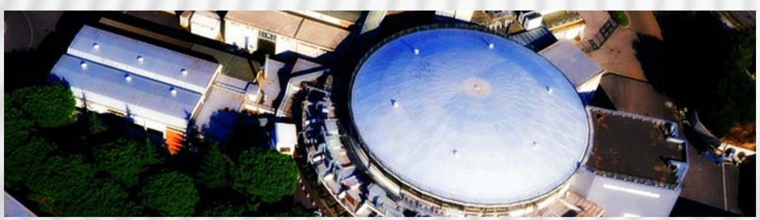
THE CERN ACCELERATOR SCHOOL

BASICS OF RF ELECTRONICS
(… OR GETTING STARTED WITH LLRF BUILDING BLOCKS)

Alessandro Gallo



INFN
Laboratori Nazionali di Frascati
Frascati (RM), Italy

CAS course on “RF for Accelerators”
Hotel Ebeltoft Strand, Ebeltoft, Denmark, June 8th to 17th 2010

SUMMARY

➢ *A. Gallo, Basics of RF Electronics*

- Attenuators
- (signal) Amplifiers
- RF Transformers
- Power Splitters/ /Combiners
- Hybrid junctions/ /Directional Couplers
- Circulators/Isolators
- Filters
- Modulation Transfer Functions

1st Lecture

- Frequency Mixers
- Phase Detectors (I&Q)
- Bi-phase attenuators/ /I&Q modulators
- Peak Detectors
- Step Recovery diodes
- PIN diode Switches/ /Attenuators
- Phase shifters
- VCOs
- PLLs

2nd Lecture

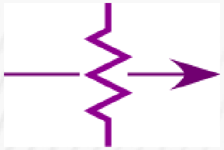
FIXED ATTENUATORS

Fixed attenuators are widely used in RF electronics to **set the proper signal level** in the various circuit branches. Proper level setting is crucial to **fully exploit** the instrumentation **dynamic range** and to avoid circuit **overload and damaging**.

Attenuators are also used as **matching pads** as they can be designed to connect lines of different impedances. Also, the insertion of **attenuators** in front of mismatched loads **reduces the VSWR** seen at the source side.

Fixed attenuators are mainly characterized by the following parameters:

- **Attenuation ΔdB;**
- **Max average power rate;**
- **Max peak power rate;**
- **Frequency range;**
- **Attenuation flatness over the specified frequency range;**
- **VSWR, size and weight, performance over the given temperature range, …**

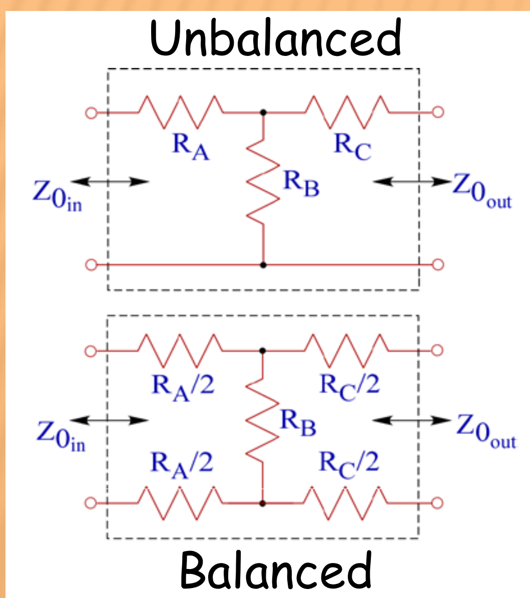


FIXED ATTENUATORS

Fixed attenuators are **passive**, 2-ports devices generally made by a **network of resistors** with a **very broadband** frequency response (dc ÷ many GHz, typical). They are designed to provide both the required attenuation and matching of the input/output lines, which might have different characteristic impedances. The attenuation ΔdB, expressed in dB units, and the linear transmission coefficient α are defined as:

$$\Delta dB = 10 \cdot \log\left(P_{in}/P_{out}\right); \quad \alpha = \sqrt{P_{out}/P_{in}} = 10^{-(\Delta dB/20)} \Rightarrow S = \begin{pmatrix} 0 & \alpha \\ \alpha & 0 \end{pmatrix}$$

T-type Attenuator



$$R_A = \frac{1+\alpha^2 - 2\alpha\sqrt{Z_{0_{out}}/Z_{0_{in}}}}{1-\alpha^2} Z_{0_{in}}$$

$$R_B = \frac{2\alpha}{1-\alpha^2}\sqrt{Z_{0_{in}} \cdot Z_{0_{out}}}$$

$$R_C = \frac{1+\alpha^2 - 2\alpha\sqrt{Z_{0_{in}}/Z_{0_{out}}}}{1-\alpha^2} Z_{0_{out}}$$

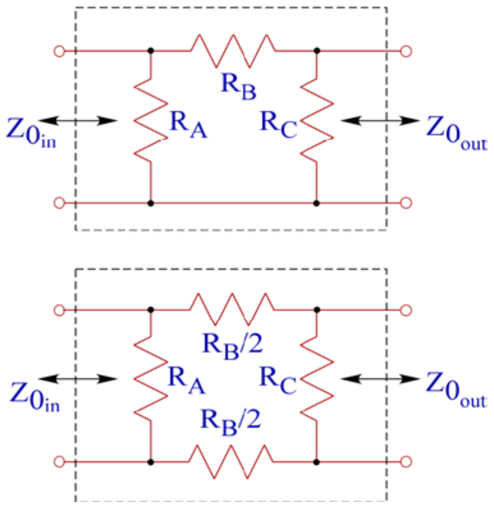
$if \ Z_{0_{in}} = Z_{0_{out}} = Z_0$

$$R_A = R_C = \frac{1-\alpha}{1+\alpha} Z_0$$

$$R_B = \frac{2\alpha}{1-\alpha^2} Z_0$$

FIXED ATTENUATORS

Unbalanced



Balanced

π-type Attenuator

$$R_A = \frac{1-\alpha^2}{1+\alpha^2 - 2\alpha\sqrt{Z_{0_{in}}/Z_{0_{out}}}} Z_{0_{in}}$$

$$R_B = \frac{1-\alpha^2}{2\alpha}\sqrt{Z_{0_{in}} \cdot Z_{0_{out}}}$$

$$R_C = \frac{1-\alpha^2}{1+\alpha^2 - 2\alpha\sqrt{Z_{0_{out}}/Z_{0_{in}}}} Z_{0_{out}}$$

if $Z_{0_{in}} = Z_{0_{out}} = Z_0$

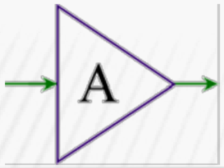
$$R_A = R_C = \frac{1+\alpha}{1-\alpha} Z_0$$

$$R_B = \frac{1-\alpha^2}{2\alpha} Z_0$$

It is important to notice that in order to **match unequal input/output line impedances** a **minimum attenuation** is **required**, according to (case $Z_{0_{out}} \geq Z_{0_{in}}$):

$$\alpha_{\max} = \sqrt{Z_{0_{out}}/Z_{0_{in}}} - \sqrt{Z_{0_{out}}/Z_{0_{in}} - 1} \quad \Rightarrow \quad \Delta dB_{\min} = 20 \cdot \log(1/\alpha_{\max})$$

Fixed attenuators are available in a huge variety of packages, power ratings (up to ≈1 kW), frequency ranges (to > 18 GHz), any attenuation value and all standard impedances used in communication electronics.



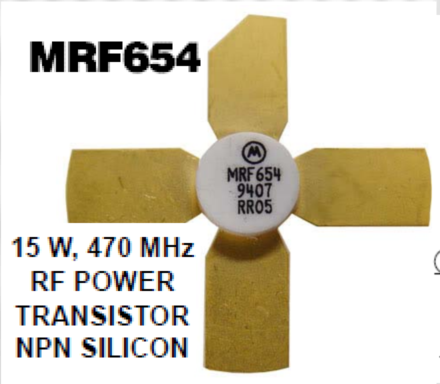
SIGNAL AMPLIFIERS

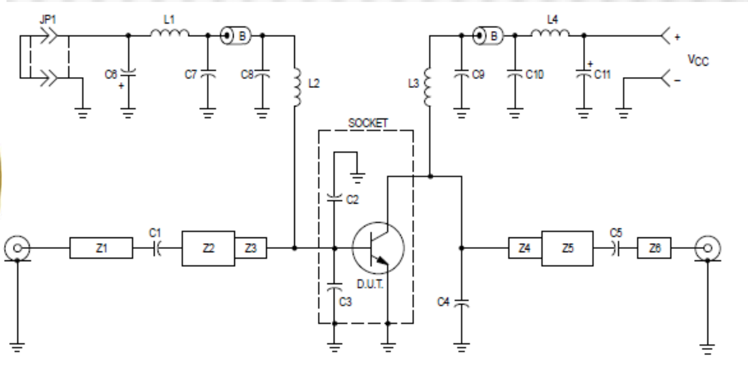
➢ *A. Gallo, Basics of RF Electronics*

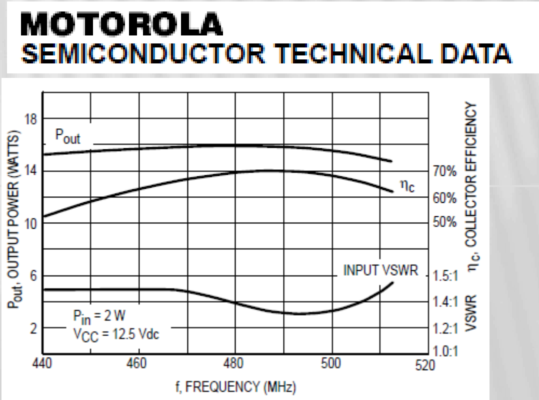
Low-level RF amplifiers are used to increase the signal level whenever it is required for **proper treatment** and/or **manipulation**. A very wide subject, can only be mentioned here. Nowadays almost only **solid state** technology (silicon or GaAs semiconductors, BJT and FET technology) is used for low/medium power (< 10 W) applications, up to ≈ 10GHz.

Construction techniques are **MIC** (Microwave Integrated Circuits) and **MMIC** (Monolithic Microwave Integrated Circuits). In MIC realization the transistor and its capacitance and resistors are soldered on microstrip lines laying on a proper substrate; MMIC are completely integrated circuits where all components (the transistors and their ancillaries) are fabricated on a common substrate.

Concerning small-signal amplifier, the operating class is generally "**A**" since power efficiency is not an issue for these kind of applications.







SIGNAL AMPLIFIERS: BASIC SPECIFICATIONS

➢ *A. Gallo, Basics of RF Electronics*

- **Frequency range**;
  from DC to > 10 GHz, multi-decades covered by a single device.
- **Output level**;
  Maximum power at the amp output.
- **Gain and gain flatness**;
  Ratio between the output (non-saturated) and input levels, typically expressed in dB. The flatness is defined as half of the gain variation over the entire specified frequency band.
- **1 dB compression point**;
  Output level corresponding to a 1 dB reduced gain because of the incipient device saturation.
- **Noise figure**;
  Ratio between the input and output signal-to–noise ratios assuming an input unilateral spectral noise power density $dP_{in}/df=kT$, (k=*Boltzmann constant*, $T=290\ K$). Being G its power gain, the amp generates an extra output spectral noise $d(\Delta P_{nout})/df= =G(NF-1)kT$.
- **Dynamic range**;
  Potential excursion of the output level, upper-limited by compression/saturation and lower-limited by the noise power integrated over the application frequency band.
- **Two-Tone Third-order intercept point** ;
  Measures the amp linearity. If two-tone fed (two equal-amplitude signals of frequencies f_1 and f_2) the amp generates intermodulation products at $m_1f_1 \pm m_2f_2$ frequencies. The amplitudes of 3rd order products ($2f_1-f_2$, $2f_2-f_1$) grow with the 3rd power of the input signals so that an input level corresponding to equal fundamental and 3rd order products amplitudes can be extrapolated (laying usually beyond the amp dynamic range).
- **Input/Output VSWR or Return Loss**;
  Measures of the input/output matching characteristics of the amplifier.

RF TRANSFORMERS

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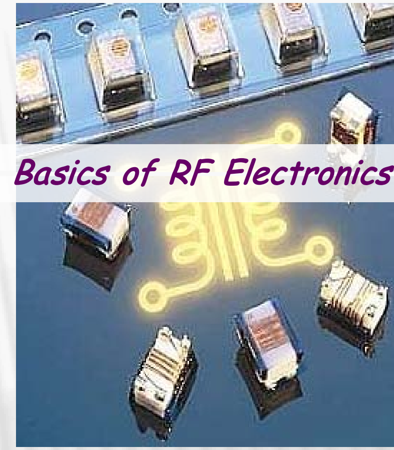
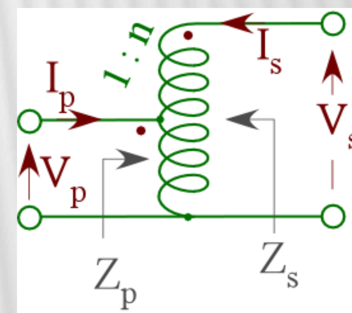


Transformers are widely used in RF electronics. They are very effective to:

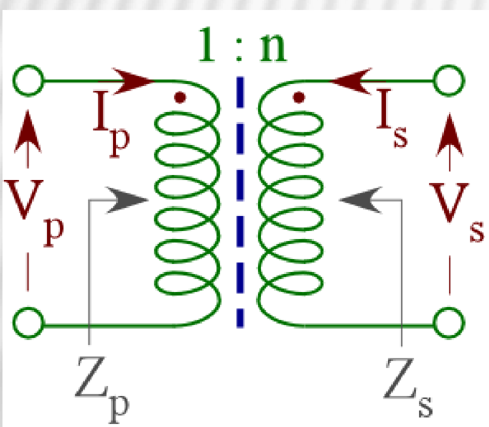
- **Match lines** of **different impedance** with negligible insertion loss;
- **De-couple ground** while transmitting RF signals;
- Connect **balanced** and **unbalanced** circuits (balun)

RF transformers are also embedded in a number of other devices (splitters/combiners, mixers, amplifiers, …)

Autotransformer ($n \geq 1$)



Ideal Transformer transfer functions

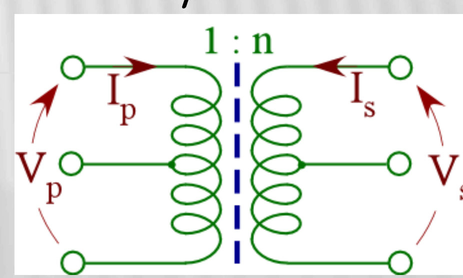


$$V_s = nV_p$$

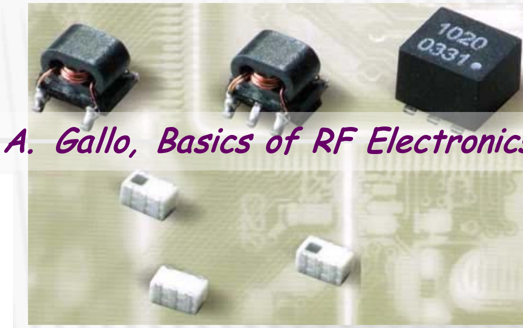
$$I_s = -I_p / n$$

$$Z_p = V_p / I_p = -Z_s / n^2$$

Center-tap transformer



RF TRANSFORMERS



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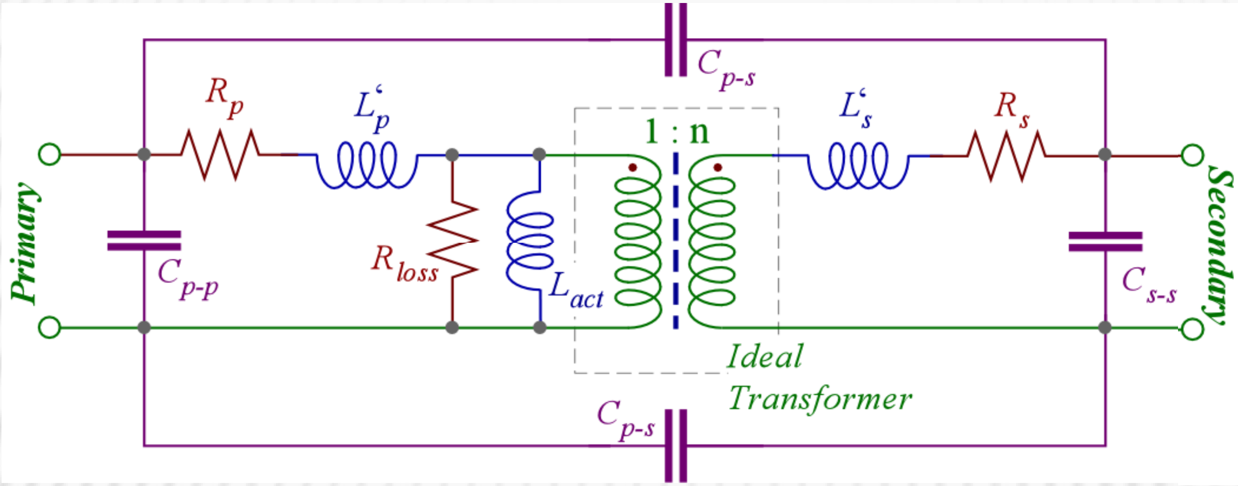
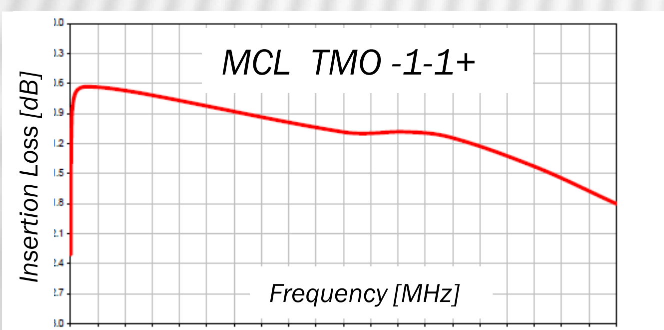
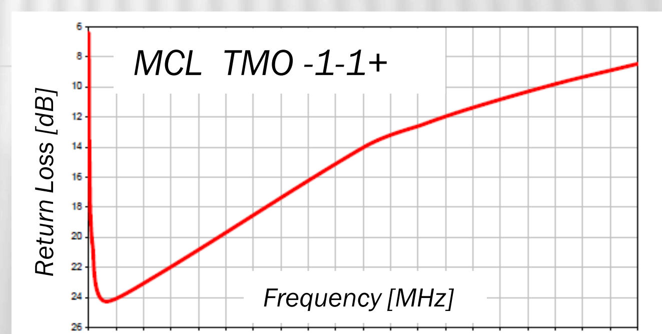


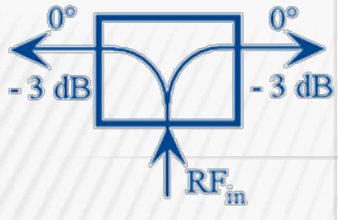
Circuital model of a real RF Transformer

Together with transform ratio n and connection topology, real transformers are characterized by operating bandwidth, insertion loss, max power rating, … The **lower cutoff** frequency is determined by the **windings active inductance** L_{act}, while the **high** frequency **cutoff** is dominated by the **inter-windings** and **intra-windings capacitances** $C_{p\text{-}p}$, $C_{s\text{-}s}$ and $C_{p\text{-}s}$.

In-band **insertion loss** is due to the **magnetic core** dissipation and to the **windings ohmic losses**, accounted by the resistances R_{loss} , R_p **and** R_s .







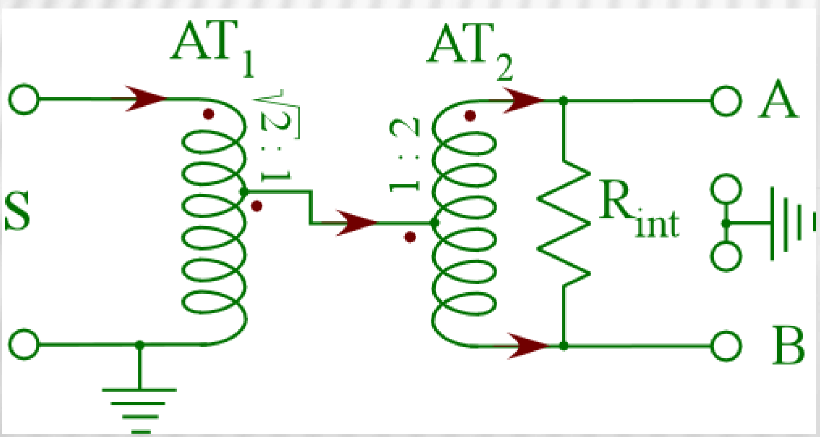
POWER SPLITTERS/COMBINERS

Power splitters/combiners are used to **divide** a signal into **N equal copies** (N = any number, preferably a power of 2), or to make a **vector sum** of N different signals.

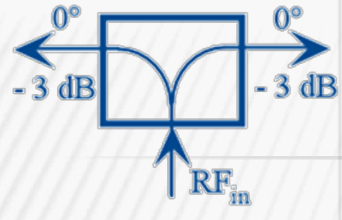
Ideally, the power into any output channel is $P_{out} = P_{in}/N$.

Basic characteristics are:

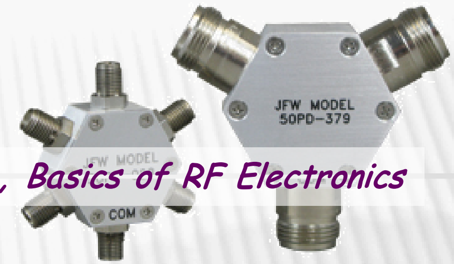
- Number of channels N;
- Operating frequency range;
- Max power ratings;
- Splitting technique (reactive or resistive);
- Insertion loss (over the nominal *10 Log N*);
- Isolation between channels;
- Phase and amplitude unbalance among output channels;
- …



A 2-way reactive splitter/combiner based on a double tapped auto-transformer provides impedance matching at all ports and isolation between A and B channels.



POWER SPLITTERS/COMBINERS



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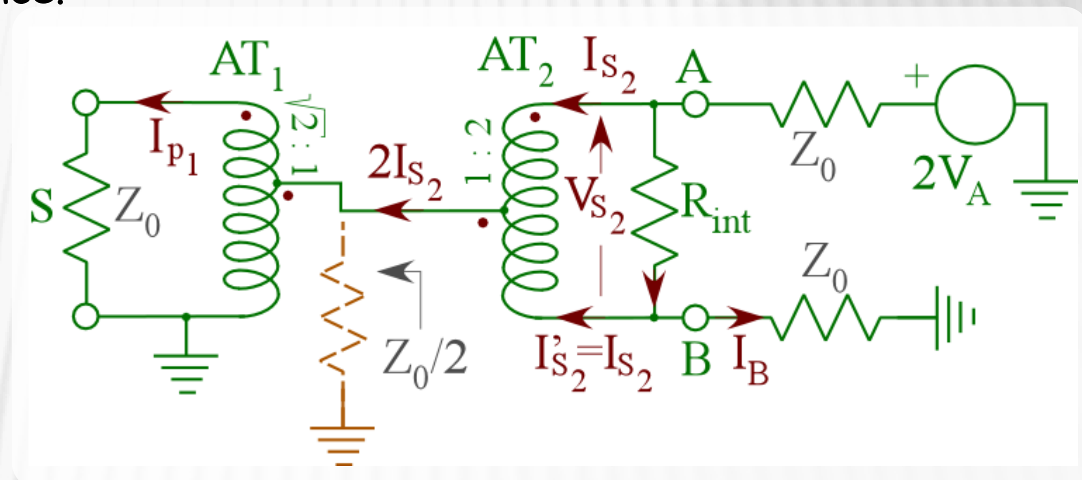
Isolation between ports A and B is obtained by a **proper choice** of the R_{int} value ($R_{int}=2Z_0$) in the 2nd auto-transformer, while the **1st transformer** is needed to **match** the characteristic input impedance.

AT_2 transformer equations

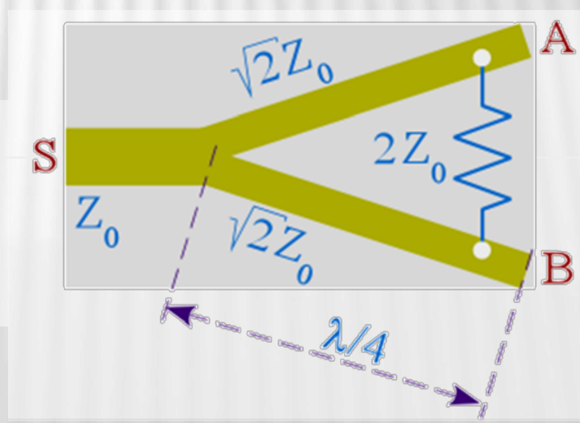
$$\boxed{V_{p_2} = V_{s_2}/2; \quad I_{p_2} = 2I_{s_2}}$$

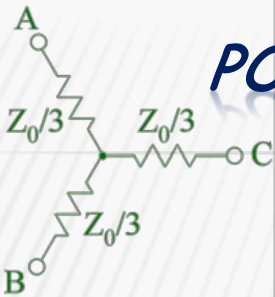
Kirchhoff current law at node B

$$\boxed{I_B = V_{s_2}/R_{int} - I_{s_2}} = $$
$$= I_{s_2}\left(2Z_0/R_{int} - 1\right) \underset{\underbrace{if}_{R_{int}=2Z_0}}{=} 0$$



Micro-strip design of a 2-way splitter/combiner with similar characteristics of a transformer based one.

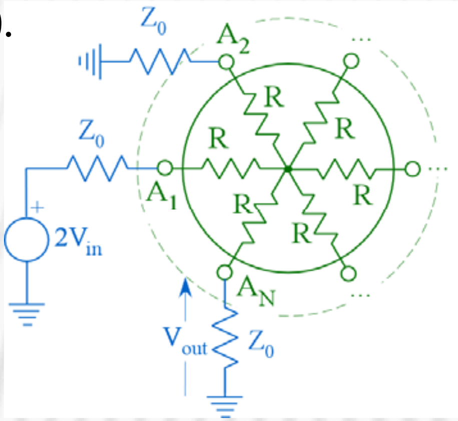




POWER SPLITTERS/COMBINERS

➢ *A. Gallo, Basics of RF Electronics*

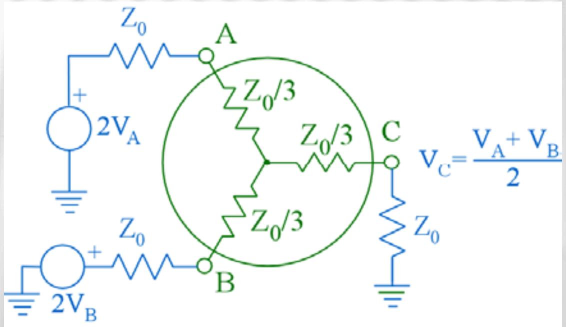
Splitter/combiners can be also **resistive**, consisting in a "star" connection of equal resistors. The **frequency response** can be much **wider** (extending from DC) and **flatter** in this case, at the expense of a **larger insertion loss** and **no isolation** (all ports equally coupled).

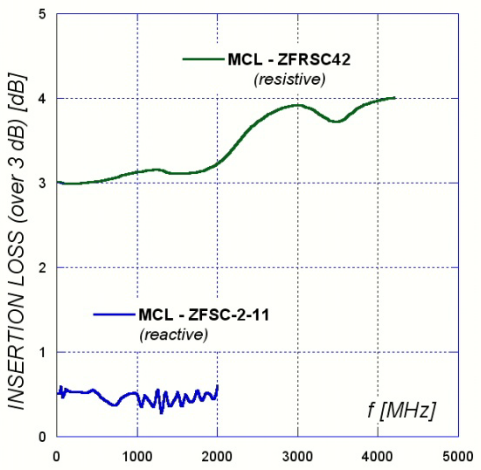


$$Matching \Rightarrow R = \frac{N-2}{N} Z_0$$

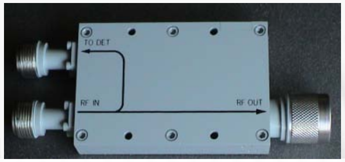
$$Transmission \Rightarrow \frac{P_{out}}{P_{in}} = \left(\frac{V_{out}}{V_{in}}\right)^2 = \frac{1}{(N-1)^2}$$

As the insertion loss grows linearly with the number of ports, practical use is restricted to 3-ports devices.





HYBRID JUNCTIONS/ DIRECTIONAL COUPLERS

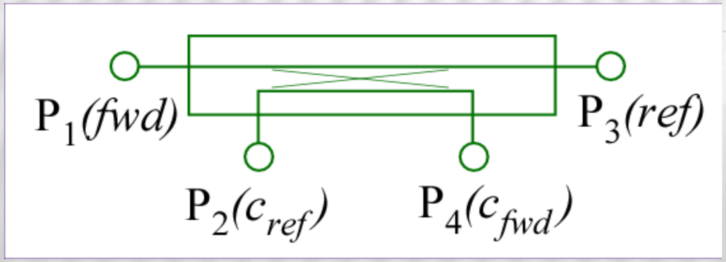


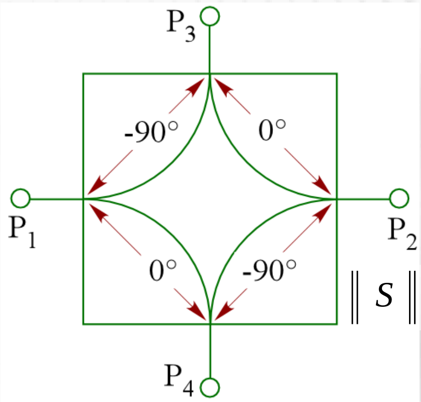
➢ *A. Gallo, Basics of RF Electronics*

Hybrid junctions and **Directional couplers** are **4-ports passive devices** based on the **same operational principles** but with **different coupling levels** between ports. The two class of devices are used for different purposes.

$$\| S \| = \begin{pmatrix} 0 & 0 & -j\sqrt{1-c^2} & c \\ 0 & 0 & c & -j\sqrt{1-c^2} \\ -j\sqrt{1-c^2} & c & 0 & 0 \\ c & -j\sqrt{1-c^2} & 0 & 0 \end{pmatrix}$$

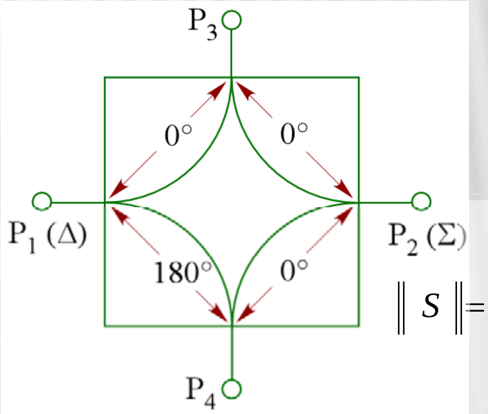
Directional Coupler





90° Hybrid

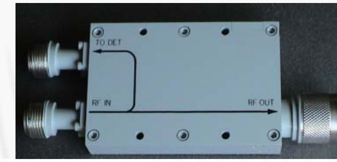
$$\| S \| = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & -j & 1 \\ 0 & 0 & 1 & -j \\ -j & 1 & 0 & 0 \\ 1 & -j & 0 & 0 \end{pmatrix}$$



180° Hybrid

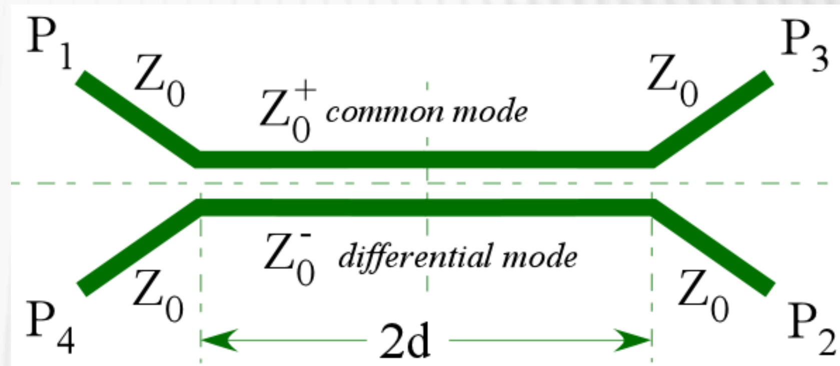
$$\| S \| = \frac{-j}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 \end{pmatrix}$$

HYBRID JUNCTIONS/DIRECTIONAL COUPLERS - COUPLED LINE



➢ *A. Gallo, Basics of RF Electronics*

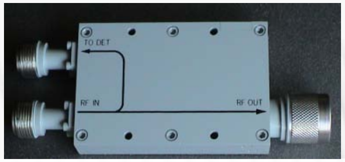
Distributed coupling between 2 lines **travelling close** each other is one of the possible hybrid/coupler configuration.
The lines have a characteristic impedance $\mathbf{Z_0}$ when travelling **separately**, while the 3-conductors system of the 2 coupled lines has **even** and **odd** **excitation** impedances Z_0^+ and Z_0^-, respectively.



The **scattering matrix** can be worked out by exploiting the **4-fold symmetry** of the network. Being $\boldsymbol{\beta^\pm}$ the **propagation constants** of the even and odd modes, we get:

$$\left.\begin{array}{l}
\beta^+ = \beta^- \equiv \beta; \; Z_0^+ Z_0^- = Z_0^2 \\[2ex]
c = \dfrac{c_{max} \sin(2\beta d)}{\sqrt{1 - c_{max}^2 \cos^2(2\beta d)}} \\[2ex]
with:: \; c_{max} = \dfrac{Z_0^+ - Z_0^-}{Z_0^+ + Z_0^-} \\[2ex]
\tan\phi_c = \dfrac{\sqrt{1 - c_{max}^2}}{\tan(2\beta d)}
\end{array}\right\} \Rightarrow \|S\| = \begin{pmatrix}
0 & 0 & -j\sqrt{1-c^2}\, e^{j\phi_c} & c\, e^{j\phi_c} \\
0 & 0 & c\, e^{j\phi_c} & -j\sqrt{1-c^2}\, e^{j\phi_c} \\
-j\sqrt{1-c^2}\, e^{j\phi_c} & c\, e^{j\phi_c} & 0 & 0 \\
c\, e^{j\phi_c} & -j\sqrt{1-c^2}\, e^{j\phi_c} & 0 & 0
\end{pmatrix}$$

HYBRID JUNCTIONS/DIRECTIONAL COUPLERS - COUPLED LINE

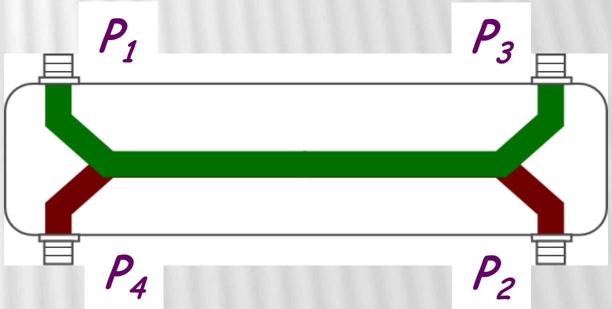


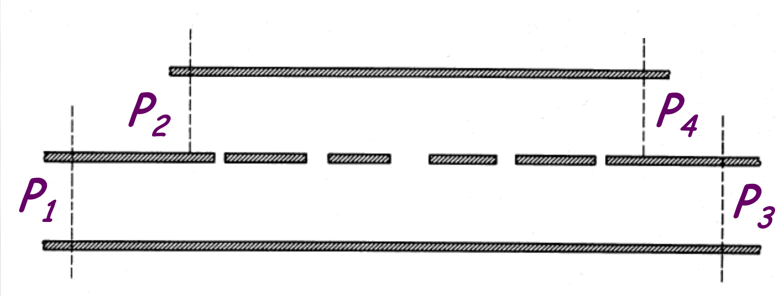
➢ *A. Gallo, Basics of RF Electronics*

If the **length** of the coupled line is exactly **2d=λ/4=π/2β**, the scattering matrix has its simplest form:

$$\left.\begin{array}{c} 2\beta d = \pi/2 \\ \Downarrow \\ c = c_{max} = \dfrac{Z_0^+ - Z_0^-}{Z_0^+ + Z_0^-} \\ \phi_c = 0 \end{array}\right\} \Rightarrow \|S\| = \begin{pmatrix} 0 & 0 & -j\sqrt{1-c_{max}^2} & c_{max} \\ 0 & 0 & c_{max} & -j\sqrt{1-c_{max}^2} \\ -j\sqrt{1-c_{max}^2} & c_{max} & 0 & 0 \\ c_{max} & -j\sqrt{1-c_{max}^2} & 0 & 0 \end{pmatrix}$$

If coupling factors **lower than 10 dB** (c_{max} > 0.3) are required, **broadside coupled strips** can be used. At c_{max} = .707 (i.e. Z_0^+=5.8 Z_0^-) the device scattering matrix is that of a 90° hybrid junction. Coupled wound coils on ferrite cores are used at low frequencies.





Another possible directional coupler structure is represented by **2 parallel lines** connected through **coupling holes**. Two equal holes separated by λ/4 are sufficient to produce the proper scattering matrix at a given frequency. Device bandwidth can be enlarged with multiple holes with different optimal dimensions.

HYBRID JUNCTIONS/DIRECTIONAL COUPLERS - BRANCH LINE

➢ *A. Gallo, Basics of RF Electronics*

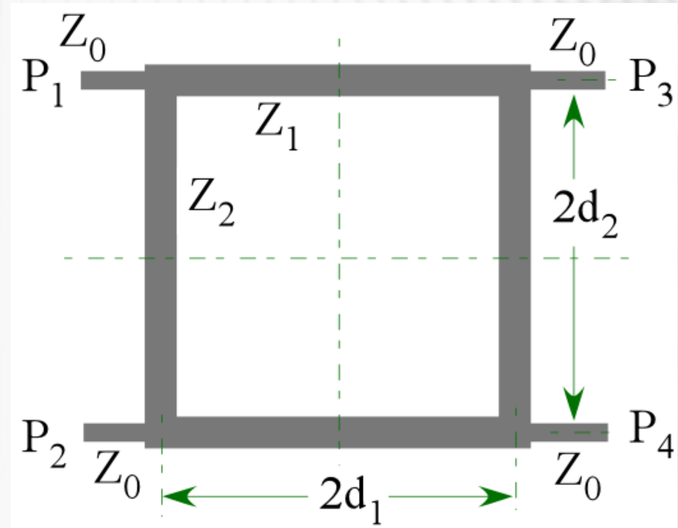
The **branch line coupler** is another geometry giving the desired port-to-port coupling.

The scattering matrix can be still worked out by exploiting the 4-fold symmetry of the network. Under the assumptions:

I) $\beta d_1 = \beta d_2 = \pi/2$;

II) $(1/Z_1)^2 - (1/Z_2)^2 = (1/Z_0)^2$

the scattering matrix has the following form:



Directional coupler

90° Hybrid

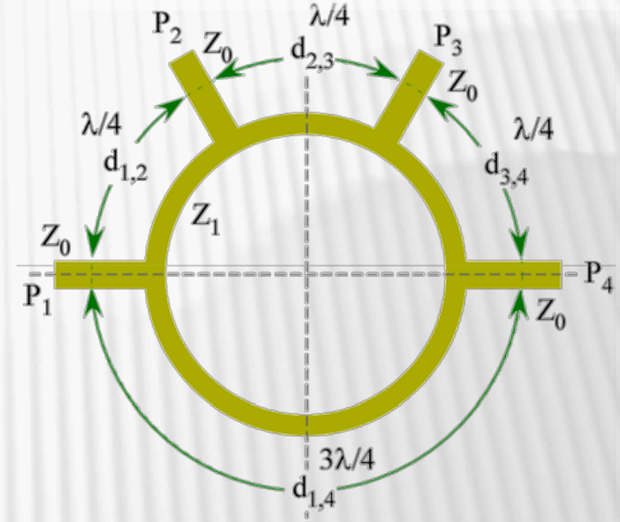
$$\| S \| = \begin{pmatrix} 0 & 0 & -jZ_1/Z_0 & -Z_1/Z_2 \\ 0 & 0 & -Z_1/Z_2 & -jZ_1/Z_0 \\ -jZ_1/Z_0 & -Z_1/Z_2 & 0 & 0 \\ -Z_1/Z_2 & -jZ_1/Z_0 & 0 & 0 \end{pmatrix} =$$

$$Z_2 = Z_0$$
$$Z_1 = Z_0/\sqrt{2}$$

$$= -\frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & j & 1 \\ 0 & 0 & 1 & j \\ j & 1 & 0 & 0 \\ 1 & j & 0 & 0 \end{pmatrix}$$

HYBRID JUNCTIONS/DIRECTIONAL COUPLERS – HYBRID RING

➢ *A. Gallo, Basics of RF Electronics*



The **hybrid ring** (also called "**rat-race**") coupler is a suitable geometry to get **180° hybrids**.

Under the assumptions:

$d_{1,2}=d_{2,3}=d_{3,4}=\pi/2\beta=\lambda/4$; $d_{1,4}=3\pi/2\beta=3\lambda/4$

the scattering matrix has the following form:

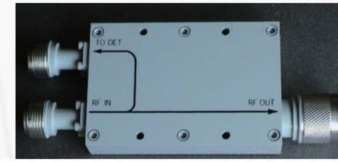
Unmatched
Isolated

180° Hybrid

$$\|S\| = \begin{pmatrix} \frac{Z_1^2-2Z_0^2}{Z_1^2+2Z_0^2} & \frac{-2jZ_1Z_0}{Z_1^2+2Z_0^2} & 0 & \frac{2jZ_1Z_0}{Z_1^2+2Z_0^2} \\ \frac{-2jZ_1Z_0}{Z_1^2+2Z_0^2} & \frac{Z_1^2-2Z_0^2}{Z_1^2+2Z_0^2} & \frac{-2jZ_1Z_0}{Z_1^2+2Z_0^2} & 0 \\ 0 & \frac{-2jZ_1Z_0}{Z_1^2+2Z_0^2} & \frac{Z_1^2-2Z_0^2}{Z_1^2+2Z_0^2} & \frac{-2jZ_1Z_0}{Z_1^2+2Z_0^2} \\ \frac{2jZ_1Z_0}{Z_1^2+2Z_0^2} & 0 & \frac{-2jZ_1Z_0}{Z_1^2+2Z_0^2} & \frac{Z_1^2-2Z_0^2}{Z_1^2+2Z_0^2} \end{pmatrix} = \xrightarrow{Z_1=\sqrt{2}\,Z_0} = \frac{-j}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 & -1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ -1 & 0 & 1 & 0 \end{pmatrix}$$

$$\|S\| \otimes \begin{bmatrix} V_1 \\ 0 \\ V_3 \\ 0 \end{bmatrix} = \frac{-j}{\sqrt{2}} \begin{bmatrix} 0 \\ V_3+V_1 \\ 0 \\ V_3-V_1 \end{bmatrix}$$

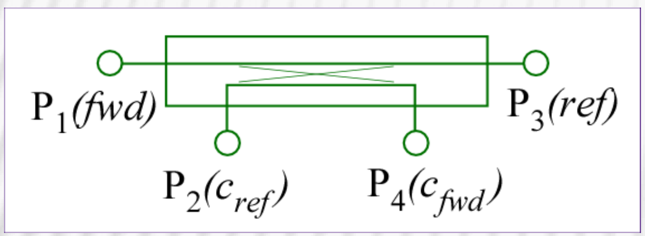
HYBRID JUNCTIONS/ DIRECTIONAL COUPLERS



➢ *A. Gallo, Basics of RF Electronics*

Directional couplers are used to **sample** or to **unequal split/sum** RF signals .

Hybrids are used whenever **splitting/ combination** of RF signals **out-of-phase** (90°) or **counter-phase** (180°) are required (I&Q modulators/ detectors, differential combination, …).



$$V_{P_4} = c\left(V_{P_1} + d\,V_{P_3}\right) \quad with \quad |d| << 1$$

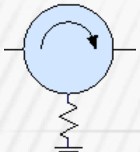
$$\| S \| = \begin{pmatrix} 0 & c\cdot d & -j\sqrt{1-c^2} & c \\ c\cdot d & 0 & c & -j\sqrt{1-c^2} \\ -j\sqrt{1-c^2} & c & 0 & c\cdot d \\ c & -j\sqrt{1-c^2} & c\cdot d & 0 \end{pmatrix}$$

Coupler Directivity: -20 Log|d|

measurement of the imperfect isolation between ideally uncoupled port.

Basic characteristics are:

- Coupling coefficient (3 dB for hybrids);
- Directivity / Isolation between un-coupled ports;
- Operating frequency range;
- Max power ratings;
- Coupling type (holes, distributed, rings, …);
- Insertion loss (over the nominal coupling factor);
- Phase and amplitude unbalance among output channels;
- Phase and amplitude flatness over frequency;
- …

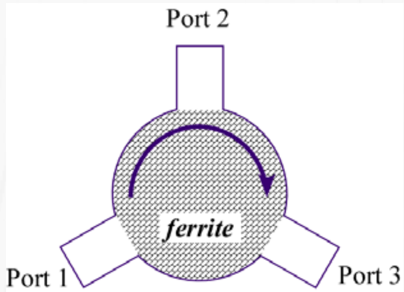


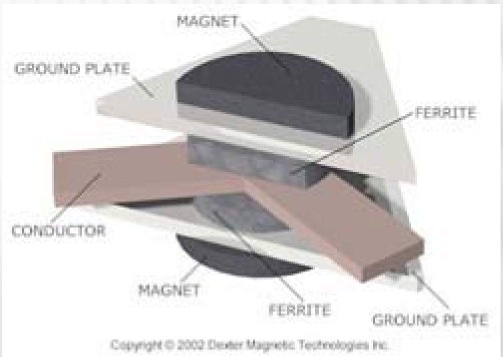
CIRCULATORS / ISOLATORS

➢ *A. Gallo, Basics of RF Electronics*

Circulators are **non-reciprocal 3-ports** (typically) devices whose scattering matrix is ideally given by:

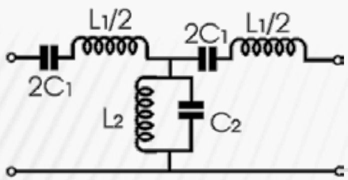
$$S = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

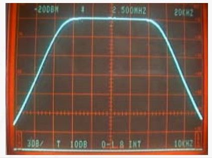




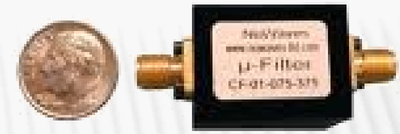
Their basic structure is a **symmetrical, 120° Y junction** with a **ferrite disk** placed at the center **biased** by an **axial magnetic field**.
Biased ferrites show a **tensor magnetic permeability**, i.e. an anisotropic behavior. The incident wave on one port excites 2 unbalanced (because of the anisotropy) waves rotating in the 2 opposite directions, so that the coupling to the output ports is also unbalanced. By proper design the junction it is possible to have almost 100% transmission in one port and no transmission in the other in a given frequency band (> 1 octave).

Isolators are **circulators** with one port **internally terminated**. Circulators and isolators are used for a number of tasks, such us matching sources and loads, protecting sources against backward power, capturing and draining reflected power from a device to another device.





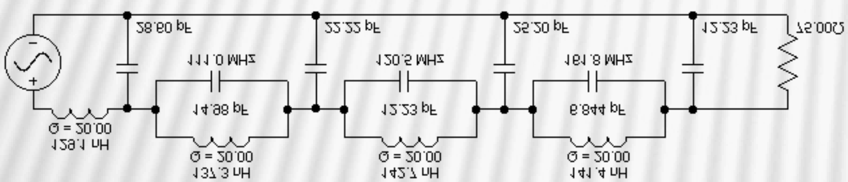
FILTERS

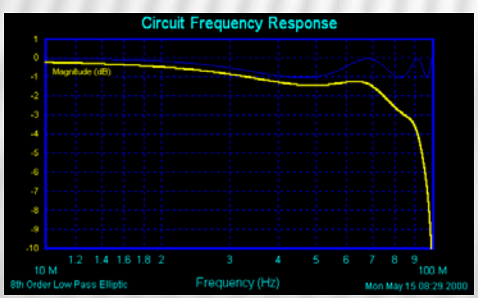


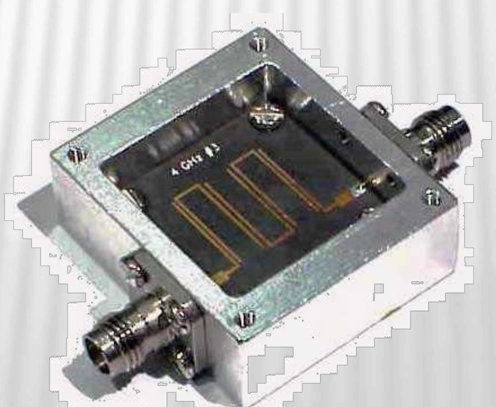
Filters are **2-ports** devices "tailored" to obtain a specific required **frequency response** (the s_{21} of the network). Typically the response is maximized at some bands of interest, and minimized at other frequency bands that have to be rejected.

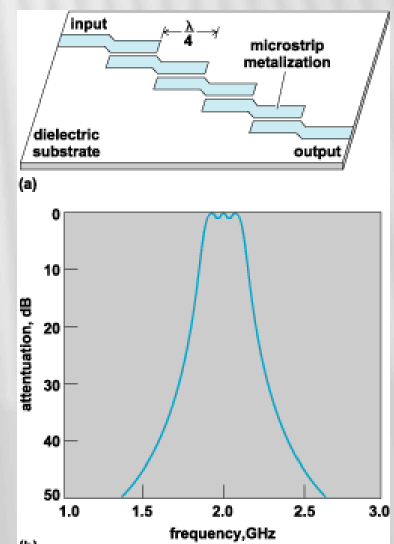
Filters are **classified** on the base of their **nature, topology, dissipation**, …

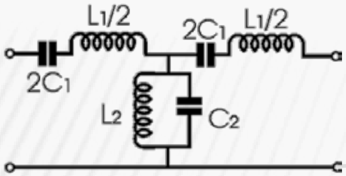
- **Analog/Digital**, depending on the nature (continuous or sampled & digitized) of the input signal;
- **Lumped/Distributed**, depending on the nature of the internal components (L-C cells, DR cavities, μ-strip cells, …);
- **Reflective/Absorbing** depending on the path of the stopbands (reflected or internally dissipated);
- **LowPass/HighPass/BandPass/Notch/Comb**, depending on the profile of the frequency response.

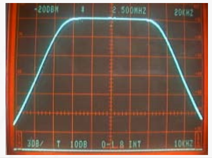




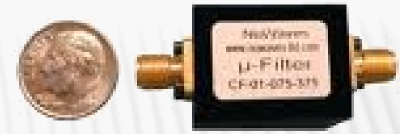








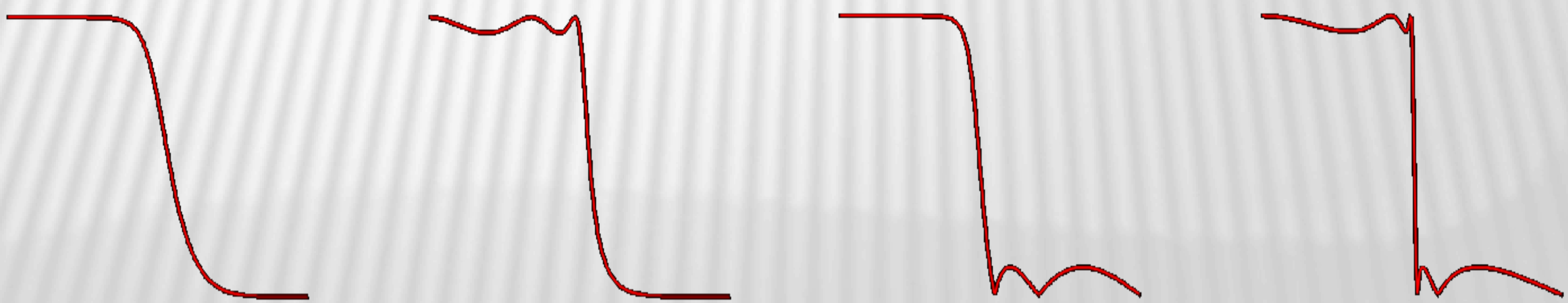
FILTERS



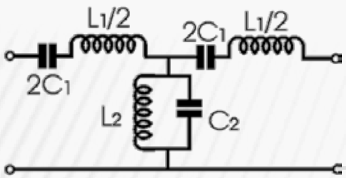
➢ *A. Gallo, Basics of RF Electronics*

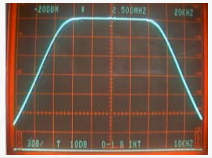
Filters with **different response** around **transition** between pass and stop bands are available for different applications. They implements **different rational complex polynomials** in their transfer functions, the most popular ones being:

- **Bessel,** for a maximally flat group delay;
- **Butterworth,** for a maximally flat frequency response in the pass-band;
- **Gaussian,** providing a gaussian response to a Dirac pulse and no overshoot for an input step function. The Gaussian filter also minimizes the group delay;
- **Chebyshev,** providing a steep transition with some passband (type I) or stopband (type II) ripples. They provide the closest possible response w.r.t. an ideal rectangular filter;
- **Elliptic,** providing the steeper possible transition between the pass-band and the stop-band by equalizing the ripple in both.

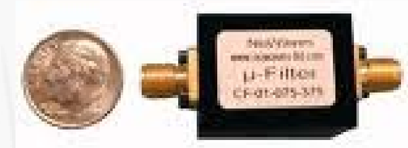


Butterworth Chebyshev I Chebyshev II Elliptic



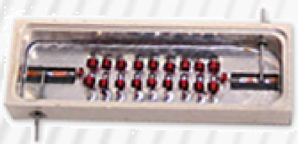


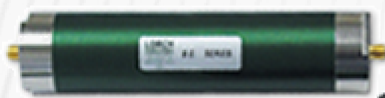
FILTERS

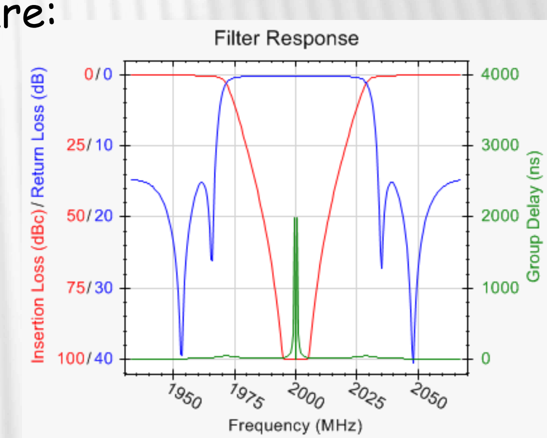


➢ *A. Gallo, Basics of RF Electronics*

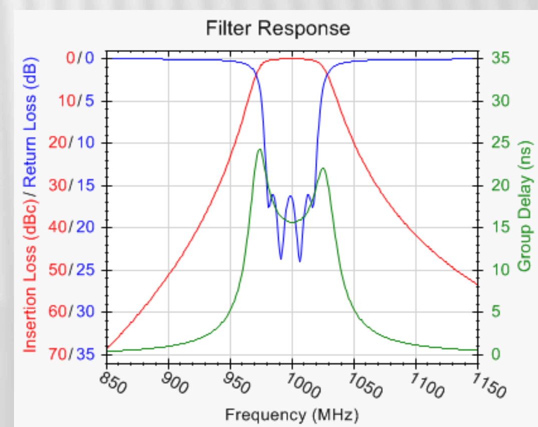
Design and construction of filters is becoming more and more a **specialized activity**, so that "home made" devices are seldom used, and mainly for very specific task. Design phase make use of **dedicated software packages** (such as Touchstone) through various iterative steps. In addition to the typologies already listed, other important characteristics defining filter performances are:



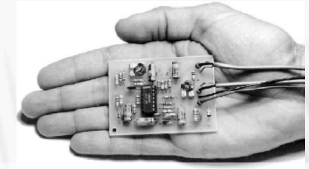




- **Insertion Loss**, defined as the in-band signal attenuation;
- **Phase linearity/Group delay**: figures of the quality of the filter phase response across the pass-band, that should present a constant negative slope to avoid distortion of time-profile of in-band pulses. The **slope** of the **response phase** is the filter **group delay**, equal to the signal latency while travelling across the device.
- **Input/output impedance** and **VSWR**: characteristic impedance of the device and reflectivity of signals transmitted (within pass-band) and rejected (within stop-band).



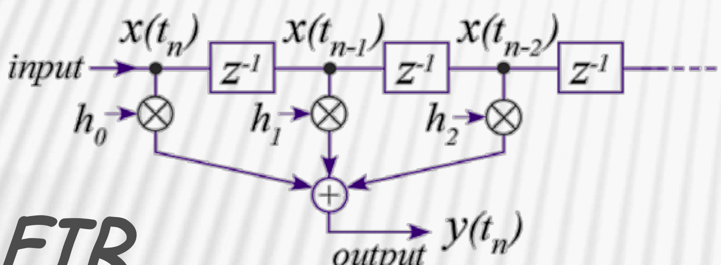
DIGITAL FILTERING



➢ *A. Gallo, Basics of RF Electronics*

Digital filters act on **sampled** and **digitized** input signals. There are 2 basic filter architectures:

- **Finite Impulsive Response** (FIR), where the output ***y*** is a linear combination of the last N sampled values of the input ***x***. The coefficients h_i of the expansion represent the discretization of the filter Green's function.
- **Infinite Impulsive Response** (IIR), where the output ***y*** is a linear combination of the last N and M sampled values of the input ***x*** and output ***y***, respectively. IIR filters directly implement a feedback architecture, which may generate sharp frequency responses with a limited number of samples.

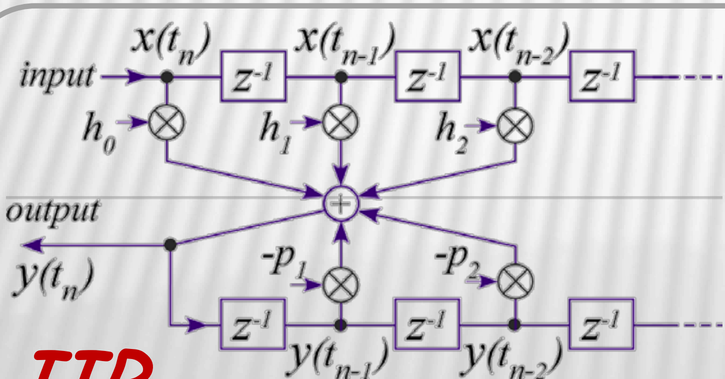


FIR

H(τ) =filter Green's function $\quad y(t)=\int_{-\infty}^{t} H(\tau)\,x(t-\tau)\,d\tau$

$$y(t_n)=h_0\,x(t_n)+h_1\,x(t_{n-1})+h_2\,x(t_{n-2})+\ldots.=\sum_{i=0}^{N} h_i\,x_{n-i}$$

$$H(z)=\frac{Y(z)}{X(z)}=h_0+h_1 z^{-1}+h_2 z^{-2}+\ldots=\sum_{i=0}^{N} h_i\,z^{-i}$$

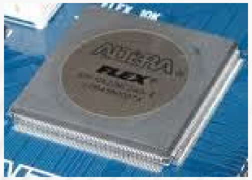


IIR

$$y(t_n)=h_0\,x(t_n)+h_1\,x(t_{n-1})+h_2\,x(t_{n-2})+\ldots.$$

$$\ldots-p_1\,y(t_{n-1})-p_2\,x(t_{n-2})+\ldots.=\sum_{i=0}^{N} h_i x_{n-i}-\sum_{k=1}^{M} p_k y_{n-k}$$

$$H(z)=\frac{Y(z)}{X(z)}=\frac{h_0+h_1 z^{-1}+h_2 z^{-2}+\ldots}{1+p_1 z^{-1}+p_2 z^{-2}+\ldots}=\frac{\sum_{i=0}^{N} h_i\,z^{-i}}{1+\sum_{k=1}^{M} p_k\,z^{-k}}$$

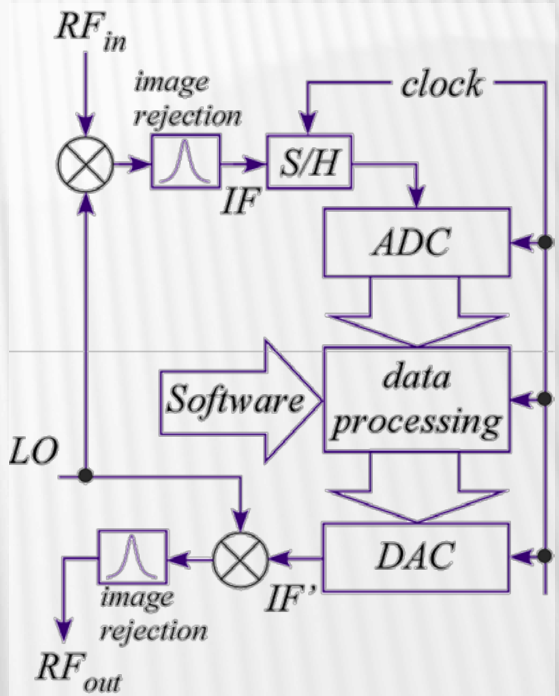


DIGITAL FILTERING

➢ *A. Gallo, Basics of RF Electronics*

The z-domain transfer function ***$H_z(z)$*** gives direct information on the filter frequency response being related to the Laplace ***$H_L(s)$*** and Fourier ***$H_F(j\omega)$*** transfer functions by the mathematical expressions:

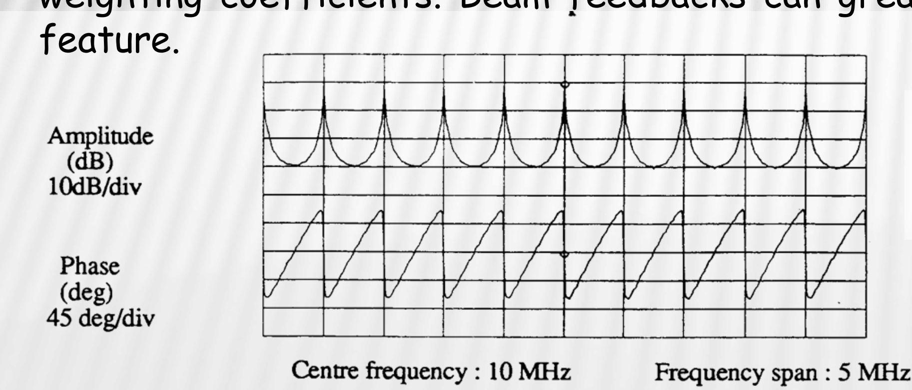
$$H_L(s) = H_z(z)\Big|_{z=e^{sT}} \qquad H_F(j\omega) = H_z(z)\Big|_{z=e^{j\omega T}}$$



Differences between analog and digital filtering are quite evident. **Digital filtering** is a complex operation requiring **many steps** such as down-conversion (necessary in most cases), A-to-D conversion, digital data manipulation, D-to-A conversion and final frequency up-conversion.

However, powerful ICs nowadays available (DSP, FPGA, …) are capable to perform various tasks in a single chip.

On the other hand, digital filtering provide **incomparable flexibility** and **operational adaptivity**, since the transfer function can be modified and optimized in real time by simply changing the weighting coefficients. Beam feedbacks can greatly benefit this feature.



$$k = 80$$

$$p_k = 1 - 2^{-4}$$

$$H_z(z) = \frac{1}{1 - p_k z^{-k}}$$

Comb filter transfer function

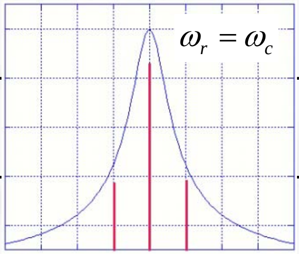
MODULATION TRANSFER FUNCTIONS

➢ *A. Gallo, Basics of RF Electronics*

LLRF servo-loops and feedback loops often need to **apply AM** and **PM modulation** to the **RF drive** signal. The response of a **resonant cavity** to AM and PM excitations depends on its **bandwidth** and **tuning** relative to the carrier:

$$v_i(t) = A_i\left[1 + a_i(t)\right]\cos\left(\omega_c t\right) \longrightarrow v_o(t) = A_o\left[1 + a_o(t)\right]\cos\left(\omega_c t\right)$$

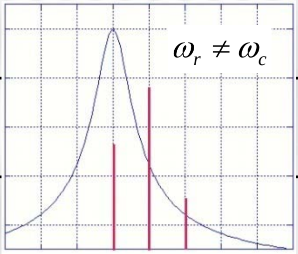
$$v_i(t) = A_i\cos\left[\omega_c t + \phi_i(t)\right] \longrightarrow v_o(t) = A_o\cos\left[\omega_c t + \Delta\phi_o + \phi_o(t)\right]$$



$$x(t) \overbrace{\Rightarrow}^{L-transform} \hat{x}(s) \quad \Longrightarrow \quad G(s) = \frac{\hat{a}_o(s)}{\hat{a}_i(s)} = \frac{\hat{\phi}_o(s)}{\hat{\phi}_i(s)} = \frac{1}{1 + s/\sigma} \quad with \quad \sigma = \frac{\omega_r}{2Q_L}$$

$$v_i(t) = A_i\left[1 + a_i(t)\right]\cos\left(\omega_c t\right) \longrightarrow v_o(t) = A_o\left[1 + a_{o,a}(t)\right]\cos\left[\omega_c t + \Delta\phi_o + \phi_{o,a}(t)\right]$$

$$v_i(t) = A_i\cos\left[\omega_c t + \phi_i(t)\right] \longrightarrow v_o(t) = A_o\left[1 + a_{o,p}(t)\right]\cos\left[\omega_c t + \Delta\phi_o + \phi_{o,p}(t)\right]$$



$$G_{aa}(s) = \frac{\hat{a}_{o,a}(s)}{\hat{a}_i(s)}; \quad G_{pp}(s) = \frac{\hat{\phi}_{o,p}(s)}{\hat{\phi}_i(s)}; \quad G_{ap}(s) = \frac{\hat{\phi}_{o,a}(s)}{\hat{a}_i(s)}; \quad G_{pa}(s) = \frac{\hat{a}_{o,p}(s)}{\hat{\phi}_i(s)}$$

MODULATION TRANSFER FUNCTION

➢ *A. Gallo, Basics of RF Electronics*

It may be demonstrated that **direct** and **cross** modulation transfer functions are given by:

$$G_{pp}(s) = G_{aa}(s) = \frac{1}{2}\left[\frac{A(s+j\omega_c)}{A(j\omega_c)} + \frac{A(s-j\omega_c)}{A(-j\omega_c)}\right]; \quad G_{ap}(s) = -G_{pa}(s) = \frac{1}{2j}\left[\frac{A(s+j\omega_c)}{A(j\omega_c)} - \frac{A(s-j\omega_c)}{A(-j\omega_c)}\right]$$

with $A(s)$ = transfer function in Laplace domain of the filter applied to the modulated signal. If the signal is filtered by a resonant cavity, one has to consider $A(s)=A_{cav}(s)$ given by:

$$A_{cav}(s) = A_0 \frac{2\sigma s}{s^2 + 2\sigma s + \omega_r^2} \quad \text{with} \quad \omega_r \approx \omega_c + \sigma \tan\phi_z$$

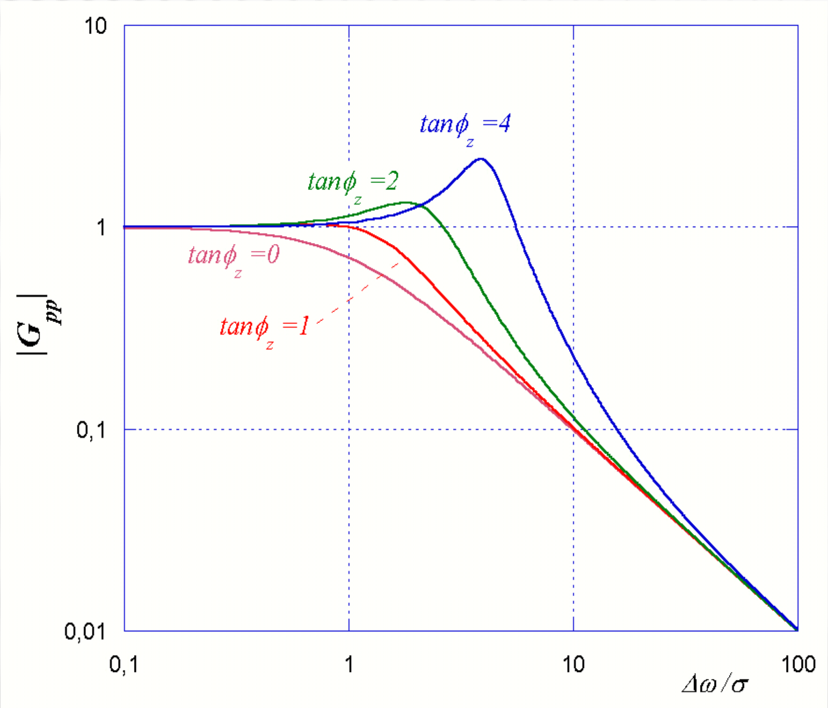
where ϕ_z is the ***cavity tuning angle***, i.e. the phase of the cavity transfer function at the carrier frequency ω_c. Finally one gets:

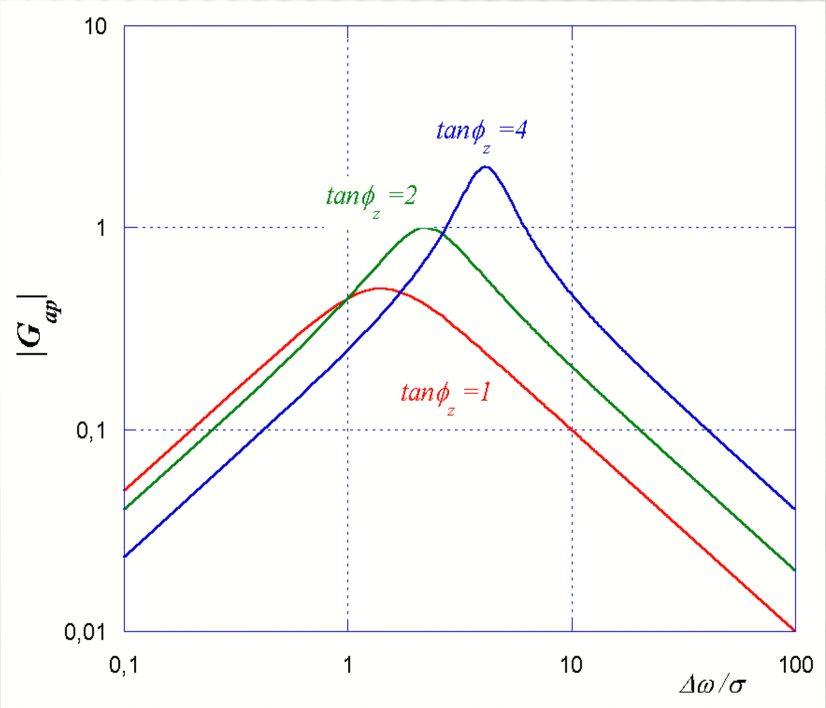
$$G_{pp}(s) = G_{aa}(s) = \frac{\sigma s + \sigma^2\left(1+\tan^2\phi_z\right)}{s^2 + 2\sigma s + \sigma^2\left(1+\tan^2\phi_z\right)}; \quad G_{ap}(s) = -G_{pa}(s) = -\frac{\sigma \tan\phi_z\, s}{s^2 + 2\sigma s + \sigma^2\left(1+\tan^2\phi_z\right)}$$

MODULATION TRANSFER FUNCTION

➢ *A. Gallo, Basics of RF Electronics*

The general form of the modulation transfer functions features **2 poles** (possibly a complex conjugate pair) and **1 zero**, and degenerates to a **single pole LPF** response if the cavity is **perfectly tuned** (cross modulation terms vanish in this case).

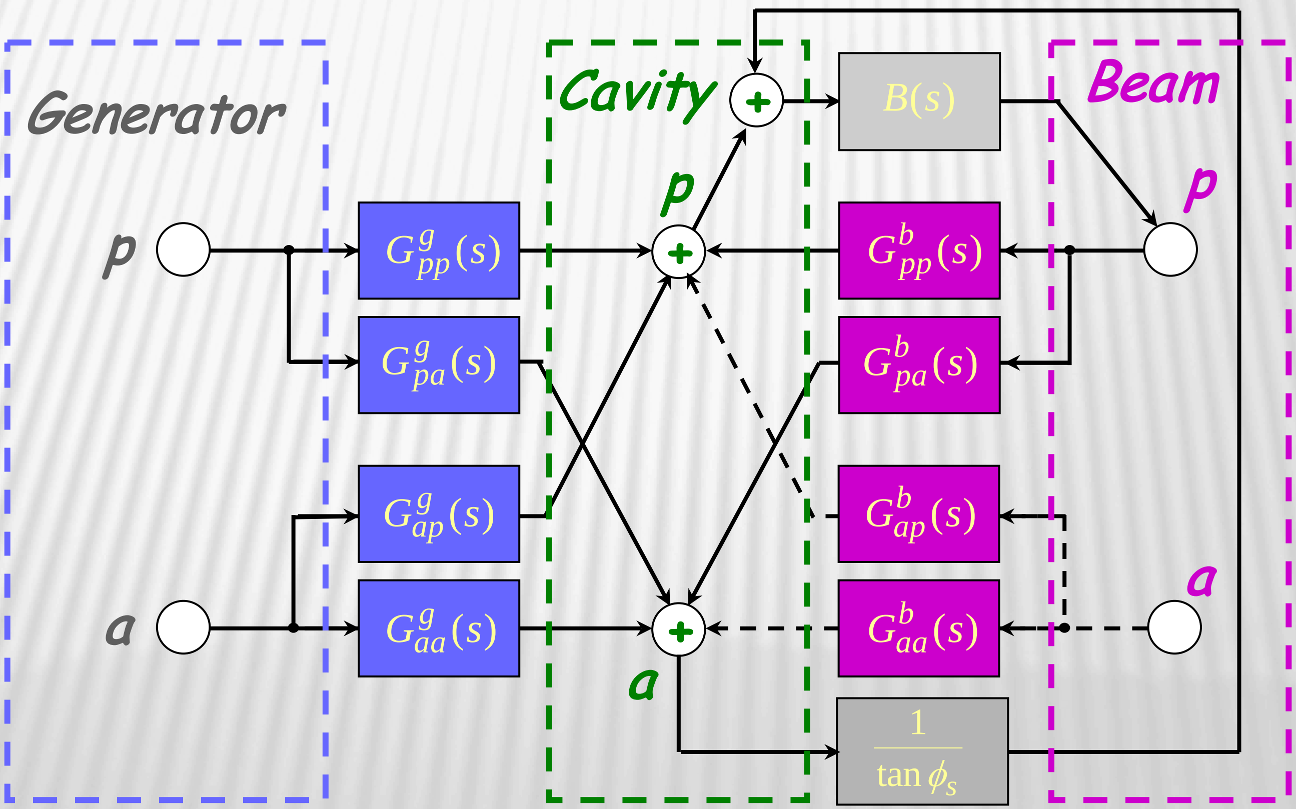




MODULATION TRANSFER FUNCTION: THE PEDERSEN MODEL

➢ *A. Gallo, Basics of RF Electronics*

In **circular accelerators** the **beam phase** depends on the **cavity RF phase** through the **beam transfer function**, while the **cavity RF amplitude** and **phase** depend on the **beam phase** through the **beam loading mechanism**. The whole **generator-cavity-beam linear system** can be graphically represented in a **diagram** called **Pedersen Model**.
The modulation transfer functions vary with the stored current and definitely couple the servo-loops and the beam loops implemented around the system.



END OF THE 1ST PART

Thank you for the moment

and …

See you tomorrow at 15.30