Beam Cooling

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Beam Cooling

Introduction

1. **Electron Cooling**

2. **Ionization Cooling**

3. **Laser Cooling**

4. **Stochastic Cooling**

Beam Cooling

Beam cooling is synonymous for a reduction of beam temperature

Temperature is equivalent to terms as
phase space volume, emittance and momentum spread

Beam Cooling processes are not following Liouville‘s Theorem:

`in a system where the particle motion is controlled by external conservative forces the phase space density is conserved´

(This neglect interactions between beam particles.)

Beam cooling techniques are non-Liouvillean processes which violate the assumption of a conservative force.

e.g. interaction of the beam particles with other particles (electrons, photons)

Cooling Force

Generic (simplest case of a) Cooling Force:

$$F_{x,y,s} = -\alpha_{x,y,s} v_{x,y,s}$$

non conservative, cannot be described by a Hamiltonian

For a 2D subspace distribution function $f(z, z', t)$

$$F_z = -\alpha_z v_z \qquad z = x, y, s$$

$$\frac{df(z, z', t)}{dt} = -\lambda_z f(z, z', t) \qquad \lambda_z \text{ cooling (damping) rate}$$

in a circular accelerator:

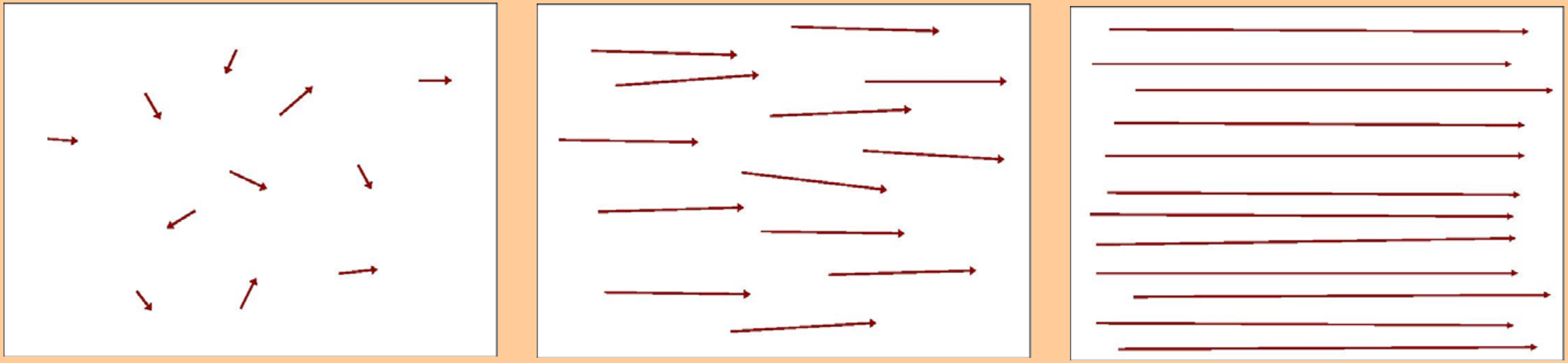
Transverse (emittance) cooling rate $\epsilon_{x,y}(t_0 + t) = \epsilon_{x,y}(t_0)\ e^{-\lambda_{x,y} t}$

Longitudinal (momentum spread) cooling rate $\frac{\delta p_\parallel}{p_0}(t_0 + t) = \frac{\delta p_\parallel}{p_0}(t_0)\ e^{-\lambda_\parallel t}$

Beam Temperature

Where does the beam temperature originate from?

The beam particles are generated in a ‘hot’ source



at rest (source) **low energy** **high energy**

In a standard accelerator the beam temperature is not reduced (thermal motion is superimposed the average motion after acceleration)

but: many processes can heat up the beam

e.g. heating by mismatch, space charge, intrabeam scattering, internal targets, residual gas

Beam Temperature Definition

Longitudinal beam temperature

$$\frac{1}{2}k_B T_{\parallel} = \frac{1}{2}mv_{\parallel}^2 = \frac{1}{2}mc^2\beta^2(\frac{\delta p_{\parallel}}{p})^2$$

Transverse beam temperature

$$\frac{1}{2}k_B T_{\perp} = \frac{1}{2}mv_{\perp}^2 = \frac{1}{2}mc^2\beta^2\gamma^2\theta_{\perp}^2 \qquad \theta_{\perp} = \frac{v_{\perp}}{\beta c}, \qquad \theta_{\perp}(s) = \sqrt{\frac{\epsilon}{\beta_{\perp}(s)}}$$

Distribution function

$$f(v_{\perp}, v_{\parallel}) \propto \exp(-\frac{mv_{\perp}^2}{2k_B T_{\perp}} - \frac{mv_{\parallel}^2}{2k_B T_{\parallel}})$$

Particle beams can be anisotropic: $k_B T_{\parallel} \neq k_B T_{\perp}$

e.g. due to laser cooling or distribution of electron beam

Don‘t confuse: beam energy $\leftrightarrow$ beam temperature

(e.g. a beam of energy 100 GeV can have a temperature of 1 eV)

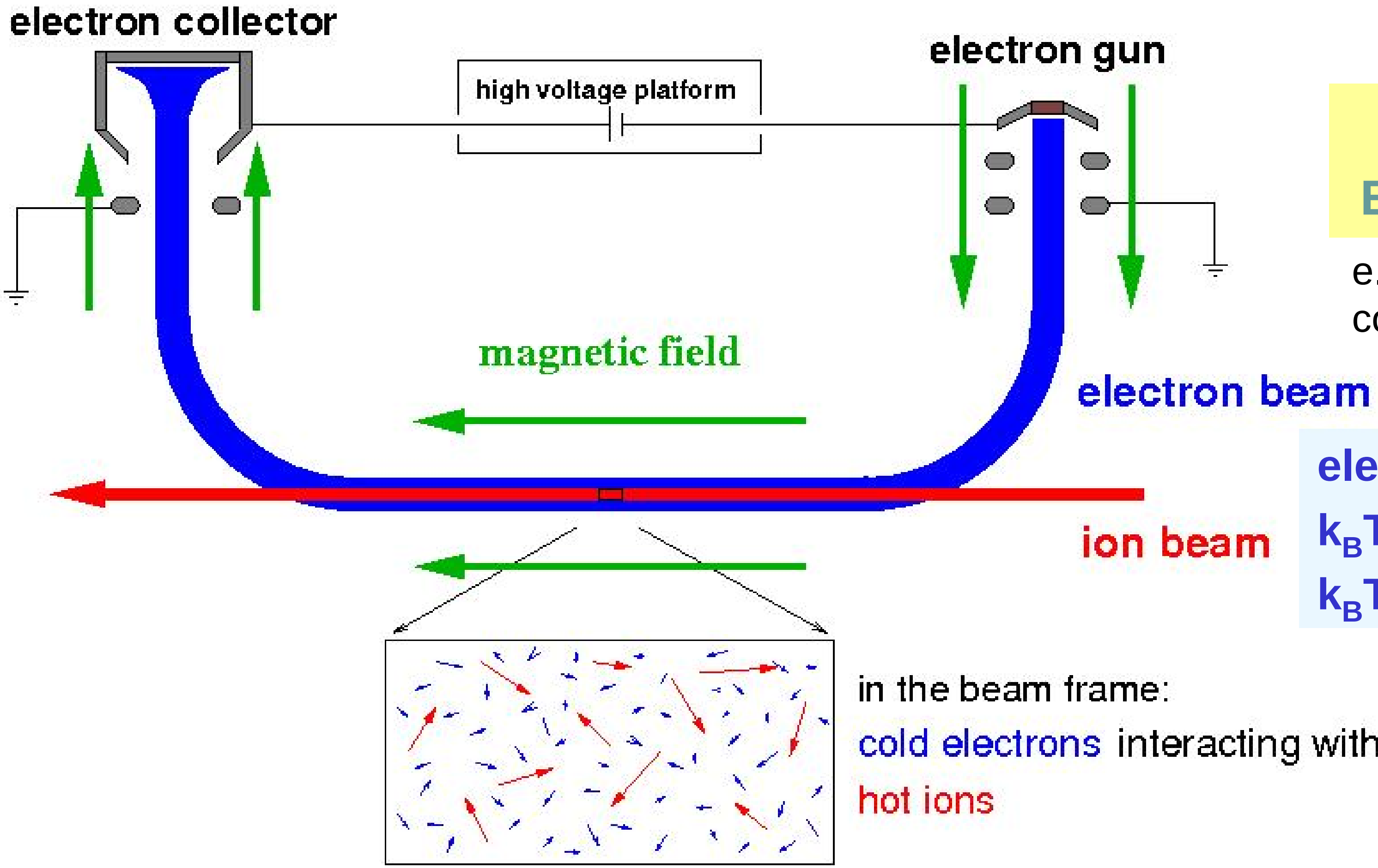
Benefits of Beam Cooling

- Improved beam quality
  - Precision experiments
  - Luminosity increase

- Compensation of heating
  - Experiments with internal target
  - Colliding beams

- Intensity increase by accumulation
  - Weak beams from source can be increased
  - Secondary beams (antiprotons, rare isotopes)

1. Electron Cooling



in the beam frame:
cold electrons interacting with
hot ions

$v_{e//} = v_{i//}$

$E_e = m_e/M_i \cdot E_i$

e.g. :220 keV electrons
cool 400 MeV/u ions

electron temperature

$k_B T_\perp \approx 0.1$ eV

$k_B T_{//} \approx 0.1 - 1$ meV

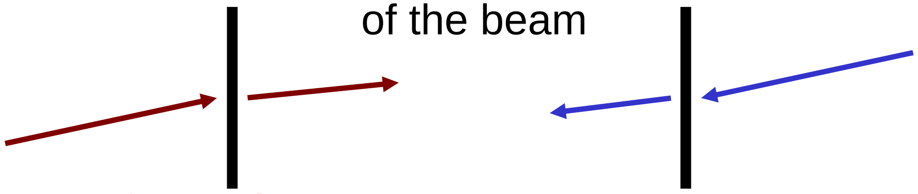
superposition of a cold intense electron beam with the same velocity

momentum transfer by Coulomb collisions

cooling force results from energy loss in the co-moving gas of free electrons

Simple Derivation of Electron Cooling Force

Analogy: energy loss in matter
(electrons in the shell)



in reference frame of the beam

faster ion

slower ion

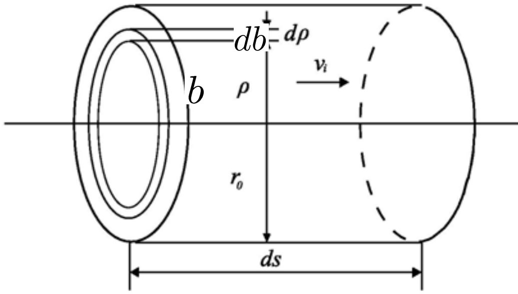
Rutherford scattering: $2\ tan(\frac{\theta}{2}) = \frac{2Z_1Z_2e^2}{4\pi\epsilon_0\Delta p v b}\quad Z_1 = Q\ (ion),\ \ Z_2 = -1\ (electron)$

Energy transfer: $\Delta E(b) = \frac{(\Delta p)^2}{2m_e} \simeq \frac{2Q^2e^4}{(4\pi\epsilon_0)^2 m_e v^2}\frac{1}{b^2}\quad (for\ \ b \gg b_{min})$

Minimum impact parameter: $b_{min} = \frac{Qe^2}{(4\pi\epsilon_0)^2 m_e v^2}$ from: $\Delta E(b_{min}) = \Delta E_{max} \simeq m_e v^2$

Energy loss:

$$-\frac{dE}{dx} = 2\pi\int_{b_{min}}^{b_{max}} b n_e \Delta E\, db = \frac{4\pi Q^2 e^4}{(4\pi\epsilon_0)^2 m_e v^2} n_e \ln\frac{b_{max}}{b_{min}}$$



Coulomb logarithm $L_C = \ln\ (b_{max}/b_{min}) \approx 10$ (typical value)

Characteristics of Electron Cooling Force

$$\boxed{\vec{F}(\vec{v_i}) = -\frac{4\pi Q^2 e^4 n_e}{(4\pi\epsilon_0)^2 m_e} \int L_C(\vec{v}_{rel}) f(\vec{v_e}) \frac{\vec{v}_{rel}}{v_{rel}^3} d^3\vec{v_e}}$$

$$\vec{v}_{rel} = \vec{v_i} - \vec{v_e}$$

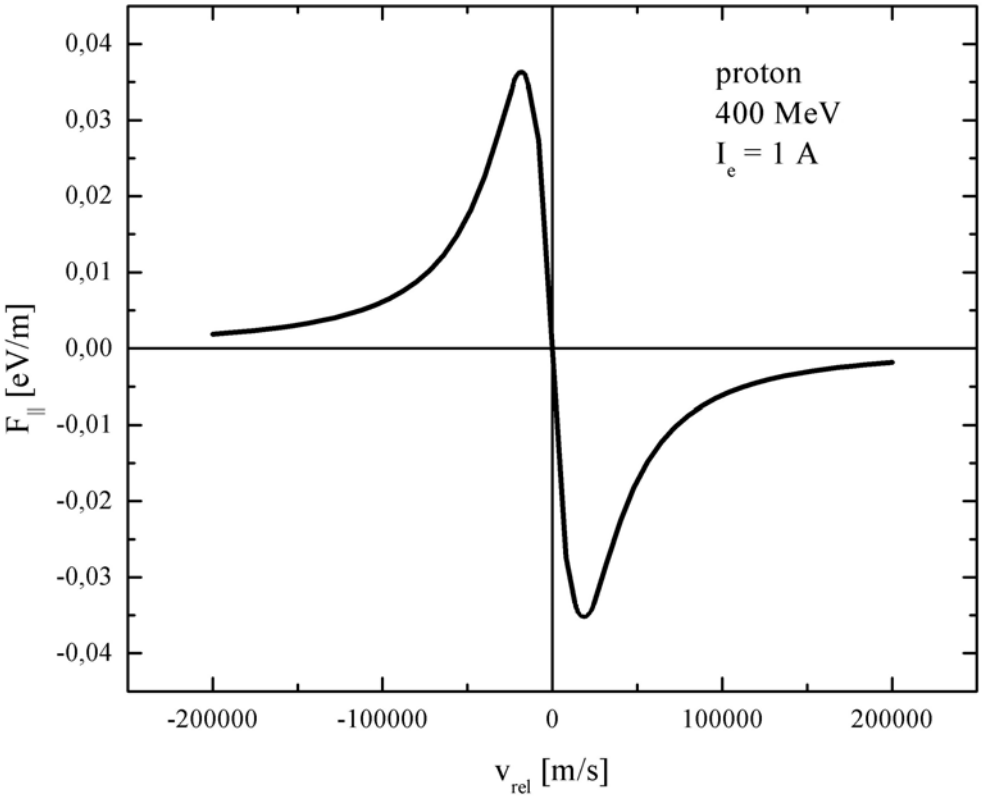
cooling force F

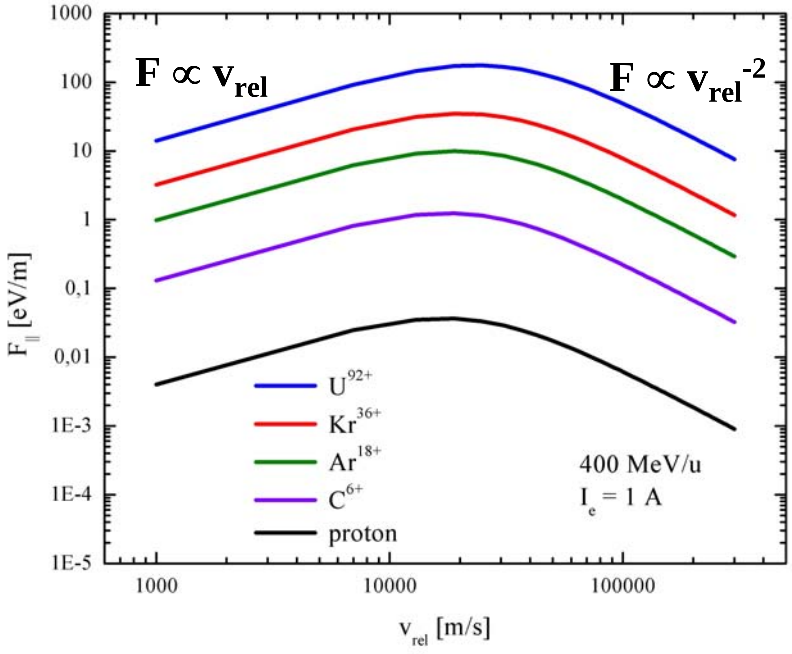
for small relative velocity: $\propto v_{rel}$

for large relative velocity: $\propto v_{rel}^{-2}$

increases with charge: $\propto Q^2$

maximum at electron temperature





Electron Cooling Time

first estimate:
(Budker 1967)

$$\tau = \frac{3}{8\sqrt{2\pi} n_e Q^2 r_e r_i c L_C} \left(\frac{k_B T_e}{m_e c^2} + \frac{k_B T_i}{m_i c^2}\right)^{3/2}$$

for large relative velocities

cooling time $\tau_z \propto \frac{A}{Q^2} \frac{1}{n_e \eta} \beta^3 \gamma^5 \theta_z^3$

$$\left\{ \begin{array}{l} \theta_{x,y} = \frac{v_{x,y}}{\gamma \beta c} \\ \theta_{\parallel} = \frac{v_{\parallel}}{\gamma \beta c} \end{array} \right.$$

<u>cooling rate:</u>

- slow for hot beams $\propto \theta^3$
- decreases with energy $\propto \gamma^{-2}$ ($\beta\gamma\theta$ is conserved)
- linear dependence on electron beam intensity n_e and cooler length $\eta = L_{ec}/C$
- favorable for highly charged ions Q^2/A
- independent of hadron beam intensity

for small relative velocities

cooling rate is constant and maximum at small relative velocity

$F \propto v_{rel} \Rightarrow \tau = \Delta t = p_{rel}/F =$ constant

Models of the Electron Cooling Force

binary collision model

description of the cooling process by successive collisions of two particles and integration over all interactions

analytic expressions become very involved, various regimes (multitude of Coulomb logarithms)

dielectric model

interaction of the ion with a continuous electron plasma (scattering off of plasma waves)

fails for small relative velocities and high ion charge

a simple empiric formula (Parkhomchuk):

$$\vec{F} = -4\frac{n_e}{m_e}\frac{(Qe^2)^2}{(4\pi\epsilon_0)^2}\ln\left(\frac{b_{max}+b_{min}+r_c}{b_{min}+r_c}\right)\frac{\vec{v}_{ion}}{(v_{ion}^2+v_{eff}^2)^{3/2}}$$

$$b_{min} = \frac{Qe^2/4\pi\epsilon_0}{m_e v_{ion}^2}; \quad b_{max} = \frac{v_{ion}}{max(\omega_{pe}, 1/T_{cool})}, \quad v_{eff}^2 = v_{e,\parallel}^2 + v_{e,\perp}^2$$

Electron Beam Properties

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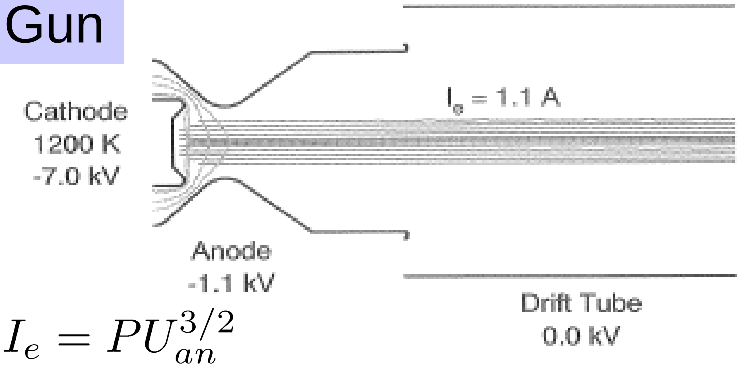
electron beam temperature

transverse $k_BT_\perp = k_BT_{cat}$, with transverse expansion ($\propto B_c/B_{gun}$)

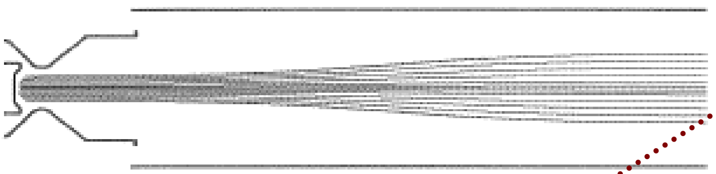
longitudinal $k_BT_{//} = (k_BT_{cat})^2/4E_0 \ll k_BT_\perp$ lower limit : $k_BT_\parallel \geq 2e\frac{n_e^{1/3}}{4\pi\epsilon_0}$

typical values: $k_BT_\perp \approx 0.1$ eV (1100 K), $k_BT_{//} \approx 0.1$ - 1 meV

Gun



$$I_e = PU_{an}^{3/2}$$

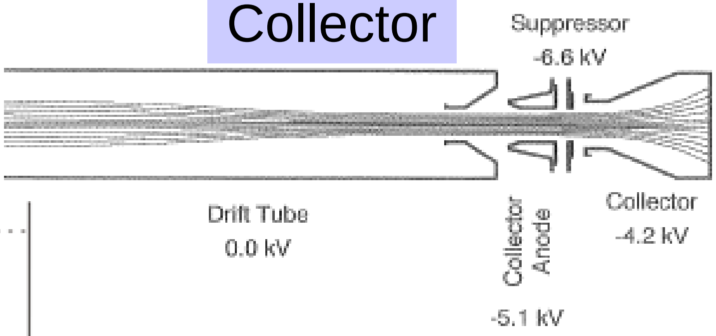


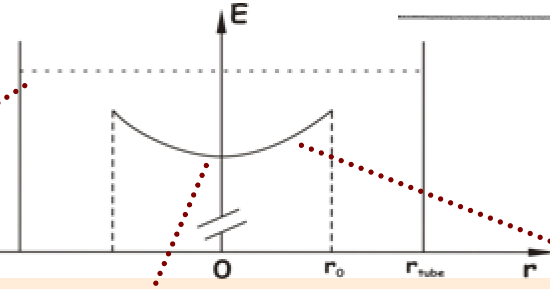
transversely expanded electron beam

electron beam confined by longitudinal magnetic field (from gun to collector)

Drift Section (Cooling)

Collector

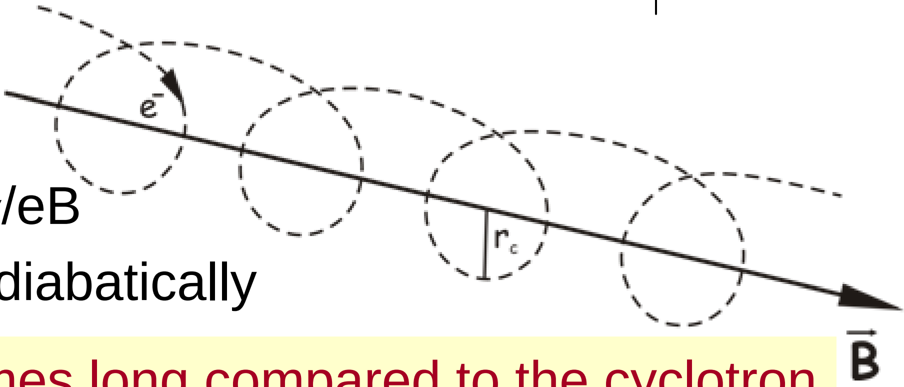




radial variation of electron energy due to space charge:

$$E(r) = eU_{cat} - n_e\pi r_0^2 r_e m_e c^2[1 + 2\ln(r_{tube}/r_0)] + n_e\pi r_e m_e c^2 r^2$$

Electron Motion in Longitudinal Magnetic Field



single particle cyclotron motion

cyclotron frequency $\omega_c = eB/\gamma m_e$

cyclotron radius $r_c = v_\perp/\omega_c = (k_B T_\perp m_e)^{1/2}\, \gamma/eB$

electrons follow the magnetic field line adiabatically

important consequence: for interaction times long compared to the cyclotron period the ion does not sense the transverse electron temperature

magnetized cooling ($T_{eff} \approx T_{//} << T_\perp$)

electron beam space charge:

transverse electric field + B-field ⇒ azimuthal drift $v_{azi} = r\omega_{azi} = r\dfrac{2\pi r_e n_e c^2}{\gamma\omega_c}$

⇒ electron and ion beam should be centered

Favorable for optimum cooling (small transverse relative velocity):

- high parallelism of magnetic field lines $\Delta B_\perp/B_0$
- large beta function (small divergence) in cooling section

Imperfections and Limiting Effects in Electron Cooling

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technical and physical issues:

ripple of accelerating voltage

magnetic field imperfections

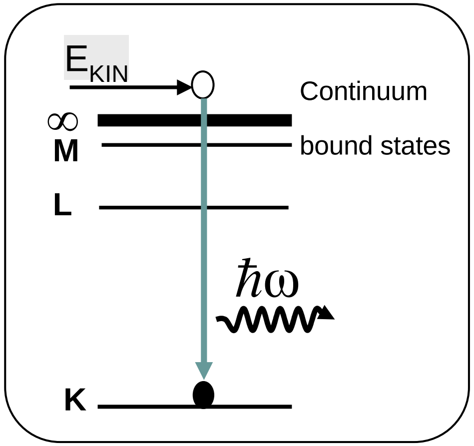
beam misalignment

space charge of electron beam and compensation

losses by recombination (REC)

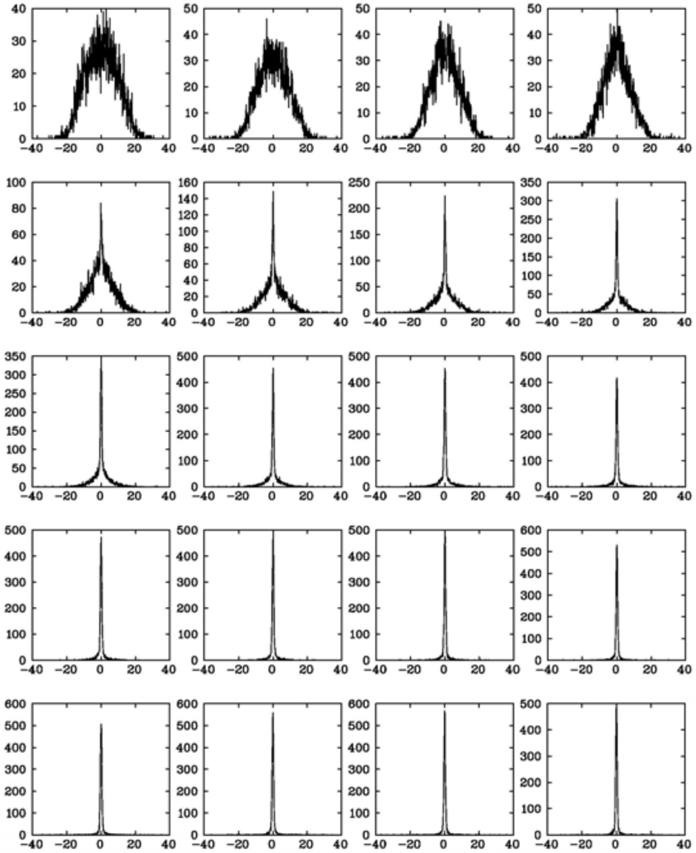
loss rate $\tau^{-1} = \gamma^{-2}\alpha_{REC} n_e \eta$

$$\alpha_{REC} = \frac{1.92\times 10^{-13} Q^2}{\sqrt{k_B T}}\left(\ln\frac{5.66Q}{\sqrt{k_B T}} + 0.196\left(\frac{k_B T}{Q^2}\right)^{1/3}\right)\ [cm^3 s^{-1}]$$



Examples of Electron Cooling

fast transverse cooling at TSR, Heidelberg



profile every 0.1 s.

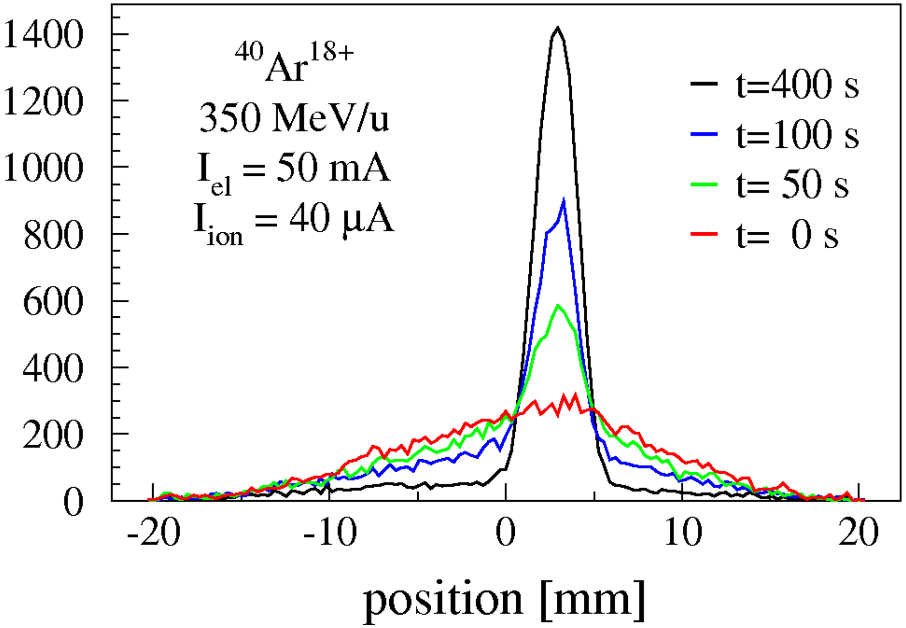
cooling of 6.1 MeV/u C^{6+} ions

0.24 A, 3.4 keV electron beam

$n_e = 1.56 \times 10^7$ cm^{-3}

measured with residual gas ionization beam profile monitor

transverse cooling at ESR, Darmstadt



cooling of 350 MeV/u Ar^{18+} ions

0.05 A, 192 keV electron beam

$n_e = 0.8 \times 10^6$ cm^{-3}

Accumulation of Heavy Ions by Electron Cooling

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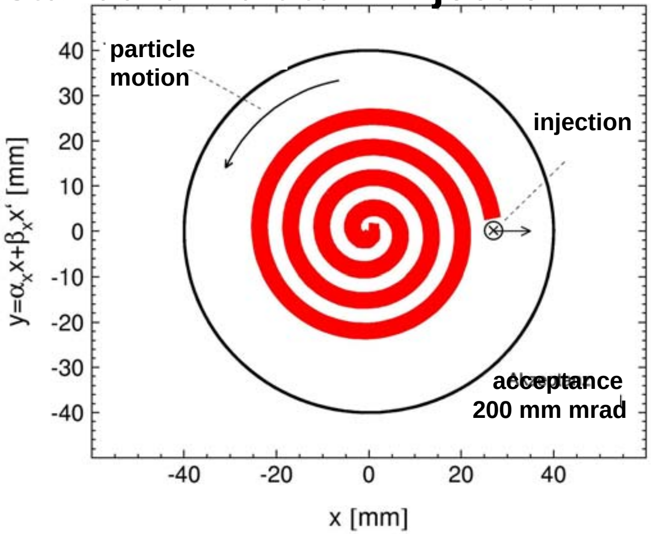
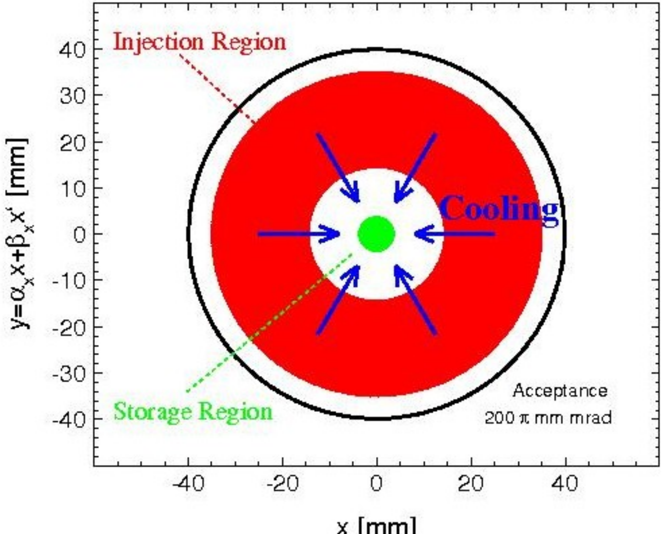
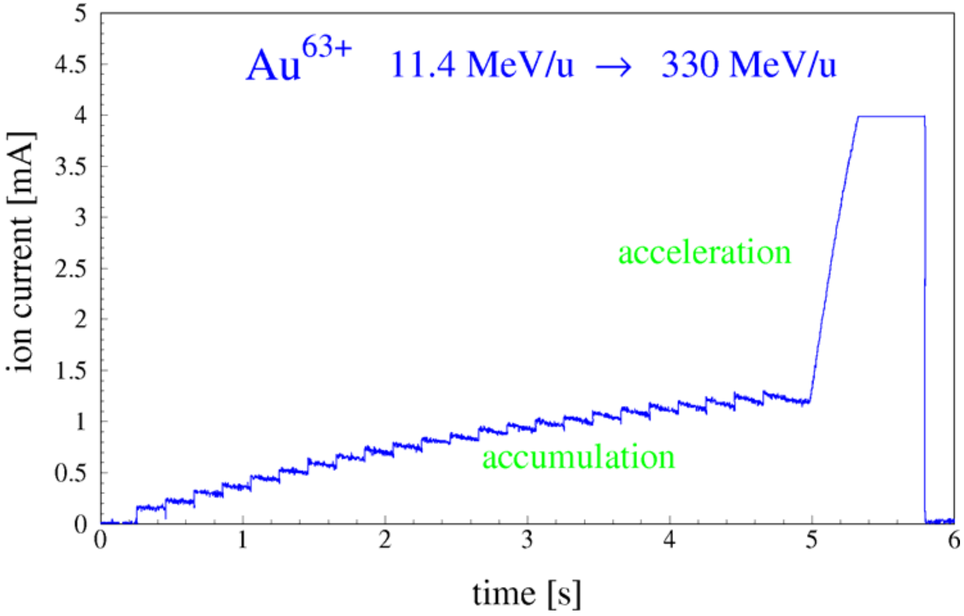
standard multiturn injection



multiturn injection with cooling

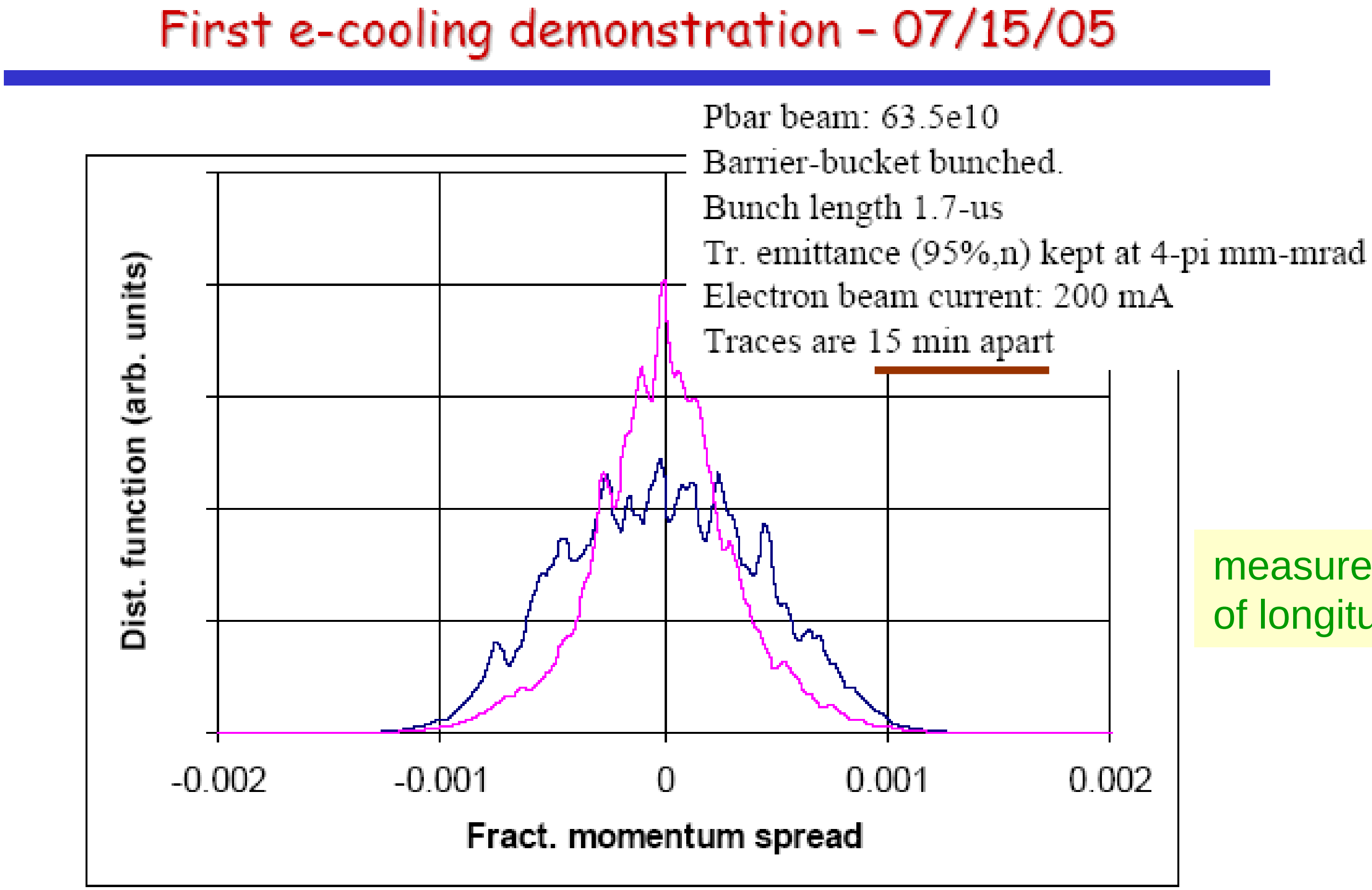


fast electron cooling of highly charged ions (Au^{63+}) at injection energy allows accumulation with a repetition rate of 5 Hz



Examples of Electron Cooling

first high energy electron cooling of 8 GeV antiprotons
longitudinal cooling with 0.2 A, 4.4 MeV electron beam

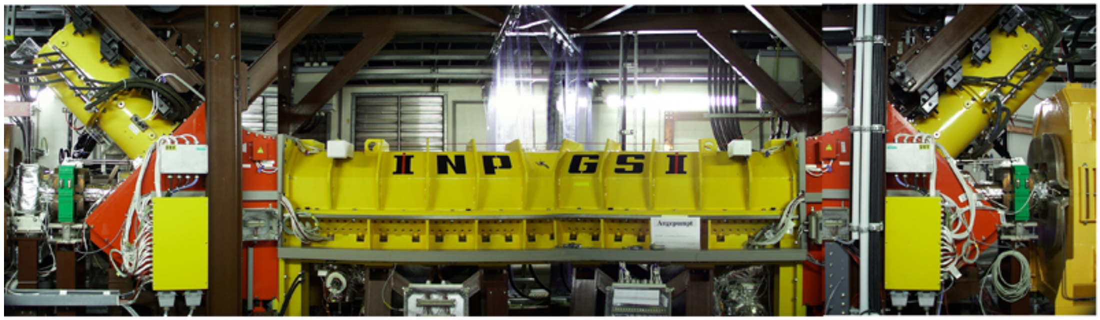


First e-cooling demonstration - 07/15/05

Pbar beam: 63.5e10
Barrier-bucket bunched.
Bunch length 1.7-us
Tr. emittance (95%,n) kept at 4-pi mm-mrad
Electron beam current: 200 mA
Traces are 15 min apart

measured by detection
of longitudinal Schottky noise

Electron Cooling at GSI



SIS Electron Cooler
35 keV

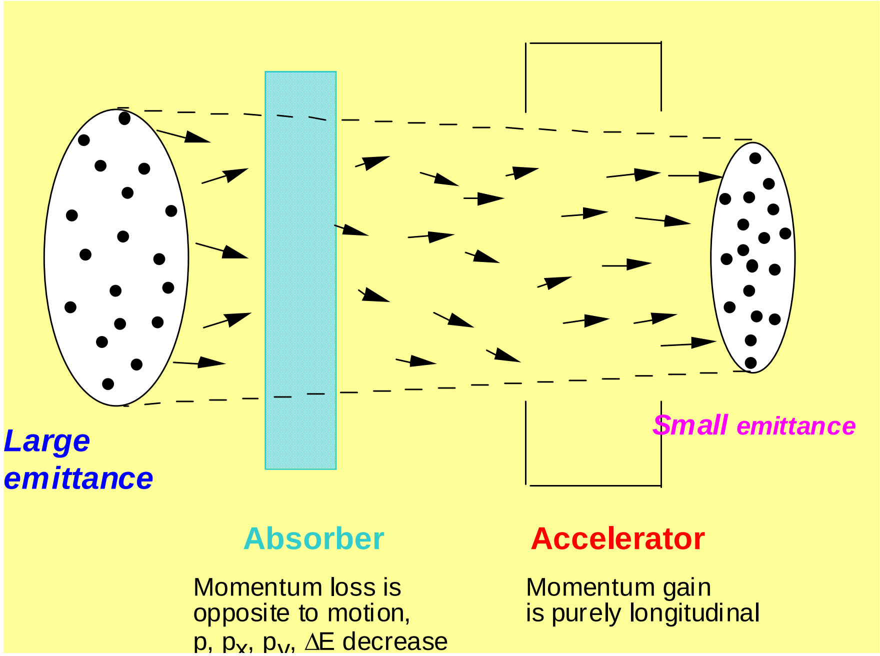
Cooling at Injection Energy of Synchrotron
Accumulation in transverse phase space
By Multiple Multiturn Injection (MMTI)

ESR Electron Cooler
300 keV

Cooling for Internal Experiments

2. Ionization Cooling

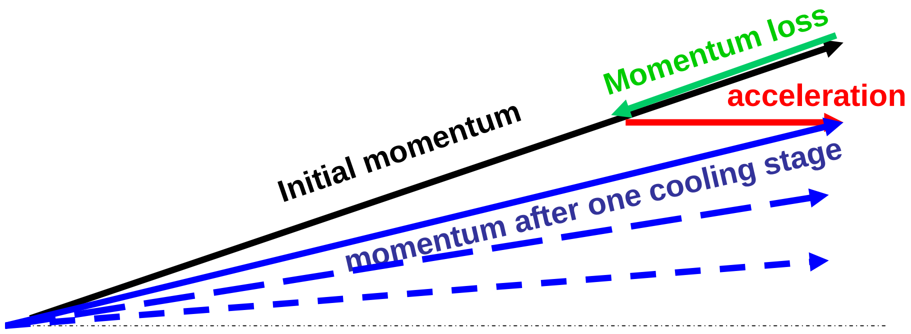
makes use of energy loss in matter



Momentum loss is opposite to motion, p, p_x, p_y, ΔE decrease

Momentum gain is purely longitudinal

proposed for muon cooling



transverse cooling

$$\frac{d\epsilon_N}{ds} = -\frac{1}{\beta^2 E}\frac{dE}{ds}\epsilon_N + \frac{\beta\gamma\beta_\perp}{2}\frac{\langle\theta_{rms}^2\rangle}{ds}$$

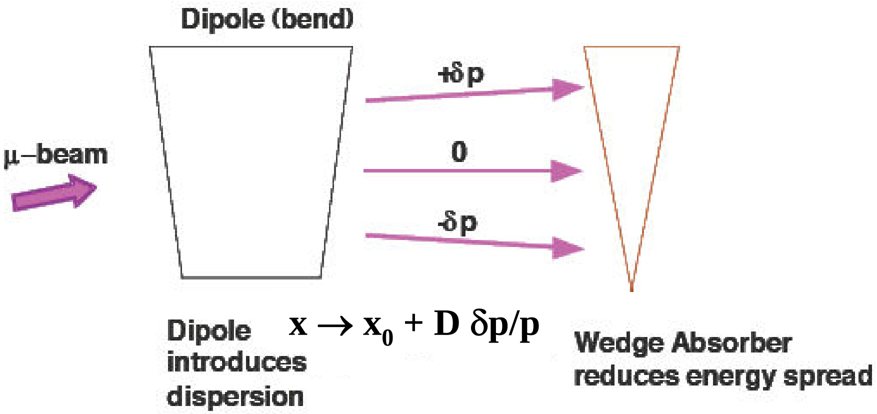
$$= -\frac{1}{\beta^2 E}\frac{dE}{ds}\epsilon_N + \frac{\beta_\perp E_s^2}{2\beta^3 m_\mu c^2 L_R E}$$

small $\beta_\perp$ at absorber in order to minimize multiple scattering

large $L_R/(dE/ds)$ ⇒ light absorbers (H_2)

Ionization Cooling

increased longitudinal cooling by longitudinal-transverse emittance exchange



$$\frac{d\sigma_E^2}{ds} = -2\frac{\partial(dE/ds)}{\partial E}\sigma_E^2 + \frac{d\langle\Delta E_{rms}^2\rangle}{ds}$$

cooling term — heating term

cooling, if $\frac{\partial(dE/ds)}{\partial E} > 0$

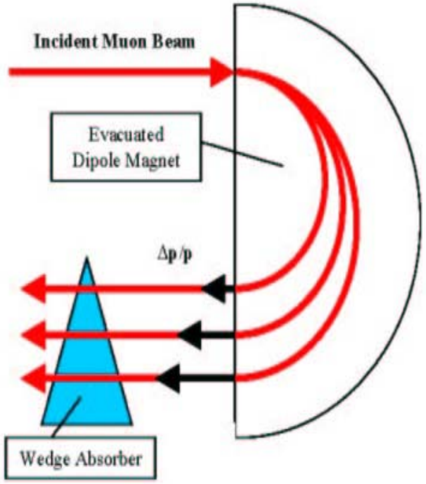


Figure 1. Use of a Wedge Absorber for Emittance Exchange

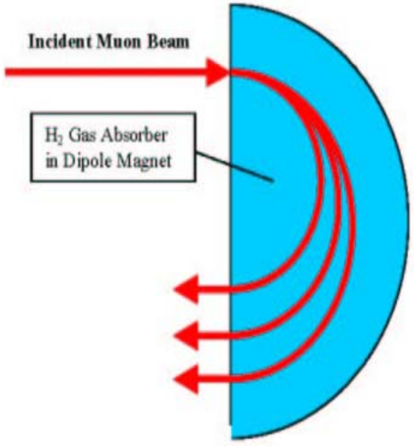


Figure 2. Use of Continuous Gaseous Absorber for Emittance Exchange

emittance exchange

increased longitudinal cooling

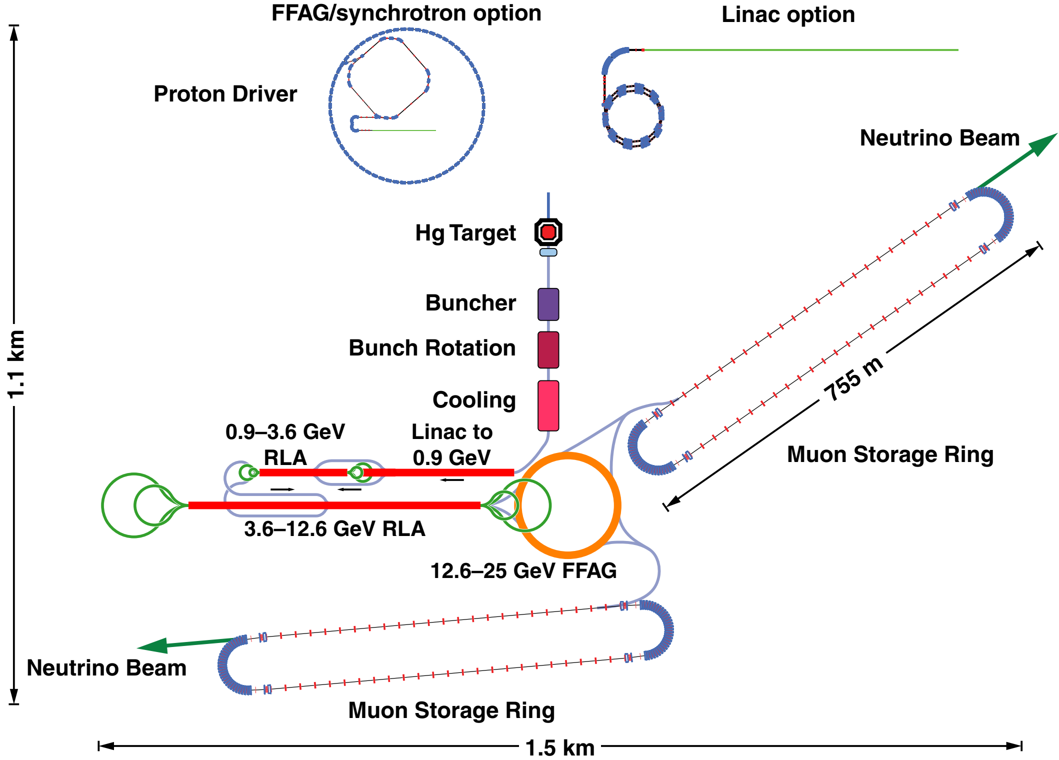
$$\frac{\partial \frac{dE}{ds}}{\partial E} \Rightarrow \frac{\partial \frac{dE}{ds}}{\partial E}\Big|_0 + \frac{dE}{ds}\,\frac{D\rho'}{\beta c p \rho_0}$$

reduced transverse cooling

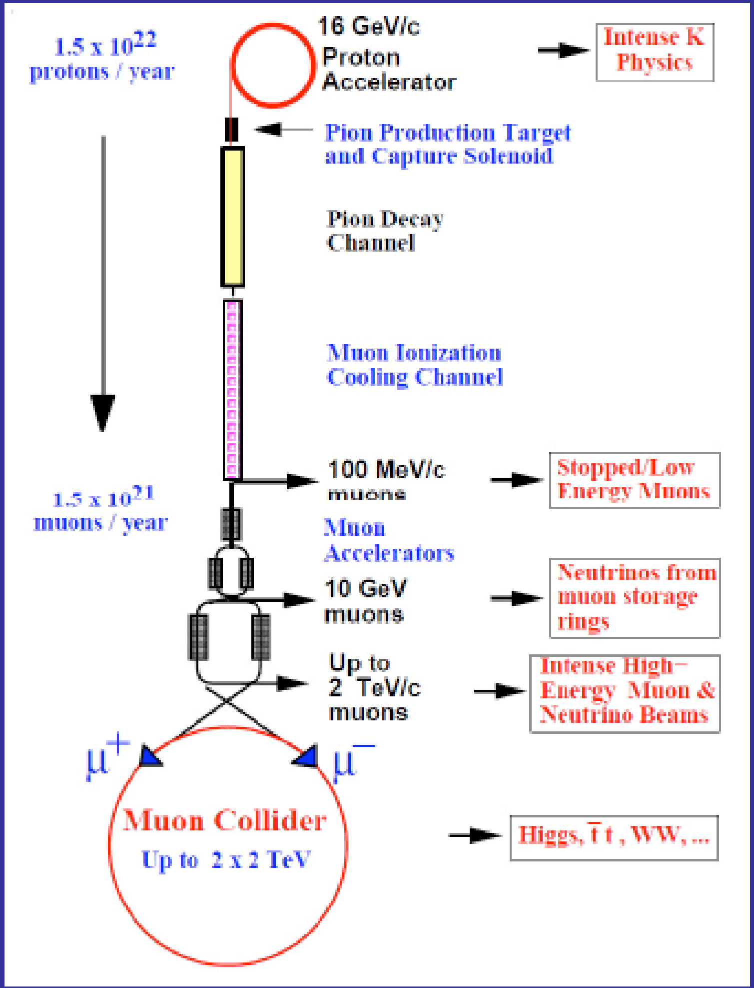
$$\frac{d\epsilon_N}{ds} = -\frac{1}{\beta^2 E}\frac{dE}{ds}\left(1 - \frac{D\rho'}{\rho_0}\right)\epsilon_N$$

Applications of Ionization Cooling

Neutrino Factory

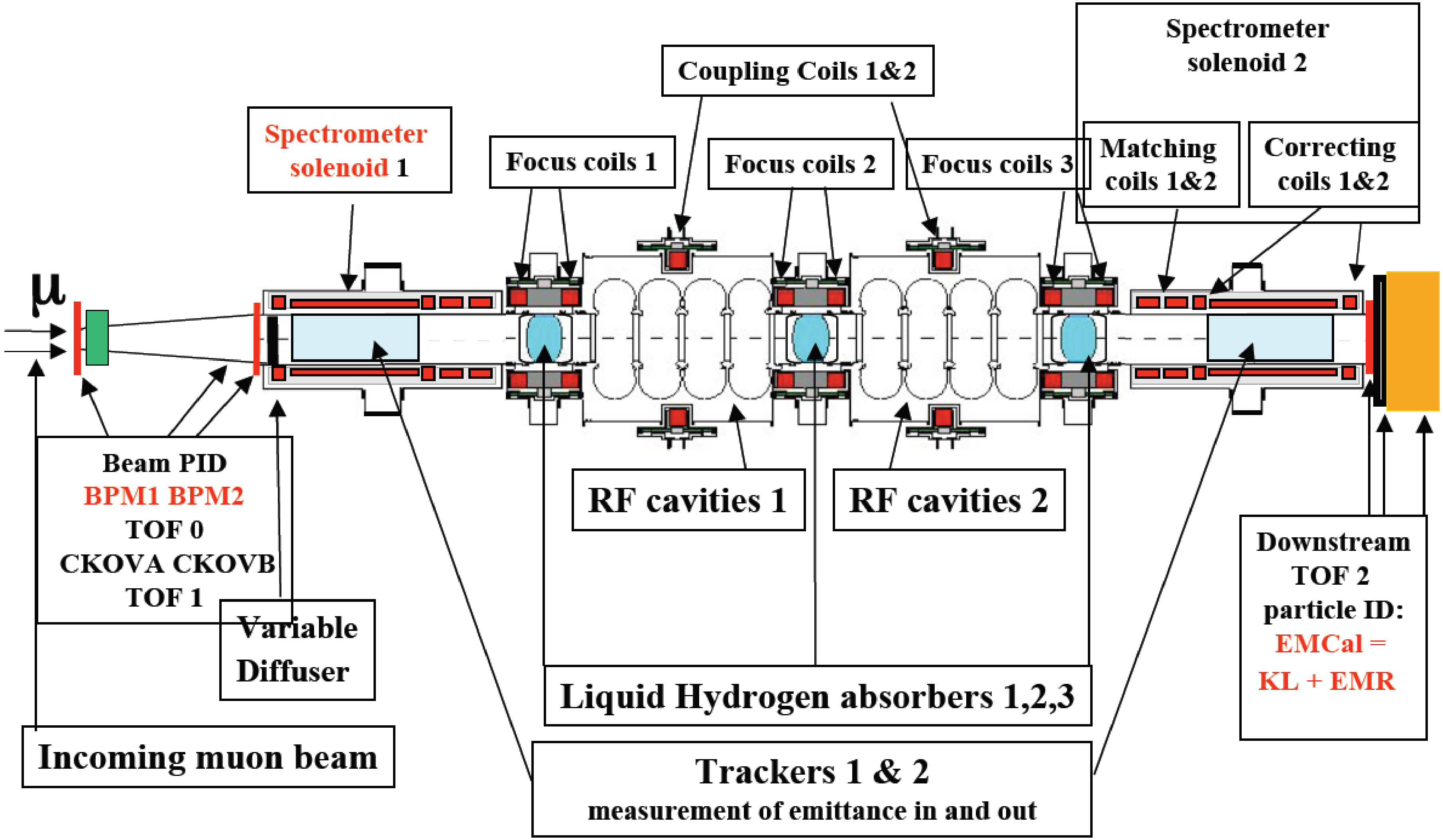


Muon Collider



MICE

Muon Ionization Cooling Experiment at RAL



Spectrometer solenoid 1

Focus coils 1

Coupling Coils 1&2

Focus coils 2

Focus coils 3

Spectrometer solenoid 2

Matching coils 1&2

Correcting coils 1&2

Beam PID
BPM1 BPM2
TOF 0
CKOVA CKOVB
TOF 1

Variable Diffuser

Incoming muon beam

RF cavities 1

RF cavities 2

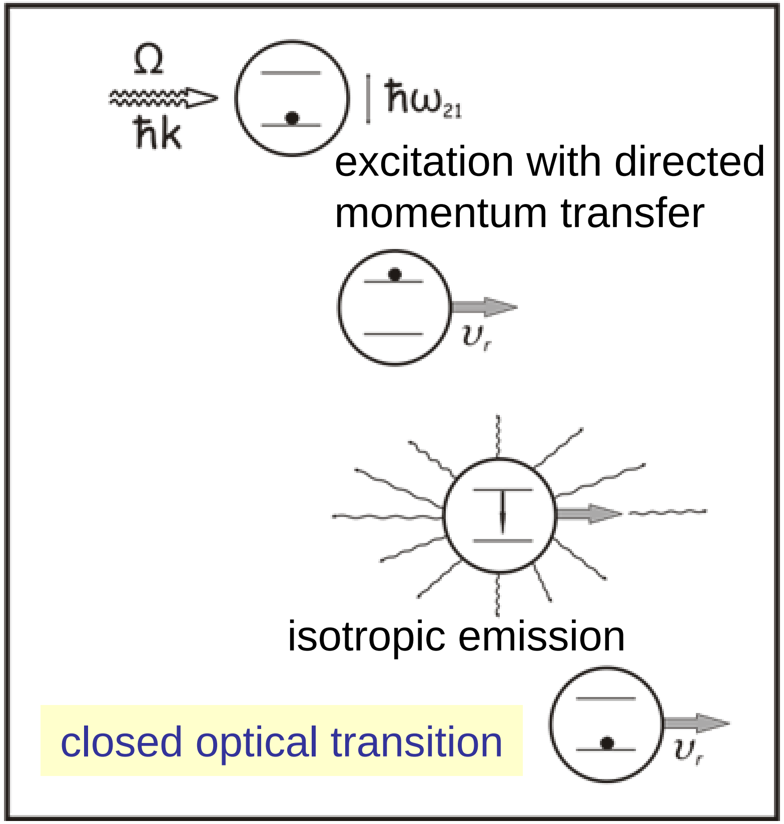
Liquid Hydrogen absorbers 1,2,3

Trackers 1 & 2
measurement of emittance in and out

Downstream
TOF 2
particle ID:
EMCal =
KL + EMR

3. Laser Cooling

$$\Omega = \gamma\omega_{21}(1 \pm \beta\cos\theta)$$

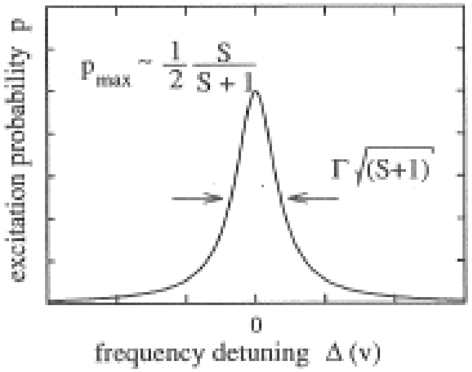


excitation with directed momentum transfer

isotropic emission

closed optical transition

the directed excitation and isotropic emission result in a transfer of velocity v_r



cooling force

$$\vec{F}(\vec{v},\vec{k}) = \frac{\hbar\vec{k}}{2}S\Gamma\frac{(\Gamma/2)^2}{(\omega-\omega_{21}-\vec{v}\vec{k}) + (\Gamma/2)^2(1+S)}$$

Lorentzian with width Γ/k ~ 10 m/s

minimum temperature $T_D = \frac{\hbar\Gamma}{2k_B}$ (Doppler limit)
typical 10^{-5} – 10^{-4} K

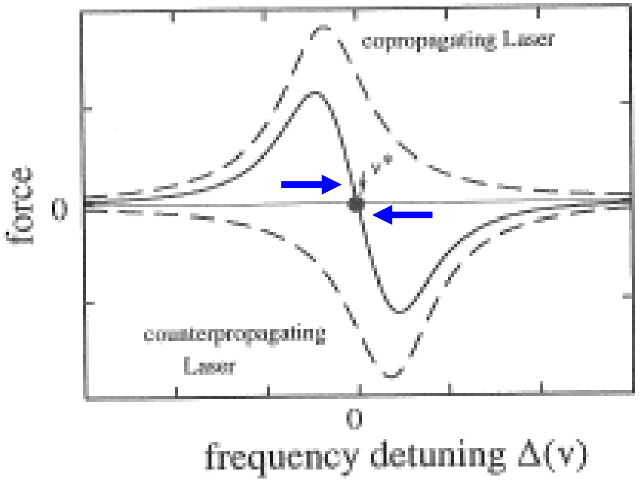
typical cooling time ~ 10 μs

only longitudinal cooling

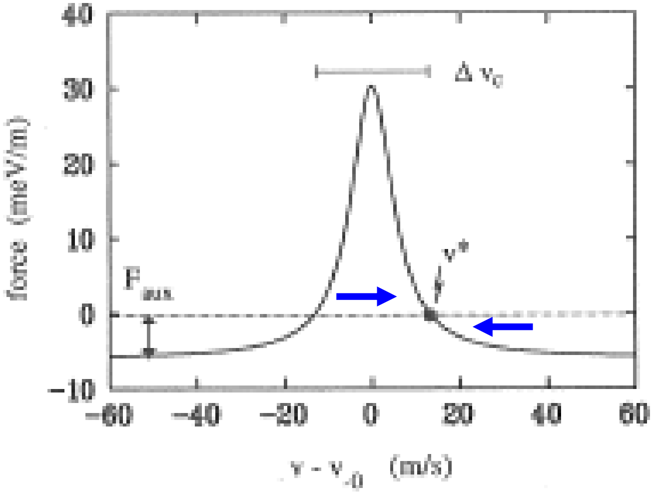
Laser Cooling

a single laser does not provide cooling (only acceleration or deceleration)

schemes for cooling



two counter-propagating lasers (matched to beam velocity)

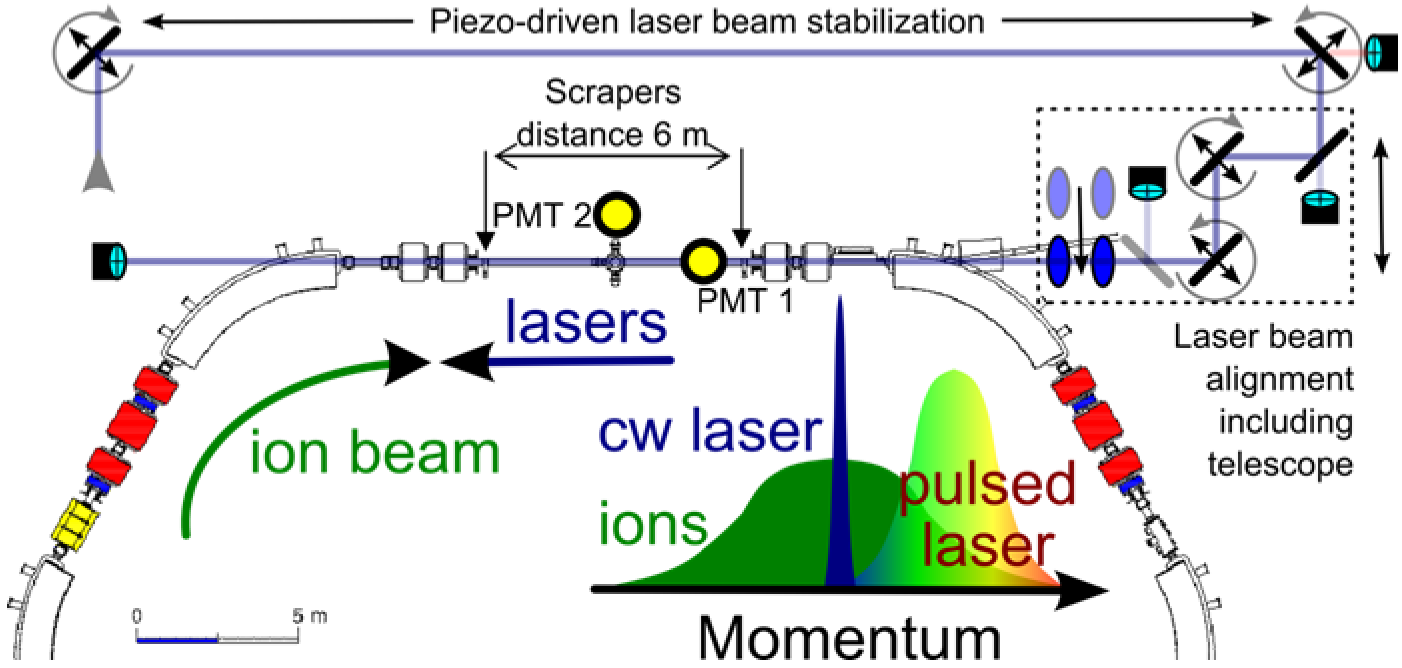


auxiliary force (betatron core, rf)

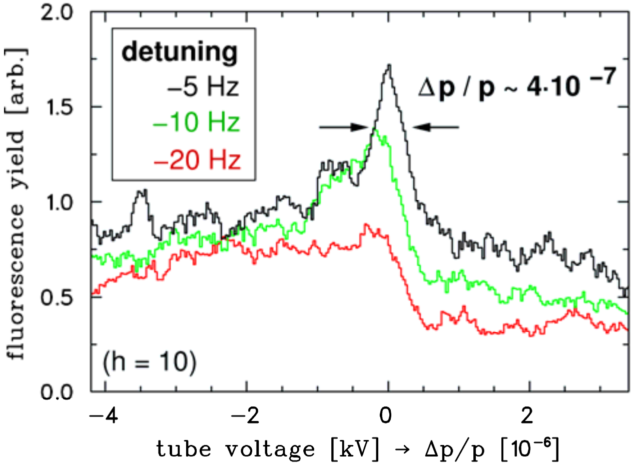
capture range of laser is limited ⇒ frequency sweep (snowplow)

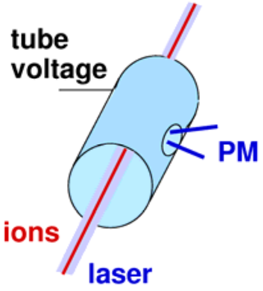
ions studies so far: **$^{7}Li^{1+}$, $^{9}Be^{1+}$, $^{24}Mg^{1+}$, $^{12}C^{3+}$** in future: Li-like heavy ions

Laser Cooling at ESR



Argon ion laser (257.3 nm) frequency doubled





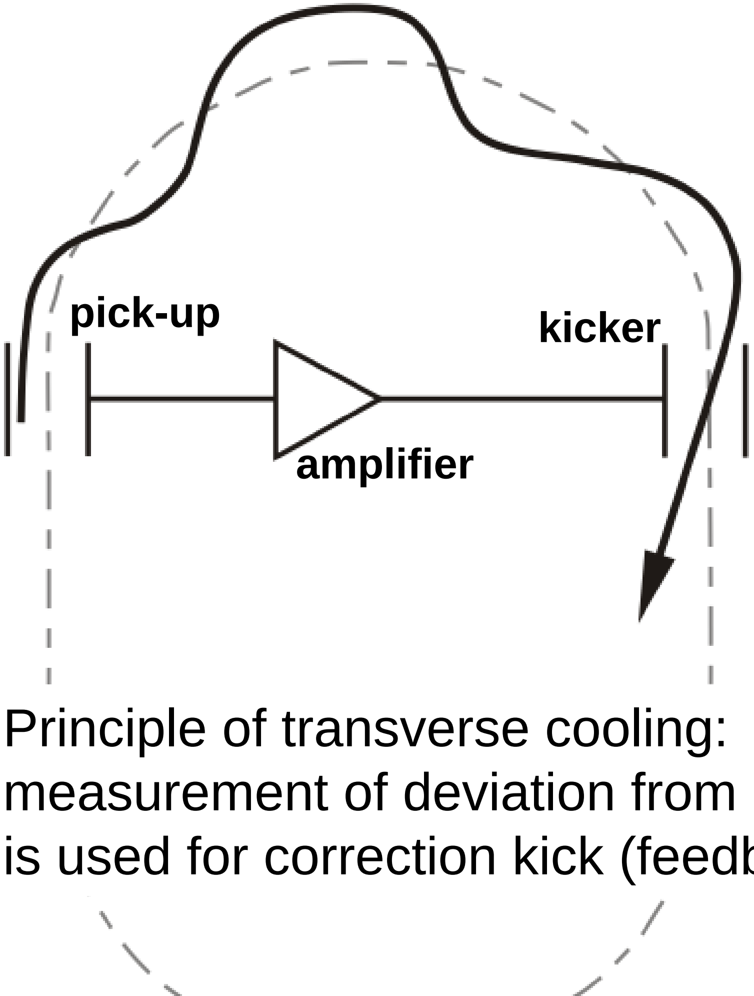
diagnostics by

fluorescence light detection

4. Stochastic Cooling

First cooling method which was successfully used for beam preparation

S. van der Meer, D. Möhl, L. Thorndahl et al.



Conditions:

Betatron phase advance
(pick-up to kicker): (n + ½) π

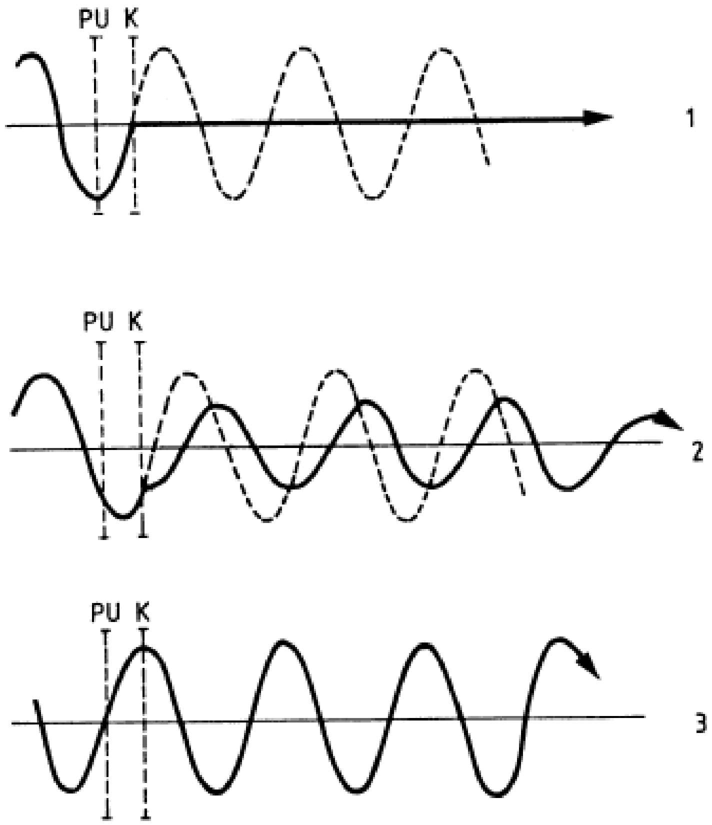
Signal travel time = time of flight of particle
(between pick-up and kicker)

Sampling of sub-ensemble of total beam

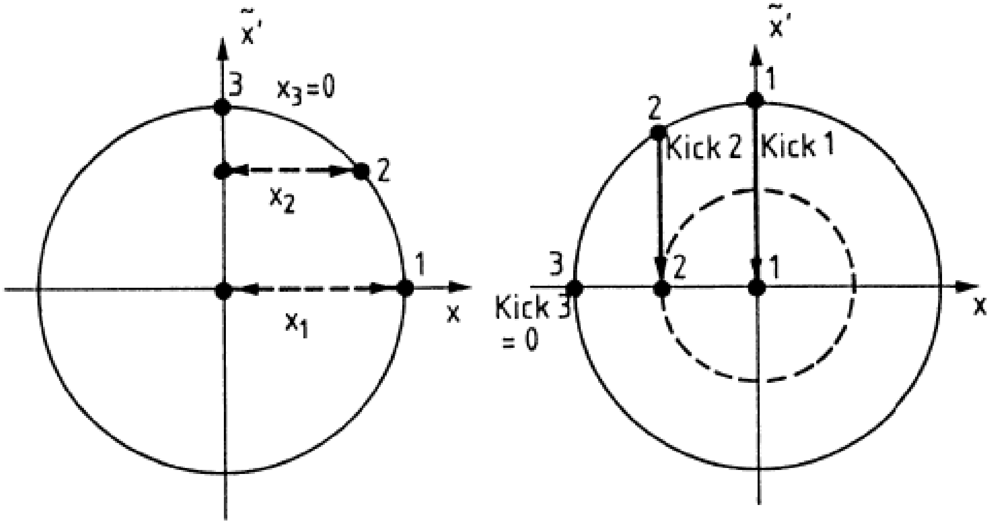
Principle of transverse cooling:
measurement of deviation from ideal orbit
is used for correction kick (feedback)

Stochastic Cooling

single particle betatron motion
along storage ring
without and with correction kick



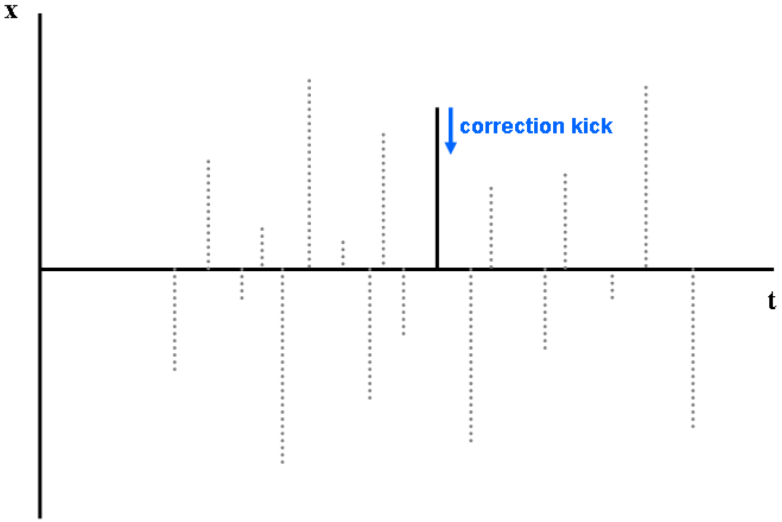
projection to two-dimensional
horizontal phase area



At pick-up

At kicker

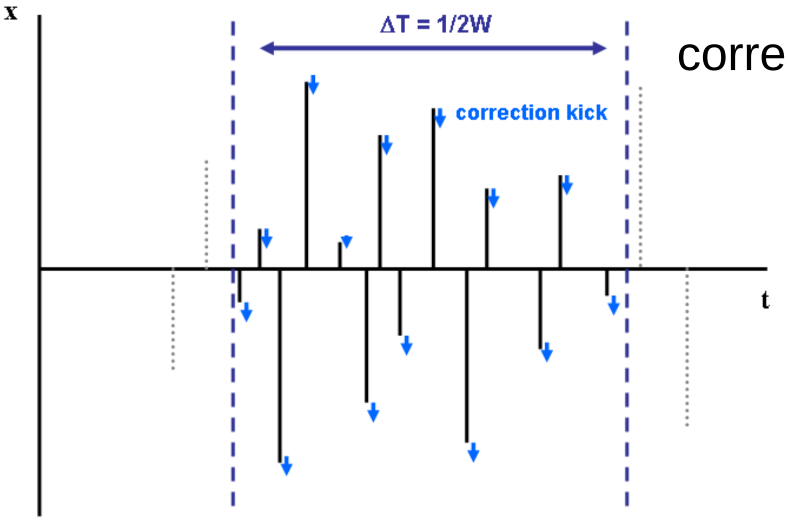
Stochastic Cooling



correction kick
(unlimited resolution)

$$\Delta x = g \times x$$

Nyquist theorem: a system with a band-width Δf = W in frequency domain can resolve a minimum time duration ΔT=1/(2W)



correction kick $\Delta x = \frac{g}{N_s} \times \sum_{i=1..N_s} x_i \ , \quad N_s = N\frac{\Delta T}{T_0} = \frac{N}{2WT_0}$

For exponential damping (x(t)=x(t$_0$) exp(-(t-t$_0$)/τ)):

$$\tau^{-1} = T_0^{-1} \times \frac{\Delta x}{x} = \frac{g2W}{N},\ if \sum_{i=1..N_s} x_i = x$$

$$\underline{\tau^{-1} \le \frac{2W}{N},}\ if\ g \le 1$$

Stochastic Cooling

some refinements of cooling rate formula

noise: thermal or electronic noise adds to beam signal

mixing: change of relative longitudinal position of particles due to momentum spread

cooling rate $$\lambda = \tau^{-1} = \frac{2W}{N}\left(\overset{\text{cooling}}{\underline{2g}} - \overset{\text{heating}}{\underline{g^2(M+U)}}\right)$$

M mixing factor

U noise to signal ratio

maximum of cooling rate

$$\boxed{\lambda_{max} = \frac{2W}{N}\frac{1}{M+U}}$$

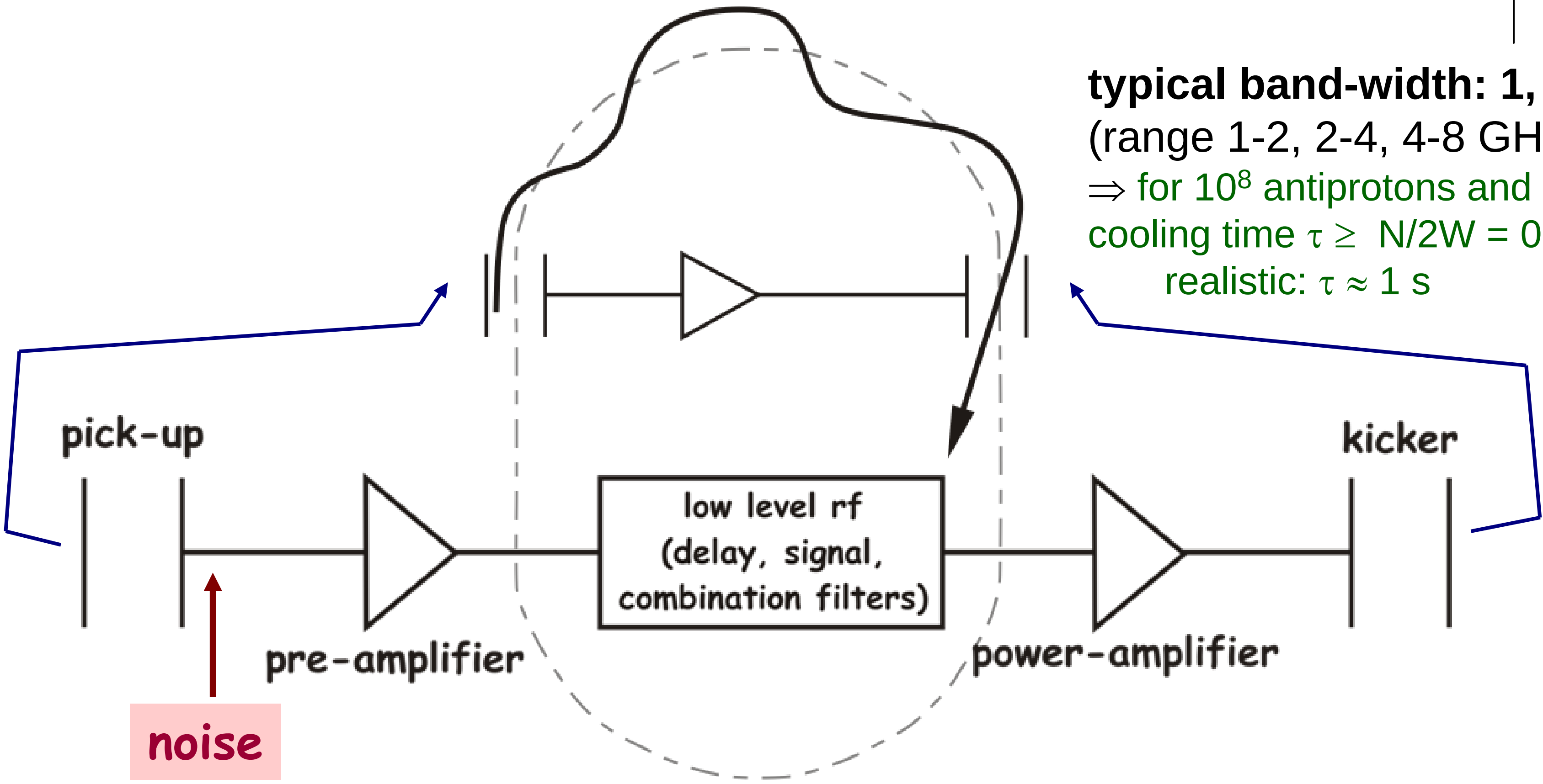
$$\frac{d\lambda}{dg} = 0 \Rightarrow g = \frac{1}{M+U}$$

further refinement (wanted ↔ unwanted mixing):

with wanted mixing M (kicker to pick-up)
and unwanted mixing $\tilde{M}$ (pick-up to kicker)

$$\lambda = \tau^{-1} = \frac{2W}{N}(2g(1-\tilde{M}^2) - g^2(M+U))$$

Stochastic Cooling Circuit



typical band-width: 1, 2 ,4 GHz
(range 1-2, 2-4, 4-8 GHz)
$\Rightarrow$ for 10^8 antiprotons and W= 1 GHz
cooling time $\tau \geq$ N/2W = 0.05 s
realistic: $\tau \approx$ 1 s

Transfer Function:

$$Z_{pick-up} \cdot G_{pick-up}(E) \cdot H(t_{delay}) \cdot F(E) \cdot G \cdot G_{kicker}(E) \cdot Z_{kicker}$$

Longitudinal Stochastic Cooling

1) **Palmer cooling**

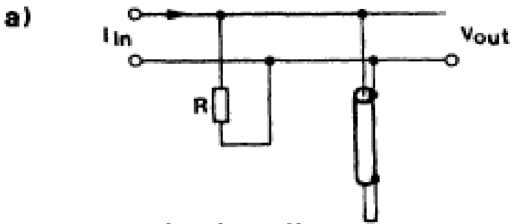
pick-up in dispersive section detects horizontal position

⇒ correcting acceleration/deceleration kick

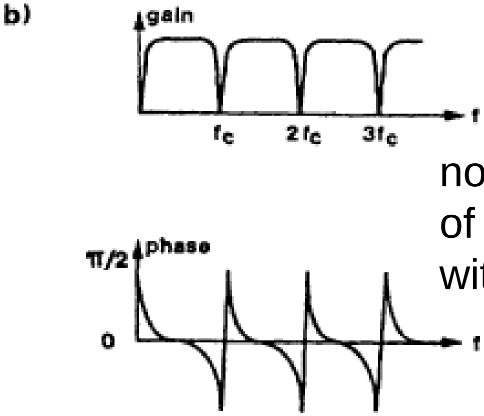
2) **Notch filter cooling**

filter creates notches at the harmonics of nominal revolution frequency

⇒ particles are forced to circulate at the nominal frequency



transmission line
short circuit at all harmonics
of the revolution frequency

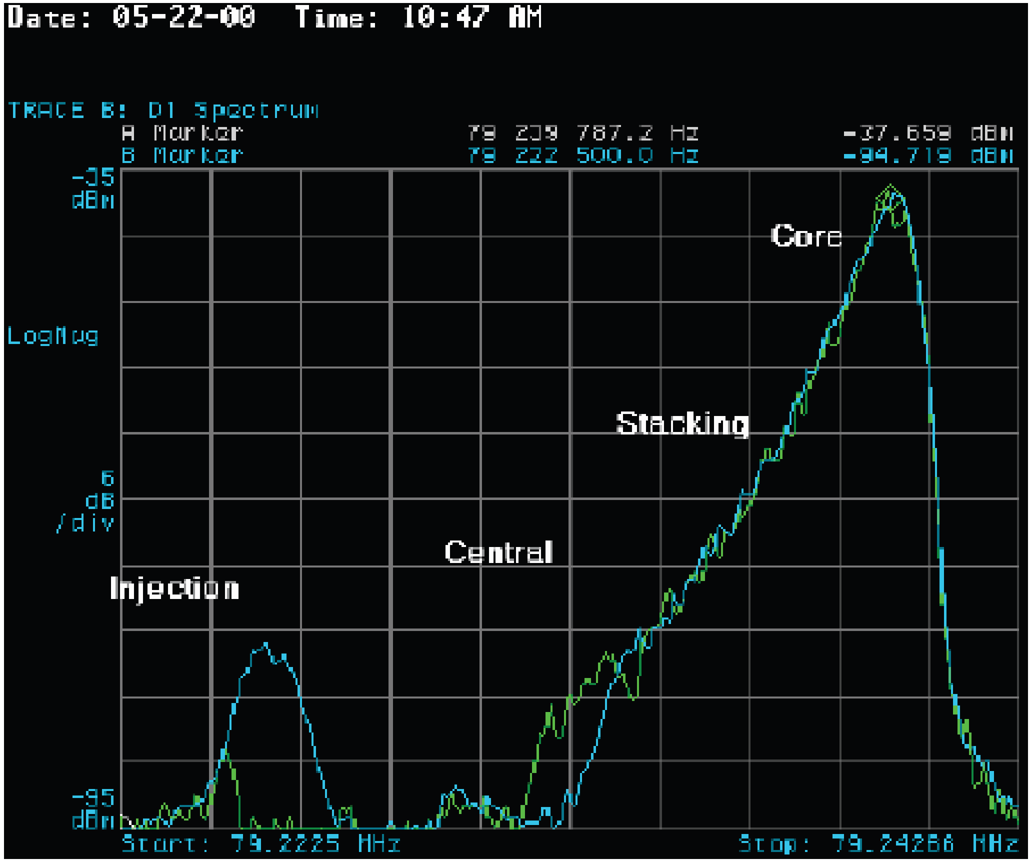


notches at harmonics
of the revolution frequency
with 180º phase jump

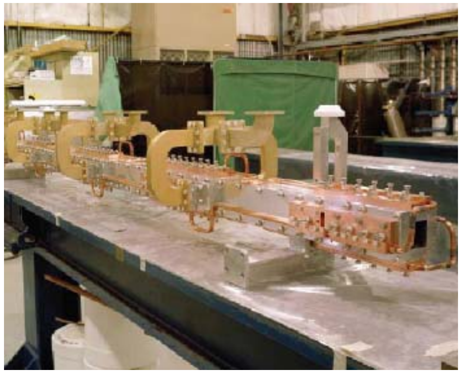
Antiproton Accumulation by Stochastic Cooling

M. Steck
CAS 2009
Darmstadt

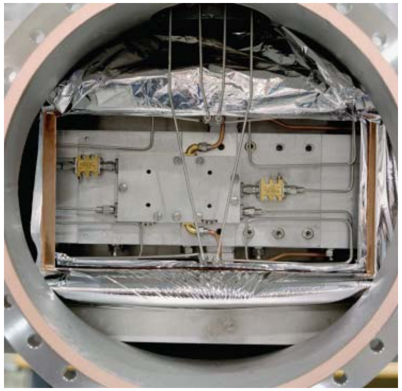
accumulation of 8 GeV antiprotons at FNAL



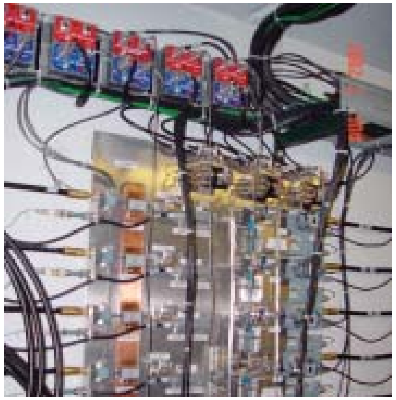
momentum distribution of accumulated antiproton beam



kicker array



cryogenic microwave amplifier



microwave electronics

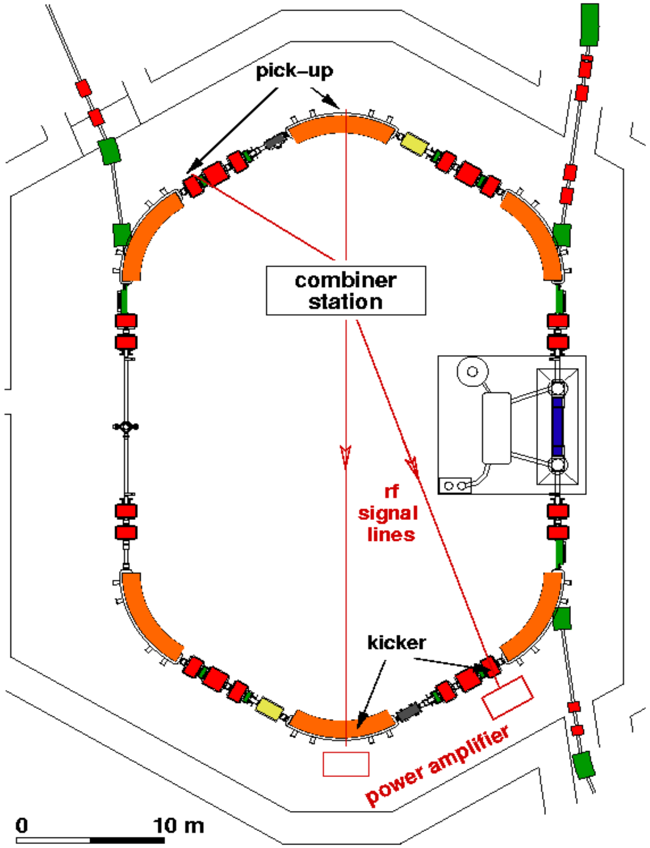
power amplifiers (TWTs)

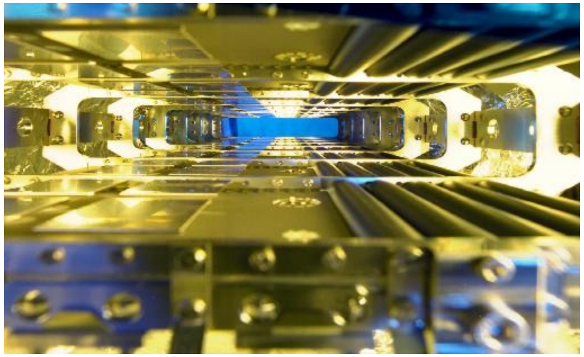
Stochastic Cooling at GSI

M. Steck
CAS 2009
Darmstadt

fast pre-cooling of hot fragment beams

energy 400 (-550) MeV/u
bandwidth 0.8 GHz (range 0.9-1.7 GHz)
$\delta p/p = \pm 0.35\ \%$ → $\delta p/p = \pm 0.01\ \%$
$\varepsilon = 10 \times 10^{-6}$ m → $\varepsilon = 2 \times 10^{-6}$ m





electrodes installed inside magnets

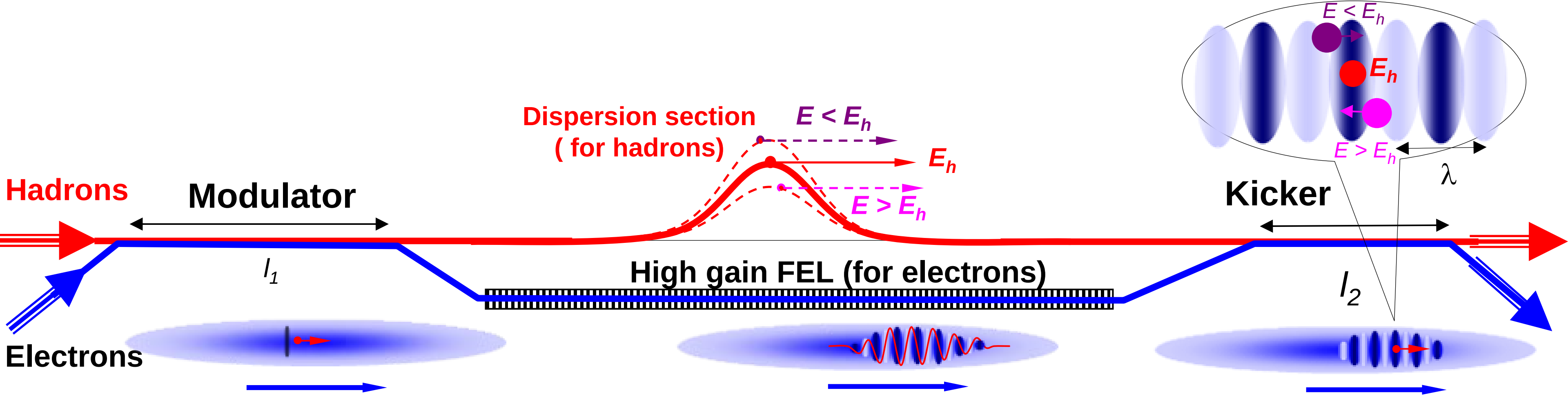
combination of signals from electrodes

power amplifiers for generation of correction kicks

Coherent Electron Cooling

A combination of electron and stochastic cooling concepts

proposed for fast cooling at highest energies (several 10 – 100 GeV)



- The Coherent Electron Cooling system has three major subsystems
  - **modulator**: the ions imprint a “density bump” on the electron distribution
  - **amplifier**: FEL interaction amplifies a density bump by orders of magnitude
  - **kicker**: the amplified & phase-shifted electron charge distribution is used to correct the velocity offset of the ions