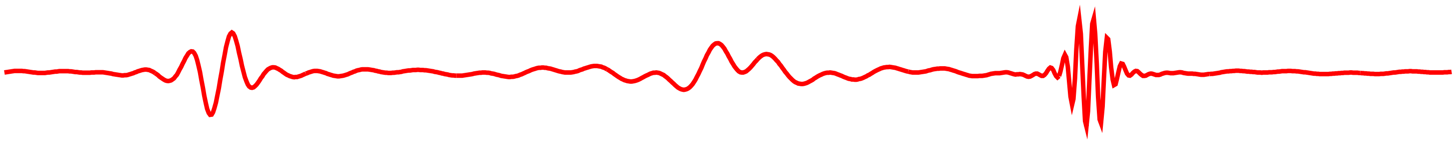
An introduction to linear imperfections



R. Tomás

CERN Accelerator School,

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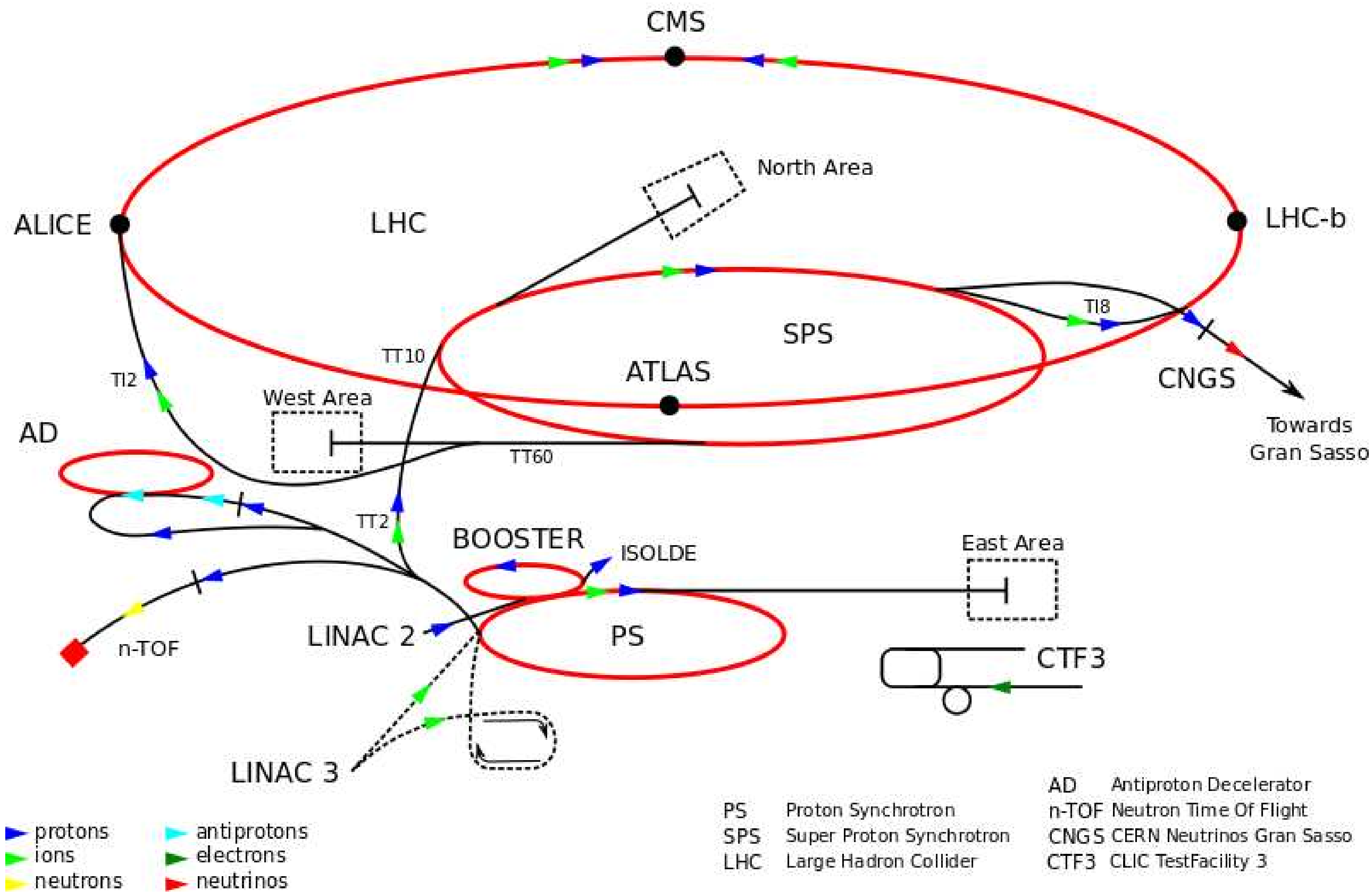
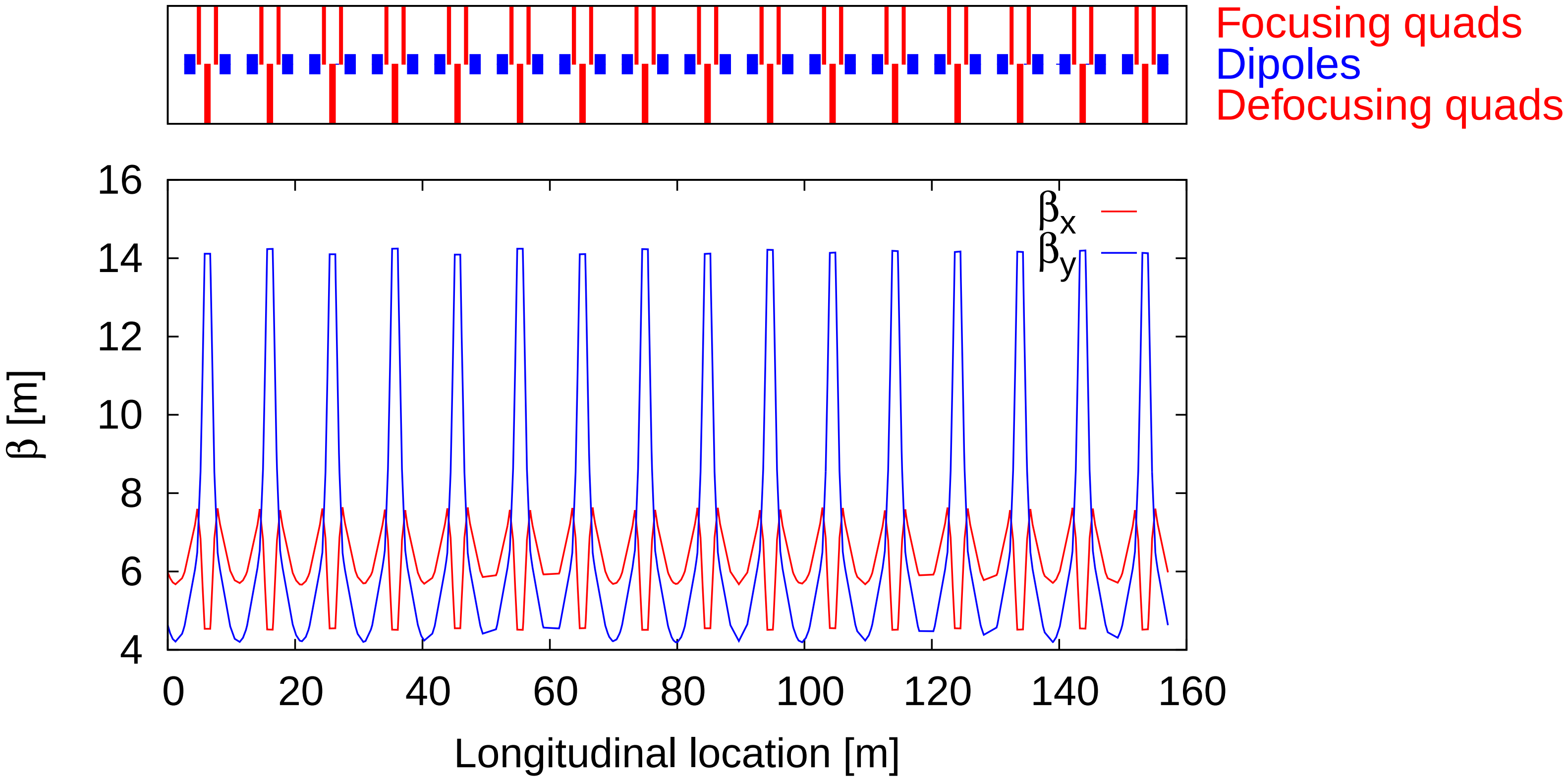
- ⋆ Some CERN lattices
- ⋆ Imperfections
- ⋆ Correction techniques
- ⋆ β-beating in the CERN synchrotrons
- ⋆ Dynamic imperfections

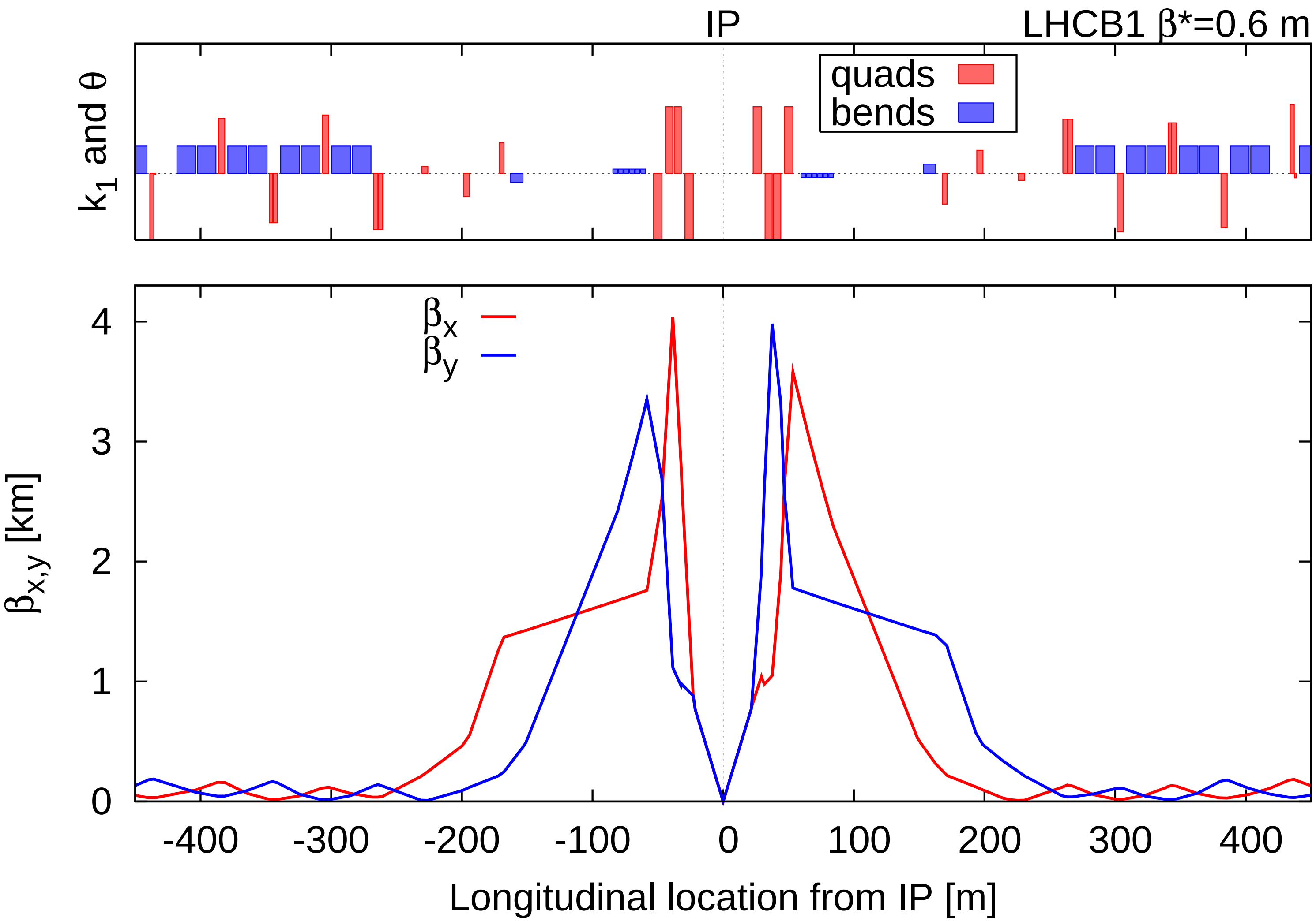
CERN accelerator complex



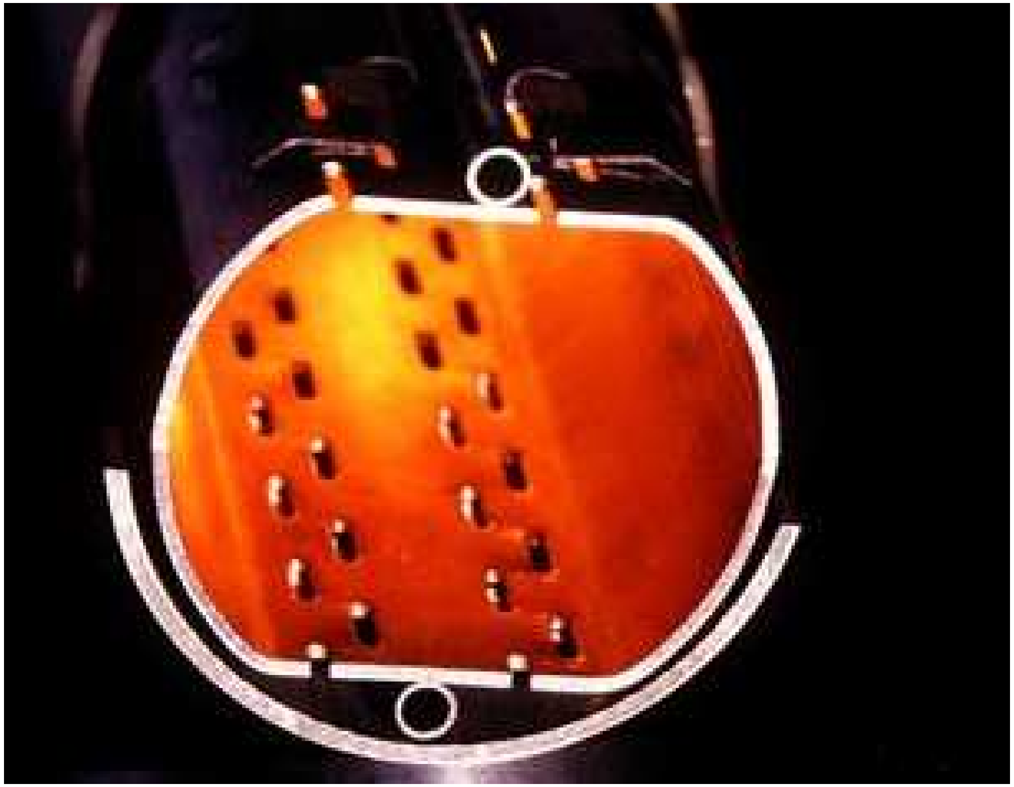
The Proton Synchrotron Booster (PSB)



The LHC Interaction Region (IR)

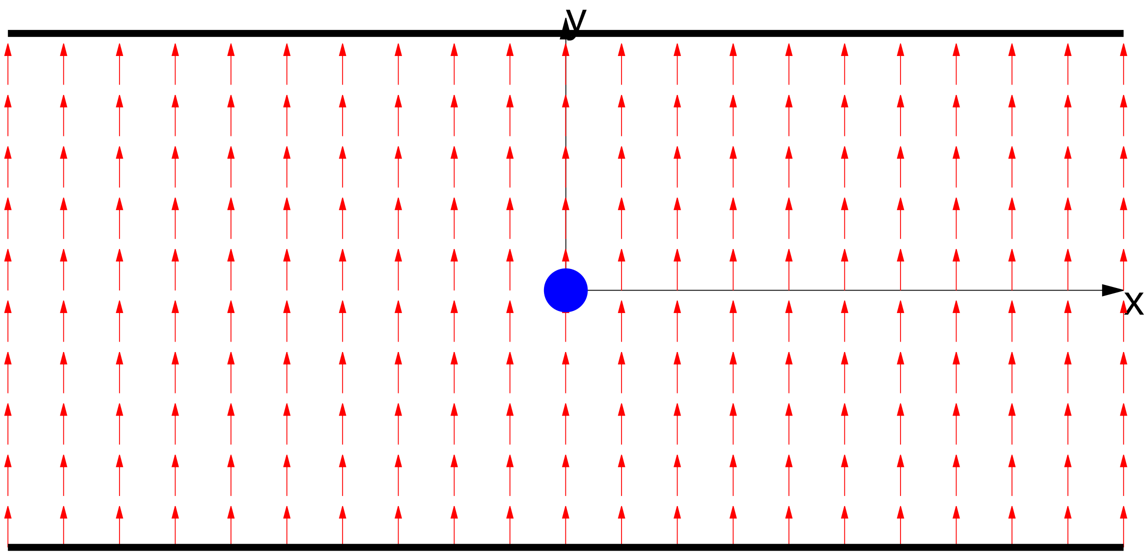


The first linear imperfection is...



...**gravity!**. The LHC vacuum chamber has a 22 mm radius. Everything takes 67 ms to fall. **Why do protons not fall?**

Dipole magnetic field

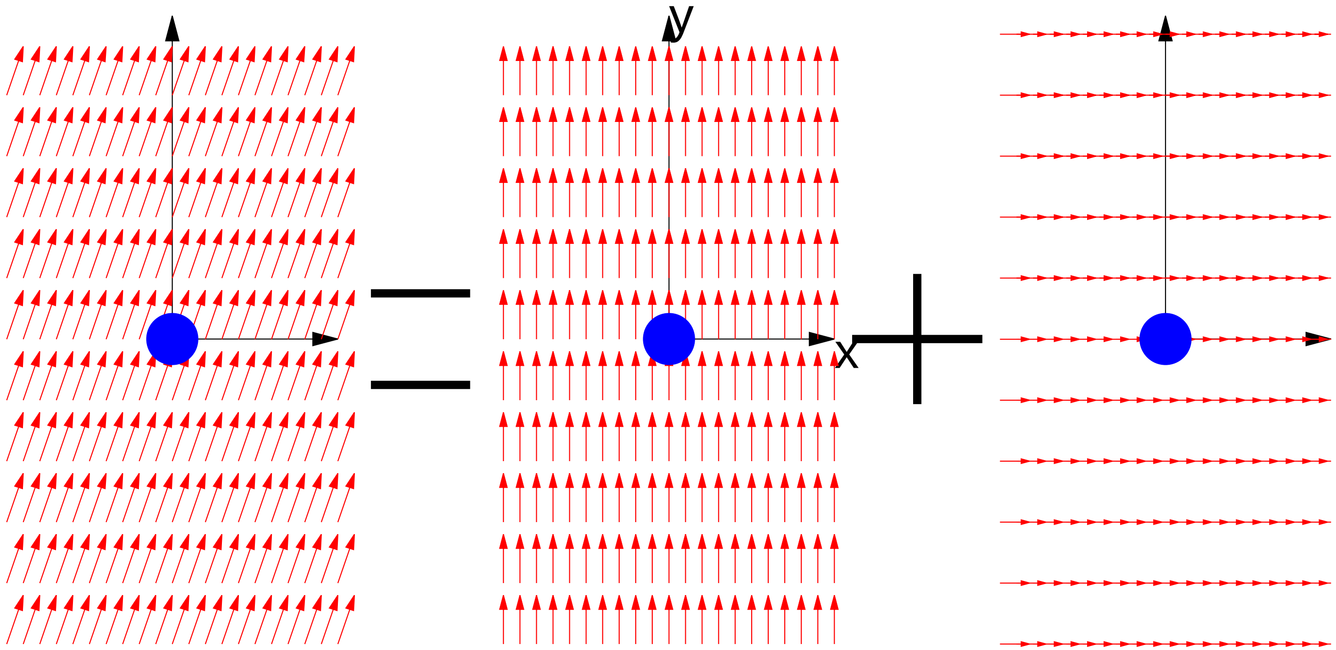


Lorentz force:

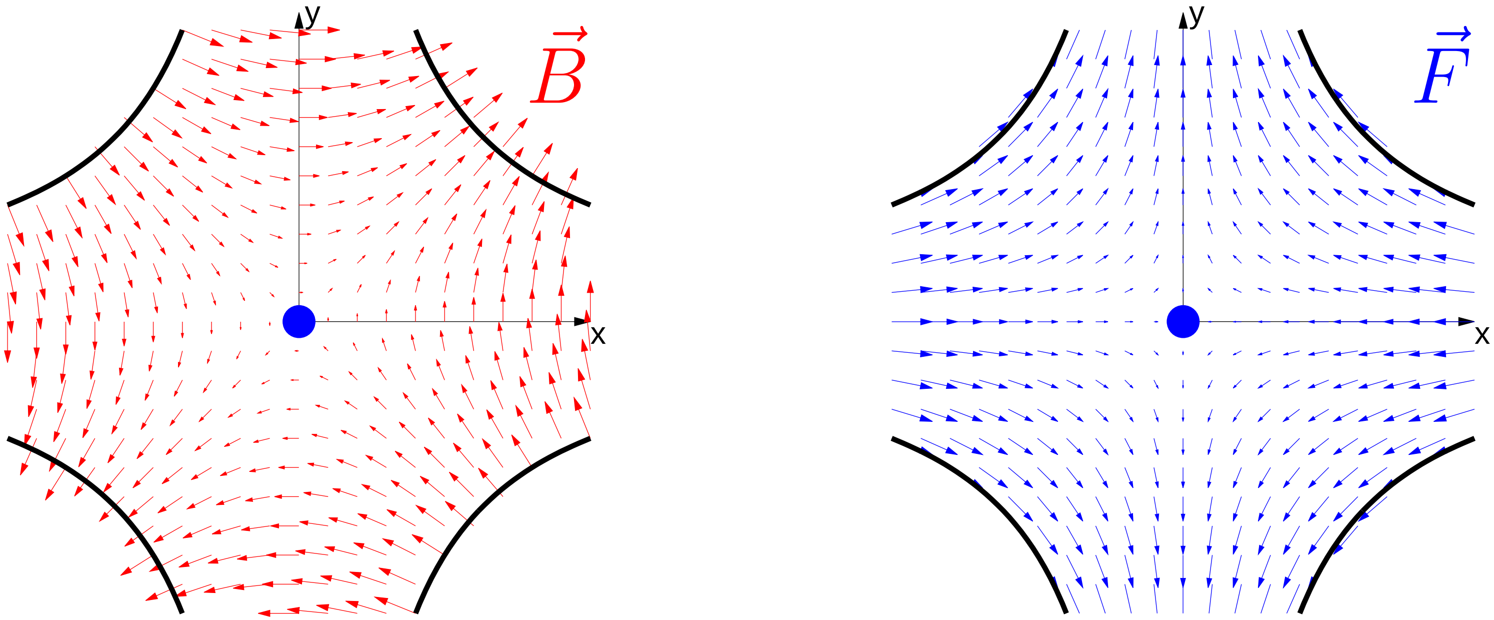
$$\vec{F} = q\vec{v} \times \vec{B}$$

Dipole errors

- An error in the strength of a main dipole causes a perturbation on the horizontal closed orbit.
- A tilt error in a main dipole causes a perturbation on the vertical closed orbit.

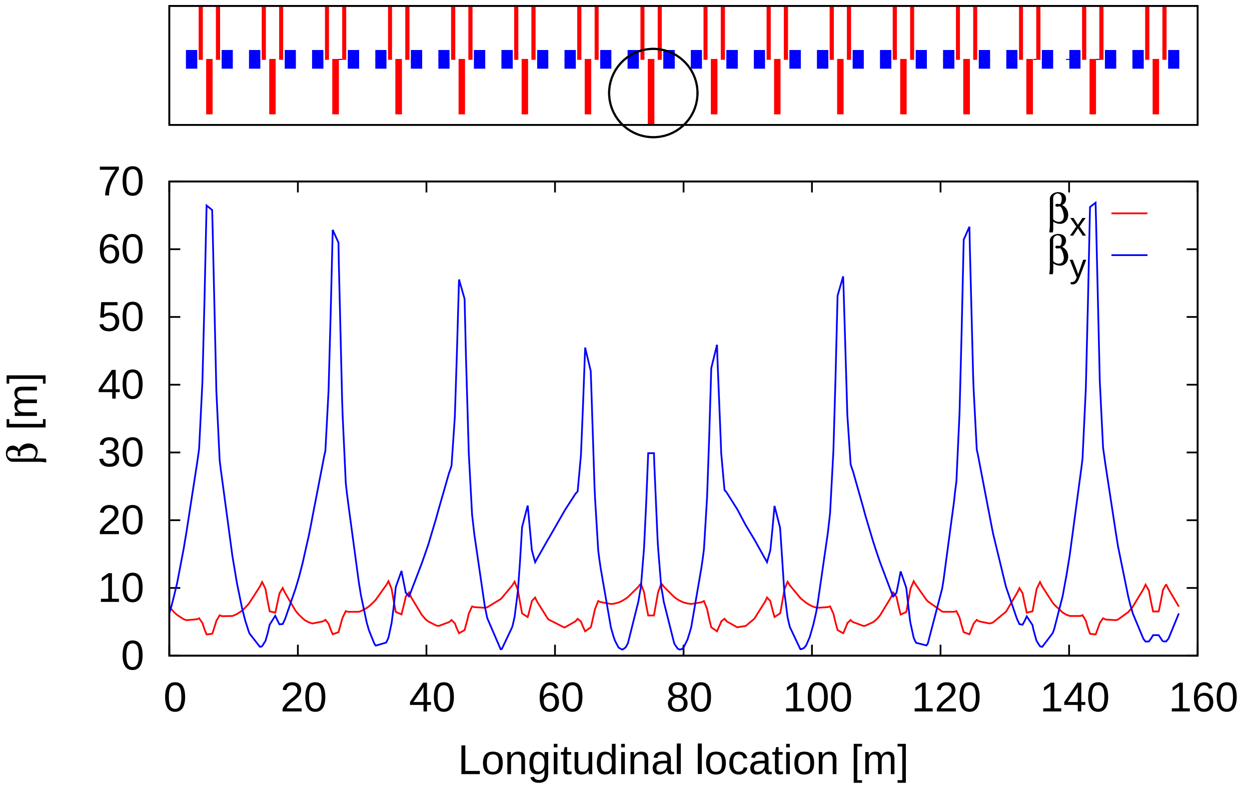


Quadrupole field and force on the beam



Note that $F_x = -kx$ and $F_y = ky$ making horizontal dynamics totally decoupled from vertical.

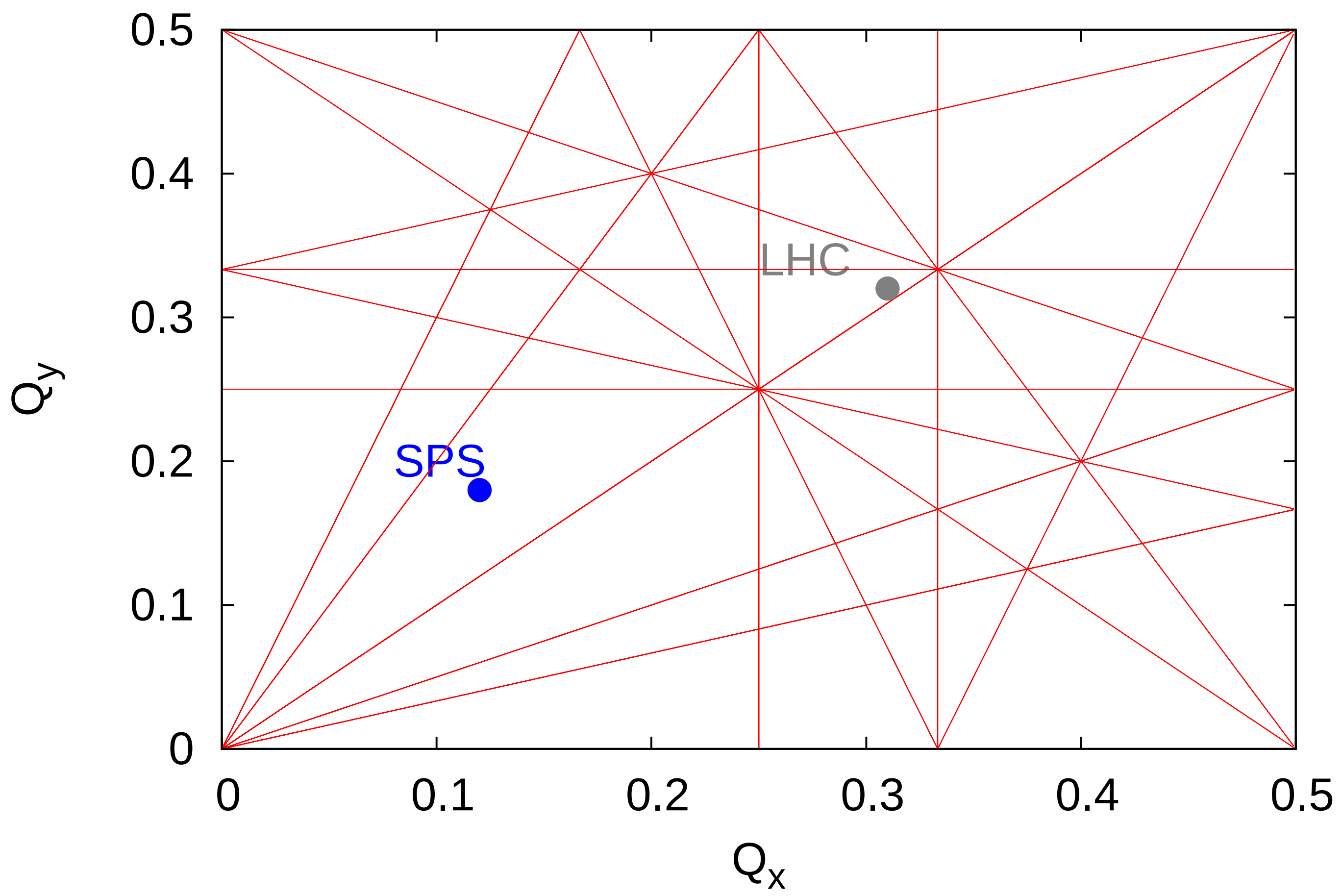
Quadrupole strength error



The β functions change, β-beating: $\frac{\Delta\beta}{\beta} = \frac{\beta_{pert} - \beta_0}{\beta_0}$.

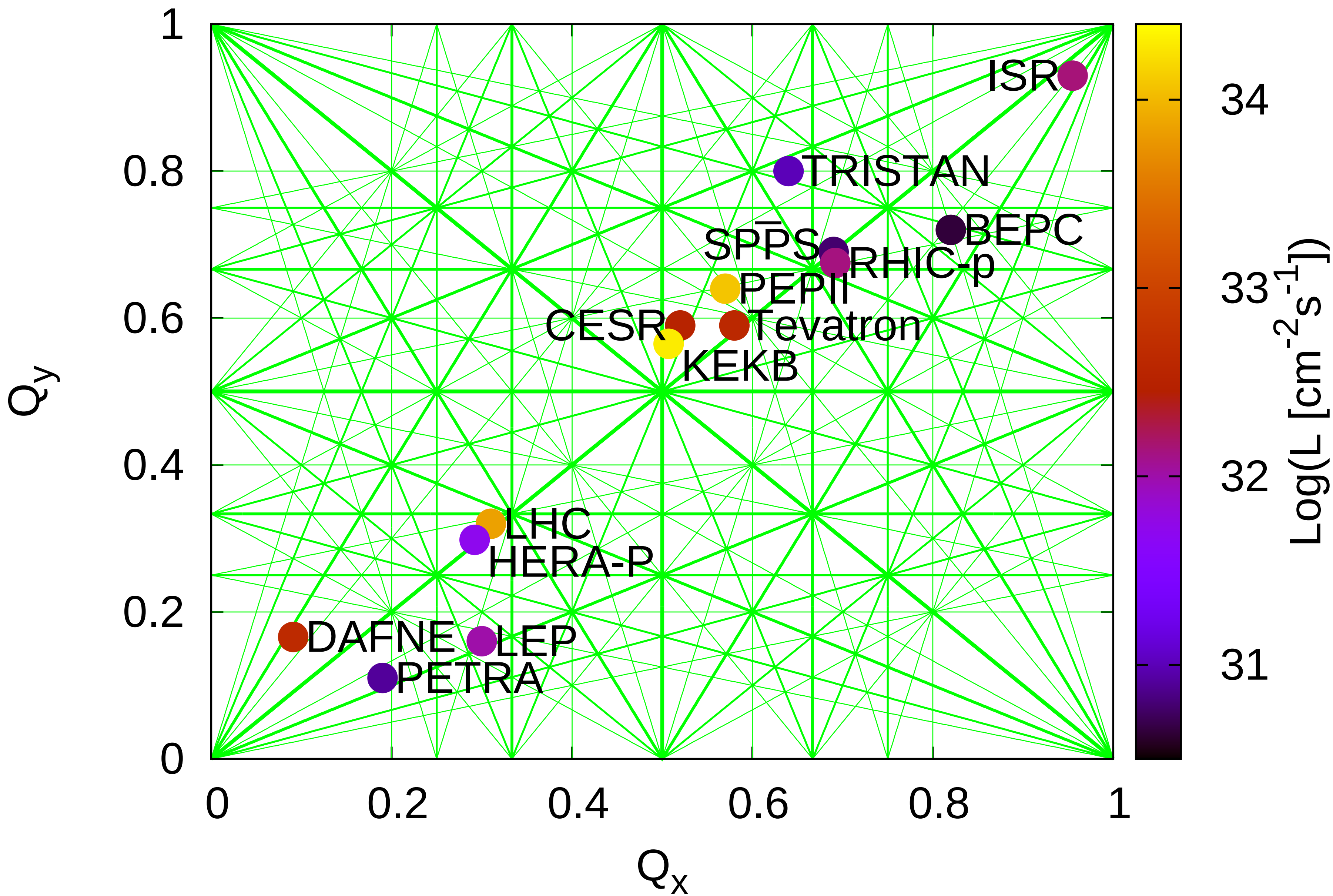
$$\Delta Q_x \approx \frac{1}{4\pi}\beta_x \Delta k, \quad \Delta Q_y \approx -\frac{1}{4\pi}\beta_y \Delta k$$

Quadrupole strength error - Tune change



Quadrupole errors can push tunes into resonances (dangerous) $nQ_x + mQ_y = N$

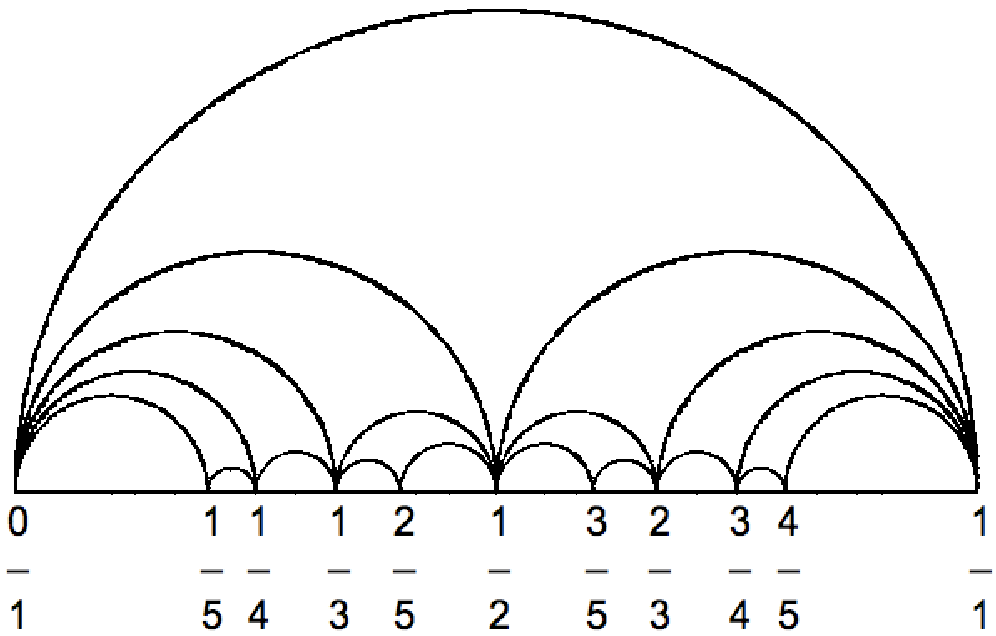
Colliders in the resonance world



Interlude: Farey sequences (1802)

The Farey sequence of order n is the sequence of completely reduced fractions between 0 and 1 which, when in lowest terms, have denominators less than or equal to $N \rightarrow$ **Resonances of order N or lower** (in one plane)

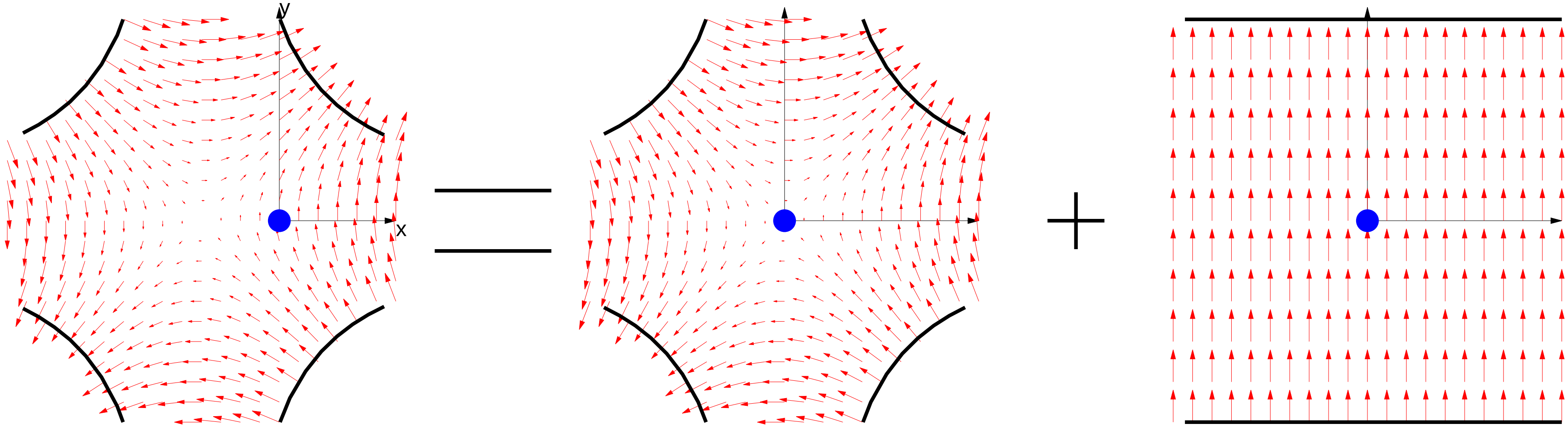
Farey diagram of order 5 (F_5)



Some properties of Farey sequences

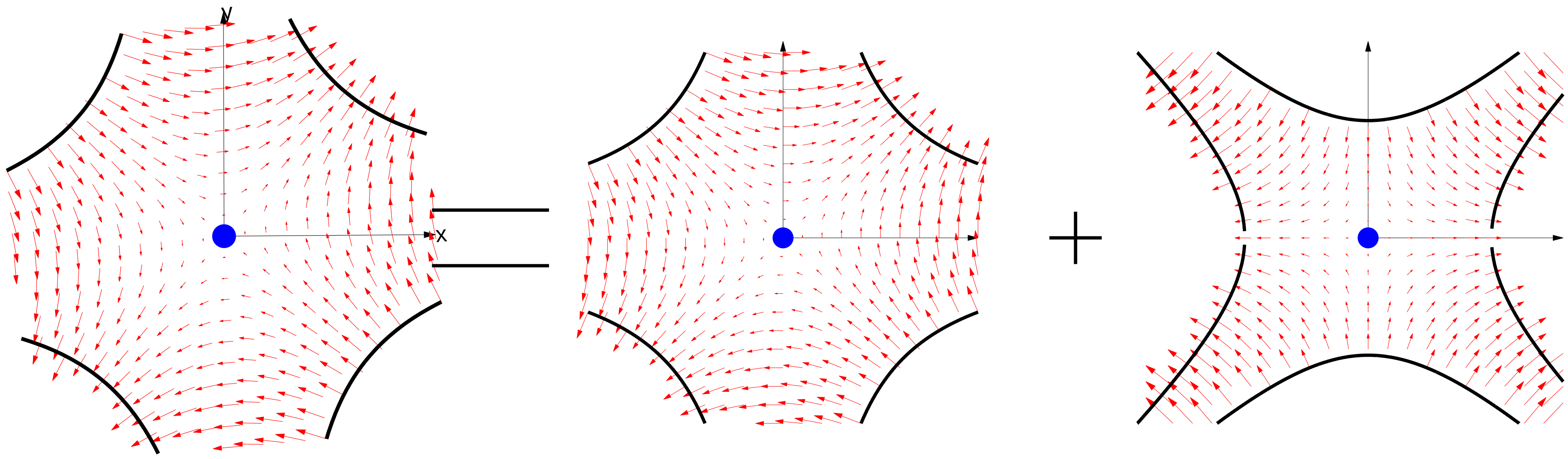
- ⋆ The distance between neighbors in a Farey sequence (aka two consecutive resonances) a/b and c/d is equal to $1/(bd)$
- ⋆ The next leading resonances in between two consecutive resonances a/b and c/d is $(a+c)/(b+d)$.
- ⋆ The number of 1D resonances of order N or lower tends asymptotically to $3N^2/\pi^2$

Offset quadrupole - Feed-down



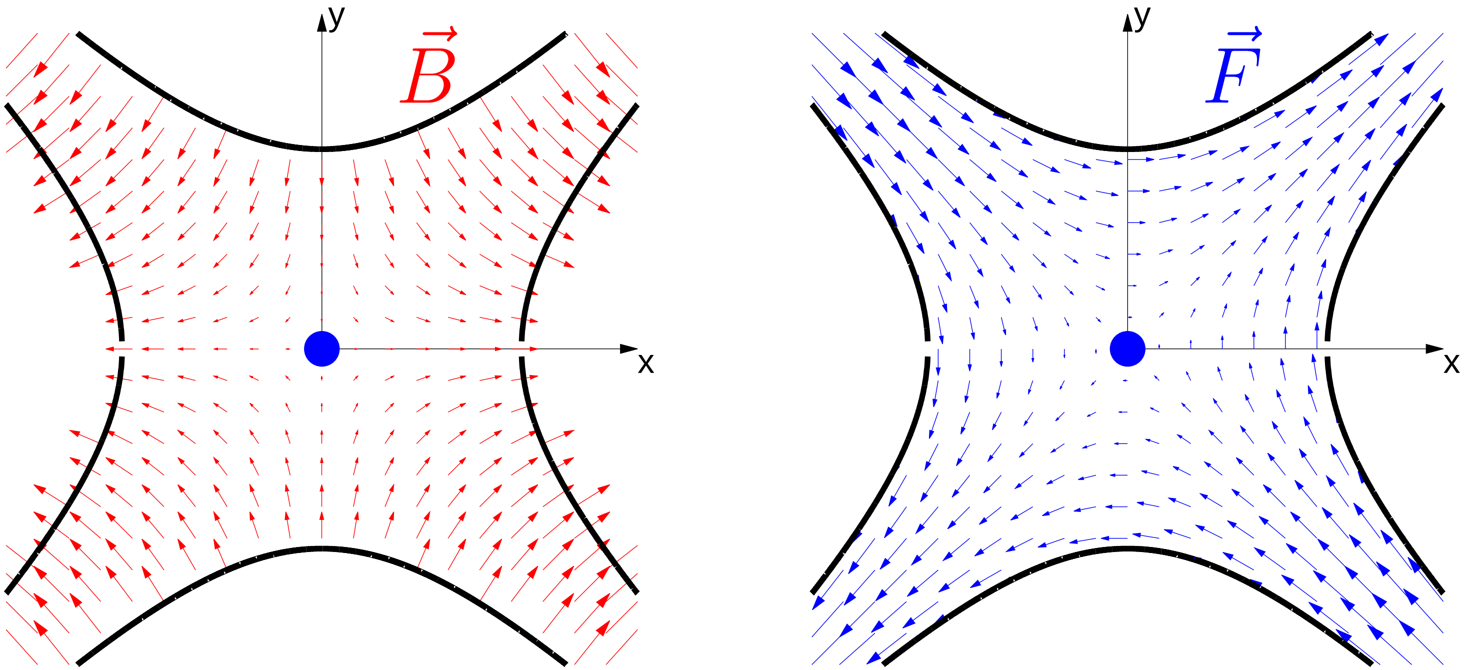
An offset quadrupole is seen as a centered quadrupole plus a dipole. This is called feed-down.

Tilted quadrupole



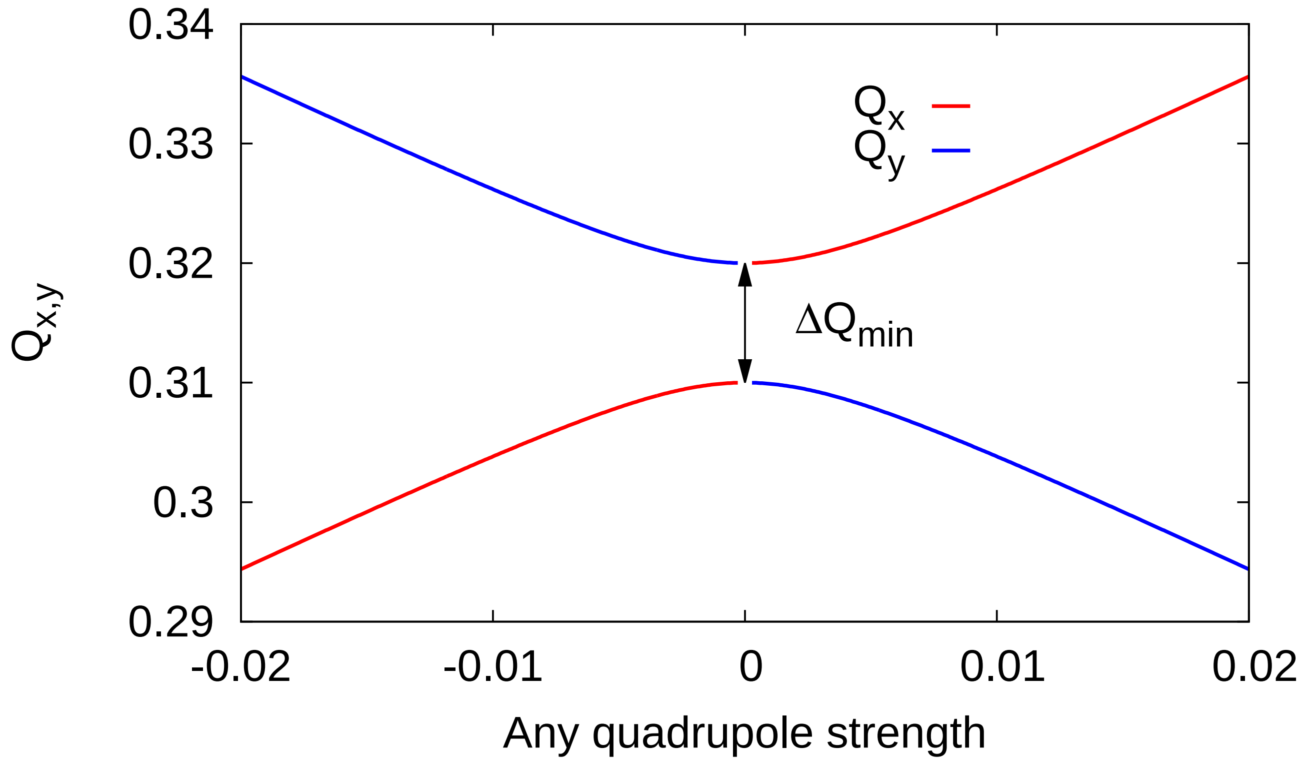
A tilted quadrupole is seen as a normal quadrupole plus another quadrupole tilted by 45° (this is called a skew quadrupole).

Skew quadrupole $\rightarrow$ x-y Coupling



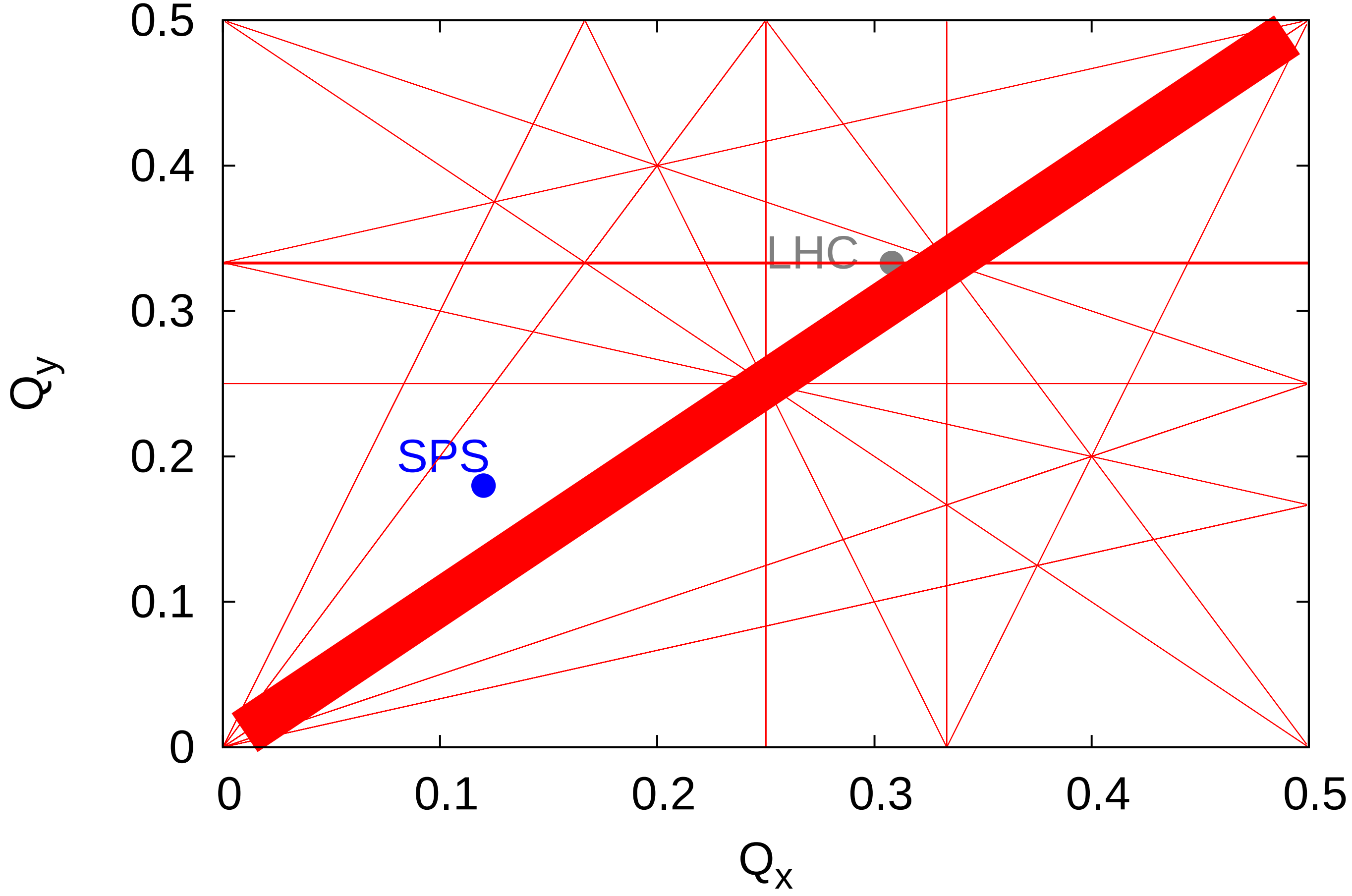
Note that $F_x = ky$ and $F_y = kx$ making horizontal and vertical dynamics to couple.

Skew quadrupole $\rightarrow$ x-y Coupling



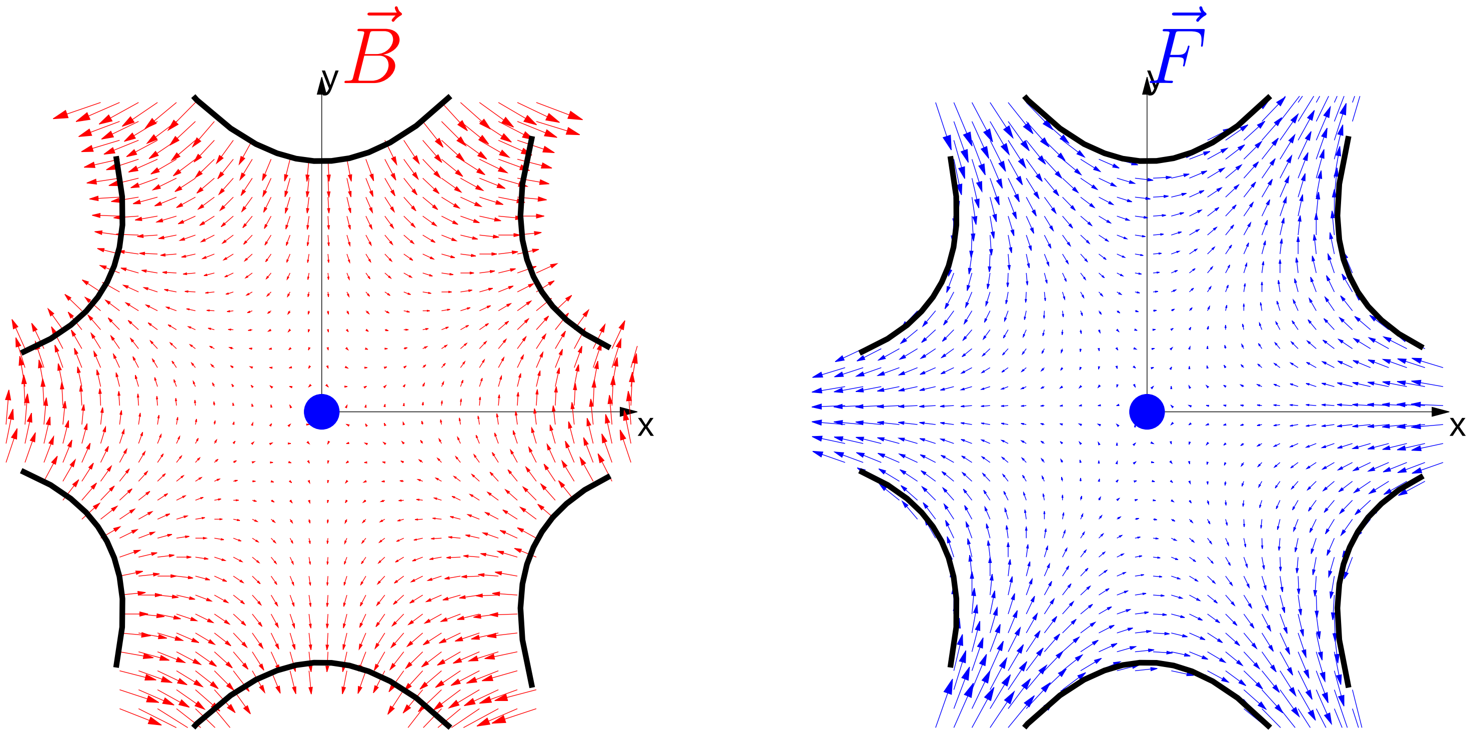
Coupling makes it impossible to approach tunes below $\Delta Q_{min} = |C^-|$, where C^- is a complex number characterizing the difference resonance $Q_x - Q_y = N$.

Coupling



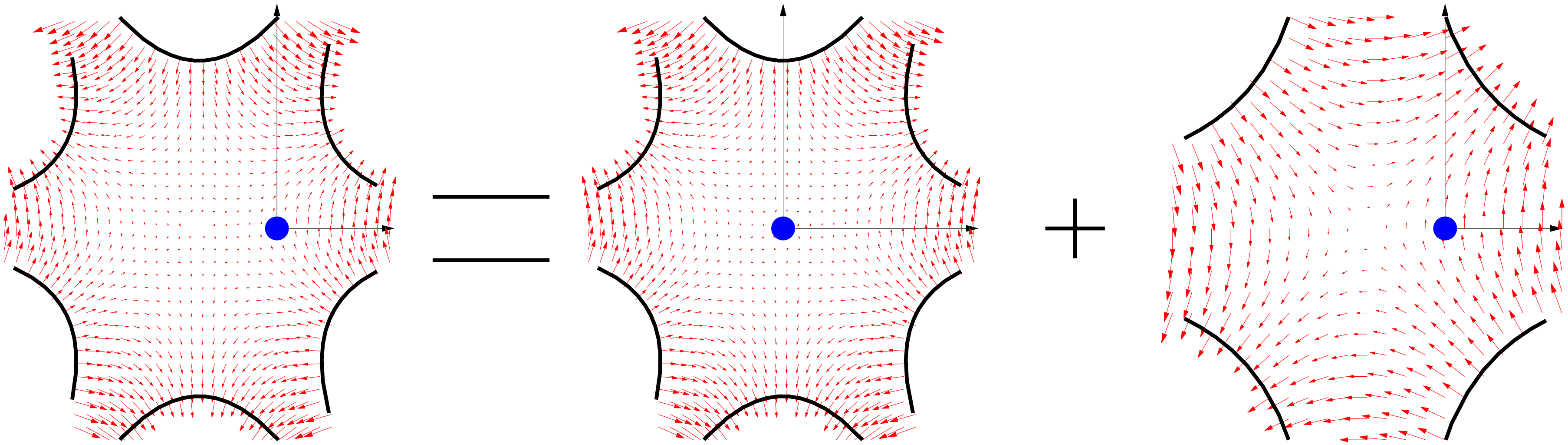
Coupling can push tunes into resonances.

Sextupole field and force



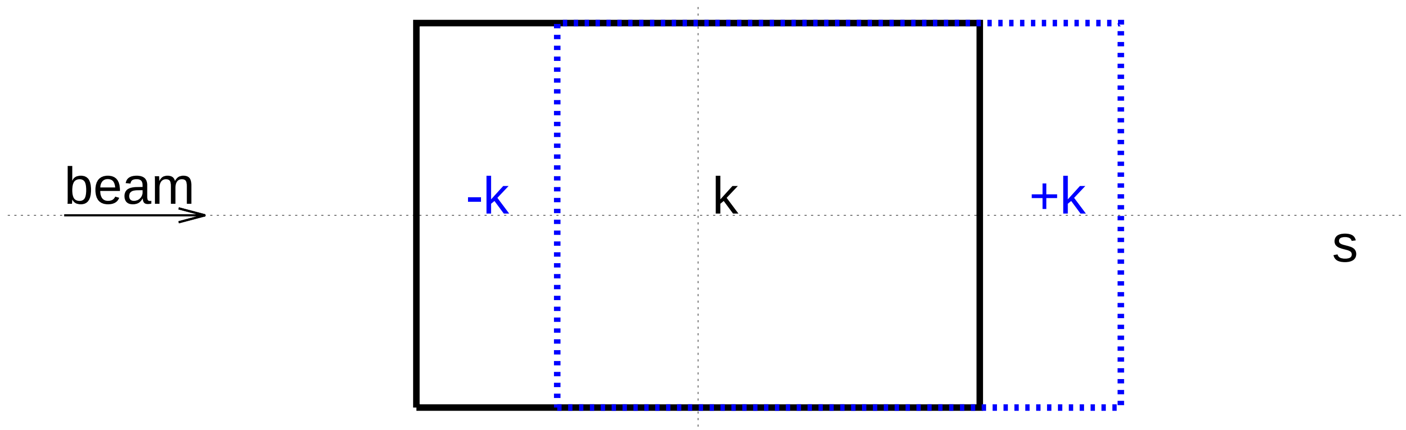
Ooops, We are entering the non-linear regime, however...

Offset sextupole



A sextupole horizontally (vertically) displaced is seen as a centered sextupole plus an off-set quadrupole (skew quadrupole). Offset sex-tupoles are also sources of quadrupole and skew quadrupole errors.

Longitudinal misalignments

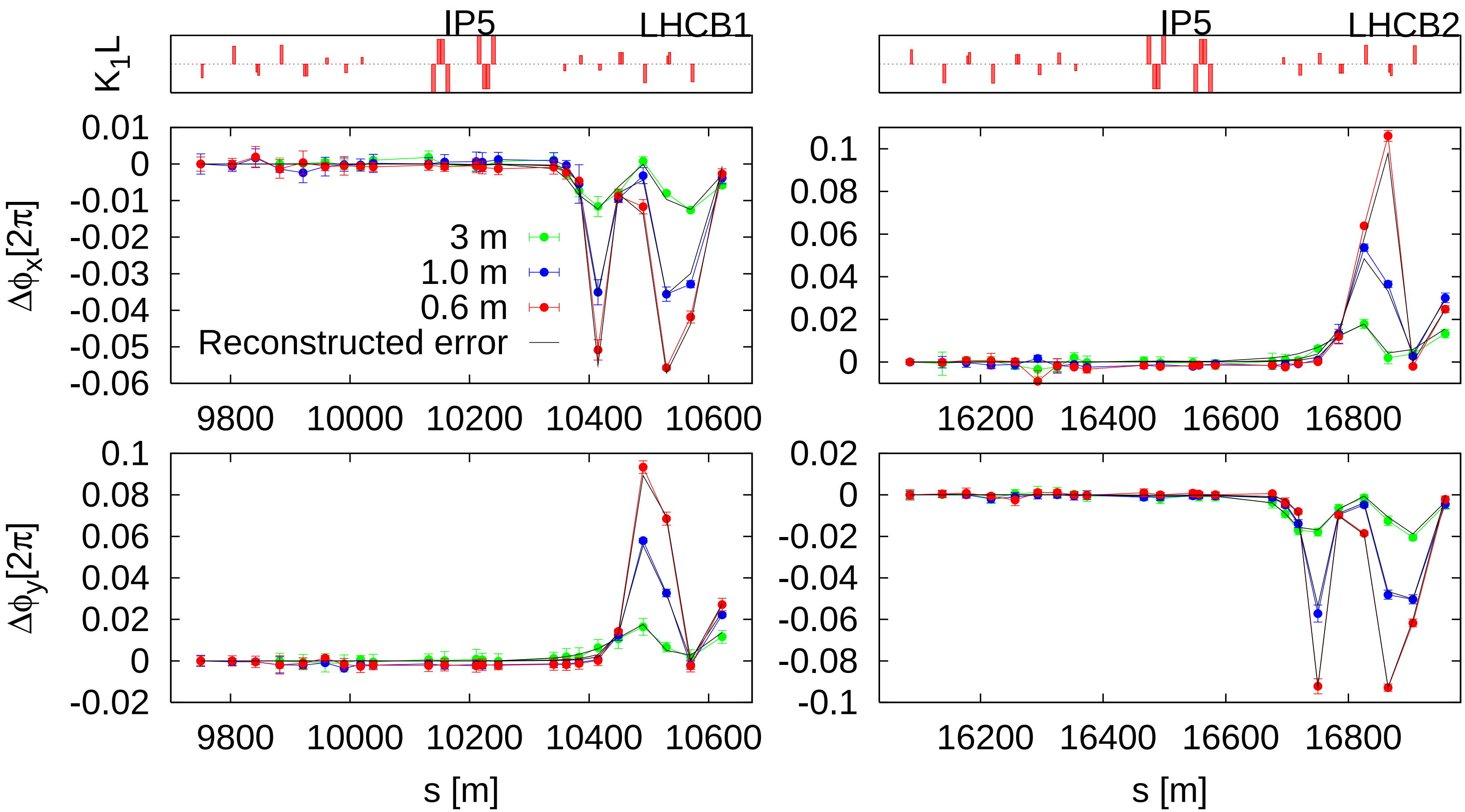


Longitudinal misalignments can be seen as perturbations at both ends of the magnet with opposite signs. Tolerances are generally larger for longitudinal misalignments.

Correction

- Local corrections
  - Ideal correction: Error source identification and repair.
  - Effective local error correction.
  - MICADO (ISR-MA/73-17): Best few correctors (no guarantee of locality).
- Global corrections
  - Pre-designed knobs for varying particular observables in the least invasive way (like tunes, coupling, β^*, etc.)
  - MICADO: Best N correctors
  - Response matrix approach

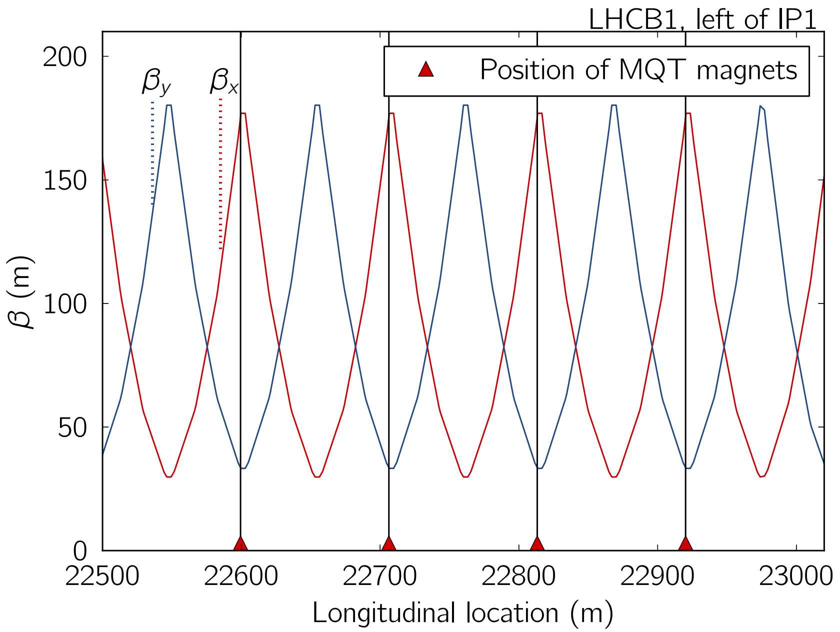
Local correction: segment-by-segment



Key point: Isolate a segment of the machine by imposing boundary conditions from measurements and find corrections.

Pre-designed knobs - Tunes

- In most machines it is OK to use all focusing quads to change Q_x and all defocusing quads for Q_y: PSB, PS, SPS
- In the LHC dedicated tune correctors (MQT) are properly placed to minimize impact on other quantities:

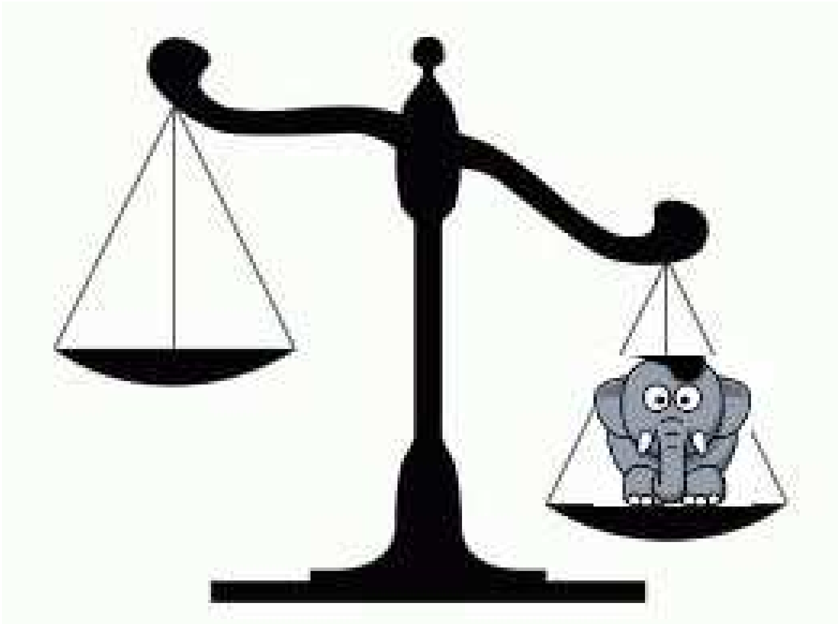


A.S. Langner

Pre-designed knobs - Coupling

- The full control of the difference resonance (C^-) needs two independent families of skew quadrupoles.
- PSB, PS and SPS can survive only with one family since $\mathrm{int}(Q_x) = \mathrm{int}(Q_y)$, making errors in phase with correctors.
- In LHC there are two families to vary the real and imaginary parts of C^- independently.

Best corrector concepts



Elephant = 80 kg. Available corrector weigths: 78 kg, 1 kg, 30 kg, 50 kg

Which is the **best** corrector?
Which is the best **second** corrector? (using the 1^{st})
Which are the **two best correctors**?

Best N-corrector challenge

- LHC has about 500 orbit correctors per plane and per beam.
- Imagine you want to find the best 20 correctors
- How many combinations of these 500 correctors taking 20 at a time exist?
- ...
- (MICADO finds a good approximations to this problem)

Response matrix approach

- ⋆ Available correctors: $\vec{c}$
- ⋆ Available observables: $\vec{a}$
- ⋆ Assume for small changes of correctors linear approximation is good: $R\Delta\vec{c} = \Delta\vec{a}$
- ⋆ Use, e.g., MADX to compute R
- ⋆ Invert or pseudo-invert R to compute an effective global correction based on measured $\Delta\vec{a}$:

$$\Delta\vec{c} = R^{-1}\Delta\vec{a}$$

- ⋆ This works for orbit, $\Delta\beta/\beta$, coupling, etc.

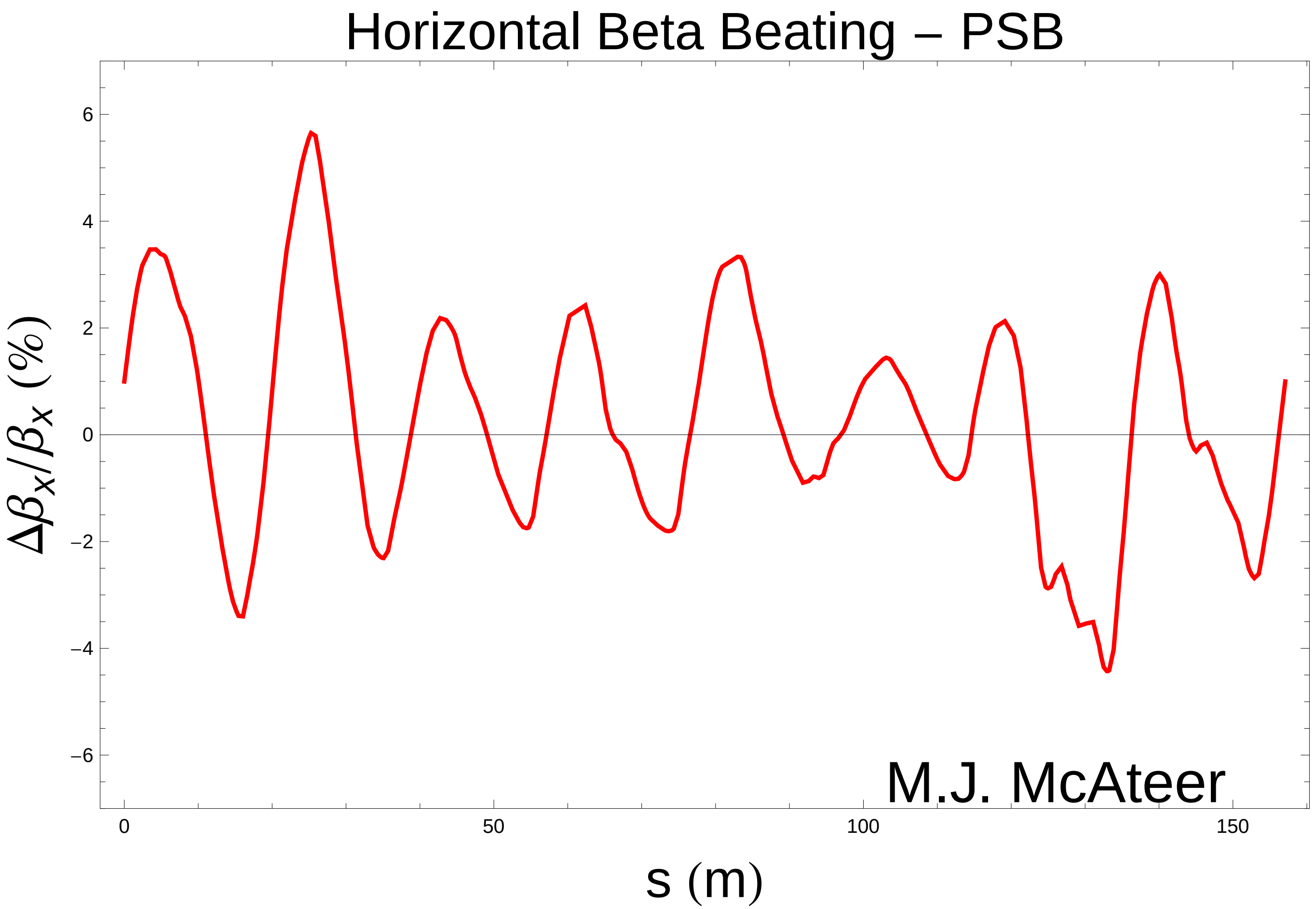
Pseudo-inverse via SVD

$$R = U \begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \\ 0 & 0 & 0 \end{pmatrix} V'$$

Imagine $\sigma_3 << \sigma_2$, then just neglect it:

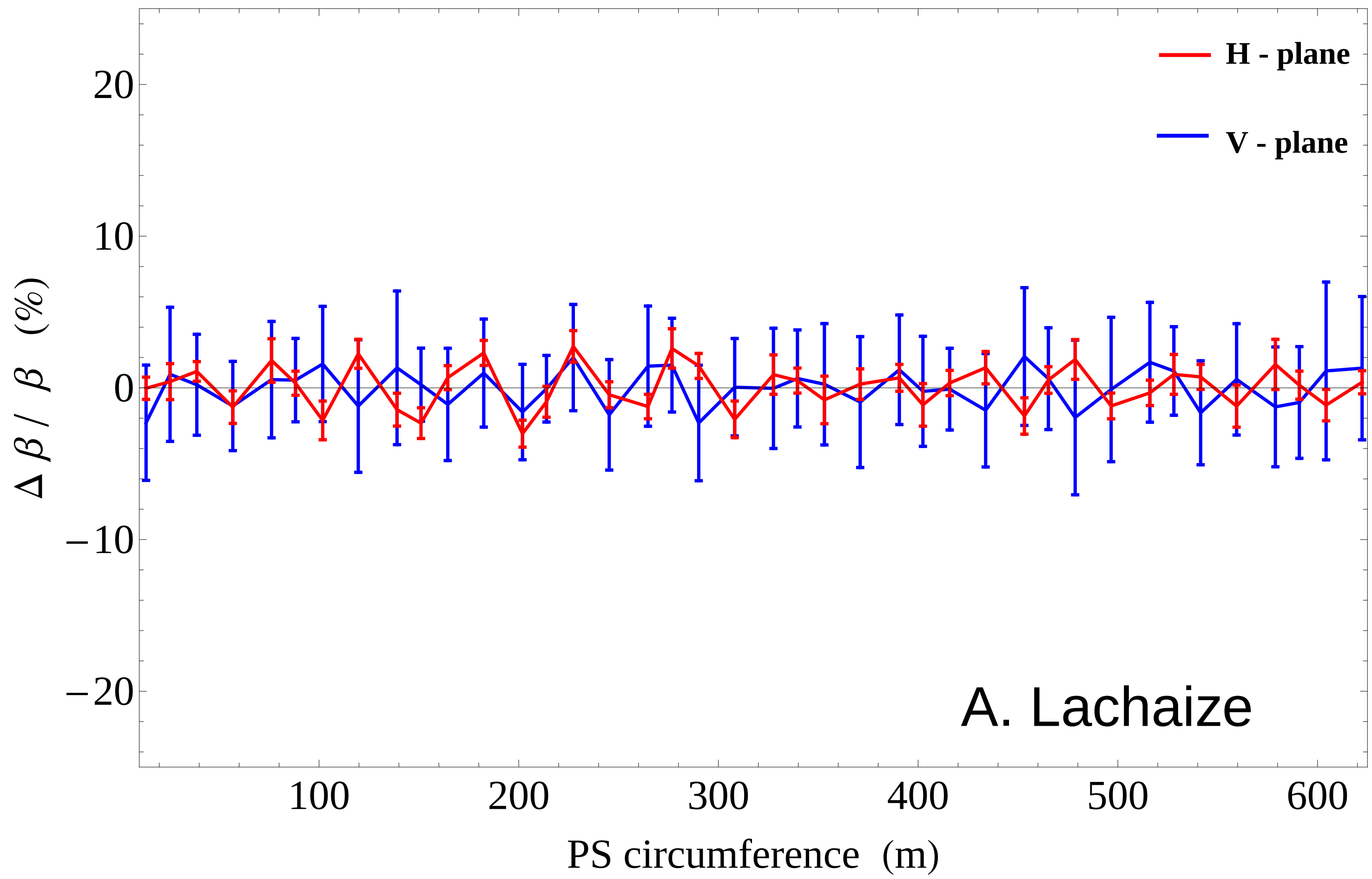
$$R^{-1} = V \begin{pmatrix} \frac{1}{\sigma_1} & 0 & 0 \\ 0 & \frac{1}{\sigma_2} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} U'$$

PSB β-beating



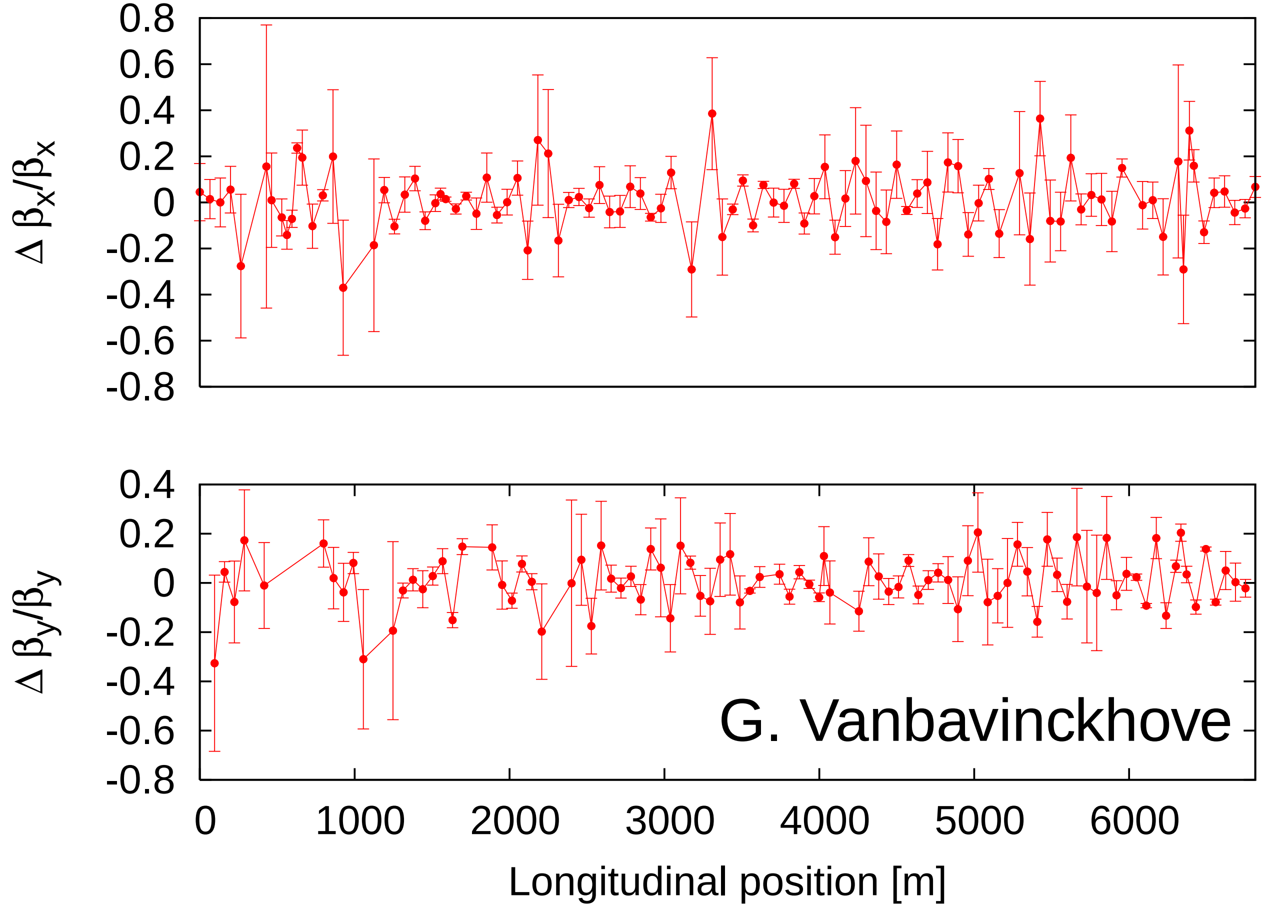
Peak β-beating of $\approx$5%

PS β-beating



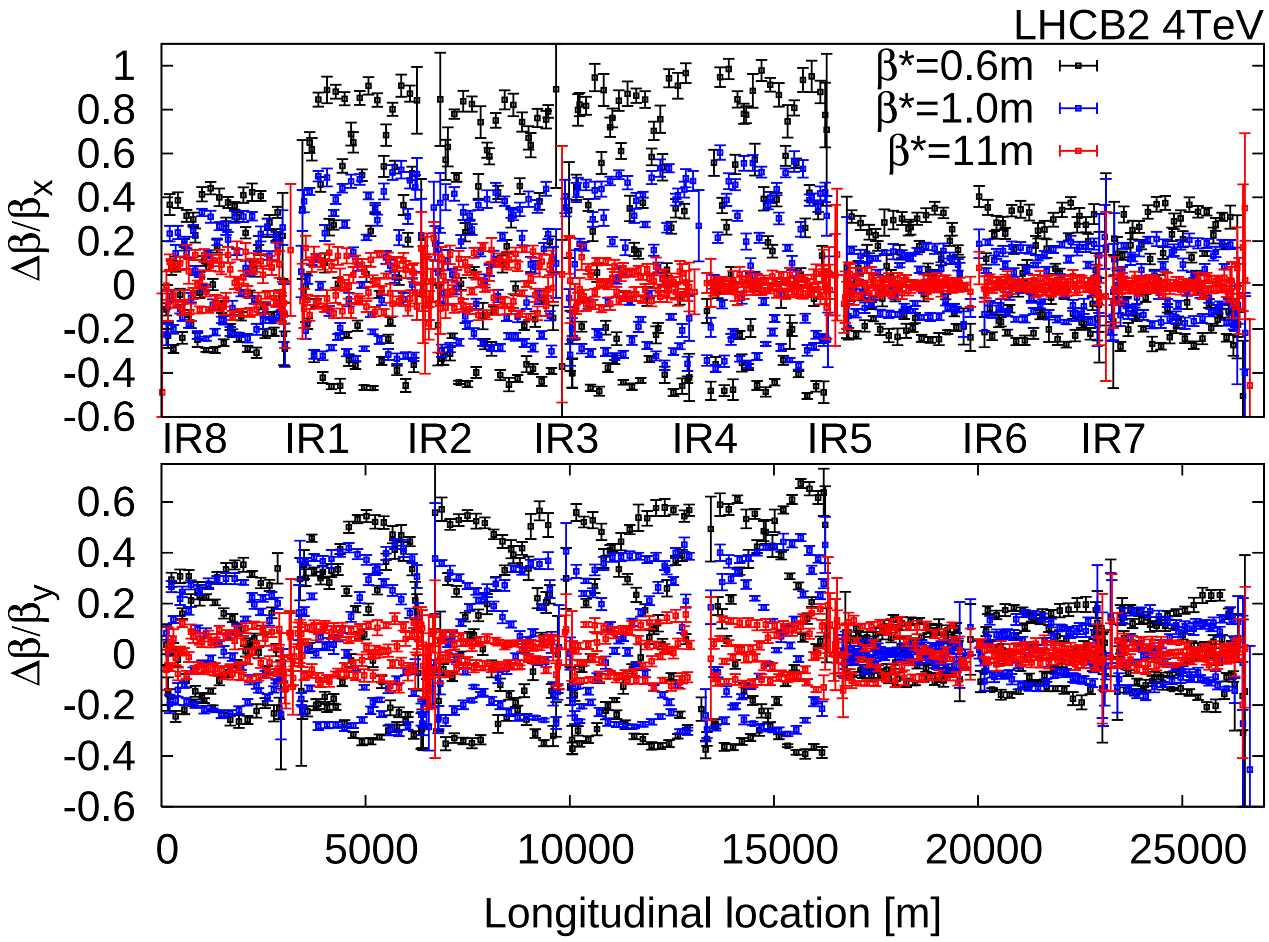
Peak β-beating of $\approx 4\%$

SPS β-beating (Q20)



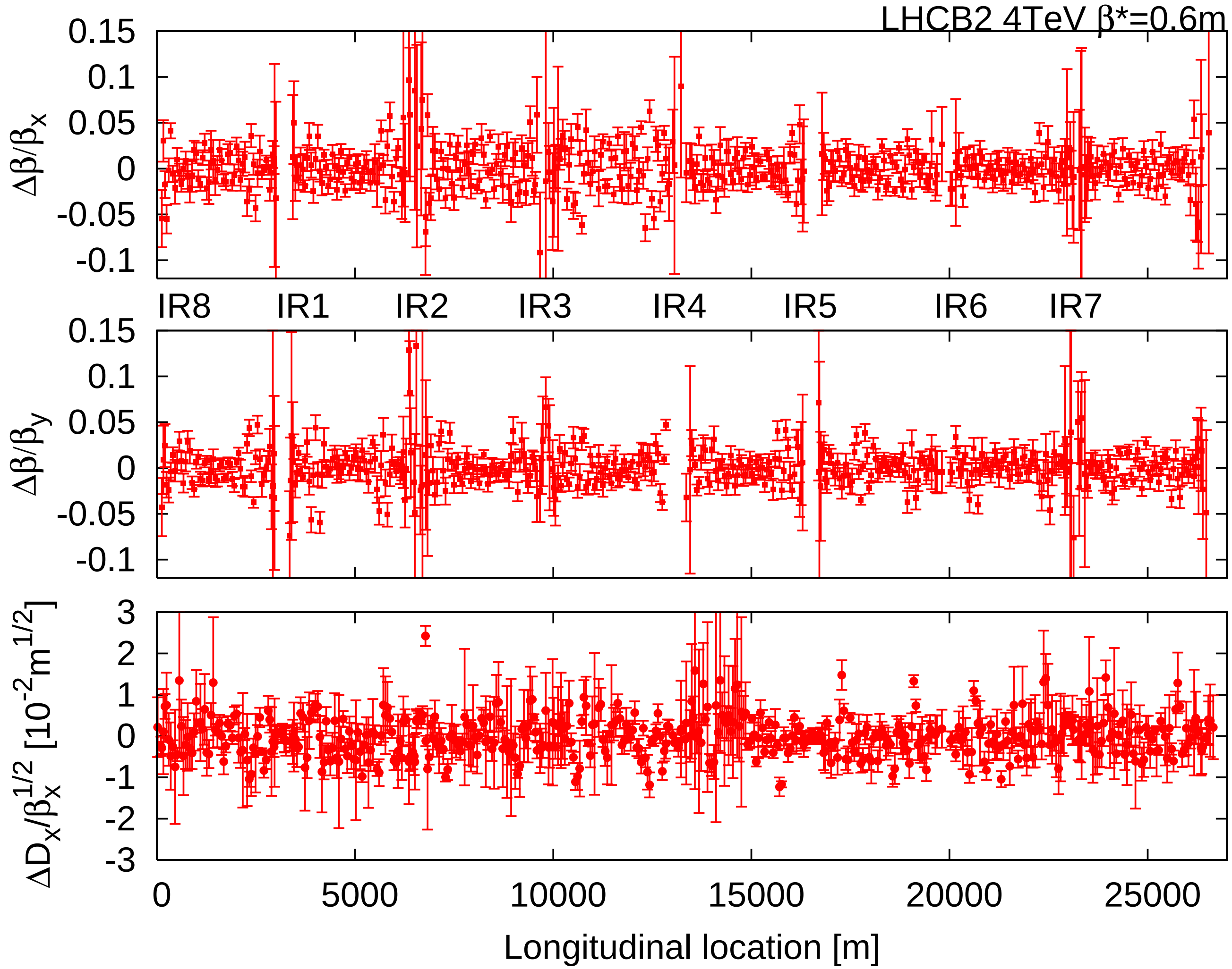
Peak β-beating of $\approx$ 25%

LHC β-beating, before correction



Peak β-beating of $\approx$ 100% !!!

LHC β-beating, after correction



Correction brings peak β-beating to $\approx$ 7%

Dynamic linear imperfections

- ⋆ Ground motion and vibrations in quadrupoles produce sinusoidal dipolar fields
- ⋆ Electrical noise can cause currents in quadrupoles and dipole to oscillate in time
- ⋆ Electromagnetic pollution can act directly on the beam.
- ⋆ Slow variations ($f << Q_{x,y} \times f_{rev}$) just cause a time varying orbit and optics
- ⋆ Fast variations ($f \approx Q_{x,y} \times f_{rev}$) can cause resonances and emittance growth

An oscillating dipolar field

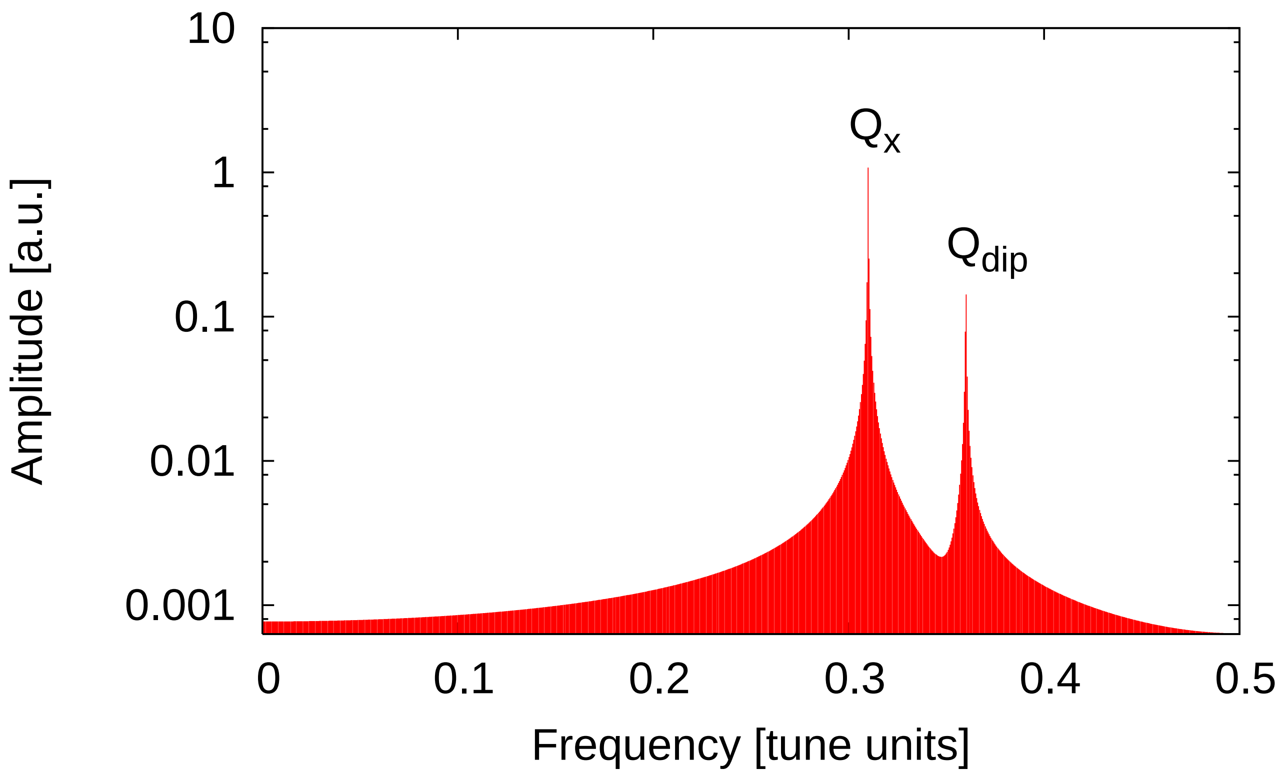
- Let $Q_{dip} = f_{dip}/f_{rev}$ be the tune of the dipolar field oscillation.
- This causes the appearance of new resonances
- Linear resonances: $Q_x \pm Q_{dip} = N$
- Non-linear resonances of sextupolar order:

$$Q_x \pm 2Q_{dip} = N$$

$$2Q_x \pm Q_{dip} = N$$

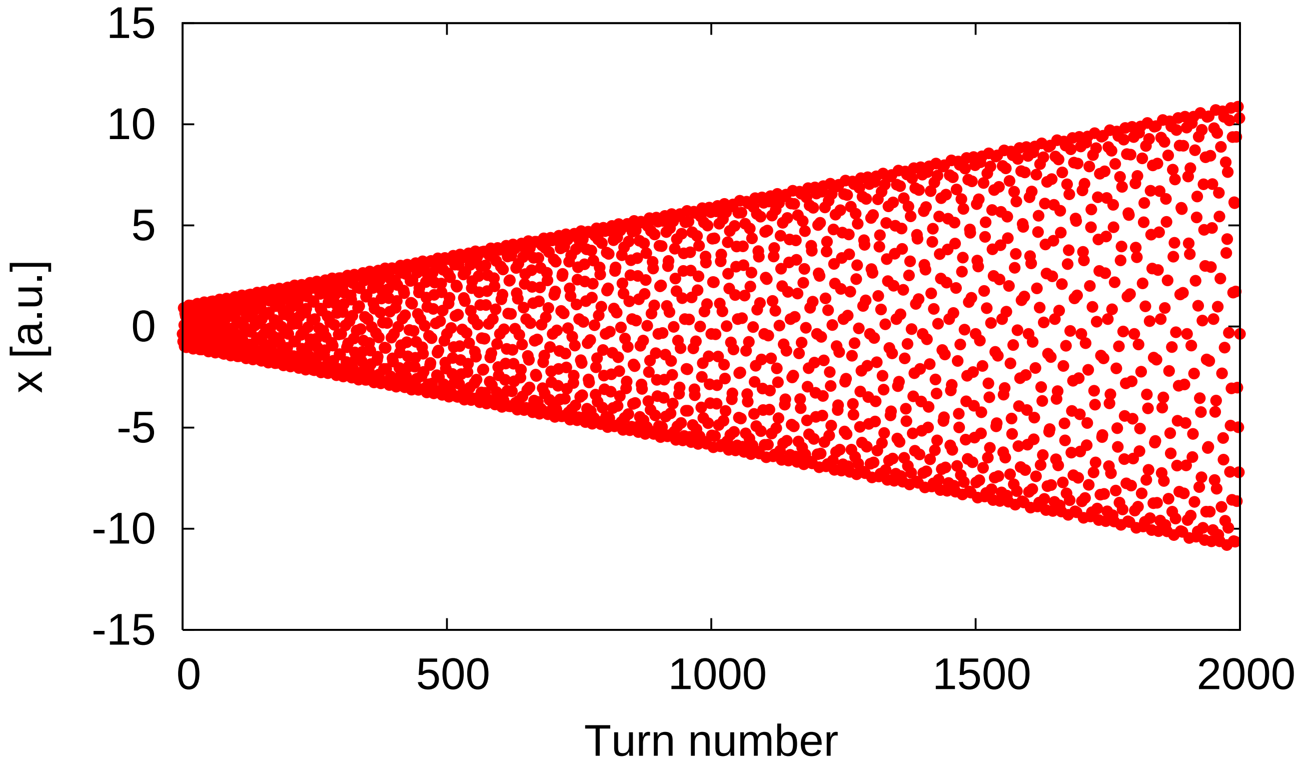
- Note that $mQ_{dip} = N$ is not a problem

Oscillating dipolar field, $Q_x \neq Q_{dip}$



Orbit oscillates with Q_{dip} but there is no emittance growth far from resonances.

Oscillating dipolar field, $Q_x = Q_{dip}$

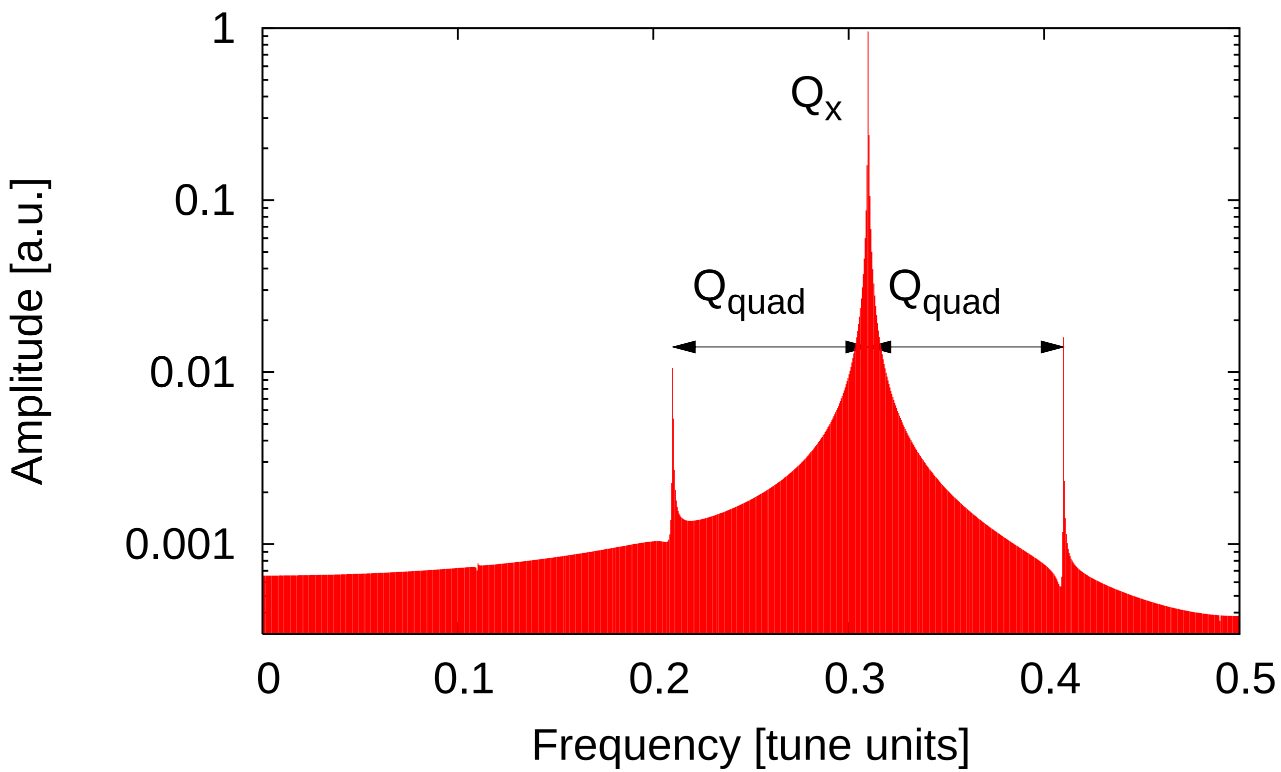


Linear growth in time $\longrightarrow$ Emittance growth.

An oscillating quadrupolar field

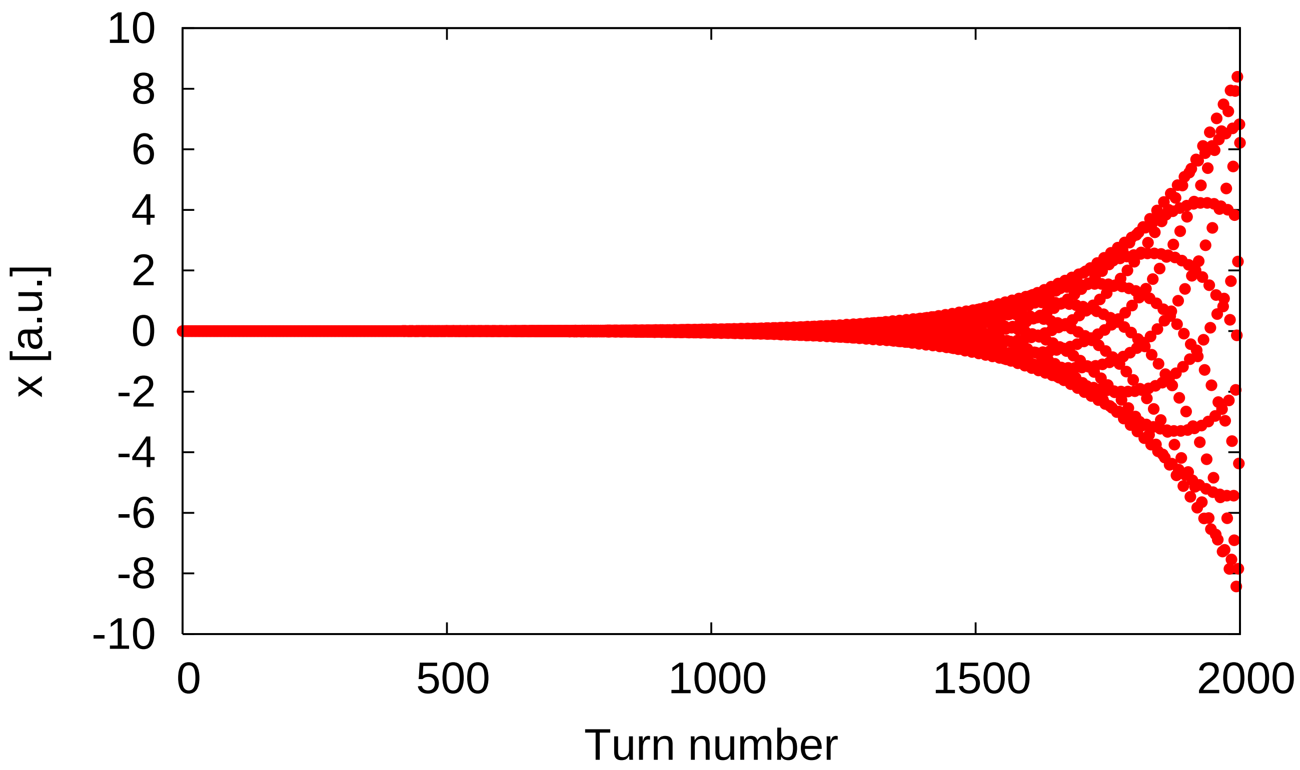
- ⋆ Let $Q_{quad} = f_{quad}/f_{rev}$ be the tune of the quadrupolar field oscillation.
- ⋆ This causes the appearance of new resonances
- ⋆ Linear resonances: $2Q_x \pm Q_{quad} = N$

Oscillating quadrupolar field, $2Q_x \neq Q_{quad}$



Tune is modulated with Q_{quad}, displaying side-bands at $Q_x \pm Q_{quad}$ but there is no emittance growth far from resonances.

Oscillating quadrupolar field, $2Q_x = Q_{quad}$



Exponential growth, clear signatures depending on the oscillating field type.

Questions?