

# RF Cavity Design

Erk Jensen

CERN BE/RF

CERN Accelerator School  
Accelerator Physics (Intermediate level)  
Chios 2011

## Overview

- DC versus RF
  - Basic equations: Lorentz & Maxwell, RF breakdown
- Some theory: from waveguide to pillbox
  - rectangular waveguide, waveguide dispersion, group and phase velocity, standing waves ...  
waveguide resonators, round waveguides, Pillbox cavity
- Accelerating gap
  - Principle, ferrite cavity, drift tube linac
- Characterizing a cavity
  - Accelerating voltage, transit time factor
  - Resonance frequency, shunt impedance,
  - Beam loading, loss factor, RF to beam efficiency,
  - Transverse effects, Panofsky-Wenzel, higher order modes, PS 80 MHz cavity (magnetic coupling)
- More examples of cavities
  - PEP II, LEP cavities, PS 40 MHz cavity (electric coupling),
- RF Power sources
- Many gaps
  - Why?
  - Example: side coupled linac, LIBO
- Travelling wave structures
  - Brillouin diagram, iris loaded structure, waveguide coupling
- Superconducting Accelerating Structures

# DC VERSUS RF

20-Sep-2011

CAS Chios 2011 — RF Cavity Design

3

## DC versus RF

DC accelerator



RF accelerator



20-Sep-2011

CAS Chios 2011 — RF Cavity Design

4

# Lorentz force

A charged particle moving with velocity  $\vec{v} = \frac{\vec{p}}{m\gamma}$  through an electromagnetic field experiences a force

$$\frac{d\vec{p}}{dt} = q(\vec{E} + \vec{v} \times \vec{B})$$

The total energy of this particle is  $W = \sqrt{(mc^2)^2 + (pc)^2} = \gamma mc^2$ , the kinetic energy is  $W_{kin} = mc^2(\gamma - 1)$ .

The role of acceleration is to increase the particle energy!

Change of  $W$  by differentiation:

$$WdW = c^2 \vec{p} \cdot d\vec{p} = qc^2 \vec{p} \cdot (\vec{E} + \vec{v} \times \vec{B}) dt = qc^2 \vec{p} \cdot \vec{E} dt$$

$$dW = q\vec{v} \cdot \vec{E} dt$$

Note: **Only the electric field can change the particle energy!**

# Maxwell's equations (in vacuum)

$$\nabla \times \vec{B} - \frac{1}{c^2} \frac{\partial \vec{E}}{\partial t} = \mu_0 \vec{J} \quad \nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0 \quad \nabla \cdot \vec{E} = \mu_0 c^2 \rho$$



why not DC?

1) DC ( $\frac{\partial}{\partial t} \equiv 0$ ):  $\nabla \times \vec{E} = 0$  which is solved by  $\vec{E} = -\nabla\Phi$

Limit: If you want to gain 1 MeV, you need a potential of 1 MV!

2) Circular machine: DC acceleration impossible since  $\oint \vec{E} \cdot d\vec{s} = 0$

With time-varying fields:

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \oint \vec{E} \cdot d\vec{s} = -\iint \frac{\partial \vec{B}}{\partial t} \cdot d\vec{A}$$

# Maxwell's equation in vacuum (contd.)

$$\nabla \times \vec{B} - \frac{1}{c^2} \frac{\partial}{\partial t} \vec{E} = 0 \quad \nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} + \frac{\partial}{\partial t} \vec{B} = 0 \quad \nabla \cdot \vec{E} = 0$$

curl of 3<sup>rd</sup> and  $\frac{\partial}{\partial t}$  of 1<sup>st</sup> equation:

$$\nabla \times \nabla \times \vec{E} + \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \vec{E} = 0$$

vector identity:

$$\nabla \times \nabla \times \vec{E} = \nabla \nabla \cdot \vec{E} - \Delta \vec{E}$$

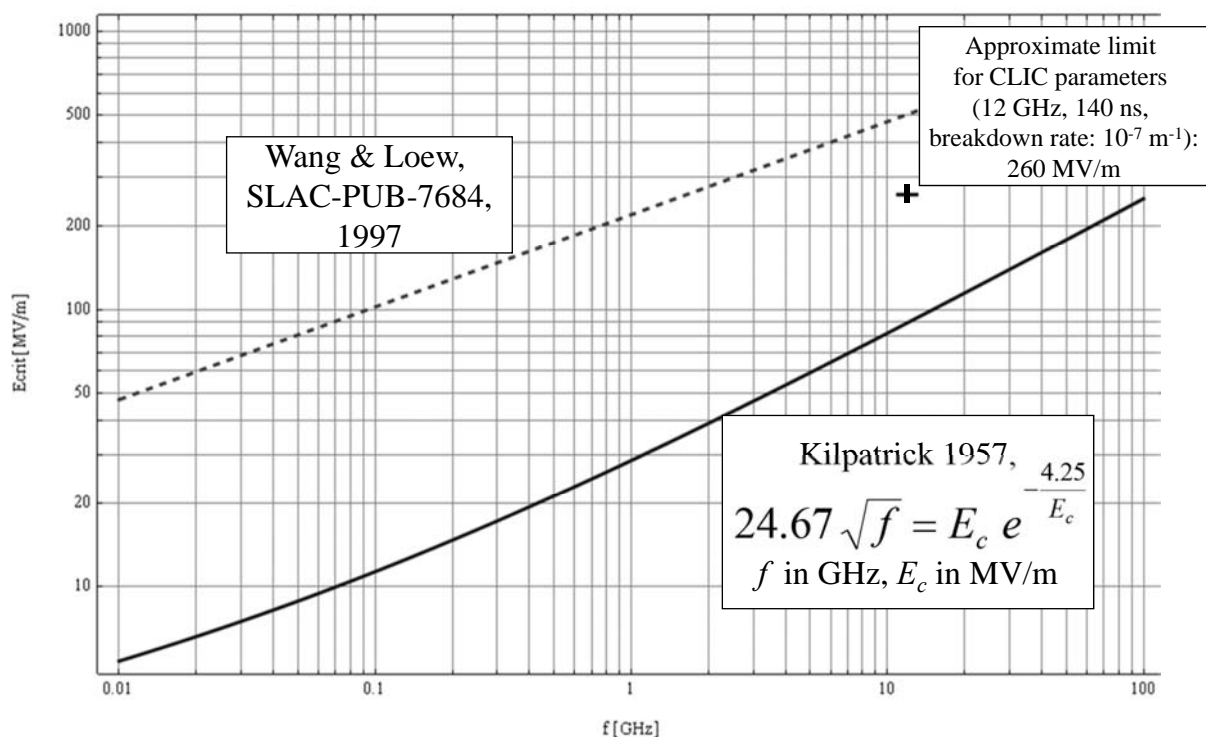
with 4<sup>th</sup> equation :

$$\Delta \vec{E} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \vec{E} = 0$$

i.e. Laplace in 4 dimensions.

## Another reason for RF: breakdown limit

surface field, in vacuum ,Cu surface, room temperature



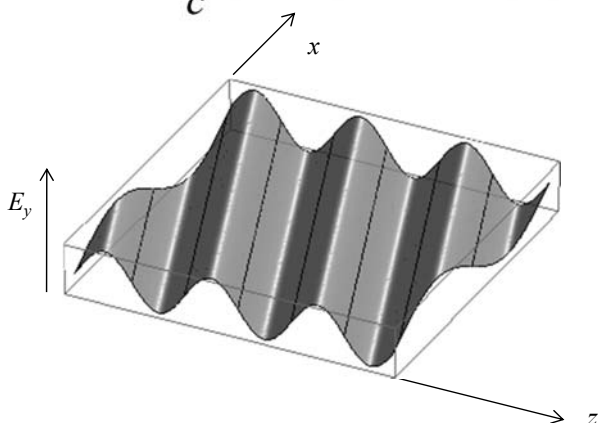
# FROM WAVEGUIDE TO PILLBOX

## Homogeneous plane wave

$$\vec{E} \propto \vec{u}_y \cos(\omega t - \vec{k} \cdot \vec{r})$$

$$\vec{B} \propto \vec{u}_x \cos(\omega t - \vec{k} \cdot \vec{r})$$

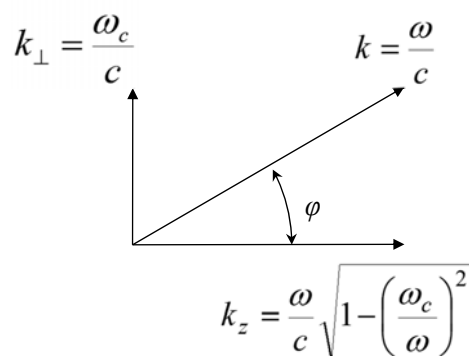
$$\vec{k} \cdot \vec{r} = \frac{\omega}{c} (\cos(\varphi)z + \sin(\varphi)x)$$



**Wave vector  $\vec{k}$ :**

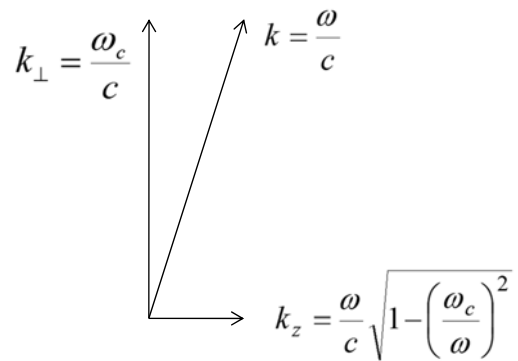
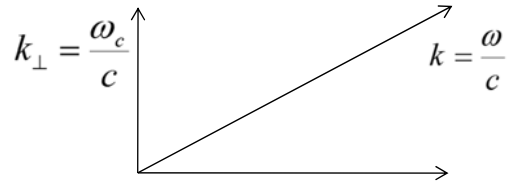
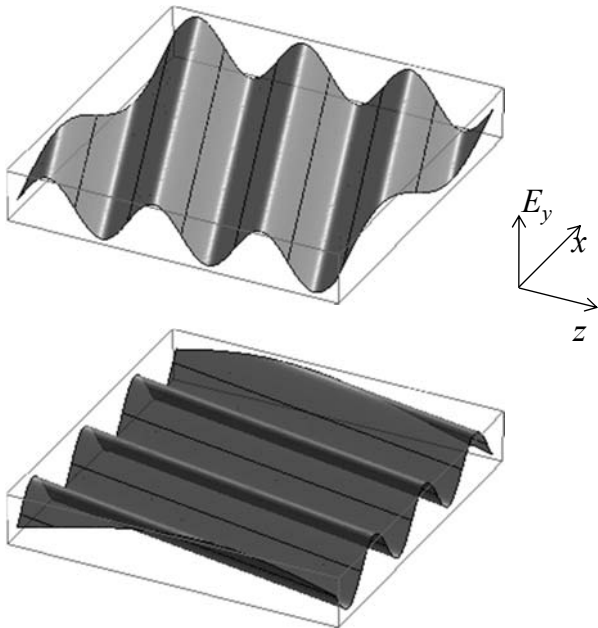
the direction of  $\vec{k}$  is the direction of propagation,  
the length of  $\vec{k}$  is the phase shift per unit length.

$\vec{k}$  behaves like a vector.



# Wave length, phase velocity

- The components of  $\vec{k}$  are related to the wavelength in the direction of that component as  $\lambda_z = \frac{2\pi}{k_z}$  etc. , to the phase velocity as  $v_{\phi,z} = \frac{\omega}{k_z} = f \lambda_z$ .

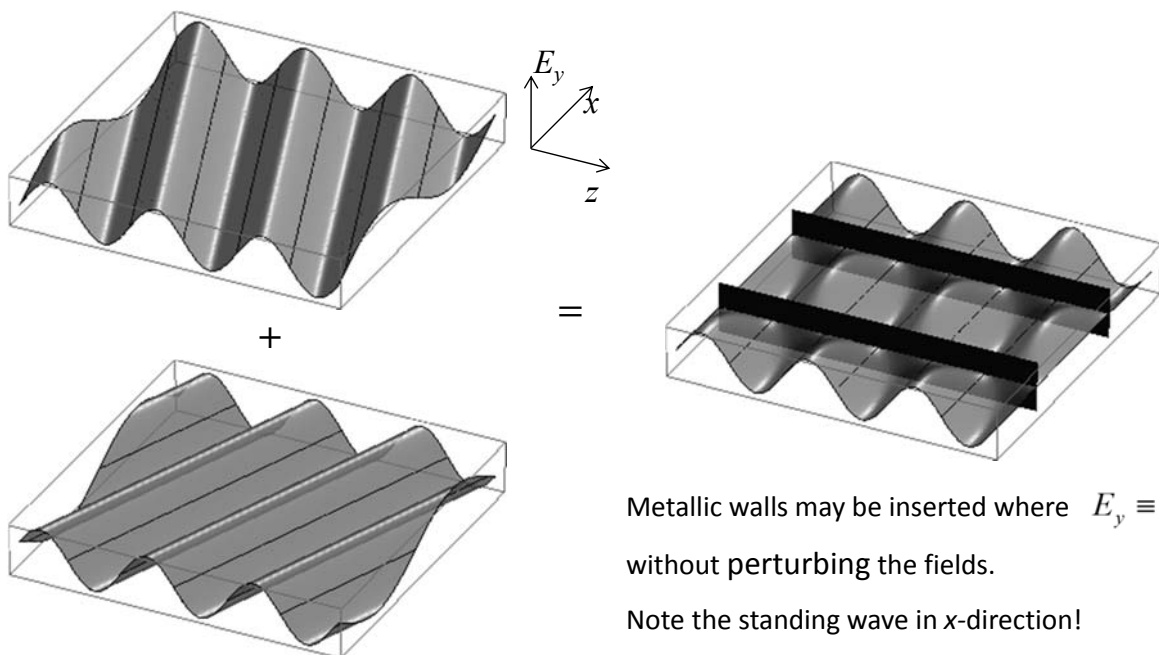


20-Sep-2011

CAS Chios 2011 — RF Cavity Design

11

## Superposition of 2 homogeneous plane waves



Metallic walls may be inserted where  $E_y \equiv 0$  without perturbing the fields.

Note the standing wave in  $x$ -direction!

This way one gets a hollow rectangular waveguide

20-Sep-2011

CAS Chios 2011 — RF Cavity Design

12

# Rectangular waveguide

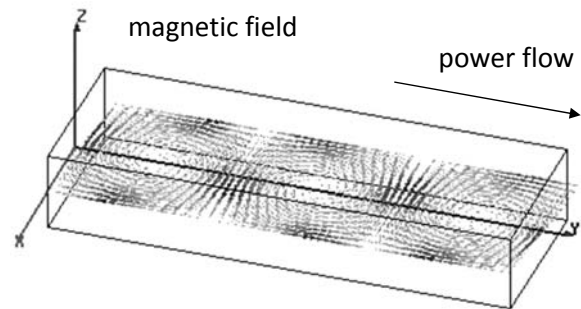
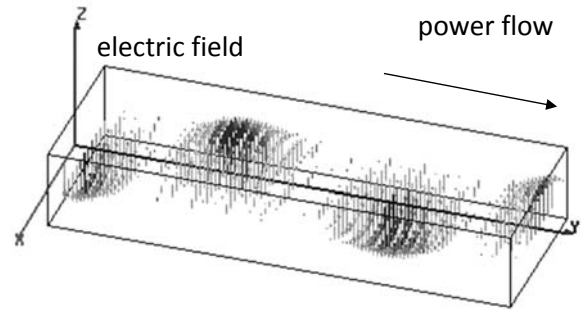
Fundamental (TE<sub>10</sub> or H<sub>10</sub>) mode in a standard rectangular waveguide.

**Example:** "S-band" : 2.6 GHz ... 3.95 GHz,

Waveguide type WR284 (2.84" wide), dimensions: 72.14 mm x 34.04 mm.

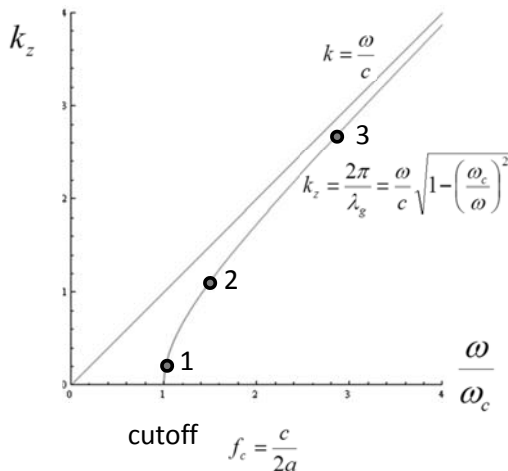
Operated at  $f = 3$  GHz.

$$\text{power flow: } \frac{1}{2} \operatorname{Re} \left\{ \iint_{\text{cross section}} \vec{E} \times \vec{H}^* \cdot d\vec{A} \right\}$$

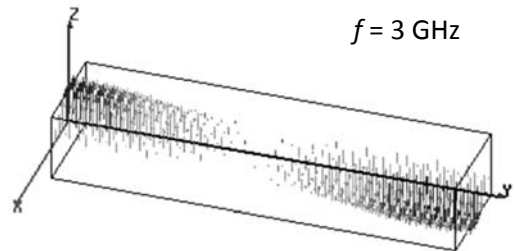


# Waveguide dispersion

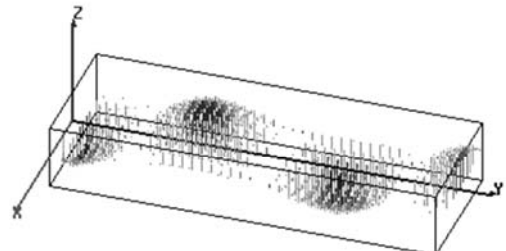
What happens with different waveguide dimensions (different width  $a$ )?



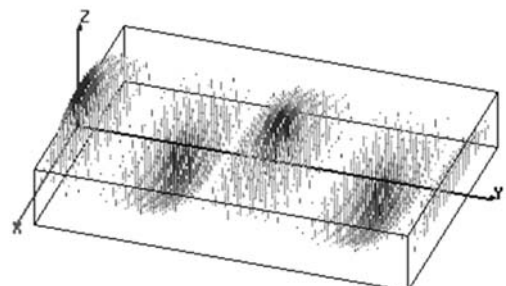
1:  
 $a = 52$  mm,  
 $f/f_c = 1.04$



2:  
 $a = 72.14$  mm,  
 $f/f_c = 1.44$



3:  
 $a = 144.3$  mm,  
 $f/f_c = 2.88$



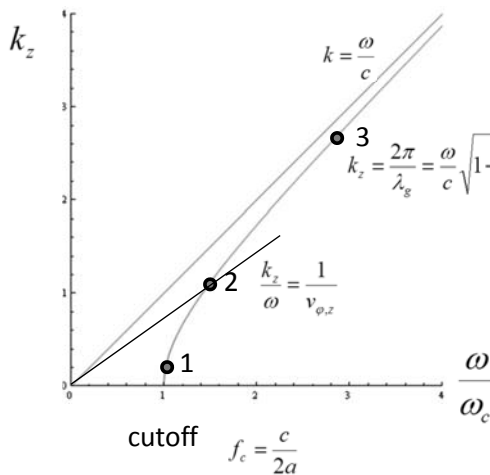
# Phase velocity

The phase velocity is the speed with which the crest or a zero-crossing travels in  $z$ -direction.

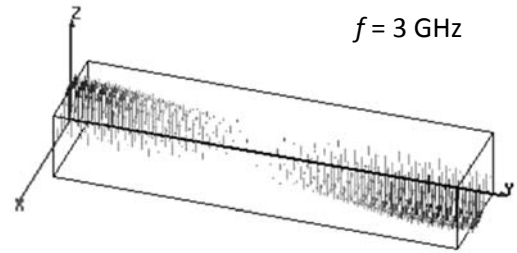
Note on the three animations on the right that, at constant  $f$ , it is  $\propto \lambda_g$

Note that at  $f = f_c$ ,  $v_{\phi,z} = \infty$ !

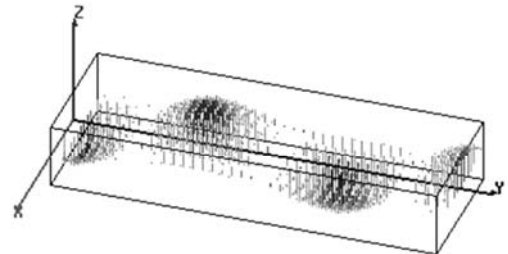
With  $f \rightarrow \infty$ ,  $v_{\phi,z} \rightarrow c$ !



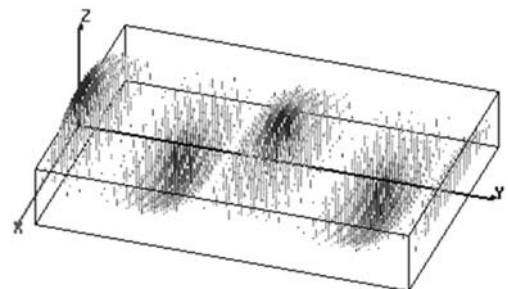
1:  
 $a = 52 \text{ mm}$ ,  
 $f/f_c = 1.04$



2:  
 $a = 72.14 \text{ mm}$ ,  
 $f/f_c = 1.44$



3:  
 $a = 144.3 \text{ mm}$ ,  
 $f/f_c = 2.88$



20-Sep-2011

CAS Chios 2011 — RF Cavity Design

15

## Summary waveguide dispersion and phase velocity:

In a **general** cylindrical waveguide:

$$\gamma = j \sqrt{\left(\frac{\omega}{c}\right)^2 - k_{\perp}^2}$$

$$Z_0 = \frac{j\omega\mu}{\gamma} \text{ for TE, } Z_0 = \frac{\gamma}{j\omega\epsilon} \text{ for TM}$$

$$k_z = \text{Im}\{\gamma\} = \frac{2\pi}{\lambda_g}$$

e.g.:  $\text{TE}_{10}$ -wave in rectangular waveguide:

$$\gamma = j \sqrt{\left(\frac{\omega}{c}\right)^2 - \left(\frac{\pi}{a}\right)^2}$$

$$Z_0 = \frac{j\omega\mu}{\gamma}$$

$$\lambda_{\text{cutoff}} = 2a$$

In a hollow waveguide: phase velocity  $> c$ , group velocity  $< c$

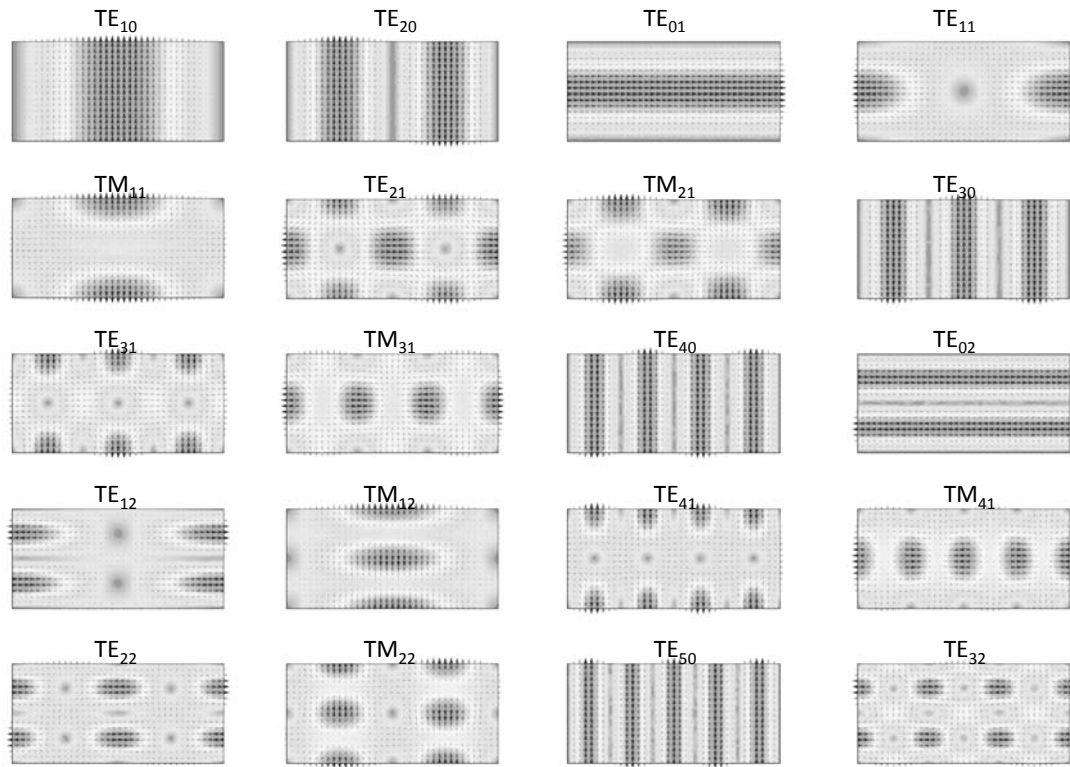
20-Sep-2011

CAS Chios 2011 — RF Cavity Design

16



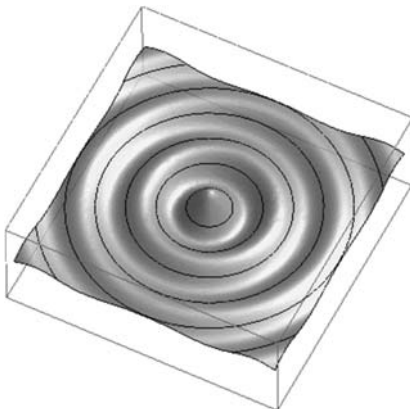
# Rectangular waveguide modes



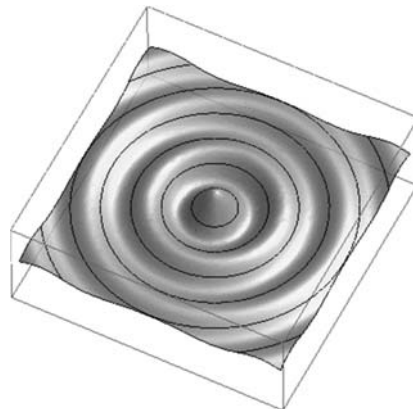
plotted:  $E$ -field

# Radial waves

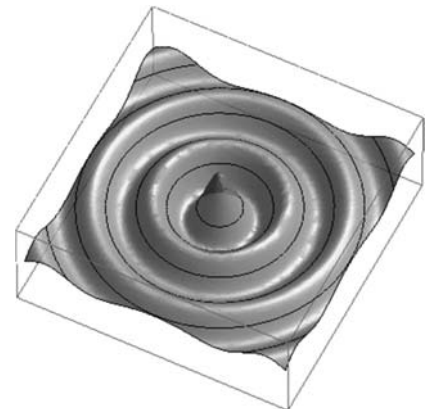
Also radial waves may be interpreted as superposition of plane waves. The superposition of an outward and an inward radial wave can result in the field of a round hollow waveguide.



$$E_z \propto H_n^{(2)}(k_\rho \rho) \cos(n\varphi)$$



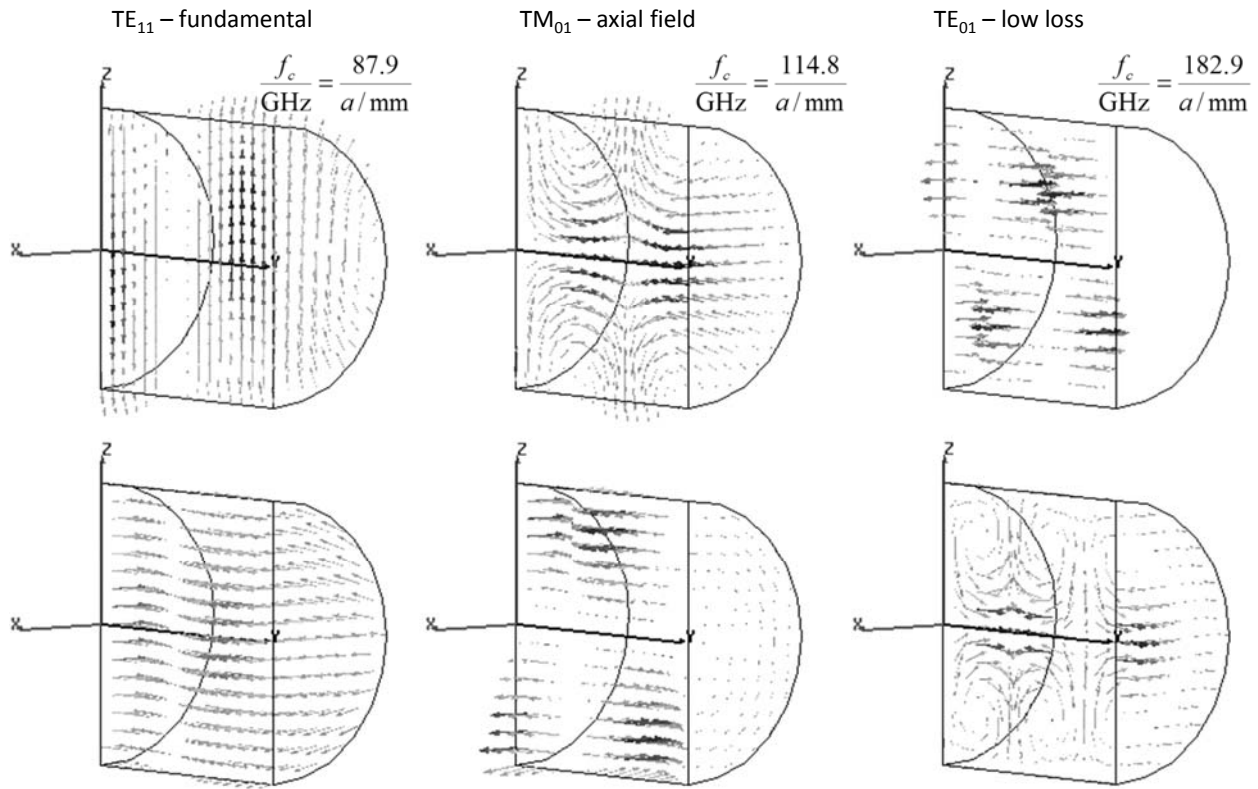
$$E_z \propto H_n^{(1)}(k_\rho \rho) \cos(n\varphi)$$



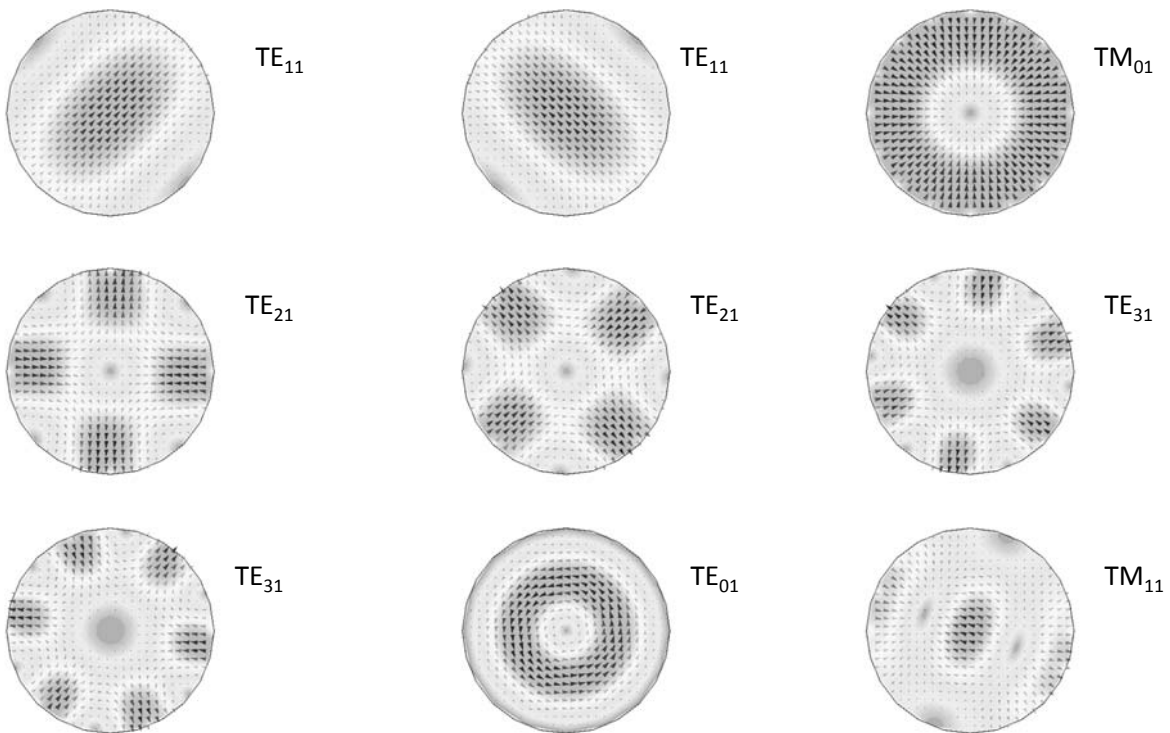
$$E_z \propto J_n(k_\rho \rho) \cos(n\varphi)$$

# Round waveguide

$$f/f_c = 1.44$$



# Circular waveguide modes



plotted: *E*-field

# General waveguide equations:

Transverse wave equation (membrane equation):  $\Delta T + \left(\frac{\omega_c}{c}\right)^2 T = 0$

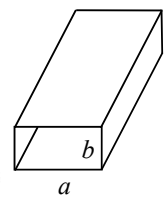
|   | TE (or H) modes  | TM (or E) modes   |
|---|--|---|
| boundary condition:   | $\vec{n} \cdot \nabla T = 0$   | $T = 0$   |
| longitudinal wave equations<br>(transmission line equations): | $\frac{dU(z)}{dz} + \gamma Z_0 I(z) = 0$<br>$\frac{dI(z)}{dz} + \frac{\gamma}{Z_0} U(z) = 0$ |   |
| propagation constant:   | $\gamma = j \frac{\omega}{c} \sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2}$              |   |
| characteristic impedance:                                     | $Z_0 = \frac{j\omega\mu}{\gamma}$  | $Z_0 = \frac{\gamma}{j\omega\epsilon}$                                  |
| ortho-normal eigenvectors:                                    | $\vec{e} = \vec{u}_z \times \nabla T$  | $\vec{e} = -\nabla T$   |
| transverse fields:  | $\vec{E} = U(z)\vec{e}$<br>$\vec{H} = I(z)\vec{u}_z \times \vec{e}$                          |   |
| longitudinal field:   | $H_z = \left(\frac{\omega_c}{c}\right)^2 \frac{TU(z)}{j\omega\mu}$                           | $E_z = \left(\frac{\omega_c}{c}\right)^2 \frac{TI(z)}{j\omega\epsilon}$ |

## Rectangular waveguide: transverse eigenfunctions

TE (H) modes:  $T_{mn}^{(H)} = \frac{1}{\pi} \sqrt{\frac{ab\epsilon_m\epsilon_n}{(mb)^2 + (na)^2}} \cos\left(\frac{m\pi}{a}x\right) \cos\left(\frac{n\pi}{b}y\right)$

TM (E) modes:  $T_{mn}^{(E)} = \frac{2}{\pi} \sqrt{\frac{ab}{(mb)^2 + (na)^2}} \sin\left(\frac{m\pi}{a}x\right) \sin\left(\frac{n\pi}{b}y\right)$

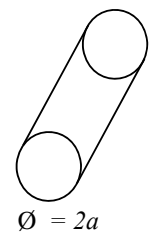
$$\frac{\omega_c}{c} = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$



## Round waveguide: transverse eigenfunctions

TE (H) modes:  $T_{mn}^{(H)} = \sqrt{\frac{\epsilon_m}{\pi(\chi_{mn}'^2 - m^2)}} \frac{J_m\left(\chi_{mn}' \frac{\rho}{a}\right)}{J_m(\chi_{mn}')} \begin{cases} \cos(m\varphi) \\ \sin(m\varphi) \end{cases}$

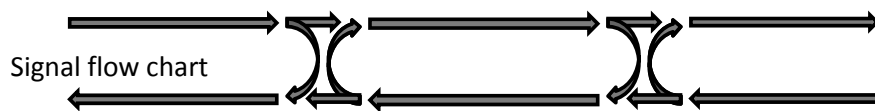
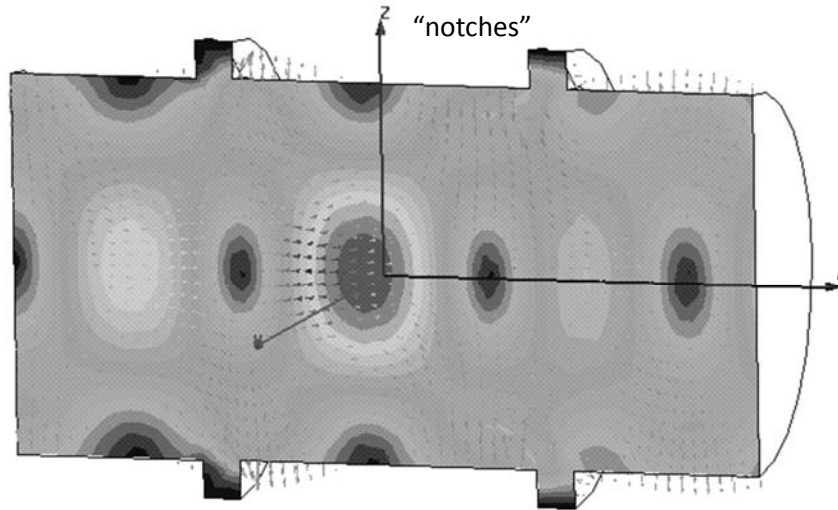
TM (E) modes:  $T_{mn}^{(E)} = \sqrt{\frac{\epsilon_m}{\pi \chi_{mn} J_{m-1}(\chi_{mn})}} \begin{cases} \sin(m\varphi) \\ \cos(m\varphi) \end{cases}$



$$\frac{\omega_c}{c} = \frac{\chi_{mn}}{a}$$

where  $\epsilon_i = \begin{cases} 1 & \text{for } i = 0 \\ 2 & \text{for } i \neq 0 \end{cases}$

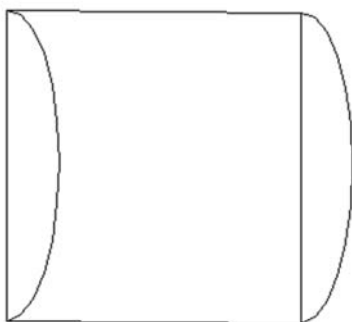
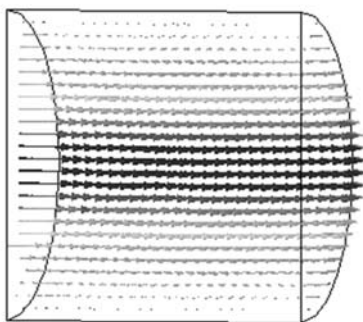
# Waveguide perturbed by notches



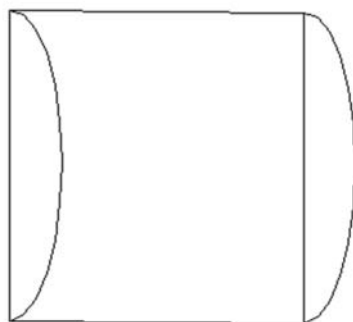
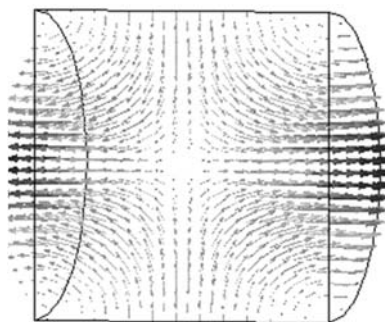
Reflections from notches lead to a superimposed standing wave pattern.  
"Trapped mode"

# Short-circuited waveguide

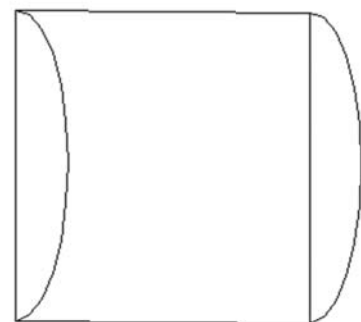
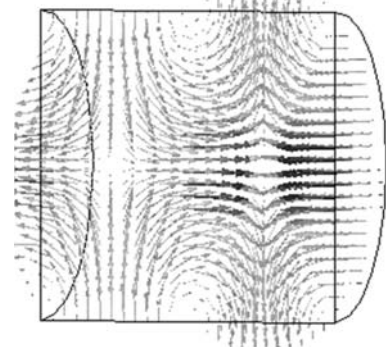
$TM_{010}$  (no axial dependence)



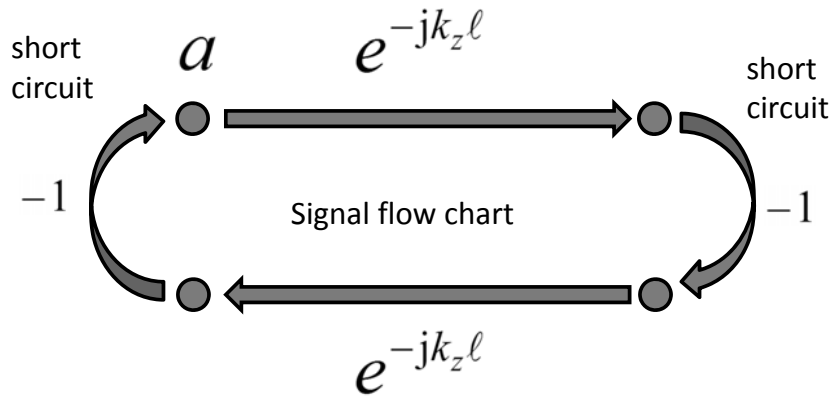
$TM_{011}$



$TM_{012}$



# Single WG mode between two shorts



Eigenvalue equation for field amplitude  $a$ :

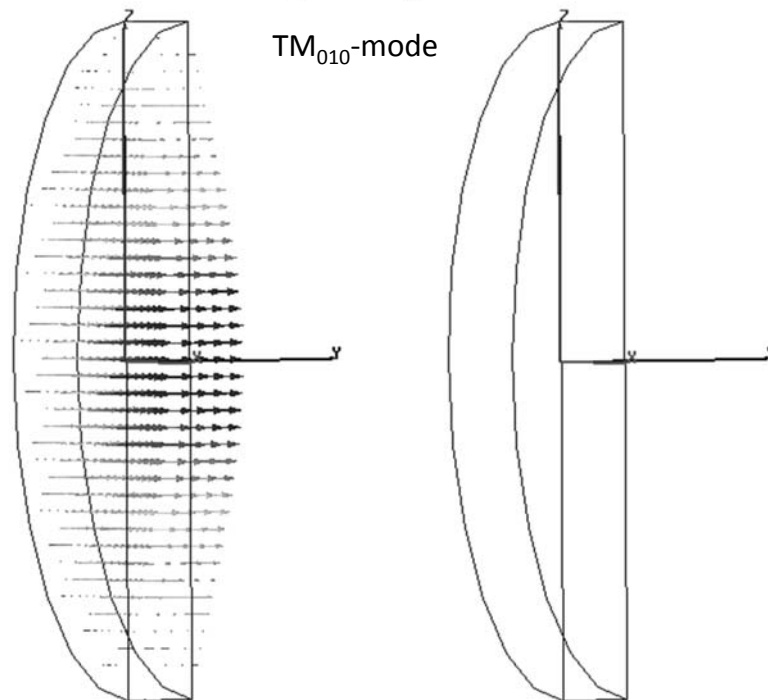
$$a = e^{-jk_z 2l} a$$

Non-vanishing solutions exist for  $2k_z l = 2\pi m$  :

With  $k_z = \frac{\omega}{c} \sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2}$ , this becomes  $f_0^2 = f_c^2 + \left(c \frac{m}{2l}\right)^2$

## Simple pillbox

(only 1/2 shown)

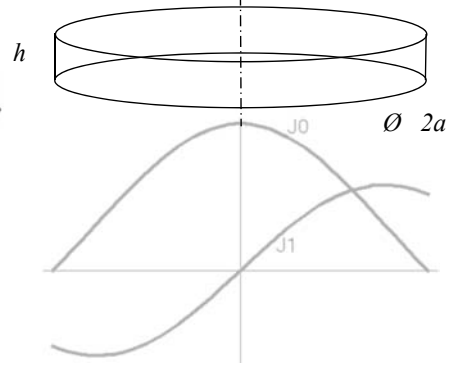


electric field (purely axial)

magnetic field (purely azimuthal)

## Pillbox cavity field (w/o beam tube)

$$T(\rho, \varphi) = \sqrt{\frac{1}{\pi}} \frac{J_0\left(\frac{\chi_{01}\rho}{a}\right)}{\chi_{01} J_1\left(\frac{\chi_{01}}{a}\right)} \quad \chi_{01} = 2.40483\dots$$



The only non-vanishing field components :

$$E_z = \frac{1}{j\omega\epsilon_0} \frac{\chi_{01}}{a} \sqrt{\frac{1}{\pi}} \frac{J_0\left(\frac{\chi_{01}\rho}{a}\right)}{a J_1\left(\frac{\chi_{01}}{a}\right)}$$

$$B_\varphi = \mu_0 \sqrt{\frac{1}{\pi}} \frac{J_1\left(\frac{\chi_{01}\rho}{a}\right)}{a J_1\left(\frac{\chi_{01}}{a}\right)}$$

$$\omega_{0|pillbox} = \frac{\chi_{01} c}{a} \quad \eta = \sqrt{\frac{\mu_0}{\epsilon_0}} = 377 \Omega$$

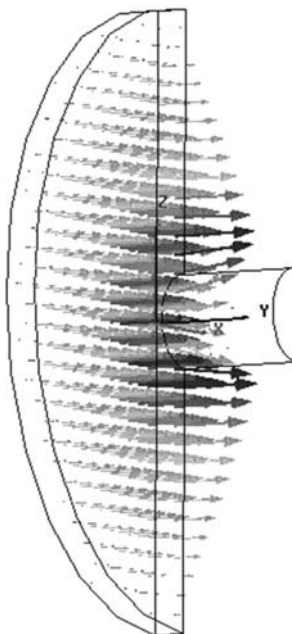
$$Q|_{pillbox} = \frac{\sqrt{2a\eta\sigma\chi_{01}}}{2\left(1 + \frac{a}{h}\right)}$$

$$\frac{R}{Q}|_{pillbox} = \frac{4\eta \sin^2\left(\frac{\chi_{01} h}{2a}\right)}{\chi_{01}^3 \pi J_1^2(\chi_{01}) h/a}$$

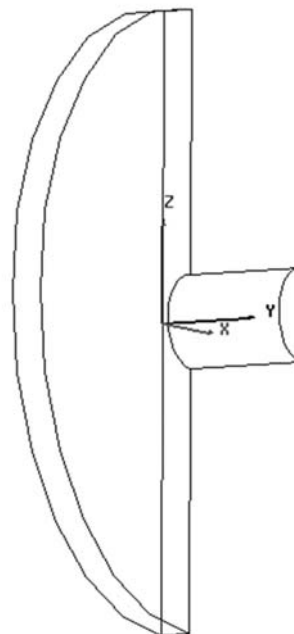
## Pillbox with beam pipe

TM<sub>010</sub>-mode (only 1/4 shown)

One needs a hole for the beam pipe – circular waveguide below cutoff



electric field

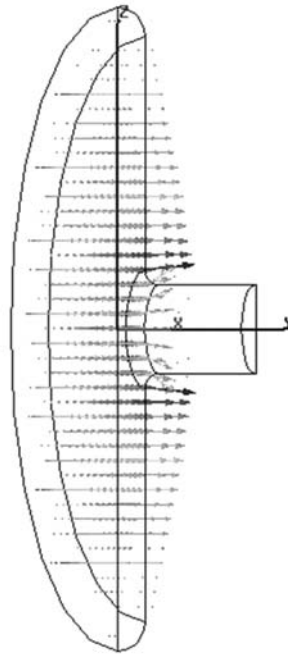


magnetic field

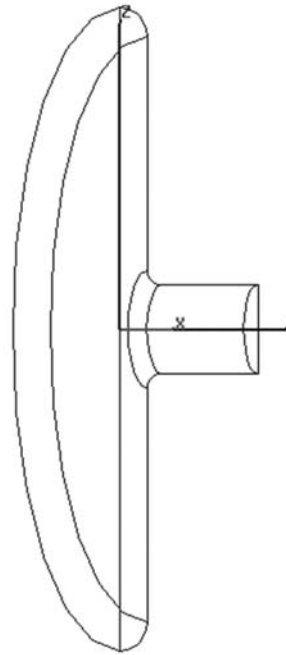
# A more practical pillbox cavity

$TM_{010}$ -mode (only 1/4 shown)

Round of sharp edges (to reduce field enhancement!)



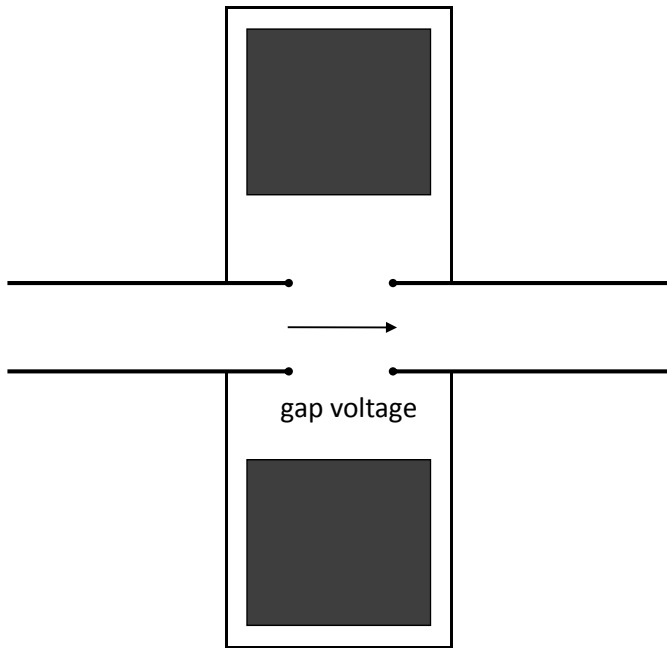
electric field



magnetic field

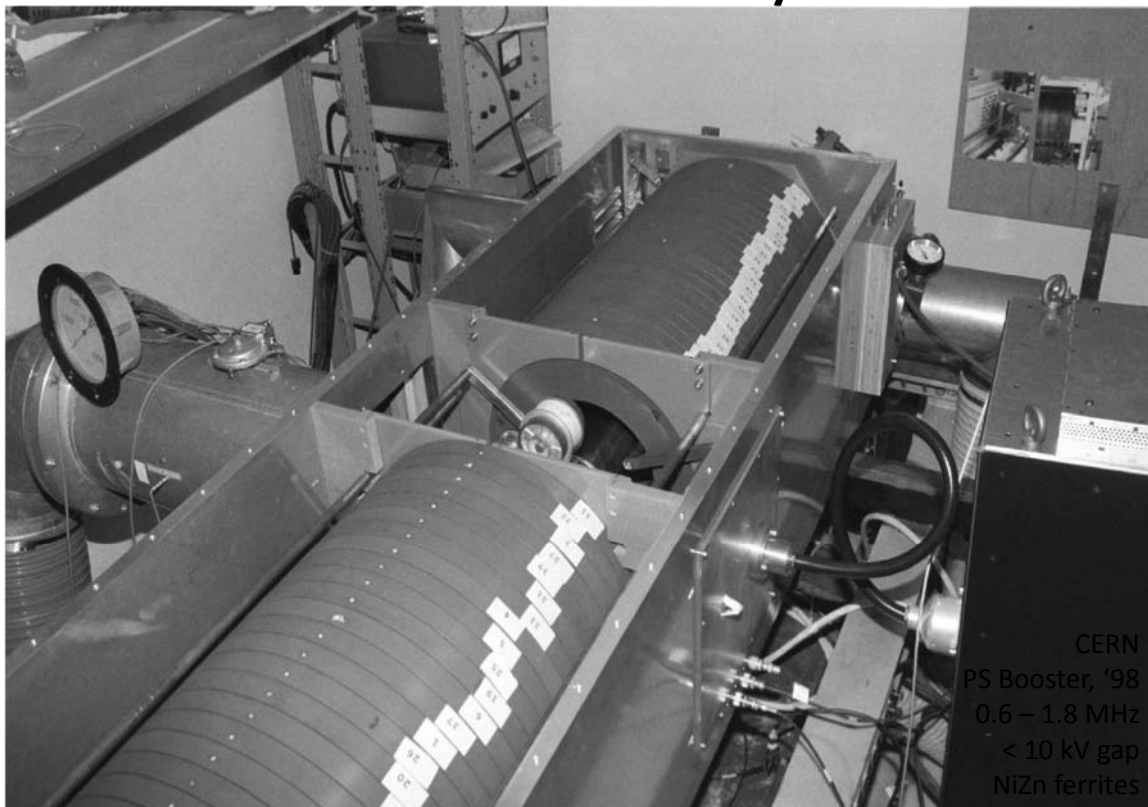
## ACCELERATING GAP

# Accelerating gap



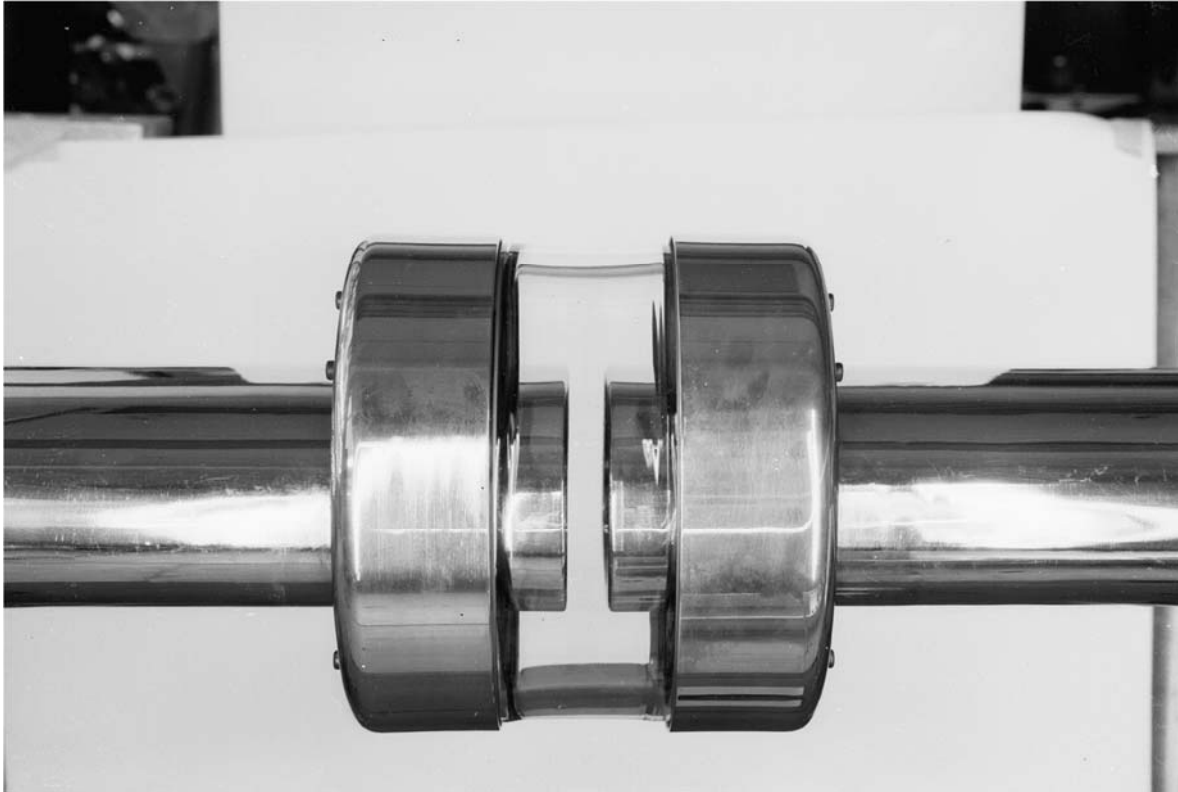
- We want a voltage across the gap!
- It cannot be DC, since we want the beam tube on ground potential.
- Use  $\oint \vec{E} \cdot d\vec{s} = -\iint \frac{d\vec{B}}{dt} \cdot d\vec{A}$
- The “shield” imposes a
  - upper limit of the voltage pulse duration or – equivalently –
  - a lower limit to the usable frequency.
- The limit can be extended with a material which acts as “open circuit”!
- Materials typically used:
  - ferrites (depending on  $f$ -range)
  - magnetic alloys (MA) like Metglas®, Finemet®, Vitrovac®...
- resonantly driven with RF (ferrite loaded cavities) – or with pulses (induction cell)

# Ferrite cavity





# Gap of PS cavity (prototype)



20-Sep-2011

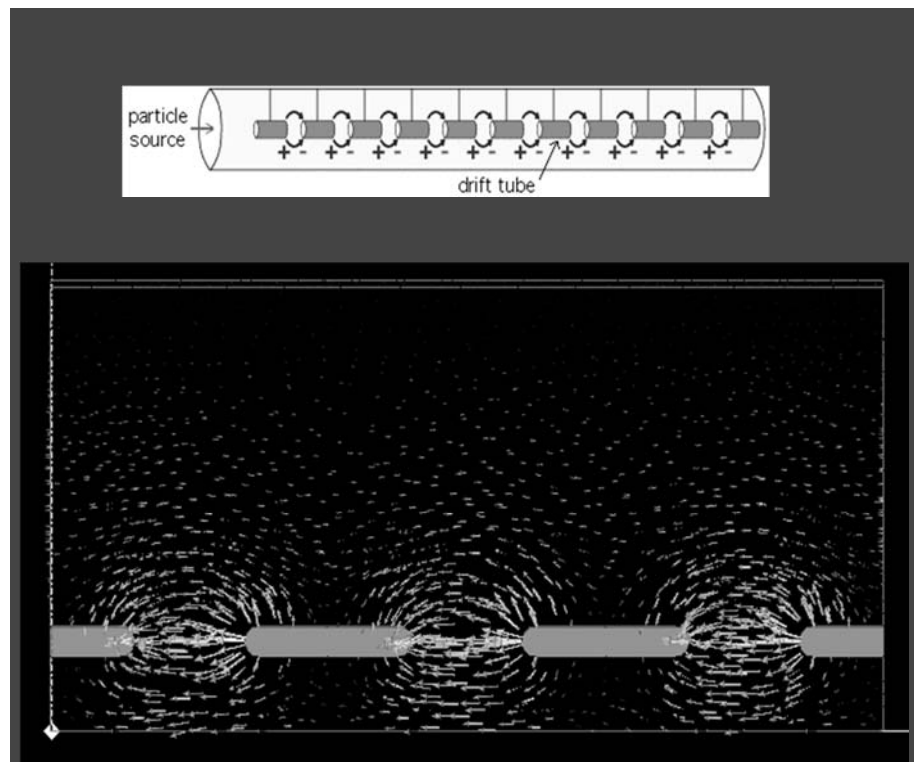
CAS Chios 2011 — RF Cavity Design

33

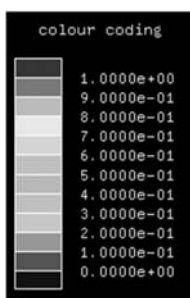
## Drift Tube Linac (DTL) – how it works

For slow particles !  
E.g. protons @ few MeV

The drift tube lengths  
can easily be adapted.



electric field

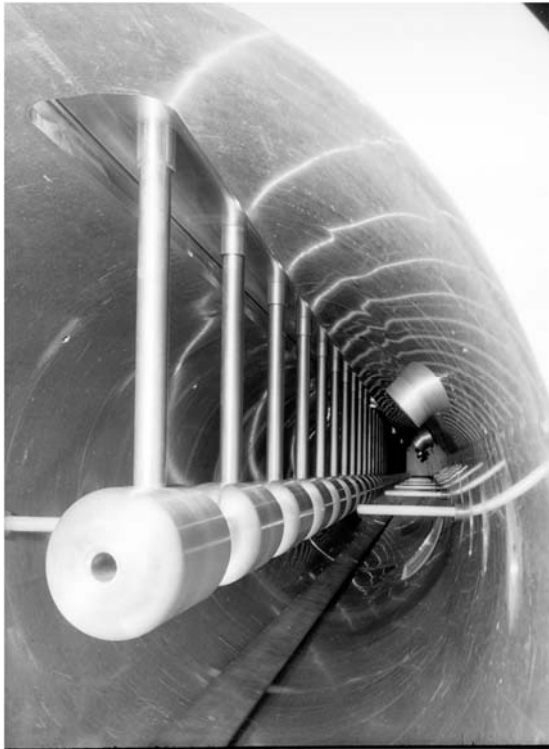


20-Sep-2011

CAS Chios 2011 — RF Cavity Design

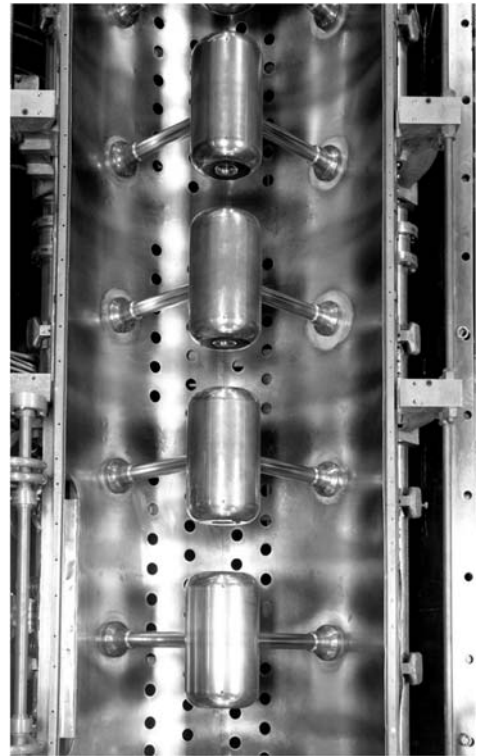
34

# Drift tube linac – practical implementations



20-Sep-2011

CAS Chios 2011 — RF Cavity Design



35

## CHARACTERIZING A CAVITY

20-Sep-2011

CAS Chios 2011 — RF Cavity Design

36

# Acceleration voltage & $R$ -upon- $Q$

I define  $V_{acc} = \int E_z e^{j\frac{\omega}{\beta c}z} dz$ . The exponential factor accounts for the variation of the field while particles with velocity  $\beta c$  are traversing the gap (see next page).

With this definition,  $V_{acc}$  is generally complex – this becomes important with more than one gap. For the time being we are only interested in  $|V_{acc}|$ .

**Attention, different definitions are used!**

The square of the acceleration voltage is proportional to the stored energy  $W$ . The proportionality constant defines the quantity called  $R$ -upon- $Q$ :

$$\frac{R}{Q} = \frac{|V_{acc}|^2}{2\omega_0 W}$$

**Attention, also here different definitions are used!**

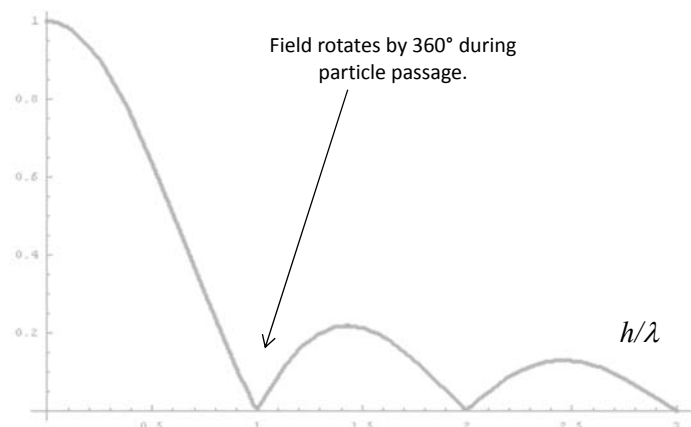
## Transit time factor

The transit time factor is the ratio of the acceleration voltage to the (non-physical) voltage a particle with infinite velocity would see.

$$TT = \frac{|V_{acc}|}{|\int E_z dz|} = \frac{\left| \int E_z e^{j\frac{\omega}{\beta c}z} dz \right|}{\left| \int E_z dz \right|}$$

The transit time factor of an ideal pillbox cavity (no axial field dependence) of height (gap length)  $h$  is:

$$TT = \sin\left(\frac{\chi_{01}h}{2a}\right) / \left(\frac{\chi_{01}h}{2a}\right)$$



# Shunt impedance

The square of the acceleration voltage is proportional to the power loss  $P_{loss}$ .  
 The proportionality constant defines the quantity "shunt impedance"

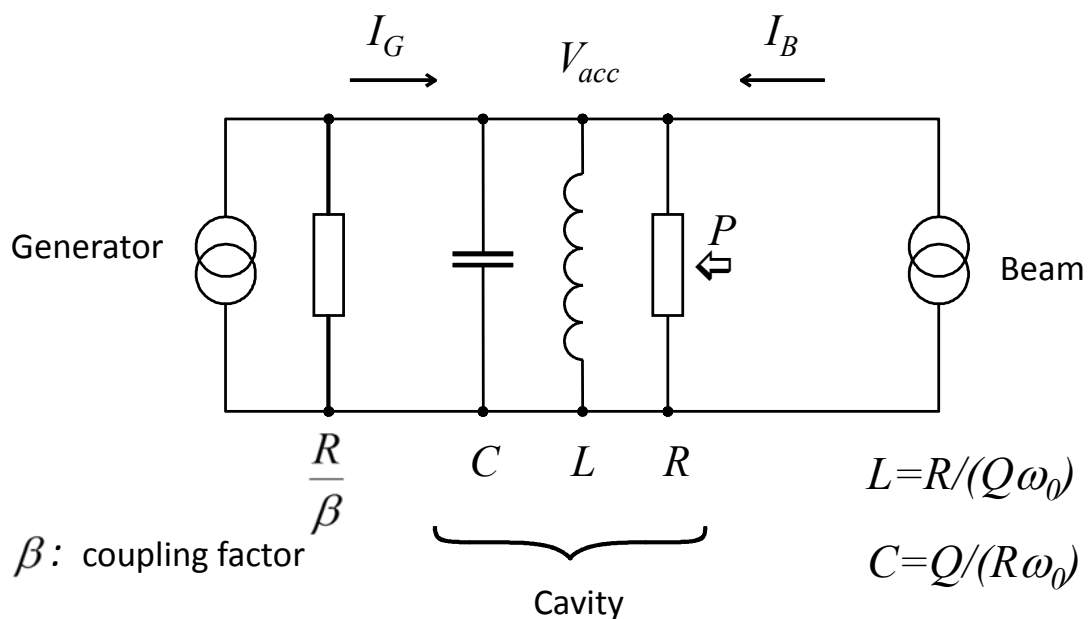
$$R = \frac{|V_{acc}|^2}{2 P_{loss}}$$

**Attention, also here different definitions are used!**

Traditionally, the shunt impedance is the quantity to optimize in order to minimize the power required for a given gap voltage.

## Cavity resonator - equivalent circuit

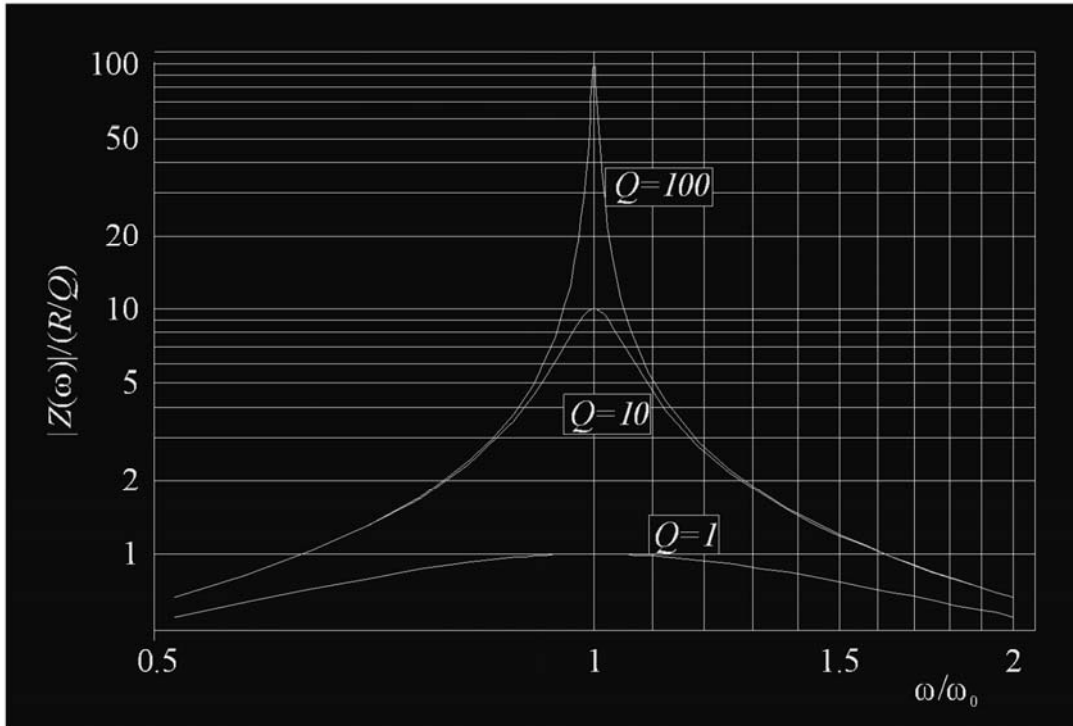
Simplification: single mode



$R$ : Shunt impedance

$\sqrt{L/C}$  :  $R$ -upon- $Q$

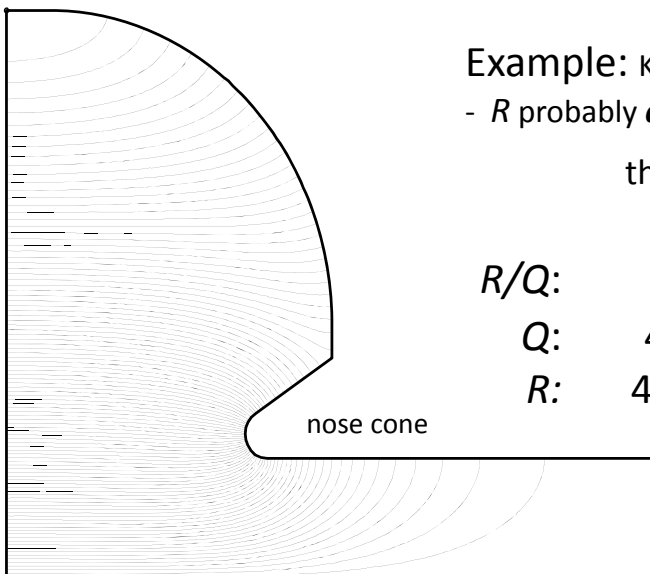
# Resonance



## Reentrant cavity

Nose cones increase transit time factor, round outer shape minimizes losses.

Nose cone example Freq = 500.003



Example: KEK photon factory 500 MHz

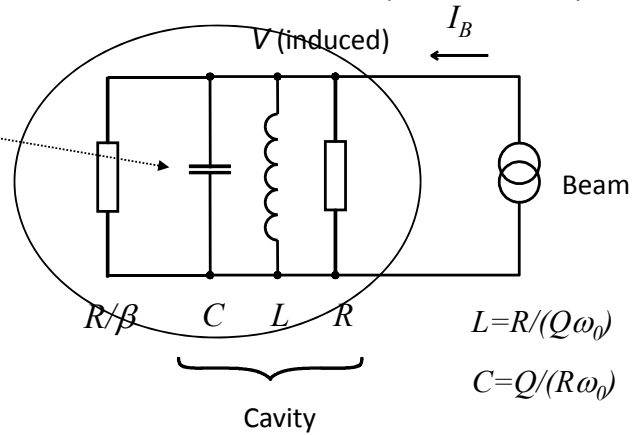
- *R* probably **as good as it gets** -

|              | this cavity | optimized pillbox |
|--------------|-------------|-------------------|
| <i>R/Q</i> : | 111 Ω       | 107.5 Ω           |
| <i>Q</i> :   | 44270       | 41630             |
| <i>R</i> :   | 4.9 MΩ      | 4.47 MΩ           |

# Loss factor

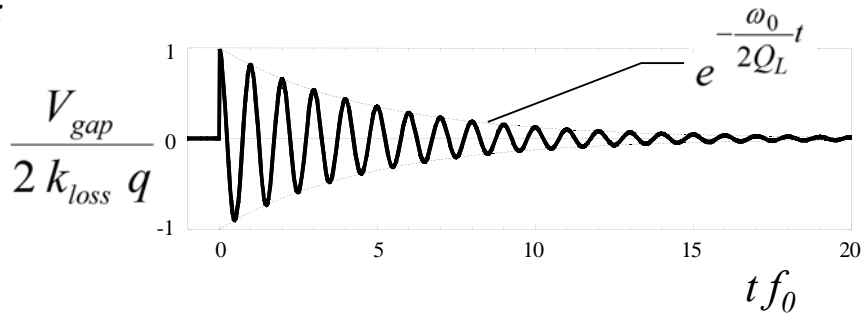
Impedance seen by the beam

$$k_{loss} = \frac{\omega_0 R}{2 Q} = \frac{|V_{gap}|^2}{4 W} = \frac{1}{2 C}$$



Energy deposited by a single charge  $q$ :  $k_{loss} q^2$

Voltage induced by a single charge  $q$ :



## Summary: relations $V_{gap}$ , $W$ , $P_{loss}$

R-upon-Q

$$\frac{R}{Q} = \frac{|V_{gap}|^2}{2 \omega_0 W}$$

$$k_{loss} = \frac{\omega_0 R}{2 Q} = \frac{|V_{gap}|^2}{4 W}$$

gap voltage

$$V_{gap}$$

Shunt impedance

$$R_{shunt} = \frac{|V_{gap}|^2}{2 P_{loss}}$$

Energy stored inside the cavity

$$W$$

Power lost in the cavity walls

$$P_{loss}$$

$$Q = \frac{\omega_0 W}{P_{loss}}$$

Q factor

# Beam loading – RF to beam efficiency

- The beam current “loads” the generator, in the equivalent circuit this appears as a resistance in parallel to the shunt impedance.
- If the generator is matched to the unloaded cavity, beam loading will cause the accelerating voltage to decrease.
- The power absorbed by the beam is  $-\frac{1}{2} \text{Re}\{V_{gap} I_B^*\}$ , the power loss  $P = \frac{|V_{gap}|^2}{2R}$ .
- For high efficiency, beam loading shall be high.
- The RF to beam efficiency is  $\eta = \frac{1}{1 + \frac{V_{gap}}{R|I_B|}} = \frac{|I_B|}{|I_G|}$ .

## Characterizing cavities

- Resonance frequency  $\omega_0 = \frac{1}{\sqrt{L \cdot C}}$
- Transit time factor  $\frac{\left| \int E_z e^{j\frac{\omega}{c}z} dz \right|}{\left| \int E_z dz \right|}$   
field varies while particle is traversing the gap  
Circuit definition
- Shunt impedance  $|V_{gap}|^2 = 2 R_{shunt} P_{loss}$   
gap voltage – power relation  
Linac definition  $|V_{gap}|^2 = R_{shunt} P_{loss}$
- $Q$  factor  $\omega_0 W = Q P_{loss}$
- $R/Q$   $\frac{R}{Q} = \frac{|V_{gap}|^2}{2 \omega_0 W} = \sqrt{\frac{L}{C}}$   
independent of losses – only geometry!  
 $\frac{R}{Q} = \frac{|V_{gap}|^2}{\omega_0 W}$
- loss factor  $k_{loss} = \frac{\omega_0 R}{2 Q} = \frac{|V_{gap}|^2}{4 W}$   
 $k_{loss} = \frac{\omega_0 R}{4 Q} = \frac{|V_{gap}|^2}{4 W}$

# Example Pillbox:

$$\omega_0|_{pillbox} = \frac{\chi_{01} c}{a}$$

$$\chi_{01} = 2.4048$$

$$Q|_{pillbox} = \frac{\sqrt{2a\eta\sigma\chi_{01}}}{2\left(1 + \frac{a}{h}\right)}$$

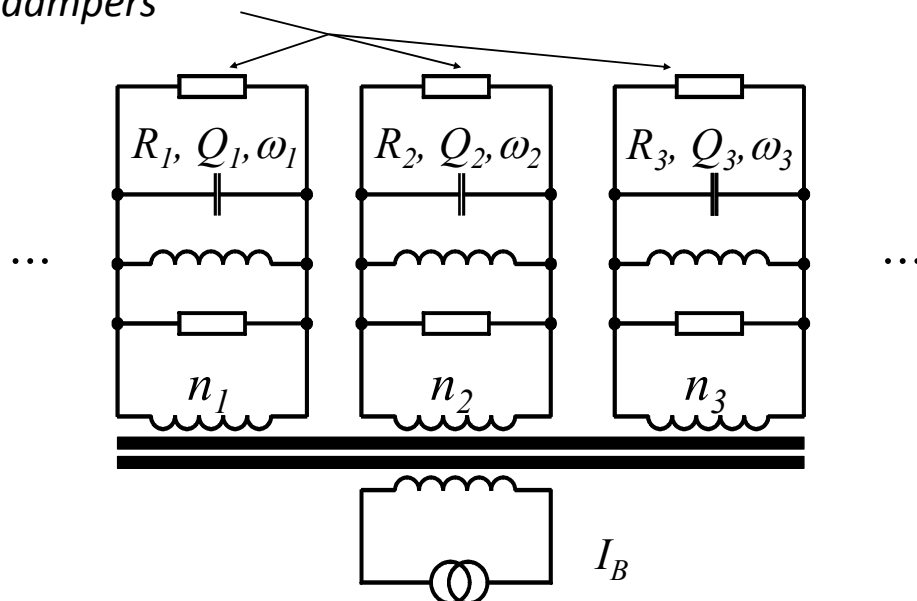
$$\eta = \sqrt{\frac{\mu_0}{\epsilon_0}} = 377 \Omega$$

$$\sigma_{Cu} = 5.8 \cdot 10^7 \text{ S/m}$$

$$\frac{R}{Q}|_{pillbox} = \frac{4\eta}{\chi_{01}^3 \pi J_1^2(\chi_{01})} \frac{\sin^2\left(\frac{\chi_{01} h}{2 a}\right)}{h/a}$$

## Higher order modes

*external dampers*

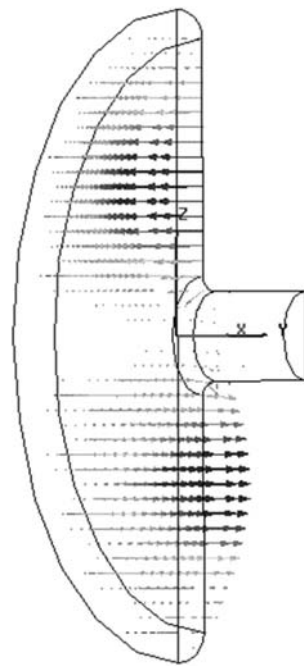




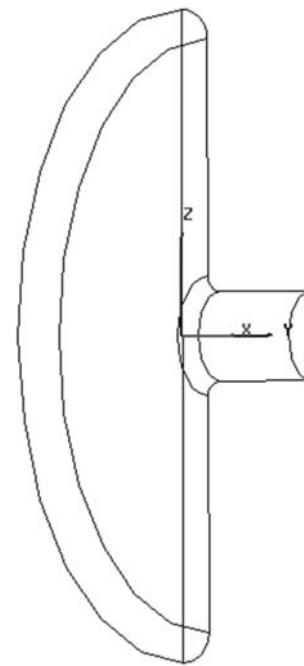
# Pillbox: dipole mode

TM<sub>110</sub>-mode

(only 1/4 shown)



electric field



magnetic field

## Panofsky-Wenzel theorem

For particles moving virtually at  $v=c$ , the integrated transverse force (kick) can be determined from the transverse variation of the integrated longitudinal force!

$$j\frac{\omega}{c}\vec{F}_{\perp} = \nabla_{\perp}F_{\parallel}$$

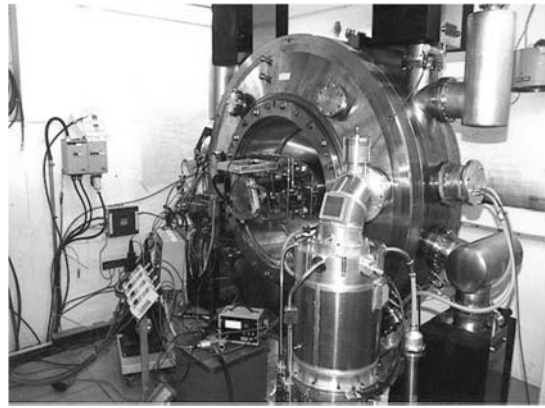
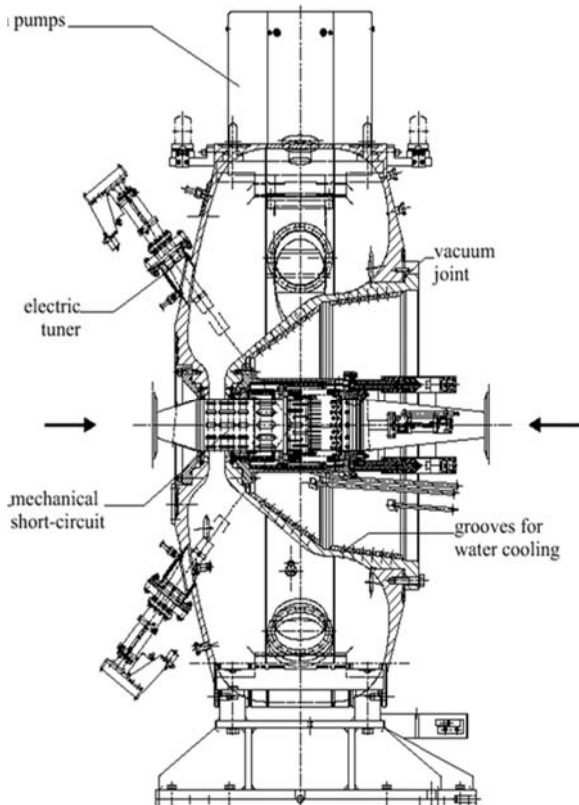
Pure TE modes: No net transverse force !

Transverse modes are characterized by

- the transverse impedance in  $\omega$ -domain
- the transverse loss factor (kick factor) in  $t$ -domain !

W.K.H. Panofsky, W.A. Wenzel: "Some Considerations Concerning the Transverse Deflection of Charged Particles in Radio-Frequency Fields", RSI **27**, 1957]

# CERN/PS 80 MHz cavity (for LHC)



20-Sep-2011

CAS Chios 2011 — RF Cavity Design

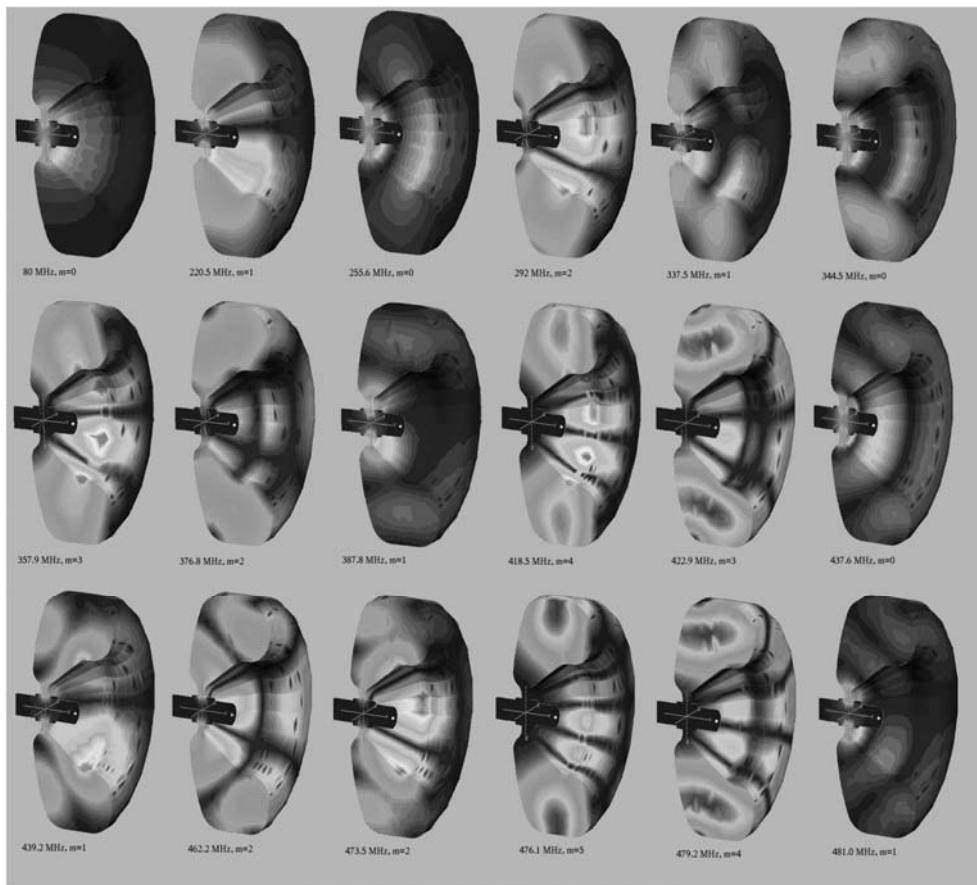
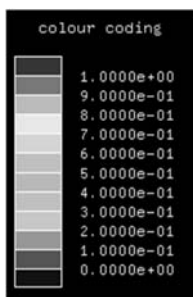
51

## Higher order modes

Example shown:  
80 MHz cavity PS  
for LHC.

Color-coded:

$$|\vec{E}|$$

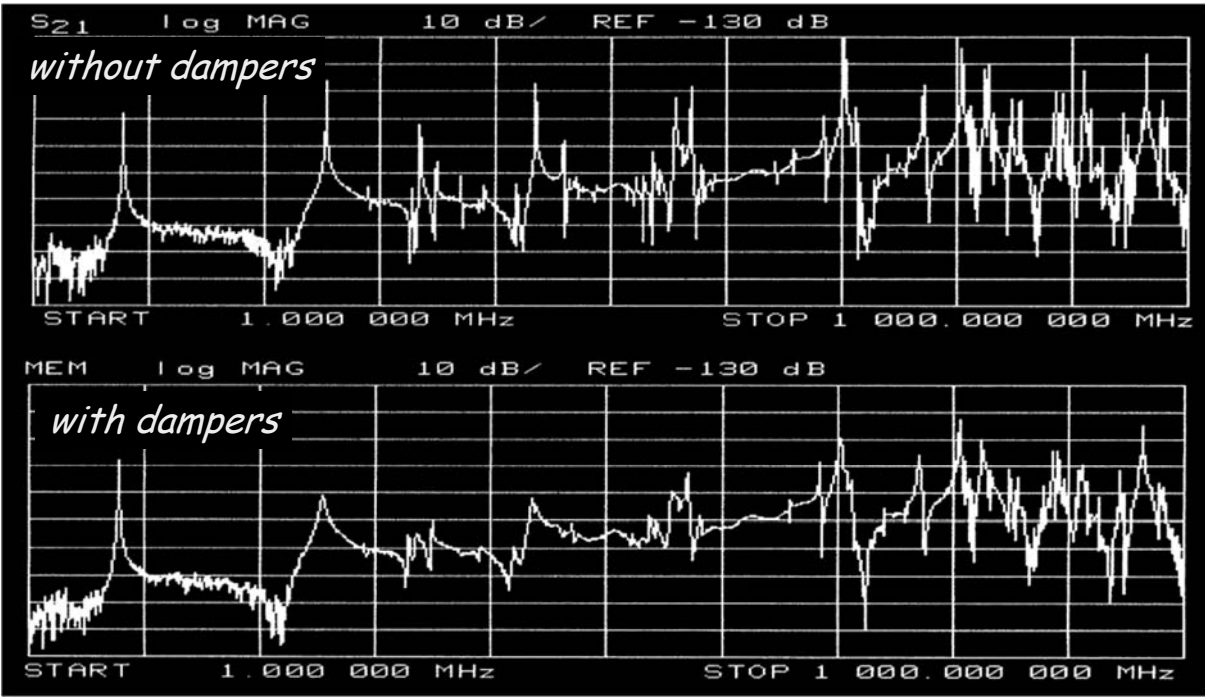


20-Sep-2011

CAS Chios 2011 — RF Cavity Design

52

# Higher order modes (measured spectrum)



## MORE EXAMPLES OF CAVITIES

# PS 19 MHz cavity (prototype, photo: 1966)

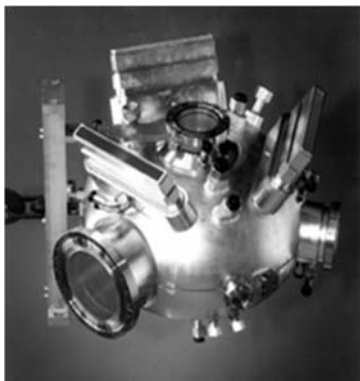


20-Sep-2011

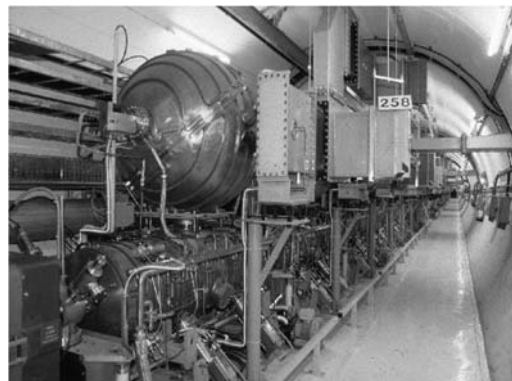
CAS Chios 2011 — RF Cavity Design

55

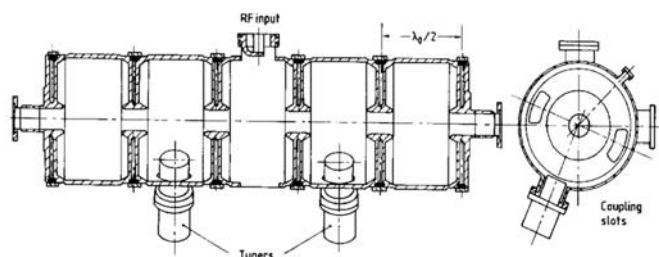
## Examples of cavities



PEP II cavity  
476 MHz, single cell,  
1 MV gap with 150 kW,  
strong HOM damping,



LEP normal-conducting Cu RF cavities,  
350 MHz. 5 cell standing wave + spherical cavity  
for energy storage, 3 MV

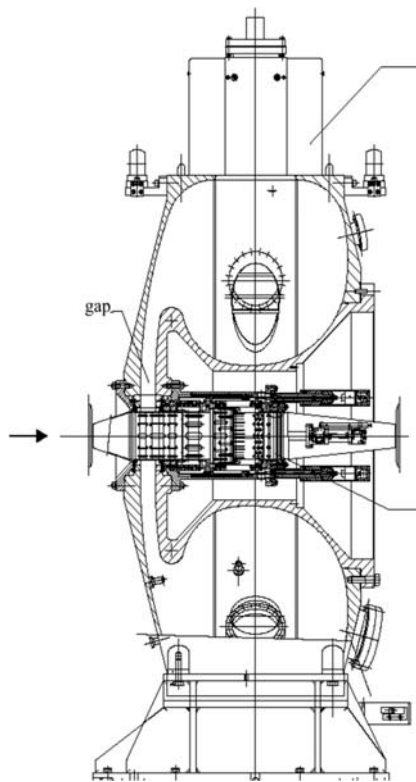


20-Sep-2011

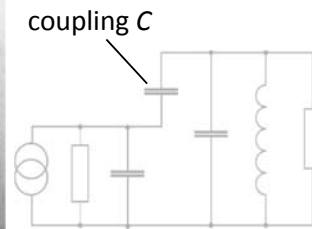
CAS Chios 2011 — RF Cavity Design

56

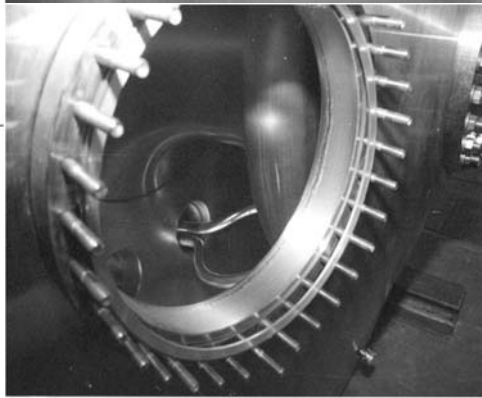
# CERN/PS 40 MHz cavity (for LHC)



example for  
capacitive coupling



cavity



## RF POWER SOURCES

# RF Power sources

> 200 MHz: Klystrons



Thales TH1801, Multi-Beam Klystron (MBK), 1.3 GHz, 117 kV. Achieved:  
48 dB gain, 10 MW peak, 150 kW average,  $\eta = 65\%$

$$48 \text{ dB: } \frac{\text{output power}}{\text{input power}} = 10^{4.8}$$

< 1000 MHz: grid tubes



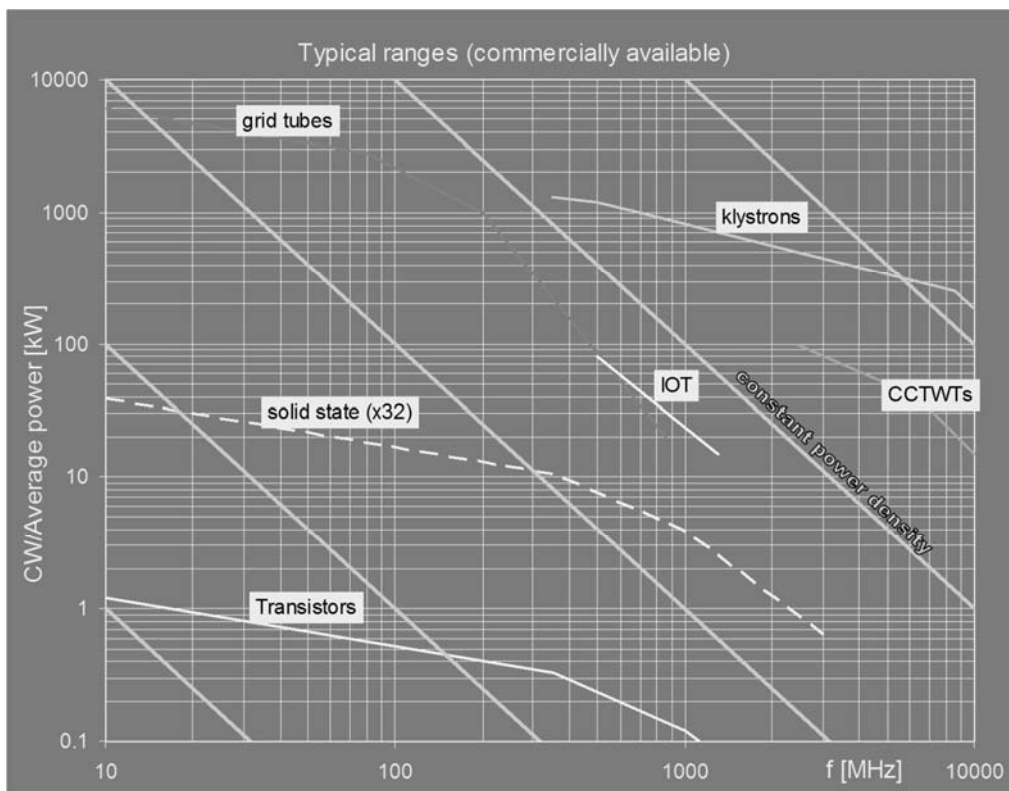
Tetrode

IOT

UHF Diacode®

pictures from <http://www.thales-electrondevices.com>

# RF power sources



# Example of a tetrode amplifier (80 MHz, CERN/PS)



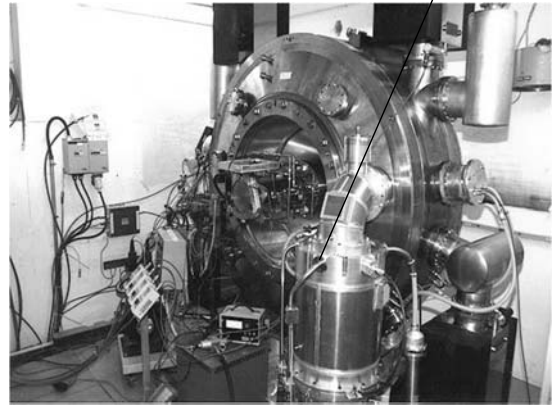
400 kW, with fast RF feedback

18  $\Omega$  coaxial output (towards cavity)

22 kV DC anode voltage feed-through with  $\lambda/4$  stub

tetrode cooling water feed-throughs

coaxial input matching circuit



20-Sep-2011

CAS Chios 2011 — RF Cavity Design

61

## MANY GAPS

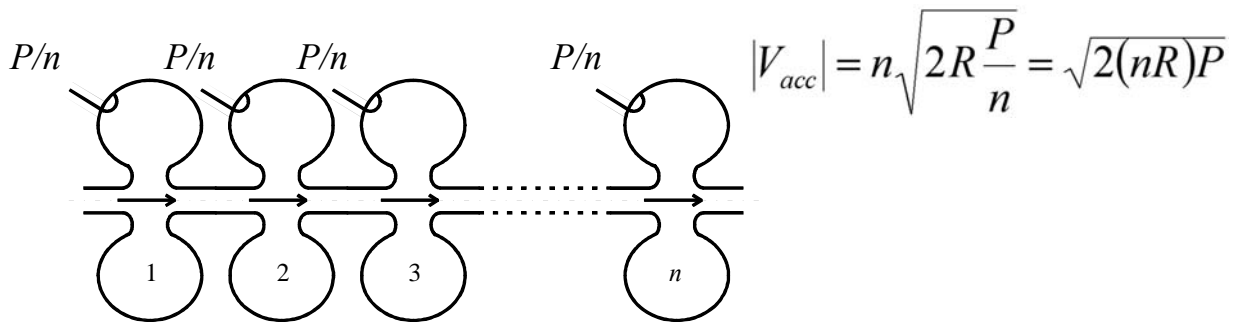
20-Sep-2011

CAS Chios 2011 — RF Cavity Design

62

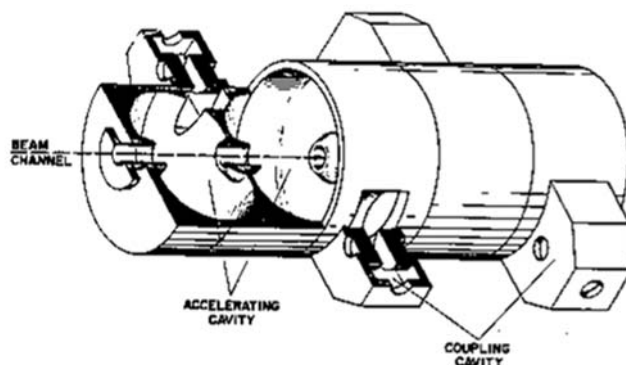
# What do you gain with many gaps?

- The  $R/Q$  of a single gap cavity is limited to some  $100 \Omega$ .  
Now consider to distribute the available power to  $n$  identical cavities: each will receive  $P/n$ , thus produce an accelerating voltage of  $\sqrt{2RP/n}$ .  
The total accelerating voltage thus increased, equivalent to a total equivalent shunt impedance of  $nR$ .



## Standing wave multicell cavity

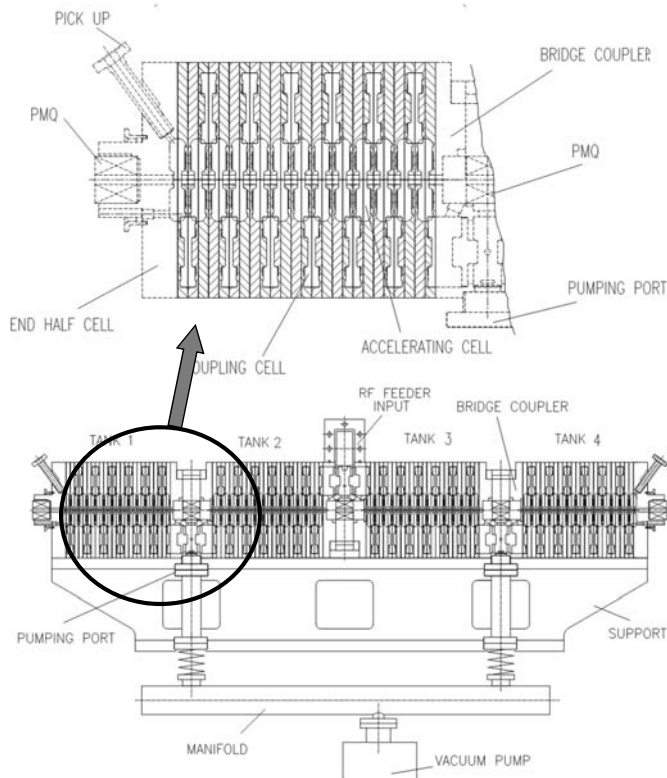
- Instead of distributing the power from the amplifier, one might as well couple the cavities, such that the power automatically distributes, or have a cavity with many gaps (e.g. drift tube linac).
- Coupled cavity accelerating structure (side coupled)



- The phase relation between gaps is important!



# Example of Side Coupled Structure



## LIBO (= Linac Booster)

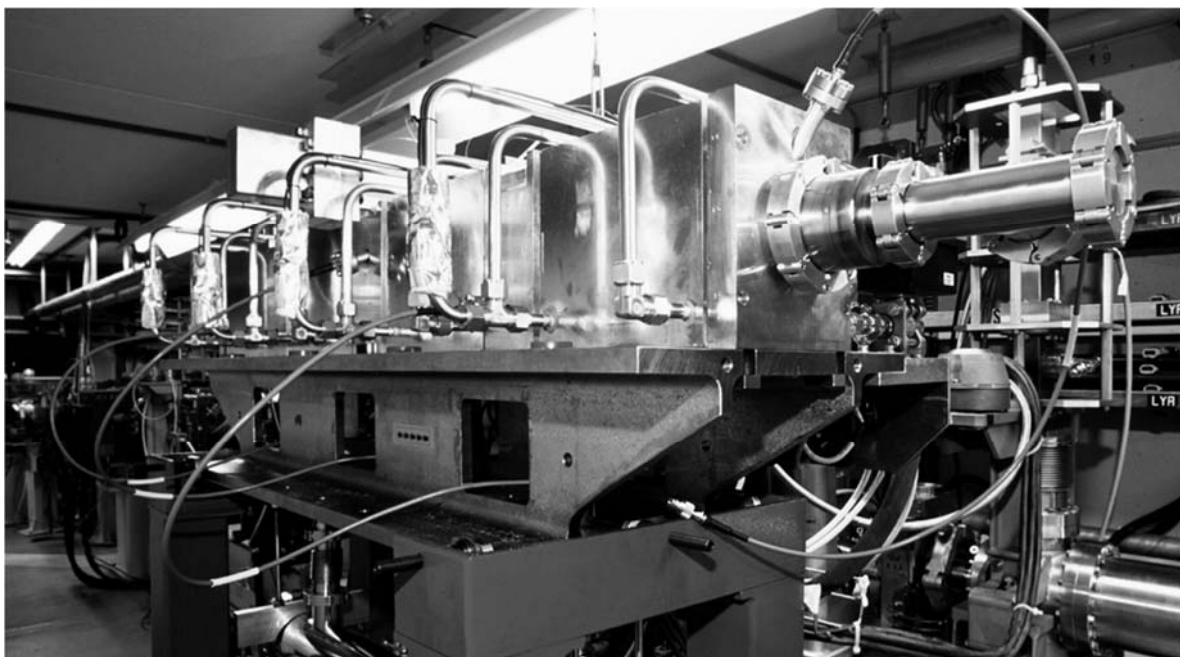
A 3 GHz Side Coupled Structure to accelerate protons out of cyclotrons from 62 MeV to 200 MeV

Medical application: treatment of tumours (proton therapy)

Prototype of Module 1 built at CERN (2000)

Collaboration CERN/INFN/TERA Foundation

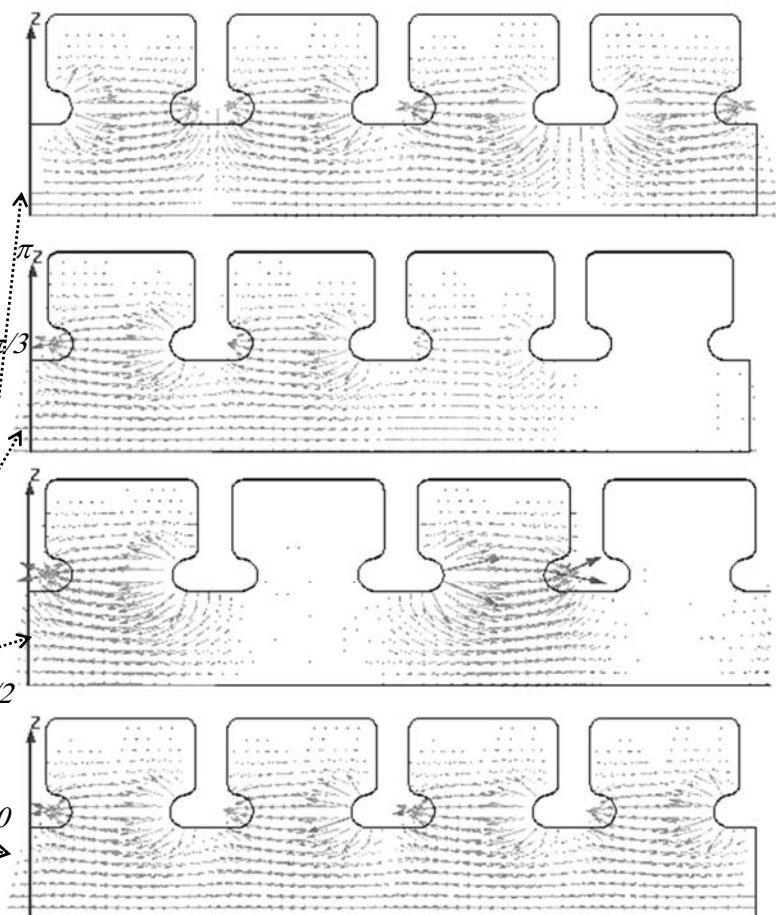
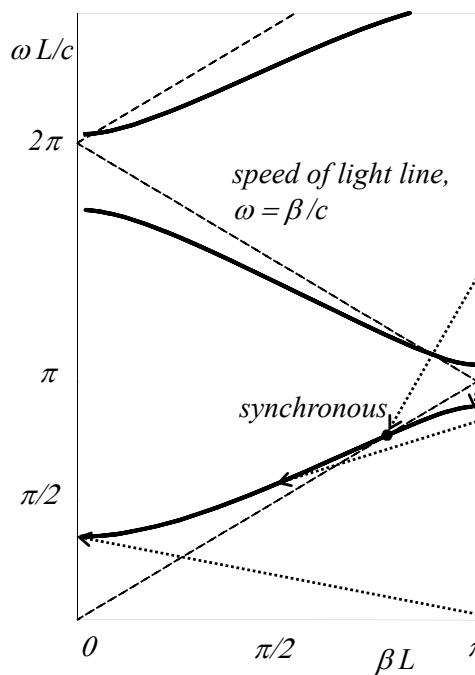
## LIBO prototype



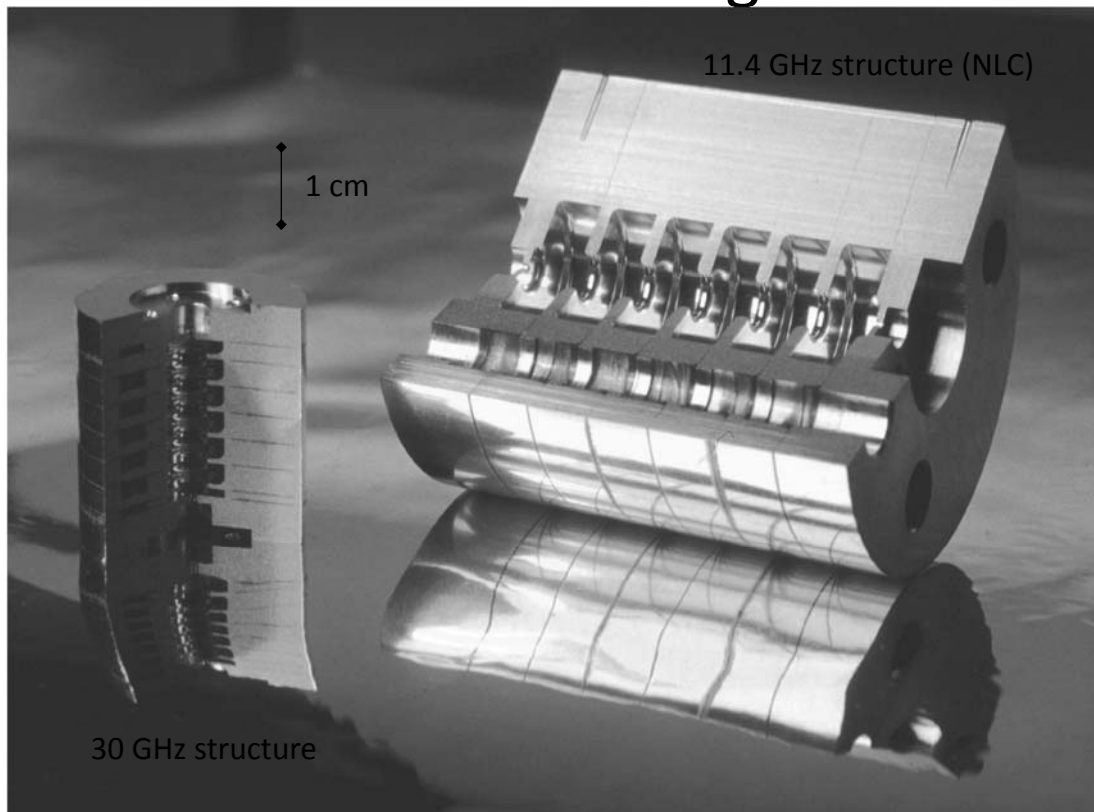
This Picture made it to the title page of CERN Courier vol. 41 No. 1 (Jan./Feb. 2001)

# TRAVELLING WAVE STRUCTURES

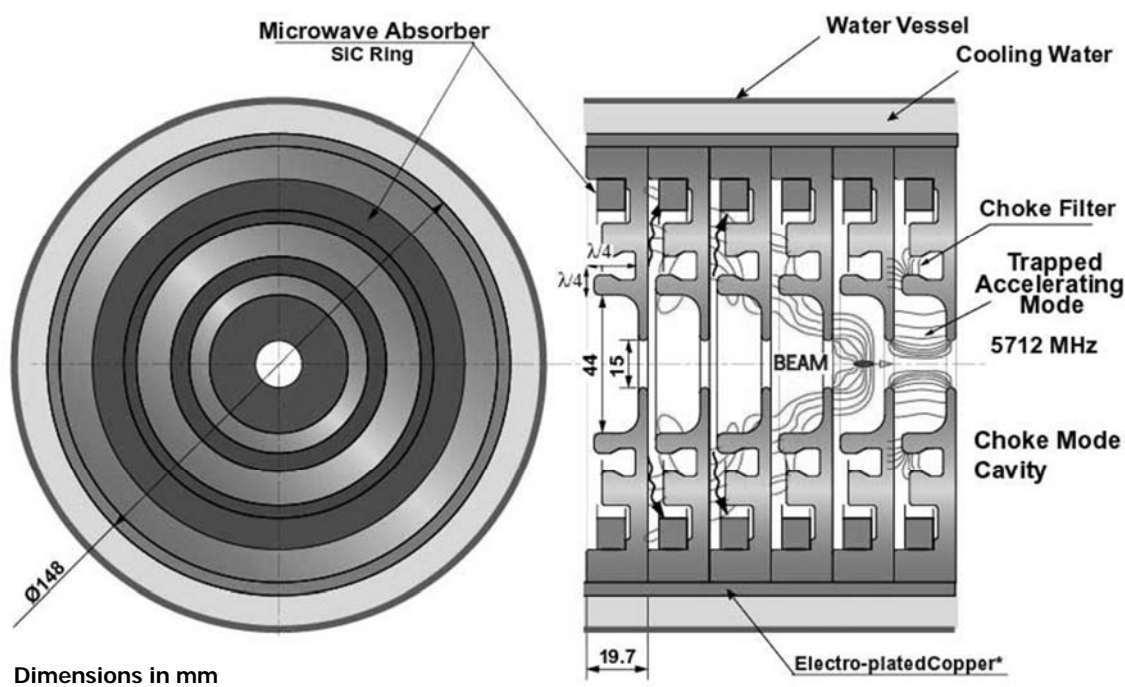
Brillouin diagram  
Travelling wave  
structure



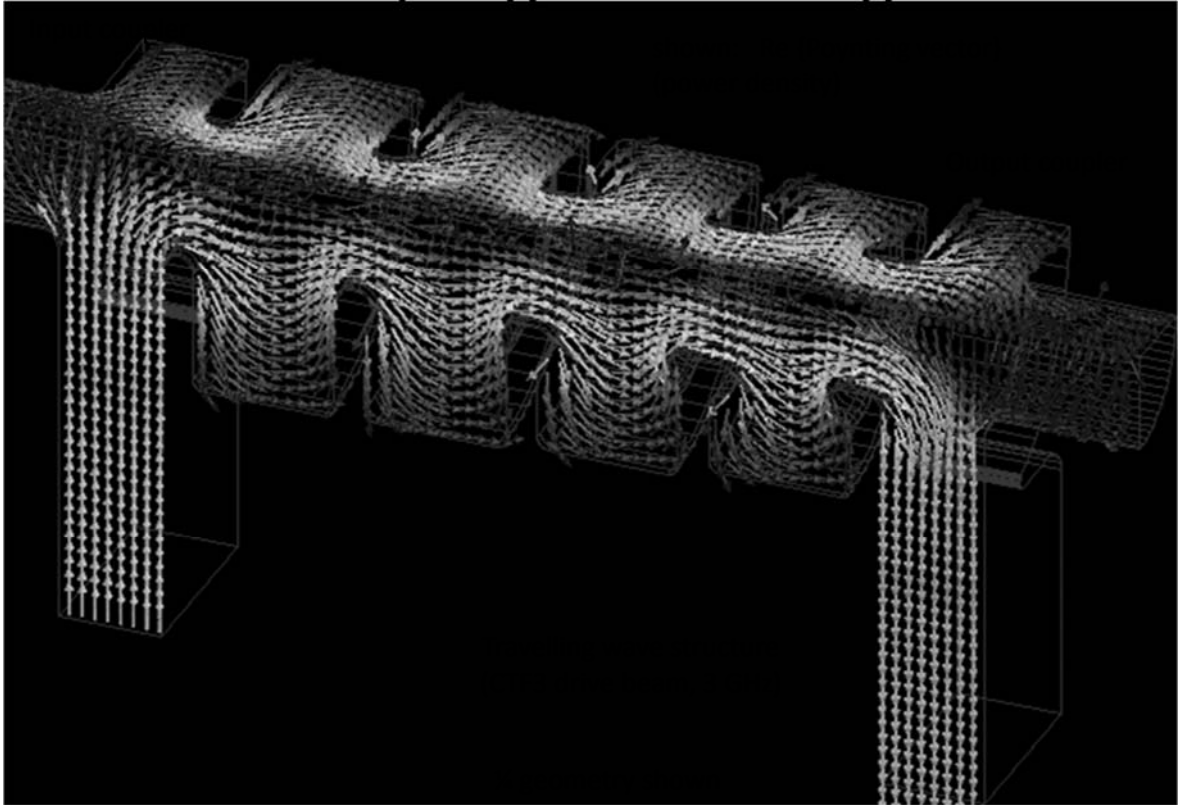
# Iris loaded waveguide



## Disc loaded structure with strong HOM damping “choke mode cavity”



# Power coupling with waveguides

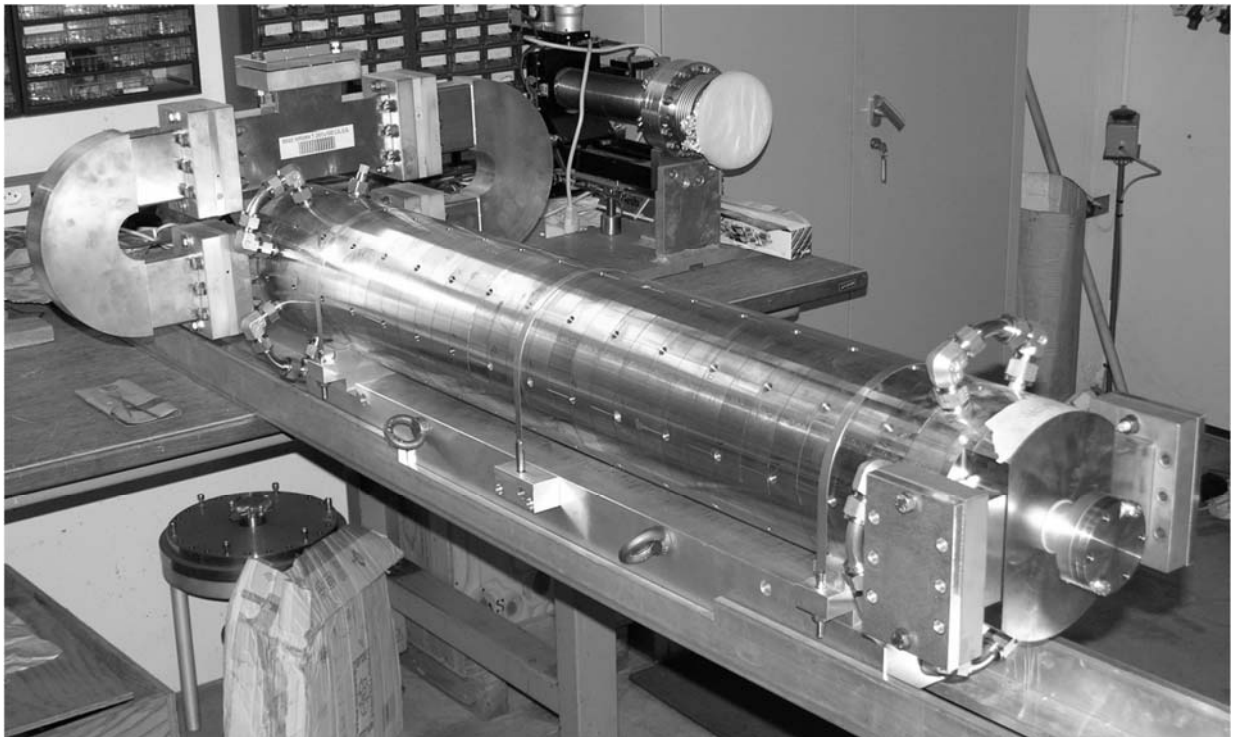


20-Sep-2011

CAS Chios 2011 — RF Cavity Design

71

# 3 GHz Accelerating structure (CTF3)

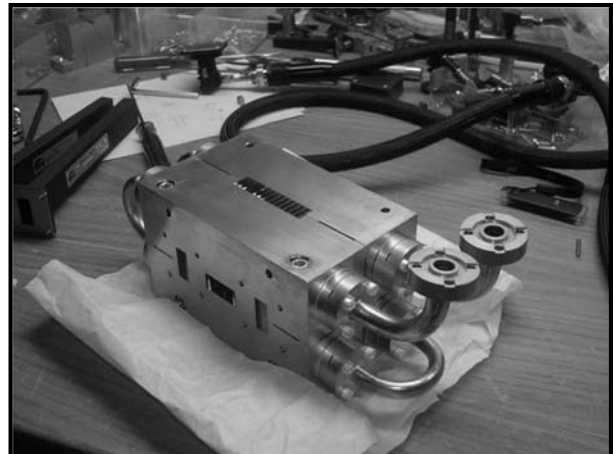
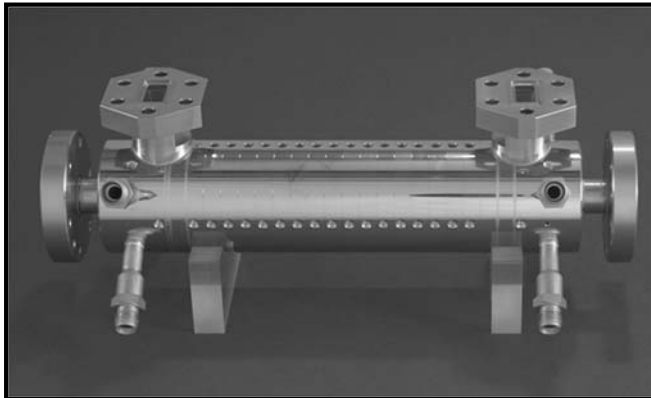
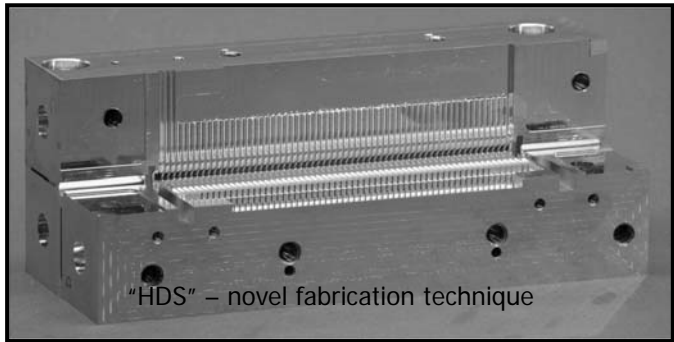
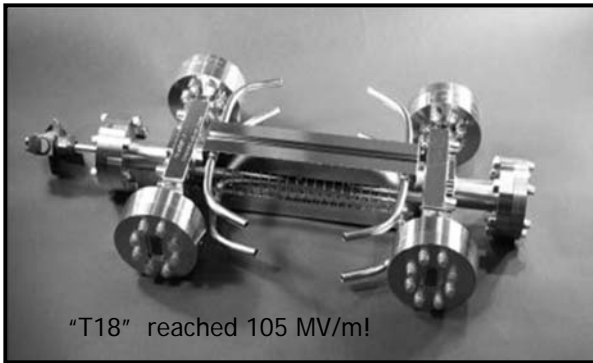


20-Sep-2011

CAS Chios 2011 — RF Cavity Design

72

## Examples (CLIC structures @ 11.4, 12 and 30 GHz)

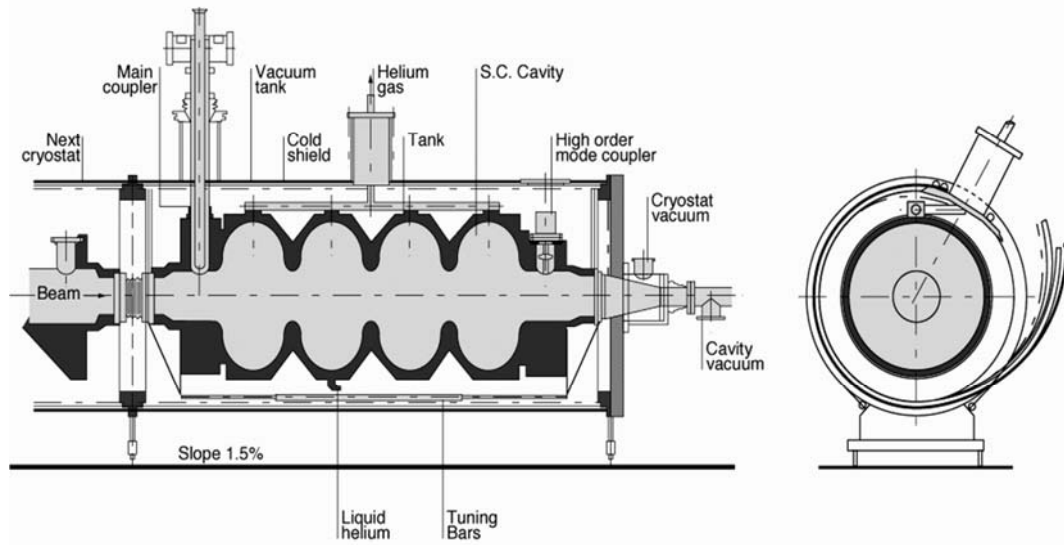


73

# SUPERCONDUCTING ACCELERATING STRUCTURES

# LEP Superconducting cavities

## SUPERCONDUCTING CAVITY WITH ITS CRYOSTAT

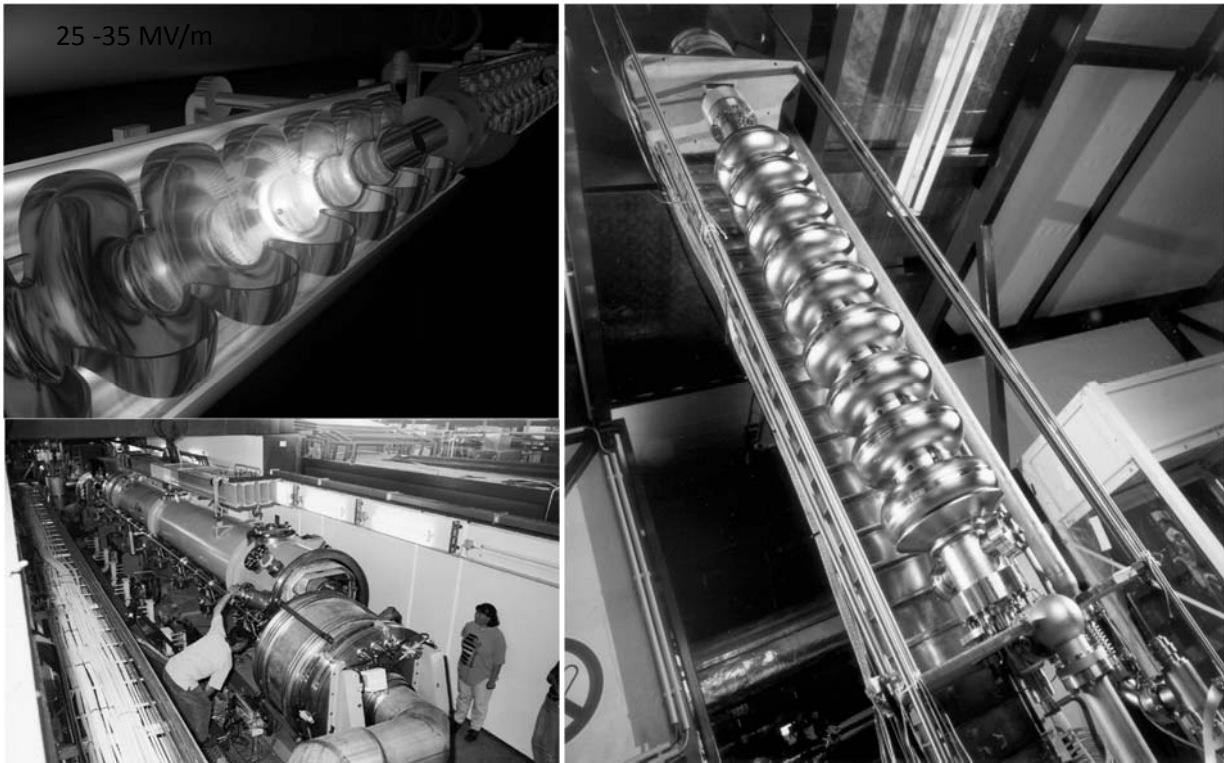


10.2 MV/ per cavity

# LHC SC RF, 4 cavity module, 400 MHz



# ILC high gradient SC structures at 1.3 GHz



20-Sep-2011

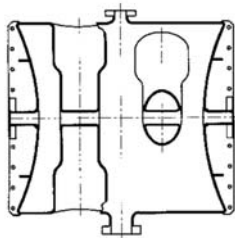
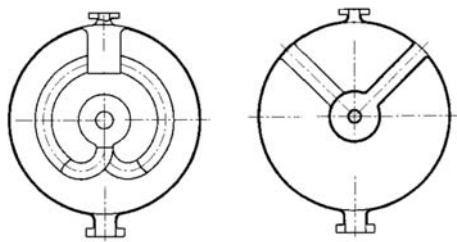
CAS Chios 2011 — RF Cavity Design

77

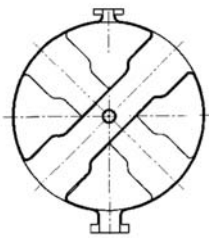
## Small $\beta$ superconducting cavities (example RIA, Argonne)

115 MHz split-ring cavity,

172.5 MHz  $\beta = 0.19$  "lollipop" cavity



345 MHz  $\beta = 0.4$  spoke cavity

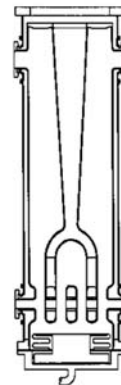


$\beta = 0.021$  fork cavity

57.5 MHz cavities:

$\beta = 0.06$  QWR  
(quarter wave resonator)

$\beta = 0.03$  fork cavity



12 IN.

pictures from Shepard et al.: "Superconducting accelerating structures for a multi-beam driver linac for RIA", Linac 2000, Monterey

20-Sep-2011

CAS Chios 2011 — RF Cavity Design

78