# FFA Accelerators
## ‘Fixed Field Alternating Gradient’

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Many thanks to Dr. S. Machida for his advice and previous lecture materials

# ‘Fixed Field Alternating Gradient’ Accelerators

- Are FFAs like a synchrotron or cyclotron?
    - EMMA non-scaling FFA
- Fixed field magnets
- Beam dynamics
- Scaling FFAs
- Advanced FFA types and optics

# Motivation

- Many challenges for future accelerators:

**High power**

Neutrons, muons, ADS

**Reliable**

Medical, ADS

**Flexible**

Is industry limited by existing technology?

**Rapid acceleration**

Muon beams, Unstable nuclei

**Cheap**

Hadron accelerators aren't known for being cheap

# Is an FFA like a synchrotron? (1)

*"Particles should be constrained to move in a circle of constant radius thus enabling the use of an annular ring of magnetic field … which would be varied in such a way that the radius of curvature remains constant as the particles gain energy through successive accelerations"* - Marcus Oliphant, 1943

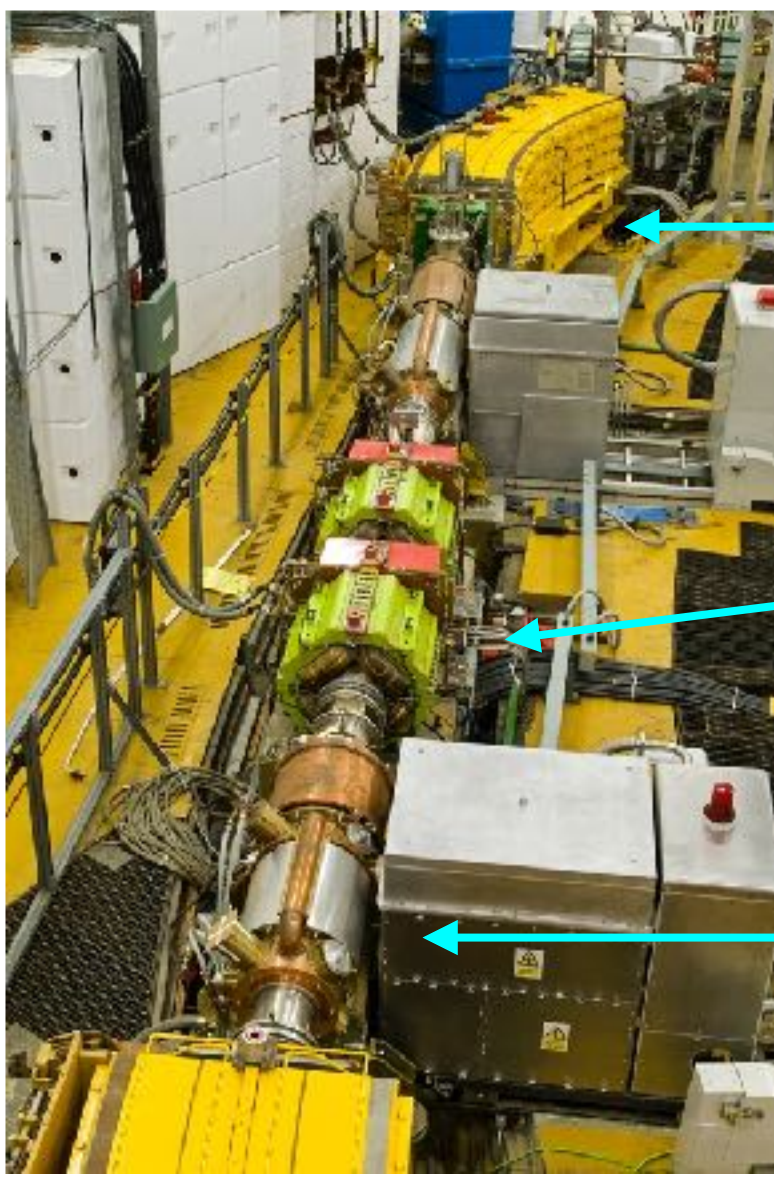
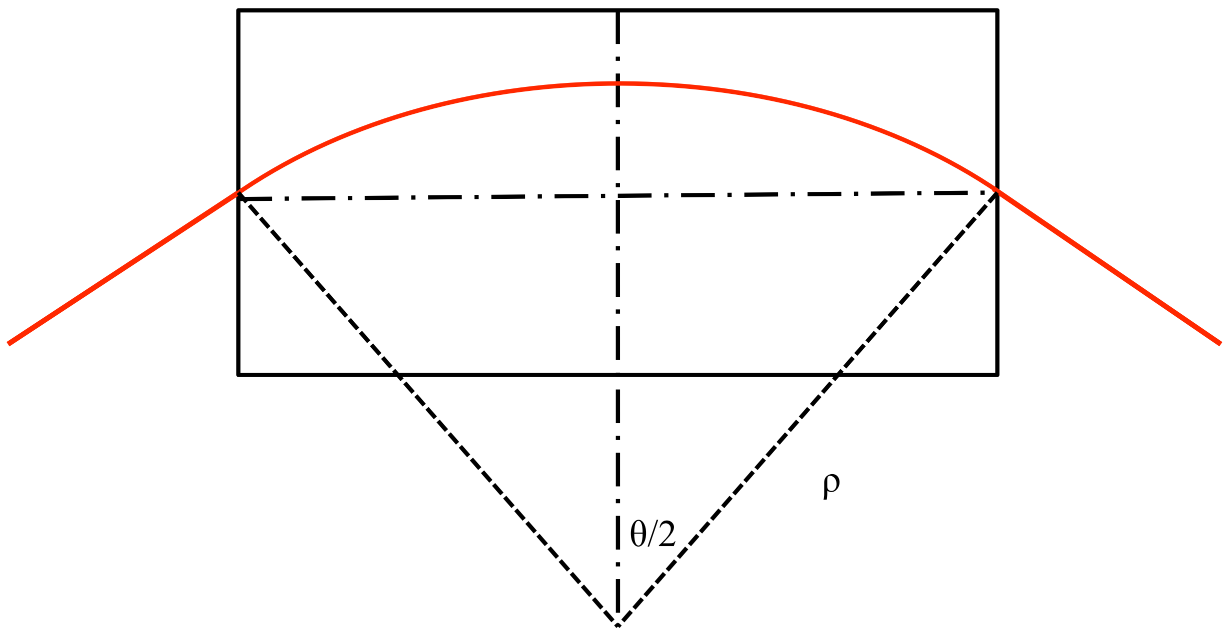


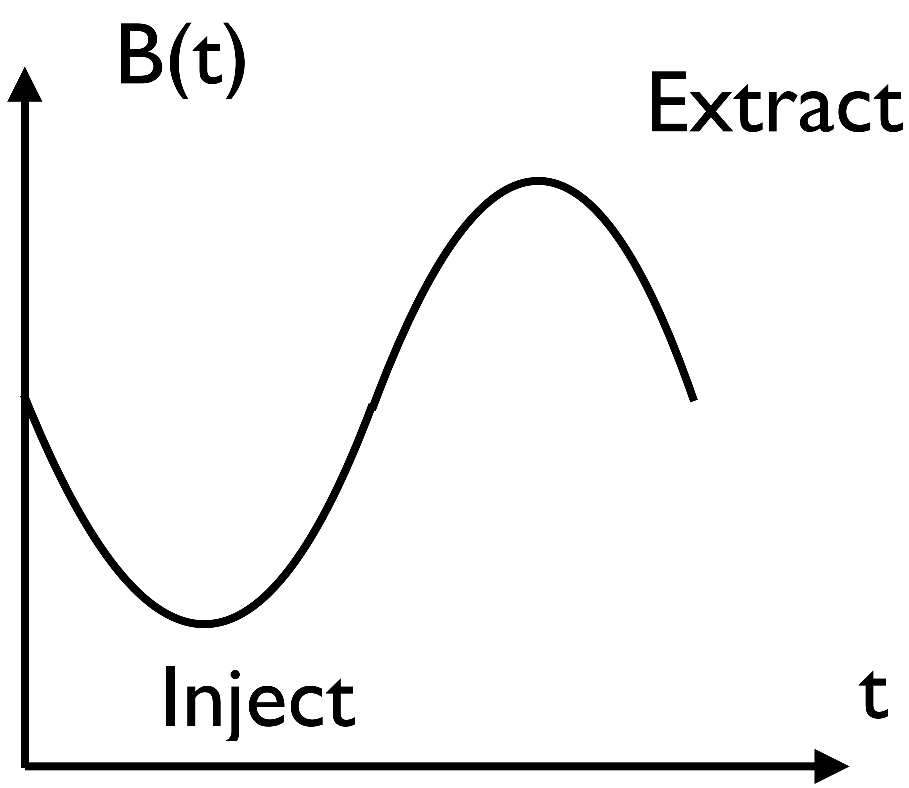

*Image courtesy of ISIS, STFC*

# Is an FFA like a synchrotron? (2)



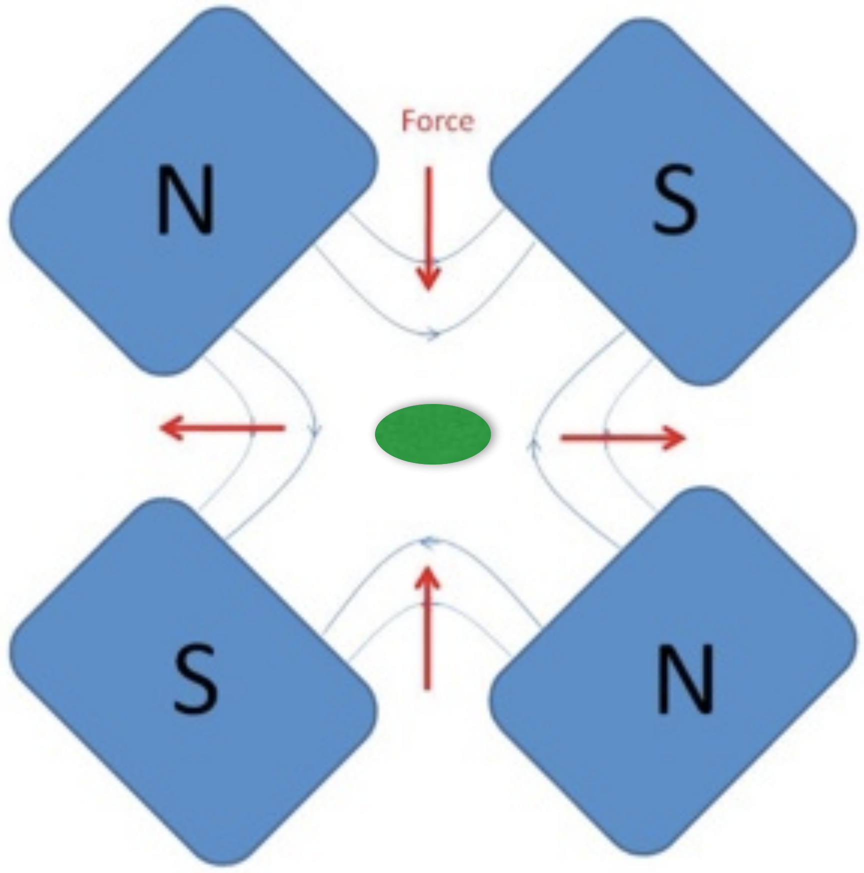

$$\sin(\theta/2) = \frac{B(t)L}{2(B(t)\rho)}$$

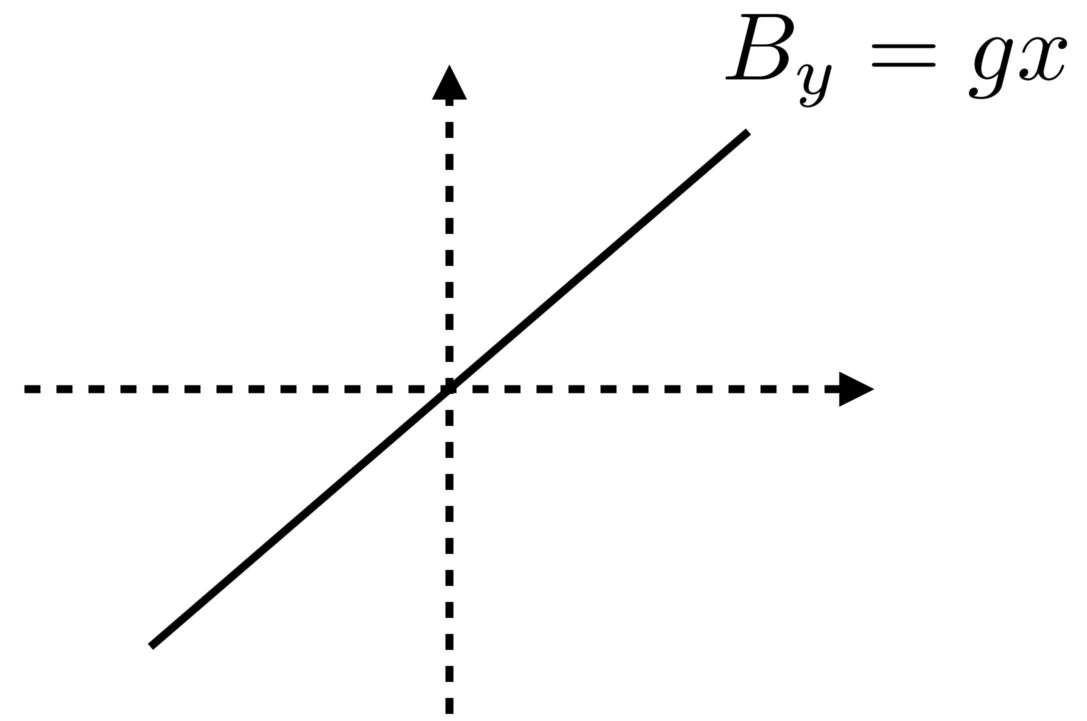
$$\theta \approx \frac{B(t)L}{p(t)/q}$$




**What happens if I don't ramp the B field with E?**

# Is an FFA like a synchrotron? (3)



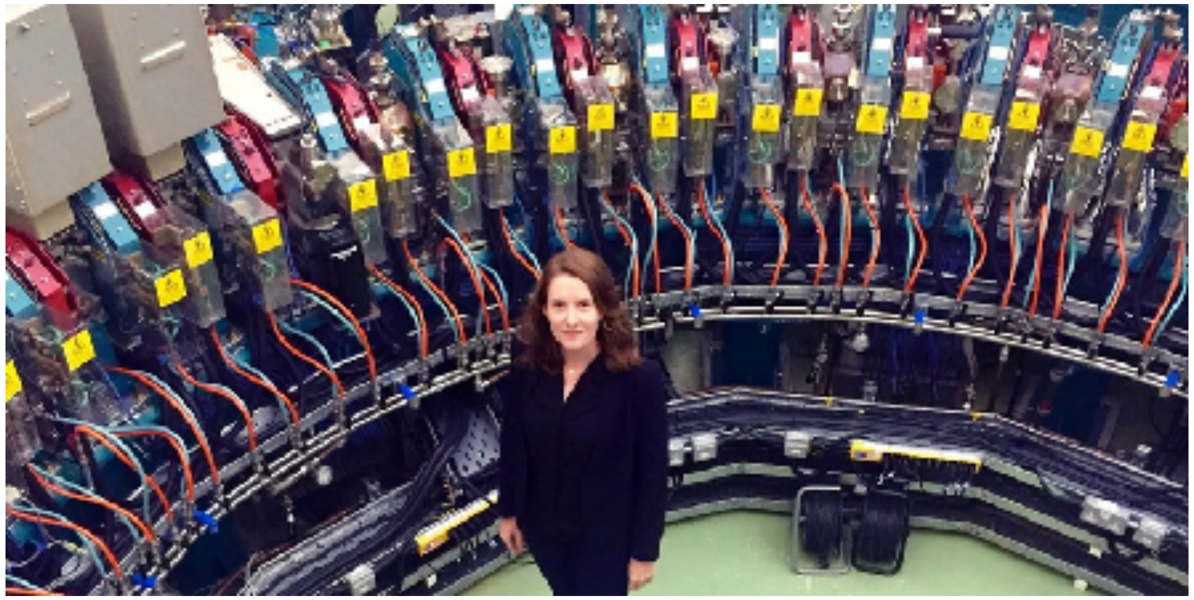

**Do we also ramp the quadrupoles in a synchrotron?**

$$k = \frac{g}{p/q}$$

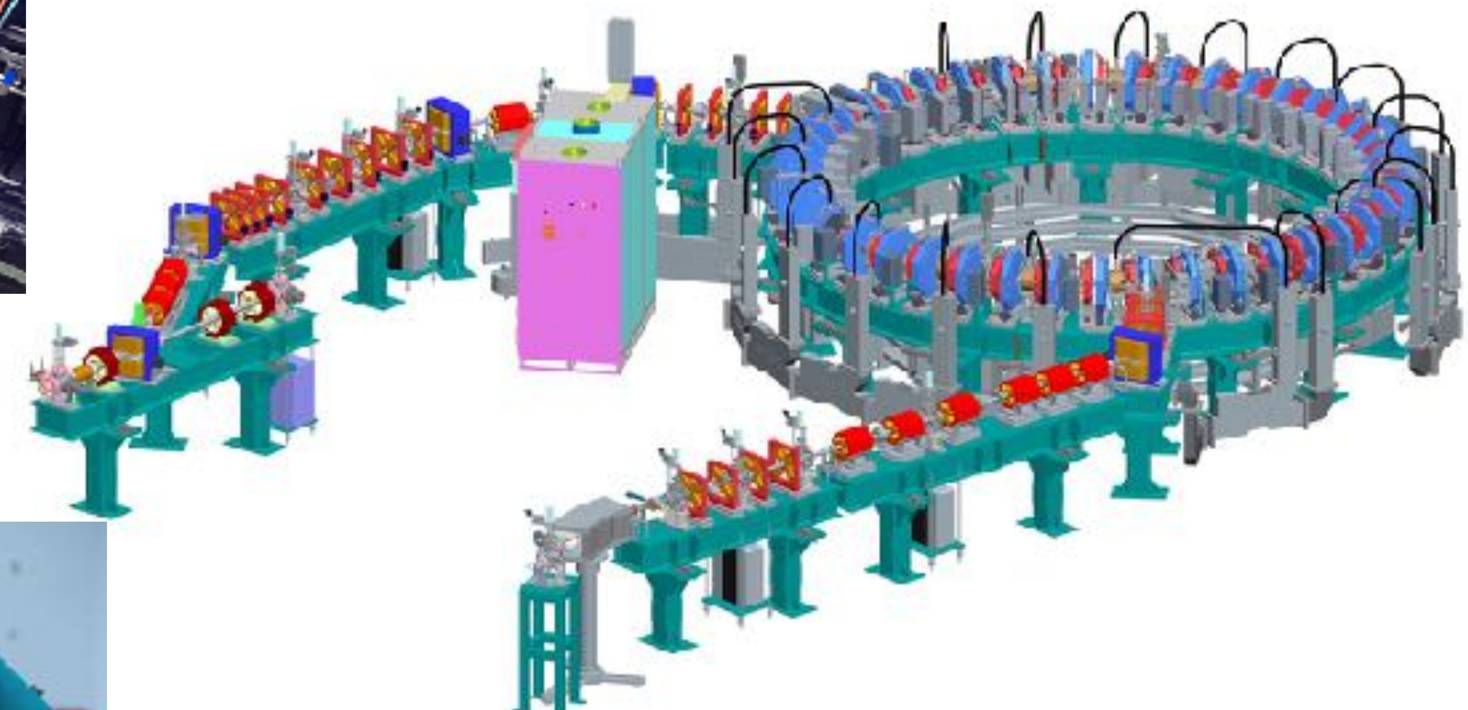
'normalised gradient' of quad

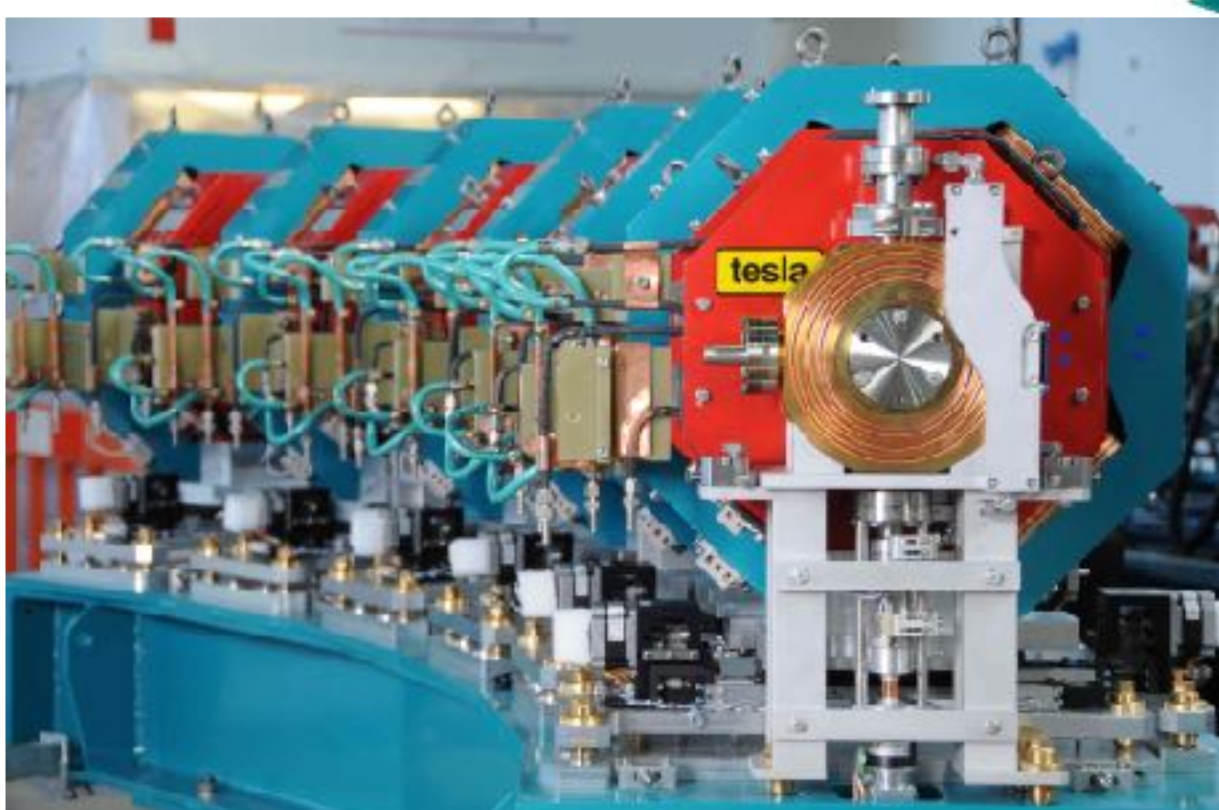
$$\frac{1}{f} = \frac{L(dB(t)/dx)}{p(t)/q}$$

# The 'EMMA' accelerator



42 Quadrupole doublets
10-20 MeV e-
Demonstrates 'non-scaling' FFA

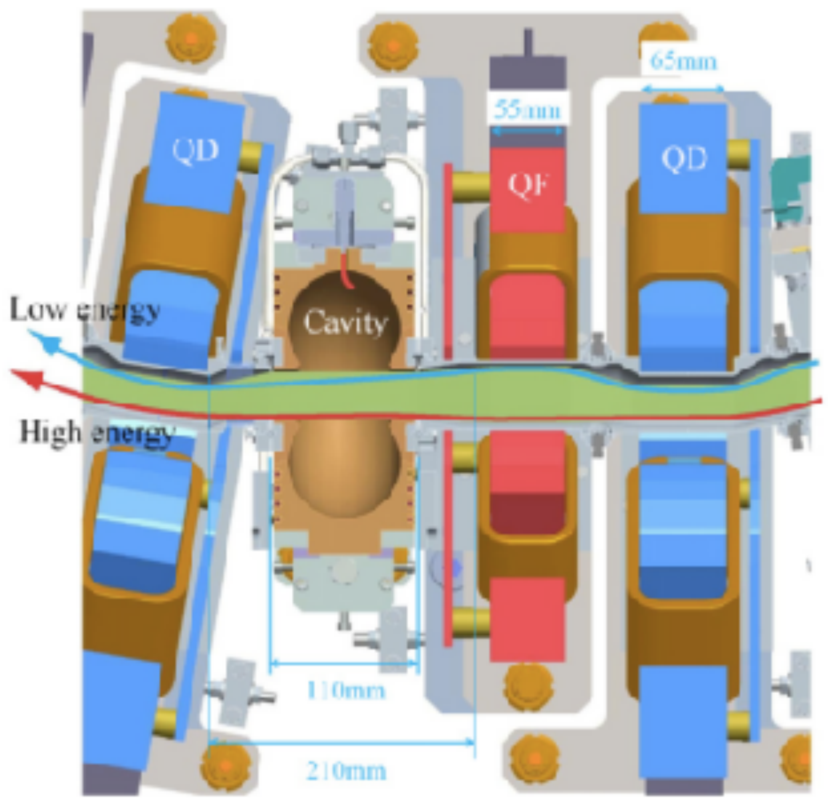




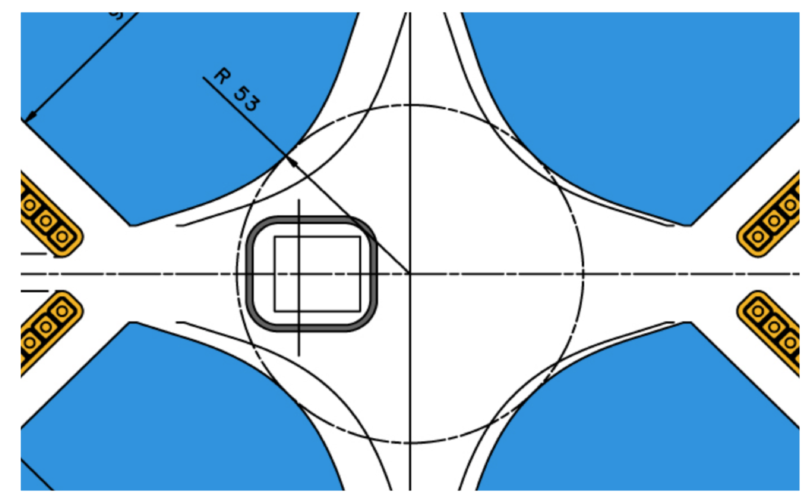

'Electron Model for Many Applications' = EMMA
Built and commissioned at STFC Daresbury Laboratory, UK

# EMMA doesn't ramp the B field with time

'Fixed Field Alternating Gradient' = FFA




Quadrupole with radial offset creates bending component




Note: this is just like a 'combined function' magnet

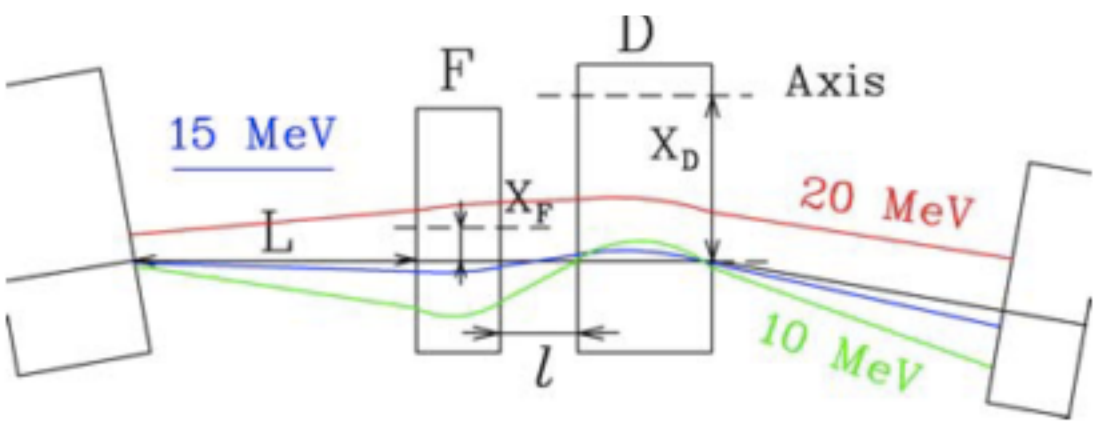



Figure 2: Orbits in a quadrupole doublet cell.

M. Craddock, PAC'07

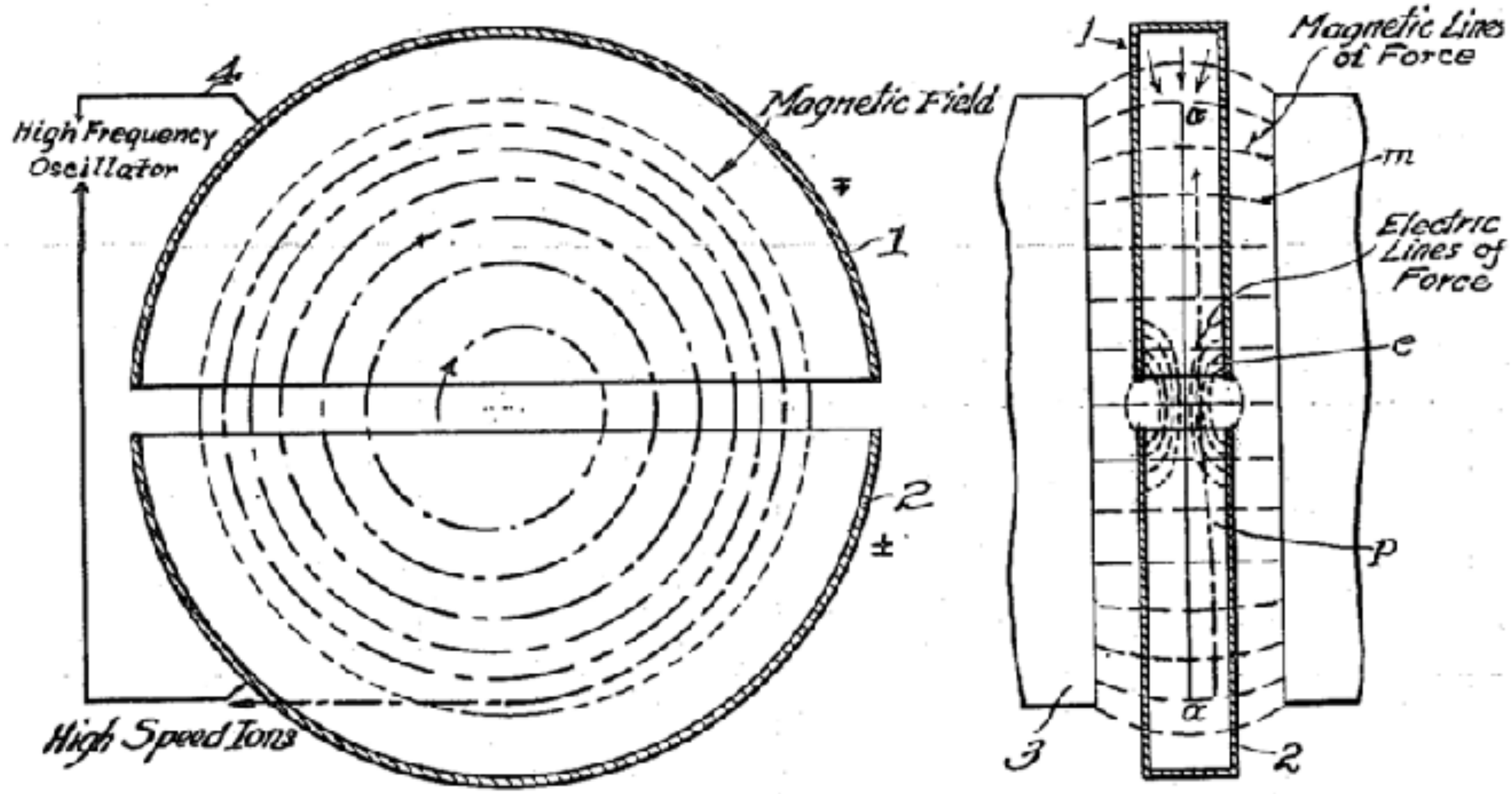
*It doesn't ramp up the magnetic field with energy*

# Fixed-field magnets have advantages

- Simple power supplies and no synchronisation issues
- You can accelerate very quickly (as fast as your RF allows…)
  - in EMMA and in muon FFAs this is ~10 turns
- Higher repetition rate, so higher average current.

# Is an FFA like a cyclotron? (1)

It has fixed field magnets too




The particles spiral outward as they gain energy

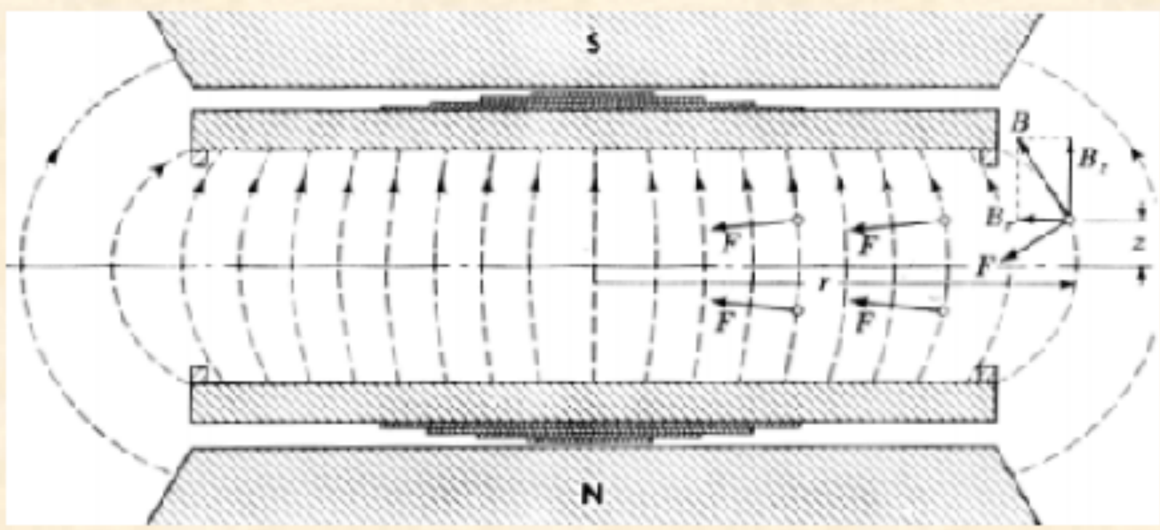
# Is an FFA like a cyclotron? (2)

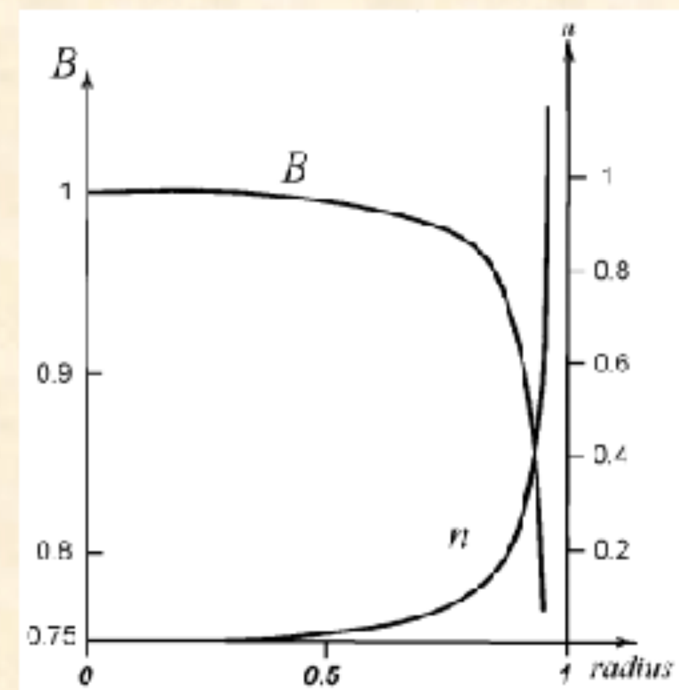
## Weak focusing

Simultaneous radial and axial focusing : Weak focusing

$$0 \leq n \approx -\frac{\partial B_z}{\partial x} \leq 1$$

slightly decreasing field




Horizontal focusing n < 1 means :

- 0 < n < 1 Bz can slightly decrease
- n < 0 Bz can increase as much as wanted

Vertical focusing n > 0 means :

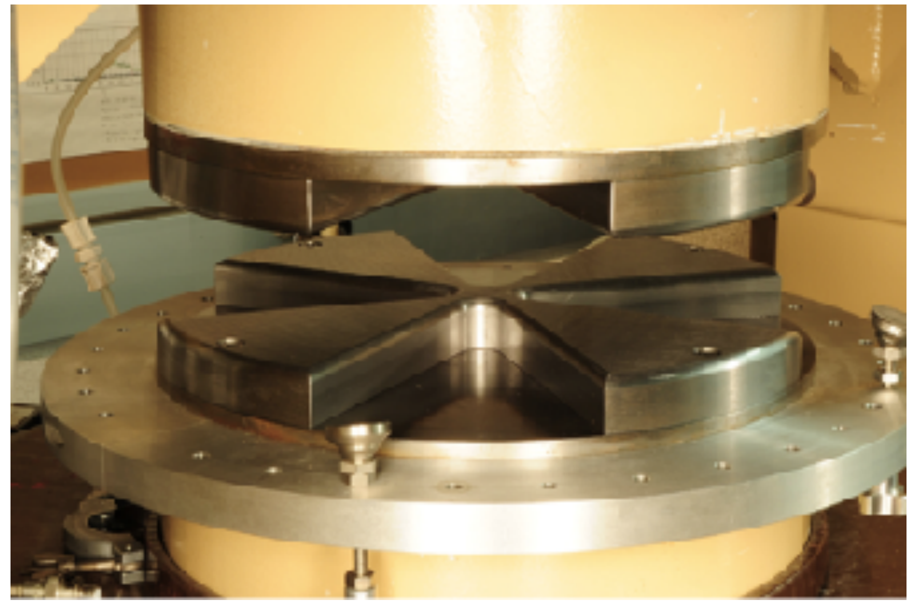
- Bz should decrease with the radius

F. Chautard

18

Slide source: F. Chautard, 2012 CAS

# Is an FFA like a cyclotron? (3)



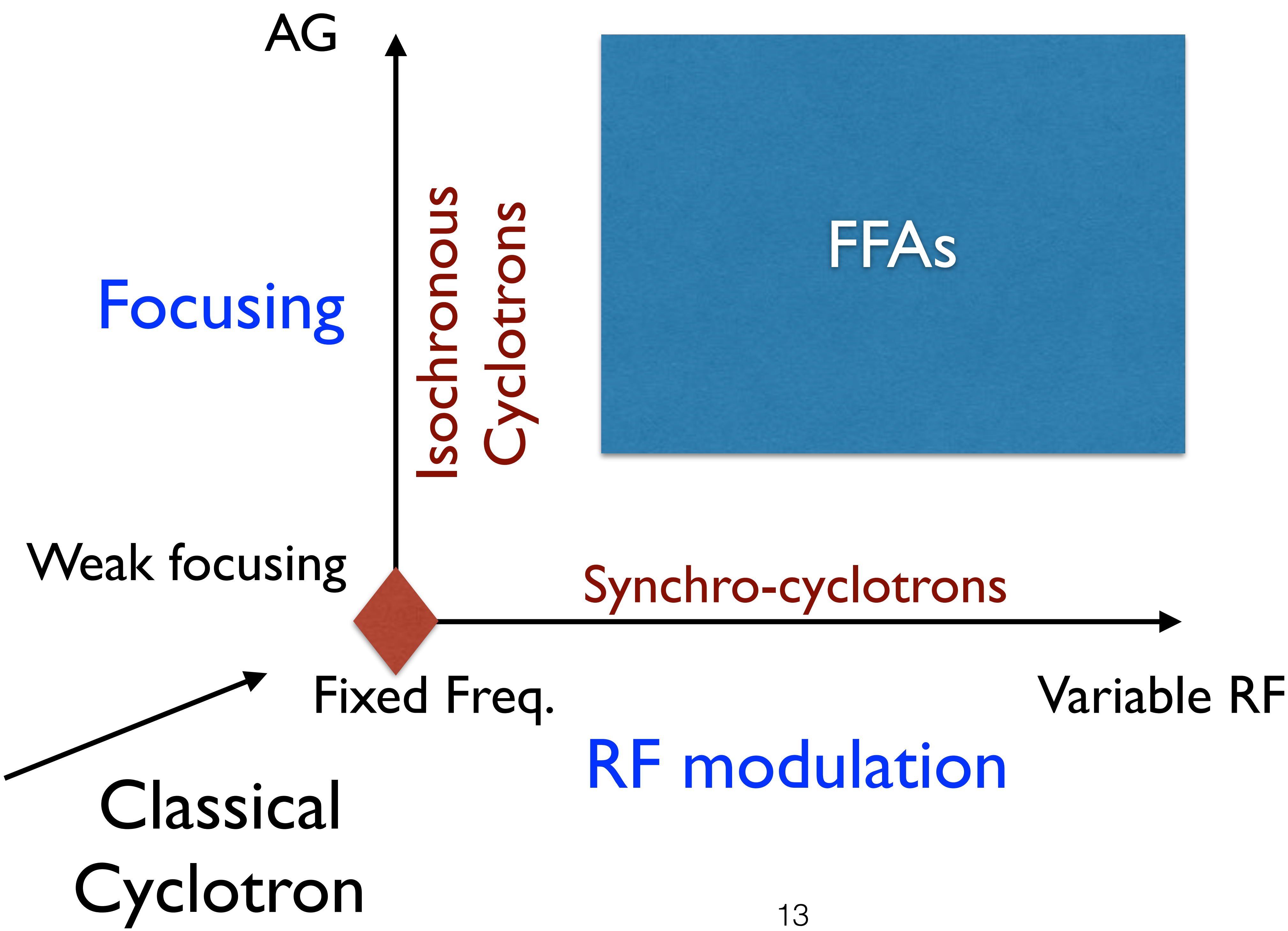
**What about the AVF cyclotron?**

You may have heard of ‘flutter’ in an AVF cyclotron
An FFA has:

- Flutter so large that the field reverses sign between ‘hills’ and ‘valleys’.
- Also: FFA has a field gradient with radius

In the AVF cyclotron the weak focusing is still important, but in the FFA the dynamics is controlled by the strong focusing

# The circular fixed-field accelerator family



# But that's not the whole story…

- So an FFA is like a synchrotron but with fixed-field magnets
- OR like a cyclotron with a field gradient and strong focusing, (and variable RF frequency**)
- But that's not all there is to it…

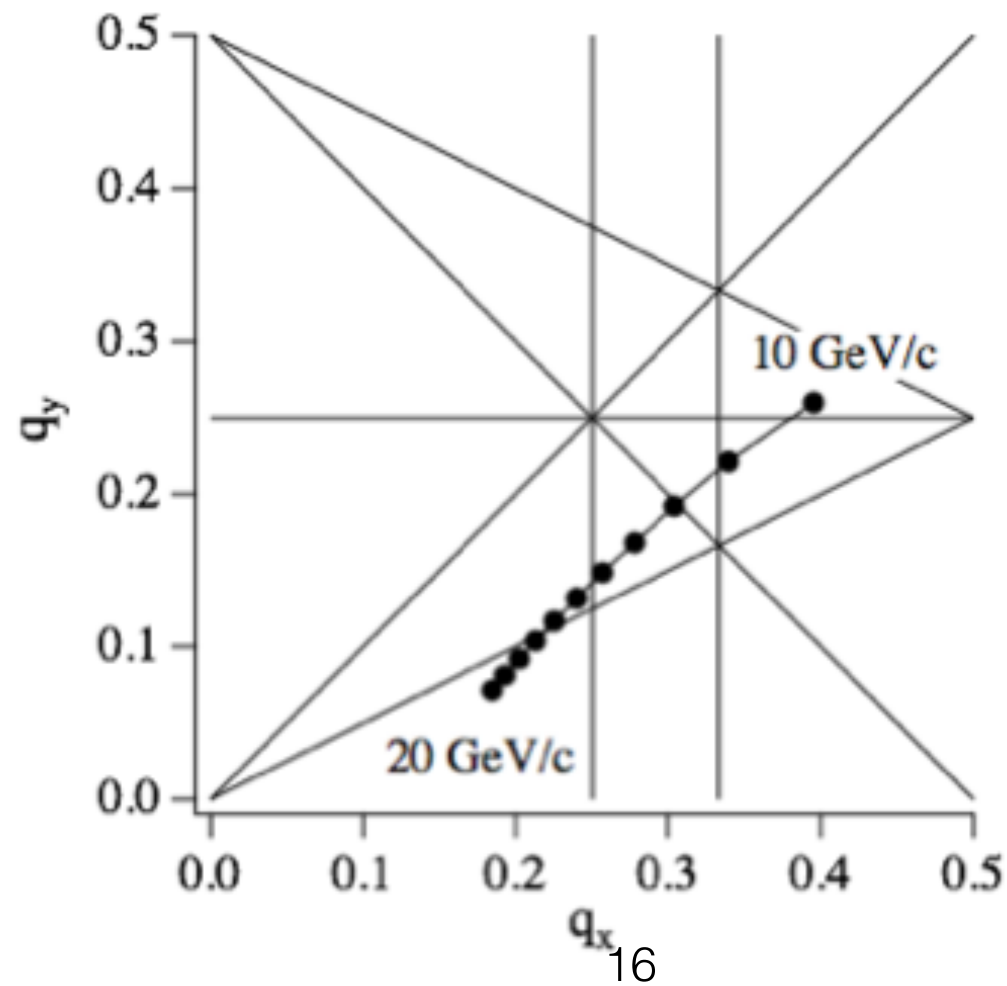
**FFAs do not always have variable RF frequency…

# Circular Accelerators

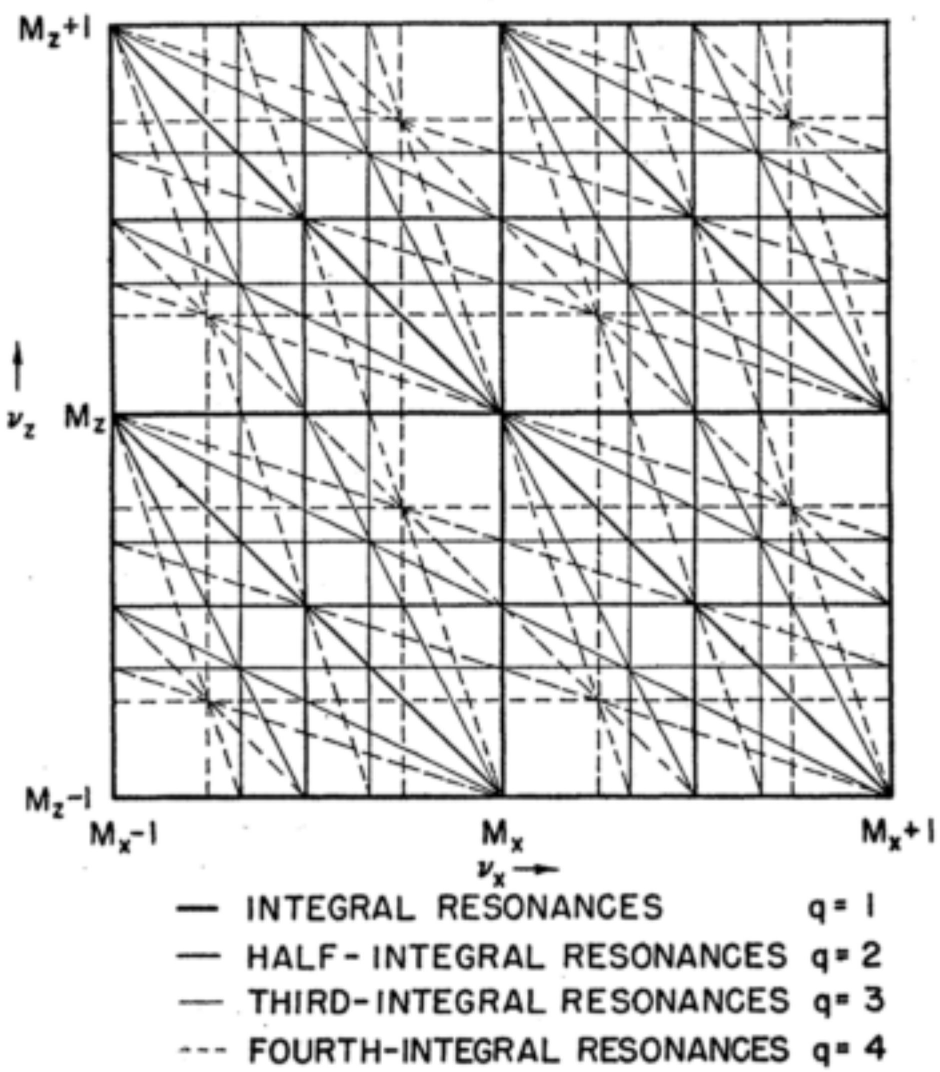
| | Cyclotron | Synchrotron | FFA |
|---|---|---|---|
| Revolution time | Constant | Variable (except relativistic) | Variable |
| Orbit radius | Variable | Constant | Variable |
| Transverse focusing | Variable | Constant | Variable |

# What does variable focusing mean?

- In a synchrotron the tune is fixed away from resonance lines
- But in an FFA, the betatron tunes can vary…



# Resonance crossing



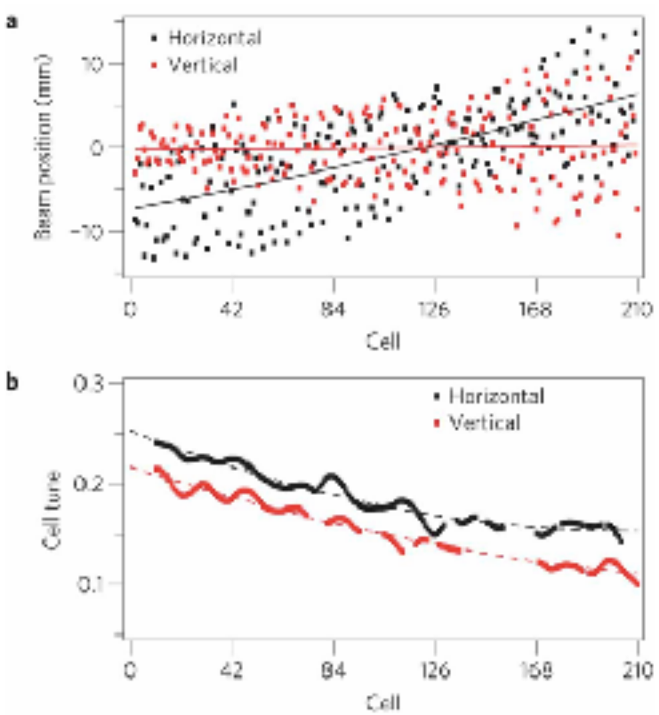

$$n\nu_x + m\nu_y = 0,1,2...$$

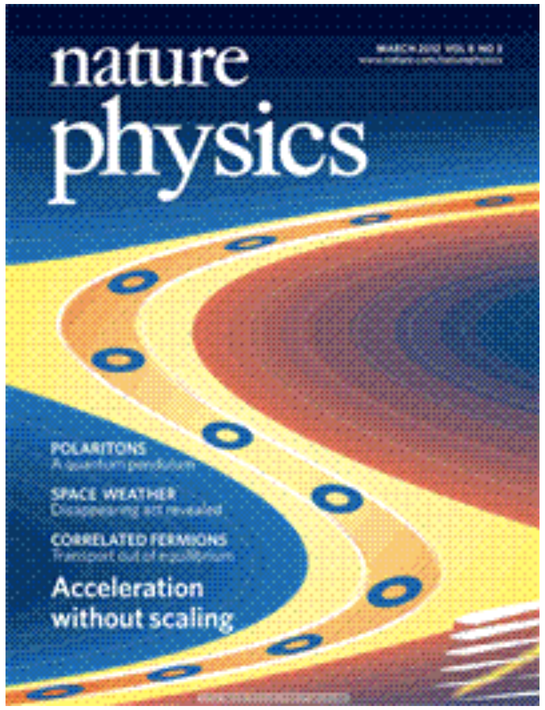
There are many resonance lines in tune space

Normally, particles would be lost on resonance, but if the resonance is weak and the crossing is fast the beam can survive.

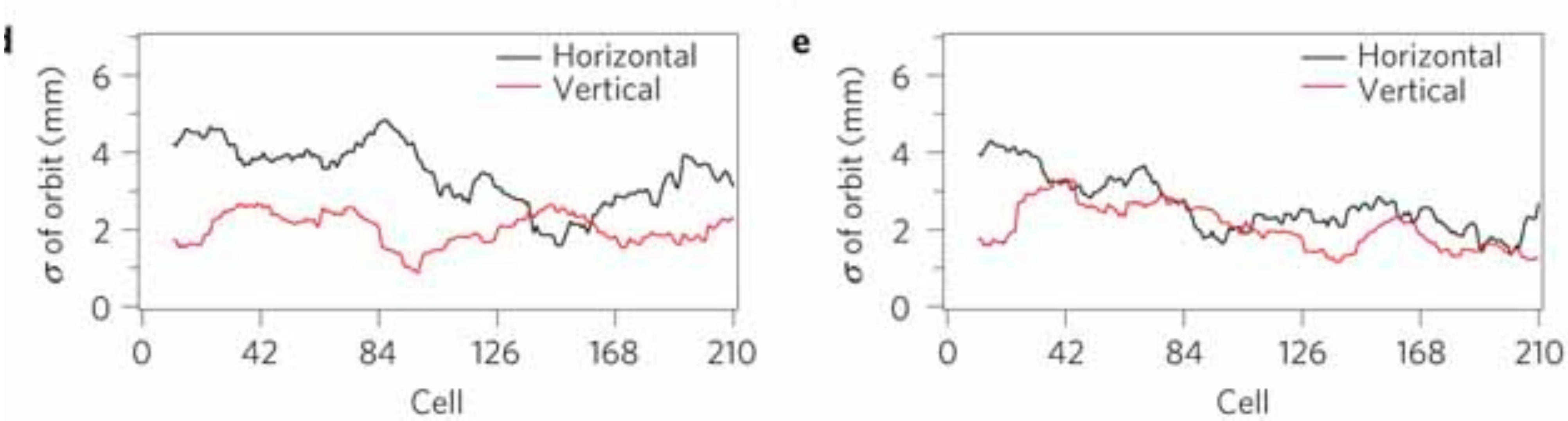
# Results from EMMA

Orbit and tune shift with momentum



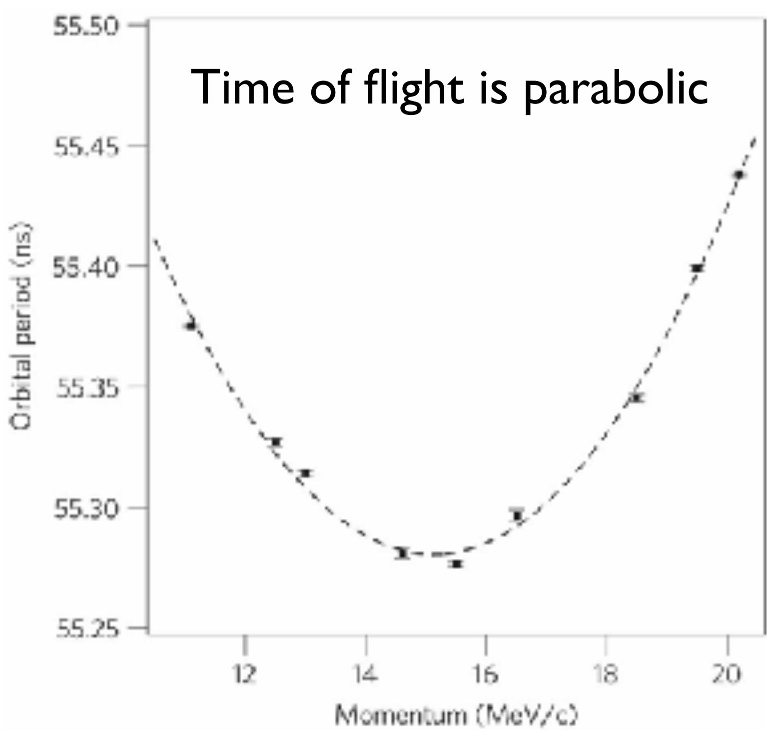

No beam 'blowup' despite resonance crossing




S. Machida et. al., Nature Physics 8, 243–247 (2012)

# EMMA - longitudinal

**Can you have an FFA with fixed RF frequency?**



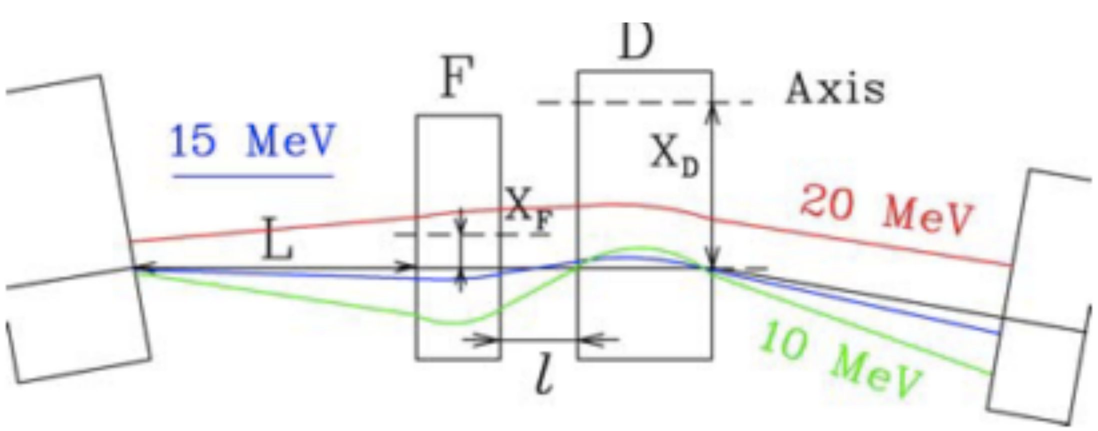
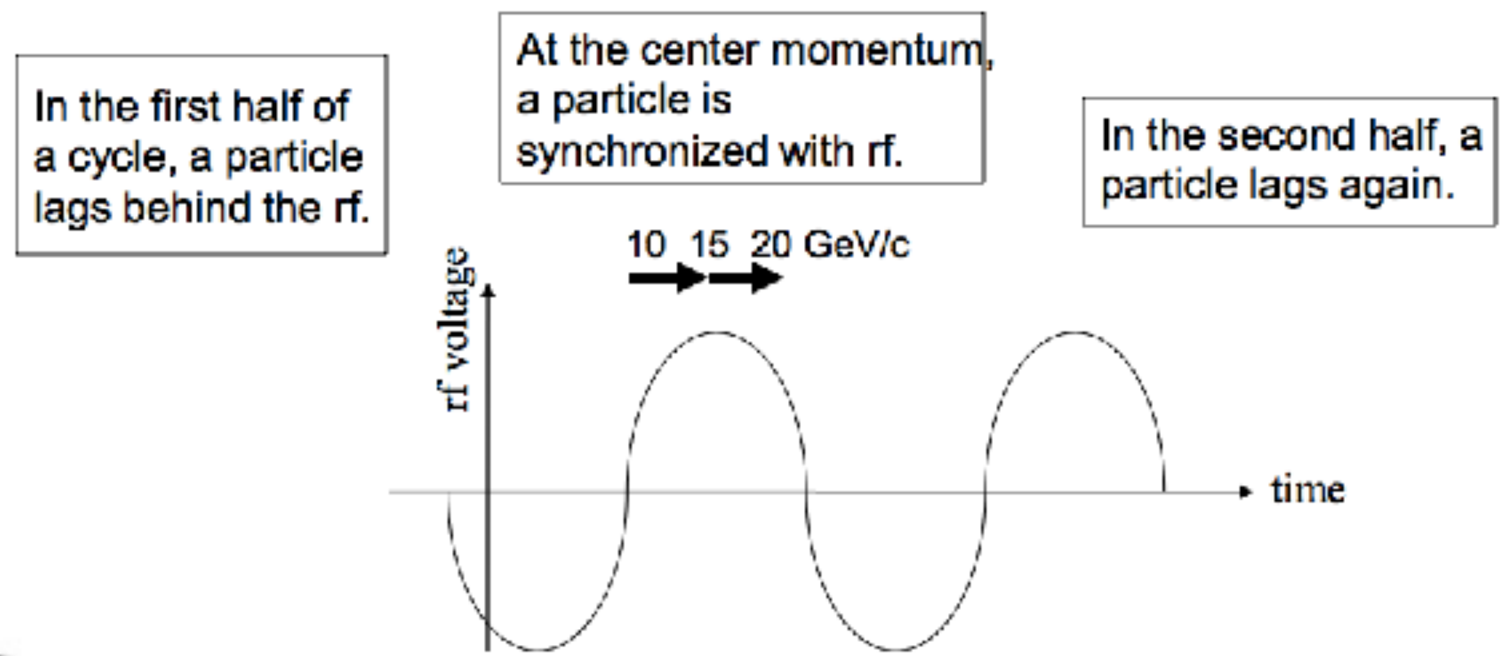




Figure 2: Orbits in a quadrupole doublet cell.

- Suppose we choose rf frequency that is synchronized with revolution frequency at the center.




If the total time lag is less than a half of rf cycle, a beam has net energy gain.

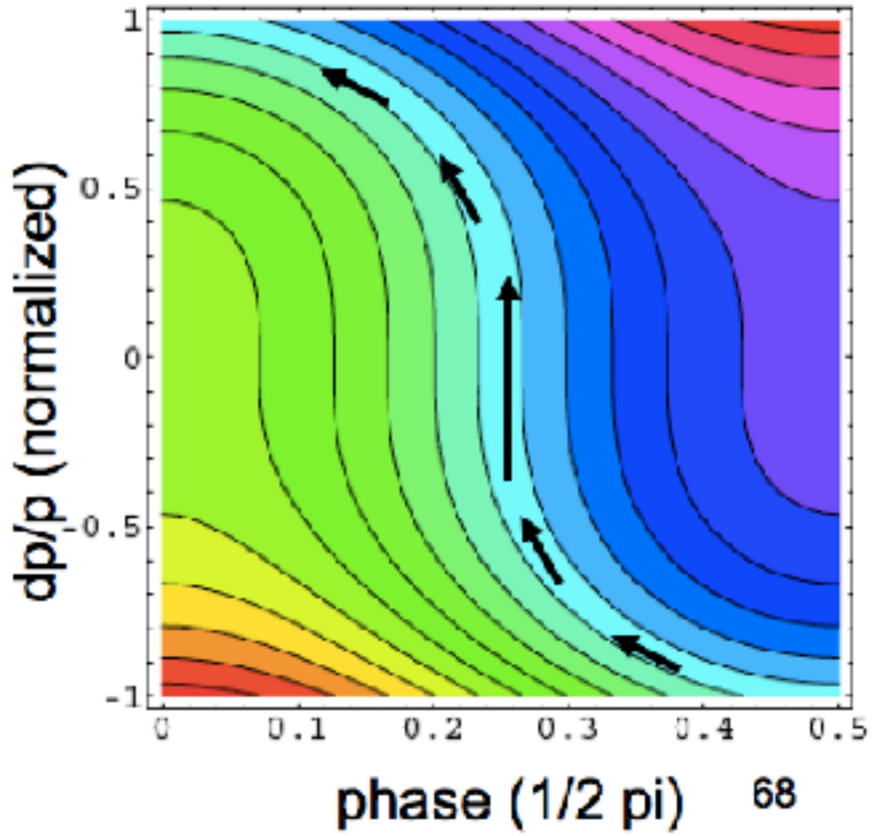
67

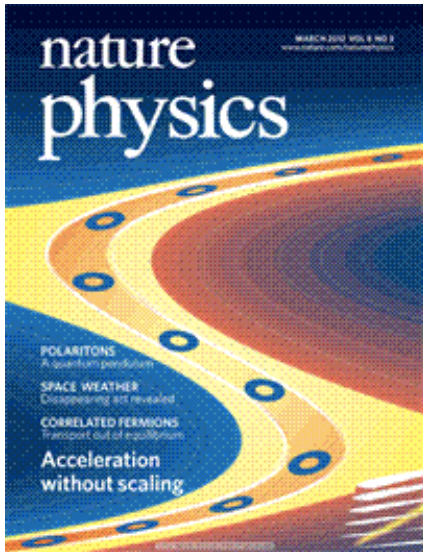
# EMMA - longitudinal

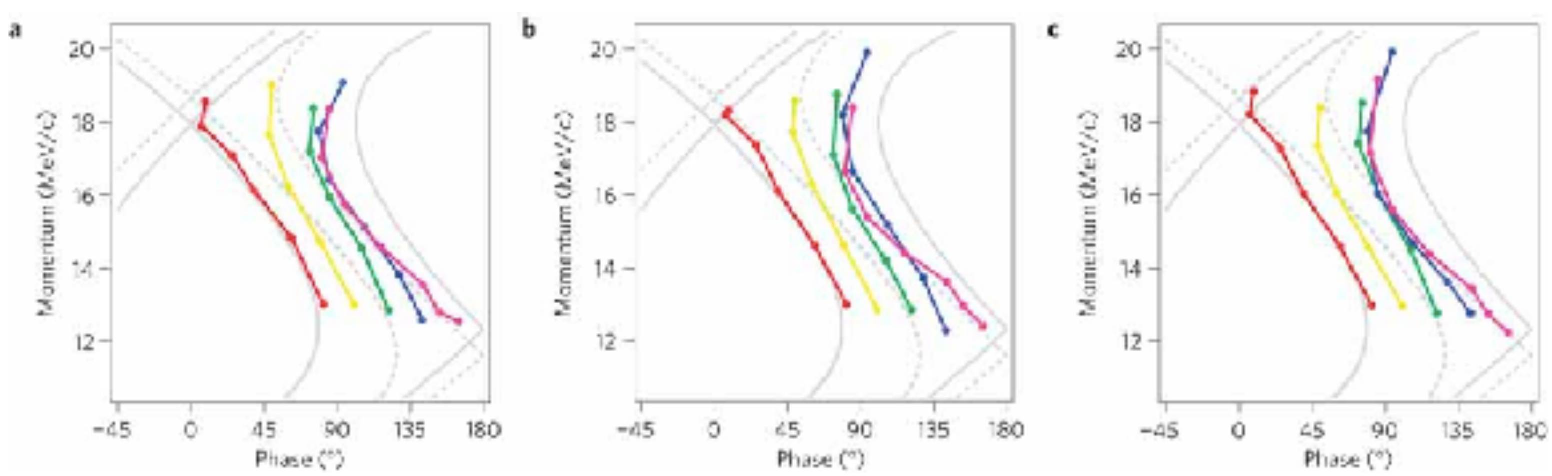
If the RF voltage is sufficient, we can accelerate over the whole energy range

Similar to acceleration in a cyclotron but with imperfect isochronicity

This is called 'serpentine' acceleration and was demonstrated in EMMA




S. Machida et. al., Nature Physics 8, 243–247 (2012)

# But that’s not the whole story…

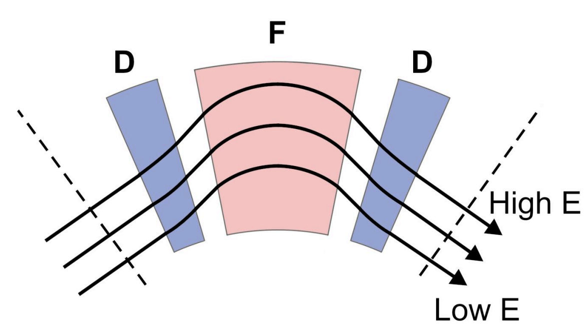
- Electrons & muons are easy to accelerate quickly, but for hadrons it’s harder…
- If resonance crossing could be harmful for hadron FFAs, what can we do to fix it?
- In a synchrotron, off-momentum tune variations = chromaticity
- Can we have stable tunes in an FFA?

# Scaling FFA

- In fact, the first FFAs had constant tunes and were designed not to cross resonances, we call them 'scaling' FFAs

## The orbits are made 'similar'

$$\frac{\partial}{\partial p}\left(\frac{\rho}{\rho_0}\right)\Bigg|_{\theta=const.} = 0$$



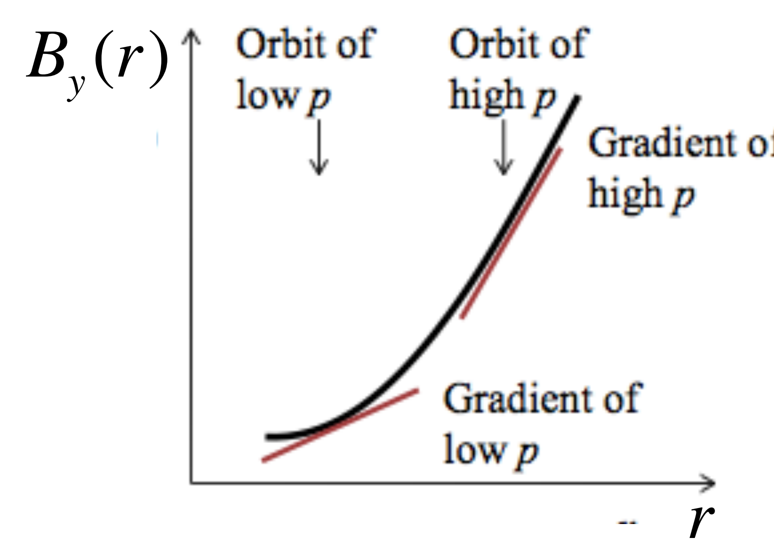

$\rho_0$ Average bending radius

$\rho$ Local bending radius

$\theta$ Generalised azimuth

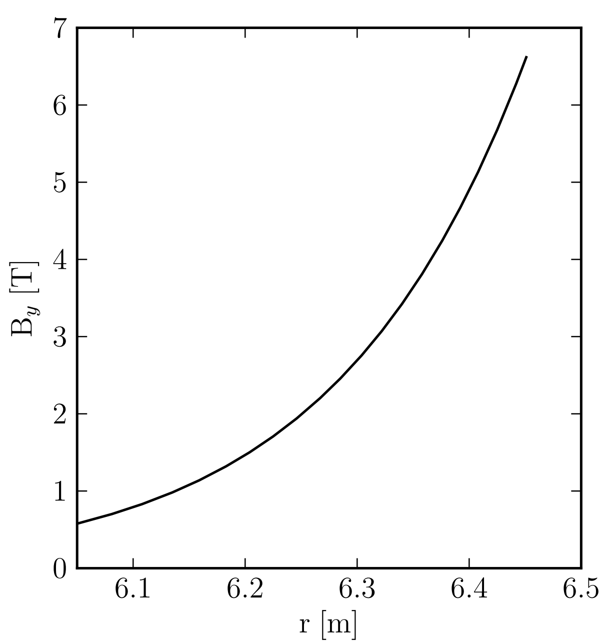
## The 'field index' is constant

$$\frac{\partial k}{\partial p}\Bigg|_{\theta=const.} = 0 \qquad k = \frac{r}{B}\left(\frac{\partial B}{\partial r}\right)$$

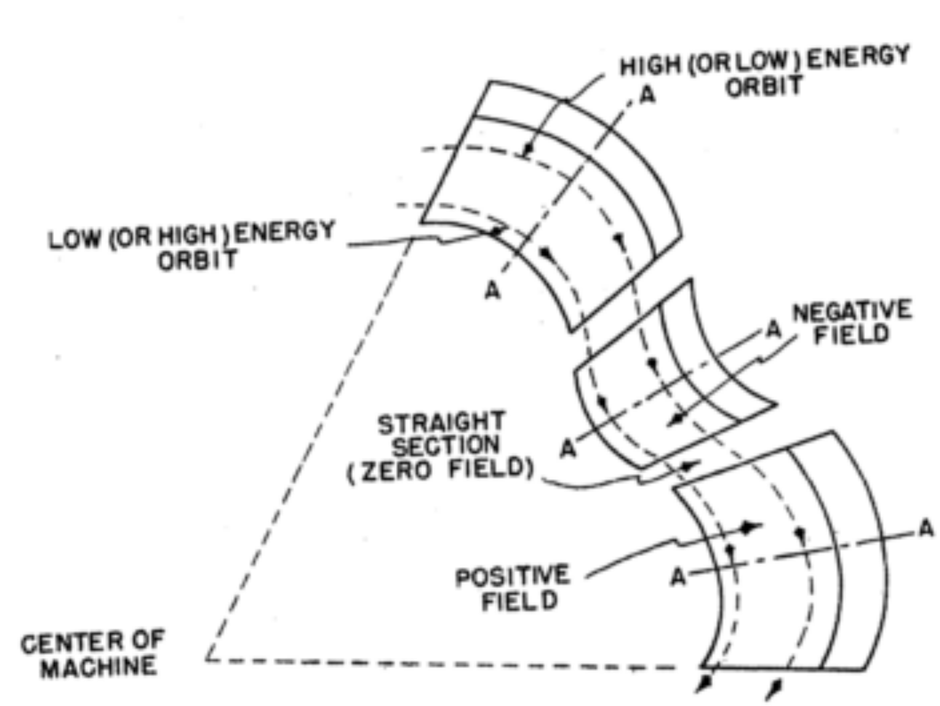


# Scaling FFA

- If the field profile is of this form, the 'cardinal conditions' are satisfied.
- We call this type of FFA a 'Scaling' type.
- Alternating magnets have opposite bending fields




$$B_y = B_0 \left( \frac{r}{r_0} \right)^k F(\theta)$$

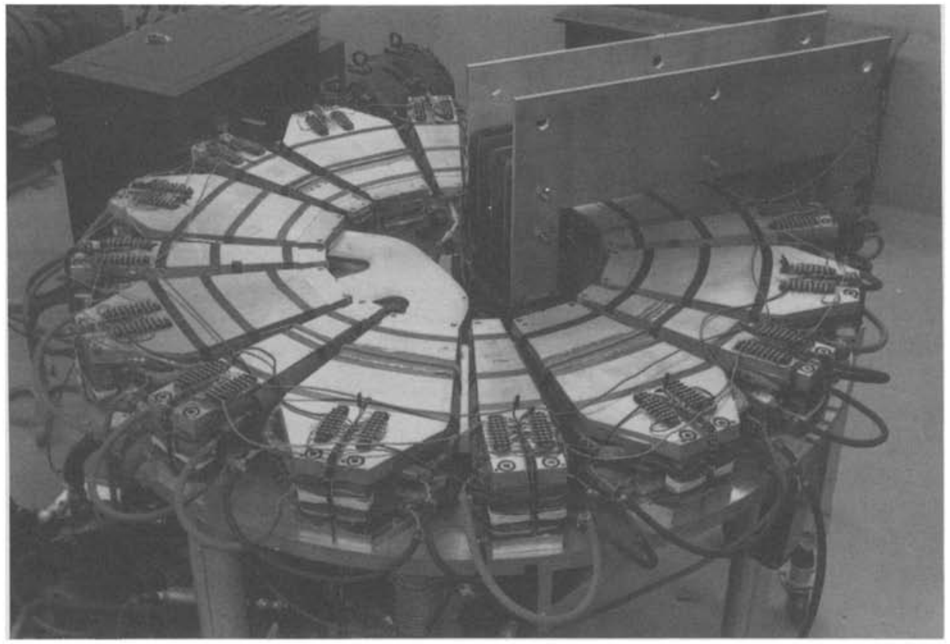


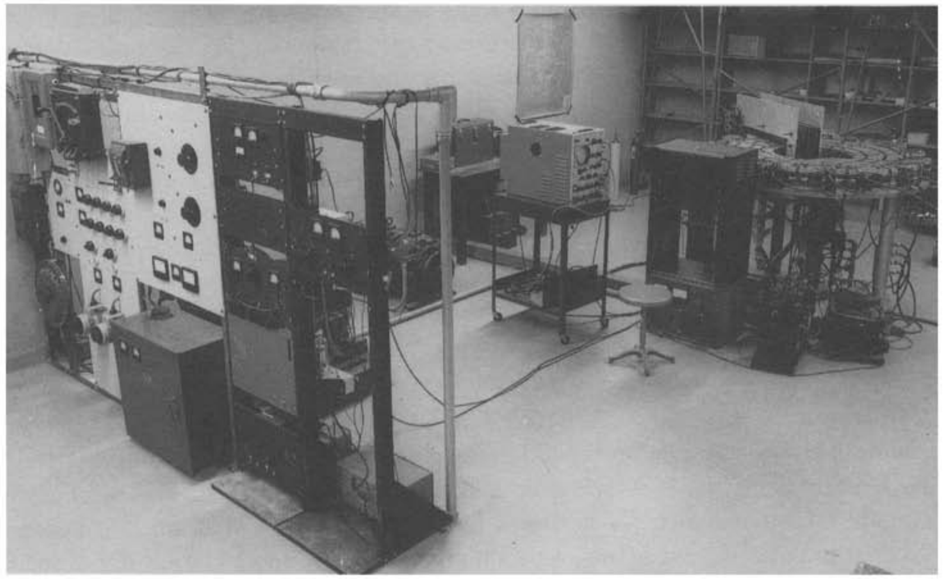

*Note that this field profile does NOT satisfy isochronicity (see M. Seidel's cyclotron lecture)*

$$\omega = \frac{eB}{m\gamma} \neq const.$$

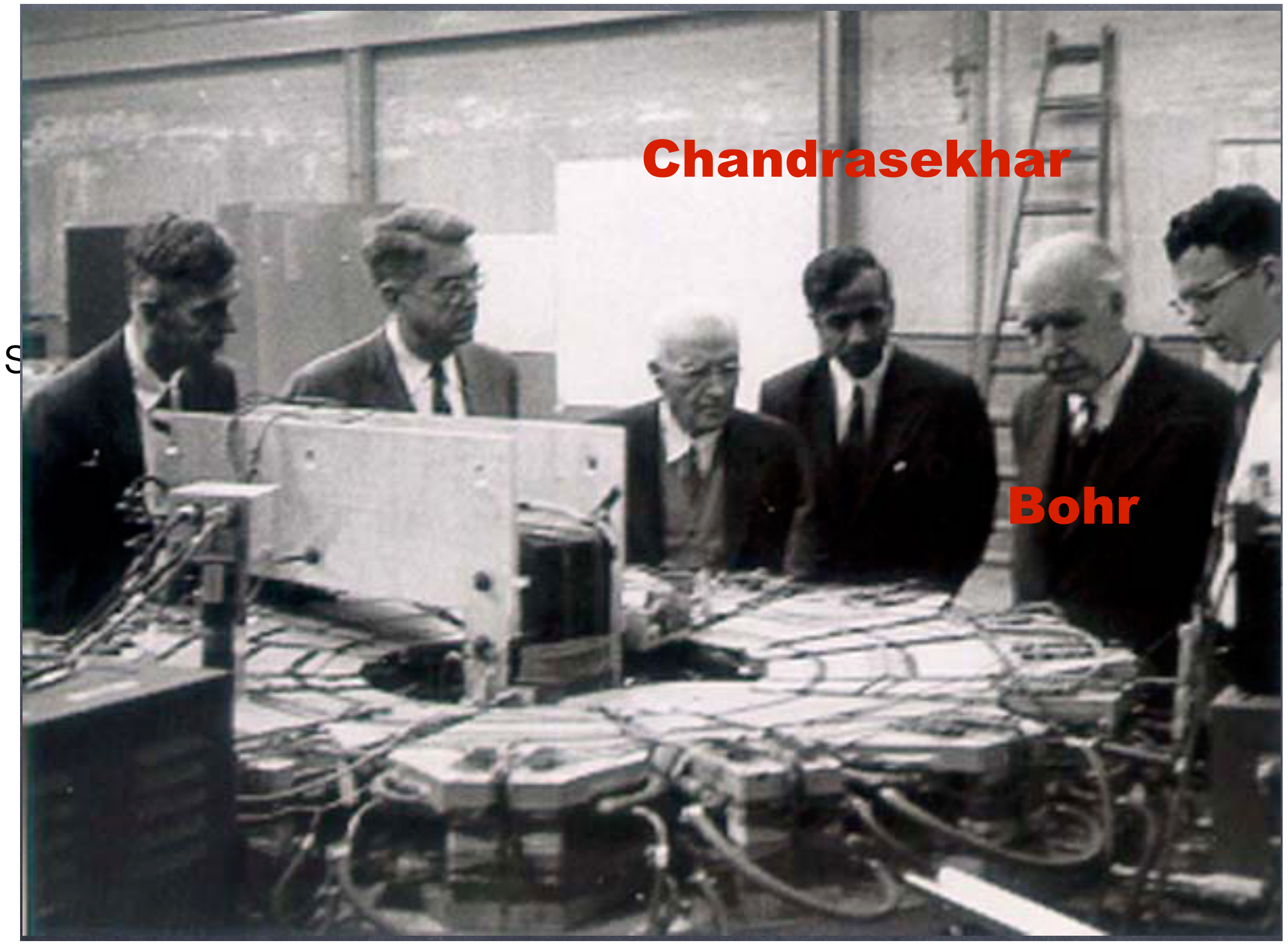
# The FFA is not so new…

## 1956





ngs



# Scaling FFA types

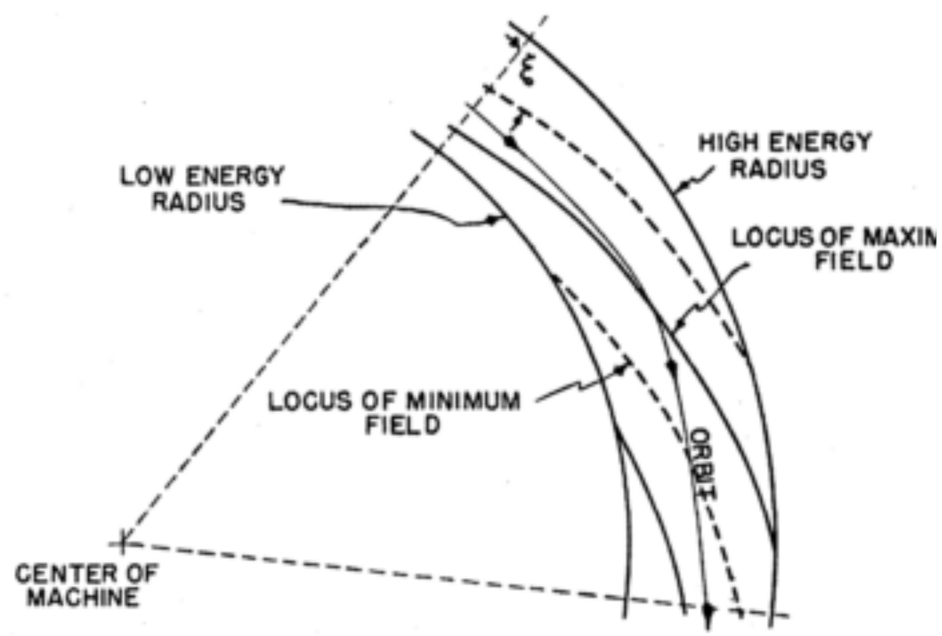



Image source: K. Symon, D. Kerst, L. Jones, L. Laslett, and K. Terwilliger, "Fixed-Field Alternating-Gradient Particle Accelerators," Phys. Rev., vol. 103, no. 6, pp. 1837–1859, Sep. 1956.

$$B(r,\theta) = B_0 \left(\frac{r}{r_0}\right)^k F(\vartheta)$$

$$F(\vartheta) = F\left(\theta - \tan\zeta \cdot \ln\frac{r}{r_0}\right)$$

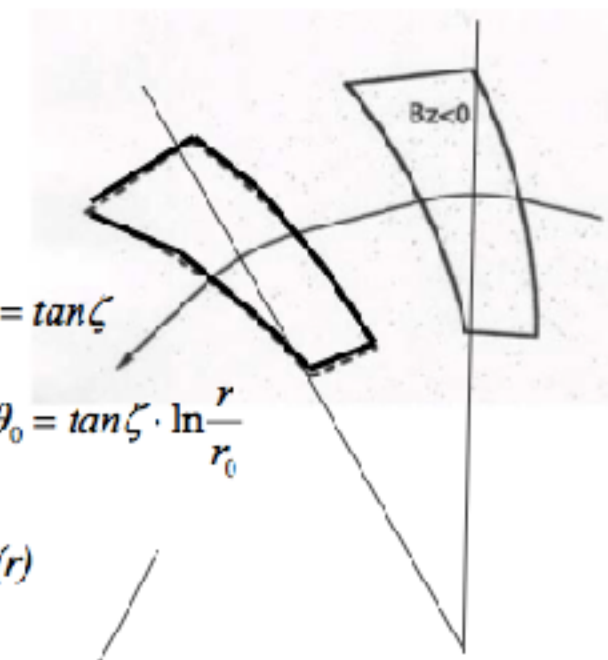
Spiral sector type

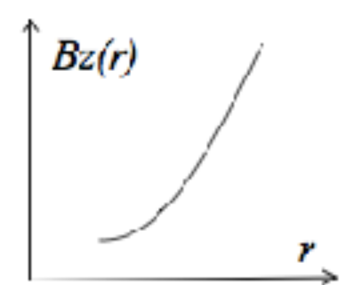
Spiral angle gives strong edge focusing.

$$\therefore \Delta p_z = \frac{e}{v_x}\int_{-\infty}^{\infty}\left(-v_y B_x\right)dx = -eB_{z0}\tan\zeta \cdot z$$

$$\frac{r\,d\theta}{dr} = \tan\zeta$$

$$\theta - \theta_0 = \tan\zeta \cdot \ln\frac{r}{r_0}$$




S. Machida, CAS 2012

# Recent Scaling FFAs

- In the late 90's and in 2000's, the FFA idea was re-awakened in Japan,
- Particular focus on hadron FFAs of scaling type

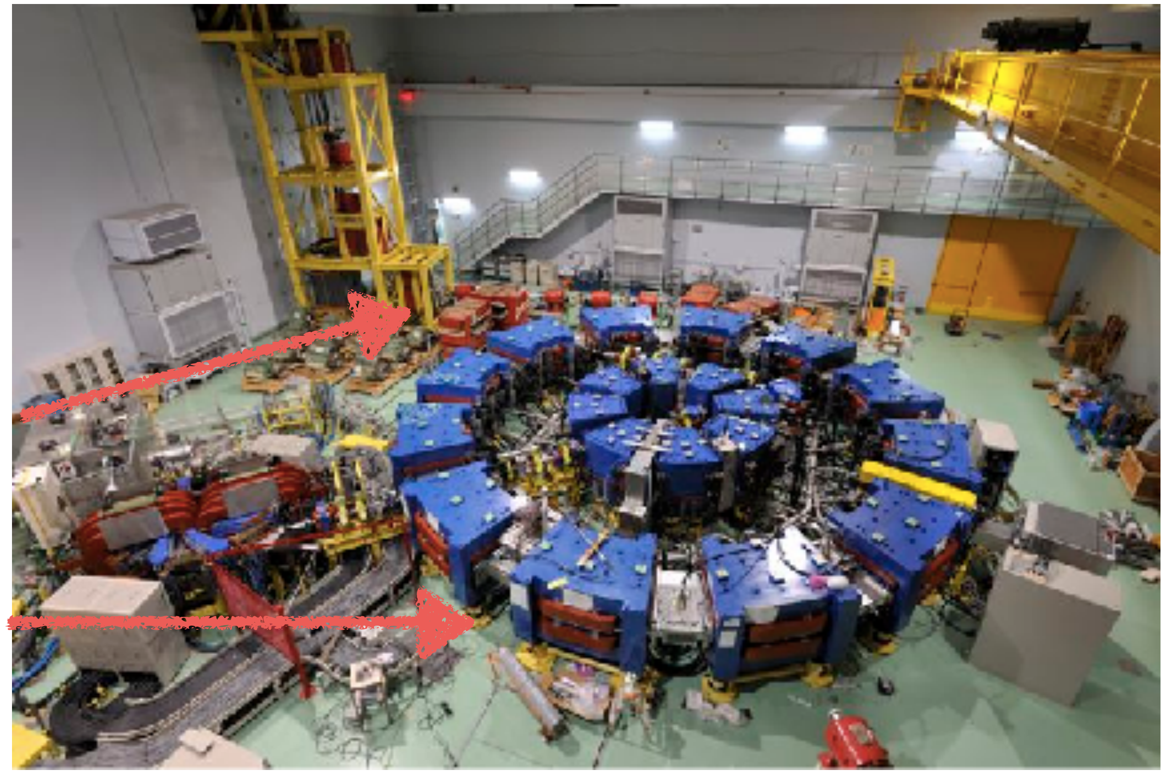
Proof of Principle machine finished in 1999 at KEK, demonstrated 1kHz rep. rate

3-stage FFA for ADSR studies:

2.5 MeV spiral (ion beta) FFA with induction cores

25 MeV radial (booster) FFA with RF

150 MeV radial (main) FFA with RF



# Technology for scaling FFAs

Image credit: A. Takagi

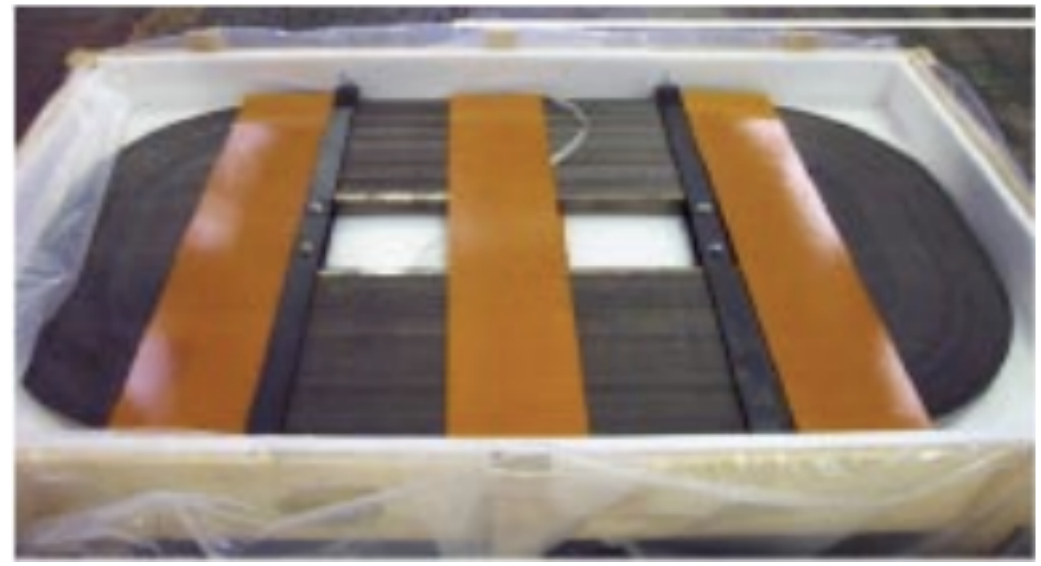


Image credit: Y. Mori,

Magnetic Alloy (MA) Cavity

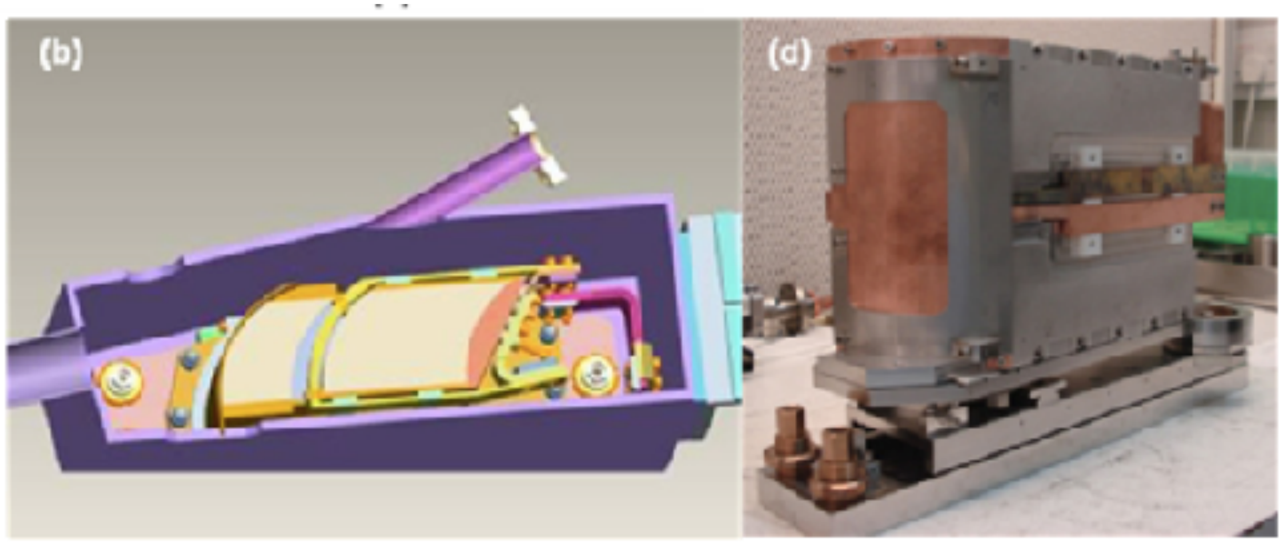
Large aperture

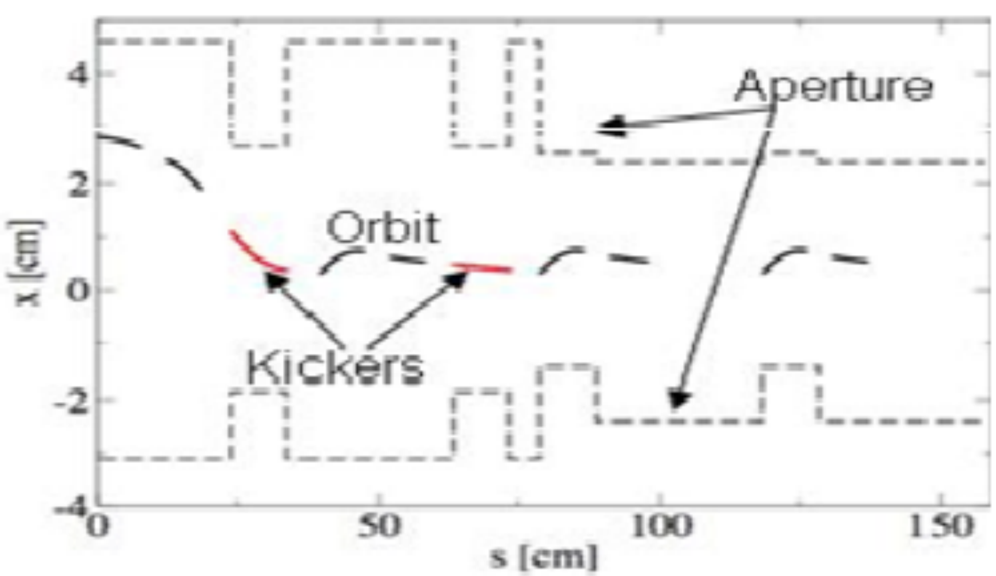
High shunt impedance

Low Q - can cover large range of frequencies.

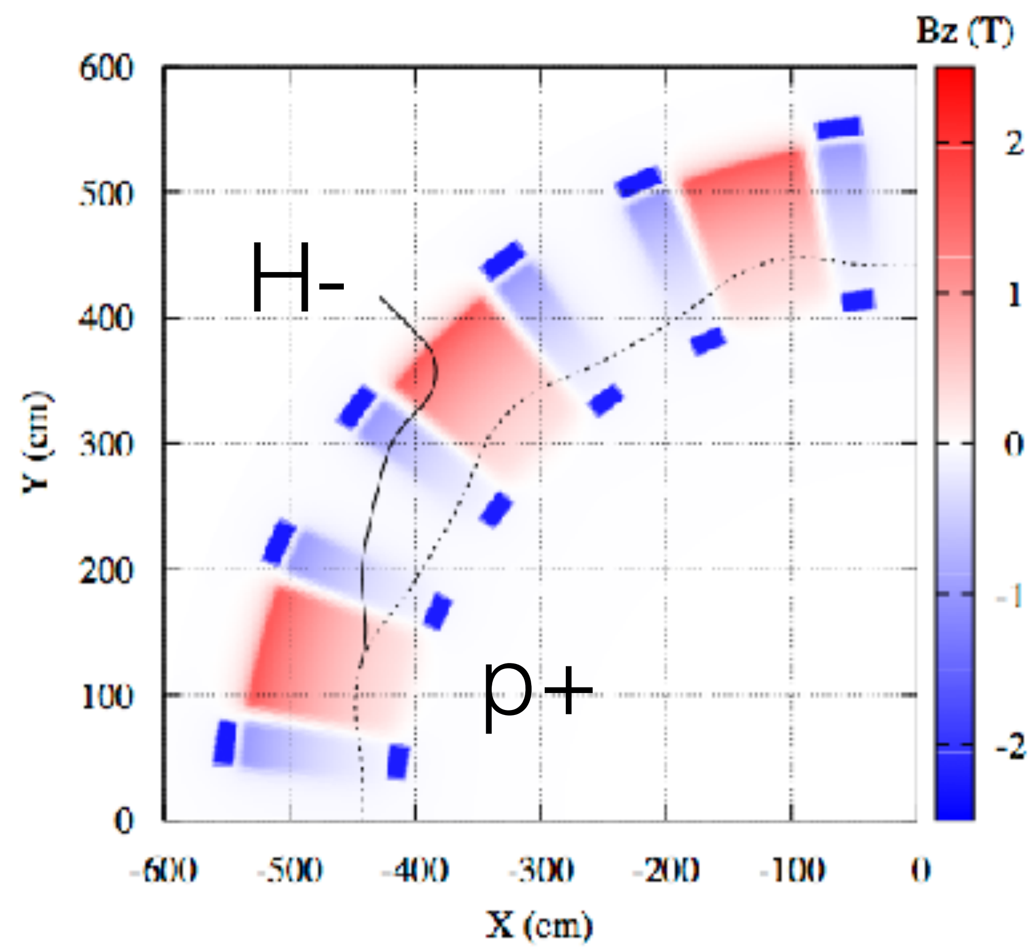
# Aside: Injection/extraction

- How do we inject/extract beams without a time dependent field?
- Well, pulsed kickers/septum can still be used.
- Can also exploit the orbit movement with acceleration




Injection/extraction in EMMA




Septum-free injection in KURRI FFA

# Circular Accelerators

| | Cyclotron | Synchrotron | Non-scaling FFA | Scaling FFA |
|---|---|---|---|---|
| Revolution time | Constant | Variable (except relativistic) | Variable (small) | Variable |
| Orbit radius | Variable | Constant | Variable (small) | Variable |
| Transverse focusing | Variable | Constant | Variable | Constant |

# A quick summary…

- ‘Scaling’ type is a very specific type of FFA. Anything else is the ‘non-scaling’ type.
- EMMA is a linear non-scaling FFA, which again is quite specific.
- …Are there any other possibilities?

# Advanced FFA optics (1)

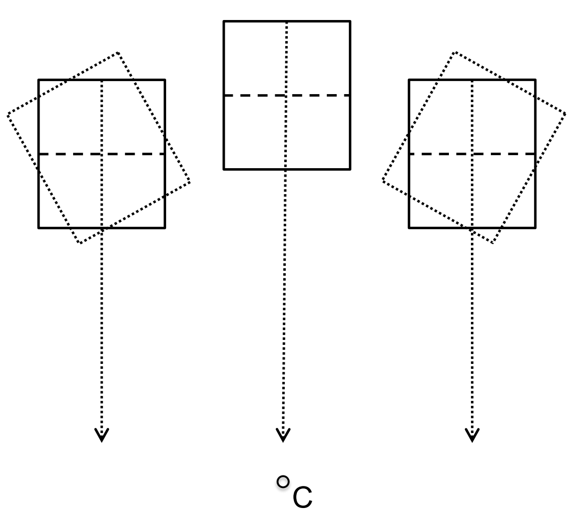
*"There are other variations of these designs which preserve betatron oscillation stability, hold $\nu_x$ and $\nu_y$ constant, but do not retain the property of similar of equilibrium orbits."*

*"The magnet edges of focusing and defocusing sectors can be made non-radial, and the fields in positive- and negative- field magnets made different functions of radius"*

- K. Symon, D. Kerst, L. Jones, L. Laslett, and K. Terwilliger, "Fixed-Field Alternating-Gradient Particle Accelerators," Phys. Rev., vol. 103, no. 6, pp. 1837–1859, Sep. 1956.

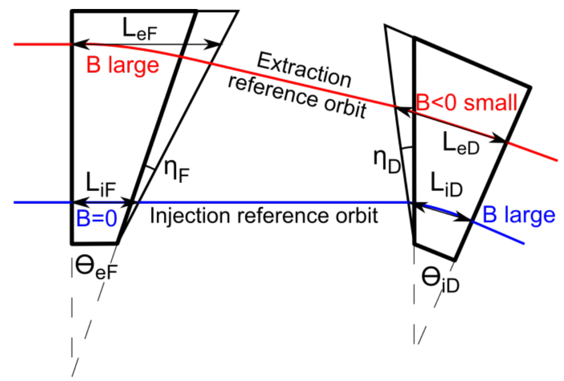
## Tune-stable non-scaling FFA designs have been developed

$$B_z = B_{z0}\left(\frac{r_0 + r}{r_0}\right)^k = B_{z0}\left(1 + \sum_{n=1} \frac{1}{n!} \frac{k(k-1)\cdots(k-n+1)}{r_0^n} r^n\right)$$



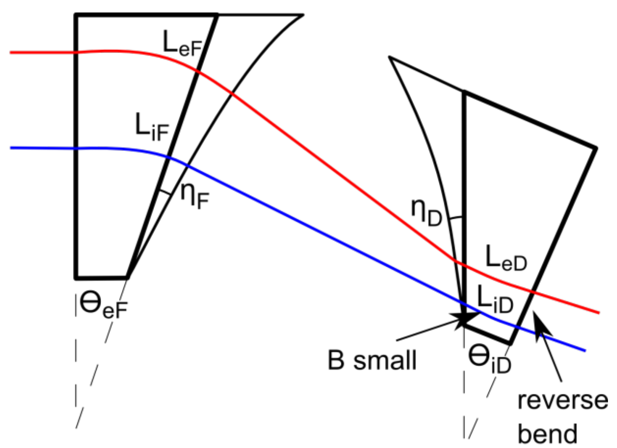

Rectangular magnets,
Simplified field profile
Higher stability region
(S. Machida, S. Sheehy)

# Advanced FFA optics (2)



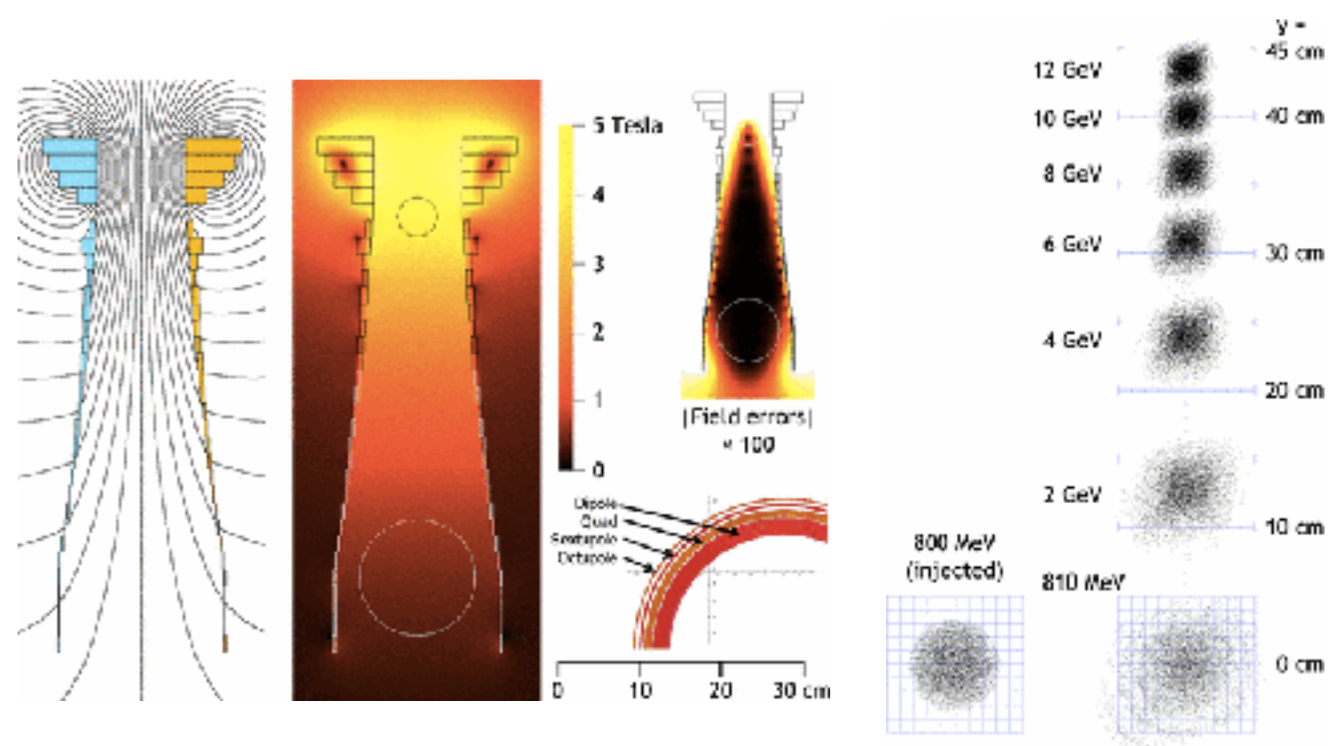

Radial designs with edge profiles

(C. Johnstone)




Vertical orbit excursion FFA

(S. Brooks)



# Current status of FFA designs

- A whole spectrum of designs have emerged in the last 5-10 years

Potential applications include:

- Accelerator Driven Subcritical Reactor
- Boron Neutron Capture Therapy
- Proton/carbon therapy
- Accelerator-based Neutron Source
- Emittance/Energy Recovery with Internal Target (ERIT)
  - e-RHIC injector
  - Muon or neutrino factory source
  - + many more…

# Summary

- FFAs are just a generalisation of synchrotrons or cyclotrons
- Two main types ‘scaling’ and ‘non-scaling’
  - Scaling: specific optics and orbit requirements put a strict requirement on the field profile (zero-chromaticity)
  - Non-scaling: removes these restrictions, very general type (chromatic)
- FFAs may be suitable for many future applications

- In my view, the next big challenge is demonstrating high power operation

# Reading List


- CERN Courier, "Rebirth of the FFAG", 2004. http://cerncourier.com/cws/article/cern/29119
- K. Symon, D. Kerst, L. Jones, L. Laslett, and K. Terwilliger, "Fixed-Field Alternating-Gradient Particle Accelerators," Phys. Rev., vol. 103, no. 6, pp. 1837–1859, Sep. 1956.
- S. Machida,"Acceleration in the linear non-scaling fixed-field alternating-gradient accelerator EMMA," Nat. Phys., vol. 8, no. 3, pp. 243–247, Jan. 2012.
- Proceedings of the FFA (formerly called FFAG) workshops


Notes on FFAs from CAS schools:


- S. Machida, FFAs, CAS Bulgaria 2010, https://cas.web.cern.ch/cas/Bulgaria-2010/Talks-web/Machida-web.pdf
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