Beam-beam effects

(an introduction)

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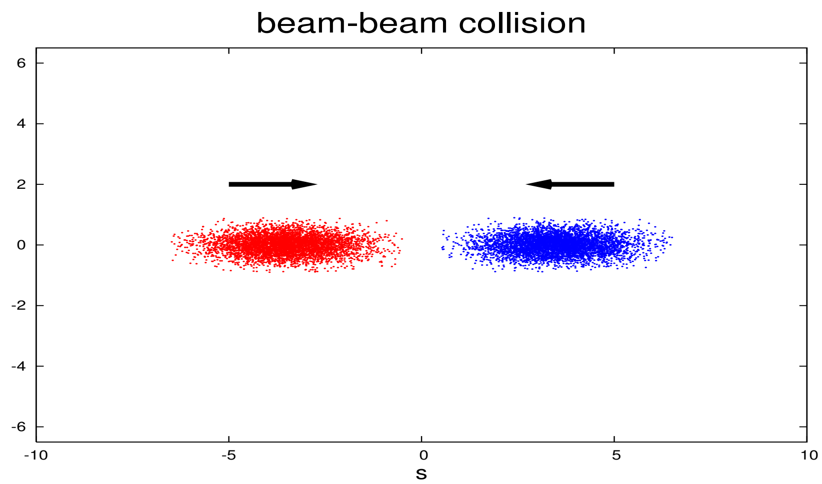
http://cern.ch/Werner.Herr/CAS2009/lectures/Darmstadt_beambeam.pdf

http://cern.ch/Werner.Herr/CAS2009/proceedings/bb_proc.pdf

What are beam-beam effects ?

- **They occur when two beams collide**
- **Two types of beam-beam effects:**
  - **High energy collisions between two particles (wanted)**
  - **Distortions of beams by electromagnetic forces (unwanted)**
- **Unfortunately: usually both go together ...**

Beam-beam collision



Typically:

- **0.001% (or less) of particles collide**
- **99.999% (or more) of particles are distorted**

Beam-beam effects

- **In circular colliders: interactions happen (at least) once per turn !**
  - **Many different effects and problems**
  - **Try to understand some of them**
- **In linear collider: VERY different problems**
- **Two main questions:**
  - **What happens to a single particle ?**
  - **What happens to the whole beam?**

BEAMS: moving charges

- **Beam is a collection of charges**
- **Represent electromagnetic potential for other charges**
  - **Forces on itself (space charge) and opposing beam (beam-beam effects)**
  - **Main limit in past, present and future colliders**
  - **Important for high density beams, i.e. high intensity and/or small beams: for high luminosity !**

Beam-beam effects

Remember:

$$\mathcal{L} = \frac{N_1 N_2 f n_B}{4\pi\sigma_x\sigma_y} \quad = \quad \frac{N_1 N_2 f n_B}{4\pi \cdot \ \sigma_x\sigma_y}$$

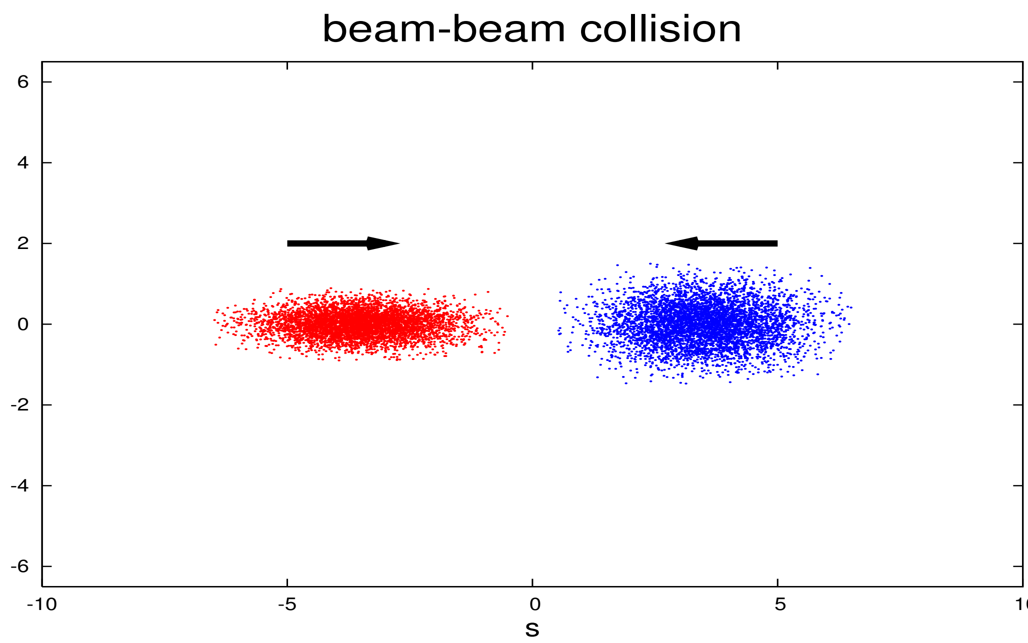
- **Overview: which effects are important for present and future machines (LEP, PEP, Tevatron, RHIC, LHC, ...)**
- **Qualitative and physical picture of the effects**
- **Mathematical derivations in:**

http://cern.ch/Werner.Herr/CAS2009/proceedings/bb_proc.pdf

Beam-beam effects

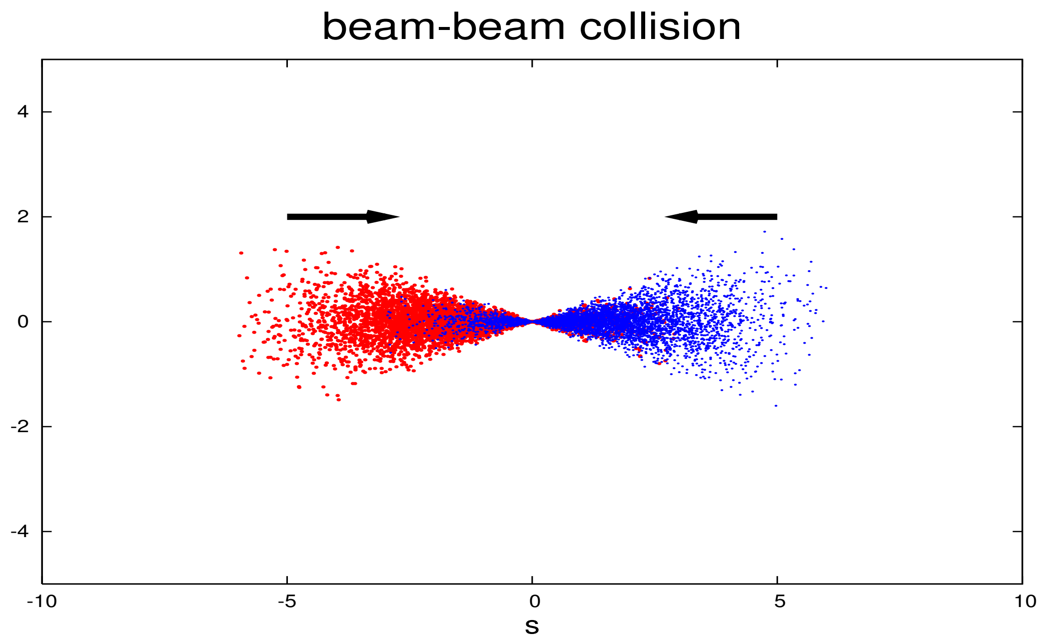
- **A beam acts on particles like an electromagnetic lens, but:**
  - **Does not represent simple form, i.e. well defined multipoles**
  - **Very non-linear form of the forces, depending on distribution**
  - **Can change distribution as result of interaction (time dependent forces ..)**

Beam-beam collision



- **Two beams can have different parameters (I, σ ..)**
- **Very detrimental effects on the beams**

Beam-beam collision



- They can change as a result of the beam-beam interaction
- Very detrimental effects on the beams

Studying beam-beam effects

- **Need knowledge of the forces**
- **Apply concepts of non-linear dynamics**
- **Apply concepts of multi-particle dynamics**
- **Analytical models and simulation techniques well developed in last 10 years**

Fields and Forces (I)

- **Need fields $\vec{E}$ and $\vec{B}$ of opposing beam with a particle distribution $\rho(x, y, z)$**
- **In rest frame only electrostatic field: $\vec{E}'$, $\vec{B}' \equiv 0$**
- **Derive potential $U(x, y, z)$ from Poisson equation:**

$$\Delta U(x, y, z) = -\frac{1}{\epsilon_0}\rho(x, y, z)$$

- **The electrostatic fields become:**

$$\vec{E}' = -\nabla U(x, y, z)$$

Fields and Forces (II)

- **Transform into moving frame and calculate Lorentz force $\vec{F}$ on particle with charge $q = Z_2$ e**

$$E_{\parallel} = E'_{\parallel}, \quad E_{\perp} = \gamma \cdot E'_{\perp} \quad \text{with :} \quad \vec{B} \;=\; \vec{\beta} \times \vec{E}/c$$

$$\vec{F} = q(\vec{E} + \vec{\beta} \times \vec{B})$$

- **Example Gaussian distribution:**

$$\rho(x,y,z) = \frac{N Z_1 e}{\sigma_x \sigma_y \sigma_z \sqrt{2\pi}^3} \exp\left(-\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2} - \frac{z^2}{2\sigma_z^2}\right)$$

Simple example: Gaussian

- **For 2D case the potential becomes (see proceedings):**

$$U(x,y,\sigma_x,\sigma_y) = \frac{NZ_1e}{4\pi\epsilon_0}\int_0^\infty \frac{\exp(-\frac{x^2}{2\sigma_x^2+q} - \frac{y^2}{2\sigma_y^2+q})}{\sqrt{(2\sigma_x^2+q)(2\sigma_y^2+q)}}dq$$

- **Can derive $\vec{E}$ and $\vec{B}$ fields and therefore forces**
- **For arbitrary distribution (non-Gaussian): difficult (or impossible, numerical solution required)**

Simple example: round Gaussian beams

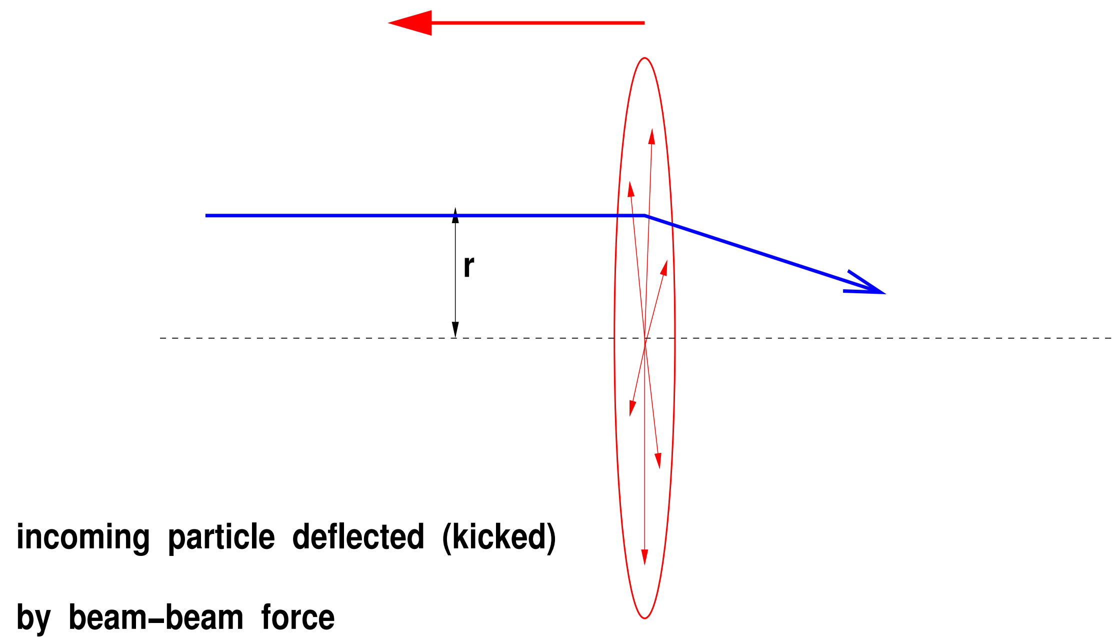
- **Assumption 1:** $\sigma_x = \sigma_y = \sigma$, $\quad Z_1 = -Z_2 = 1$
- **Assumption 2: very relativistic** ➡ $\beta \approx 1$

- **Only components** E_r **and** B_Φ **are non-zero**
- **Force has only radial component, i.e. depends only on distance r from bunch centre** *(where:* $r^2 = x^2 + y^2$*)* **(see proceedings)**

$$\boxed{F_r(r) = -\frac{Ne^2(1+\beta^2)}{2\pi\epsilon_0 \cdot r}\left[1 - \exp\left(-\frac{r^2}{2\sigma^2}\right)\right]}$$

Beam-beam kick:

- **We use (x, x', y, y') as coordinates**
- **We need the deflections (kicks $\Delta x'$, $\Delta y'$) of the particles:**



Beam-beam kick:

- **Kick ($\Delta r'$): angle by which the particle is deflected during the passage**

- **Integration of force over the collision, i.e. time of passage Δt (assuming: m$_1$=m$_2$ and Z_1=$-Z_2$= 1):**

$$\text{Newton's law}: \qquad \Delta r' = \frac{1}{mc\beta\gamma}\int_{-\frac{\Delta t}{2}}^{+\frac{\Delta t}{2}} F_r(r,s,t)dt$$

with:

$$F_r(r,s,t) = -\frac{Ne^2(1+\beta^2)}{\sqrt{(2\pi)^3}\epsilon_0 r\sigma_s}\left[1-\exp(-\frac{r^2}{2\sigma^2})\right]\cdot\left[\exp(-\frac{(s+vt)^2}{2\sigma_s^2})\right]$$

Beam-beam kick:

- **Using the classical particle radius (implies $Z_1 = \pm Z_2$):**

$$r_0 = e^2/4\pi\epsilon_0 mc^2$$

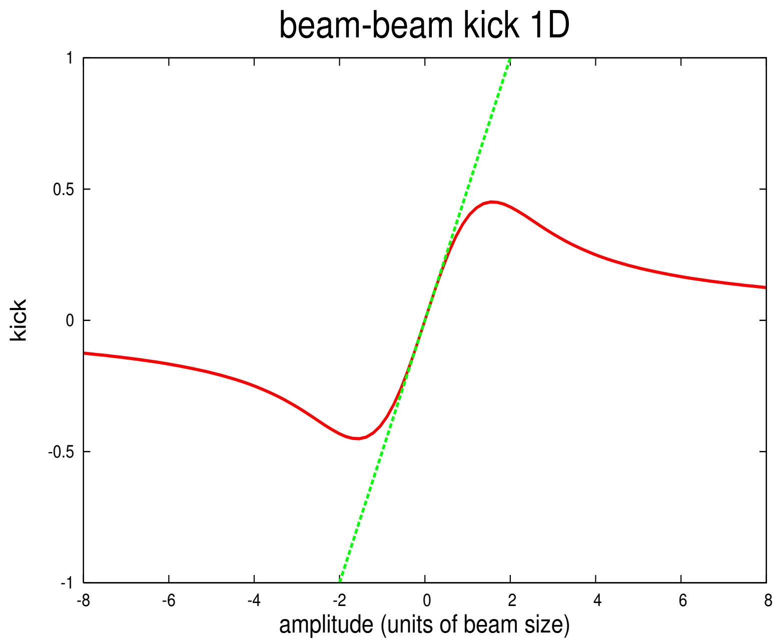
we have (radial kick and in Cartesian coordinates):

$$\Delta r' = -\frac{2Nr_0}{\gamma} \cdot \frac{r}{r^2} \cdot \left[1 - \exp(-\frac{r^2}{2\sigma^2})\right]$$

$$\Delta x' = -\frac{2Nr_0}{\gamma} \cdot \frac{x}{r^2} \cdot \left[1 - \exp(-\frac{r^2}{2\sigma^2})\right]$$

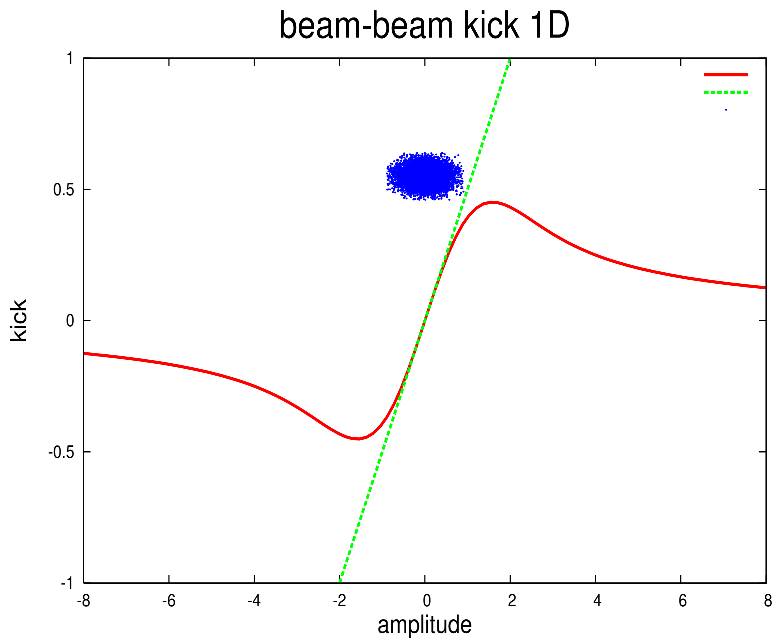
$$\Delta y' = -\frac{2Nr_0}{\gamma} \cdot \frac{y}{r^2} \cdot \left[1 - \exp(-\frac{r^2}{2\sigma^2})\right]$$

Beam-beam force/kick



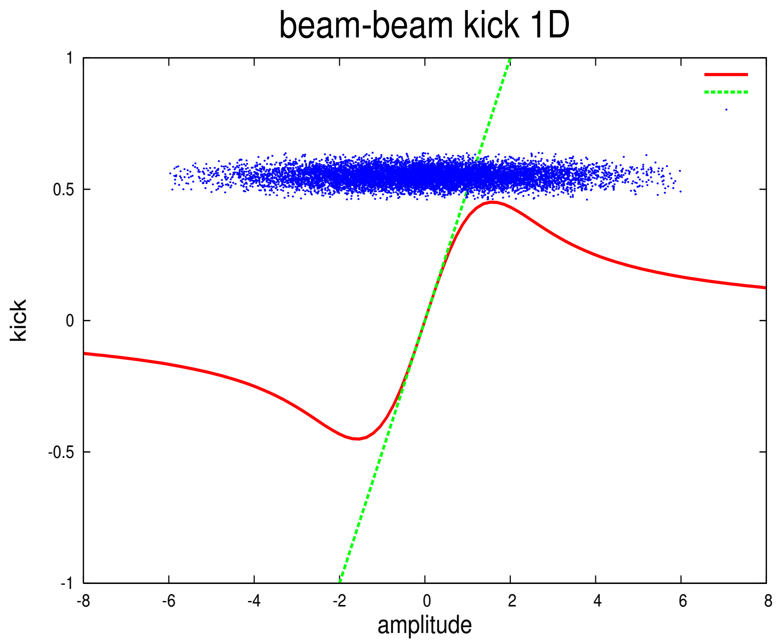
- **For small amplitude: linear force (like quadrupole)**
- **For large amplitude: very non-linear force**

Beam-beam force/kick



- **For small amplitude: tune shift**

Beam-beam force/kick



- **For small amplitude: tune shift**
- **For large amplitude: amplitude dependent tune shift**

Can we quantify the beam-beam strength ?

- **Try the slope of force (kick $\Delta r'$) at zero amplitude**
- **This defines: beam-beam parameter ξ**
- **For head-on interactions and round beams ($\beta^* = \beta^*_x = \beta^*_y$) we get:**

$$\boxed{\xi = \frac{\beta^*}{4\pi} \cdot \frac{\delta(\Delta r')}{\delta r} = \frac{N \cdot r_o \cdot \beta^*}{4\pi\gamma\sigma^2}}$$

LEP - LHC

	LEP (e^+e^-)	LHC (pp)
Beam sizes	160 - 200μm · 2 - 4μm	16.6μm · 16.6μm
Intensity N	4.0 · 10^{11}/bunch	1.15 · 10^{11}/bunch
Energy	100 GeV	7000 GeV
$\epsilon_x \cdot \epsilon_y$	($\approx$) 20 nm · 0.2 nm	0.5 nm · 0.5 nm
$\beta_x^* \cdot \beta_y^*$	($\approx$) 1.25 m · 0.05 m	0.55 m · 0.55 m
Crossing angle	0.0	285 μrad
Beam-beam parameter(ξ)	0.0700	0.0037

Can we quantify the beam-beam strength ?

- **In general for non-round beams ($\beta_x^* \neq \beta_y^*$):**

$$\boxed{\xi_{x,y} = \frac{N \cdot r_o \cdot \beta^*_{x,y}}{2\pi\gamma\sigma_{x,y}(\sigma_x + \sigma_y)}}$$

- **Proportional to (linear) tune shift $\Delta\, Q_{bb}$ from beam-beam interaction:** $\Delta\, Q_{bb} \propto \pm\, \xi$
- **Good measure for strength of beam-beam interaction**
- **BUT: does not describe**
  - **changes to optical functions**
  - **non-linear part of beam-beam force**

Linear beam-beam tune shift

- **For small amplitudes linear force like a quadrupole with focal length f**

$$\frac{1}{f} = \frac{\Delta x'}{x} = \frac{N r_0}{\gamma \sigma^2} = \left[\frac{\xi \cdot 4\pi}{\beta^*}\right]$$

- **Transformation matrix over the interaction becomes:**

$$\begin{pmatrix} 1 & 0 \\ \frac{1}{-f} & 1 \end{pmatrix}$$

Linear beam-beam tune shift

- **Full turn matrix including the tune shift ΔQ computed from unperturbed full turn matrix plus interaction**

$$\begin{pmatrix} cos(2\pi(Q+\Delta Q)) & \beta^* sin(2\pi(Q+\Delta Q)) \\ -\frac{1}{\beta^*} sin(2\pi(Q+\Delta Q)) & cos(2\pi(Q+\Delta Q)) \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ \frac{1}{-2f} & 1 \end{pmatrix} \circ \begin{pmatrix} cos(2\pi Q) & \beta_0^* sin(2\pi Q) \\ -\frac{1}{\beta_0^*} sin(2\pi Q) & cos(2\pi Q) \end{pmatrix} \circ \begin{pmatrix} 1 & 0 \\ \frac{1}{-2f} & 1 \end{pmatrix}$$

Linear beam-beam tune shift

- **Solving this equation gives us:**

$$cos(2\pi(Q+\Delta Q)) = cos(2\pi Q) - \frac{\beta_0^*}{2f} sin(2\pi Q)$$

and

$$\frac{\beta^*}{\beta_0^*} = sin(2\pi Q)/sin(2\pi(Q+\Delta Q))$$

- **Tune is changed by ΔQ**
- **β-function is changed (β-beating)**

Linear beam-beam tune shift

- **For small ξ and Q not too close to 0.0 and 0.5 we have:**

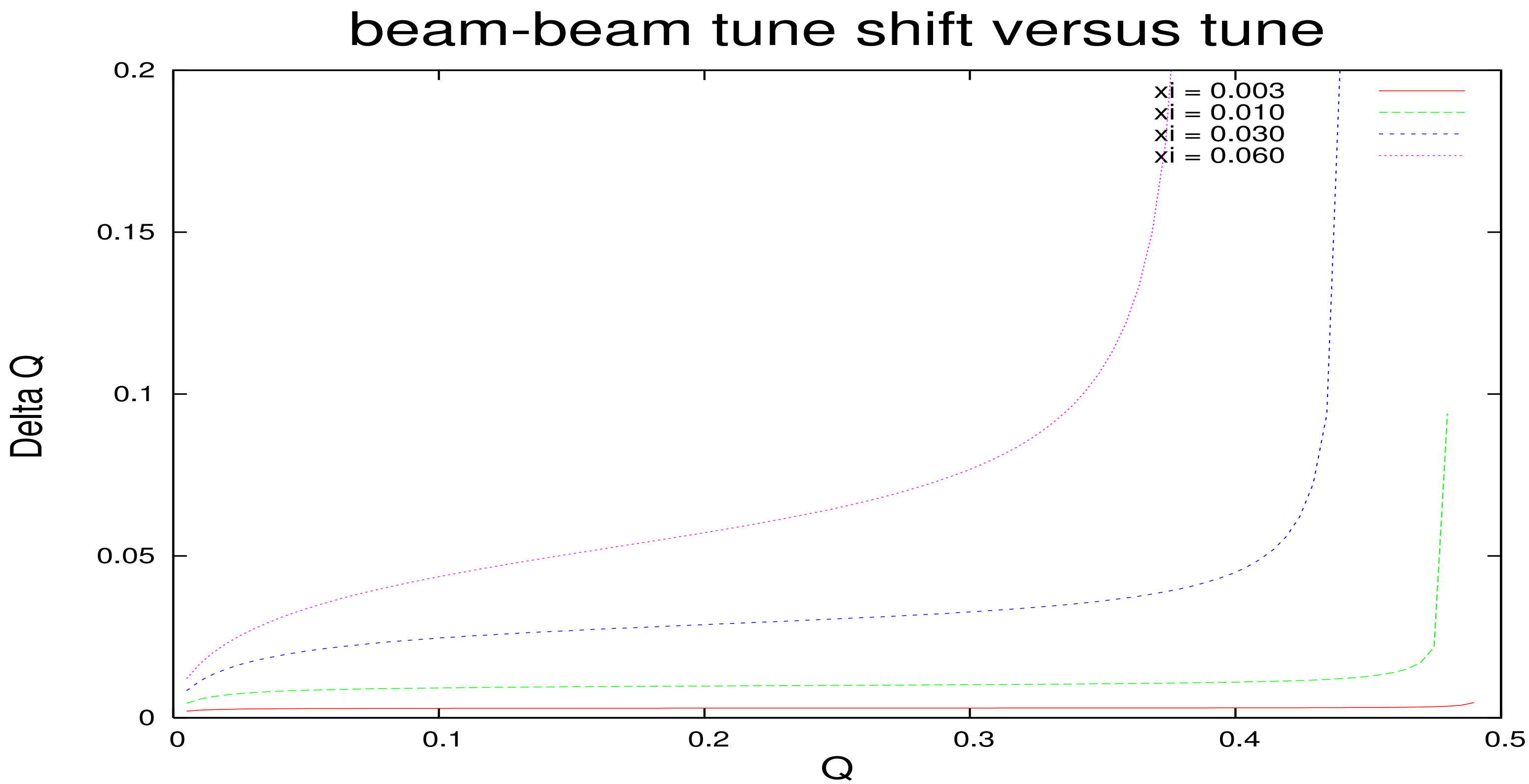
$$\Delta Q \approx \xi$$

and

$$\frac{\beta^*}{\beta_0^*} = \frac{sin(2\pi Q)}{sin(2\pi(Q + \Delta Q))} = \frac{\beta_0}{\sqrt{1 + 4\pi\xi cot(2\pi Q) - 4\pi^2\xi^2}}$$

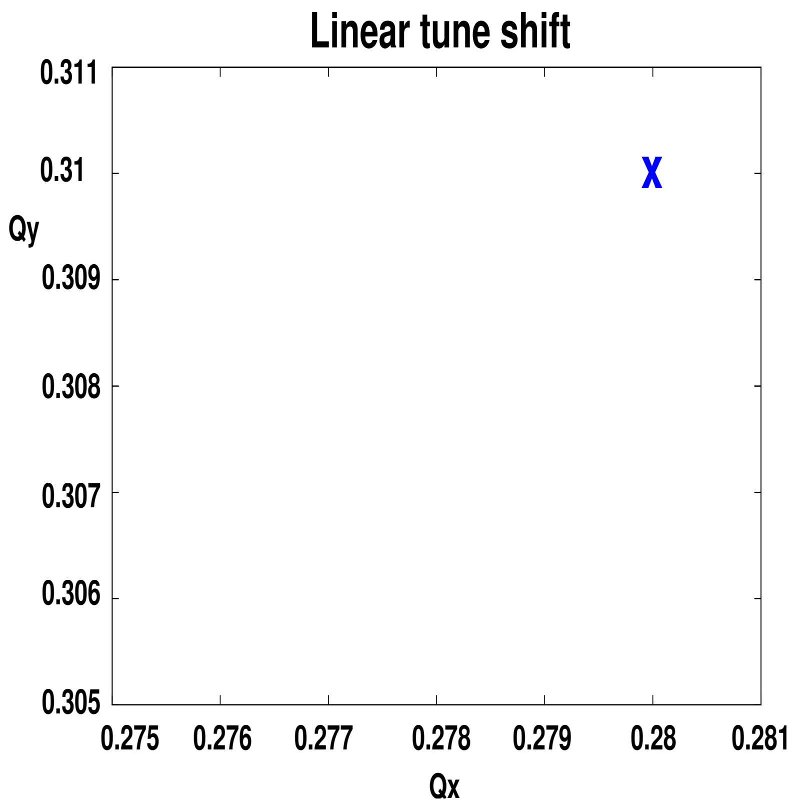
- **β can become smaller or larger at interaction point (dynamic β)**

Tune dependence of tune shift



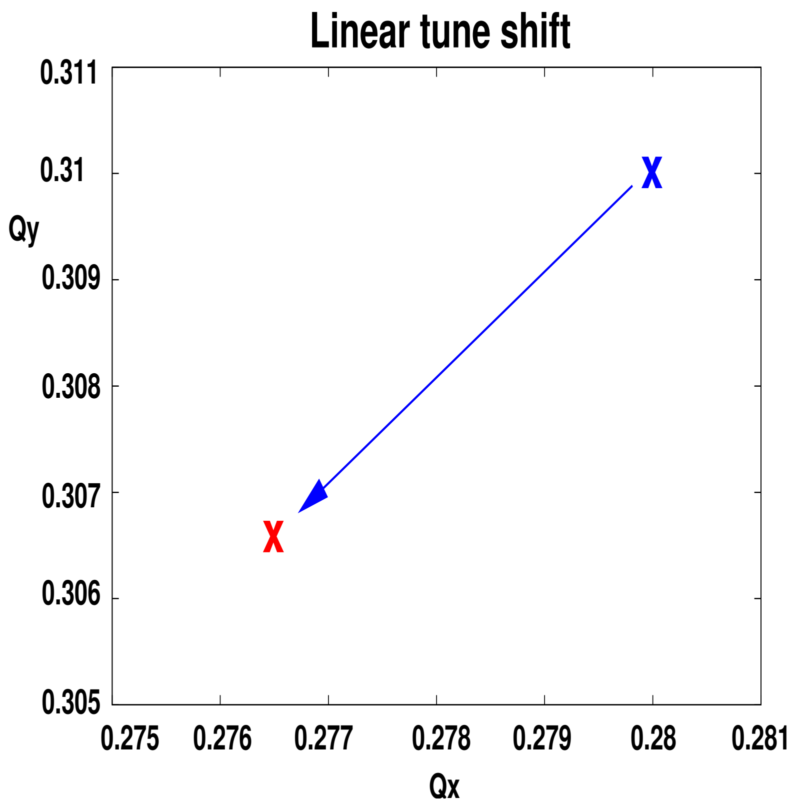
- **Strong dependence on Q for larger ξ (dynamic β)**

Linear tune shift - two dimensions



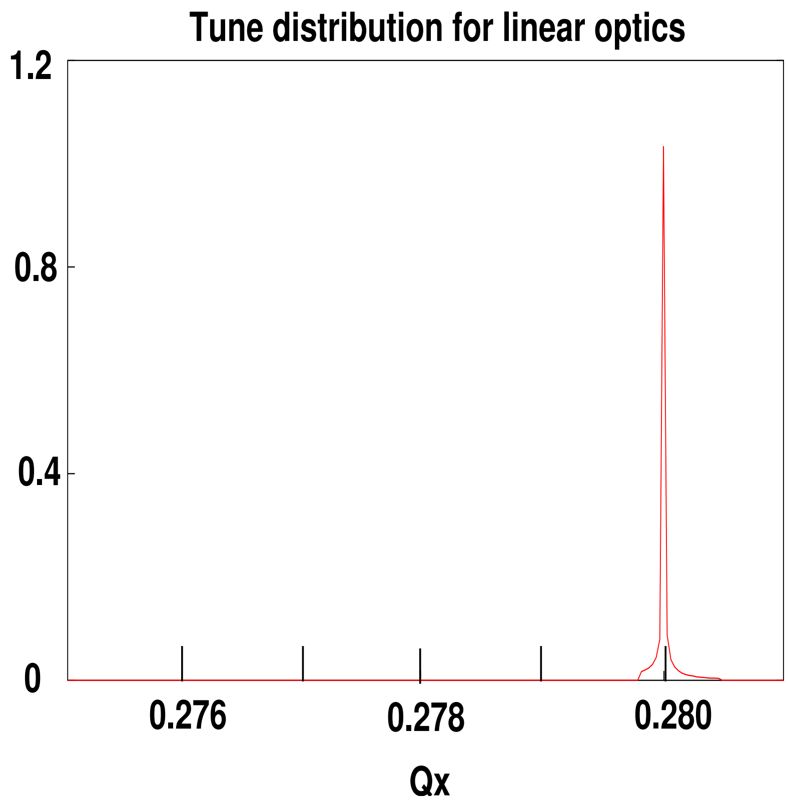
- Start with standard working point
- LHC (equally charged beams)
- Beam–beam shifts tune in both planes

Linear tune shift - two dimensions



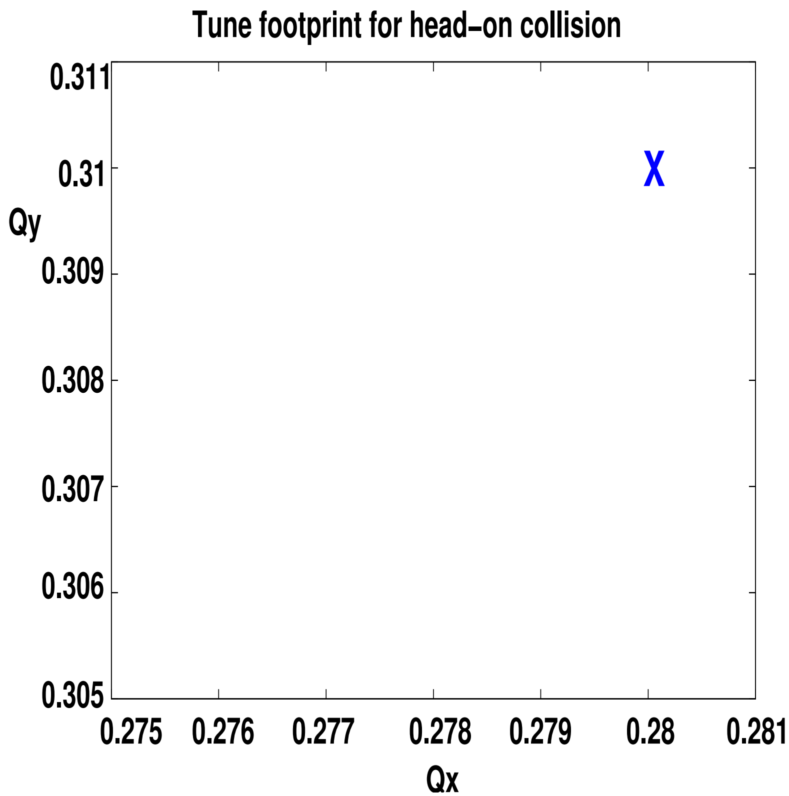
- Start with standard working point
- LHC (equally charged beams)
- Beam–beam shifts tune in both planes

Tune measurement: linear optics



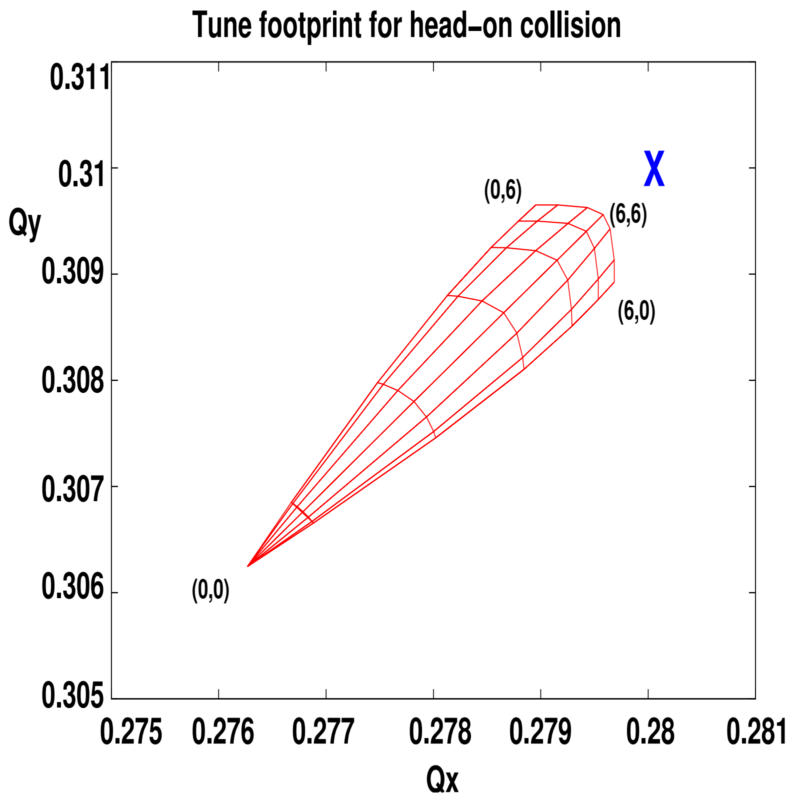
- Linear force: →
- all particles have same tune
- ▶ Only one frequency (tune) visible

Non-linear tune shift - two dimensions



- Tunes depend on x and y amplitudes
- No single tune in the beam
- Compute and plot for every amplitude (pair) the tunes in both planes
- In 2 dimensions:
  plotted as footprint

Non-linear tune shift - two dimensions



- Tunes depend on x and y amplitudes
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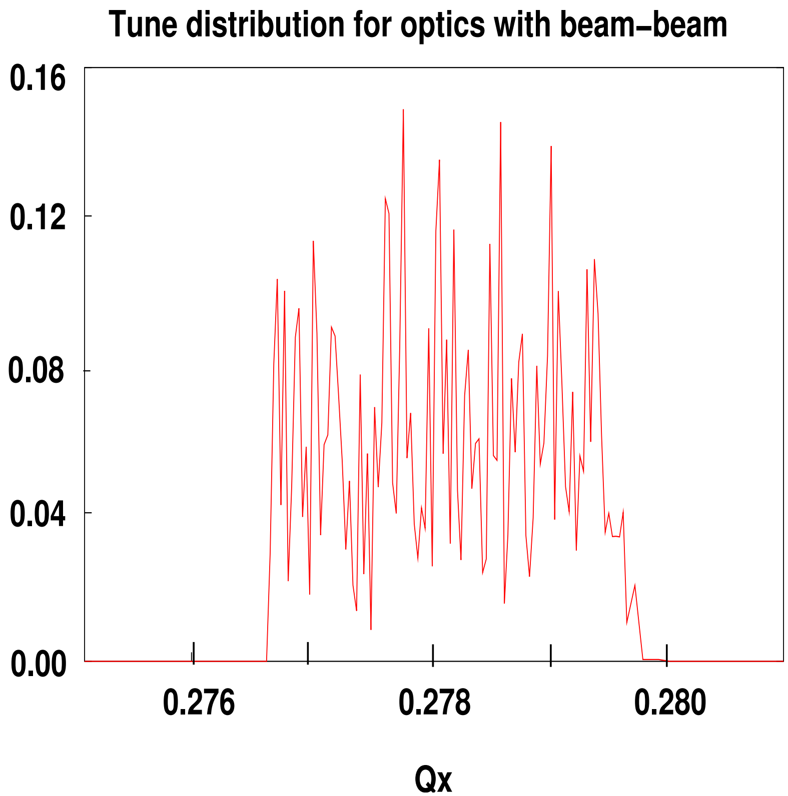
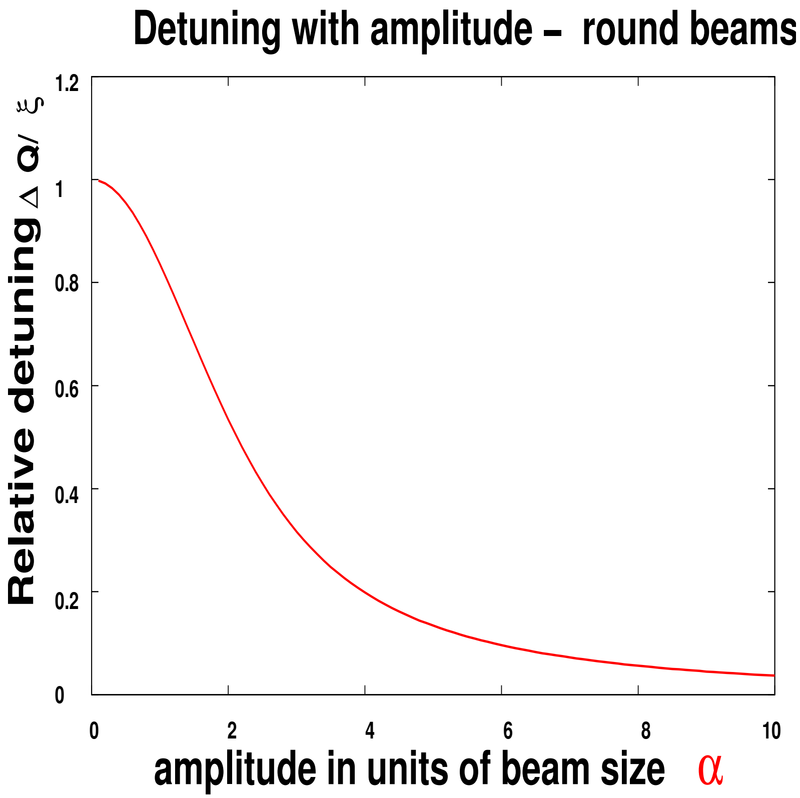
Tune measurement: with beam-beam



- Non−linear force:
- particles with different amplitudes have different frequencies (tunes)
- We get frequency (tune) spectra
- Width of the spectra: about ξ

Amplitude detuning



- Non–linear force: → tune depends on amplitude
- Largest effect for **small** amplitudes
- Calculation in the **proceedings**

→ **with** $\alpha = \frac{a}{\sigma_*}$ **we get:** $\Delta Q/\xi = \frac{4}{\alpha^2}\left[1 - I_0\left(\frac{\alpha^2}{4}\right)\cdot e^{\frac{-\alpha^2}{4}}\right]$

Weak-strong and strong-strong

- **Both beams are very strong (strong-strong):**
  - **Both beam are affected and change due to beam-beam interaction**
  - **Examples: LHC, LEP, RHIC, ...**
  - **Evaluation of effects challenging**
- **One beam much stronger (weak-strong):**
  - **Only the weak beam is affected and changed due to beam-beam interaction**
  - **Examples: SPS collider, Tevatron, ...**

Incoherent effects

(single particle effects)

- Single particle dynamics: treat as a particle through a static electromagnetic lens
- Basically non-linear dynamics
- All single particle effects observed:
  - Unstable and/or irregular motion
  - Beam blow up
  - Bad lifetime, particle loss

Observations hadrons

- **Non-linear motion can become chaotic**
  - **reduction of "dynamic aperture"**
  - **particle loss and bad lifetime**
- **Strong effects in the presence of noise or ripple**
- **Very bad: unequal beam sizes (studied at SPS, HERA)**
- **Evaluation is done by simulation**

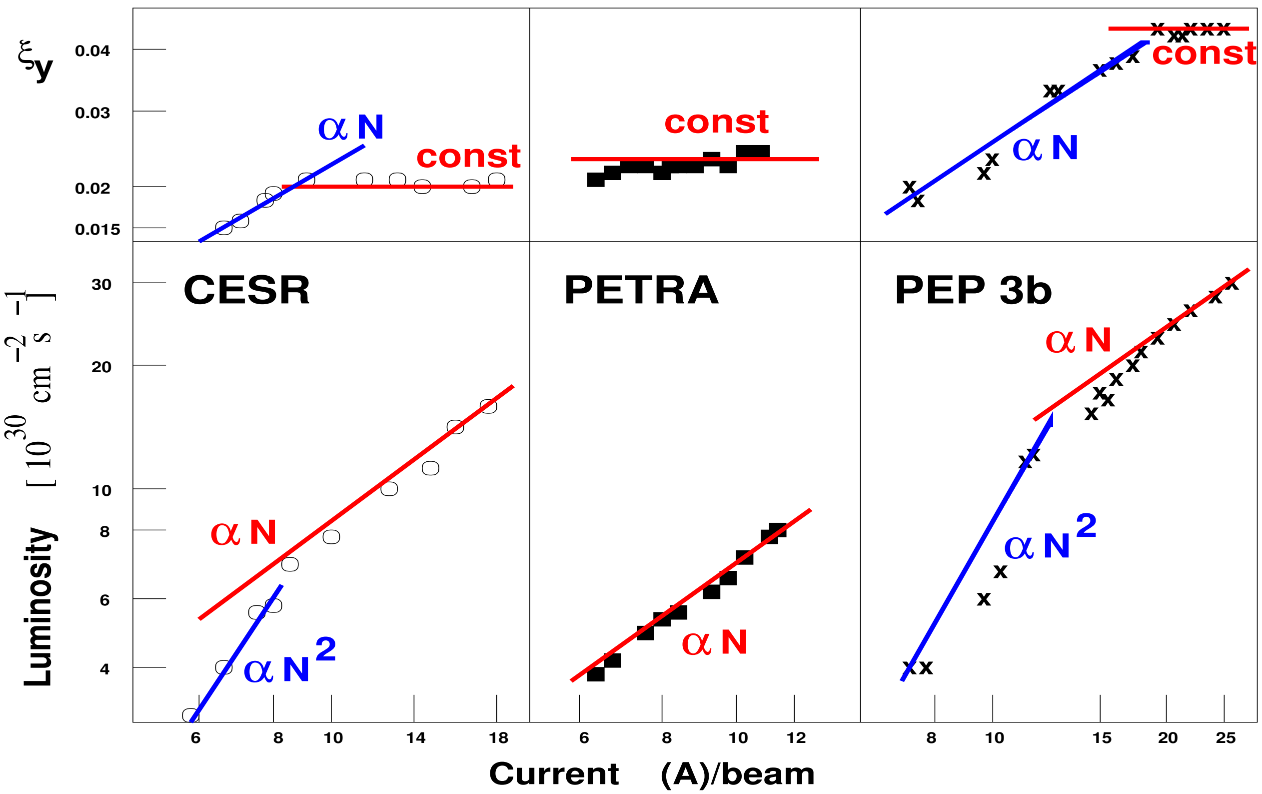
Observations leptons

Remember:

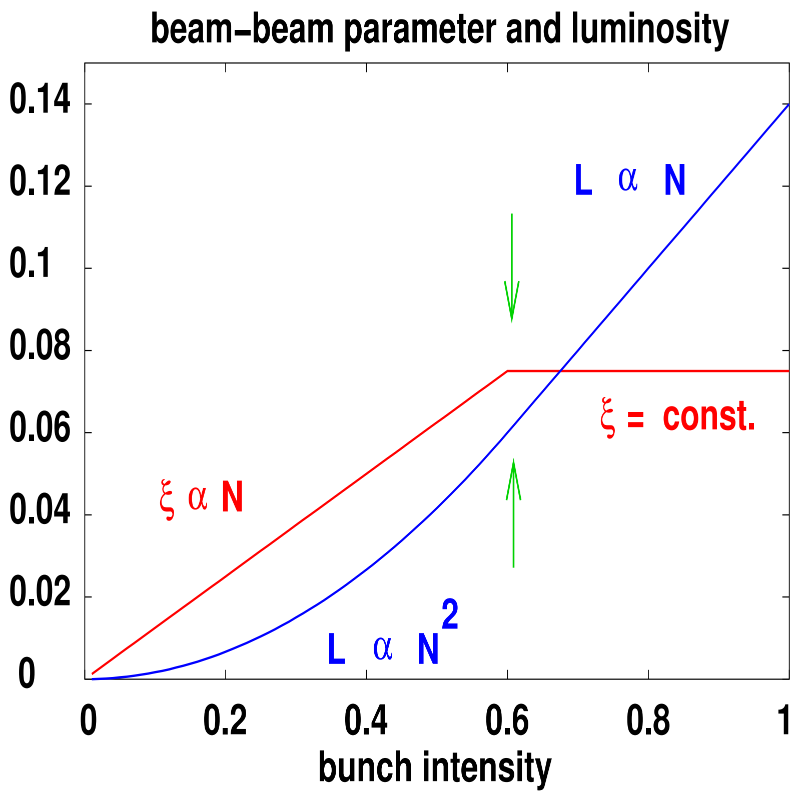
$$\Longrightarrow \quad \mathcal{L} = \frac{N_1 N_2 f n_B}{4\pi\sigma_x\sigma_y}$$

- **Luminosity should increase** $\propto \; N_1 N_2$
  - → **for:** $N_1 = N_2 = N$ → $\propto \; N^2$
- **Beam-beam parameter should increase** $\propto \; N$
- **But:**

Examples: beam-beam limit



Beam-beam limit (schematic)



- Beam−beam parameter increases linearly with intensity
- Saturation above some intensity
- Luminosity increases linearly
- So−called **beam−beam limit**

What is happening ?

we have
$$\xi_y = \frac{N r_0 \beta_y}{2\pi\gamma\sigma_y(\sigma_x + \sigma_y)} \overset{(\sigma_x \gg \sigma_y)}{\approx} \frac{r_0 \beta_y}{2\pi\gamma(\sigma_x)} \cdot \frac{N}{\sigma_y}$$

and
$$\mathcal{L} = \frac{N^2 f n_B}{4\pi\sigma_x\sigma_y} = \frac{N f n_B}{4\pi\sigma_x} \cdot \frac{N}{\sigma_y}$$

- **Above beam-beam limit: σ_y increases when N increases to keep ξ constant ➡ equilibrium emittance !**
- **Therefore: $\mathcal{L} \propto N$ and $\xi \approx$ constant**
- **ξ_{limit} is NOT a universal constant !**
- **Difficult to predict**

What is happening ?

- **Where does it come from ?**
  - **From synchrotron radiation: vertical plane damped, horizontal plane excited**
  - **Horizontal beam size usually (much) larger**
  - **Vertical beam-beam effect depends on horizontal (large) amplitude**
  - **Coupling from horizontal to vertical plane**
- **Equilibrium between this excitation and damping determines** ξ_{limit}

Lesson: **Keep the coupling small !**

The next problem

Remember:

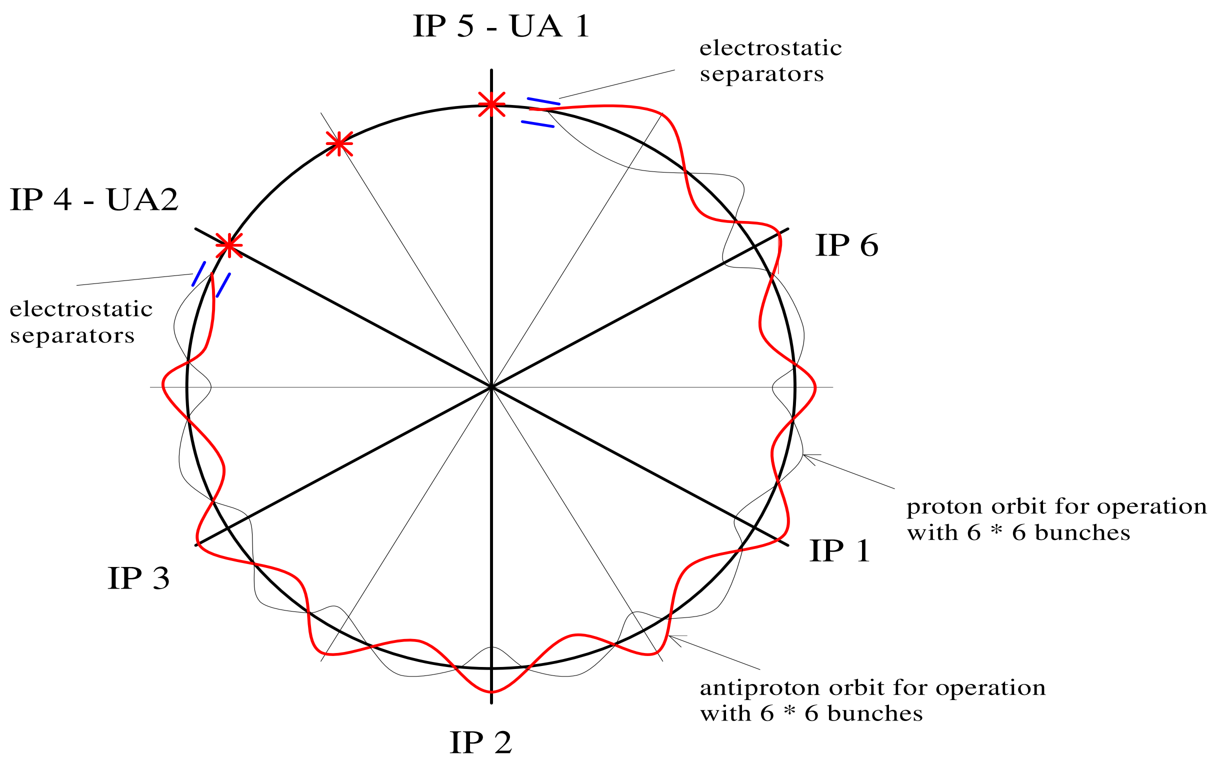
$$\Longrightarrow \quad \mathcal{L} = \frac{N_1 N_2 f \cdot n_B}{4\pi\sigma_x\sigma_y}$$

- **How to collide many bunches (for high $\mathcal{L}$) ??**
- **Must avoid unwanted collisions !!**
- **Separation of the beams:**
  - **Pretzel scheme (SPS,LEP,Tevatron)**
  - **Bunch trains (LEP,PEP)**
  - **Crossing angle (LHC)**

Separation: SPS

- Few equidistant bunches
  (6 against 6)
- Beams travel in same beam pipe
  (12 collision points !)
  - Two experimental areas
  - Need global separation
  - Horizontal pretzel around most of the circumference

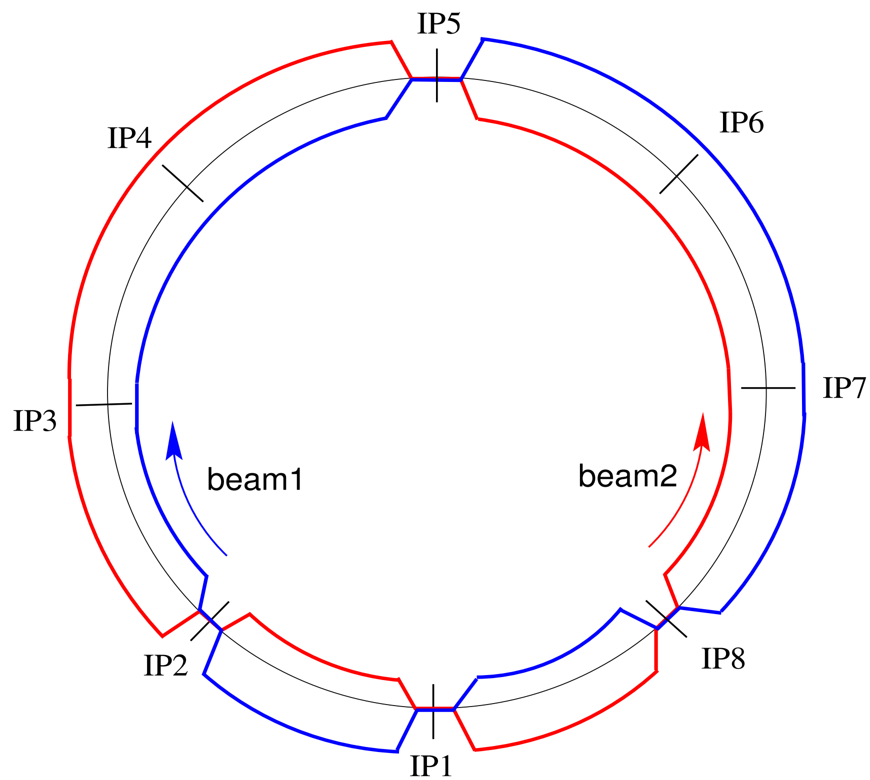
Separation: SPS



Separation: LHC

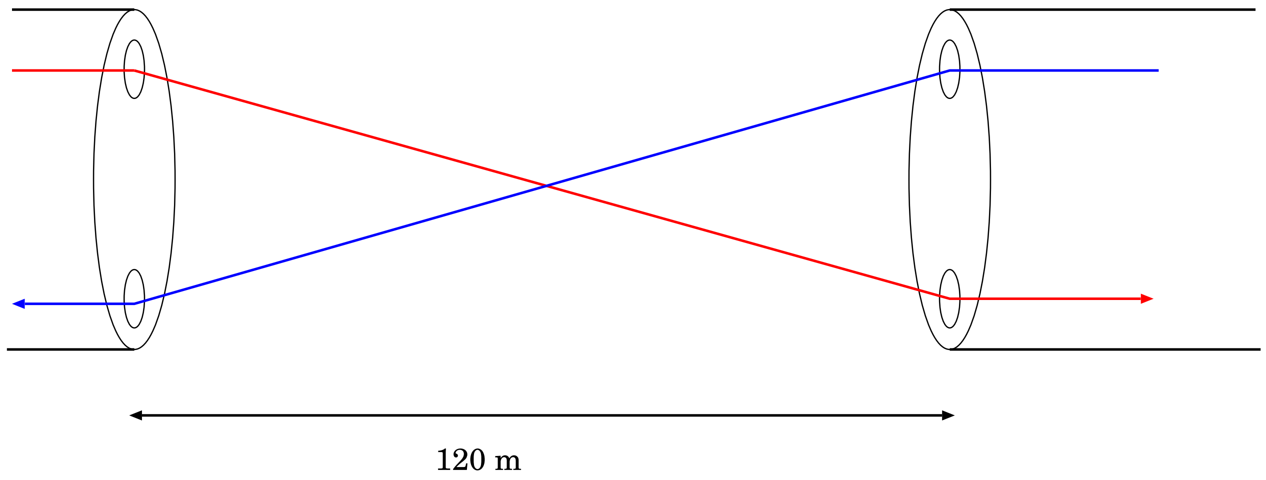
- Many equidistant bunches (2808 per beam)
- Two beams already separated in two separate beam pipes except:
  - Four experimental areas
  - Need local separation
- Two horizontal and two vertical crossing angles

Layout of LHC



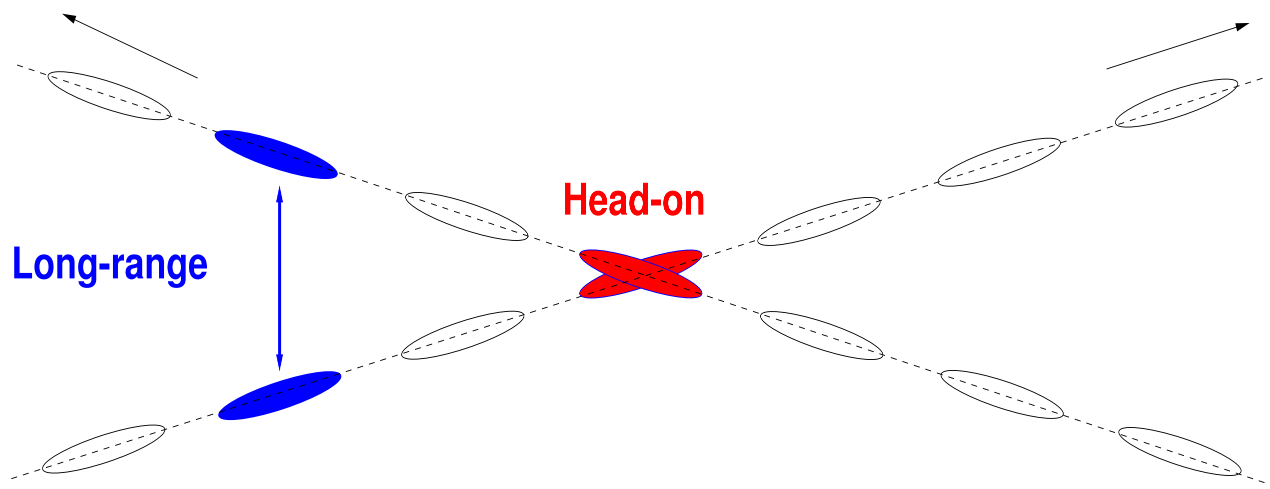
Example: LHC

- Two beams, 2808 bunches each, every 25 ns
- In common chamber around experiments



- Over 120 m: about 30 parasitic interactions

Crossing angles (example LHC)



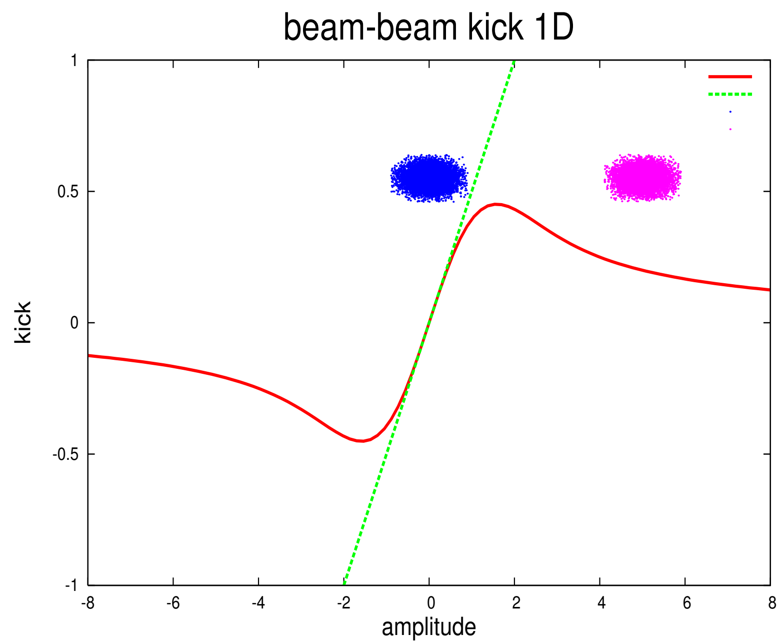
- Particles experience distant (weak) forces
- Separation typically 6 - 12 σ

➡ We get so-called long range interactions

What is special about them ?

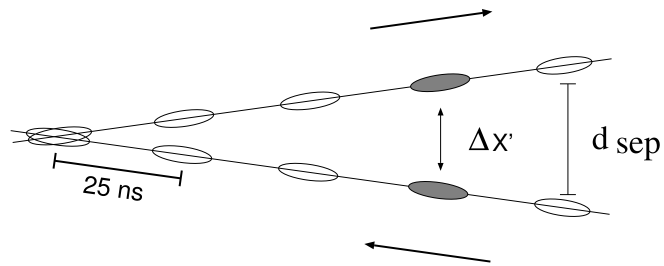
- Break symmetry between planes, stronger resonance excitation
- Mostly affect particles at large amplitudes
- Cause effects on closed orbit
- PACMAN effects
- Tune shift has opposite sign in plane of separation

Why opposite tuneshift ???



- **Local slope has opposite sign for large separation**
- **Opposite sign for focusing !**

Long range interactions (LHC)

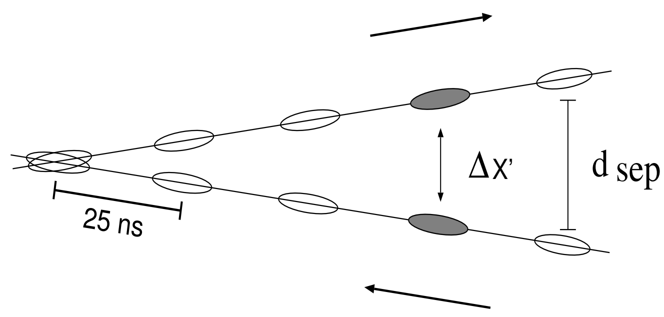


➡ **For horizontal separation d:**

$$\Delta x'(x+d, y, r) = -\frac{2Nr_0}{\gamma} \cdot \frac{(x+d)}{r^2}\left[1 - \exp(-\frac{r^2}{2\sigma^2})\right]$$

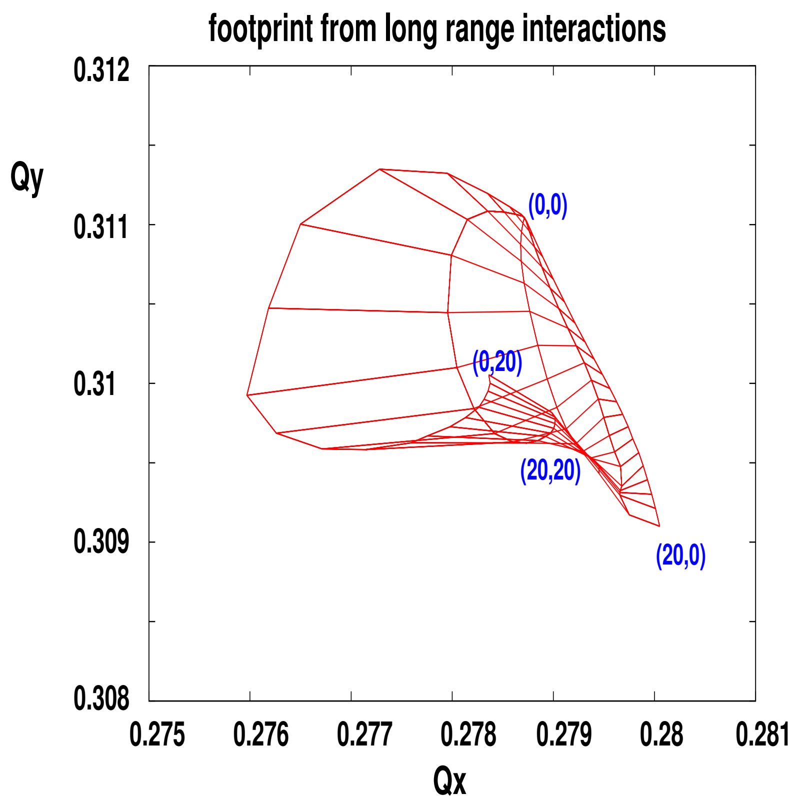
(with: $r^2 = (x+d)^2 + y^2$***)***

Long range interactions (LHC)



- **Number of long range interactions depends on spacing and length of common part**
- **In LHC 15 collisions on each side, 120 in total !**
- **Effects depend on separation:** $\Delta Q \propto -\frac{N}{d^2}$ **(for large enough d !)** **footprints ??**

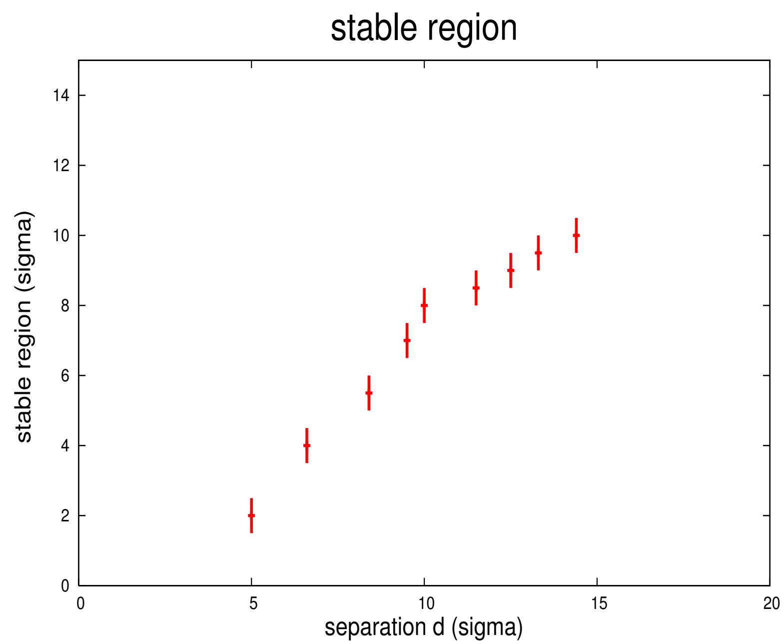
Footprints



- **Large for largest amplitudes where non–linearities are strong**
- **Size proportional to** $\frac{1}{d^2}$
- **Must expect problems at small separation**
- **Footprint very asymmetric**

Particle losses

- **Small crossing angle $\Longleftrightarrow$ small separation**
- **Small separation: particles become unstable and get lost**



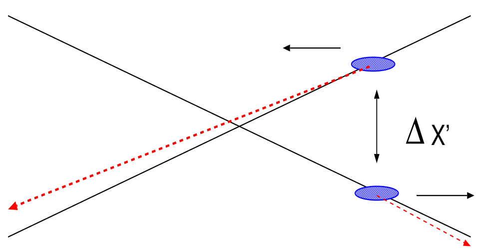
- **Minimum separation for LHC: $\approx 10\ \sigma$**

Closed orbit effects

$$\Delta x'(x+d, y, r) = -\frac{2Nr_0}{\gamma} \cdot \frac{(x+d)}{r^2}\left[1 - \exp(-\frac{r^2}{2\sigma^2})\right]$$

For well separated beams ($d \gg \sigma$) the force (kick) has an amplitude independent contribution: ➡ **orbit kick**

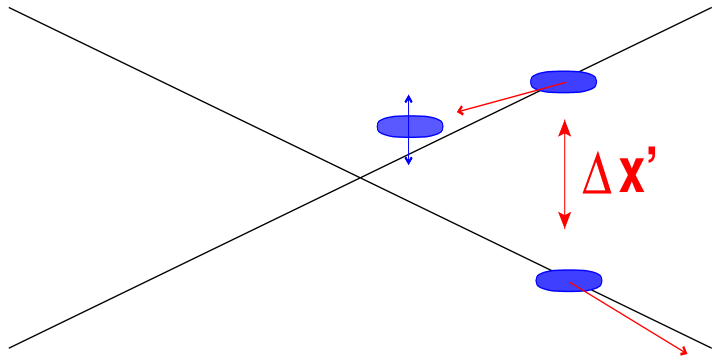
$$\Delta x' \;=\; \underbrace{\frac{const.}{d} \;\cdot\; [\;1}\; - \;\frac{x}{d}\; + \;O\left(\frac{x^2}{d^2}\right)\; + \;...$$



Closed orbit effects

- Beam-beam kick from long range interactions changes the orbit
  - Has been observed in LEP with bunch trains
  - Self-consistent calculation necessary
  - Effects can add up and become important
  - The two beams separate, more than 1σ not unusual !

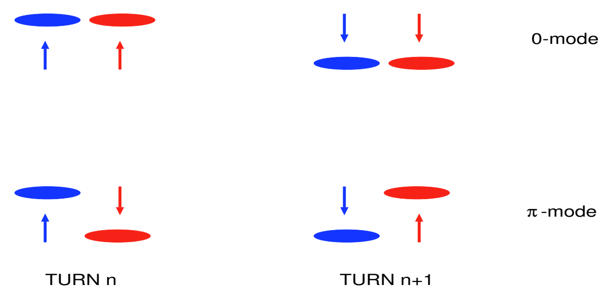
Coherent beam-beam effect



- Whole bunch sees a kick as an entity (coherent kick)
- The coherent kick of separated beams can excite coherent dipole oscillations
- All bunches couple because each bunch "sees" many opposing bunches: many coherent modes possible !

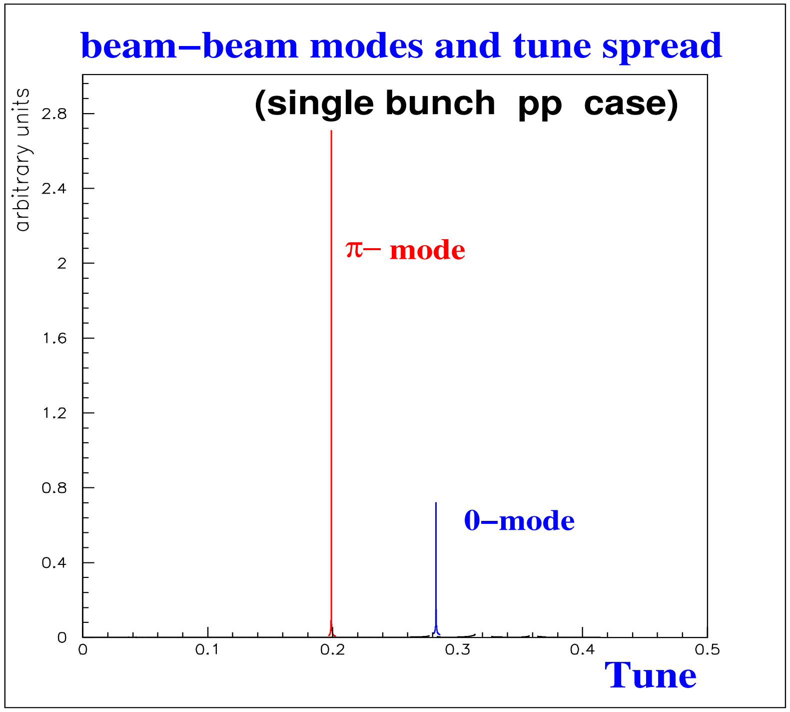
Coherent beam-beam effect

Simplest case: one bunch per beam:



- **Coherent mode: two bunches are "locked" in a coherent oscillation**
- **0-mode is stable (Mode with NO tune shift)**
- **π-mode can become unstable (Mode with LARGEST tune shift)**

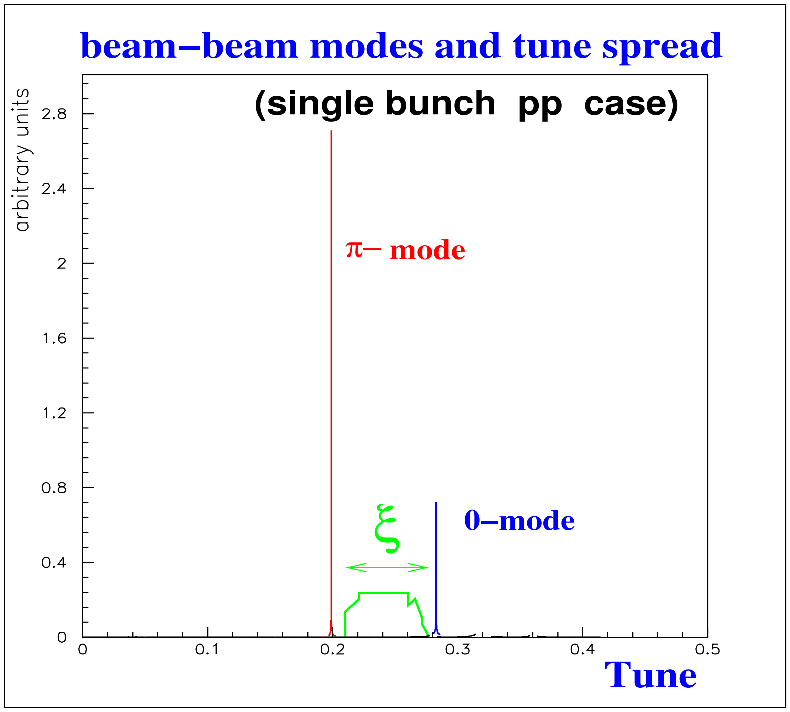
Coherent beam-beam frequencies (schematic)



- **0−mode is at unperturbed tune**
- **π−mode is shifted by 1.1 – 1.3 · ξ**

- Two separate modes visible
- But we have many particles and tune spread ... !

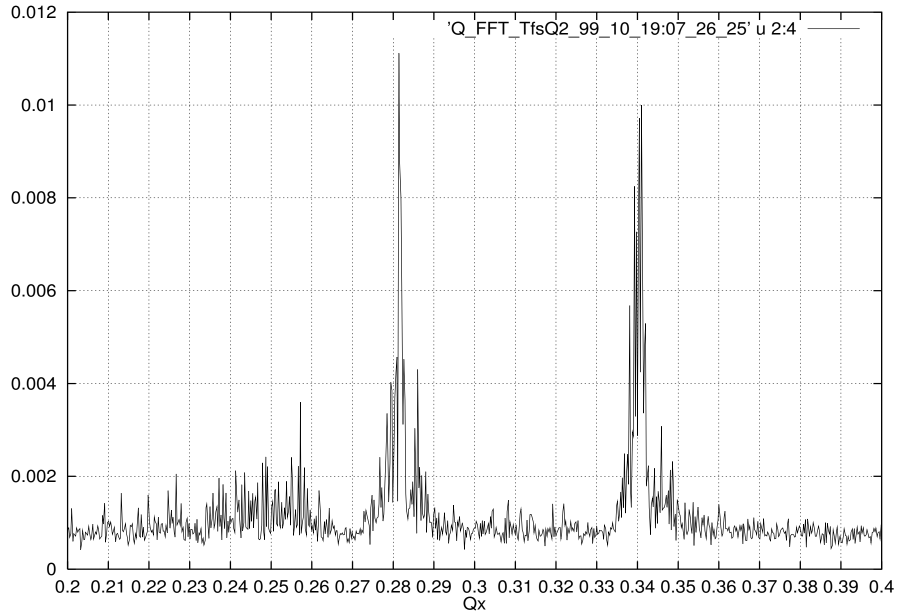
Coherent beam-beam frequencies (schematic)



- **0−mode is at unperturbed tune**
- **π−mode is shifted by 1.1 – 1.3 · ξ**
- **Incoherent spread between [0.0,1.0] · ξ**

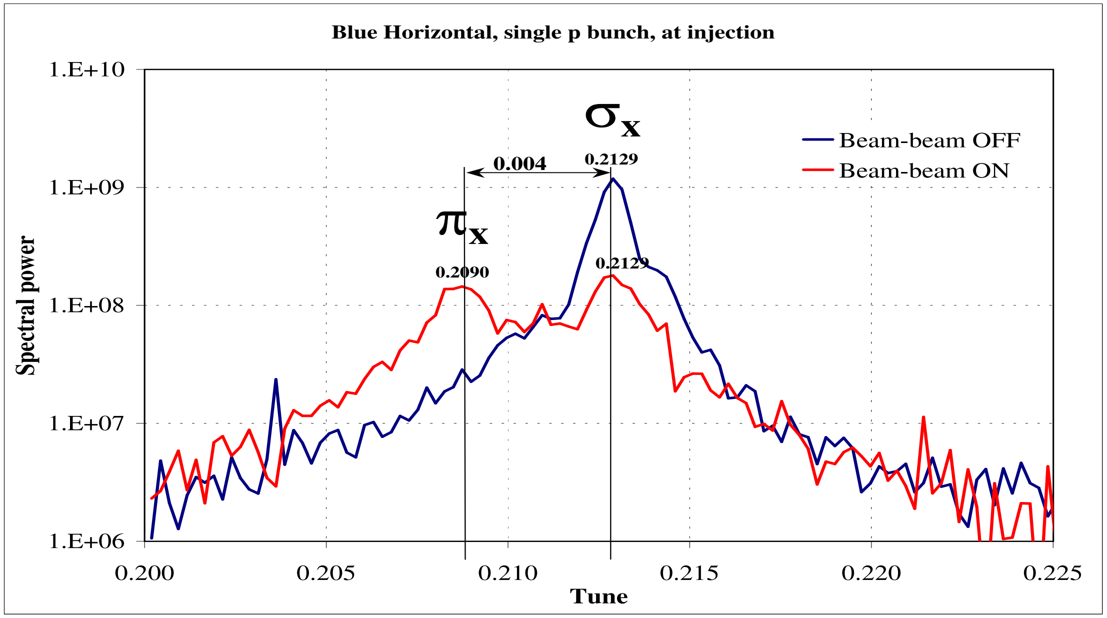
- **Strong-strong case: π-mode shifted outside tune spread**
- **No Landau damping possible**

What we measure: LEP



- **Two modes clearly visible**
- **Can be distinguished by phase relation, i.e. sum and difference signals**

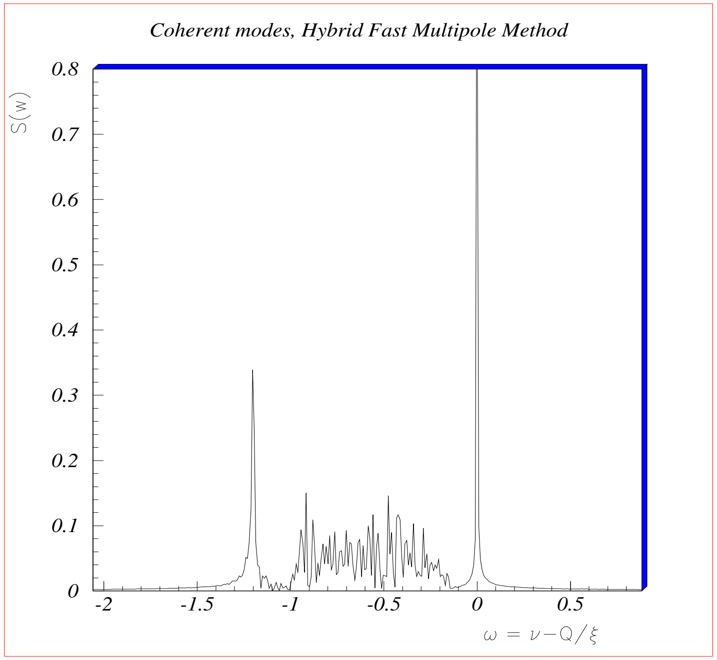
What we measure: RHIC



Courtesy W. Fischer (BNL)

➡ **Compare spectra with and without beams : two modes visible with beams**

Simulation of coherent spectra



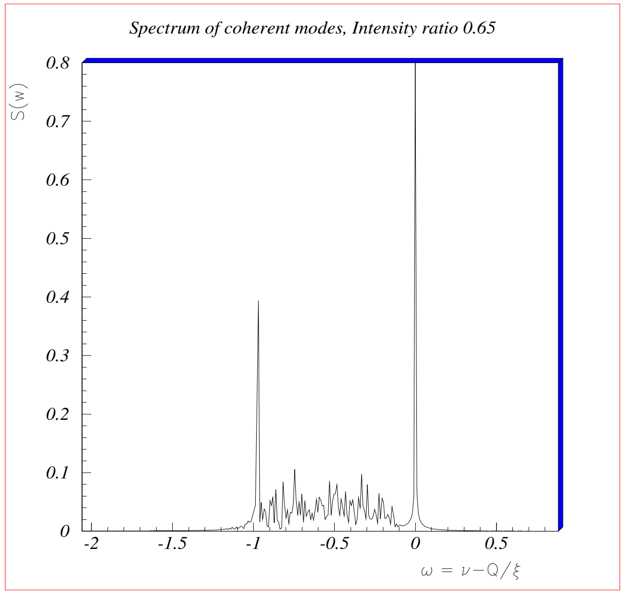
- **Full simulation of both beams required**
- **Use up to 10^8 particles in simulations**
- **Must take into account changing fields**
- **Requires computation of arbitrary fields**

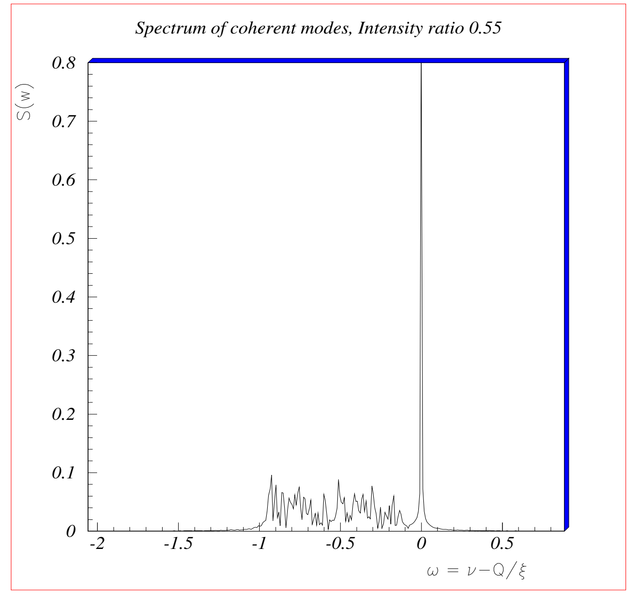
- Time consuming for many particles ..

What can be done to avoid problems ?

- **Coherent motion requires 'organized' motion of many particles**
- **Therefore high degree of symmetry required**
- **Possible countermeasure: (symmetry breaking)**
  - **Different bunch intensity**
  - **Different tunes in the two beams**

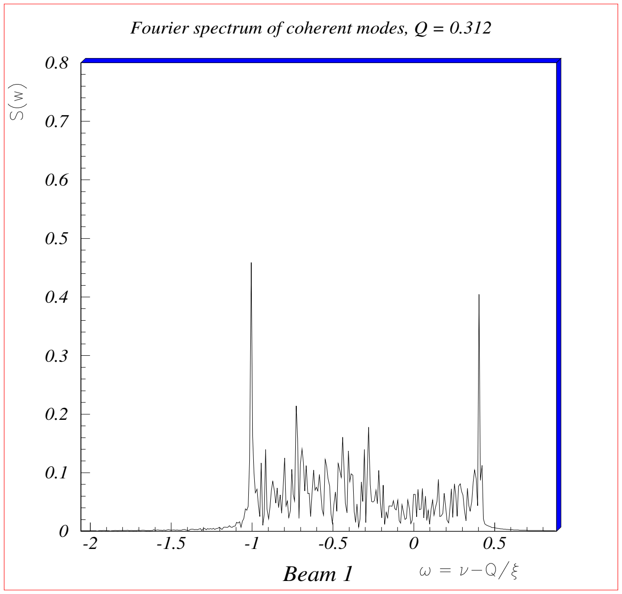
Beams with different intensity

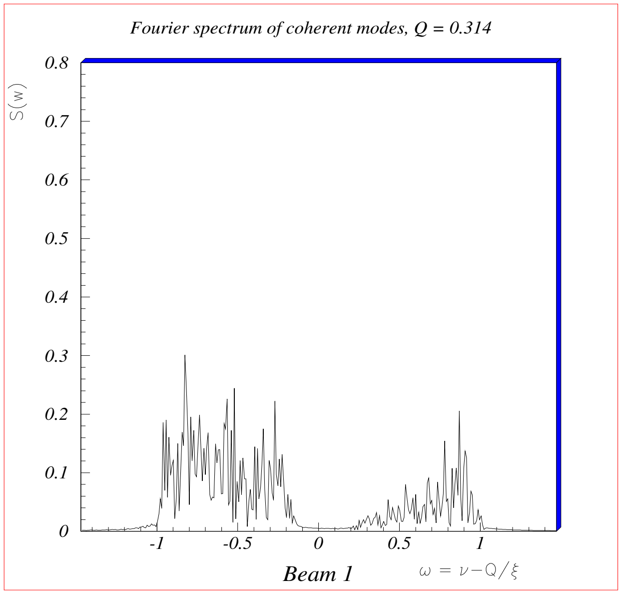




- **Bunches with different intensities cannot maintain coherent motion**
- **Landau damping restored**

Beams with different tunes





- **Bunches with different tunes cannot maintain coherent motion**
- **Landau damping restored**

Can we suppress beam-beam effects ?

- **Find 'lenses' to correct beam-beam effects**
  - **Head on effects:**
    - **Linear "electron lens" to shift tunes**
    - **Non-linear "electron lens" to reduce spread**
    - **Tests in progress at Tevatron and RHIC**
  - **Long range effects:**
    - **At very large distance: force is 1/r**
    - **Same force as a wire !**
- **So far: mixed success with active compensation**

Others: Möbius lattice

- **Principle:**
  - **Interchange horizontal and vertical plane each turn**
- **Effects:**
  - **Round beams (even for leptons)**
  - **Some compensation effects for beam-beam interaction**
  - **First test at CESR at Cornell**

Not mentioned:

- Effects in linear colliders
- Asymmetric beams
- Coasting beams
- Beamstrahlung
- Synchrobetatron coupling
- Monochromatization
- Beam-beam experiments
- ... and many more

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