Concept of Luminosity

(in particle colliders)

Werner Herr, CERN, AB Department
Bruno Muratori, Daresbury Laboratory

(http://cern.ch/lhc-beam-beam/talks/Daresbury_luminosity.pdf)

Why colliding beams ?

- **Two beams:** $E_1, \vec{p_1}, E_2, \vec{p_2}, m_1 = m_2 = m$
- $E_{cm} = \sqrt{(E_1 + E_2)^2 - (\vec{p_1} + \vec{p_2})^2}$
- **Collider versus fixed target:**

  Fixed target: $\vec{p_2} = \mathbf{0}$ ➡ $E_{cm} = \sqrt{2m^2 + 2E_1 m}$

  Collider: $\vec{p_1} = -\vec{p_2}$ ➡ $E_{cm} = E_1 + E_2$
- **LHC (pp): 14000 GeV versus ≈ 115 GeV**
- **LEP (e^+e^-): 210 GeV versus ?**

Collider performance issues

- Available energy
- Number of interactions per second (useful collisions)
- Total number of interactions
- Secondary issues:
  - Time structure of interactions (how often and how many at the same time)
  - Space structure of interactions (size of interaction region)
  - Quality of interactions (background, dead time etc.)

Luminosity:

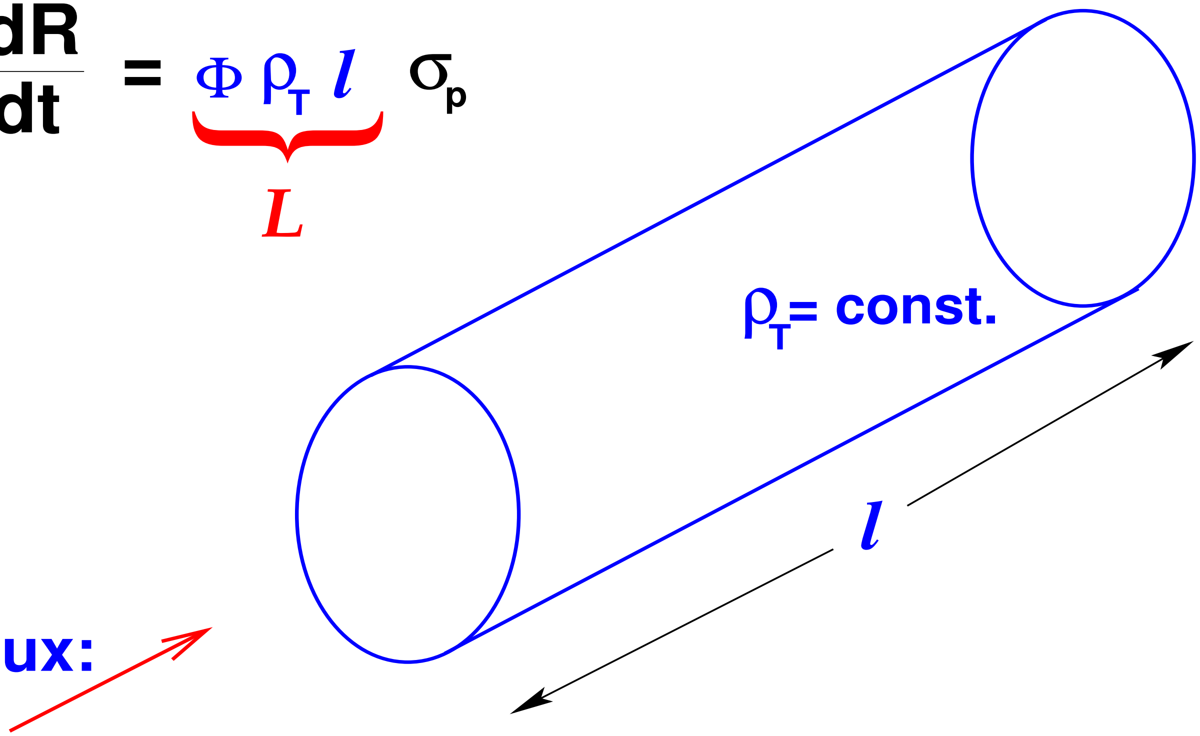
- **We want:**
  - **Proportionality factor between cross section σ_p and number of interactions per second $\frac{dR}{dt}$**

$$\frac{dR}{dt} = \mathcal{L} \times \sigma_p \qquad (\rightarrow \quad \mathrm{units:}\ \ \mathrm{cm}^{-2}\mathrm{s}^{-1})$$

  - **Relativistic invariant**
  - **Independent of the physical reaction**
  - **Reliable procedures to compute and measure**

Fixed target luminosity

$$\frac{dR}{dt} = \underbrace{\Phi\, \rho_T\, l}_{L}\, \sigma_p$$

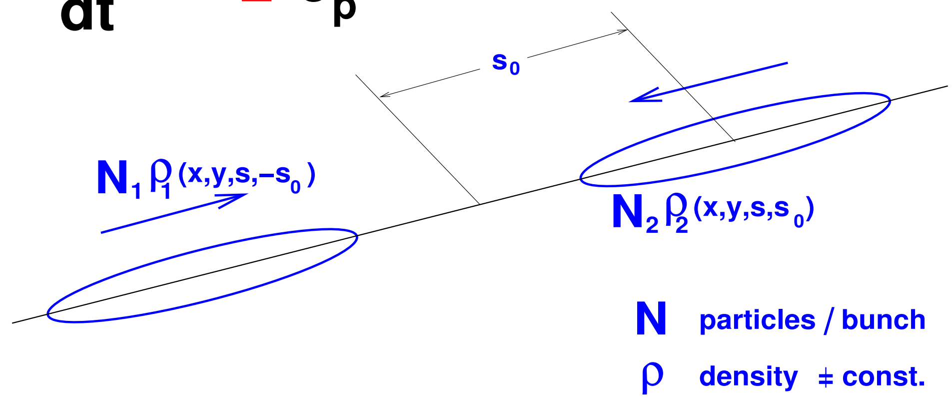


Flux:

$\Phi = \mathrm{N/s}$

Collider luminosity (per bunch crossing)

$$\frac{dR}{dt} = L\,\sigma_p$$



$$\mathcal{L} \propto K N_1 N_2 \int\int\int\int_{-\infty}^{+\infty} \rho_1(x,y,s,-s_0)\rho_2(x,y,s,s_0)\,dx\,dy\,ds\,ds_0$$

s_0 is "time"-variable: $s_0 = c \cdot t$

Kinematic factor: $K = \sqrt{(\vec{v_1} - \vec{v_2})^2 - (\vec{v_1} \times \vec{v_2})^2/c^2}$

Collider luminosity (per beam)

- **Assume uncorrelated densities in all planes**

→ **factorize:** $\rho(x, y, s, s_0) = \rho_x(x) \cdot \rho_y(y) \cdot \rho_s(s \pm s_0)$

- **For head-on collisions ($\vec{v_1} = -\vec{v_2}$) we get:**

$$\mathcal{L} = 2 \cdot N_1 N_2 \cdot f \cdot n_b \cdot \int\int\int\int_{-\infty}^{+\infty} dx dy ds ds_0$$

$$\rho_{1x}(x)\rho_{1y}(y)\rho_{1s}(s - s_0) \cdot \rho_{2x}(x)\rho_{2y}(y)\rho_{2s}(s + s_0)$$

- **In principle: should know all distributions**

→ **Mostly use Gaussian ρ for analytic calculation (in general: it is a good approximation)**

Gaussian distribution functions

- $$\rho_{iz}(z) = \frac{1}{\sigma_z\sqrt{2\pi}} \exp\left(-\frac{z^2}{2\sigma_z^2}\right) \quad i = 1, 2, \quad z = x, y$$

- $$\rho_{is}(s \pm s_0) = \frac{1}{\sigma_s\sqrt{2\pi}} \exp\left(-\frac{(s \pm s_0)^2}{2\sigma_s^2}\right)$$

- **For non-Gaussian profiles not always possible to find analytic form, need a numerical integration**

Luminosity for two beams (1 and 2)

- **Simplest case : equal beams**
  - $\sigma_{1x} = \sigma_{2x}, \quad \sigma_{1y} = \sigma_{2y}, \quad \sigma_{1s} = \sigma_{2s}$
  - **but:** $\sigma_{1x} \neq \sigma_{1y}, \quad \sigma_{2x} \neq \sigma_{2y}$ **is allowed**
- **Further: no dispersion at collision point**

Integration (head-on)

for $\sigma_1 = \sigma_2 \quad \rightarrow \quad \rho_1\rho_2 \;=\; \rho^2$ **:**

$$\mathcal{L} = \frac{2 \cdot N_1 N_2 f n_b}{(\sqrt{2\pi})^6 \sigma_s^2 \sigma_x^2 \sigma_y^2} \int\int e^{-\frac{x^2}{\sigma_x^2}} e^{-\frac{y^2}{\sigma_y^2}} e^{-\frac{s^2}{\sigma_s^2}} e^{-\frac{s_0^2}{\sigma_s^2}} dx dy ds ds_0$$

integrating over s **and** s_0**, using:**

$$\int_{-\infty}^{+\infty} e^{-at^2} dt \quad = \quad \sqrt{\pi/a}$$

$$\mathcal{L} = \frac{2 \cdot N_1 N_2 f n_b}{8(\sqrt{\pi})^4 \sigma_x^2 \sigma_y^2} \int\int e^{-\frac{x^2}{\sigma_x^2}} e^{-\frac{y^2}{\sigma_y^2}} dx dy$$

finally after integration over x and y: $\Longrightarrow \quad \mathcal{L} = \dfrac{N_1 N_2 f n_b}{4\pi\sigma_x\sigma_y}$

Luminosity for two (equal) beams (1 and 2)

- **Simplest case :** $\sigma_{1x} = \sigma_{2x}, \sigma_{1y} = \sigma_{2y}, \sigma_{1s} = \sigma_{2s}$

  or: $\sigma_{1x} \neq \sigma_{2x} \neq \sigma_{1y} \neq \sigma_{2y}$, $\quad$ *but* : $\sigma_{1s} \approx \sigma_{2s}$

$$\Longrightarrow \quad \mathcal{L} = \frac{N_1 N_2 f n_b}{4\pi\sigma_x\sigma_y} \left(\mathcal{L} = \frac{N_1 N_2 f n_b}{2\pi\sqrt{\sigma_{1x}^2 + \sigma_{2x}^2}\sqrt{\sigma_{1y}^2 + \sigma_{2y}^2}} \right)$$

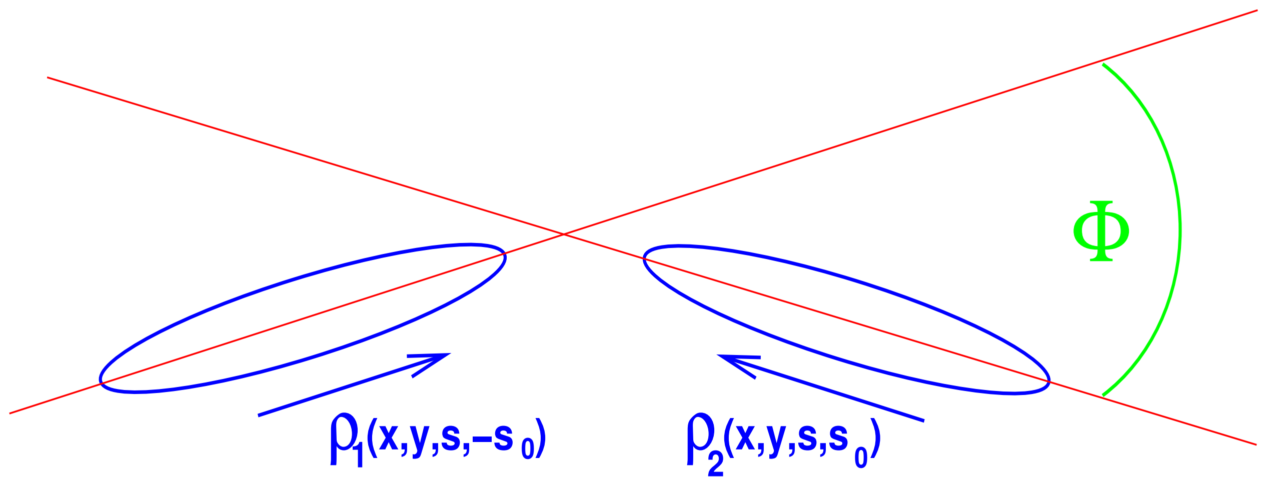
Examples

	Energy (GeV)	$\mathcal{L}_{max}$ $cm^{-2}s^{-1}$	rate s^{-1}	σ_x/σ_y $\mu m/\mu m$	Particles per bunch
SPS ($p\bar{p}$)	**315x315**	$6\ 10^{30}$	$4\ 10^{5}$	**60/30**	$\approx 10\ 10^{10}$
Tevatron ($p\bar{p}$)	**1000x1000**	$100\ 10^{30}$	$7\ 10^{6}$	**30/30**	$\approx 30/8\ 10^{10}$
HERA (e^+p)	**30x920**	$40\ 10^{30}$	**40**	**250/50**	$\approx 3/7\ 10^{10}$
LHC (pp)	**7000x7000**	$10000\ 10^{30}$	10^{9}	**17/17**	$\approx 11\ 10^{10}$
LEP (e^+e^-)	**105x105**	$100\ 10^{30}$	≤ 1	**200/2**	$\approx 50\ 10^{10}$
PEP (e^+e^-)	**9x3**	$8000\ 10^{30}$	**NA**	**150/5**	$\approx 2/6\ 10^{10}$

Complications

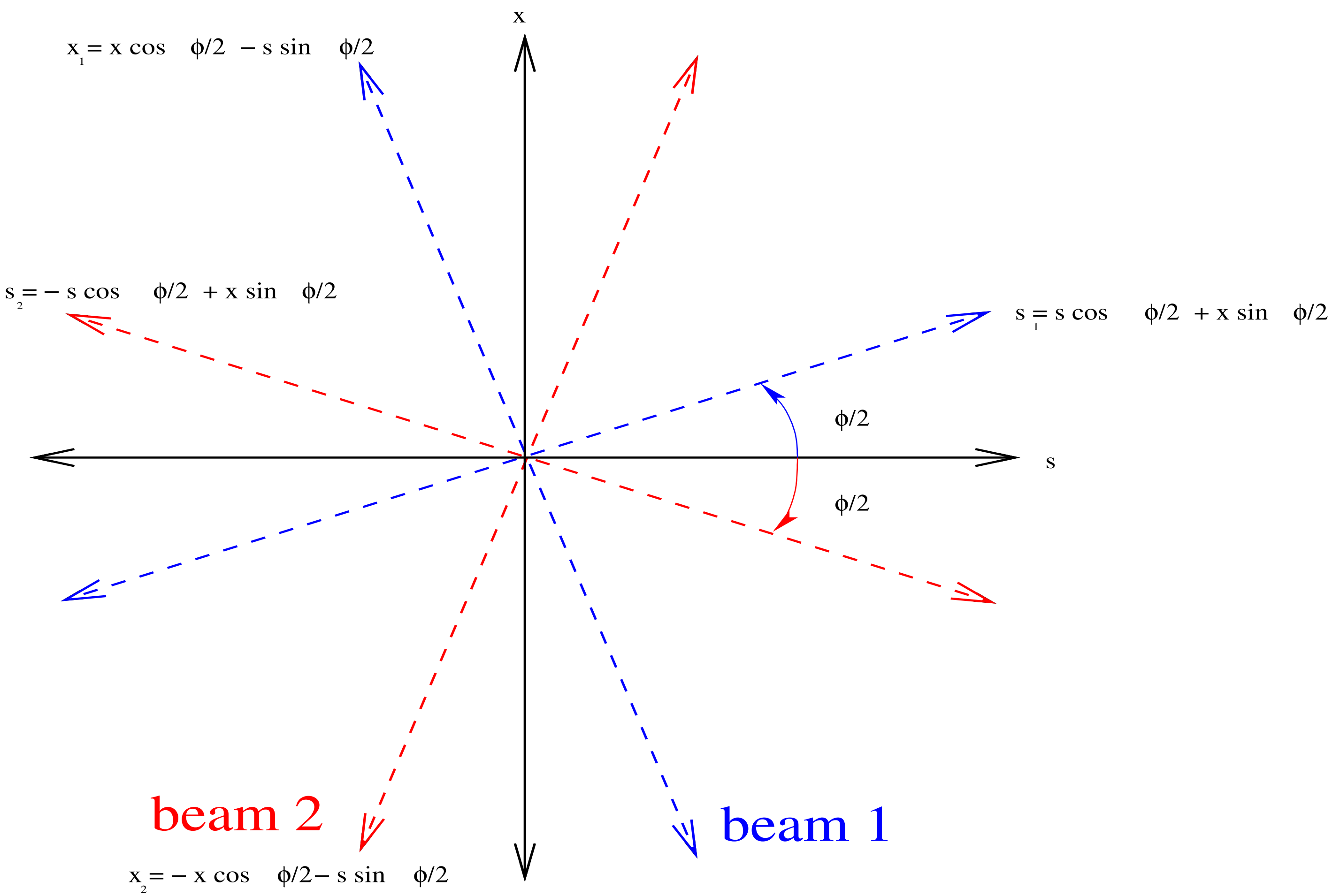
- **Crossing angle**
- **Hour glass effect**
- **Collision offset (wanted or unwanted)**
- **Non-Gaussian profiles**
- **Dispersion at collision point**
- $\delta\beta^*/\delta s = \alpha^* \neq 0$
- **Strong coupling**
- **etc.**

Collisions at crossing angle



- Needed to avoid unwanted collisions
  - For colliders with many bunches: LHC, CESR, KEKB
  - For colliders with coasting beams

Collisions angle geometry (horizontal plane)



Crossing angle

Assume crossing in horizontal (x, s)- plane.
Transform to new coordinates:

$$\begin{cases} x_1 = x\cos\frac{\phi}{2} - s\sin\frac{\phi}{2}, & s_1 = s\cos\frac{\phi}{2} + x\sin\frac{\phi}{2}, \\ x_2 = x\cos\frac{\phi}{2} + s\sin\frac{\phi}{2}, & s_2 = s\cos\frac{\phi}{2} - x\sin\frac{\phi}{2} \end{cases}$$

$$\mathcal{L} = 2\cos^2\frac{\phi}{2} N_1 N_2 f n_b \int\int\int\int_{-\infty}^{+\infty} dx dy ds ds_0$$
$$\rho_{1x}(x_1)\rho_{1y}(y_1)\rho_{1s}(s_1 - s_0)\rho_{2x}(x_2)\rho_{2y}(y_2)\rho_{2s}(s_2 + s_0)$$

Integration (crossing angle)

use as before:

$$\int_{-\infty}^{+\infty} e^{-at^2} dt \quad = \quad \sqrt{\pi/a}$$

and:

$$\int_{-\infty}^{+\infty} e^{-(at^2+bt+c)} dt \quad = \quad \sqrt{\pi/a} \cdot e^{\frac{b^2-ac}{a}}$$

Further:

- **Since** σ_x**,** x **and sin**$(\phi/2)$ **are small: drop all terms** ${\sigma^k}_x sin^l(\phi/2)$ **or** $x^k sin^l(\phi/2)$ **for all: k+l** $\geq$ 4
- **Approximate: sin**$(\phi/2) \approx$ **tan**$(\phi/2) \approx \phi/2$

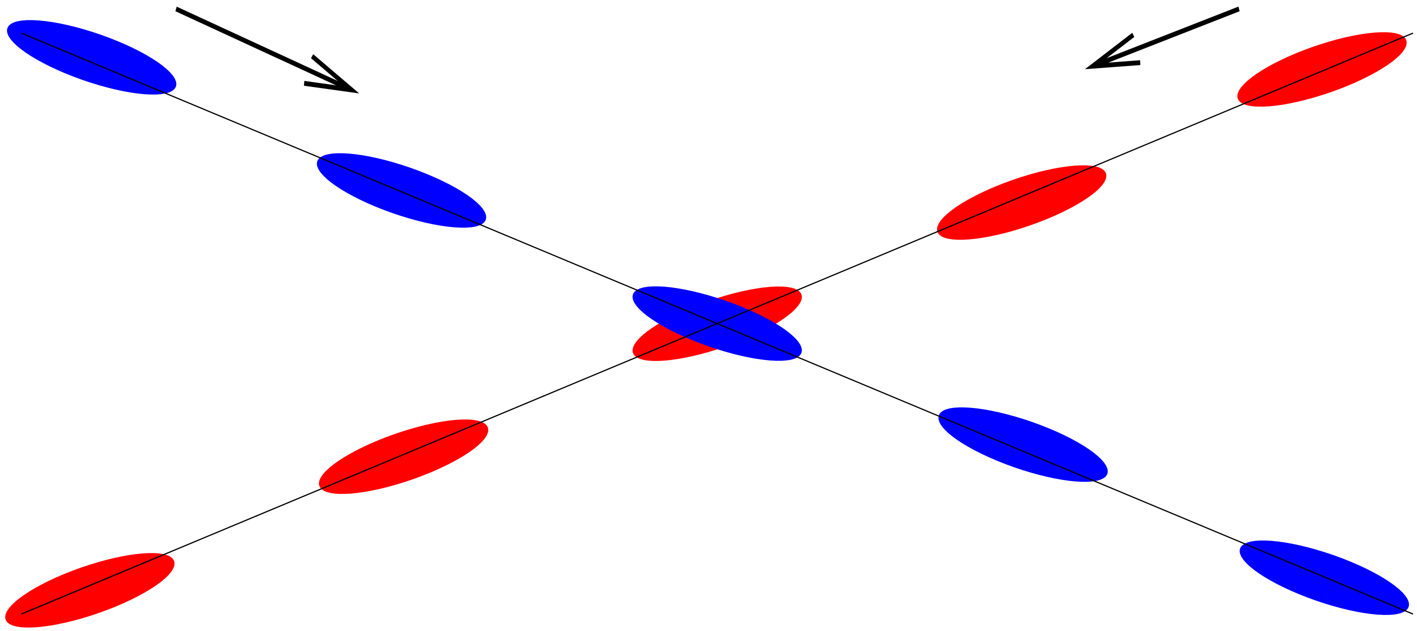
Crossing angle

- **Crossing Angle** $\Rightarrow$ $$\mathcal{L} = \frac{N_1 N_2 f n_b}{4\pi\sigma_x\sigma_y} \cdot S$$
- S **is the reduction factor**
- **For small crossing angles and** $\sigma_s \gg \sigma_{x,y}$
- $$\Rightarrow S = \frac{1}{\sqrt{1 + (\frac{\sigma_s}{\sigma_x}\tan\frac{\phi}{2})^2}} \approx \frac{1}{\sqrt{1 + (\frac{\sigma_s}{\sigma_x}\frac{\phi}{2})^2}}$$

Example LHC:

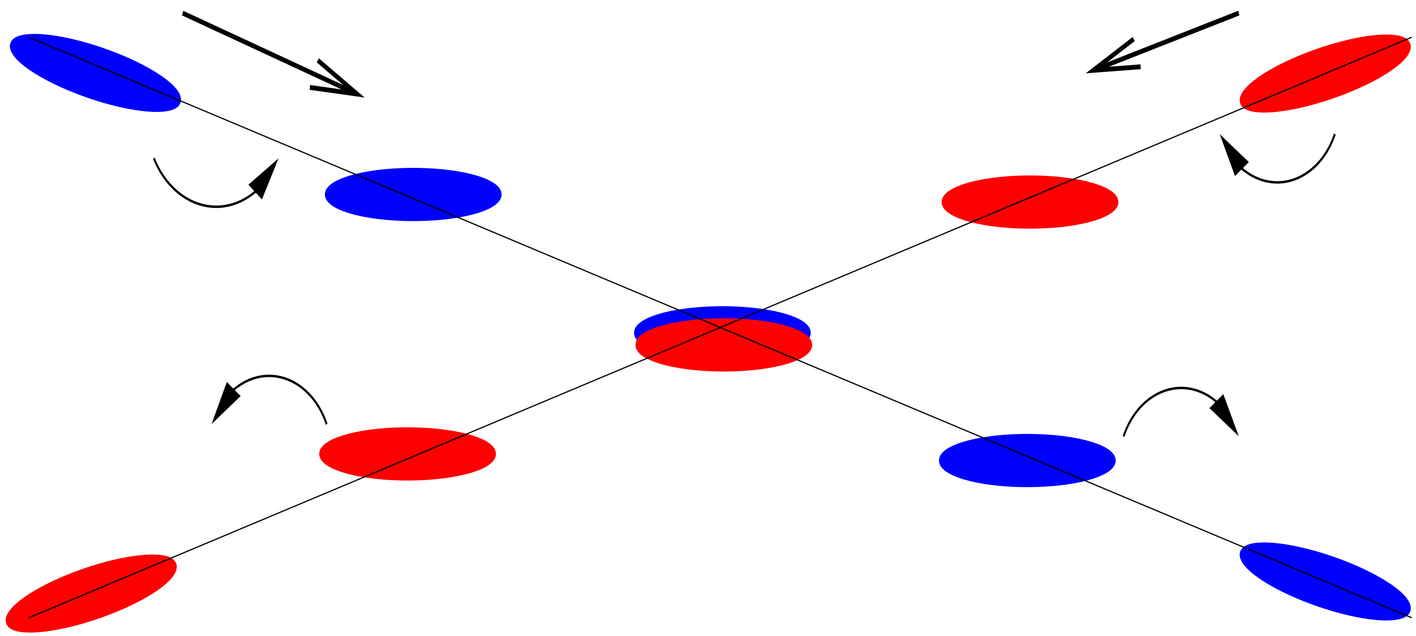
Φ **= 285** μ**rad,** σ_s **= 7.5 cm, S = 0.84**

Large crossing angle



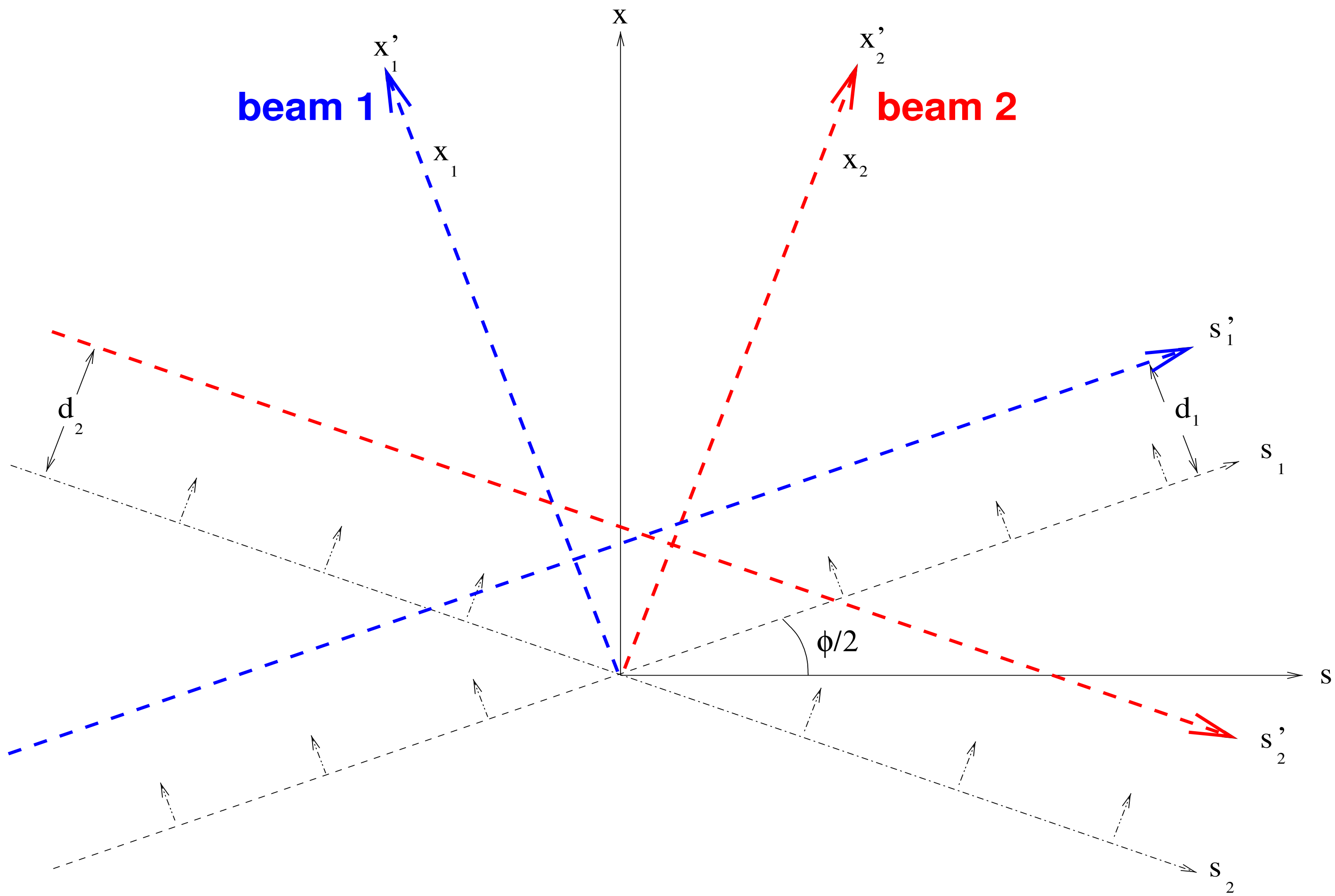
➡ **Large crossing angle: large loss of luminosity**

"crab" crossing scheme



- "crab" crossing recovers geometric loss factor
- feasibility needs to be demonstrated

Offset and crossing angle



Offset and crossing angle

- **Transformations with offsets in crossing plane:**

$$\begin{cases} x_1 = d_1 + x\cos\frac{\phi}{2} - s\sin\frac{\phi}{2}, \quad & s_1 = s\cos\frac{\phi}{2} + x\sin\frac{\phi}{2}, \\ x_2 = d_2 + x\cos\frac{\phi}{2} + s\sin\frac{\phi}{2}, \quad & s_2 = s\cos\frac{\phi}{2} - x\sin\frac{\phi}{2} \end{cases}$$

- **Gives after integration over y and s_0:**

$$\mathcal{L} = \frac{\mathcal{L}_0}{2\pi\sigma_s\sigma_x} 2\cos^2\frac{\phi}{2} \int\int e^{-\frac{x^2\cos^2(\phi/2)+s^2\sin^2(\phi/2)}{\sigma_x^2}} e^{-\frac{x^2\sin^2(\phi/2)+s^2\cos^2(\phi/2)}{\sigma_s^2}}$$

$$\times \quad e^{-\frac{d_1^2+d_2^2+2(d_1+d_2)x\cos(\phi/2)-2(d_2-d_1)s\sin(\phi/2)}{2\sigma_x^2}} dxds.$$

Offset and crossing angle

After integration over x:

$$\mathcal{L} = \frac{N_1 N_2 f n_b}{8\pi^{\frac{3}{2}}\sigma_s} \quad 2\cos\frac{\phi}{2} \quad \int_{-\infty}^{+\infty} W \cdot \frac{e^{-(As^2+2Bs)}}{\sigma_x \sigma_y} ds$$

with:

$$A = \frac{\sin^2\frac{\phi}{2}}{\sigma_x^2} + \frac{\cos^2\frac{\phi}{2}}{\sigma_s^2} \qquad B = \frac{(d_2 - d_1)\sin(\phi/2)}{2\sigma_x^2}$$

and $$W = e^{-\frac{1}{4\sigma_x^2}(d_2-d_1)^2}$$

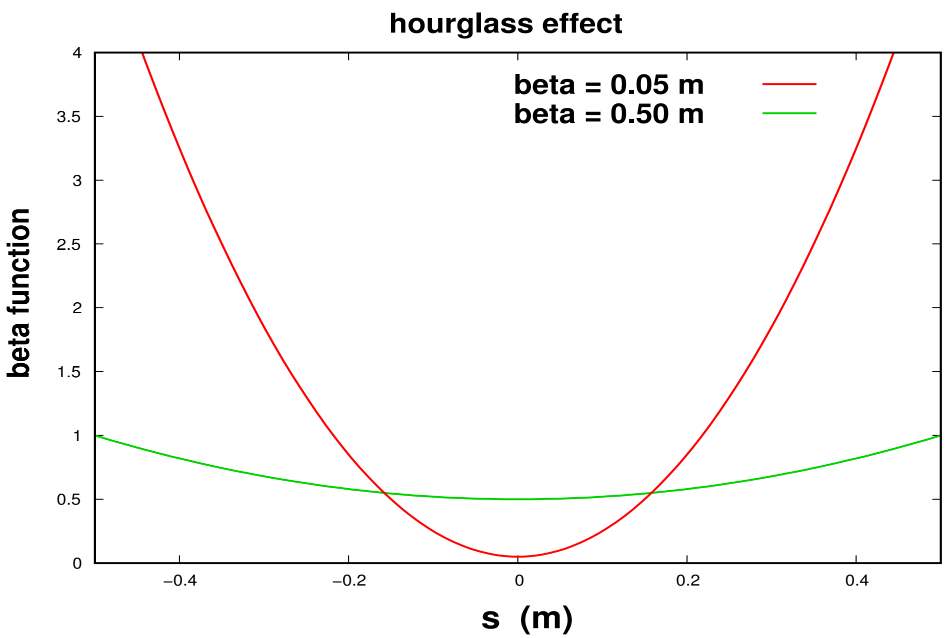
$\Longrightarrow$ **Luminosity with correction factors**

Luminosity with correction factors

$$\mathcal{L} = \frac{N_1 N_2 f n_b}{4\pi\sigma_x\sigma_y} \cdot W \cdot e^{\frac{B^2}{A}} \cdot S$$

- W: **correction for beam offset**
- S: **correction for crossing angle**
- $e^{\frac{B^2}{A}}$: **correction for crossing angle and offset**

Hour glass effect

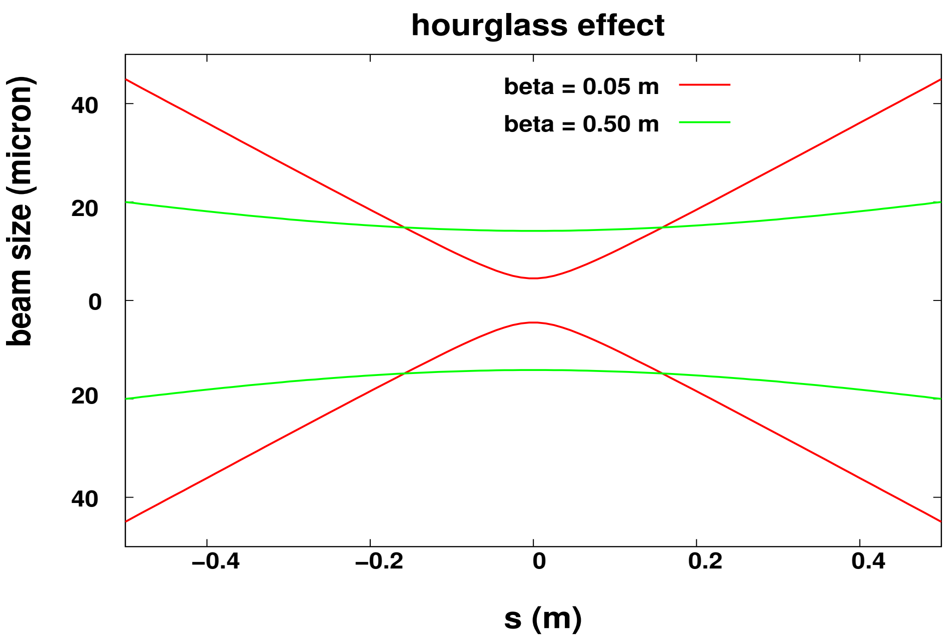


- β-functions depends on position s
- $\beta(s) \approx \beta^*(1 + \left(\frac{s}{\beta^*}\right)^2)$

Hour glass effect

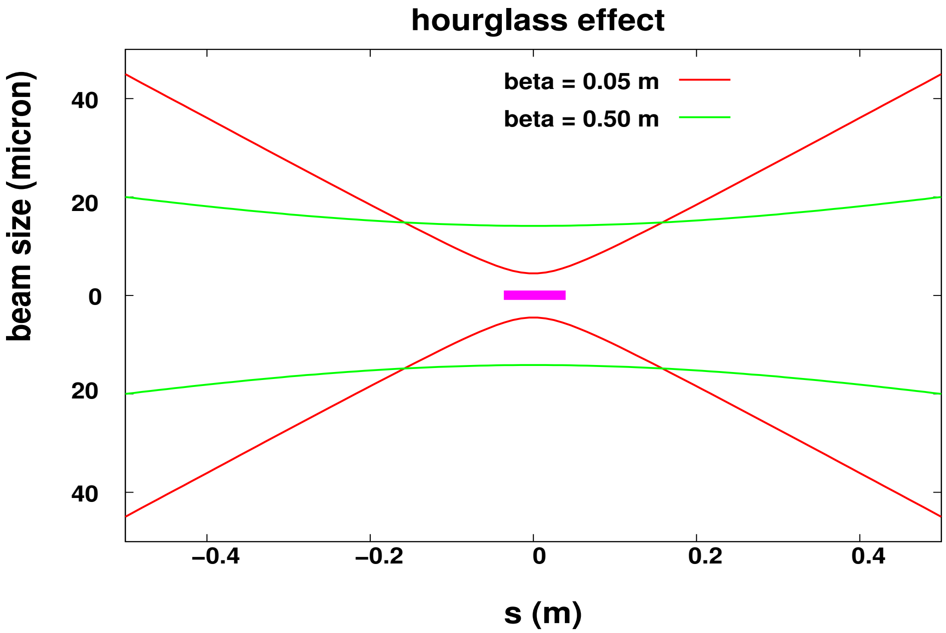
- β-functions depends on position s

Hour glass effect



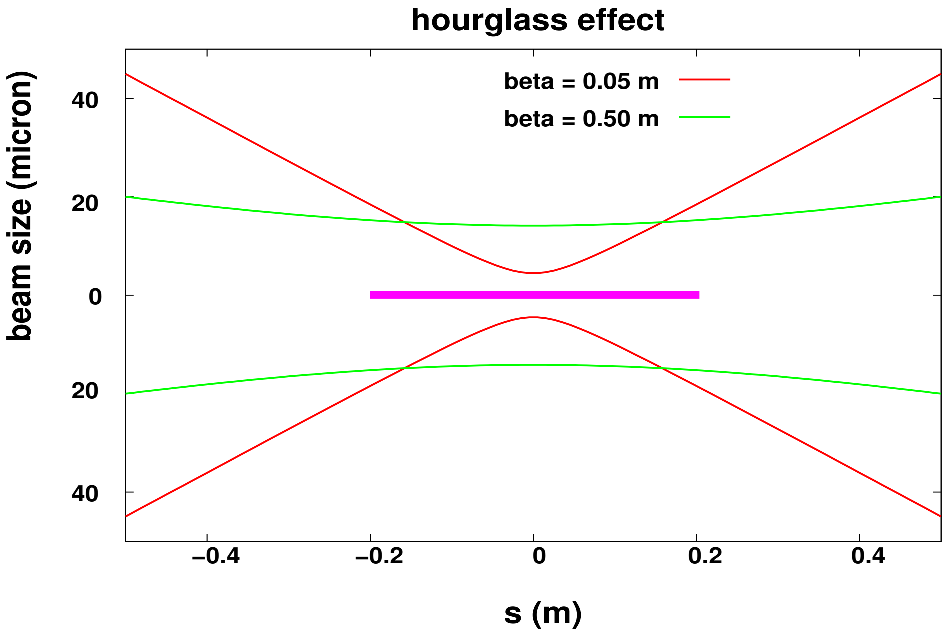
- **Beam size** $\sigma \quad (\propto \sqrt{\beta^*(s)})$ **depends on position s**

Hour glass effect - short bunches



- Small variation of beam size along bunch

Hour glass effect - long bunches



- Significant effect for long bunches and small β^*

Hour glass effect

- β**-functions depends on position s**
- **Usually:** $\beta(s) = \beta^*(1 + \left(\frac{s}{\beta^*}\right)^2)$
  - **i.e.** $\sigma \implies \sigma(s) \neq$ **const.**
  - $\sigma(s) = \sigma^* \sqrt{(1 + \left(\frac{s}{\beta^*}\right)^2)}$
- **Important when** β^* **comparable to the r.m.s. bunch length** σ_s **(or smaller !)**

Hour glass effect

- Take it easy: $\beta_x^* = \beta_y^*$, crossing angle, but no offset
- Replace σ by $\sigma(\mathbf{s})$ in standard formulae

- $$\mathcal{L} = \left(\frac{N_1 N_2 f n_b}{8\pi}\right) \frac{2\cos\frac{\phi}{2}}{\sqrt{\pi}\sigma_s} \int_{-\infty}^{+\infty} \frac{e^{-s^2 A}}{\sigma_x^* \sigma_y^* [1 + (\frac{s}{\beta^*})^2]} ds$$

- $$A = \frac{\sin^2\frac{\phi}{2}}{(\sigma_x^*)^2[1 + (\frac{s}{\beta^*})^2]} + \frac{\cos^2\frac{\phi}{2}}{\sigma_s^2}$$

- $\rightarrow$ **Numerical Integration**

Calculations for the LHC

- $N_1 = N_2 = 1.15 \times 10^{11}$ **particles/bunch**

- $n_b = 2808$ **bunches/beam**

- $f = 11.2455$ kHz, $\phi = 285$ μ**rad**

- $\beta_x^* = \beta_y^* = 0.55$ m

- $\sigma_x^* = \sigma_y^* = 16.6$ μm, $\sigma_s = 7.7$ cm

- **Simplest case (Head on collision):**

$$\mathcal{L} = 1.200 \times 10^{34}\ \mathrm{cm}^{-2}\mathrm{s}^{-1}$$

- **Effect of crossing angle:**

$$\mathcal{L} = 0.973 \times 10^{34}\ \mathrm{cm}^{-2}\mathrm{s}^{-1}$$

- **Effect of crossing angle & Hourglass:**

$$\mathcal{L} = 0.969 \times 10^{34}\ \mathrm{cm}^{-2}\mathrm{s}^{-1}$$

If the beams are not Gaussian ??

Exercise:

- **Assume flat distributions (normalized to 1)**

  $\rho_1 = \rho_2 = \frac{1}{2a}$, **for** $[-a \leq z \leq a]$, $z = x, y$

  Calculate r.m.s. in x and y:

$$< (x,y)^2 > = \int_{-\infty}^{+\infty} (x,y)^2 \cdot \rho(x,y) dx dy$$

and

$$\mathcal{L} = \int_{-\infty}^{+\infty} \rho_1(x,y) \rho_2(x,y) \; dx dy$$

- **Compute:** $\mathcal{L} \cdot \sqrt{< x^2 > \cdot < y^2 >}$
- **Repeat for various distributions and compare**

Integrated luminosity

- $$\mathcal{L}_{\text{int}} = \int_0^T \mathcal{L}(t)dt$$
- The figure of merit:

  $\mathcal{L}_{\text{int}} \cdot \sigma_p =$ **number of events**
- **Experiments: continuous recording of $\mathcal{L}$**
- **For studies: assume some life time behaviour.**

  E.g. $\mathcal{L}(t) \longrightarrow \mathcal{L}_0 \exp\left(-\frac{t}{\tau}\right)$
- **Contributions to life time from: intensity decay, emittance growth etc.**

Integrated luminosity

- **Knowledge of preparation time allows optimization of $\mathcal{L}_{\mathrm{int}}$**

Integrated luminosity

- **Typical run times LEP:**
  t_r **≈ 8 - 10 hours**
- **For LHC long preparation time t_p expected**

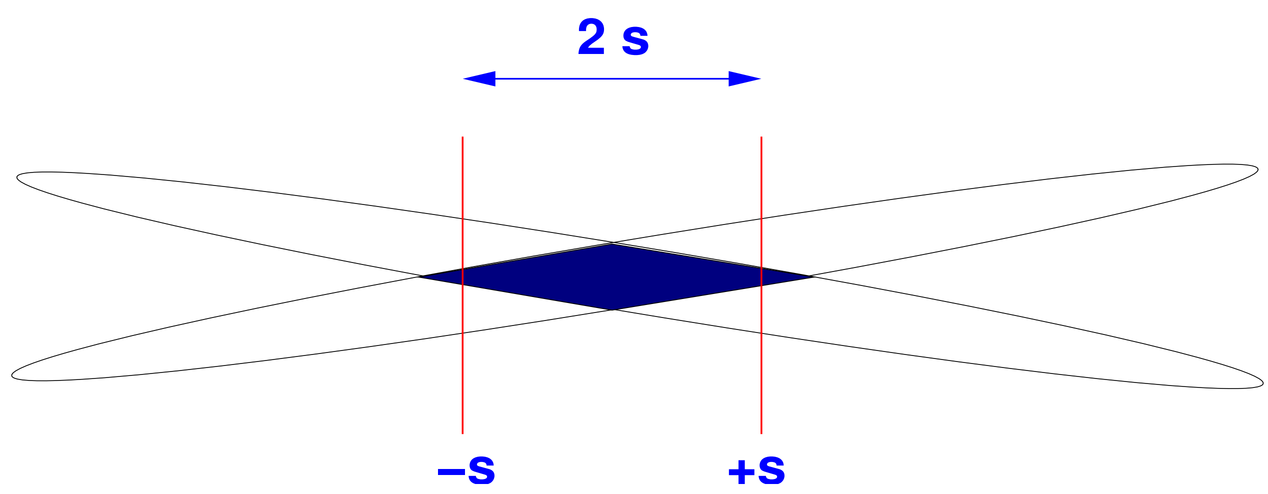
→ **Optimum combination of t_r and t_p gives maximum luminosity**

→ t_r **is usually a "free" parameter, i.e. can be chosen**

Maximising Integrated Luminosity

- **Assume exponential decay of luminosity**
  $\mathcal{L}(t) = \mathcal{L}_0 \cdot e^{t/\tau}$
- **Average luminosity** $< \mathcal{L} >$
  $< \mathcal{L} >= \frac{\int_0^{t_r} dt \mathcal{L}(t)}{t_r + t_p} = \mathcal{L}_0 \cdot \tau \cdot \frac{1 - e^{-t_r/\tau}}{t_r + t_p}$
- **(Theoretical) maximum for:**
  $t_r \approx \tau \cdot \ln(1 + \sqrt{2 t_p/\tau} + t_p/\tau)$
- **Example LHC:** $t_p \approx$ **10h,** $\tau \approx$ **15h,** $\Rightarrow t_r \approx$ **15h**
- **Exercise: Would you improve** τ **(long** t_r**) or** t_p **?**

Luminous region



- Density distribution of interaction vertices
- Fraction of collisions occur $\pm$ s from the IP ?
- Important for experiments !

Luminous region

- **Depends on σ_x, σ_y, σ_s and crossing angle ϕ**
- **Integrate only along a finite longitudinal length:**

$$\mathcal{L}_0 = \int_{-\infty}^{+\infty} \mathcal{L}(s')ds' \longrightarrow \mathcal{L}(S) = \int_{-S}^{+S} \mathcal{L}(s')ds'$$

$$\mathcal{L}(S) = \left(\frac{N_1 N_2 f n_b}{8\pi\sigma_x^* \sigma_y^*} \right) \frac{2\cos\frac{\phi}{2}}{\sqrt{\pi}\sigma_s} \sqrt{\frac{\pi}{A}} \ \mathrm{erf}\left(\sqrt{A} \ S \right)$$

- **Real figure of merit:** $\mathcal{L}_{\mathrm{int}}(S) = \int_0^T \int_{-S}^{+S} \mathcal{L}(s', t) ds' dt$

Some results for LHC

- $\sigma_s = 7.7$ cm, $\beta^* = 0.55$ m, $\phi = 285$ μrad:

- 100% **lumi** $\rightarrow$ $S = \pm$ 12 cm

- 90% **lumi** $\rightarrow$ $S = \pm$ 7 cm

- 80% **lumi** $\rightarrow$ $S = \pm$ 5.5 cm

Interactions per crossing

- **Luminosity/$fn_b \propto$ N_1N_2**
- **In LHC: crossing every 25 ns**
- **Per crossing approximately 20 interactions**
- **May be undesirable (pile up in detector)**
- $\Longrightarrow$ **more bunches n_b, or smaller N ??**

→ **Beware: maximum (peak) luminosity $\mathcal{L}_{max}$ is not the whole story ... !**

Luminosity measurement

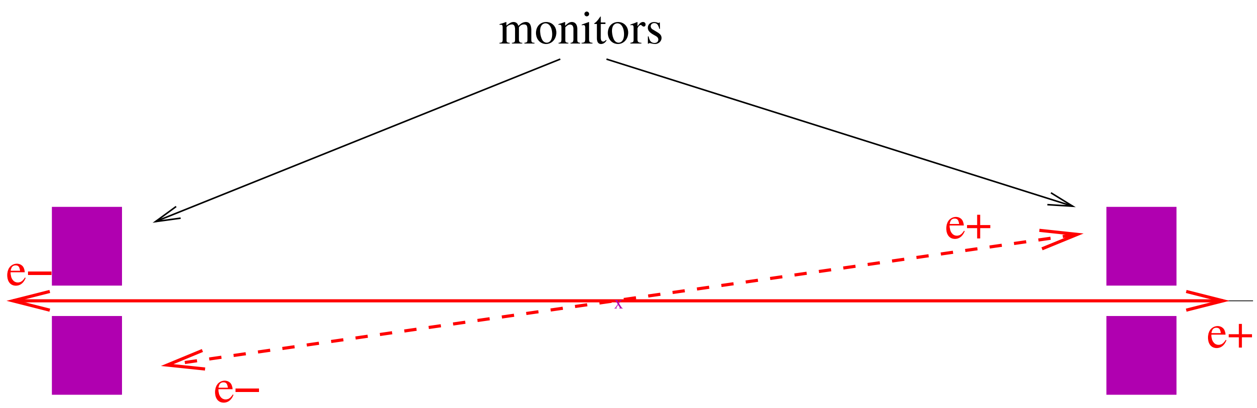
- One needs to get a signal proportional to interaction rate → Beam diagnostics
- Large dynamic range:
  10^{27} cm^{-2}s^{-1} to 10^{34} cm^{-2}s^{-1}
- Very fast, if possible for individual bunches
- Used for optimization
- For absolute luminosity need calibration

Luminosity calibration

(e^+e^-)

- **Use well known and calculable process**
- $e^+e^- \rightarrow e^+e^-$ **elastic scattering (Bhabha scattering)**
- **Have to go to small angles** $(\sigma_{el} \propto \Theta^{-3})$
- **Small rates at high energy** $(\sigma_{el} \propto \frac{1}{E^2})$

Luminosity calibration



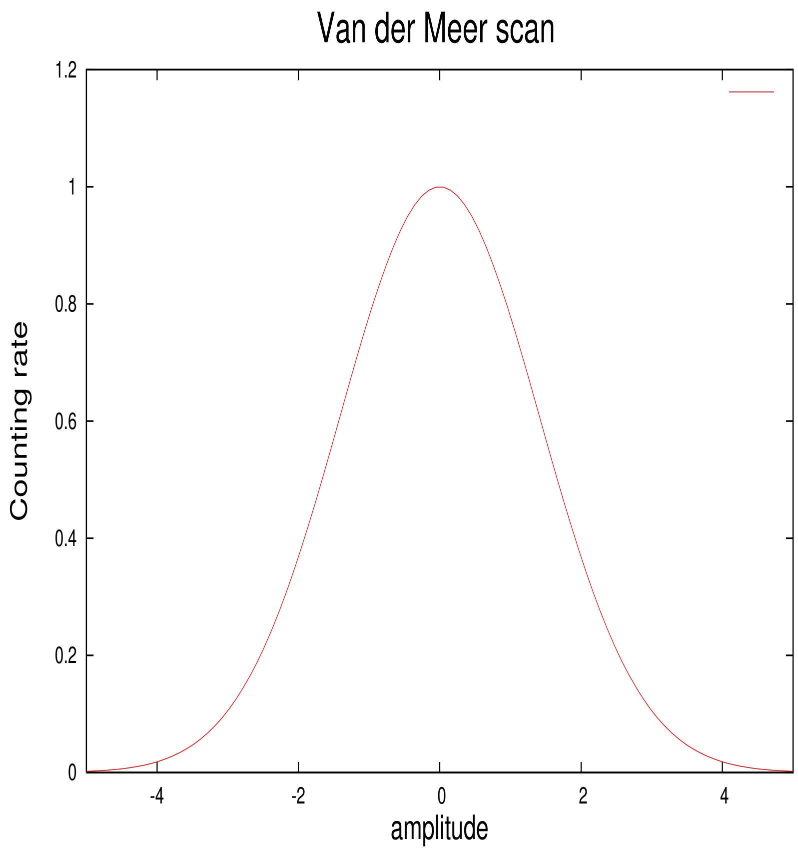
- Measure coincidence at small angles
- Low counting rates, in particular for high energy !
- Background may be problematic

Luminosity calibration

(hadrons, e.g. pp or $p\bar{p}$)

- **Must measure beam current and beam sizes**
- **Beam size measurement:**
  - **Wire scanner or synchrotron light monitors**
  - **Measurement with beam ... $\rightarrow$ remember luminosity with offset**
  - **Move the two beams against each other in transverse planes (van der Meer scan)**

Luminosity optimization



- Record counting rates ***R(d)*** as function of movement ***d***
- Since ***R(d)*** is proportional to luminosity ***L(d)***
- Get ratio of luminosity ***L(d)/L(0)***

Luminosity optimization

- **From ratio of luminosity $\mathcal{L}(\mathrm{d})/\mathcal{L}_0$**
- **Remember:** $W = e^{-\frac{1}{4\sigma^2}(d_2-d_1)^2}$
- **Determines σ**
- **... and centres the beams !**
- **"beam-beam deflection scans ..."** $\Longrightarrow$

Absolute value of $\mathcal{L}$ (pp or $p\bar{p}$)

- **By total rate and optical theorem**

  (also: luminosity independent determination of σ_{tot}):

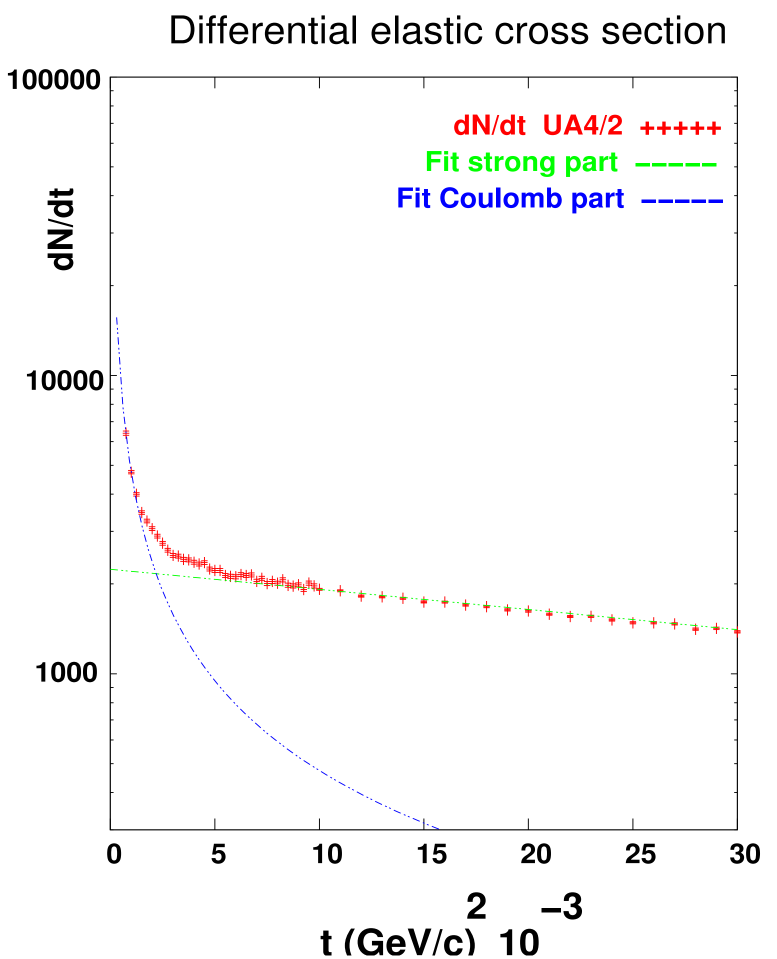
  - $\sigma_{tot} \cdot \mathcal{L} = N_{inel} + N_{el}$ **(Total counting rate)**
  - $$\lim_{t \to 0} \frac{d\sigma_{el}}{dt} = (1+\rho^2)\frac{\sigma_{tot}^2}{16\pi} = \frac{1}{\mathcal{L}}\frac{dN_{el}}{dt}|_{t=0}$$

  → $$\mathcal{L} = \frac{(1+\rho^2)}{16\pi}\frac{(N_{inel}+N_{el})^2}{(dN_{el}/dt)_{t=0}}$$

- **Luminosity determined from experimental rates**

Absolute value of $\mathcal{L}$ (pp or $p\bar{p}$)

- **By Coulomb normalization:**
  - **Coulomb amplitude exactly calculable:**
  - $$\lim_{t\to 0} \frac{d\sigma_{el}}{dt} = \frac{1}{\mathcal{L}} \frac{dN_{el}}{dt}\Big|_{t=0} = \pi \mid f_C + f_N \mid^2$$
    $$\simeq \pi \mid \frac{2\alpha_{em}}{-t} + \frac{\sigma_{tot}}{4\pi}(\rho + i)e^{b\frac{t}{2}} \mid^2 \simeq \frac{4\pi\alpha_{em}^2}{t^2} \Big|_{|t|\to 0}$$
  - **Fit gives:** σ_{tot}, ρ, b **and** $\mathcal{L}$
- **Can be done measuring only elastic scattering (No N$_{inel}$ needed !)**
- $\Longrightarrow$ **Roman pots**



- Measure dN/dt at small t (0.01 < (GeV/c)**2) and extrapolate to t = 0.0
- Needs special optics to allow measurement at very small t
- Measure total counting rate N_{el} + N_{inel}
  Needs good detector coverage
- Often use slightly modified method, precision 1 – 2 %

Not treated :

- **Coasting beams (e.g. ISR)**
- **Asymmetric colliders (e.g. PEP, HERA)**
- **Linear colliders (SLC, TESLA etc.)**

How to cook high Luminosity ?

- **Get high intensity**
- **Get small beam sizes (small ϵ and β^*)**
- **Get many bunches**
- **Get small crossing angle (if any)**
- **Get exact head-on collisions**
- **Get short bunches**