Beam Transfer Lines

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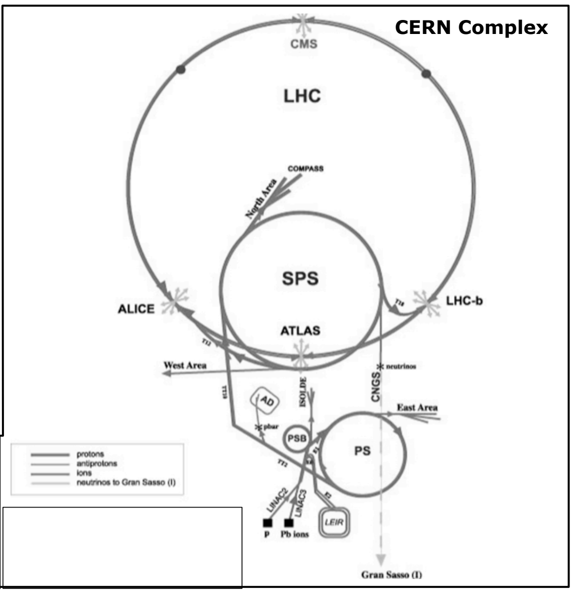
(based on lecture by B. Goddard and M. Meddahi)

What is the purpose of "transfer lines"?

- An accelerator has limited dynamic range
- Chain of stages needed to reach high energy
- Periodic re-filling of storage rings, like LHC
- External experiments, like CNGS

Transfer lines transport the beam between accelerators, and onto targets, dumps, instruments etc.



LHC:	**Large Hadron Collider**
SPS:	**Super Proton Synchrotron**
AD:	**Antiproton Decelerator**
ISOLDE:	**Isotope Separator Online Device**
PSB:	**Proton Synchrotron Booster**
PS:	**Proton Synchrotron**
LINAC:	**LINear Accelerator**
LEIR:	**Low Energy Ring**
CNGS:	**CERN Neutrino to Gran Sasso**

Transport of good quality beams

- Transfer lines transport beams

- The challenge: preserve "GOOD QUALITY"

Examples:

- **Position stability** on a target
  - Trajectory in transfer line needs to be under control
- Not below/above certain **beam size** at a window/target
  - Optics in transfer line AND exit of last machine need to be under control
- Preserve **emittance** between machines
  - Trajectory, optics, tilt angles etc. need to be under control

Challenges with high energy/intensity

- Machine protection becomes design constraint for extraction/ injection/ transfer lines

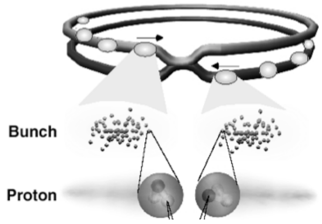
- Sophisticated and reliable active protection with surveillance of power supply currents, beam losses, etc.

- Passive protection with absorbers
  - Dedicated areas in the transfer lines with optics requirements to install a collimation system

Example: CNGS Target

- CNGS transfer line from SPS is ~ 1 km long
- At the end of the line the CNGS target needs to be hit

Example: Emittance and LHC Luminosity



Want to measure rare events with high statistics

Collision rate = Luminosity x cross-section

$$\mathcal{L} \propto \frac{N_1 N_2 n_b}{\sigma^2} \qquad \sigma^2 = \frac{\varepsilon_n \cdot \beta^*}{\gamma}$$

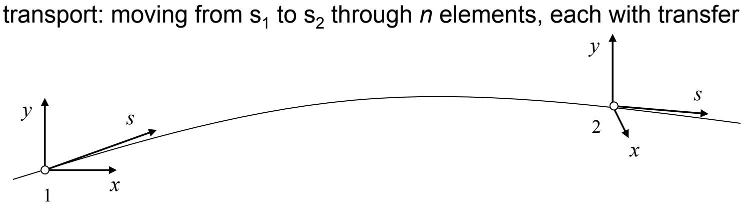
- Proton machines ➔ emittance can only grow
- Emittance has to be preserved through the LHC cycle
- One of most critical moments for emittance:
  - Beam Transfer from Injectors
  - ➔ NO ERRORS FROM THE BEAM TRANSFER

Contents of this lecture

- Distinctions between transfer lines and circular machines
- Linking machines together
- Emittance Blow-up from steering errors
- Correction of injection oscillations
- Emittance Blow-up from optics mismatch
- Optics measurement
- Blow-up from thin screens

General transport

Beam transport: moving from s_1 to s_2 through *n* elements, each with transfer matrix M_i

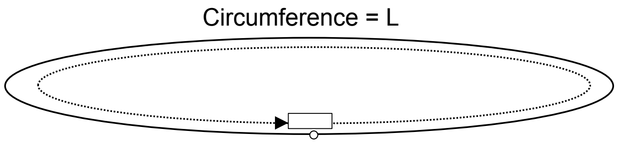


$$\begin{bmatrix} x_2 \\ x'_2 \end{bmatrix} = \mathbf{M}_{1\rightarrow 2} \cdot \begin{bmatrix} x_1 \\ x'_1 \end{bmatrix} = \begin{bmatrix} C & S \\ C' & S' \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x'_1 \end{bmatrix} \qquad \mathbf{M}_{1\rightarrow 2} = \prod_{i=1}^{n} \mathbf{M}_n$$

Twiss parameterisation

$$\mathbf{M}_{1\rightarrow 2} = \begin{bmatrix} \sqrt{\beta_2/\beta_1}\left(\cos\Delta\mu + \alpha_1 \sin\Delta\mu\right) & \sqrt{\beta_1\beta_2}\sin\Delta\mu \\ \sqrt{1/\beta_1\beta_2}\left[(\alpha_1 - \alpha_2)\cos\Delta\mu - (1 + \alpha_1\alpha_2)\sin\Delta\mu\right] & \sqrt{\beta_1/\beta_2}\left(\cos\Delta\mu - \alpha_2 \sin\Delta\mu\right) \end{bmatrix}$$

Circular Machine

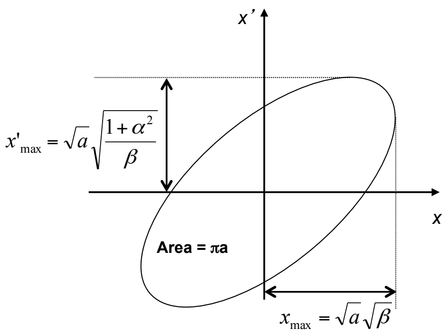


One turn $\quad \mathrm{M}_{1\to 2} = \mathrm{M}_{0\to L} = \begin{bmatrix} \cos 2\pi Q + \alpha \sin 2\pi Q & \beta \sin 2\pi Q \\ -1/\beta\left(1+\alpha^2\right)\sin 2\pi Q & \cos 2\pi Q - \alpha \sin 2\pi Q \end{bmatrix}$

- The solution is *periodic*
- Periodicity condition for one turn (closed ring) imposes $\alpha_1=\alpha_2$, $\beta_1=\beta_2$, $D_1=D_2$
- This condition *uniquely* determines $\alpha(s)$, $\beta(s)$, $\mu(s)$, $D(s)$ around the whole ring

Circular Machine

- Periodicity of the structure leads to regular motion
  - Map single particle coordinates on each turn at any location
  - Describes an ellipse in phase space, defined by one set of α and β values $\Rightarrow$ Matched Ellipse (for this location)

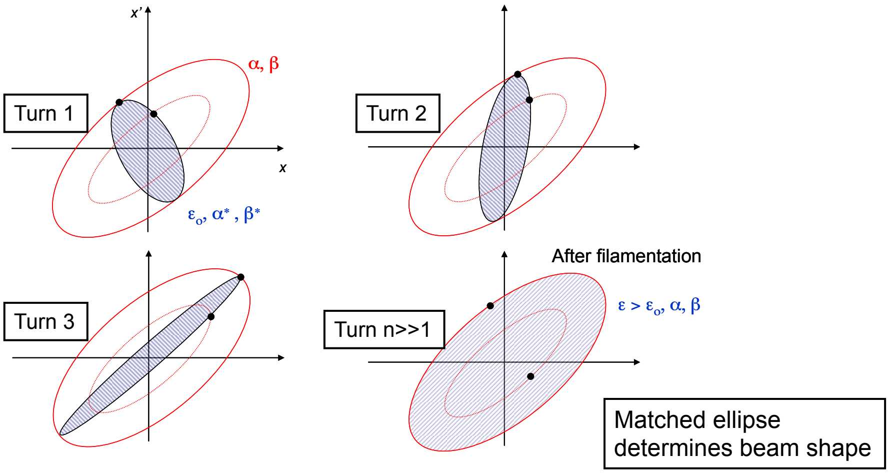


$$a = \gamma \cdot x^2 + 2\alpha \cdot x \cdot x' + \beta \cdot x'^2$$

$$\gamma = \frac{1+\alpha^2}{\beta}$$

Circular Machine

- For a location with matched ellipse (α, β), an injected beam of emittance ε, characterised by a different ellipse (α^*, β^*) generates (via filamentation) a large ellipse <u>with the original α, β, but larger ε</u>



Transfer line

One pass: $\begin{bmatrix} x_2 \\ x_2' \end{bmatrix} = \mathbf{M}_{1\to 2} \cdot \begin{bmatrix} x_1 \\ x_1' \end{bmatrix}$

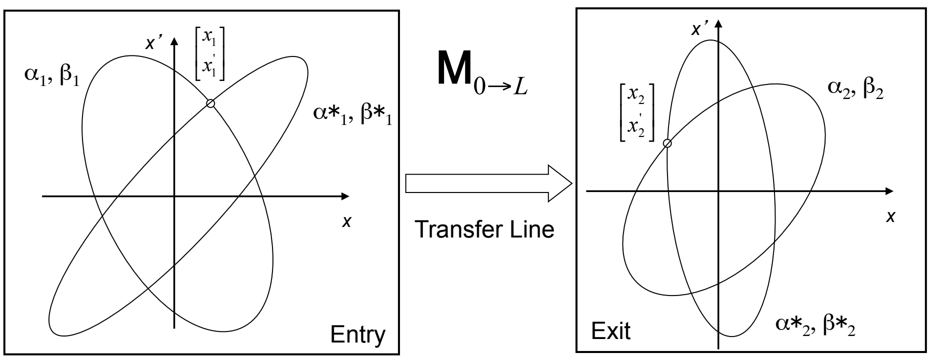
$$\begin{bmatrix} x_1 \\ x_1' \end{bmatrix} \qquad \longrightarrow \qquad \begin{bmatrix} x_2 \\ x_2' \end{bmatrix}$$

$$\mathbf{M}_{1\to 2} = \begin{bmatrix} \sqrt{\beta_2/\beta_1}\left(\cos\Delta\mu + \alpha_1 \sin\Delta\mu\right) & \sqrt{\beta_1\beta_2}\,\sin\Delta\mu \\ \sqrt{1/\beta_1\beta_2}\left[(\alpha_1-\alpha_2)\cos\Delta\mu - (1+\alpha_1\alpha_2)\sin\Delta\mu\right] & \sqrt{\beta_1/\beta_2}\left(\cos\Delta\mu - \alpha_2 \sin\Delta\mu\right) \end{bmatrix}$$

- No periodic condition exists
- The Twiss parameters are simply propagated from beginning to end of line
- At any point in line, $\alpha(s)$ $\beta(s)$ are functions of α_1 β_1

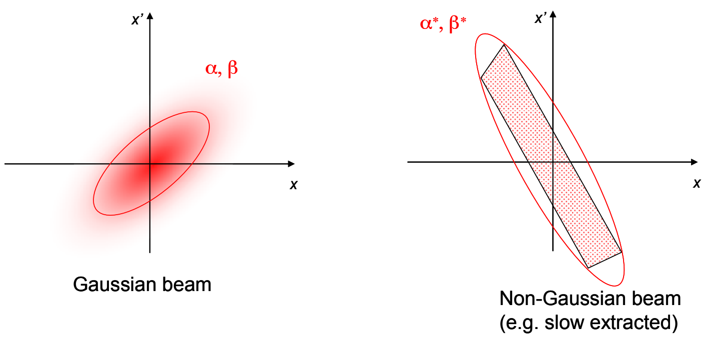
Transfer line

- On a single pass…
  - Map single particle coordinates at entrance and exit.
  - Infinite number of equally valid possible starting ellipses for single particle ……transported to infinite number of final ellipses…



Transfer Line

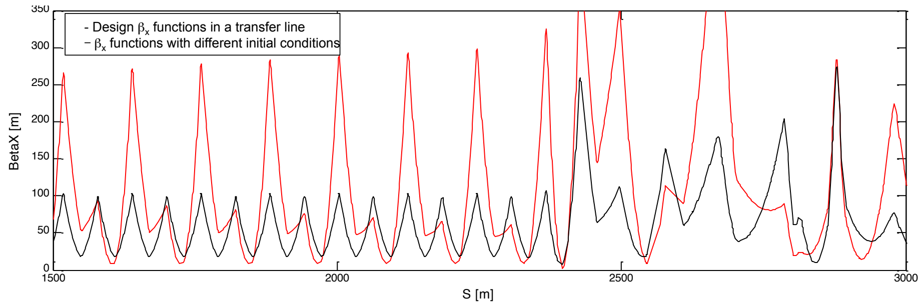
- Initial α, β defined for transfer line by beam shape at entrance



- Propagation of this beam ellipse depends on line elements
- A transfer line optics is different for different input beams

Transfer Line

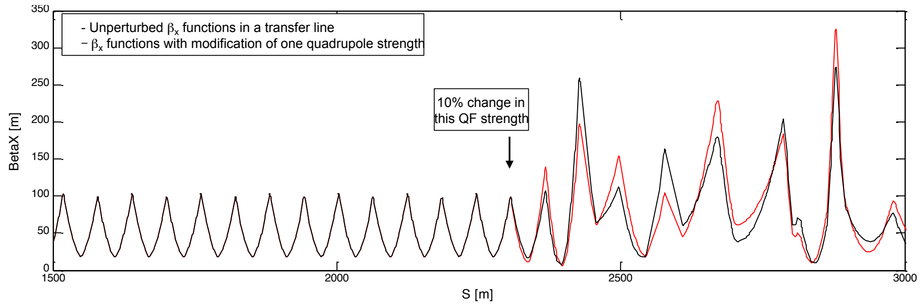
- <u>The optics functions in the line depend on the initial values</u>



- Same considerations are true for Dispersion function:
  - Dispersion in ring defined by periodic solution → ring elements
  - Dispersion in line defined by initial D and D' and line elements

Transfer Line

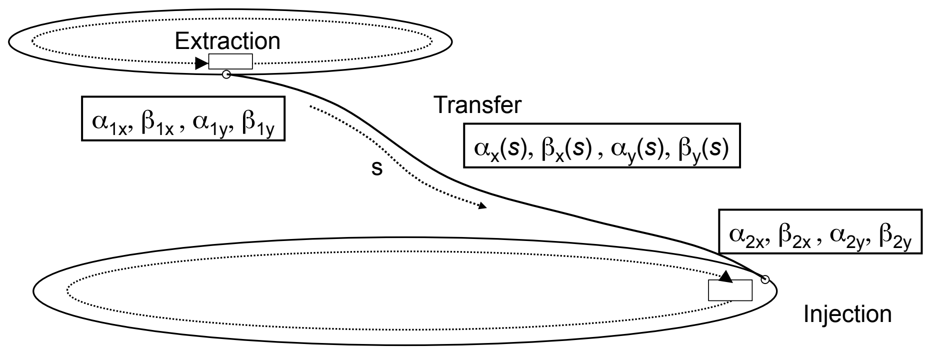
- Another difference….unlike a circular ring, <u>a change of an element in a line affects *only* the downstream Twiss values</u> (including dispersion)



Linking Machines

- Beams have to be transported from extraction of one machine to injection of next machine
  - Trajectories must be matched, ideally in all 6 geometric degrees of freedom (x,y,z,θ,ϕ,ψ)
  - Otherwise emittance blow-up
- Other important constraints can include
  - Minimum bend radius, maximum quadrupole gradient, magnet aperture, cost, geology

Linking Machines



The Twiss parameters can be propagated when the transfer matrix **M** is known

$$\begin{bmatrix} x_2 \\ x_2' \end{bmatrix} = \mathbf{M}_{1\to2} \cdot \begin{bmatrix} x_1 \\ x_1' \end{bmatrix} = \begin{bmatrix} C & S \\ C' & S' \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_1' \end{bmatrix}$$

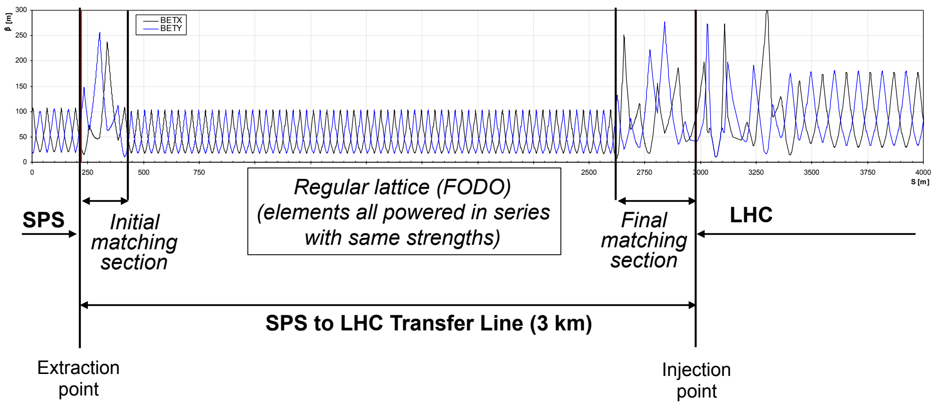
$$\begin{bmatrix} \beta_2 \\ \alpha_2 \\ \gamma_2 \end{bmatrix} = \begin{bmatrix} C^2 & -2CS & S^2 \\ -CC' & CS'+SC' & -SS' \\ C'^2 & -2C'S' & S'^2 \end{bmatrix} \cdot \begin{bmatrix} \beta_1 \\ \alpha_1 \\ \gamma_1 \end{bmatrix}$$

Linking Machines

- Linking the optics is a complicated process
  - Parameters at start of line have to be propagated to matched parameters at the end of the line
  - Need to "match" 8 variables (α_x β_x D_x D'_x and α_y β_y D_y D'_y)
  - Maximum β and D values are imposed by magnet apertures
  - Other constraints can exist
    - phase conditions for collimators,
    - insertions for special equipment like stripping foils
  - Need to use a number of independently powered ("matching") quadrupoles
  - Matching with computer codes and relying on mixture of theory, experience, intuition, trial and error, …

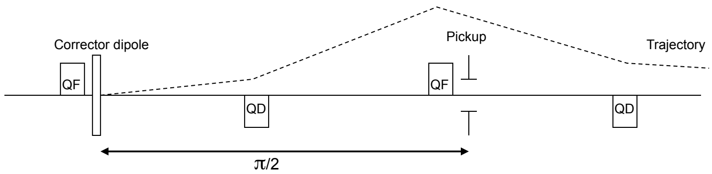
Linking Machines

- For long transfer lines we can simplify the problem by designing the line in separate sections
  - Regular central section – e.g. FODO or doublet, with quads at regular spacing, (+ bending dipoles), with magnets powered in series
  - Initial and final matching sections – independently powered quadrupoles, with sometimes irregular spacing.



Trajectory correction

- Magnet misalignments, field and powering errors cause the trajectory to deviate from the design
- Use small independently powered dipole magnets (correctors) to steer the beam
- Measure the response using monitors (pick-ups) downstream of the corrector ($\pi/2$, $3\pi/2$, …)

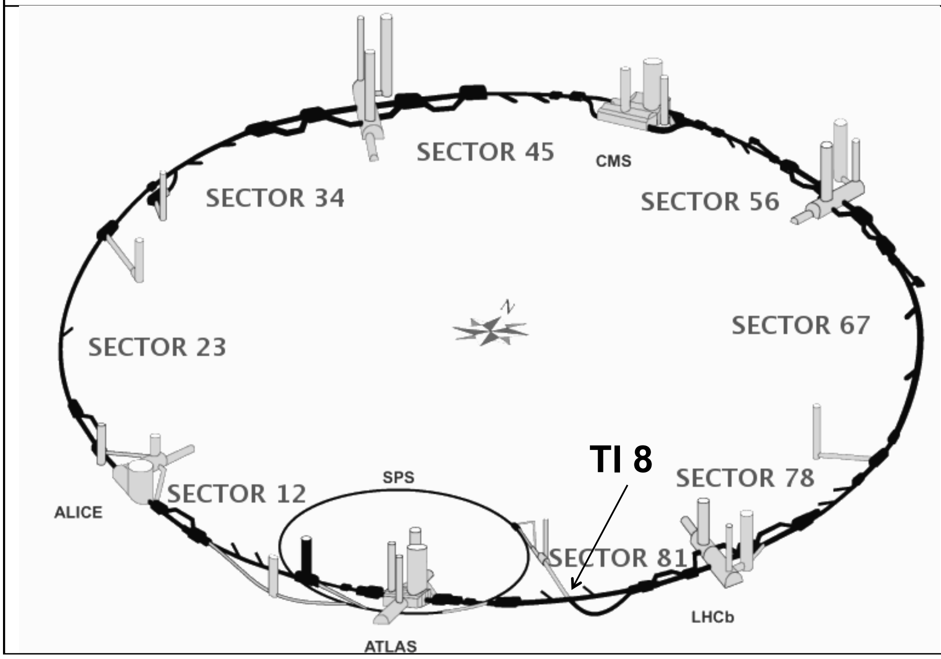


- Horizontal and vertical elements are separated
- H-correctors and pick-ups located at F-quadrupoles (large β_x)
- V-correctors and pick-ups located at D-quadrupoles (large β_y)

Trajectory correction

- Global correction can be used which attempts to minimise the RMS offsets at the BPMs, using all or some of the available corrector magnets.
- Steering in matching sections, extraction and injection region requires particular care
  - D and β functions can be large → bigger beam size
  - Often very limited in aperture
  - Injection offsets can be detrimental for performance

Some pictures – LHC Transfer Line TI 8



Some pictures – LHC Transfer Line TI 8

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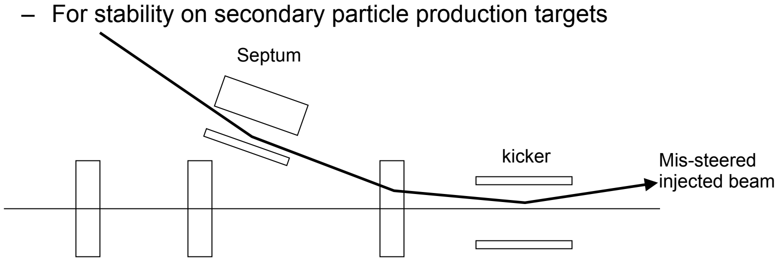
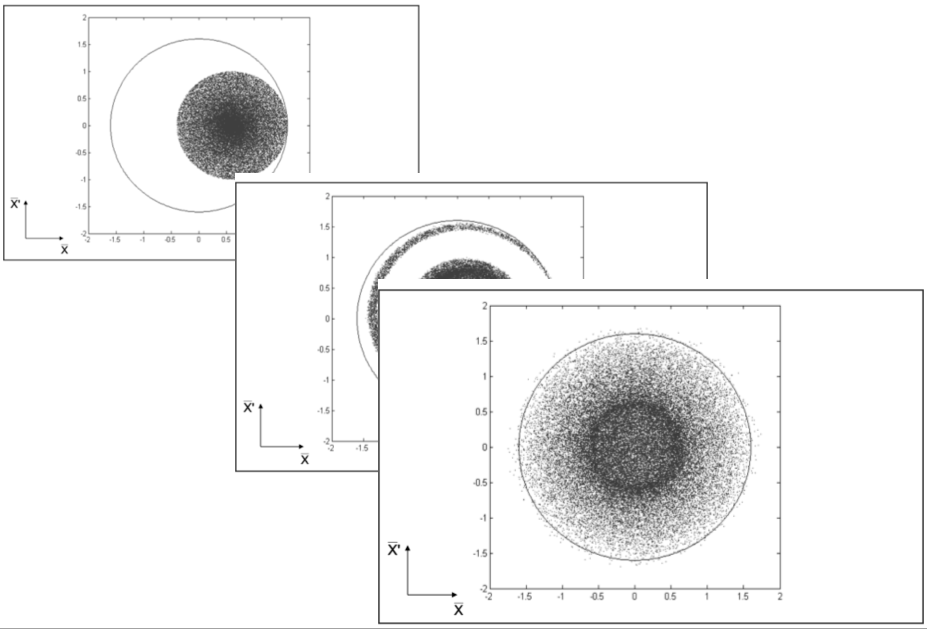
Steering (dipole) errors

- Precise delivery of the beam is important.
  - To avoid **injection oscillations** and emittance growth in rings
  - For stability on secondary particle production targets



– Injection oscillations = if beam is not injected on the closed orbit, beam oscillates around closed orbit and eventually filaments (if not damped)

Reminder – Filamentation – Emittance Growth



Reminder - Normalised phase space

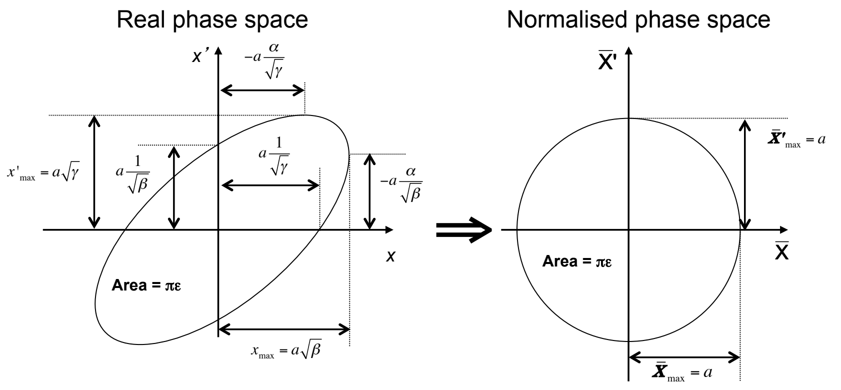
- Transform real transverse coordinates x, x' by

$$\begin{bmatrix} \overline{X} \\ \overline{X}' \end{bmatrix} = \mathbf{N} \cdot \begin{bmatrix} x \\ x' \end{bmatrix} = \sqrt{\frac{1}{\beta_S}} \cdot \begin{bmatrix} 1 & 0 \\ \alpha_S & \beta_S \end{bmatrix} \cdot \begin{bmatrix} x \\ x' \end{bmatrix}$$

$$\overline{X} = \sqrt{\frac{1}{\beta_S}} \cdot x$$

$$\overline{X}' = \sqrt{\frac{1}{\beta_S}} \cdot \alpha_S x + \sqrt{\beta_S} x'$$

Reminder - Normalised phase space

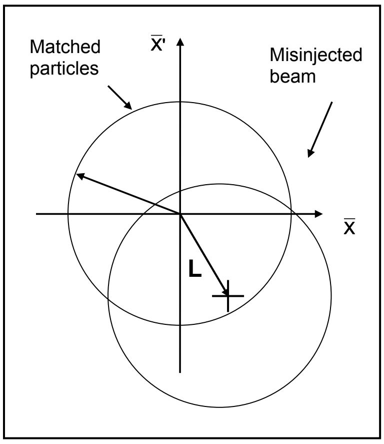


$$a = \gamma \cdot x^2 + 2\alpha \cdot x \cdot x' + \beta \cdot x'^2$$

$$a = \bar{X}^2 + \bar{X}'^2$$

Blow-up from steering error

- Consider a collection of particles
- The beam can be injected with a error in angle and position.
- For an injection error Δa (in units of sigma = √βε) the mis-injected beam is offset in normalised phase space by L = Δa√ε



Blow-up from steering error

- The new particle coordinates in normalised phase space are

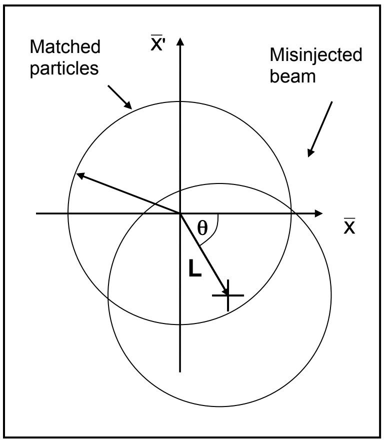
$$\overline{X}_{new} = \overline{X}_0 + L\cos\theta$$

$$\overline{X}'_{new} = \overline{X}'_0 + L\sin\theta$$

- For a general particle distribution, where A denotes amplitude of a paricle in normalised phase space

$$A^2 = \overline{X}^2 + \overline{X}'^2$$

$$\varepsilon = \langle A^2 \rangle / 2$$



Blow-up from steering error

- So if we plug in the new coordinates….

$$A^2_{new} = \overline{X}^2_{new} + \overline{X}'^2_{new} = \left(\overline{X}_0 + L\cos\theta\right)^2 + \left(\overline{X}'_0 + L\sin\theta\right)^2$$

$$= \overline{X}_0^2 + \overline{X}_0'^2 + 2L\left(\overline{X}_0\cos\theta + \overline{X}'_0\sin\theta\right) + L^2$$

$$\langle A^2_{new} \rangle = \langle \overline{X}_0^2 \rangle + \langle \overline{X}_0'^2 \rangle + \langle 2L\left(\overline{X}_0\cos\theta + \overline{X}'_0\sin\theta\right) \rangle + \langle L^2 \rangle$$

$$= 2\varepsilon_0 + 2L\left(\cancelto{0}{\langle \cos\theta\overline{X}_0 \rangle} + \cancelto{0}{\langle \sin\theta\overline{X}'_0 \rangle}\right) + L^2$$

$$= 2\varepsilon_0 + L^2$$

- Giving for the emittance increase

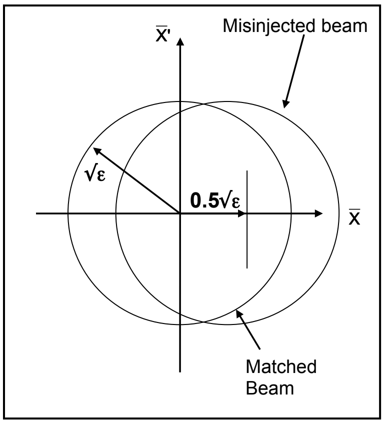
$$\boxed{\varepsilon_{new} = \langle A_{new}{}^2 \rangle / 2 = \varepsilon_0 + L^2/2}$$

$$= \varepsilon_0\left(1 + \Delta a^2/2\right)$$

Blow-up from steering error

A numerical example….

Consider an offset Δa of 0.5 sigma for injected beam

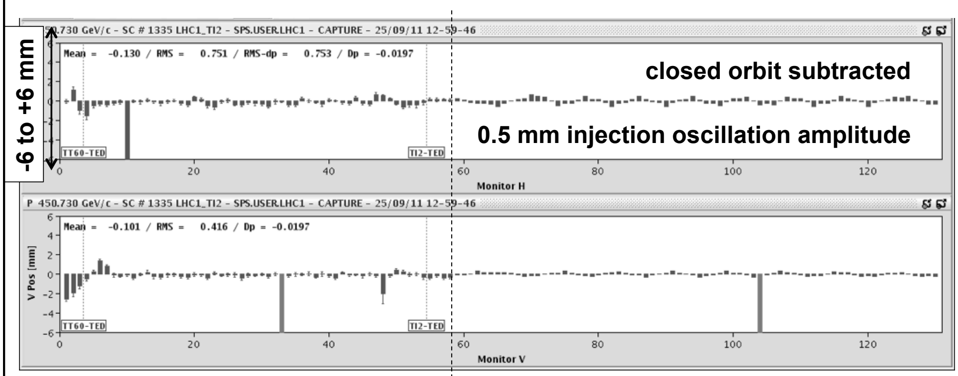


$$\varepsilon_{new} = \varepsilon_0 \left(1 + \Delta a^2 / 2\right)$$
$$= 1.125\,\varepsilon_0$$

For nominal LHC beam:
ε_{norm} = 3.5 μm
allowed growth through LHC cycle ~ 10 %

Example: LHC injection of beam 1

Display of injection oscillations in LHC control room



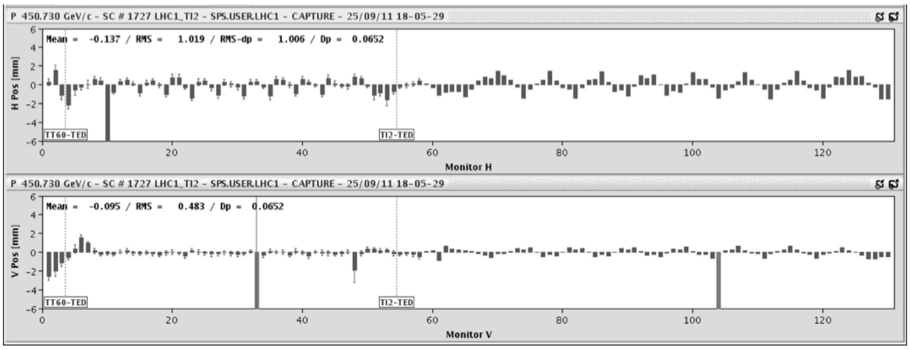
Transfer line TI 2 ~3 km **LHC arc 23 ~3 km**

Injection point in LHC IR2

0.5 mm injection oscillation is GOOD. Don't touch. ➔ Transverse Damper

Example: LHC injection of beam 1

- Oscillation down the line has developed in horizontal plane
- Injection oscillation amplitude > 1.5 mm
- Good working range of LHC transverse damper +/- 2 mm



- Aperture margin for injection oscillation is 2 mm
- ➔ correct injection oscillations before continue LHC filling
- Transfer line steering

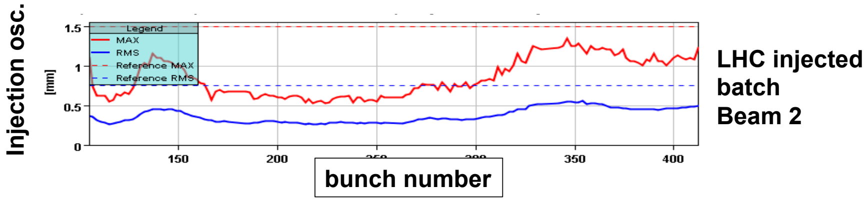
Injection oscillation correction

- x, x' and y, y' at injection point need to be corrected.
- Minimum diagnostics: 2 pickups per plane, 90° phase advance apart
- Pickups need to be triggered to measure on the first turn
- **Correctors in the transfer lines** are used to minimize offset at these pickups.

- Best strategy:
  - Acquire many BPMs in circular machine (e.g. one octant/sextant of machine)
  - Combine acquisition of transfer line and of BPMs in circular machine
    - Transfer line: difference trajectory to reference
    - Circular machine: remove closed orbit from first turn trajectory ➔ pure injection oscillation
  - Correct combined trajectory with correctors in transfer line with typical correction algorithms. Use correctors of the line only.

Steering (dipole) errors

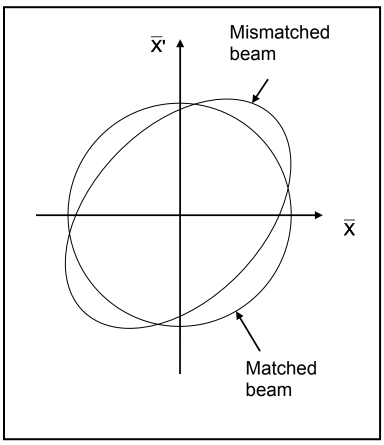
- Static effects (e.g. from errors in alignment, field, calibration, …) are dealt with by trajectory correction (steering).
- But there are also dynamic effects, from:
  - Power supply ripples
  - Temperature variations
  - Non-trapezoidal kicker waveforms
- These dynamic effects produce a variable injection offset which can vary from batch to batch, or even within a batch.



- An **injection damper system** is used to minimize effect on emittance

Blow-up from betatron mismatch

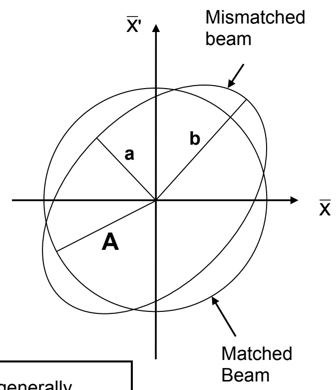
- Optical errors occur in transfer line and ring, such that the beam can be injected with a mismatch.
- Filamentation will produce an emittance increase.
- In normalised phase space, consider the matched beam as a circle, and the mismatched beam as an ellipse.



Blow-up from betatron mismatch

Define parameter λ to transform original coordinates:

$$\overline{X}_{new} = \lambda \cdot A\sin(\phi+\phi_1), \qquad \overline{X}'_{new} = \frac{1}{\lambda} A\cos(\phi+\phi_1)$$



Frequently also use H as mismatch parameter:

$$a = \frac{A}{\sqrt{2}}\left(\sqrt{H+1}+\sqrt{H-1}\right), \qquad b = \frac{A}{\sqrt{2}}\left(\sqrt{H+1}-\sqrt{H-1}\right)$$

$$\lambda = \frac{1}{\sqrt{2}}\left(\sqrt{H+1}+\sqrt{H-1}\right), \qquad \frac{1}{\lambda} = \frac{1}{\sqrt{2}}\left(\sqrt{H+1}-\sqrt{H-1}\right)$$

generally

$$a = A/\lambda$$
$$b = A\cdot\lambda$$

Blow-up from betatron mismatch

We can evaluate the square of the distance of a particle from the origin as

$$A_{new}^2 = \overline{X}_{new}^2 + \overline{X}'^2_{new} = \lambda^2 \cdot A_0^2 \sin^2(\phi+\phi_1) + \frac{1}{\lambda^2} A_0^2 \cos^2(\phi+\phi_1)$$

The new emittance is the average over all phases

$$\varepsilon_{new} = \frac{1}{2}\left\langle A_{new}^2 \right\rangle = \frac{1}{2}\left(\lambda^2 \left\langle A_0^2 \sin^2(\phi+\phi_1)\right\rangle + \frac{1}{\lambda^2}\left\langle A_0^2 \cos^2(\phi+\phi_1)\right\rangle\right)$$

$$= \frac{1}{2}\left\langle A_0^2\right\rangle\left(\lambda^2 \overset{0.5}{\cancel{\left\langle \sin^2(\phi+\phi_1)\right\rangle}} + \frac{1}{\lambda^2}\overset{0.5}{\cancel{\left\langle \cos^2(\phi+\phi_1)\right\rangle}}\right)$$

$$= \frac{1}{2}\varepsilon_0\left(\lambda^2 + \frac{1}{\lambda^2}\right)$$

If we're feeling diligent, we can substitute back for λ to give

$$\varepsilon_{new} = \frac{1}{2}\varepsilon_0\left(\lambda^2 + \frac{1}{\lambda^2}\right) = H\varepsilon_0 = \frac{1}{2}\varepsilon_0\left(\frac{\beta_1}{\beta_2} + \frac{\beta_2}{\beta_1}\left(\alpha_1 - \alpha_2\frac{\beta_1}{\beta_2}\right)^2 + \frac{\beta_2}{\beta_1}\right)$$

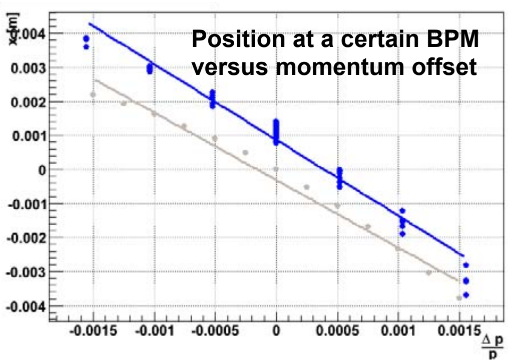
where subscript 1 refers to matched ellipse, 2 to mismatched ellipse.

OPTICS AND EMITTANCE MEASUREMENT IN TRANSFER LINES

Dispersion measurement

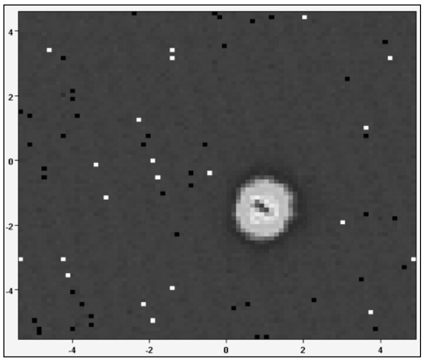
- Introduce ~ few permille momentum offset at extraction into transfer line
- Measure position at different monitors for different momentum offset
  - Linear fit of position versus dp/p at each BPM/screens.
  - ➔ Dispersion at the BPMs/screens

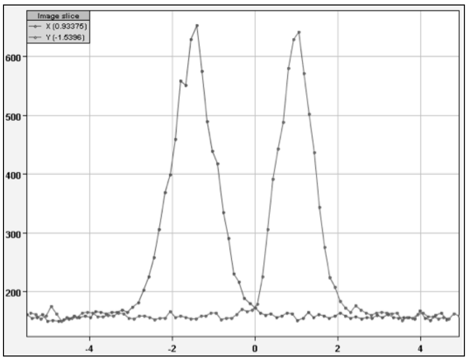
$$x(s) = x_\beta(s) + D(s) \cdot \frac{dp}{p}$$



Optics measurement with screens

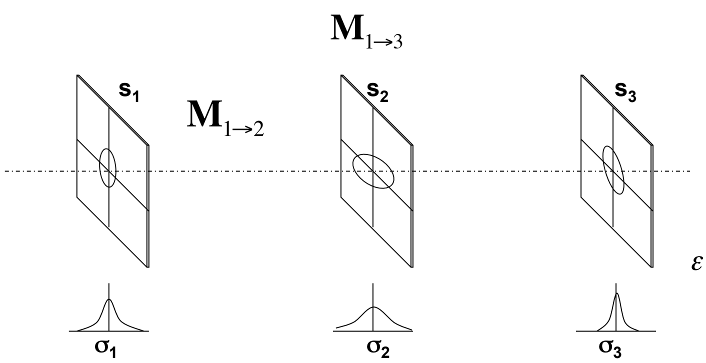
- A profile monitor is needed to measure the beam size
  - e.g. beam screen (luminescent) provides 2D density profile of the beam
- Profile fit gives transverse beam sizes σ.
- In a ring, β is 'known' so ε can be calculated from a single screen





Optics Measurement with 3 Screens

- Assume 3 screens in a dispersion free region
- Measurements of σ_1,σ_2,σ_3, plus the two transfer matrices M_{12} and M_{13} allows determination of ε, α and β



$$\varepsilon = \frac{\sigma_1^2}{\beta_1} = \frac{\sigma_2^2}{\beta_2} = \frac{\sigma_3^2}{\beta_3}$$

Optics Measurement with 3 Screens

- Remember:

$$\begin{bmatrix} x_2 \\ x_2' \end{bmatrix} = \mathbf{M}_{1\to 2} \cdot \begin{bmatrix} x_1 \\ x'_1 \end{bmatrix} = \begin{bmatrix} C & S \\ C' & S' \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x'_1 \end{bmatrix}$$

$$\begin{bmatrix} \beta_2 \\ \alpha_2 \\ \gamma_2 \end{bmatrix} = \begin{bmatrix} C_2^2 & -2C_2S_2 & S_2^2 \\ -C_2C_2' & C_1S_1' + S_1C_1' & -S_2S_2' \\ C_2'^2 & -2C_2'S_2' & S_2'^2 \end{bmatrix} \cdot \begin{bmatrix} \beta_1 \\ \alpha_1 \\ \gamma_1 \end{bmatrix}$$

$$\Rightarrow \left. \begin{aligned} \beta_2 &= C_2^2 \cdot \beta_1 - 2C_2S_2 \cdot \alpha_1 + S_2^2 \cdot \gamma_1 \\ \beta_3 &= C_3^2 \cdot \beta_1 - 2C_3S_3 \cdot \alpha_1 + S_3^2 \cdot \gamma_1 \end{aligned} \right| \times \varepsilon$$

$$\begin{aligned} \boxed{\sigma_2^2} &= C_2^2 \cdot \boxed{\beta_1\varepsilon} - 2C_2S_2 \cdot \boxed{\alpha_1\varepsilon} + S_2^2 \cdot \boxed{\gamma_1\varepsilon} \\ \boxed{\sigma_3^2} &= C_3^2 \cdot \boxed{\beta_1\varepsilon} - 2C_3S_3 \cdot \boxed{\alpha_1\varepsilon} + S_3^2 \cdot \boxed{\gamma_1\varepsilon} \end{aligned}$$

Square of beam sizes as function of optical functions at first screen

Optics Measurement with 3 Screens

$$\begin{aligned} \sigma_1^2 &= 1 \cdot \beta_1\varepsilon - 0 \cdot \alpha_1\varepsilon + 0 \cdot \gamma_1\varepsilon \\ \sigma_2^2 &= C_2^2 \cdot \beta_1\varepsilon - 2C_2S_2 \cdot \alpha_1\varepsilon + S_2^2 \cdot \gamma_1\varepsilon \\ \sigma_3^2 &= C_3^2 \cdot \beta_1\varepsilon - 2C_3S_3 \cdot \alpha_1\varepsilon + S_3^2 \cdot \gamma_1\varepsilon \end{aligned}$$

- Build matrix

$$\Sigma = \mathbf{N} \cdot \Pi$$

$$\Sigma = \begin{pmatrix} \sigma_1^2 \\ \sigma_2^2 \\ \sigma_3^2 \end{pmatrix}_{meas} \qquad \mathbf{N} = \begin{pmatrix} 1 & 0 & 0 \\ C_2^2 & -2C_2S_2 & S_2^2 \\ C_3^2 & -2C_3S_3 & S_3^2 \end{pmatrix} \qquad \Pi = \begin{pmatrix} \beta_1\varepsilon \\ \alpha_1\varepsilon \\ \gamma_1\varepsilon \end{pmatrix}$$

- We want to know Π

$$\boxed{\Pi = \mathbf{N}^{-1} \cdot \Sigma}$$

Optics Measurement with 3 Screens

- Measure beam sizes and want to calculate β_1, α_1, ε
- with $\beta_1\gamma_1-\alpha_1^2 = 1$ get 3 equations for β_1, α_1 and ε

$$\begin{aligned} \beta_1 &= A/\sqrt{AC-B^2} \\ \alpha_1 &= B/\sqrt{AC-B^2} \\ \varepsilon &= \sqrt{AC-B^2} \end{aligned} \quad \text{with} \quad \begin{aligned} A &= \Pi_1 \\ B &= \Pi_2 \\ C &= \Pi_3 \end{aligned}$$

…and if there is dispersion at the screens

- Beam size at screen i $\quad \sigma_i^2 = \beta_i \cdot \varepsilon + D_i^2 \cdot \delta^2$
- D_i …dispersion, δ…momentum spread
- ➔ Measure first momentum spread in circular machine before extraction and dispersion at every screen
- If you have more than 3 screens, can try to measure δ or D with screens

Remember:

- Trajectory transforms with M_i transport matrix for $\delta \neq 0$
  - ξ_i is the contribution to the dispersion between the first and the i[th] screen

$$\begin{pmatrix} x_i \\ x_i' \\ \delta \end{pmatrix} = \begin{pmatrix} C_i & S_i & \xi_i \\ C_i' & S_i' & \xi_i' \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_1' \\ \delta \end{pmatrix} \qquad D_i = C_i D_1 + S_i D_1' + \xi_i$$

6 screens with dispersion

- Your measurements are $\sigma_i^2 = \beta_i \cdot \varepsilon + D_i^2 \cdot \delta^2$
- You know how β_i and D_i transform depending on the optical functions of screen 1.
- Build again system $\Sigma = N\Pi$

$$\Sigma = \begin{pmatrix} \sigma_1^2 \\ \sigma_2^2 \\ \sigma_3^2 \\ \sigma_4^2 \\ \sigma_5^2 \\ \sigma_6^2 \end{pmatrix}, \Pi = \begin{pmatrix} \beta_1\varepsilon + D_1^2\delta^2 \\ \alpha_1\varepsilon - D_1 D_1'\delta^2 \\ \gamma_1\varepsilon + D_1'^2\delta^2 \\ D_1\delta^2 \\ D_1'\delta^2 \\ \delta^2 \end{pmatrix}, \mathbf{N} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ C_2^2 & -2C_2S_2 & S_2^2 & 2C_2\xi_2 & 2S_2\xi_2 & \xi_2 \\ C_3^2 & -2C_3S_3 & S_3^2 & 2C_3\xi_3 & 2S_3\xi_3 & \xi_3 \\ C_4^2 & -2C_4S_4 & S_4^2 & 2C_4\xi_4 & 2S_4\xi_4 & \xi_4 \\ C_5^2 & -2C_5S_5 & S_5^2 & 2C_5\xi_5 & 2S_5\xi_5 & \xi_5 \\ C_6^2 & -2C_6S_6 & S_6^2 & 2C_6\xi_6 & 2S_6\xi_6 & \xi_6 \end{pmatrix}$$

- Result is again $\Pi = N^{-1}\Sigma$
- ➔ Can measure β, α, ε, D, D' and δ with 6 screens without any other measurements.

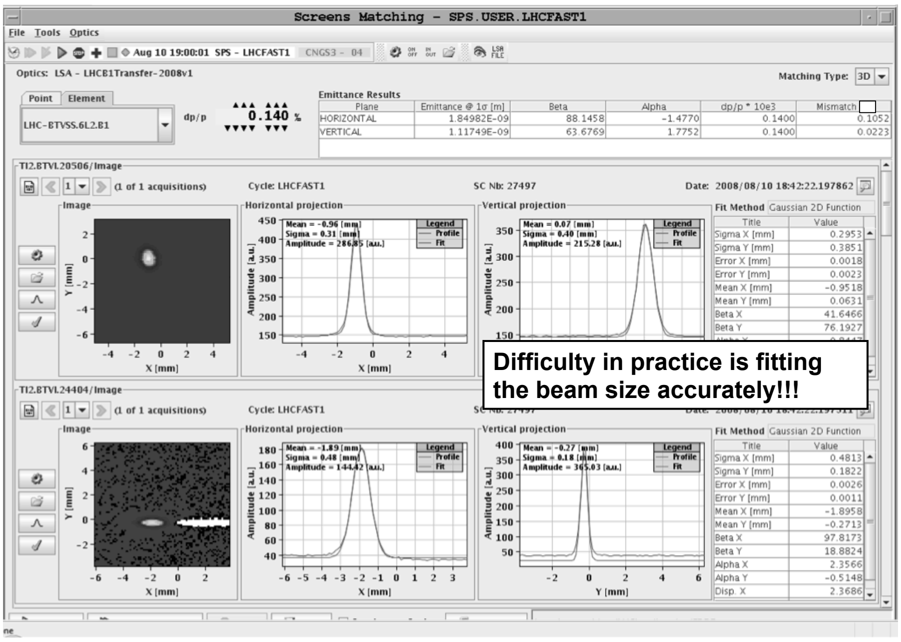
More than 6 screens…

- Fit procedure…
- Function to be minimized: Δ_i…measurement error

$$\chi^2(\Pi) = \sum_{i=1}^{N_{mon}} \left[\frac{\Sigma_i - (N\Pi)_i}{\Delta_i} \right]^2 \qquad \frac{\partial \chi^2}{\partial \Pi_i} = 0 \quad (*)$$

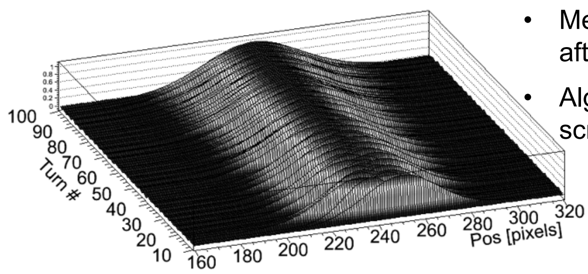
- Equation (*) can be solved analytically see
  - G. Arduini et al., "New methods to derive the optical and beam parameters in transport channels", Nucl. Instrum. Methods Phys. Res., 2001.

In Practice….



Difficulty in practice is fitting the beam size accurately!!!

Matching screen



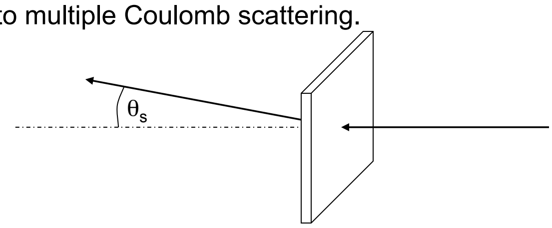
- 1 screen in the circular machine
- Measure turn-by-turn profile after injection
- Algorithm same as for several screens in transfer line

Profiles at matching monitor after injection with steering error.

- Only allowed with low intensity beam
- Issue: radiation hard fast cameras

Blow-up from thin scatterer

- Scattering elements are sometimes required in the beam
  - Thin beam screens (Al_2O_3,Ti) used to generate profiles.
  - Metal windows also used to separate vacuum of transfer lines from vacuum in circular machines.
  - Foils are used to strip electrons to change charge state
- The emittance of the beam increases when it passes through, due to multiple Coulomb scattering.



rms angle increase: $\sqrt{\langle \theta_s^2 \rangle}[mrad] = \frac{14.1}{\beta_c p[MeV/c]} Z_{inc} \sqrt{\frac{L}{L_{rad}}} \left(1 + 0.11 \cdot \log_{10} \frac{L}{L_{rad}}\right)$

β_c = v/c, p = momentum, Z_{inc} = particle charge /e, L = target length, L_{rad} = radiation length

Blow-up from thin scatterer

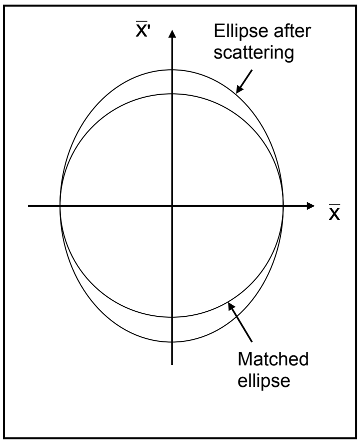
Each particles gets a random angle change θ_s but there is no effect on the positions at the scatterer

$$\overline{X}_{new} = \overline{X}_0$$

$$\overline{X}'_{new} = \overline{X}'_0 + \sqrt{\beta}\theta_s$$

After filamentation the particles have different amplitudes and the beam has a larger emittance

$$\varepsilon = \left\langle A_{new}^2 \right\rangle / 2$$



Blow-up from thin scatterer

$$A^2_{new} = \overline{X}^2_{new} + \overline{X}'^2_{new}$$

$$= \overline{X}_0^2 + \left(\overline{X}'_0 + \sqrt{\beta}\theta_s\right)^2$$

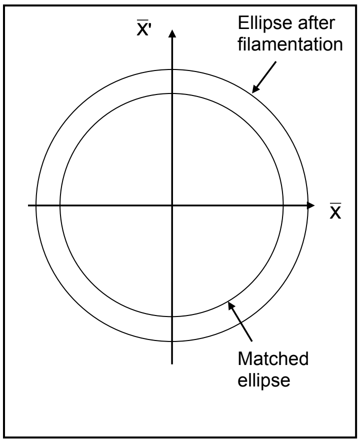
$$= \overline{X}_0^2 + \overline{X}_0'^2 + 2\sqrt{\beta}\left(\overline{X}'_0\theta_s\right) + \beta\theta_s^2$$

uncorrelated

$$\left\langle A^2_{new}\right\rangle = \left\langle \overline{X}_0^2\right\rangle + \left\langle \overline{X}_0'^2\right\rangle + 2\sqrt{\beta}\left\langle \overline{X}'_0\theta_s\right\rangle + \beta\left\langle \theta_s^2\right\rangle$$

$$= 2\varepsilon_0 + 2\sqrt{\beta}\left\langle \cancelto{0}{\overline{X}'_0}\right\rangle\left\langle \theta_s\right\rangle + \beta\left\langle \theta_s^2\right\rangle$$

$$= 2\varepsilon_0 + \beta\left\langle \theta_s^2\right\rangle$$

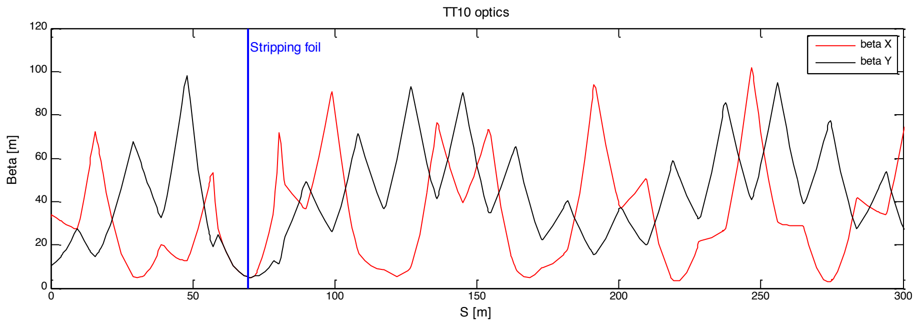


$$\boxed{\varepsilon_{new} = \varepsilon_0 + \frac{\beta}{2}\left\langle \theta_s^2\right\rangle}$$

<u>Need to keep β small to minimise blow-up</u> (small β means large spread in angles in beam distribution, so additional angle has small effect on distn.)

Blow-up from charge stripping foil

- For LHC heavy ions, Pb^{53+} is stripped to Pb^{82+} at 4.25GeV/u using a 0.8mm thick Al foil, in the PS to SPS line
- $\Delta\varepsilon$ is minimised with low-β insertion (β_{xy} ~5 m) in the transfer line
- Emittance increase expected is about 8%



Kick-response measurement

- The observable during kick-response measurement are the elements of the response matrix R

$$R_{ij} = \frac{u_i}{\delta_j} \qquad R_{ij}^{model} = \begin{cases} \sqrt{\beta_i \beta_j} \sin(\mu_i - \mu_j) & \text{for } \mu_i > \mu_j \\ 0 & \text{otherwise} \end{cases}$$

  - u_i is the position at the i^{th} monitor
  - δ_j is the kick of the j^{th} corrector

- Cannot read off optics parameters directly
- A fit varies certain parameters of a machine model to reproduce the measured data ➔ LOCO principle
- The fit minimizes the quadratic norm of a difference vector V

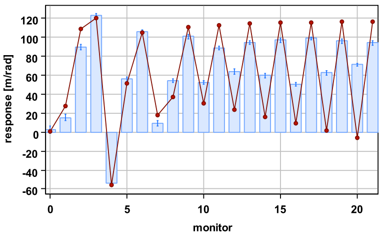
$$V_k = \frac{R_{ij}^{meas} - R_{ij}^{model}}{\sigma_i} \qquad k = i \cdot (N_c - 1) + j$$

σ_i… BPM rms noise
N_c… number of correctors

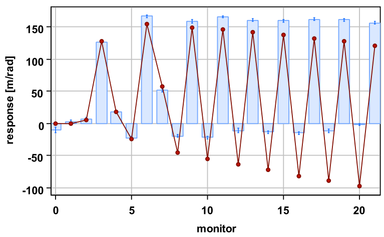
Reference: K. Fuchsberger, CERN-THESIS-2011-075

Example: LHC transfer line TI 8

- Phase error in the vertical plane

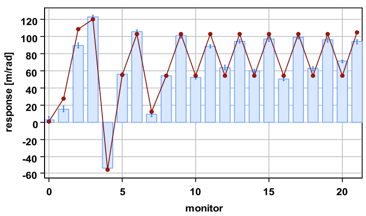


(a) MDMV.400097

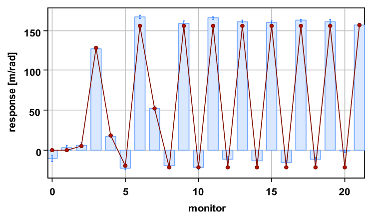


(b) MDSV.400293

- Traced back to error in QD strength in transfer line arc:



(a) MDMV.400097



(b) MDSV.400293

Summary

- Transfer lines present interesting challenges and differences from circular machines
  - No periodic condition mean optics is defined by transfer line element strengths <u>and by initial beam ellipse</u>
  - Matching at the extremes is subject to many constraints
  - Emittance blow-up is an important consideration, and arises from several sources
  - The optics of transfer line has to be well understood
  - Several ways of assessing optics parameters in the transfer line have been shown

EXTRA SLIDES

Blow-up from betatron mismatch

General betatron motion

$$x_2 = \sqrt{a_2\beta_2}\,\sin(\varphi+\varphi_o), \qquad x'_2 = \sqrt{a_2/\beta_2}\left[\cos(\varphi+\varphi_o) - \alpha_2\sin(\varphi+\varphi_o)\right]$$

applying the normalising transformation for the matched beam

$$\begin{bmatrix} \overline{X}_2 \\ \overline{X}'_2 \end{bmatrix} = \sqrt{\frac{1}{\beta_1}} \cdot \begin{bmatrix} 1 & 0 \\ \alpha_1 & \beta_1 \end{bmatrix} \cdot \begin{bmatrix} x_2 \\ x'_2 \end{bmatrix}$$

an ellipse is obtained in <u>normalised</u> phase space

$$A^2 = \overline{X}_2^2\left[\frac{\beta_1}{\beta_2} + \frac{\beta_2}{\beta_1}\left(\alpha_1 - \alpha_2\frac{\beta_1}{\beta_2}\right)^2\right] + \overline{X}'^2_2\frac{\beta_2}{\beta_1} - 2\overline{X}_2\overline{X}'_2\left[\frac{\beta_2}{\beta_1}\left(\alpha_1 - \alpha_2\frac{\beta_1}{\beta_2}\right)\right]$$

characterised by γ_{new}, β_{new} *and* α_{new}, where

$$\alpha_{new} = \frac{-\beta_2}{\beta_1}\left(\alpha_1 - \alpha_2\frac{\beta_1}{\beta_2}\right), \qquad \beta_{new} = \frac{\beta_2}{\beta_1}, \quad \gamma_{new} = \frac{\beta_1}{\beta_2} + \frac{\beta_2}{\beta_1}\left(\alpha_1 - \alpha_2\frac{\beta_1}{\beta_2}\right)^2$$