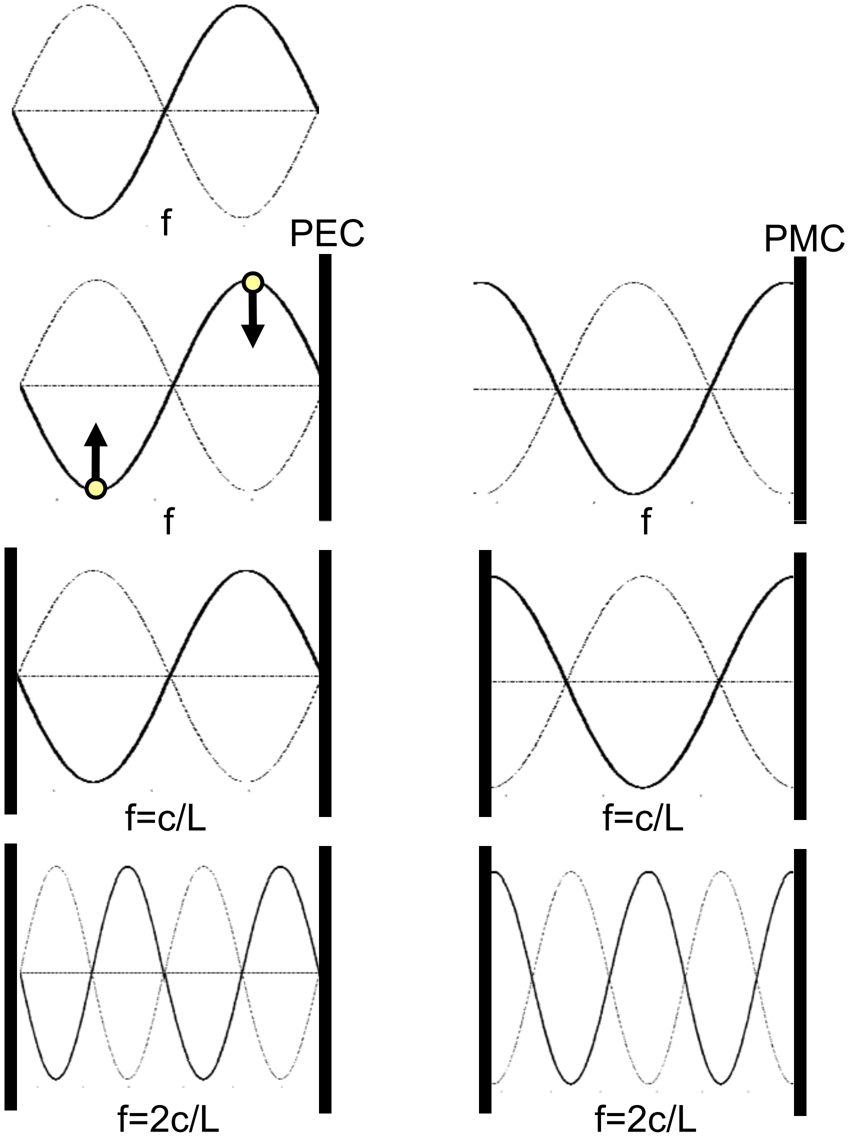
3D EM Simulations

Overview

- Cavities
  - Eigenmodes
  - Q Factor
  - Time Domain and Resonances
- Cavities and Particles
- Electron Guns
- Collector
- Wakefields

Eigenmode Solver
Theoretical Background



In free space *travelling waves* can exist for any frequencies.

If such a plane wave is reflected at a perfect wall (electric or magnetic) there will be a standing wave. This *standing wave* exists independently from the frequency.

The insertion of a second wall does not affect those standing wave as long as the distance L between the walls fits perfectly to the wave-length. The standing wave of this closed structure is called an *eigenmode*.

The frequency and the shape of the next eigenmode fitting between those two walls is predictable.

Eigenmode Computation

$$rot\,\vec{E} = -\,j\omega\mu\,\vec{H}$$
$$rot\,\vec{H} = j\omega\varepsilon\,\vec{E} + \sigma\,\vec{E} = j\omega(\varepsilon + \frac{\sigma}{j\omega})\vec{E} = j\omega\,\underline{\underline{\varepsilon}}\,\vec{E}$$

$$\boxed{rot\,\frac{1}{\mu}\,rot\,\vec{E} = \omega^2\,\underline{\underline{\varepsilon}}\,\vec{E}}$$

Eigenvalue equation for the resonant structure modes and resonance frequencies.

Eigenvalues ω and eigenvectors $\vec{E}$

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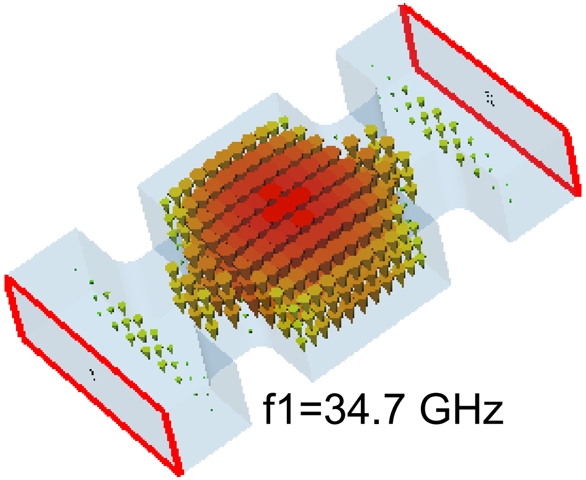
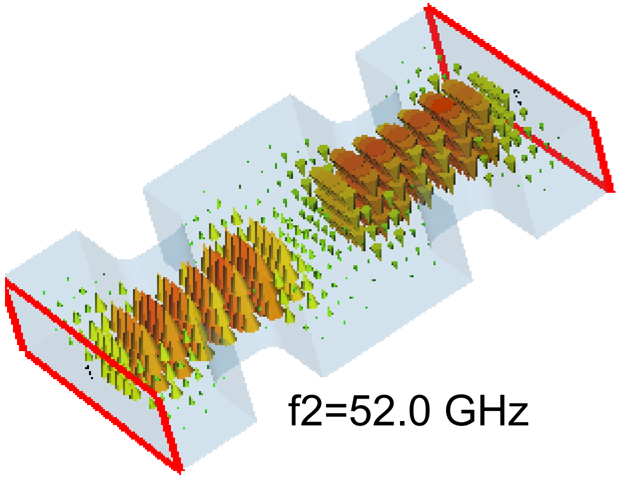
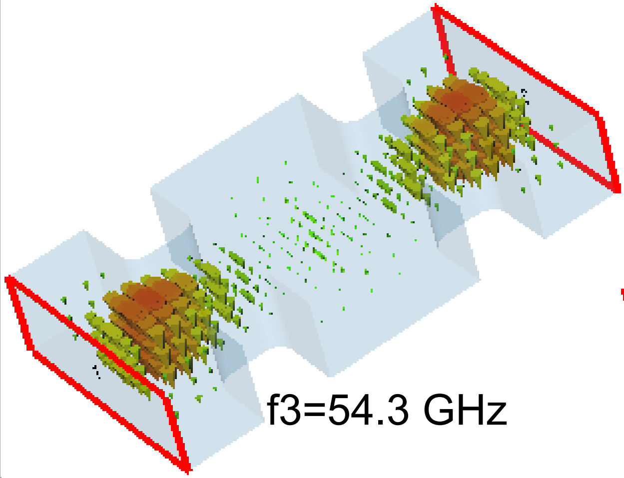
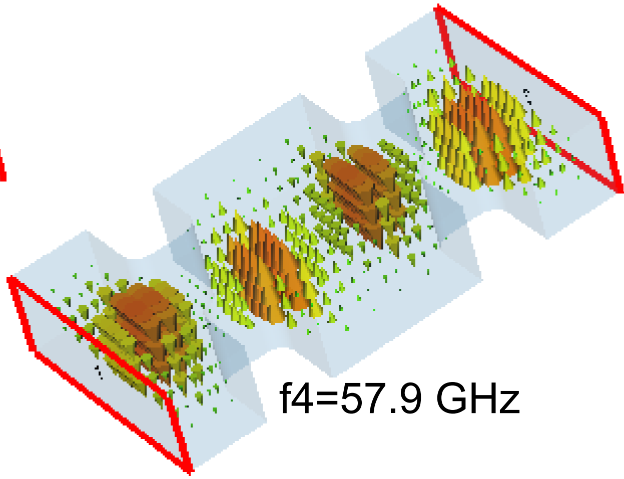
Eigenmodes

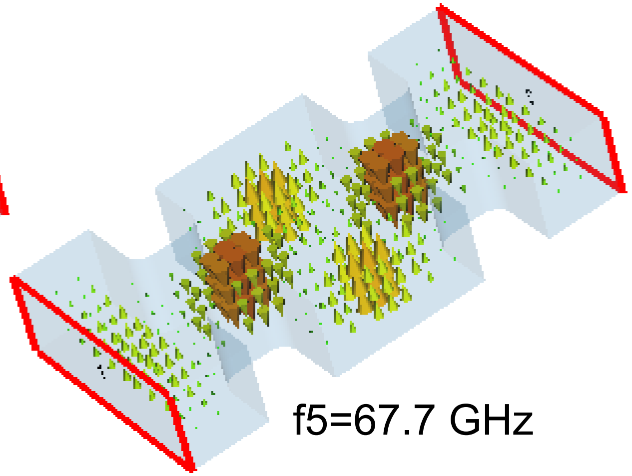
Mode	Frequency
1	34.678795659
2	52.0134908652
3	54.2886537464
4	57.8927542991
5	67.7081900015
6	74.7479841682
7	76.5247777167
8	84.8114666276
9	87.0611153307
10	92.3143483109


f1=34.7 GHz

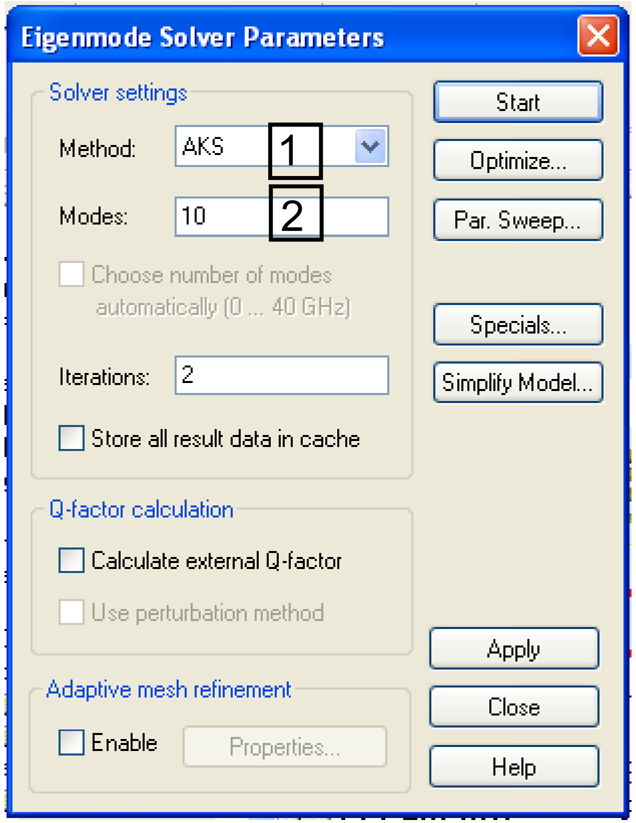

f2=52.0 GHz


f3=54.3 GHz


f4=57.9 GHz


f5=67.7 GHz

Eigenmode Solver

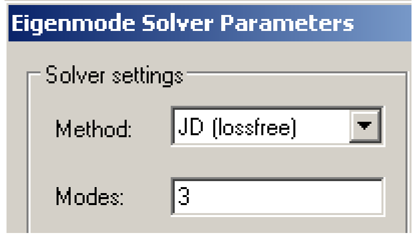


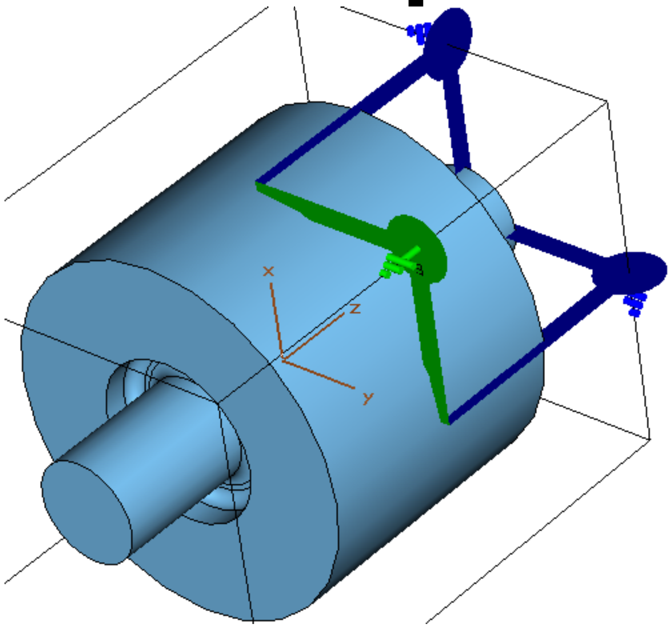
Main user input:

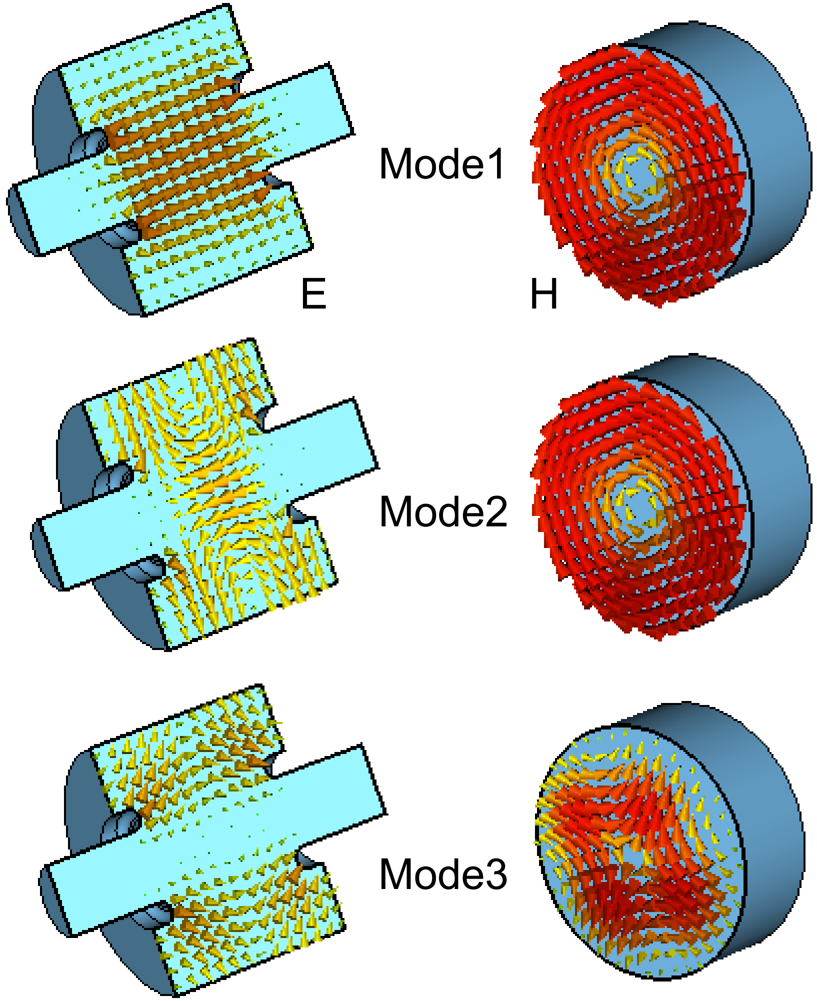
1 **Which eigenmode method should be used?**
(*AKS* might be faster for well behaved examples, *JD* is more robust and might even find good solutions for bad conditioned problems)

2 **How many modes are required?**
The *AKS* method always calculates internally at least 10 modes. (Therefore nearly no difference in cpu time between 1 and 10 modes).
The *JD* method calculates one mode after another, therefore 10 modes need roughly 10times the simulation time of 1 mode.
Therefore the number of modes should be decreased when using the JD method.

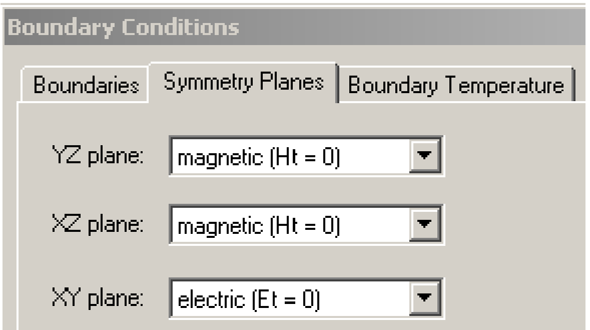
A simple lossfree Cavity

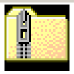






3 symmetry planes ➔ only 1/8
of the volume needs to be calculated




cavity_03.zip

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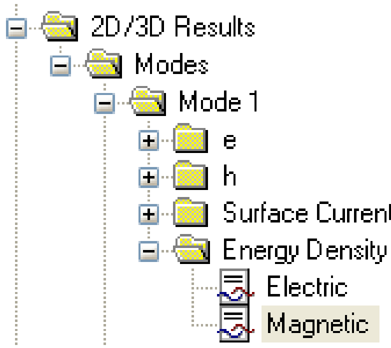
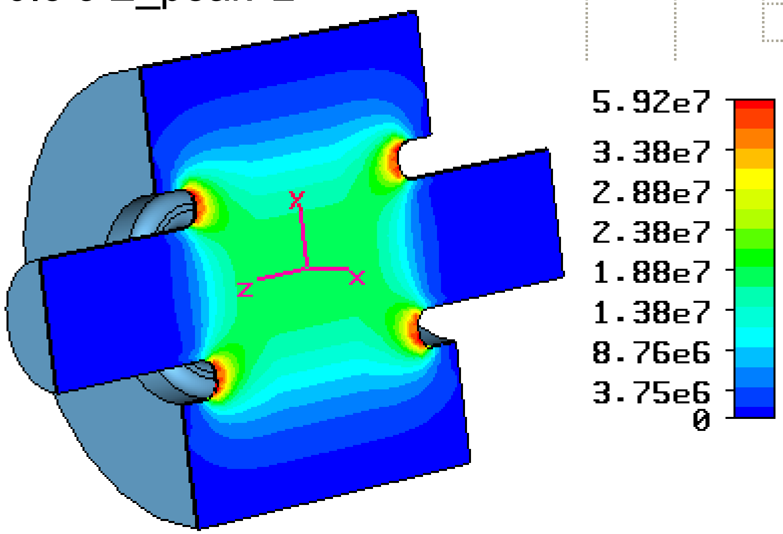
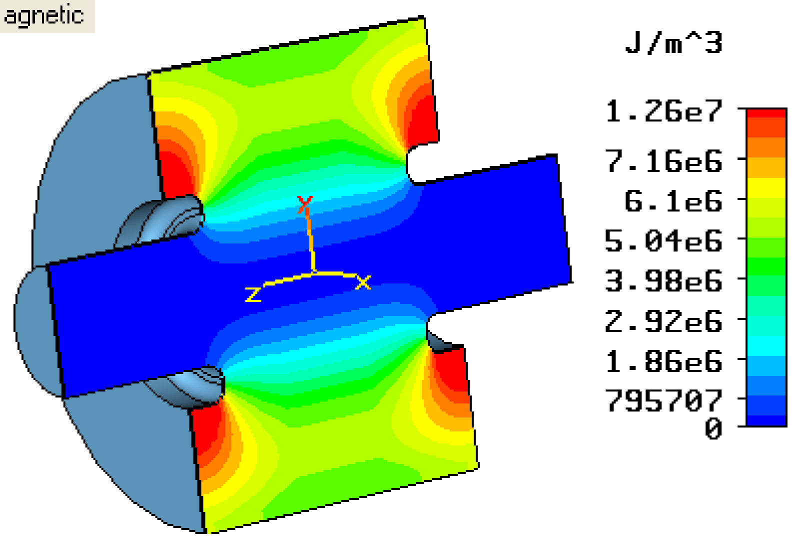
Energy Densities



electric energy density
0.5 ε E_peak^2



magnetic energy density
0.5 μ H_peak^2



Volume Integration of Energy Density is possible via Result Template
0D / Evaluate Field in arbitrary Coordinates (0D, 1D, 2D, 3D)
Note: for a lossfree eigenmode both integrals (el. + mag.energy) will be 1 Joule,
since the energy is oscillating between electric and magnetic field.

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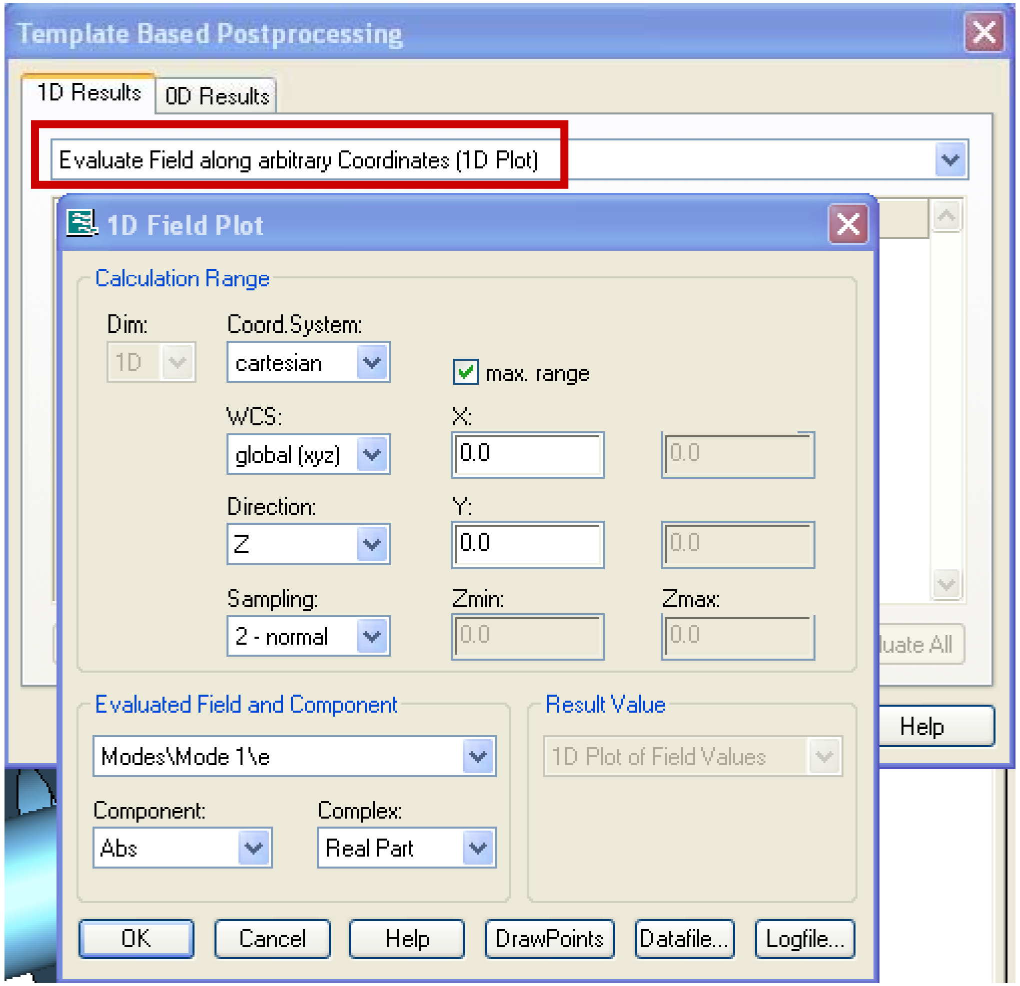
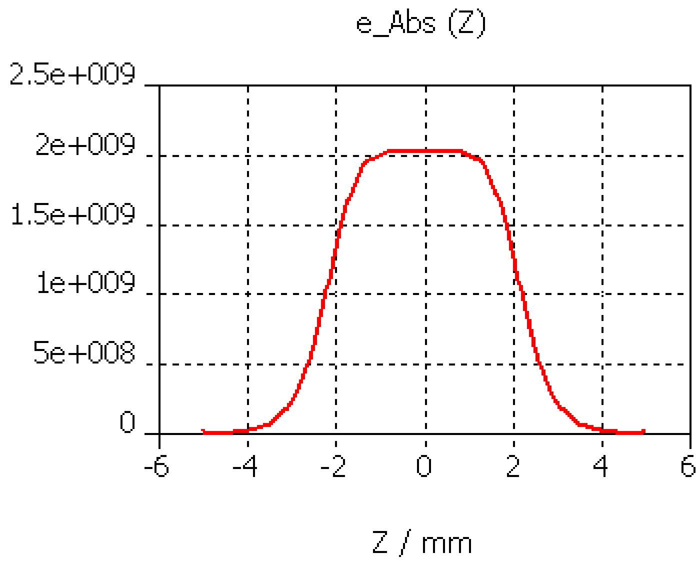
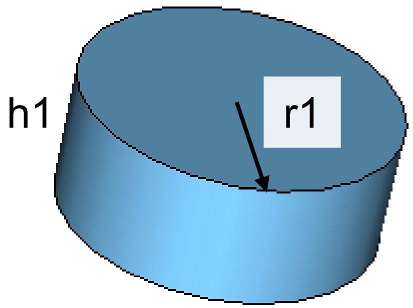
Plotting Fields and getting Field Values



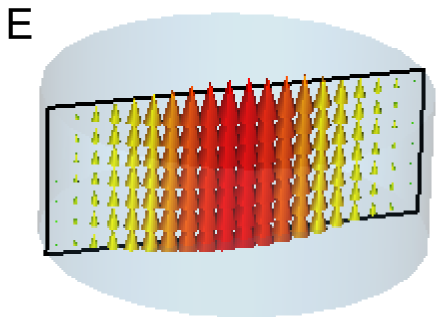


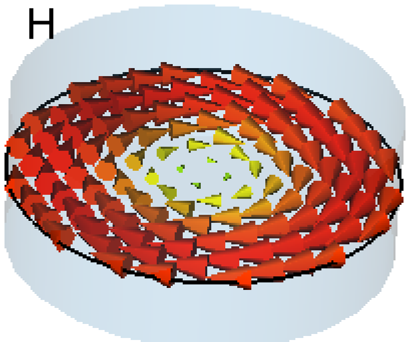
All modes are normed
to 1 Joule stored energy.
E / H / surface current
are stored as peak values.

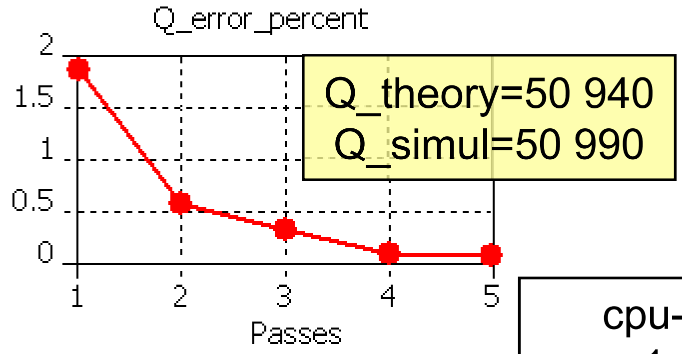
Benchmark Pillbox



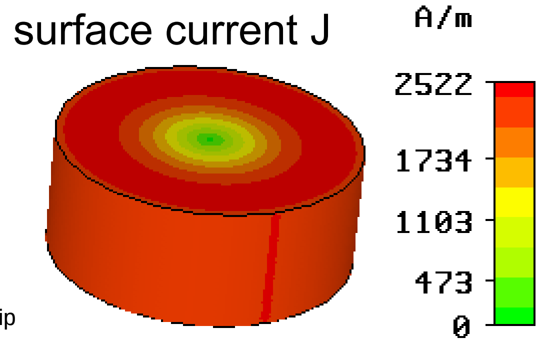
Influence of Meshing

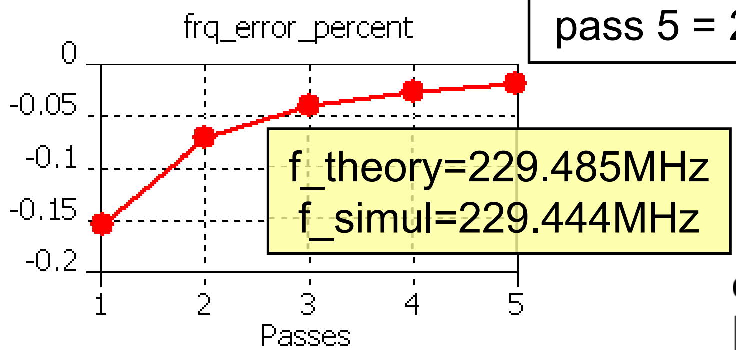






cpu-time
pass 1 = 16sec
pass 5 = 2 min





pillbox_01.zip

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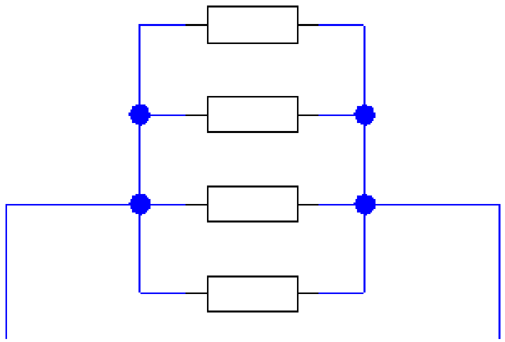
The Quality Factor Q

$$Q = \frac{2\pi \bullet StoredEnergy}{Energy\_consumed\_per\_period} = \frac{2\pi \cdot f \cdot W}{P_{rms}}$$

the higher Q, the longer the energy is kept

- for a lossfree structure P=0 ➔ infinite Q

- different kind of losses exist:
  - skin effect (surface) losses: Q_{wall}
  - dielectric (volume) losses: Q_{diel}
  - losses due to connected feeding lines: Q_{ext}
  - losses due to energy transfer between beam and mode: Q_{beam}

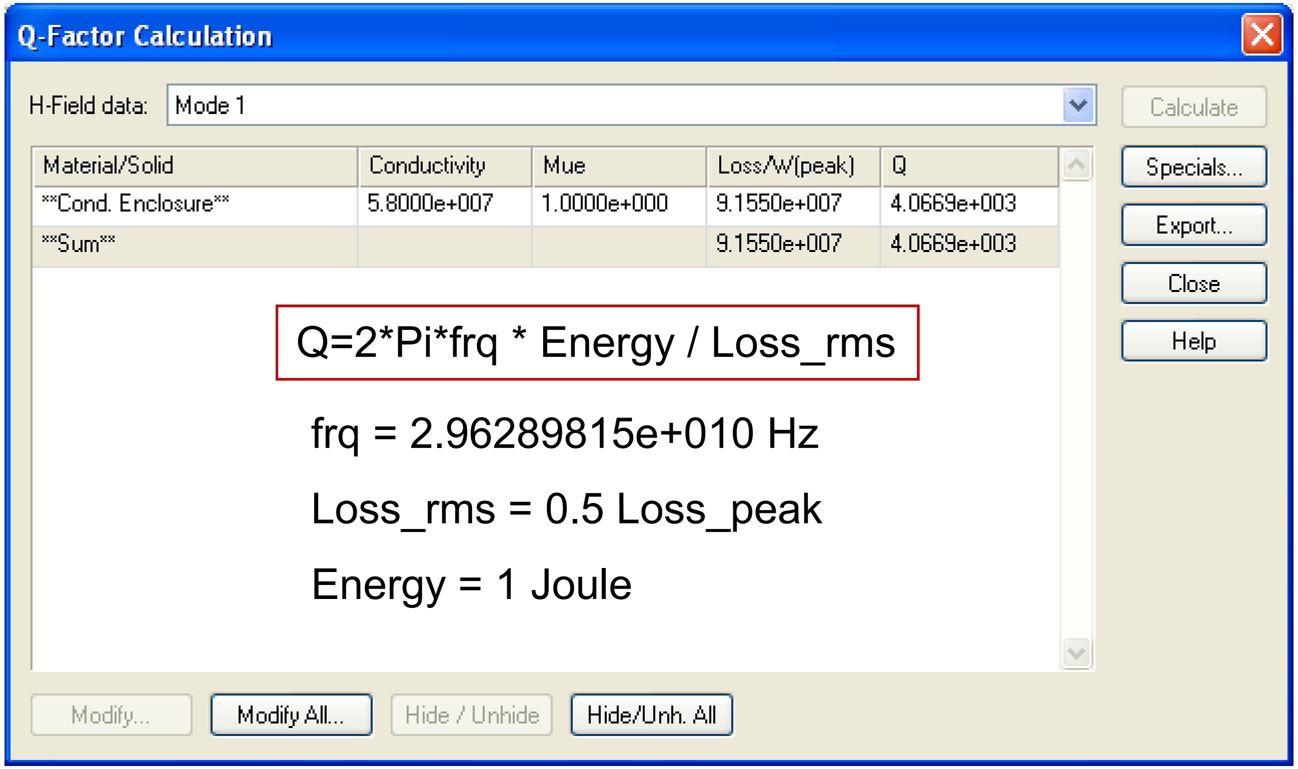


$$\frac{1}{Q_{tot}} = \frac{1}{Q_{wall}} + \frac{1}{Q_{diel}} + \frac{1}{Q_{ext}} + \frac{1}{Q_{beam}}$$

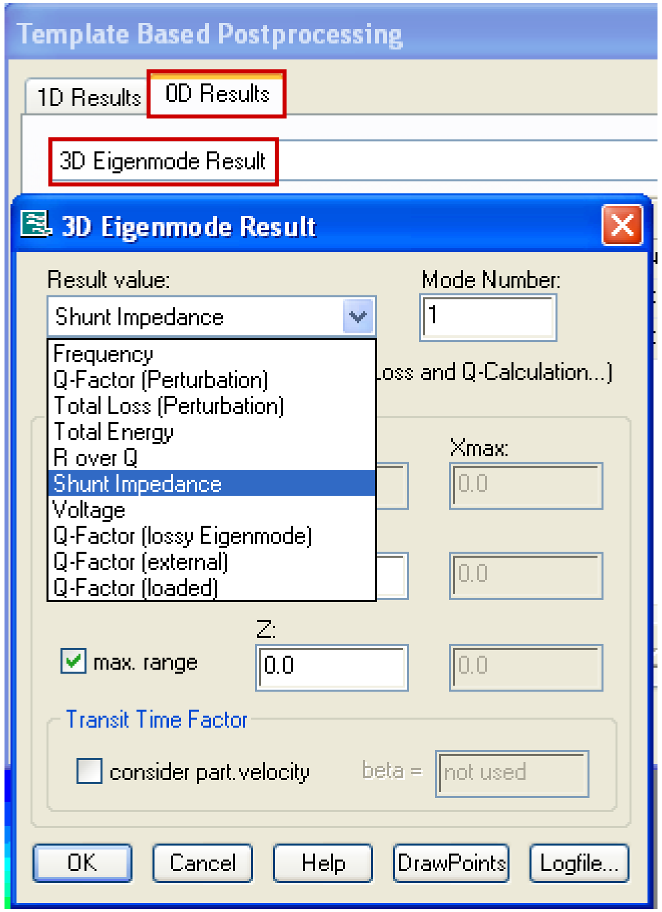
As a circuit model, all losses can be seen as a parallel circuit, acting on the same mode.

Calculation of Q_{wall} and Q_{diel}

Results -> Loss and Q-Calculation performs a loss calculation in the postprocessing based on perturbation theory. It handles metallic losses due to finite conductivity (skin effect) as well as dielectric losses.



Integration of Voltage, Calculation of Shunt Impedance & R/Q



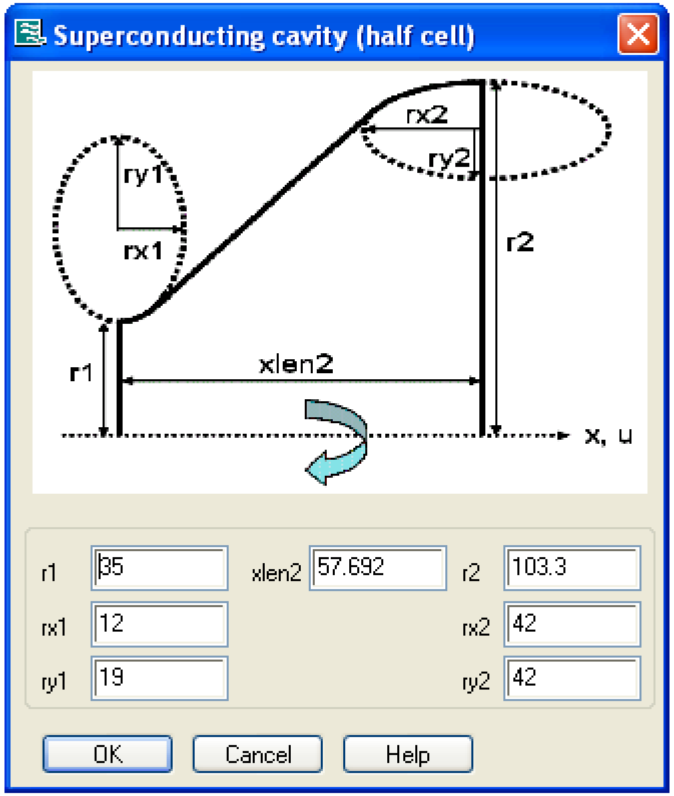
Shunt Impedance Rs=Vo^2/(2W)
with Vo : voltage „seen“ by a charged interacting particle.

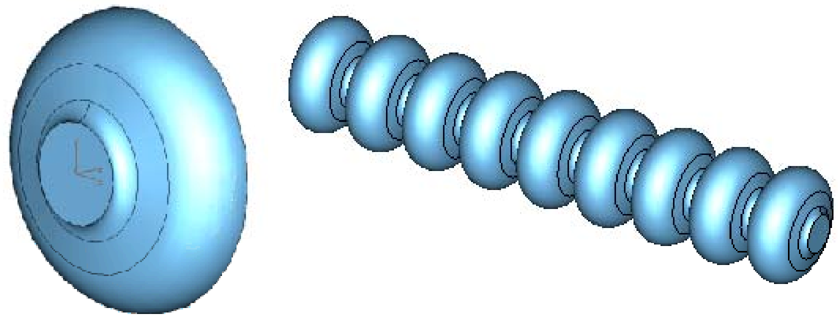
For voltage integration also the ***Transit Time Factor*** can be specified, which defines the speed of the particle (beta=v/c) and guarantees a phase-correct integration of electr. field.

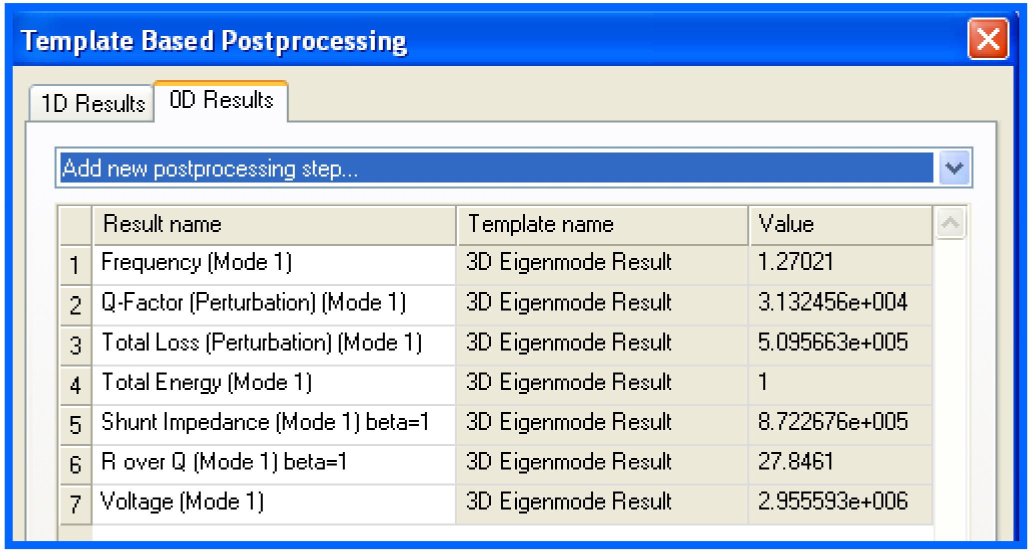
R/Q (R over Q) only depends on the geometry (not on the loss mechanism) and is therefore often used to compare different cavity structures.

Example Superconducting Cavity

easy construction via Macros -> Construct -> Superconducting Cavity





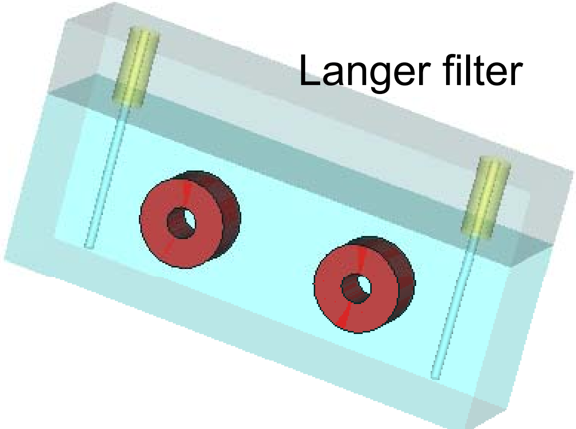


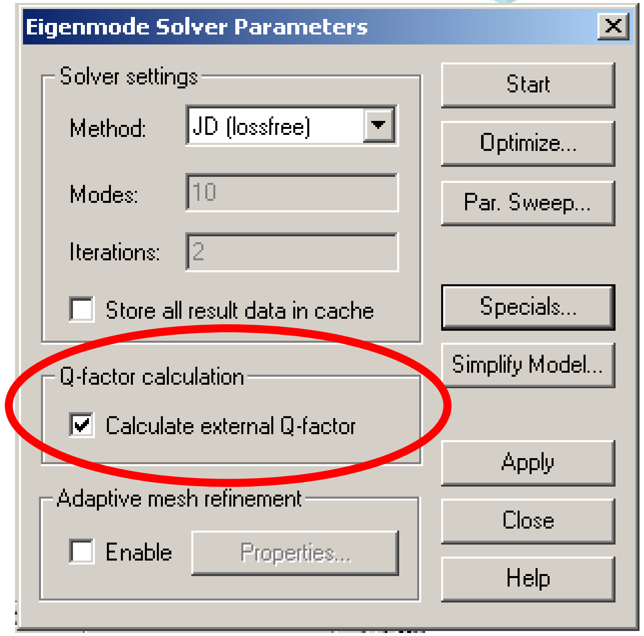
	Result name	Template name	Value
1	Frequency (Mode 1)	3D Eigenmode Result	1.27021
2	Q-Factor (Perturbation) (Mode 1)	3D Eigenmode Result	3.132456e+004
3	Total Loss (Perturbation) (Mode 1)	3D Eigenmode Result	5.095663e+005
4	Total Energy (Mode 1)	3D Eigenmode Result	1
5	Shunt Impedance (Mode 1) beta=1	3D Eigenmode Result	8.722676e+005
6	R over Q (Mode 1) beta=1	3D Eigenmode Result	27.8461
7	Voltage (Mode 1)	3D Eigenmode Result	2.955593e+006

ellipt_cav_01.zip

Eigenmode Solver
Calculation of Q_{ext}

Langer filter





Results of the Q-Factor calculation: Log File

```
--------------------------------------------------------------------------------
External Q-factors:

--------------------------------------------------------------------------------
Mode     Frequency                | Q-factor
--------------------------------------------------------------------------------
  1      0                        |    ----
  2      0                        |    ----
  3      3.2586148445             |  2.83e+000
  4      3.2586148445             |  2.83e+000
  5      4.54620112319            |  3.32e+002
  6      4.57202070385            |  3.05e+002
  7      6.85412046144            |  3.95e+002
  8      6.96493323077            |  5.62e+002
  9      7.08016640849            |  2.62e+004
 10      7.18545391085            |  9.07e+003
 11      7.73471051767            |  2.91e+002
 12      7.90217291982            |  2.01e+002
 13      8.03194365865            |  2.97e+004
--------------------------------------------------------------------------------
```

E.g. external Q-factor comparison for the Langer filter

f [GHz]	CST Q_ext	Steiglitz-McBride
4.546	332	332
4.572	305	305
7.080	26200	24132
7.185	9070	8472

Other methods for loaded Q calculation

- Using the transient analysis
- Amplitude E-field inside a resonator is given:

$$E(t) = E_0 \cdot e^{-\frac{\omega_0 \cdot t}{2 \cdot Q_{load}}} \quad \text{or} \quad \frac{E(t)}{E_0} = e^{-\frac{\omega_0 \cdot t}{2 \cdot Q_{load}}}$$

- Monitored using an E-field probe
- Measure the time difference Δt in which the E-field is damped by a factor of 1/e then Q load is given by

$$Q_{load} = \pi \cdot \Delta t \cdot f_0$$

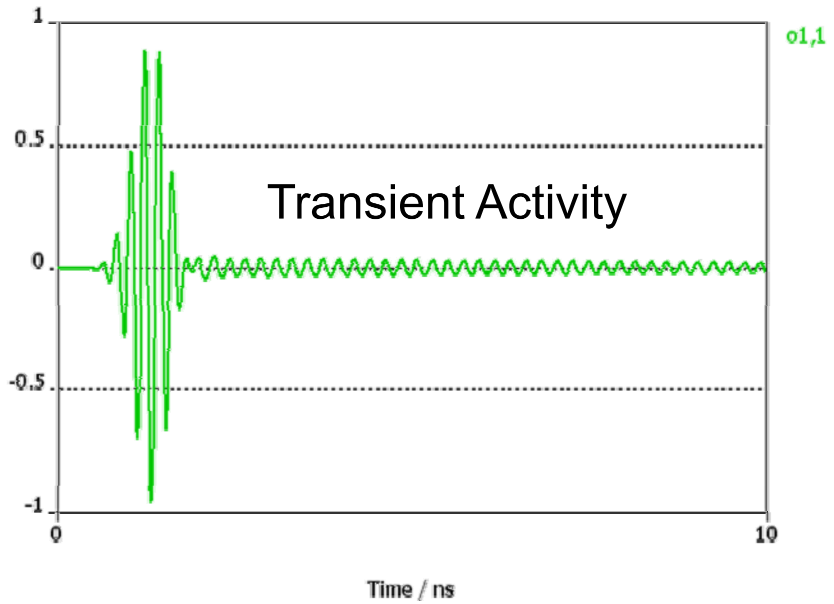
Other methods for loaded Q calculation

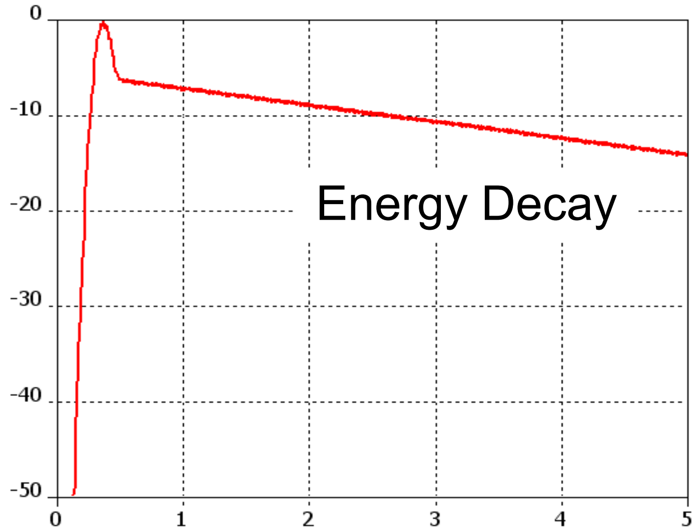
$$Q_{load} = \pi \cdot \Delta t \cdot f_0$$

Signal: 503.7 at t1=200 ns
Signal: 503.7/e = at t2= 423.6ns
Δt = t2 –t1 = 223.6ns
f_0 = 2.4615 GHz
Q_{load} = 1729

Time Domain Simulation and Resonant Structures

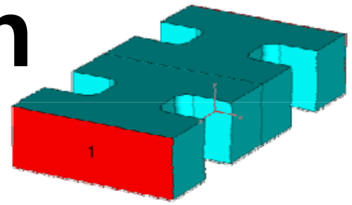
Slow energy decay since energy is kept in the resonance
- Long Simulation Time
- Prediction of signal by Autoregressive Filter
- Usage of Frequency Domain Solver

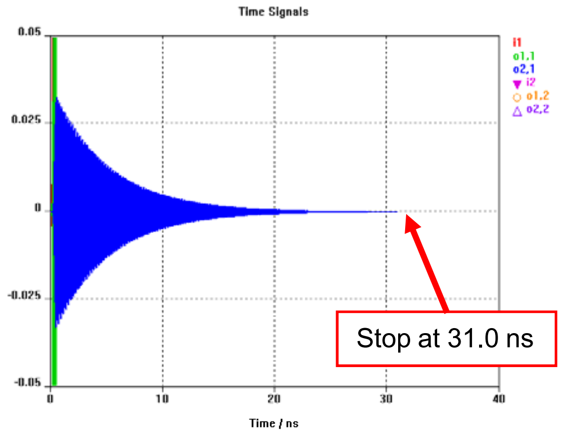




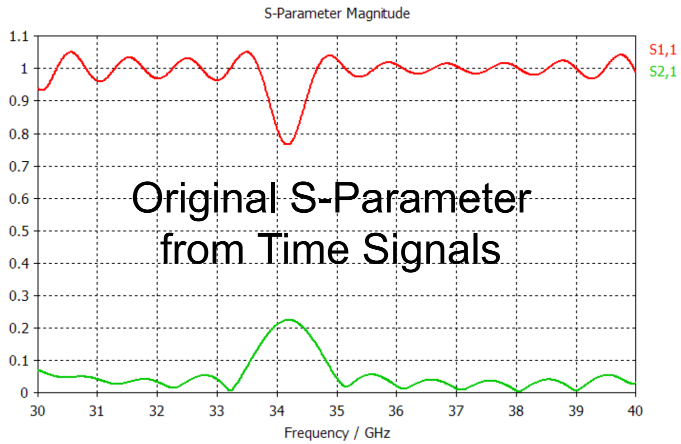
Time Domain Simulation

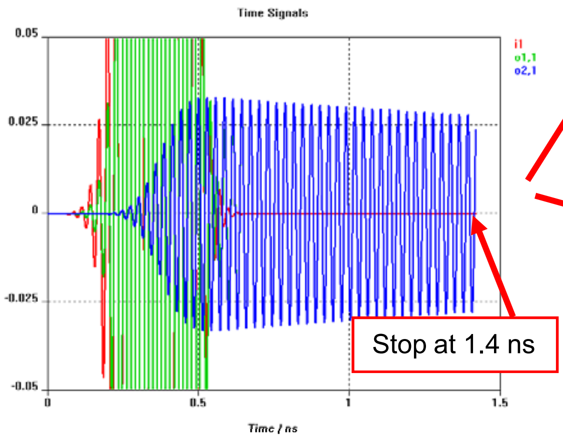
AR Filter

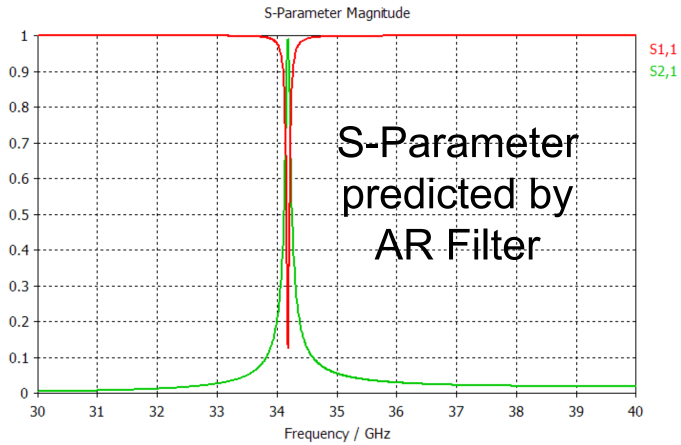




Without online-AR cpu=100 sec



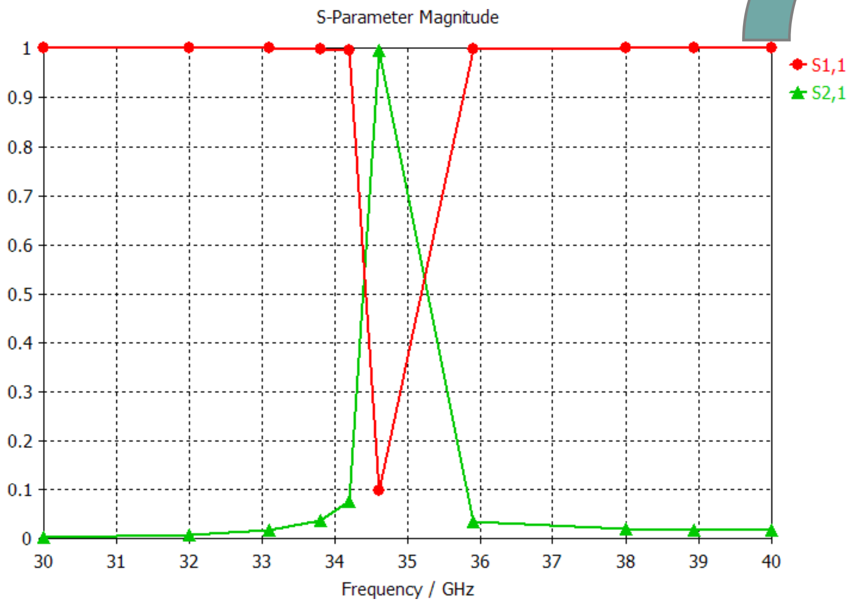


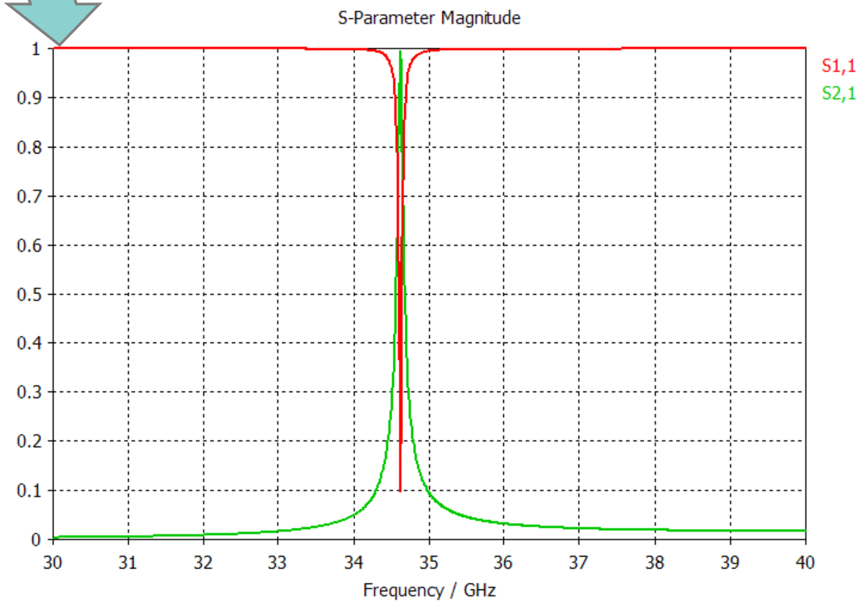


With online-AR-Filter cpu=15 sec (1port)

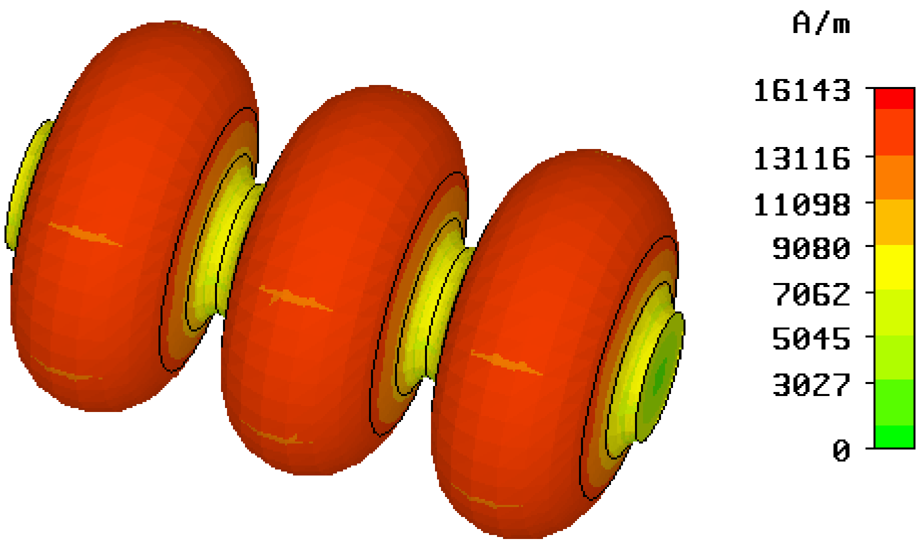
Frequency Domain Solver

- Simulation performed at single frequencies
- Simulation of Steady State
- Broadband Frequency Sweep

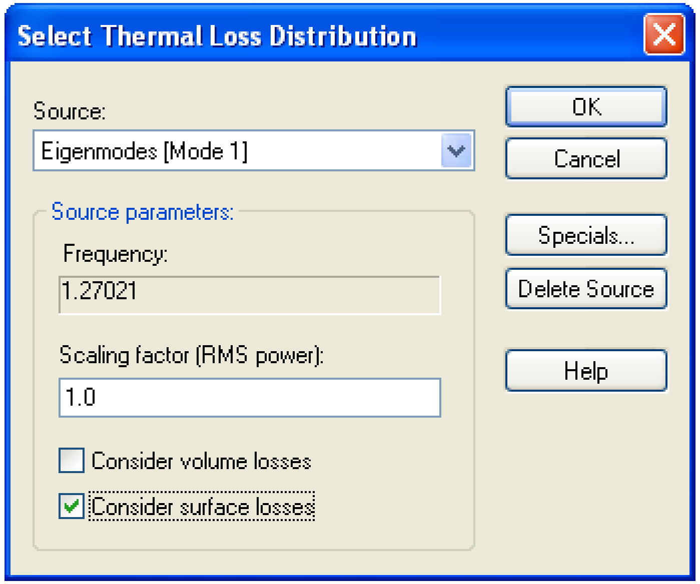




Thermal Calculation



Surface and volume losses from previous eigenmode simulation can be used to perform a thermal analysis



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Tracking Algorithm

Workflow:

1. Calculate electro- and magnetostatic fields
2. Calculate force on charged particles
3. Move particles according to the previously calculated force ⇨ Trajectory

$$\frac{d}{dt}\left(m\vec{v}\right) = q(\vec{E} + \vec{v} \times \vec{B})$$

Velocity update

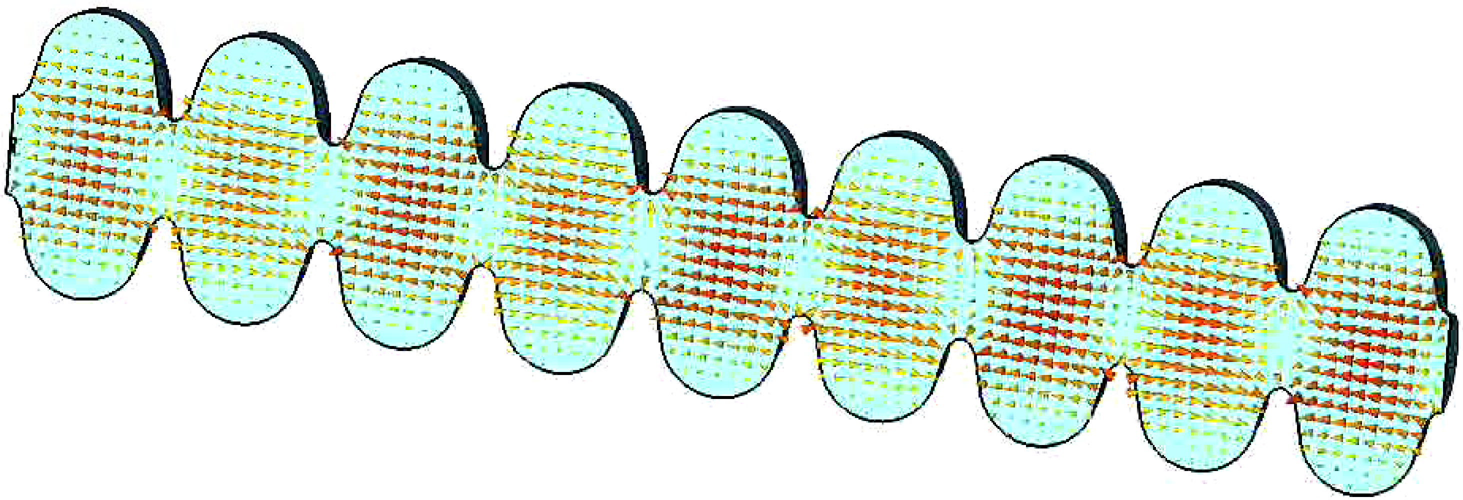
$$m^{n+1}v^{n+1} = m^{n}v^{n} + q\Delta t\left(\vec{E}^{n+1/2} + \vec{v}^{n+1} \times \vec{B}^{n+1/2}\right)$$

$$\frac{dr}{dt} = v$$

Position update

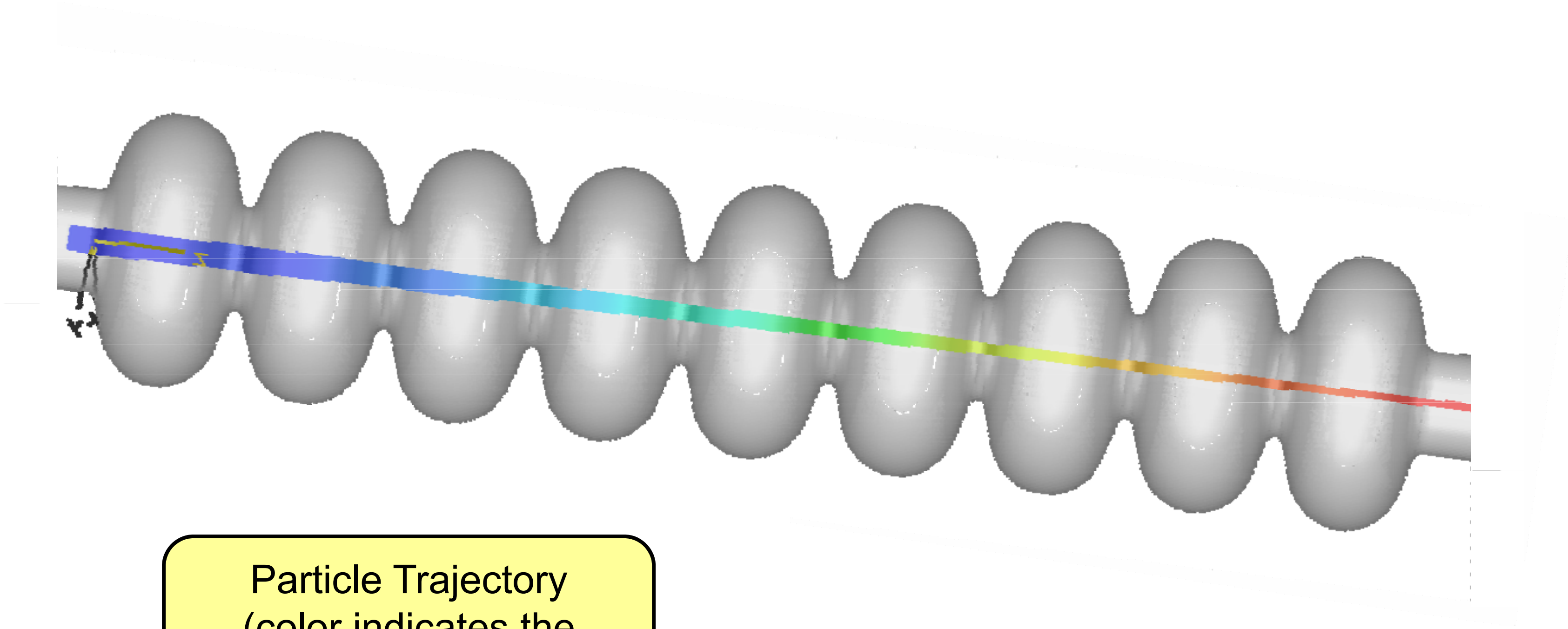
$$\vec{r}^{\,n+3/2} = \vec{r}^{\,n+1/2} + \Delta t\vec{v}^{\,n+1}$$

Tesla-Type 9-Cell Cavity



Electric Field calculated by MWS-E

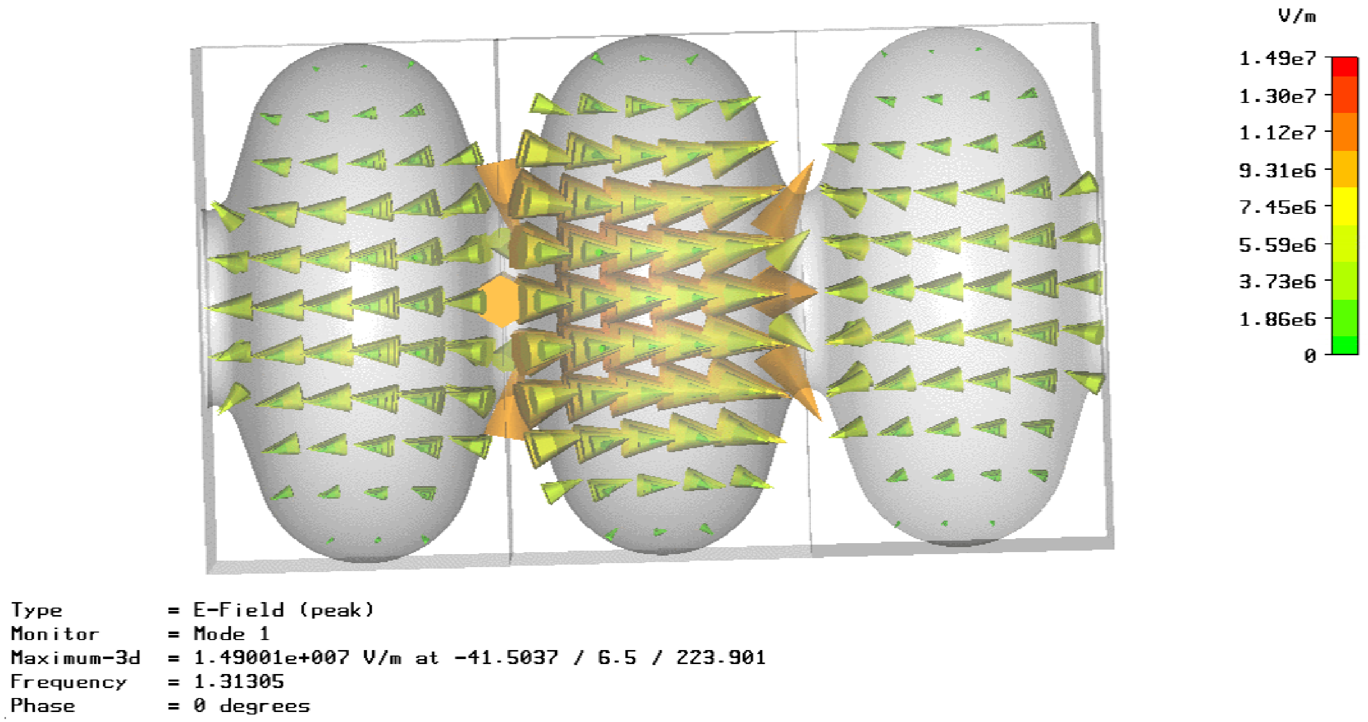
Tesla-Type 9-Cell Cavity



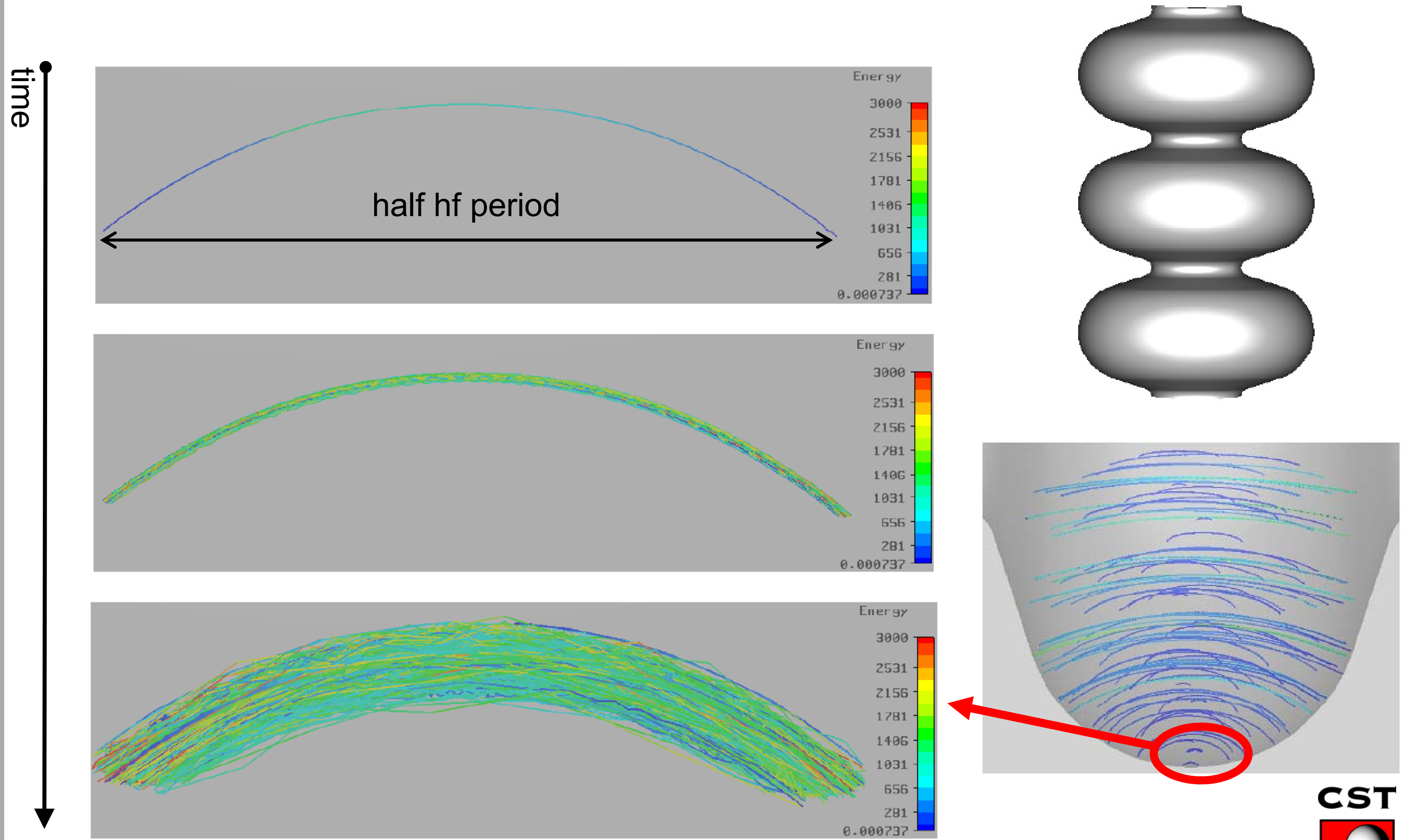
Particle Trajectory
(color indicates the
energy of the particles)

TESLA – Two-Point-Multipacting

- Eigenmode at 1.3 GHz
- E_{max} = 45 MV/m



TESLA – Two-Point-Multipacting



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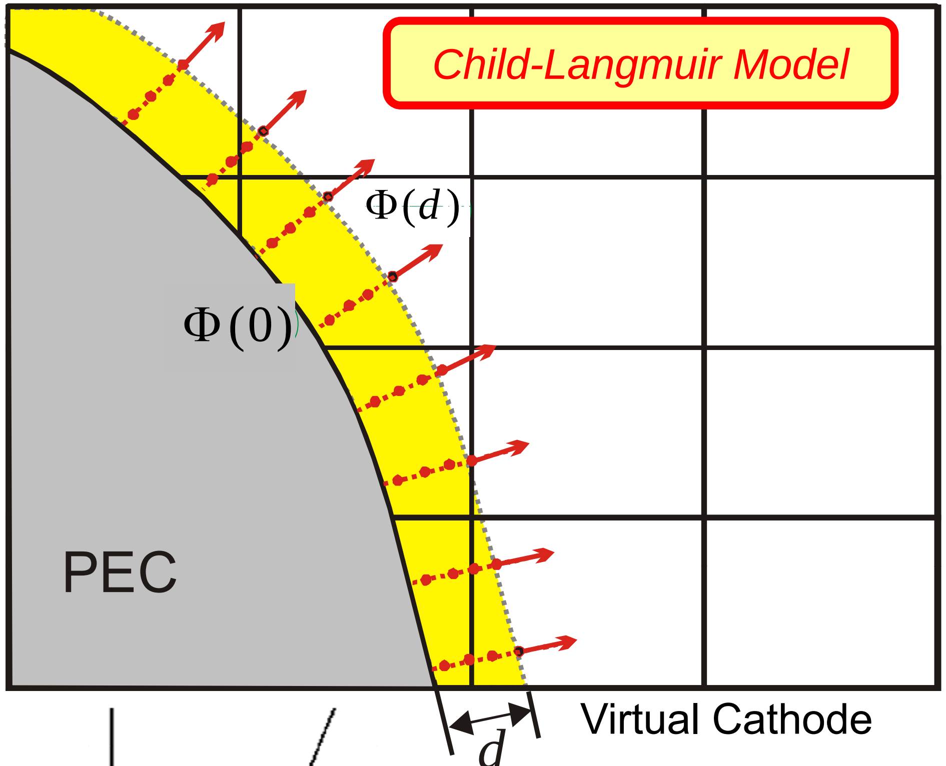
Emission Models

Space Charge Limited Emission:

Assumption:

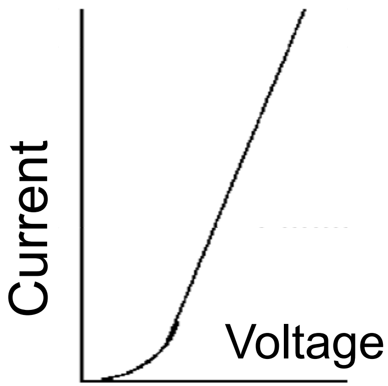
•Unlimited number of particles

•Particle extraction depends on field close to emitting surface



Childs Law:

$$J_s = \frac{4}{9}\varepsilon\sqrt{2\frac{q}{m}}\frac{\left(\Phi(d)-\Phi(0)\right)^{3/2}}{d^2}$$



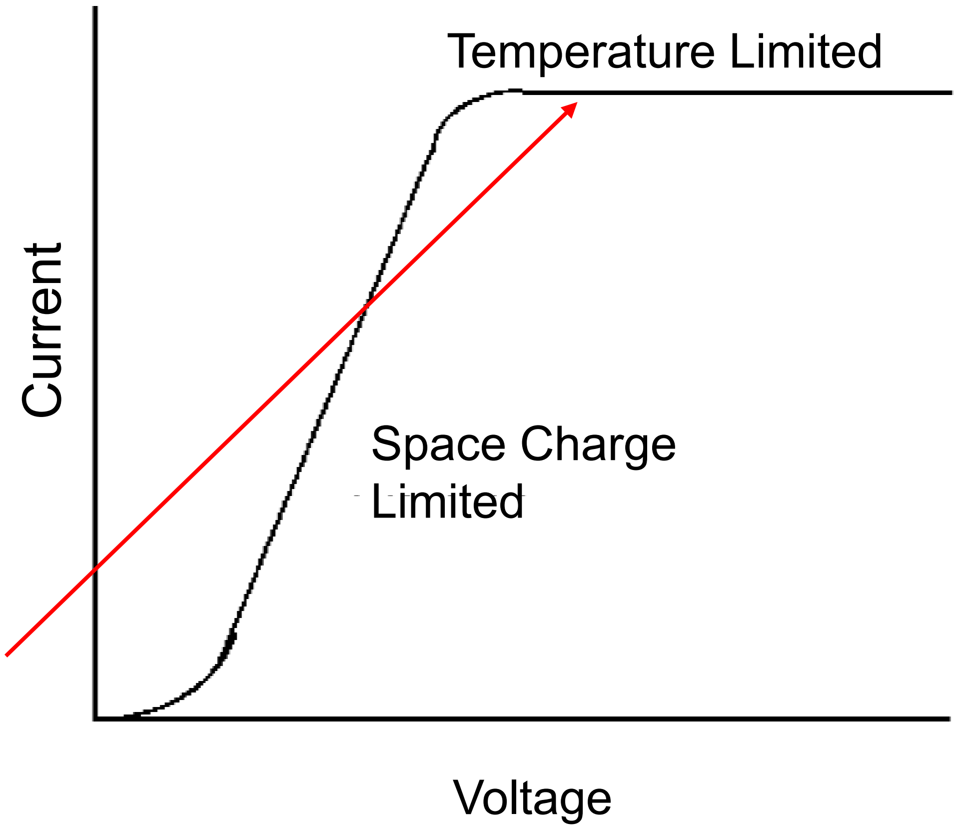
Emission Models

Thermionic Emission:

Assumption:

•Limited number of particles

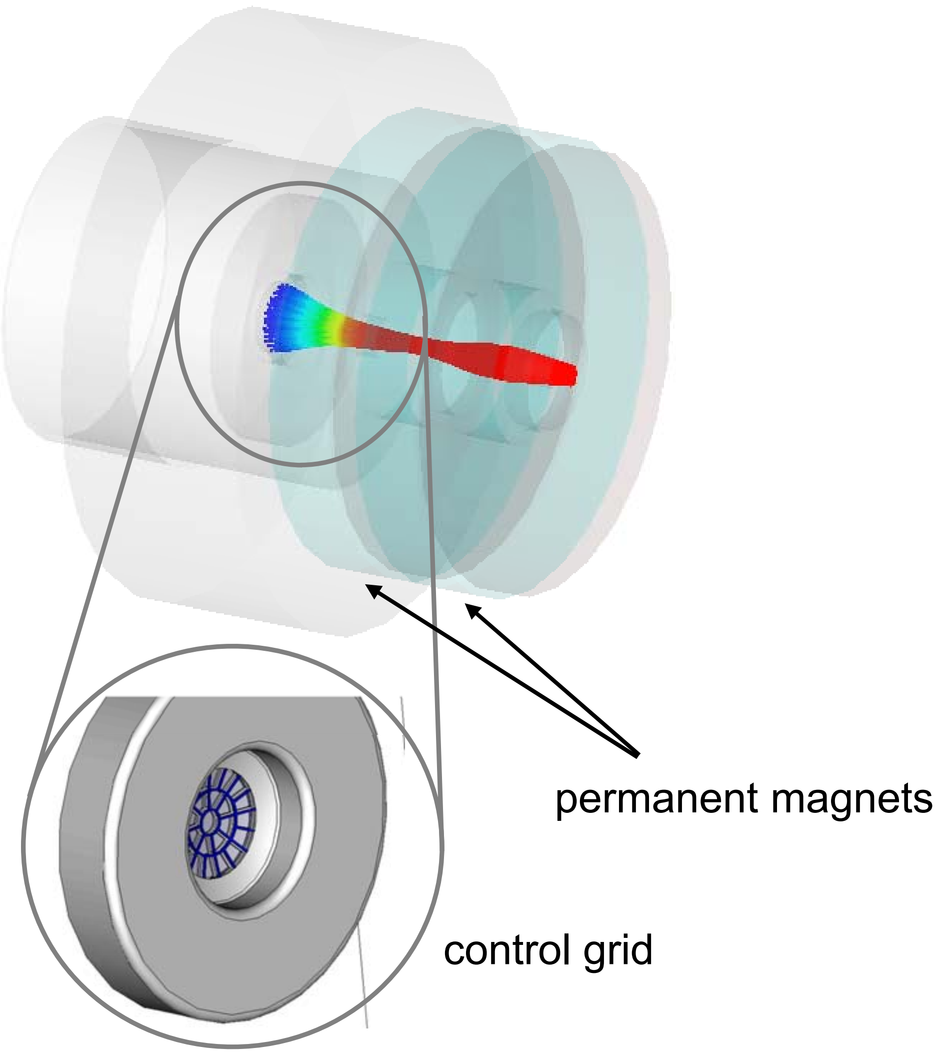
•Particle extraction depends on field close to emitting surface until all particles are emitted

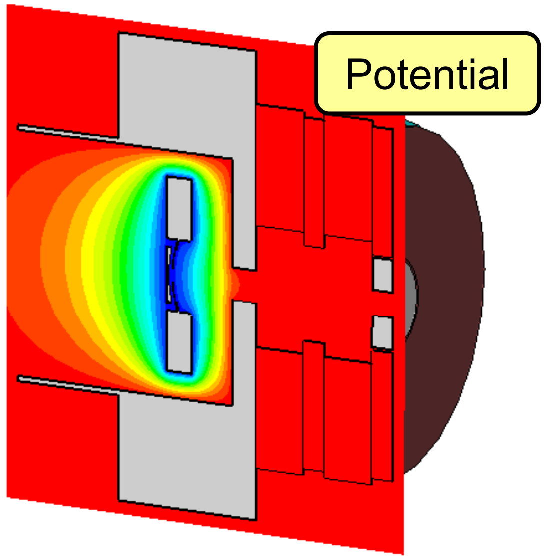


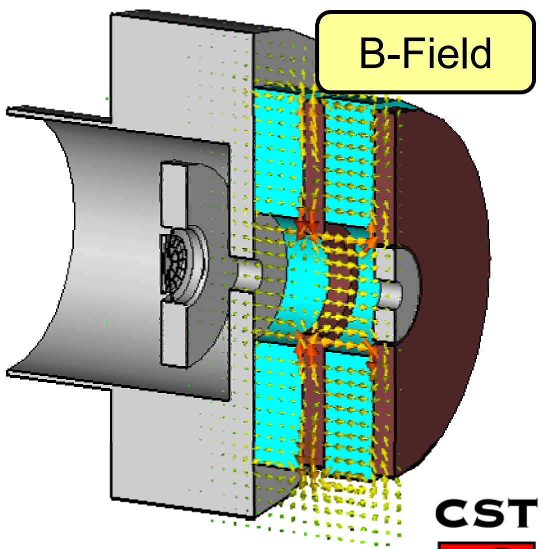
Richardson-Dushman Equation:

$$J_s = AT^2 e^{-\frac{e\Phi}{kT}}$$

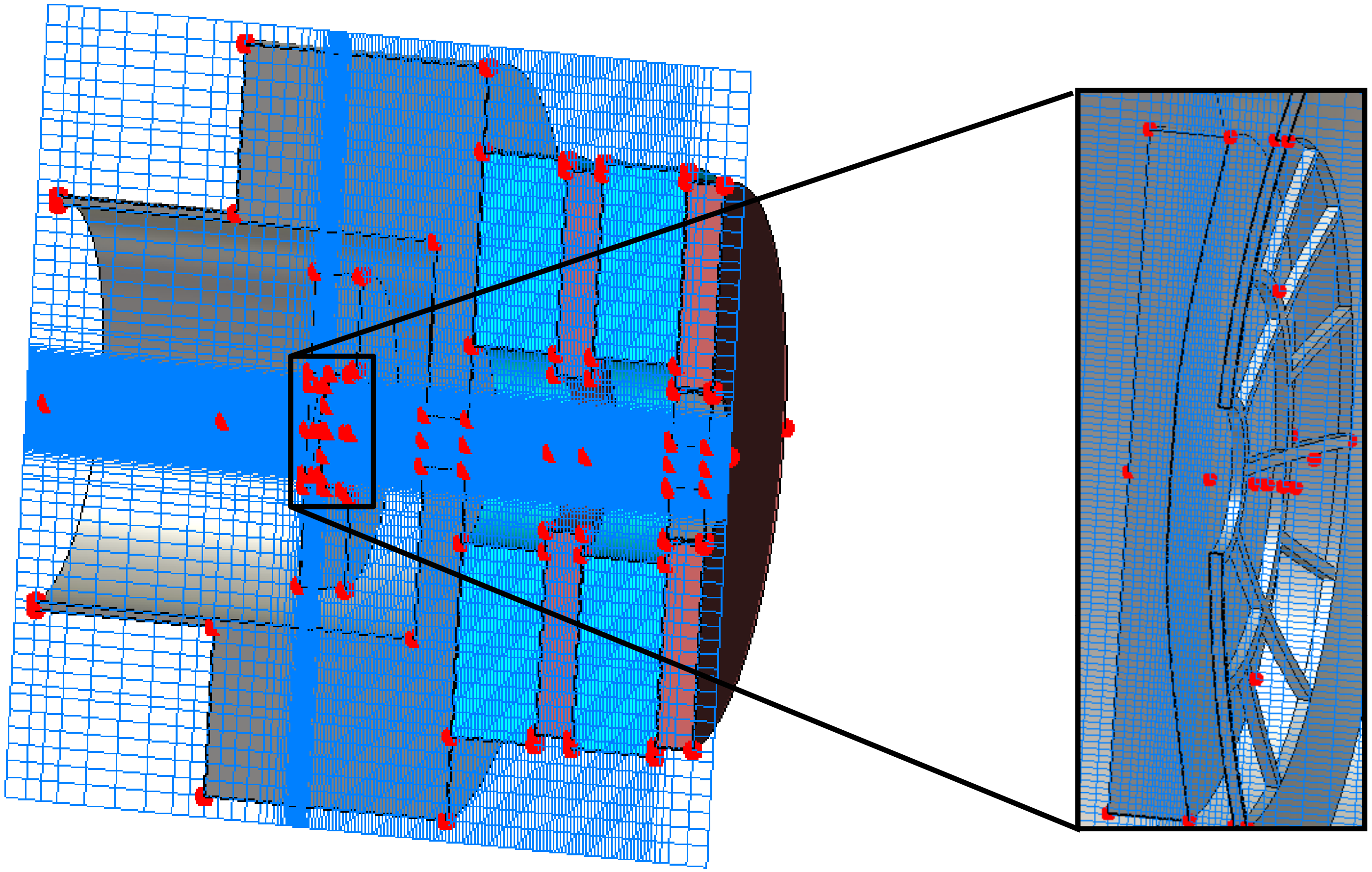
Gridded Gun



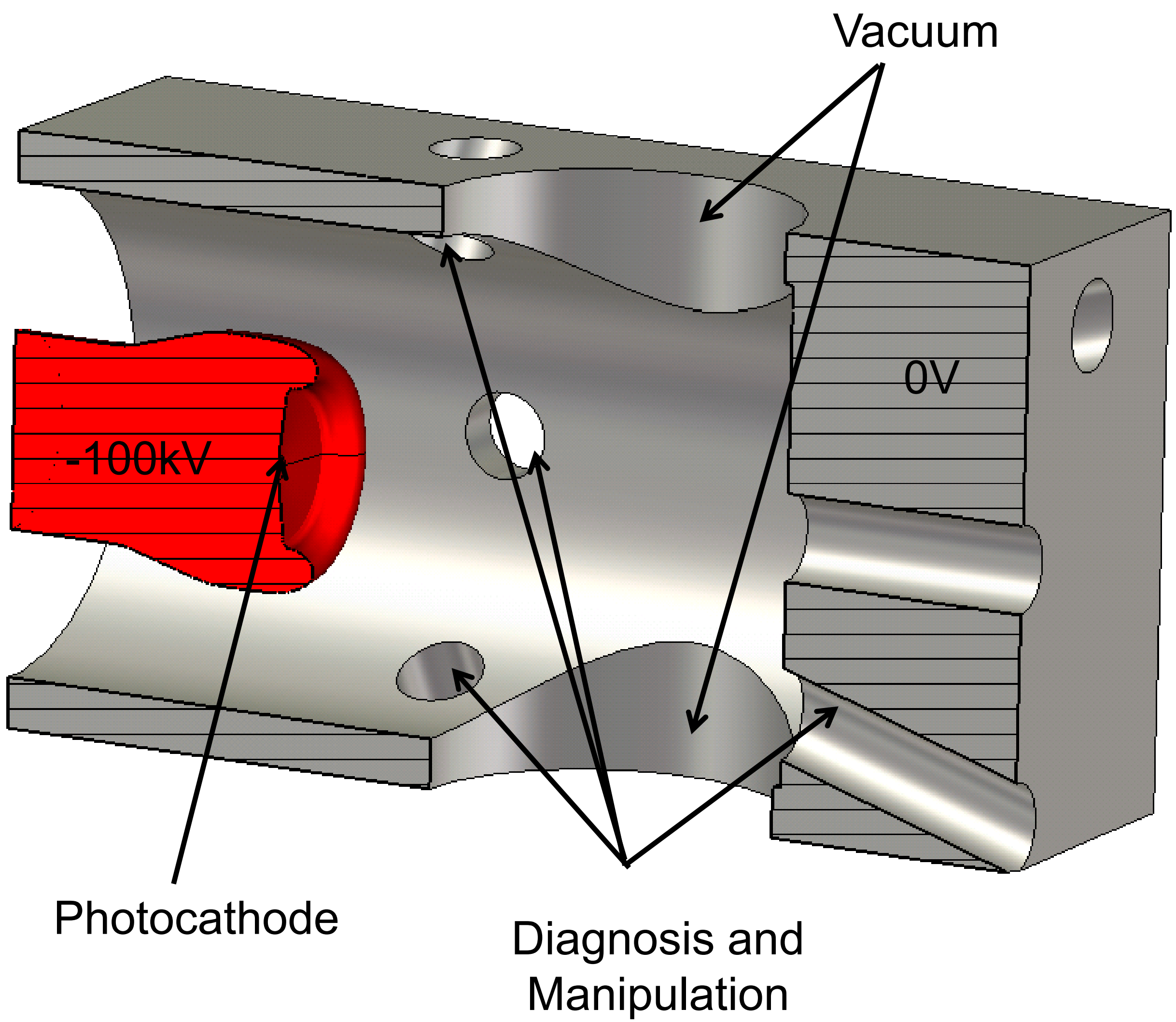




Gridded Gun - Mesh



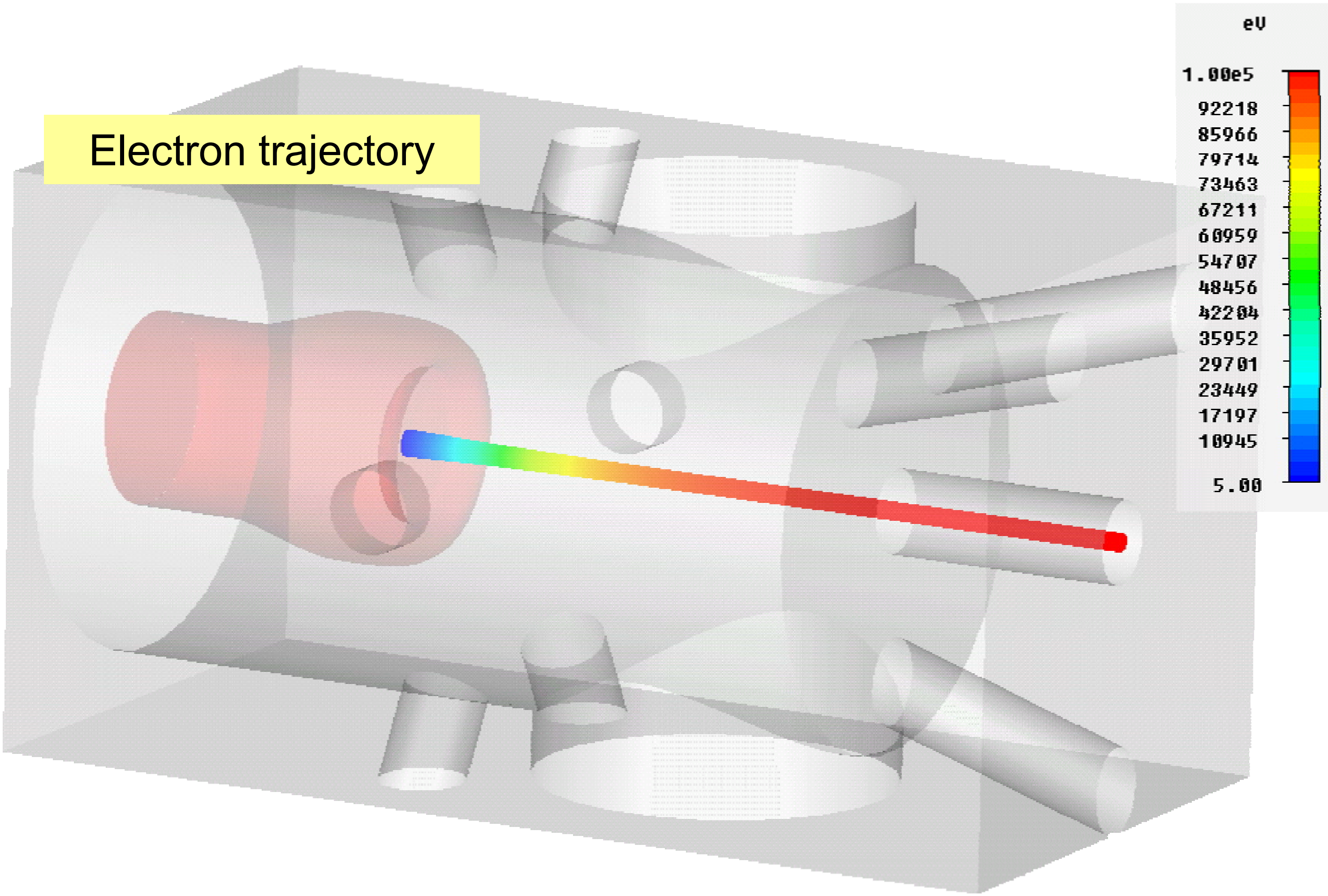
S-DALINAC Electron Source



 See also Bastian Steiner, PhD Thesis, TEMF, TU Darmstadt

CST

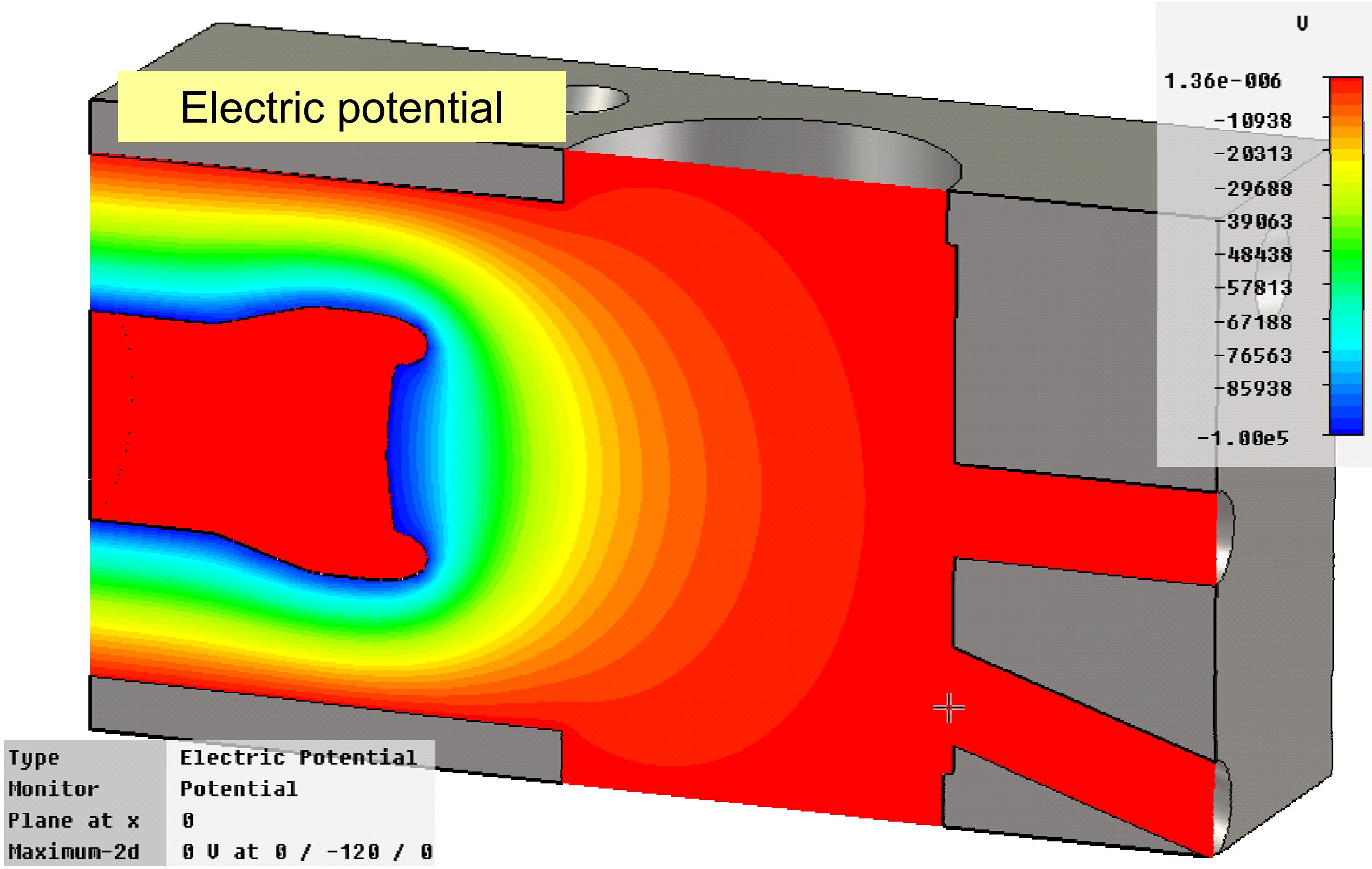
S-DALINAC Electron Source



 See also Bastian Steiner, PhD Thesis, TEMF, TU Darmstadt

CST

S-DALINAC Electron Source

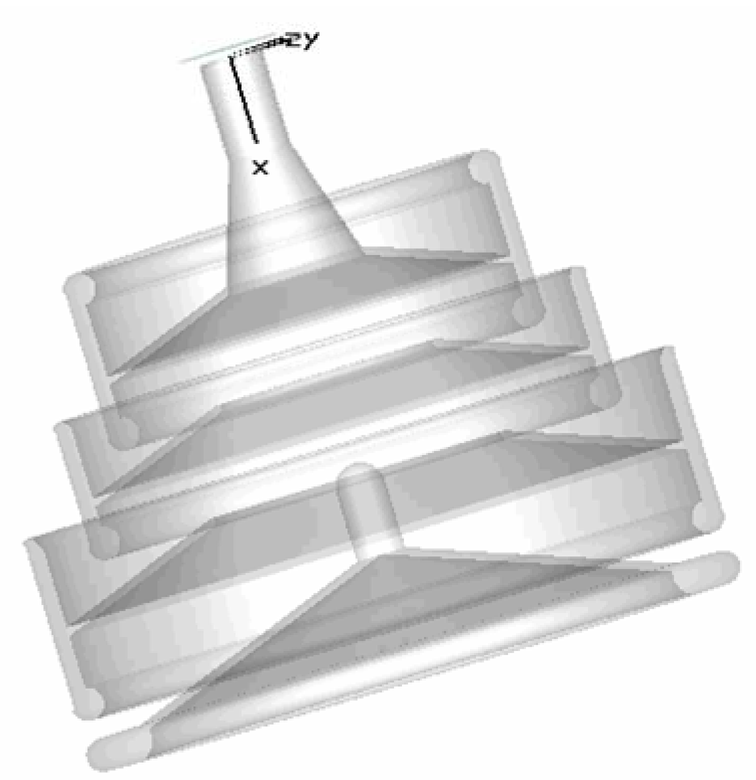


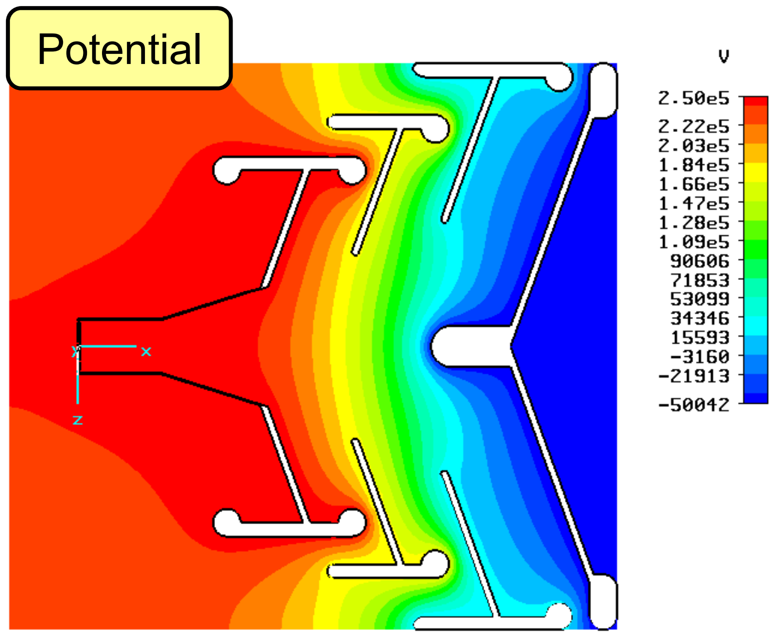
 See also Bastian Steiner, PhD Thesis, TEMF, TU Darmstadt

Overview

- Cavities
  - Eigenmodes
  - Q Factor
  - Time Domain and Resonances
- Cavities and Particles
- Electron Guns
- Collector
- Wakefields

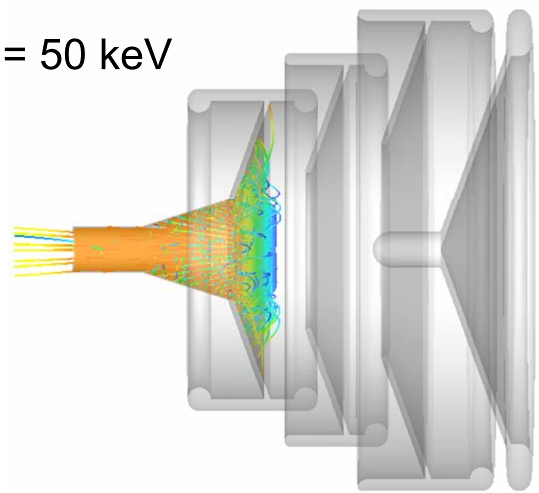
Collector



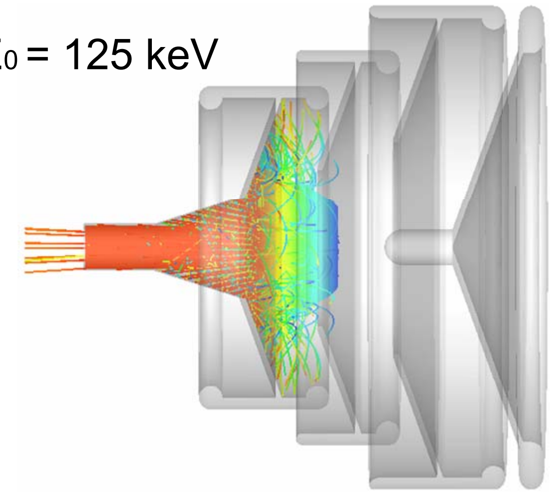

Potential
V
2.50e5
2.22e5
2.03e5
1.84e5
1.66e5
1.47e5
1.28e5
1.09e5
90606
71853
53099
34346
15593
-3160
-21913
-50042

Collector, including secondary electron emission

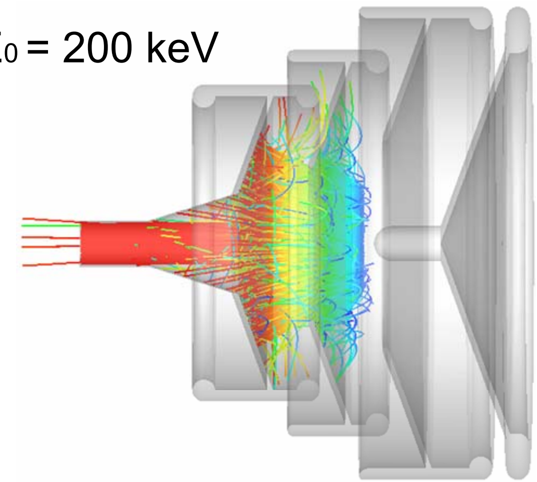
E_0 = 50 keV



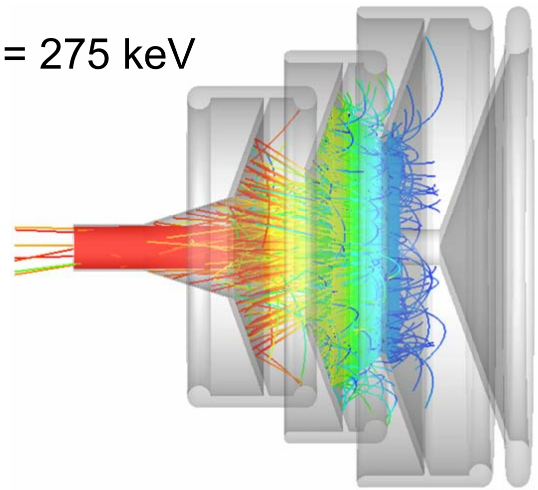
E_0 = 125 keV



E_0 = 200 keV



E_0 = 275 keV



E_0 = initial energy of the electrons

Overview

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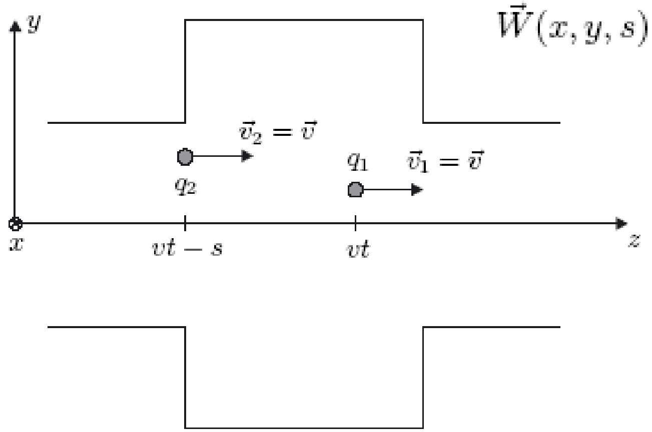
Wakefields

What does the Wakefield Solver do?

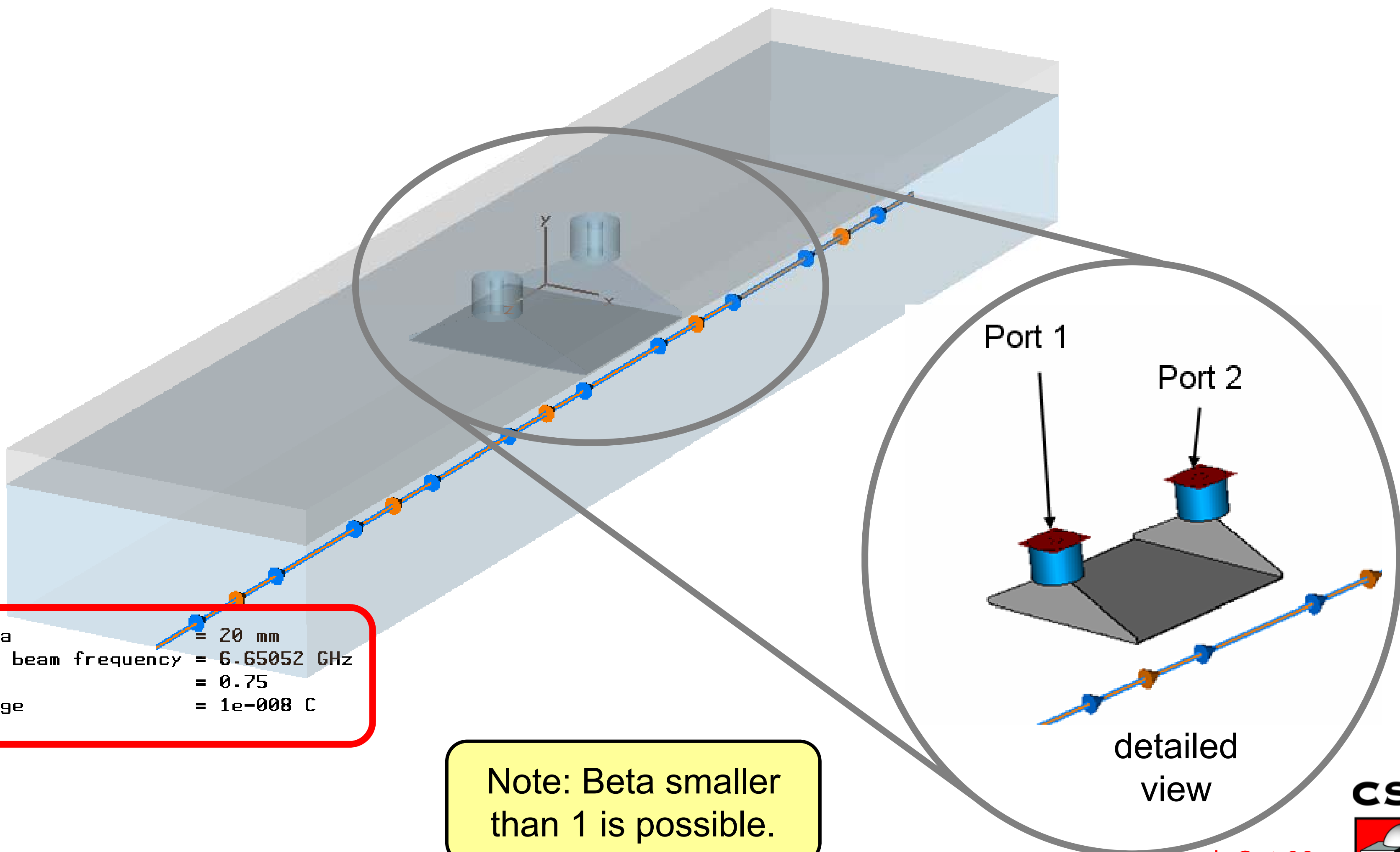
- simple explanation: Special current excitation of CST MWS T-Solver.
- more complex explanation:
  - Moving charged particles are represented as Gaussian current density
  - At structure discontinuities the intrinsic electromagnetic fields of the moving charged particles causes the appearance of „Wakefields“
  - These Wakefields can act back on the particles which is expressed in terms of a Wakepotential



$$\vec{W}(x,y,s) = \frac{1}{q_1}\int_{-\infty}^{\infty}\left(\vec{E}(x,y,z,t=\frac{s+z}{v}) + \vec{v}\times\vec{B}(x,y,z,t=\frac{s+z}{v})\right)\mathrm{d}z$$

Wakefields

Beam Position Monitor

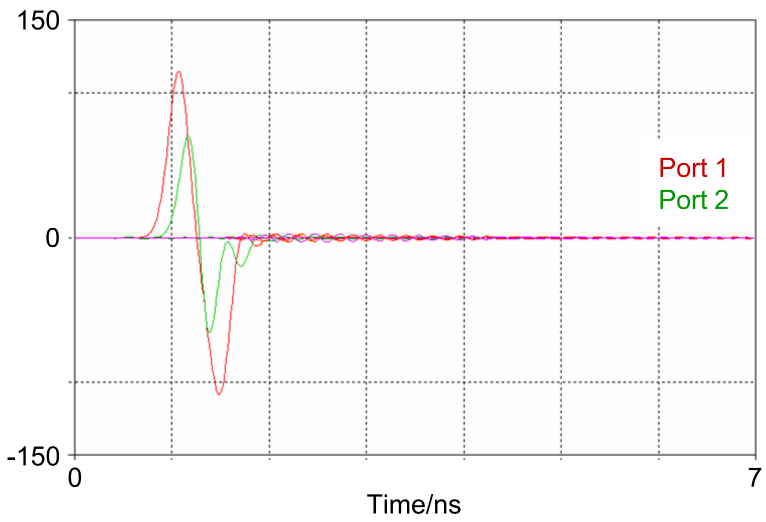



```
Sigma                  = 20 mm
Max. beam frequency    = 6.65052 GHz
Beta                   = 0.75
Charge                 = 1e-008 C
```

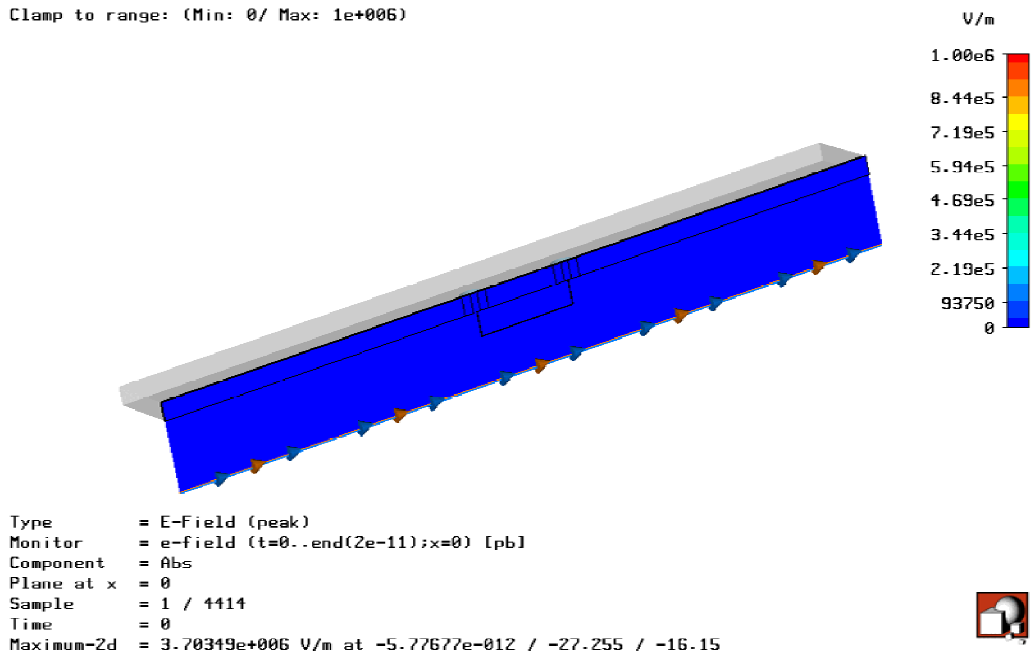
Note: Beta smaller than 1 is possible.

Wakefields

Beam Position Monitor



Output Signals at the electrode ports excited by the beam



Electric field vs. time

Wakefields

$$\underbrace{\frac{U_{PU}(f)}{\sqrt{Z_L}Q(f)}}_{o(f)_{PU}} = \underbrace{-\frac{\sqrt{Z_L}}{2}\,\frac{\int_z \vec{E}_K(0,0,z,f)\,e^{-j\frac{2\pi f}{v}z}\,dz}{U_K(f)}}_{o(f)_K}$$

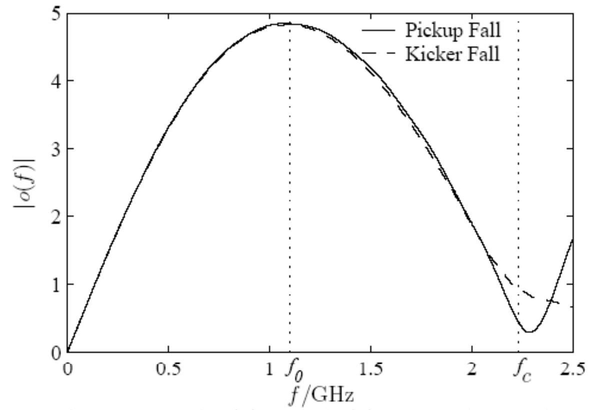
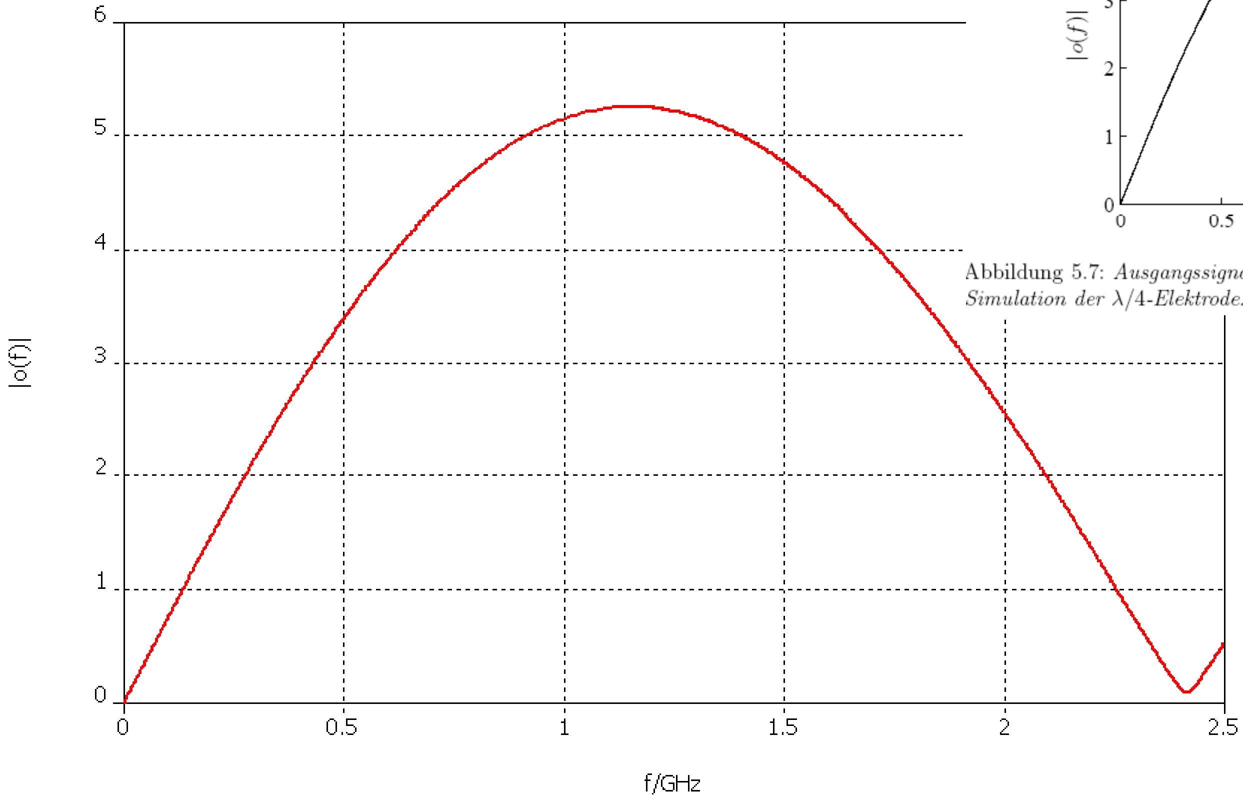


Abbildung 5.7: *Ausgangssignale $o(f)_{PU}$ und $o(f)_K$ im Pickup- und Kicker-Fall bei Simulation der $\lambda/4$-Elektrode.*

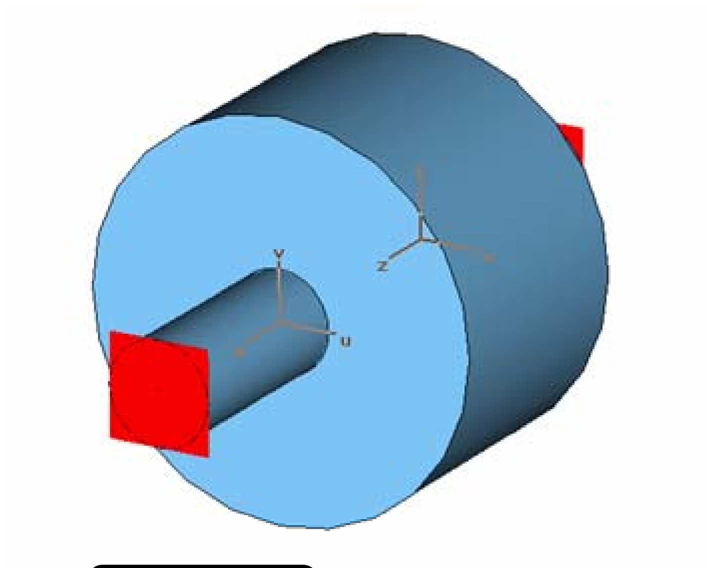


Normalized output at the electrode

See also: P. Raabe, VDI Fortschrittsberichte, Reihe 21, Nr. 128, 1993

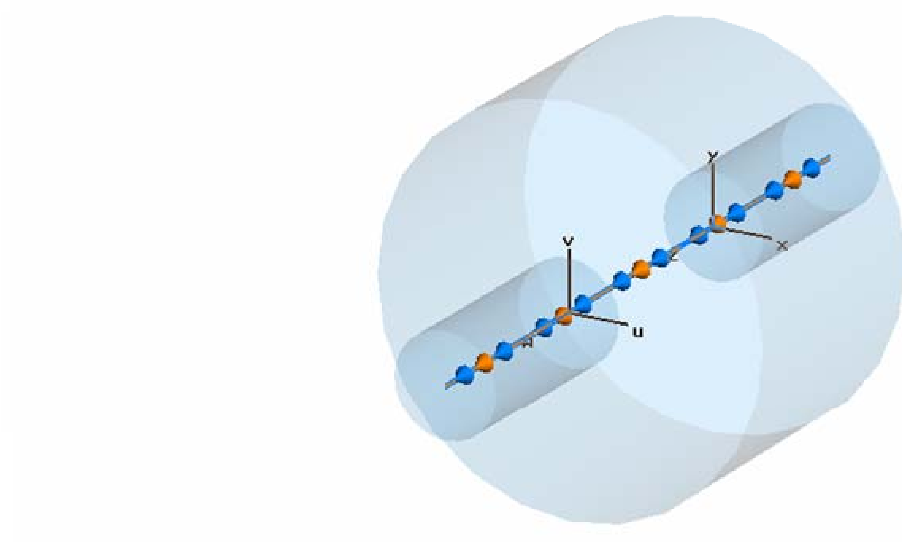
Wakefields

Pillbox Cavity



Structure

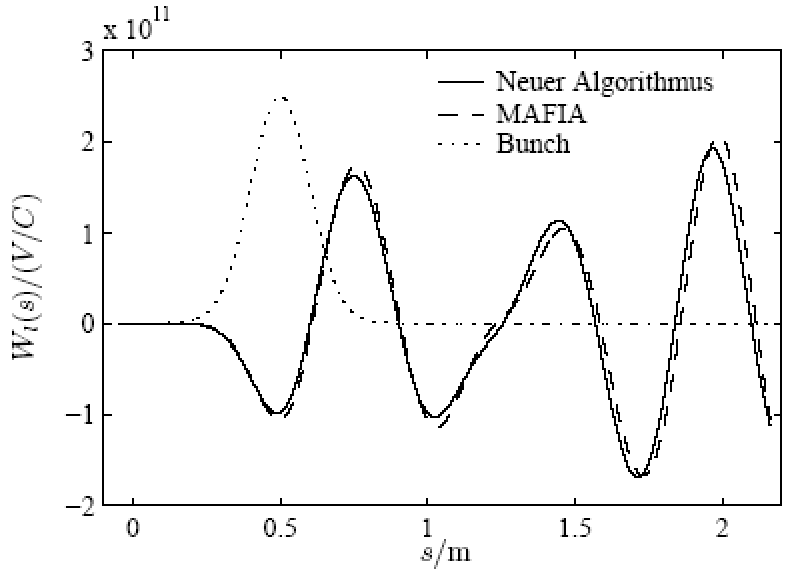
Beam definition

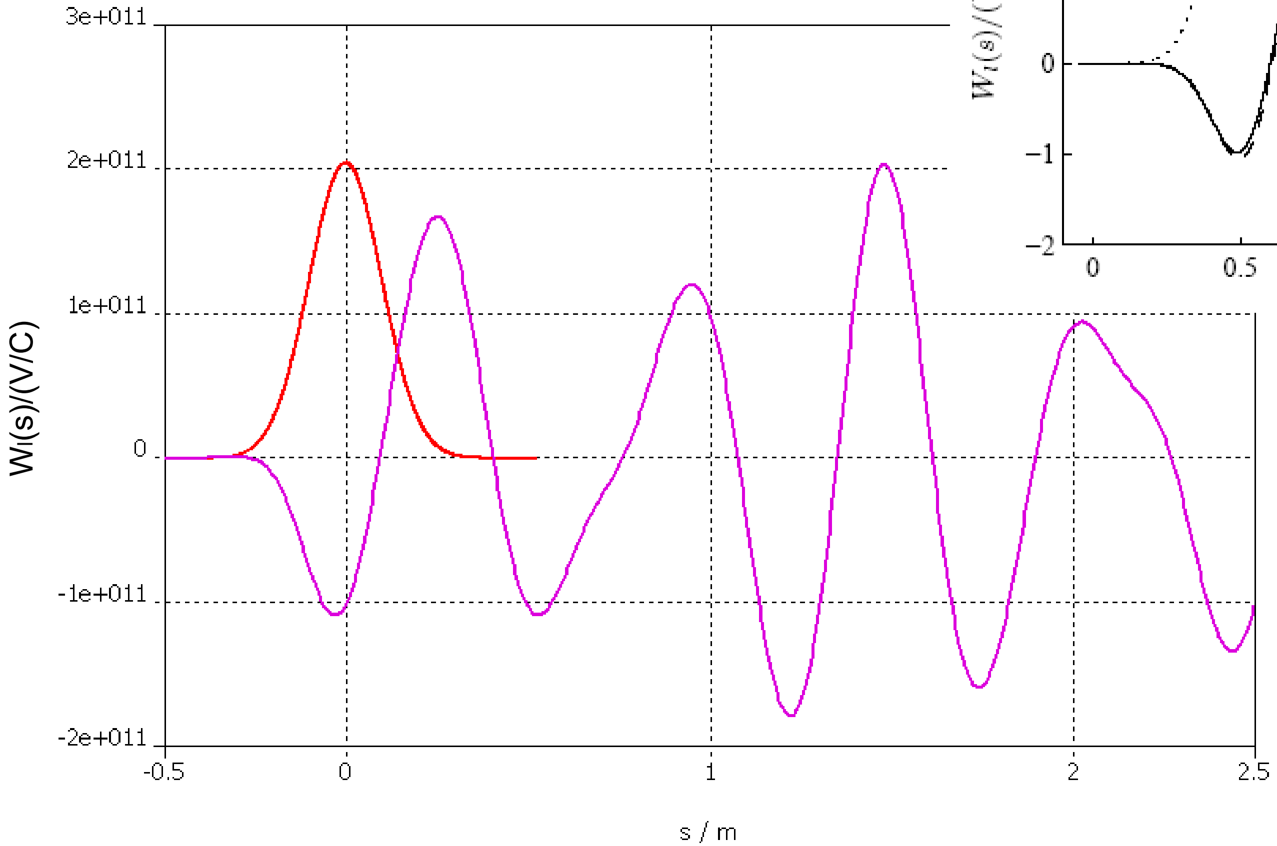


Sigma	=	10 cm
Max. beam frequency	=	1.77347 GHz
Beta	=	1
Charge	=	1e-009 C

Wakefields

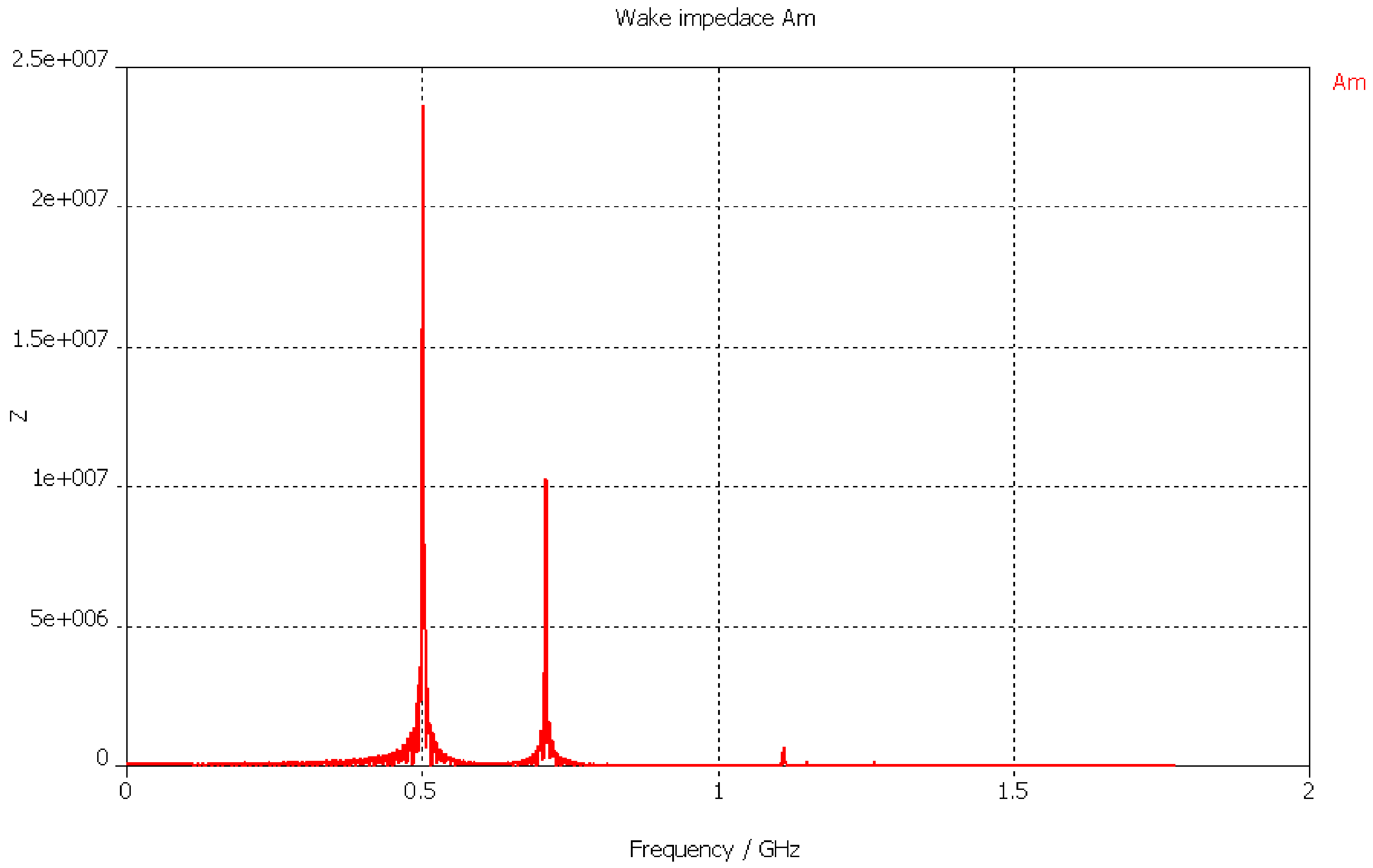
$$W_l(x,y,s) = \frac{1}{q_1}\int_{-\infty}^{\infty} E_z\left(x,y,z,t=\frac{s+z}{v}\right)\mathrm{d}z$$





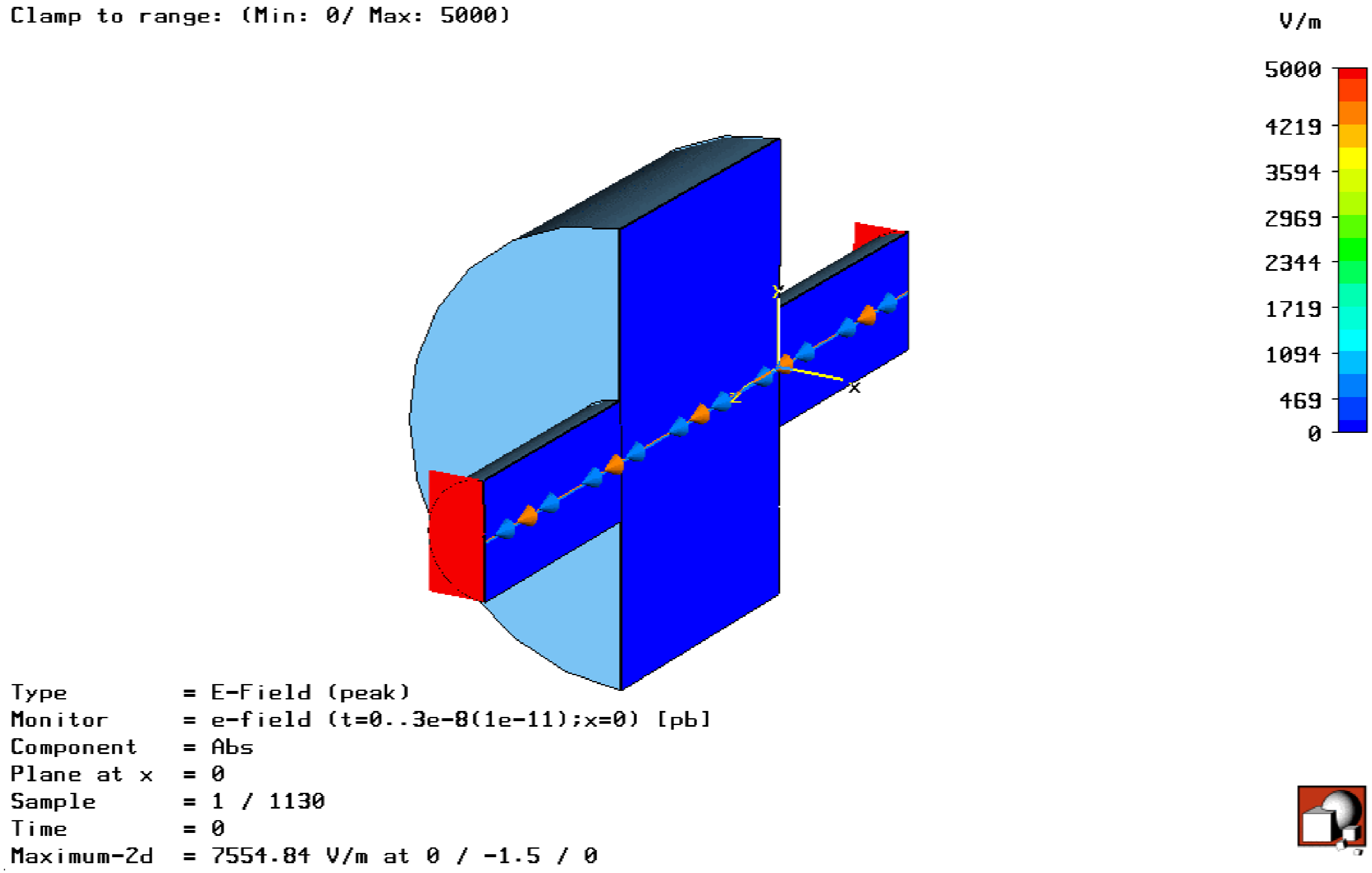
Reference Pulse
Wl(s)

Wakefields



Wake impedace Am
2.5e+007
2e+007
1.5e+007
1e+007
5e+006
0
Z
Am
0
0.5
1
1.5
2
Frequency / GHz

Wakefields


Clamp to range: (Min: 0/ Max: 5000)
V/m
5000
4219
3594
2969
2344
1719
1094
469
0
Type = E-Field (peak)
Monitor = e-field (t=0..3e-8(1e-11);x=0) [pb]
Component = Abs
Plane at x = 0
Sample = 1 / 1130
Time = 0
Maximum-2d = 7554.84 V/m at 0 / -1.5 / 0

SUMMARY

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