Beam Transfer Lines

- Distinctions between transfer lines and circular machines
- Linking machines together
- Blow-up from steering errors
- Correction of injection oscillations
- Blow-up from optics mismatch
- Optics measurement
- Blow-up from thin screens

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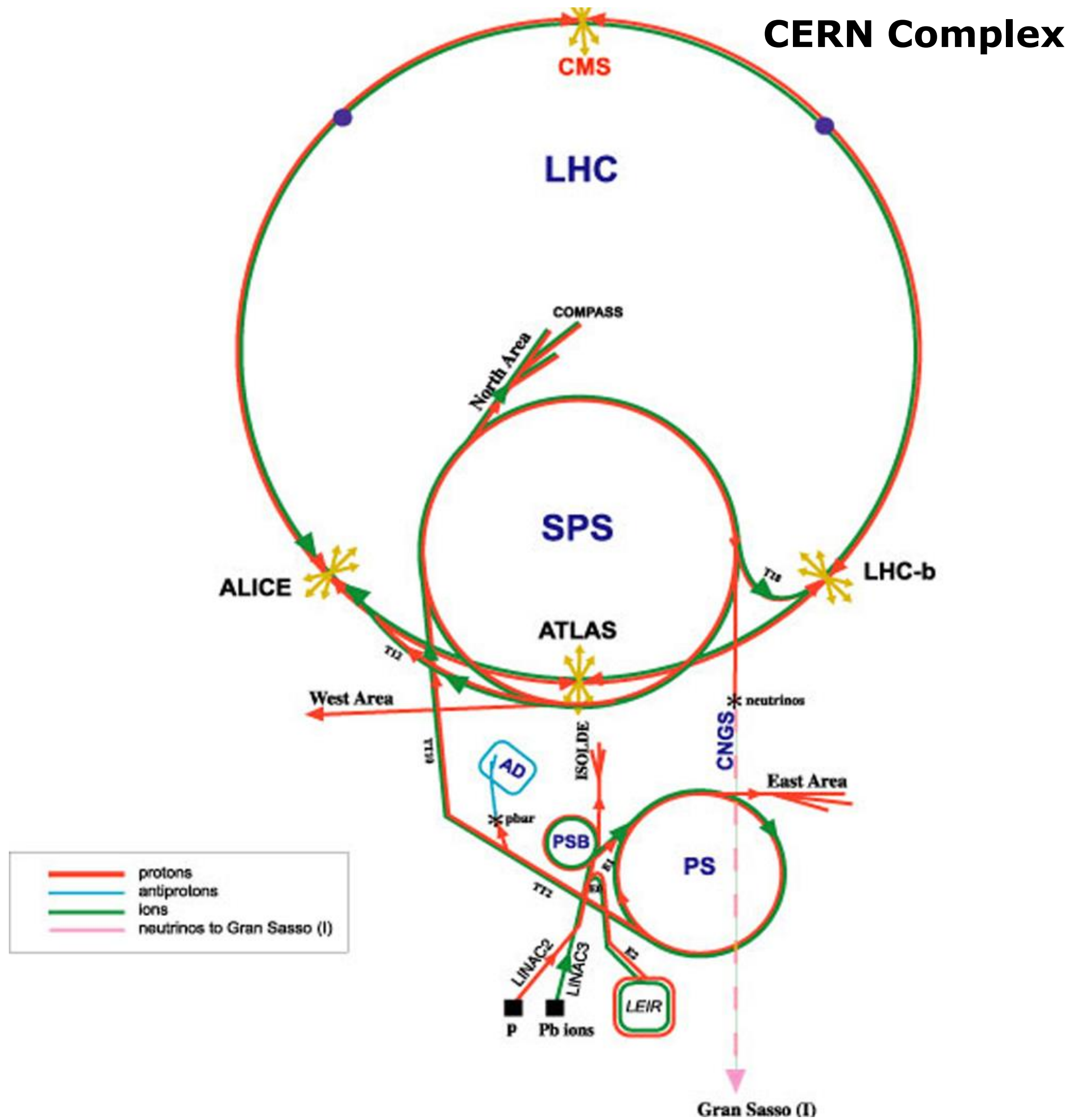
(based on lecture by B. Goddard and M. Meddahi)

Injection, extraction and transfer

- An accelerator has limited dynamic range
- Chain of stages needed to reach high energy
- Periodic re-filling of storage rings, like LHC
- External experiments, like CNGS



Transfer lines transport the beam between accelerators, and onto targets, dumps, instruments etc.

LHC:	Large Hadron Collider
SPS:	Super Proton Synchrotron
AD:	Antiproton Decelerator
ISOLDE:	Isotope Separator Online Device
PSB:	Proton Synchrotron Booster
PS:	Proton Synchrotron
LINAC:	LINear Accelerator
LEIR:	Low Energy Ring
CNGS:	CERN Neutrino to Gran Sasso

Normalised phase space

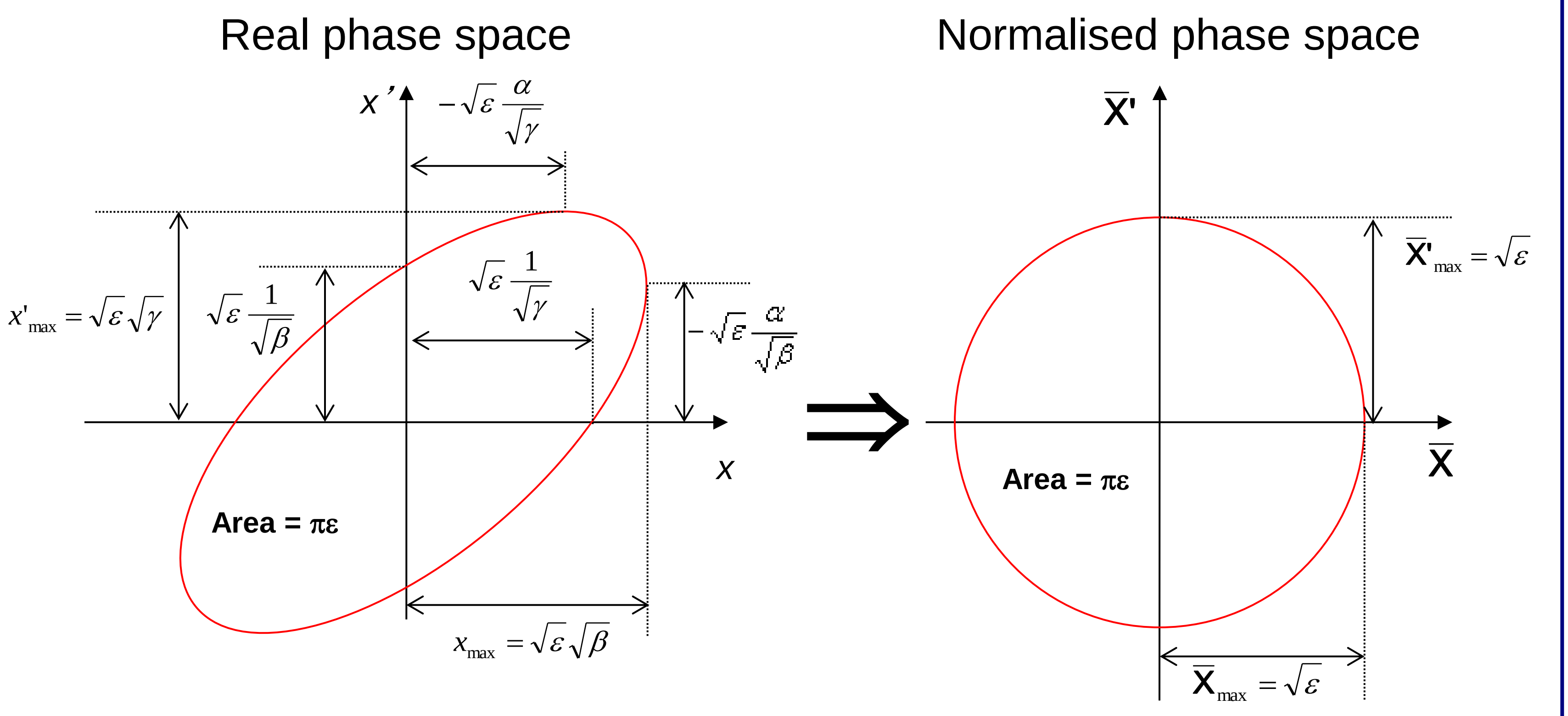
- Transform real transverse coordinates x, x' by

$$\begin{bmatrix} \overline{\mathsf{X}} \\ \overline{\mathsf{X}}' \end{bmatrix} = \mathbf{N} \cdot \begin{bmatrix} x \\ x' \end{bmatrix} = \sqrt{\frac{1}{\beta_S}} \cdot \begin{bmatrix} 1 & 0 \\ \alpha_S & \beta_S \end{bmatrix} \cdot \begin{bmatrix} x \\ x' \end{bmatrix}$$

$$\overline{\mathsf{X}} = \sqrt{\frac{1}{\beta_S}} \cdot x$$

$$\overline{\mathsf{X}}' = \sqrt{\frac{1}{\beta_S}} \cdot \alpha_S x + \sqrt{\beta_S}\, x'$$

Normalised phase space

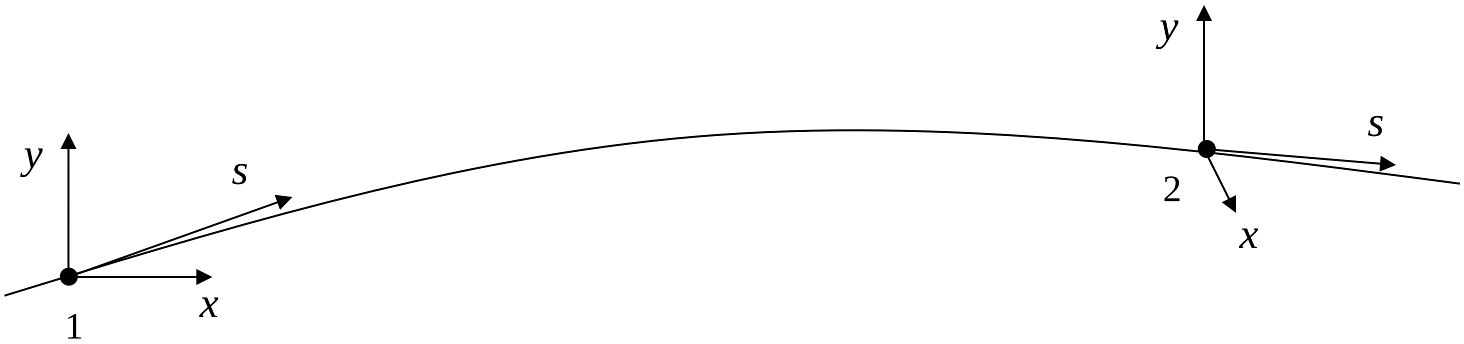


$$\varepsilon = \gamma \cdot x^2 + 2\alpha \cdot x \cdot x' + \beta \cdot x'^2$$

$$\varepsilon = \overline{\mathbf{X}}^2 + \overline{\mathbf{X}}'^2$$

General transport

Beam transport: moving from s_1 to s_2 through *n* elements, each with transfer matrix M_i

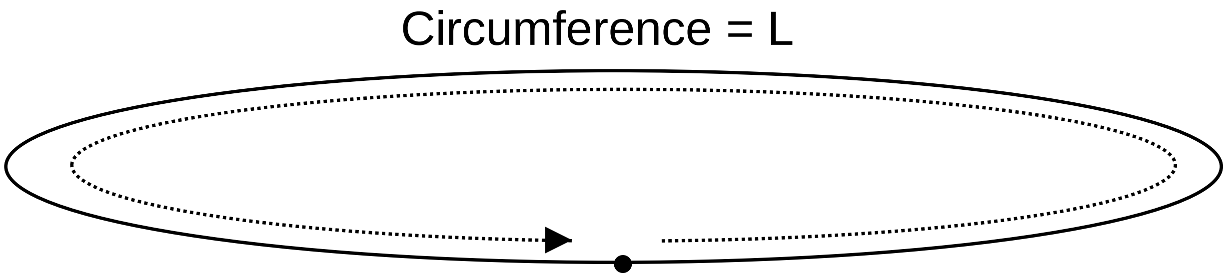


$$\begin{bmatrix} x_2 \\ x'_2 \end{bmatrix} = \mathbf{M}_{1\to 2} \cdot \begin{bmatrix} x_1 \\ x'_1 \end{bmatrix} = \begin{bmatrix} C & S \\ C' & S' \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x'_1 \end{bmatrix} \qquad \mathbf{M}_{1\to 2} = \prod_{i=1}^{n} \mathbf{M}_n$$

Twiss parameterisation

$$\mathbf{M}_{1\to 2} = \begin{bmatrix} \sqrt{\beta_2/\beta_1}\left(\cos\Delta\mu + \alpha_1 \sin\Delta\mu\right) & \sqrt{\beta_1\beta_2}\,\sin\Delta\mu \\ \sqrt{1/\beta_1\beta_2}\left[\left(\alpha_1 - \alpha_2\right)\cos\Delta\mu - \left(1 + \alpha_1\alpha_2\right)\sin\Delta\mu\right] & \sqrt{\beta_1/\beta_2}\left(\cos\Delta\mu - \alpha_2 \sin\Delta\mu\right) \end{bmatrix}$$

Circular Machine



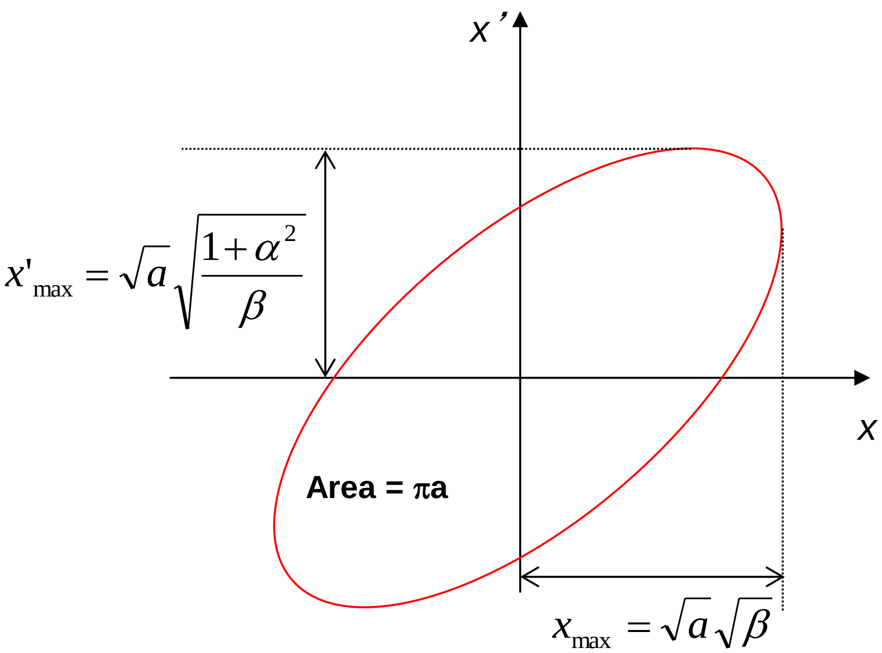
One turn

$$\mathbf{M}_{1\rightarrow 2} = \mathbf{M}_{0\rightarrow L} = \begin{bmatrix} \cos 2\pi Q + \alpha \sin 2\pi Q & \beta \sin 2\pi Q \\ -\frac{1}{\beta}\left(1+\alpha^2\right)\sin 2\pi Q & \cos 2\pi Q - \alpha \sin 2\pi Q \end{bmatrix}$$

- The solution is *periodic*
- Periodicity condition for one turn (closed ring) imposes $\alpha_1=\alpha_2$, $\beta_1=\beta_2$, $D_1=D_2$
- This condition *uniquely* determines $\alpha(s)$, $\beta(s)$, $\mu(s)$, $D(s)$ around the whole ring

Circular Machine

- Periodicity of the structure leads to regular motion
  - Map single particle coordinates on each turn at any location
  - Describes an ellipse in phase space, defined by one set of α and β values $\Rightarrow$ Matched Ellipse (for this location)

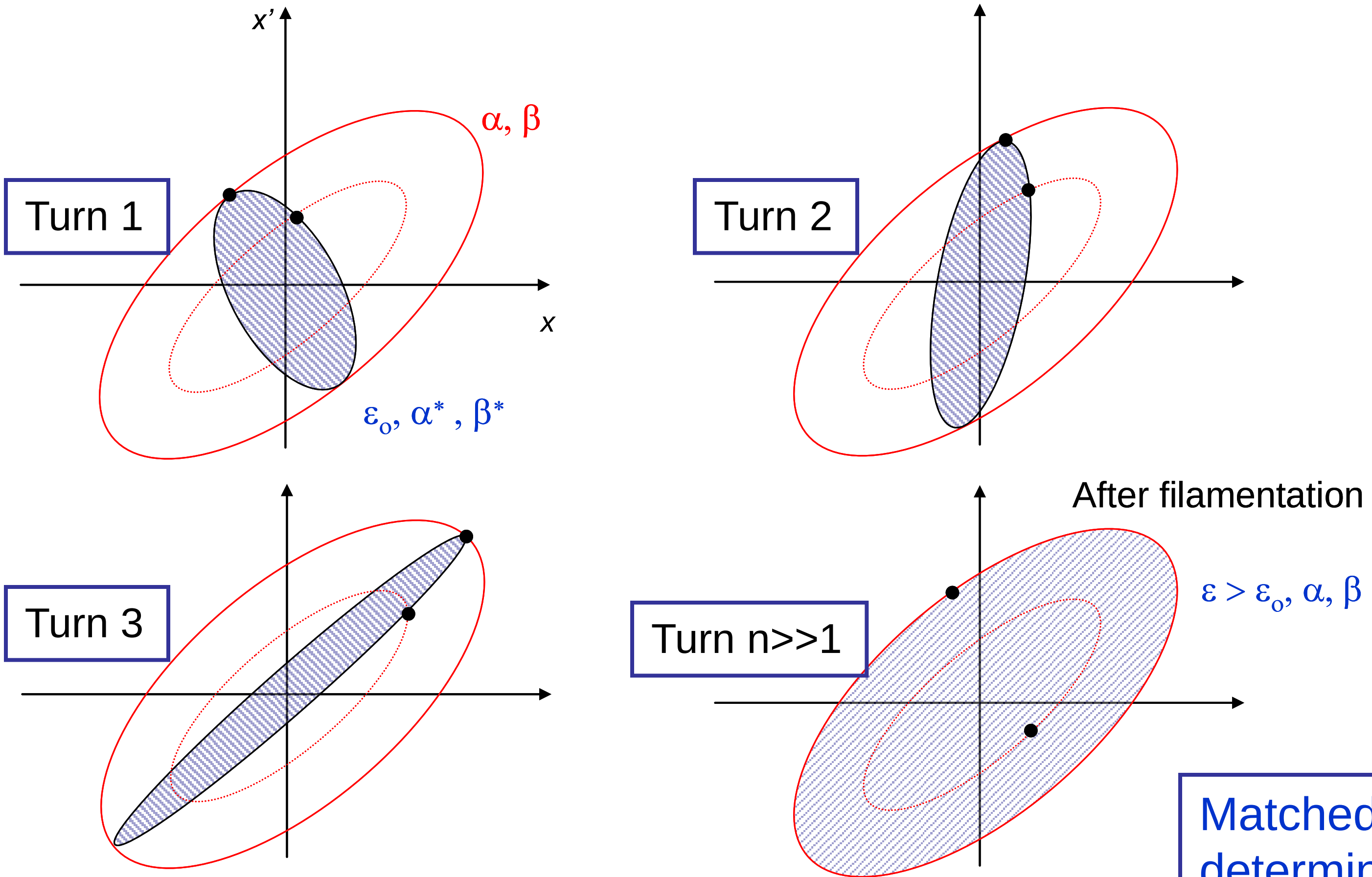


$$a = \gamma \cdot x^2 + 2\alpha \cdot x \cdot x' + \beta \cdot x'^2$$

$$\gamma = \frac{1+\alpha^2}{\beta}$$

Circular Machine

- For a location with matched ellipse (α, β), an injected beam of emittance ε, characterised by a different ellipse (α^*, β^*) generates (via filamentation) a large ellipse with the original α, β, but larger ε

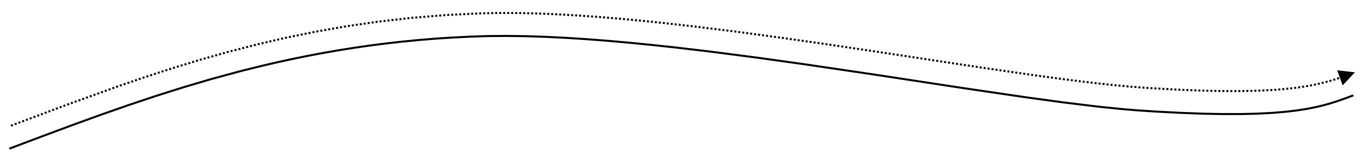


Matched ellipse determines beam shape

Transfer line

One pass: $$\begin{bmatrix} x_2 \\ x_2' \end{bmatrix} = \mathbf{M}_{1\to 2} \cdot \begin{bmatrix} x_1 \\ x_1' \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_1' \end{bmatrix}$$



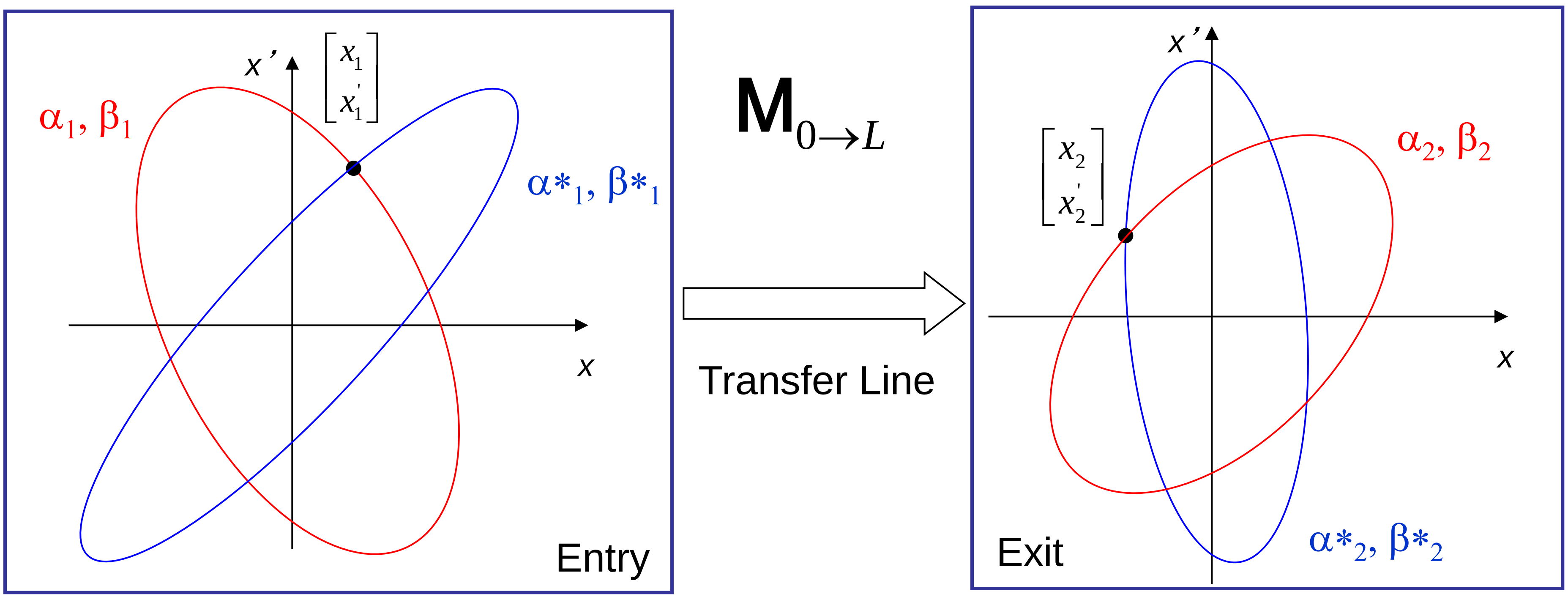
$$\begin{bmatrix} x_2 \\ x_2' \end{bmatrix}$$

$$\mathbf{M}_{1\to 2} = \begin{bmatrix} \sqrt{\beta_2/\beta_1}\left(\cos\Delta\mu + \alpha_1 \sin\Delta\mu\right) & \sqrt{\beta_1\beta_2}\sin\Delta\mu \\ \sqrt{1/\beta_1\beta_2}\left[(\alpha_1-\alpha_2)\cos\Delta\mu - (1+\alpha_1\alpha_2)\sin\Delta\mu\right] & \sqrt{\beta_1/\beta_2}\left(\cos\Delta\mu - \alpha_2\sin\Delta\mu\right) \end{bmatrix}$$

- No periodic condition exists
- The Twiss parameters are simply propagated from beginning to end of line
- At any point in line, $\alpha(s)$ $\beta(s)$ are functions of α_1 β_1

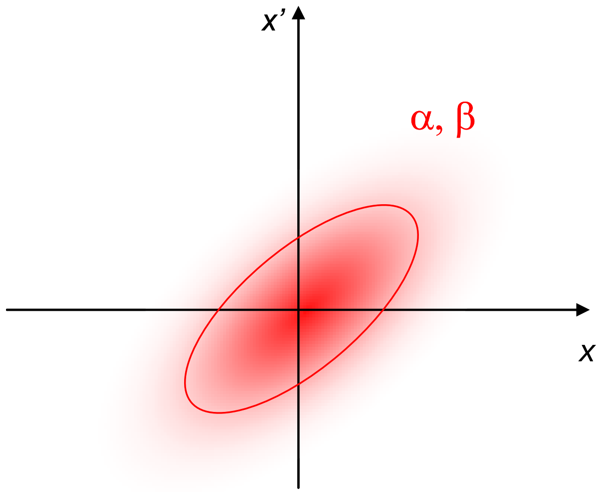
Transfer line

- On a single pass…
  - Map single particle coordinates at entrance and exit.
  - Infinite number of equally valid possible starting ellipses for single particle
    ……transported to infinite number of final ellipses…

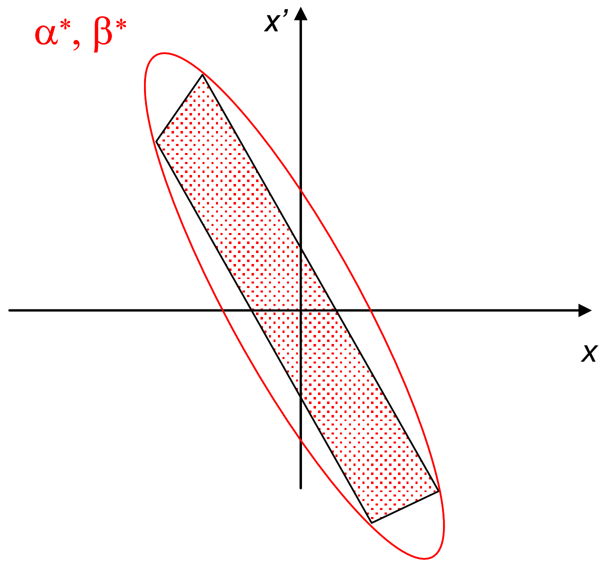


Transfer Line

- Initial α, β defined for transfer line by beam shape at entrance



Gaussian beam

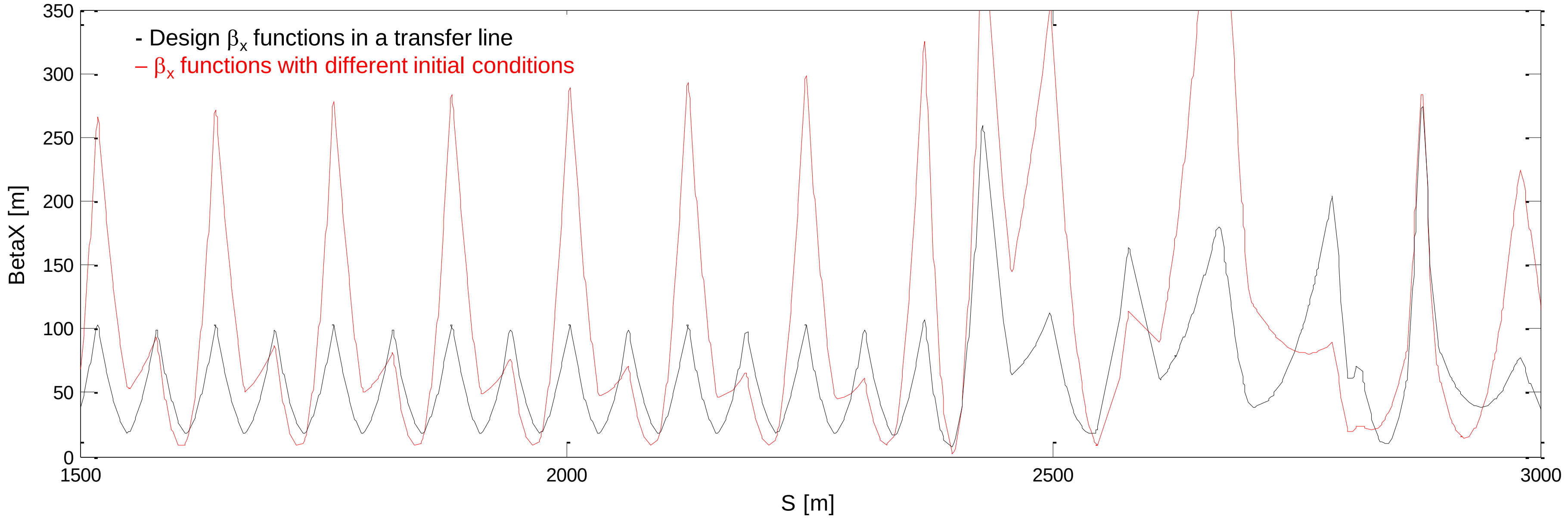


Non-Gaussian beam
(e.g. slow extracted)

- Propagation of this beam ellipse depends on line elements
- A transfer line optics is different for different input beams

Transfer Line

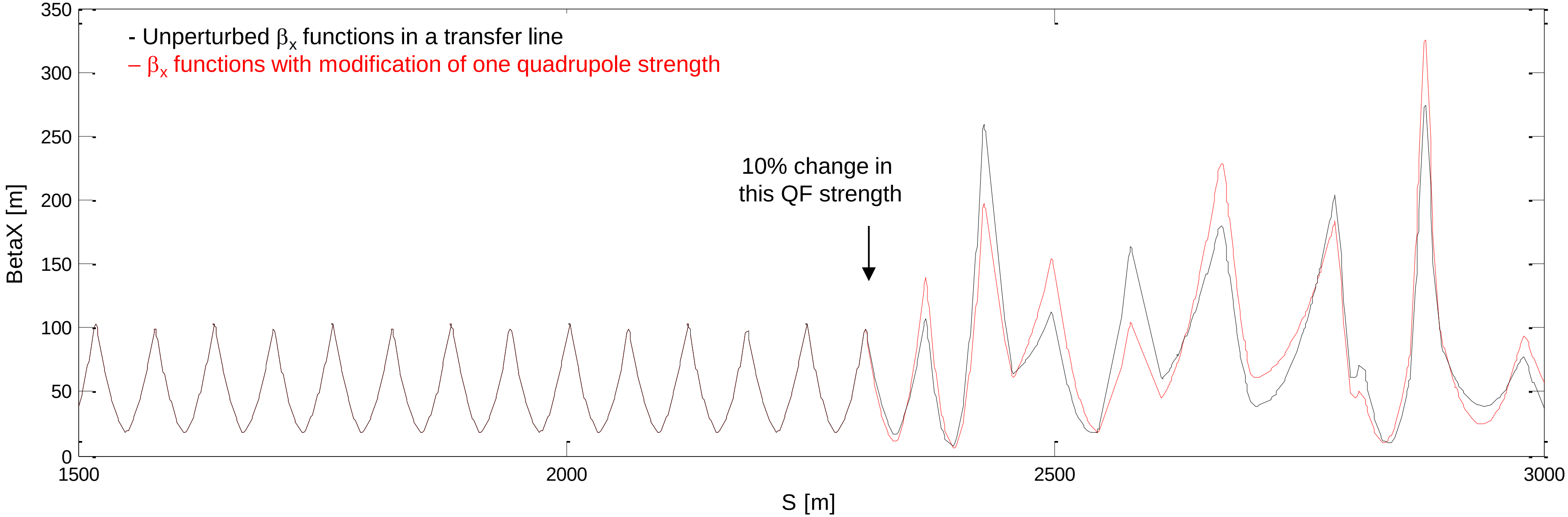
- The optics functions in the line depend on the initial values



- Same considerations are true for Dispersion function:
  - Dispersion in ring defined by periodic solution → ring elements
  - Dispersion in line defined by initial D and D' and line elements

Transfer Line

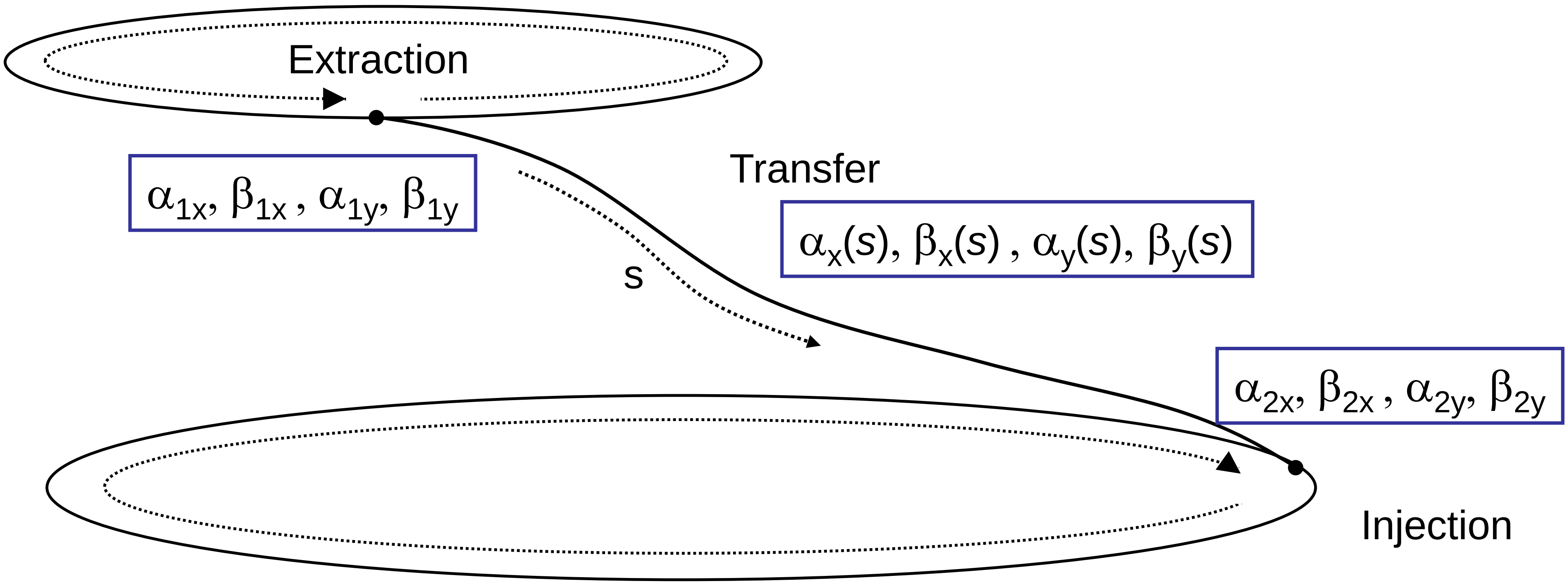
- Another difference….unlike a circular ring, a change of an element in a line affects *only* the downstream Twiss values (including dispersion)



Linking Machines

- Beams have to be transported from extraction of one machine to injection of next machine
  - Trajectories must be matched, ideally in all 6 geometric degrees of freedom (x,y,z,θ,ϕ,ψ)
- Other important constraints can include
  - Minimum bend radius, maximum quadrupole gradient, magnet aperture, cost, geology

Linking Machines



The Twiss parameters can be propagated when the transfer matrix **M** is known

$$\begin{bmatrix} x_2 \\ x_2' \end{bmatrix} = \mathbf{M}_{1\to 2} \cdot \begin{bmatrix} x_1 \\ x_1' \end{bmatrix} = \begin{bmatrix} C & S \\ C' & S' \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_1' \end{bmatrix}$$

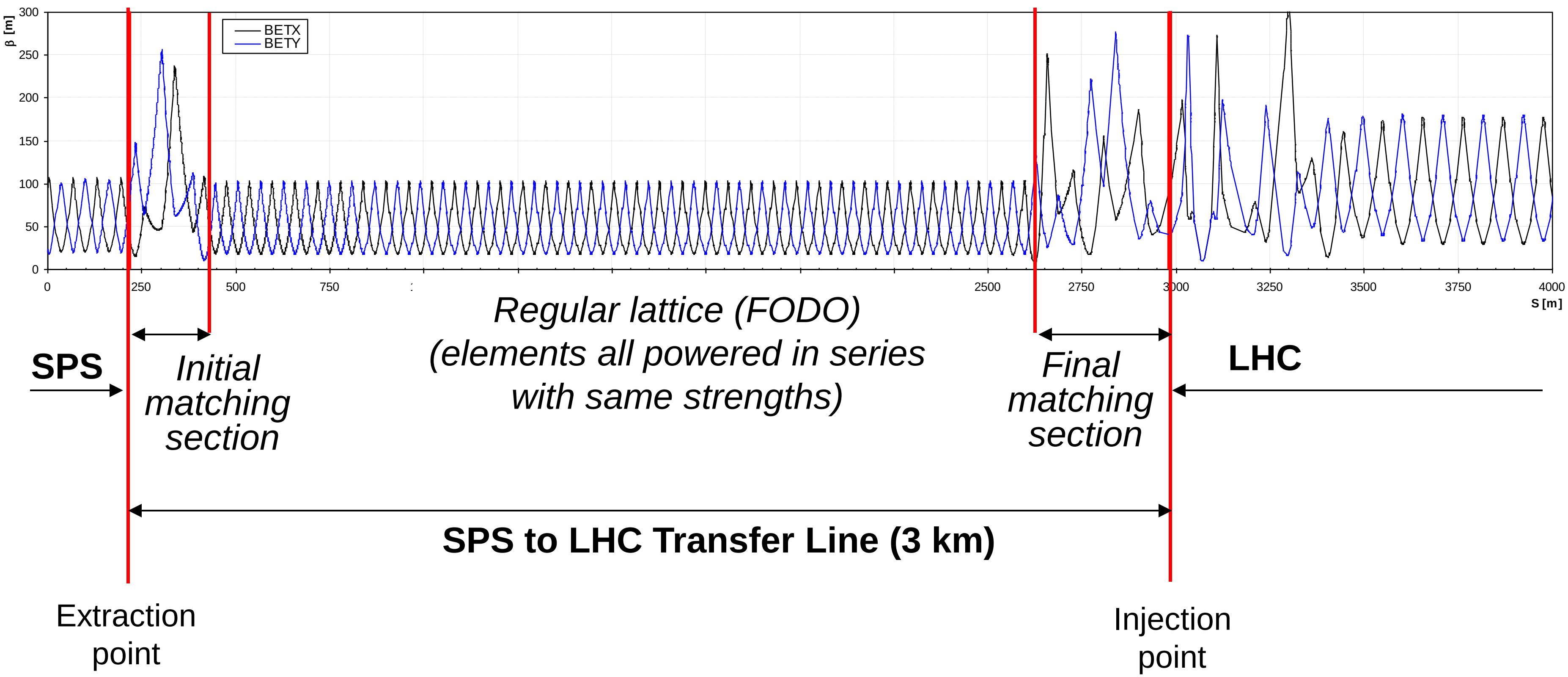
$$\begin{bmatrix} \beta_2 \\ \alpha_2 \\ \gamma_2 \end{bmatrix} = \begin{bmatrix} C^2 & -2CS & S^2 \\ -CC' & CS'+SC' & -SS' \\ C'^2 & -2C'S' & S'^2 \end{bmatrix} \cdot \begin{bmatrix} \beta_1 \\ \alpha_1 \\ \gamma_1 \end{bmatrix}$$

Linking Machines

- Linking the optics is a complicated process
  - Parameters at start of line have to be propagated to matched parameters at the end of the line
  - Need to “match” 8 variables (α_x β_x D_x D'_x and α_y β_y D_y D'_y)
  - Maximum β and D values are imposed by magnet apertures
  - Other constraints can exist
    - phase conditions for collimators,
    - insertions for special equipment like stripping foils
  - Need to use a number of independently powered (“matching”) quadrupoles
  - Matching with computer codes and relying on mixture of theory, experience, intuition, trial and error, …

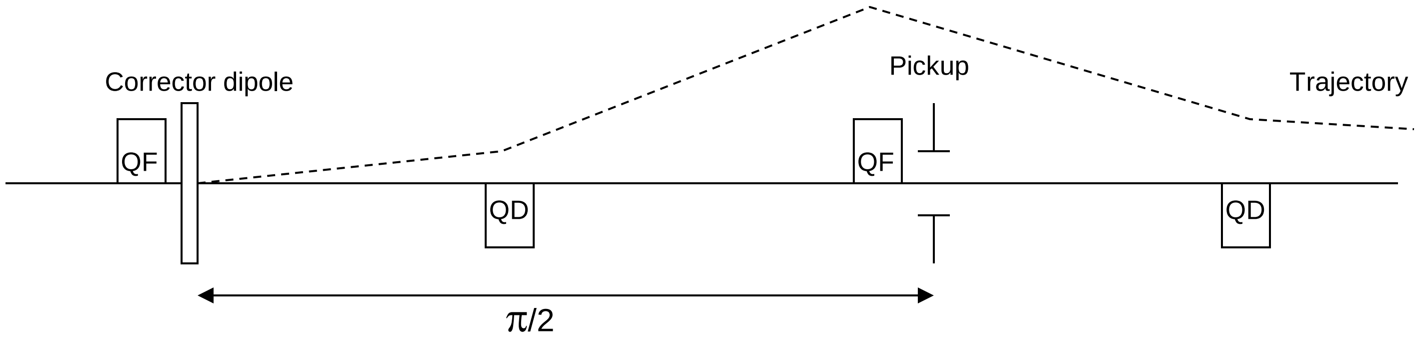
Linking Machines

- For long transfer lines we can simplify the problem by designing the line in separate sections
  - Regular central section – e.g. FODO or doublet, with quads at regular spacing, (+ bending dipoles), with magnets powered in series
  - Initial and final matching sections – independently powered quadrupoles, with sometimes irregular spacing.



Trajectory correction

- Magnet misalignments, field and powering errors cause the trajectory to deviate from the design
- Use small independently powered dipole magnets (correctors) to steer the beam
- Measure the response using monitors (pick-ups) downstream of the corrector ($\pi/2$, $3\pi/2$, …)



- Horizontal and vertical elements are separated
- H-correctors and pick-ups located at F-quadrupoles (large β_x)
- V-correctors and pick-ups located at D-quadrupoles (large β_y)

Trajectory correction

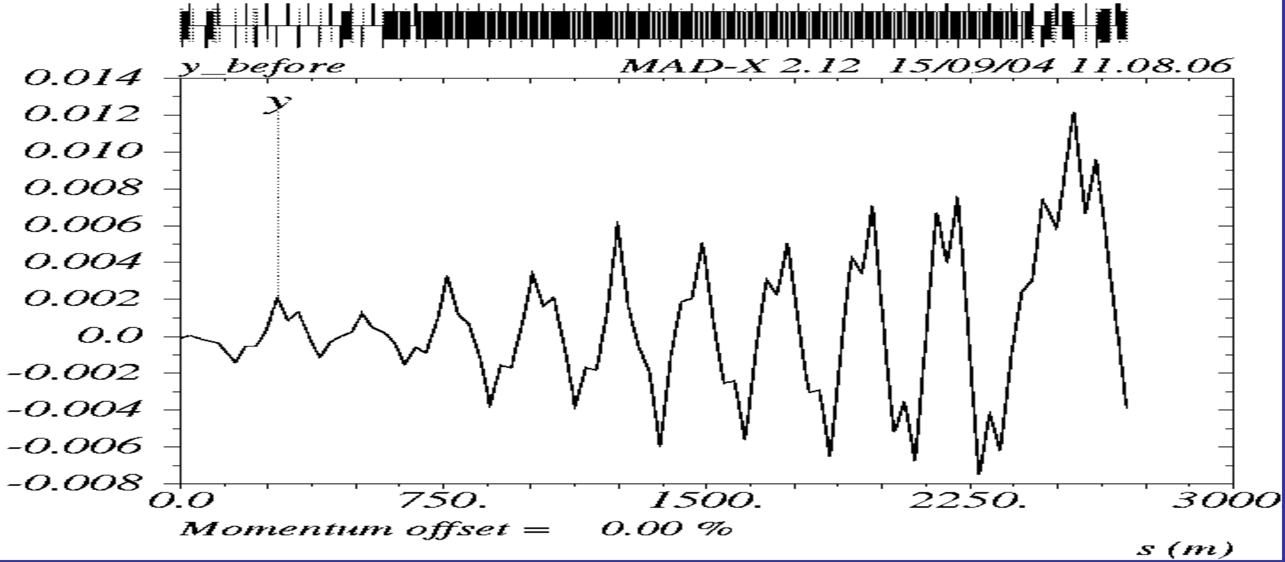
- Global correction can be used which attempts to minimise the RMS offsets at the BPMs, using all or some of the available corrector magnets.
- Steering in matching sections, extraction and injection region requires particular care
  - D and β functions can be large $\rightarrow$ bigger beam size
  - Often very limited in aperture
  - Injection offsets can be detrimental for performance

Trajectory correction

Uncorrected trajectory.

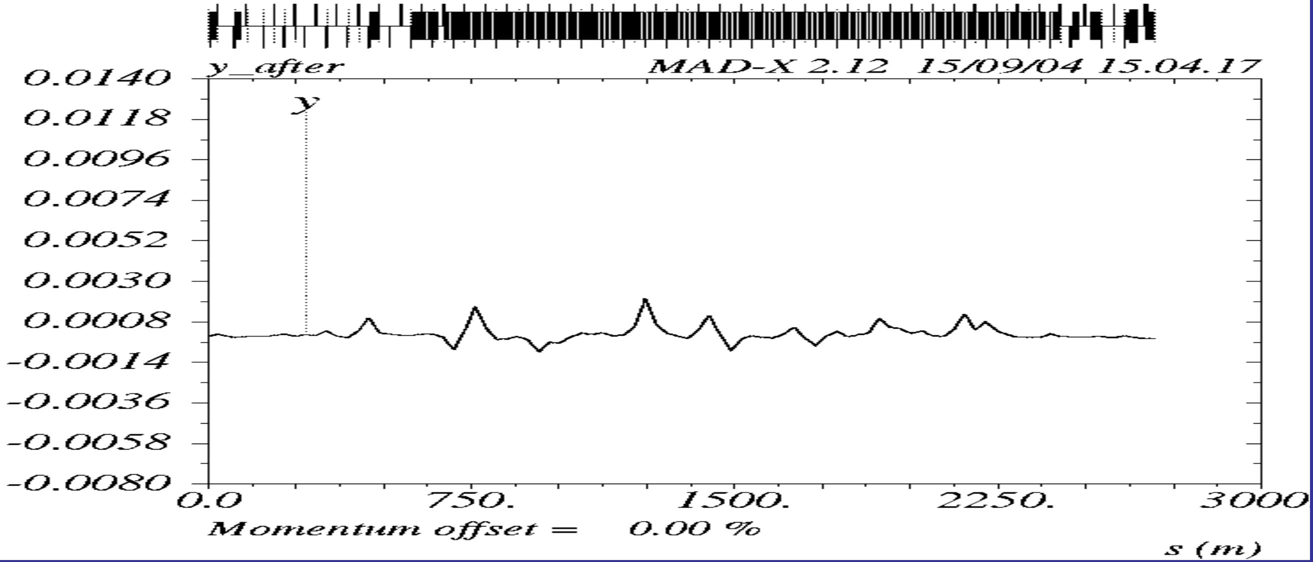
y growing as a result of random errors in the line.

The RMS at the BPMs is 3.4 mm, and y_{max} is 12.0mm



Corrected trajectory.

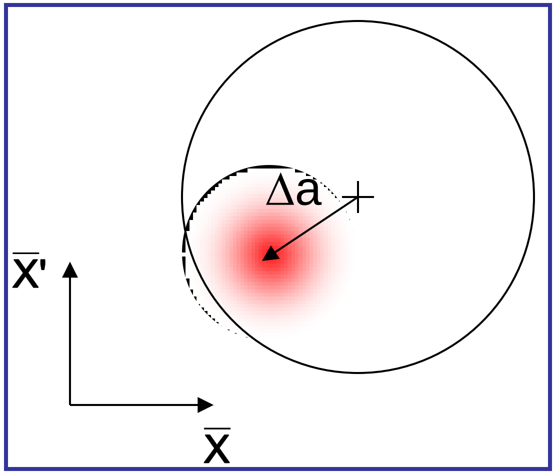
The RMS at the BPMs is 0.3mm and y_{max} is 1mm

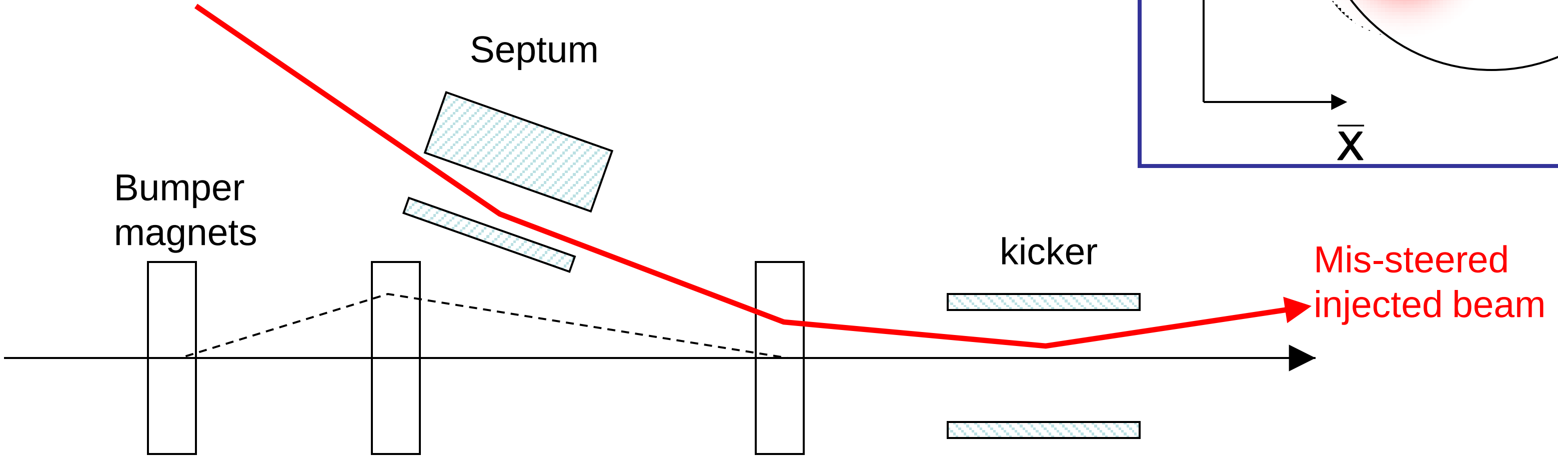


Steering (dipole) errors

- Precise delivery of the beam is important.
  - To avoid **injection oscillations** and emittance growth in rings
  - For stability on secondary particle production targets
- Convenient to express injection error in σ (includes x and x' errors)

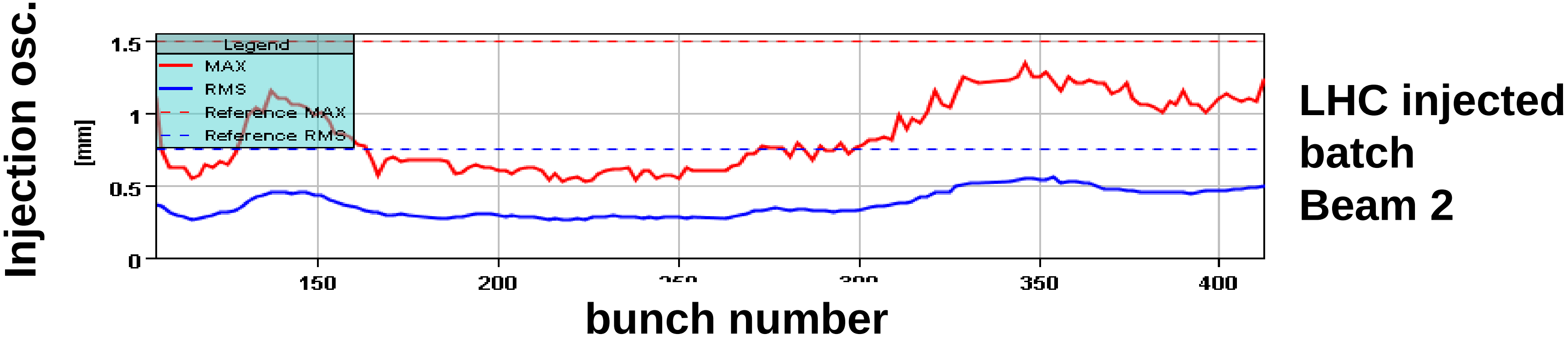
$$\Delta a\ [\sigma] = \sqrt{(\mathbf{X}^2+\mathbf{X}'^2)/\varepsilon} = \sqrt{(\gamma x^2 + 2\alpha x x' + \beta x'^2)/\varepsilon}$$





Steering (dipole) errors

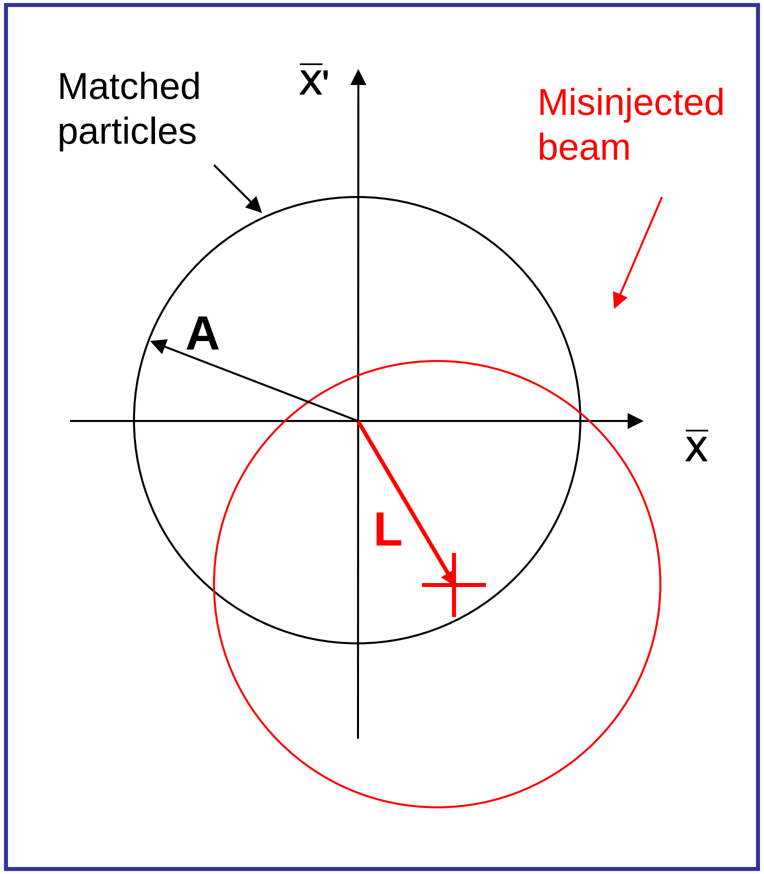
- Static effects (e.g. from errors in alignment, field, calibration, …) are dealt with by trajectory correction (steering).
- But there are also dynamic effects, from:
  - Power supply ripples
  - Temperature variations
  - Non-trapezoidal kicker waveforms
- These dynamic effects produce a variable injection offset which can vary from batch to batch, or even within a batch.



- An **injection damper system** is used to minimize effect on emittance

Blow-up from steering error

- Consider a collection of particles with max. amplitudes A
- The beam can be injected with a error in angle and position.
- For an injection error Δa_y (in units of sigma = $\sqrt{\beta\varepsilon}$) the mis-injected beam is offset in normalised phase space by $L = \Delta a_y \sqrt{\varepsilon}$



Blow-up from steering error

- The new particle coordinates in normalised phase space are

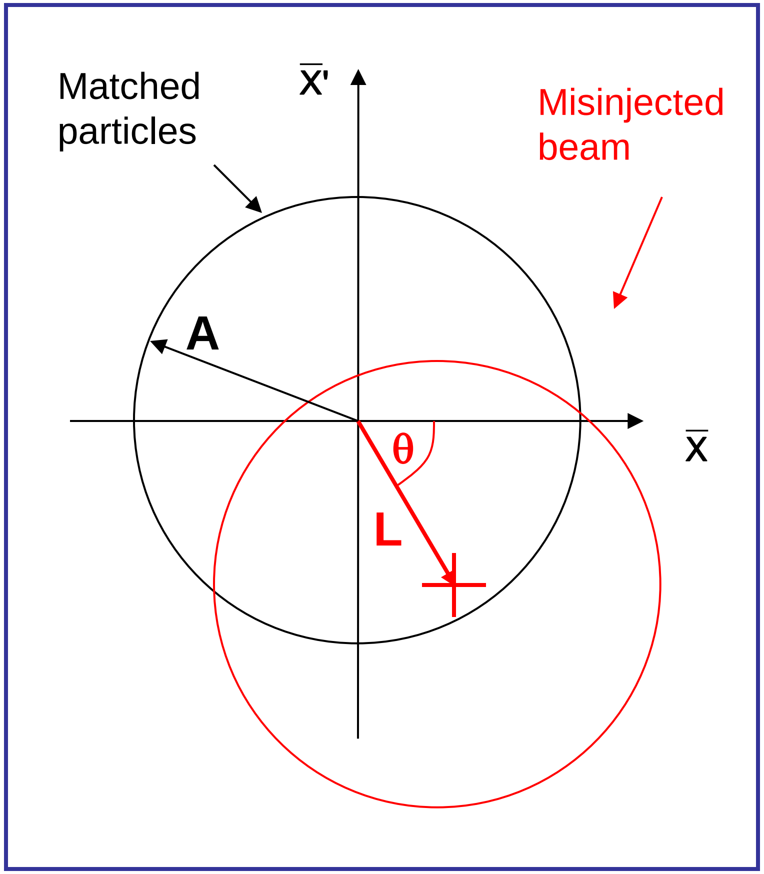
$$\overline{X}_{new} = \overline{X}_0 + L\cos\theta$$

$$\overline{X}'_{new} = \overline{X}'_0 + L\sin\theta$$

- For a general particle distribution, where A denotes amplitude in normalised phase space

$$A^2 = \overline{X}^2 + \overline{X}'^2$$

$$\varepsilon = \langle A^2 \rangle / 2$$



Blow-up from steering error

- So if we plug in the new coordinates….

$$A_{new}^2 = \bar{X}_{new}^2 + \bar{X}'^2_{new} = \left(\bar{X}_0 + L\cos\theta\right)^2 + \left(\bar{X}'_0 + L\sin\theta\right)^2$$

$$= \bar{X}_0^2 + \bar{X}_0'^2 + 2L\left(\bar{X}_0\cos\theta + \bar{X}'_0\sin\theta\right) + L^2$$

$$\left\langle A_{new}^2 \right\rangle = \left\langle \bar{X}_0^2 \right\rangle + \left\langle \bar{X}_0'^2 \right\rangle + \left\langle 2L\left(\bar{X}_0\cos\theta + \bar{X}'_0\sin\theta\right)\right\rangle + \left\langle L^2 \right\rangle$$

$$= 2\varepsilon_0 + 2L\left(\left\langle \cos\theta \bar{X}_0 \right\rangle + \left\langle \sin\theta \bar{X}'_0 \right\rangle\right) + L^2$$

(Both averaged terms struck through and marked 0.)

$$= 2\varepsilon_0 + L^2$$

- Giving for the emittance increase

$$\varepsilon_{new} = \left\langle \mathbf{A}_{new}{}^2 \right\rangle / 2 = \varepsilon_0 + \mathbf{L}^2 / 2$$

$$= \varepsilon_0\left(1 + \Delta\mathbf{a}^2 / 2\right)$$

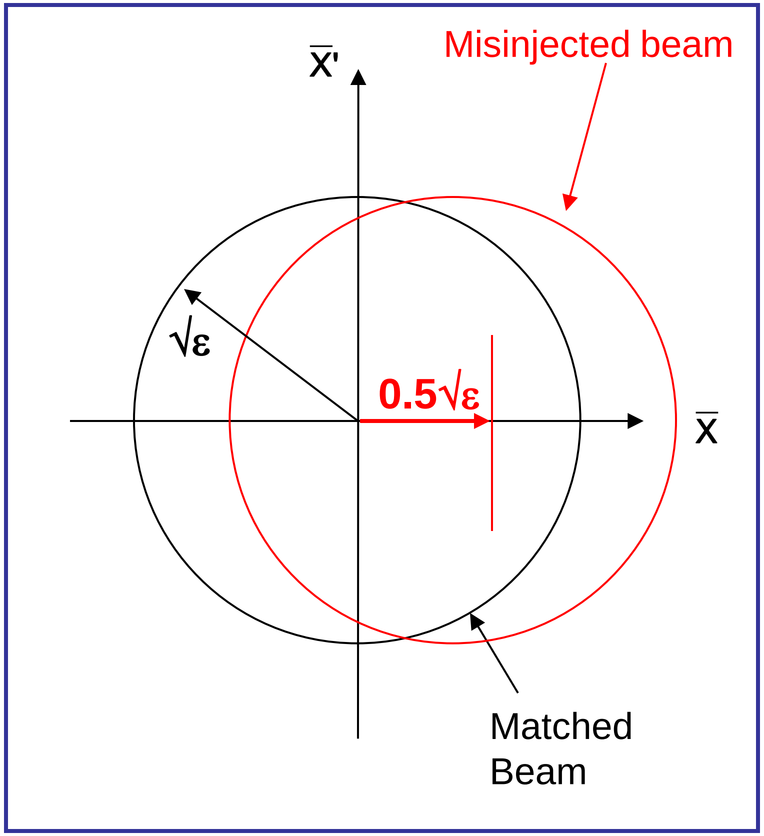
Blow-up from steering error

A numerical example….

Consider an offset Δa of 0.5 sigma for injected beam

$$\varepsilon_{new} = \varepsilon_0 \left(1 + \Delta \mathbf{a}^2 / 2\right)$$

$$= 1.125 \varepsilon_0$$



For nominal LHC beam:
ε_{norm} = 3.5 µm
allowed growth through LHC cycle ~ 10 %

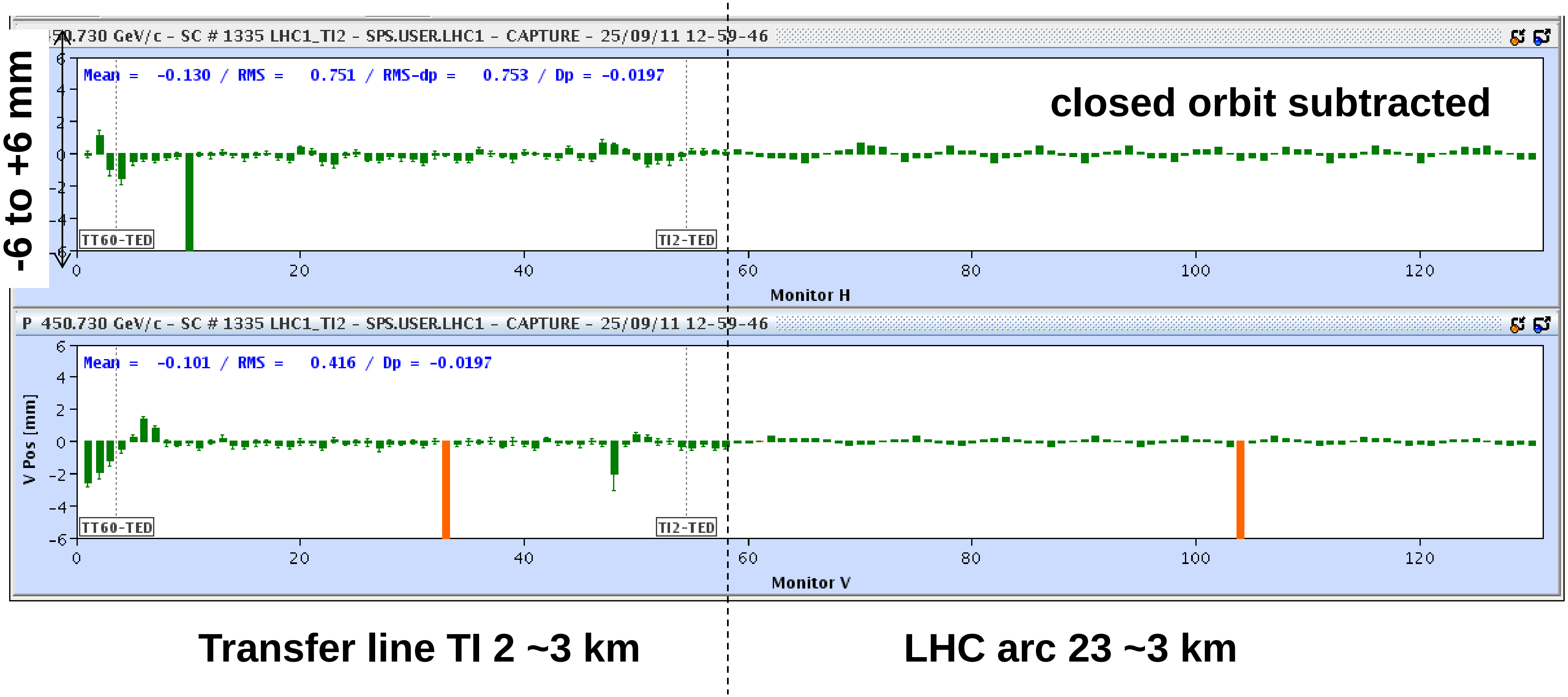
Injection oscillation correction

- x, x' and y, y' at injection point need to be corrected.
- Minimum diagnostics: 2 pickups per plane, 90° phase advance apart
- Pickups need to be triggered to measure on the first turn
- Correctors in the transfer lines are used to minimize offset at these pickups.

- Best strategy:
  - Acquire many BPMs in circular machine (e.g. one octant/sextant of machine)
  - Combine acquisition of transfer line and of BPMs in circular machine
    - Transfer line: difference trajectory to reference
    - Circular machine: remove closed orbit from first turn trajectory ➔ pure injection oscillation
  - Correct combined trajectory with correctors in transfer line with typical correction algorithms. Use correctors of the line only.

Example: LHC injection of beam 1

Display from the LHC control room to correct injection oscillations



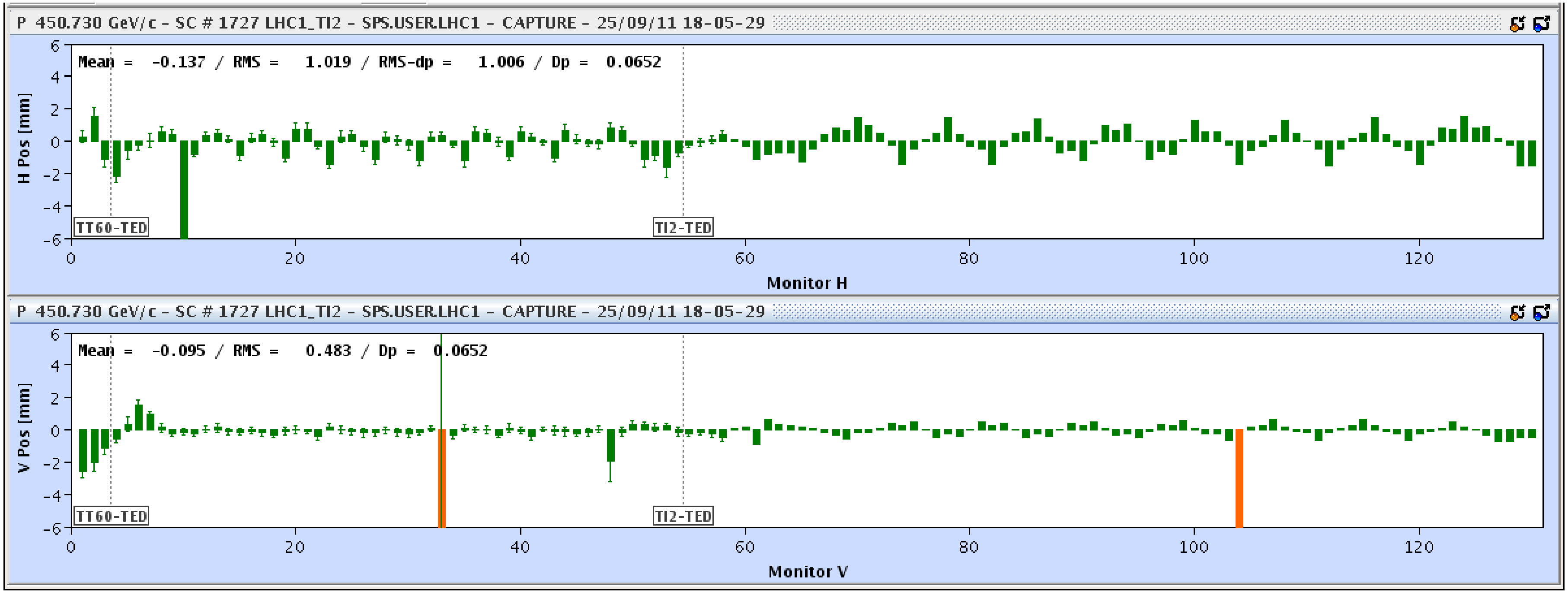
Transfer line TI 2 ~3 km

LHC arc 23 ~3 km

Injection point in LHC IR2

Example: LHC injection of beam 1

- Oscillation down the line has developed in horizontal plane
- Injection oscillation amplitude > 1.5 mm
- Good working range of LHC transverse damper +/- 2 mm

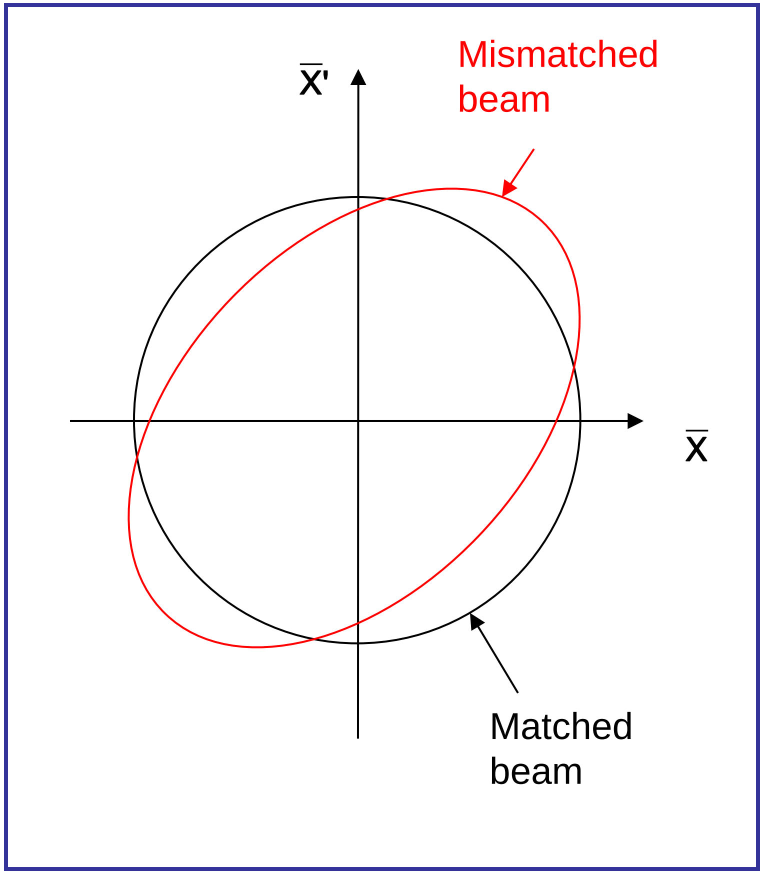


- Aperture margin for injection oscillation is 2 mm

➓ ➔ correct trajectory in line before continue LHC filling

Blow-up from betatron mismatch

- Optical errors occur in transfer line and ring, such that the beam can be injected with a mismatch.
- Filamentation will produce an emittance increase.
- In normalised phase space, consider the matched beam as a circle, and the mismatched beam as an ellipse.



Blow-up from betatron mismatch

General betatron motion

$$x_2 = \sqrt{a_2 b_2}\,\sin(j + j_o), \qquad x'_2 = \sqrt{a_2 / b_2}\,\left[\cos(j + j_o) - a_2 \sin(j + j_o)\right]$$

applying the normalising transformation for the matched beam

$$\begin{bmatrix} \overline{X}_2 \\ \overline{X}'_2 \end{bmatrix} = \sqrt{\frac{1}{\beta_1}} \cdot \begin{bmatrix} 1 & 0 \\ \alpha_1 & \beta_1 \end{bmatrix} \cdot \begin{bmatrix} x_2 \\ x'_2 \end{bmatrix}$$

an ellipse is obtained in normalised phase space

$$A^2 = \overline{X}_2^2 \left[\frac{\beta_1}{\beta_2} + \frac{\beta_2}{\beta_1} \left(\alpha_1 - \alpha_2 \frac{\beta_1}{\beta_2} \right)^2 \right] + \overline{X}'^2_2 \frac{\beta_2}{\beta_1} - 2 \overline{X}_2 \overline{X}'_2 \left[\frac{\beta_2}{\beta_1} \left(\alpha_1 - \alpha_2 \frac{\beta_1}{\beta_2} \right) \right]$$

characterised by γ_{new}, β_{new} *and* α_{new}, where

$$\alpha_{new} = \frac{-\beta_2}{\beta_1} \left(\alpha_1 - \alpha_2 \frac{\beta_1}{\beta_2} \right), \qquad \beta_{new} = \frac{\beta_2}{\beta_1}, \qquad \gamma_{new} = \frac{\beta_1}{\beta_2} + \frac{\beta_2}{\beta_1} \left(\alpha_1 - \alpha_2 \frac{\beta_1}{\beta_2} \right)^2$$

Blow-up from betatron mismatch

From the general ellipse properties

$$a = \frac{A}{\sqrt{2}}\left(\sqrt{H+1} + \sqrt{H-1}\right), \qquad b = \frac{A}{\sqrt{2}}\left(\sqrt{H+1} - \sqrt{H-1}\right)$$

where

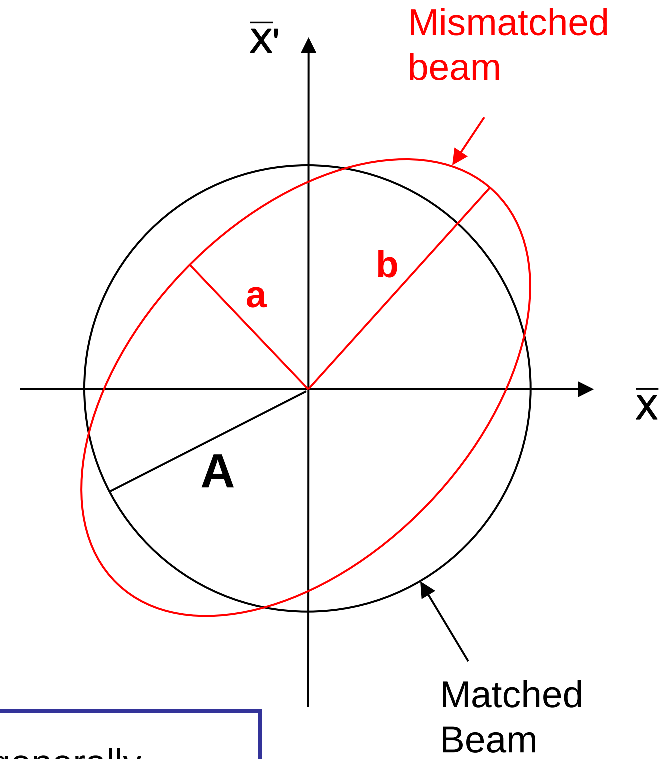
$$H = \frac{1}{2}\left(\gamma_{new} + \beta_{new}\right)$$

$$= \frac{1}{2}\left(\frac{\beta_1}{\beta_2} + \frac{\beta_2}{\beta_1}\left(\alpha_1 - \alpha_2\frac{\beta_1}{\beta_2}\right)^2 + \frac{\beta_2}{\beta_1}\right)$$

giving

$$\lambda = \frac{1}{\sqrt{2}}\left(\sqrt{H+1} + \sqrt{H-1}\right), \qquad \frac{1}{\lambda} = \frac{1}{\sqrt{2}}\left(\sqrt{H+1} - \sqrt{H-1}\right)$$

$$\overline{\mathbf{X}}_{new} = \lambda \cdot \mathbf{A}\sin(\phi + \phi_1), \qquad \overline{\mathbf{X}}'_{new} = \frac{1}{\lambda}\mathbf{A}\cos(\phi + \phi_1)$$



generally

$$a = \mathrm{A}/\lambda$$
$$b = \mathrm{A}\cdot\lambda$$

Blow-up from betatron mismatch

We can evaluate the square of the distance of a particle from the origin as

$$\mathbf{A}^2_{new} = \overline{\mathbf{X}}^2_{new} + \overline{\mathbf{X}}'^2_{new} = \lambda^2 \cdot \mathbf{A}_0^2 \sin^2(\phi + \phi_1) + \frac{1}{\lambda^2} \mathbf{A}_0^2 \cos^2(\phi + \phi_1)$$

The new emittance is the average over all phases

$$\varepsilon_{new} = \frac{1}{2}\left\langle \mathbf{A}^2_{new} \right\rangle = \frac{1}{2}\left(\lambda^2 \left\langle \mathbf{A}_0^2 \sin^2(\phi + \phi_1) \right\rangle + \frac{1}{\lambda^2} \left\langle \mathbf{A}_0^2 \cos^2(\phi + \phi_1) \right\rangle \right)$$

$$= \frac{1}{2}\left\langle \mathbf{A}_0^2 \right\rangle \left(\lambda^2 \left\langle \sin^2(\phi + \phi_1) \right\rangle + \frac{1}{\lambda^2} \left\langle \cos^2(\phi + \phi_1) \right\rangle \right)$$

(each averaged term $\left\langle \sin^2(\phi+\phi_1) \right\rangle$ and $\left\langle \cos^2(\phi+\phi_1) \right\rangle$ struck out and replaced by 0.5)

$$= \frac{1}{2}\varepsilon_0 \left(\lambda^2 + \frac{1}{\lambda^2} \right)$$

If we' re feeling diligent, we can substitute back for λ to give

$$\varepsilon_{new} = \frac{1}{2}\varepsilon_0 \left(\lambda^2 + \frac{1}{\lambda^2} \right) = H\varepsilon_0 = \frac{1}{2}\varepsilon_0 \left(\frac{\beta_1}{\beta_2} + \frac{\beta_2}{\beta_1}\left(\alpha_1 - \alpha_2 \frac{\beta_1}{\beta_2} \right)^2 + \frac{\beta_2}{\beta_1} \right)$$

where subscript 1 refers to matched ellipse, 2 to mismatched ellipse.

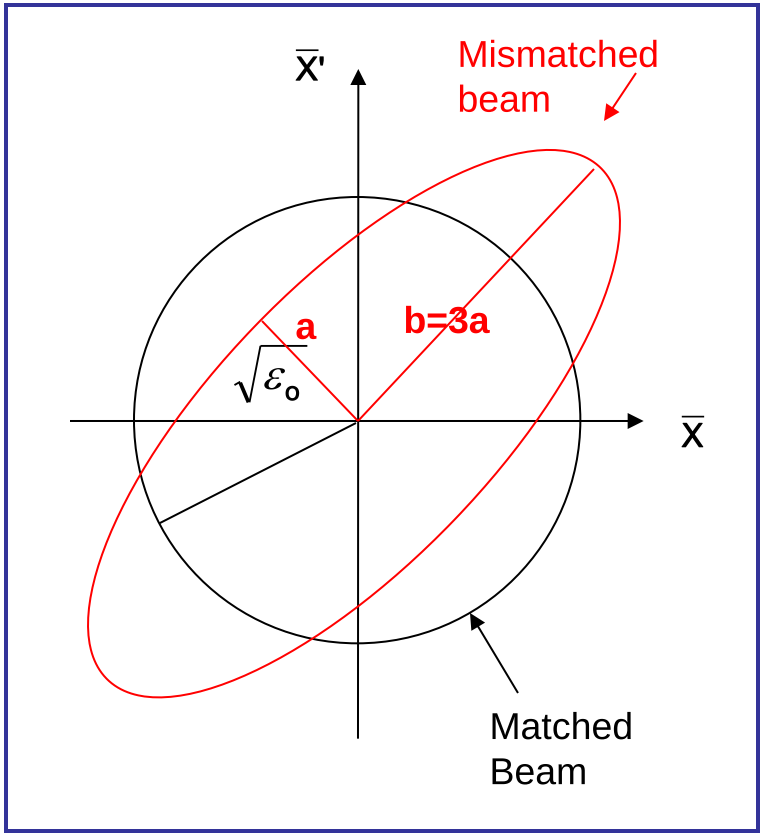
Blow-up from betatron mismatch

A numerical example….consider *b* = *3a* for the mismatched ellipse

$$\lambda = \sqrt{b/a} = \sqrt{3}$$

Then

$$\varepsilon_{new} = \frac{1}{2}\varepsilon_0\left(\lambda^2 + 1/\lambda^2\right)$$
$$= 1.67\varepsilon_0$$



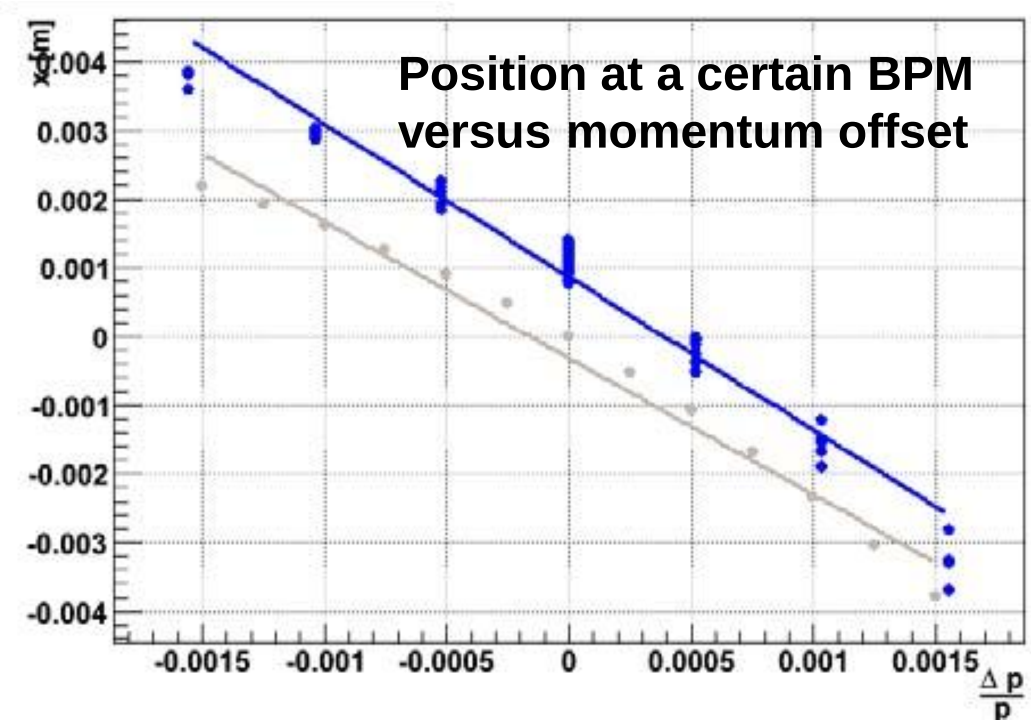
OPTICS AND EMITTANCE MEASUREMENT IN TRANSFER LINES

Dispersion measurement

- Introduce ~ few permille momentum offset at extraction into transfer line

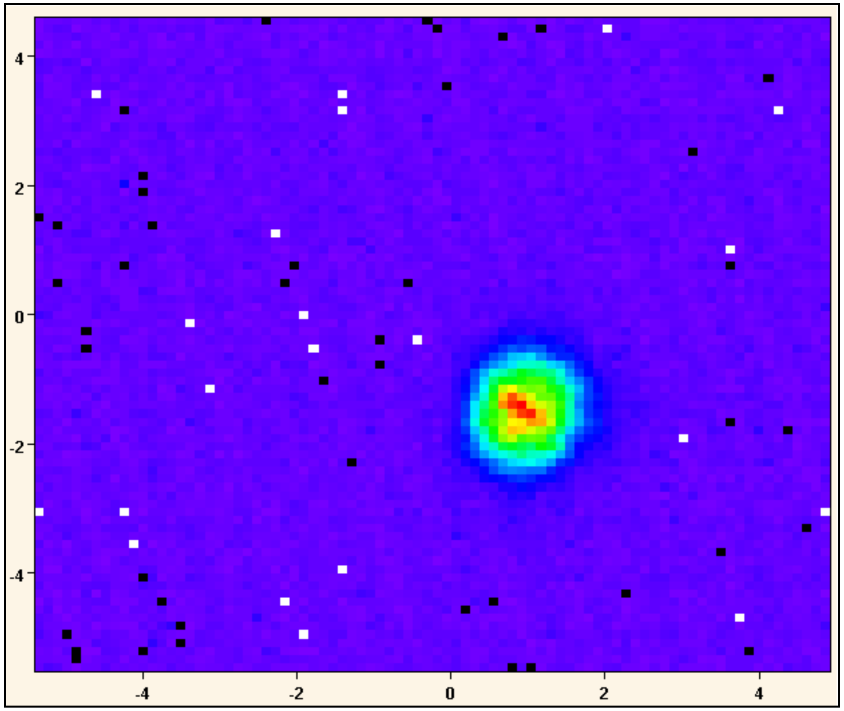
- Measure position at different monitors for different momentum offset
  - Linear fit of position versus dp/p at each BPM/screens.
  - ➔ Dispersion at the BPMs/screens

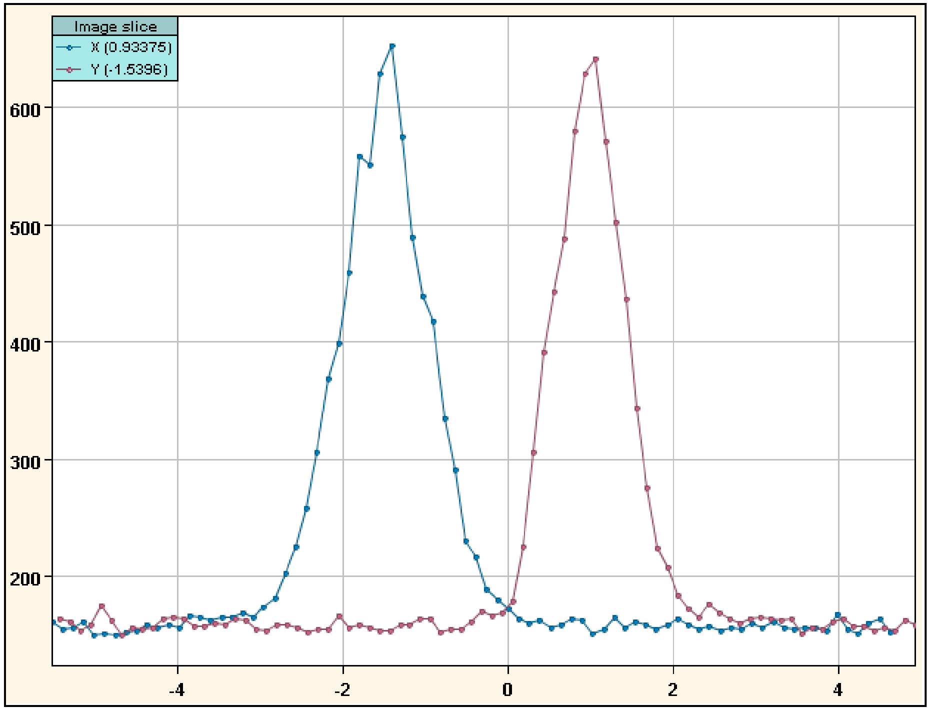
$$x(s) = x_b(s) + D(s) \times \frac{dp}{p}$$



Optics measurement with screens

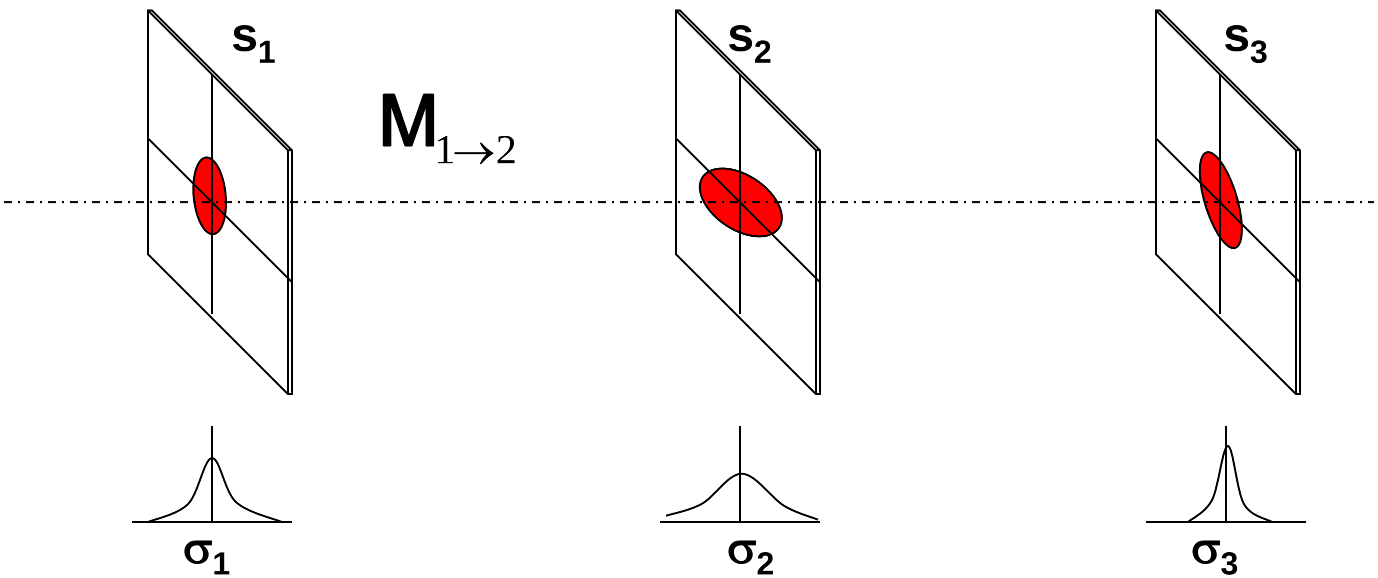
- A profile monitor is needed to measure the beam size
  - e.g. beam screen (luminescent) provides 2D density profile of the beam
- Profile fit gives transverse beam sizes σ.
- In a ring, β is ‘known’ so ε can be calculated from a single screen





Optics Measurement with 3 Screens

- Assume 3 screens in a dispersion free region
- Measurements of $\sigma_1, \sigma_2, \sigma_3$, plus the two transfer matrices M_{12} and M_{13} allows determination of ε, α and β



$$e = \frac{s_1^2}{b_1} = \frac{s_2^2}{b_2} = \frac{s_3^2}{b_3}$$

Optics Measurement with 3 Screens

- Remember:

$$\begin{bmatrix} x_2 \\ x_2' \end{bmatrix} = \mathbf{M}_{1\to 2} \cdot \begin{bmatrix} x_1 \\ x'_1 \end{bmatrix} = \begin{bmatrix} C & S \\ C' & S' \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x'_1 \end{bmatrix}$$

$$\begin{bmatrix} \beta_2 \\ \alpha_2 \\ \gamma_2 \end{bmatrix} = \begin{bmatrix} C_2^2 & -2C_2S_2 & S_2^2 \\ -C_2C_2' & C_1S_1' + S_1C_1' & -S_2S_2' \\ C_2'^2 & -2C_2'S_2' & S_2'^2 \end{bmatrix} \cdot \begin{bmatrix} \beta_1 \\ \alpha_1 \\ \gamma_1 \end{bmatrix}$$

$$\Rightarrow \quad \left. \begin{aligned} \beta_2 &= C_2^2 \cdot \beta_1 - 2C_2S_2 \cdot \alpha_1 + S_2^2 \cdot \gamma_1 \\ \beta_3 &= C_3^2 \cdot \beta_1 - 2C_3S_3 \cdot \alpha_1 + S_3^2 \cdot \gamma_1 \end{aligned} \right| \times \varepsilon$$

$$\begin{aligned} \sigma_2^2 &= C_2^2 \cdot \beta_1\varepsilon - 2C_2S_2 \cdot \alpha_1\varepsilon + S_2^2 \cdot \gamma_1\varepsilon \\ \sigma_3^2 &= C_3^2 \cdot \beta_1\varepsilon - 2C_3S_3 \cdot \alpha_1\varepsilon + S_3^2 \cdot \gamma_1\varepsilon \end{aligned}$$

Square of beam sizes as function of optical functions at first screen

Optics Measurement with 3 Screens

- Define matrix N where $\Sigma = N\Pi$

$$\Sigma = \begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{pmatrix}_{meas} \qquad \mathcal{N} = \begin{pmatrix} 1 & 0 & 0 \\ C_2^2 & -2C_2S_2 & S_2^2 \\ C_3^2 & -2C_3S_3 & S_3^2 \end{pmatrix} \qquad \Pi = \begin{pmatrix} \beta_1\epsilon \\ \alpha_1\epsilon \\ \gamma_1\epsilon \end{pmatrix}$$

- Measure beam sizes and want to calculate β_1, α_1, ϵ
- Solution to our problem $\Sigma' = N^{-1}\Sigma$
  - with $\beta_1\gamma_1-\alpha_1^2 = 1$ get 3 equations for β_1, α_1 and ϵ – the optical functions at the first screen

$$\beta_1 = A/\sqrt{AC - B^2} \qquad A = \Sigma'_1$$

$$\alpha_1 = B/\sqrt{AC - B^2} \quad \text{with} \quad B = \Sigma'_2$$

$$\epsilon = \sqrt{AC - B^2} \qquad C = \Sigma'_3$$

…and if there is dispersion at the screens

- Dispersion and momentum spread need to be measured independently at the different screens
- Trajectory transforms with T_i transport matrix for $\delta \neq 0$
  - ξ_i is the contribution to the dispersion between the first and the i[th] screen

$$\begin{pmatrix} x_i \\ x'_i \\ \delta \end{pmatrix} = \begin{pmatrix} C_i & S_i & \xi_i \\ C'_i & S'_i & \xi'_i \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x'_1 \\ \delta \end{pmatrix} \qquad D_i = C_i D_i + S_i D'_i + \xi_i$$

- Define 6 dimensional Σ and Π and respective N and then same procedure as before

$$\Sigma = \begin{pmatrix} \sigma_1^2 \\ \sigma_2^2 \\ \sigma_3^2 \\ D_1\delta^2 \\ D'_1\delta^2 \\ \delta^2 \end{pmatrix} \quad \mathcal{N} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ C_2^2 & -2C_2S_2 & S_2^2 & 2C_2\xi_2 & 2S_2\xi_2 & \xi_2 \\ C_3^2 & -2C_3S_3 & S_3^2 & 2C_3\xi_3 & 2S_3\xi_3 & \xi_3 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \Pi = \begin{pmatrix} \beta_1\varepsilon + D_1^2\delta^2 \\ \alpha_1\varepsilon - D_1D'_1\delta^2 \\ \gamma_1\varepsilon + D'^2_1\delta^2 \\ D_1\delta^2 \\ D'_1\delta^2 \\ \delta^2 \end{pmatrix}$$

6 screens with dispersion

- Can measure β, α, ε, D, D’ and δ with 6 screens without any other measurements.

$$\Sigma = \begin{pmatrix} \sigma_1^2 \\ \sigma_2^2 \\ \sigma_3^2 \\ \sigma_4^2 \\ \sigma_5^2 \\ \sigma_6^2 \end{pmatrix}, \Pi = \begin{pmatrix} \beta_1\varepsilon + D_1^2\delta^2 \\ \alpha_1\varepsilon - D_1 D_1'\delta^2 \\ \gamma_1\varepsilon + D_1'^2\delta^2 \\ D_1\delta^2 \\ D_1'\delta^2 \\ \delta^2 \end{pmatrix}, \mathcal{N} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ C_2^2 & -2C_2S_2 & S_2^2 & 2C_2\xi_2 & 2S_2\xi_2 & \xi_2 \\ C_3^2 & -2C_3S_3 & S_3^2 & 2C_3\xi_3 & 2S_3\xi_3 & \xi_3 \\ C_4^2 & -2C_4S_4 & S_4^2 & 2C_4\xi_4 & 2S_4\xi_4 & \xi_4 \\ C_5^2 & -2C_5S_5 & S_5^2 & 2C_5\xi_5 & 2S_5\xi_5 & \xi_5 \\ C_6^2 & -2C_6S_6 & S_6^2 & 2C_6\xi_6 & 2S_6\xi_6 & \xi_6 \end{pmatrix}$$

- Invert N, multiply with Σ to get Σ’

$$\delta^2 = \Sigma'_6$$

$$D_1 = \Sigma'_4 \Sigma'_6$$

$$D'_1 = \Sigma'_5 \Sigma'_6$$

$$\beta_1 = A / \sqrt{AC - B^2}$$

$$\alpha_1 = B / \sqrt{AC - B^2}$$

$$\varepsilon = \sqrt{AC - B^2}$$

$$A = \Sigma'_1 - \Sigma'^2_4 \Sigma'_6$$

$$B = \Sigma'_2 + \Sigma'_4 \Sigma'_5 \Sigma'_6$$

$$C = \Sigma'_3 - \Sigma'^2_5 \Sigma'_6$$

More than 6 screens…

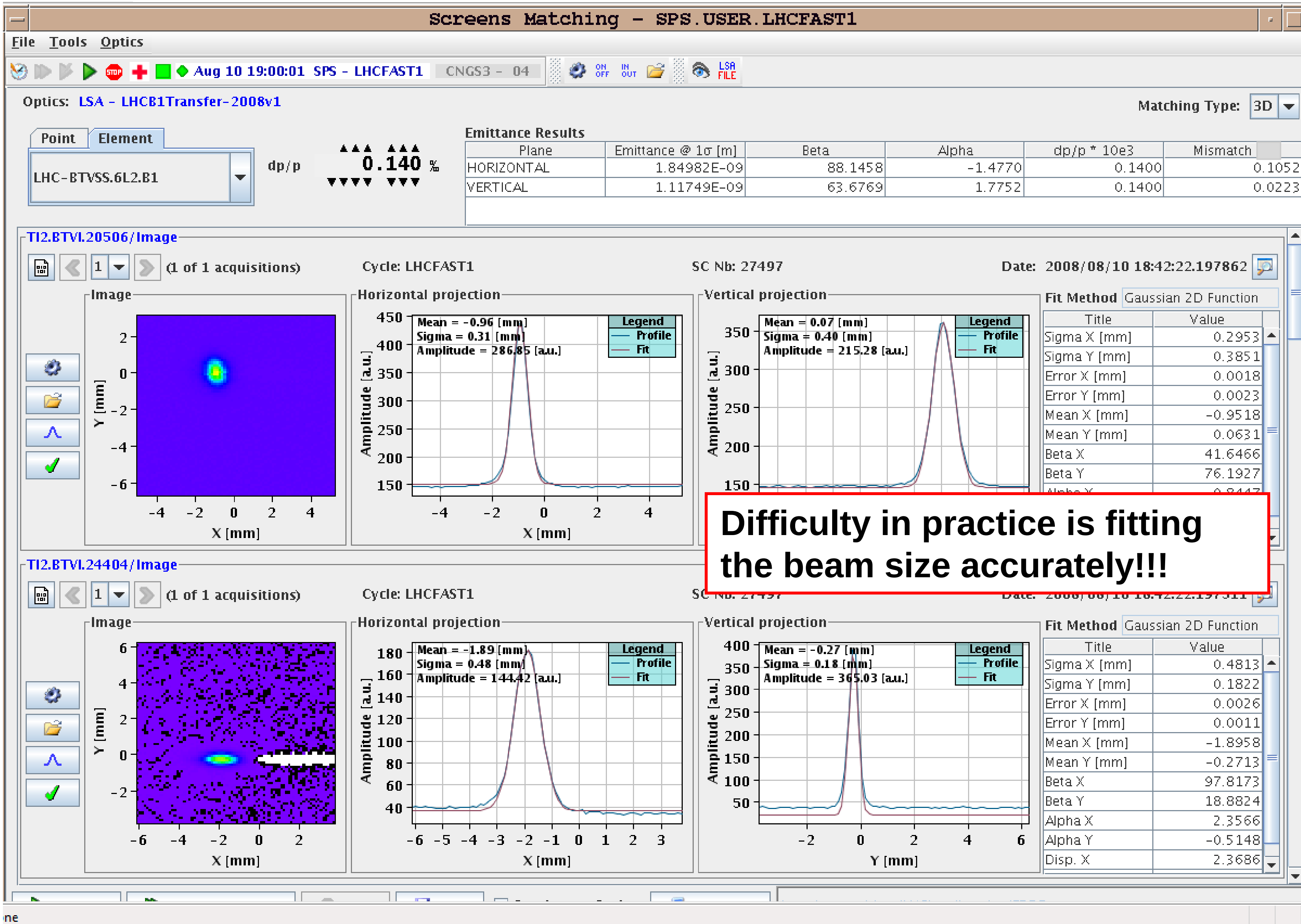
- Fit procedure…

- Function to be minimized: Δ_i…measurement error

$$\chi^2(\Pi) = \sum_{i=1}^{N_{mon}} \left[\frac{\Sigma_i - (\mathcal{N}\Pi)_i}{\Delta_i} \right]^2 \qquad \frac{\partial \chi^2}{\partial \Pi_i} = 0 \quad \textbf{(*)}$$

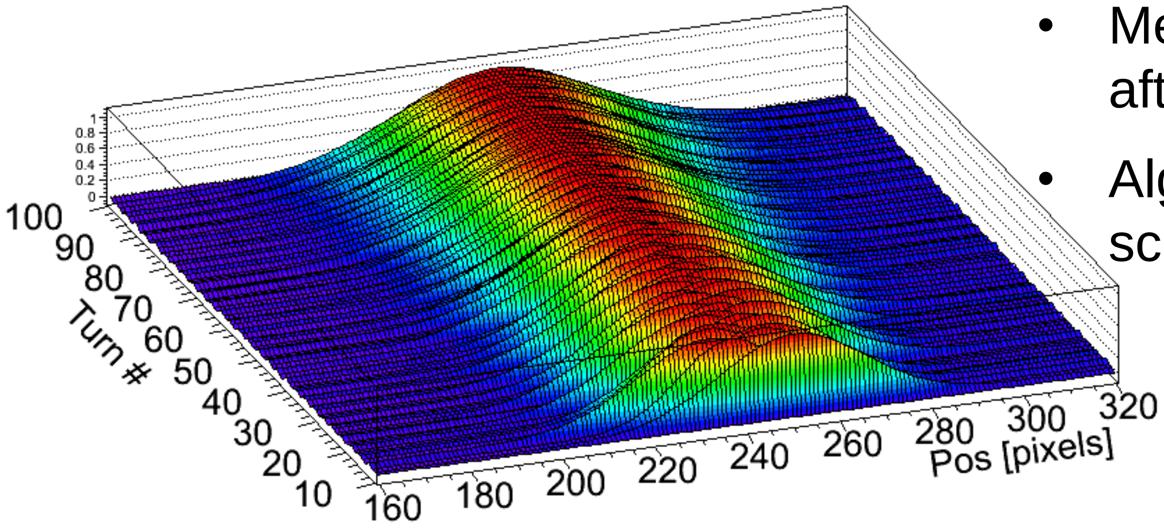
- Equation (*) can be solved analytically see
  - G. Arduini et al., “New methods to derive the optical and beam parameters in transport channels”, Nucl. Instrum. Methods Phys. Res., 2001.

In Practice….



Screens Matching – SPS.USER.LHCFAST1
File Tools Optics
Aug 10 19:00:01 SPS – LHCFAST1
CNGS3 – 04
Optics: LSA – LHCB1Transfer-2008v1
Matching Type: 3D
Point Element
LHC-BTVSS.6L2.B1
dp/p 0.140 %
Emittance Results
Plane | Emittance @ 1σ [m] | Beta | Alpha | dp/p * 10e3 | Mismatch
HORIZONTAL | 1.84982E-09 | 88.1458 | -1.4770 | 0.1400 | 0.1052
VERTICAL | 1.11749E-09 | 63.6769 | 1.7752 | 0.1400 | 0.0223
TI2.BTVI.20506/Image
(1 of 1 acquisitions)
Cycle: LHCFAST1
SC Nb: 27497
Date: 2008/08/10 18:42:22.197862
Image
Horizontal projection
Vertical projection
Mean = -0.96 [mm]
Sigma = 0.31 [mm]
Amplitude = 286.85 [a.u.]
Mean = 0.07 [mm]
Sigma = 0.40 [mm]
Amplitude = 215.28 [a.u.]
Legend
Profile
Fit
Amplitude [a.u.]
Y [mm]
X [mm]
Fit Method Gaussian 2D Function
Title | Value
Sigma X [mm] | 0.2953
Sigma Y [mm] | 0.3851
Error X [mm] | 0.0018
Error Y [mm] | 0.0023
Mean X [mm] | -0.9518
Mean Y [mm] | 0.0631
Beta X | 41.6466
Beta Y | 76.1927
TI2.BTVI.24404/Image
(1 of 1 acquisitions)
Cycle: LHCFAST1
Mean = -1.89 [mm]
Sigma = 0.48 [mm]
Amplitude = 144.42 [a.u.]
Mean = -0.27 [mm]
Sigma = 0.18 [mm]
Amplitude = 365.03 [a.u.]
Fit Method Gaussian 2D Function
Title | Value
Sigma X [mm] | 0.4813
Sigma Y [mm] | 0.1822
Error X [mm] | 0.0026
Error Y [mm] | 0.0011
Mean X [mm] | -1.8958
Mean Y [mm] | -0.2713
Beta X | 97.8173
Beta Y | 18.8824
Alpha X | 2.3566
Alpha Y | -0.5148
Disp. X | 2.3686
Difficulty in practice is fitting the beam size accurately!!!

Matching screen



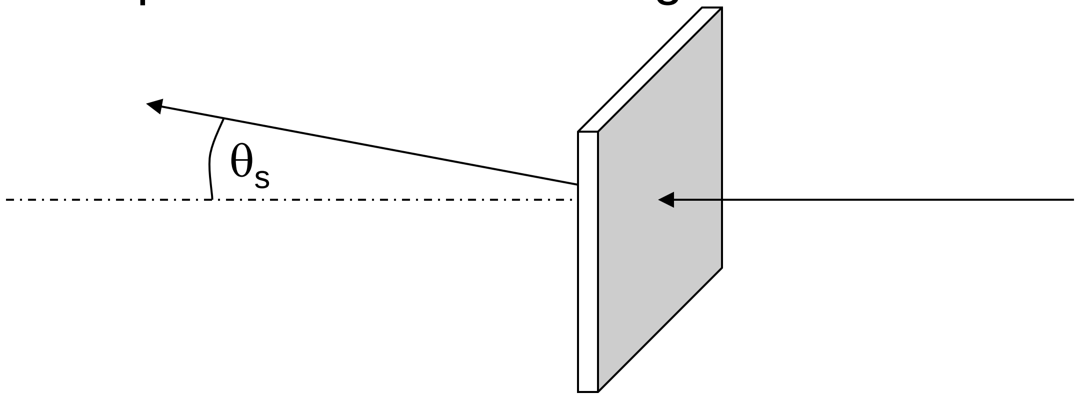
Profiles at matching monitor after injection with steering error.

- 1 screen in the circular machine
- Measure turn-by-turn profile after injection
- Algorithm same as for several screens in transfer line

- Only allowed with low intensity beam
- Issue: radiation hard fast cameras

Blow-up from thin scatterer

- Scattering elements are sometimes required in the beam
  - Thin beam screens (Al_2O_3,Ti) used to generate profiles.
  - Metal windows also used to separate vacuum of transfer lines from vacuum in circular machines.
  - Foils are used to strip electrons to change charge state
- The emittance of the beam increases when it passes through, due to multiple Coulomb scattering.



rms angle increase: $\sqrt{\langle \theta_s^2 \rangle}[mrad] = \frac{14.1}{\beta_c p[MeV/c]} Z_{inc} \sqrt{\frac{L}{L_{rad}}} \left(1 + 0.11 \cdot \log_{10} \frac{L}{L_{rad}}\right)$

β_c = v/c, p = momentum, Z_{inc} = particle charge /e, L = target length, L_{rad} = radiation length

Blow-up from thin scatterer

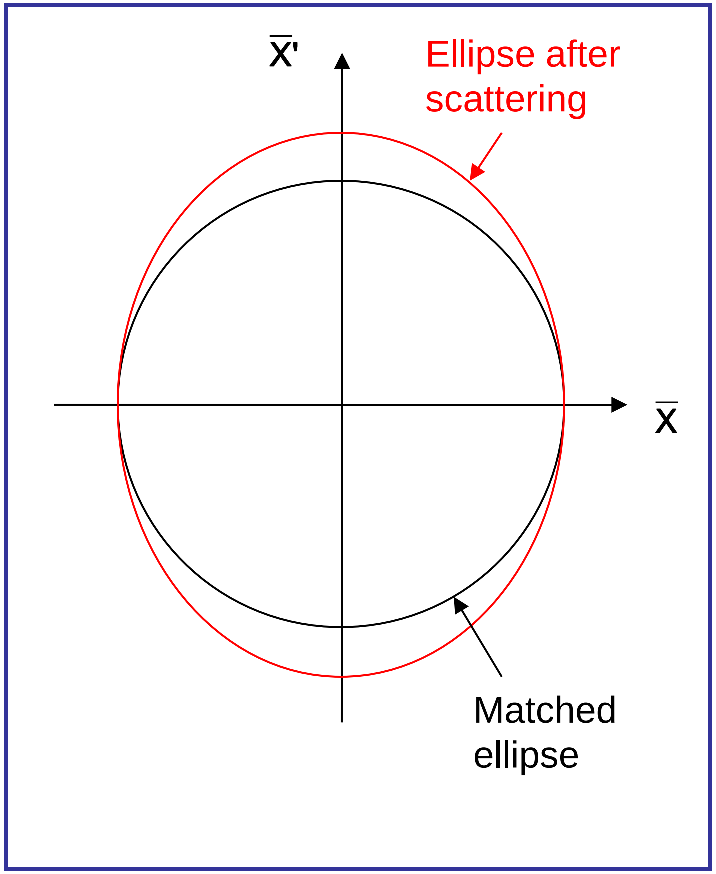
Each particles gets a random angle change θ_s but there is no effect on the positions at the scatterer

$$\overline{\mathbf{X}}_{new} = \overline{\mathbf{X}}_0$$

$$\overline{\mathbf{X}}'_{new} = \overline{\mathbf{X}}'_0 + \sqrt{\beta}\theta_s$$

After filamentation the particles have different amplitudes and the beam has a larger emittance

$$\varepsilon = \left\langle \mathbf{A}^2_{new} \right\rangle / 2$$



Blow-up from thin scatterer

$$A_{new}^2 = \overline{X}_{new}^2 + \overline{X}_{new}'^2$$

$$= \overline{X}_0^2 + \left(\overline{X}_0' + \sqrt{\beta}\theta_s\right)^2$$

$$= \overline{X}_0^2 + \overline{X}_0'^2 + 2\sqrt{\beta}\left(\overline{X}_0'\theta_s\right) + \beta\theta_s^2$$

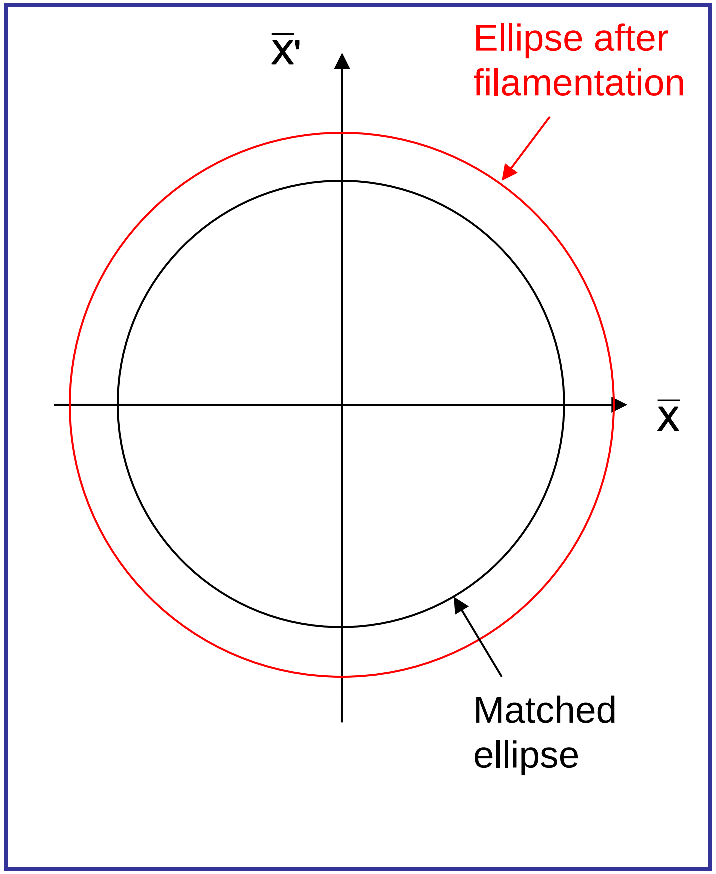
uncorrelated

$$\left\langle A_{new}^2 \right\rangle = \left\langle \overline{X}_0^2 \right\rangle + \left\langle \overline{X}_0'^2 \right\rangle + 2\sqrt{\beta}\left\langle \overline{X}_0'\theta_s \right\rangle + \beta\left\langle \theta_s^2 \right\rangle$$

$$= 2\varepsilon_0 + 2\sqrt{\beta}\left\langle \overline{X}_0' \right\rangle \left\langle \theta_s \right\rangle + \beta\left\langle \theta_s^2 \right\rangle$$

(with $\left\langle \overline{X}_0' \right\rangle$ crossed out = 0)

$$= 2\varepsilon_0 + \beta\left\langle \theta_s^2 \right\rangle$$

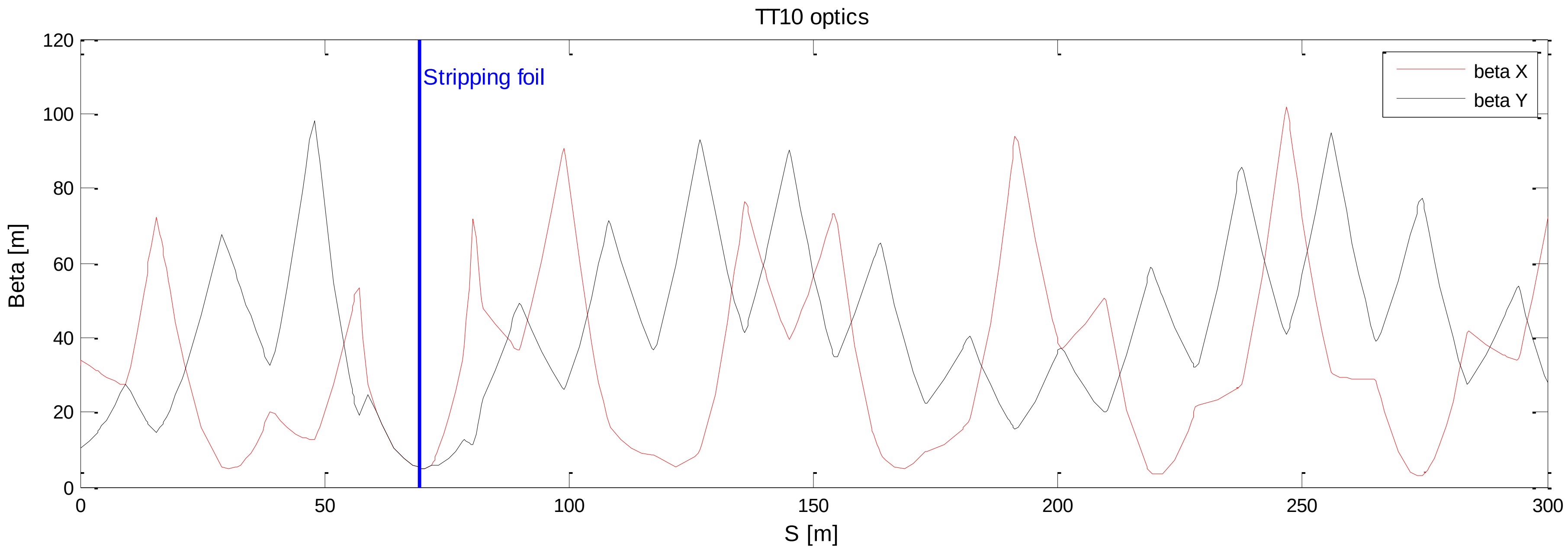


$$\varepsilon_{new} = \varepsilon_0 + \frac{\beta}{2}\left\langle \theta_s^2 \right\rangle$$

Need to keep β small to minimise blow-up (small β means large spread in angles in beam distribution, so additional angle has small effect on distn.)

Blow-up from charge stripping foil

- For LHC heavy ions, Pb^{53+} is stripped to Pb^{82+} at 4.25GeV/u using a 0.8mm thick Al foil, in the PS to SPS line
- $\Delta\varepsilon$ is minimised with low-β insertion (β_{xy} ~5 m) in the transfer line
- Emittance increase expected is about 8%



Kick-response measurement

- The observable during kick-response measurement are the elements of the response matrix R

$$R_{ij} = \frac{u_i}{\delta_j} \qquad R_{ij}^{model} = \begin{cases} \sqrt{\beta_i \beta_j} \sin(\mu_i - \mu_j) & \text{for } \mu_i > \mu_j \\ 0 & \text{otherwise} \end{cases}$$

  - u_i is the position at the ith monitor
  - δ_j is the kick of the jth corrector

- Cannot read off optics parameters directly
- A fit varies certain parameters of a machine model to reproduce the measured data ➔ LOCO principle
- The fit minimizes the quadratic norm of a difference vector V

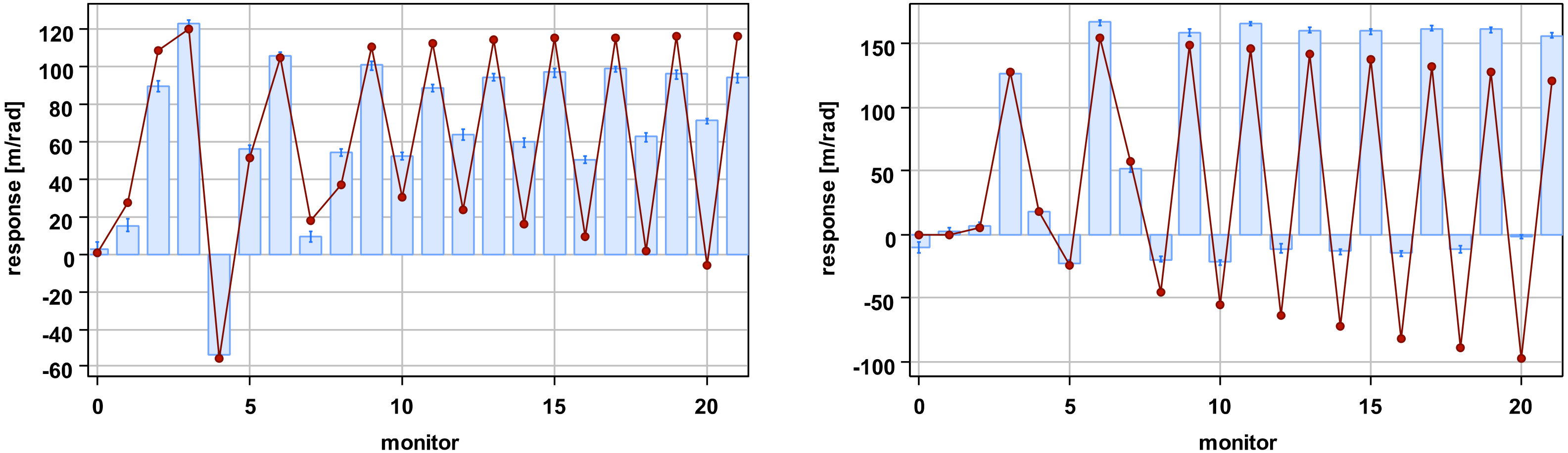
$$V_k = \frac{R_{ij}^{meas} - R_{ij}^{model}}{\sigma_i} \qquad k = i \cdot (N_c - 1) + j$$

σ_i… BPM rms noise
N_c… number of correctors

Reference: K. Fuchsberger, CERN-THESIS-2011-075

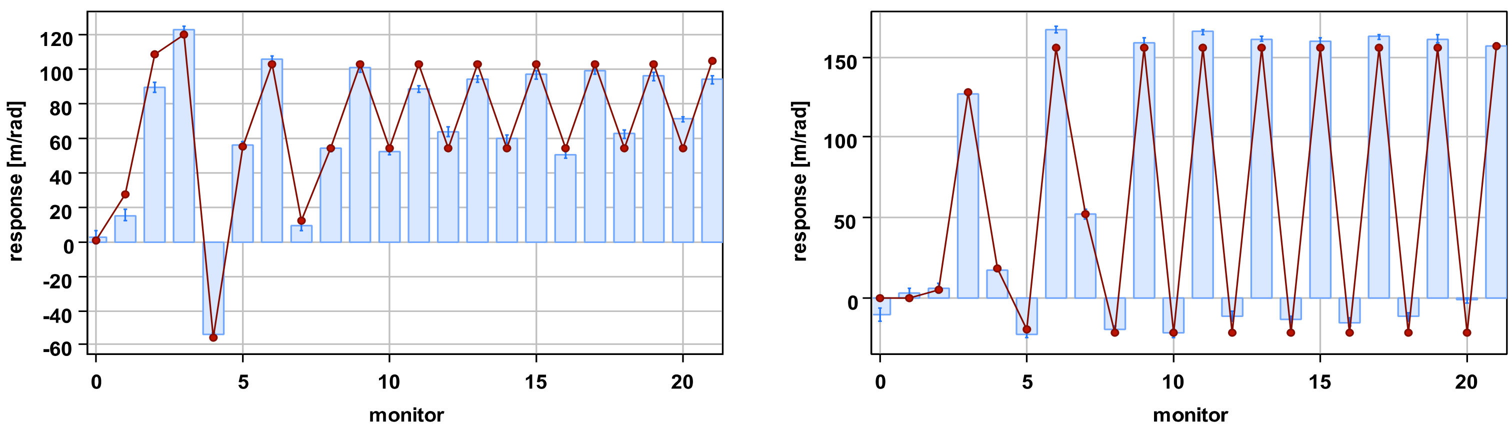
Example: LHC transfer line TI 8

- Phase error in the vertical plane



(a) MDMV.400097 (b) MDSV.400293

- Traced back to error in QD strength in transfer line arc:



(a) MDMV.400097 (b) MDSV.400293

Summary

- Transfer lines present interesting challenges and differences from circular machines
  - No periodic condition mean optics is defined by transfer line element strengths and by initial beam ellipse
  - Matching at the extremes is subject to many constraints
  - Emittance blow-up is an important consideration, and arises from several sources
  - The optics of transfer line has to be well understood
  - Several ways of assessing optics parameters in the transfer line have been shown

Keywords for related topics

- Transfer lines
  - Achromat bends
  - Algorithms for optics matching
  - The effect of alignment and gradient errors on the trajectory and optics
  - Trajectory correction algorithms
  - SVD trajectory analysis
  - Phase-plane exchange insertion solutions