Beam Instrumentation & Diagnostics Part 1

CAS Introduction to Accelerator Physics

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Beam Instrumentation: **Functionality of devices & basic applications**

Beam Diagnostics: **Usage of devices for complex measurements**

Diagnostics is the ’sensory organs’ for the beam in the real environment.

(Referring to Volker Ziemann’s lecture: ‘Detecting imperfections to enable corrections’)

Different demands lead to different installations:

- Quick, non-destructive measurements leading to a single number or simple plots
  Used as a check for online information. Reliable technologies have to be used
  Example: Current measurement by transformers
- Complex instruments for severe malfunctions, accelerator commissioning & development
  The instrumentation might be destructive and complex
  Example: Emittance determination, chromaticity measurement

General usage of beam instrumentation:

- Monitoring of beam parameters for operation, beam alignment & accelerator development
- Instruments for automatic, active beam control
  Example: Closed orbit feedback at synchrotrons using position measurement by BPMs

Non-destructive (’non-intercepting’ or ‘non-invasive’) methods are preferred:

- The beam is not influenced $\Rightarrow$ the **same** beam can be measured at several locations
- The instrument is not destroyed due to high beam power

Outline of the Lectures

The ordering of the subjects is oriented by the beam quantities:

Part 1 of the lecture on electro-magnetic monitors:

- **Current measurement:** Transformers, Faraday cups, particle detectors
- **Pick-ups for bunched beams**: Principle of rf pick-ups& relevant beam measurements

Part 2 of the lecture on transverse and longitudinal diagnostics:

- **Profile measurement:** Various methods depending on the beam properties
- **Transverse emittance measure:** Destructive devices, linear transformations
- **Measurement of longitudinal parameters:** time structure of bunches, beam energy spread energies, longitudinal emittance

Lecture on Machin Protection System on Friday:

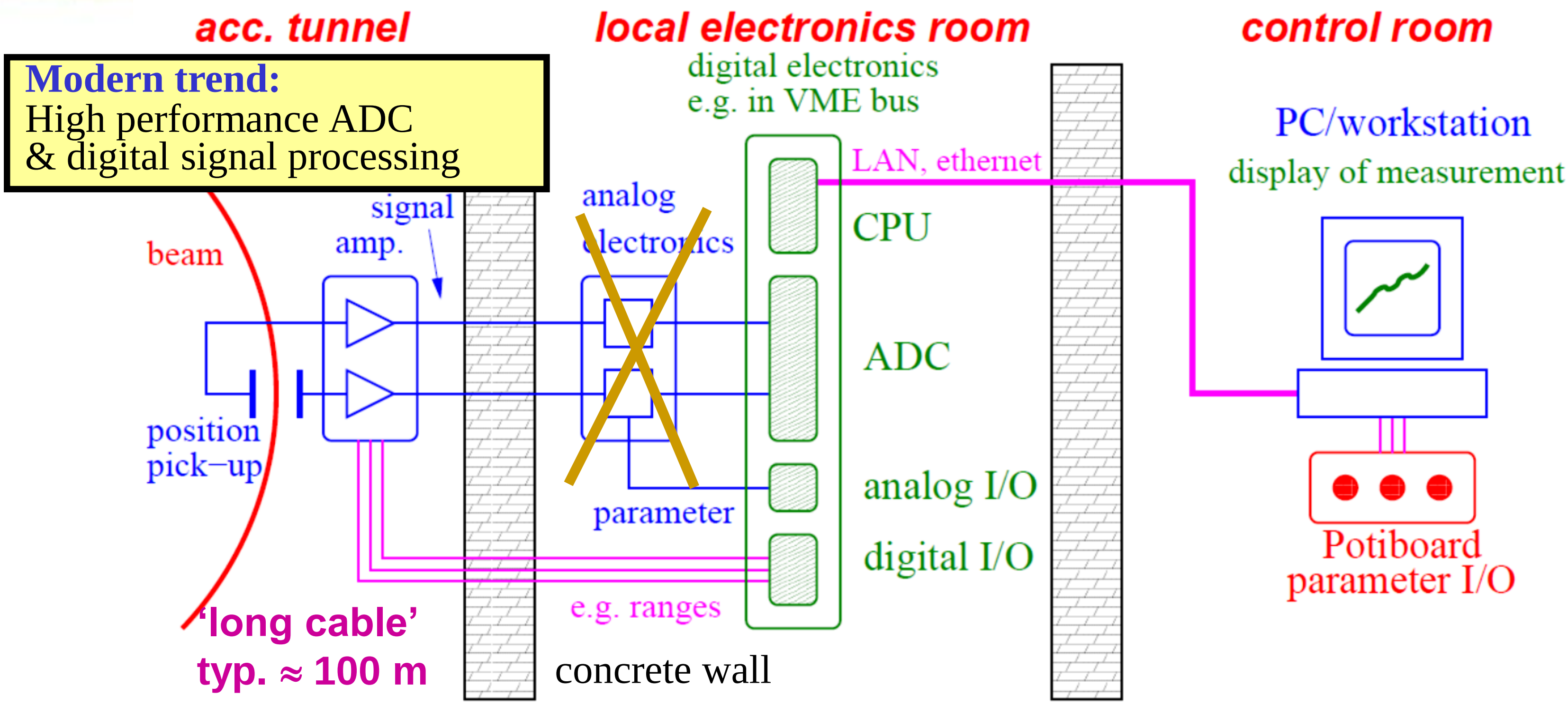
- **Beam loss detection:** Secondary particle detection for optimization and protection

Some instruments must be different for:

- Transfer lines with single pass ↔ synchrotrons with multi-pass
- Electrons are (nearly) always relativistic ↔ protons are at the beginning non-relativistic

Remark: Most instrumentation is installed outside of rf-cavities to prevent for signal disturbance

Typical Installation of a Beam Instrument



accelerator tunnel:
→ action of the beam to the detector
→ low noise pre-amplifier and first signal shaping

local electronics room:
→ analog treatment, partly combining other parameters
→ digitalization, data bus systems (GPIB, VME, cPCI, µTCA...)

control room:
→ visualization and storage on PC farm
→ parameter setting of the beam and the instruments

Measurement of Beam Current

The beam current and its time structure the basic quantity of the beam.

- It this the first check of the accelerator functionality
- It has to be determined in an absolute manner
- Important for transmission measurement and to prevent for beam losses.

Different devices are used:

- **Transformers:** Measurement of the beam's **magnetic field**

  They are non-destructive. No dependence on beam energy

  They have lower detection threshold.

- **Faraday cups:** Measurement of the beam's **electrical charges**

Magnetic field of the beam and the ideal Transformer

➢Beam current of N_{part} charges with velocity β

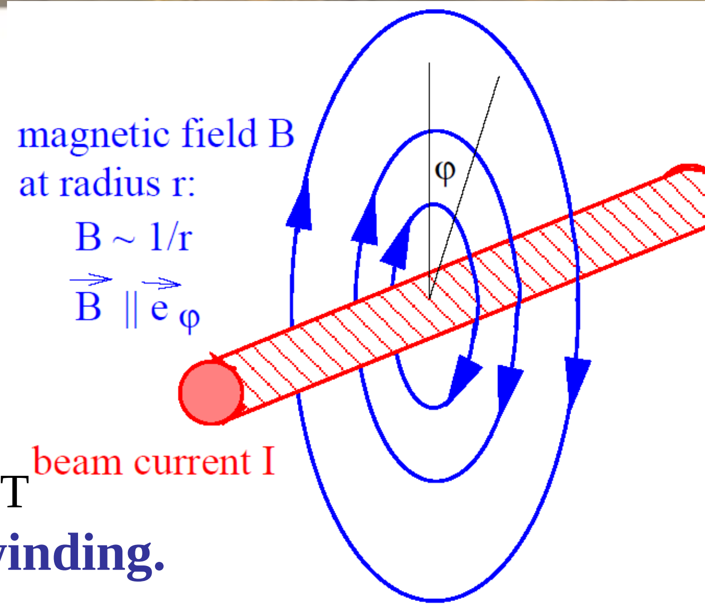
$$I_{beam} = qe \cdot \frac{N_{part}}{t} = qe \cdot \beta c \cdot \frac{N_{part}}{l}$$

➢ cylindrical symmetry

→ only azimuthal component

$$\vec{B} = \mu_0 \frac{I_{beam}}{2\pi r} \cdot \vec{e_\varphi}$$

Example: I =1µA, r =10cm ⇒ B_{beam}= 2pT, earth B_{earth}= 50µT



Idea: Beam as primary winding and sense by sec. winding.

⇒ Loaded current transformer

$$I_1/I_2 = N_2/N_1 \Rightarrow I_{sec} = 1/N \cdot I_{beam}$$

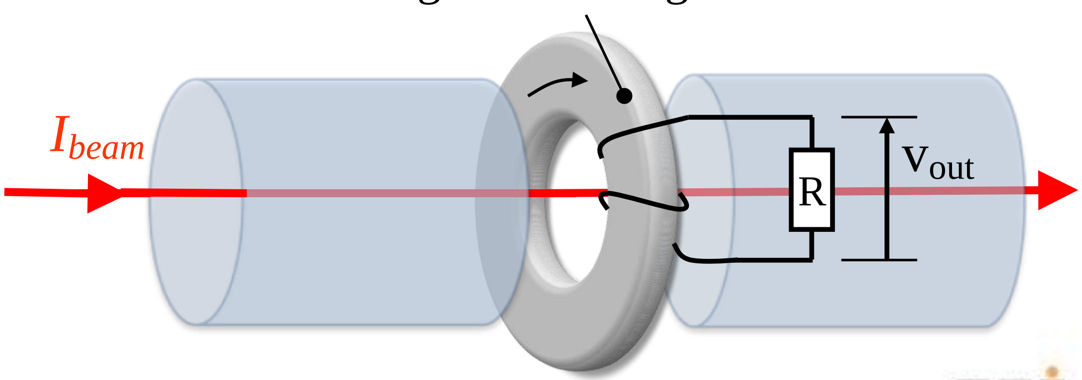
➢ Inductance of a torus of μ_r

$$L = \frac{\mu_0 \mu_r}{2\pi} \cdot lN^2 \cdot \ln \frac{r_{out}}{r_{in}}$$

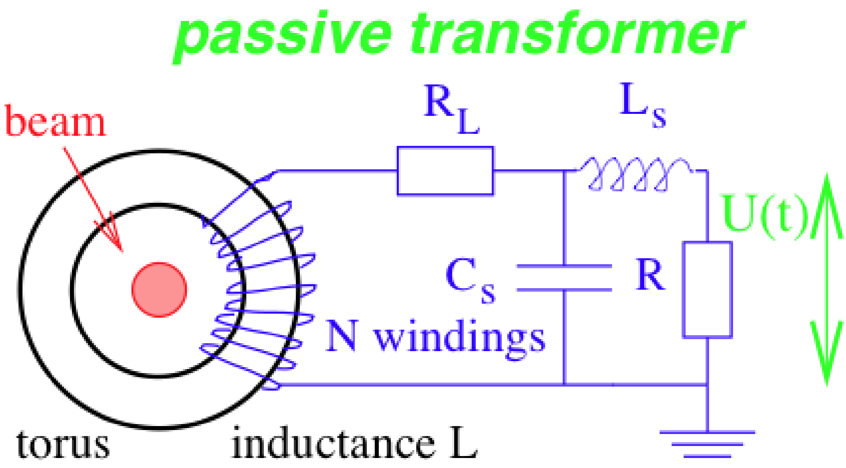
➢ Goal of torus: Large inductance ***L*** ***and*** guiding of field lines.

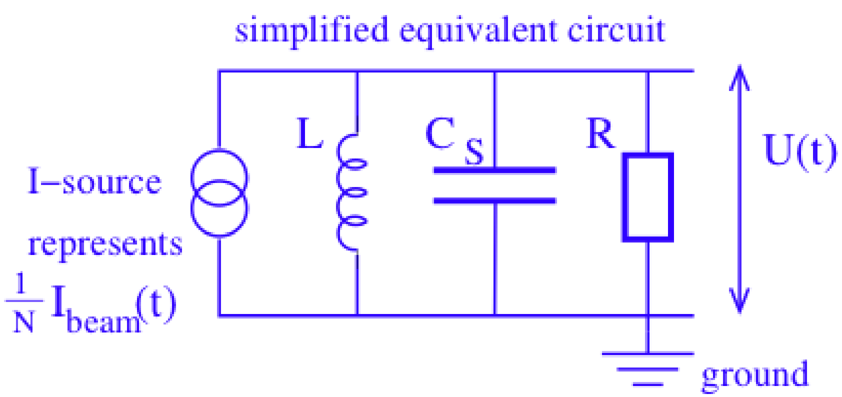
Definition: $U = L \cdot dI/dt$

Torus to guide the magnetic field



Simplified electrical circuit of a passively loaded transformer:





A voltages is measured: $U = R \cdot I_{sec} = R/N \cdot I_{beam} \equiv S \cdot I_{beam}$

with ***S* sensitivity [V/A],** equivalent to transfer function or transfer impedance ***Z***

Equivalent circuit for analysis of sensitivity and bandwidth

(disregarding the loss resistivity R_L)

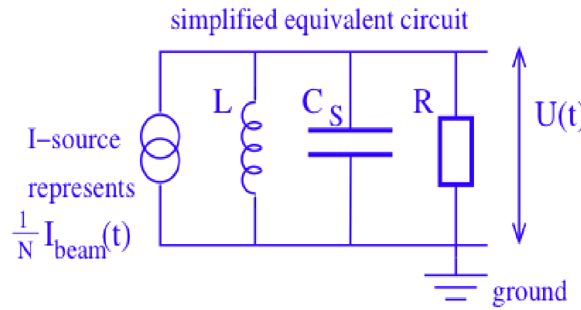
Time domain description:

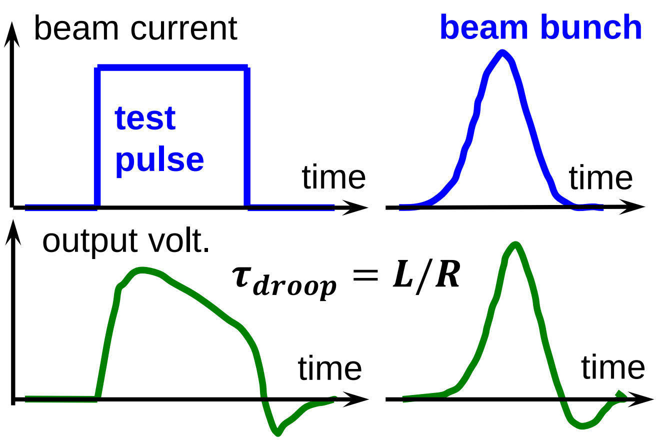
Droop time: $\tau_{droop} = 1/(2\pi f_{low}) = L/R$

Rise time: $\tau_{rise} = 1/(2\pi f_{high}) = 1/RC_S$ (ideal without cables)

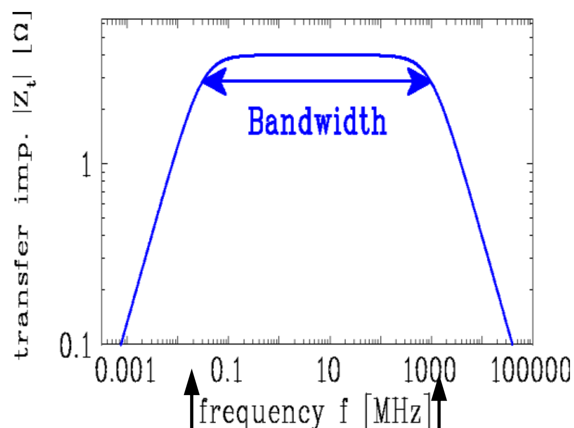
Rise time: $\tau_{rise} = 1/(2\pi f_{high}) = \sqrt{L_S C_s}$ (with cables)

R_L: loss resistivity, R: for measuring.



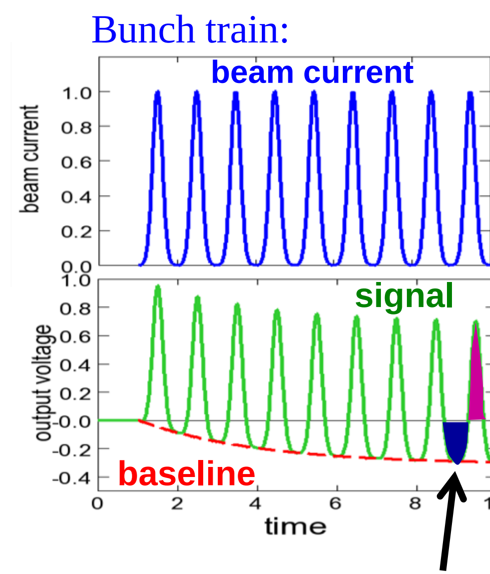


$\tau_{rise} = \sqrt{L \cdot C_S}$



$2\pi f_{low} = R/L$

$2\pi f_{high} = 1/RC_S$



Baseline: $U_{base} \propto 1 - \exp(-t/\tau_{droop})$
positive & negative areas are equal

Example for Fast Current Transformer

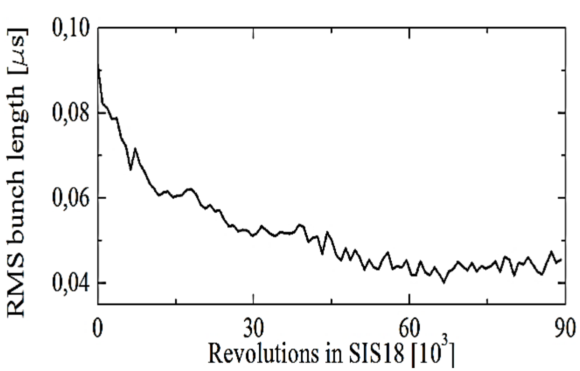
For bunch beams e.g. during accel. in a synchrotron typical bandwidth of 2 kHz < f < 1 GHz

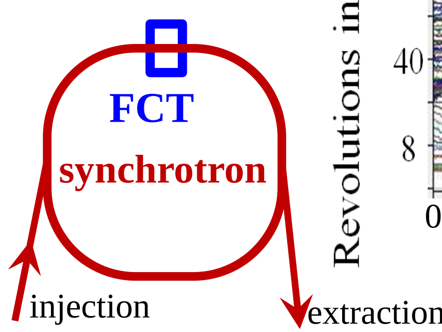
⇔ 10 ns < t_{bunch} < 1 µs is well suited

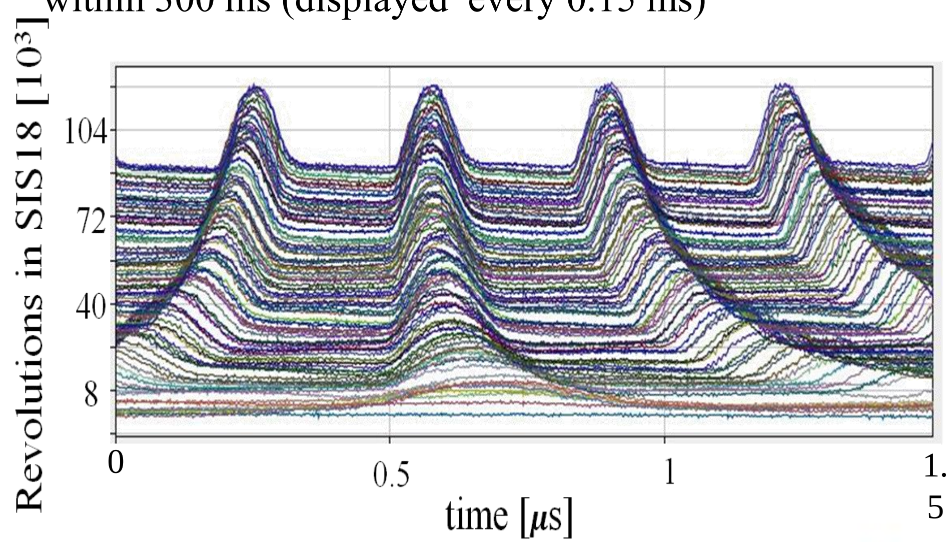
Example GSI type:

Inner / outer radius	70 / 90 mm
Torus thickness	16 mm
Permeability	$\mu_r \approx 10^5$ for f < 100kHz $\mu_r \propto 1/f$ above
Windings	10
Sensitivity	4 V/A for R = 50 Ω
Droop time τ_{droop} = L/R	0.2 ms
Rise time $\tau_{rise} = \sqrt{L_S C_S}$	1 ns
Bandwidth	2 kHz ... 500 MHz

Example: U^{73+} from 11 MeV/u (β = 15 %) to 350 MeV/u within 300 ms (displayed every 0.15 ms)







Example for Fast Current Transformer

For bunch beams e.g. transfer between synchrotrons typical bandwidth of 2 kHz < f < 1 GHz

⇔ 1 ns < t_{batch} < 200 μs is well suited

Example GSI type:

Inner / outer radius	70 / 90 mm
Torus thickness	16 mm
Permeability	$\mu_r \approx 10^5$ for f < 100kHz $\mu_r \propto$ 1/f above
Windings	10
Sensitivity	4 V/A for R = 50 Ω
Droop time τ_{droop} = L/R	0.2 ms
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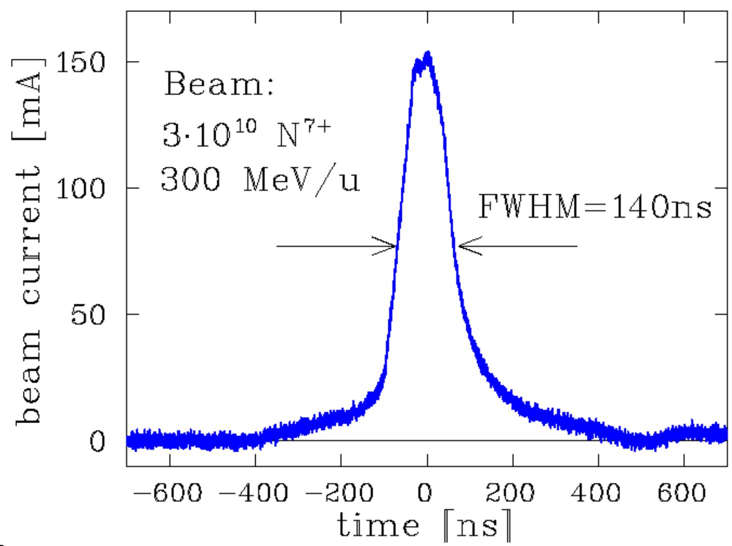
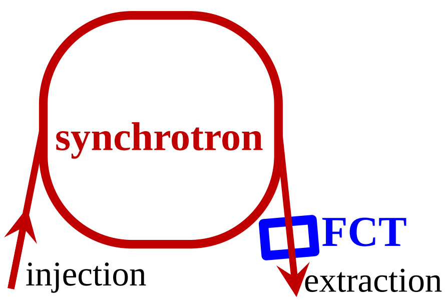
Fast extraction from GSI synchrotron:



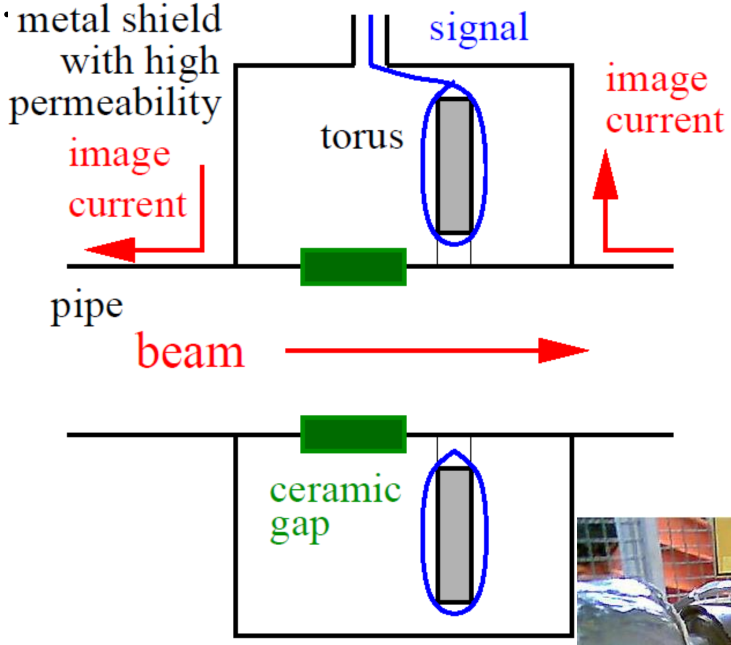
Numerous application e.g.:

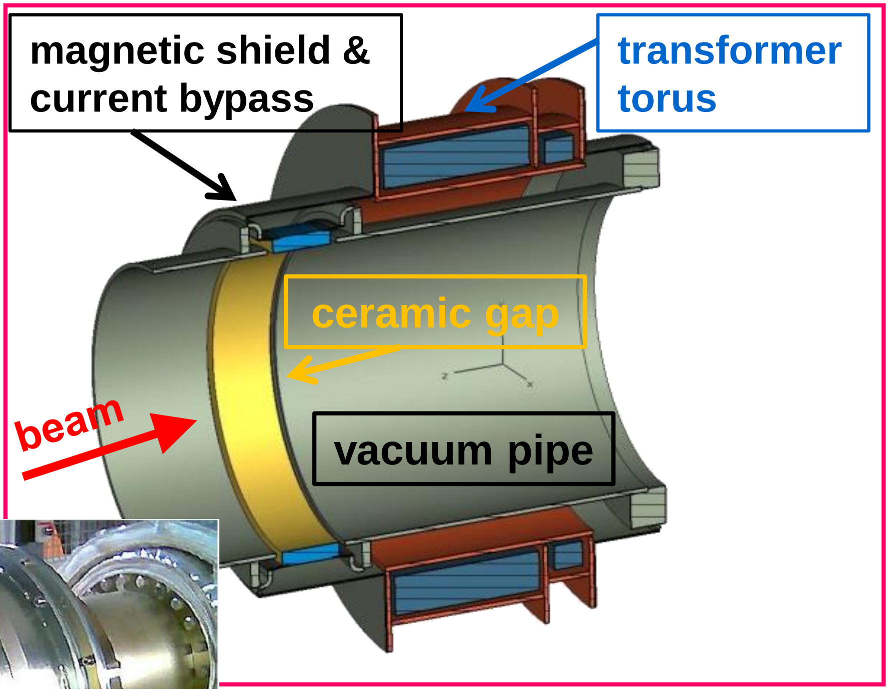
- Transmission optimization
- Bunch shape measurement
- Input for synchronization of 'beam phase'



Task of the shield:

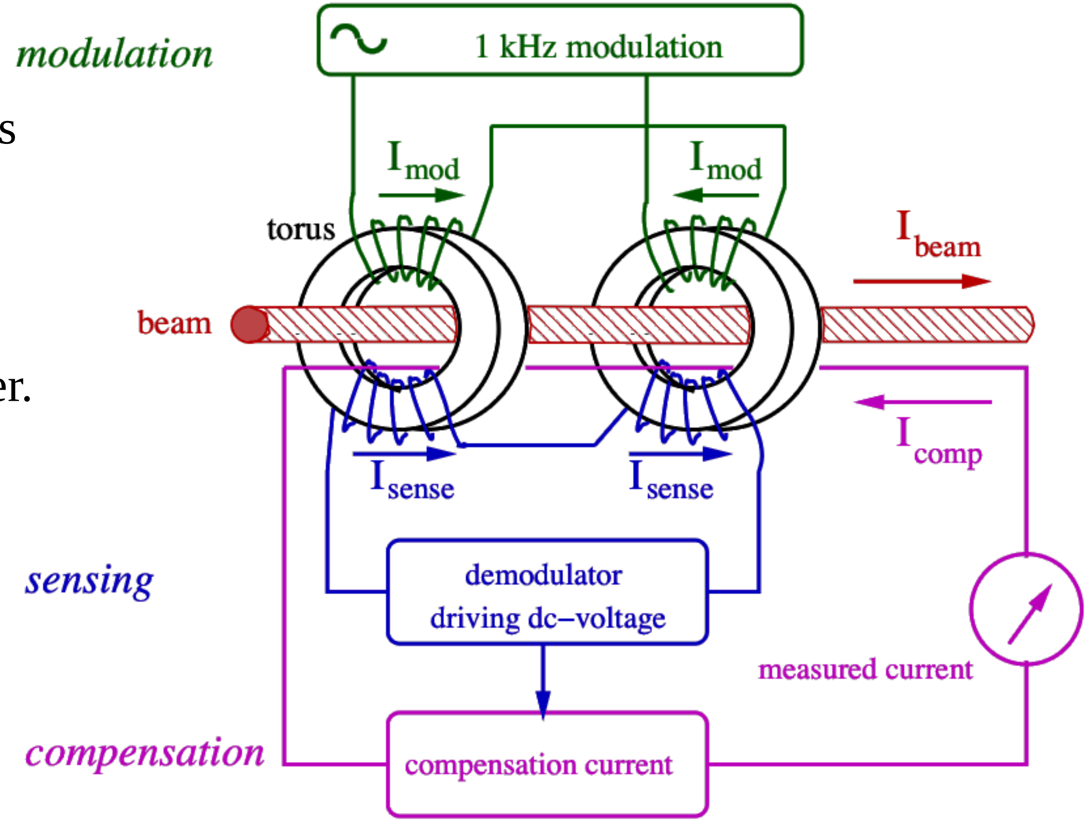
- The image current of the walls have to be bypassed by a gap and a metal housing.
- This housing uses μ-metal and acts as a shield of external B-field
  (remember: I_{beam} = 1 μA, r = 10 cm ⇒ B_{beam} = 2pT, earth field B_{earth} = 50 μT)



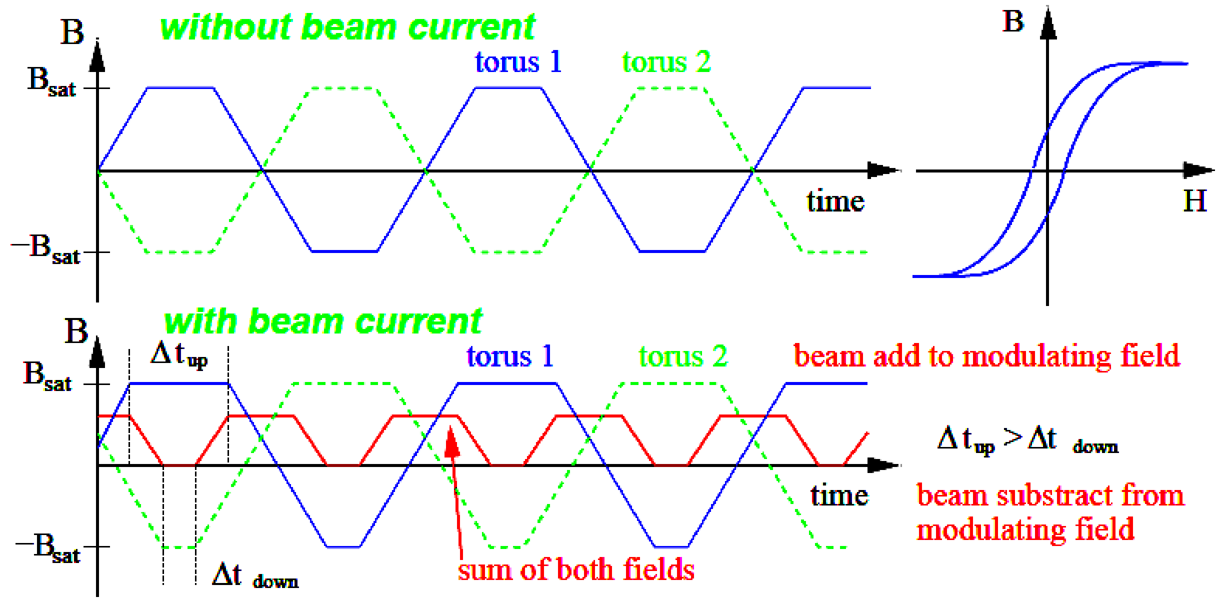


How to measure the DC current? The current transformer discussed sees only B-flux ***changes.***
The DC Current Transformer (DCCT) → look at the magnetic saturation of two torii.

- **Modulation** of the primary windings forces both torii into saturation twice per cycle
- **Sense windings** measure the modulation signal and cancel each other.
- But with the I_{beam}, the saturation is shifted and I_{sense} is not zero
- **Compensation current** adjustable until I_{sense} is zero once again



The dc Transformer

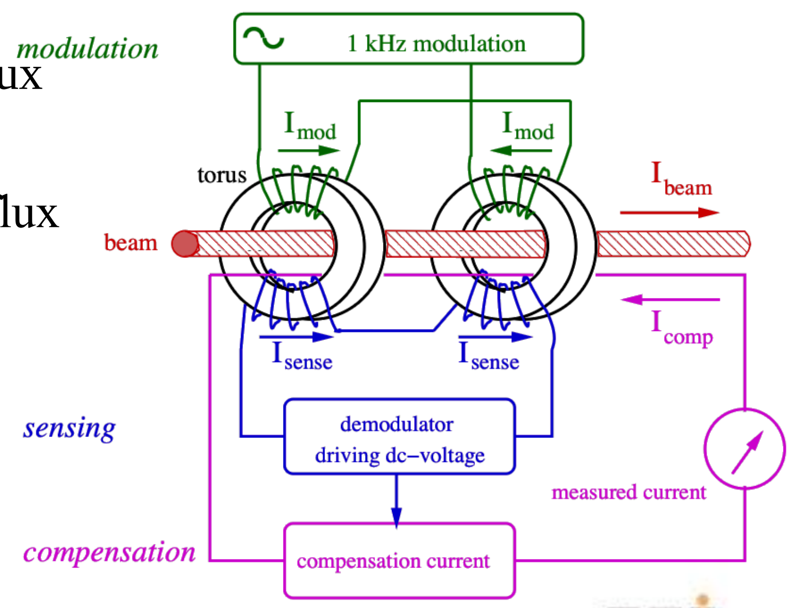


- **Modulation without beam:**

  typically about 9 kHz to saturation → **no** net flux
- **Modulation with beam:**

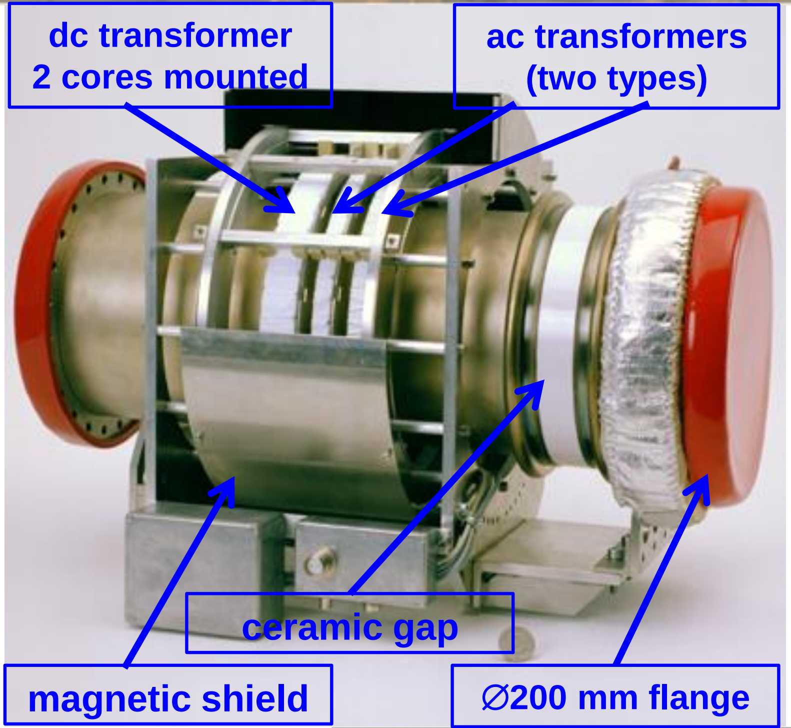
  saturation is reached at different times, → net flux
- **Net flux:** double frequency than modulation
- **Feedback:** Current fed to compensation winding

  for larger sensitivity
- **Two magnetic cores:** Must be very similar.



Example: The DCCT at GSI synchrotron

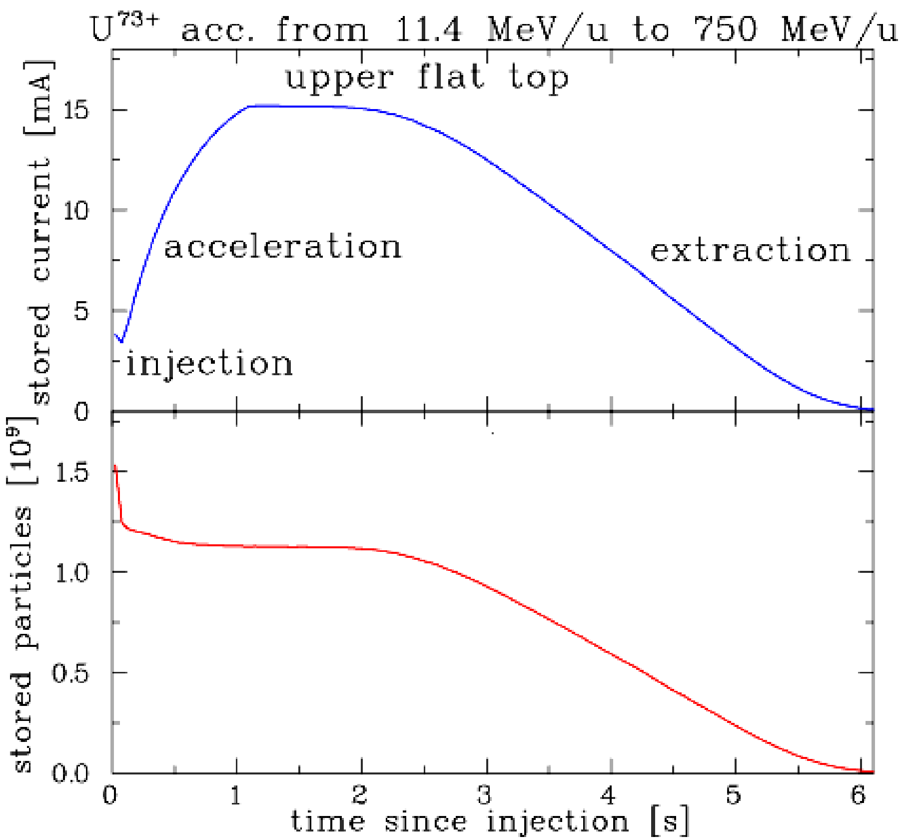
Torus radii	r_i = 135 mm r_o=145 mm
Torus thickness	d = 10 mm
Torus permeability	$\mu_r = 10^5$
Saturation inductance	B_{sat} = 0.6 T
Number of windings	16 for modulation & sensing 12 for feedback
Resolution	I^{min}_{beam} = 2 μA
Bandwidth	Δf = dc .... 20 kHz
Rise time constant	τ_{rise} = 10 μs
Temperature drift	1.5 μA/^{o}C



***Application for dc transformer*:**

⇒ Observation of beam behavior with typ. 20 μs time resolution → **the basic operation tool**

Example: The DCCT at GSI synchrotron:



Important parameter:

Detection threshold: ≈ 1 μA

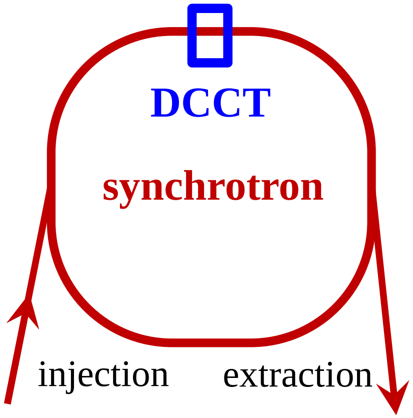
(= resolution)

Bandwidth: Δf = dc to 20 kHz

Rise-time: t_{rise} = 20 μs

Temperature drift: 1.5 μA/^{0}C

⇒ compensation required.



Measurement of Beam Current

The beam current is the basic quantity of the beam.

- It this the first check of the accelerator functionality
- It has to be determined in an absolute manner
- Important for transmission measurement and to prevent for beam losses.

Different devices are used:

- **Transformers:** Measurement of the beam's **magnetic field**

  They are non-destructive. No dependence on beam energy

  They have lower detection threshold.

- **Faraday cups:** Measurement of the beam's **electrical charges**

  They are destructive. For low energies only
  Low currents can be determined.

Bethe Bloch formula:
(simplest formulation)

$$-\frac{dE}{dx} = 4\pi N_A r_e m_e c^2 \cdot \frac{Z_t}{A_t}\rho_t \cdot \frac{Z_p^{\ 2}}{\beta^2}\left(\ln \frac{2m_e c^2\gamma^2\beta^2}{I} - \beta^2 \right)$$

Semi-classical approach:

- Projectiles of mass ***M*** collide with free electrons of mass ***m***
- If ***M*** **>>** ***m*** then the relative energy transfer is low

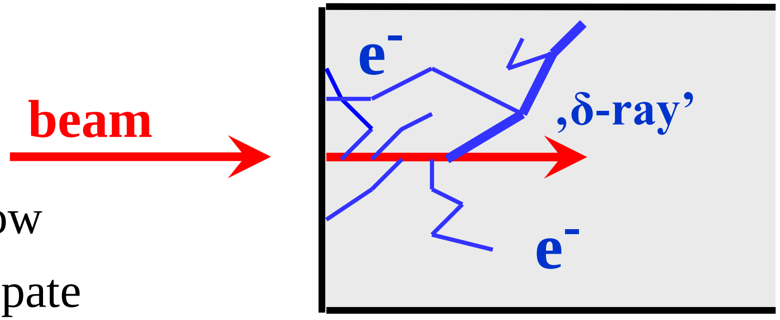
⇒ many collisions required many elections participate

proportional to electron density $n_e = \frac{Z_t}{A_t}\rho_t$

⇒ low straggling for the heavy projectile i.e. 'straight trajectory'

- If projectile velocity $\beta \approx 1$ low relative energy change of projectile (γ is Lorentz factor)
- I is mean ionization potential including kinematic corrections $I \approx Z_t \cdot 10\ eV$ for most metals
- Strong dependence an projectile charge Z_p

Constants: N_A Advogadro number, r_e classical e$^-$ radius, m_e electron mass, c velocity of light



Energy Loss of Protons & Ions in Copper

Bethe Bloch formula: $$-\frac{dE}{dx} = 4\pi N_A r_e m_e c^2 \cdot \frac{Z_t}{A_t}\rho_t \cdot \frac{Z_p^{\,2}}{\beta^2}\left(\ln\frac{2m_e c^2\gamma^2\beta^2}{I} - \beta^2\right)$$

Range: $$R = \int_0^{E_{\max}} \left(\frac{dE}{dx}\right)^{-1} dE$$

with approx. scaling $\boldsymbol{R \propto E_{max}^{1.75}}$

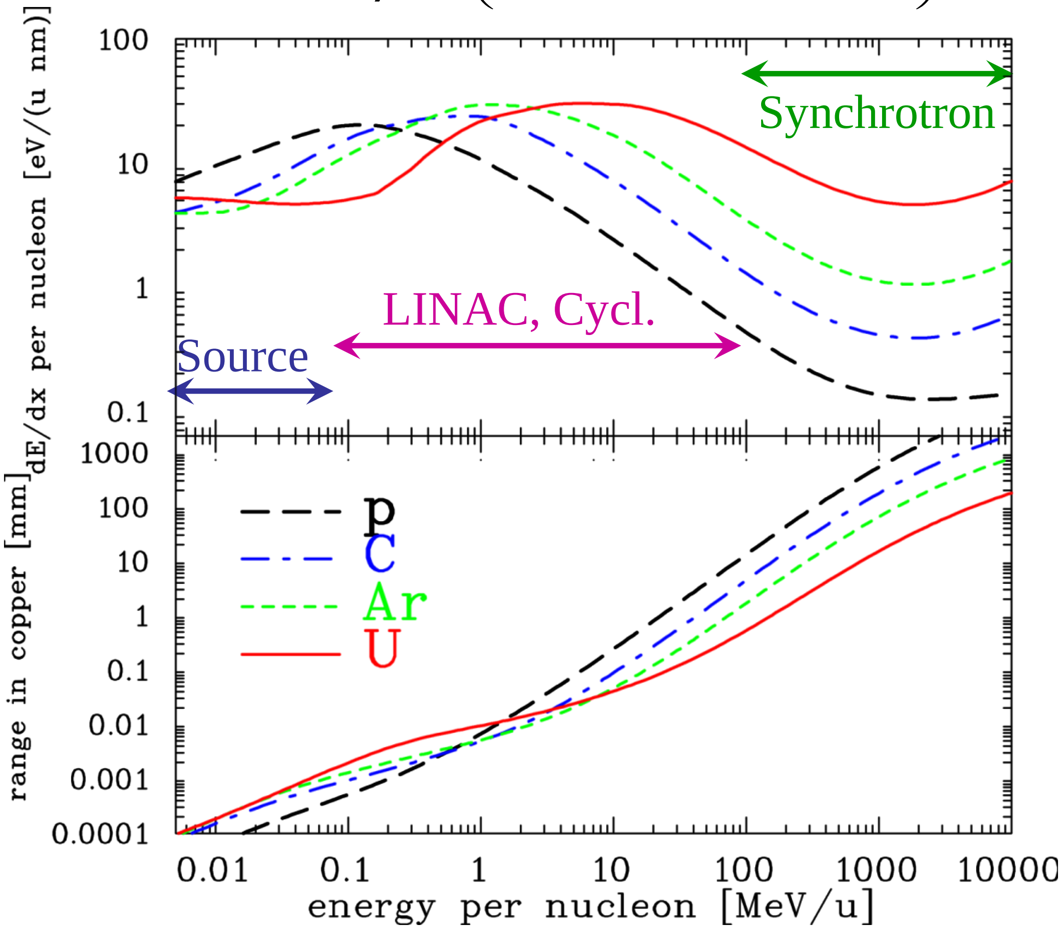
Numerical calculation for **ions**

with semi-empirical model e.g. SRIM

Main modification $Z_p \longrightarrow Z^{eff}_p(E_{kin})$

⇒ Cups only for

E_{kin} < 100 MeV/u due to R < 10 mm



Secondary Electron Emission caused by Ion Impact

Energy loss of ions in metals close to a surface:

Closed collision with large energy transfer: $\rightarrow$ fast e^- with E_{kin} > 100 eV

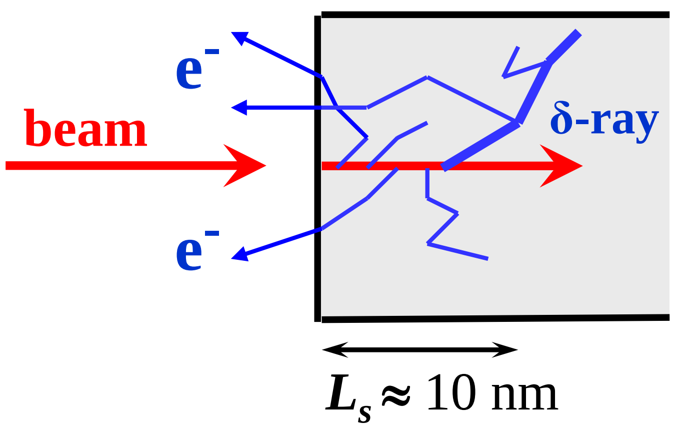
Distant collision with low energy transfer $\rightarrow$ slow e^- with $E_{kin} \leq$ 10 eV

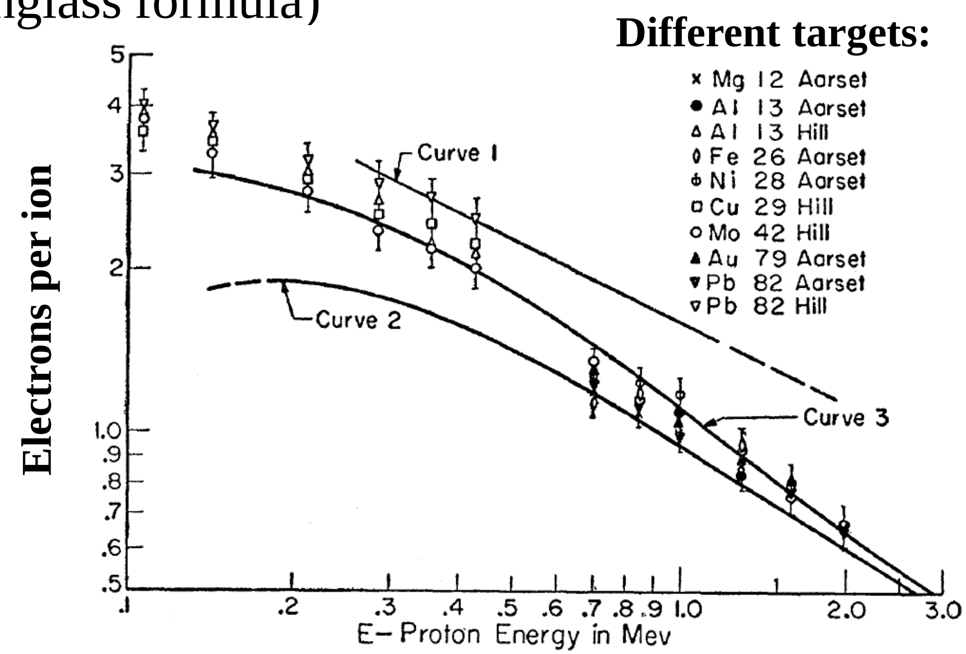
$\rightarrow$ 'diffusion' & scattering with other e^-: scattering length $L_s \approx$ 1 - 10 nm

$\rightarrow$ at surface $\approx$ 90 % probability for escape

Secondary **electron yield** and energy distribution comparable for all metals!

$\Rightarrow$ $Y = const. * dE/dx$ (Sternglass formula)

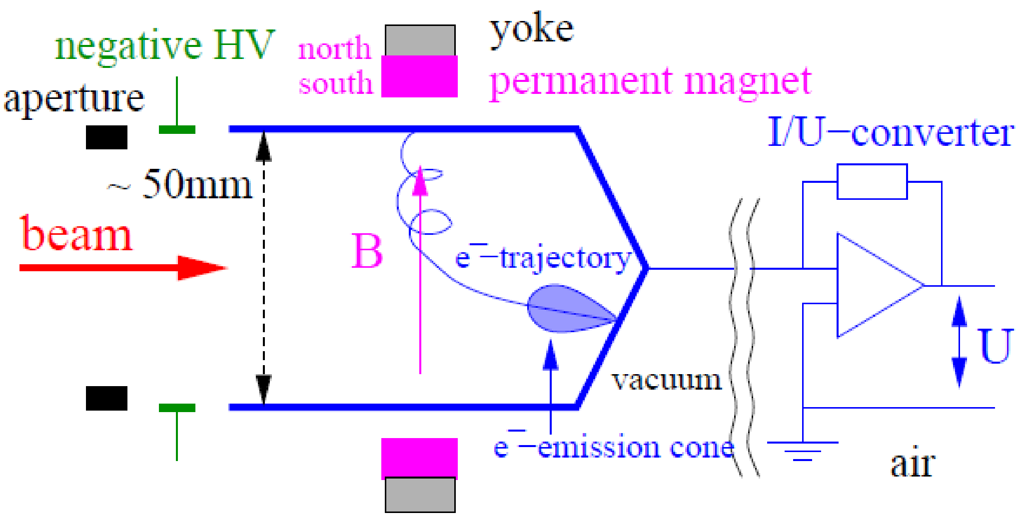


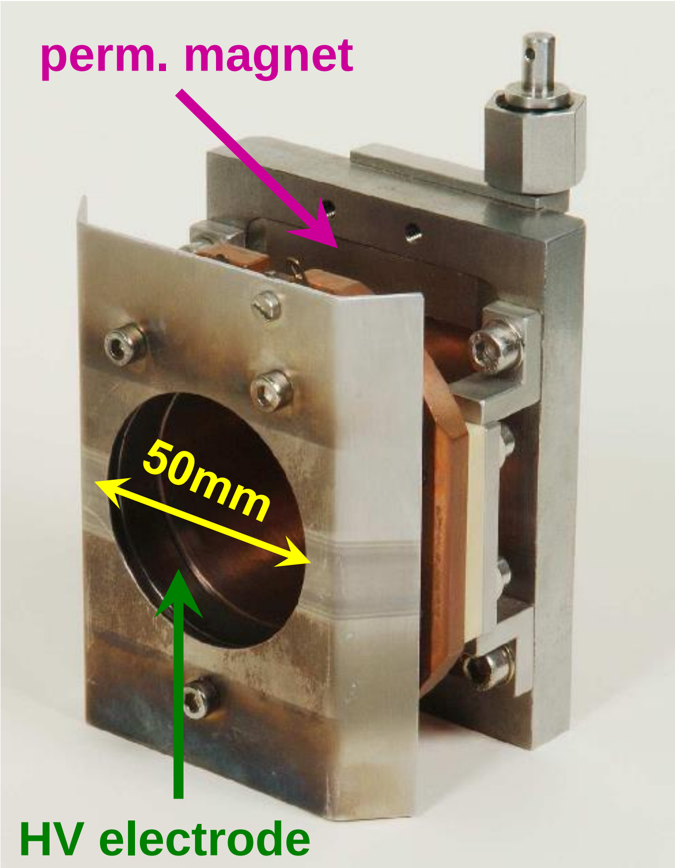


E.J. Sternglass, Phys. Rev. 108, 1 (1957)

The beam particles are collected inside a metal cup
⇒ The beam's charge are recorded as a function of time.

The cup is moved in the beam pass
→ destructive device





Currents down to 10 pA with bandwidth of 100 Hz!

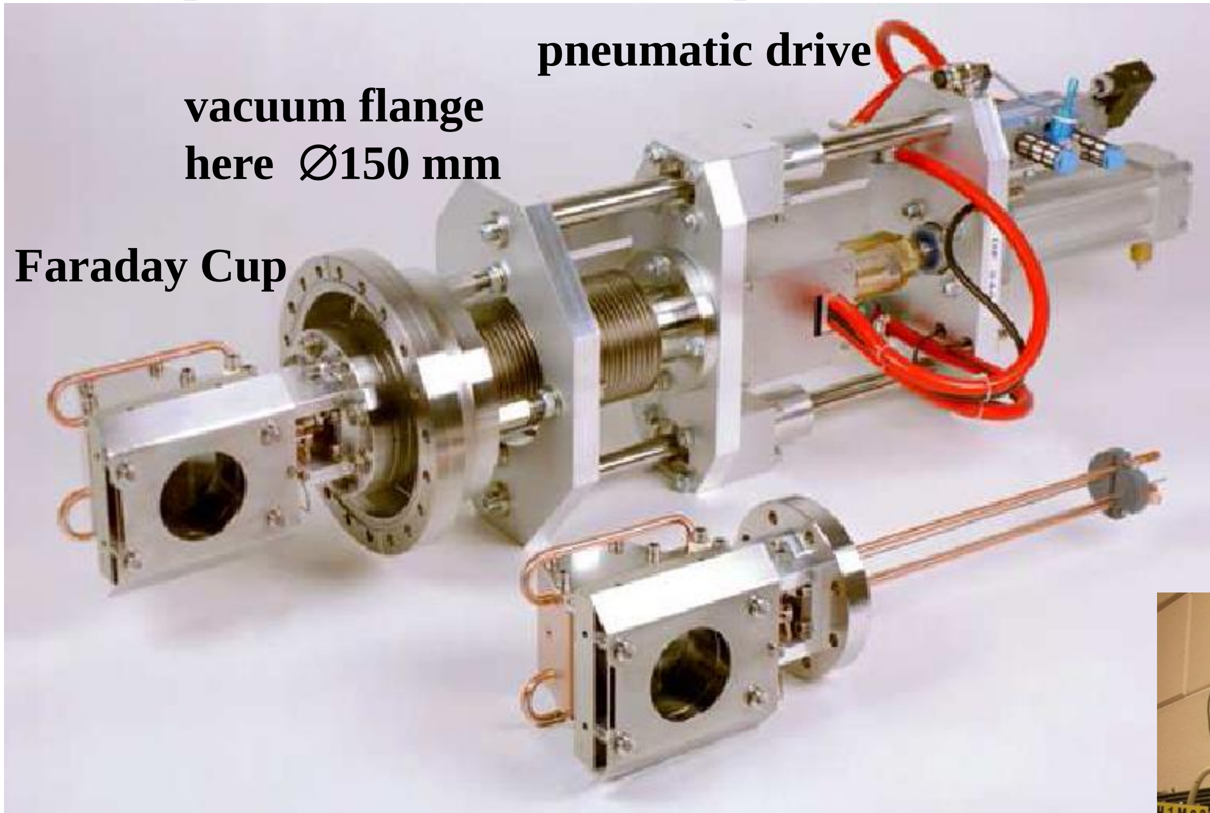
To prevent for secondary electrons leaving the cup

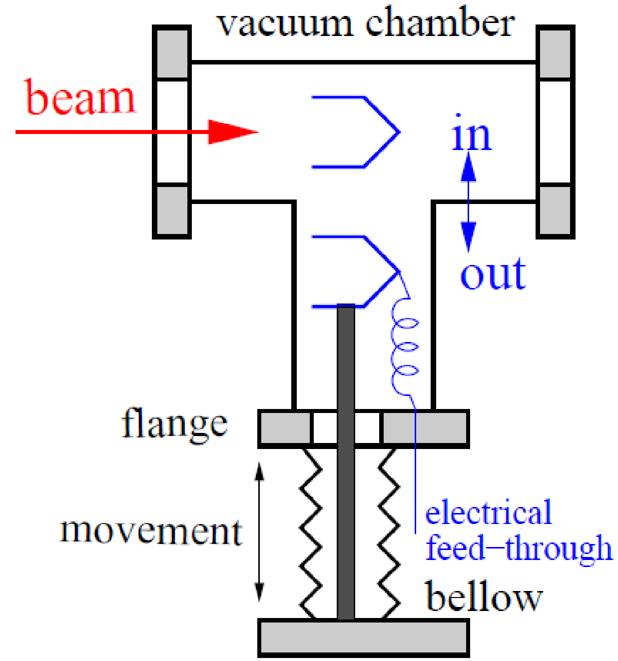
Magnetic field:

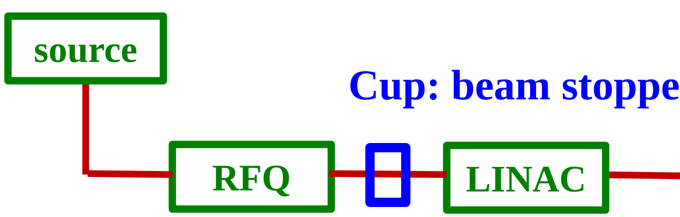
The central field is B ≈ 10 mT ⇒ $r_c = \frac{mB}{e} \cdot v_\perp \approx 1 \text{ mm}$.

or **Electric field:** Potential barrier at the cup entrance U ≈ 1 kV.

The Cup is moved into the beam pass.







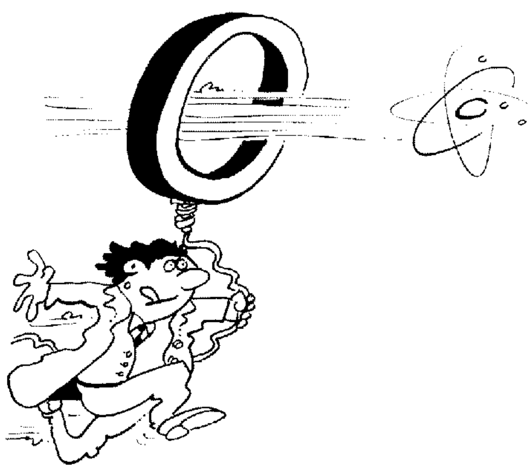
***Transformer:* → measurement of the beam's magnetic field**

- Magnetic field is guided by a high μ toroid
- **Types:** FCT → large bandwidth, $I_{min} \approx 30$ μA, BW = 10 kHz ... 500 MHz
  [ACT : $I_{min} \approx 0.3$ μA, BW = 10 Hz .... 1 MHz, used at proton LINACs]
  DCCT: two toroids + modulation, $I_{min} \approx 1$ μA, BW = dc ... 20 kHz
- non-destructive, used for all beams

***Faraday cup:* → measurement of beam's charge,**

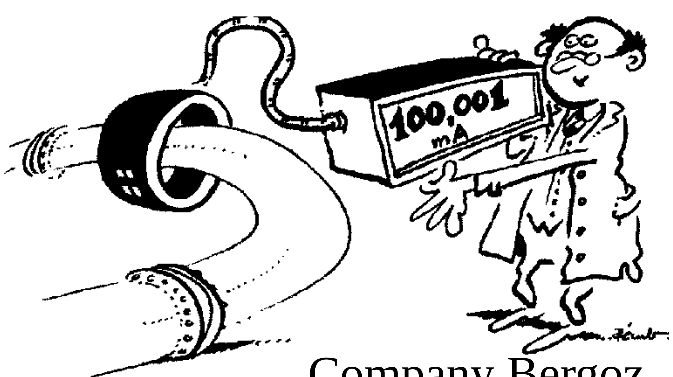
- low threshold by I/U-converter: $I_{beam} > 10$ pA
- totally destructive, used for low energy beams only

Fast Transformer FCT



Active transformer ACT

DC transformer DCCT



Company Bergoz

Outline:

- **Signal generation → transfer impedance**
- **Capacitive *button* BPM for high frequencies**
- **Capacitive *linear-cut* BPM for low frequencies**
- **Electronics for position evaluation**
- **BPMs for measurement**
- **Summary**

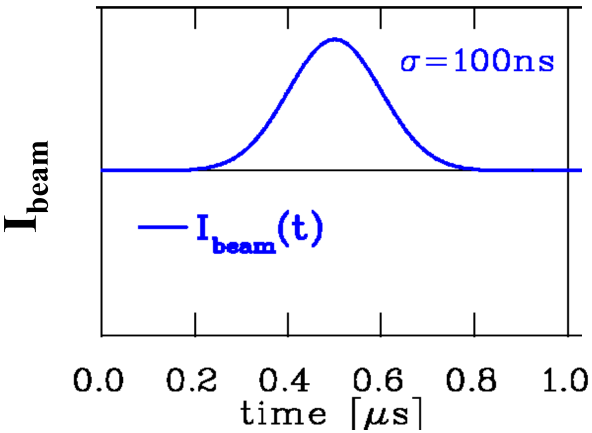
A Beam Position Monitor is an non-destructive device for bunched beams

1. It delivers information about the transverse center of the beam

- ***Trajectory:*** Position of an individual bunch within a transfer line or synchrotron
- ***Closed orbit***: Central orbit averaged over a period much longer than a betatron oscillation
- ***Single bunch position:*** Determination of parameters like tune, chromaticity, $\boldsymbol{\beta}$-function

Remarks: BPMs have a low cut-off frequency ⇔. dc-beam behavior can't be monitored
The abbreviation **BPM** and pick-up **PU** are synonyms

Time domain: **Recording of a voltage as a function of time:**



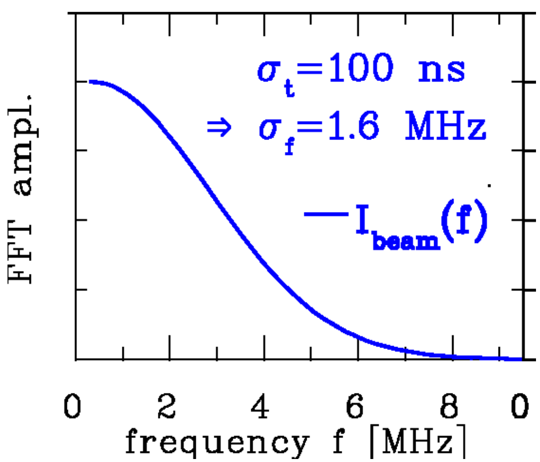
Instrument:

Oscilloscope

Fourier Transformation:

$$\tilde{f}(\omega) = \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt$$

Frequency domain: **Displaying of a voltage as a function of frequency:**



Instrument:

Spectrum Analyzer

Fourier Transformation

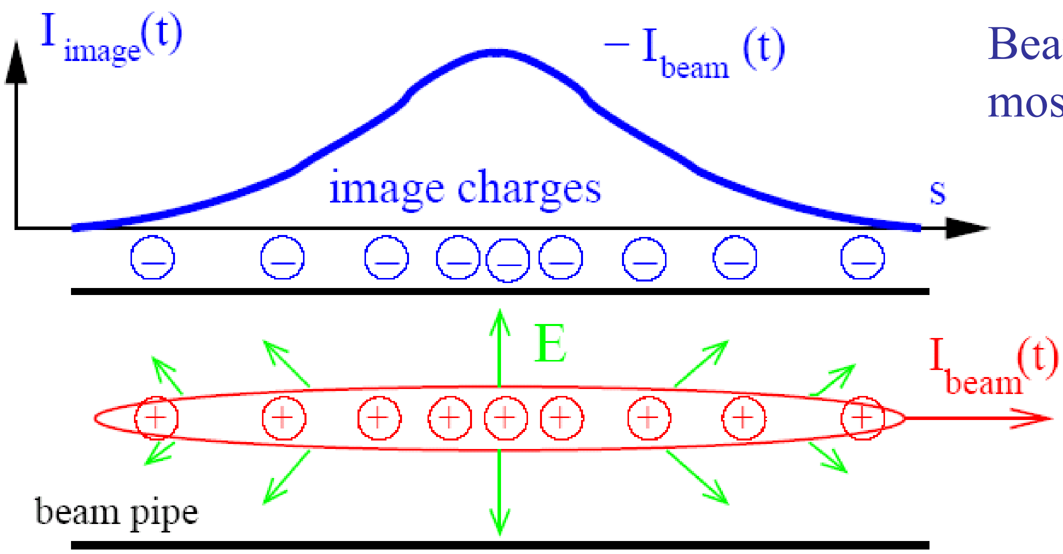
Care: Contains amplitude & phase

The same information

is differently displayed

Pick-Ups for bunched Beams

The image current at the beam pipe is monitored on a high frequency basis i.e. the ac-part given by the bunched beam.



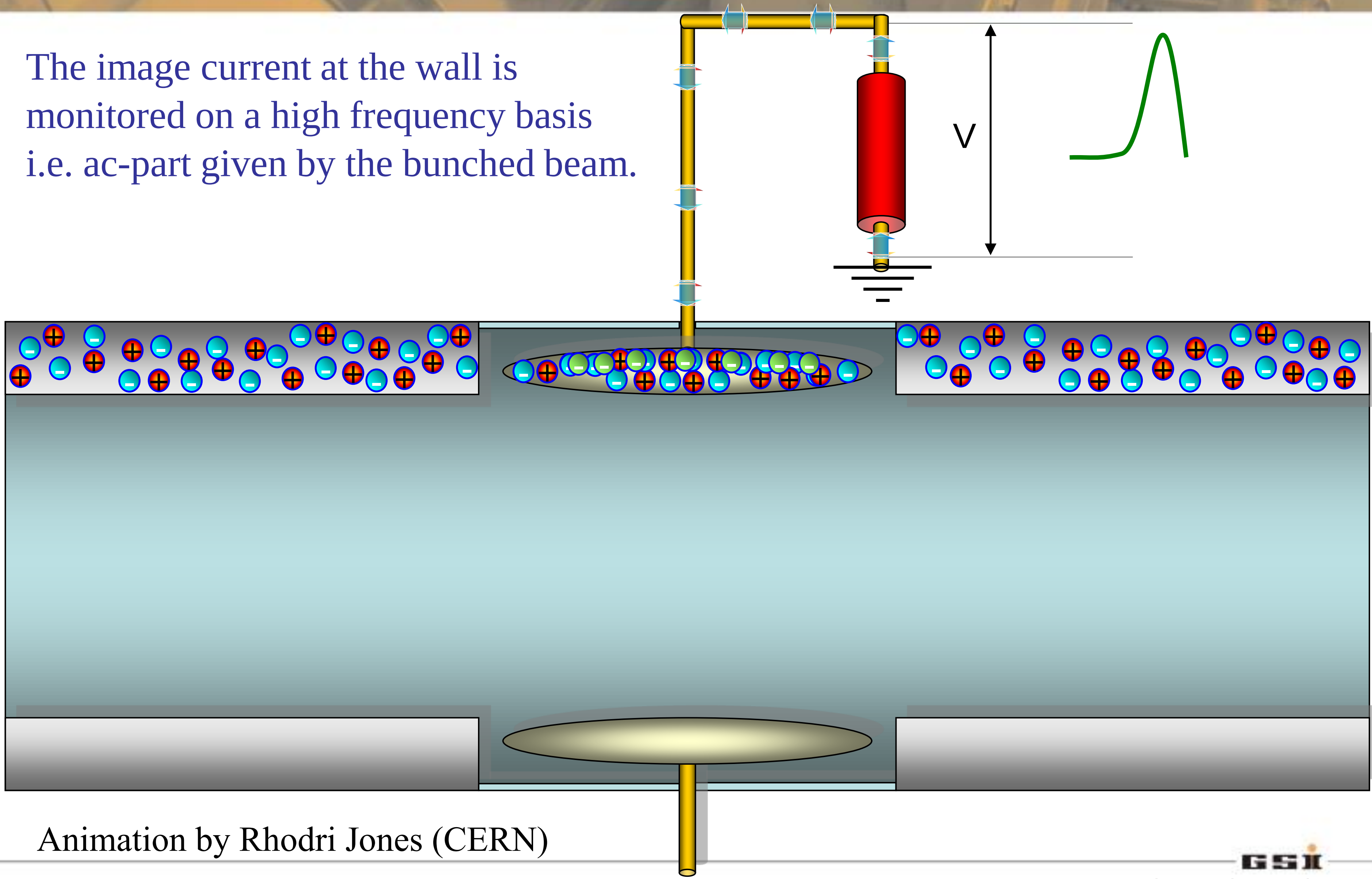
Beam Position Monitor **BPM** is the most frequently used instrument!

For relativistic velocities,
the electric field is transversal:

$$E_{\perp,lab}(t) = \gamma \cdot E_{\perp,rest}(t')$$

Principle of Signal Generation of a BPMs, centered Beam

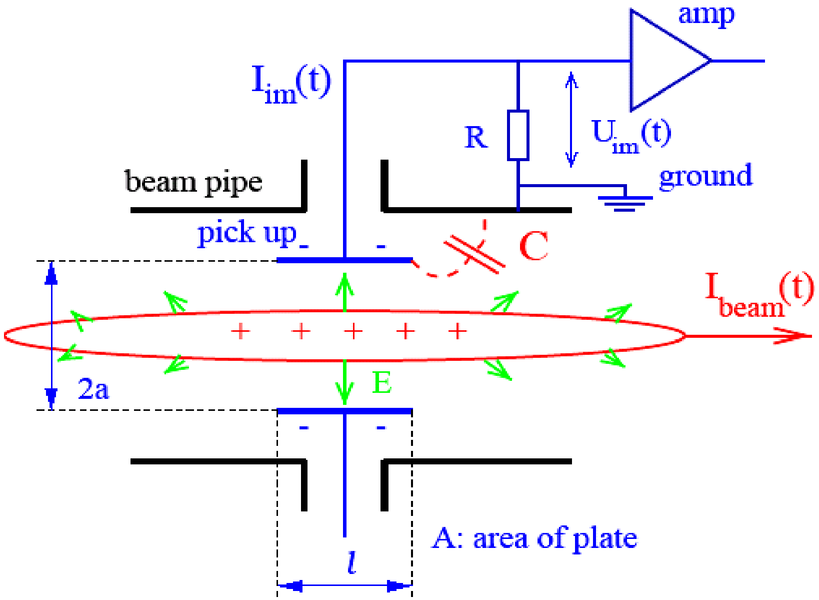
The image current at the wall is monitored on a high frequency basis i.e. ac-part given by the bunched beam.



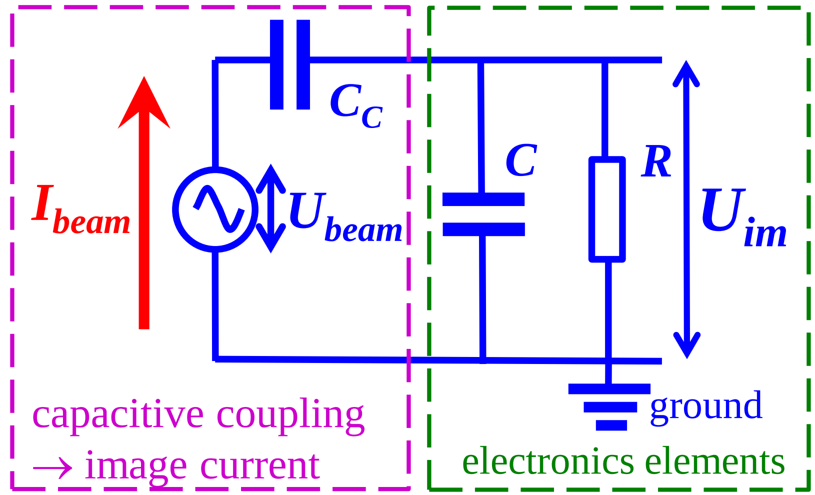
Animation by Rhodri Jones (CERN)

Model for Signal Treatment of capacitive BPMs

The wall current is monitored by a plate or ring inserted in the beam pipe:



Equivalent circuit



At a resistor $\boldsymbol{R}$ the voltage $\boldsymbol{U_{im}}$ from the image current is measured.

Goal: Connection from beam current to signal strength by transfer impedance $Z_t(\omega)$

in frequency domain: $U_{im}(\omega) = R \cdot I_{im}(\omega) = Z_t(\omega) \cdot I_{beam}(\omega)$

Result: $$Z_t(\omega) = \frac{A}{2\pi a} \cdot \frac{1}{\beta c} \cdot \frac{1}{C} \cdot \frac{i\omega RC}{1+i\omega RC}$$

geometry

stray capacitance

frequency response

The high-pass characteristic for typical synchrotron BPM:

$$U_{im}(\omega)=Z_t(\omega)\cdot I_{beam}(\omega)$$

$$|Z_t| = \frac{A}{2\pi a}\cdot\frac{1}{\beta c}\cdot\frac{1}{C}\cdot\frac{\omega/\omega_{cut}}{\sqrt{1+\omega^2/\omega^2_{cut}}}$$

$$\varphi = \arctan(\omega_{cut}/\omega)$$

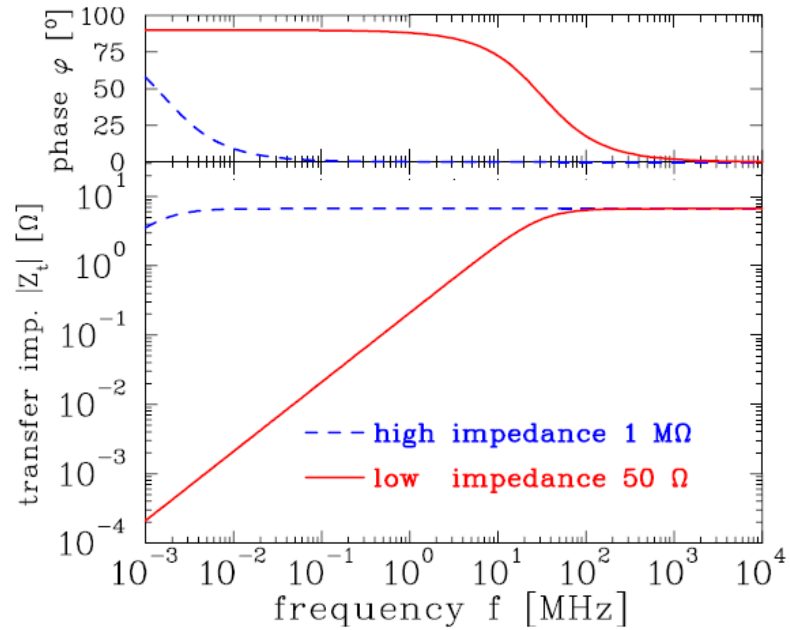
Parameter linear-cut BPM at proton synchr.:

C=100pF, l=10cm, β=50%

$f_{cut}=\omega/2\pi=(2\pi RC)^{-1}$

for R=50 Ω $\Rightarrow f_{cut}$= 32 MHz

for R=1 MΩ $\Rightarrow f_{cut}$= 1.6 kHz



Large signal strength for long bunches $\rightarrow$ **high impedance**

Smooth signal transmission important for short bunches $\rightarrow$ **50 Ω**

Remark: For $\omega \rightarrow 0$ it is $Z_t \rightarrow 0$ i.e. **no** signal is transferred from dc-beams e.g.

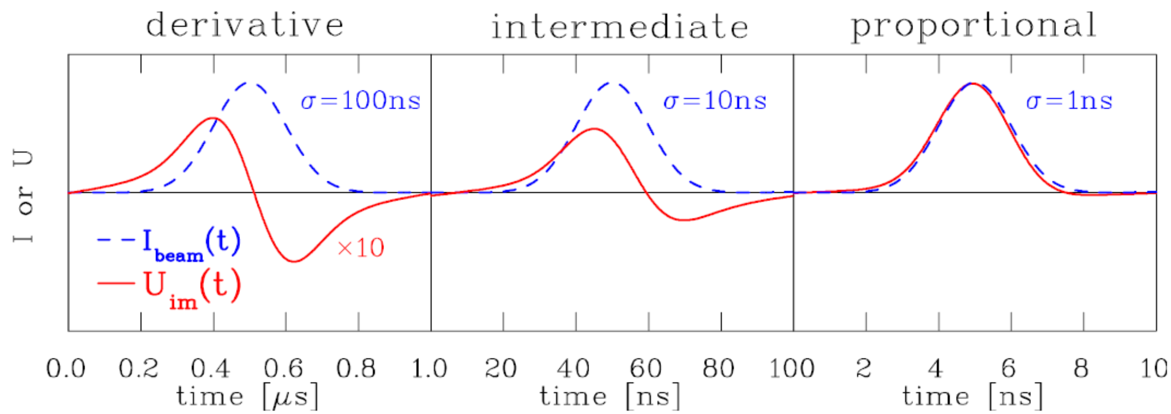
- de-bunched beam inside a synchrotron
- for slow extraction through a transfer line

Calculation of Signal Shape (here single Bunch)

The transfer impedance is used in frequency domain! The following is performed:

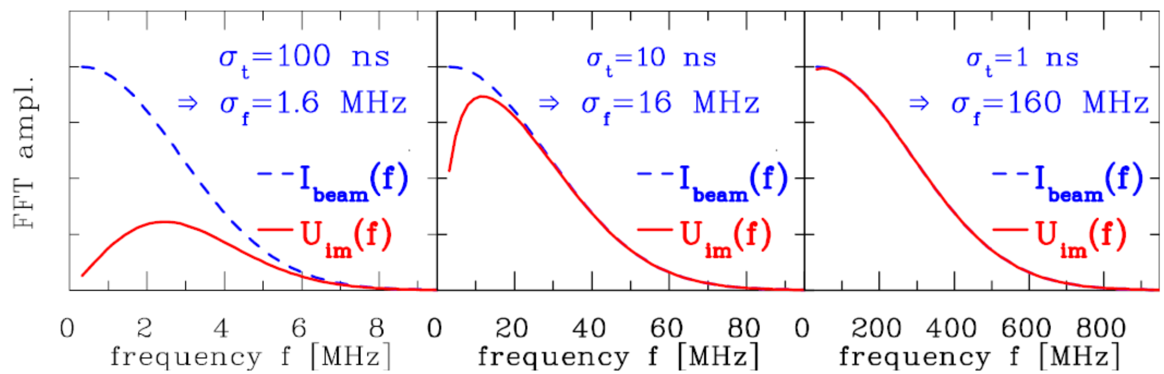
1. Start: Time domain Gaussian function $I_{beam}(t)$ having a width of σ_t

Fourier trans.



inverse Fourier trans.

2. FFT of $I_{beam}(t)$ leads to the frequency domain Gaussian $I_{beam}(f)$ with $\sigma_f = (2\pi\sigma_t)^{-1}$

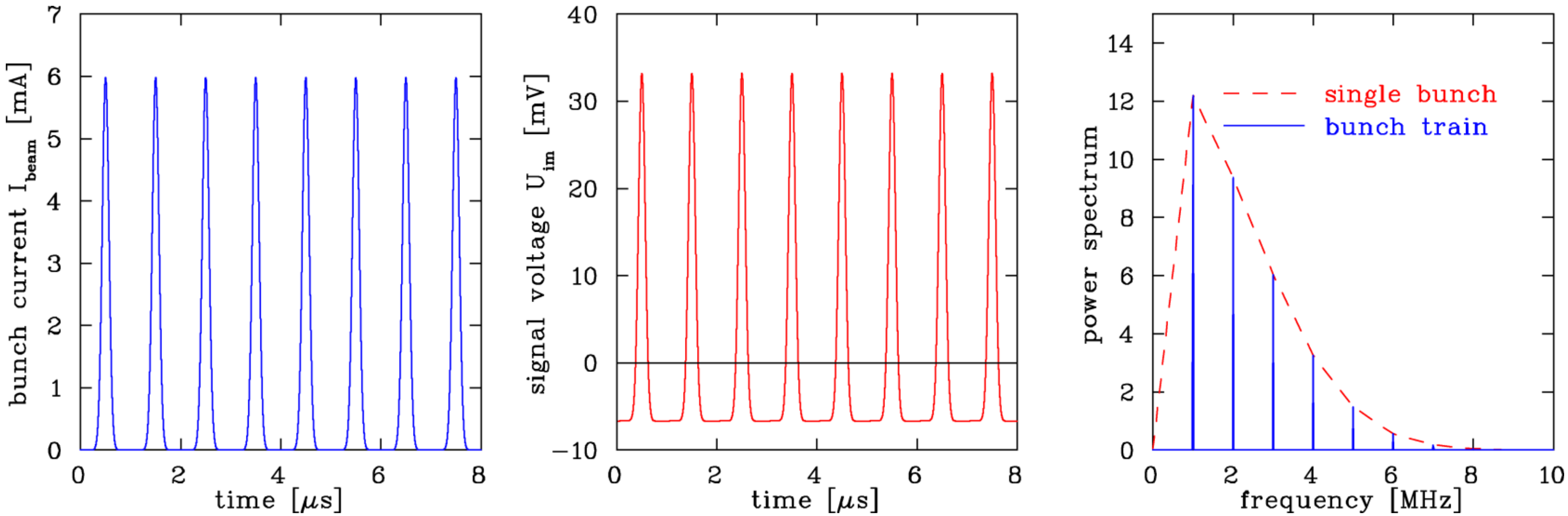


3. Multiplication with $Z_t(f)$ with f_{cut}=32 MHz leads to $U_{im}(f) = Z_t(f) \cdot I_{beam}(f)$

4. Inverse FFT leads to $U_{im}(t)$

Synchrotron filled with 8 bunches accelerated with f_{acc}=1 MHz

BPM terminated with R= 1 MΩ $\Rightarrow f_{acc} >> f_{cut}$:



Parameter: R=1 MΩ $\Rightarrow f_{cut}$=2 kHz, Z_t=5 Ω , all buckets filled

C=100pF, l=10cm, β=50%, σ_t=100 ns $\Rightarrow \sigma_l$=15m

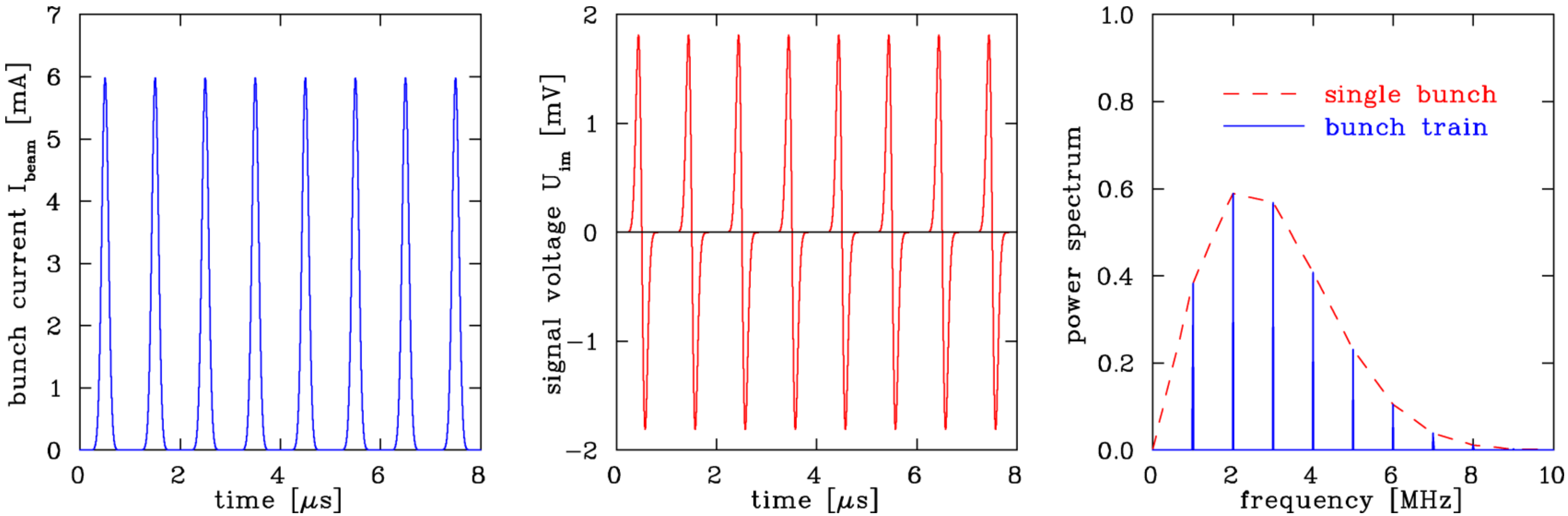
- Fourier spectrum is composed of lines separated by acceleration f_{rf}
- Envelope given by single bunch Fourier transformation
- Baseline shift due to ac-coupling

Remark: 1 MHz< f_{rf} <10MHz $\Rightarrow$ Bandwidth ≈100MHz=10·f_{rf} for broadband observation

Calculation of Signal Shape: repetitive Bunch in a Synchrotron

Synchrotron filled with 8 bunches accelerated with $\boldsymbol{f_{acc}}$=1 MHz

BPM terminated with $\boldsymbol{R}$=50 Ω $\Rightarrow f_{acc} \ll f_{cut}$:

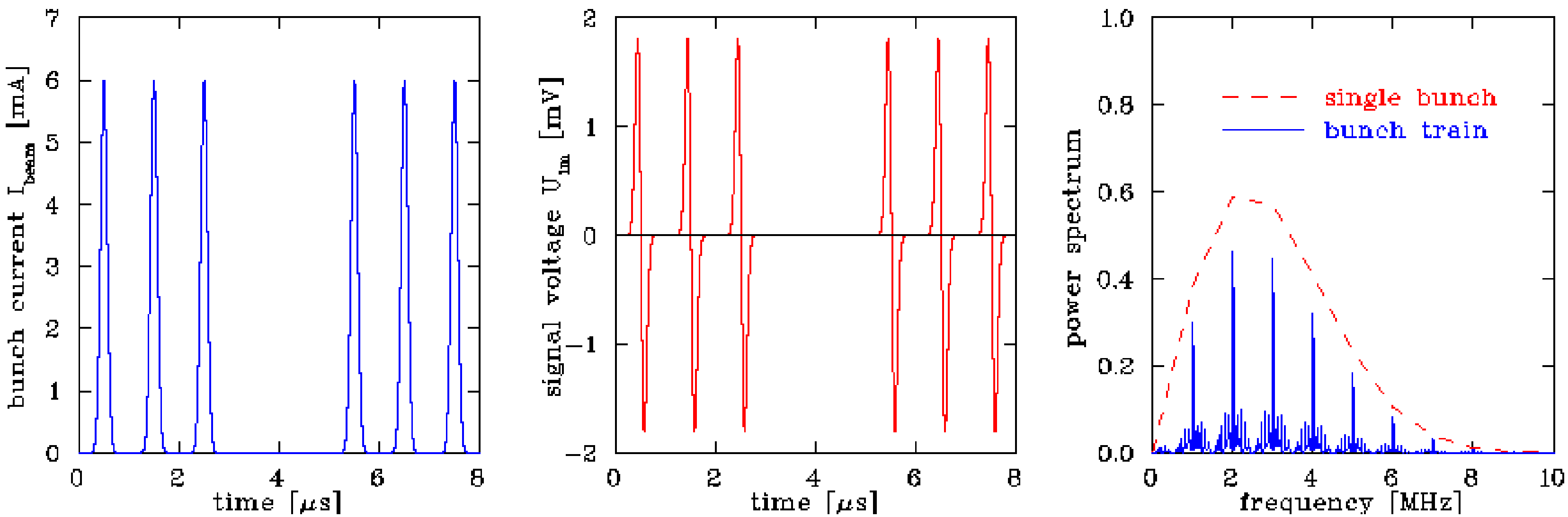


Parameter: $\boldsymbol{R}$=50 Ω ⇒ $\boldsymbol{f_{cut}}$=32 MHz, all buckets filled

C=100pF, l=10cm, β=50%, σ_t=100 ns ⇒ σ_l=15m

- Fourier spectrum is concentrated at acceleration harmonics with single bunch spectrum as an envelope.
- Bandwidth up to typically 10*$\boldsymbol{f_{acc}}$

Synchrotron during filling: Empty buckets, R=50 Ω:

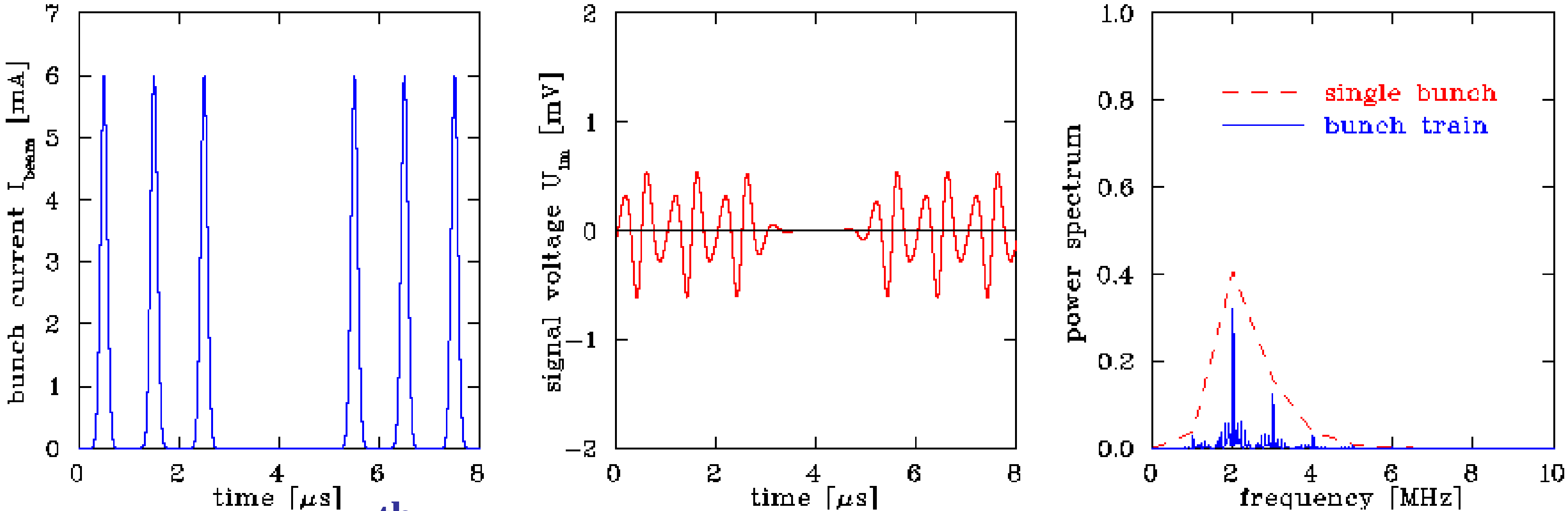


Parameter: R=50 Ω ⇒ f_{cut}=32 MHz, 2 empty buckets

C=100pF, l=10cm, β=50%, σ_t=100 ns ⇒ σ_l=15m

- Fourier spectrum is more complex, harmonics are broader due to sidebands

Effect of filters, here bandpass:



Parameter: *R*=50 Ω, 4th order Butterworth filter at f_{cut}=2 MHz

C=100pF, *l*=10cm, *β*=50%, σ=100 ns

- Ringing due to sharp cutoff
- Other filter types more appropriate

*n^{th} order Butterworth filter, math. simple, but **not** well suited:*

$$|H_{low}| = \frac{1}{\sqrt{1+(\omega/\omega_{cut})^{2n}}} \quad \text{and} \quad |H_{high}| = \frac{(\omega/\omega_{cut})^n}{\sqrt{1+(\omega/\omega_{cut})^{2n}}}$$

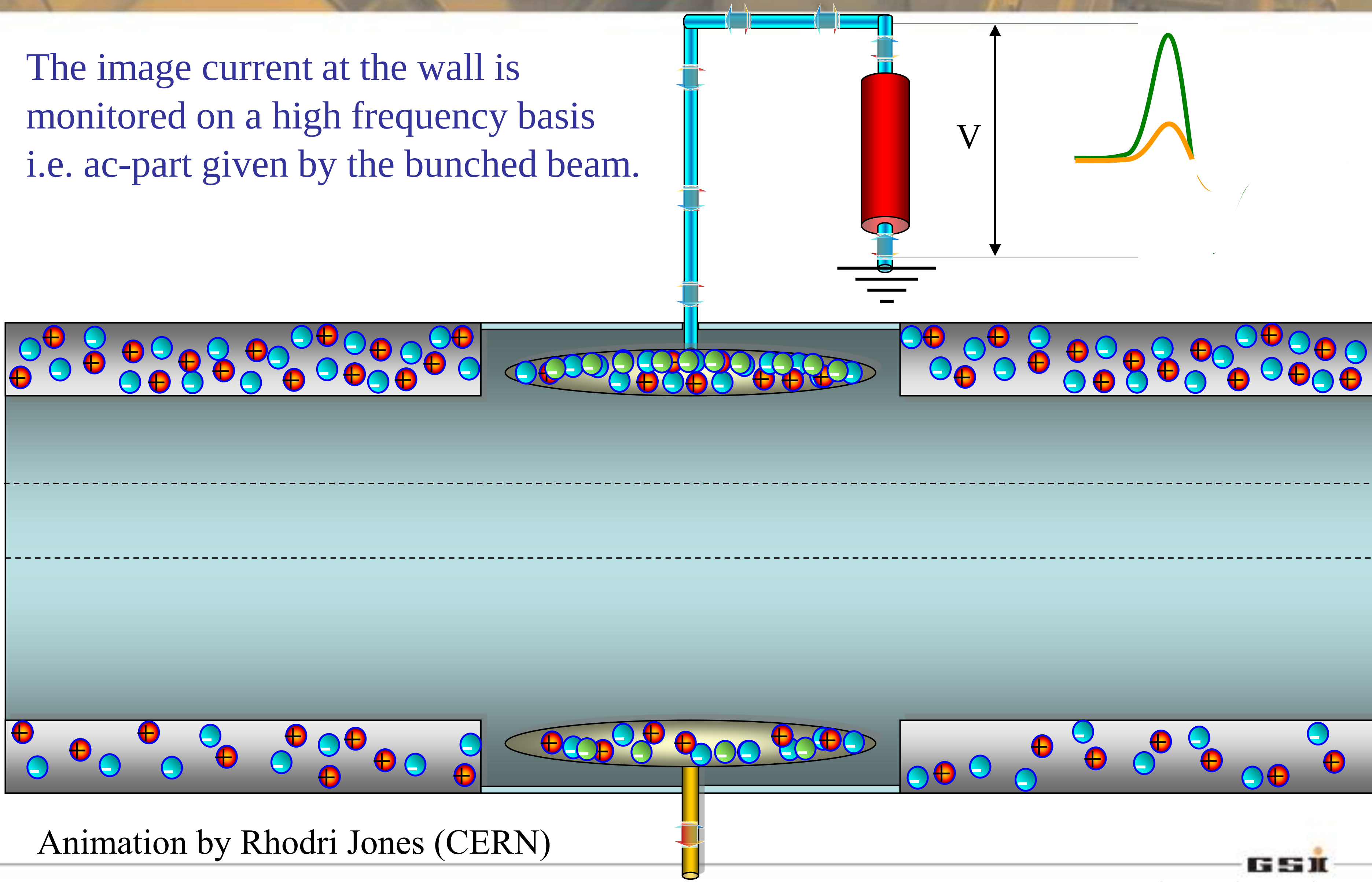
$$H_{filter} = H_{high} \cdot H_{low}$$

Generally: $Z_{tot}(\omega) = H_{cable}(\omega) \cdot H_{filter}(\omega) \cdot H_{amp}(\omega) \cdot \ldots \cdot Z_t(\omega)$

Remark: For numerical calculations, time domain filters (FIR and IIR) are more appropriate

Principle of Signal Generation of a BPMs: off-center Beam

The image current at the wall is monitored on a high frequency basis i.e. ac-part given by the bunched beam.



Animation by Rhodri Jones (CERN)

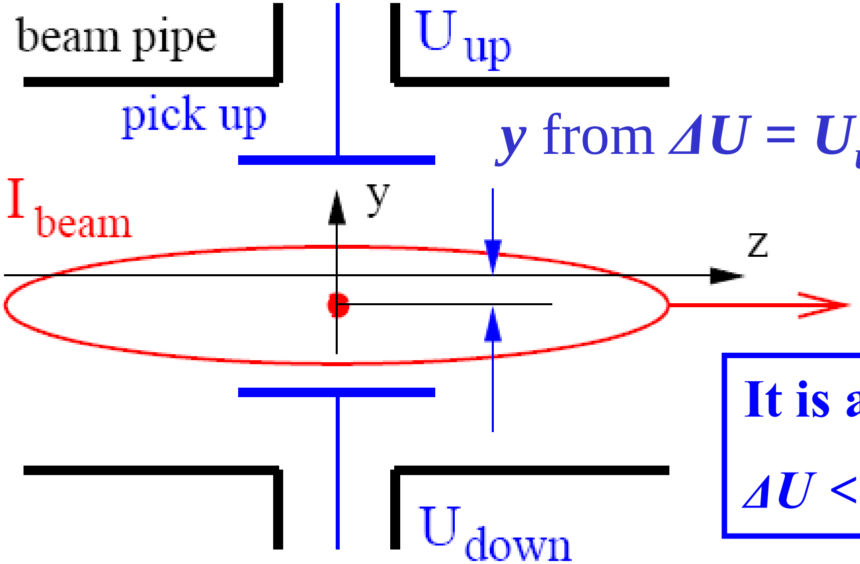
The difference voltage between plates gives the beam's center-of-mass
→**most frequent application**

'Proximity' effect leads to different voltages at the plates:

$$y = \frac{1}{S_y(\omega)} \cdot \frac{U_{up} - U_{down}}{U_{up} + U_{down}} + \delta_y(\omega)$$

$$\equiv \frac{1}{S_y} \cdot \frac{\Delta U_y}{\Sigma U_y} + \delta_y$$

$$x = \frac{1}{S_x(\omega)} \cdot \frac{U_{right} - U_{left}}{U_{right} + U_{left}} + \delta_x(\omega)$$



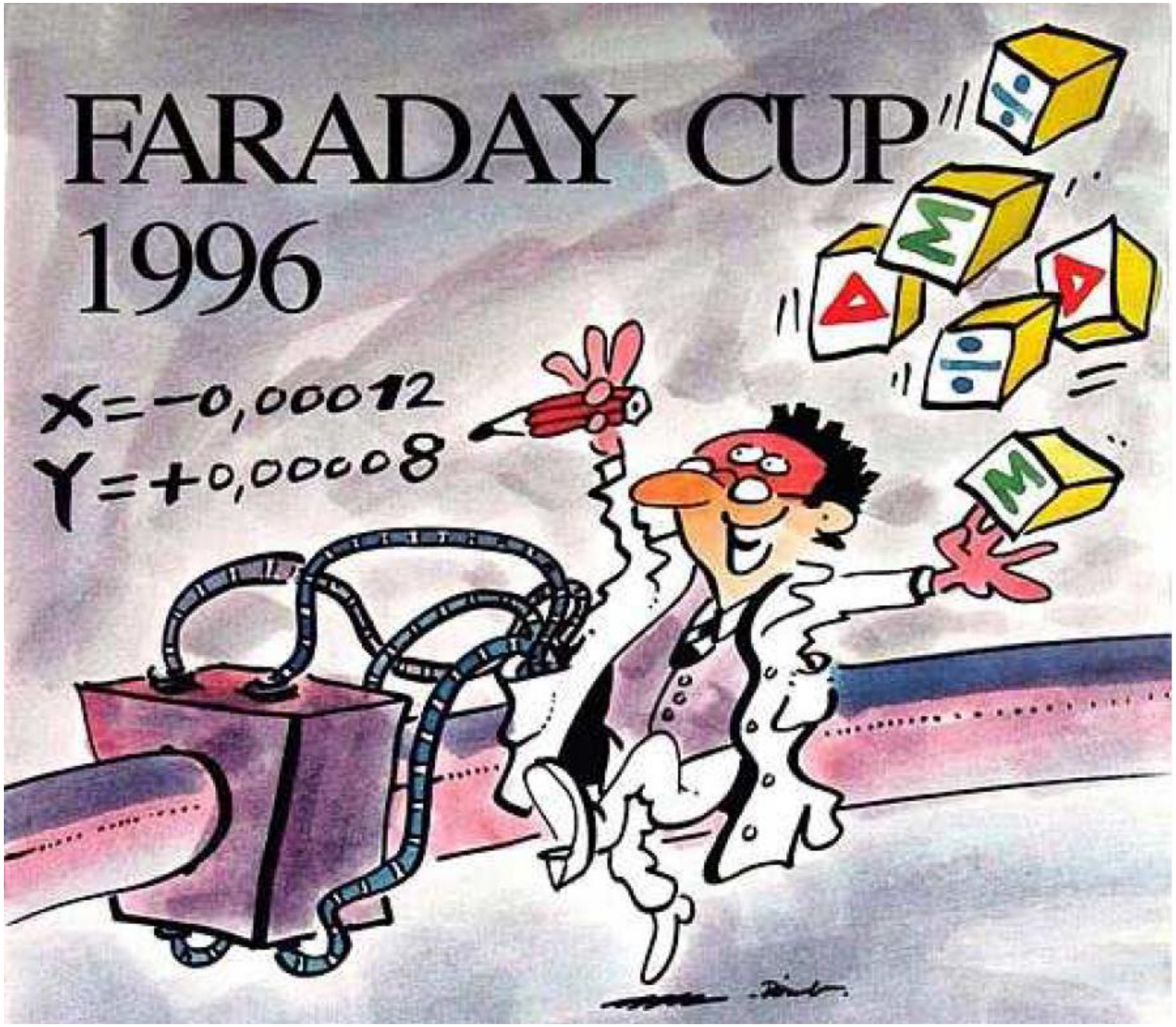
***y* from $\Delta U = U_{up} - U_{down}$**

It is at least:
$\Delta U \ll \Sigma U / 10$

S(ω,x) is called **position sensitivity,** sometimes the inverse is used ***k(ω,x)=1/S(ω,x)***
S is a geometry dependent, non-linear function, which have to be optimized
Units: ***S***=[%/mm] and sometimes ***S***=[dB/mm] or ***k***=[mm].

Typical desired position resolution: Δ*x* ≈ 0.3 ...0.1 · σ_x of beam width

The Artist View of a BPM



Outline:

- Signal generation $\rightarrow$ transfer impedance
- Capacitive *button* BPM for high frequencies

  used at most proton LINACs and electron accelerators
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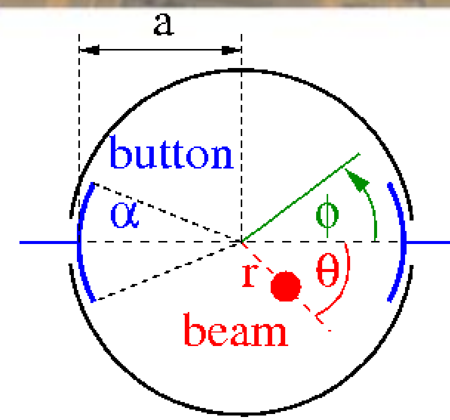
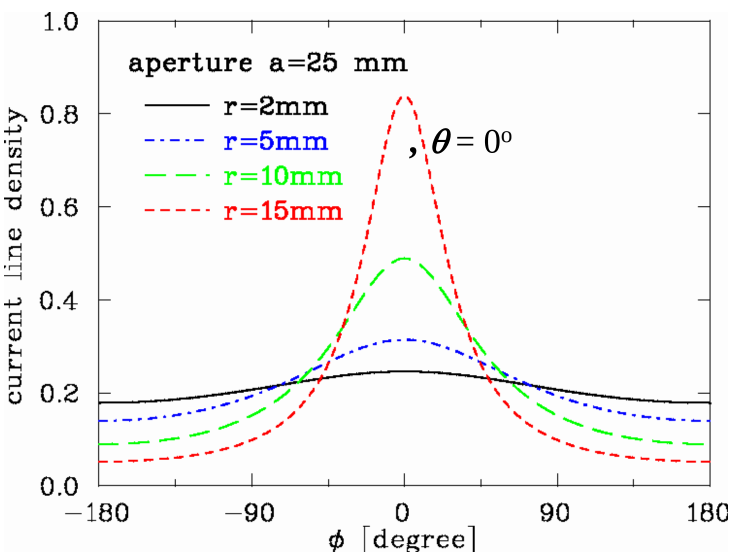
'Proximity effect': larger signal for closer plate

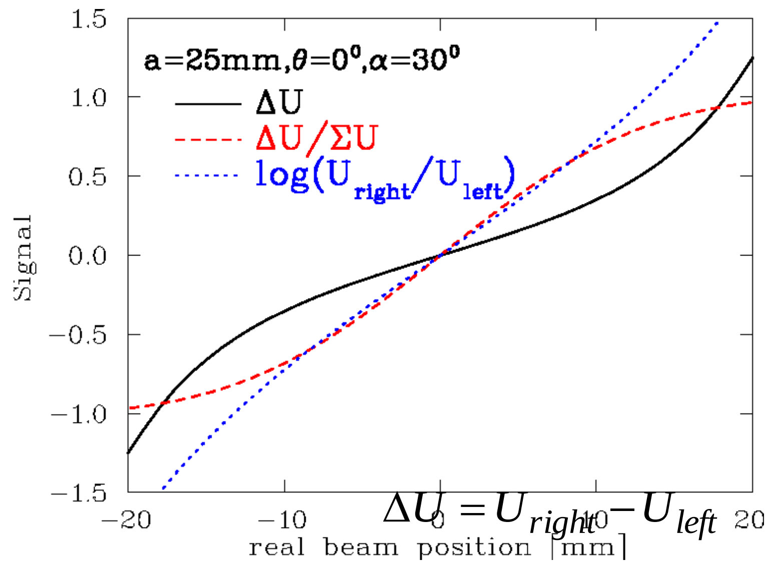


Ideal 2-dim model: Cylindrical pipe → image current density via 'image charge method' for 'pensile' beam:

$$j_{im}(\phi) = \frac{I_{beam}}{2\pi a} \cdot \left(\frac{a^2 - r^2}{a^2 + r^2 - 2ar \cdot \cos(\phi - \theta)} \right)$$

Image current: Integration of finite BPM size: $I_{im} = a \cdot \int_{-\alpha/2}^{\alpha/2} j_{im}(\phi) d\phi$





$\Delta U = U_{right} - U_{left}$

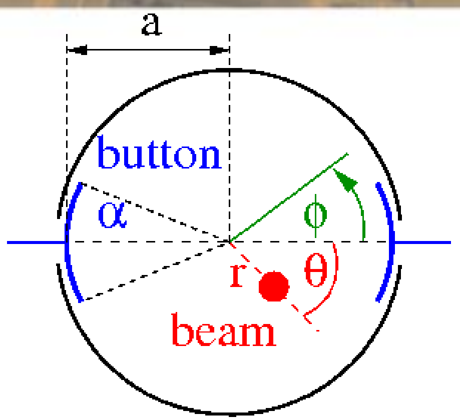
Ideal 2-dim model: Non-linear behavior and hor-vert coupling:

Sensitivity ***S*** is converts signal to position $x = \frac{1}{S} \cdot \frac{\Delta U}{\Sigma U}$

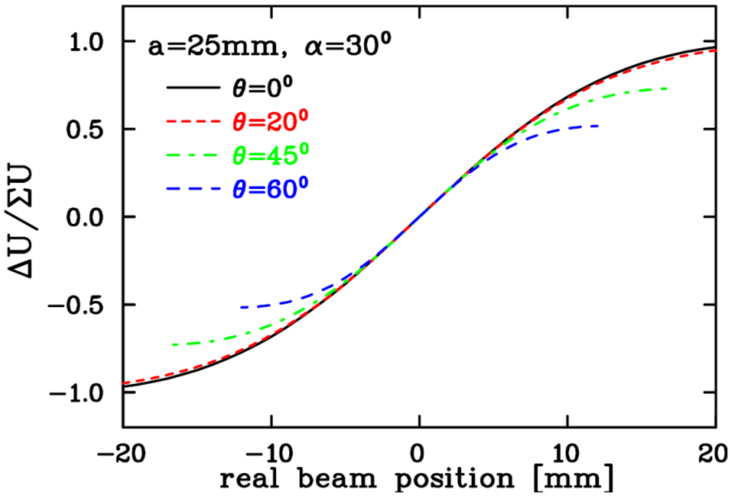
with *S* [%/mm] or [dB/mm]

i.e. ***S*** is the derivative of the curve $S_x = \frac{\partial(\frac{\Delta U}{\Sigma U})}{\partial x}$, here $S_x = S_x(x, y)$ i.e. non-linear.

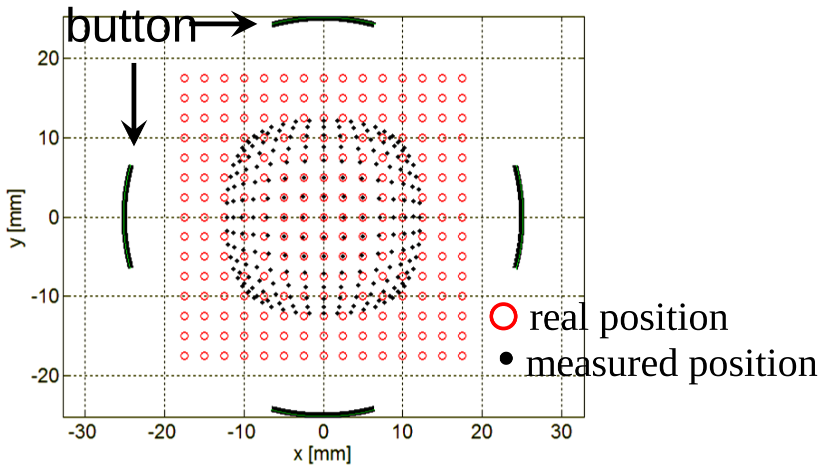
For this example: center part *S*=7.4%/mm ⇔ *k*=*1*/*S*=14mm



Horizontal plane



Position Map



Button BPM Realization

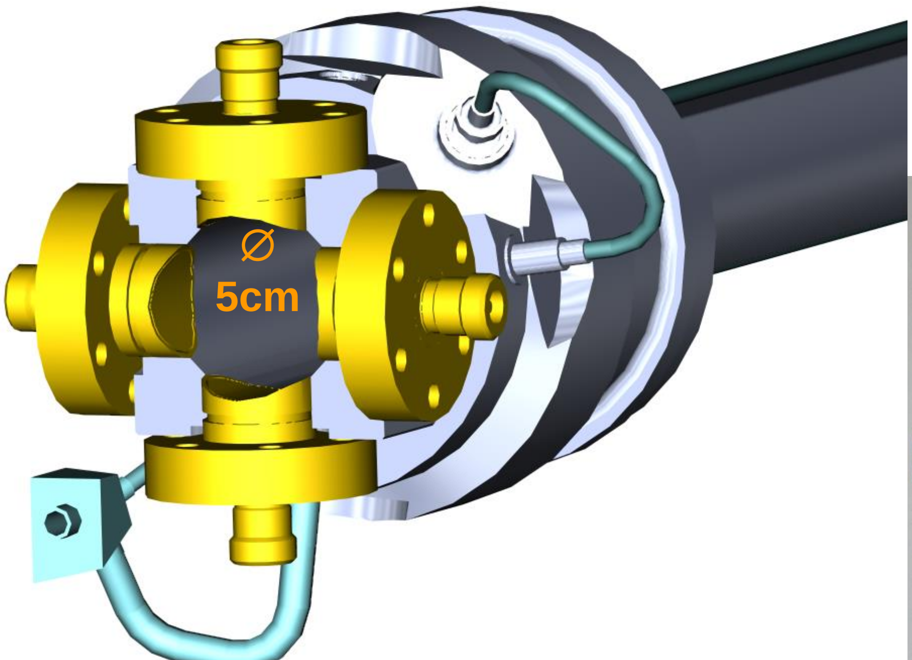
LINACs, e⁻-synchrotrons: 100 MHz < f_{rf} < 3 GHz → bunch length ≈ BPM length

→ 50 Ω signal path to prevent reflections

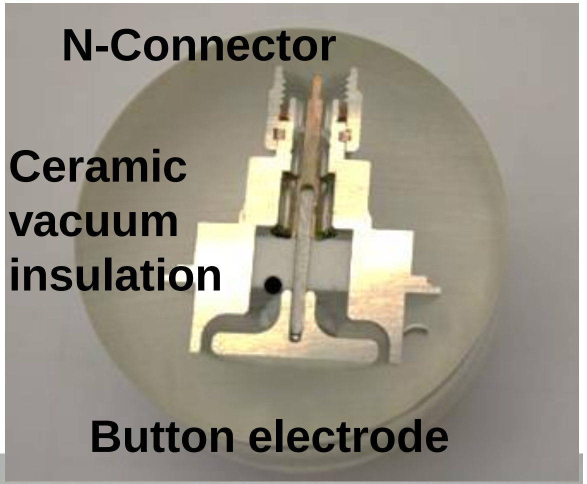
Example: LHC-type inside cryostat:

∅24 mm, half aperture a=25 mm, C=8 pF

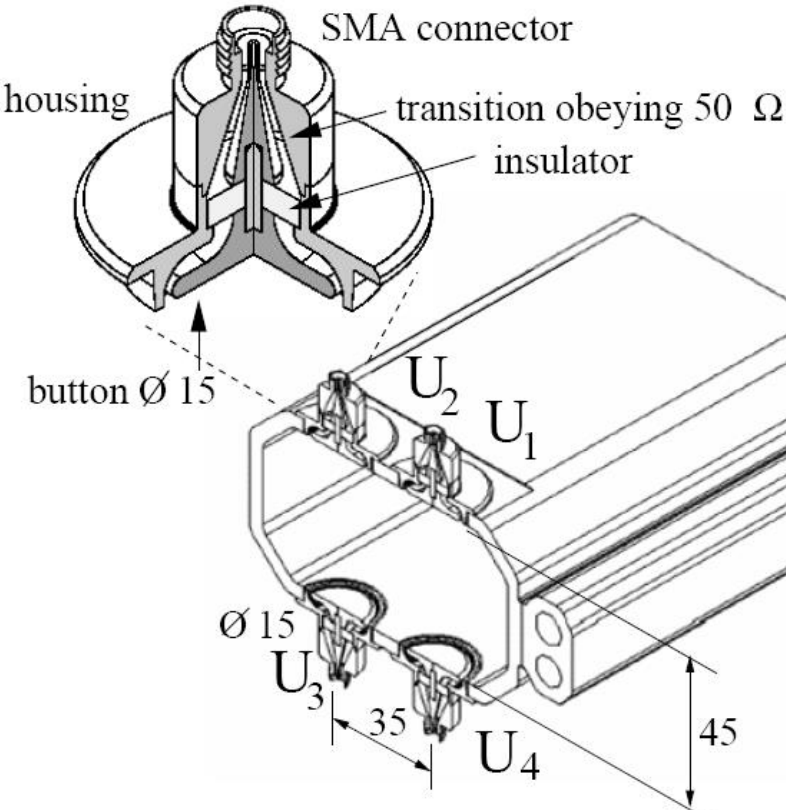
⇒ f_{cut}=400 MHz, Z_t = 1.3 Ω above f_{cut}



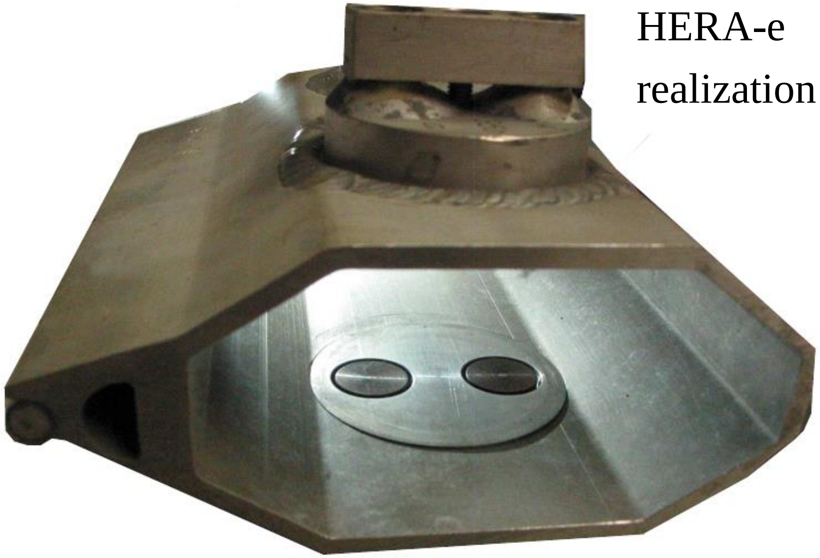
Courtesy C. Boccard (CERN)



Due to synchrotron radiation, the button insulation might be destroyed
⇒buttons only in vertical plane possible ⇒ increased non-linearity



PEP-realization: N. Kurita et al., PAC 1995



HERA-e realization

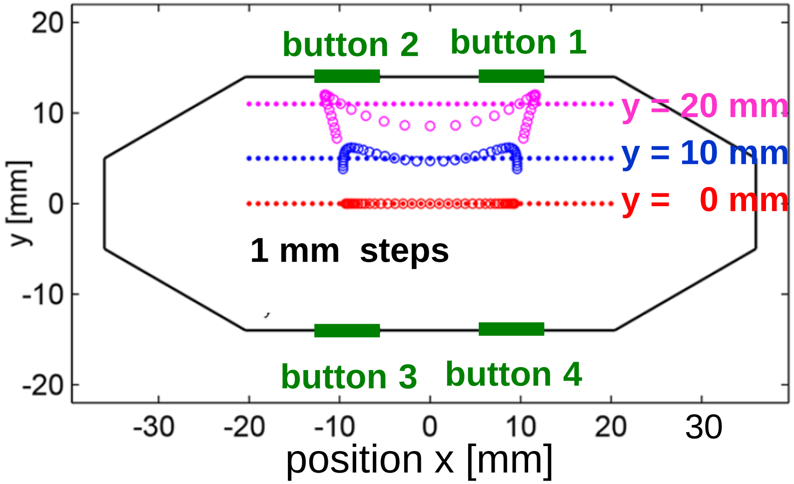
$$\text{horizontal}: x = \frac{1}{S_x} \cdot \frac{(U_1 + U_4) - (U_2 + U_3)}{U_1 + U_2 + U_3 + U_4}$$

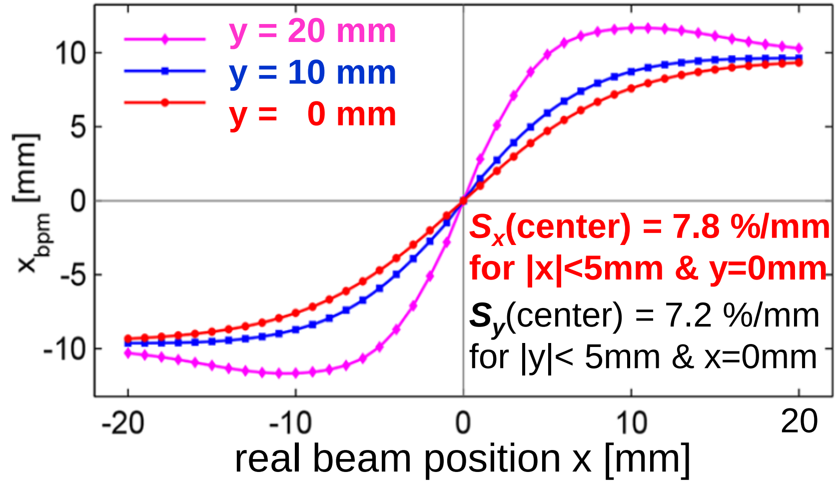
$$\text{vertical}: y = \frac{1}{S_y} \cdot \frac{(U_1 + U_2) - (U_3 + U_4)}{U_1 + U_2 + U_3 + U_4}$$

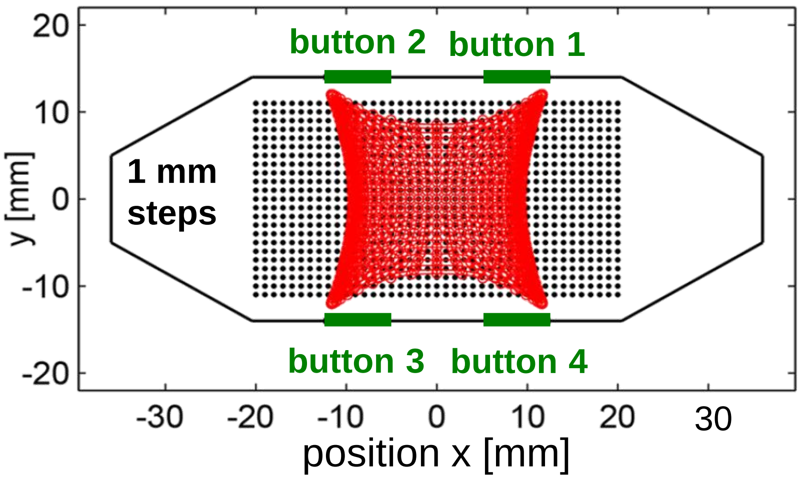
Example: Simulation for ALBA light source for 72 x 28 mm² chamber

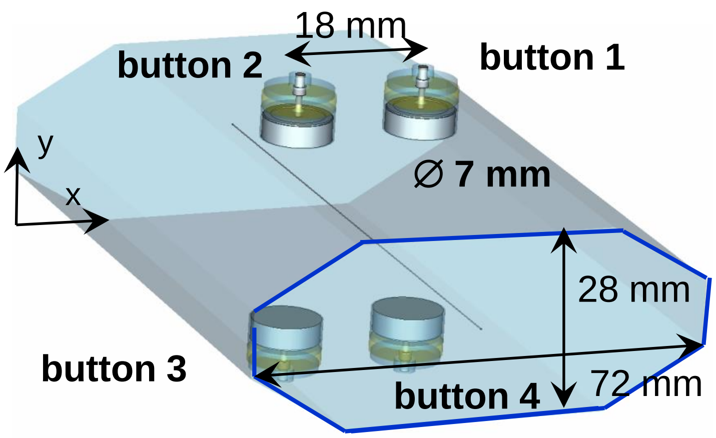
Optimization: horizontal distance and size of buttons

from A.A. Nosych et al., IBIC'14









Result: non-linearity and ***xy***-coupling occur in dependence of button size and position

Outline:

- Signal generation → transfer impedance
- Capacitive *button* BPM for high frequencies
  used at most proton LINACs and electron accelerators
- Capacitive *linear-cut* BPM for low frequencies
  used at most proton synchrotrons due to linear position reading
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- Summary

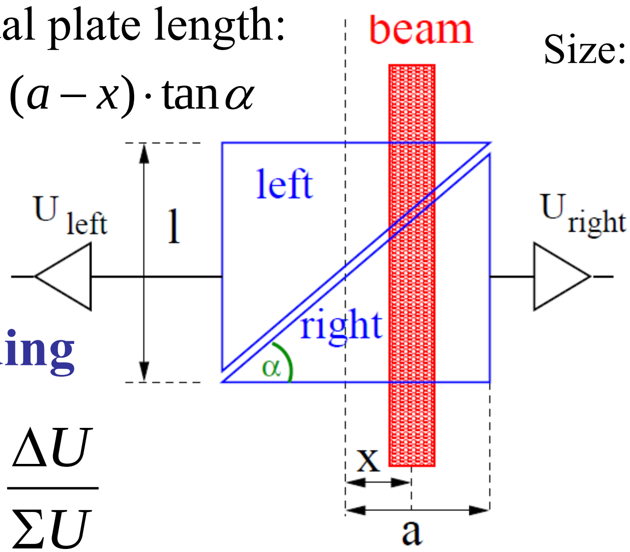
Linear-cut BPM for Proton Synchrotrons

Frequency range: 1 MHz < f_{rf} < 10 MHz $\Rightarrow$ bunch-length >> BPM length.

Signal is proportional to actual plate length:

$$l_{\text{right}} = (a + x) \cdot \tan\alpha, \quad l_{\text{left}} = (a - x) \cdot \tan\alpha$$

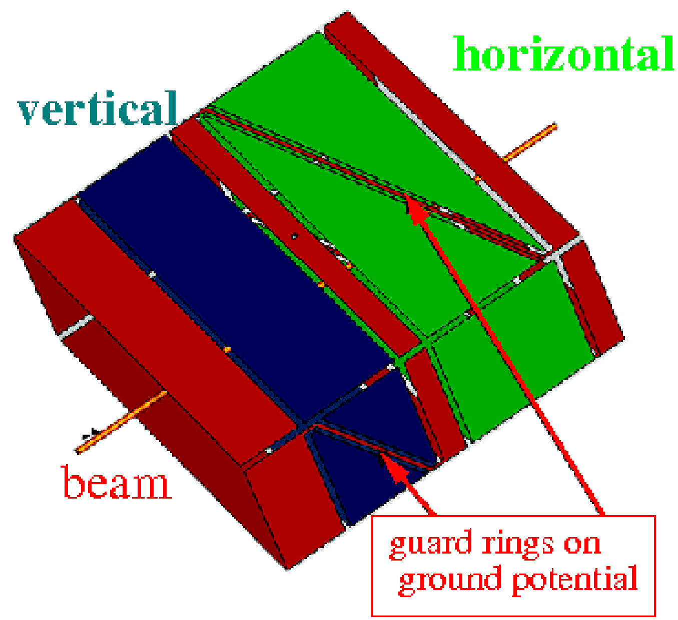
$$\Rightarrow \quad x = a \cdot \frac{l_{\text{right}} - l_{\text{left}}}{l_{\text{right}} + l_{\text{left}}}$$

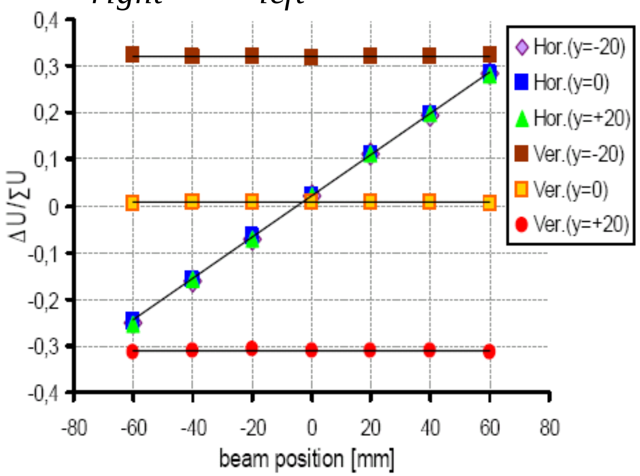


In ideal case: linear reading

$$x = a \cdot \frac{U_{right} - U_{left}}{U_{right} + U_{left}} \equiv a \cdot \frac{\Delta U}{\Sigma U}$$

Size: 200x70 mm^2





Linear-cut BPM:

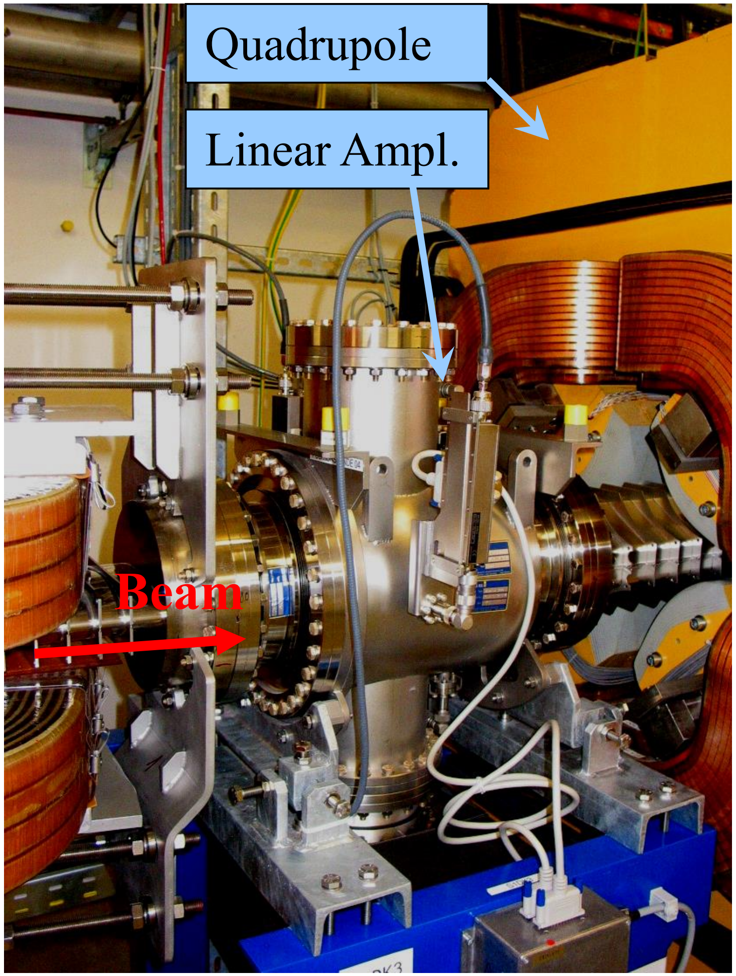
Advantage: Linear, i.e. constant position sensitivity ***S***

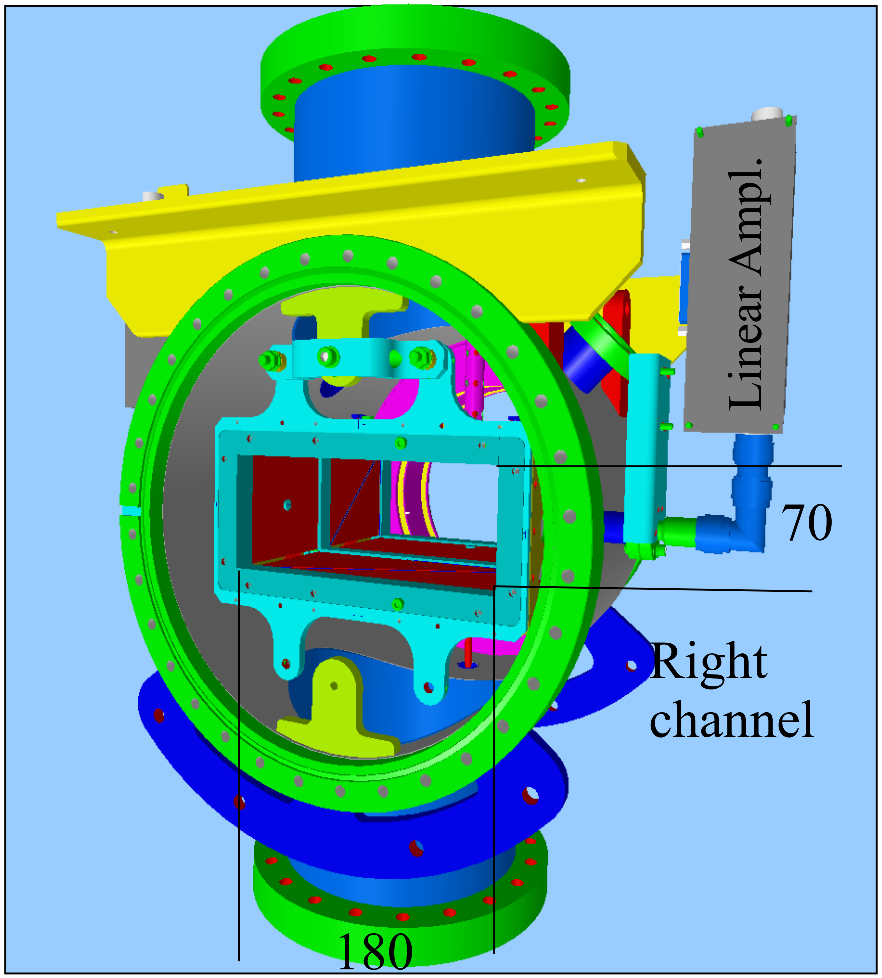
$\Leftrightarrow$ no beam size dependence

Disadvantage: Large size, complex mechanics

high capacitance

Technical realization at HIT synchrotron of 46 m length for 7 MeV/u→ 440 MeV/u
BPM clearance: 180x70 mm^2, standard beam pipe diameter: 200 mm.





Technical Realization of a linear-cut BPM

Technical realization at HIT synchrotron of 46 m length for 7 MeV/u→ 440 MeV/u

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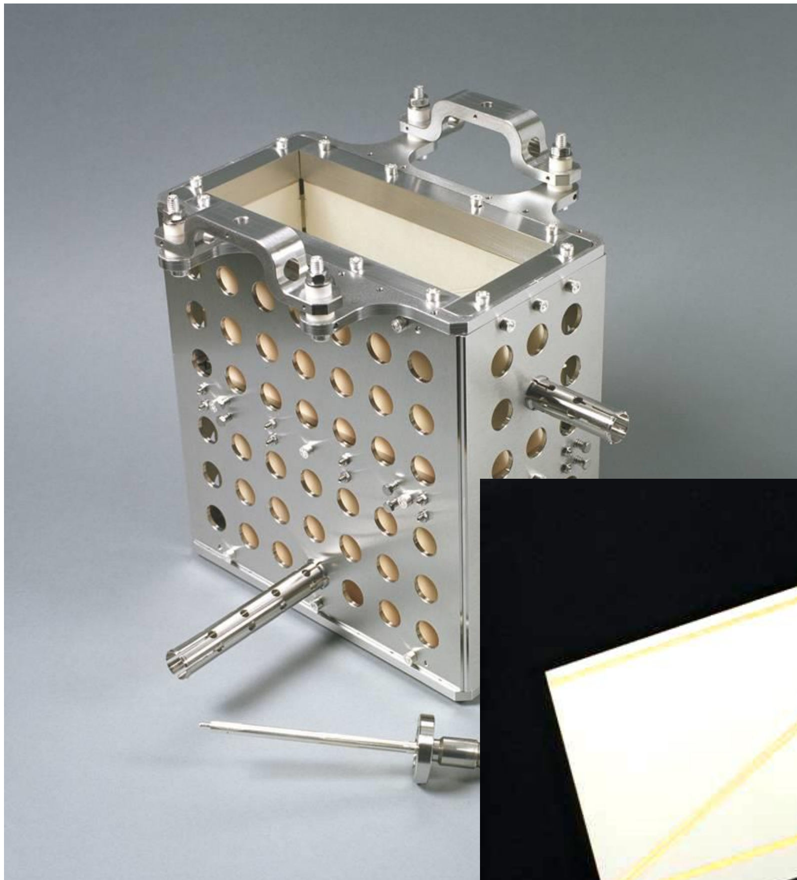
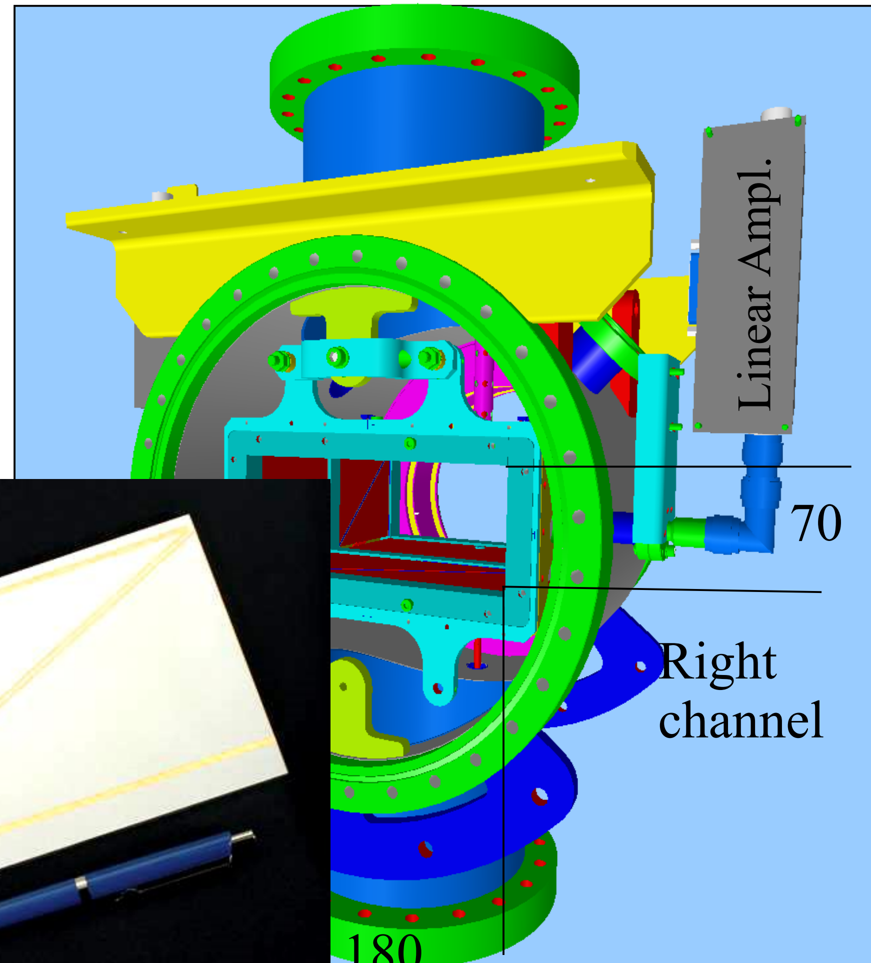
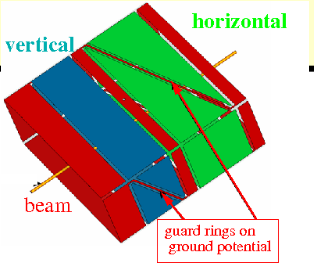
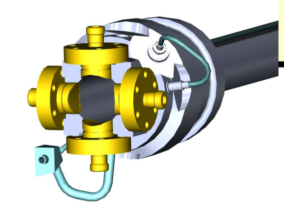




Comparison linear-cut and Button BPM

	Linear-cut BPM	Button BPM
Precaution	Bunches longer than BPM	Bunch length comparable to BPM
BPM length (typical)	10 to 20 cm length per plane	∅1 to 5 cm per button
Shape	Rectangular or cut cylinder	Orthogonal or planar orientation
Bandwidth (typical)	0.1 to 100 MHz	100 MHz to 5 GHz
Coupling	1 MΩ or ≈1 kΩ (transformer)	50 Ω
Cutoff frequency (typical)	0.01… 10 MHz (C=30…100pF)	0.3… 1 GHz (C=2…10pF)
Linearity	Very good, no x-y coupling	Non-linear, x-y coupling
Sensitivity	Good, care: plate cross talk	Good, care: signal matching
Usage	At proton synchrotrons, f_{rf}< 10 MHz	All electron acc., proton Linacs, f_{rf}> 100 MHz





Remark: Other types are also some time used: e.g. wall current monitors, inductive antenna, BPMs with external resonator, cavity BPM, slotted wave-guides for stochastic cooling etc.

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  analog signal conditioning to achieve small signal processing
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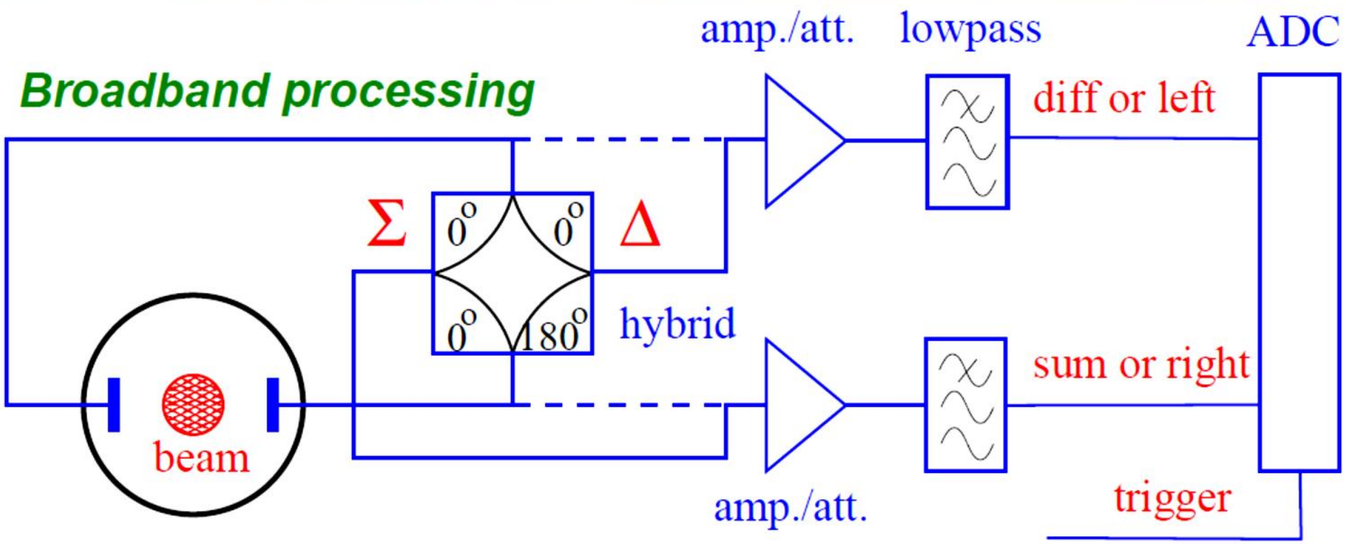
Broadband Signal Processing



- Hybrid or transformer close to beam pipe for analog ΔU & ΣU generation or U_{left} & U_{right}
- Attenuator/amplifier
- Filter to get the wanted harmonics and to suppress stray signals
- ADC: digitalization $\longrightarrow$ followed by calculation of of $\Delta U / \Sigma U$

Advantage: Bunch-by-bunch observation possible, versatile post-processing possible

Disadvantage: Resolution down to ≈ 100 µm for shoe box type , i.e. ≈0.1% of aperture, resolution is worse than narrowband processing, see below

General: Noise Consideration

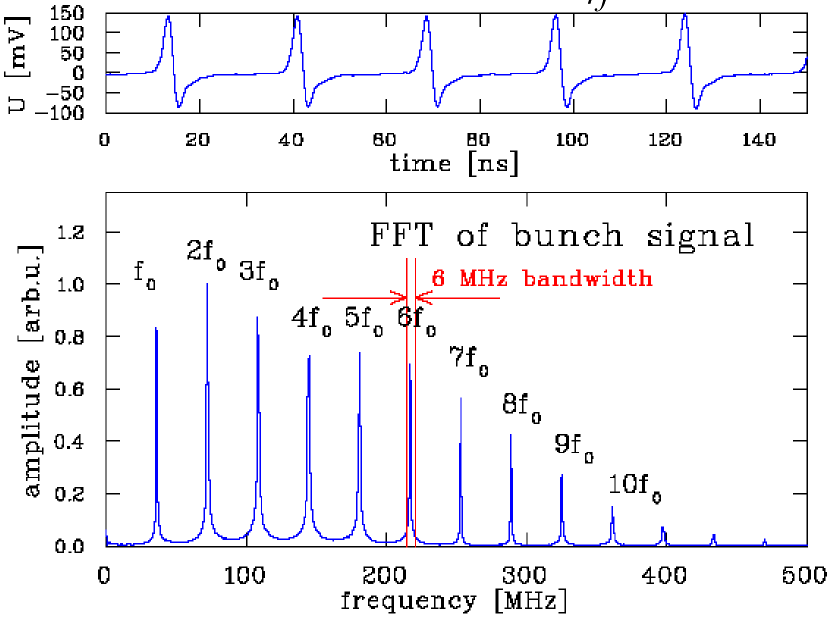
1. Signal voltage given by: $U_{im}(f) = Z_t(f) \cdot I_{beam}(f)$
2. Position information from voltage difference: $x = 1/S \cdot \Delta U / \Sigma U$
3. Thermal noise voltage given by: $U_{noise}(R, \Delta f) = \sqrt{4k_B \cdot T \cdot R \cdot \Delta f}$

1Signal-to-noise $\Delta U_{im}/U_{noise}$ is influenced by:

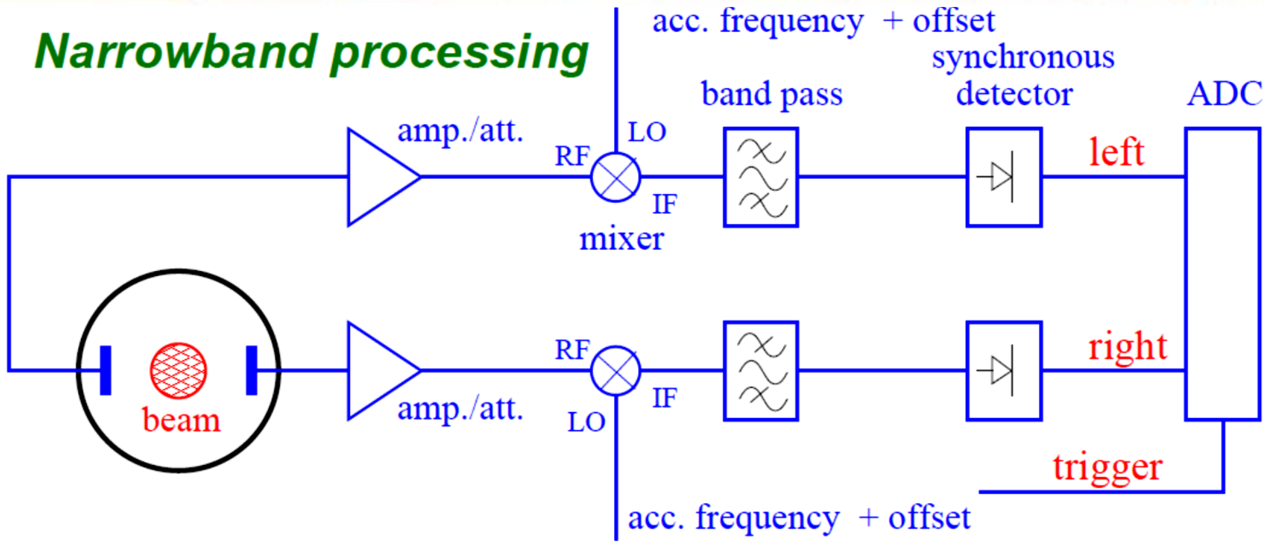
- Input signal amplitude
- Thermal noise from amplifiers etc.
- Bandwidth Δf

$\Rightarrow$ Restriction of frequency width as the power is concentrated at harm. nf_{rf}

Example: GSI-LINAC with f_{rf}=36 MHz



Narrowband Processing for improved Signal-to-Noise



Narrowband processing equals heterodyne receiver (e.g. AM-radio or spectrum analyzer)

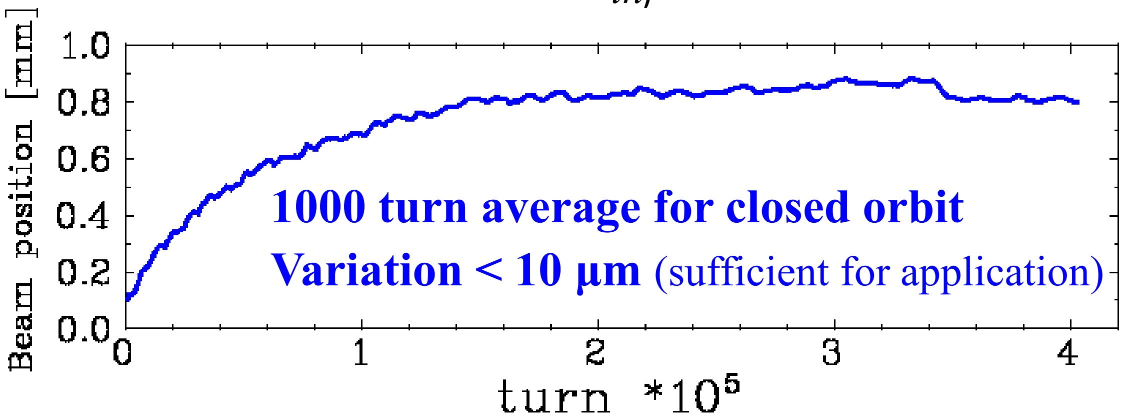
- Attenuator/amplifier
- Mixing with accelerating frequency $f_{rf} \Rightarrow$ signal with difference frequency
- Bandpass filter of the mixed signal (e.g at 10.7 MHz)
- Rectifier: synchronous detector
- ADC: digitalization → followed calculation of $\Delta U/\Sigma U$

Digital correspondence: I/Q demodulation

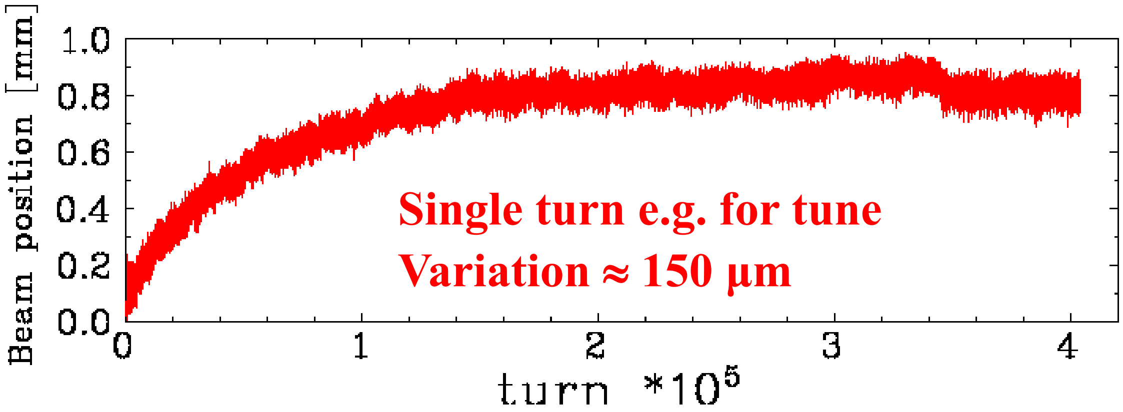
Advantage: spatial resolution about 100 time better than broadband processing
Disadvantage: No turn-by-turn diagnosis, due to mixing = 'long averaging time'

Comparison: Filtered Signal ↔ Single Turn

Example: GSI Synchr.: U^{73+}, E_{inj}=11.5 MeV/u→ 250 MeV/u within 0.5 s, 10^9 ions



- Position resolution < 30 µm (BPM diameter d=180 mm)
- average over 1000 turns corresponding to ≈1 ms
  or ≈1 kHz bandwidth



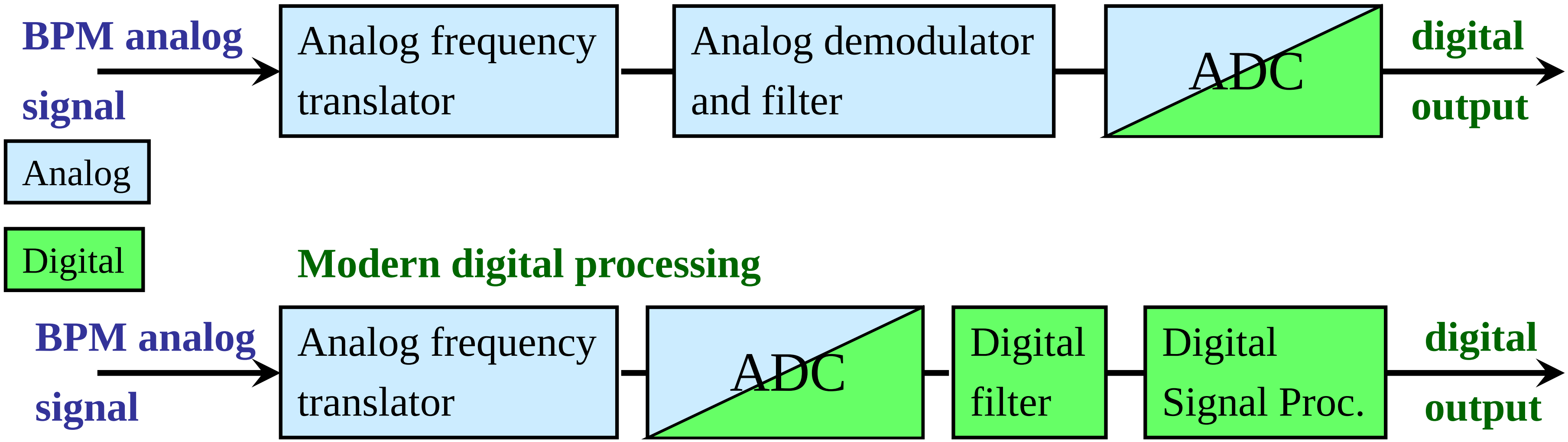
- Turn-by-turn data have much larger variation

However: not only noise contributes but additionally **beam movement** by betatron oscillation
⇒ broadband processing i.e. turn-by-turn readout for tune determination.

Analog versus Digital Signal Processing

Modern instrumentation uses **digital** techniques with extended functionality.

Traditional analog processing



Digital receiver as modern successor of super heterodyne receiver

- Basic functionality is preserved but implementation is very different
- Digital transition just after the amplifier & filter or mixing unit
- Signal conditioning (filter, decimation, averaging) on FPGA

Advantage of DSP: Versatile operation, flexible adoption without hardware modification

Disadvantage of DSP: **non**, good engineering skill requires for development, expensive

Comparison of BPM Readout Electronics (simplified)

Type	Usage	Precaution	Advantage	Disadvantage
Broadband	p-sychr.	Long bunches	Bunch structure signal Post-processing possible **Required for transfer lines with few bunches**	Resolution limited by noise
Narrowband	all synchr.	Stable beams >100 rf-periods	High resolution	No turn-by-turn Complex electronics
Digital Signal Processing	all	ADC sample typ. 250 MS/s	Very flexible & versatile High resolution **Trendsetting technology for future demands**	**Basically non!** Limited time resolution by ADC $\rightarrow$ under-sampling Man-power intensive

Outline:

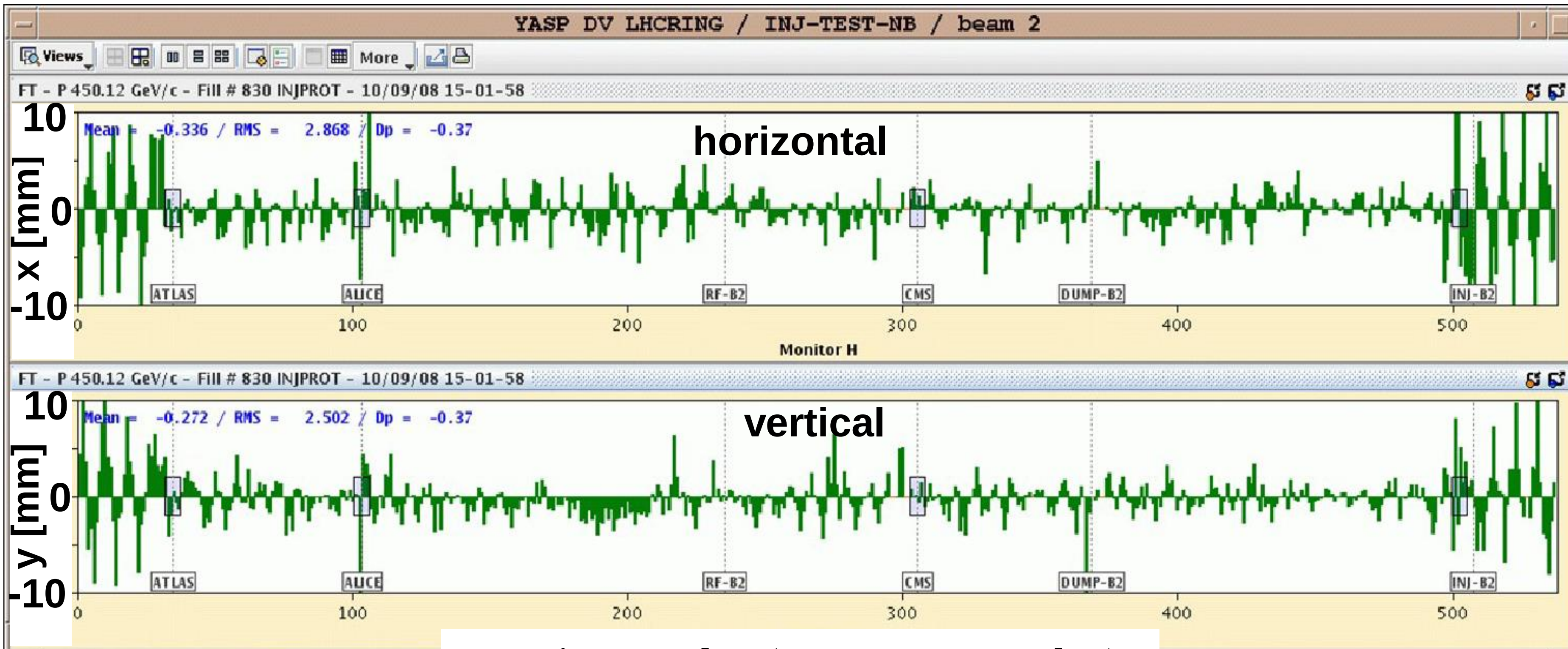
- Signal generation $\rightarrow$ transfer impedance
- Capacitive *button* BPM for high frequencies
  used at most proton LINACs and electron accelerators
- Capacitive *linear-cut* BPM for low frequencies
  used at most proton synchrotrons due to linear position reading
- Electronics for position evaluation
  analog signal conditioning to achieve small signal processing
- BPMs for measurement of closed orbit, tune and further lattice functions
  frequent application of BPMs
- Summary

Trajectory:

The position delivered by an **individual bunch** within a transfer line or a synchrotron.

Main task: Control of matching (center and angle), first-turn diagnostics

Example: LHC injection 10/09/08 i.e. first day of operation !

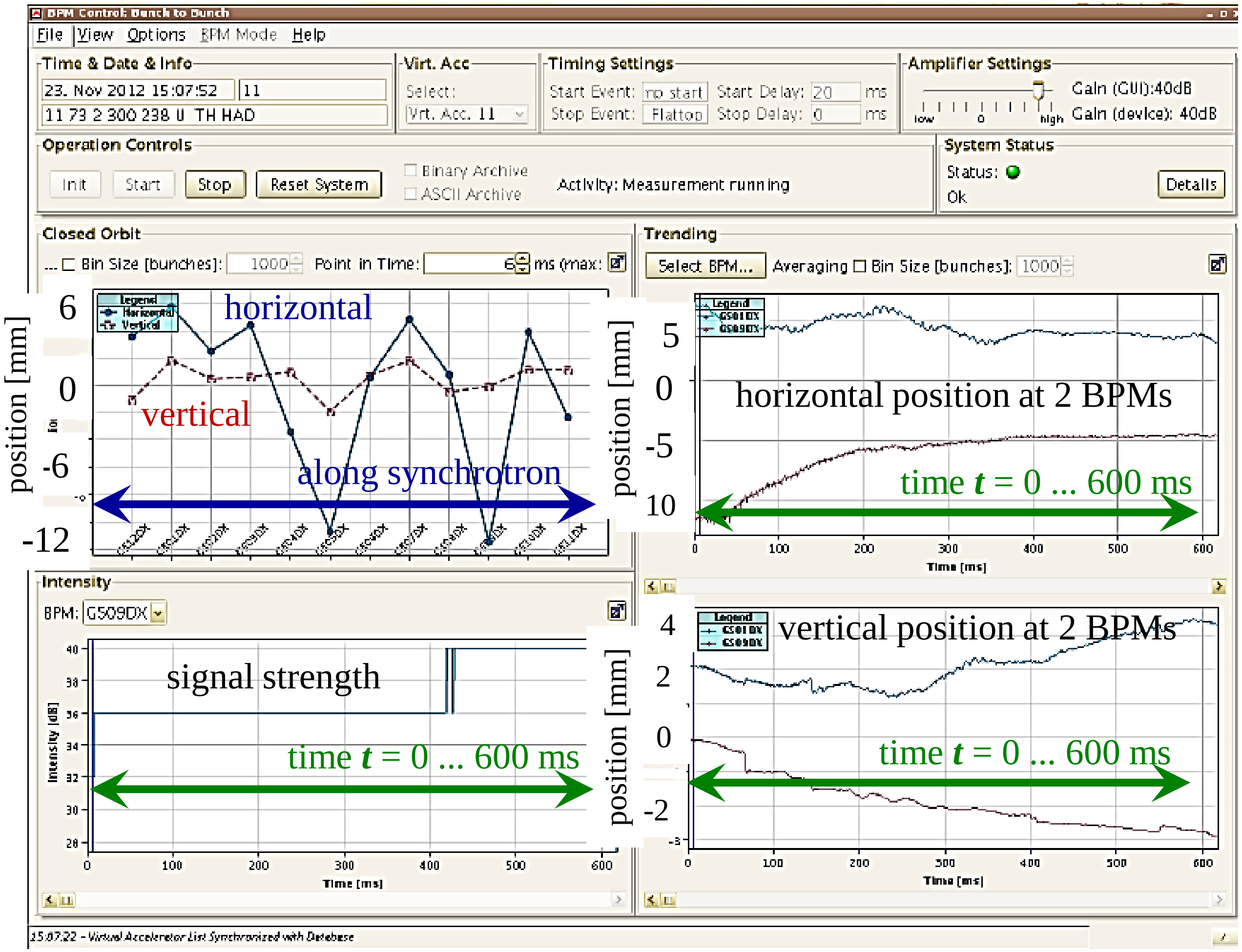


Monitor number (530 BPMs on 27 km)

From R. Jones (CERN)

Tune values: $Q_h = 64.3$, $Q_v = 59.3$

Single bunch position averaged over 1000 bunches → closed orbit with ms time steps.
It differs from ideal orbit by misalignments of the beam or components.
Example: GSI-synchrotron at two BPM locations, 1000 turn average during acceleration:



Closed orbit:
Beam position averaged over many turns (i.e. betatron oscillations).
The result is the basic tool for alignment & stabilization

Closed Orbit Feedback: Typical Noise Sources

Beam movement:

Short term (min to 10 ms):

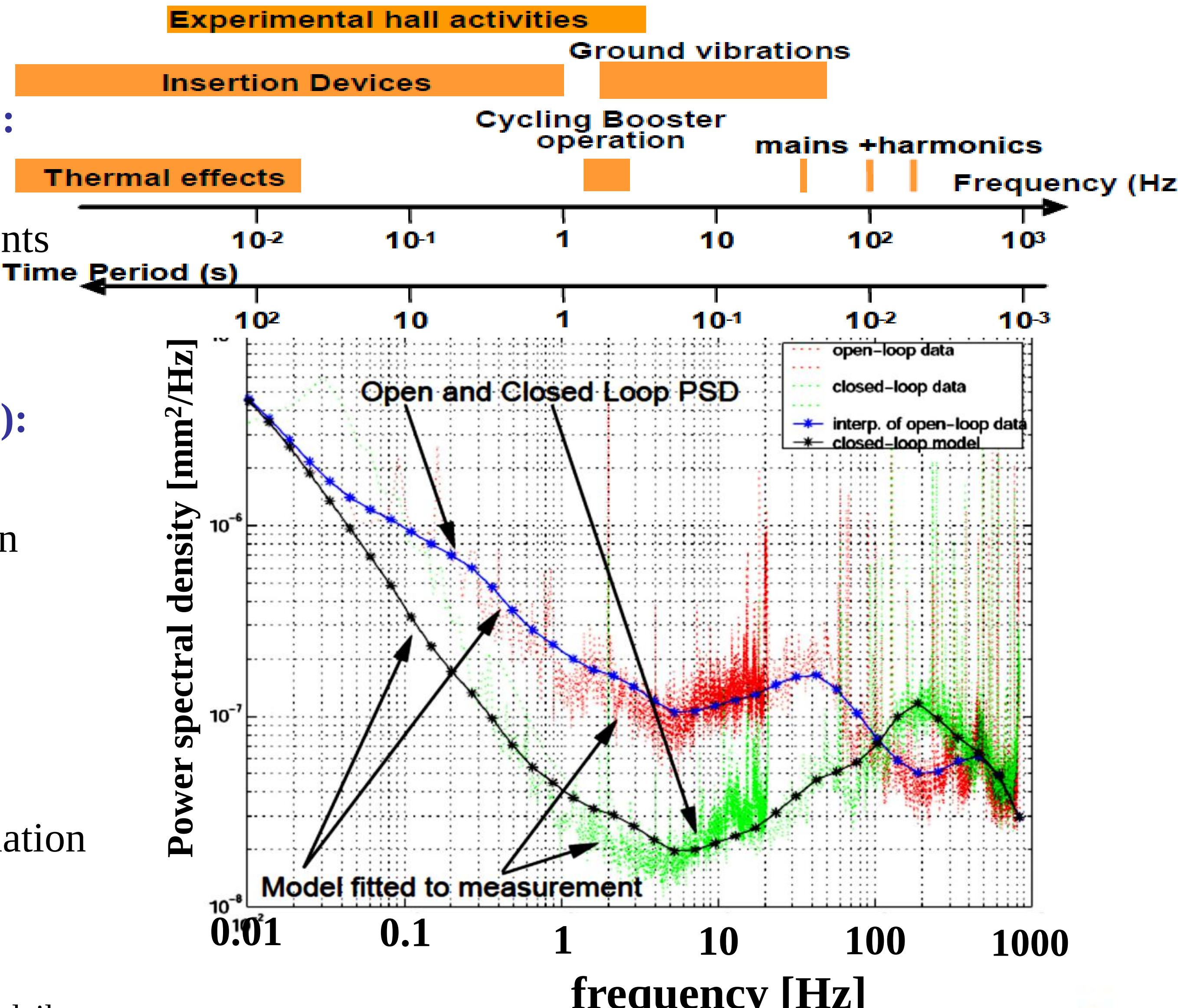
- Traffic
- Machine (crane) movements
- Water & vacuum pumps
- 50 Hz main power net

Medium term (day to min):

- Movement of chambers due to heating by radiation
- Day-night variation
- tide, moon cycle

Lang term (> days):

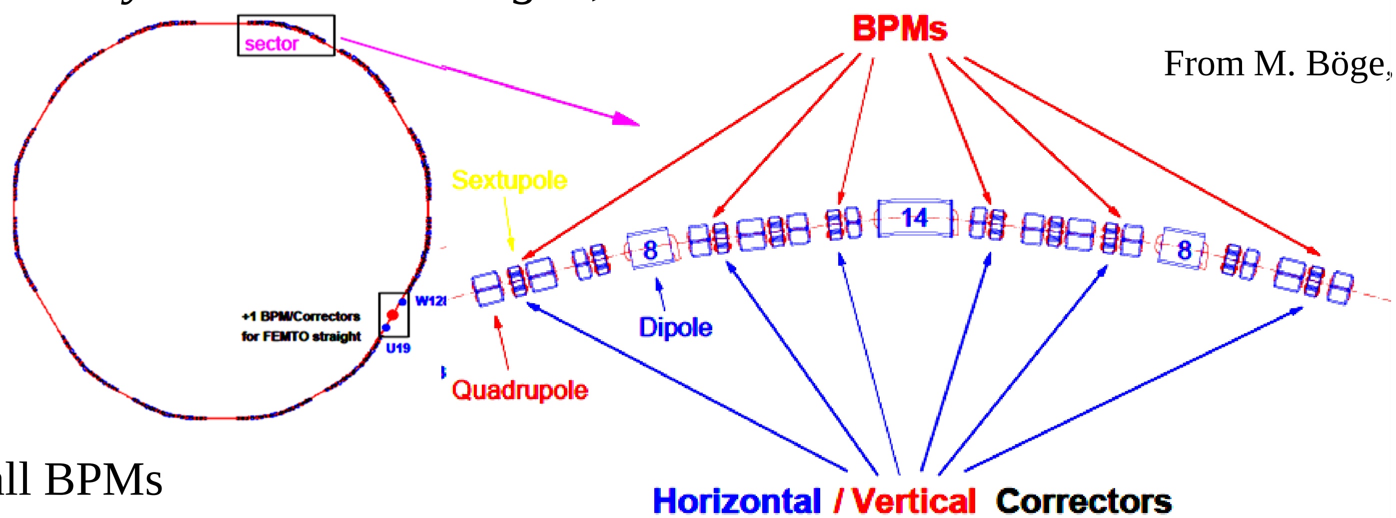
- Ground settlement
- Seasons, temperature variation



From M. Böge, PSI, N. Hubert, Soleil

Orbit feedback: Synchrotron light source → spatial stability of light beam

Example from SLS-Synchrotron at Villigen, Swiss:



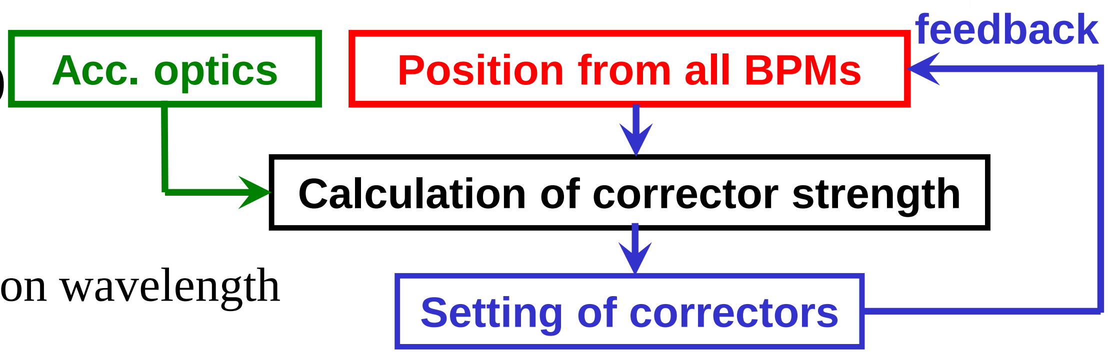
From M. Böge, PSI

Procedure:

1. Position from all BPMs
2. Calculation of corrector setting via Orbit Response Matrix (→ V. Ziemann)
3. Digital feedback loop

⇒ regulation time down to 10 ms

⇒ Role od thumb: ≈ 4 BPMs per betatron wavelength



Uncorrected orbit: typ. $\langle x^2 \rangle_{rms} \approx 1$ mm

Corrected orbit: $\langle x^2 \rangle_{rms} \approx 1$ μm up to 100 Hz bandwidth!

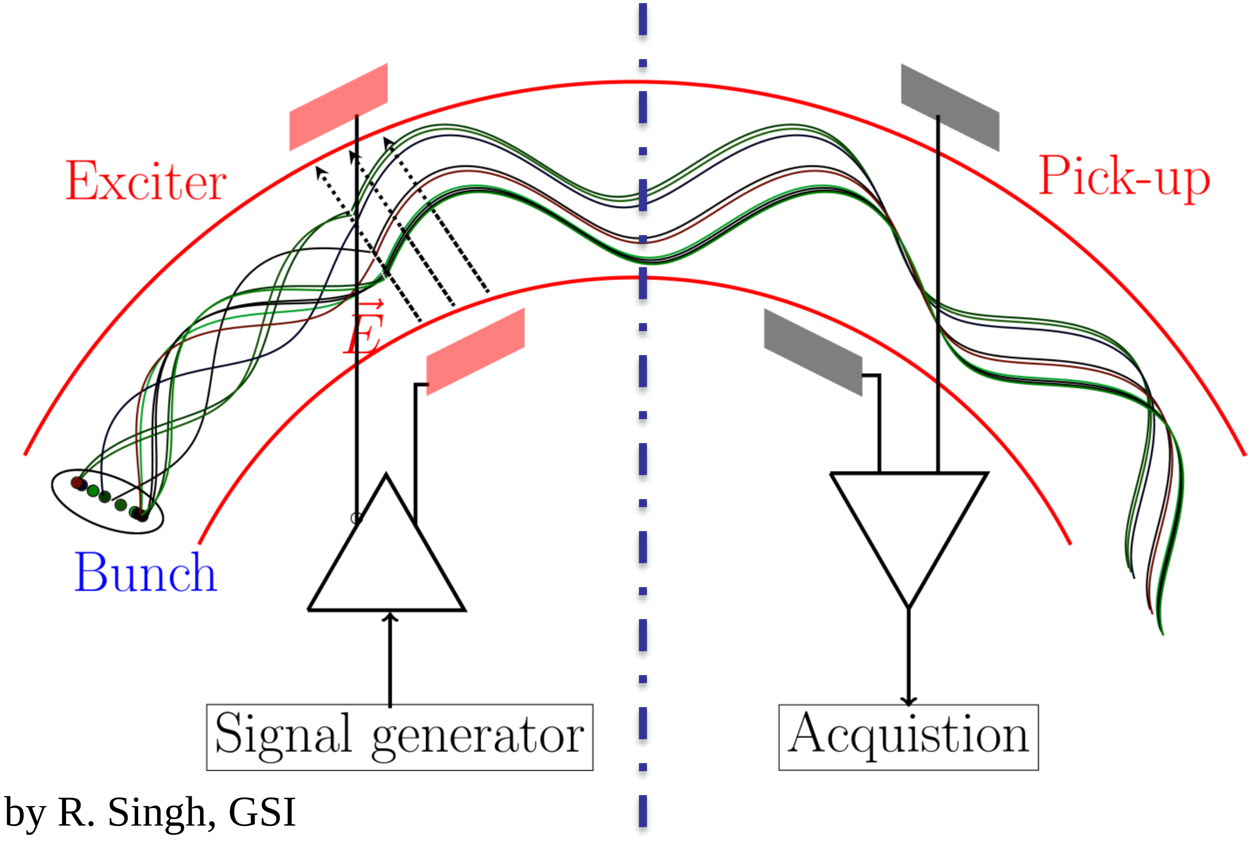
Tune Measurement: General Considerations

Coherent excitations are required for the detection by a BPM

Beam particle's ***in-coherent*** motion ⇒ center-of-mass stays constant

Excitation of **all** particles by rf ⇒ ***coherent*** motion

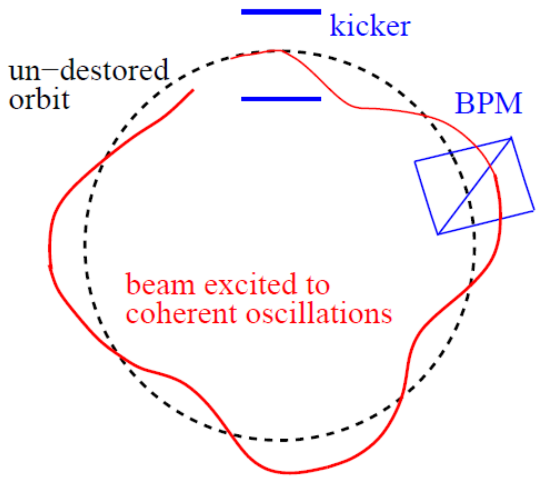
⇒ center-of-mass variation turn-by-turn i.e. center acts as **one** macro-particle

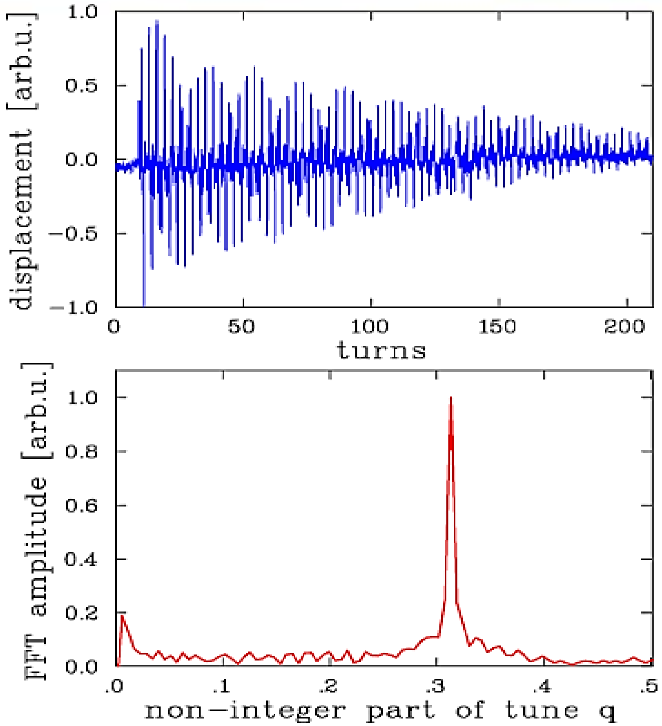


Graphics by R. Singh, GSI

Tune Measurement: The Kick-Method in Time Domain

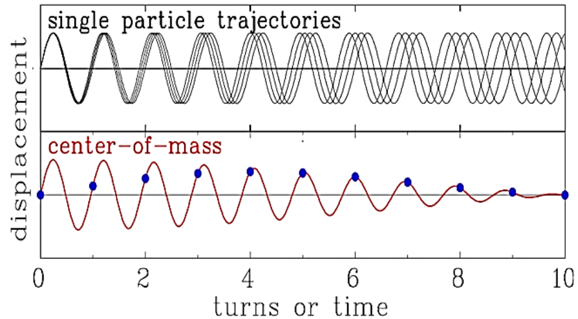
The beam is excited to coherent betatron oscillation
$\rightarrow$ the beam position measured each revolution ('turn-by-turn')
$\rightarrow$ Fourier Trans. gives the non-integer tune q.
Short kick compared to revolution.





The de-coherence time limits the **resolution**:
N non-zero samples
$\Rightarrow$ General limit of discrete FFT: $\Delta q > \frac{1}{2N}$

Here: N = 200 turn $\Rightarrow$ $\Delta q > 0.003$
(tune spreads can be $\Delta q \approx 0.001$!)



Decay is caused by de-phasing, **not** by decreasing single particle amplitude.

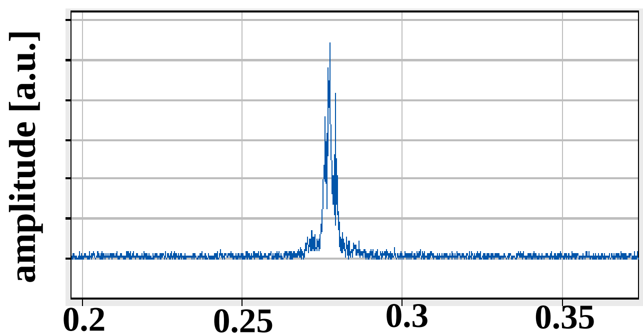
Instead of a sine wave, noise with adequate bandwidth can be applied

→ beam picks out its resonance frequency:

- broadband excitation with white noise of ≈ 10 kHz bandwidth
- turn-by-turn position measurement
- Fourier transformation of the recorded data

⇒ Continues monitoring with low disturbance

vertical tune at fixed time ≈ 15ms

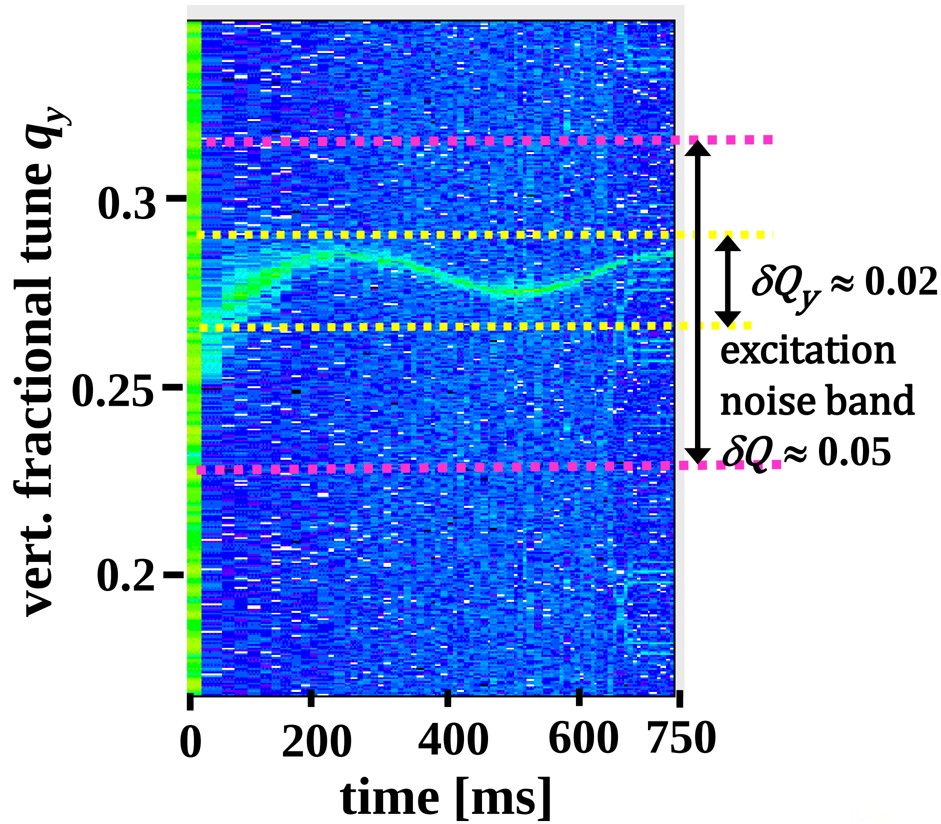


Advantage:

Fast scan with good time resolution

Example: Vertical tune within 4096 turn duration ≃ 15 ms at GSI synchrotron 11 → 300 MeV/u in 0.7 s

vertical tune versus time



U. Rauch et al., DIPAC 2009

Chromaticity ξ: Change of tune for off-momentum particle $$\frac{\Delta Q}{Q} = \xi \cdot \frac{\Delta p}{p}$$

Two step measurement procedure:

1. Change of momentum ***p*** by detuned rf-frequency $$\frac{\Delta p}{p} = \eta^{-1} \cdot \frac{\Delta f_{acc}}{f_{acc}}$$

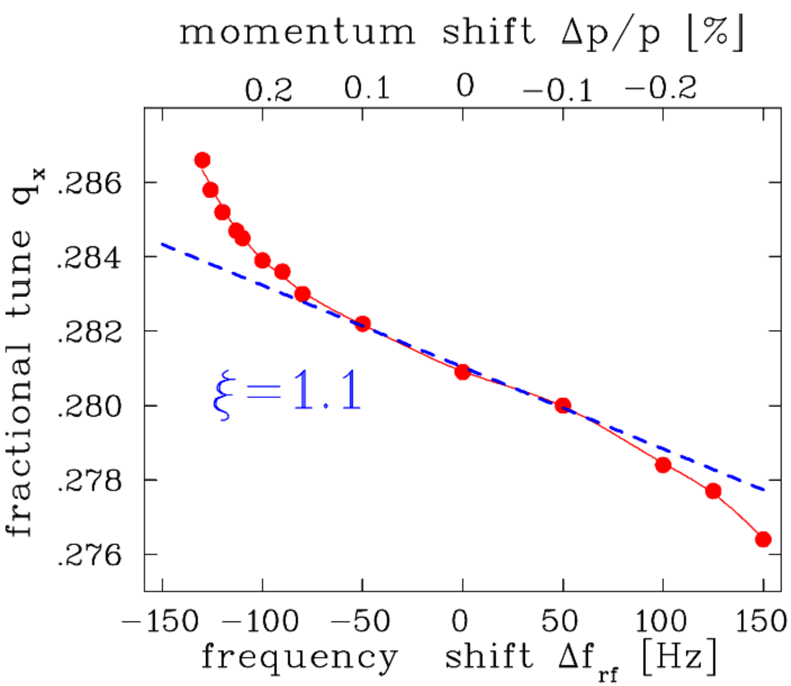
2. Excitation of coherent betatron oscillations and tune measurement

(kick-method, BTF, noise excitation):

Plot of *ΔQ/Q* as a function of *Δp/p*

$\Rightarrow$ slope is dispersion ξ.

Example: Measurement at LEP:



From M Minty, F. Zimmermann,
Measurement and Control of charged Particle Beam,
Springer Verlag 2003

β-Function Measurement from Bunch-by-Bunch BPM Data

Excitation of **coherent** betatron oscillations:

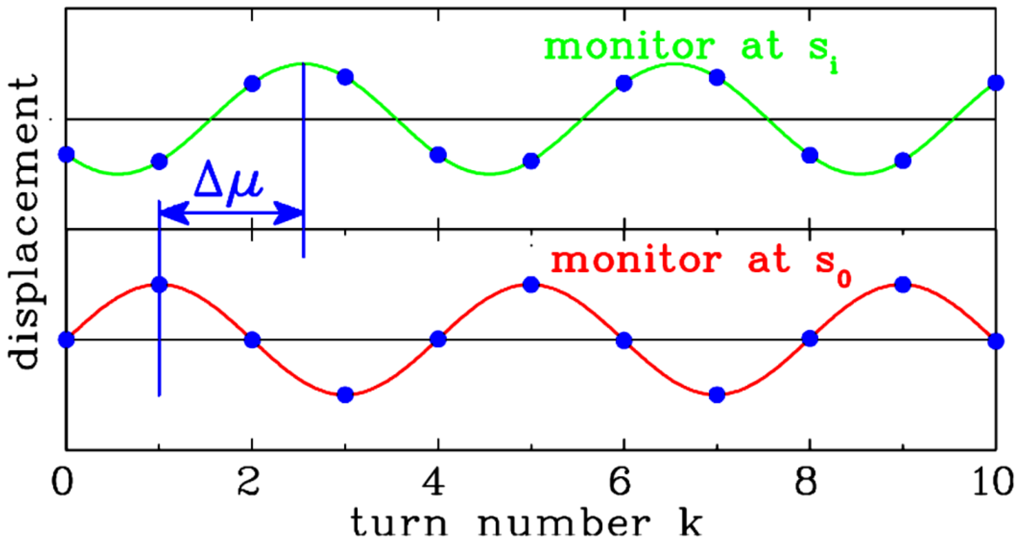
$\rightarrow$ Time-dependent position reading results the phase advance between BPMs

The phase advance is:

$$\Delta\mu = \mu_i - \mu_0$$

β-function from

$$\Delta\mu = \int_{S0}^{Si} \frac{ds}{\beta(s)}$$



Example: 'Beta-beating' at BPM $\Delta\beta = \beta_{meas} - \beta_{model}$ with measured β_{meas} & calculated β_{model} for each BPM at BNL for RHIC (proton-proton or ions circular collider with 3.8 km length)

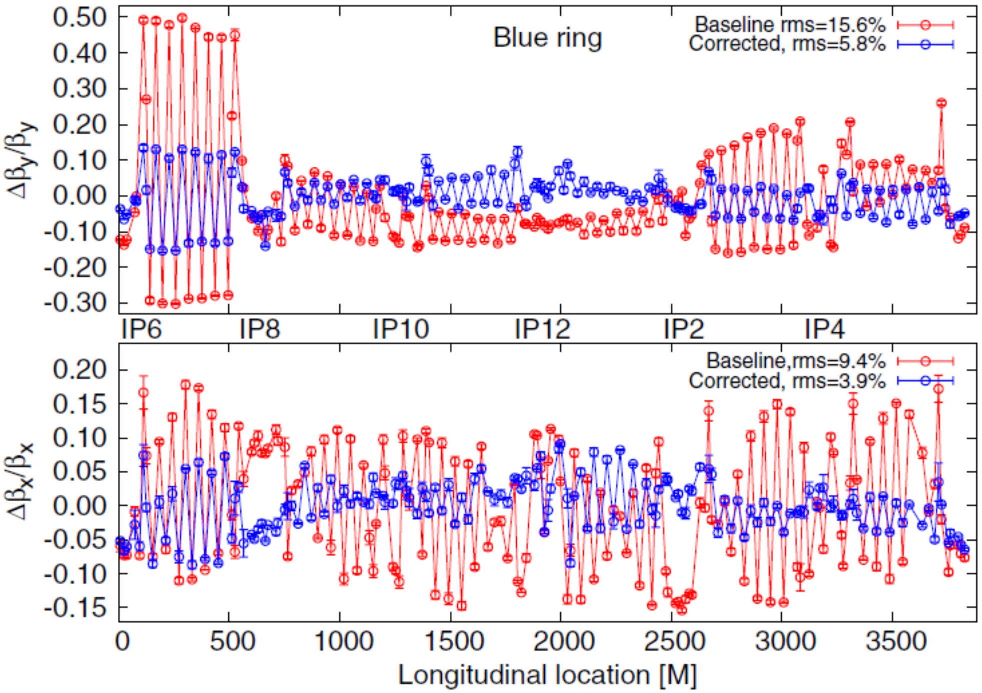
Result concerning 'beta-beating':

- Model doesn't fit reality completely e.g. caused by misalignments
- Corrections executed
- Increase of the luminosity

Remark:

Measurement accuracy depends on

- BPM accuracy
- Numerical evaluation method



Determination of β-function with 3 BPMs:

$$\beta_{meas}(BPM_1) = \beta_{model}(BPM_1) \cdot \frac{\cot[\mu_{meas}(1\to2)] - \cot[(\mu_{meas}(1\to3)]}{\cot[\mu_{model}(1\to2)] - \cot[\mu_{model}(1\to3)]}$$

See e.g.: R. Tomas et al., Phys. Rev. Acc. Beams **20,** 054801 (2017)
A. Wegscheider et al., Phys. Rev. Acc. Beams **20,** 111002 (2017)

From X. Shen et al.,
Phys. Rev. Acc. Beams **16**, 111001 (2013)

Dispersion Measurement from Closed Orbit Data

Dispersion D(s$_i$): Change of momentum ***p*** by detuned rf-cavity $\frac{\Delta p}{p} = \eta^{-1} \cdot \frac{\Delta f_{acc}}{f_{acc}}$

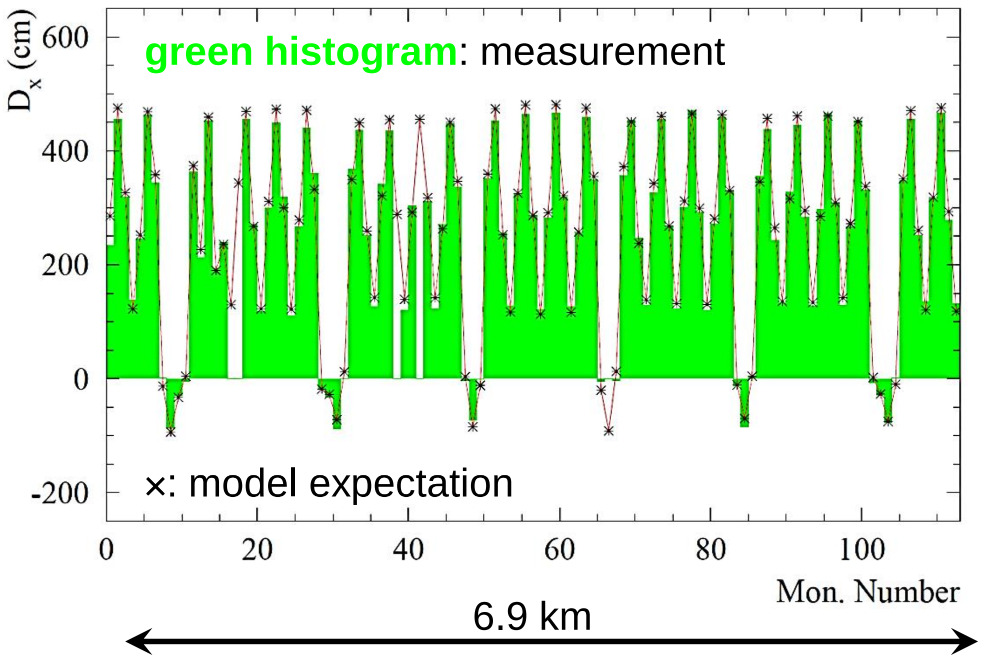
→ Position reading at one location $x_i = D(s_i) \cdot \frac{\Delta p}{p}$: ***η***: frequency slip factor

→ Result from plot of $\boldsymbol{x_i}$ as a function of **Δ*p*/*p*** ⇒ slope is local dispersion ***D(s$_i$)***

Example: Dispersion measurement ***D(s)*** at BPMs at CERN SPS

Theory-experiment correspondence after correction of

- BPM calibration
- quadrupole calibration



From J. Wenninger: CAS on BD, CERN-2009-005 & J. Wenninger CERN-AB-2004-009

Summary Pick-Ups for bunched Beams

The electric field is monitored for bunched beams using rf-technologies ('frequency domain'). Beside transformers they are the most often used instruments!

Differentiated or proportional signal: rf-bandwidth ↔ beam parameters

Proton synchrotron: 1 to 100 MHz, mostly 1 MΩ → proportional shape

LINAC, e−-synchrotron: 0.1 to 3 GHz, 50 Ω → differentiated shape

Important quantity: transfer impedance $Z_t(\omega, \beta)$.

Types of capacitive pick-ups:

Linear-cut (p-synch.), button (p-LINAC, e−-LINAC and synch.)

Position reading: difference signal of four pick-up plates (BPM):

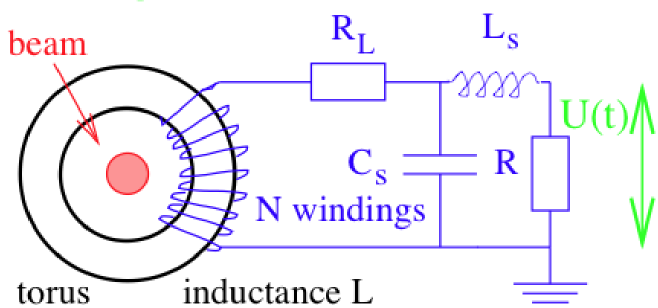
- Non-intercepting reading of center-of-mass → online measurement and control
  Synchrotron: slow reading → closed orbit, ***fast bunch-by-bunch***→ trajectory
- ***Synchrotron:*** Excitation of ***coherent*** betatron oscillations ⇒ tune q, ξ, $\beta(s)$, $D(s)$...

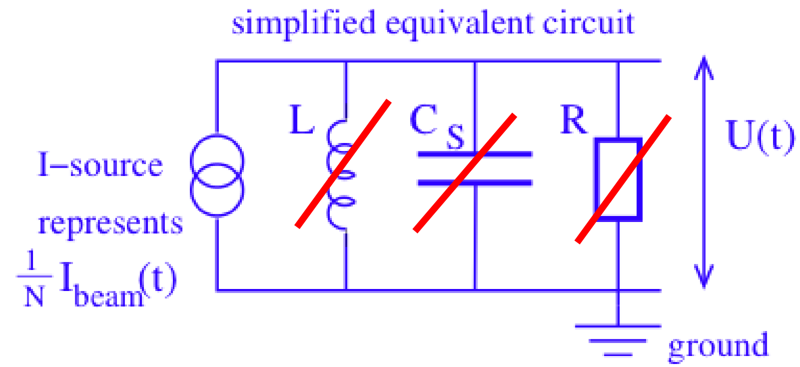
Remark: BPMs have high pass characteristic ⇒ no signal for dc-beams

Thank you for your attention!

Backup slides

Analysis of a simplified electrical circuit of a passively loaded transformer:





For this parallel shunt:

$$\frac{1}{Z} = \frac{1}{i\omega L} + \frac{1}{R} + i\omega C_S \Leftrightarrow Z = \frac{i\omega L}{1 + i\omega L/R - \omega L/R \cdot \omega R C_S}$$

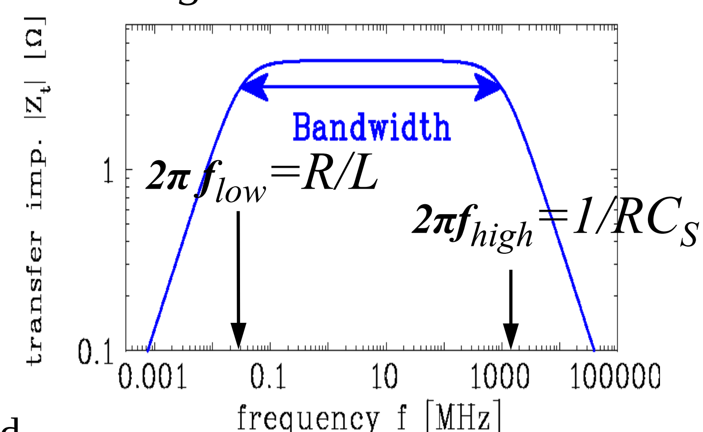
- ***Low frequency*** $\omega \ll R/L$: $Z \rightarrow i\omega L$

  i.e. no dc-transformation
- ***High frequency*** $\omega \gg 1/RC_S$: $Z \rightarrow 1/i\omega C_S$

  i.e. current flow through C_S
- ***Working region*** $R/L < \omega < 1/RC_S$: $Z \simeq R$

  i.e. voltage drop at R and sensitivity $S=R/N$.

No oscillations due to over-damping by low R = 50 Ω to ground.

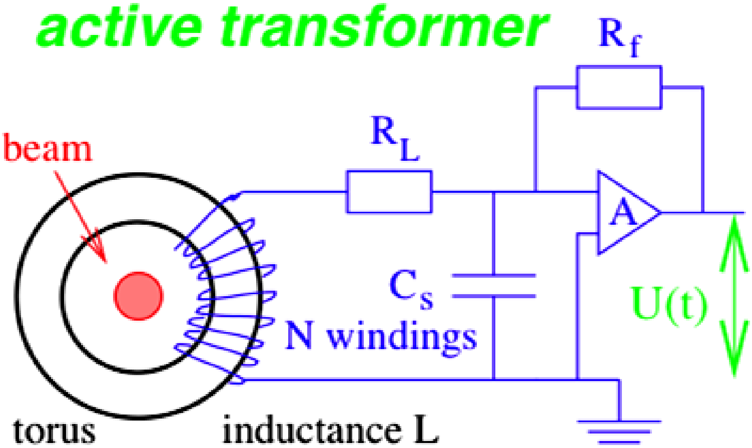


Active Transformer or Alternating Current Transformer ACT:

uses a trans-impedance amplifier (I/U converter) to $\boldsymbol{R} \approx \mathbf{0}$ Ω load impedance i.e. a current sink

+ compensation feedback ⇒ longer droop time $\boldsymbol{\tau_{droop}}$

Application: measurement of longer $\boldsymbol{t}$ > 10 μs e.g. at pulsed LINACs



The input resistor is for an op-amp: $\boldsymbol{R_f/A << R_L}$

⇒ $\boldsymbol{\tau_{droop} = L/(R_f/A+R_L) \simeq L/R_L}$

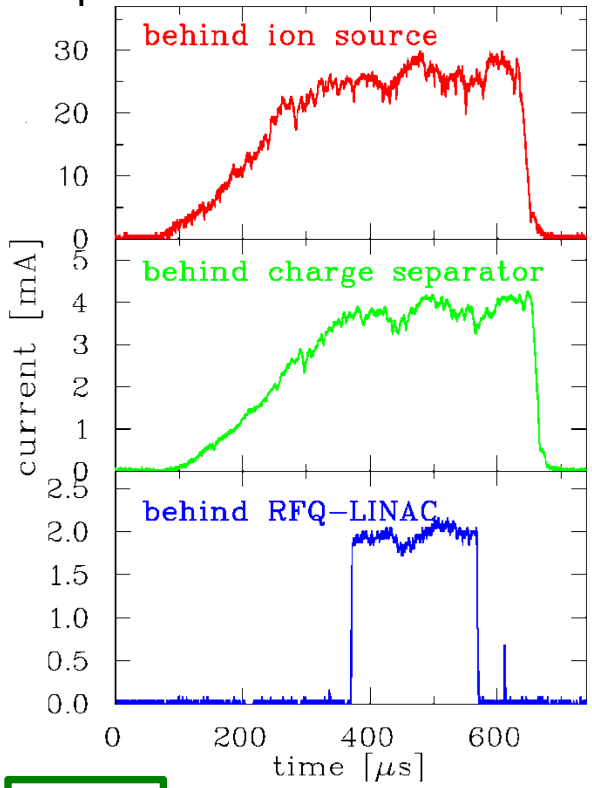
Droop time constant can be up to 1 s!

Feedback resistor is also used for range switching.

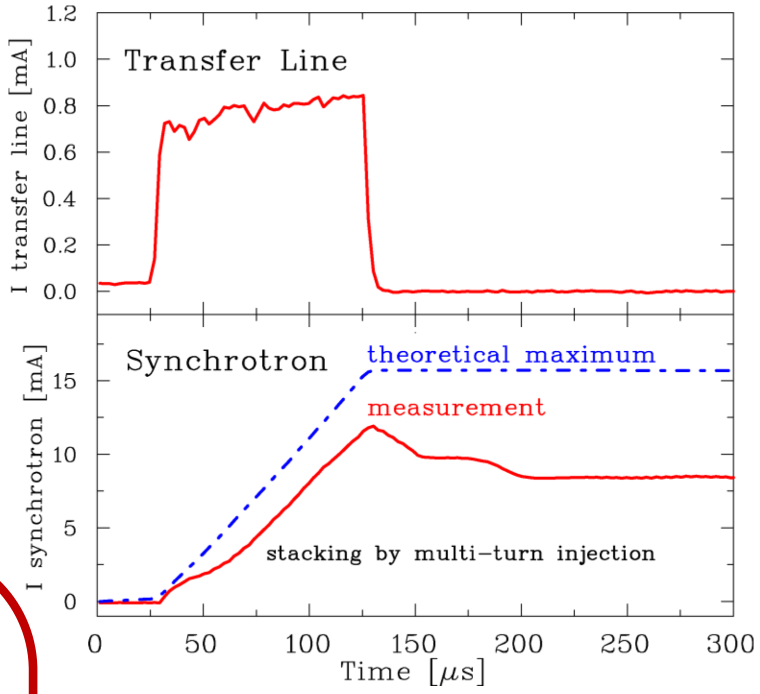
Torus inner radius	r_i=30 mm
Torus outer radius	r_o=45 mm
Core thickness	l=25 mm
Core material	**Vitrovac 6025** $(CoFe)_{70\%}(MoSiB)_{30\%}$
Core permeability	$u_r=10^5$
Number of windings	**2x10 crossed**
Max. sensitivity	10^6 **V/A**
Beam current range	**10 μA to 100 mA**
Bandwidth	**1 MHz**
Droop	**0.5 % for 5 ms**
rms resolution	**0.2 μA for full bw**

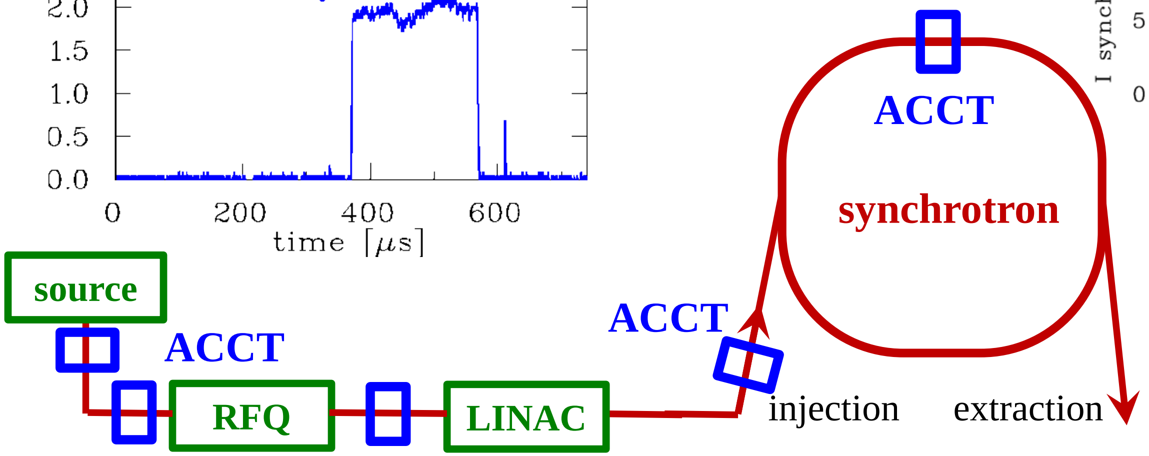
Active transformer for the measurement of long ***t*** > 10 µs pulses e.g. at pulsed LINACs

Example: Transmission and macro-pulse shape for Ni^{2+} beam at GSI LINAC



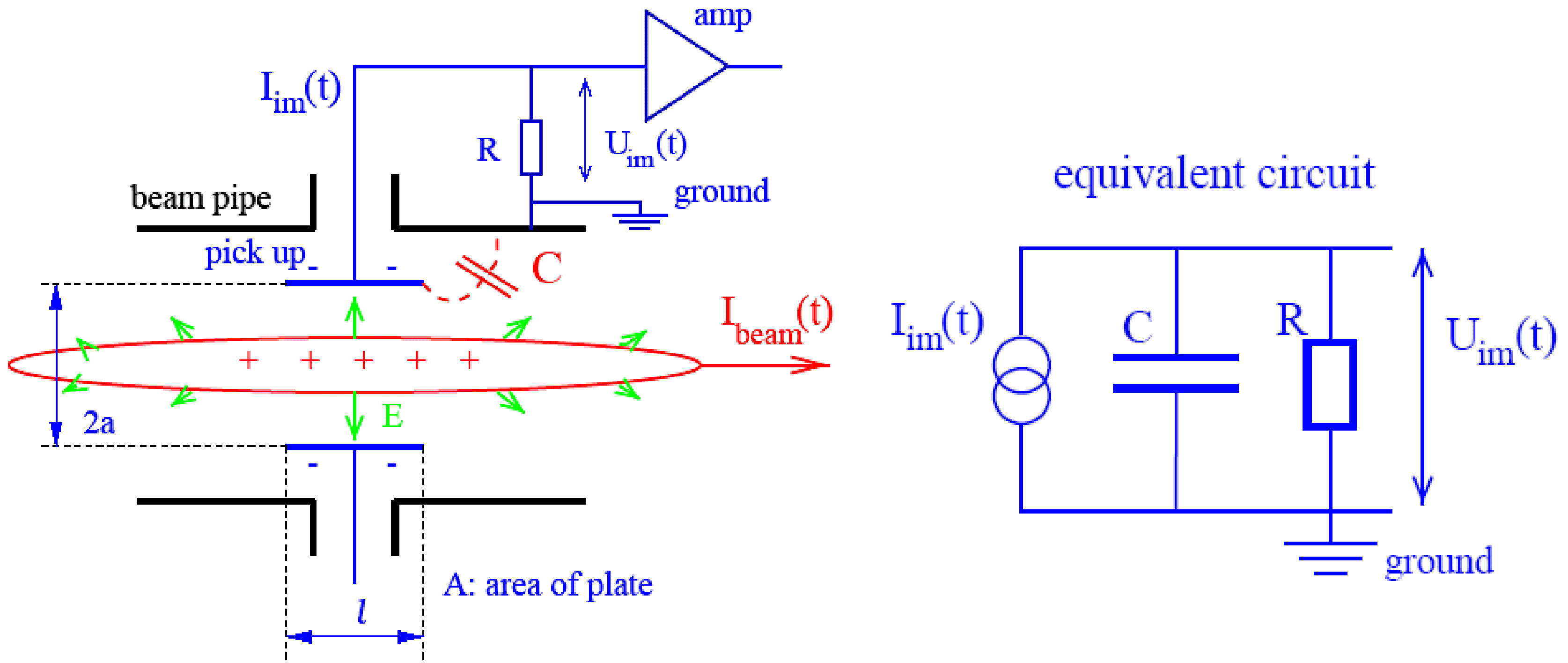
***Example*:** Multi-turn injection of a Ni^{26+} beam into GSI Synchrotron, 5 µs per turn





→ Transformer are frequently used for operation.

The wall current is monitored by a plate or ring inserted in the beam pipe:



The image current I_{im} at the plate is given by the beam current and geometry:

$$I_{im}(t) = -\frac{dQ_{im}(t)}{dt} = \frac{-A}{2\pi a l} \cdot \frac{dQ_{beam}(t)}{dt} = \frac{-A}{2\pi a} \cdot \frac{1}{\beta c} \cdot \frac{dI_{beam}(t)}{dt} = \frac{A}{2\pi a} \cdot \frac{1}{\beta c} \cdot i\omega I_{beam}(\omega)$$

Using a relation for Fourier transformation: $I_{beam} = I_0 e^{-i\omega t} \Rightarrow dI_{beam}/dt = -i\omega I_{beam}$.

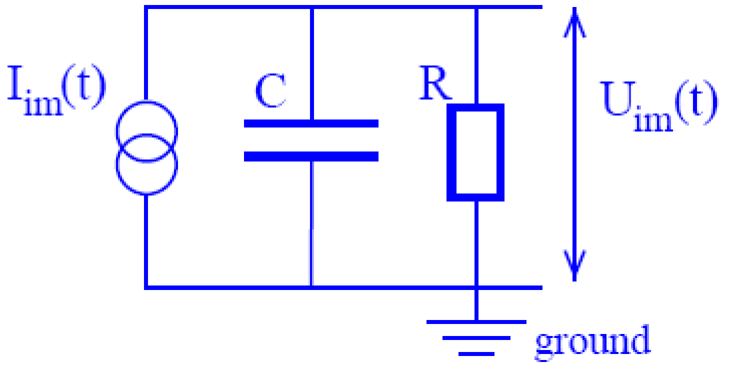
At a resistor ***R*** the voltage U_{im} from the image current is measured.
The transfer impedance Z_t is the ratio between voltage U_{im} and beam current I_{beam} in ***frequency domain***: $U_{im}(\omega) = R \cdot I_{im}(\omega) = Z_t(\omega, \beta) \cdot I_{beam}(\omega)$.

Capacitive BPM:

- The pick-up capacitance ***C***: plate ↔ vacuum-pipe and cable.
- The amplifier with input resistor ***R***.
- The beam is a high-impedance current source:

$$U_{im} = \frac{R}{1+i\omega RC} \cdot I_{im}$$
$$= \frac{A}{2\pi a} \cdot \frac{1}{\beta c} \cdot \frac{1}{C} \cdot \frac{i\omega RC}{1+i\omega RC} \cdot I_{beam}$$
$$\equiv Z_t(\omega, \beta) \cdot I_{beam}$$

equivalent circuit



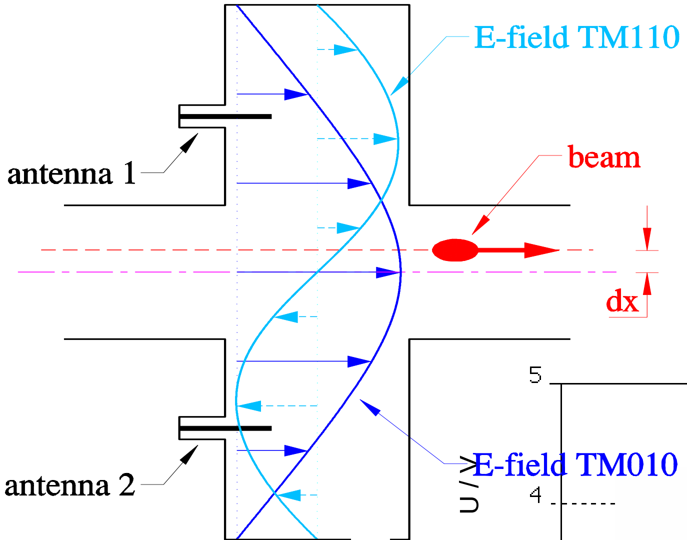
$$\frac{1}{Z} = \frac{1}{R} + i\omega C \Leftrightarrow Z = \frac{R}{1+i\omega RC}$$

This is a high-pass characteristic with $\omega_{cut} = 1/RC$:

Amplitude: $|Z_t(\omega)| = \frac{A}{2\pi a} \cdot \frac{1}{\beta c} \cdot \frac{1}{C} \cdot \frac{\omega/\omega_{cut}}{\sqrt{1+\omega^2/\omega^2_{cut}}}$ **Phase:** $\varphi(\omega) = \arctan(\omega_{cut}/\omega)$

Cavity BPM: Principle

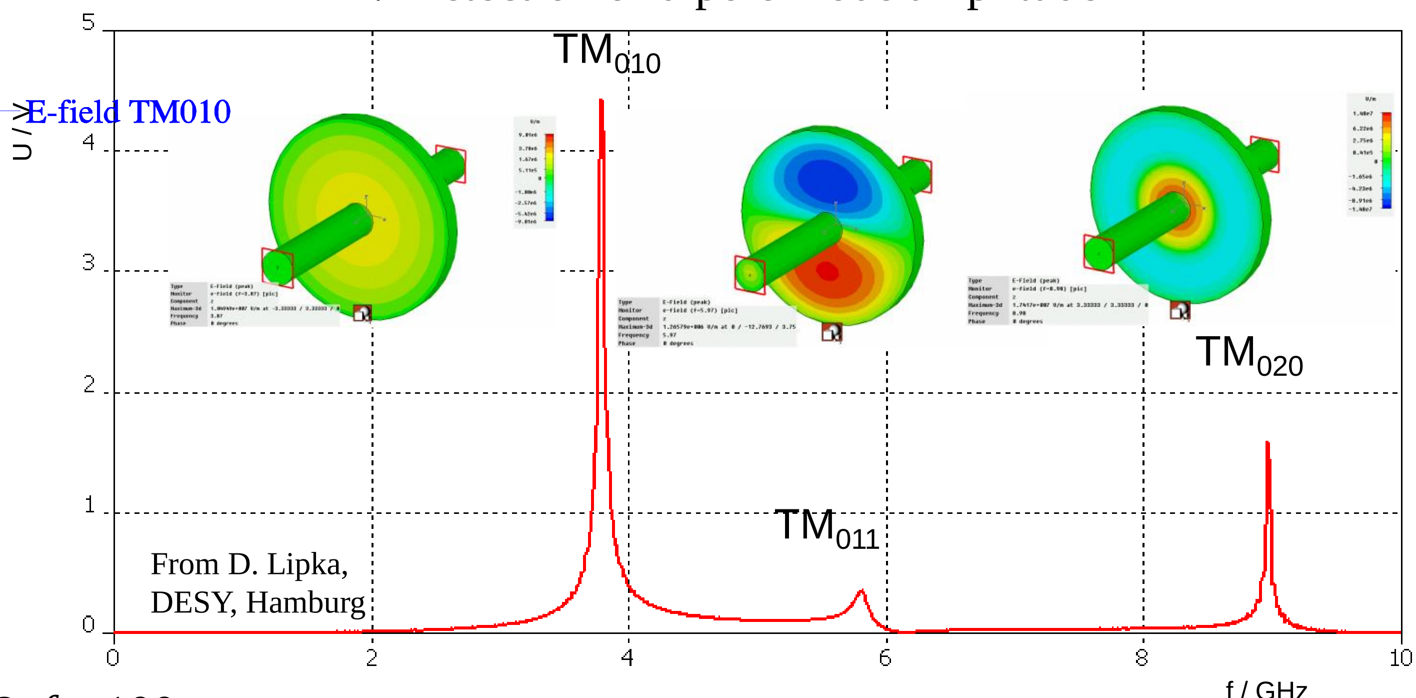
High resolution on t < 1 μs time scale can be achieved by excitation of a dipole mode:



For pill box the resonator modes given by geometry:

- monopole TM_{010} with f_{010}
  → maximum at beam center ⇒ strong excitation
- Dipole mode TM_{011} with f_{011}
  → minimum at center ⇒ excitation by beam offset

⇒ Detection of dipole mode amplitude



Application:

small e⁻ beams

and short pulses < ns

(ILC, X-FEL…)

'δ-excitation'

→oscillation with

Q ≈ 1000 and $\tau = 2Q/2\pi f \approx 100$ ns

Cavity BPM: Example of Realization

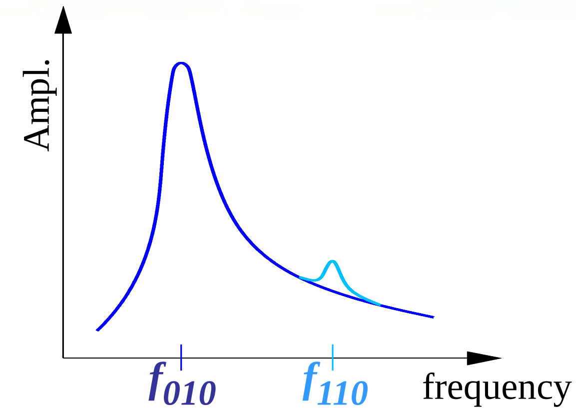
Basic consideration for detection of eigen-frequency amplitudes:

Dipole mode f_{110} separated from monopole mode

due to finite quality factor $\boldsymbol{Q} \Rightarrow \boldsymbol{\Delta f = f/Q}$

- Frequency $f_{110} \approx 1\ldots10$ GHz
- Waveguide house the antennas

Task: suppression of TM_{010} mono-pole mode



FNAL realization:

Cavity: ∅ 113 mm

Gap 15 mm

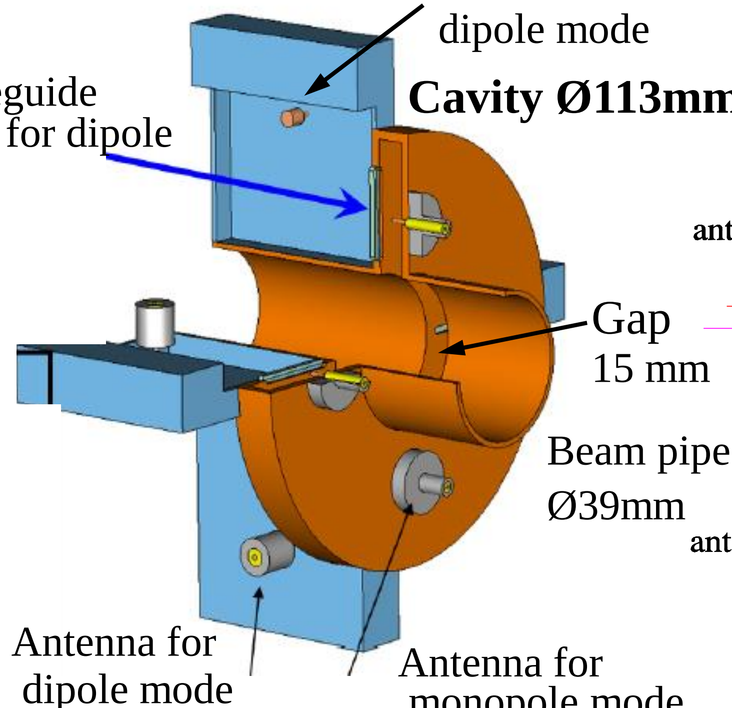
Mono. f_{010}=1.1GHz

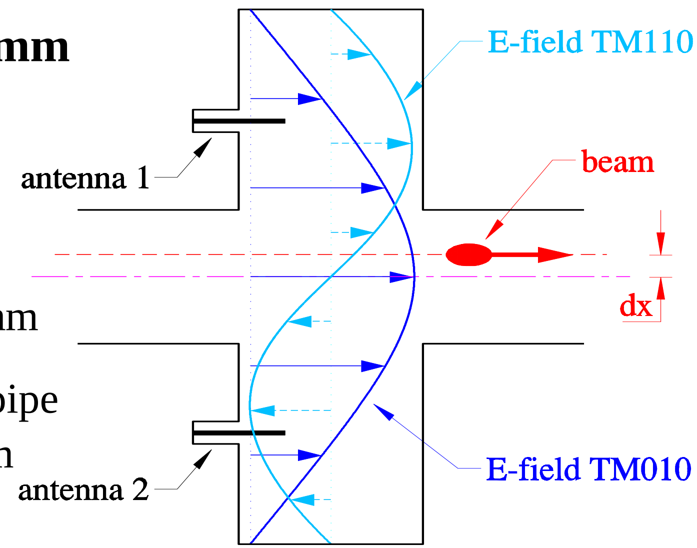
Dipole. f_{110}=1.5GHz

$Q_{load} \approx 600$

With comparable BPM

⇒0.1 µm resolution within 1 µs



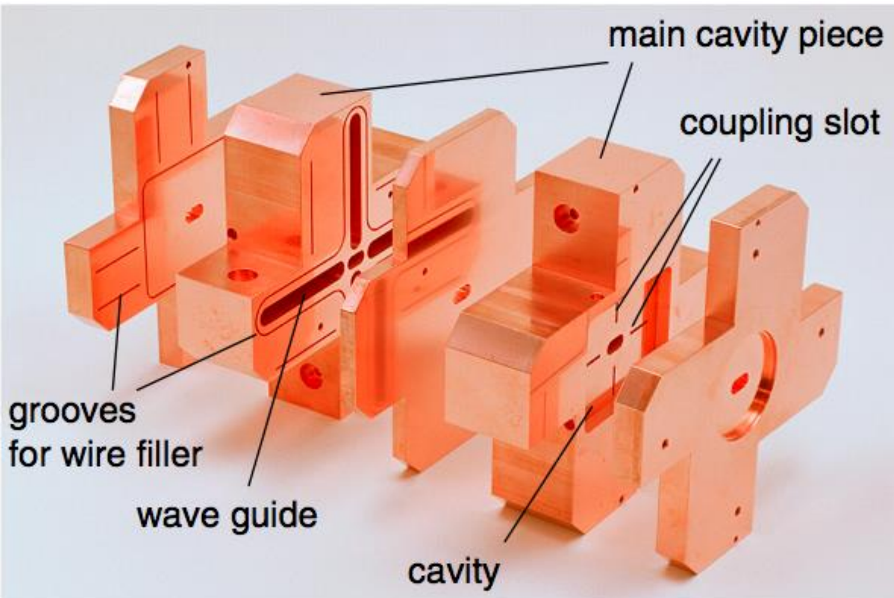


From M. Wendt (FNAL)

Cavity BPM: Suppression of monopole Mode

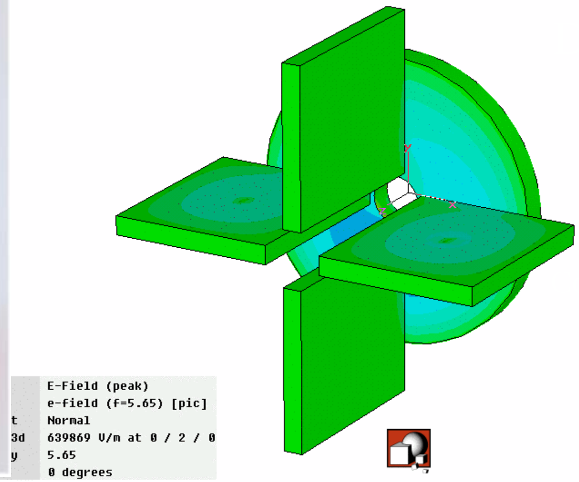
Suppression of mono-pole mode: waveguide that couple only to dipole-mode

due to $f_{mono} < f_{cut} < f_{dipole}$



Courtesy of D. Lipka and Y. Honda

Dipole-pole mode



Courtesy of D. Lipka, DESY, Hamburg

Prototype BPM for ILC Final Focus

- Required resolution of 2 nm in a 6 × 12 mm diameter beam pipe
- Achieved World Record so far: **resolution** of 8.7 nm at ATF2 (KEK, Japan)

Instead of one kick, the beam can be excited by a sweep of a sine wave, called 'chirp'

→ **B**eam **T**ransfer **F**unction (BTF) Measurement

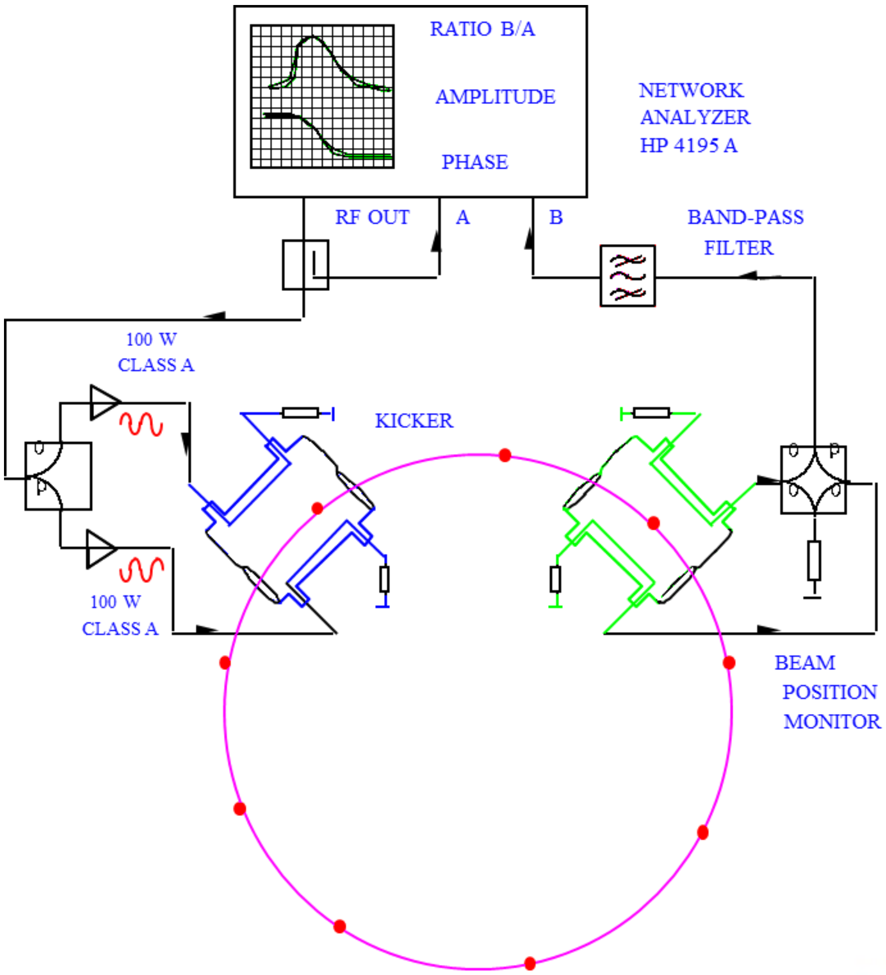
as the velocity response to a kick

Principle:

Beam acts like a driven oscillator!

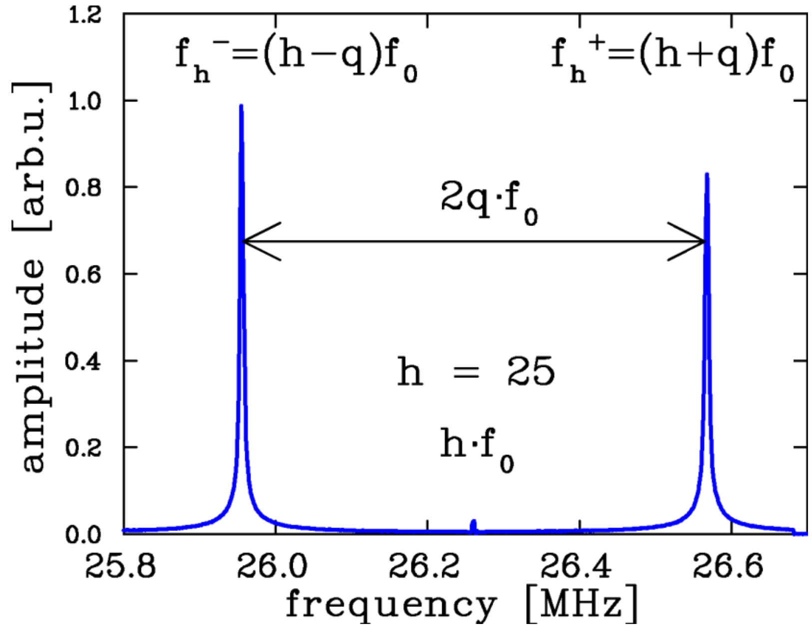
Using a network analyzer:

- RF OUT is feed to the beam by a kicker (reversed powered as a BPM)
- The position is measured at one BPM
- Network analyzer: amplitude and phase of the response
- Sweep time up to seconds due to de-coherence time per band
- resolution in tune: up to 10^{-4}

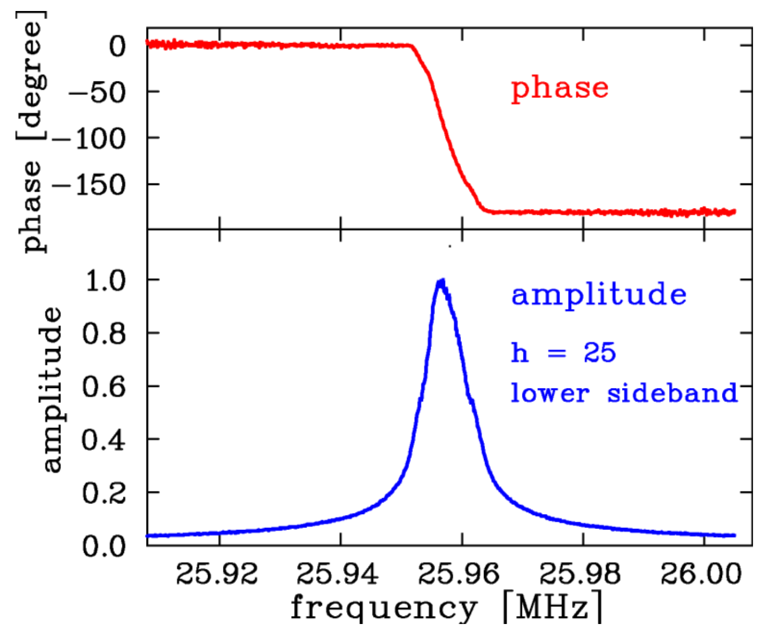


BTF measurement at the GSI synchrotron, recorded at the 25th harmonics.

A wide scan with both sidebands at $h=25^{th}$-harmonics:



*A detailed scan for the **lower** sideband $\rightarrow$ beam acts like a driven oscillator:*



From the position of the sidebands $\boldsymbol{q}$ = 0.306 is determined. From the width $\boldsymbol{\Delta f/f} \approx 5 \cdot 10^{-4}$ the tune spread can be calculated via $\Delta f_h^- = \eta \frac{\Delta p}{p} \cdot h f_0 \left(h - q + \frac{\xi}{\eta} Q \right)$

Advantage: High resolution for tune and tune spread (also for de-bunched beams)

Disadvantage: Long sweep time (up to several seconds).

Excitation of **coherent** betatron oscillations:

$\rightarrow$ Time-dependent position reading results the phase advance between BPMs

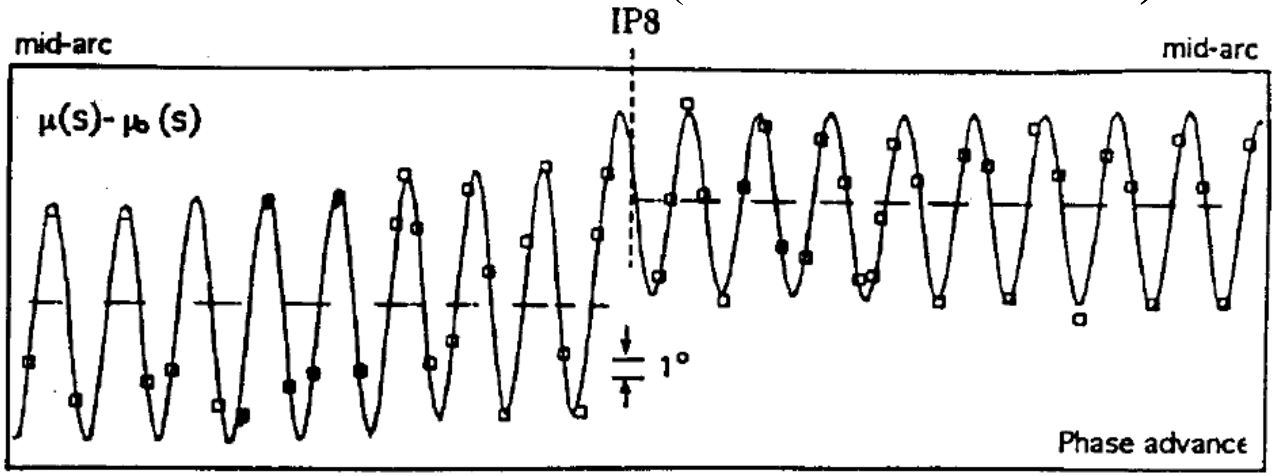
The phase advance is:

$$\Delta\mu = \mu_i - \mu_0$$

β-function from

$$\Delta\mu = \int_{S0}^{Si} \frac{ds}{\beta(s)}$$

Example: Phase advance $\boldsymbol{\mu(s)}$ compared to the expected $\boldsymbol{\mu_0(s)}$ at each BPM at CERN's at LEP (e^+ - e^- collider of 27 km)



Result:

- Model does not describes the reality completely, corrections required
- At interaction point IP (detector location) an additional phase shift is originated
- Alignment by correction dipoles (steerer), quadrupoles or sextupoles.

From J. Borer et al, EPAC'92