



Introduction to RF Linear Accelerators

Maurizio Vretenar - CERN BE/RF

CAS Darmstadt 2009

- 1. Linac building blocks, synchronicity
- 2. Periodic accelerating structures
- 3. Beam dynamics, linac architecture





Linear Accelerators are used for:

- Low-Energy acceleration (injectors to synchrotrons or stand-alone): 1. for protons and ions, linacs can be synchronous with the RF fields in the range where velocity increases with energy. When velocity is ~constant, synchrotrons are more efficient (multiple crossings instead of single crossing). Protons : $\beta = v/c = 0.51$ at 150 MeV, 0.95 at 2 GeV.
- <u>High-Energy acceleration</u> in the case of: 2.
 - Production of <u>high-intensity proton beams</u> in comparison with synchrotrons, linacs can go to higher repetition rate, are less affected by resonances and have more distributed beam losses. Higher injection energy from linacs to synchrotrons leads to lower space charge effects in the synchrotron and allows increasing the beam intensity.
 - High energy linear colliders for leptons, where the main advantage is the absence of synchrotron radiation.

Basic linac structure





Acceleration \rightarrow the beam has to pass in each cavity on a phase ϕ near the crest of the wave The beam must to be <u>bunched</u> at frequency ω
 <u>distance</u> between cavities and <u>phase</u> of each cavity must be correlated

Phase change from cavity *i* to *i*+1 is

$$\Delta \phi = \omega t = \omega \frac{d}{\beta c} = 2\pi \frac{d}{\beta \lambda}$$

For the beam to be synchronous with the RF wave ("ride on the crest") phase must be related to distance by the relation:

 $\frac{\Delta\phi}{d} = \frac{2\pi}{\beta\lambda}$

... and on top of acceleration, we need to introduce in our "linac" some focusing elements?







Protons and ions: at the beginning of the acceleration, beta is changing rapidly \rightarrow we need to keep the synchronism between the particle beam and the RF wave in our accelerating cells.

Electrons:

velocity=c above few keV \rightarrow constant ratio between phase and distance.



For our array of cavities to work with a beam of increasing velocity, we have 2 options:

 2π

- Keep <u>d constant</u>, and <u>change the phase</u> of each cavity → individual cavities, with individual RF amplifiers.
- Keep <u>A optimized constant</u> and <u>change the distance</u>, i.e. increase progressively the distance between cavities → cavities with a <u>fixed phase relation</u> between them → the cavities can be <u>coupled together and use a single</u> large RF amplifier → coupled cell linac structures.



Individual cavity linac – distance between cavities constant, each cavity fed by an individual RF source, phase of each cavity adjusted to keep synchronism – used for linac required to operate with different ions or at different energies. Flexible but expensive!



Coupled cell linac – a single RF source feeds a large number of cells (up to ~100!), the phase between adjacent cells is fixed by the coupling, the distance between cells increases to keep synchronism – is the standard accelerating structure for linacs. Once the geometry is defined, it can accelerate only one type of ion for a given energy range. Effective but not flexible.





The goal is flexibility: acceleration of different ions (e/m) at different energies \rightarrow need to change phase relation for each ion-energy $\underline{Cryomodule High Beta (5 Cavités Ø320)}$



Synchronism condition in coupled cell linacs

 ϕ = const. d variable

 Φ = phase difference between 2 adjacent accelerating cells, defined by the type of coupling

DC

particle

injector

buncher

 $d \propto \beta \lambda$

$$E_{i}(t) = E_{i}^{0} \cos(\omega t)$$

$$E_{i+1}(t) = E_{i+1}^{0} \cos(\omega t + \Phi)$$

$$\begin{cases}
Condition for acceleration is \Phi + \Delta \phi = 2\pi \\
(when a particle that was on the crest in the cell i arrives in the cell i+1, the change of phase due to the time of flight adds to the phase difference due to the coupling to give the new phase at the cell i+1, which must correspond again to the crest)
$$\frac{\Delta \phi}{d} = \frac{2\pi}{\beta \lambda}$$

$$\Phi + \frac{2\pi d}{\beta \lambda} = 2\pi \rightarrow d = \beta \lambda (1 - \frac{\Phi}{2\pi})$$
Synchronism conditions !
$$\Phi + \Delta \phi = 2\pi$$
All cells oscillate at the same phase, $\Phi = 0$
The phase is reversed from one cell to the next, $\Phi = \pi$

$$d = \beta \lambda / 2$$$$



The first synchronous multi-cell linac: the Wideröe



Rolf Wideroe for his Thesis at Aachen University (1928) built the first synchronous multi-gap linac:



Fig. 3.5: The principle of the 'drift-tube' as illustrated in Wideröe's thesis [Wi28].

Fig. 3.6: Acceleration tube and switching circuits [Wi28].

Potassium ions 1+, 25kV of RF at 1 MHz \rightarrow 50 keV acceleration ("at a cost of four to five hundred marks"...)

In the Wideröe, there are already all the main principles of modern linacs:

- 1. acceleration with variable RF fields (here low frequency \rightarrow LC circuit, no need of resonators)
- 2. cell length increasing with β (here $\Phi=\pi \rightarrow$ cell length $\beta\lambda/2$)



A modern synchronous multi-cell linac: the Drift Tube Linac





Tank 2 and 3 of the new Linac4 at CERN: 57 coupled accelerating gaps Frequency 352.2 MHz, λ = 85 cm Cell length (= $\beta \lambda$) goes from 12.3 cm to 26.4 cm (factor 2 !)

Linear and circular accelerators





Linear accelerator:

Particles accelerated by a sequence of gaps (all at the same RF phase).

Distance between gaps increases proportionally to the particle velocity, to keep synchronicity.

Used in the range where $\boldsymbol{\beta}$ increases. "Newton" machine

Circular accelerator:

Particles accelerated by one (or more) gaps at given positions in the ring.

Distance between gaps is fixed. Synchronicity only for β ~const, or varying (in a limited range!) the RF frequency.

Used in the range where β is nearly constant. "Einstein" machine









What did we learn?

- 1. A linac is composed of an array of accelerating gaps, interlaced with focusing magnets (quadrupoles or solenoids), following an ion source with a DC extraction and a bunching section.
- 2. When beam velocity is increasing with energy ("Newton" regime), we have to match to the velocity (or to the relativistic β) either the phase difference or the distance between two subsequent gaps.
- 3. A linac structure is usually composed of a number of accelerating gaps coupled together and fed by a single RF source, designed for a well defined ion, energy range and energy gain (voltage). To operate it, are needed a number of ancillary equipments (vacuum, RF, focusing supplies, etc.)





2 - Linac Accelerating Structures







The effect of the coupling is that the cells no longer resonate independently, but will have common resonances with well defined field patterns.









This structure is composed of 7 accelerating cells, magnetically coupled.

The cells in a cavity have the same length, but they are longer from one cavity to the next, to follow the increase in beam velocity.



15



Linac cavities as chains of coupled resonators



What is the relative phase and amplitude between cells in a chain of coupled cavities?

Anti An-L COUPLED CAVITIES COUPLED CIRCUITS

NEAR LATTICE

B-field, etc.

A linear chain of accelerating cells can be represented as a chain of resonant circuits magnetically coupled.

Individual cavity resonating at $\omega_0 \rightarrow$ frequenci(es) of the coupled system ?

Resonant circuit equation for circuit i (R≅0):

$$\mathbf{L} = \frac{\mathbf{M} \mathbf{K} \mathbf{M}}{\mathbf{L} \mathbf{L}} = \frac{\mathbf{M} \mathbf{K} \mathbf{M}}{\mathbf{L} \mathbf{L}}$$

$$I_{i}(2j\omega L + \frac{1}{j\omega C}) + j\omega kL(I_{i-1} + I_{i+1}) = 0$$

Dividing both terms by $2j\omega L$:

 $X_{i}(1 - \frac{\omega_{0}^{2}}{\omega^{2}}) + \frac{k}{2}(X_{i-1} + X_{i+1}) = 0$ General response term, Contribution from General \propto (stored energy)^{1/2,} adjacent oscillators can be voltage, E-field, resonance term



A chain of N+1 resonators is described by a (N+1)x(N+1) matrix:

$$\begin{vmatrix} 1 - \frac{\omega_0^2}{\omega^2} & \frac{k}{2} & 0 & \cdots \\ \frac{k}{2} & 1 - \frac{\omega_0^2}{\omega^2} & \frac{k}{2} & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \frac{k}{2} & 1 - \frac{\omega_0^2}{\omega^2} \end{vmatrix} | \begin{array}{c} X_0 \\ X_2 \\ \vdots \\ X_N \\ \end{array} | = 0 \quad \text{or} \quad M X = 0$$

This matrix equation has solutions only if $\det M = 0$

Eigenvalue problem!

- 1. System of order (N+1) in $\omega \rightarrow$ only N+1 frequencies will be solution of the problem ("eigenvalues", corresponding to the resonances) \rightarrow a system of N coupled oscillators has N resonance frequencies \rightarrow an *individual resonance opens up into a band of frequencies*.
- 2. At each frequency ω_1 will correspond a set of relative amplitudes in the different cells ($X_0, X_2, ..., X_N$): the "eigenmodes" or "<u>modes</u>".

i = 0, ..., NCOUPLED CAVITIES COUPLED CIRCUITS

 $X_{i}(1 - \frac{\omega_{0}^{2}}{\omega^{2}}) + \frac{k}{2}(X_{i-1} + X_{i+1}) = 0$



Modes in a linear chain of oscillators



We can find an analytical expression for eigenvalues (frequencies) and eigenvectors (modes):

Frequencies of the coupled system :

$$q^{2} = \frac{\omega_{0}^{2}}{1 + k \cos \frac{\pi q}{N}}, \quad q = 0, ..., N$$

the index q defines the number of the solution \rightarrow is the "mode index"

→ Each mode is characterized by a phase $\pi q/N$. Frequency vs. phase of each mode can be plotted as a "dispersion curve" $\omega=f(\phi)$:

ω

- 1.each mode is a point on a sinusoidal curve.
- 2.modes are equally spaced in phase.

The "eigenvectors = relative amplitude of the field in the cells are:

$$\omega_0/\sqrt{1-k}$$

 ω_0
 ω_0
 $\sqrt{1+k}$
 0
 $\pi/2$
 π
 π

$$X_i^{(q)} = (const) \cos \frac{\pi q i}{N} e^{j\omega_q t} \quad q = 0, ..., N$$



STANDING WAVE MODES, defined by a phase $\pi q/N$ corresponding to the phase shift between an oscillator and the next one $\rightarrow \pi q/N = \Phi$ is the phase difference 18 between adjacent cells that we have introduces in the 1st part of the lecture.



Example: Acceleration on the normal modes of a 7-cell structure





Note: Field always maximum in first and last cell!



Practical linac accelerating structures



Note: our relations depend only on the cell frequency ω , not on the cell length d !!!

$$\omega_q^2 = \frac{\omega_0^2}{1 + k \cos \frac{\pi q}{N}}, \quad q = 0, \dots, N$$

$$X_n^{(q)} = (const) \cos \frac{\pi qn}{N} e^{j\omega_q t} \quad q = 0, ..., N$$

 \rightarrow As soon as we keep the frequency of each cell constant, we can change the cell length following any acceleration (β) profile!



 $d \uparrow \rightarrow (L \uparrow , C \downarrow) \rightarrow LC \sim const \rightarrow \omega \sim const$

Example:

The Drift Tube Linac (DTL)

Chain of many (up to 100!) accelerating cells operating in the 0 mode. The ultimate coupling slot: no wall between the cells!

Each cell has a different length, but the cell frequency remains constant \rightarrow *"the EM fields don't see that the cell length is changing!"* 20



The DTL





Standing wave linac structure for protons and ions, β =0.1-0.5, f=20-400 MHz

Drift tubes are suspended by stems (no net RF current on stem)

Coupling between cells is maximum (no slot, fully open !)

The O-mode allows a long enough cell $(d=\beta\lambda)$ to house <u>focusing</u> <u>quadrupoles inside the drift tubes</u>!





Examples of DTL







Top; CERN Linac2 Drift Tube Linac accelerating tank 1 (200 MHz). The tank is 7m long (diameter 1m) and provides an energy gain of 10 MeV.

Left: DTL prototype for CERN Linac4 (352 MHz). Focusing is provided by (small) quadrupoles inside drift tubes. Length of drift tubes (cell length) increases with proton velocity.



The Linac4 DTL







Traveling wave accelerating structures (electrons)



What happens if we have an infinite chain of oscillators?

$$\omega_q^2 = \frac{\omega_0^2}{1 + k \cos \frac{\pi q}{N}}, \quad q = 0, ..., N$$
 becomes (N \rightarrow h) $\omega^2 = \frac{\omega_0^2}{1 + k \cos \varphi}$
$$X_n^{(q)} = (const) \cos \frac{\pi qn}{N} e^{j\omega_q t} \quad q = 0, ..., N$$
 becomes (N \rightarrow h) $X_i = (const) e^{j\omega_q t}$



All modes in the dispersion curve are allowed, the original frequency degenerates into a continuous band. The field is the same in each cell, there are no more standing wave modes \rightarrow only "traveling wave modes", if we excite the EM field at one end of the structure it will propagate towards the other end.

But: our dispersion curve remains valid, and defines the velocity of propagation of the travelling wave, v_{\phi} = $\omega d/\Phi$

For acceleration, the wave must propagate at $v_{\phi} = c$

 \rightarrow for each frequency ϖ and cell length d we can find a phase Φ where the apparent velocity of the wave v_{_{\!\!\!\!\!\!\!\!\!\!\!\!\!}} is equal to c



Traveling wave accelerating structures



How to "simulate" an infinite chain of resonators? Instead of a singe input, exciting a standing wave mode, use an input + an output for the RF wave at both ends of the structure.



"Disc-loaded waveguide" or chain of electrically coupled cells characterized by a continuous band of frequencies. In the chain is excited a "traveling wave mode" that has a propagation velocity $v_{ph} = \omega/k$ given by the dispersion relation.

For a given frequency ω , v_{ph} = c and the structure can be used for particles traveling at β =1 The "traveling wave" structure is the standard linac for electrons from β ~1.

- \rightarrow Can <u>not</u> be used for protons at v<c:
 - 1. constant cell length does not allow synchronism
 - 2. structures are long, without space for transverse focusing



Examples: a TW accelerating structure





A 3 GHz LIL accelerating structure used for CTF3. It is 4.5 meters long and provides an energy gain of 45 MeV. One can see 3 quadrupoles around the RF structure.







The old CERN LIL (LEP Injector Linac) accelerating structures (3 GHz). The TW structure is surrounded by focusing solenoids, required for the positrons.







What did we learn?

- 1. Coupling together accelerating cells (via the magnetic or electric field) is a way to fix their phase relation.
- 2. A chain of N coupled resonators will always have N modes of oscillation. Each mode will have a resonance frequency and a field pattern with a corresponding phase shift from cell to cell.
- 3. Choosing the excitation frequency, we can decide in which mode to operate the structure, and we can select a mode with a phase advance between cells suitable for acceleration. If we change the length of a cell without changing its frequency, we can follow the increase the particle velocity.
- 4. Practical linac structures operate either on mode 0 (DTL), less efficient but leaving space for internal focusing elements, or on mode π , standard for multi-cell cavities.
- 5. Electron linacs operate with long chains of identical cells excited by a traveling wave, propagating at the (constant) velocity of the beam.





3 - Beam Dynamics in Linacs

Longitudinal dynamics

 \rightarrow





- Ions are accelerated around a (negative = linac definition) synchronous phase.
- Particles around the synchronous one perform oscillations in the longitudinal phase space.
- Frequency of small oscillations:

$$\omega_l^2 = \omega_0^2 \frac{qE_0T\sin(-\varphi)\lambda}{2\pi mc^2\beta\gamma^3}$$

- Tends to zero for relativistic particles $\gamma \gg 1$.
- → Note phase damping of oscillations:

$$\Delta \varphi = \frac{const}{\left(\beta \gamma\right)^{3/4}} \qquad \Delta W = const \times \left(\beta \gamma\right)^{3/4}$$

At relativistic velocities phase oscillations stop, and the beam is compressed in phase around the initial phase. The crest of the wave can be used for acceleration.







- \rightarrow Large numbers of particles per bunch (~10¹⁰).
- → Coulomb repulsion between particles (space charge) plays an important role.
- \rightarrow But space charge forces ~ $1/\gamma^2$ disappear at relativistic velocity



$$F = e(E_r - vB_{\varphi}) = eE_r(1 - \frac{v^2}{c^2}) = eE_r(1 - \beta^2) = \frac{eE_r}{\gamma^2}$$



Transverse dynamics - RF defocusing





- RF defocusing experienced by particles crossing a gap on a longitudinally stable phase.
 - In the rest frame of the particle, only electrostatic forces \rightarrow no stable points (maximum or minimum) \rightarrow radial defocusing.
 - Lorentz transformation and calculation of radial momentum impulse per period (from electric and magnetic field contribution in the laboratory frame):

$$\Delta p_r = -\frac{\pi e E_0 T L r \sin \varphi}{c \beta^2 \gamma^2 \lambda}$$

→ Transverse defocusing ~ $1/\gamma^2$ disappears at relativistic velocity (transverse magnetic force cancels the transverse RF electric force).

Transverse beam equilibrium in linacs



The equilibrium between external focusing force and internal defocusing forces defines the frequency of beam oscillations.

Oscillations are characterized in terms of phase advance per focusing period σ_t or phase advance per unit length k_t

Ph. advance = Ext. quad focusing - RF defocusing - space charge

$$k_t^2 = \left(\frac{\sigma_t}{N\beta\lambda}\right)^2 = \left(\frac{q\,Gl}{2\,mc\,\beta\gamma}\right)^2 - \frac{\pi\,q\,E_0T\sin(-\varphi)}{mc^2\lambda\,\beta^3\gamma^3} - \frac{3q\,I\,\lambda(1-f)}{8\pi\varepsilon_0\,r_0^3mc^3\beta^2\gamma^3}$$

q=charge G=quad gradient I=length foc. element f=bunch form factor r_0 =bunch radius λ =wavelength

Approximate expression valid for:

FODO lattice, smooth focusing approximation, space charge of a uniform 3D ellipsoidal bunch.

A "low-energy" linac is dominated by space charge and RF defocusing forces !!

Phase advance per period must stay in reasonable limits (30-80 deg), phase advance per unit length must be continuous (smooth variations) \rightarrow at low β , we need a strong focusing term to compensate for the defocusing, but the limited space limits the achievable G and I \rightarrow needs to use short focusing periods N $\beta\lambda$.

Note that the RF defocusing term ∞f sets a higher limit to the basic linac frequency (whereas for shunt impedance considerations we should aim to the highest possible frequency, $Z \propto \sqrt{f}$). 333







Focusing provided by quadrupoles (but solenoids for low β !).

Different distance between focusing elements (=1/2 length of a FODO focusing period) ! For the main linac accelerating structure (after injector):

Protons, (high beam current and high space charge) require short distances:

- $\beta\lambda$ in the DTL, from ~70mm (3 MeV, 352 MHz) to ~250mm (40 MeV),
- can be increased to 4-10 $\beta\lambda$ at higher energy (>40 MeV).
- longer focusing periods require special dynamics (example: the IH linac).

Heavy ions (low current, no space charge): 2-10 $\beta\lambda$ in the main linac (>~150mm).

Electrons (no space charge, no RF defocusing):

up to several meters, depending on the required beam conditions. Focusing is mainly required to control the emittance.



High-intensity protons - the case of Linac4



Transverse (x) r.m.s. beam envelope along Linac4



Example: beam dynamics design for Linac4@CERN.

High intensity protons (60 mA bunch current, duty cycle could go up to 5%), 3 - 160 MeV

Beam dynamics design minimising emittance growth and halo development in order to: 1. avoid uncontrolled beam loss (activation of machine parts)

2. preserve small emittance (high luminosity in the following accelerators)



Proton linac architecture cell length, focusing period



EXAMPLE: the Linac4 project at CERN. H-, 160 MeV energy, 352 MHz. A 3 MeV injector + 22 multi-cell standing wave accelerating structures of 3 types

DTL: every cell is different, focusing quadrupoles in each drift tubeCCDTL: sequences of 2 identical cells, quadrupoles every 3 cellsPIMS: sequences of 7 identical cells, quadrupoles every 7 cells



Two basic principles to remember:

1. As beta increases, phase error between cells of identical length becomes small \rightarrow we can have short sequences of identical cells (lower construction costs).

2. As beta increases, the distance between focusing elements can increase.







What did we learn?

- 1. Transverse beam dynamics in linacs is dominated by space charge and RF defocusing forces.
- 2. In order to keep the transverse phase advance within reasonable limits, focusing has to be strong (large focusing gradients, short focusing periods) at low energy, and can then be relaxed at higher energy.
- 3. A usual linac is made of a sequence of structures, matched to the beam velocity, and where the length of the focusing period increases with energy.
- 4. The very low energy section remains a special problem \rightarrow next lecture







1. Reference Books:

- T. Wangler, Principles of RF Linear Accelerators (Wiley, New York, 1998).
- P. Lapostolle, A. Septier (editors), Linear Accelerators (Amsterdam, North Holland, 1970).
- I.M. Kapchinskii, Theory of resonance linear accelerators (Harwood, Chur, 1985).

2. General Introductions to linear accelerators

- M. Puglisi, The Linear Accelerator, in E. Persico, E. Ferrari, S.E. Segré, Principles of Particle Accelerators (W.A. Benjamin, New York, 1968).
- P. Lapostolle, Proton Linear Accelerators: A theoretical and Historical Introduction, LA-11601-MS, 1989.
- P. Lapostolle, M. Weiss, Formulae and Procedures useful for the Design of Linear Accelerators, CERN-PS-2000-001 (DR), 2000.
- P. Lapostolle, R. Jameson, Linear Accelerators, in Encyclopaedia of Applied Physics (VCH Publishers, New York, 1991).

3. CAS Schools

- S. Turner (ed.), CAS School: Cyclotrons, Linacs and their applications, CERN 96-02 (1996).
- M. Weiss, Introduction to RF Linear Accelerators, in CAS School: Fifth General Accelerator Physics Course, CERN-94-01 (1994), p. 913.
- N. Pichoff, Introduction to RF Linear Accelerators, in CAS School: Basic Course on General Accelerator Physics, CERN-2005-04 (2005).
- M. Vretenar, Differences between electron and ion linacs, in CAS School: Small Accelerators, CERN-2006-012.