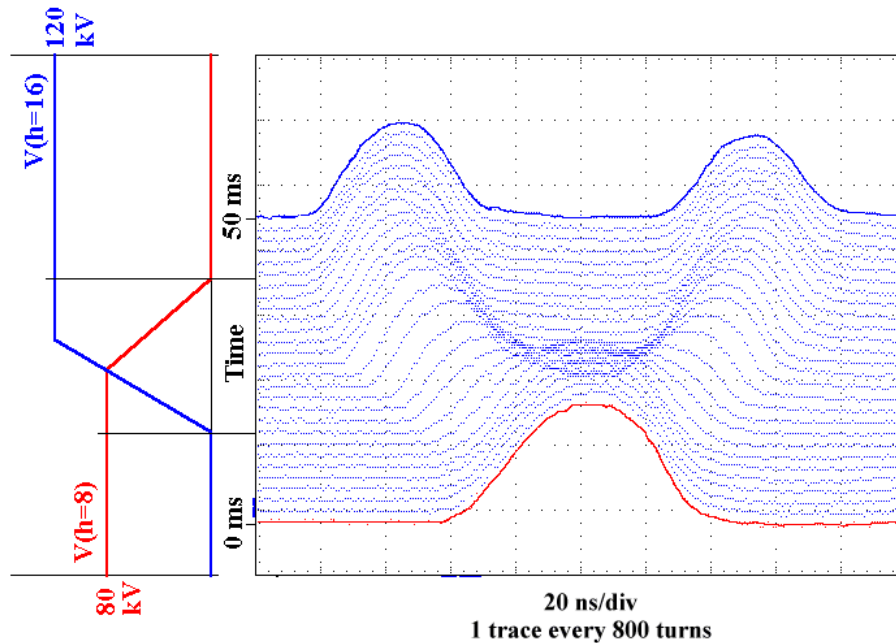


# 4. Multi-bunch gymnastics

## Splitting in two (measurement)

### BUNCH DOUBLE SPLITTING

CERN-PS @ 3.57 GeV/c - 3E12 protons/bunch

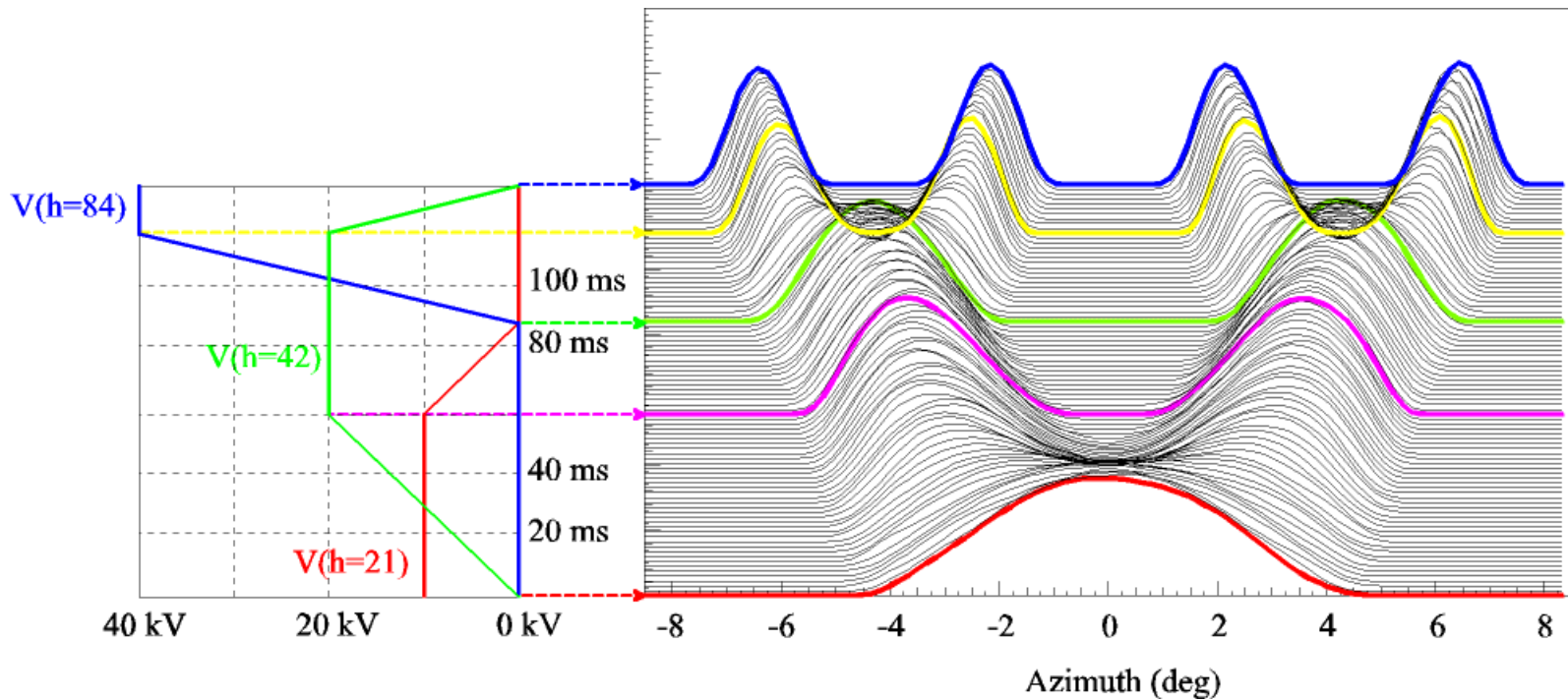




# 4. Multi-bunch gymnastics

## Splitting in four (simulation)

Splitting bunches in four at 26 GeV/c

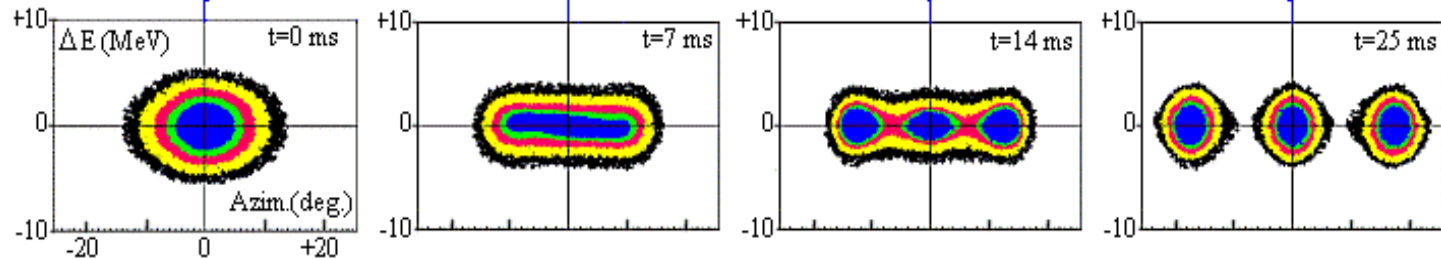
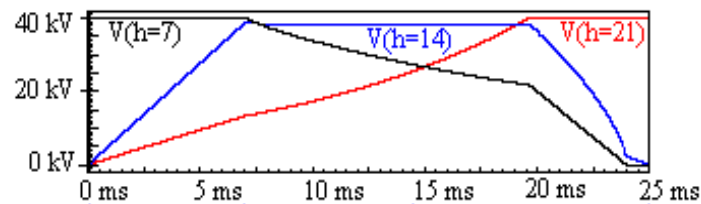


# 4. Multi-bunch gymnastics

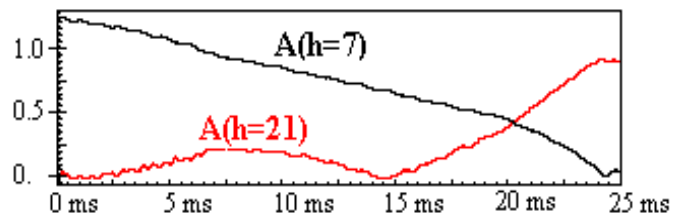
## Triple splitting [simulation (1)]

Splitting bunches in three at 3.57 GeV/c in the PS

Beam voltages  
on h=7, 14 and 21  
during triple splitting



Beam spectral amplitudes  
on h=7 and 21  
during triple splitting

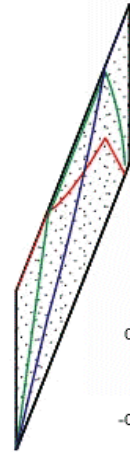


# 4. Multi-bunch gymnastics

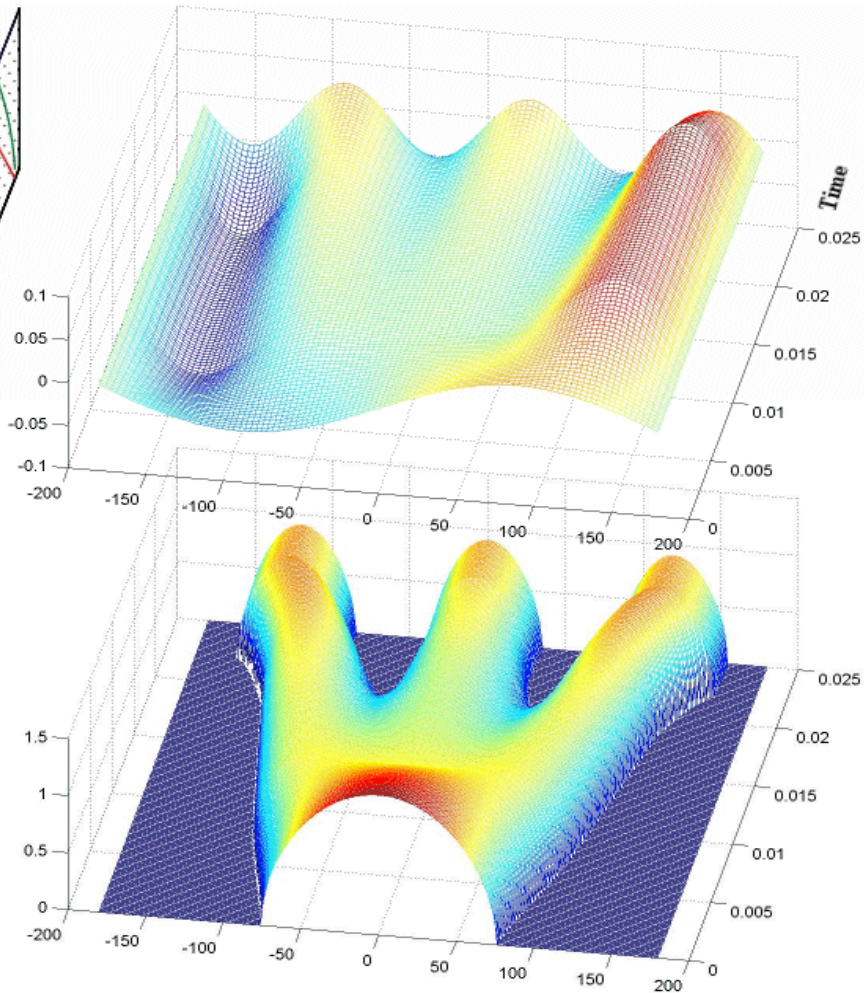
## Triple splitting [simulation (2)]

Time evolution of the RF voltage

$V(h=7)$   
 $V(h=14)$   
 $V(h=21)$



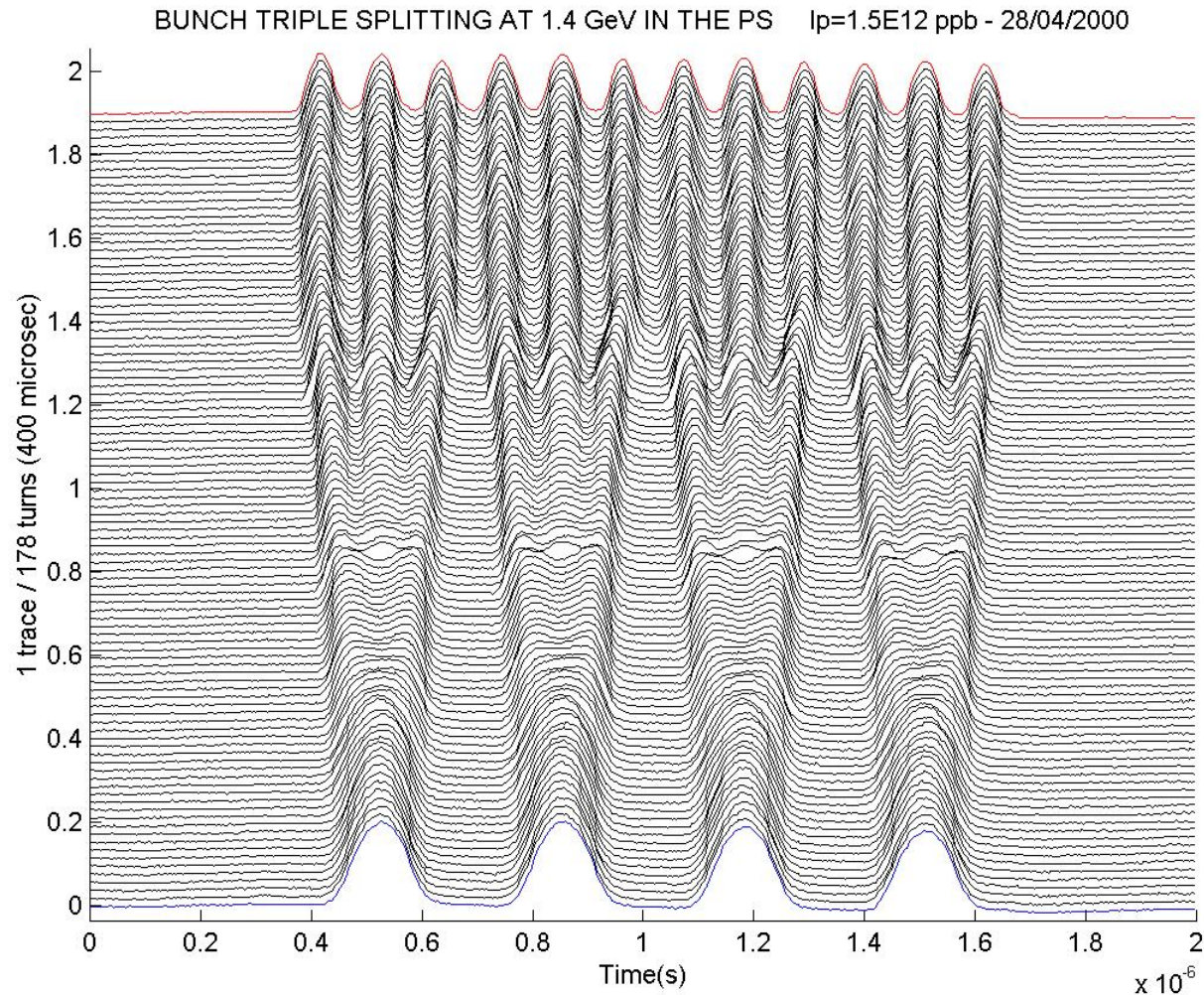
Time evolution of the bunch(es)





# 4. Multi-bunch gymnastics

## Triple splitting (measurement)



## 4. Multi-bunch gymnastics

### Splitting (Merging)

#### Advantages with respect to debunching / rebunching:

- Capability to preserve longitudinal emittance: no need for very low RF voltages to minimise blow-up,
- Capability to keep a gap without particles over a fraction of the circumference,
- Good reproducibility: beam is always confined by RFs, and feedback loops can be used for stabilisation,
- Lower risk of microwave instability thanks to the larger  $(\Delta p/p)^2/l$  in the beam.