Linear Imperfections

October, 2016. Budapest

R. Tomás

The CERN Accelerator School

E.D. Courant and H.S. Snyder 1957

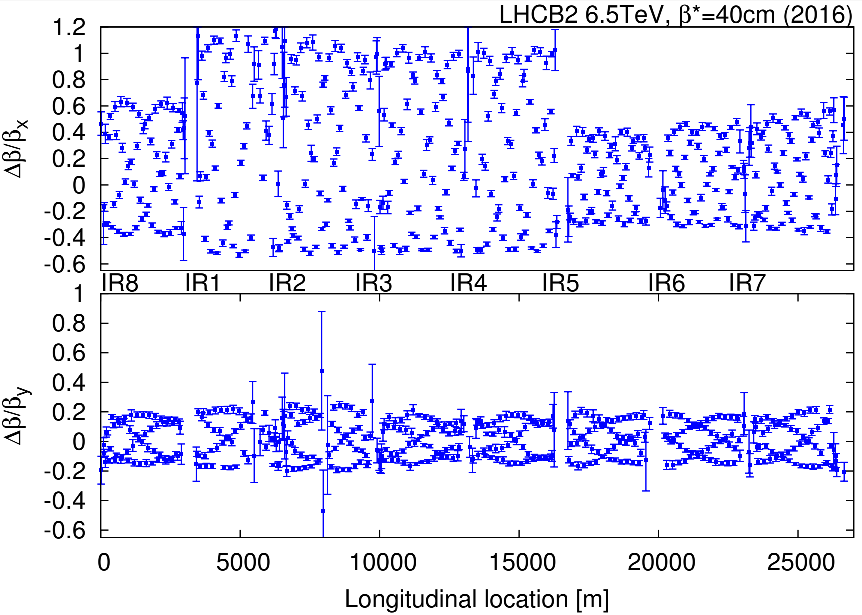
Theory of the Alternating-Gradient Synchrotron [1]:

$$\left(\frac{\Delta\beta}{\beta}\right)_{\max} = 4.0\left(\frac{\Delta k}{k}\right)_{\mathrm{rms}}$$

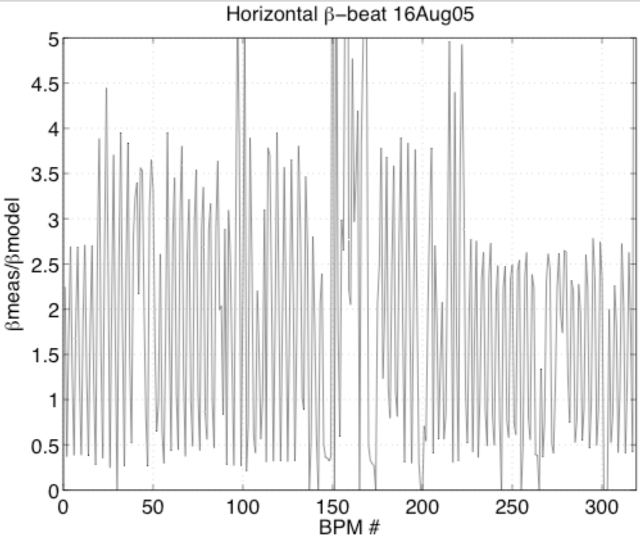
"Thus if the variation in k from magnet to magnet were 1% (...) we would have a β-**beating of 4%**. Any particular machine (...) would be unlikely to be worse by more than factor of 2."

$\rightarrow$ Expected β-beating below 8% for *any machine*

120% in LHC, commissioning 2016



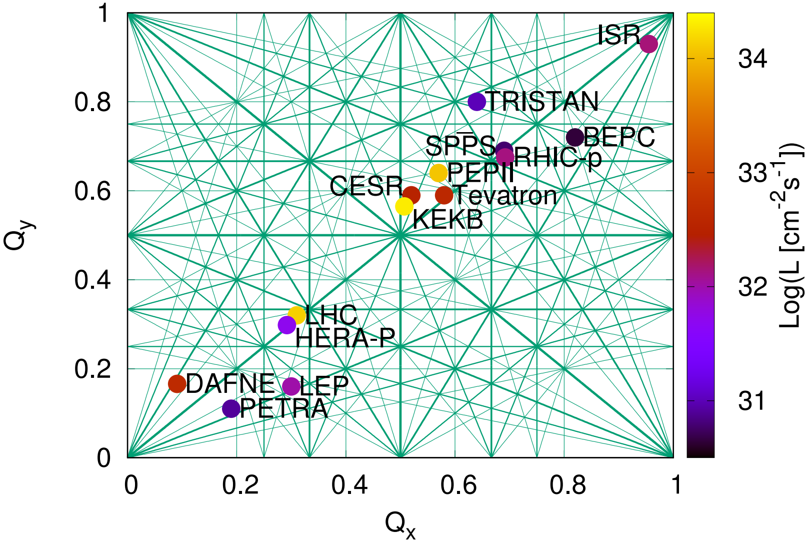
≈400% in PEP-II, commissioning 2005



G. Yocky,
SLAC-PUB-12523

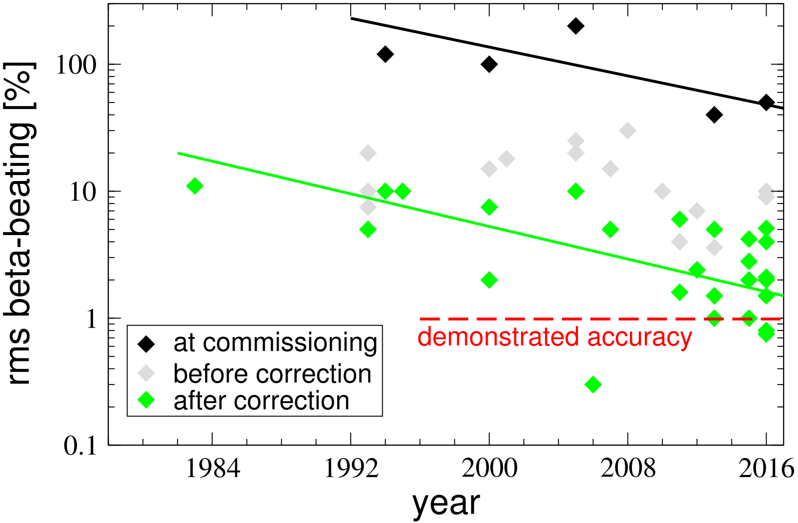
Even $\Delta\beta/\beta \approx 700\%$ was reached when LER tune was pushed closer to the half integer

Colliders in the tune space

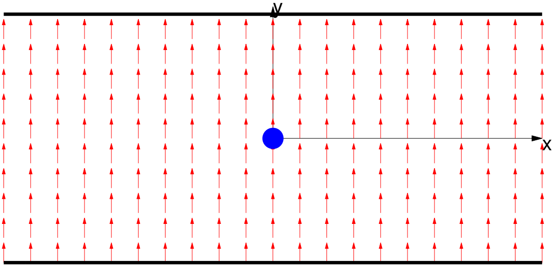


β-beating versus time

Courtesy Andrea Franchi



Dipole magnetic field

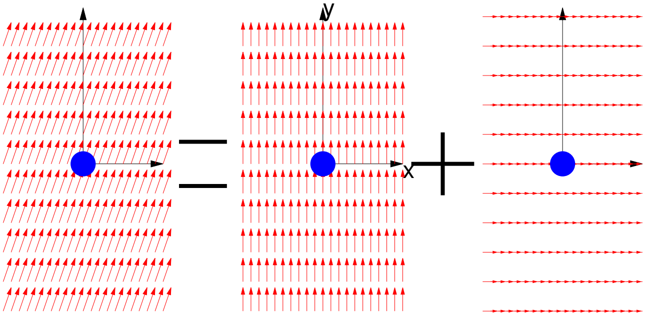


Lorentz force:

$$\vec{F} = q\vec{v} \times \vec{B}$$

Dipole errors

- An error in the strength of a main dipole causes a perturbation on the horizontal closed orbit.
- A tilt error in a main dipole causes a perturbation on the vertical closed orbit.



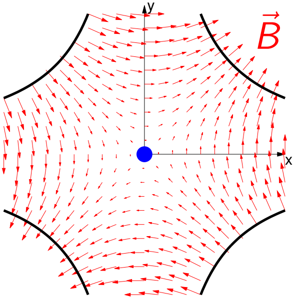
Orbit perturbation formula

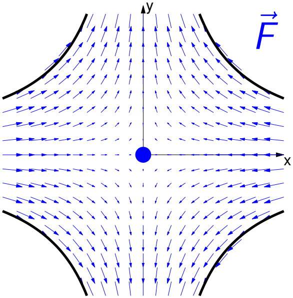
From distributed angular kicks θ_i the closed orbit results in:

$$CO(s) = \frac{\sqrt{\beta(s)}}{2 \sin \pi Q} \sum_i \sqrt{\beta_i} \theta_i \cos(\pi Q - |\phi(s) - \phi_i|)$$

Attention to the denominator $\sin(\pi Q)$ that makes closed orbit to diverge at the integer resonance $Q \in \mathbb{N}$.
Another source of orbit errors is offset quadrupoles.

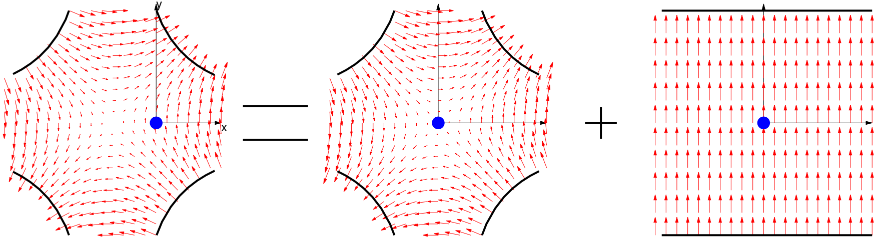
Quadrupole field and force on the beam





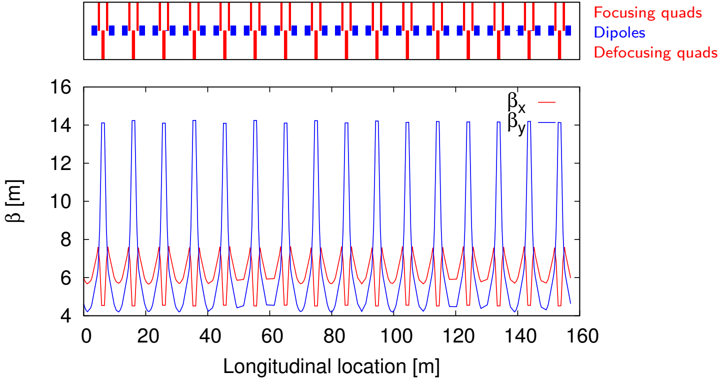
Note that $F_x = -kx$ and $F_y = ky$ making horizontal dynamics totally decoupled from vertical.

Offset quadrupole - Feed-down



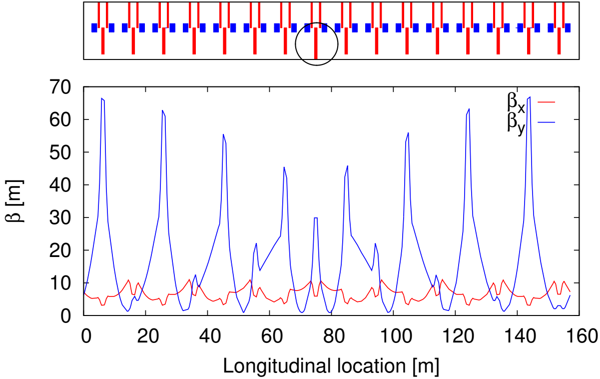
An offset quadrupole is seen as a centered quadrupole plus a dipole. This is called feed-down.

Quadrupole strength error



Ideal PSB lattice without errors.

Quadrupole strength error



β functions change (β-beating$=\frac{\Delta\beta}{\beta} = \frac{\beta_{pert}-\beta_0}{\beta_0}$).
Tunes change too (ΔQ).

Quadrupole strength error - Formulae

Tune change (single source):

$$\Delta Q_x \approx \frac{1}{4\pi}\overline{\beta_x}\Delta k_i L_i, \quad \Delta Q_y \approx -\frac{1}{4\pi}\overline{\beta_y}\Delta k_i L_i$$

β-beating from many sources:

$$\frac{\Delta\beta}{\beta}(s) \approx \pm \sum_i \frac{\Delta k_i L_i \overline{\beta_i}}{2\sin(2\pi Q)} \cos(2\pi Q - 2|\phi(s) - \phi_i|)$$

Attention to the denominator $\sin(2\pi Q)$ that makes β-beating diverge at the integer and half integer resonances, $2Q \in \mathbb{N}$.

Phase beating *and higher orders*

$$\Delta\phi(s_0, s) = \int_{s_0}^{s} \frac{\mathrm{d}s'}{\beta(s')} \left(\frac{1}{1 + \frac{\Delta\beta}{\beta}(s')} - 1 \right)$$

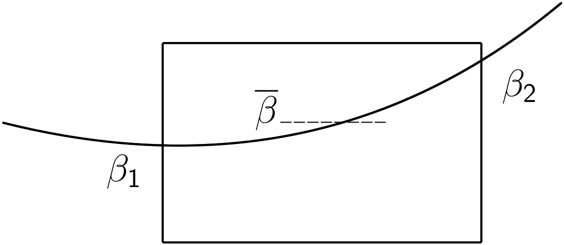
For first and higher order expansions see [2, 3, 4].
$\Delta\phi$ is given in [5] as function of Hill's determinant:

$$D = \left| \delta_{nm} + \frac{\theta_{n-m} |1 - \delta_{0(n-m)}|}{\theta_0 - (2n)^2} \right|_{-\infty}^{\infty}, \ \theta_m = \frac{2Q \oint \mathrm{d}s e^{-im\phi/Q} \beta \Delta k}{\pi}$$

or using RDTs [4], $f = \left| \frac{\oint ds \Delta k \beta_x e^{i2\phi_x}}{1 - e^{2\pi i Q_x}} \right| + \mathcal{O}(\Delta k^2)$,

$$\frac{\Delta\beta_x}{\beta_x}(s) = 2 \sinh f \left[\sinh f + \cosh f \sin \phi_f \right]$$

Average beta function in a quad ($\overline{\beta}$)

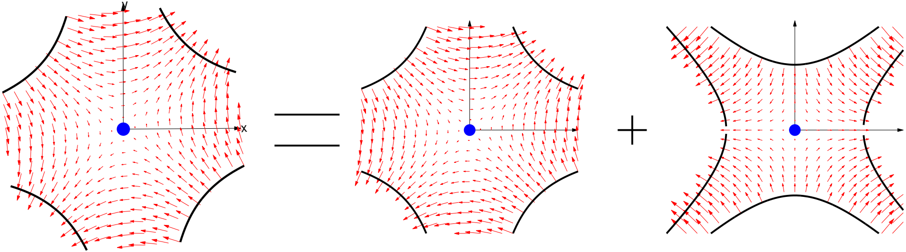


quadrupole (k, L)

$$\overline{\beta} \approx \frac{1}{3}\left(\beta_1 + \beta_2 + \sqrt{\beta_1\beta_2 - L^2}\right)$$

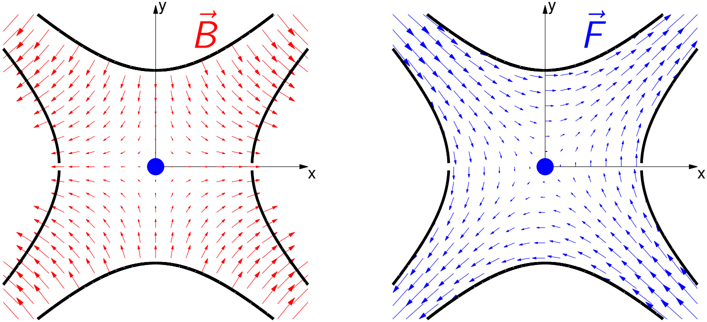
A. Hofmann and B. Zotter [6]

Tilted quadrupole



A tilted quadrupole is seen as a normal quadrupole plus another quadrupole tilted by 45° (this is called a skew quadrupole).

Skew quadrupole → x-y Coupling



Note that $F_x = k_s y$ and $F_y = k_s x$ making horizontal and vertical dynamics to couple.

Transverse coupling in the 1-turn map

In the ideal uncoupled case:

$$\begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix}_f = \begin{pmatrix} M_{11} & M_{12} & 0 & 0 \\ M_{21} & M_{22} & 0 & 0 \\ 0 & 0 & M_{33} & M_{34} \\ 0 & 0 & M_{43} & M_{44} \end{pmatrix} \begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix}_i$$

In presence of coupling:

$$\begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix}_f = \begin{pmatrix} M_{11} & M_{12} & M_{13} & M_{14} \\ M_{21} & M_{22} & M_{23} & M_{24} \\ M_{31} & M_{32} & M_{33} & M_{34} \\ M_{41} & M_{42} & M_{43} & M_{44} \end{pmatrix} \begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix}_i$$

Motion with coupling

To first order in the coupling the transverse motion can be approximated as [7, 8]

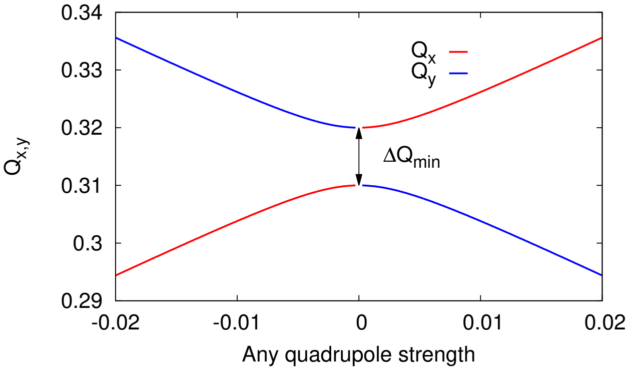
$$
\begin{aligned}
x(N,s) \;\approx\; & \sqrt{\beta_x(s)}\Re\Big\{\sqrt{2J_x}e^{i(2\pi Q_x N+\phi_x(s)+\phi_{x0})} \\
& -2if_{1010}\sqrt{2J_y}e^{-i(2\pi Q_y N+\phi_y(s)+\phi_{y0})} \\
& -2if_{1001}\sqrt{2J_y}e^{i(2\pi Q_y N+\phi_y(s)+\phi_{y0})}\Big\} \\
f_{\substack{1010\\1001}} \;=\; & \frac{\oint ds k_s \sqrt{\beta_x\beta_y}e^{i(\phi_x\pm\phi_y)}}{4(1-e^{2\pi i(Q_x\pm Q_y)})}
\end{aligned}
$$

f_{1001} drives the difference resonance $Q_x - Q_y = N$ and f_{1010} the sum resonance $Q_x + Q_y = N$

Bothering effects of coupling

Lepton machines: increases the vertical equilibrium emittance.

Hadron machines: makes it impossible to approach tunes below $\Delta Q_{\min}$



ΔQ_{min} formula

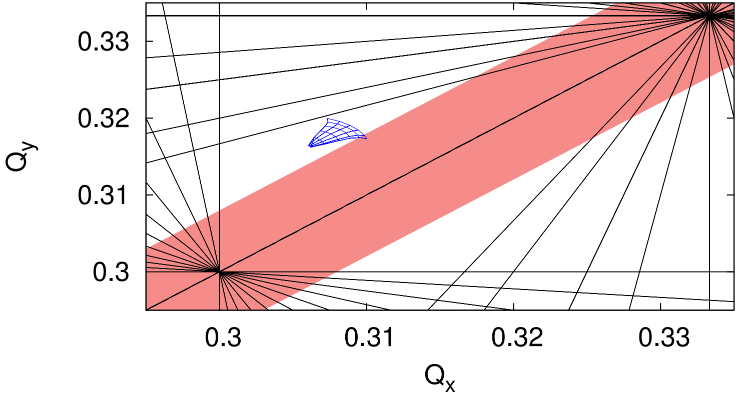
$$\Delta Q_{\mathrm{min}} = \left| \frac{1}{2\pi} \sum_j k_{s,j} L_j \sqrt{\beta_x \beta_y} e^{-i(\phi_x - \phi_y) + is(\hat{Q}_x - \hat{Q}_y)/R} \right|$$

$$\begin{aligned} \Delta Q_{\mathrm{min}} &= \left| \frac{4(\hat{Q}_x - \hat{Q}_y)}{2\pi R} \oint \mathrm{d}s f_{1001} \mathrm{e}^{-i(\phi_x - \phi_y) + is(\hat{Q}_x - \hat{Q}_y)/R} \right| \\ &\lesssim 4 |\hat{Q}_x - \hat{Q}_y| \overline{|f_{1001}|} \end{aligned}$$

f_{1001} defines both the phase space and the stopband. See [9, 10, 11] for further details.

$\Delta Q_{\rm min}$ limits the resonance-free space

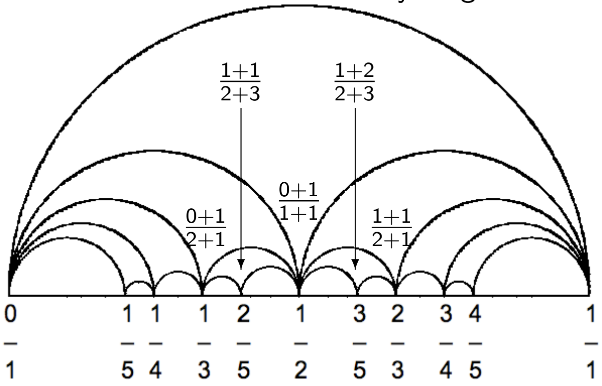
LHC beam-beam tune footprint and a hypothetical large coupling:



Interlude: Farey sequences (1802)

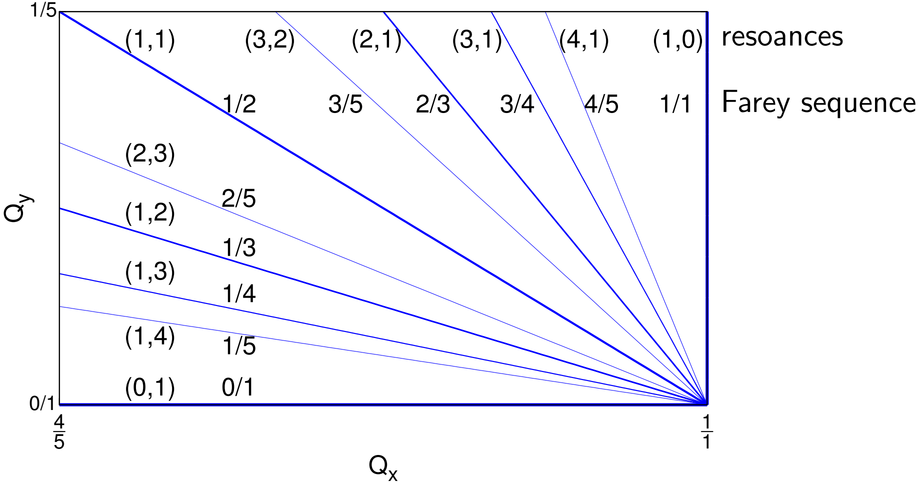
The Farey sequence of order n is the sequence of completely reduced fractions between 0 and 1 which have denominators less than or equal to $N \rightarrow$ **Resonances of order N or lower** (in one plane)

Farey diagram of order 5



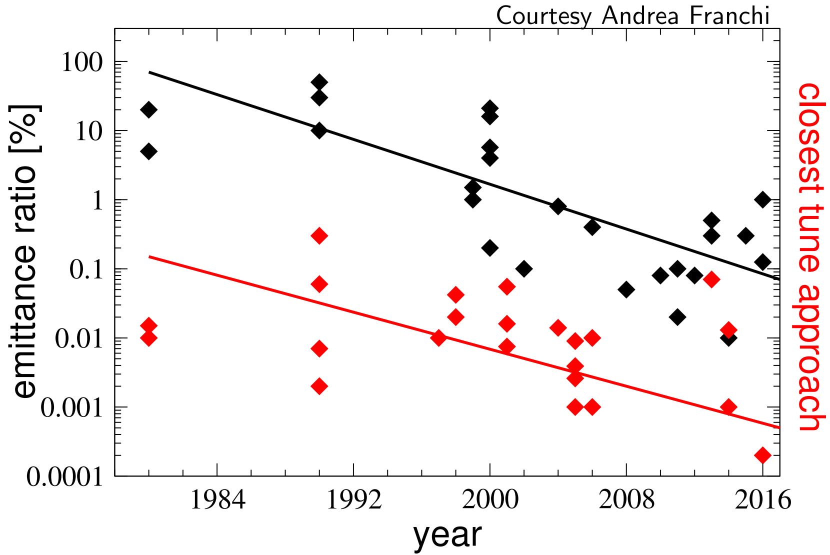
Farey sequences also in 2D res. diagram

Example, node at 1/1, order 5: $\frac{h}{k} \mapsto (h,\ k-h)$

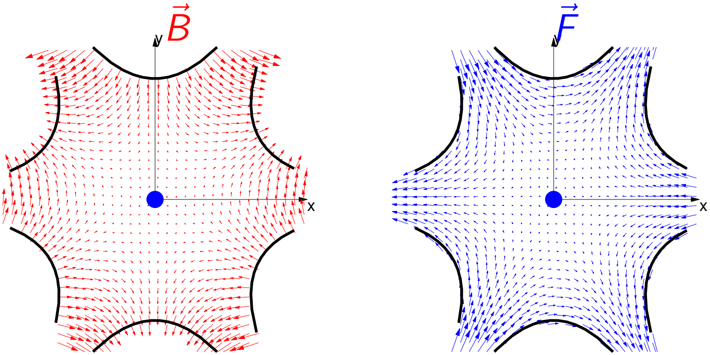


Phys. Rev. ST Accel. Beams 17, 014001 (2014)

Coupling control versus time



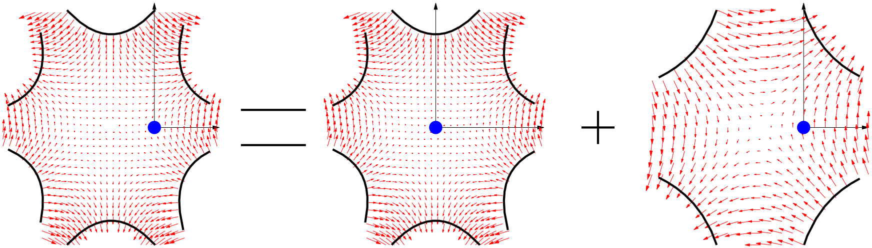
Sextupole field and force



$$F_x = \frac{1}{2}K_2(x^2 - y^2)\ , \quad F_y = -K_2 xy$$

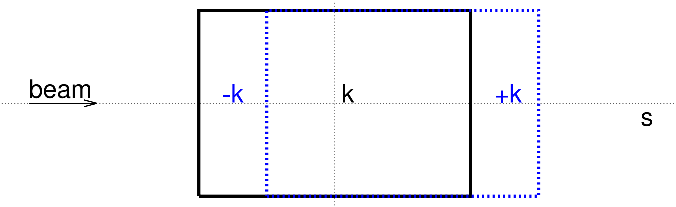
Ooops, We are entering the non-linear regime, however...

Offset sextupole



A sextupole horizontally (vertically) displaced is seen as a centered sextupole plus an offset quadrupole (skew quadrupole). Offset sextupoles are also sources of quadrupole and skew quadrupole errors.

Longitudinal misalignments



Longitudinal misalignments can be seen as perturbations at both ends of the magnet with opposite signs. Tolerances are generally larger for longitudinal misalignments.

Correction

★ Local corrections

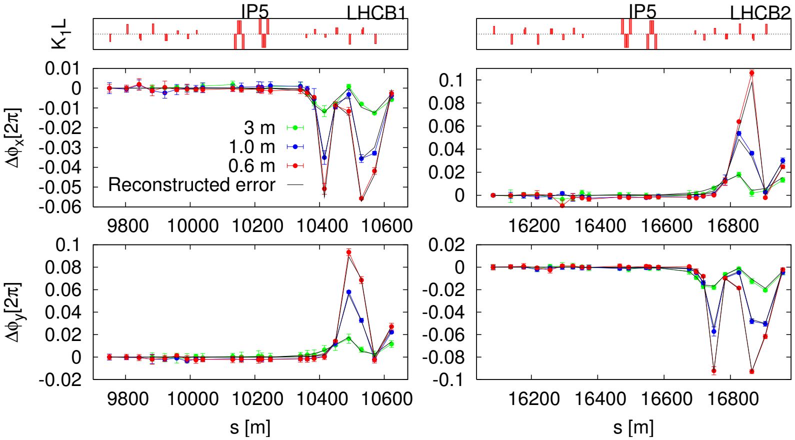
- Ideal correction: Error source identification and repair.
- Effective local error correction.
- MICADO (ISR-MA/73-17): Best few correctors (no guarantee of locality).

★ Global corrections

- Pre-designed knobs for varying particular observables in the least invasive way (like tunes, coupling, β^*, etc.)
- MICADO: Best N correctors
- Response matrix approach

★ Passive corrections (optimizing, scanning, etc.)

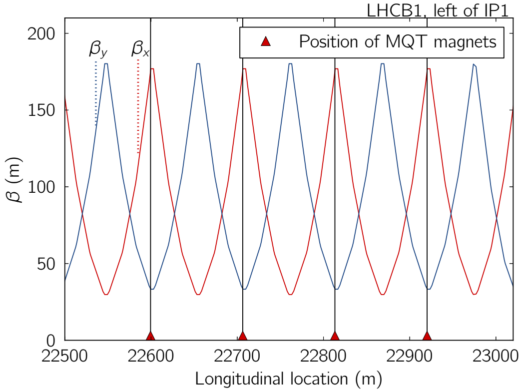
Local correction: segment-by-segment



Key point: Isolate a segment of the machine by imposing boundary conditions from measurements and find corrections [12].

Pre-designed knobs - Tunes

- In most machines it is OK to use all focusing quads to change Q_x and all defocusing quads for Q_y: PSB, PS, SPS
- In the LHC dedicated tune correctors (MQT) are properly placed to minimize impact on β-beating:



A.S. Langner

Pre-designed knobs - Coupling

- The full control of the difference resonance (f_{1001}) needs two independent families of skew quadrupoles.
- PSB, PS and SPS can survive only with one family since $\mathrm{int}(Q_x) = \mathrm{int}(Q_y)$, making errors in phase with correctors.
- In LHC there are two families to vary the real and imaginary parts of f_{1001} independently.

Best N-corrector challenge

- LHC has about 500 orbit correctors per plane and per beam.
- Imagine you want to find the best 20 correctors
- How many combinations of these 500 correctors taking 20 at a time exist?
- ...
- (MICADO finds a good approximation to this problem)

Response matrix approach

- Available correctors: $\vec{c}$
- Available observables: $\vec{a}$
- Assume for small changes of correctors linear approximation is good:

$$R\Delta\vec{c} = \Delta\vec{a}$$

- Use, e.g., MADX to compute R
- Invert or pseudo-invert R to compute an effective global correction based on measured $\Delta\vec{a}$:

$$\Delta\vec{c} = R^{-1}\Delta\vec{a}$$

- This works for orbit, $\Delta\beta/\beta$, coupling, etc.

Correcting optics and coupling

$$\begin{pmatrix} \Delta\vec{\phi}_x \\ \Delta\vec{\phi}_y \\ \frac{\Delta\vec{\beta}_x}{\beta_x} \\ \frac{\Delta\vec{\beta}_y}{\beta_y} \\ \Delta\vec{D}_x \\ \Delta\vec{Q} \end{pmatrix}_{\mathrm{meas}} = \mathbf{P}_{\mathrm{(theo)}} \cdot \Delta\vec{k}$$

$$\begin{pmatrix} \vec{f}_{1001} \\ \vec{f}_{1010} \\ \vec{D}_y \end{pmatrix}_{\mathrm{meas}} = \mathbf{T}_{\mathrm{(theo)}} \cdot \Delta\vec{k}_s$$

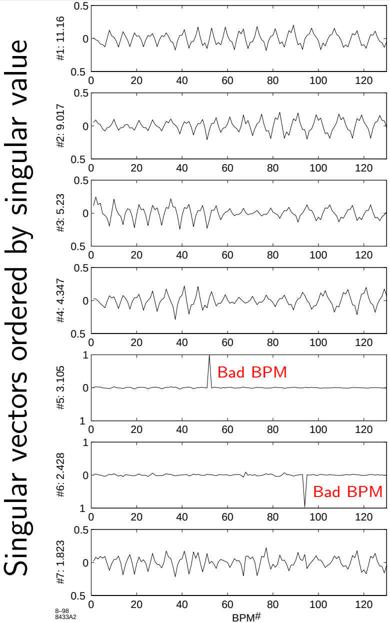
Pseudo-inverse via SVD

$$R = U \begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \\ 0 & 0 & 0 \end{pmatrix} V^T$$

Imagine $\sigma_3 \ll \sigma_2 \leq \sigma_1$, then just neglect σ_3:

$$R^{-1} = V \begin{pmatrix} \frac{1}{\sigma_1} & 0 & 0 \\ 0 & \frac{1}{\sigma_2} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} U^T$$

SLC: Cleaning BPM data with SVD, 1999



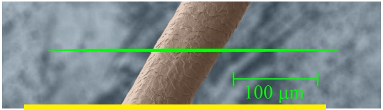
$$\underbrace{B_{t-b-t}}_{\substack{bpm\\matrix}} = USV^T$$

Bad BPMs easily identified as uncorrelated signals.

Noise removed by cutting low singular values

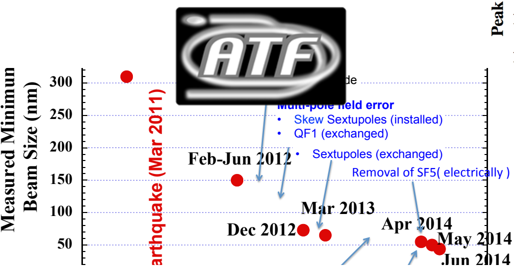
J. Irwin et al., Phys. Rev. Letters **82**, 8

Three world records via passive corrs.



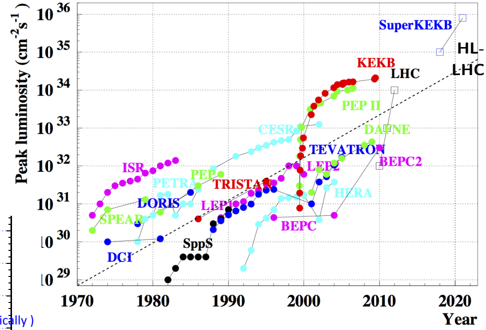
$\epsilon_y = 0.9 \pm 0.4$ pm

via random walk optimization



$\sigma_y = 44 \pm 3$ nm

via scanning orthogonal knobs



$L = 2.1 \times 10^{34}$ cm^{-2}s^{-1}

Luminosity optimized via downhill Simplex

Dynamic linear imperfections

- ★ Ground motion and vibrations in quadrupoles produce sinusoidal dipolar fields
- ★ Electrical noise can cause currents in quadrupoles and dipoles to oscillate in time
- ★ Electromagnetic pollution can act directly on the beam.
- ★ Slow variations ($f << Q_{x,y} f_{rev}$) just cause a time varying orbit and optics
- ★ Fast variations ($f \approx Q_{x,y} f_{rev}$) can cause resonances and emittance growth

An oscillating dipolar field

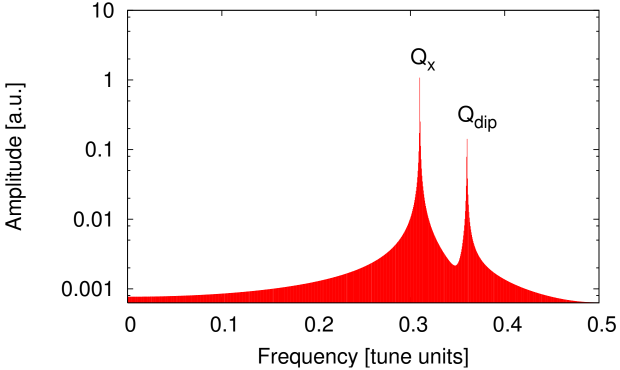
- Let $Q_{dip} = f_{dip}/f_{rev}$ be the tune of the dipolar field oscillation.
- This causes the appearance of new resonances
- Linear resonances: $Q_x \pm Q_{dip} = N$
- Non-linear resonances of sextupolar order:

$$Q_x \pm 2Q_{dip} = N$$

$$2Q_x \pm Q_{dip} = N$$

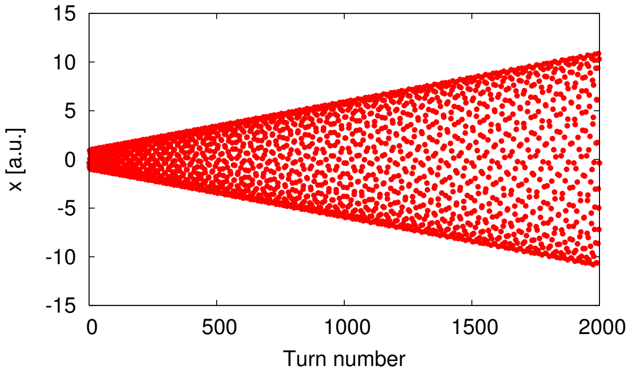
- Note that $mQ_{dip} = N$ is not a problem

Oscillating dipolar field, $Q_x \neq Q_{dip}$



Orbit oscillates with Q_{dip} but there is no emittance growth far from resonances.

Oscillating dipolar field, $Q_x = Q_{dip}$

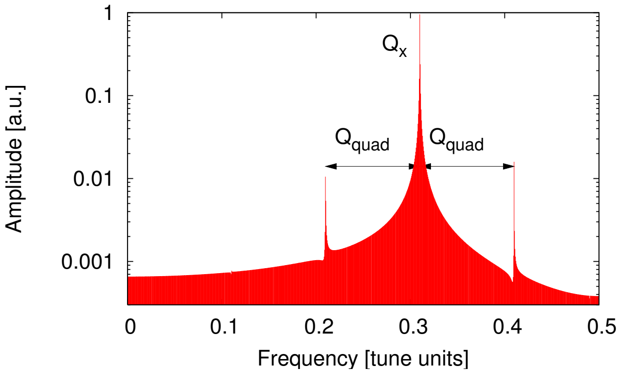


Linear growth in time $\rightarrow$ Emittance growth.

An oscillating quadrupolar field

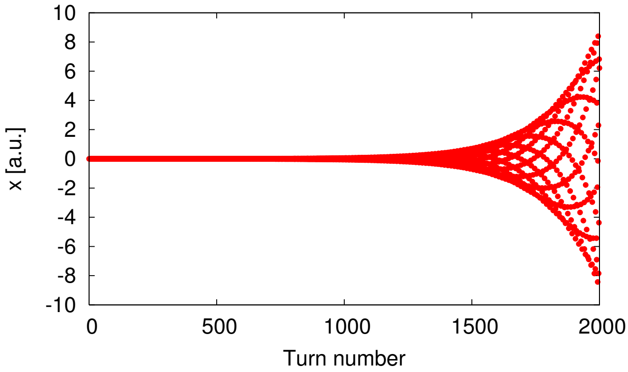
- Let $Q_{quad} = f_{quad}/f_{rev}$ be the tune of the quadrupolar field oscillation.
- This causes the appearance of new resonances
- Linear resonances: $2Q_x \pm Q_{quad} = N$

Oscillating quadrupolar field, $2Q_x \neq Q_{quad}$



Tune is modulated with Q_{quad}, displaying sidebands at $Q_x \pm Q_{quad}$ but there is no emittance growth far from resonances.

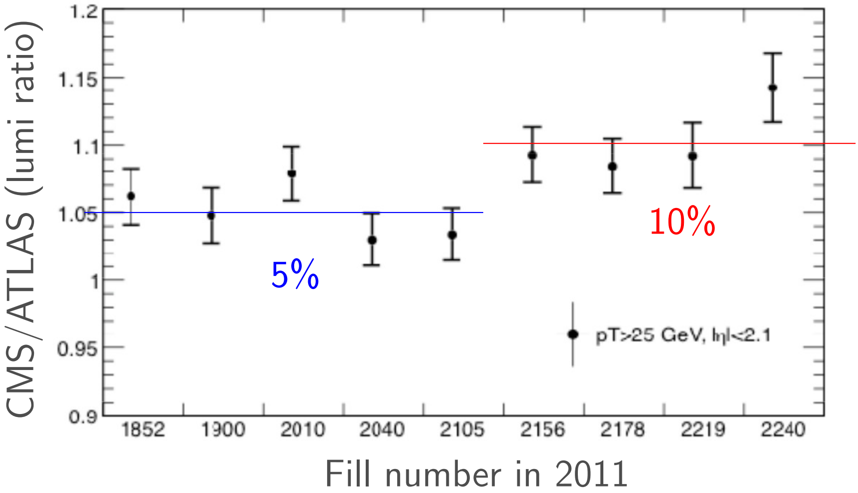
Oscillating quadrupolar field, $2Q_x = Q_{quad}$



Exponential growth, clear signatures depending on the oscillating field type.

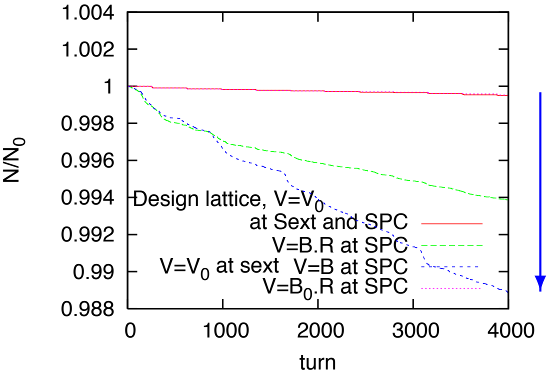
Concluding...

Luminosity imbalance CMS/ATLAS



ATLAS was not happy to get lower luminosity. This was due to β-beating at the IPs.

Space charge simulations with measured optics in J-PARC, K. Ohmi et al., IPAC 2013



Beam loss due to introducing measured $\frac{\Delta\beta}{\beta}$=5% in simulations

K. Ohmi et al.: "Estimation of errors of accelerator elements is inevitable to study beam loss."

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