RF, part I

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Definitions & basic concepts

dB

t-domain vs. *ω*-domain

phasors

Decibel (dB)

- Convenient logarithmic measure of a power ratio.
- A "Bel" (= 10 dB) is defined as a power ratio of 10^1. Consequently, 1 dB is a power ratio of $10^{0.1} \approx 1.259$
- If *rdb* denotes the measure in dB, we have:

$$rdb = 10\ \text{dB} \log\left(\frac{P_2}{P_1}\right) = 10\ \text{dB} \log\left(\frac{A_2^2}{A_1^2}\right) = 20\ \text{dB} \log\left(\frac{A_2}{A_1}\right)$$

$$\frac{P_2}{P_1} = \frac{A_2^2}{A_1^2} = 10^{rdb/(10\ \text{dB})} \qquad \frac{A_2}{A_1} = 10^{rdb/(20\ \text{dB})}$$

rdb	-30 dB	-20 dB	-10 dB	-6 dB	-3 dB	0 dB	3 dB	6 dB	10 dB	20 dB	30 dB
P_2/P_1	0.001	0.01	0.1	0.25	.50	1	2	3.98	10	100	1000
A_2/A_1	0.0316	0.1	0.316	0.50	.71	1	1.41	2	3.16	10	31.6

- Related: dBm (relative to 1 mW), dBc (relative to carrier)

Time domain – frequency domain (1)

- An arbitrary signal *g(t)* can be expressed in *ω*-domain using the ***Fourier transform*** (FT).

$$g(t) \circ\!\!-\!\!\bullet\; G(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} g(t) e^{j\omega t} dt$$

- The inverse transform (IFT) is also referred to as ***Fourier Integral***

$$G(\omega) \bullet\!\!-\!\!\circ\; g(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} G(\omega) e^{-j\omega t} d\omega$$

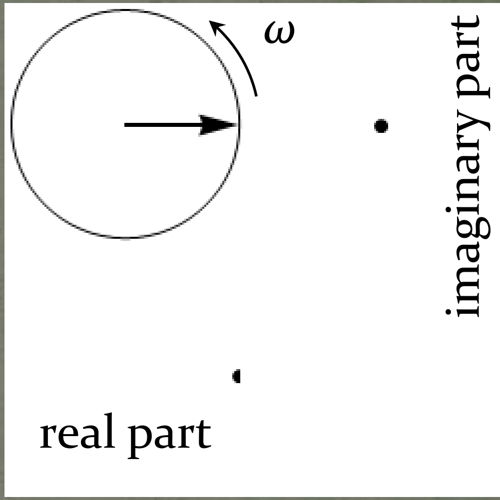
- The advantage of the *ω*-domain description is that linear time-invariant (LTI) systems are much easier described.
- The mathematics of the FT requires the extension of the definition of a *function* to allow for infinite values and non-converging integrals.
- The FT of the signal can be understood at looking at "what frequency components it's composed of".

Time domain – frequency domain (2)

- For *T*-periodic signals, the FT becomes the Fourier-Series, $d\omega$ becomes $2\pi/T$, $\int$ becomes Σ.
- The cousin of the FT is the ***Laplace transform***, which uses a complex variable (often *s*) instead of $j\omega$; it has generally a better convergence behaviour.
- Numerical implementations of the FT require discretisation in *t* (sampling) and in ω. There exist very effective algorithms (FFT).
- In digital signal processing, one often uses the related *z*-Transform, which uses the variable $z = e^{j\omega\tau}$, where τ is the sampling period. A delay of $k\tau$ becomes z^{-k}.

Fixed frequency oscillation (steady state, CW)
Definition of phasors

- General: $A\cos(\omega t - \varphi) = A\cos(\omega t)\cos(\varphi) + A\sin(\omega t)\sin(\varphi)$
- This can be interpreted as the projection on the real axis of a circular motion in the complex plane. $\mathrm{Re}\{A(\cos(\varphi) + j\sin(\varphi))e^{j\omega t}\}$



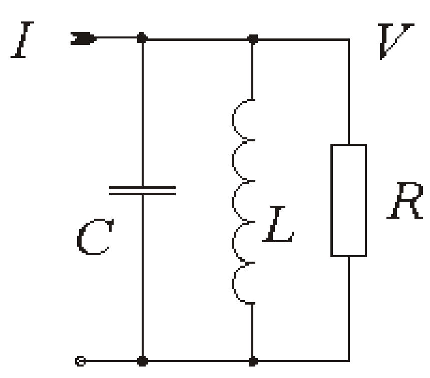
- The complex amplitude $\tilde{A}$ is called "phasor".

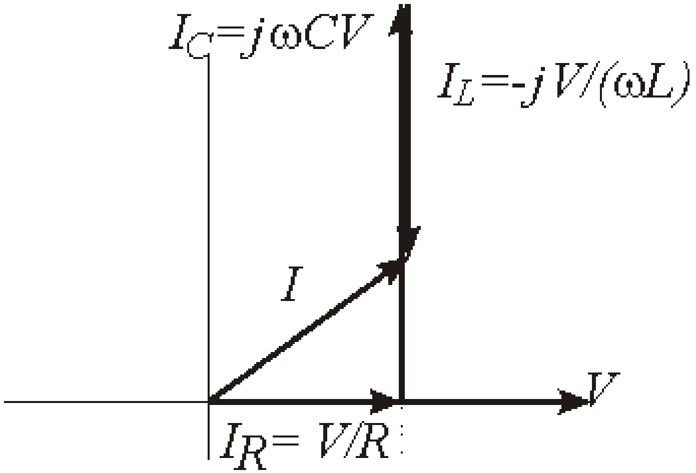
$$\tilde{A} = A(\cos(\varphi) + j\sin(\varphi))$$

Calculus with phasors

- Why this seeming "complication"?:
  Because things become easier!
- Using $\frac{\mathrm{d}}{\mathrm{d}t} \equiv j\omega$, one may now forget about the rotation with ω and the projection on the real axis, and do the complete analysis making use of complex algebra!

Example:





$$I = V\left(\frac{1}{R} + j\omega C - \frac{j}{\omega L}\right)$$

Slowly varying amplitudes

- For band-limited signals, one may conveniently use "slowly varying" phasors and a fixed frequency RF oscillation
- So-called in-phase (I) and quadrature (Q) "baseband envelopes" of a modulated RF carrier are the real and imaginary part of a slowly varying phasor

On Modulation

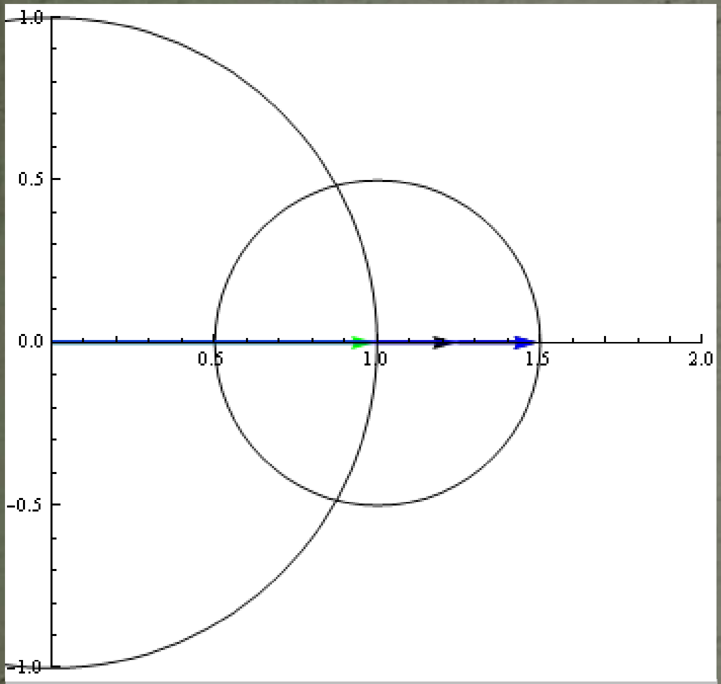
AM

PM

I-Q

Amplitude modulation

$$(1+m\cos(\varphi))\cdot\cos(\omega_c t)=\mathrm{Re}\left\{\left(1+\frac{m}{2}e^{j\varphi}+\frac{m}{2}e^{-j\varphi}\right)e^{j\omega_c t}\right\}$$

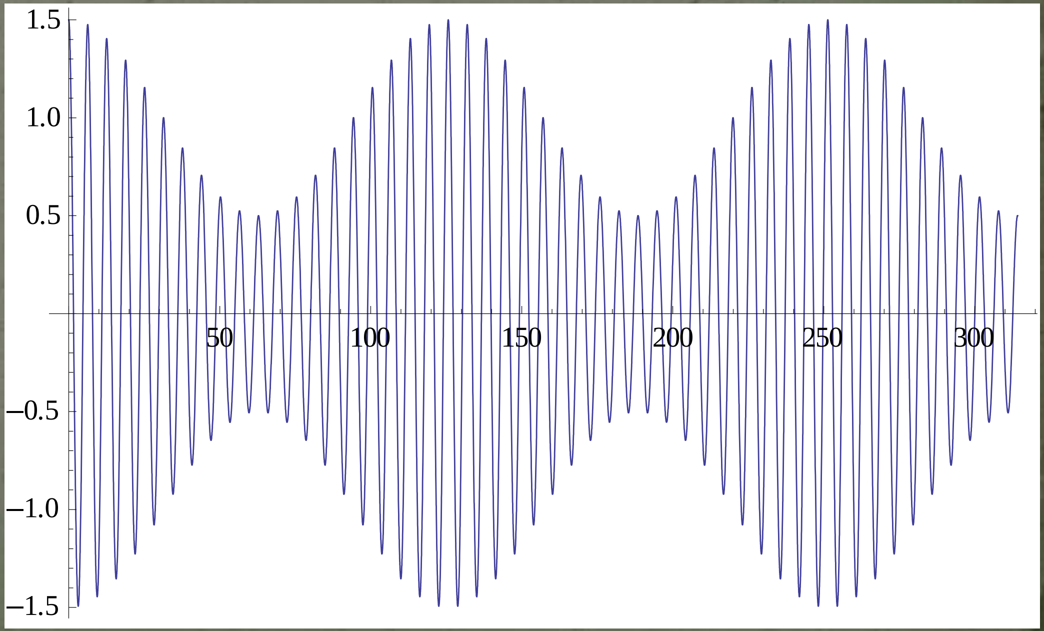


green: carrier
black: sidebands at ± f_m
blue: sum

m: modulation index or modulation depth

example: $\varphi=\omega_m t=0.05\,\omega_c t$

$m=0.5$



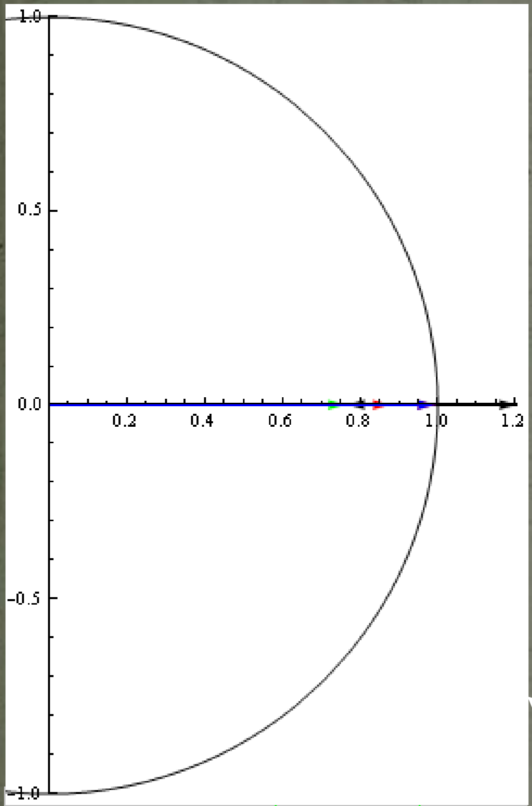
Phase modulation

$$\mathrm{Re}\left\{e^{j\omega_c t + M\sin(\varphi)}\right\} = \mathrm{Re}\left\{\sum_{n=-\infty}^{\infty} J_n(M)\, e^{j(n\varphi+\omega_c t)}\right\}$$

M: modulation index
(= max. phase deviation)

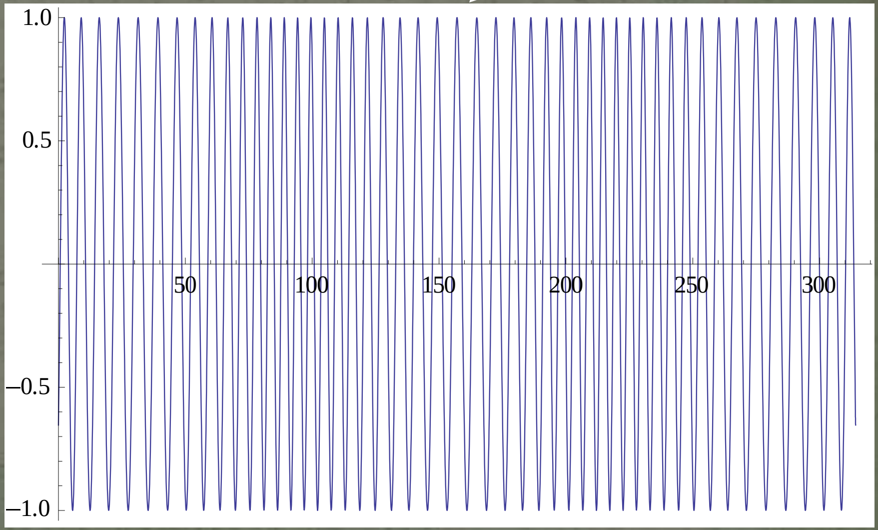
$$\varphi = \omega_m t = 0.05\,\omega_c t$$

$$M = 4$$



Green: n=0 (carrier)
black: n=1 sidebands
red: n=2 sidebands
blue: sum

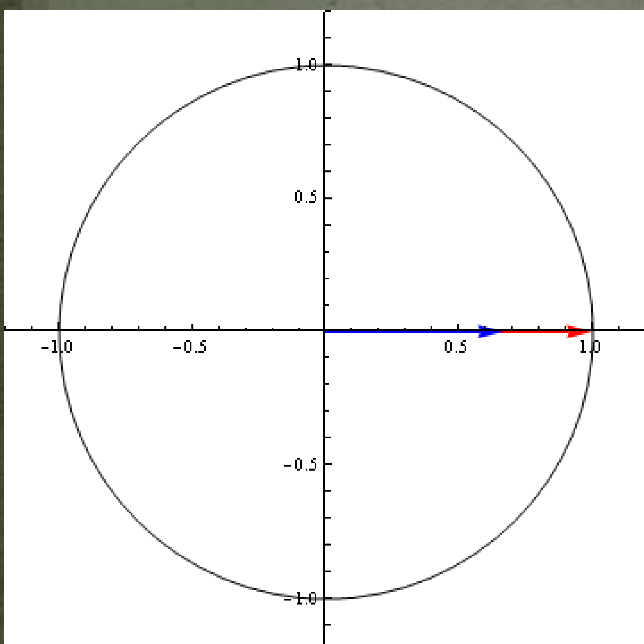
$$M = 1$$



Spectrum of phase modulation

Plotted: spectral lines for sinusoidal PM at f_m

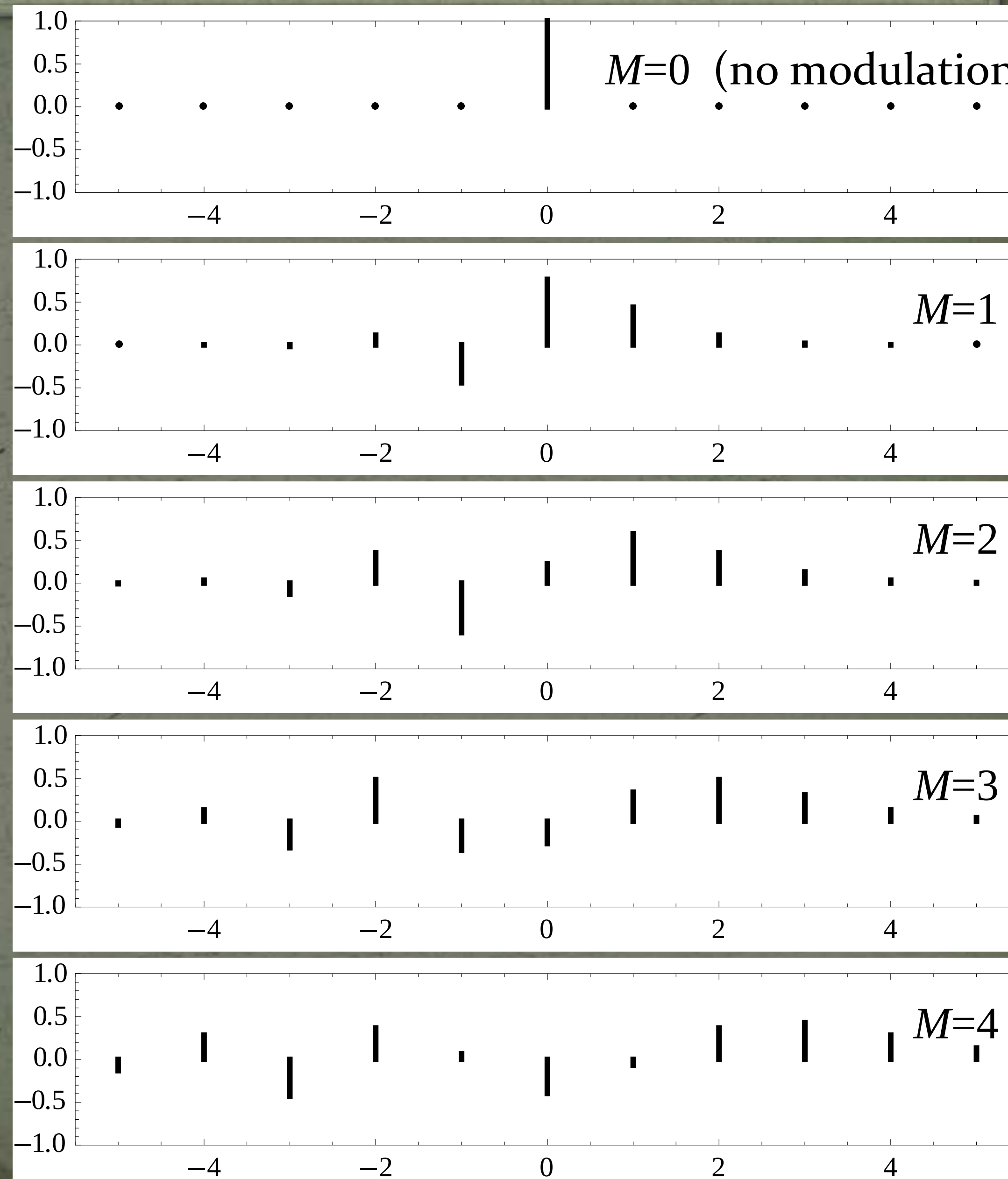
Abscissa: $(f-f_c)/f_m$



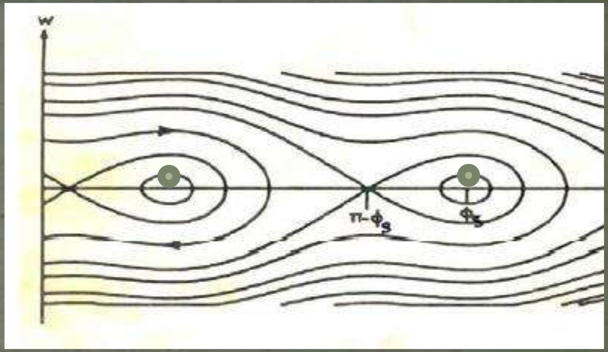
Phase modulation with $M=\pi$:

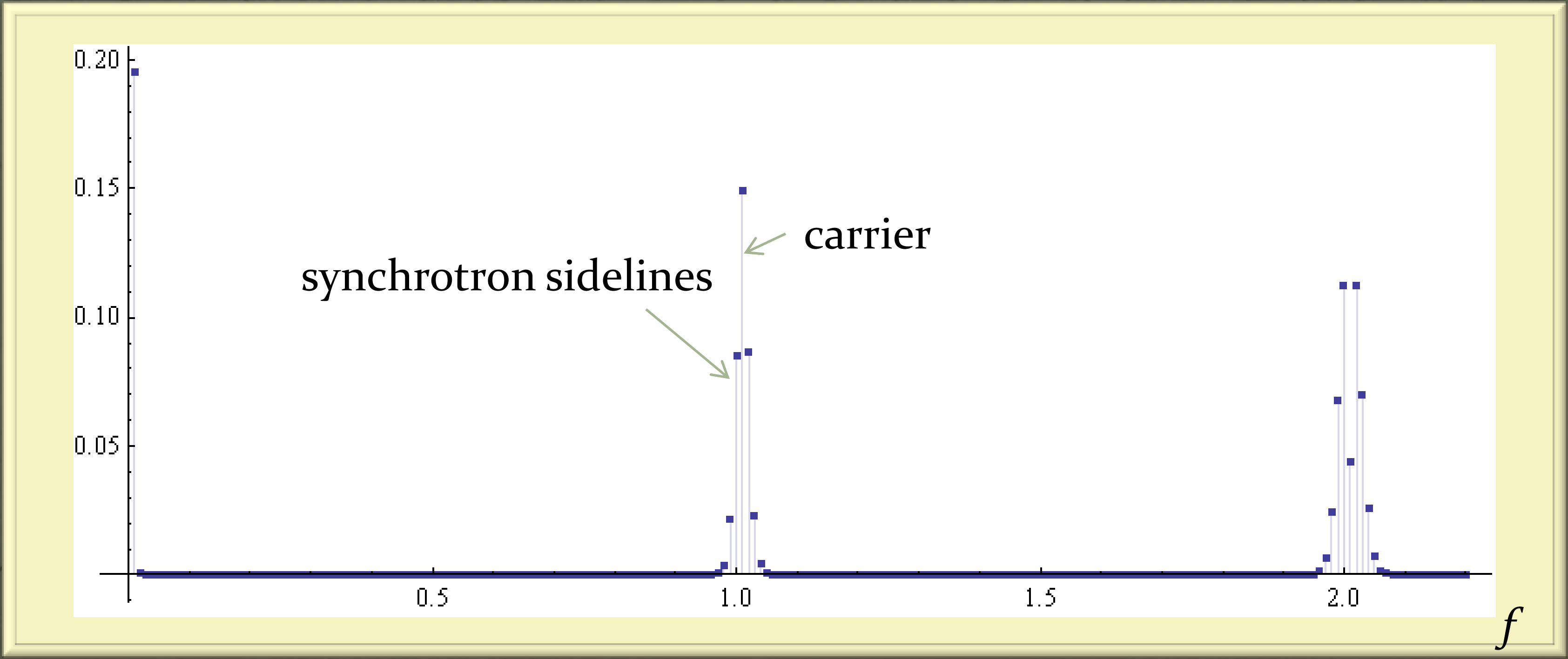
red: real phase modulation

blue: sum of sidebands $n \leq 3$

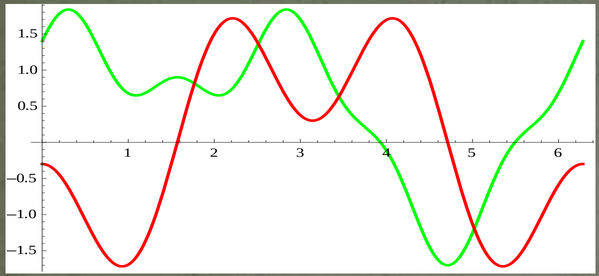


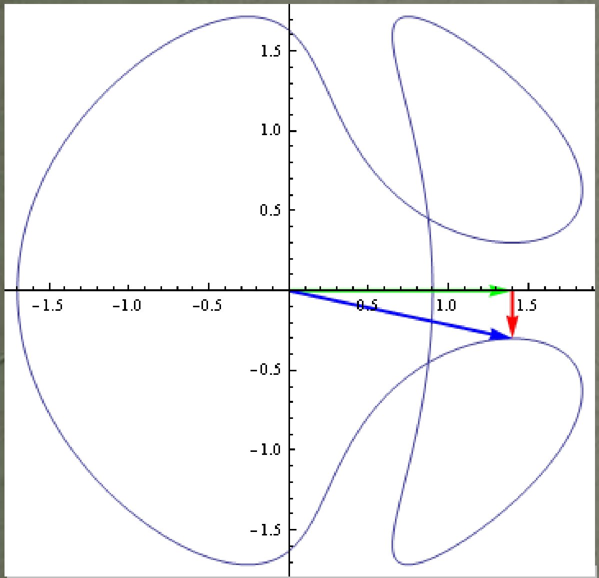
Spectrum of a beam with synchrotron oscillation, M =1 (=57°)





Vector (I-Q) modulation





I-Q modulation:
green: *I* component
red: *Q* component
blue: vector-sum

More generally, a modulation can have both amplitude and phase modulating components. They can be described as the in-phase (I) and quadrature (Q) components in a chosen reference, $\cos(\omega_r t)$. In complex notation, the modulated RF is:

$$\mathrm{Re}\left\{(I(t)+jQ(t))e^{j\omega_r t}\right\}=$$

$$\mathrm{Re}\left\{(I(t)+jQ(t))\left(\cos(\omega_r t)+j\sin(\omega_r t)\right)\right\}$$

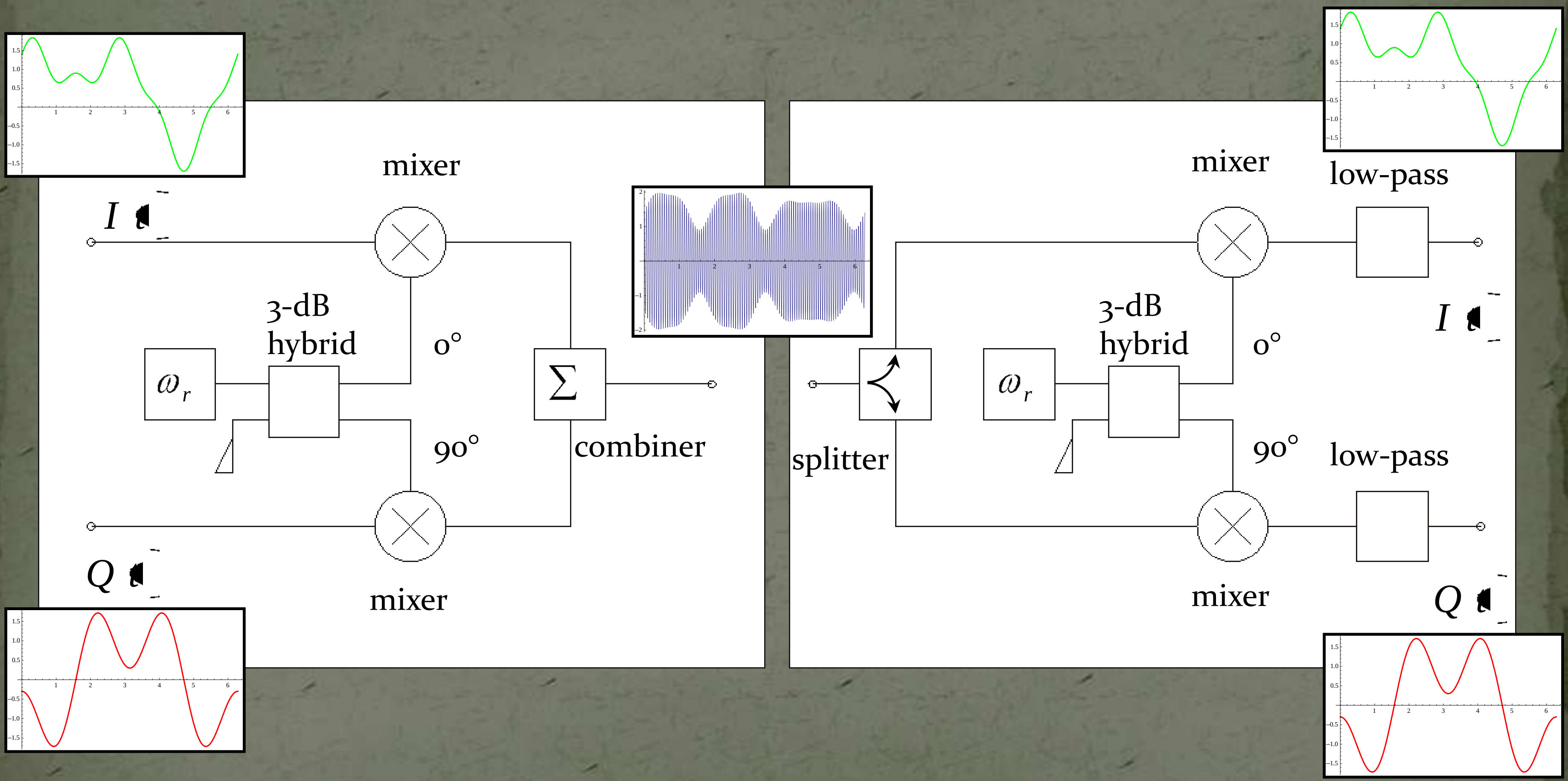
$$I(t)\cos(\omega_r t)-Q(t)\sin(\omega_r t)$$

So *I* and *Q* are the cartesian coordinates in the complex "Phasor" plane, where amplitude and phase are the corresponding polar coordinates.

$$I(t)=A(t)\cdot\cos(\varphi)$$

$$Q(t)=A(t)\cdot\sin(\varphi)$$

Vector modulator/demodulator

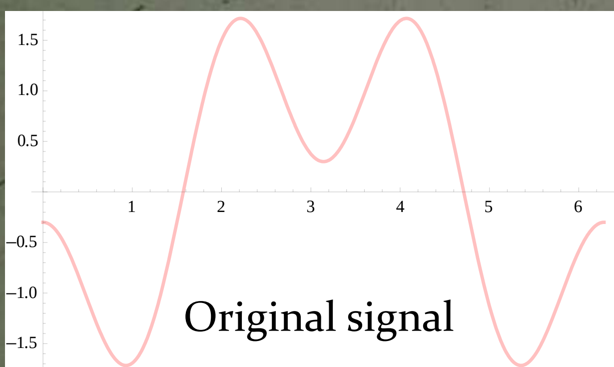


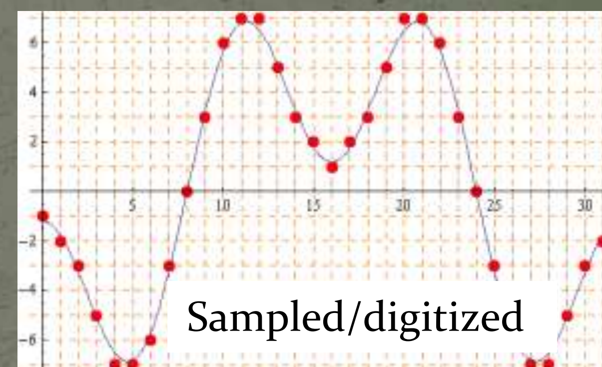
Digital Signal Processing

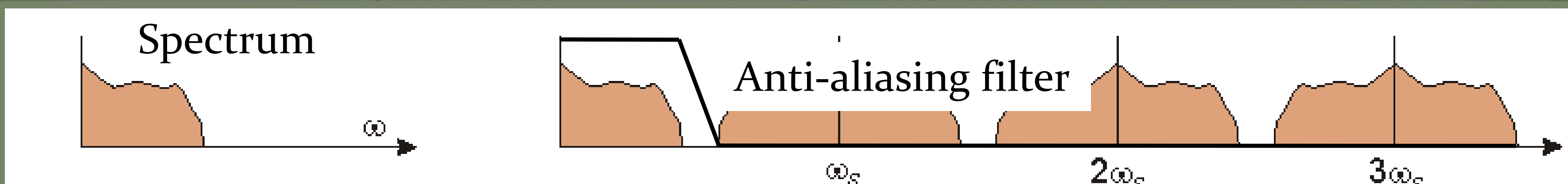
Just some basics

Sampling and quantization

- Digital Signal Processing is very powerful – note recent progress in digital audio, video and communication!
- Concepts and modules developed for a huge market; highly sophisticated modules available "off the shelf".
- The "slowly varying" phasors are ideal to be sampled and quantized as needed for digital signal processing.
- Sampling (at $1/\tau_s$) and quantization (n bit data words – here 4 bit):



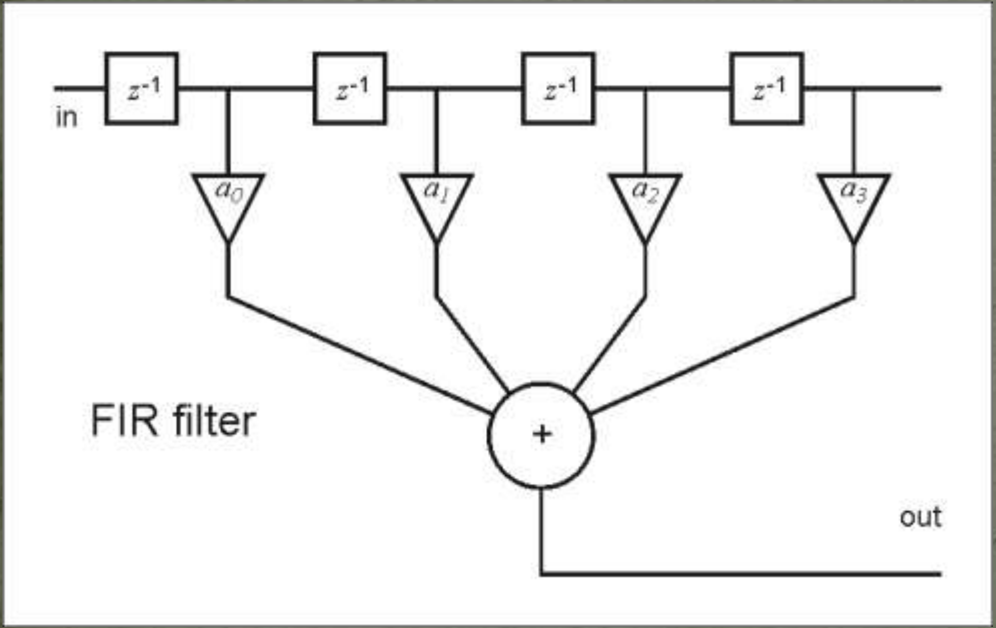


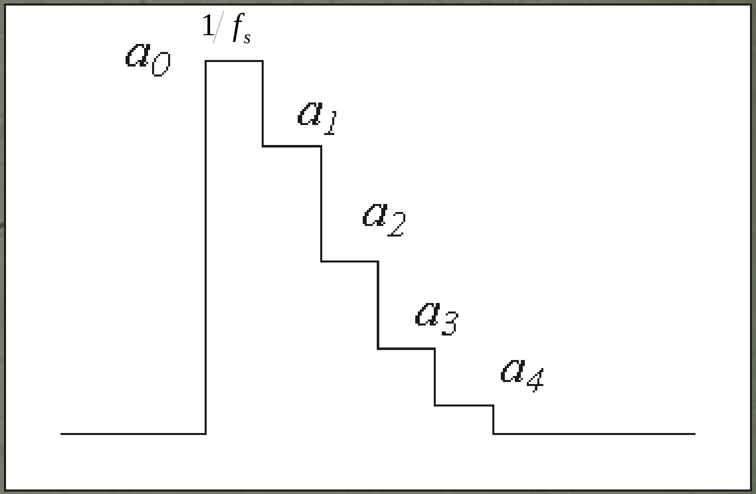


The "baseband" is limited to half the sampling rate!

Digital filters (1)

- Once in the digital realm, signal processing becomes "computing"!
- In a "finite impulse response" (FIR) filter, you directly program the coefficients of the impulse response.





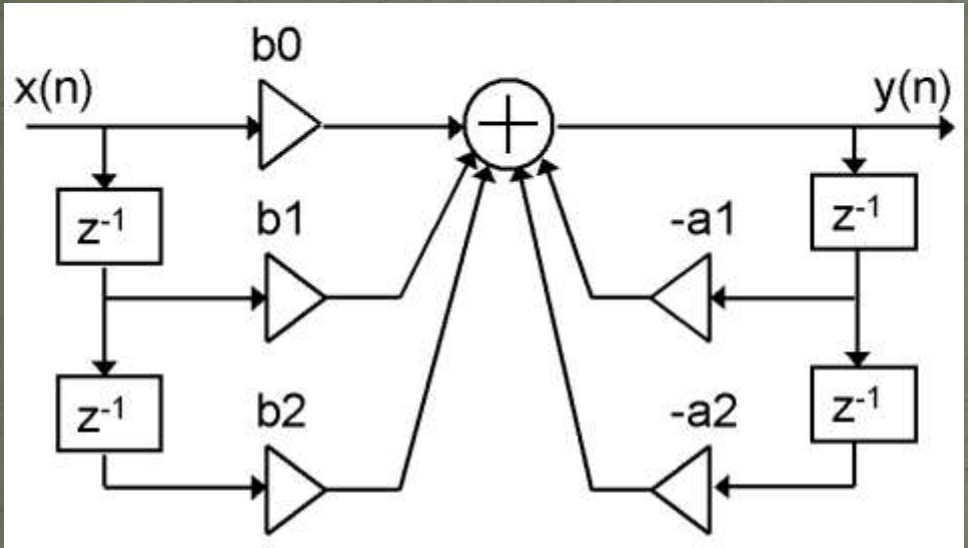
$$z = e^{j\omega\tau_s}$$

Transfer function:

$$a_0 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3} + a_4 z^{-4}$$

Digital filters (2)

- An "infinite impulse response" (IIR) filter has built-in recursion, e.g. like

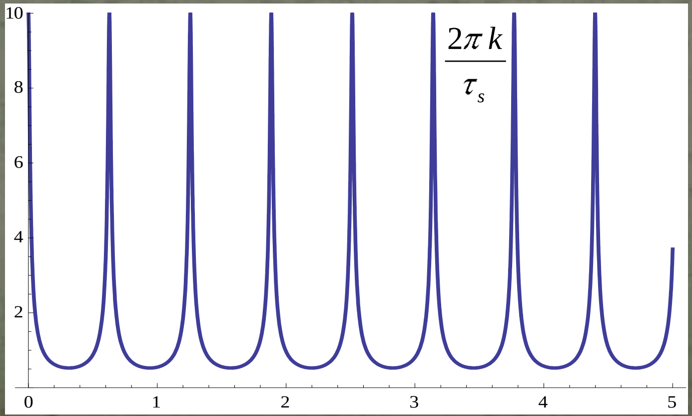


Transfer function:

$$\frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}}$$

Example:

$$\frac{b_0}{1 + b_k z^{-k}}$$

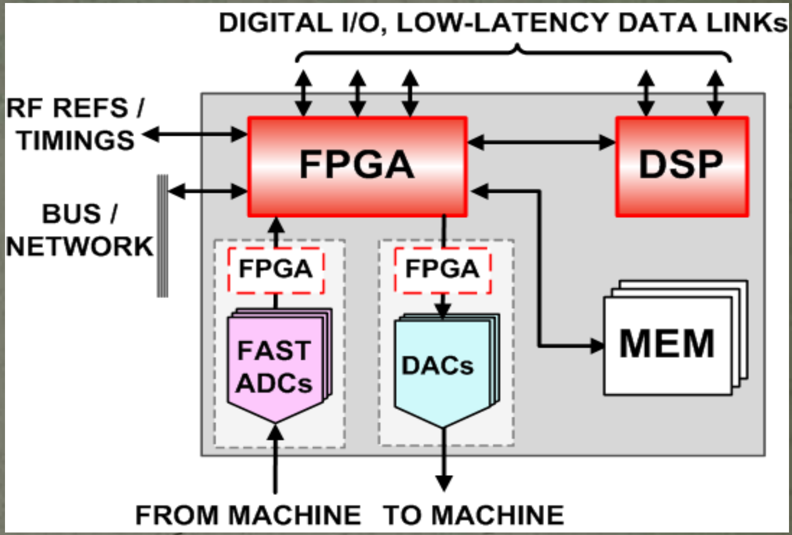


… is a comb filter

Digital LLRF building blocks – examples

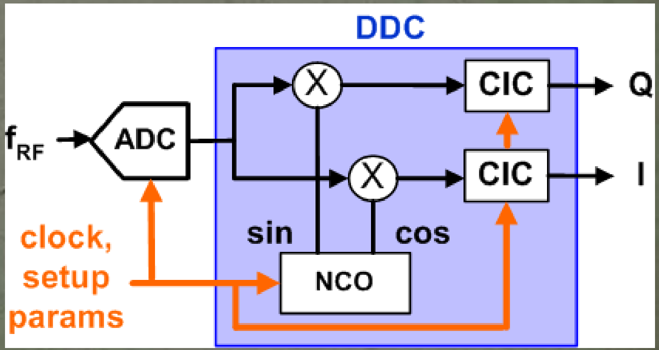
- General D-LLRF board:
  - modular!

FPGA: Field-programmable gate array
DSP: Digital Signal Processor



- DDC (Digital Down Converter)
  - Digital version of the I-Q demodulator

CIC: cascaded integrator-comb
(a special low-pass filter)



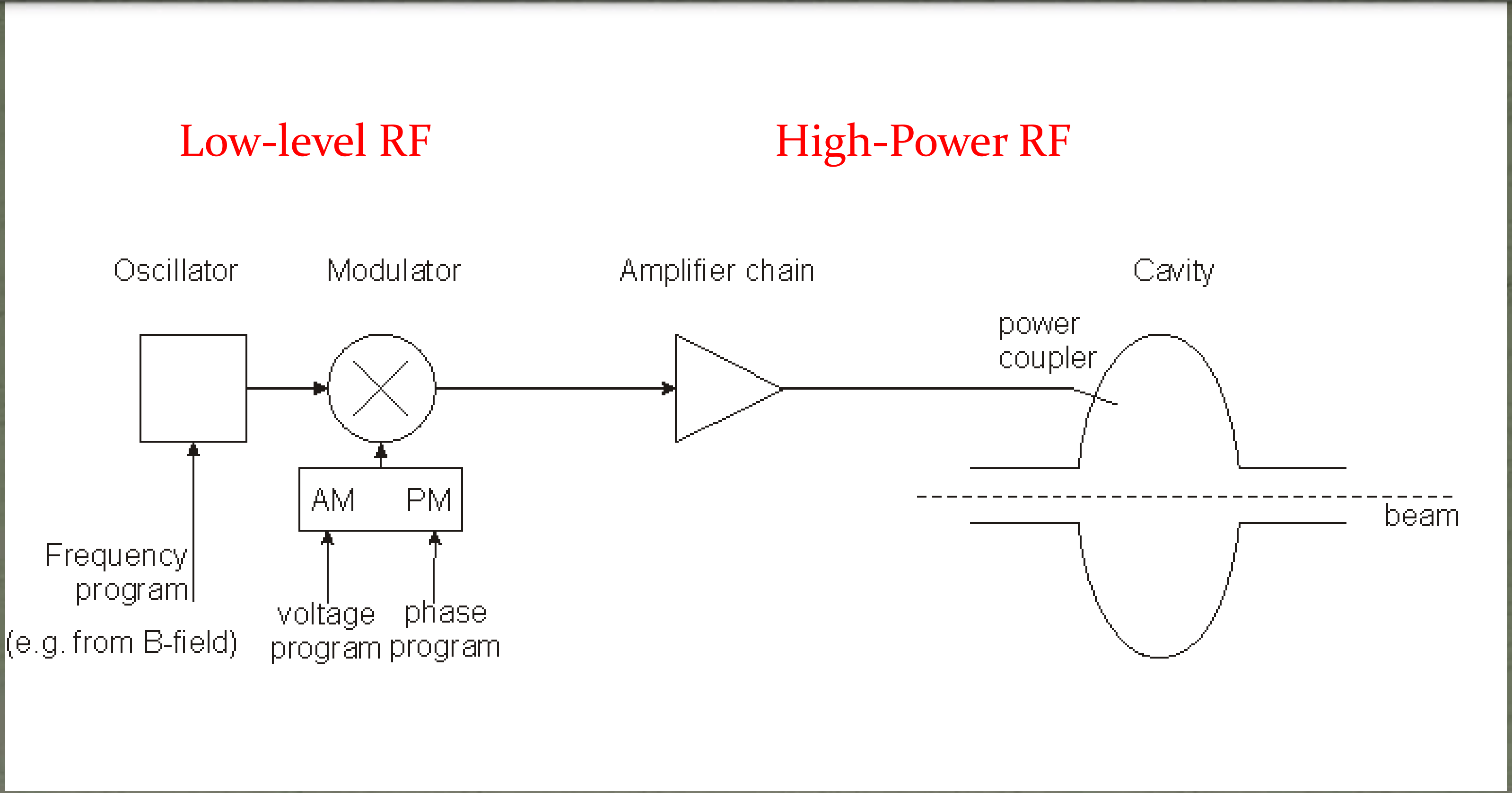
RF system & control loops

e.g.: … for a synchrotron:

Cavity control loops

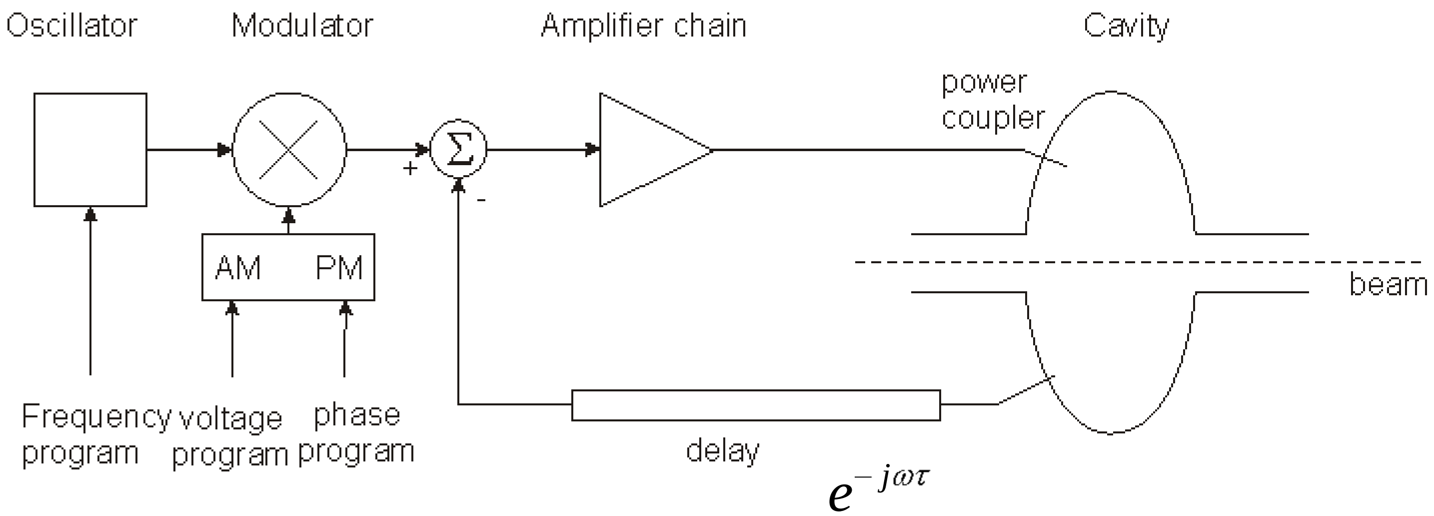
Beam control loops

Minimal RF system (of a synchrotron)



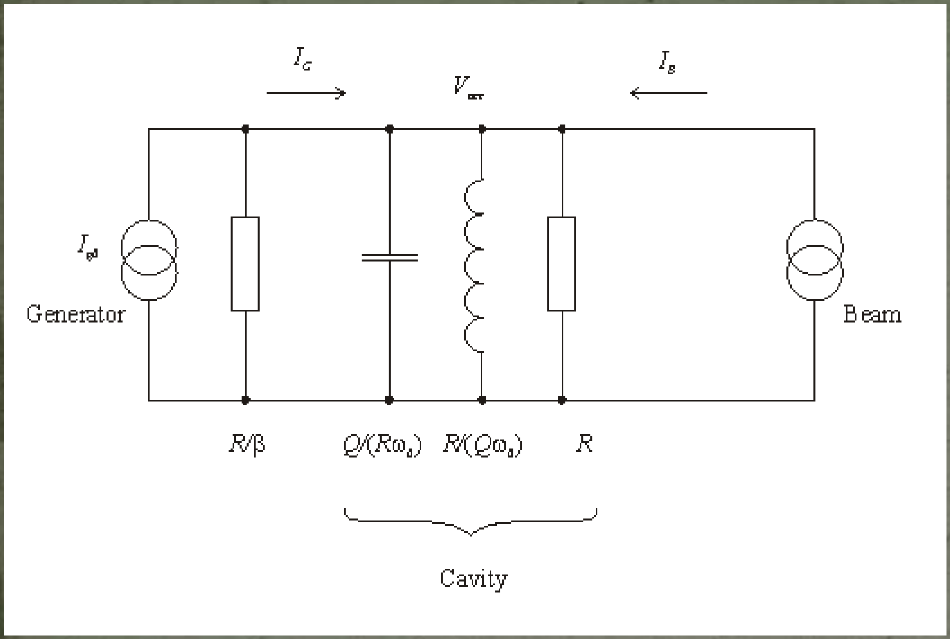
- The frequency has to be controlled to follow the magnetic field such that the beam remains in the centre of the vacuum chamber.
- The voltage has to be controlled to allow for capture at injection, a correct bucket area during acceleration, matching before ejection; phase may have to be controlled for transition crossing and for synchronisation before ejection.

Fast RF Feed-back loop



- Compares actual RF voltage and phase with desired and corrects.
- Rapidity limited by total group delay (path lengths) (some 100 ns).
- Unstable if loop gain =1 with total phase shift 180 ° – design requires to stay away from this point (stability margin)
- The group delay limits the gain·bandwidth product.
- Works also to keep voltage at zero for strong beam loading, i.e. it reduces the beam impedance.

Fast feedback loop at work



- Without feedback, $V_{acc} = \left(I_{G0} + I_B\right) \cdot Z(\omega)$

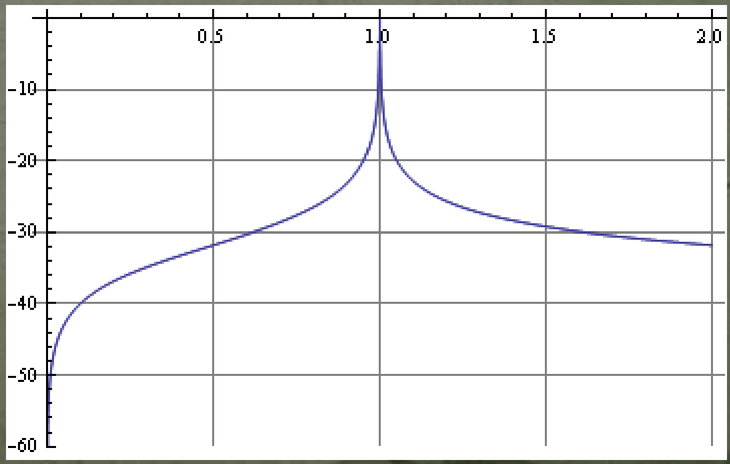
  where $$Z(\omega) = \frac{R/(1+\beta)}{1 + jQ\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)}$$

- Detect the gap voltage, feed it back to I_{G0} such that $I_{G0} = I_{drive} - G \cdot V_{acc}$

  where G is the total loop gain (pick-up, cable, amplifier chain …)
- Result: $$V_{acc} = \left(I_{drive} + I_B\right) \cdot \frac{Z(\omega)}{1 + G \cdot Z(\omega)}$$

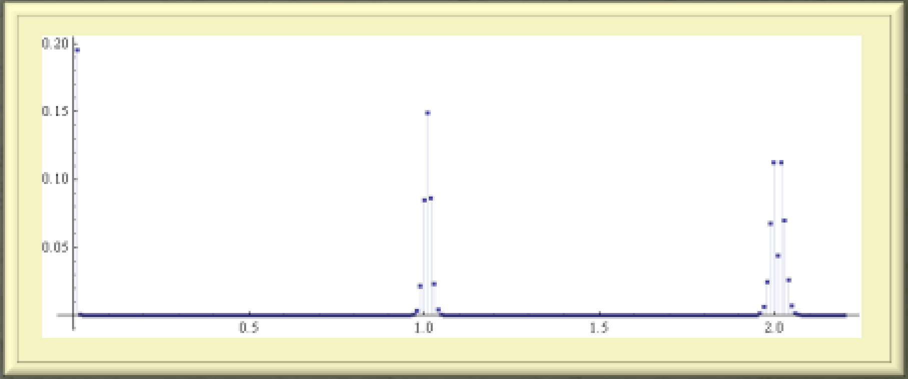
- Gap voltage is stabilised!
- Impedance seen by the beam is reduced by the loop gain!
- Plot on the right: $\frac{1+\beta}{R}\left|\frac{Z(\omega)}{1+G \cdot Z(\omega)}\right|$ vs. ω

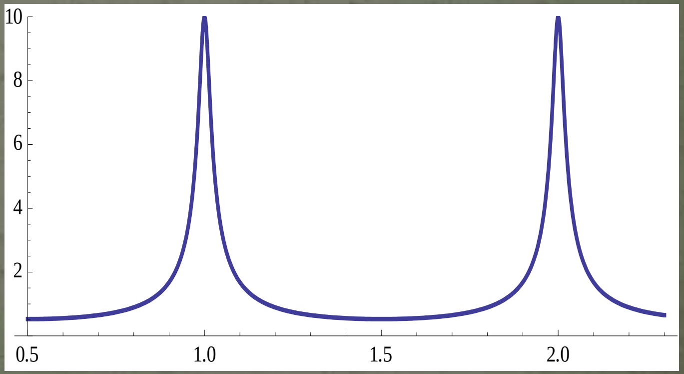
  with the loop gain varying from 0 to 50 dB



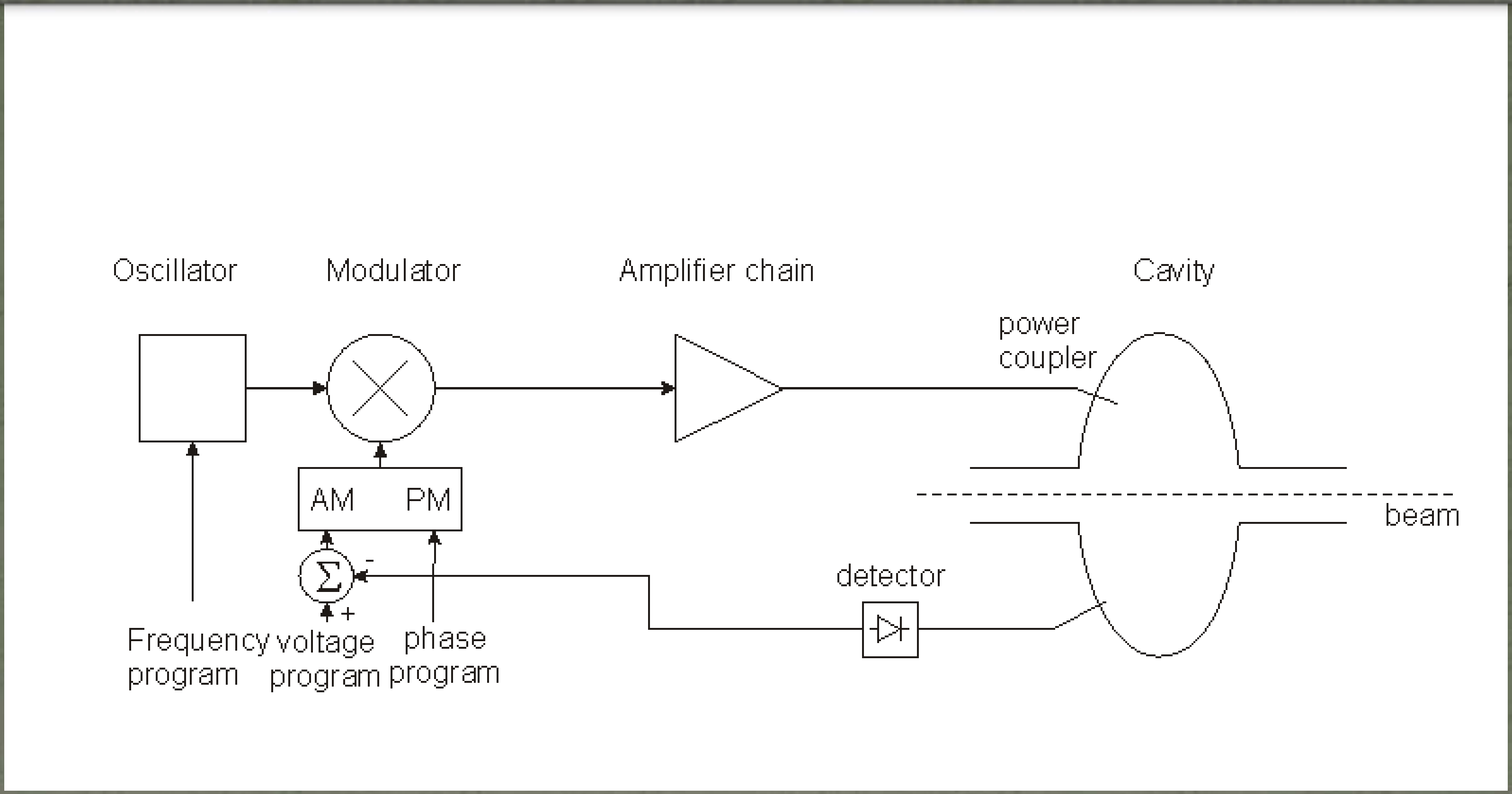
1-turn delay feed-back loop

- The speed of the "fast RF feedback" is limited by the group delay – this is typically a significant fraction of the revolution period.
- How to lower the impedance over many harmonics of the revolution frequency?
- Remember: the beam spectrum is limited to relatively narrow bands around the multiples of the revolution frequency!
- Only in these narrow bands the loop gain must be high!
- Install a comb filter! ... and extend the group delay to exactly 1 turn – in this case the loop will have the desired effect and remain stable!



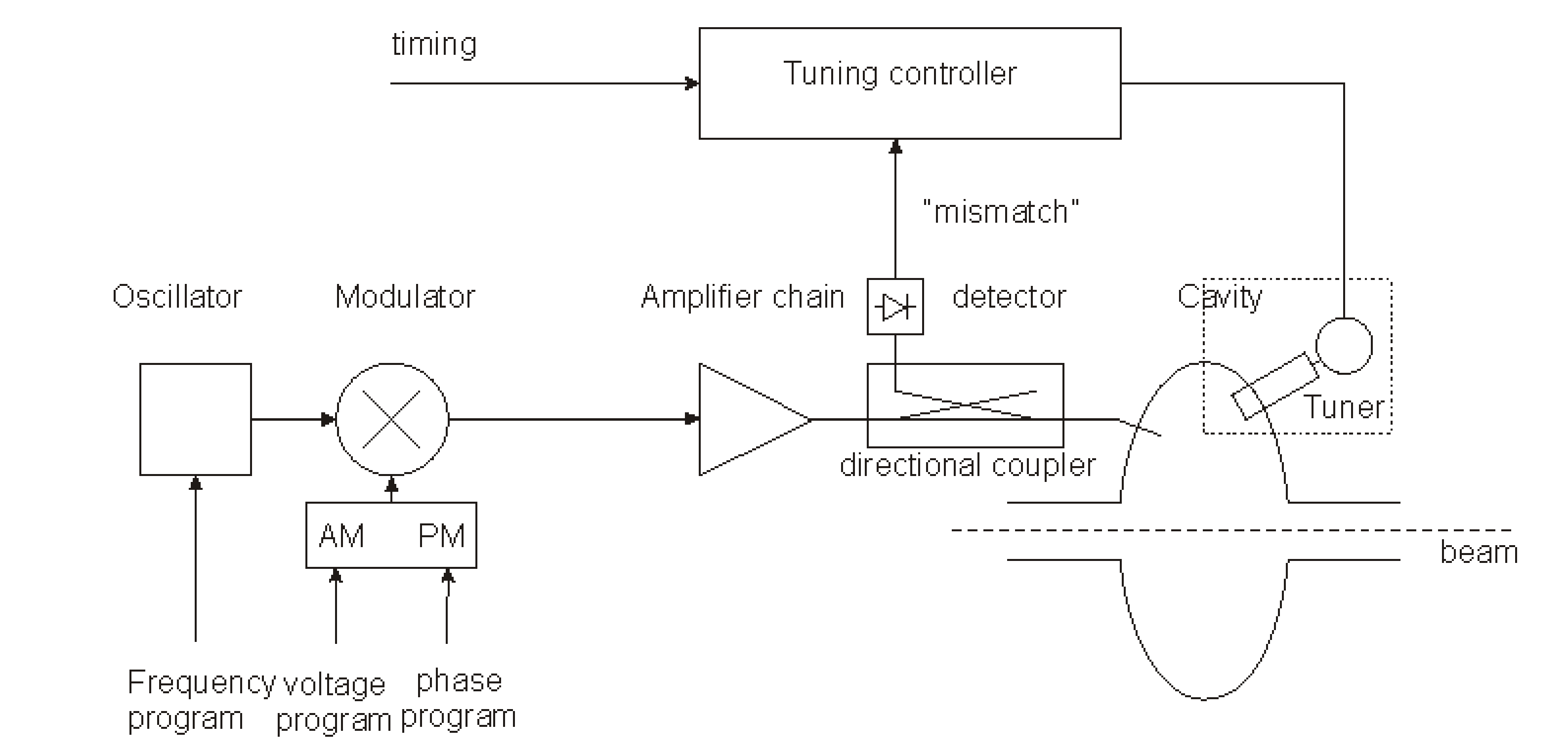


Field amplitude control loop (AVC)



- Compares the detected cavity voltage to the voltage program. The error signal serves to correct the amplitude

Tuning loop

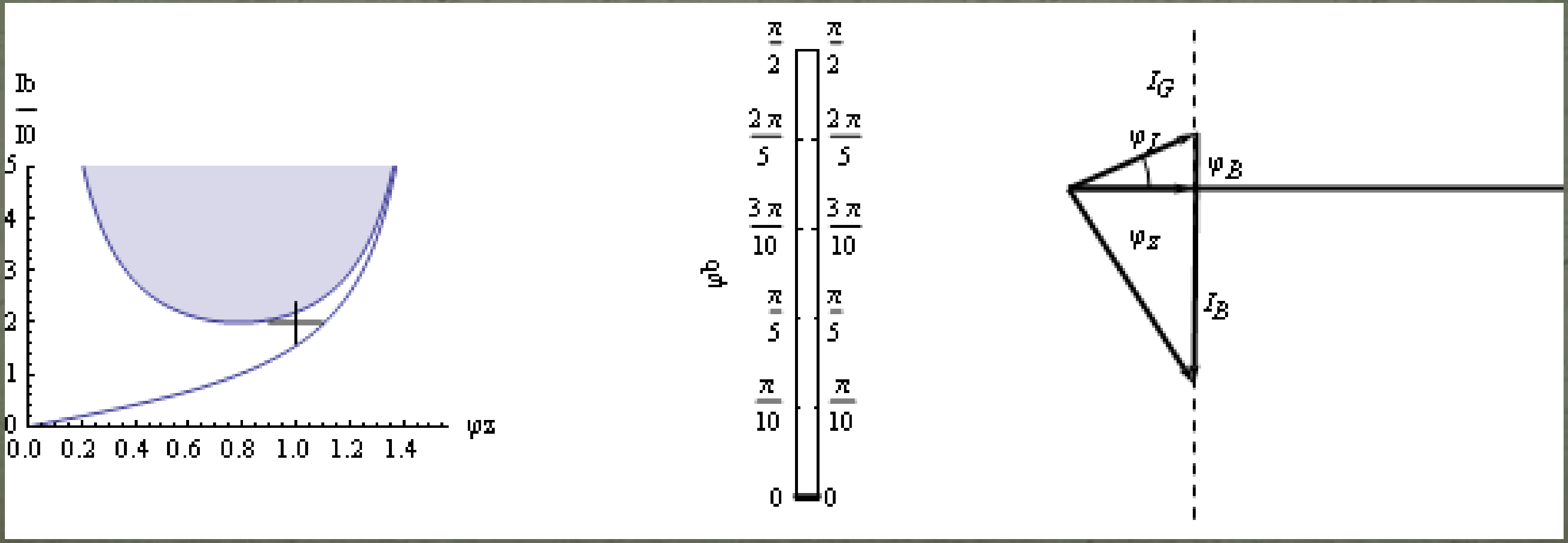


- Tunes the resonance *f* of the cavity to minimize the mismatch of the PA.
- In the presence of beam loading, this may mean $f_r \neq f$.
- In an ion ring accelerator, the tuning range might be > octave!
- For fixed *f* systems, tuners are needed to compensate for slow drifts.
- Examples for tuners:
  - controlled power supply driving ferrite bias (varying μ),
  - stepping motor driven plunger,
  - motorized variable capacitor, …

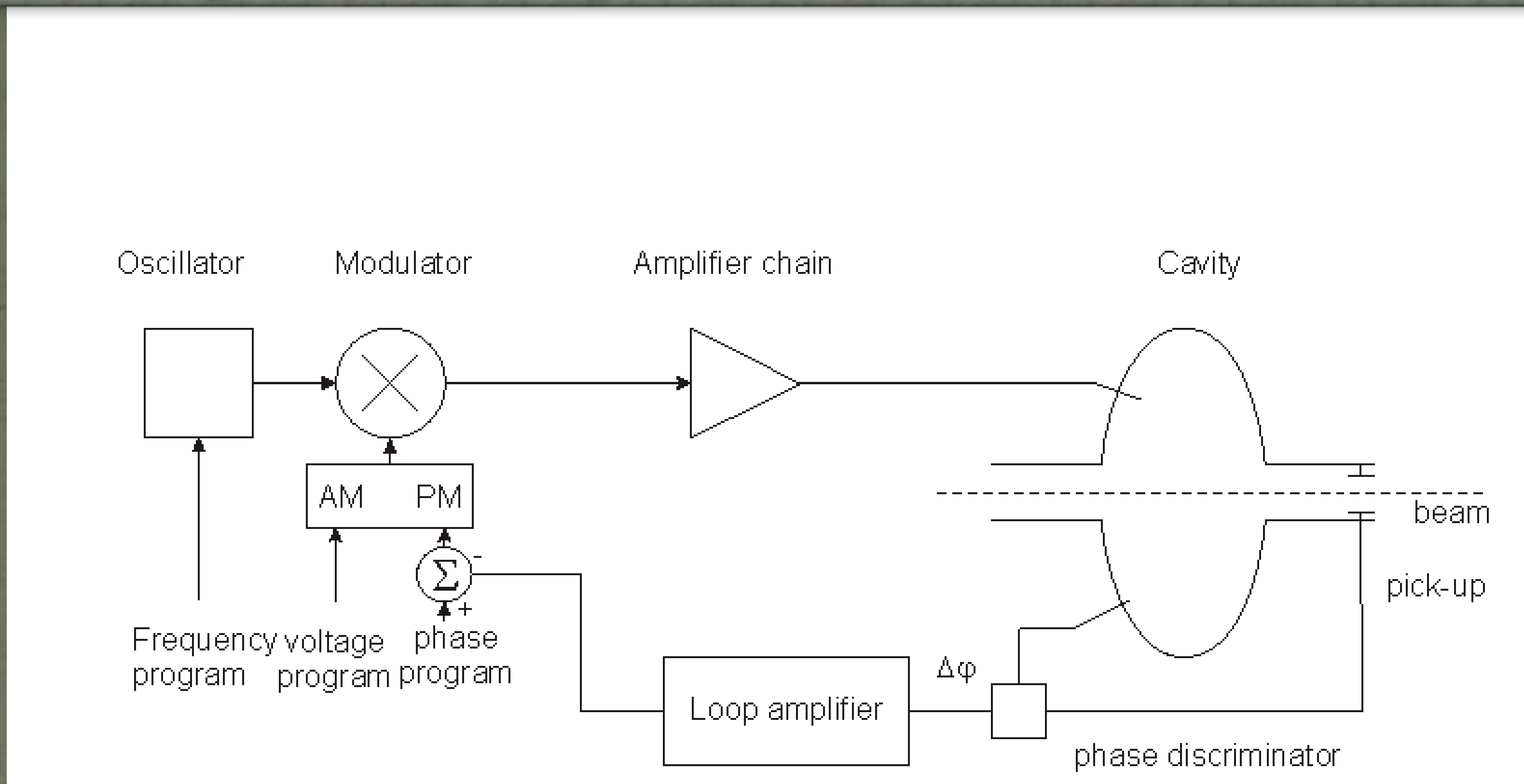
Example: how tuning may depend on beam current

- Horizontal axis: the tuning angle
- Vertical axis: the beam current
- Hashed: unstable area (Robinson criterion)
- Line: $\varphi_L = 0$ (matching condition)
- Parameter: φ_B

Phasor diagram for point marked (fixed I_B and φz)

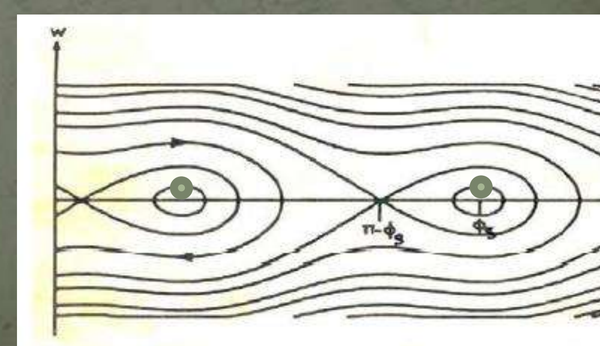


Beam phase loop



- Longitudinal motion: $$\frac{d^2(\Delta\phi)}{dt^2} + \Omega_s^2(\Delta\phi)^2 = 0$$
- Loop amplifier transfer function designed to damp
- synchrotron oscillation. Modified equation:

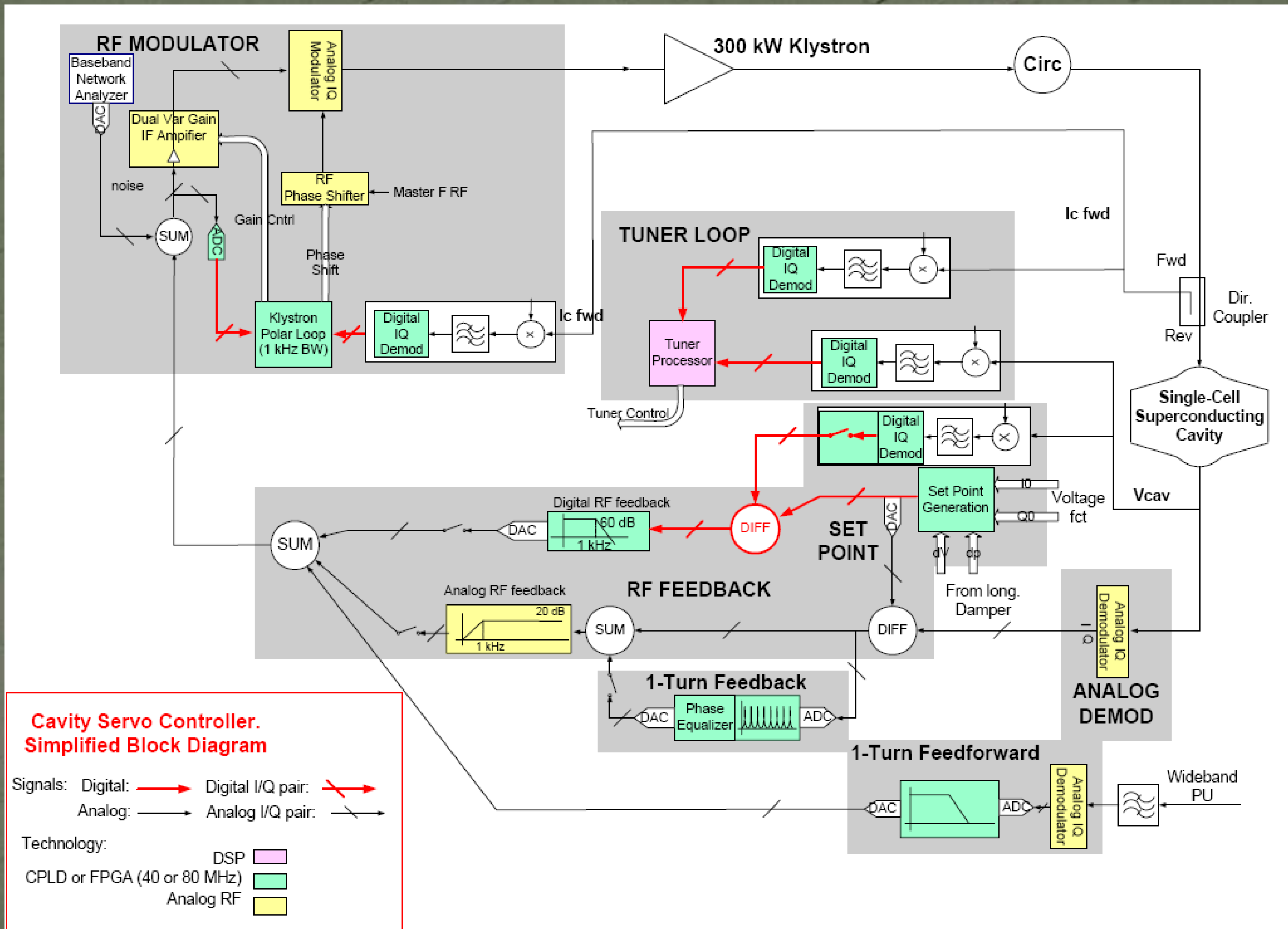
$$\frac{d^2(\Delta\phi)}{dt^2} + \alpha\frac{d(\Delta\phi)}{dt} + \Omega_s^2(\Delta\phi)^2 = 0$$



Other loops

- Radial loop:
  - Detect average radial position of the beam,
  - Compare to a programmed radial position,
  - Error signal controls the frequency.
- Synchronisation loop:
  - 1st step: Synchronize *f* to an external frequency (will also act on radial position!).
  - 2nd step: phase loop
- …

A real implementation: LHC LLRF



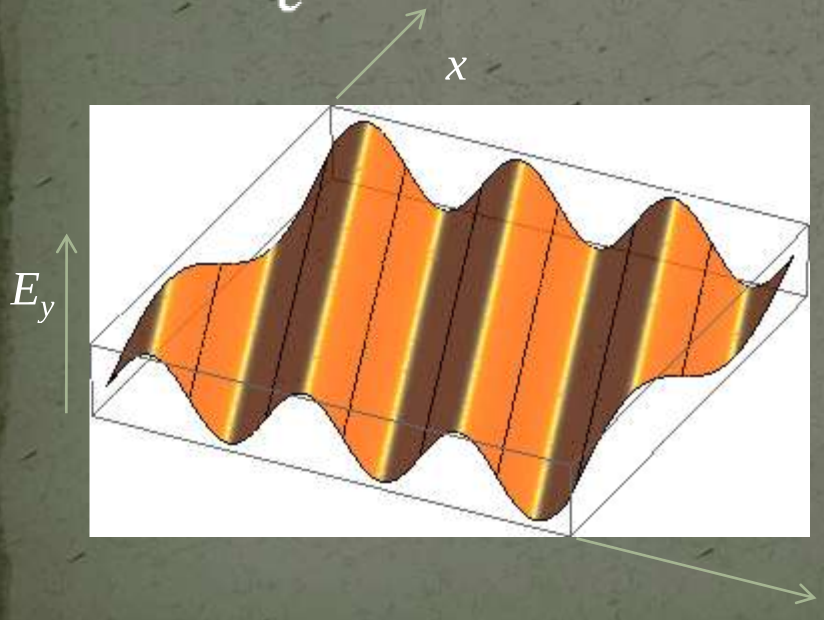
Fields in a waveguide

Homogeneous plane wave

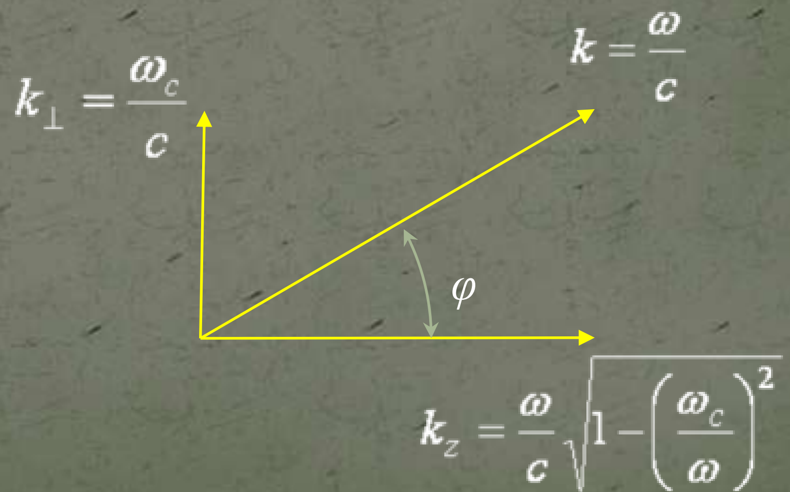
$$\vec{E} \propto \vec{u}_y \cos\left(\omega t - \vec{k} \cdot \vec{r}\right)$$

$$\vec{B} \propto \vec{u}_x \cos\left(\omega t - \vec{k} \cdot \vec{r}\right)$$

$$\vec{k} \cdot \vec{r} = \frac{\omega}{c}\left(\cos(\varphi) z + \sin(\varphi) x\right)$$

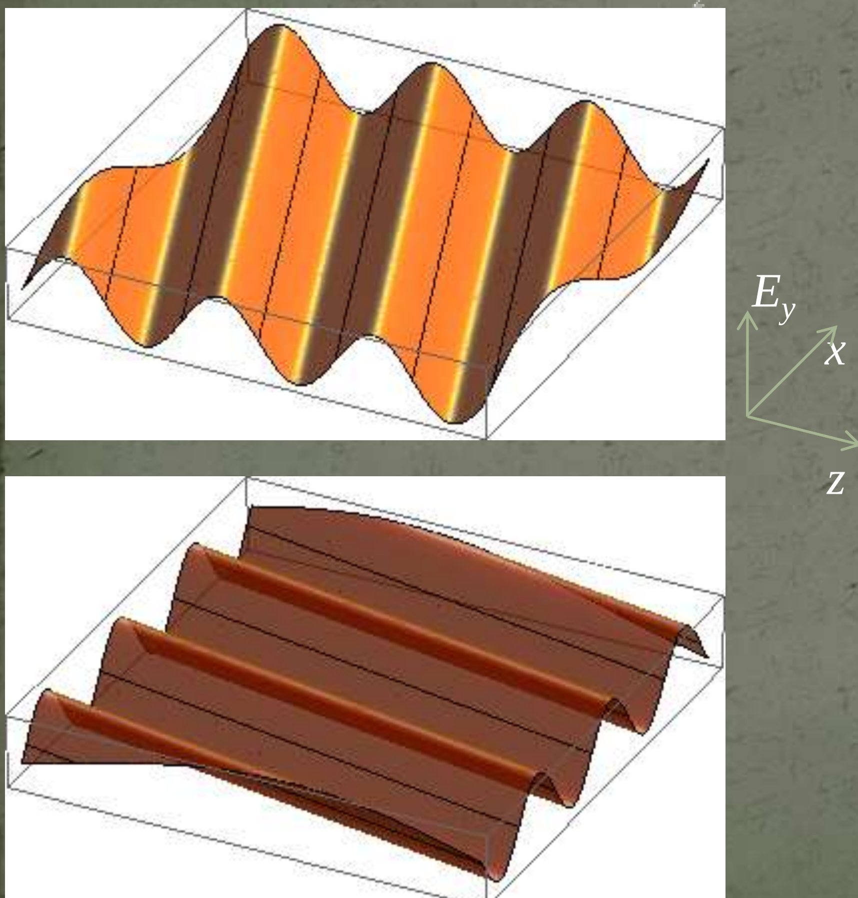


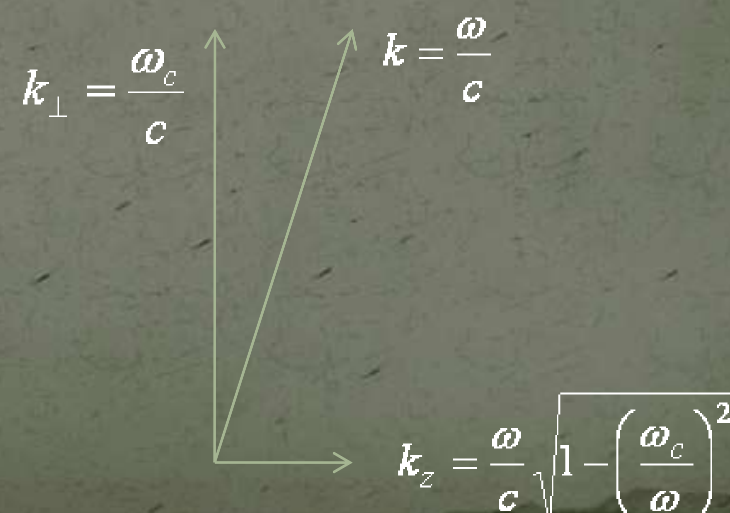
Wave vector $\vec{k}$:
the direction of $\vec{k}$ is the direction of propagation,
the length of $\vec{k}$ is the phase shift per unit length.
$\vec{k}$ behaves like a vector.



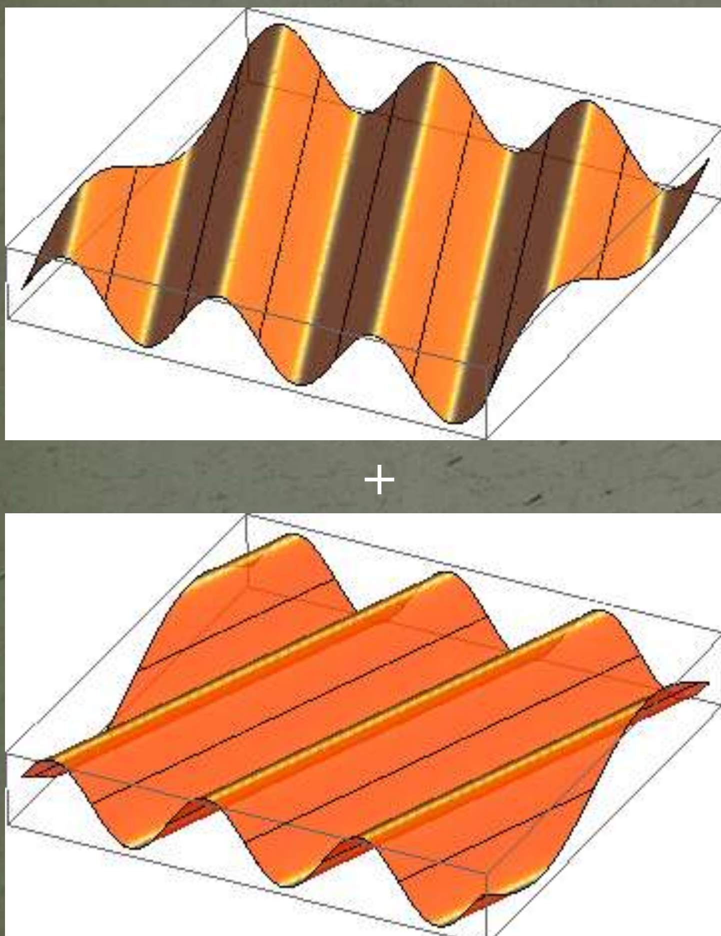
Wave length, phase velocity

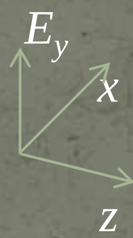
- The components of $\vec{k}$ are related to the wavelength in the direction of that component as $\lambda_z = \frac{2\pi}{k_z}$ etc. , to the phase velocity as $v_{\varphi,z} = \frac{\omega}{k_z} = f\,\lambda_z$.





Superposition of 2 homogeneous plane waves





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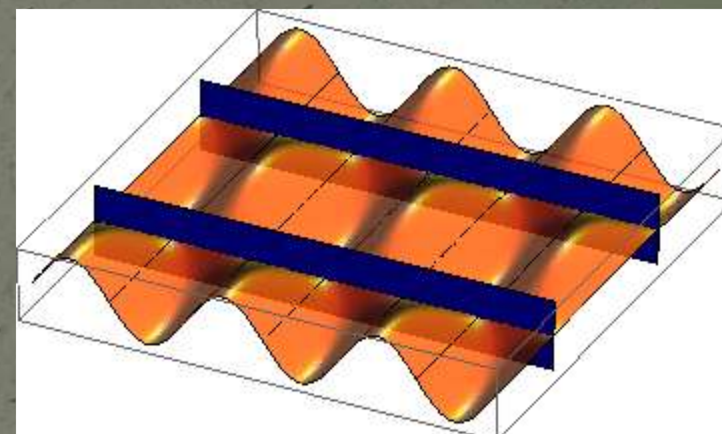


Metallic walls may be inserted where $E_y \equiv 0$ without perturbing the fields. Note the standing wave in *x*-direction!

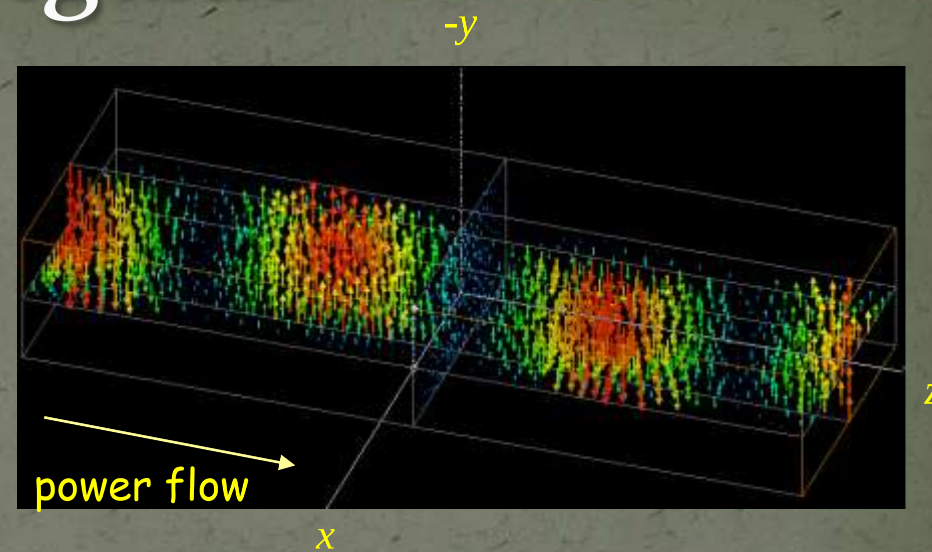
This way one gets a hollow rectangular waveguide

Rectangular waveguide

Fundamental (TE_{10} or H_{10}) mode
in a standard rectangular waveguide.

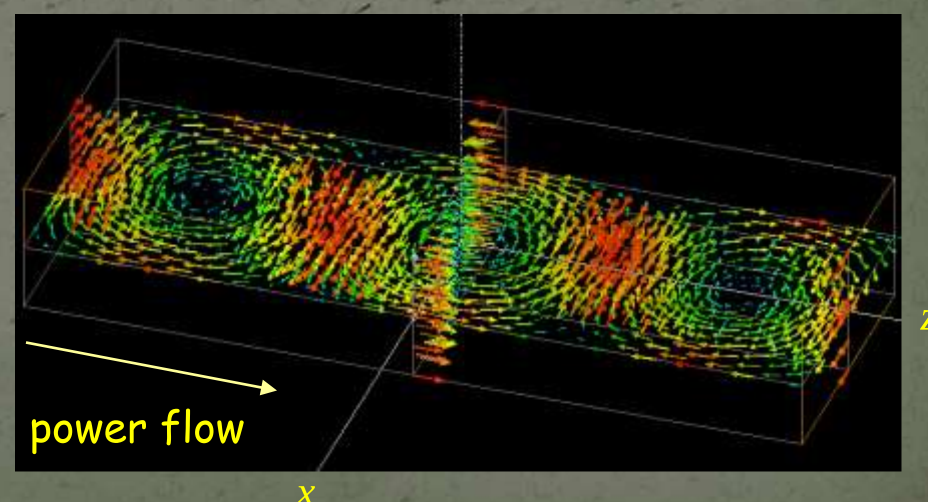
E.g. forward wave

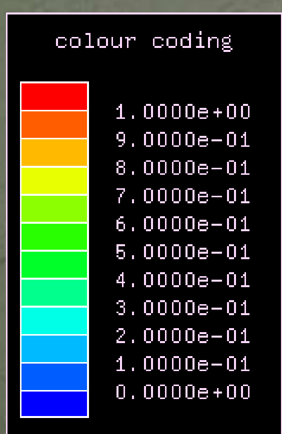
electric field



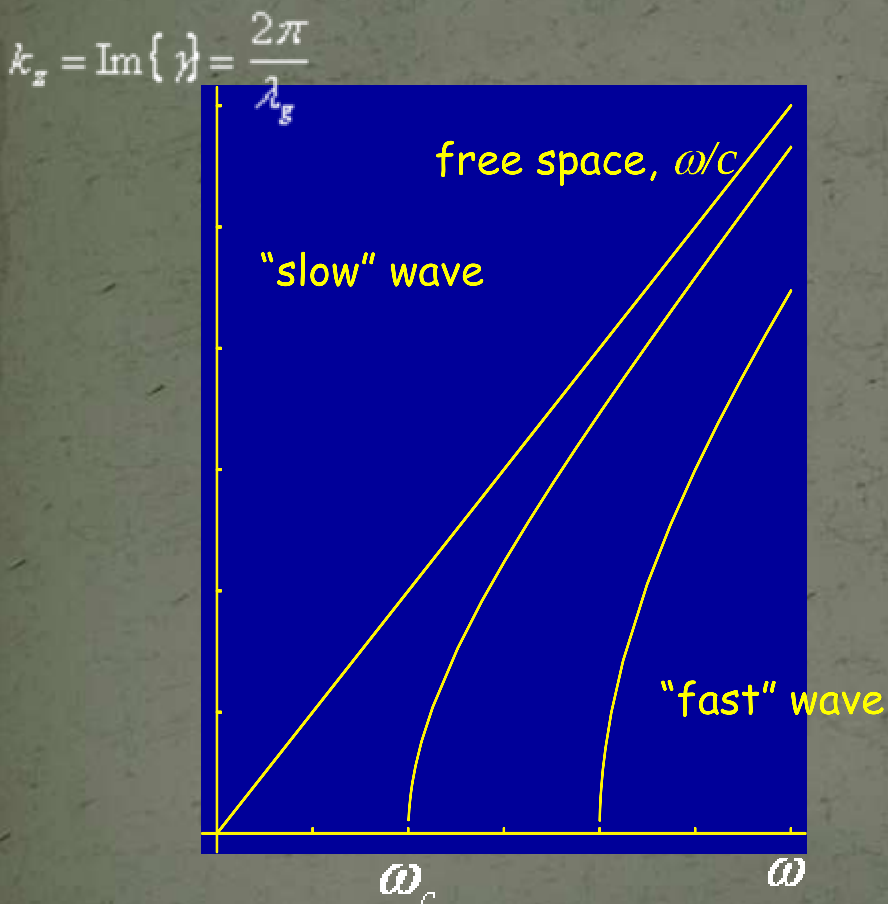
power flow: $\frac{1}{2}\mathrm{Re}\left\{\iint_{\substack{\text{cross}\\ \text{section}}} \vec{E}\times\vec{H}^{*}\cdot \mathrm{d}\vec{A}\right\}$

magnetic field





Waveguide dispersion



e.g.: TE_{10}-wave in rectangular waveguide:

$$\gamma = \mathrm{j}\sqrt{\left(\frac{\omega}{c}\right)^2 - \left(\frac{\pi}{a}\right)^2}$$

$$Z_0 = \frac{\mathrm{j}\omega\mu}{\gamma}$$

$$\lambda_{\mathrm{cutoff}} = 2a$$

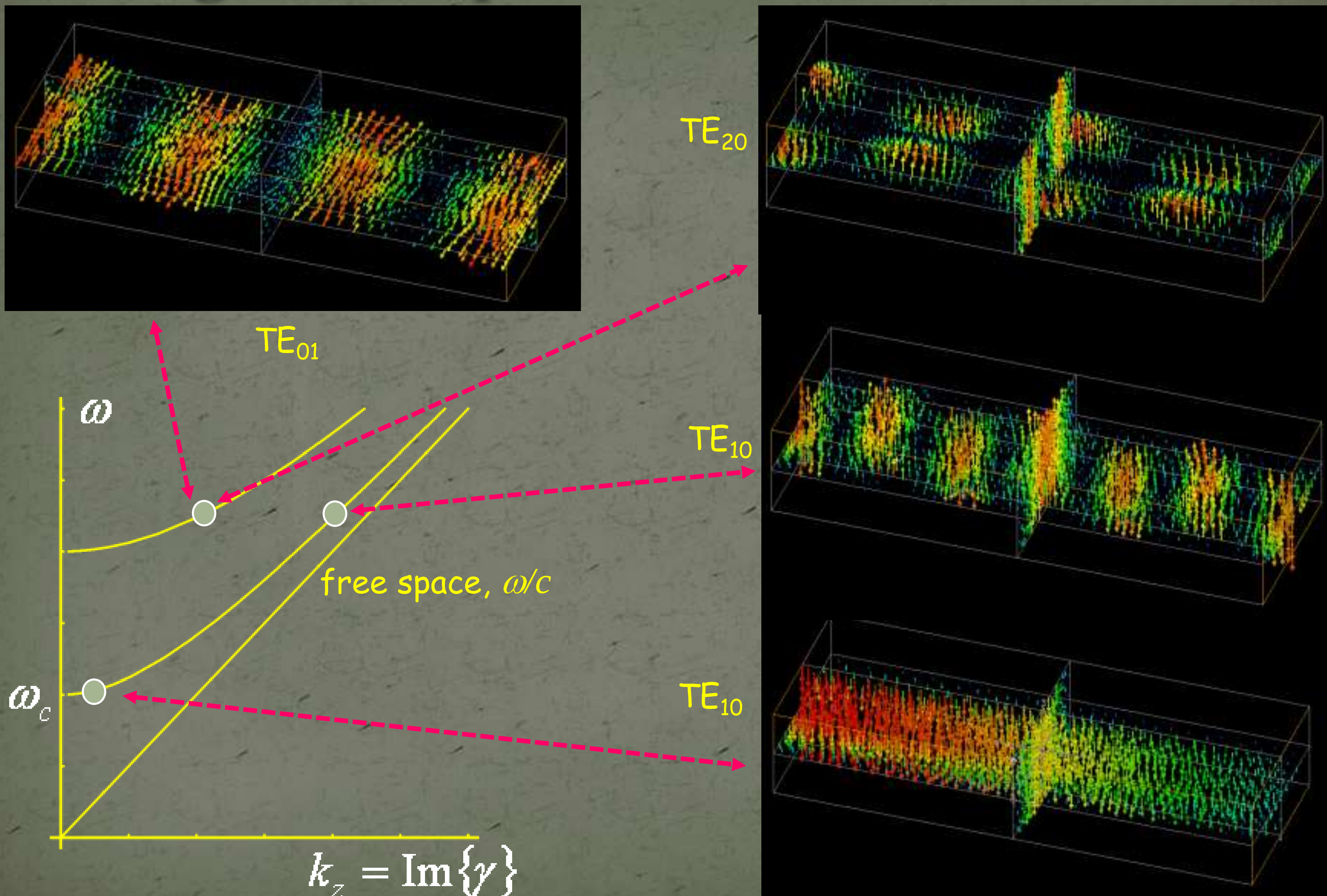
general cylindrical waveguide:

$$\gamma = \mathrm{j}\sqrt{\left(\frac{\omega}{c}\right)^2 - k_\perp{}^2}$$

$$Z_0 = \frac{\mathrm{j}\omega\mu}{\gamma} \text{ for TE}, \quad Z_0 = \frac{\gamma}{\mathrm{j}\omega\varepsilon} \text{ for TM}$$

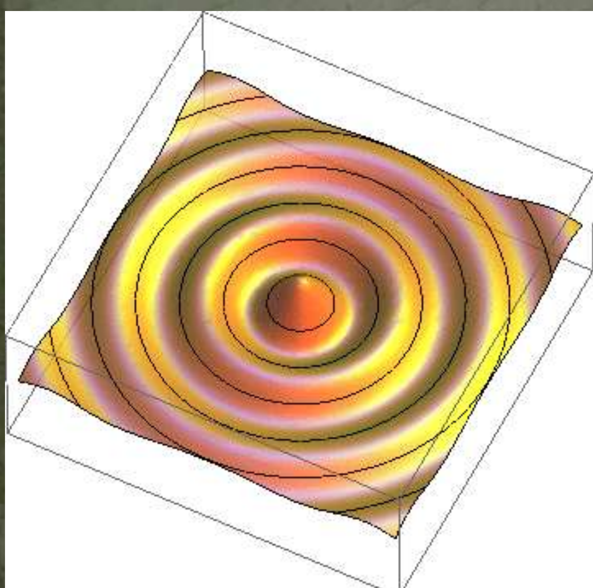
In a hollow waveguide: phase velocity > c, group velocity < c

Waveguide dispersion (continued)

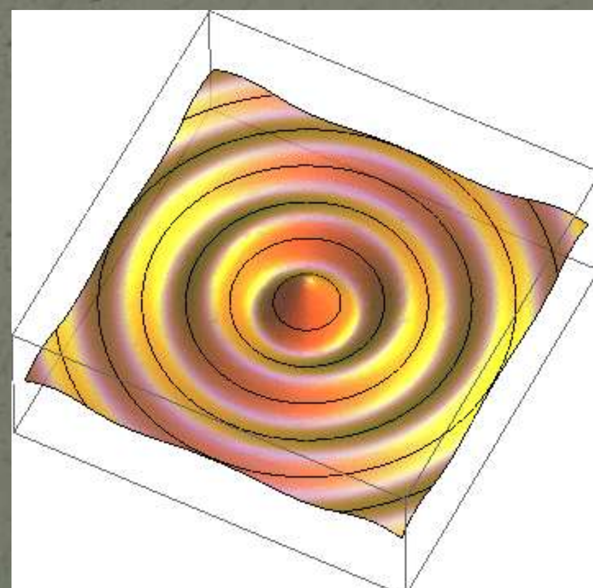


Radial waves

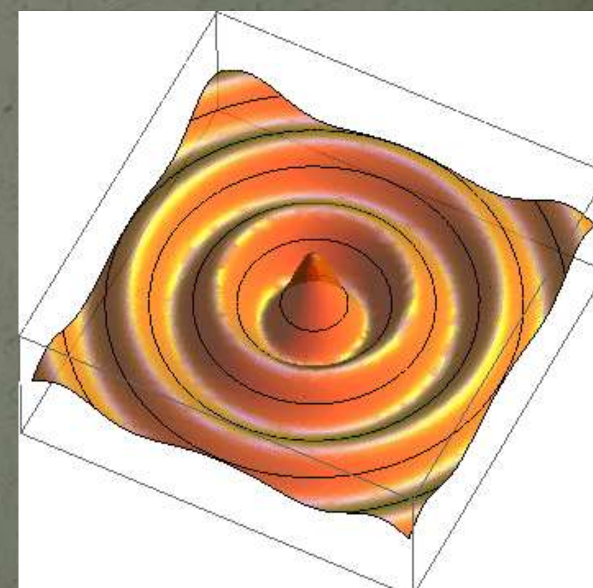
- Also radial waves may be interpreted as superpositions of plane waves.
- The superposition of an outward and an inward radial wave can result in the field of a round hollow waveguide.



$E_z \propto H_n^{(2)}(k_\rho \rho)\cos(n\varphi)$



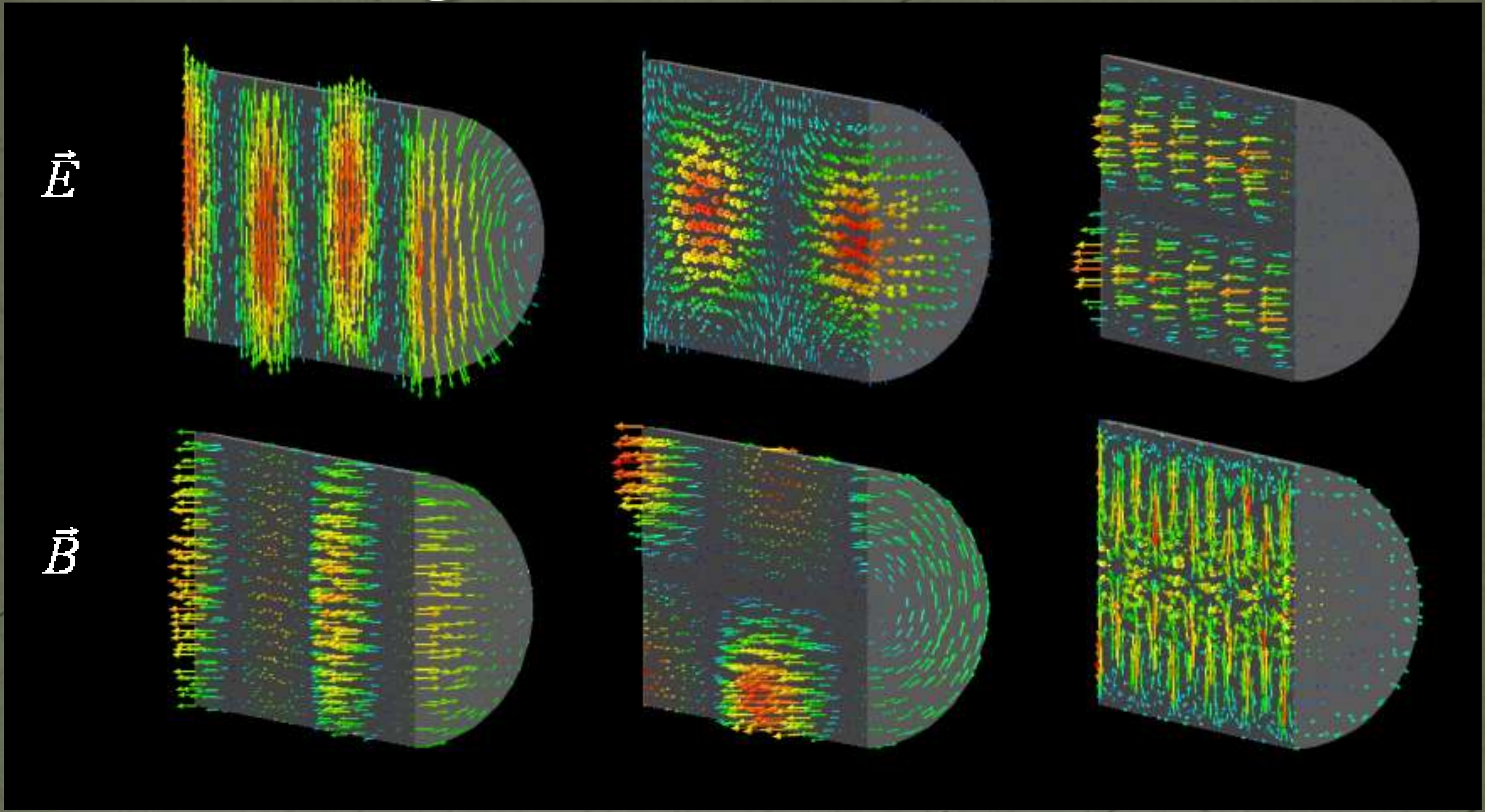
$E_z \propto H_n^{(1)}(k_\rho \rho)\cos(n\varphi)$



$E_z \propto J_n(k_\rho \rho)\cos(n\varphi)$

Round waveguide modes

parameters used in calculation:
f = 1.43, 1.09, 1.13 f_c, a: radius



TE$_{11}$: fundamental mode

$$\frac{f_c}{\text{GHz}} = \frac{87.85}{a/\text{mm}}$$

TM$_{01}$: axial electric field

$$\frac{f_c}{\text{GHz}} = \frac{114.74}{a/\text{mm}}$$

TE$_{01}$: lowest losses!

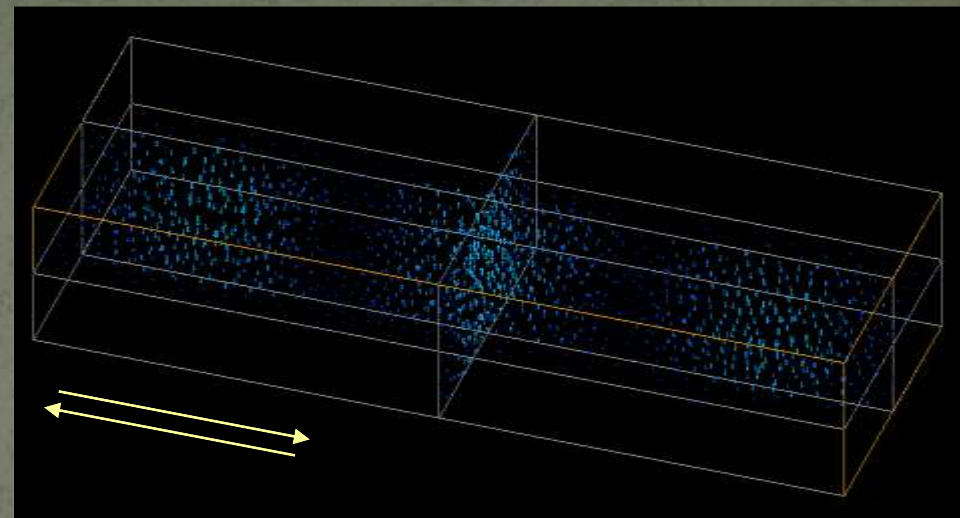
$$\frac{f_c}{\text{GHz}} = \frac{334.74}{a/\text{mm}}$$

From waveguide to cavity

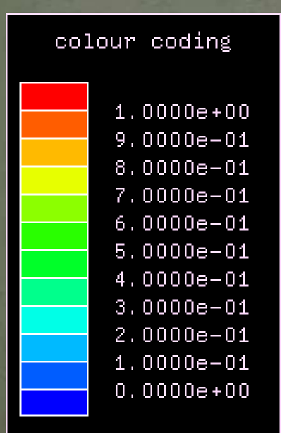
Standing wave – resonator

Same as above, but two counter-running waves of identical amplitude.

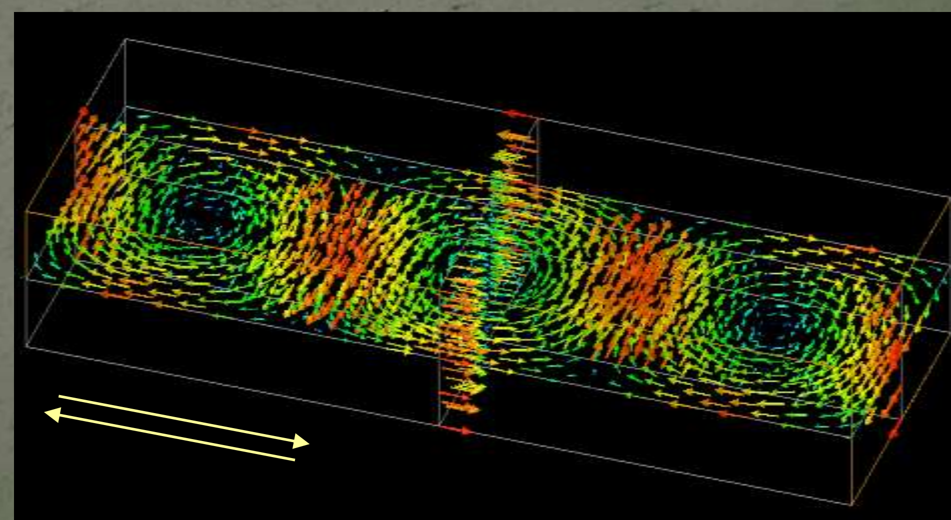
electric field



no net power flow: $\frac{1}{2}\mathrm{Re}\left\{\iint_{\substack{\text{cross}\\\text{section}}} \vec{E}\times\vec{H}^{*}\cdot \mathrm{d}\vec{A}\right\}=0$

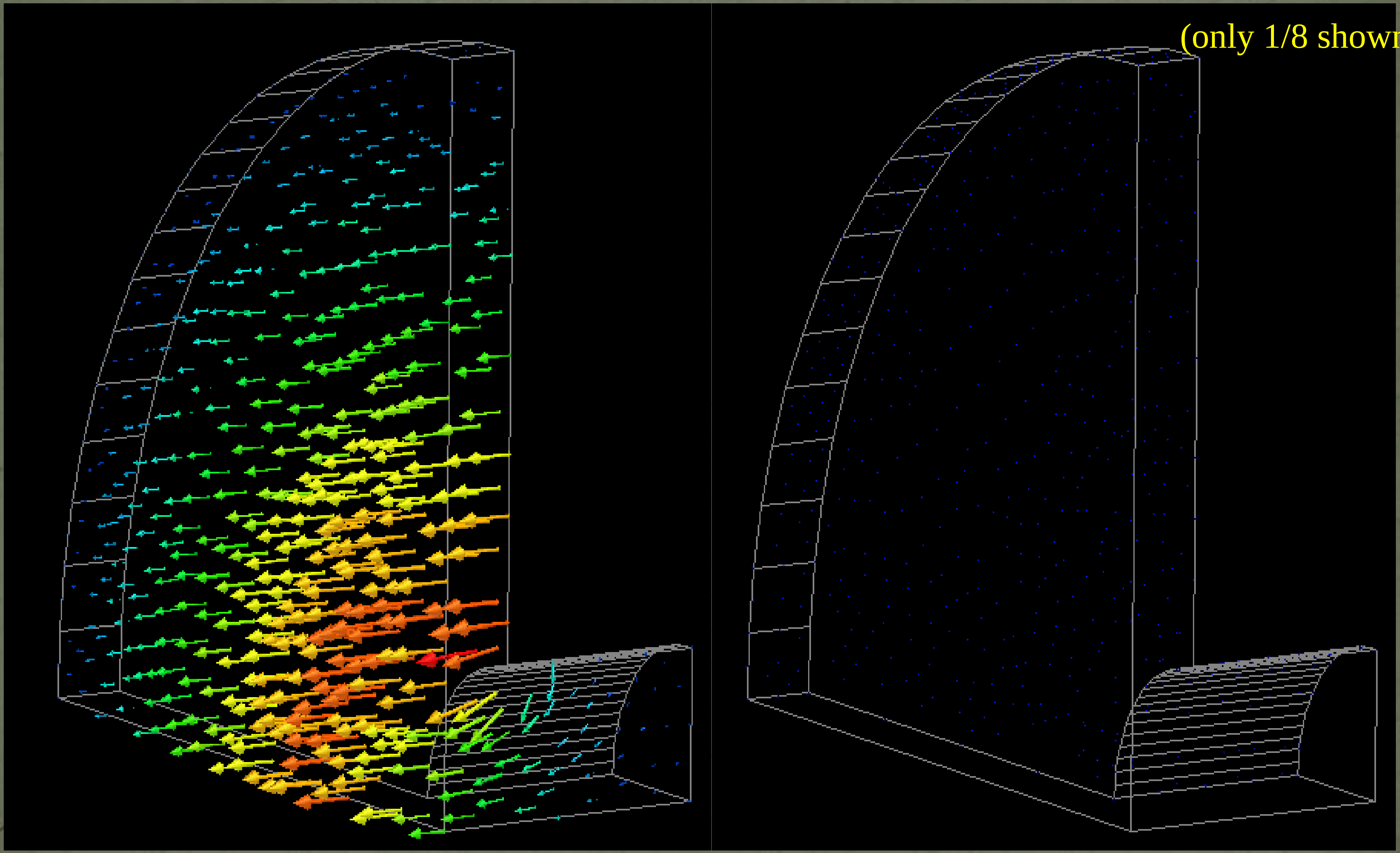


magnetic field (90° out of phase)



A piece of round waveguide – pillbox cavity

TM$_{010}$-mode

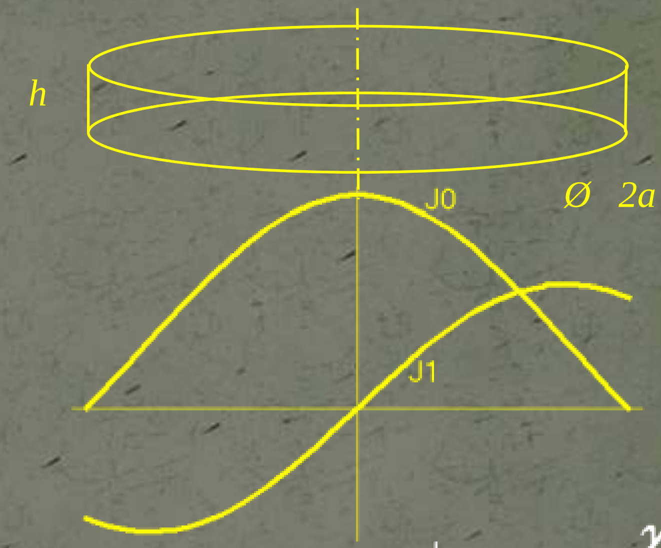


electric field | magnetic field

Pillbox cavity field (w/o beam tube)

The only non-vanishing field components :

$$E_z = \frac{1}{j\omega\varepsilon_0}\frac{\chi_{01}}{a}\sqrt{\frac{1}{\pi}}\frac{J_0\left(\frac{\chi_{01}\rho}{a}\right)}{a\,J_1\left(\frac{\chi_{01}}{a}\right)}$$



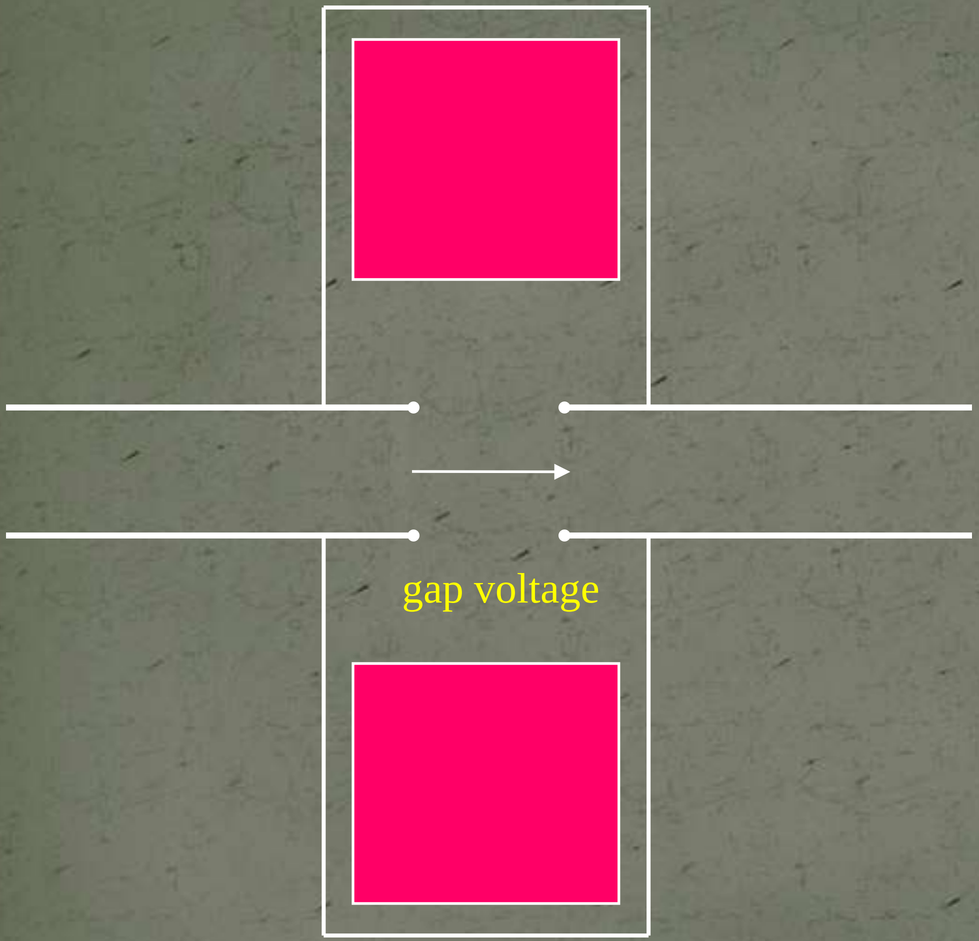
$$B_\varphi = \mu_0\sqrt{\frac{1}{\pi}}\frac{J_1\left(\frac{\chi_{01}\rho}{a}\right)}{a\,J_1\left(\frac{\chi_{01}}{a}\right)}$$

$$\omega_0\big|_{pillbox} = \frac{\chi_{01}\,c}{a}$$

$$\chi_{01} = 2.40483\ldots$$

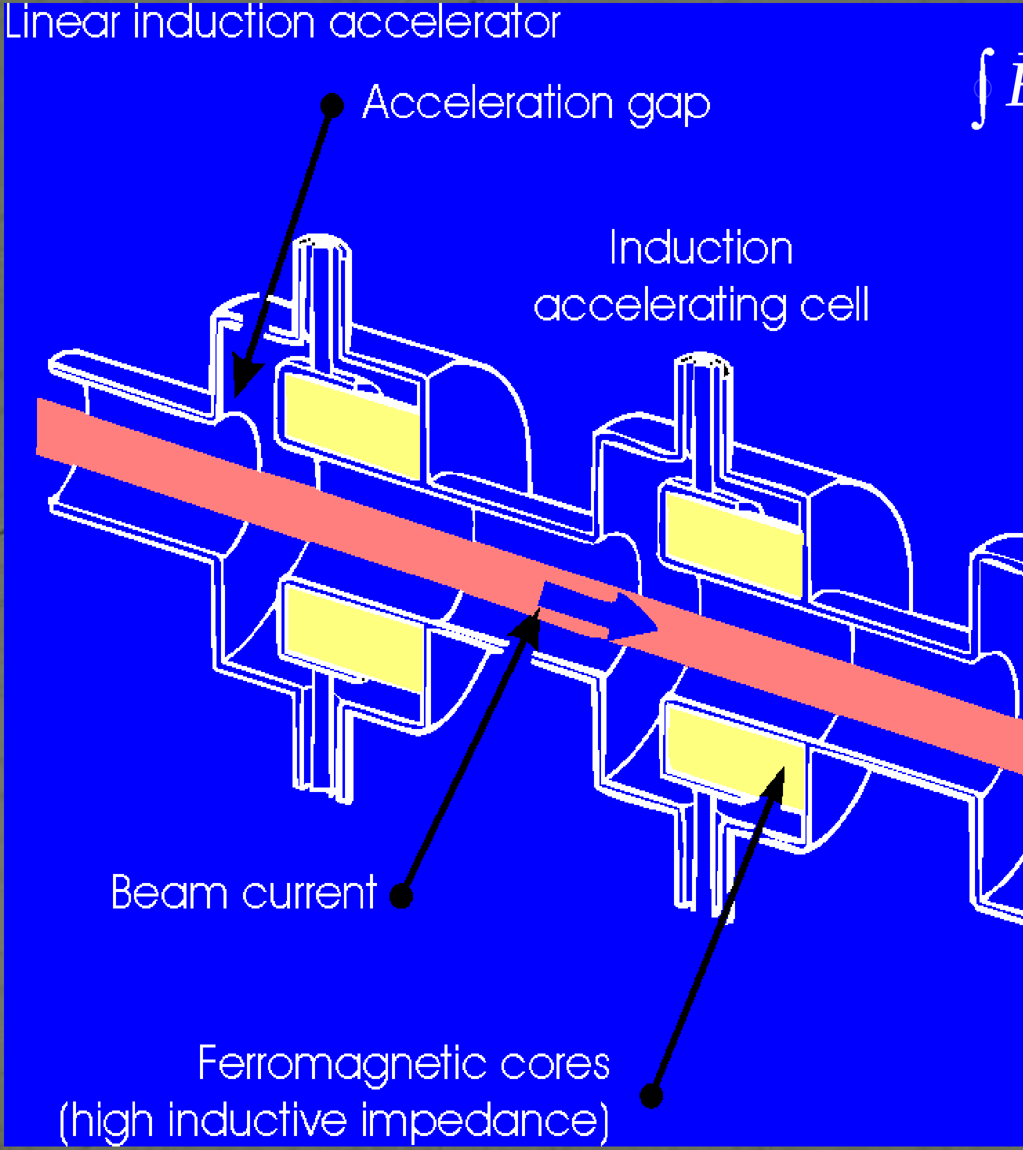
Accelerating gap

Accelerating gap



- We want a voltage across the gap!
- It cannot be DC, since we want the beam tube on ground potential.
- Use $\oint \vec{E} \cdot \mathrm{d}\vec{s} = -\iint \frac{\mathrm{d}\vec{B}}{\mathrm{d}t} \cdot \mathrm{d}\vec{A}$
- The "shield" imposes a
  - upper limit of the voltage pulse duration or – equivalently –
  - a lower limit to the usable frequency.
- The limit can be extended with a material which acts as "open circuit"!
- Materials typically used:
  - ferrites (depending on *f*-range)
  - magnetic alloys (MA) like Metglas®, Finemet®, Vitrovac®…
- resonantly driven with RF (ferrite loaded cavities) – or with pulses (induction cell)

Linear induction accelerator

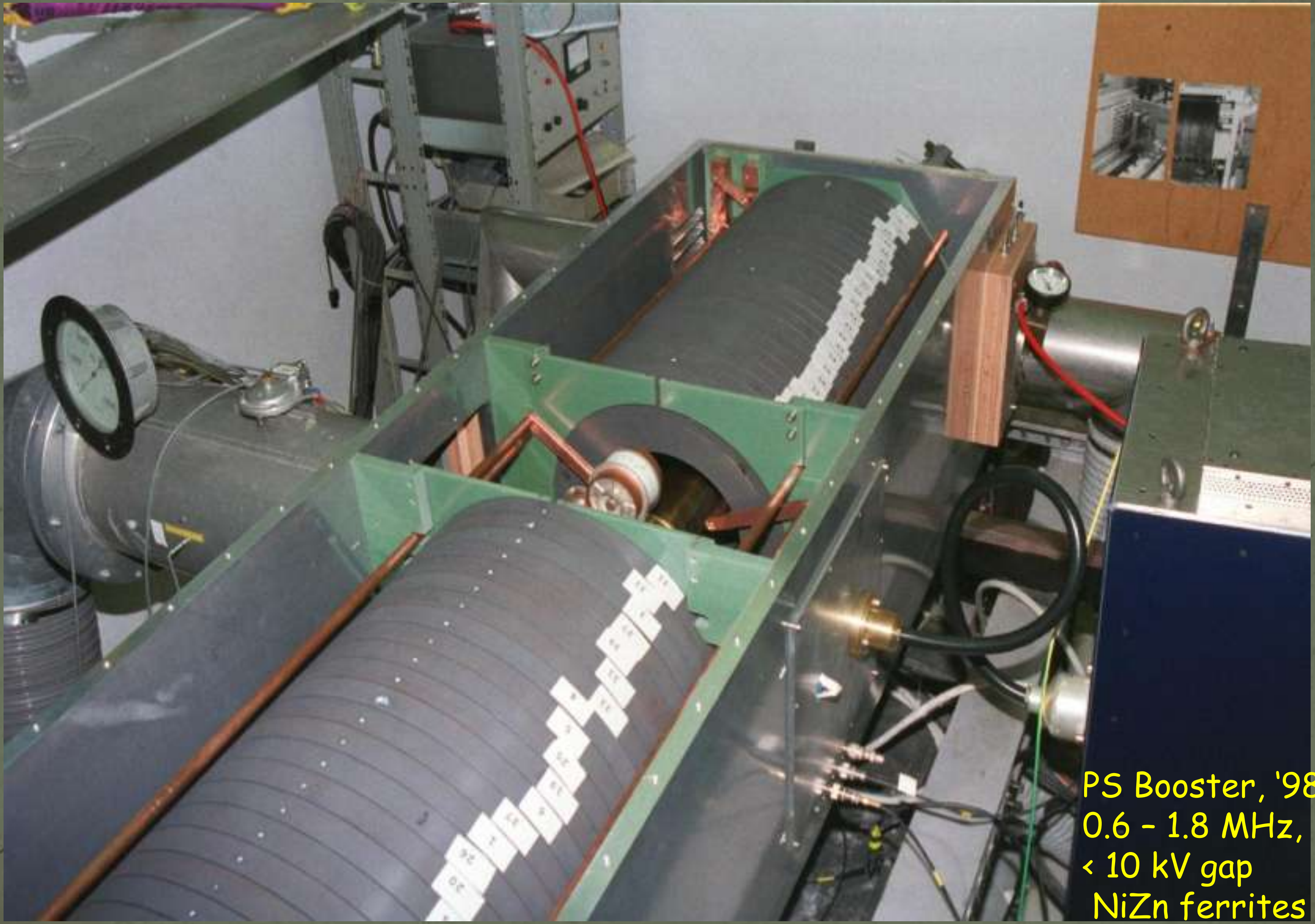


$$\oint \vec{E} \cdot d\vec{s} = -\iint \frac{\partial \vec{B}}{\partial t} \cdot d\vec{A}$$

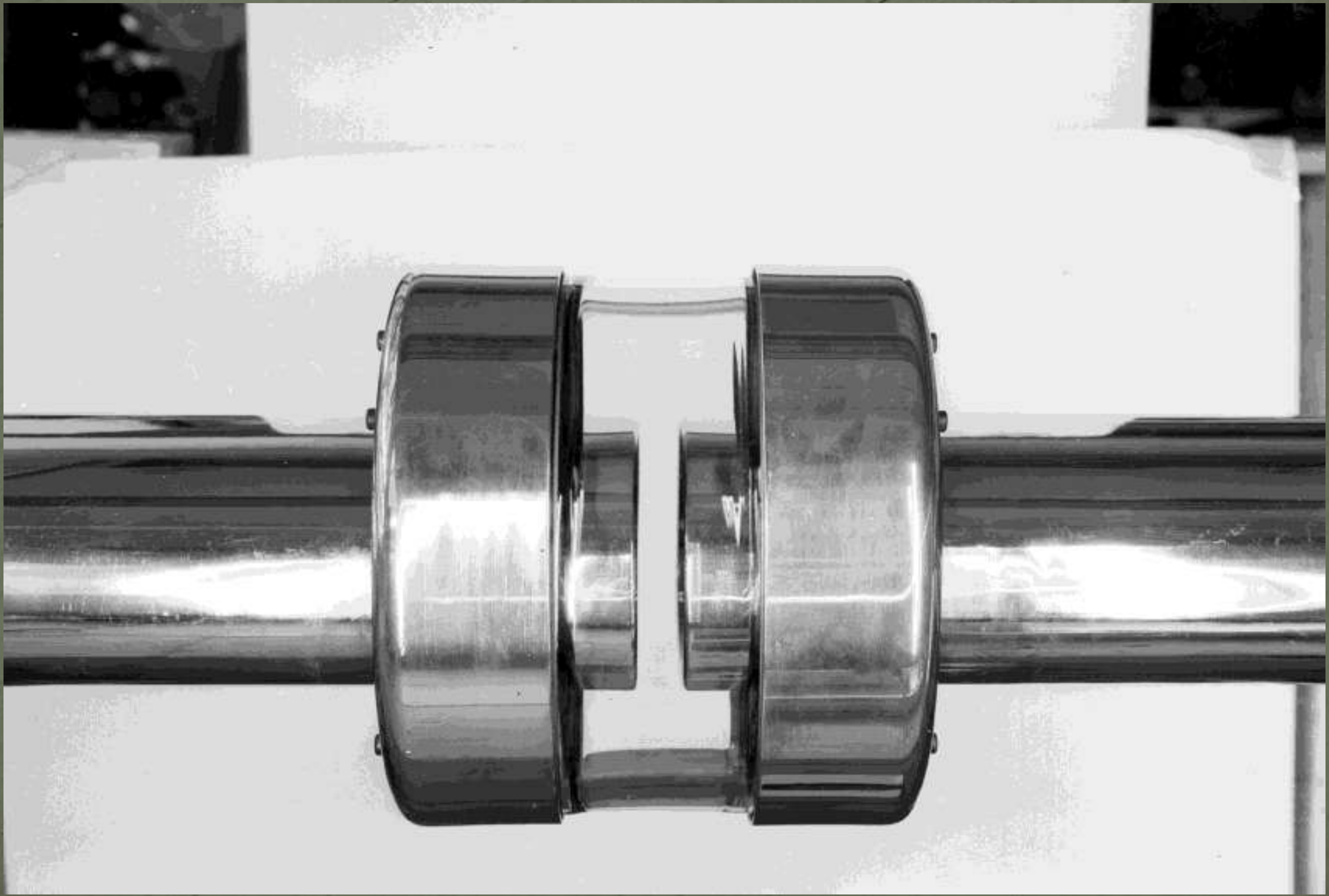
compare: transformer, secondary = beam

Acc. voltage during B ramp.

Ferrite cavity

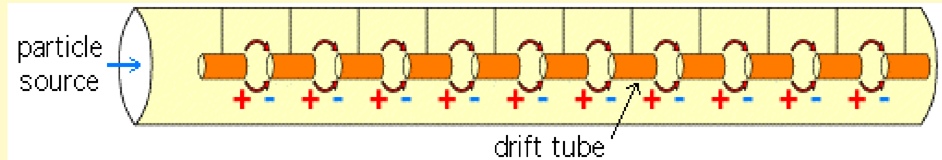


Gap of PS cavity (prototype)

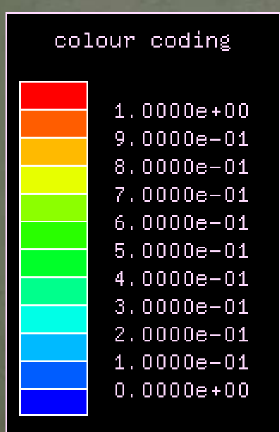


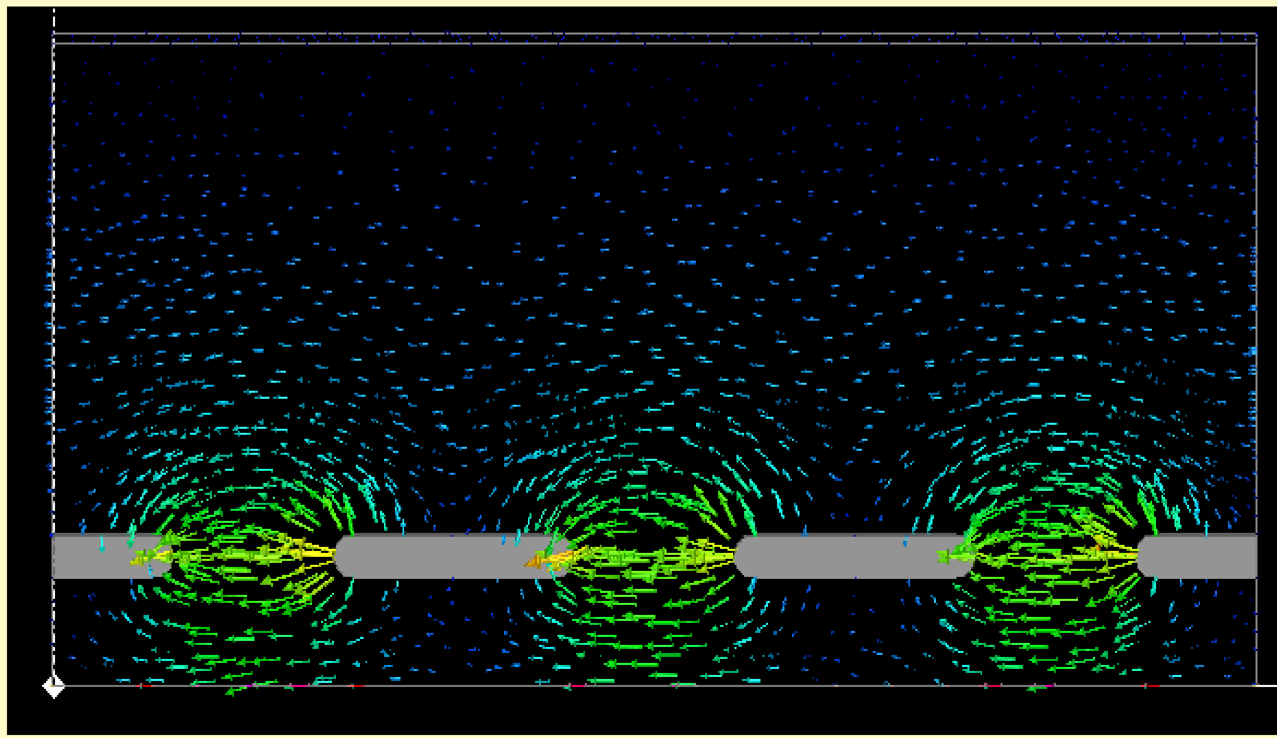
Drift Tube Linac (DTL) – how it works

For slow particles – protons @ few MeV e.g. – the drift tube lengths can easily be adapted.

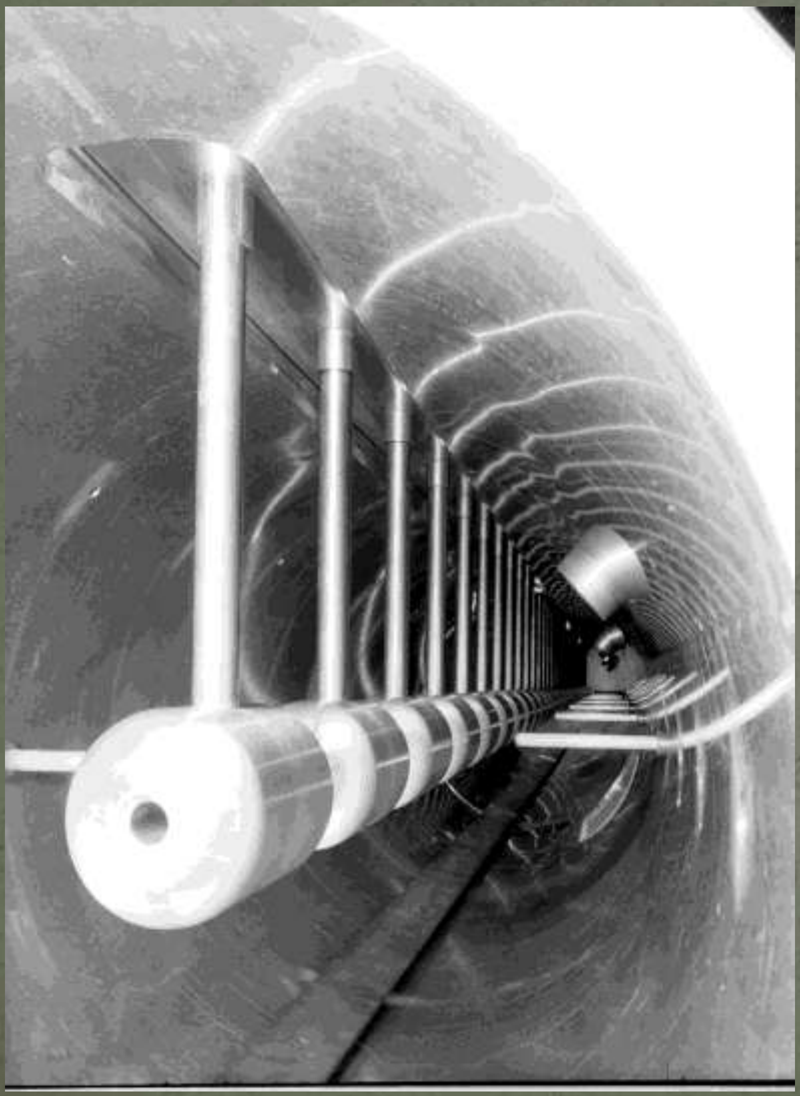


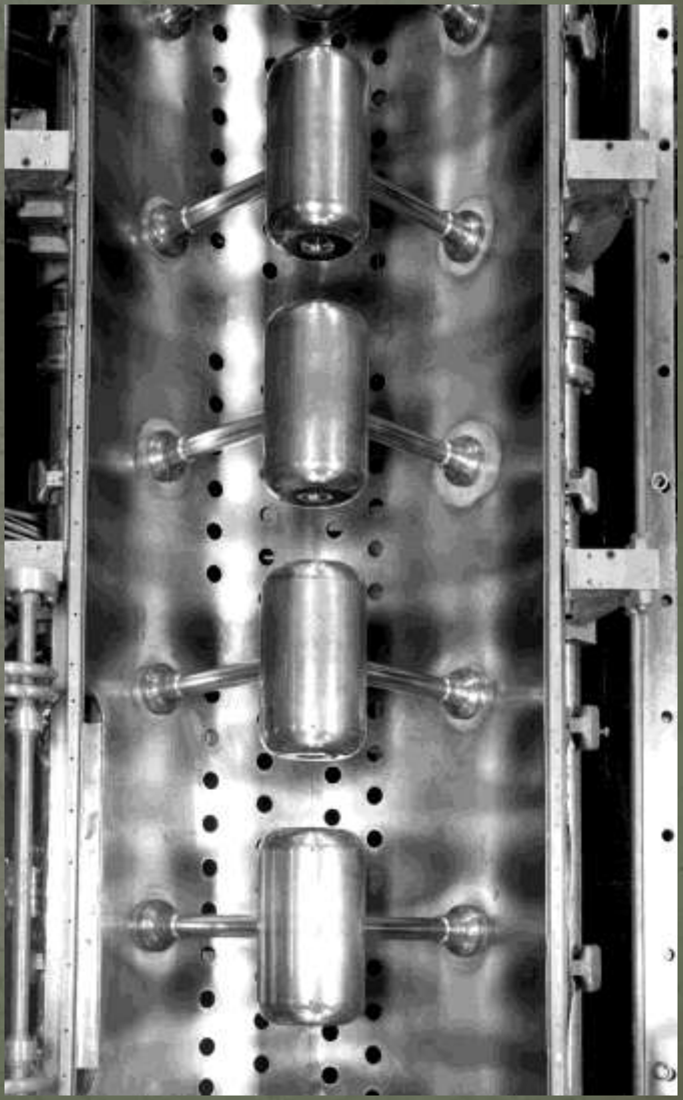
electric field





Drift tube linac – practical implementations





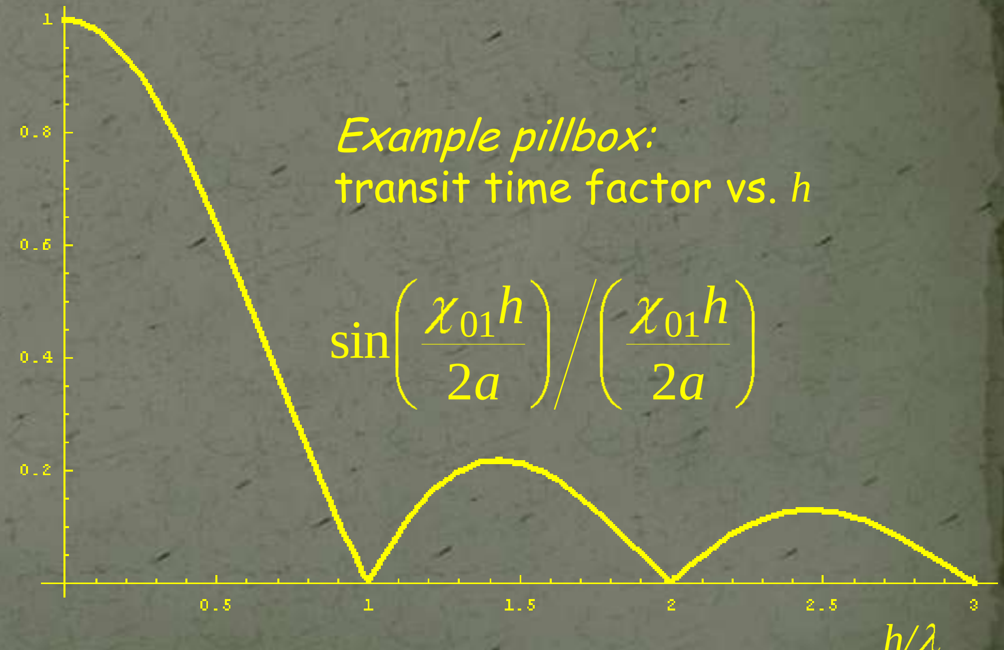
Transit time factor

If the gap is small, the voltage $\int E_z \mathrm{d}z$ is small.

If the gap large, the RF field varies notably while the particle passes.

Define the accelerating voltage $V_{gap} = \left| \int E_z e^{\mathrm{j}\frac{\omega}{c}z} \mathrm{d}z \right|$

Transit time factor $\dfrac{\left| \int E_z e^{\mathrm{j}\frac{\omega}{c}z} \mathrm{d}z \right|}{\left| \int E_z \mathrm{d}z \right|}$



Example pillbox:
transit time factor vs. h

$$\sin\left(\frac{\chi_{01} h}{2a}\right) \Big/ \left(\frac{\chi_{01} h}{2a}\right)$$