RF Power Transportation

Part I

Stefan Choroba, DESY, Hamburg, Germany

HELMHOLTZ ASSOCIATION

DESY

Overview

Part I

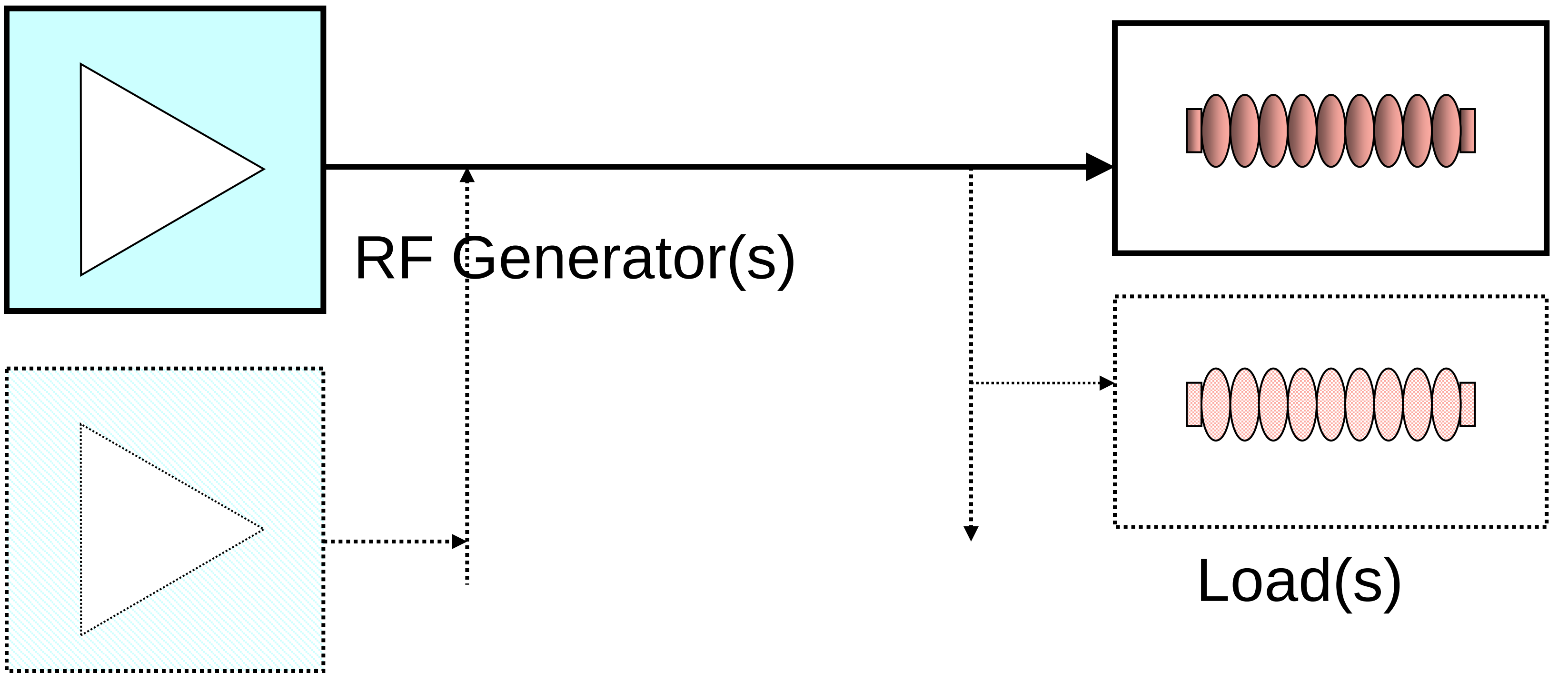
- Introduction
- Theory of Electromagnetic Waves in Waveguides
- TE_{10}-Mode

Part II

- Reminder
- Waveguide Elements
- Waveguide Distributions
- Limitations, Problems and Countermeasures

Introduction

RF Power Transportation

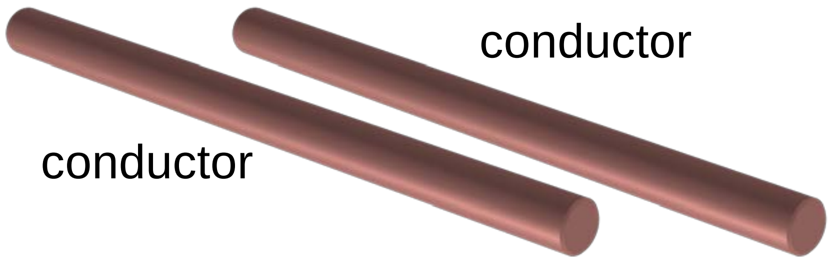


•Purpose: Transmission of RF power of several kW up to several MW at frequencies from the MHz to GHz range. The RF power generated by an RF generator or a number of RF generators must be combined, transported and distributed to a load or cavity or a number of loads or cavities.

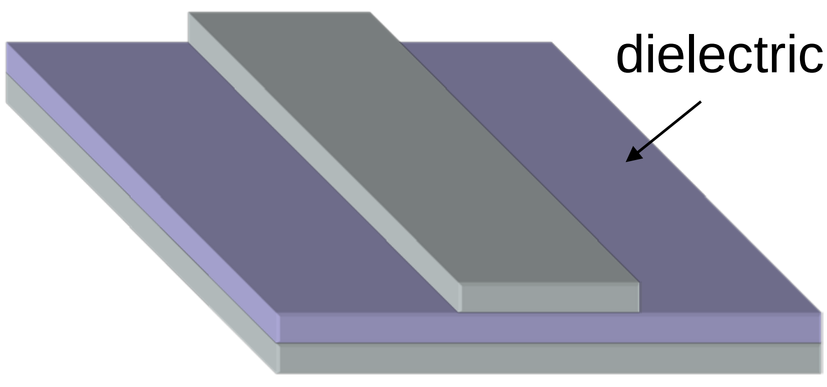
•Requirements: low loss, high efficieny, low reflections, high reliability, adjustment of phase and amplitude, ….

Transmission Lines for RF Transportation

- Two-wire lines (Lecher Leitung)
  - often used for indoor antenna (e.g. radio or TV)
  - problem: radiation to the environment, can not be used for high power transportation

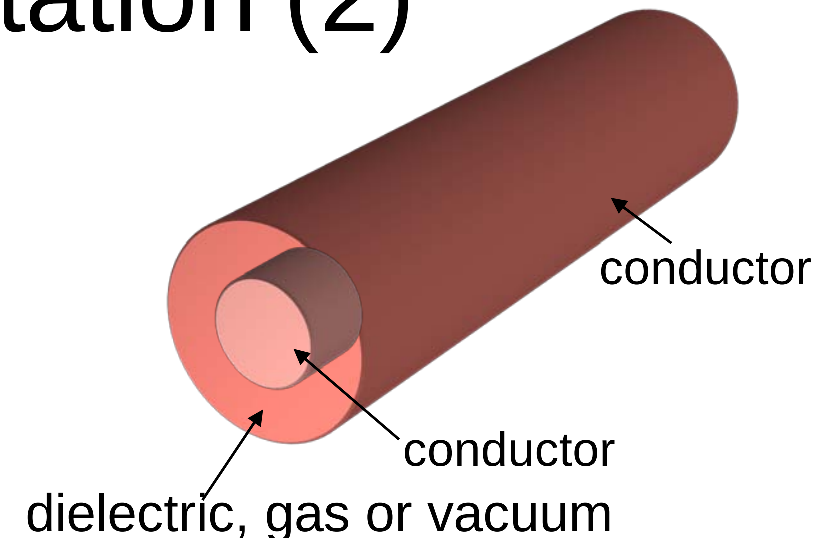


- Strip-lines
  - often used for microwave integrated circuits
  - problem: radiation to the environment and limited power capability, can not be used for high power transportation

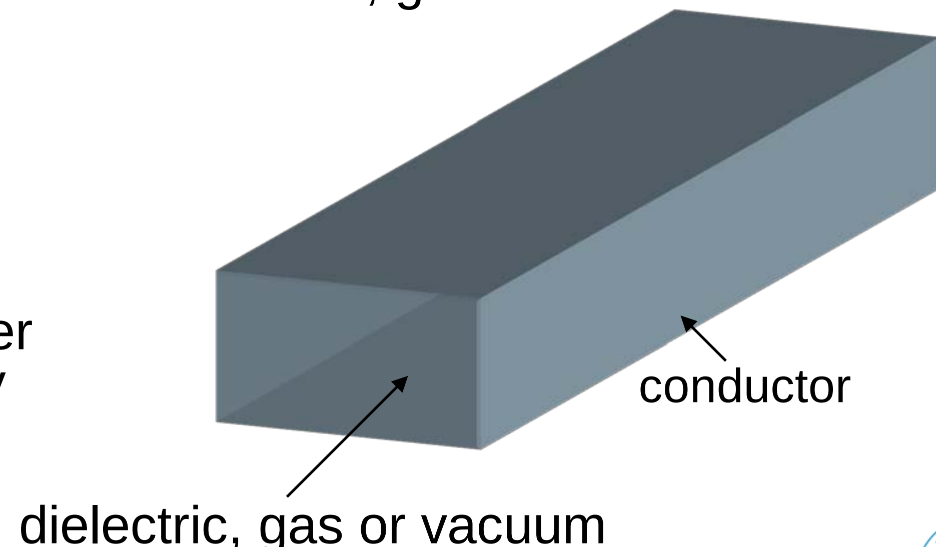


Transmission Lines for RF Transportation (2)

- Coaxial transmission lines
  - often used for power RF transmission and connection of RF components
  - problem: high loss above a certain frequency due to heating of inner conductor and dielectric material and limited power capability at higher frequencies due to small dimensions



- Waveguides (rectangular,cylindrical or elliptical)
  - often used for high power RF transmission (mostly rectangular)
  - problem: waveguide plumbing, rigidity



Theory of Electromagnetic Waves in Waveguides

Strategy for Calculation of Fields in Lines for Power Transportation

- start with Maxwell equation
- derive wave equation
- Ansatz: separation into transversal and longitudinal field components
- wave equation for transversal and longitudinal components
- rewrite Maxwell equation in transversal and longitudinal components
- solve eigenvalue problem for three cases

  TEM (E_z=H_z =0), TE (E_z=0, $H_z \neq$0), TM (H_z=0, $E_z \neq$0)
- derive properties of the the solutions

Maxwell Equations

$$\nabla \times \mathbf{H} = \partial/\partial t\, \mathbf{D} + \mathbf{j}$$ Amperes law

$$\nabla \times \mathbf{E} = -\partial/\partial t\, \mathbf{B}$$ Faradays law

$$\nabla \cdot \mathbf{D} = \rho$$ Gauss law

$$\nabla \cdot \mathbf{B} = 0$$

$\mathbf{B}$ magnetic field, $\mathbf{H}$ magnetic intensity, $\mathbf{D}$ electric displacement,

$\mathbf{E}$ electric field

with $\rho = 0$ (no external charges), $\mathbf{j} = 0$ (no external current),

$\mathbf{B} = \mu\mathbf{H}$ (μ permeability) and $\mathbf{D} = \varepsilon\mathbf{E}$ (ε permittivity) one gets

$$\nabla \times \mathbf{H} = \varepsilon\, \partial/\partial t\, \mathbf{E}$$

$$\nabla \times \mathbf{E} = -\mu\, \partial/\partial t\, \mathbf{H}$$

$$\nabla \cdot \mathbf{E} = 0$$

$$\nabla \cdot \mathbf{H} = 0$$

Wave Equation

The wave equation can be derived from the Maxwell equations.

start with $\nabla \times \mathbf{E} = -\mu \partial / \partial t \, \mathbf{H}$ and apply curl

$$\Rightarrow \quad \nabla \times \nabla \times \mathbf{E} = -\mu \nabla \times \partial / \partial t \, \mathbf{H}$$

use of $\nabla \times \mathbf{H} = \varepsilon \partial / \partial t \, \mathbf{E}$ and $\nabla \times \nabla \times = \nabla(\nabla \cdot) - \nabla^2$

$$\Rightarrow \quad \nabla(\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E} = -\mu\varepsilon \partial^2 / \partial t^2 \, \mathbf{E}$$

use $\nabla \cdot \mathbf{E} = 0$

$$\Rightarrow \quad \boxed{\nabla^2 \mathbf{E} - \mu\varepsilon \partial^2 / \partial t^2 \, \mathbf{E} = 0}$$ wave equation for **E**

in the same way for H

$$\boxed{\nabla^2 \mathbf{H} - \mu\varepsilon \partial^2 / \partial t^2 \, \mathbf{H} = 0}$$ wave equation for **H**

Ansatz for Wave Equation

Separation of fields in a function depending on transversal coordinates only and a wave moving to the right depending on longitudinal coordinate and time.

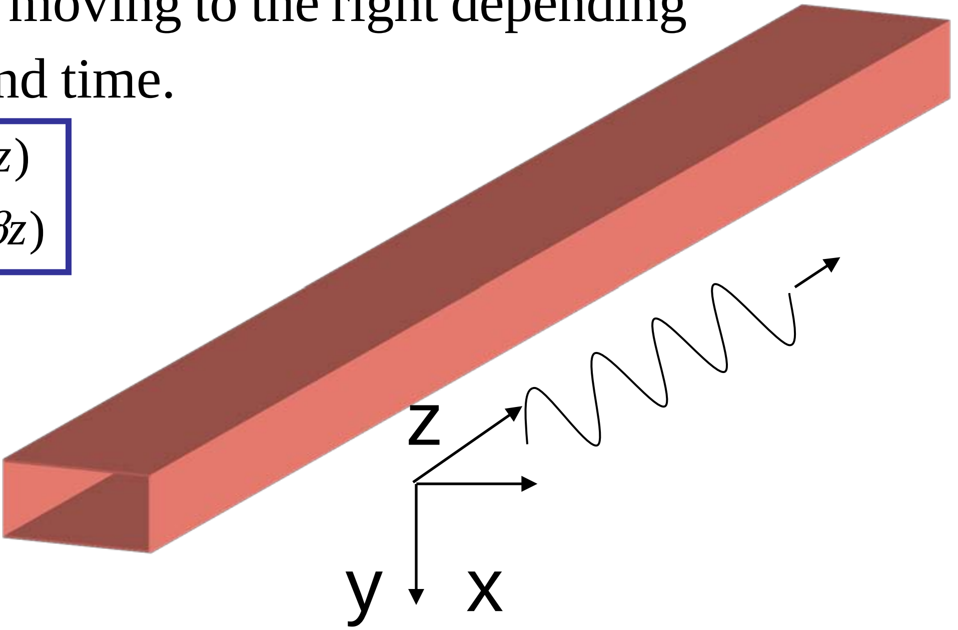
$$\mathbf{E}(x,y,z,t) = \mathbf{E}(x,y)\exp i(\omega t - \beta z)$$
$$\mathbf{H}(x,y,z,t) = \mathbf{H}(x,y)\exp i(\omega t - \beta z)$$

results in :

$$\nabla^2\mathbf{E} + \mu\varepsilon\omega^2\mathbf{E} = 0$$

with : $k^2 = \mu\varepsilon\omega^2$ and $\gamma = i\beta$

$$\nabla_t^2\mathbf{E} + (\gamma^2 + k^2)\mathbf{E} = 0$$



Note :

∇_t operates on transversal coordinates only, e.g. x, y (or r, Φ).

$\mathbf{E}(x,y), \mathbf{H}(x,y)$ are still vectors with longitudinal and transversal components but they depend only on transversal coordinates.

Derivation of Maxwell Equation for transversal and longitudinal Components

With $\nabla \times \mathbf{E} = -i\omega\mu\mathbf{H}$

$$\nabla \times \mathbf{E} = \left(\nabla_t - i\beta\mathbf{e}_z\right)\times\left(\mathbf{E}_t + \mathbf{E}_z\right) = -i\omega\mu\left(\mathbf{H}_t + \mathbf{H}_z\right)$$

$$\nabla_t \times \mathbf{E}_t - i\beta\mathbf{e}_z \times \mathbf{E}_t + \nabla_t \times \mathbf{E}_z - i\beta\mathbf{e}_z \times \mathbf{E}_z = -i\omega\mu\mathbf{H}_t - i\omega\mu\mathbf{H}_z$$

$$\nabla_t \times \mathbf{E}_z = \nabla_t \times \mathbf{e}_z E_z = -\mathbf{e}_z \times \nabla_t E_z$$

Now one can see

$\nabla_t \times \mathbf{E}_t \parallel \mathbf{e}_z$

$\mathbf{e}_z \times \mathbf{E}_t \perp \mathbf{e}_z$

$\nabla_t \times \mathbf{E}_z \perp \mathbf{e}_z$

$\mathbf{e}_z \times \mathbf{E}_z = 0$

Separation of longitudinal and transversal components

$$\nabla_t \times \mathbf{E}_t = -i\omega\mu\mathbf{H}_z$$

$$-i\beta\mathbf{e}_z \times \mathbf{E}_t + \nabla_t \times \mathbf{E}_z =$$

$$-i\beta\mathbf{e}_z \times \mathbf{E}_t - \mathbf{e}_z \times \nabla_t E_z = -i\omega\mu\mathbf{H}_t$$

Derivation Maxwell Equation for transversal and longitudinal Components (2)

With $\nabla \times \mathbf{H} = i\omega\varepsilon\mathbf{E}$ in the same manner

$$\nabla_t \times \mathbf{H}_t = i\omega\varepsilon\mathbf{E}_z$$

$$i\beta\mathbf{e}_z \times \mathbf{H}_t + \mathbf{e}_z \times \nabla_t H_z = -i\omega\varepsilon\mathbf{E}_t$$

With $\nabla \cdot \mathbf{H} = 0$

$$\nabla_t \cdot \mathbf{H}_t = i\beta H_z$$

With $\nabla \cdot \mathbf{E} = 0$

$$\nabla_t \cdot \mathbf{E}_t = i\beta E_z$$

Equations of transversal and longitudinal Components as Function of transversal Coordinates

$$\nabla_t^2 \mathbf{E} + (\gamma^2 + k^2)\mathbf{E} = 0$$

$$\nabla_t^2 \mathbf{H} + (\gamma^2 + k^2)\mathbf{H} = 0$$

Wave equation

$$\nabla_t \times \mathbf{E}_t = -i\omega\mu\mathbf{H}_z$$

$$-i\beta\mathbf{e}_z \times \mathbf{E}_t - \mathbf{e}_z \times \nabla_t E_z = -i\omega\mu\mathbf{H}_t$$

$$\nabla_t \times \mathbf{H}_t = i\omega\varepsilon\mathbf{E}_z$$

$$i\beta\mathbf{e}_z \times \mathbf{H}_t + \mathbf{e}_z \times \nabla_t H_z = -i\omega\varepsilon\mathbf{E}_t$$

$$\nabla_t \cdot \mathbf{H}_t = i\beta H_z$$

$$\nabla_t \cdot \mathbf{E}_t = i\beta E_z$$

Maxwell equation for transversal and longitudinal components

TEM-, TE-, TM- Waves

1. $E_z = H_z = 0$ TEM
2. $E_z = 0$ and $H_z \neq 0$ TE (or H-wave)
3. $H_z = 0$ and $E_z \neq 0$ TM (or E-wave)

On the next slides we will try to find solutions for TE-waves. The treatment for TM- modes is similar. For TEM modes the treatment is even easier, but TEM-modes do not exist in hollow transmission lines, because transversal E components require longitudinal H components and transversal H components require longitudinal E components. These are 0 in TEM fields. TEM-modes exist in coaxial lines since on the inner conductor we might have j≠0.

Derivation of TE-Wave Equations

With wave equation

$$\nabla_t^2 \mathbf{H} + (\gamma^2 + k^2)\mathbf{H} = 0 \text{ and } \gamma^2 + k^2 = k_c^2$$

$$\nabla_t^2 \mathbf{H} + k_c^2 \mathbf{H} = 0$$

or for transversal and longitudinal components

$$\nabla_t^2 \mathbf{H}_t + k_c^2 \mathbf{H}_t = 0$$

$$\nabla_t^2 H_z + k_c^2 H_z = 0$$

With $E_z = 0$ in Maxwell equations

$$\nabla_t \times \mathbf{E}_t = -i\omega\mu \mathbf{H}_z$$

$$-i\beta \mathbf{e}_z \times \mathbf{E}_t = -i\omega\mu \mathbf{H}_t$$

$$\nabla_t \times \mathbf{H}_t = 0$$

$$i\beta \mathbf{e}_z \times \mathbf{H}_t + \mathbf{e}_z \times \nabla_t H_z = -i\omega\varepsilon \mathbf{E}_t$$

$$\nabla_t \cdot \mathbf{H}_t = i\beta H_z$$

$$\nabla_t \cdot \mathbf{E}_t = 0$$

Derivation of TE-Wave Equations(2)

Application of curl on $\nabla_t \times \mathbf{H}_t = 0$ gives $\nabla_t \times \nabla_t \times \mathbf{H}_t = 0$

With $\nabla_t \times \nabla_t \times = \nabla_t \left(\nabla_t \cdot \right) - \nabla_t^2$ and $\nabla_t^2 \mathbf{H}_t + k_c^2 \mathbf{H}_t = 0$

$$\Rightarrow \nabla_t \left(\nabla_t \cdot \mathbf{H}_t \right) + k_c^2 \mathbf{H}_t = 0$$

and with $\nabla_t \cdot \mathbf{H}_t = i\beta H_z$

$$\nabla_t \left(i\beta H_z \right) + k_c^2 \mathbf{H}_t = 0$$

$$\mathbf{H}_t = -\frac{i\beta}{k_c^2} \nabla_t H_z$$

Now

$$\mathbf{e}_z \times \left(\beta \mathbf{e}_z \times \mathbf{E}_t \right) = \beta \left(\mathbf{e}_z \left(\mathbf{e}_z \cdot \mathbf{E}_t \right) - \mathbf{E}_t \left(\mathbf{e}_z \cdot \mathbf{e}_z \right) \right) = -\beta \mathbf{E}_t = \omega \mu \mathbf{e}_z \times \mathbf{H}_t$$

$$\mathbf{E}_t = -\frac{\omega\mu}{\beta} \mathbf{e}_z \times \mathbf{H}_t = -\sqrt{\frac{\mu}{\varepsilon}} \frac{\sqrt{\mu\varepsilon}\,\omega}{\beta} \mathbf{e}_z \times \mathbf{H}_t = -Z_F \frac{k}{\beta} \mathbf{e}_z \times \mathbf{H}_t$$

$\mathbf{H}_t$ can be calculated from H_z and $\mathbf{E}_t$ from $\mathbf{H}_t$ and therefore from H_z too.

Derivation of TE-Wave Equations(3)
Impedance of a TE-Wave

$$Z_F = \sqrt{\frac{\mu}{\varepsilon}} \quad \text{impedance of an electromagnetic wave in free space}$$

$$Z_{TE} = Z_F \frac{k}{\beta} \quad \text{impedance of a TE-wave (H-wave)}$$

$$Z_{TE} = \frac{E_x}{H_y} = -\frac{E_y}{H_x} \quad \text{for TE-wave moving to the right}$$

TE-Wave Equation in rectangular Waveguides

$$\nabla_t^2 H_z + k_c^2 H_z = 0$$

$$\mathbf{H}_t = -\frac{i\beta}{k_c^2}\nabla_t H_z$$

$$\mathbf{E}_t = -Z_{TE}\mathbf{e}_z \times \mathbf{H}_t$$

TE wave equation

$$\partial^2/\partial x^2 H_z + \partial^2/\partial y^2 H_z + k_c^2 H_z = 0$$ Eigenvalue problem

$$H_x = -\frac{i\beta}{k_c^2}\partial/\partial x \; H_z$$

$$H_y = -\frac{i\beta}{k_c^2}\partial/\partial y \; H_z$$

$$E_x = -Z_{TE}\frac{i\beta}{k_c^2}\partial/\partial y \; H_z$$

$$E_y = Z_{TE}\frac{i\beta}{k_c^2}\partial/\partial x \; H_z$$

TE wave equation written in components

Solution of TE- Wave Equation

$$\frac{\partial^2}{\partial x^2} H_z + \frac{\partial^2}{\partial y^2} H_z + k_c^2 H_z = 0$$

Ansatz : Separation into x and y coordinates.

$$H_z = f \cdot g \quad \text{with} \quad f = f(x) \text{ and } g = g(y)$$

$$\frac{1}{f}\frac{d^2}{dx^2} f + \frac{1}{g}\frac{d^2}{dy^2} g + k_c^2 = 0$$

$$\frac{1}{f}\frac{d^2}{dx^2} f = -k_x^2$$

$$\frac{1}{g}\frac{d^2}{dy^2} g = -k_y^2$$

$$k_x^2 + k_y^2 = k_c^2$$

Solution of TE-Wave Equation(2)

$\Rightarrow$

$$f = A_1 \cos k_x x + A_2 \sin k_x x$$

$$g = B_1 \cos k_y y + B_2 \sin k_y y$$

Boundary conditions:

Normal component of H: $H_\perp = 0$ on surface

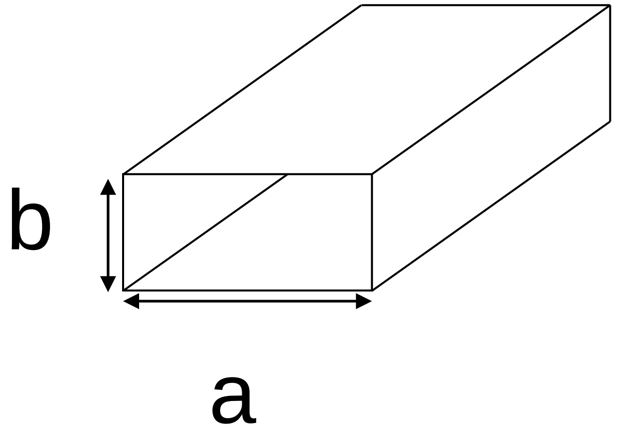
$$H_x = 0 \quad \text{for} \quad x = 0 \quad \text{and} \quad x = a$$

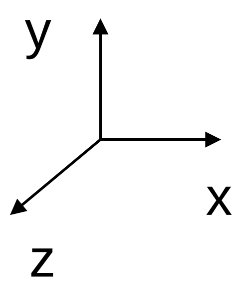
$$H_y = 0 \quad \text{for} \quad y = 0 \quad \text{and} \quad y = b$$

$\Rightarrow$

$$\partial / \partial x \, H_z = 0 \ \text{for} \quad x = 0 \quad \text{and} \quad x = a$$

$$\partial / \partial y \, H_z = 0 \ \text{for} \quad y = 0 \quad \text{and} \quad y = b$$





Solution of TE-Wave Equation(3)

$$-A_1 k_x \sin k_x x + A_2 k_x \cos k_x x = 0 \quad \text{for} \quad x = 0 \quad \text{and} \quad x = a$$

$$\Rightarrow$$

$$A_2 = 0 \quad \text{from} \quad x = 0$$

$$k_x = \frac{n\pi}{a} \text{ from} \quad x = a$$

$$n = 0,1,2,....$$

in the same manner

$$B_2 = 0 \quad \text{from} \quad y = 0$$

$$k_y = \frac{m\pi}{b} \text{ from} \quad y = b$$

$$m = 0,1,2,....$$

$$H_z(x,y) = H_{nm} \cos\frac{n\pi x}{a} \cos\frac{m\pi y}{b} \text{ with n} = 0,1,2.. \quad \text{m} = 0,1,2,.. \text{but n} = \text{m} \neq 0$$

Solution of TE-Wave Equation(3)

$$k_{cnm} = \left[\left(\frac{n\pi}{a} \right)^2 + \left(\frac{m\pi}{b} \right)^2 \right]^{1/2} \text{ cut off wave number}$$

$$k_c^2 = \gamma^2 + k^2$$

$$k_{cnm}^2 = \gamma_{nm}^2 + k^2$$

with

$$\gamma = \mathrm{i}\beta$$

$$\exp i(\omega t - \beta z)$$

$$k_{cnm}^2 = k^2 - \beta_{nm}^2$$

$$\beta_{nm}^2 = k^2 - k_{cnm}^2$$

for $k > k_{cnm}$ β_{nm} is real $\Rightarrow$ propagation

for $k < k_{cnm}$ β_{nm} is imaginary $\Rightarrow$ exponential decay

TE_{nm} -Fields

$$E_z(x,y,z,t)=0$$

$$E_x(x,y,z,t)=-\frac{\beta_{nm}m\pi}{bk_{cnm}^2}Z_{TE,nm}H_{nm}\cos\left(\frac{n\pi x}{a}\right)\sin\left(\frac{m\pi y}{b}\right)\sin(\omega t-\beta_{nm}z)$$

$$E_y(x,y,z,t)=-\frac{\beta_{nm}n\pi}{ak_{cnm}^2}Z_{TE,nm}H_{nm}\sin\left(\frac{n\pi x}{a}\right)\cos\left(\frac{m\pi y}{b}\right)\sin(\omega t-\beta_{nm}z)$$

$$H_z(x,y,z,t)=H_{nm}\cos\left(\frac{n\pi x}{a}\right)\cos\left(\frac{m\pi y}{b}\right)\cos(\omega t-\beta_{nm}z)$$

$$H_x(x,y,z,t)=-\frac{\beta_{nm}n\pi}{ak_{cnm}^2}H_{nm}\sin\left(\frac{n\pi x}{a}\right)\cos\left(\frac{m\pi y}{b}\right)\sin(\omega t-\beta_{nm}z)$$

$$H_y(x,y,z,t)=-\frac{\beta_{nm}m\pi}{bk_{cnm}^2}H_{nm}\cos\left(\frac{n\pi x}{a}\right)\sin\left(\frac{m\pi y}{b}\right)\sin(\omega t-\beta_{nm}z)$$

Cut Off Frequency and Wavelength

$$\text{for } \beta_{nm} = 0 \text{ is } k = k_{cnm}$$

$$\text{since } k_{cnm} = \frac{2\pi}{\lambda_{cnm}} = \frac{2\pi}{c} f_{cnm}$$

$$f_{cnm} = \frac{c}{2\pi}\left[\left(\frac{n\pi}{a}\right)^2 + \left(\frac{m\pi}{b}\right)^2\right]^{1/2} \quad \text{cut off frequency}$$

$$\lambda_{cnm} = \frac{2}{\left[\left(\frac{n}{a}\right)^2 + \left(\frac{m}{b}\right)^2\right]^{1/2}} \quad \text{cut off wavelength}$$

Waves with frequency lower than the cut off frequency ($f<f_{cnm}$) or wavelength longer than the cut off wavelength ($\lambda>\lambda_{cnm}$) can not propagte in nm-mode.

Guide Wavelength

$\beta_{nm}^2 = k^2 - k_{cnm}^2$ propagation constant of mode nm

$$\lambda_{gnm} = \frac{2\pi}{\beta_{nm}} = \frac{2\pi}{\sqrt{k^2 - k_{cnm}^2}} = \frac{1}{\sqrt{\frac{1}{\lambda^2} - \frac{1}{\lambda_{cnm}^2}}} = \frac{\lambda}{\sqrt{1 - \frac{\lambda^2}{\lambda_{cnm}^2}}}$$

λ_{gnm} is called guide wavelength of mode nm.

It gives the distance after which the mode pattern repeats in the waveguide.

$\lambda_{gnm} > \lambda$

TM-Waves

$H_z = 0 \text{ and } E_z \neq 0$

in the same manner as for TE – Waves one calculates

$$E_z = E_{nm} \sin\frac{n\pi x}{a} \sin\frac{n\pi y}{b} \quad \text{with n} = 1,2,\ldots \text{ and m} = 1,2,\ldots$$

and the other field components. One also obtains:

$$k_{cnm} = \left[\left(\frac{n\pi}{a}\right)^2 + \left(\frac{m\pi}{b}\right)^2\right]^{1/2} \quad \text{cut off wave number}$$

$$f_{cnm} = \frac{c}{2\pi}\left[\left(\frac{n\pi}{a}\right)^2 + \left(\frac{m\pi}{b}\right)^2\right]^{1/2} \quad \text{cut off frequency}$$

$$\lambda_{cnm} = \frac{2}{\left[\left(\frac{n}{a}\right)^2 + \left(\frac{m}{b}\right)^2\right]^{1/2}} \quad \text{cut off wavelength}$$

$$Z_{\mathrm{TMnm}} = \frac{\beta_{nm}}{k} Z_0 \quad \text{impedance of the TM wave}$$

TM_{nm} -Fields

$$E_z(x,y,z,t) = E_{nm} \sin\left(\frac{n\pi x}{a}\right) \sin\left(\frac{m\pi y}{b}\right) \cos(\omega t - \beta_{nm} z)$$

$$E_x(x,y,z,t) = \frac{\beta_{nm} n\pi}{a k_{cnm}^2} E_{nm} \cos\left(\frac{n\pi x}{a}\right) \sin\left(\frac{m\pi y}{b}\right) \sin(\omega t - \beta_{nm} z)$$

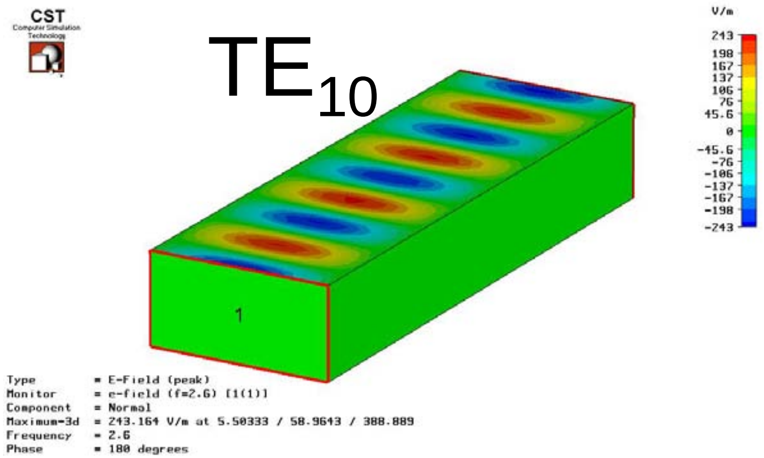
$$E_y(x,y,z,t) = \frac{\beta_{nm} n\pi}{b k_{cnm}^2} E_{nm} \sin\left(\frac{n\pi x}{a}\right) \cos\left(\frac{m\pi y}{b}\right) \sin(\omega t - \beta_{nm} z)$$

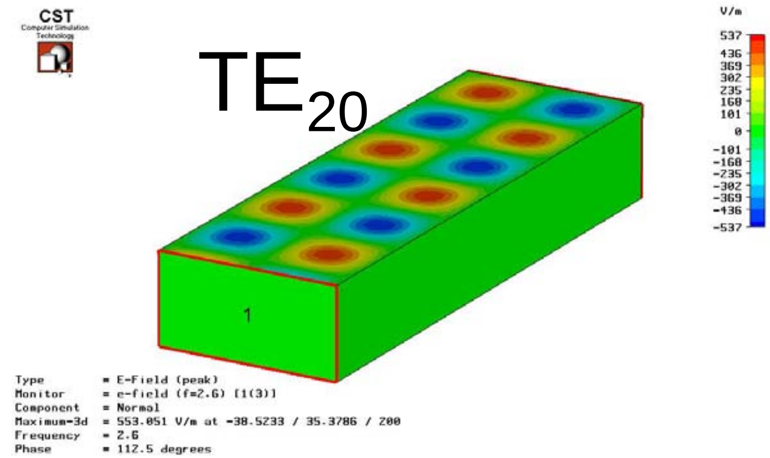
$$H_z(x,y,z,t) = 0$$

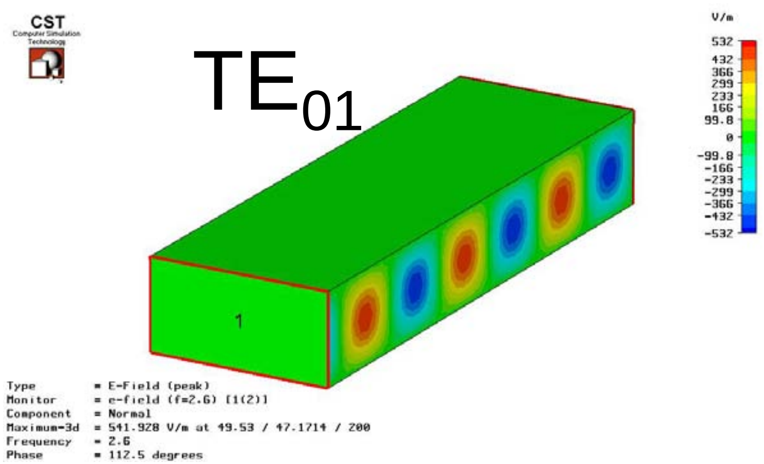
$$H_x(x,y,z,t) = -\frac{\beta_{nm} n\pi}{b k_{cnm}^2} \frac{1}{Z_{TMnm}} E_{nm} \sin\left(\frac{n\pi x}{a}\right) \cos\left(\frac{m\pi y}{b}\right) \sin(\omega t - \beta_{nm} z)$$

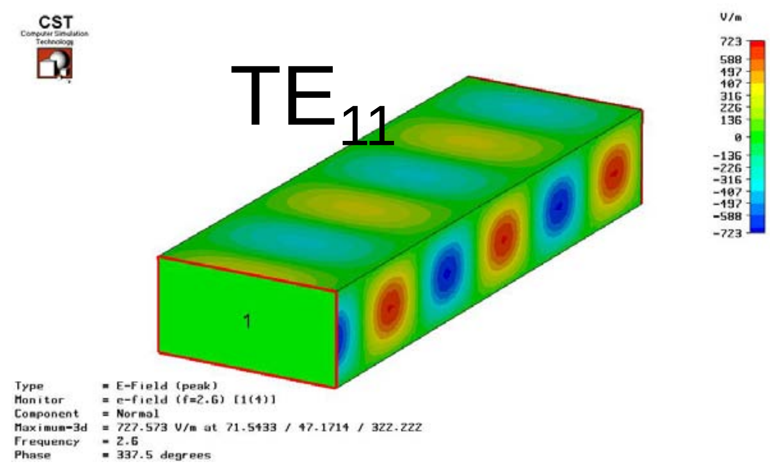
$$H_y(x,y,z,t) = \frac{\beta_{nm} n\pi}{a k_{cnm}^2} \frac{1}{Z_{TMnm}} E_{nm} \cos\left(\frac{n\pi x}{a}\right) \sin\left(\frac{m\pi y}{b}\right) \sin(\omega t - \beta_{nm} z)$$

Rectangular Waveguide Mode Pattern



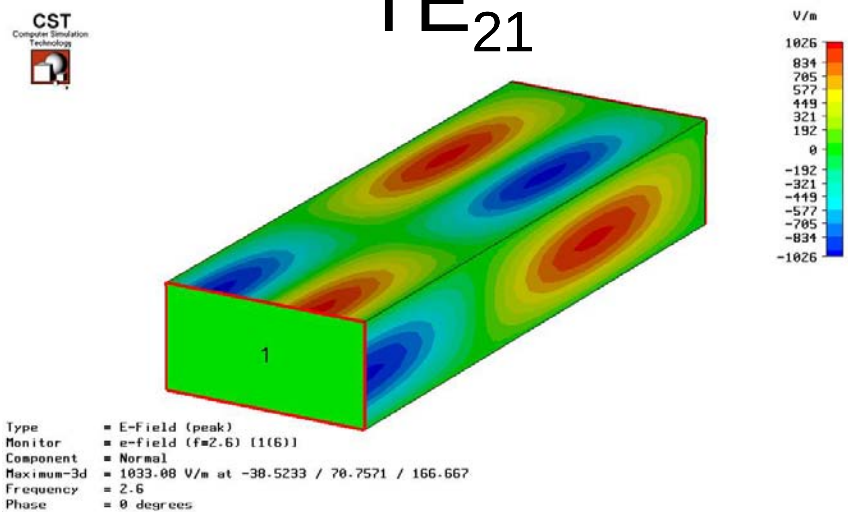




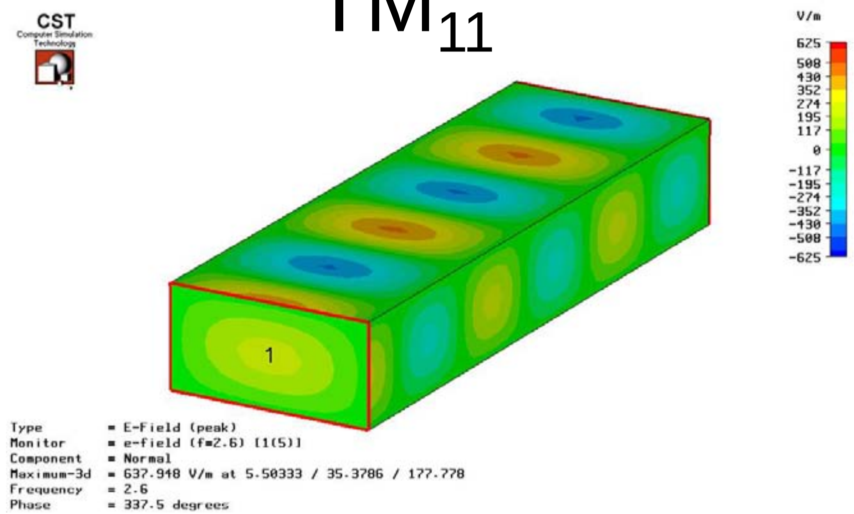


Rectangular Waveguide Mode Pattern(2)

TE_{21}



TM_{11}



TE- and TM- Mode Pattern

Mode pattern images can be found for instance in

- N. Marcuvitz, Waveguide Handbook, MIT Radiation Laboratory Series, Vol. 10, McGraw Hill 1951
- H. J. Reich, P. F. Ordung, H. L.Krauss, J. G. Skalnik, Microwave Theory and Techniques, D. van Nostrand 1953

and probably in other books, too.

TE_{10} (H_{10})-Mode

Waveguide Size and Modes

It is common to use $a = 2b$ in a retangular waveguide system.

$$\Rightarrow \lambda_{cnm} = \frac{2a}{\left(n^2 + 4m^2\right)^{1/2}}$$

TE – modes cutoff wavelengths

$$\lambda_{c10} = 2a, \lambda_{c01} = a, \lambda_{c11} = 2a/\sqrt{5}, \lambda_{c20} = a,$$

$$\lambda_{c02} = a/2, \lambda_{c21} = 2a/\sqrt{8}, ...$$

$\Rightarrow$ for $\lambda_{c01} = a < \lambda < \lambda_{c10} = 2a$ only TE_{10} can propagate.

TM – modes cutoff wavelength

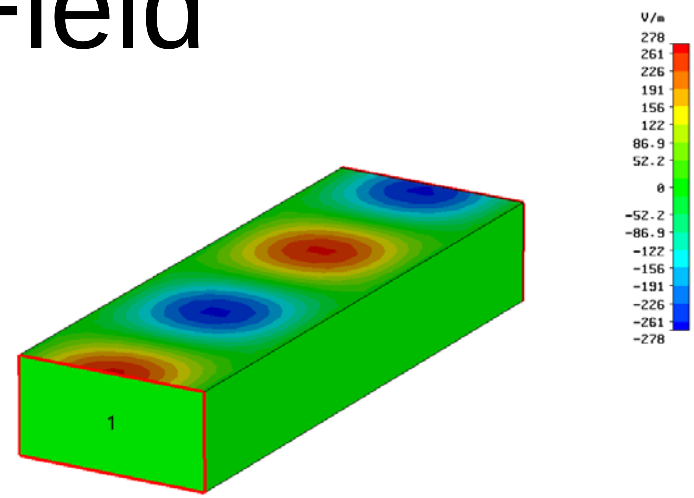
$$\lambda_{c11} = 2a/\sqrt{5}, \lambda_{c21} = 2a/\sqrt{8}, ...$$

TM - mode with lowest frequency (longest wavegenth), which can propagate in a half height waveguide is TM_{11} with cut off wavelength $\lambda_{c11} = 2a/\sqrt{5}$

$\Rightarrow$ for $a < \lambda < 2a$ only the TE_{10} – mode can propagate

TE$_{10}$ (H$_{10}$)-Field

- The mode with lowest frequency propagating in the waveguide is the TE$_{10}$ (H$_{10}$) mode. For a<λ<2a only this mode can propagate.



$$E_z(x,y,z,t) = 0$$

$$E_x(x,y,z,t) = 0$$

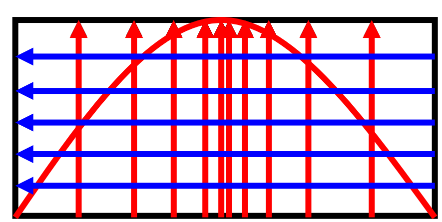
$$E_y(x,y,z,t) = Z_{TE}\, H_{nm} \frac{\beta}{k_c} \sin\left(\frac{\pi x}{a}\right) \sin(\omega t - \beta z)$$

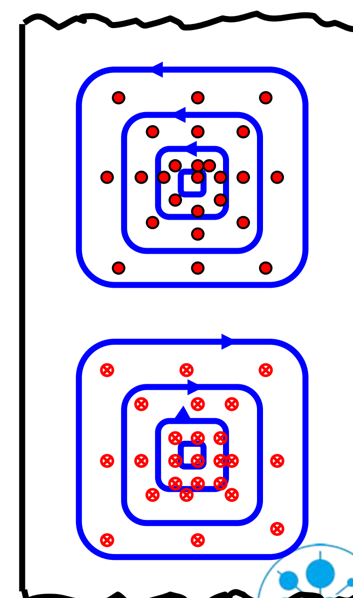
$$H_z(x,y,z,t) = H_{nm} \cos\left(\frac{\pi x}{a}\right) \cos(\omega t - \beta z)$$

$$H_x(x,y,z,t) = H_{nm} \frac{\beta}{k_c} \sin\left(\frac{\pi x}{a}\right) \sin(\omega t - \beta z)$$

$$H_y(x,y,z,t) = 0$$

E-Field
H-Field





Some TE_{10} Properties

cut off frequency $f_c = \frac{c}{2a}$

cut off wavelength $\lambda_c = 2a$

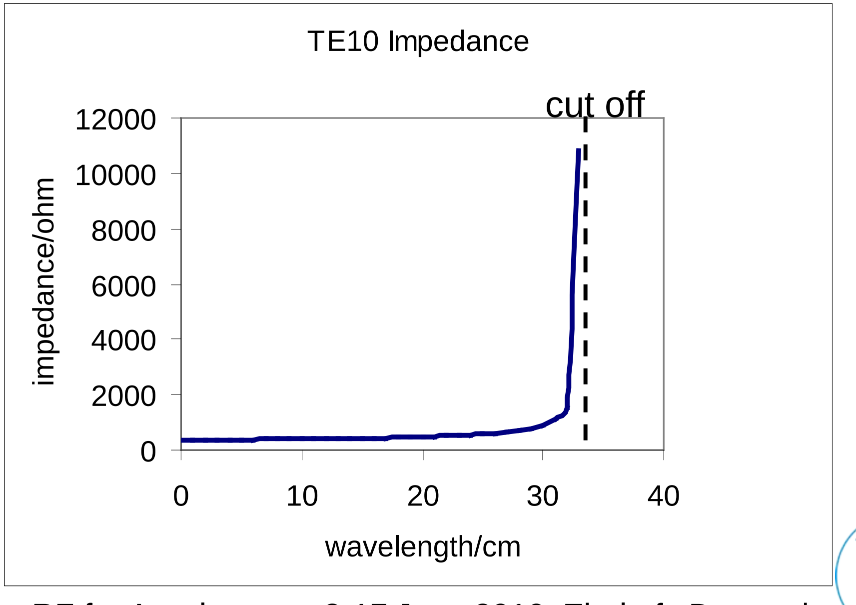
guide wavelength $\lambda_g = \frac{\lambda}{\sqrt{1-\left(\frac{\lambda}{\lambda_c}\right)^2}}$

$\Rightarrow$ guide wavelength $\lambda_g > \lambda$ free space wavelength

example: WR650 at f = 1.3GHz

$\lambda = 23.1cm$ but $\lambda_g = 32.2cm$

impedance $Z = \frac{377\Omega}{\sqrt{1-\left(\frac{\lambda}{\lambda_c}\right)^2}}$



Some TE_{10} Properties (2)

propagation constant $\beta_g = \frac{2\pi}{\lambda}\sqrt{1-\left(\frac{\lambda}{\lambda_c}\right)^2} = \sqrt{\left(\frac{2\pi}{\lambda}\right)^2 - \left(\frac{\pi}{a}\right)^2}$

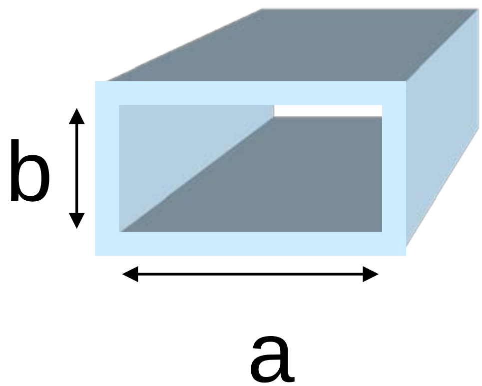
$\Rightarrow \beta_g$ depends on λ : dispersion

phase velocity $v_{ph} = \frac{\omega}{\beta_g} = \frac{\frac{2\pi c}{\lambda}}{\frac{2\pi}{\lambda}\sqrt{1-\left(\frac{\lambda}{\lambda_c}\right)^2}} = \frac{c}{\sqrt{\left(\frac{2\pi}{\lambda}\right)^2 - \left(\frac{\pi}{a}\right)^2}} > c$

but

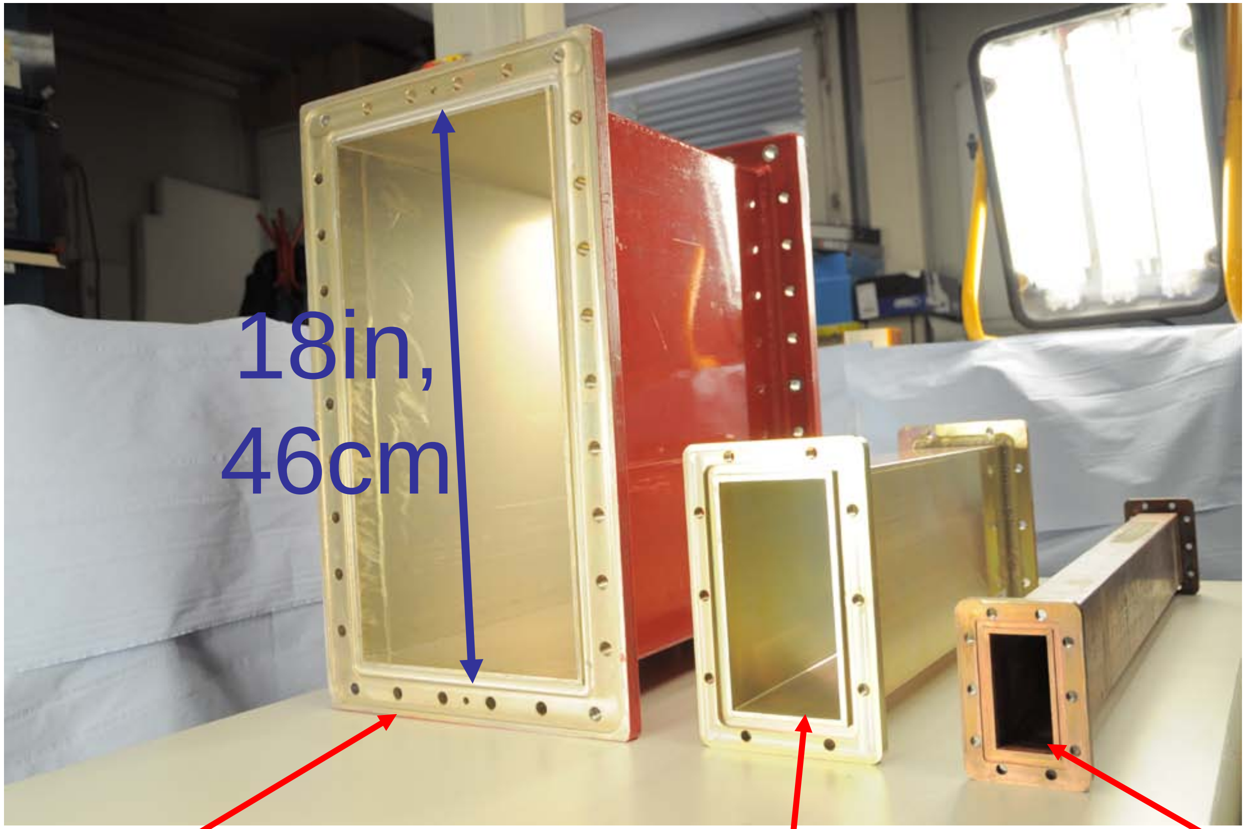
group velocity $v_g = \left(\frac{d\,\beta_g}{d\omega}\right)^{-1} = \frac{c^2}{\omega}\beta_g = \frac{c^2}{v_{ph}} < c$

Some Standard Waveguide Sizes



Waveguide type	a (in)	b (in)	f_{c10} (GHz)	frequency range (GHz)
WR 2300	23.000	11.500	.256	.32–.49
WR 2100	21.000	10.500	.281	.35 –.53
WR 1800	18.000	9.000	.328	.41 –.62
WR975	9.750	4.875	.605	.75 – 1.12
WR770	7.700	3.850	.767	.96 – 1.45
WR650	6.500	3.250	.908	1.12 – 1.70
WR430	4.300	2.150	1.375	1.70 – 2.60
WR284	2.84	1.34	2.08	2.60 – 3.95
WR187	1.872	.872	3.16	3.95 – 5.85
WR137	1.372	.622	4.29	5.85 – 8.20
WR90	.900	.450	6.56	8.2 – 12.4
WR62	.622	.311	9.49	12.4 - 18

Some Waveguides of different Size



WR1800
e.g. for 500MHz
P-Band

WR650
e.g. for 1.3GHz
L-Band

WR284
e.g. for 3GHz
S-Band

Power in TE_{10}-Mode

$$\text{Poynting Vector}: \mathrm{S} = \mathbf{E} \times \mathbf{H}$$

$$\text{Power transported in } \mathrm{TE}_{\mathrm{nm}} \quad P_{nm} = \frac{1}{T} \int_0^T \int_0^a \int_0^b \left(\mathbf{E}_{nm} \times \mathbf{H}_{nm} \right) \cdot \mathbf{e}_z \, dx dy dt$$

$$\Rightarrow$$

$$P_{nm} = \frac{\left| H_{nm} \right|^2 ab}{2 \varepsilon_{0n} \varepsilon_{0m}} \left(\frac{\beta_{nm}}{k_{cnm}} \right)^2 Z_{TEnm}$$

$$\text{with } \varepsilon_{0n} = 1 \text{ for } n = 0 \text{ and } \varepsilon_{0n} = 2 \text{ for } n \neq 0$$

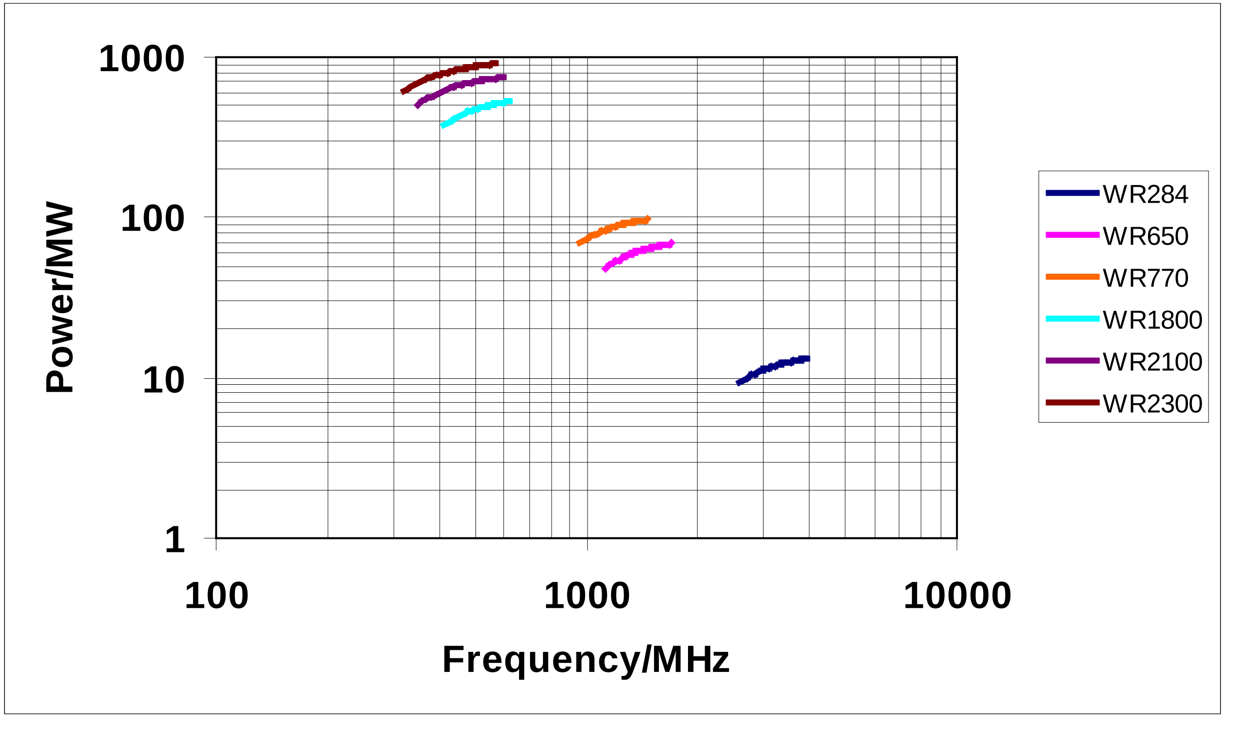
$$\text{with } \varepsilon_{0m} = 1 \text{ for } m = 0 \text{ and } \varepsilon_{0m} = 2 \text{ for } m \neq 0$$

$$\text{for } \mathrm{TE}_{10} \text{ one can caluclate}:$$

$$P_{10} = 6.63 \cdot 10^{-4} a[cm] b[cm] \left(\lambda / \lambda_g \right) \left(E[V/cm] \right)^2$$

Theoretical Power Limit in TE_{10}

The maximum power which can be transmitted theoretically in a waveguide of certain size a, b and frequency f is determined by the electrical breakdown limit E_{max}.
In air it is E_{max}=30kV/cm. From this one can find the maximum handling power in air filled waveguides.



Attenuation in TE_{10}

- The walls of the waveguides are not perfect conductors. They have finite conductivity σ, resulting in a skin depth of

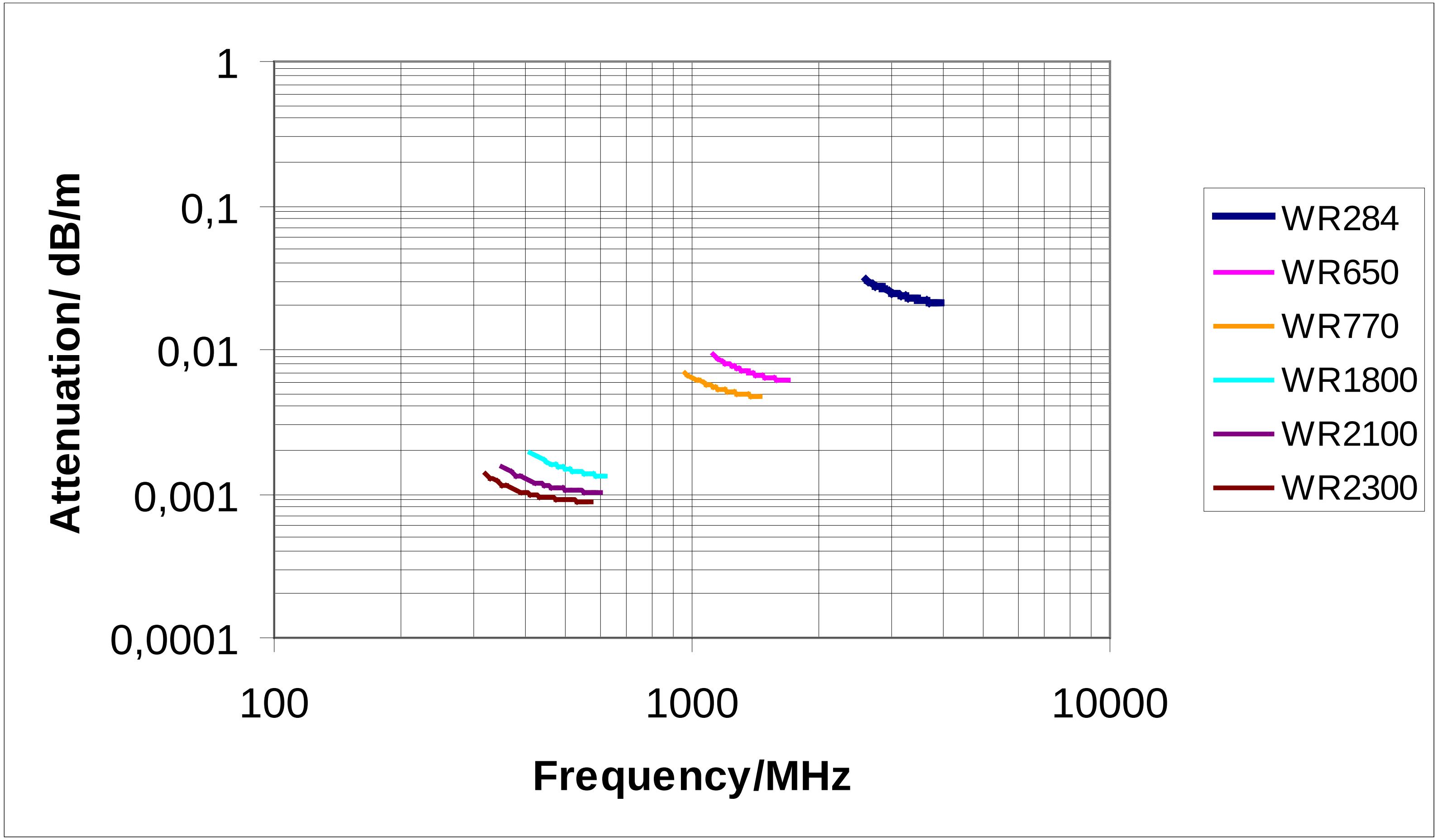
$$\delta_s = (\omega\mu\sigma/2)^{-1/2}$$

- Due to current in the walls of the waveguides loss appears and the waves are attenuated.
- The attenuation constant for the TE_{10} is:

$$\alpha[dB/m] = 0.2026\, k_1 \frac{1}{b[cm]\sqrt{\lambda\,[cm]}} \frac{\frac{1}{2} + \frac{b}{a}\left(\frac{\lambda}{2a}\right)^2}{\left(1 - \left(\frac{\lambda}{2a}\right)^2\right)^{1/2}}$$

k_1= 1.00 Ag, 1.03 Cu, 1.17 Au, 1.37 Al, 2.2 Brass

Attenuation in Al-Waveguides in TE_{10}



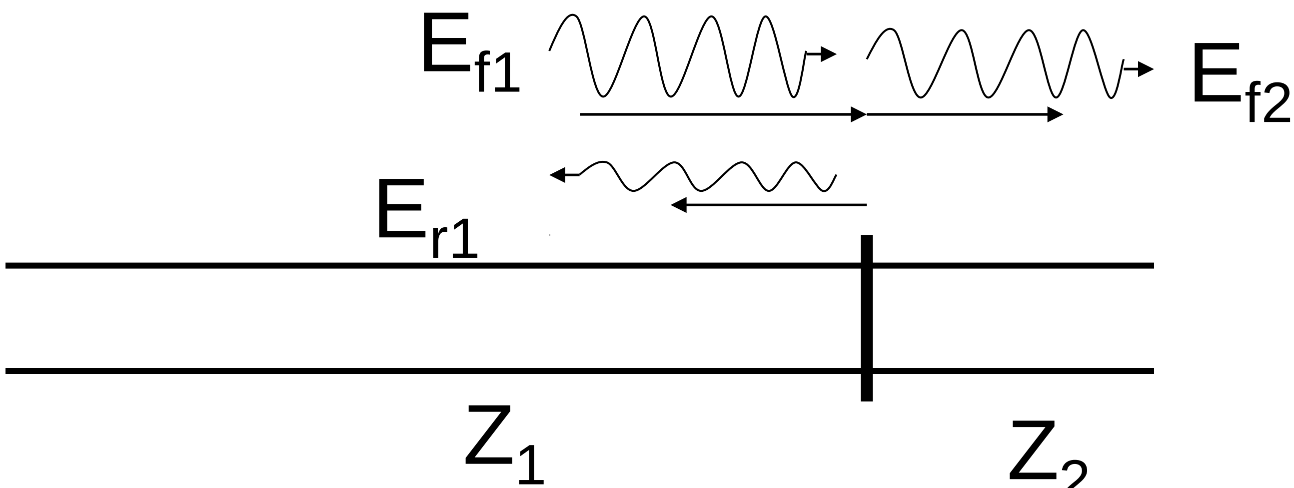
Reflection and Impedance

$$\rho = \frac{E_r}{E_f} \text{ reflection coefficient,}$$

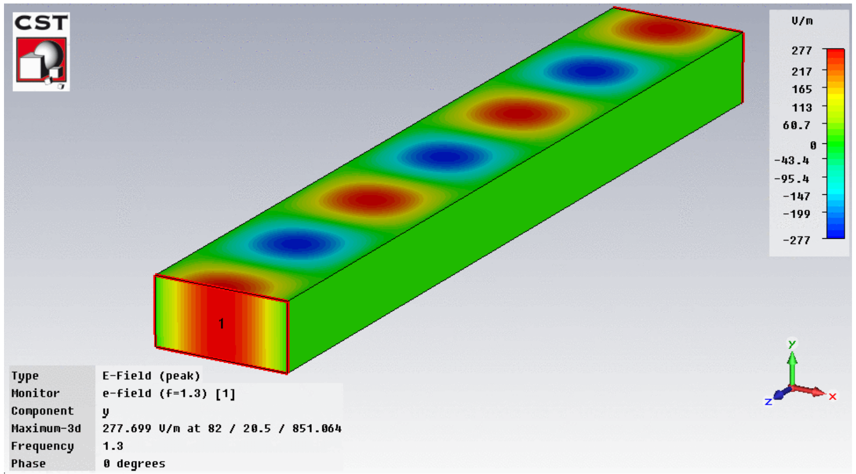
E_r and E_f amlitude of the reflected and incoming wave

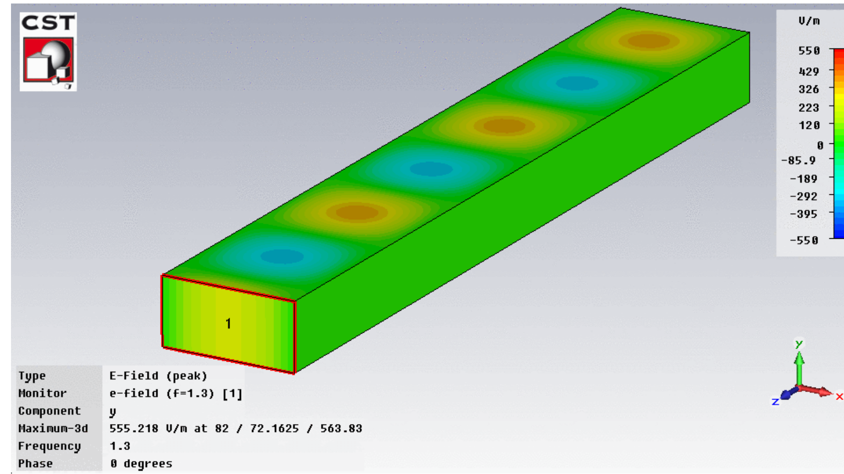
$$\rho = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

$$VSWR = \frac{|E_f| + |E_r|}{|E_f| - |E_r|} = \frac{1+\rho}{1-\rho} \text{ voltage standing wave ratio}$$



Travelling and Standing Wave





TE_{10} travelling wave

TE_{10} standing wave due to full reflection ρ=1.

The maximum electrical field strength in the standing wave is double the strength of the travelling wave. The same field strength can only be found in a travelling wave of 4-times power.