Linear Accelerators

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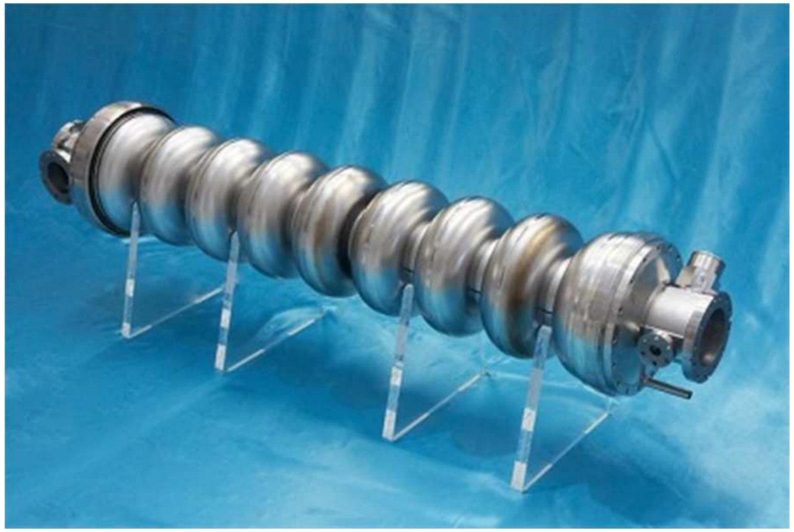

The CERN Accelerator School

Introduction to Accelerator Physics

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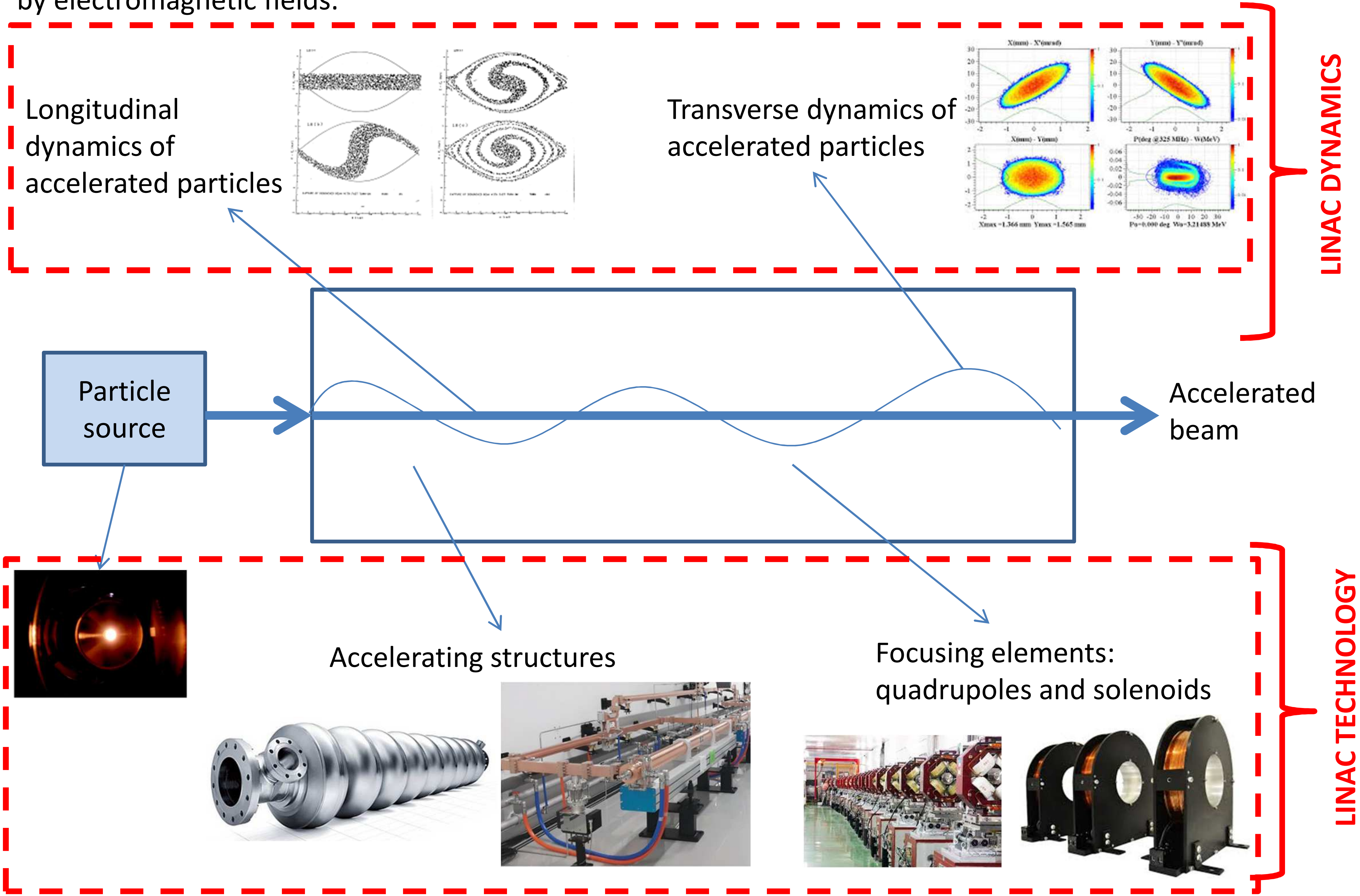
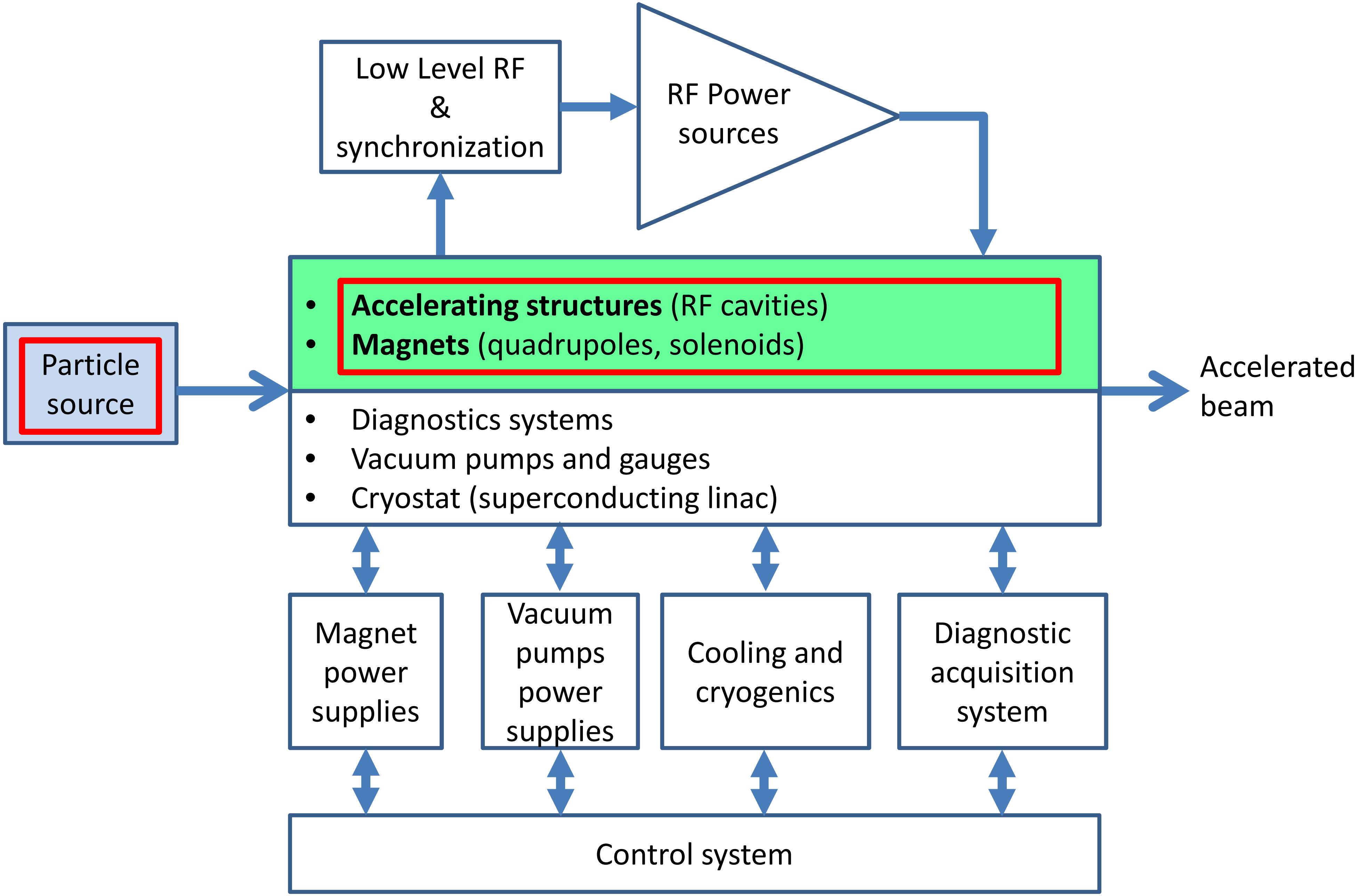
LINAC: BASIC DEFINITION AND MAIN COMPONENTS

LINAC (linear accelerator) is a **system that allows to accelerate charged particles through a linear trajectory** by electromagnetic fields.



LINAC TECHNOLOGY COMPLEXITY



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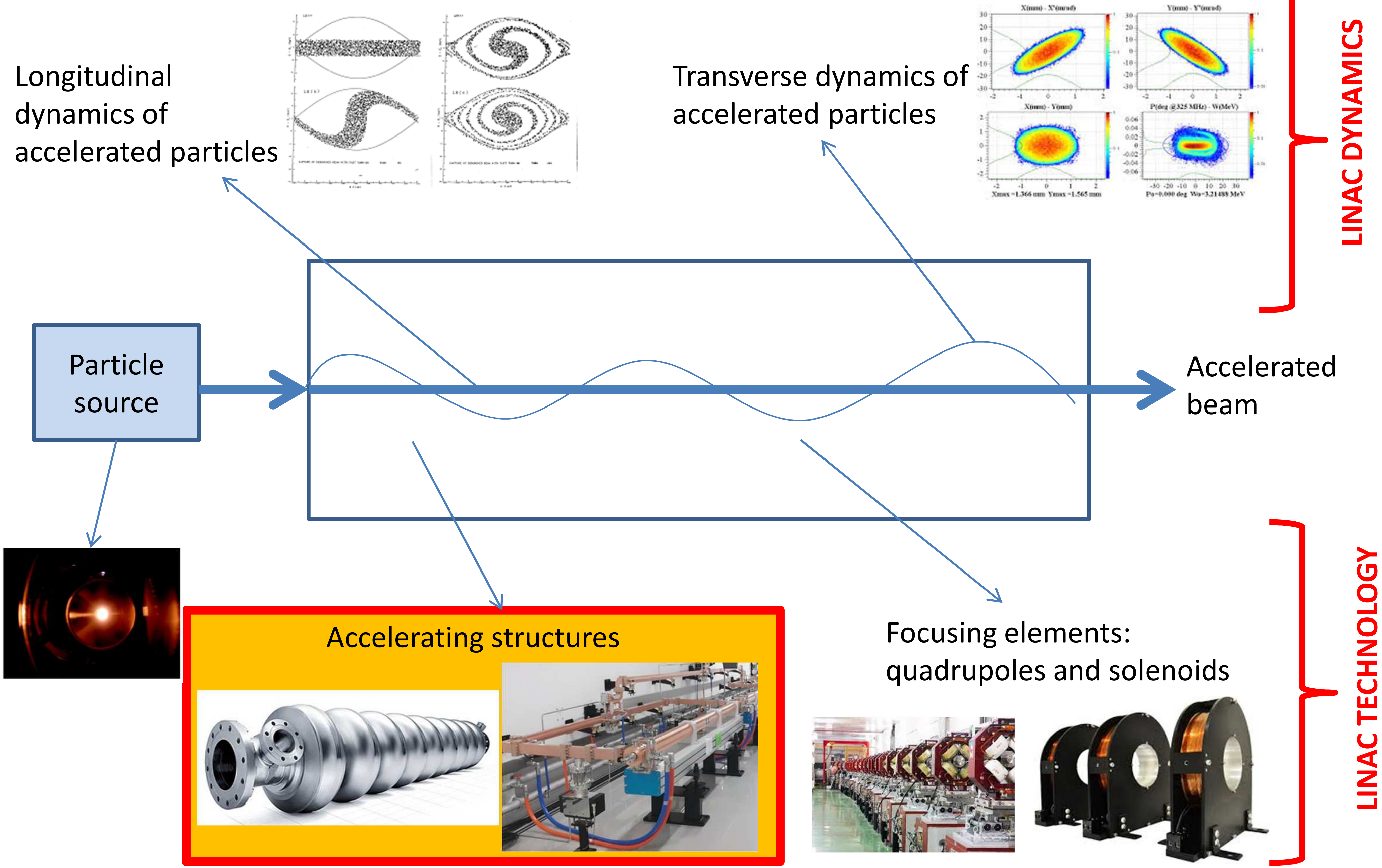


LORENTZ FORCE: ACCELERATION AND FOCUSING

The basic equation that describes the acceleration/bending/focusing processes is the **Lorentz Force**. Particles are **accelerated through electric** fields and are **bended and focused through magnetic** fields.

$\vec{p} = momentum$

$m = mass$

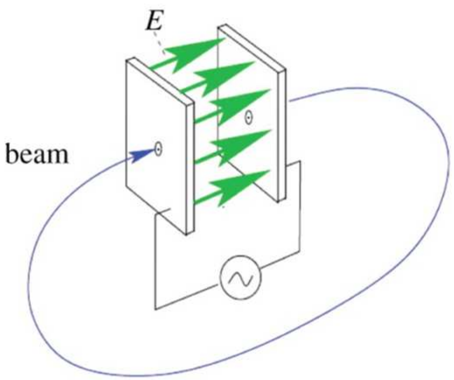
$\vec{v} = velocity$

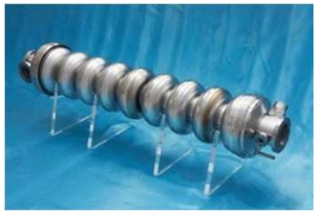
$q = charge$

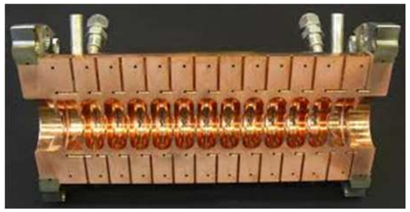
$$\frac{d\vec{p}}{dt} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

ACCELERATION

To accelerate, we need a force in the direction of motion

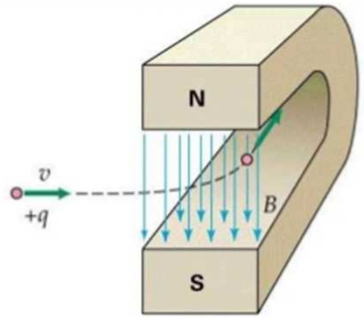


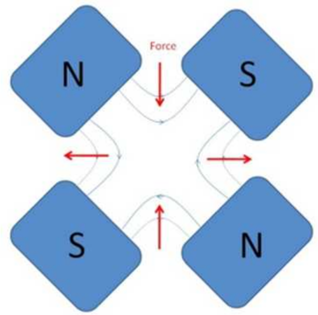


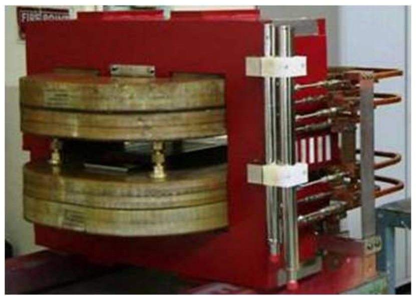


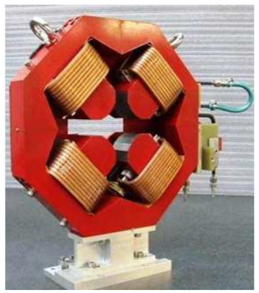
BENDING AND FOCUSING

2^{nd} term always perpendicular to motion => no energy gain







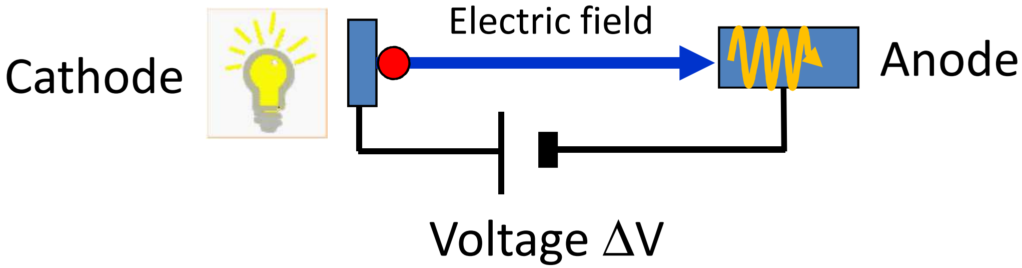


Longitudinal Dynamics

Transverse Dynamics

ACCELERATION: SIMPLE CASE

The **first historical linear particle accelerator** was built by the Nobel prize Wilhelm Conrad Röntgen (1900). It consisted in a vacuum tube containing a cathode connected to the negative pole of a DC voltage generator. **Electrons emitted by the heated cathode** were accelerated while flowing to another electrode connected to the positive generator pole (anode). Collisions between the energetic electrons and the anode produced **X-rays**.



Bertha Röntgen's Hand
8 Nov, 1895

The **energy gained** by the electrons travelling from the cathode to the anode is equal to their charge multiplied the DC voltage between the two electrodes.

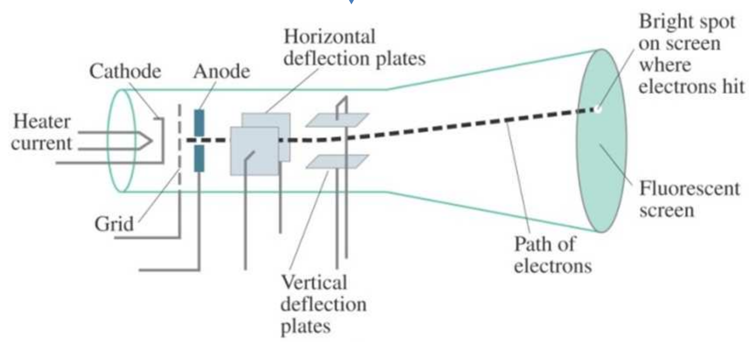
$$\frac{d\vec{p}}{dt} = q\vec{E} \quad \Rightarrow \quad \Delta E = q\Delta V$$

$$\vec{p} = momentum$$

$$q = charge$$

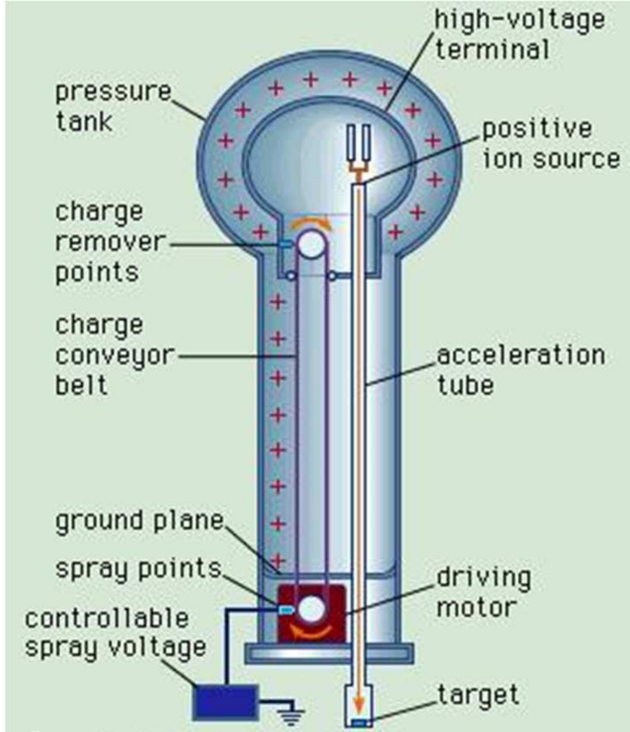
$$E = energy$$

Particle energies are typically expressed in electron-volt [eV], equal to the energy gained by 1 electron accelerated through an electrostatic potential of 1 volt:
1 eV=1.6x10^{-19} J



ELECTROSTATIC ACCELERATORS

To increase the achievable maximum energy, Van de Graaff invented an electrostatic generator based on a **dielectric belt** transporting positive charges to an isolated electrode hosting an **ion source**. The positive ions generated in a large positive potential were accelerated toward ground by the static electric field.



LIMITS OF ELECTROSTATIC ACCELERATORS

DC voltage as large as ~10 MV can be obtained (E~10 MeV). The main limit in the achievable voltage is the **breakdown** due to **insulation** problems.

APPLICATIONS OF DC ACCELERATORS

DC particle accelerators are in operation worldwide, typically at V<15MV (E_{max}=15 MeV), I<100mA.

They are used for:

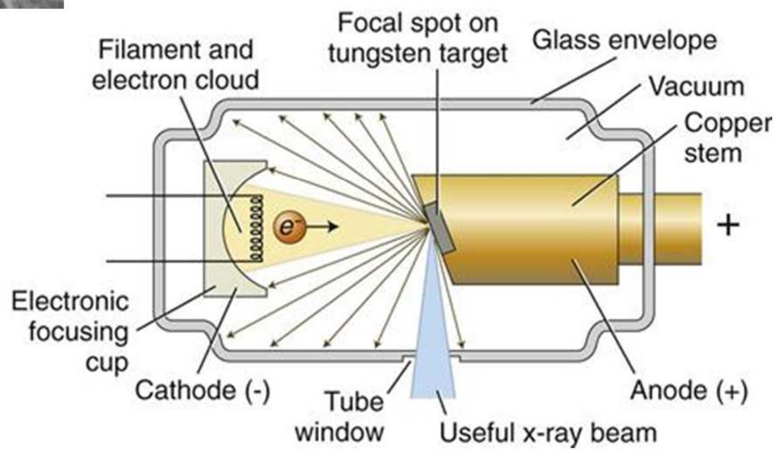
⇒ *material analysis*

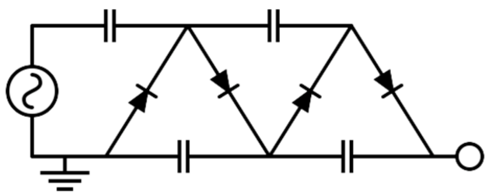
⇒ *X-ray production,*

⇒ *ion implantation for semiconductors*

⇒ *first stage of acceleration (particle sources)*

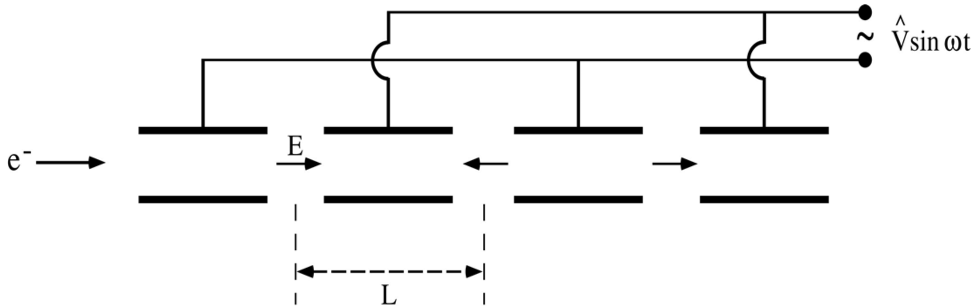
750 kV Cockcroft-Walton Linac2 injector at CERN from 1978 to 1992



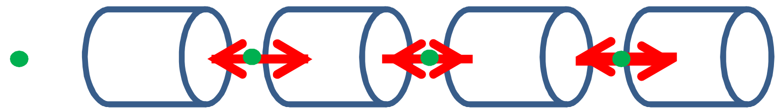


RF ACCELERATORS : WIDERÖE "DRIFT TUBE LINAC" (DTL)

Basic idea: the particles are accelerated by the electric field in the gap between electrodes connected alternatively to the poles of an AC generator. This original idea of **Ising** (1924) was implemented by **Wideroe** (1927) who applied a sine-wave voltage to a sequence of **drift tubes**. The particles **do not experience any force while travelling inside the tubes** (equipotential regions) and are **accelerated across the gaps**. This kind of structure is called **Drift Tube LINAC (DTL).**



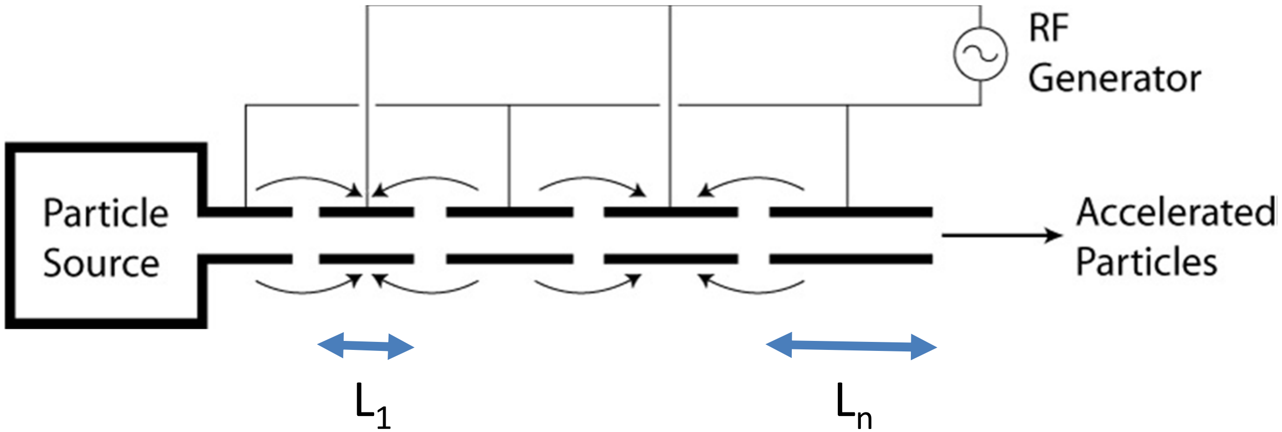
Rolf Wideroe



⇒If the **length of the tubes** increases with the particle velocity during the acceleration such that the time of flight is kept constant and equal to half of the RF period, the particles are subject to a **synchronous accelerating voltage** and experience an energy gain of $\Delta E = q\Delta V$ at each gap crossing.

⇒In principle a single **RF generator** can be used to indefinitely accelerate a beam, **avoiding the breakdown limitation** affecting the electrostatic accelerators.

⇒The Wideroe LINAC is the **first RF LINAC**



PARTICLE VELOCITY VS ENERGY: LIGHT AND HEAVY PARTICLES

Single particle:
- rest mass m_0
- rest energy E_0 $(=m_0c^2)$
- total energy E
- mass m
- velocity v
- momentum p $(=mv)$
- Kinetic energy $W=E-E_0$

Relativistic factor $\beta=v/c$ (<1)

Relativistic factor $\gamma=E/E_0$ (≥1)

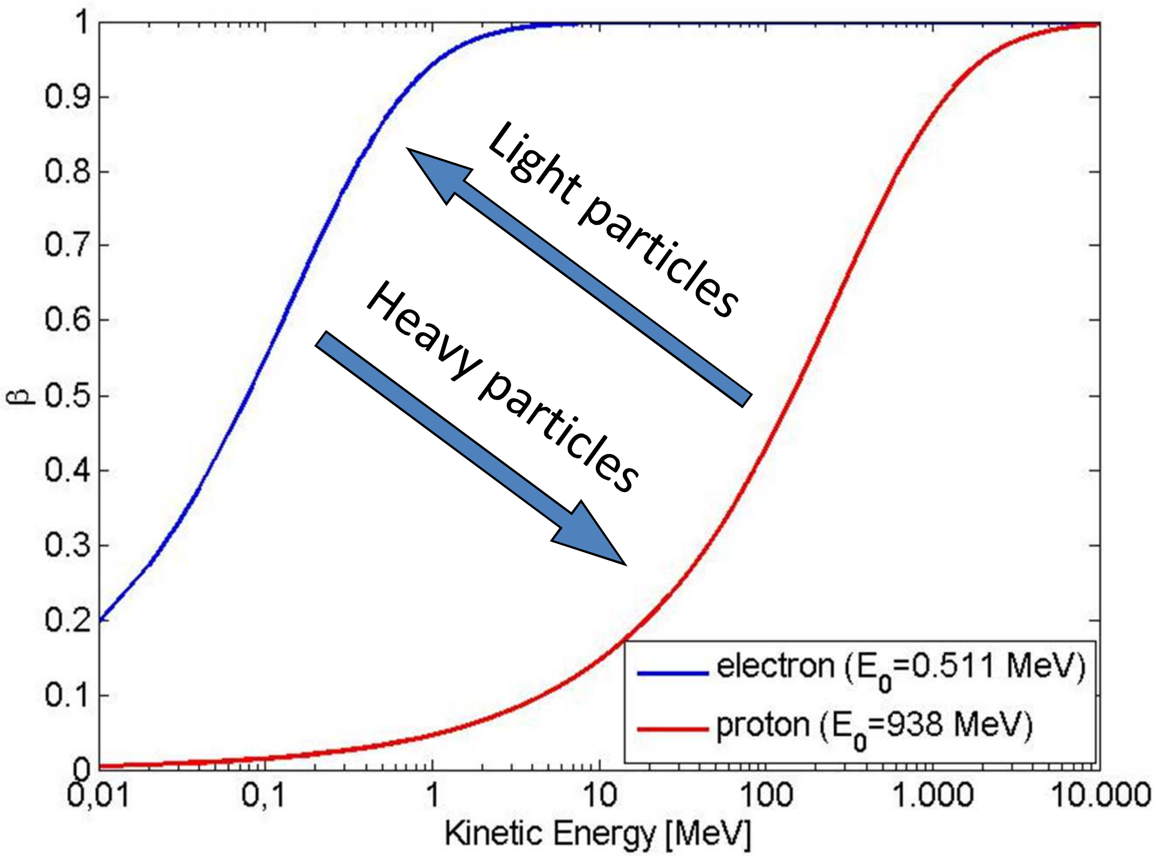
+

$$E^2 = E_0^2 + p^2c^2$$

⇒

$$\beta = \sqrt{1-1/\gamma^2}$$
$$\gamma = 1/\sqrt{1-\beta^2}$$
$$W = (\gamma-1)m_0c^2 \approx \frac{1}{2}m_0v^2 \quad if\ \beta << 1$$

$$\beta = \sqrt{1-\frac{1}{\gamma^2}} = \sqrt{1-\left(\frac{E_0}{E}\right)^2} = \sqrt{1-\left(\frac{E_0}{E_0+W}\right)^2}$$



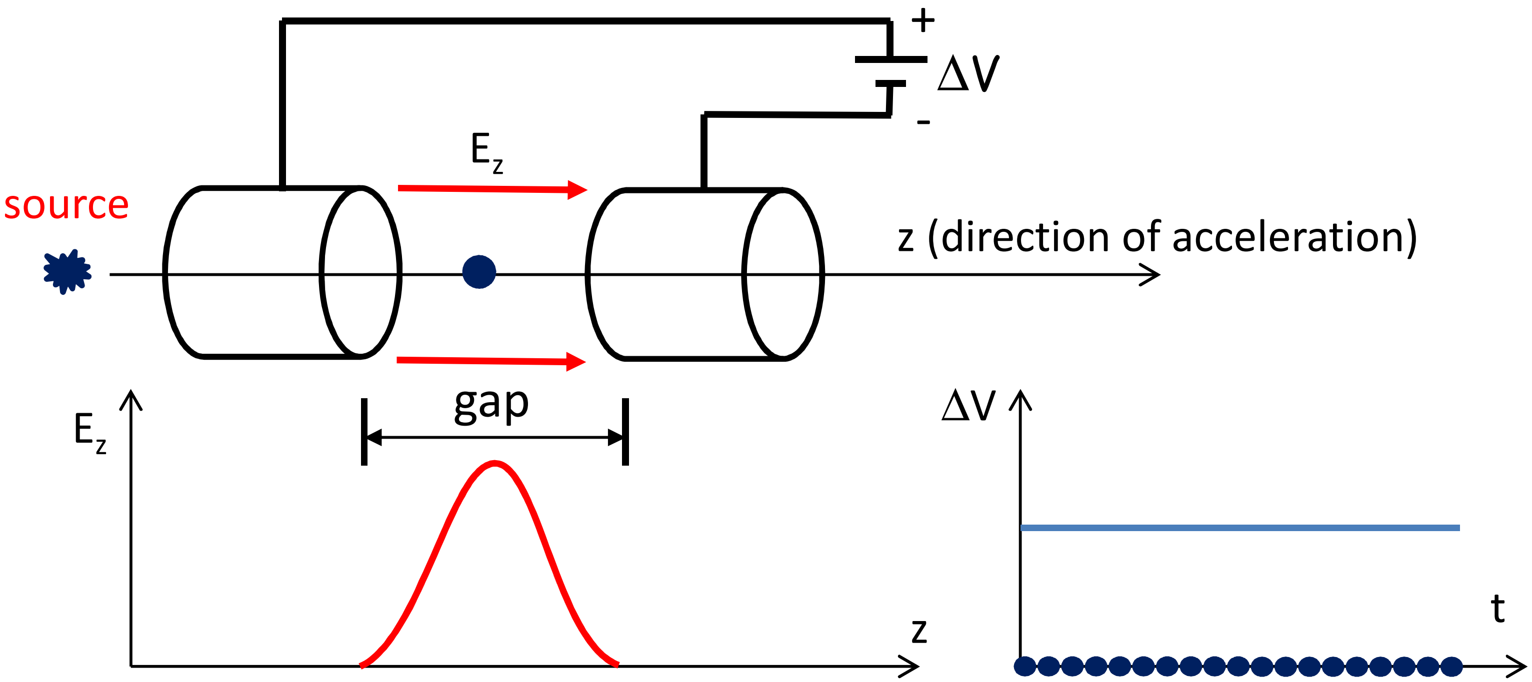
⇒**Light particles** (as **electrons**) are practically fully relativistic (β≅1, γ>>1) at relatively low energy and **reach a constant velocity** (~c). The acceleration process occurs at constant particle velocity

⇒**Heavy particles** (**protons and ions**) are typically weakly relativistic and **reach a constant velocity only at very high energy.** The velocity changes a lot during acceleration process.

⇒This implies **important differences** in the technical characteristics of the **accelerating structures**. In particular for protons and ions we need different types of accelerating structures, **optimized for different velocities** and/or the accelerating structure has to vary its geometry to take into account the velocity variation.

ACCELERATION: ENERGY GAIN

We consider the acceleration between two electrodes in DC.



$$E^2 = E_0^2 + p^2c^2 \Rightarrow 2EdE = 2pdpc^2 \Rightarrow dE = v\frac{mc^2}{E}dp \Rightarrow dE = vdp$$

$$\frac{dp}{dt} = qE_z \Rightarrow v\frac{dp}{dz} = qE_z \Rightarrow \boxed{\frac{dE}{dz} = qE_z} \quad \left(\text{and also}\,\frac{dW}{dz} = qE_z\right) \qquad W=E-E_0$$

→ rate of energy gain per unit length

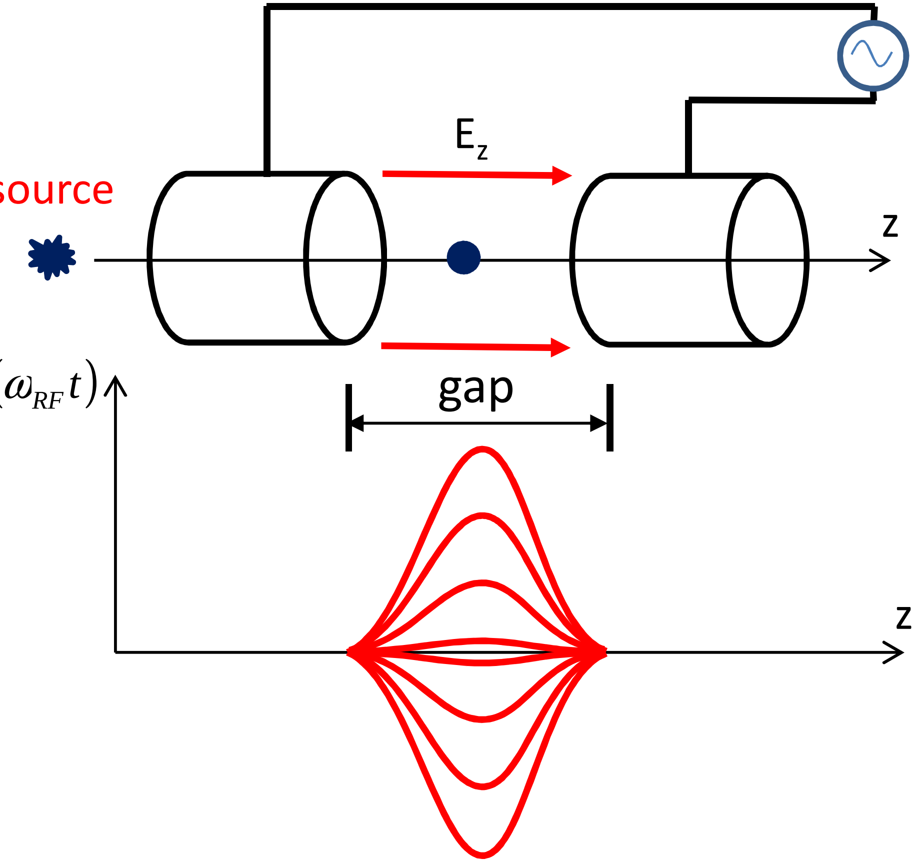
$$\Rightarrow \Delta E = \int_{gap}\frac{dE}{dz}dz = \int_{gap} qE_z \Rightarrow \boxed{\Delta E = q\Delta V}$$

→ energy gain per electrode

RF ACCELERATION: BUNCHED BEAM

We consider now the acceleration between two electrodes fed by an RF generator

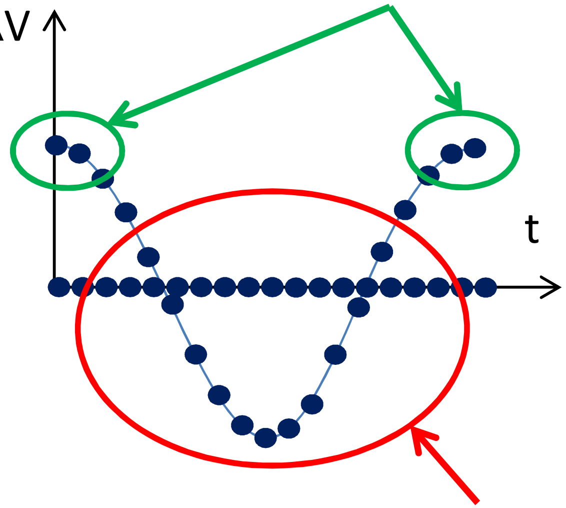
$$\Delta V = V_{RF}\cos(\omega_{RF}t) \qquad \omega_{RF} = 2\pi f_{RF} = \frac{2\pi}{T_{RF}}$$



$$E_z(z,t) = E_{RF}(z)\cos(\omega_{RF}t)$$

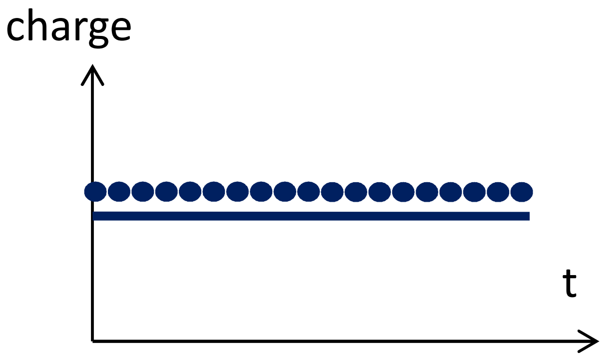
$$\Rightarrow \Delta E = q\Delta V = qV_{RF}\cos(\omega_{RF}t)$$

Only these particles are accelerated



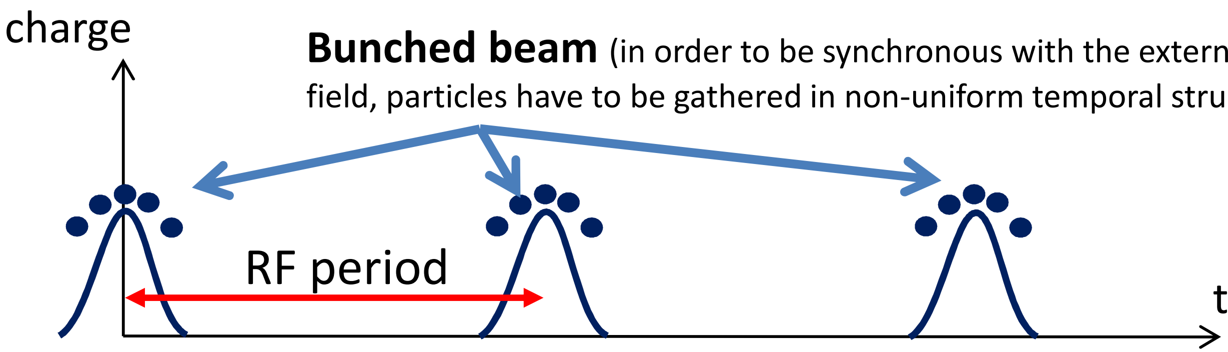
These particles are not accelerated and basically are lost during the acceleration process

DC acceleration



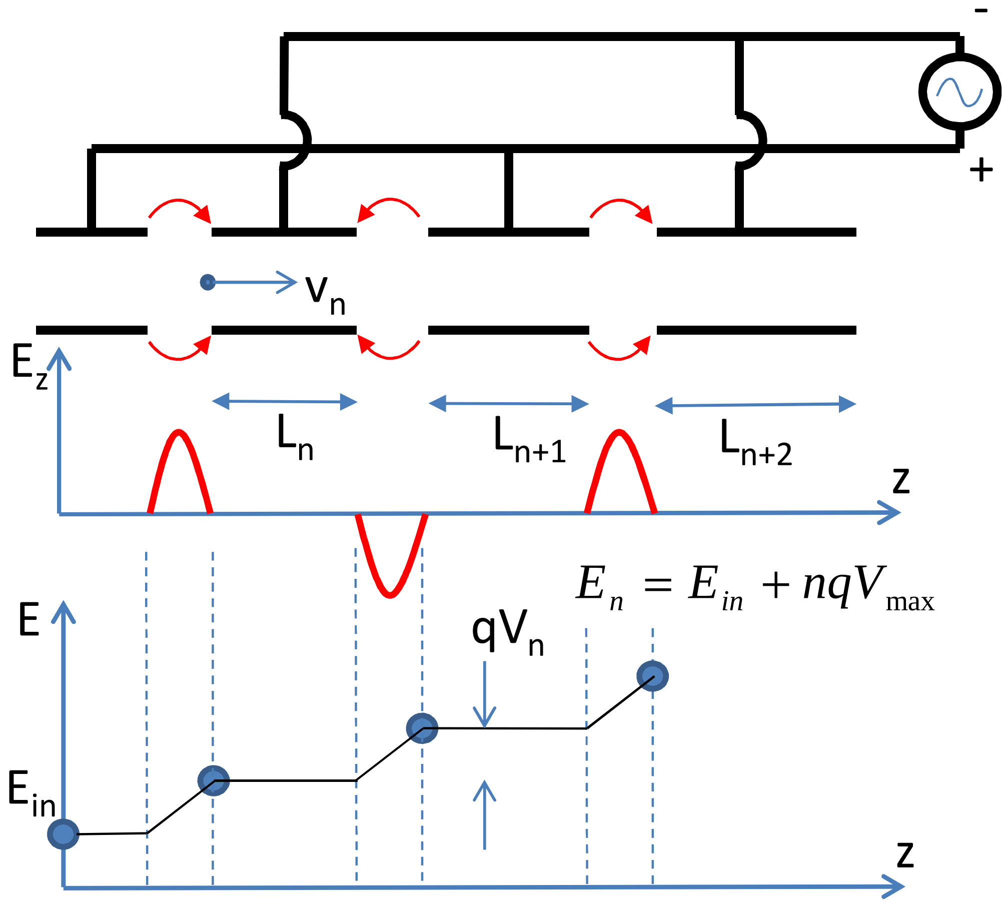
RF acceleration

Bunched beam (in order to be synchronous with the external AC field, particles have to be gathered in non-uniform temporal structure)



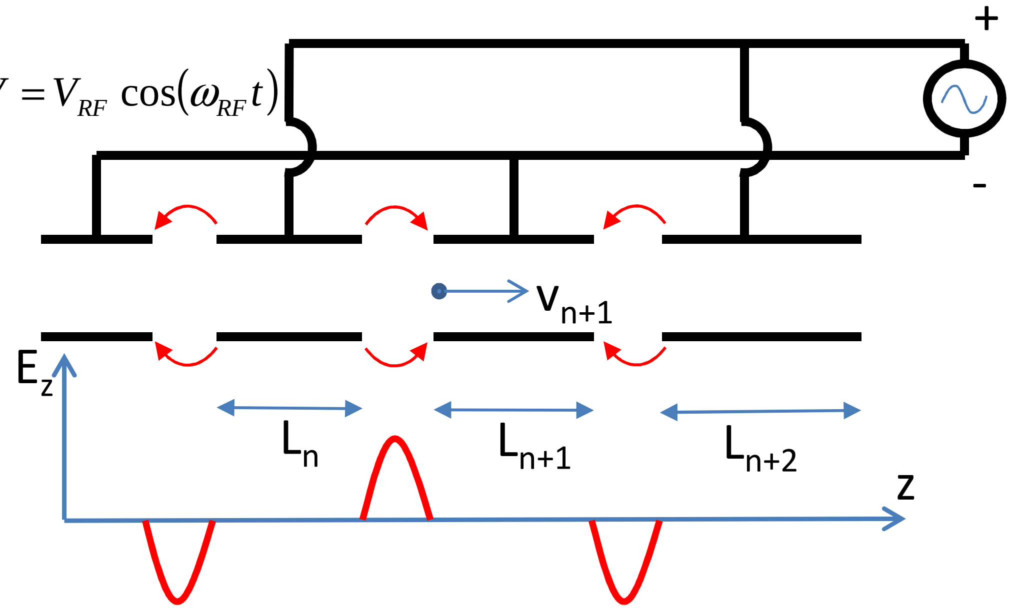
DRIFT TUBE LENGTH AND FIELD SYNCHRONIZATION

If now we consider a DTL structure with an injected particle at an energy E_{in}, we have that at each gap the energy gain is $\Delta E_n = qV_{RF}$ and the particle increase its velocity accordingly to the previous relativistic formulae.

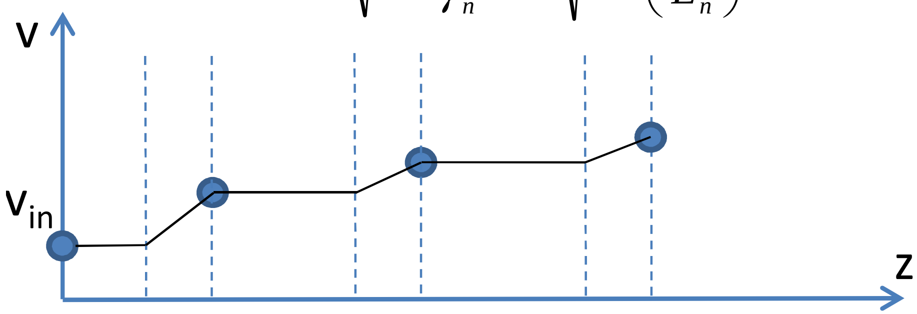


$$E_n = E_{in} + nqV_{max}$$

$$\Delta V = V_{RF}\cos(\omega_{RF} t)$$



$$v_n = c\beta_n = c\sqrt{1-\frac{1}{\gamma_n^2}} = c\sqrt{1-\left(\frac{E_o}{E_n}\right)^2}$$



⇒ In order to be synchronous with the accelerating field at each gap the **length of the n-th drift tube** has to be L_n:

$$t_n = \frac{L_n}{v_n} = \frac{T_{RF}}{2} \Rightarrow L_n = \frac{1}{2}v_n T_{RF} = \frac{1}{2}\beta_n c T_{RF}$$

$$L_n = \frac{1}{2}\beta_n \lambda_{RF}$$

⇒The **energy gain per unit length** (i.e. the **average accelerating gradient**) is given by:

$$\frac{\Delta E}{\Delta L} = \frac{qV_{RF}}{L_n} = \frac{2qV_{RF}}{\lambda_{RF}\beta_n}$$

[eV/m]

ACCELERATION WITH HIGH RF FREQUENCIES: RF CAVITIES

There are two important **consequences** of the previous obtained formulae:

$$L_n = \frac{1}{2}\beta_n\lambda_{RF}$$

The condition **$L_n \ll \lambda_{RF}$** (necessary to model the tube as an **equipotential region**) requires $\beta \ll 1$. ⇒The Wideröe technique can **not be applied to relativistic particles**.

$$\frac{\Delta E}{\Delta L} = \frac{qV_{RF}}{L_n} = qE_{RF} = \frac{2qV_{RF}}{\lambda_{RF}\beta_n}$$

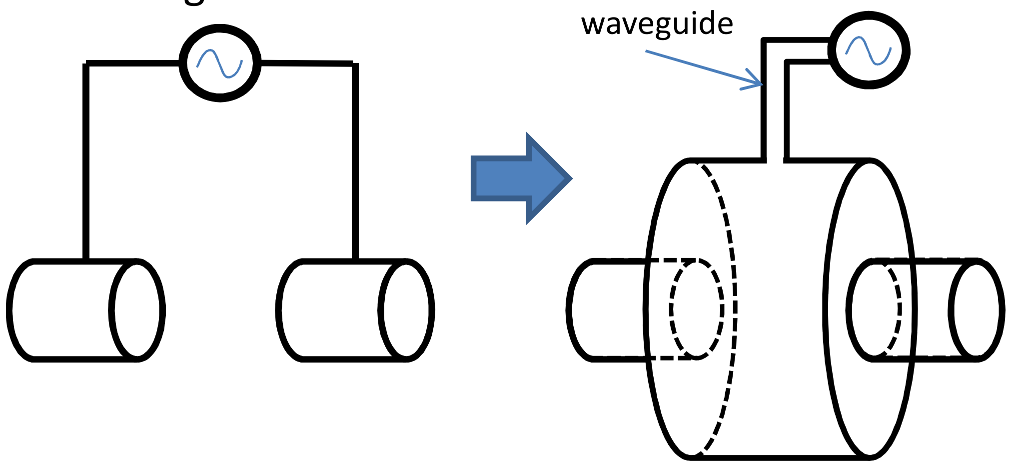
Moreover when particles get high velocities the drift spaces get longer and one looses on the efficiency. The **average accelerating gradient (E_{RF} [V/m]) increase pushes towards small λ_{RF}** (high frequencies).

High frequency high power **sources** became available after the **2nd world war** pushed by **military** technology needs (such as **radar**). However, the concept of equipotential DT can not be applied at small λ_{RF} and the **power lost by radiation is proportional to the RF frequency.**

As a consequence we must consider **accelerating structures different from drift tubes**.

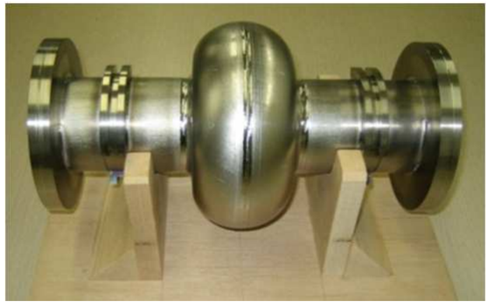
⇒The solution consists of **enclosing the system in a cavity** which resonant frequency matches the RF generator frequency.

⇒Each cavity can be independently powered from the RF generator

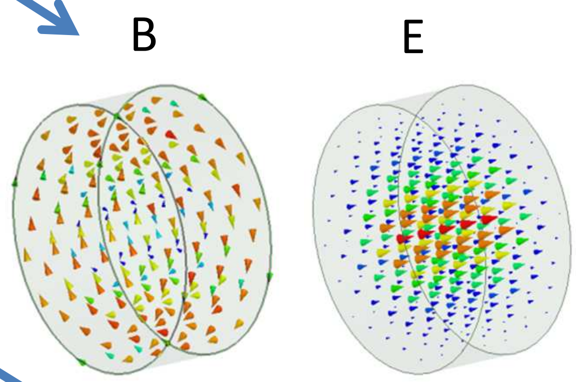


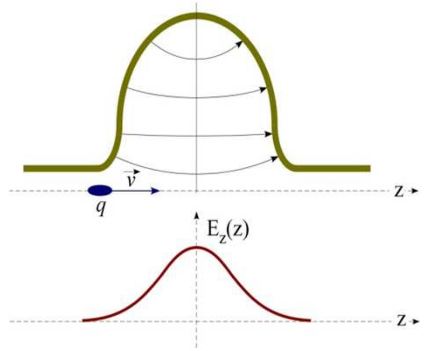
RF CAVITIES

⇒High frequency RF accelerating fields are confined in **cavities**.

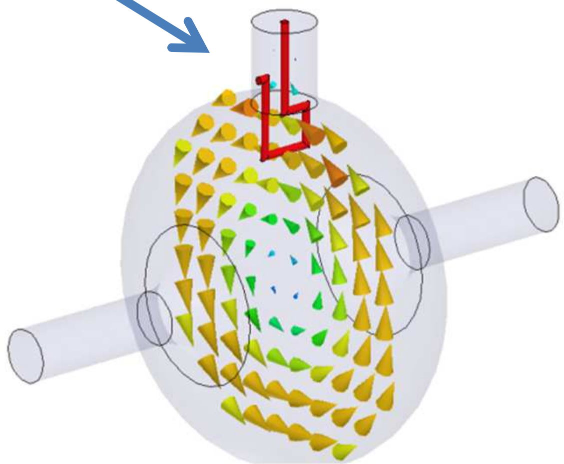


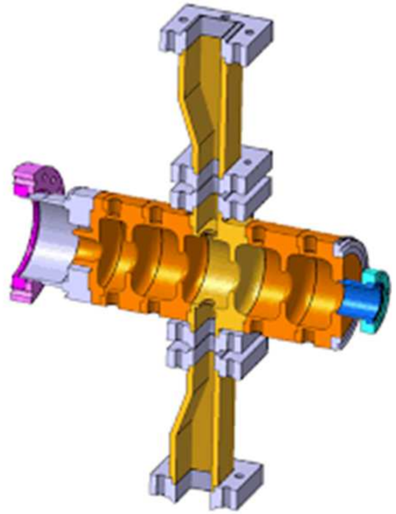
⇒The cavities are **metallic closed volumes** were the e.m fields has a particular spatial configuration (**resonant modes**) whose components, including the accelerating field **E_z**, oscillate at some specific frequencies **f_{RF}** (resonant frequency) characteristic of the mode.





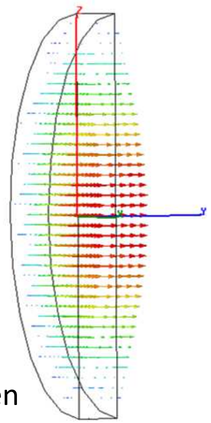
⇒The modes are excited by **RF generators** that are **coupled to the cavities** through waveguides, coaxial cables, etc…

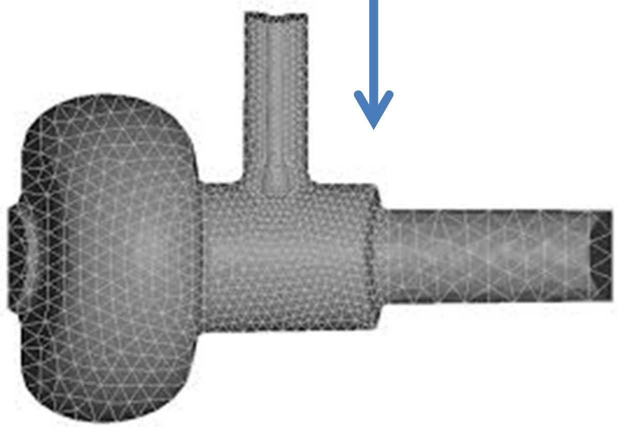




⇒The resonant modes are called **Standing Wave (SW) modes** (spatial fixed configuration, oscillating in time).

⇒The spatial and temporal field profiles in a cavity have to be computed (analytically or numerically) **by solving the Maxwell equations** with the proper boundary conditions.

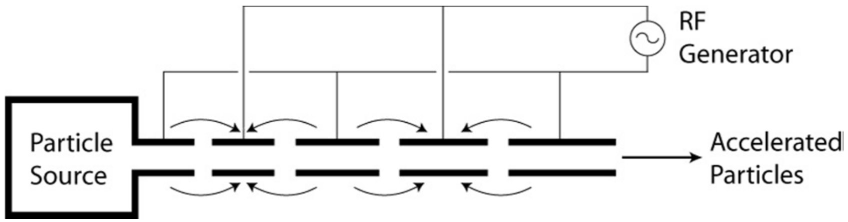


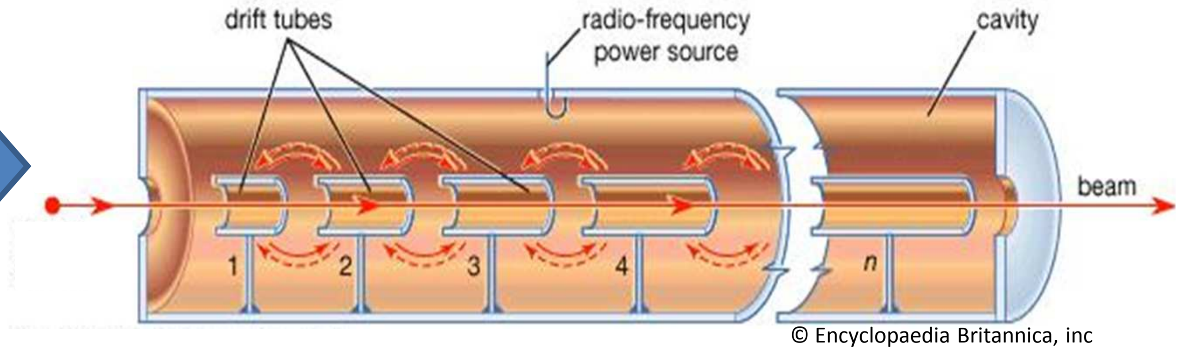


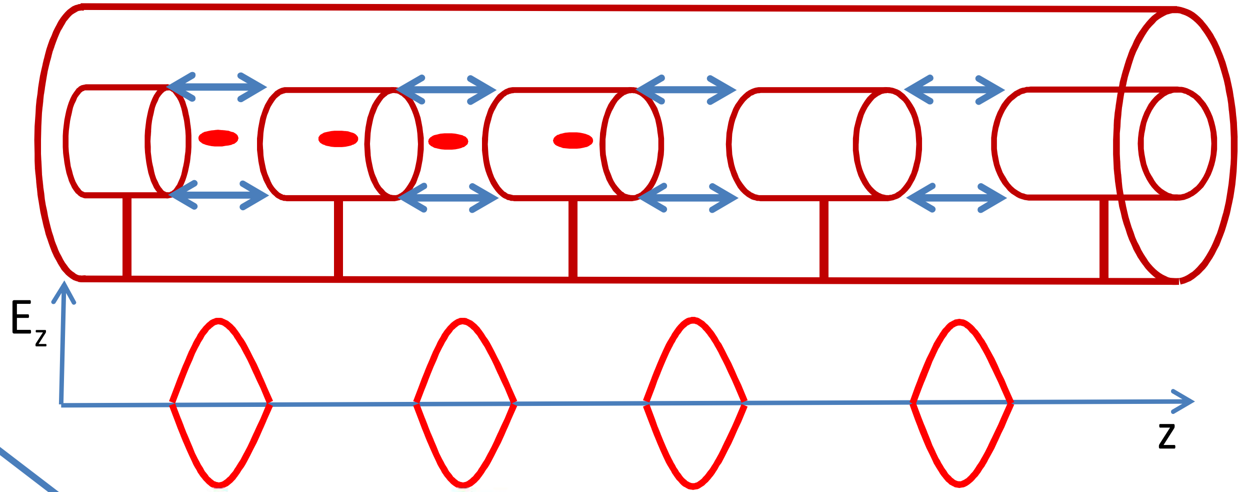
Courtesy E. Jensen

ALVAREZ STRUCTURES

Alvarez's structure can be described as a special DTL in which the electrodes are part of a **resonant macrostructure.**



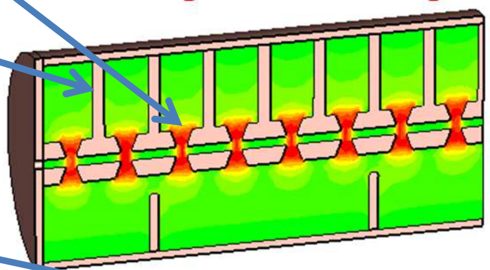




⇒The DTL operates in **0 mode** for protons and ions in the range β=0.05-0.5 (f_{RF}=50-400 MHz) 1-100 MeV;

⇒The beam is inside the "**drift tubes**" when the electric field is decelerating. The **electric field** is concentrated between gaps;

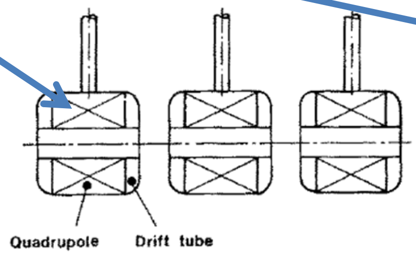
⇒The drift tubes are suspended by **stems**;

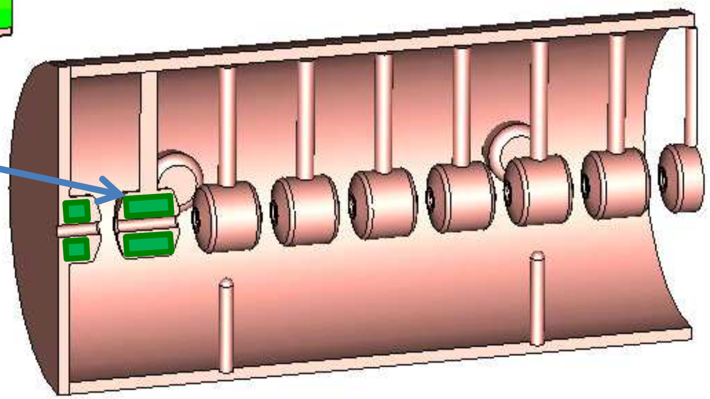


⇒**Quadrupole** (for transverse focusing) can fit inside the drift tubes.

⇒In order to be synchronous with the accelerating field at each gap the **length of the n-th drift tube** L_n has to be:

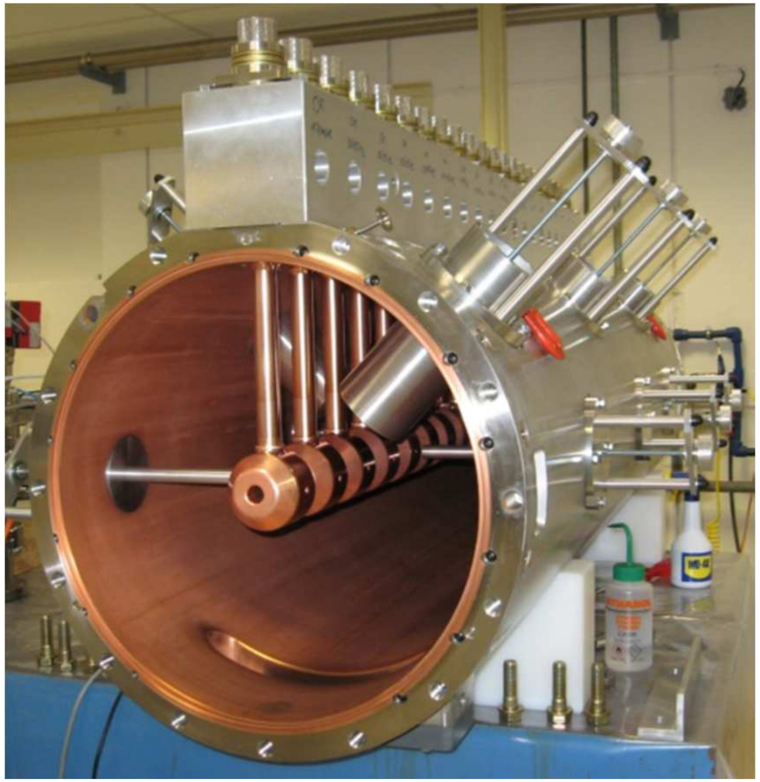
$$L_n = \beta_n \lambda_{RF}$$



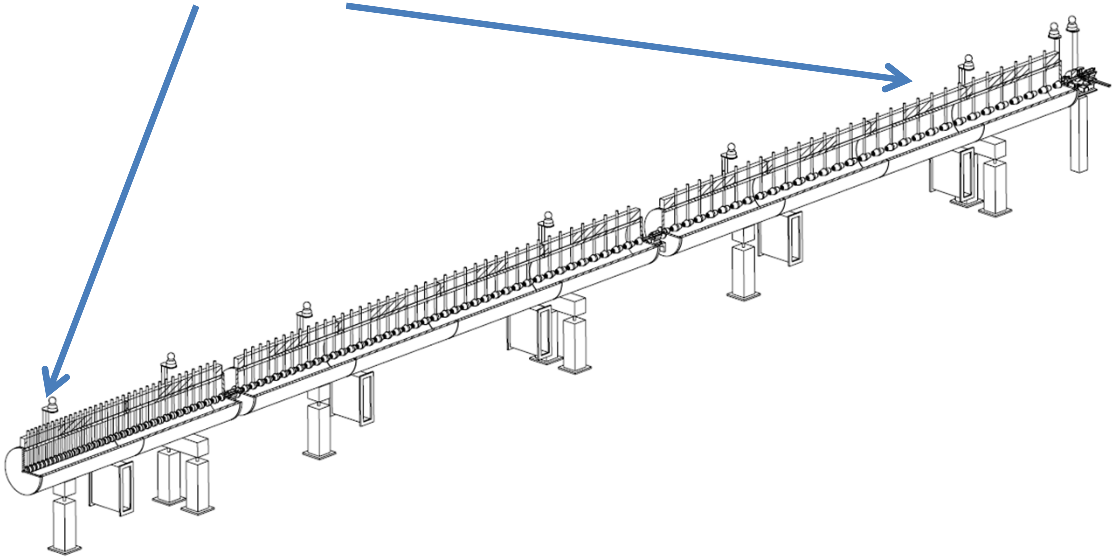


ALVAREZ STRUCTURES: EXAMPLES

CERN LINAC 2 tank 1:
200 MHz 7 m x 3 tanks, 1 m diameter, final energy 50 MeV.



CERN LINAC 4: 352 MHz frequency, Tank diameter 500 mm, 3 resonators (tanks), Length 19 m, 120 Drift Tubes, Energy: 3 MeV to 50 MeV, β=0.08 to 0.31 $\rightarrow$ cell length from 68mm to 264mm.



HIGH β CAVITIES: CYLINDRICAL STRUCTURES

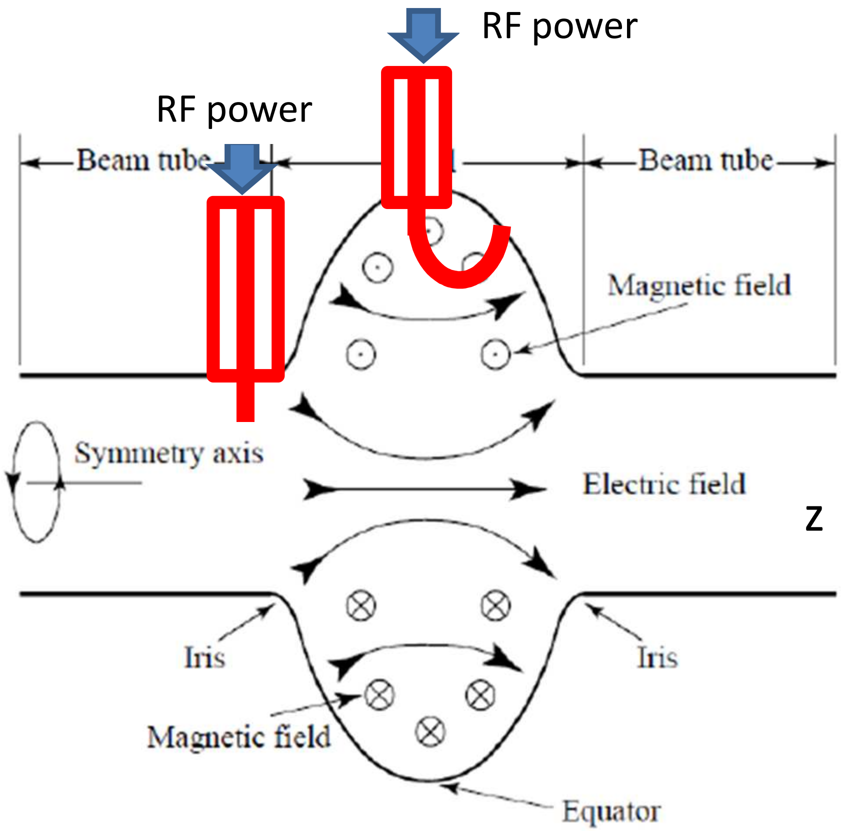
⇒When the β of the particles increases (>0.5) one has to use **higher RF frequencies** (>400-500 MHz) to increase the accelerating gradient per unit length

⇒the **DTL structures became less efficient** (effective accelerating voltage per unit length for a given RF power);

Cylindrical single or multiple cavities working on the $\mathbf{TM_{010}}$**-like** mode are used

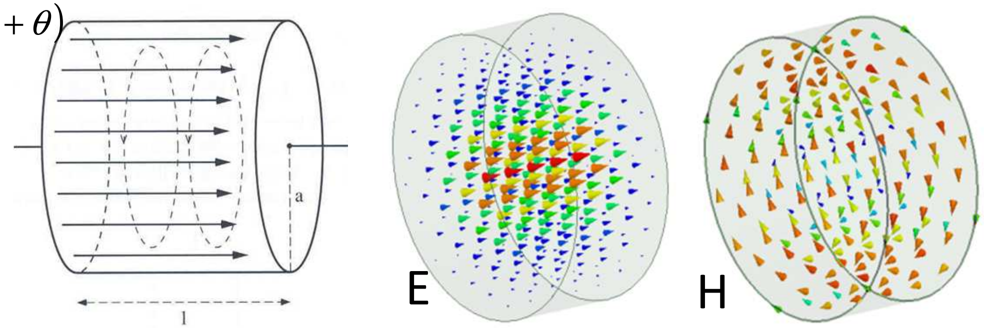
Real cylindrical cavity

(TM_{010}-like mode because of the shape and presence of beam tubes)

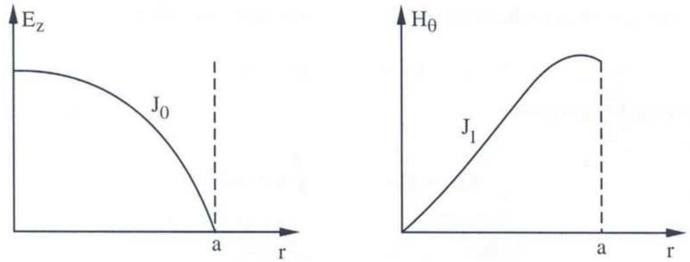


For a **pure cylindrical structure** (also called **pillbox cavity**) the first accelerating mode (i.e. with non zero longitudinal electric field on axis) is the $\mathbf{TM_{010}}$ **mode**. It has a well known analytical solution from Maxwell equation.

$V_{RF}\cos(\phi_s+\theta)$

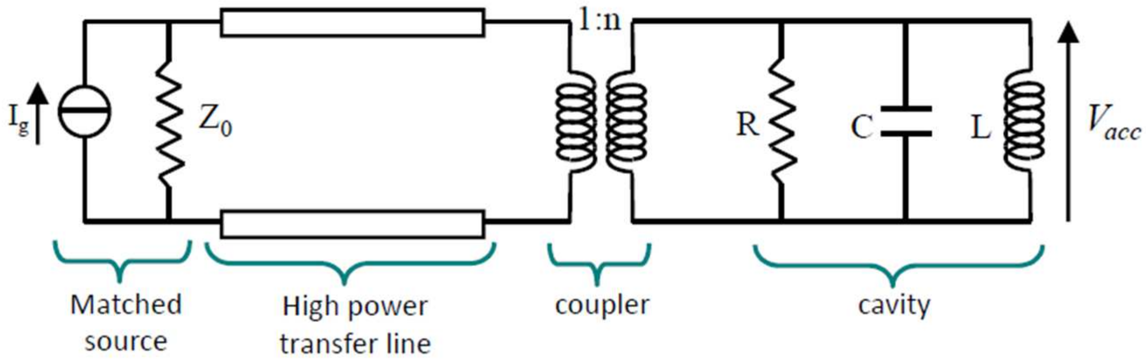


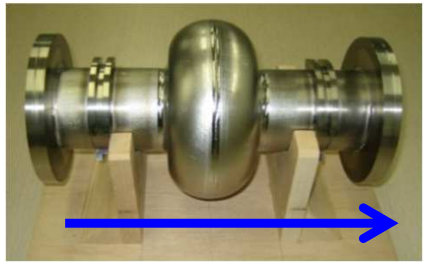
$$f_{res}=\frac{c}{2.62a}$$



$$E_z=AJ_0\left(\frac{2\pi}{2.62a}r\right)\cos(\omega_{RF}t)\qquad H_\theta=A\frac{1}{Z_0}J_1\left(\frac{2\pi}{2.62a}r\right)\sin(\omega_{RF}t)$$

SW CAVITIES PARAMETERS: R, Q





V_{acc}

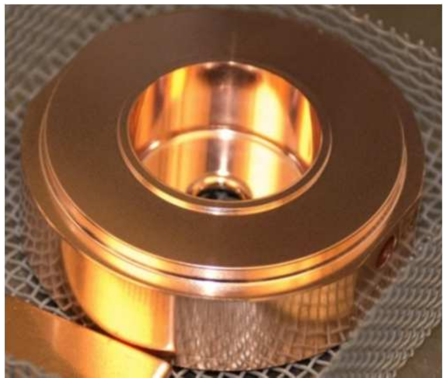
ACCELERATING VOLTAGE (V_{acc}) ***DISSIPATED POWER (P_{diss})*** ***STORED ENERGY (W)***

SHUNT IMPEDANCE

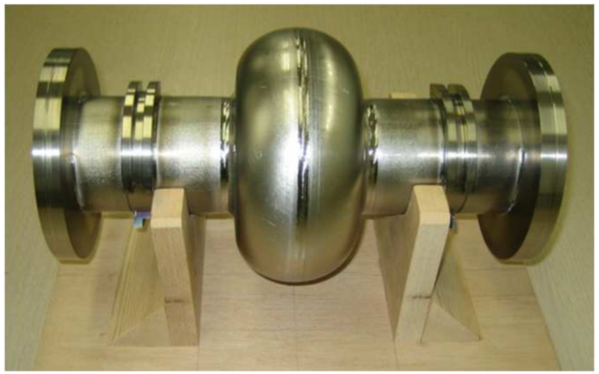
The shunt impedance is the parameter that qualifies the **efficiency of an accelerating mode**. The higher is its value, the larger is the obtainable accelerating voltage for a given power. Traditionally, it is the quantity to optimize in order to **maximize the accelerating field for a given dissipated power**:

$$R = \frac{V_{acc}^2}{2P_{diss}} \quad [\Omega]$$

NC cavity R~1MΩ



SC cavity R~1TΩ



QUALITY FACTOR

$$Q = \omega_{RF} \frac{W}{P_{diss}}$$

NC cavity $Q \sim 10^4$

SC cavity $Q \sim 10^{10}$

Example:

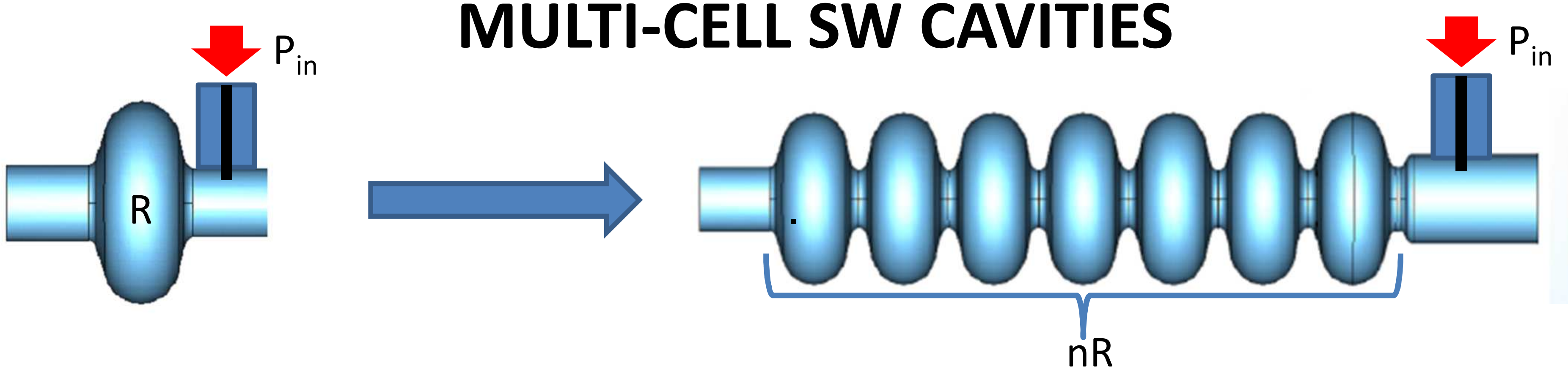
R~0.5MΩ

P_{diss}=1 MW

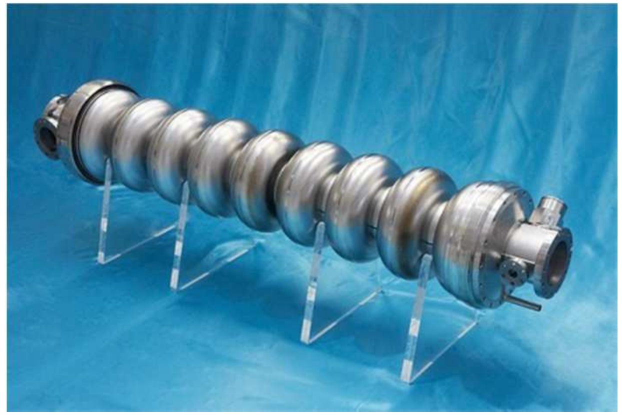
V_{acc}=1MV

For a cavity working at 1 GHz with a structure length of 10 cm we have an average accelerating field of 10 MV/m

MULTI-CELL SW CAVITIES



- In a multi-cell structure there is **one RF input coupler**. As a consequence the **total number of RF sources is reduced**, with a **simplification of the layout and reduction of the costs;**



- The **shunt impedance is n time** the impedance of a single cavity

- They are **more complicated** to fabricate than single cell cavities;

- The fields of adjacent cells couple through the cell **irises** and/or through properly designed coupling **slots**.

MULTI-CELL SW CAVITIES: π MODE STRUCTURES

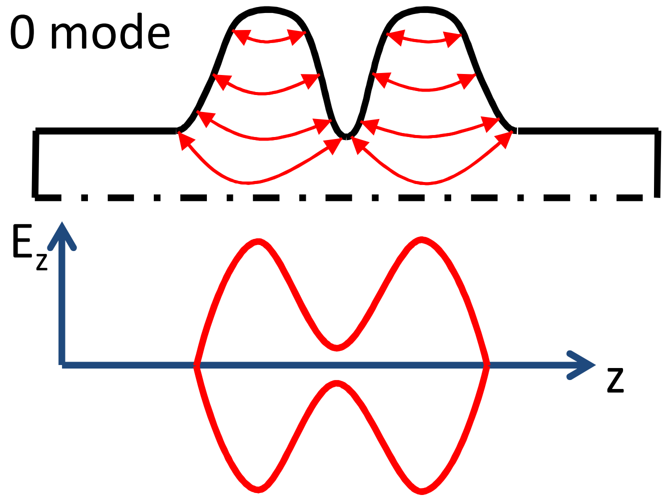
- The N-cell structure behaves like a system composed by **N coupled oscillators** with **N coupled multi-cell resonant modes**.

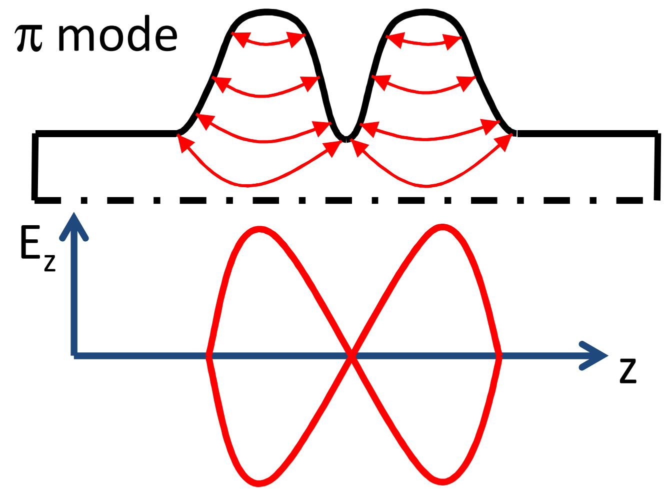
- The modes are characterized by a cell-to-cell phase advance given by:

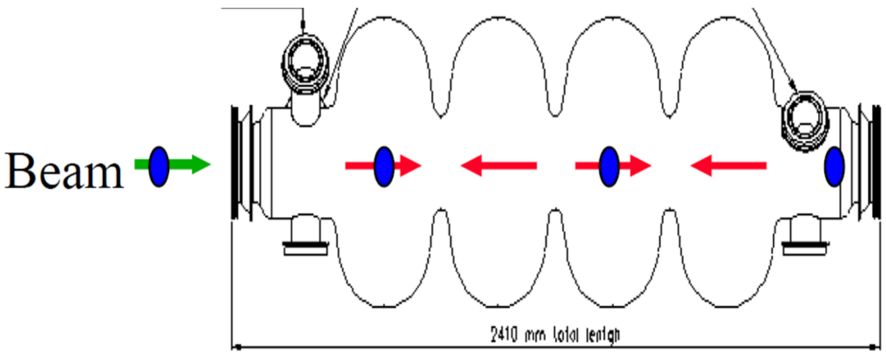
$$\Delta\phi_n = \frac{n\pi}{N-1} \qquad n = 0,1,\ldots,N-1$$

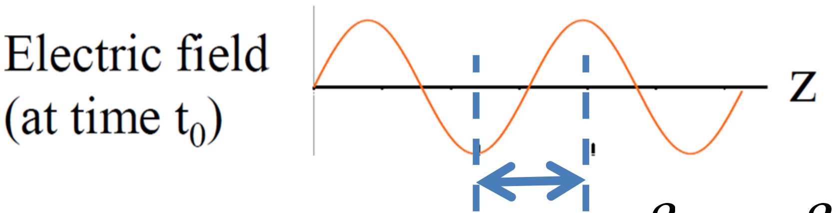
- The multi cell mode generally used for acceleration is the **π, π/2 and 0 mode** (DTL as example operate in the 0 mode).

- In this case as done for the DTL structures the cell length has to be chosen in order to synchronize the accelerating field with the particle traveling into the structure at a certain velocity









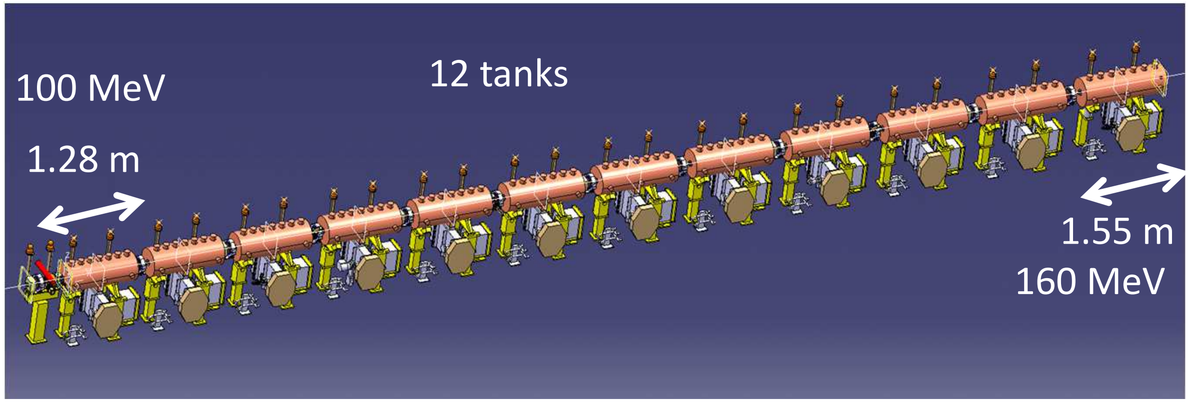
$$d = \frac{\beta c}{2 f_{RF}} = \frac{\beta \lambda_{RF}}{2}$$

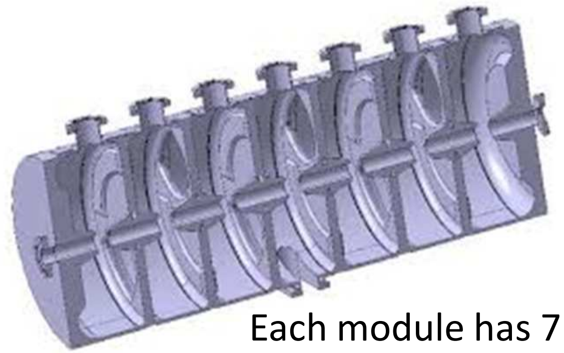
⇒For **ions and protons** the cell length has to be increased and the linac will be made of a sequence of different accelerating structures matched to the ion/proton velocity.

⇒For **electron**, β=1, $d=\lambda_{RF}/2$ and the linac will be made of an injector followed by a series of identical accelerating structures, with cells all the same length.

π MODE STRUCTURES: EXAMPLES

LINAC 4 (CERN) PIMS (PI Mode Structure) for protons: f_{RF}=352 MHz, β>0.4

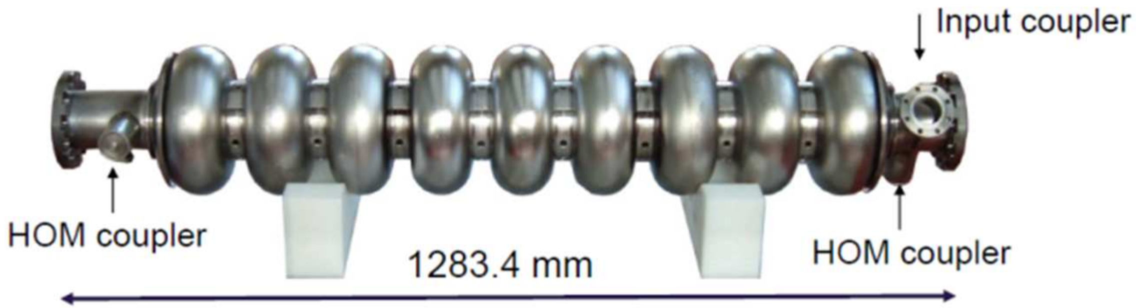




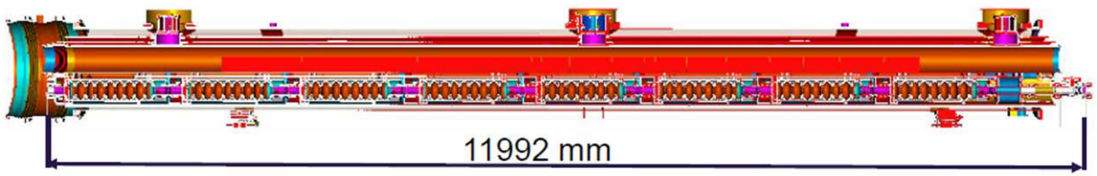
Each module has 7 identical cells

European XFEL (Desy): electrons

800 accelerating cavities
1.3 GHz / 23.6 MV/m



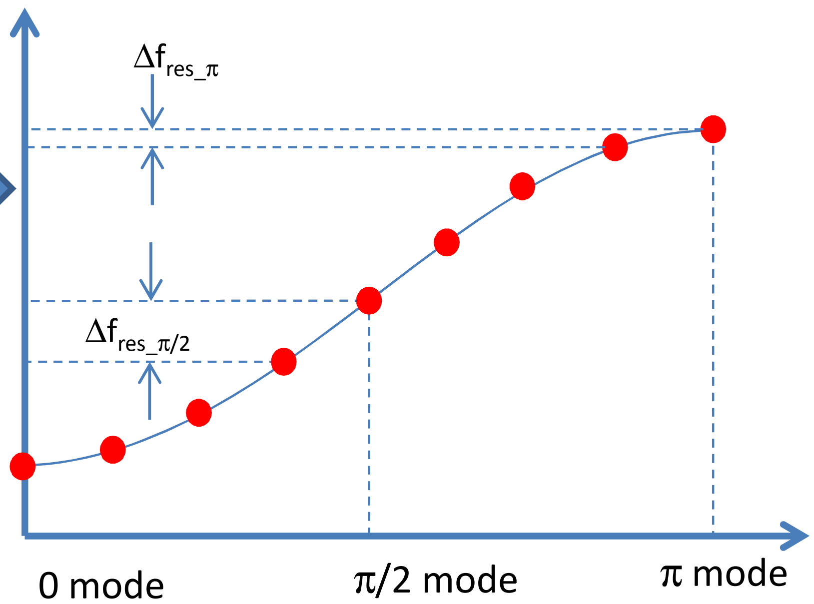
Cryomodule housing: 8 cavities, quadrupole and BPM



All identical β=1
Superconducting cavities

MULTI-CELL SW CAVITIES: π/2 MODE STRUCTURES

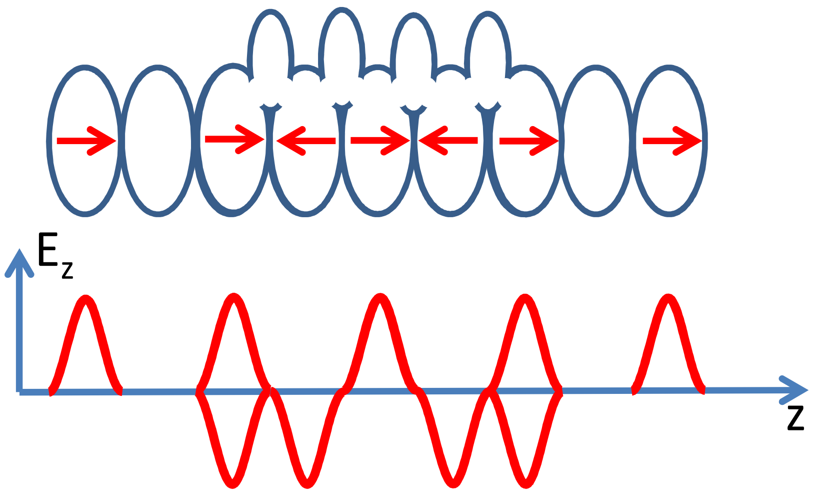
⇒It is possible to demonstrate that **over a certain number of cavities** (>10) working on the π mode, the **overlap between adjacent modes** can be a problem (as example the field uniformity due to machining errors is difficult to tune).



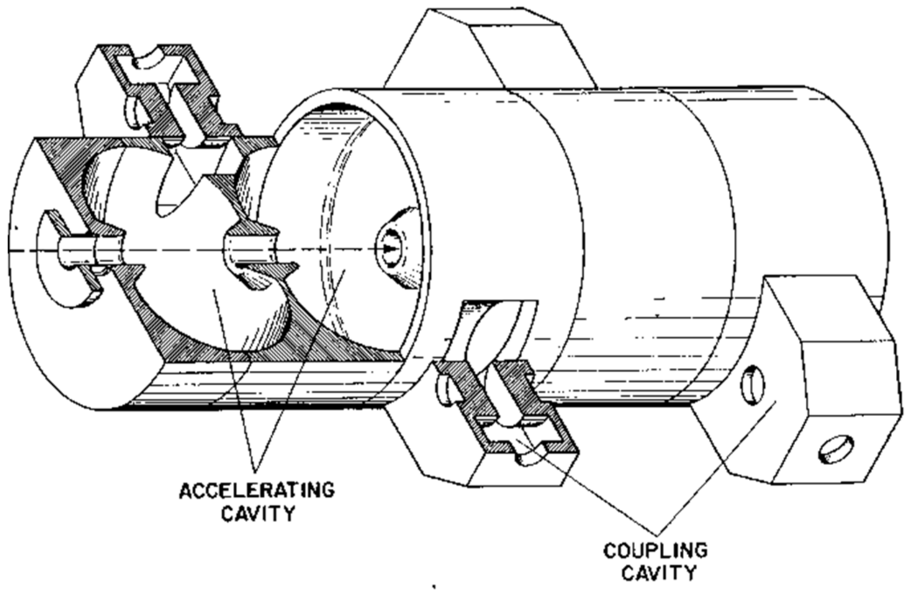
⇒The criticality of a working mode depend on the **frequency separation between the working mode and the adjacent mode**

⇒the **π/2 mode from this point of view is the most stable mode**. For this mode it is possible to demonstrate that the accelerating field is zero every two cells. For this reason the empty cells are put of axis and coupling slots are opened from the accelerating cells to the empty cells.

⇒this allow to increase the number of cells to >20-30 without problems



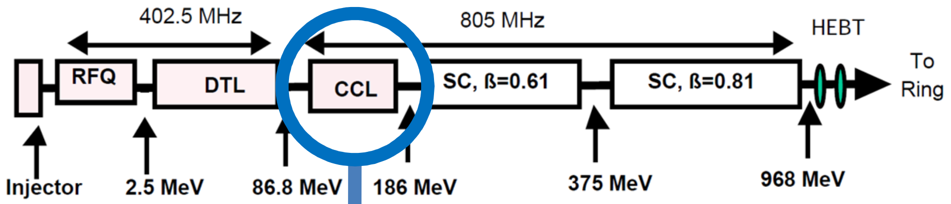
Side Coupled Cavity (SCC)



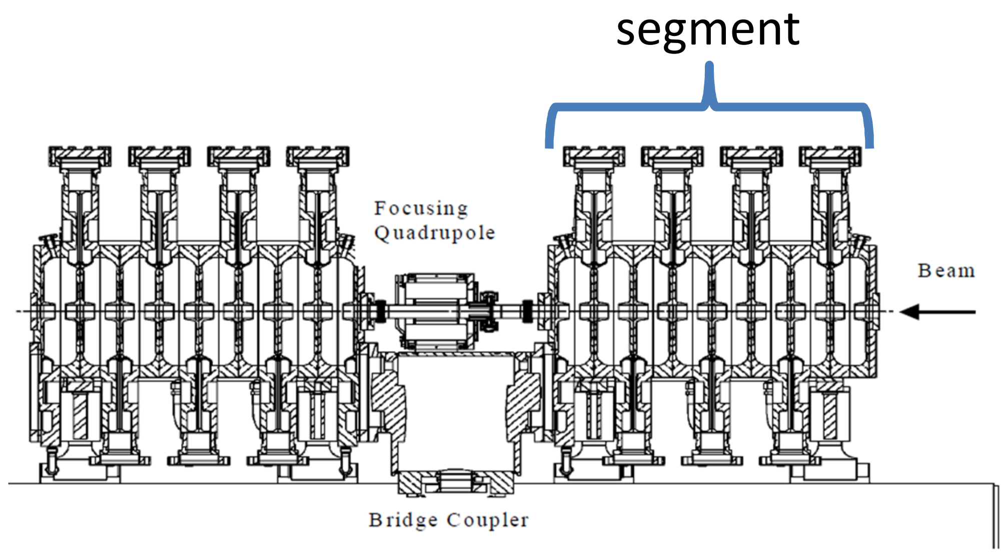
f_{RF}=800 - 3000 MHz for proton (β=0.5-1) and electrons

SCC STRUCTURES: EXAMPLES

Spallation Neutron Source Coupled Cavity Linac (protons)

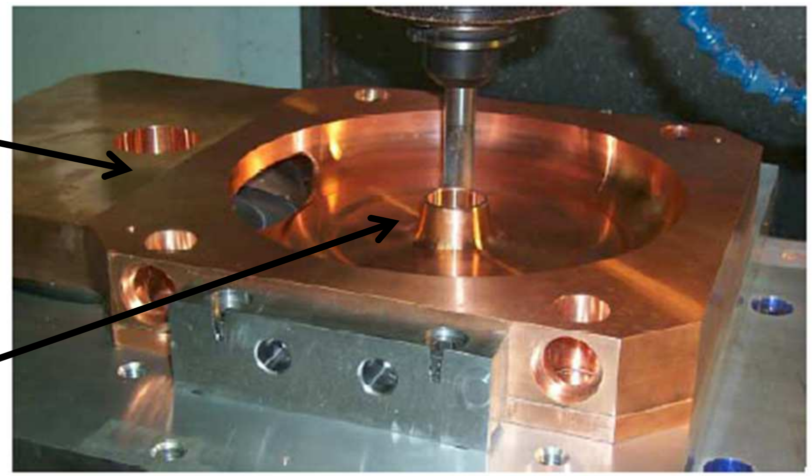


4 modules, each containing 12 accelerator segments CCL and 11 bridge couplers. The CCL section is a RF Linac, operating at **805 MHz** that accelerates the beam **from 87 to 186 MeV** and has a physical installed length of slightly over **55 meters.**



Coupling cell

Accelerating cell



TRAVELLING WAVE (TW) STRUCTURES

⇒To accelerate charged particles, the electromagnetic field must have an **electric field along the direction of propagation of the particle**.

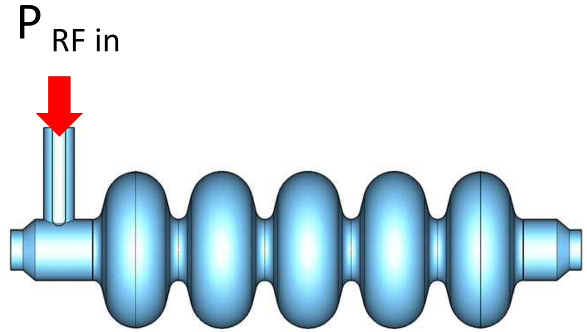
⇒The field has to be synchronous with the particle velocity.

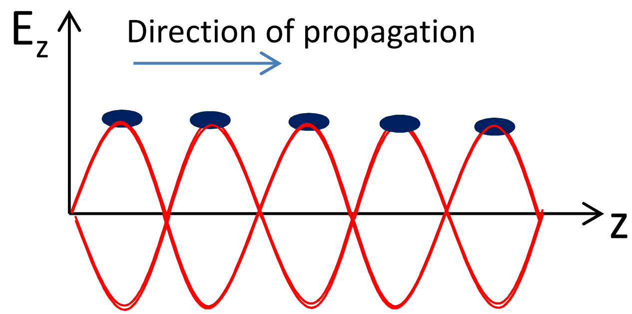
⇒Up to now we have analyzed the cases standing **standing wave (SW)** structures in which the field has basically a given profile and oscillate in time (as example in DTL or **resonant cavities operating on the** TM_{010}-like).

Time oscillation

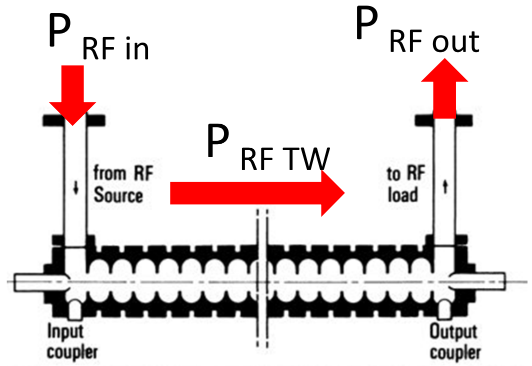
$$E_z(z,t)=E_{RF}(z)\cos(\omega_{RF}t)$$

field profile

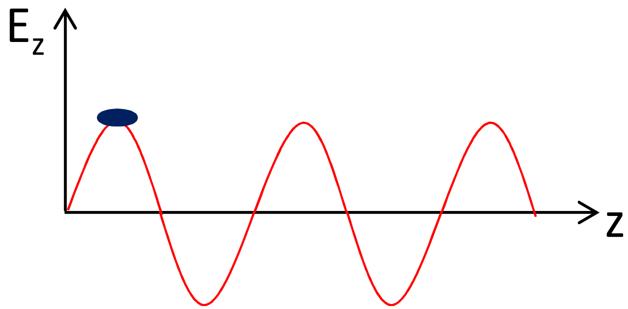




⇒There is another possibility to accelerate particles: using a **travelling wave (TW)** structure in which the RF wave is **co-propagating** with the beam with a **phase velocity equal to the beam velocity**.



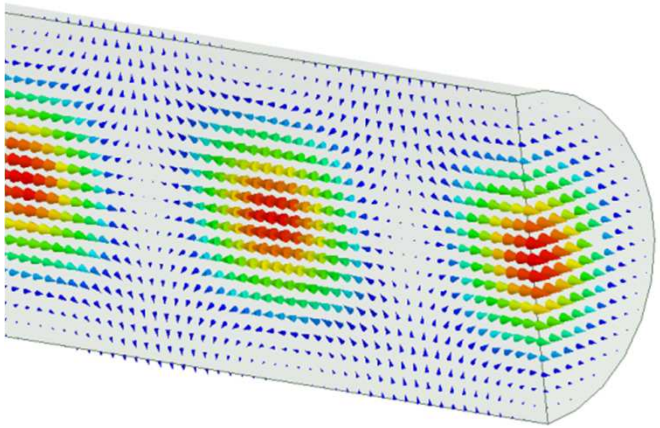
⇒Typically these structures **are used for electrons** because in this case the **phase velocity can be constant** all over the structure and equal to c. On the other hand it is difficult to modulate the phase velocity itself very quickly for a low β particle that changes its velocity during acceleration.

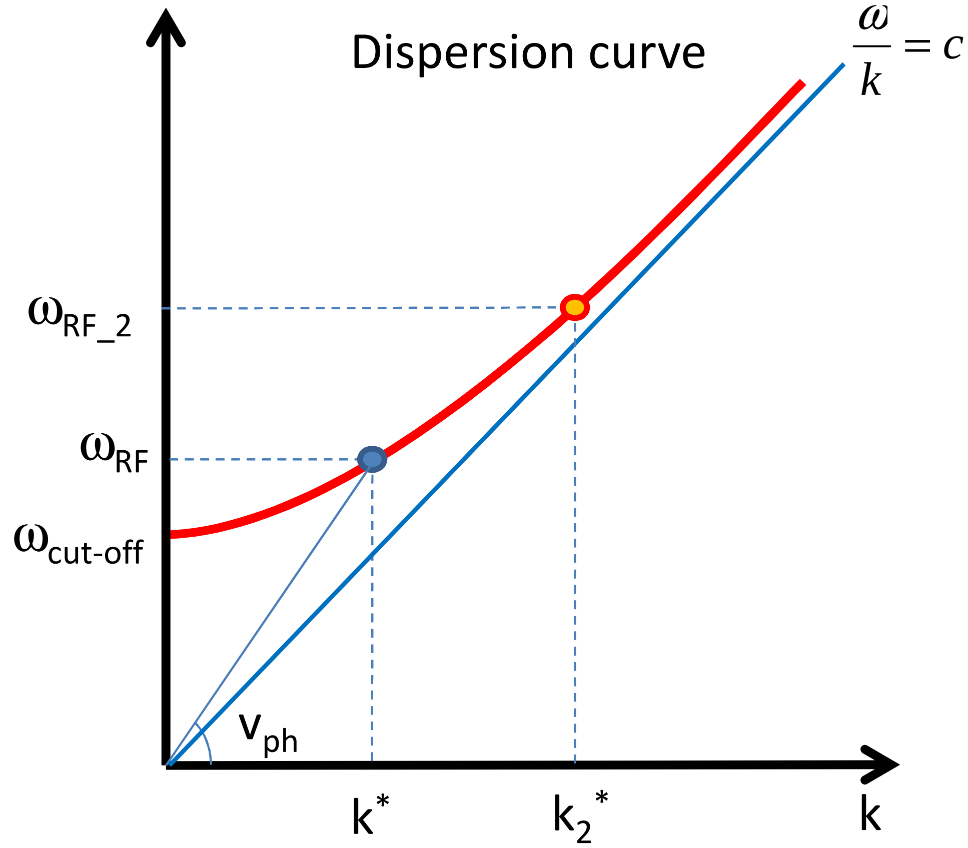


TW CAVITIES: CIRCULAR WAVEGUIDE AND DISPERSION CURVE

In **TW structures** an e.m. wave with **$E_z \neq 0$** travel together with the beam in a special guide in which the **phase velocity of the wave matches the particle velocity (v)**. In this case the beam absorbs energy from the wave and it is **continuously accelerated**.

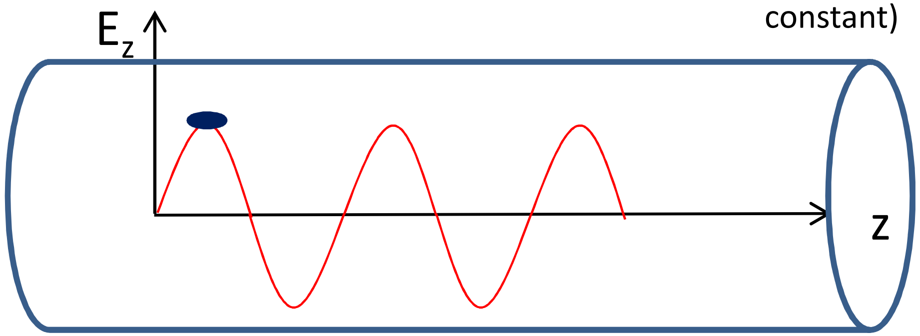
CIRCULAR WAVEGUIDE





As example if we consider a simple circular waveguide the first propagating mode with **$E_z \neq 0$** is the TM_{01} mode. Nevertheless by solving the wave equation it turns out that an e.m. wave propagating in this **constant cross section waveguide** will **never be synchronous with a particle beam** since the **phase velocity is always larger than the speed of light c**.

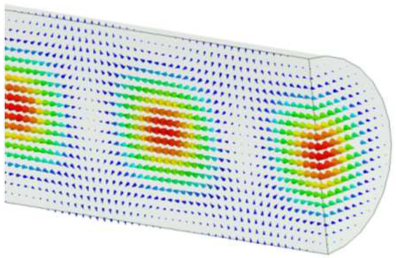
$$E_z\big|_{TM_{01}} = \underbrace{E_0(r)}_{J_0\left(\frac{p_{01}}{a} r\right)} \cos\left(\omega_{RF} t - k^* z\right) \quad \Rightarrow \quad v_{ph} = \frac{\omega_{RF}}{k^*} > c$$

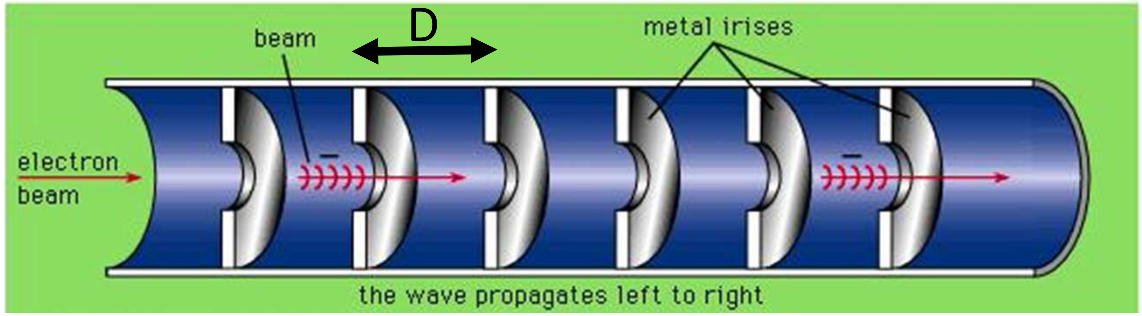


TW CAVITIES: IRIS LOADED STRUCTURES

In order to **slow-down the wave phase velocity, iris-loaded periodic structure have to be used.**

CIRCULAR WAVEGUIDE → ***IRIS LOADED STRUCTURE***





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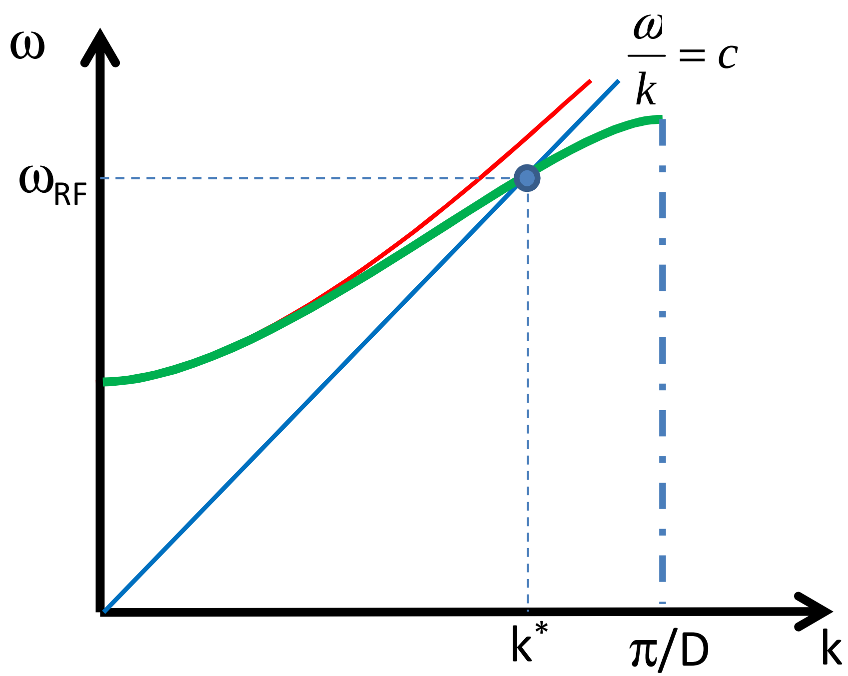
MODE TM_{01} → MODE TM_{01}-like

Periodic in z of period D

$$E_z\big|_{TM_{01}} = E_0(r)\cos\left(\omega_{RF} t - k^* z\right) \longrightarrow E_z\big|_{TM_{01}-like} = \underbrace{E_P(r,z)}\cos\left(\omega_{RF} t - k^* z\right)$$

⇒The field in this kind of structures is that of a special wave travelling within a spatial periodic profile.

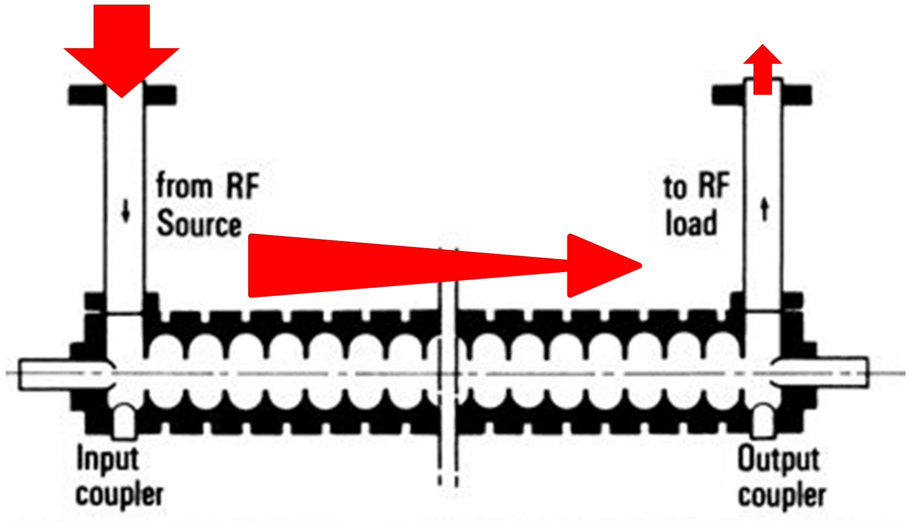


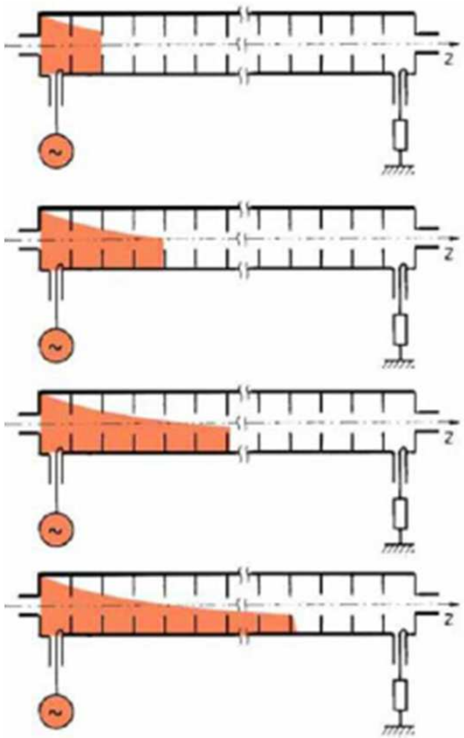
⇒The structure can be designed to have the **phase velocity equal to the speed of the particles**.

⇒This allows **acceleration over large distances** (few meters, hundred of cells) with just an input coupler and a relatively **simple geometry**.

TW CAVITIES: CONSTANT GRADIENT STRUCTURES

In a TW structure, the **RF power enters** into the cavity through an **input coupler**, flows (travels) through the cavity in the same direction as the beam and an **output coupler at the end** of the structure is connected to a **matched power load**.
If there is no beam, the input power reduced by the cavity losses goes to the power load where it is dissipated.
In the presence of a large beam current, however, a fraction of the TW power is transferred to the beam.

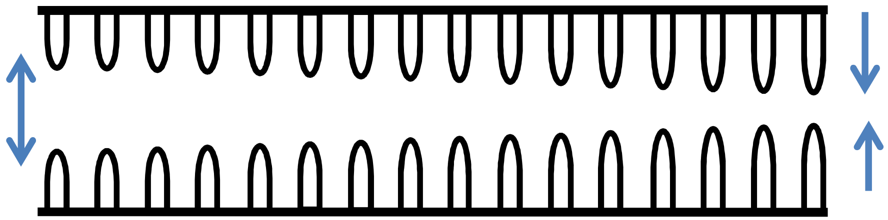


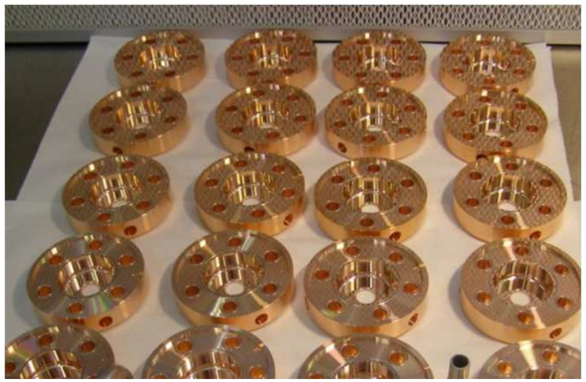


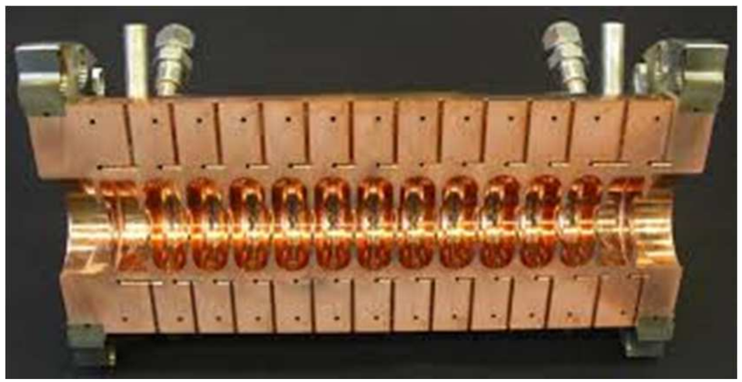
In a purely periodic structure, made by a sequence of **identical cells** (also called "**constant impedance structure**"), the RF power flux and the intensity of the accelerating field decay exponentially along the structure :

$$E_z(z) = E_0 e^{-\alpha z}$$

It is possible to demonstrate that, in order to keep the **accelerating field constant** along the structure, the **iris apertures have to decrease along the structure.**

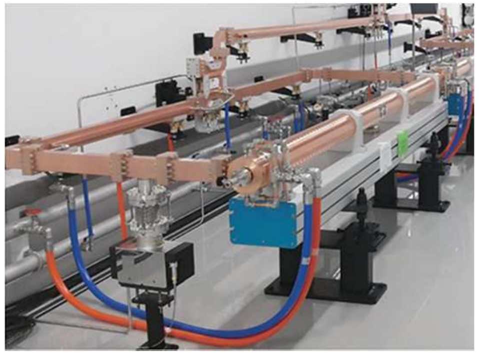






LINAC TECHNOLOGY

ACCELERATING CAVITY TECHNOLOGY



⇒The structures are powered by RF generators (like **klystrons**).

⇒The cavities (and the related LINAC technology) can be of different material:

- **copper** for **normal conducting (NC, both SW than TW)** cavities;
- **Niobium** for **superconducting cavities (SC, SW)**;

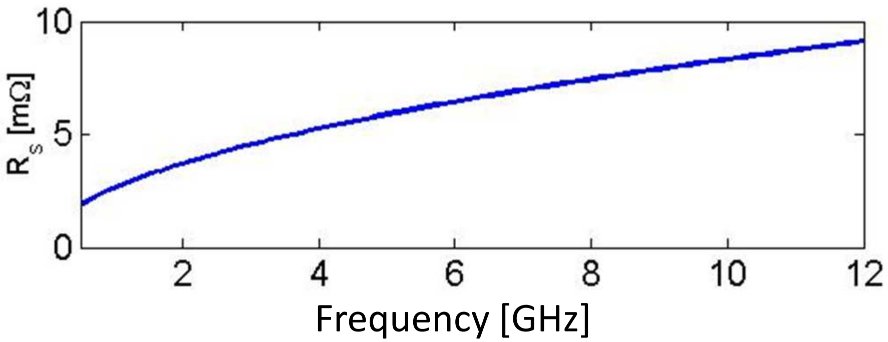
⇒We can choose between NC or the SC technology depending on the required performances in term of:

- **accelerating gradient** (MV/m);
- **RF pulse length** (how many bunches we can contemporary accelerate);
- **Duty cycle**: pulsed operation (i.e. 10-100 Hz) or continuous wave (CW) operation;
- **Average beam current**.
- …

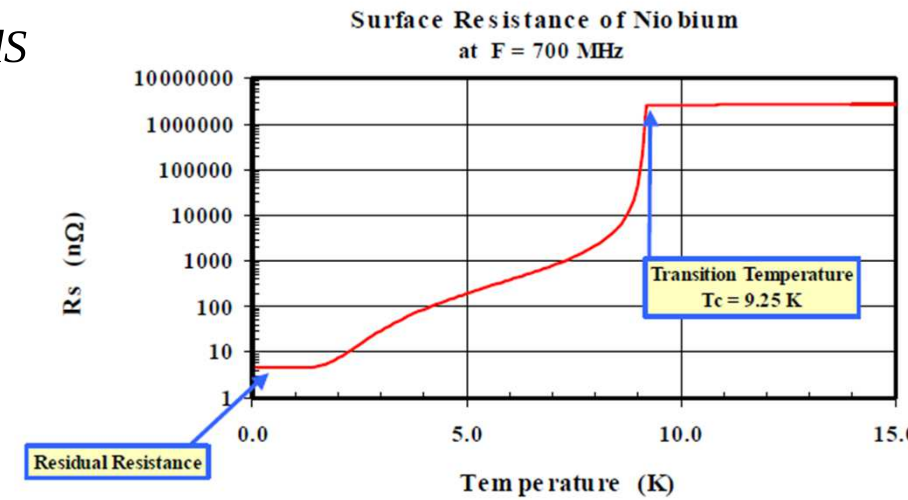
Dissipated power into the cavity walls is related to the surface currents

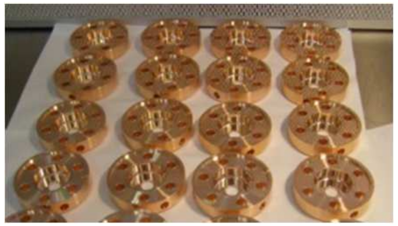
$$P_{diss} = \int_{\text{cavity wall}} \overbrace{\frac{1}{2} R_s H_{tan}^2}^{\text{power density}} dS$$

COPPER



NIOBIUM

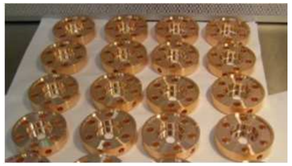




Between copper and Niobium there is a factor 10^5-10^6

NORMAL CONDUCTING AND SUPER CONDUCTING

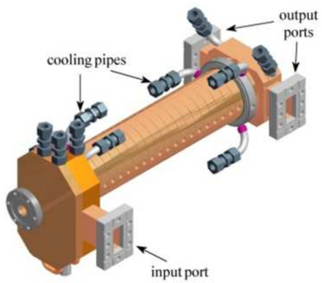
NC: COPPER



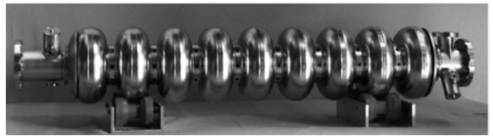
The most widely used NC metal for RF structures is **OFHC copper** (Oxigen free high conductivity) for several reasons:

1) Easy to machine (good achievable roughness at the few nm level)
2) Easy to braze/weld
3) Easy to find at relatively low cost
4) Very good electrical (and thermal) conductivity
5) Low SEY (multipacting phenomena)
6) Good performances at high accelerating gradient

-**Higher dissipation**
-**Pulsed** operation
-**Higher peak accelerating gradient** (up to 50-100 MV/m)
-**Standard cleaning** procedures for the cavity fabrication
-**Cooling** of dissipated power with pipes



SC: NIOBIUM



The **most common material for SC cavities is Nb** because:

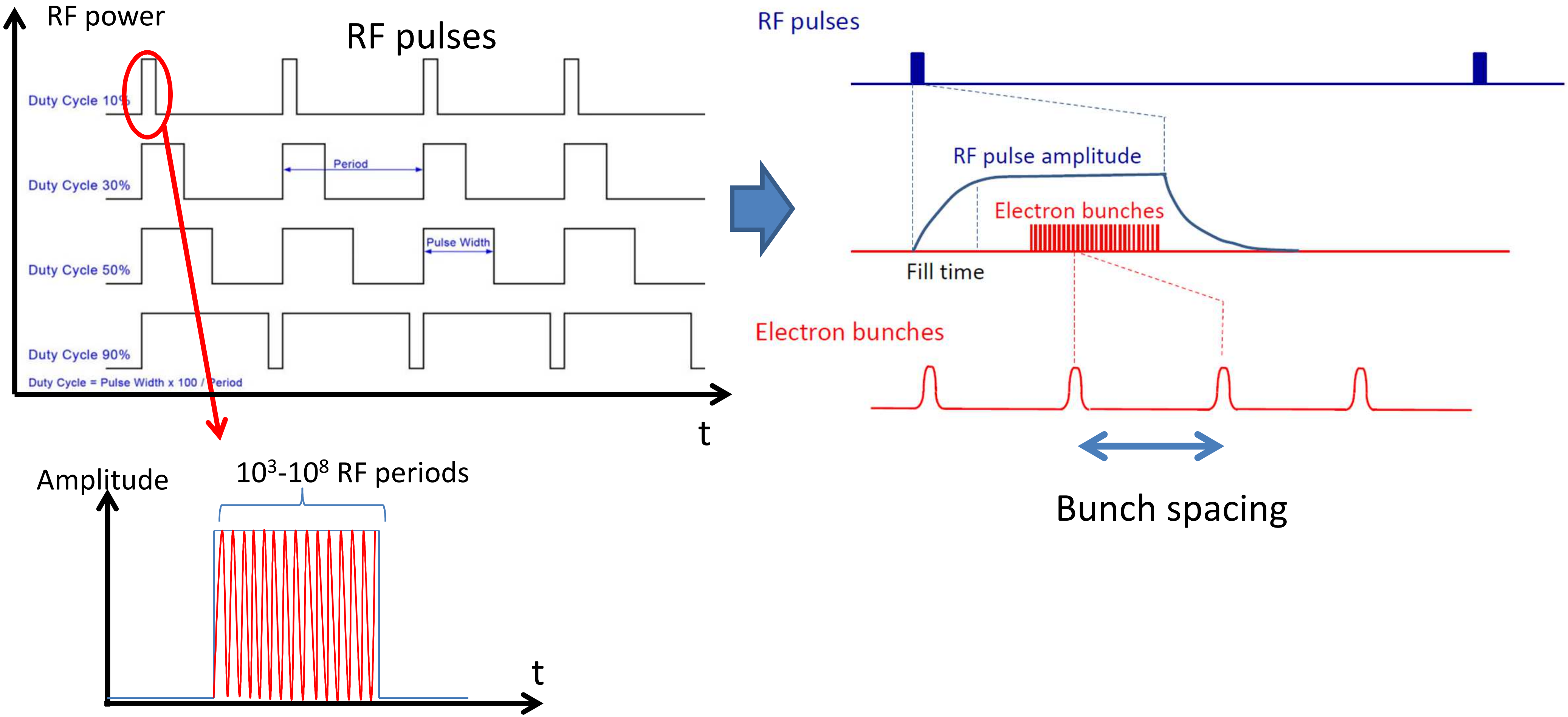
1) Nb has a relatively **high transition temperature** (Tc=9.25 K).
2) SC can be destroyed by magnetic field greater than a critical field $H_c \Rightarrow$ Pure Nb has a **relatively high critical magnetic field Hc**=170-180 mT.
3) It is chemically inert
4) It can be **machined and deep-drawn**
5) It is available as bulk and sheet material in any size, fabricated by forging and rolling….

-lower dissipation
-Allow **continuous operation**
-**lower peak accelerating gradient** (max 30-40 MV/m)
-**Special cleaning procedures** for the cavity fabrication
-They need a **cryostat** to reach the SC temperature of few K

RF STRUCTURE AND BEAM STRUCTURE

The "**beam structure**" in a LINAC is directly related to the "**RF structure**". There are basically two possible type of operations:

- CW (continuous wave) ⇒ allow, in principle, to operate with a continuous beam
- PULSED OPARATION ⇒ there are RF pulses at a certain repetition rate (Duty Cycle (DC)=pulsed width/period)



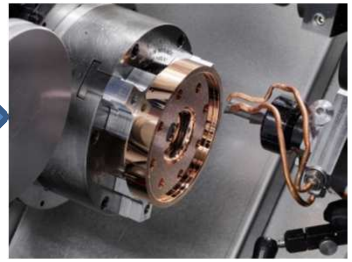
⇒Because of the very low power dissipation and low RF power required to achieve a certain accelerating voltage the **SC structures allow operation at very high Duty Cycle (DC) up to a CW operation with high gradient (>20 MV/m)**.

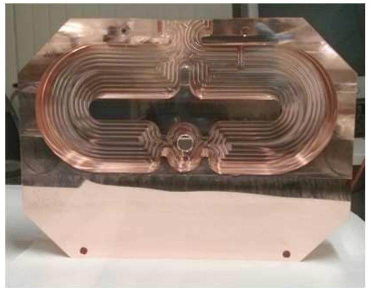
⇒On the other hand **NC structures can operate in pulsed mode** at very low DC with **higher peak field** (TW structures can >50-80 MV/m peak field).

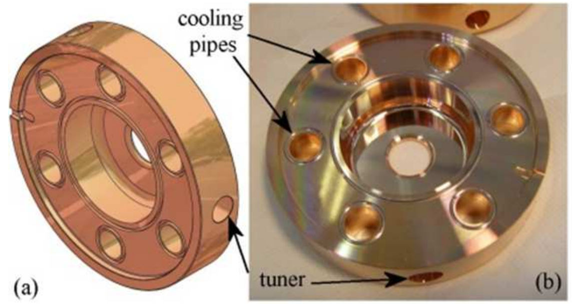
⇒NC structures can also operate in CW but at very low gradient because of the dissipated power.

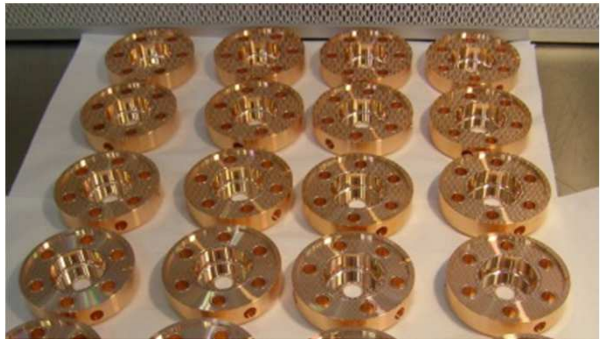
EXAMPLE FABRICATION PROCESS:NC TW STRUCTURES

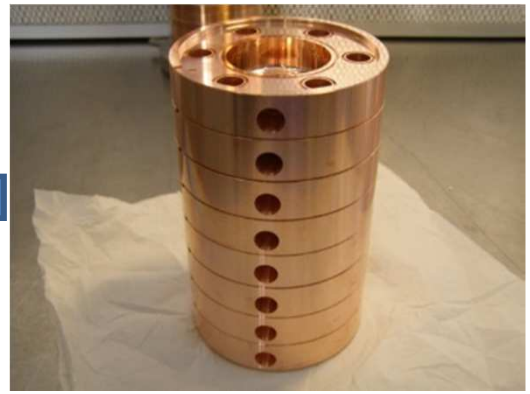
The cells and couplers are fabricated with milling machines and lathes starting from **OFHC forged or laminated copper** with precisions that can be of the order of few um and surface roughness <50 nm.



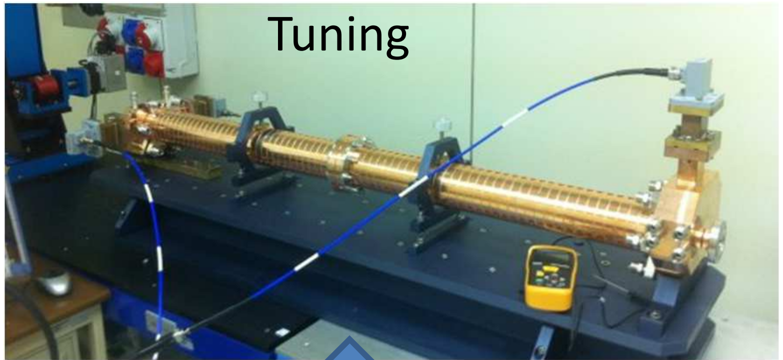








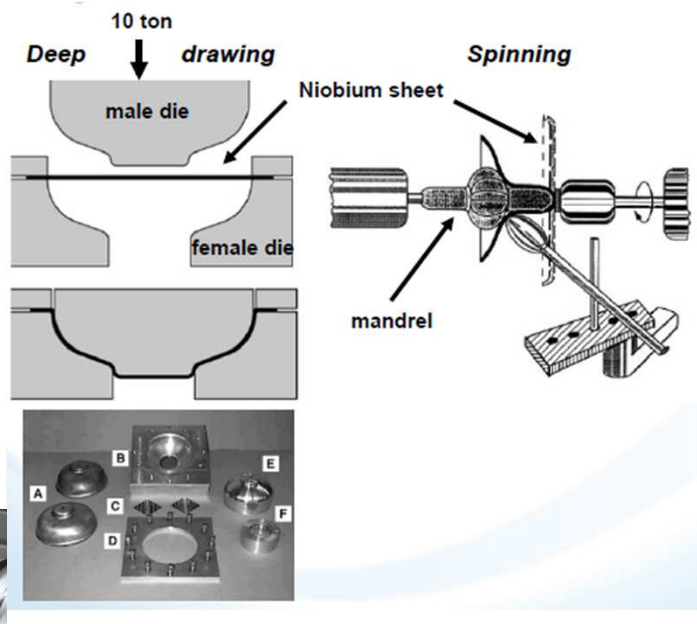
The cells are then piled up and **brazed** together in vacuum or hydrogen furnace using different alloys at different temperatures (700-1000 C) and/or in different steps.

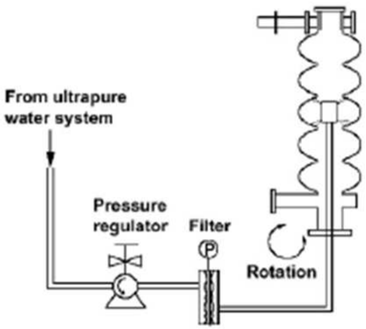


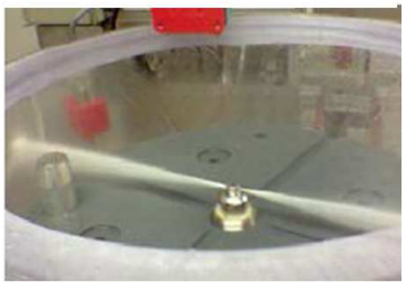
EXAMPLE FABRICATION PROCESS:SC SW STRUCTURES

Nb is available as **bulk and sheet material** in any size, fabricated by forging and rolling. **High Purity Nb** is made by **electron beam melting** under good vacuum.
The most common fabrication techniques for the cavities are to **deep draw or spin half-cells**.
Alternative techniques are: hydroforming, spinning an entire cavity out of single sheet or tube and Nb sputtering

After forming the parts are **electron beam welded** together



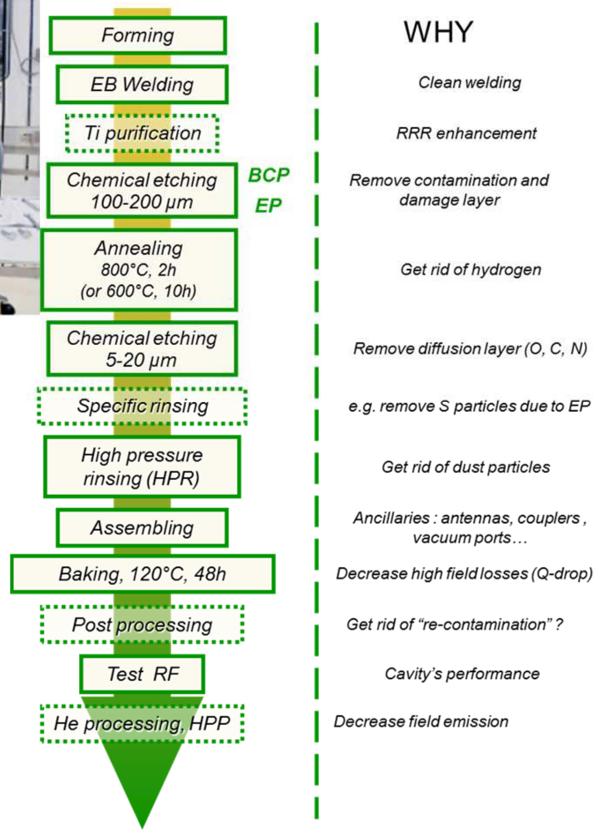




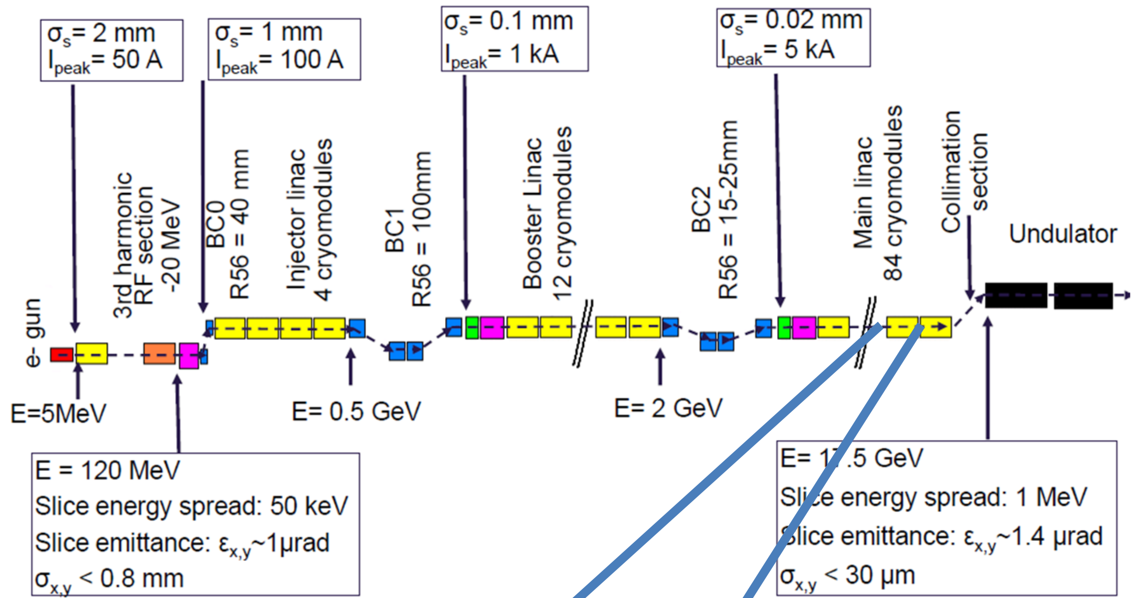
CAVITY TREATMENT

The cavity treatment after the welding is quite complicated and require several steps between:

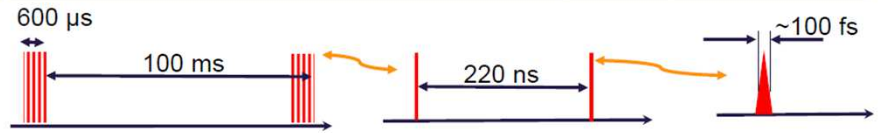
- buffered **chemical polishing** (BCP), **electropolishing** and etching to remove surface damaged layers of the order of 100 μm
- **rinsed with ultraclean water** also at high pressure (100 bar)
- **Thermal treatments** up to >1000 C to diffuse H_2 out of the material increasing the Nb purity (RRR)
- **high-temperature treatment** with Ti getter (post-purification)
- RF tuning



EXAMPLES: EUROPEAN XFEL

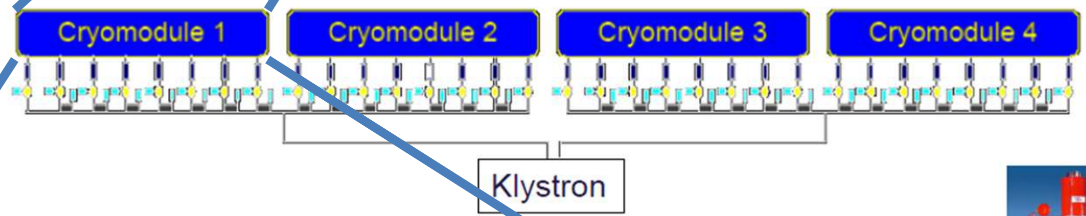


Nominal Energy	GeV	17.5
Beam pulse length	ms	0.60
Repetition rate	Hz	10
Max. # of bunches per pulse		2700
Min. bunch spacing	ns	220
Bunch charge	nC	1
Bunch length, σ_z	µm	< 20
Emittance (slice) at undulator	µrad	< 1.4
Energy spread (slice) at undulator	MeV	1



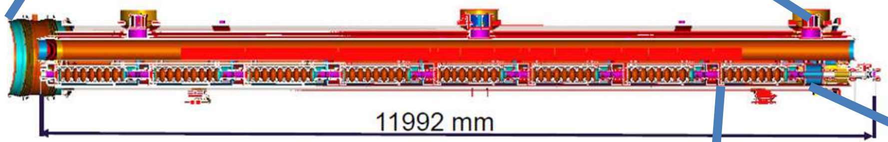
101 cryomodules in total

RF- system: 25 RF units. The unit = 4 cryomodules + RF-power source (klystron)



25 RF stations
5.2 MW each

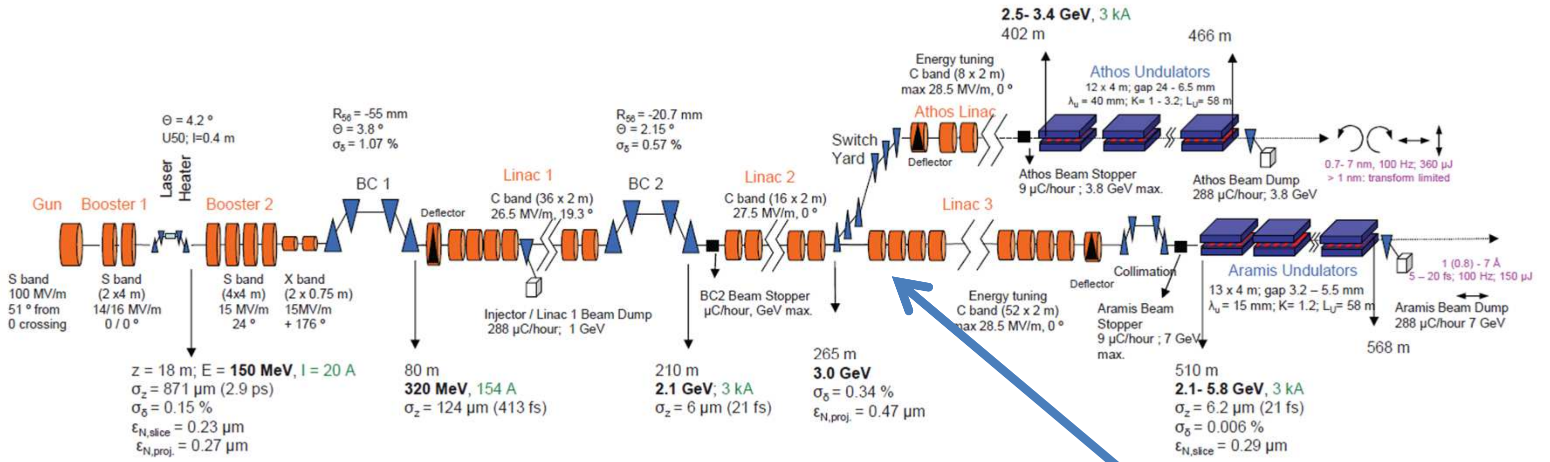
Cryomodule housing: 8 cavities, quadrupole and BPM

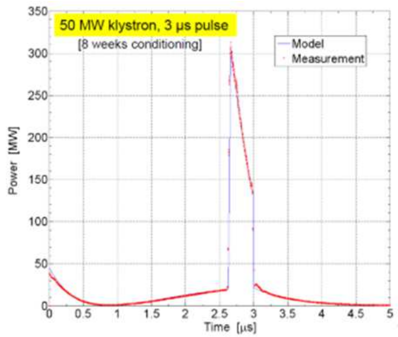


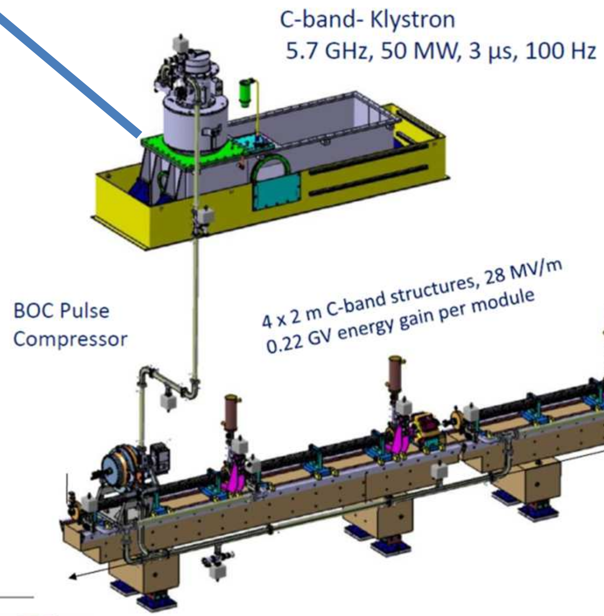
800 accelerating cavities

1.3 GHz / 23.6 MV/m

EXAMPLE: SWISSFEL LINAC (PSI)

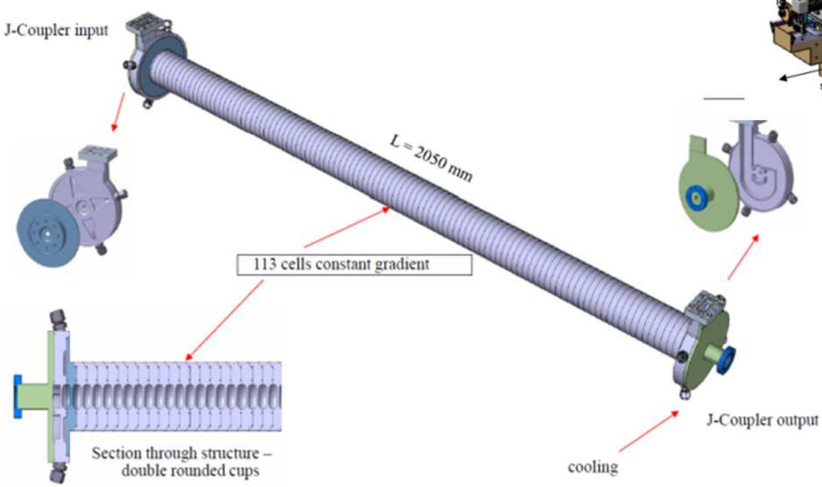


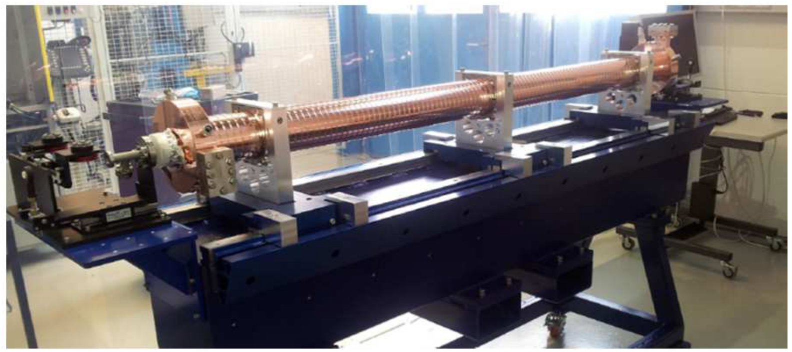




Main LINAC	#
LINAC modules	26
Modulator	26
Klystron	26
Pulse compressor	26
Accelerating structures	104
Waveguide splitter	78
Waveguide loads	104

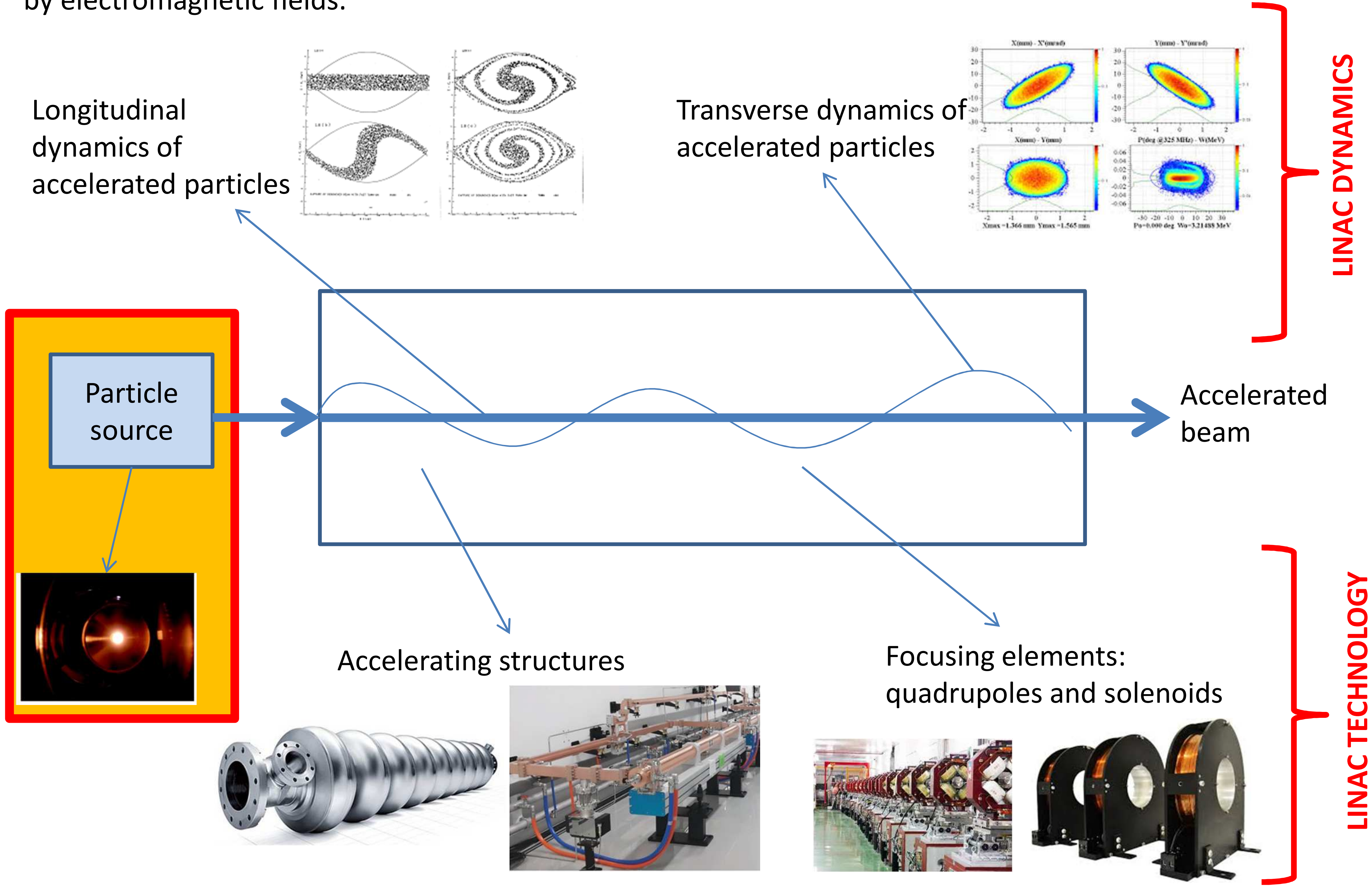
Courtesy T. Garvey





LINAC: BASIC DEFINITION AND MAIN COMPONENTS

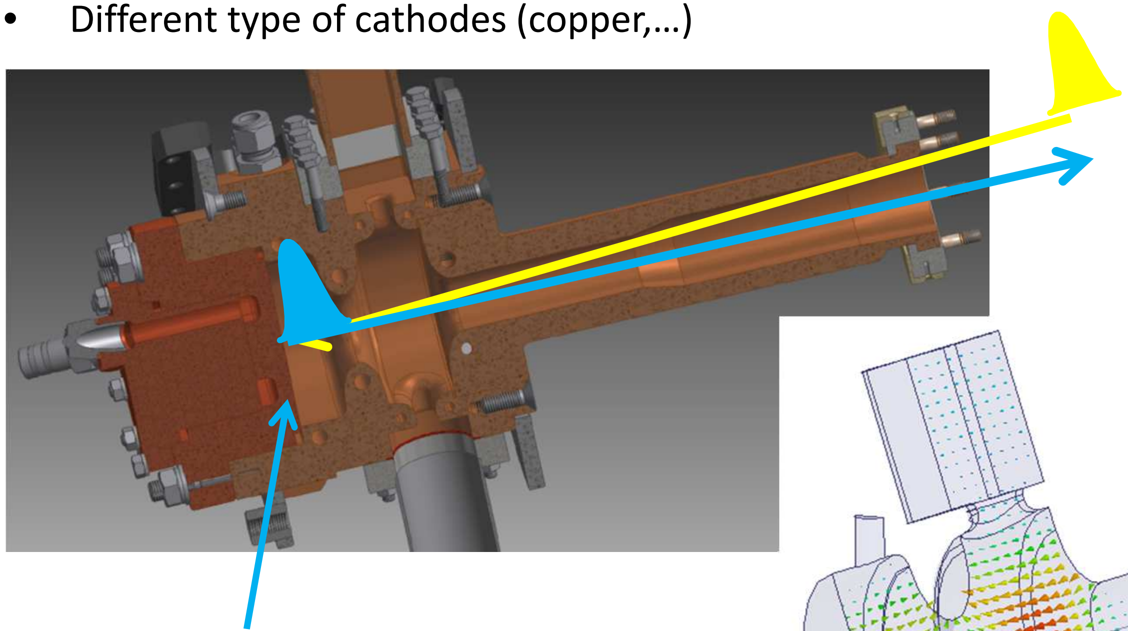
LINAC (linear accelerator) is a **system that allows to accelerate charged particles through a linear trajectory** by electromagnetic fields.

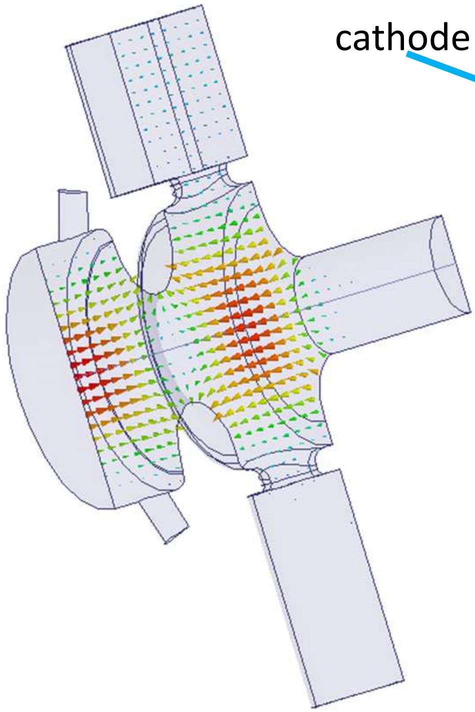


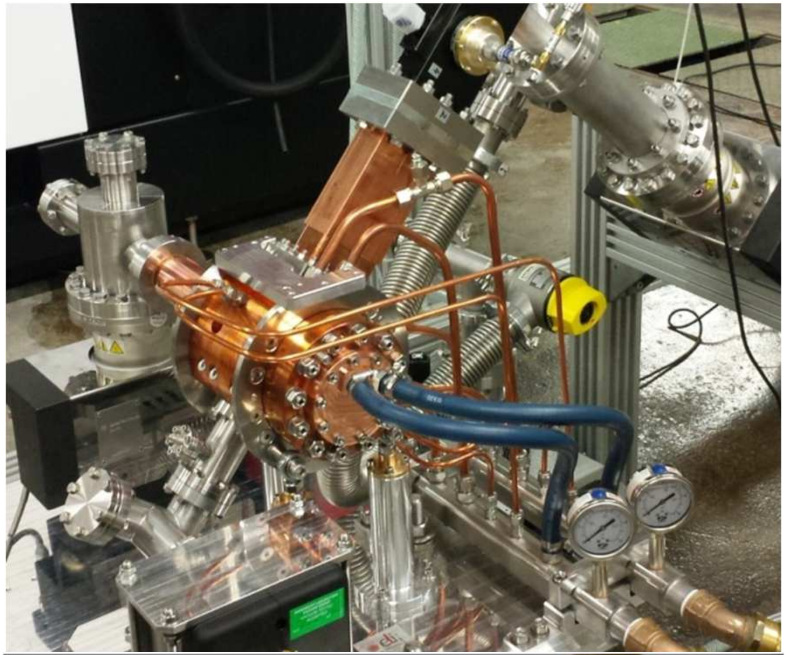
ELECTRON SOURCES: RF PHOTO-GUNS

RF guns are used in the first stage of electron beam generation in FEL and acceleration.

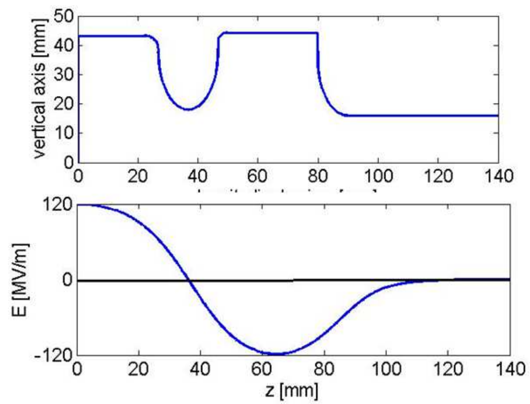
- Multi cell: typically 2-3 cells
- SW π mode cavities
- operate in the range of 60-120 MV/m cathode peak accelerating field with up to 10 MW input power.
- Typically in L-band- S-band (1-3 GHz) at 10-100 Hz.
- Single or multi bunch (L-band)
- Different type of cathodes (copper,…)





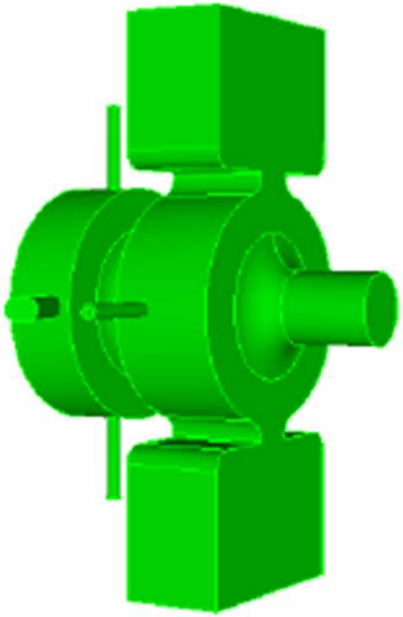


cathode



The electrons are emitted on the **cathode** through a laser that hit the surface. They are then accelerated trough the electric field that has a longitudinal component on axis TM_{010}.

RF PHOTO-GUNS: EXAMPLES

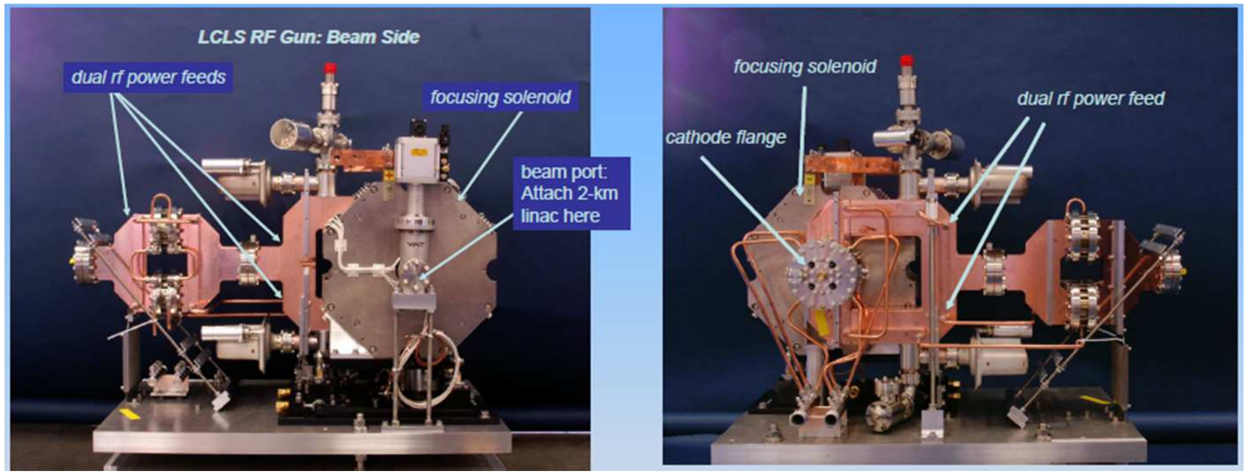


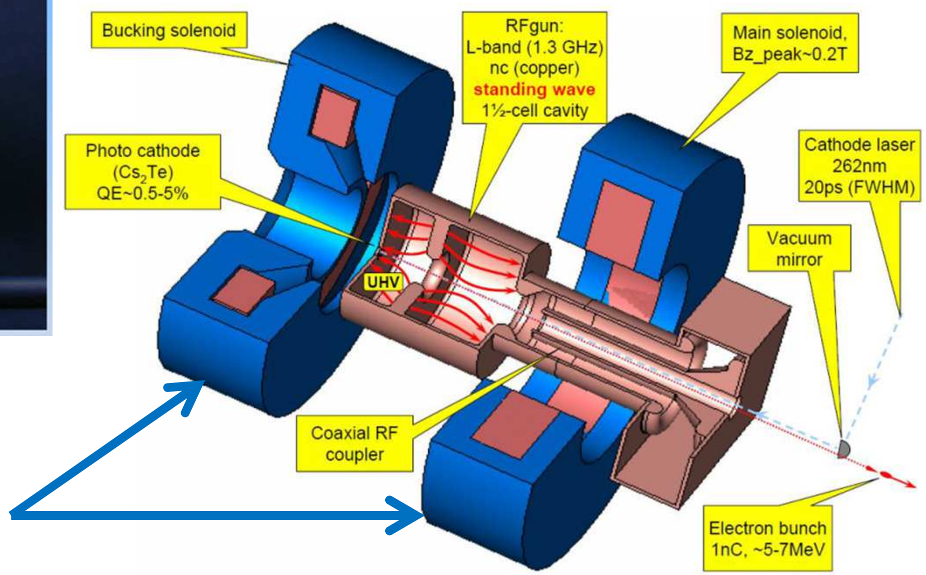
LCLS

Frequency = 2,856 MHz
Gradient = 120 MV/m
Exit energy = 6 MeV
Copper photocathode
RF pulse length ~2 μs
Bunch repetition rate = 120 Hz
Norm. rms emittance
0.4 mm·mrad at 250 pC

PITZ L-band Gun

Frequency = 1,300 MHz
Gradient = up to 60 MV/m
Exit energy = 6.5 MeV
Rep. rate 10 Hz
Cs_2Te photocathode
RF pulse length ~1 ms
800 bunches per macropulse
Normalized rms emittance
1 nC 0.70 mm·mrad
0.1 nC 0.21 mm·mrad

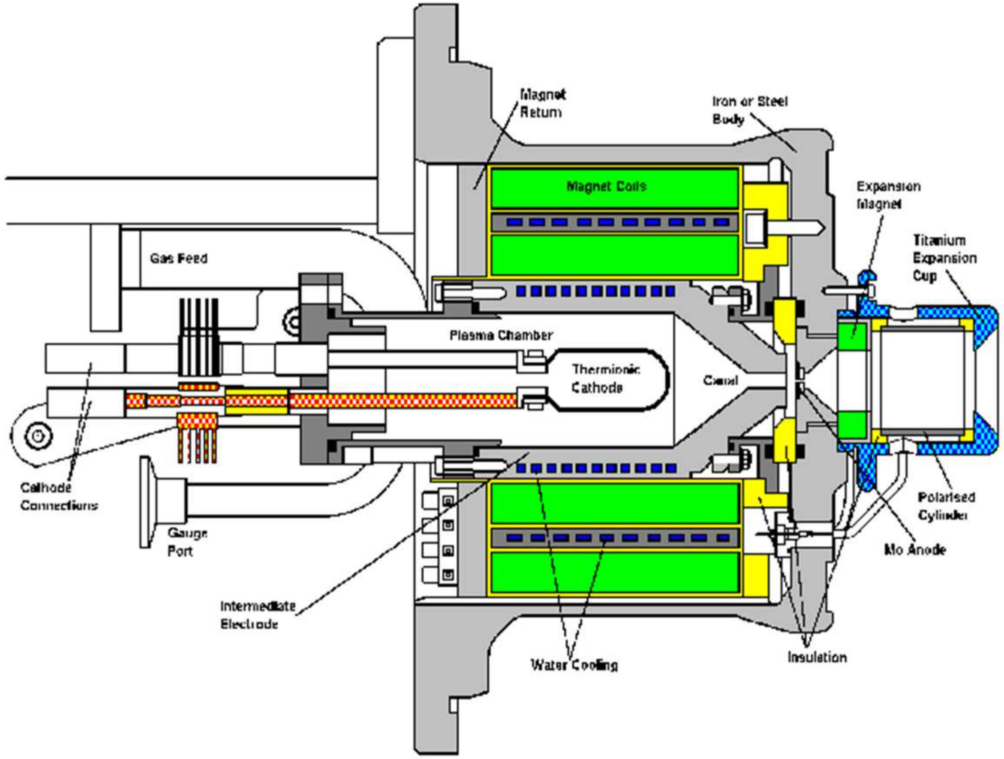


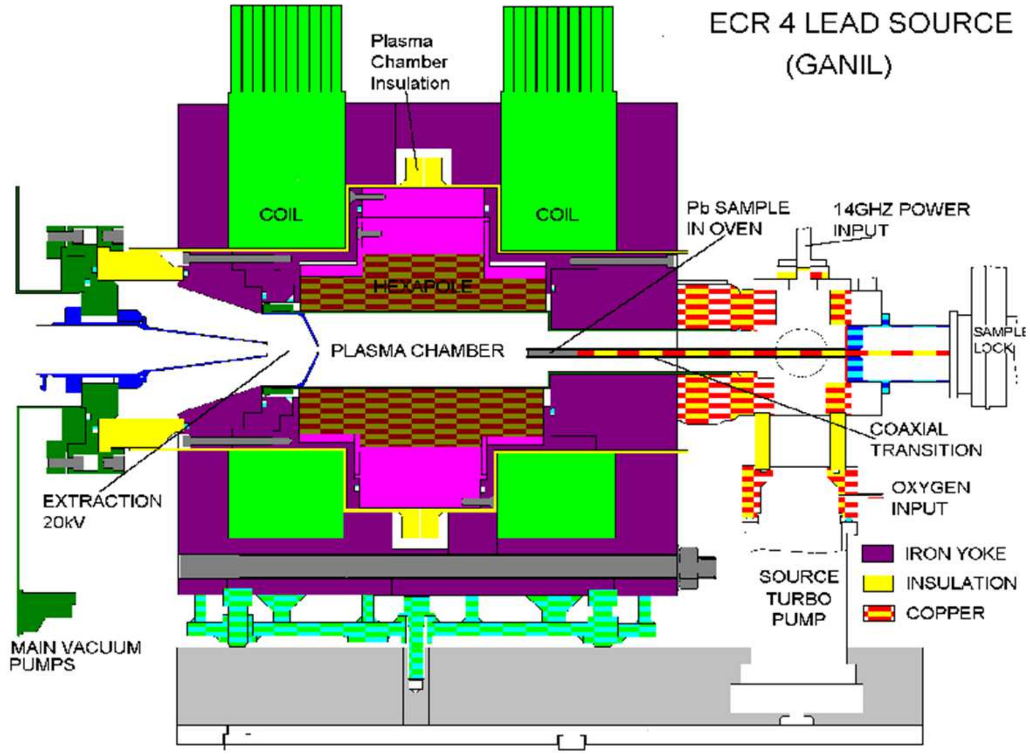


Solenoids field are used to compensate the space charge effects in low energy guns. The configuration is shown in the picture

ION SOURCES

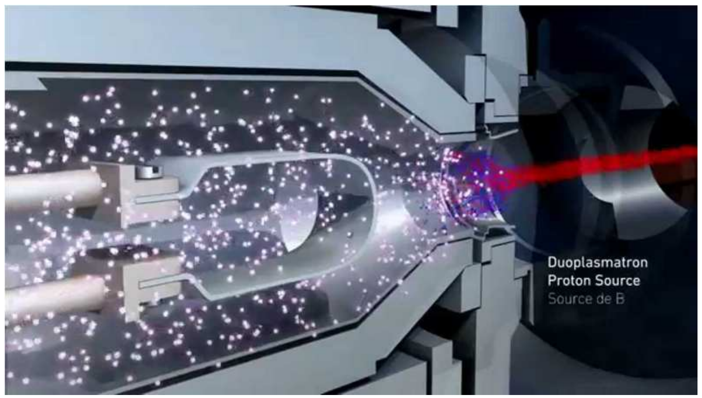
Basic principle: create a plasma and optimize its conditions (heating, confinement and loss mechanisms) to produce the desired ion type. Remove ions from the plasma via an aperture and a strong electric field.

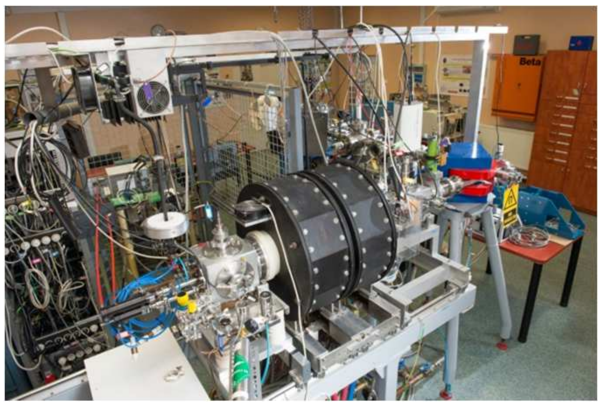




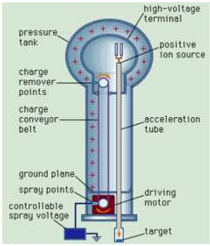
CERN Duoplasmatron proton Source

Electron Cyclotron Resonance (ECR) ECR

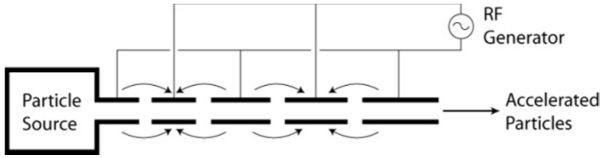




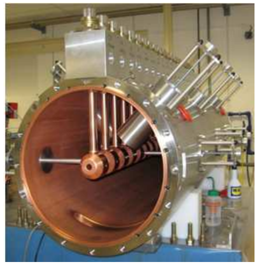
SUMMARIZING THE FIRST PART

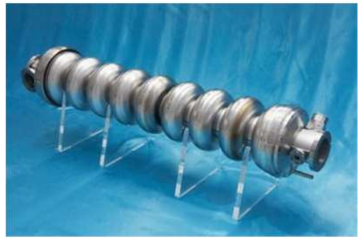


ELECTROSTATIC ACCELERATORS



WIDERÖE RF ACCELERATION



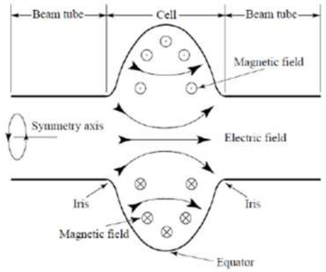


HIGH FREQUENCIES: RF CAVITIES

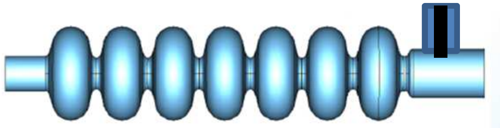
$$L_n = \beta_n \lambda_{RF}$$

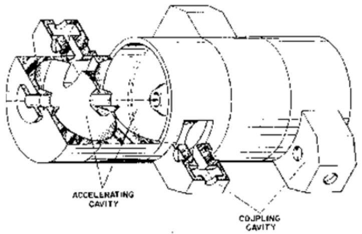
$$\Delta E = q\Delta V = qV_{RF}\cos(\omega_{RF}t + \theta)$$

CYLINDRICAL PILL-BOX STRUCTURES

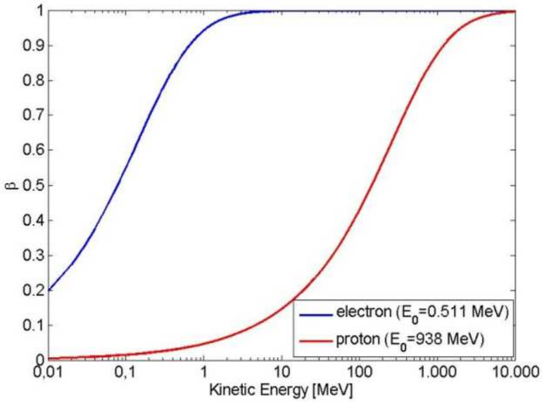


MULTI-CELL SW CAVITIES:
π,π/2,0 MODE STRUCTURES

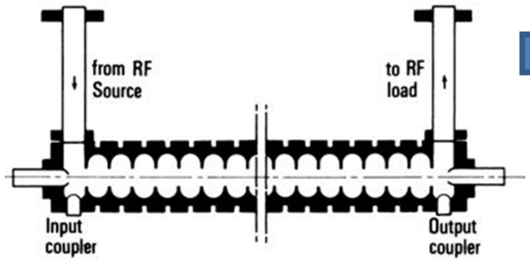




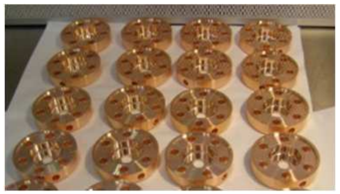
LIGHT/HEAVY PARTICLES



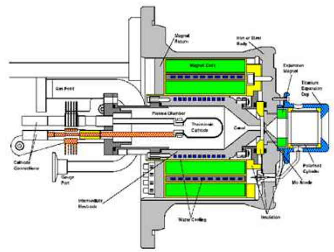
TRAVELLING WAVE (TW) STRUCTURES FOR e-

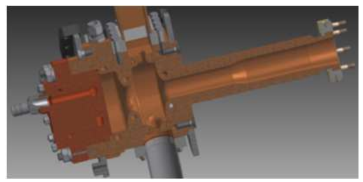


NORMAL CONDUCTING AND SUPER CONDUCTING



ELECTRON/ION SOURCES





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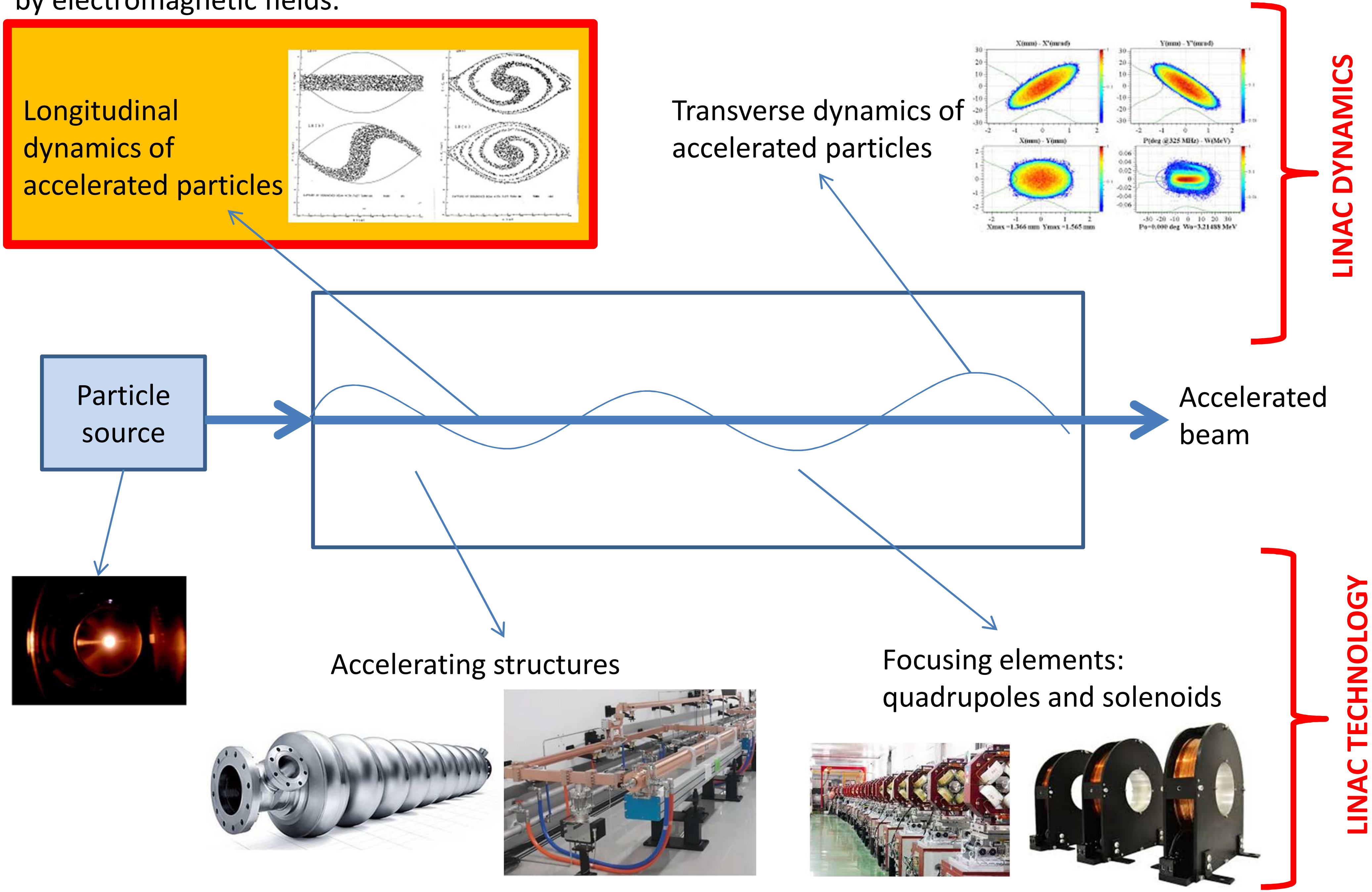


SYNCHRONOUS PARTICLE/PHASE

⇒Let us consider a **SW linac structure** made by accelerating **gaps** (like in DTL) or **cavities**.

⇒In **each gap we have an accelerating field** oscillating in time and an integrated accelerating voltage (V_{acc}) still oscillating in time than can be expressed as:

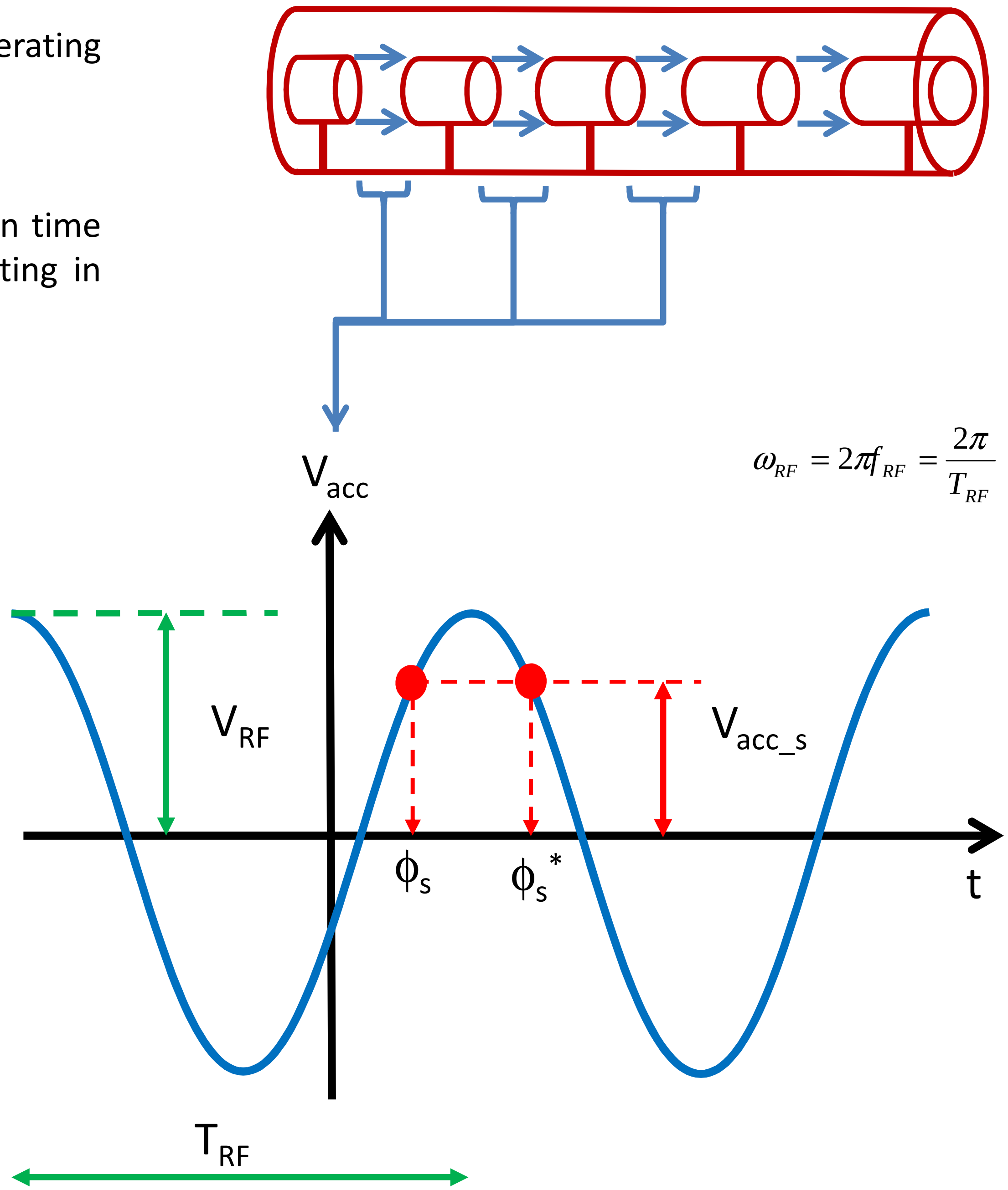
$$V_{acc} = V_{RF} \cos(\omega_{RF} t + \theta)$$

⇒Let's assume that **the "perfect" synchronism condition is fulfilled for a phase ϕ_s** (called ***synchronous phase***). This means that a particle (called ***synchronous particle***) entering in a gap with a phase ϕ_s ($\phi_s = \omega_{RF} t_s$) with respect to the RF voltage receive a **energy gain** (and a consequent change in velocity) that allow entering in the subsequent gap with the **same phase ϕ_s** and so on.

⇒for this particle the energy gain in each gap is:

$$\Delta E = q \underbrace{V_{RF} \cos(\phi_s + \theta)}_{V_{acc\_s}} = q V_{acc\_s}$$

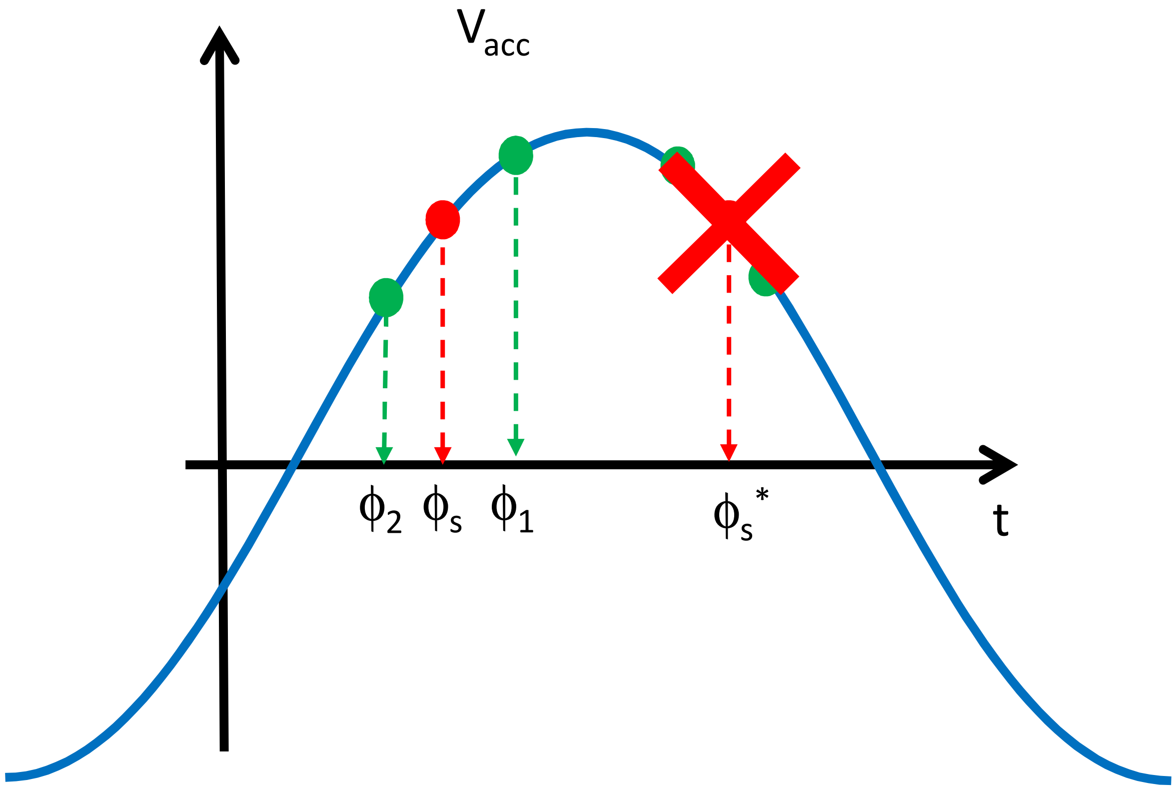
⇒obviously both ϕ_s and ϕ_s^* are synchronous phases.



$$\omega_{RF} = 2\pi f_{RF} = \frac{2\pi}{T_{RF}}$$

PRINCIPLE OF PHASE STABILITY

⇒Let us consider now the first synchronous phase ϕ_s (on the positive slope of the RF voltage). If we consider **another particle** "near" to the synchronous one **that arrives later in the gap** ($t_1>t_s$, $\phi_1>\phi_s$), it will see an higher voltage, it will gain an higher energy and an higher velocity with respect to the synchronous one. As a consequence its time of flight to next gap will be shorter, partially **compensating its initial delay**.



⇒**Similarly** if we consider another particle "near" to the synchronous one that arrives before in the gap ($t_1<t_s$, $\phi_1<\phi_s$), it will see a smaller voltage, it will gain a smaller energy and a smaller velocity with respect to the synchronous one. As a consequence its time of flight to next gap will be longer, compensating the initial advantage.

⇒**On the contrary** if we consider now the synchronous particle at phase ϕ_s^* and another particle "near" to the synchronous one that arrives later or before in the gap, it will receive an energy gain that will increase further its distance form the synchronous one

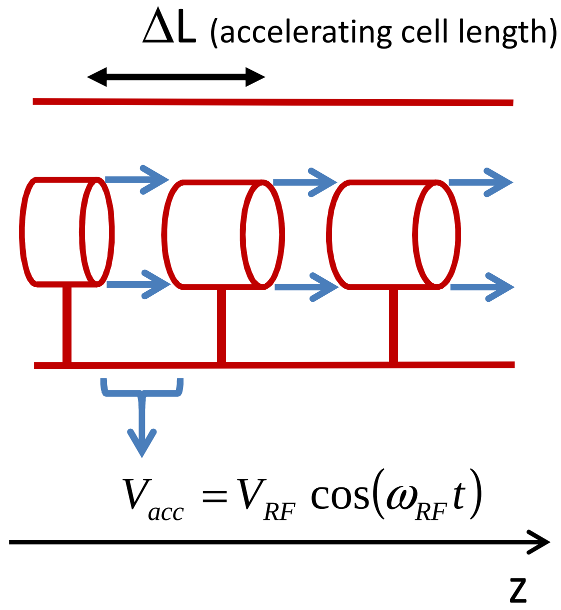
⇒The choice of the synchronous phase in the positive slope of the RF voltage provides longitudinal focusing of the beam: **phase stability principle**.

⇒The synchronous phase on the negative slope of the RF voltage is, on the contrary, **unstable**

⇒Relying on particle velocity variations, **longitudinal focusing does not work for fully relativistic beams** (electrons). In this case acceleration "on crest" is more convenient.

ENERGY-PHASE EQUATIONS (1/2)

In order to study the **longitudinal dynamics in a LINAC**, the following variables are used, which describe the generic particle **phase** (time of arrival) and **energy with respect to the synchronous particle**:



Arrival time (phase) of a **generic particle** at a certain gap (or cavity)

Arrival time (phase) of the **synchronous particle** at a certain gap (or cavity)

$$\begin{cases} \varphi = \phi - \phi_s = \omega_{RF}(t - t_s) \\ w = E - E_s \end{cases}$$

Energy of the **synchronous particle** at a certain position along the linac

Energy of a **generic particle** at a certain position along the linac

The **energy gain per cell (one gap + tube in case of a DTL)** of a generic particle and of a synchronous particle are (we put θ=0 in the generic expression of the accelerating voltage just for simplicity):

$$\begin{cases} \Delta E_s = qV_{RF}\cos\phi_s \\ \Delta E = qV_{RF}\cos\phi = qV_{RF}\cos(\phi_s + \varphi) \end{cases}$$

subtracting

$$\Delta w = \Delta E - \Delta E_s = qV_{RF}\left[\cos(\phi_s + \varphi) - \cos\phi_s\right]$$

Dividing by the accelerating cell length ΔL and assuming that:

$$\frac{V_{acc}}{\Delta L} = E_{acc} \Rightarrow \frac{V_{RF}}{\Delta L}\cos(\phi) = E_{RF}\cos(\phi)$$

Average accelerating field over the cell (or accelerating gradient)

$$\frac{\Delta w}{\Delta L} = qE_{RF}\left[\cos(\phi_s + \varphi) - \cos\phi_s\right]$$

Approximating

$$\frac{\Delta w}{\Delta L} \approx \frac{dw}{dz}$$

$$\frac{dw}{dz} = qE_{RF}\left[\cos(\phi_s + \varphi) - \cos\phi_s\right]$$

ENERGY-PHASE EQUATIONS (2/2)

On the other hand we have that the **phase variation per cell** of a generic particle and of a synchronous particle are:

$$\begin{cases} \Delta\phi_s = \omega_{RF}\Delta t_s \\ \Delta\phi = \omega_{RF}\Delta t \end{cases}$$

Δt is basically the time of flight between two accelerating cells

subtracting

$$\Delta\varphi = \omega_{RF}\left(\Delta t - \Delta t_s\right)$$

Dividing by the accelerating cell length ΔL

v, v_s are the average particles velocities

$$\frac{\Delta\varphi}{\Delta L} = \omega_{RF}\left(\frac{\Delta t}{\Delta L} - \frac{\Delta t_s}{\Delta L}\right) = \omega_{RF}\left(\frac{1}{v} - \frac{1}{v_s}\right) \underbrace{\cong}_{MAT} -\frac{\omega_{RF}}{cE_0\beta_s^3\gamma_s^3}w$$

Approximating

$$\frac{\Delta\varphi}{\Delta L} \cong \frac{d\varphi}{dz}$$

$$\begin{cases} \dfrac{d\varphi}{dz} = -\dfrac{\omega_{RF}}{cE_0\beta_s^3\gamma_s^3}w \\ \dfrac{dw}{dz} = qE_{RF}\left[\cos\left(\phi_s + \varphi\right) - \cos\phi_s\right] \end{cases}$$

This system of coupled (non linear) differential equations **describe the motion of a non synchronous particles** in the longitudinal plane with respect to the synchronous one.

MAT

$$\omega_{RF}\left(\frac{1}{v} - \frac{1}{v_s}\right) = \omega_{RF}\left(\frac{v_s - v}{vv_s}\right) \underbrace{\cong}_{\substack{vv_s \cong v_s^2 \\ v - v_s \cong \Delta v}} -\frac{\omega_{RF}}{v_s^2}\Delta v = -\frac{\omega_{RF}}{c}\frac{\Delta\beta}{\beta_s^2} \text{ remembering that } \beta = \sqrt{1\text{-}1/\gamma^2} \Rightarrow \beta d\beta = d\gamma/\gamma^3 \Rightarrow -\frac{\omega_{RF}}{c}\frac{\Delta\beta}{\beta_s^2} \cong -\frac{\omega_{RF}}{c}\frac{\Delta\gamma}{\beta_s^3\gamma_s^3} = -\frac{\omega_{RF}}{c}\frac{\overbrace{\Delta E}^{w}}{E_0\beta_s^3\gamma_s^3}$$

SMALL AMPLITUDE ENERGY-PHASE OSCILLATIONS

$$\begin{cases} \dfrac{dw}{dz} = qE_{RF}\left[\cos(\phi_s+\varphi)-\cos\phi_s\right] \\[2em] \dfrac{d\varphi}{dz} = -\dfrac{\omega_{RF}}{cE_0\beta_s^3\gamma_s^3}w \end{cases}$$

Assuming small oscillations around the synchronous particle that allow to approximate $\cos(\phi_s+\varphi)-\cos\phi_s \cong \varphi\sin\phi_s$

Deriving both terms with respect to z and assuming an **adiabatic acceleration** process i.e. a particle energy and speed variations that allow to consider

$$\frac{d\left(\frac{\omega_{RF}}{cE_0\beta_s^3\gamma_s^3}\right)}{dz}w \ll \frac{\omega_{RF}}{cE_0\beta_s^3\gamma_s^3}\frac{dw}{dz}$$

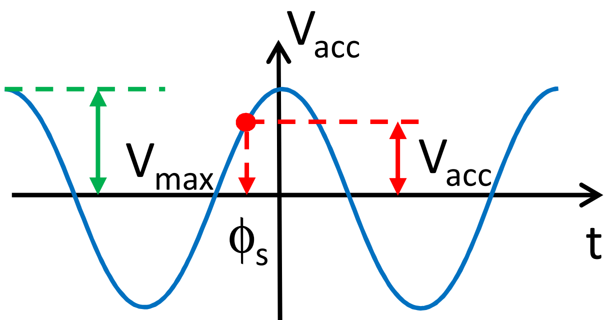
$$\frac{d^2\varphi}{dz^2} = -\frac{\omega_{RF}}{cE_0\beta_s^3\gamma_s^3}\frac{dw}{dz}$$

$$\frac{d^2\varphi}{dz^2} + \underbrace{q\frac{\omega_{RF}E_{RF}\sin(-\phi_s)}{cE_0\beta_s^3\gamma_s^3}}_{\Omega_s^2}\varphi = 0$$

harmonic oscillator equation

$\Rightarrow$The condition to have stable longitudinal oscillations and acceleration at the same time is:

$$\left.\begin{aligned} \Omega_s^2 > 0 &\Rightarrow \sin(-\phi_s) > 0 \\ V_{acc} > 0 &\Rightarrow \cos\phi_s > 0 \end{aligned}\right\} \Rightarrow -\frac{\pi}{2} < \phi_s < 0$$



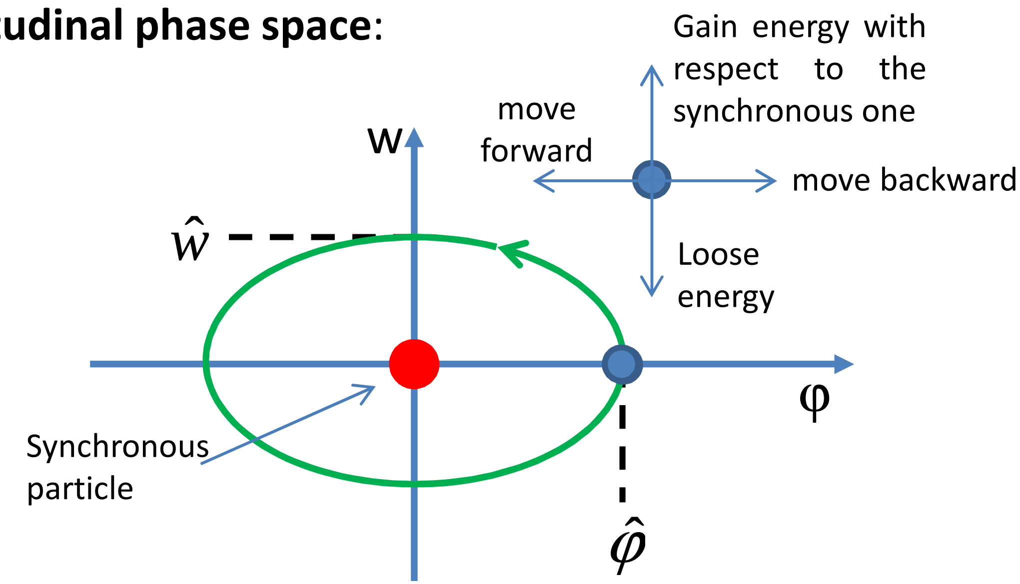
if we accelerate on the rising part of the positive RF wave we have a **longitudinal force keeping the beam bunched** around the synchronous phase.

$$\begin{cases} \varphi = \hat{\varphi}\cos(\Omega_s z) \\ w = \hat{w}\sin(\Omega_s z) \end{cases}$$

ENERGY-PHASE OSCILLATIONS IN PHASE SPACE

The energy-phase oscillations can be drawn in the **longitudinal phase space**:

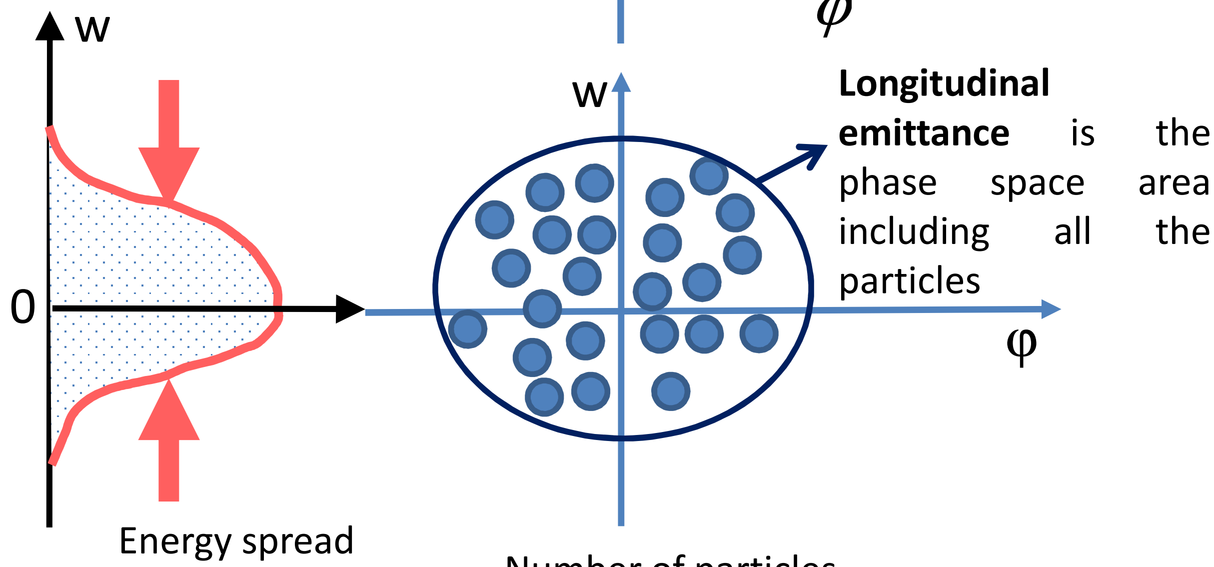
$$\begin{cases} \varphi = \hat{\varphi}\cos(\Omega_s z) \\ w = \hat{w}\sin(\Omega_s z) \end{cases}$$



⇒The trajectory of a generic particle in the longitudinal phase space is an **ellipse**.

⇒The **maximum energy deviation** is reached at φ=0 while the **maximum phase excursion** corresponds to w=0.

⇒the bunch occupies an area in the longitudinal phase space called **longitudinal emittance** and the projections of the bunch in the energy and phase planes give the **energy spread** and the **bunch length**.



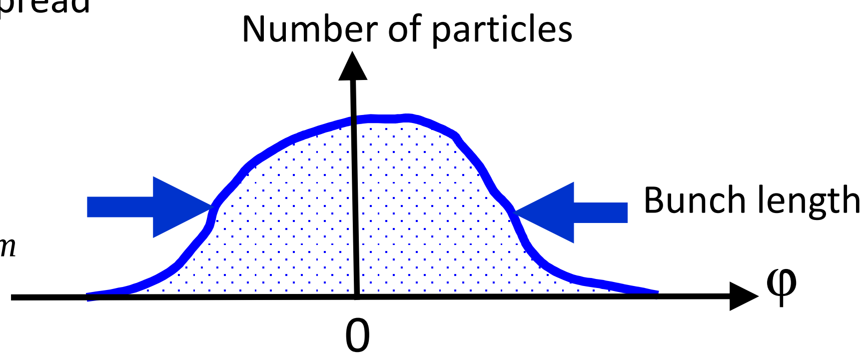
⇒It is easy to shown that

$$\frac{\hat{w}}{\hat{\varphi}} = \left(-\frac{cqE_{RF}E_0\sin(-\phi_s)\beta_s^3\gamma_s^3}{\omega_{RF}}\right)^{1/2}$$

The parameter of the ellipse vary (slowly) during acceleration

⇒numerical example: Protons @ 100 MeV, E_{RF} = 5 MV/m; ϕ_s = -30°; f_{RF} = 300 MHz

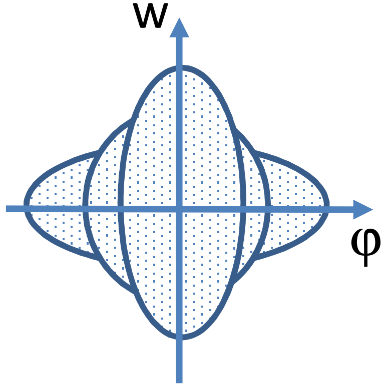
$$\Omega_s \approx 0.4\,rad/m \;\; ; \;\; \lambda_s = \frac{2\pi}{\Omega_s} = 15.7\,m$$



ADIABATIC PHASE DAMPING

⇒According to **Liouville's theorem**, under the action of conservative forces, the **ellipse area in phase space is constant**.

⇒The amplitude of the synchrotron oscillations remains **rigorously constant** only at **constant energy** (which is not the case in accelerators!), while in case of slow acceleration (low gradients) the **oscillation amplitudes** will **smoothly change** to satisfy the **Liouville's theorem**. In fact:

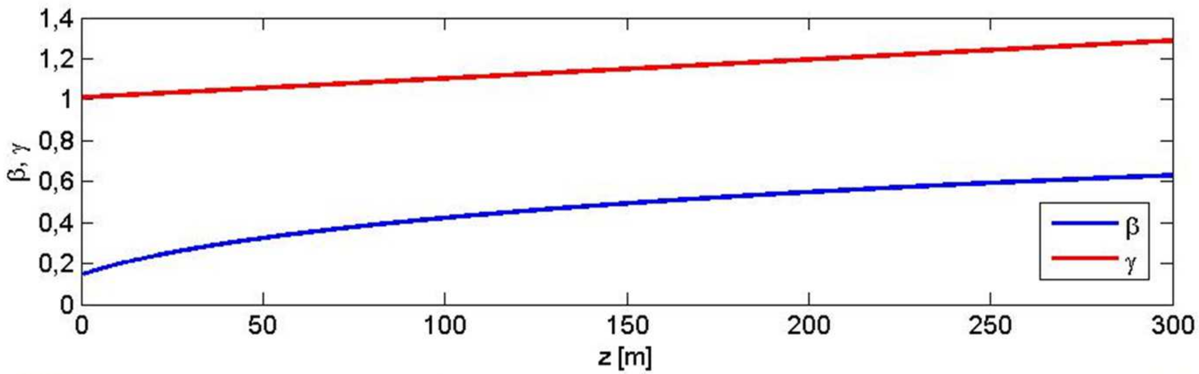


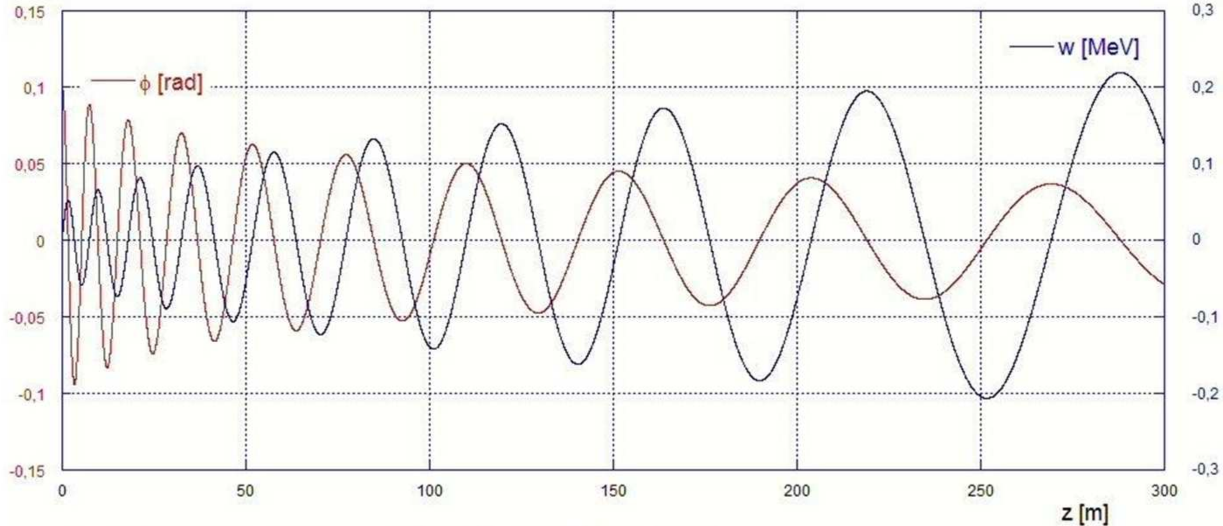
$$\begin{cases} \hat{w}\cdot\hat{\varphi} = \overbrace{\text{const}}^{Liouville} \\ \hat{w}/\hat{\varphi} \propto (\beta_s \cdot \gamma_s)^{3/2} \end{cases} \Rightarrow \begin{cases} \hat{w} \propto (\beta_s \cdot \gamma_s)^{3/4} \\ \hat{\varphi} \propto (\beta_s \cdot \gamma_s)^{-3/4} \end{cases}$$

$\hat{w} \propto (\beta_s \cdot \gamma_s)^{3/4}$ → The energy spread increases

$\hat{\varphi} \propto (\beta_s \cdot \gamma_s)^{-3/4}$ → The result is that during acceleration the bunch gets shorter: adiabatic **phase damping**

Protons
W_{in}=10 MeV
W_{fin}=260 MeV
E_{RF}=1 MV/m
ϕ_s=-30°
f_{RF}=300 MHz
Linac length 300 m





LARGE OSCILLATIONS

To study the longitudinal dynamics **at large oscillations**, we have to consider the **non linear system of differential equations** without approximations. By neglecting particle energy and speed variations along the LINAC (**adiabatic acceleration**) it is possible to obtain easily the following relation between w and φ that is the **Hamiltonian of the system** related to the total particle energy.

$$\frac{1}{2}\left(\frac{\omega_{RF}}{cE_0\beta_s^3\gamma_s^3}\right)^2 w^2 + \frac{\omega_{RF}\, qE_{RF}}{cE_0\beta_s^3\gamma_s^3}\left[\sin(\phi_s+\varphi)-\varphi\cos\phi_s-\sin(\phi_s)\right] = \mathrm{const} = \mathrm{H}$$

⇒**For each H we have different trajectories** in the longitudinal phase space

⇒the oscillations are **stable** within a region bounded by a special curve called **separatrix**: its equation is:

$$\frac{1}{2}\frac{\omega_{RF}}{cE_0\beta_s^3\gamma_s^3} w^2 + qE_{RF}\left[\sin(\phi_s+\varphi)-(2\varphi_s+\varphi)\cos\phi_s+\sin(\phi_s)\right] = 0$$

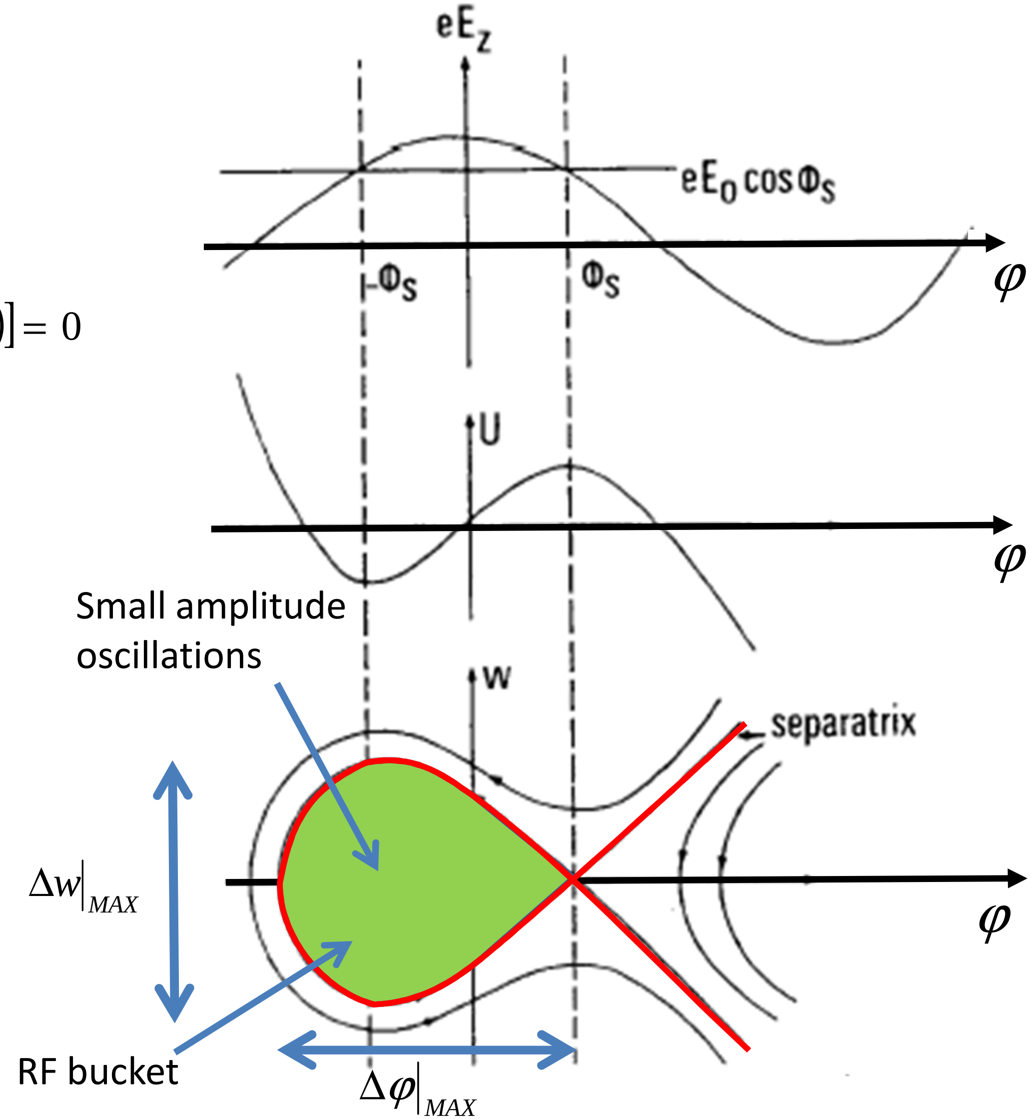
⇒the region inside the separatrix is called **RF bucket**. The dimensions of the bucket shrinks to zero if ϕ_s=0.

⇒trajectories outside the RF buckets are **unstable**.

⇒we can define the **RF acceptance** as the maximum extension in phase and energy that we can accept in an accelerator:

$$\Delta\varphi\big|_{MAX} \cong 3\phi_s$$

$$\Delta w\big|_{MAX} = \pm 2\left[\frac{qcE_o\beta_s^3\gamma_s^3 E_{RF}(\phi_s\cos\phi_s-\sin\phi_s)}{\omega_{RF}}\right]^{\frac{1}{2}}$$



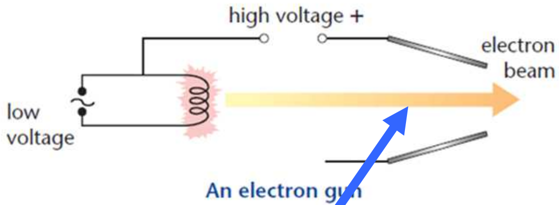
LONGITUDINAL DYNAMICS OF LOW ENERGY ELECTRONS (1/3)

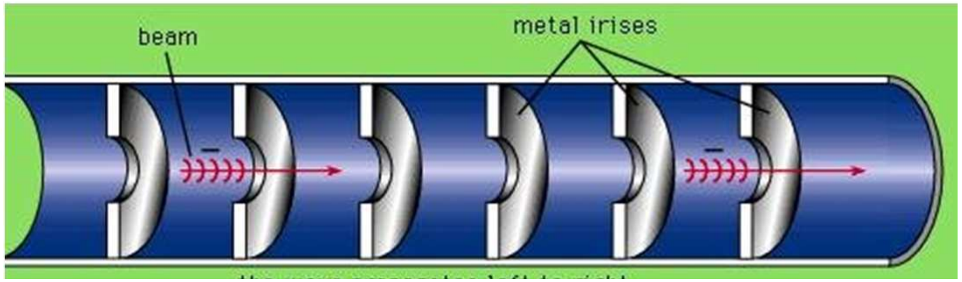
From previous formulae it is clear that there is **no motion in the longitudinal phase plane for ultrarelativistic particles** (γ>>1).

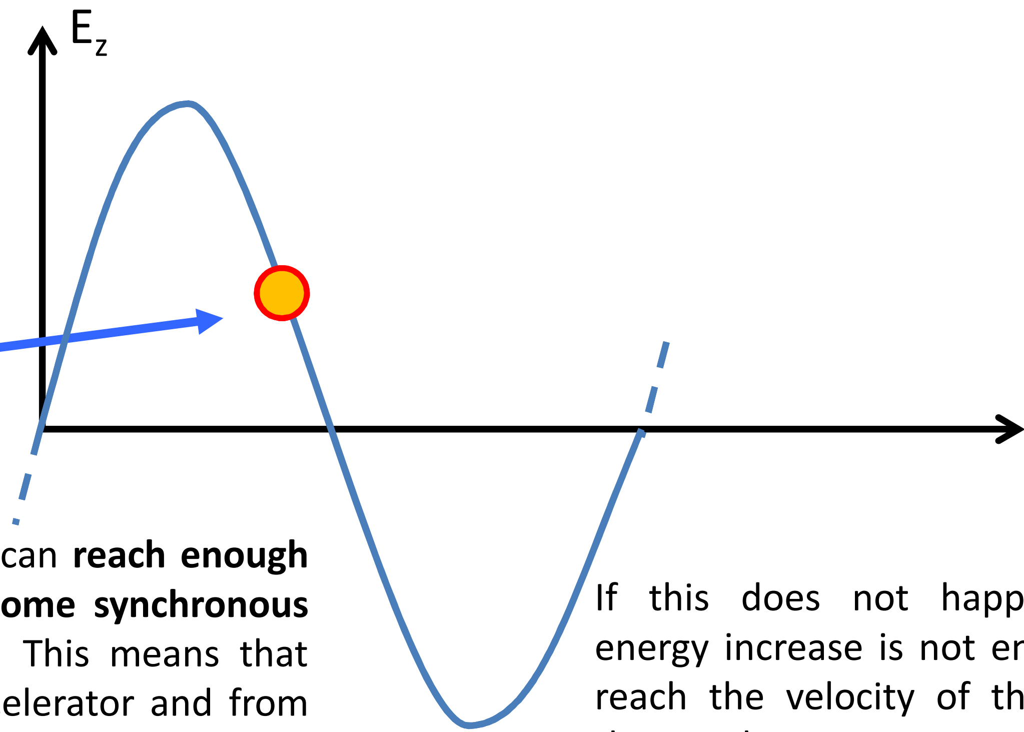
⇒This is the case of **electrons** whose **velocity is always close to speed of light c** even at low energies.

⇒Accelerating structures are designed to provide an accelerating field synchronous with particles moving at v=c. like **TW structures with phase velocity equal to c**.

It is interesting to analyze what happen if we **inject an electron beam produced by a cathode (at low energy) directly in a TW structure** (with v_{ph}=c) and the conditions that allow to **capture** the beam (this is equivalent to consider instead of a TW structure a SW designed to accelerate ultrarelativistic particles at v=c).







Particles enter the structure with velocity **v<c** and, initially, they are **not synchronous with the accelerating field** and there is a so called slippage.

After a certain distance they can **reach enough energy (and velocity) to become synchronous** with the accelerating wave. This means that they are captured by the accelerator and from this point they are stably accelerated.

If this does not happen (the energy increase is not enough to reach the velocity of the wave) they are lost

LONGITUDINAL DYNAMICS OF LOW ENERGY ELECTRONS (2/3)

The **accelerating field** of a TW structure can be expressed by

$$E_{acc} = E_{RF} \cos\underbrace{\left(\omega_{RF} t - kz\right)}_{\phi(z,t)}$$

The **equation of motion of a particle with a position z at time t accelerated by the TW** is then

$$\frac{d}{dt}(mv) = qE_{RF}\cos\phi(z,t) \Rightarrow m_0 c \frac{d}{dt}(\gamma\beta) = m_0 c\gamma^3 \frac{d\beta}{dt} = qE_{RF}\cos\phi$$

This is **the phase of the TW wave seen by the particle at a certain time t and position z**

The **phase motion during acceleration** is then

$$\frac{d\phi}{dt} = \omega_{RF} - k\frac{dz}{dt} = \omega_{RF} - \frac{\omega_{RF}}{c}\frac{dz}{dt} \Rightarrow \frac{d\phi}{dt} = \omega_{RF}(1-\beta) > 0$$

The phase of the wave “seen” by the particle always increase because $\beta<1$.

It is useful to find which is the relation between β and ϕ

$$m_0 c\gamma^3 \frac{d\beta}{dt} = m_0 c\gamma^3 \frac{d\beta}{d\phi}\frac{d\phi}{dt} = \frac{1}{(1+\beta)\sqrt{1-\beta^2}}\frac{d\beta}{d\varphi} = \frac{qE_{RF}}{\omega_{RF} m_0 c}\cos\phi$$

MAT

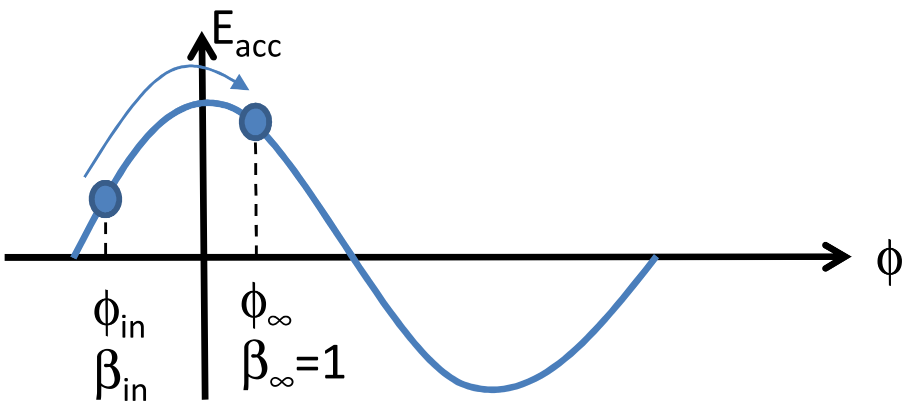
$$\sin\phi_{fin} = \sin\phi_{in} + \frac{2\pi E_0}{\lambda_{RF} qE_{RF}}\left(\sqrt{\frac{1-\beta_{in}}{1+\beta_{in}}} - \sqrt{\frac{1-\beta_{fin}}{1+\beta_{fin}}}\right)$$

Suppose that the particle reach asymptotically the value β_{fin}=1 we have:

$$\sin\phi_{\infty} = \sin\phi_{in} + \frac{2\pi E_0}{\lambda_{RF} qE_{RF}}\sqrt{\frac{1-\beta_{in}}{1+\beta_{in}}}$$

$\Rightarrow$1st Result

$$\sin\phi_{\infty} > \sin\phi_{in} \Rightarrow \phi_{\infty} > \phi_{in}$$



LONGITUDINAL DYNAMICS OF LOW ENERGY ELECTRONS (3/3)

$\Rightarrow$2nd Result

For a given injection energy (β_{in}) and phase (ϕ_{in}) we can find which is the electric field (E_{RF}) that is necessary to have the completely relativistic beam at phase ϕ_∞ (that is necessary to **capture the beam at phase ϕ_∞**)

$$E_{RF} = \frac{2\pi E_0}{\lambda_{RF} q(\sin\phi_\infty - \sin\phi_{in})}\sqrt{\frac{1-\beta_{in}}{1+\beta_{in}}}$$

Example:

E_{in} = 50 keV, (kinetic energy), ϕ_{in} =-π/2, ϕ_∞ =0$\Rightarrow$ $\gamma_{in} \approx 1.1$; $\beta_{in} \approx 0.41$

f_{RF}= 2856 MHz$\Rightarrow \lambda_{RF} \approx$ 10.5 cm

We obtain $E_{RF} \cong$20MV/m;

$\Rightarrow$3rd Result

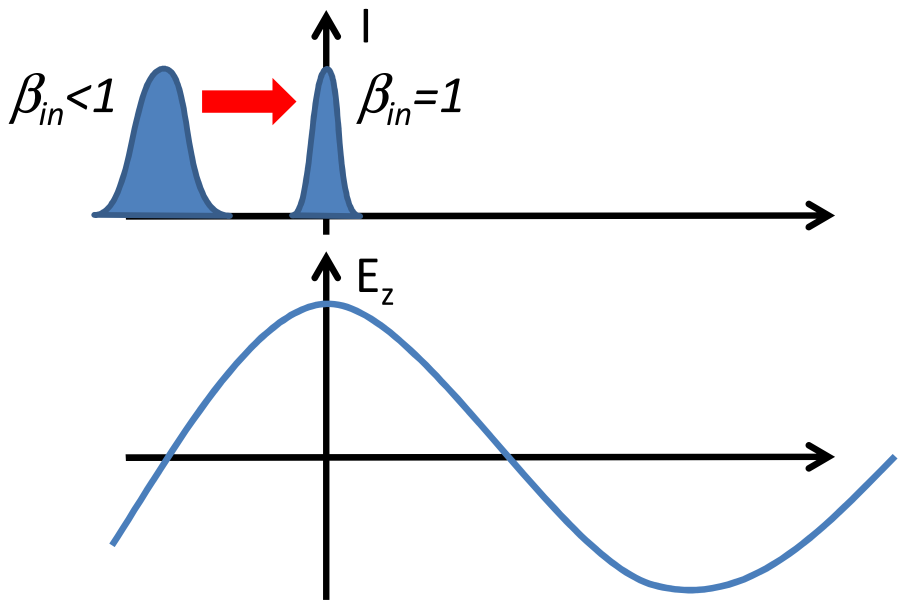
For a given injection energy we can find which is the **minimum value of the electric field (E_{RF}) that allow to capture a beam**. Obviously this correspond to an injection phase ϕ_{in} =-π/2 and ϕ_∞=π/2.

$$E_{RF\_MIN} = \frac{\pi E_0}{\lambda_{RF} q}\sqrt{\frac{1-\beta_{in}}{1+\beta_{in}}}$$

Example: For the previous case we obtain: $E_{RF_min} \cong$10MV/m;

$\Rightarrow$4th Result

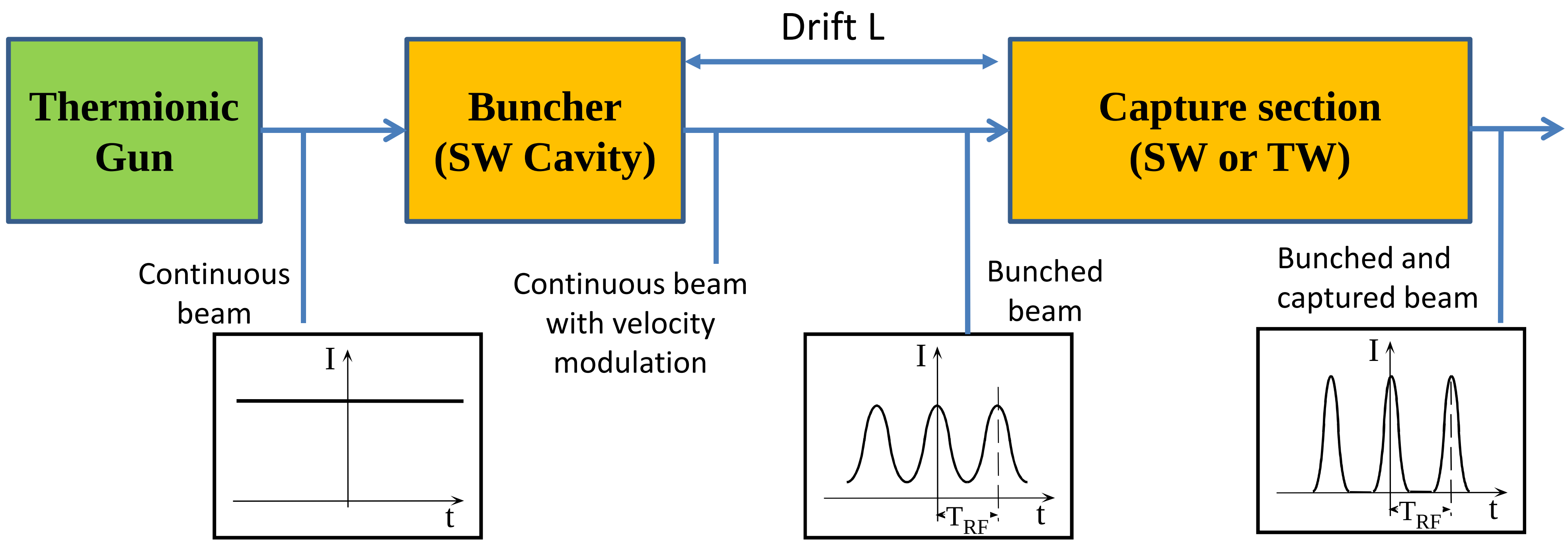
It is possible to demonstrate that if we inject the beam at ϕ_{in} =-π/2 (and E_{RF}>E_{RF_MIN}) we have that the captured beam is compressed (shortened) by the RF.



BUNCHER AND CAPTURE SECTIONS

Once the capture condition $E_{RF}>E_{RF\_MIN}$ is fulfilled the fundamental equation of previous slide sets the **ranges of the injection phases ϕ_{in} actually accepted**. Particles whose injection phases are within this range can be **captured** the other are **lost**.

In order **to increase the capture efficiency** of a traveling wave section, **pre-bunchers** are often used. They are SW cavities aimed at **pre-forming particle bunches gathering particles continuously emitted by a source.**

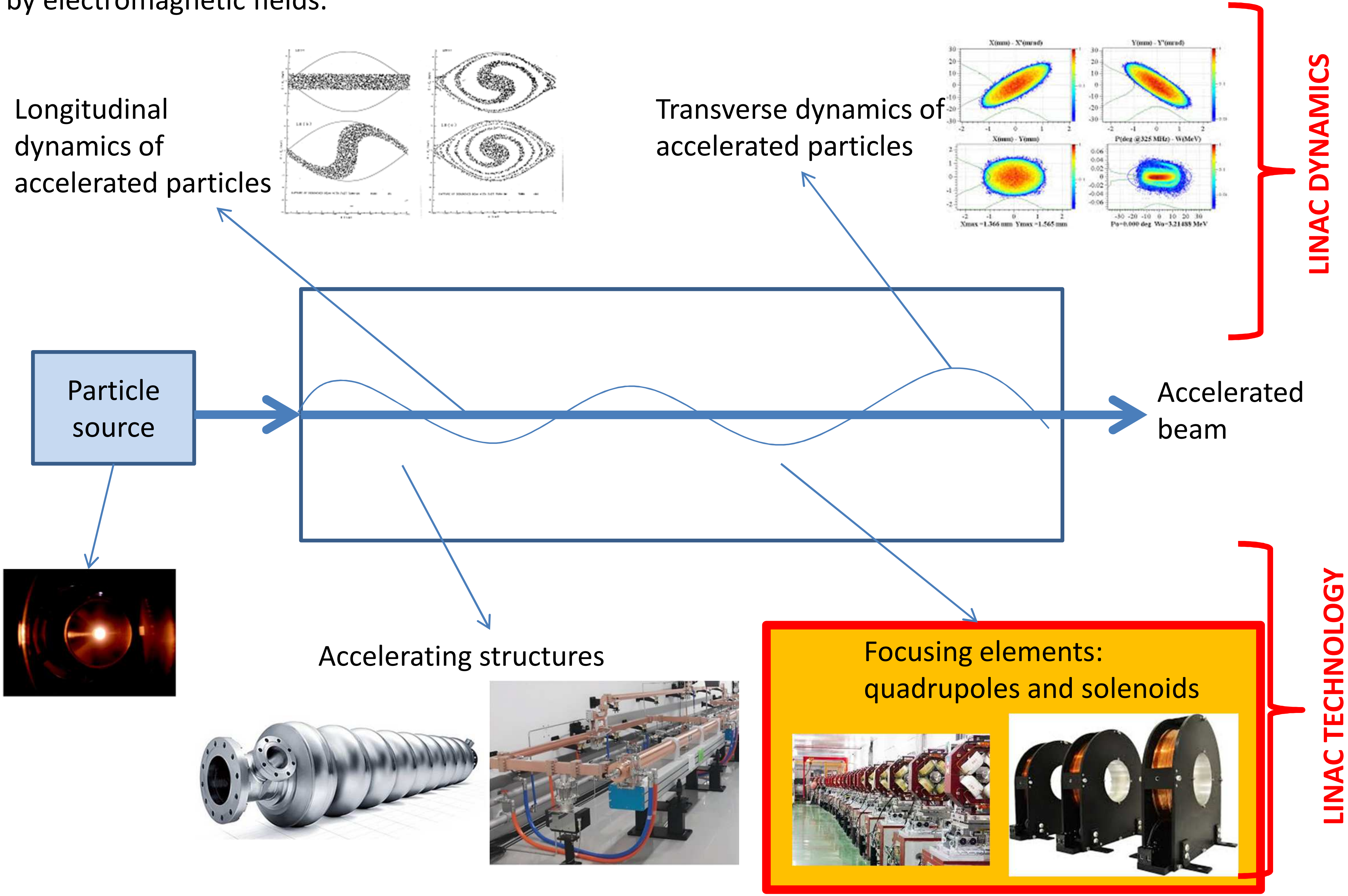


⇒**Bunching** is obtained by modulating the energy (and therefore the velocity) of a continuous beam using the longitudinal E-field of a SW cavity. After a certain **drift space** the **velocity modulation is converted in a density charge modulation**. The density modulation depletes the regions corresponding to injection phase values incompatible with the capture process

⇒A TW accelerating structure (**capture section**) is placed at an **optimal distance from the pre-buncher**, to capture a large fraction of the charge and accelerate it till relativistic energies. The **amount of charge lost is drastically reduced**, while the capture section provide also further beam bunching.

LINAC: BASIC DEFINITION AND MAIN COMPONENTS

LINAC (linear accelerator) is a **system that allows to accelerate charged particles through a linear trajectory** by electromagnetic fields.



LORENTZ FORCE: ACCELERATION AND FOCUSING

Particles are accelerated through electric field and are bended and focalized through magnetic field. The basic equation that describe the acceleration/bending /focusing processes is the **Lorentz Force**.

$\vec{p} = momentum$

$m = mass$

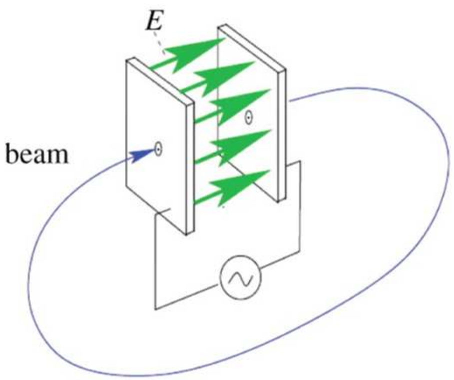
$\vec{v} = velocity$

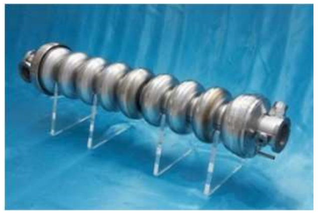
$q = charge$

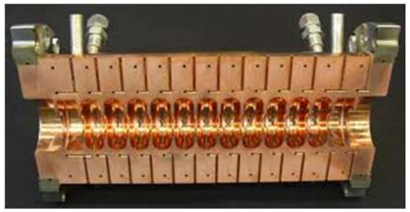
$$\frac{d\vec{p}}{dt} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

ACCELERATION

To accelerate, we need a force in the direction of motion



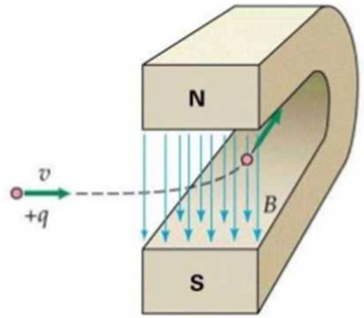


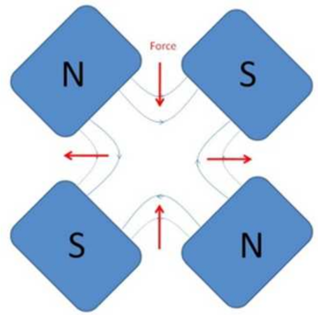


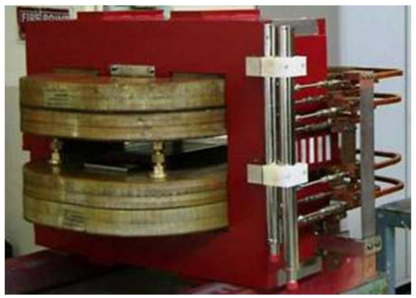
Longitudinal Dynamics

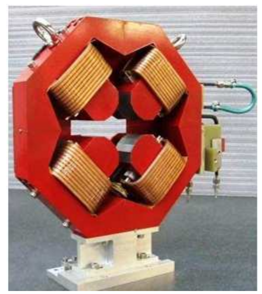
BENDING AND FOCUSING

2^{nd} term always perpendicular to motion => no energy gain









Transverse Dynamics

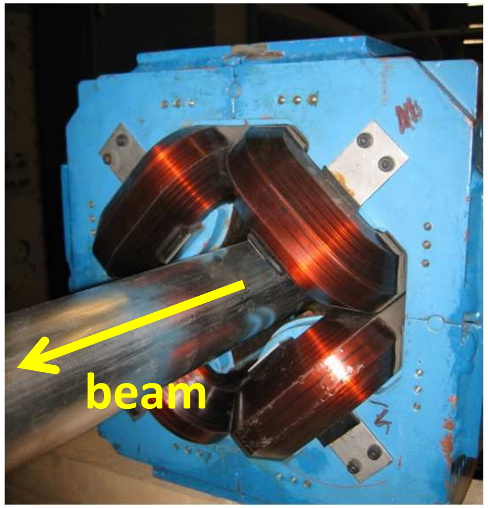
MAGNETIC QUADRUPOLE

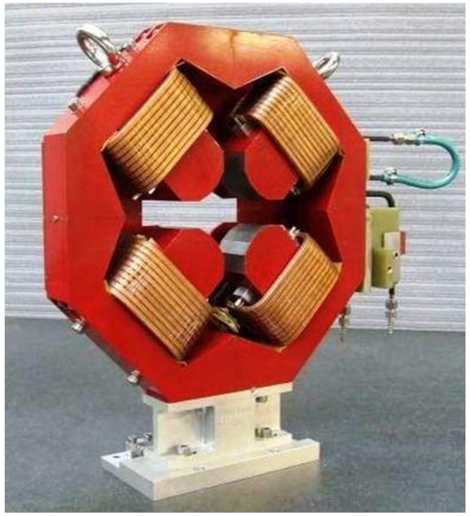
Quadrupoles are used to **focalize the beam in the transverse plane**. It is a **4 poles magnet**:

⇒**B=0 in the center of the quadrupole**

⇒The **B intensity increases linearly** with the off-axis displacement.

⇒If the quadrupole is **focusing in one plane is defocusing in the other plane**



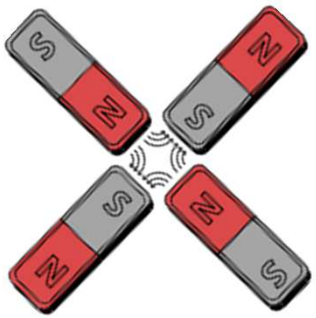


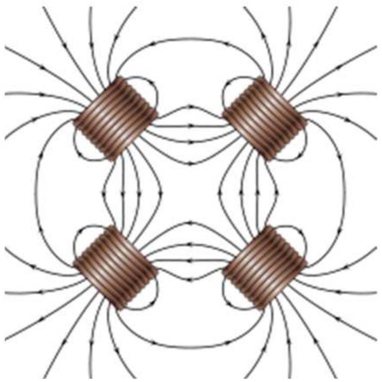
$$\begin{cases} B_x = G \cdot y \\ B_y = G \cdot x \end{cases} \Rightarrow \begin{cases} F_y = qvG \cdot y \\ F_x = -qvG \cdot x \end{cases}$$

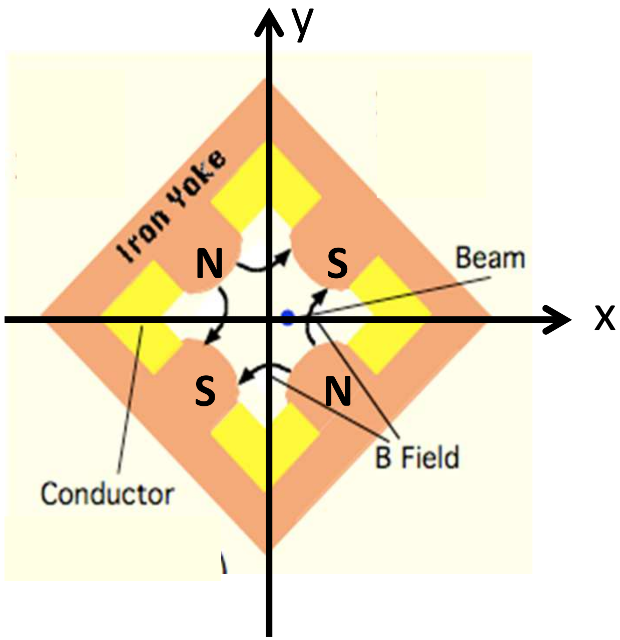
$$G = \text{quadrupole gradient} \left[\frac{T}{m} \right]$$

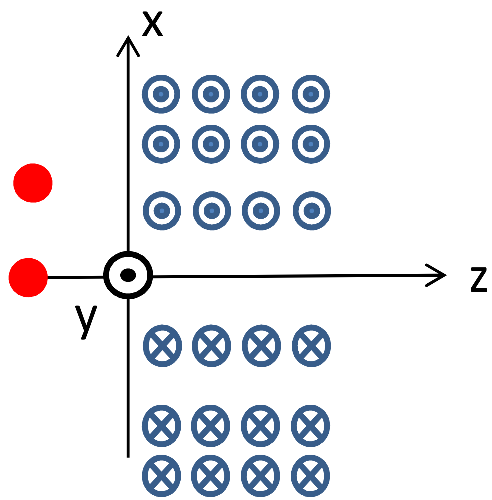
Electromagnetic quadrupoles G <50-100 T/m

$$\frac{F_B}{F_E} = v \Rightarrow \begin{cases} F_B(1T) = F_E\left(300\frac{MV}{m}\right) @ \beta = 1 \\ F_B(1T) = F_E\left(3\frac{MV}{m}\right) @ \beta = 0.01 \end{cases}$$



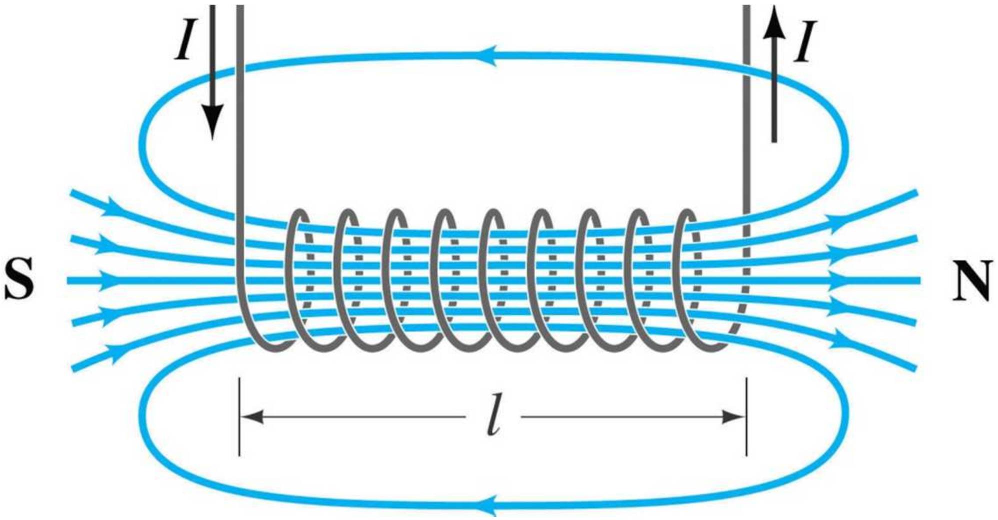


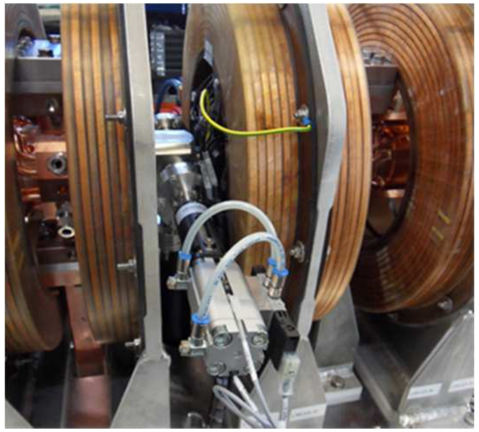


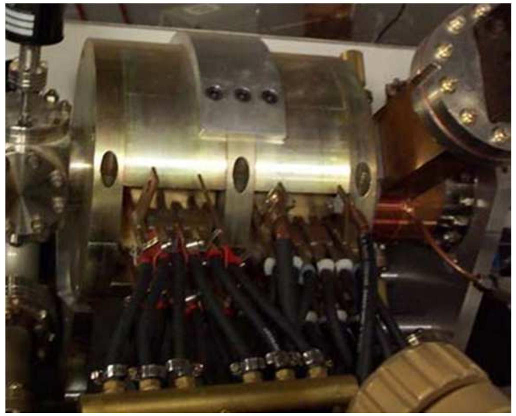


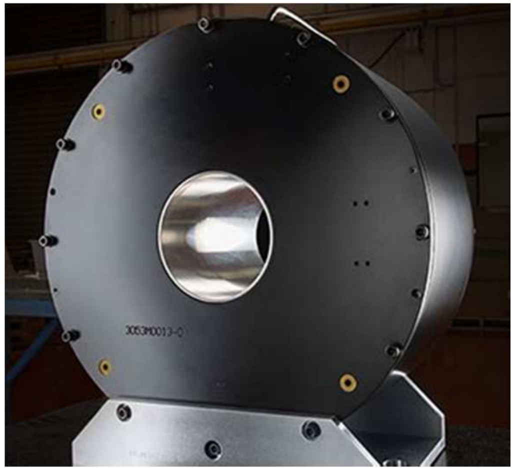
SOLENOID

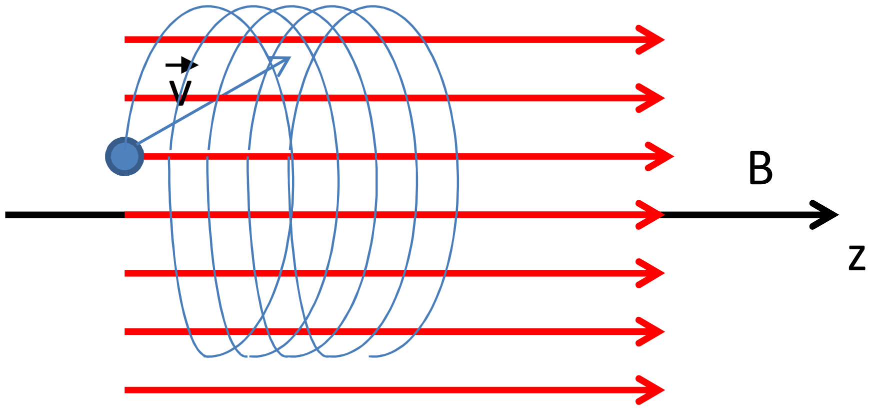
Also solenoids can be used for focalization of beams (in particular electron beams).









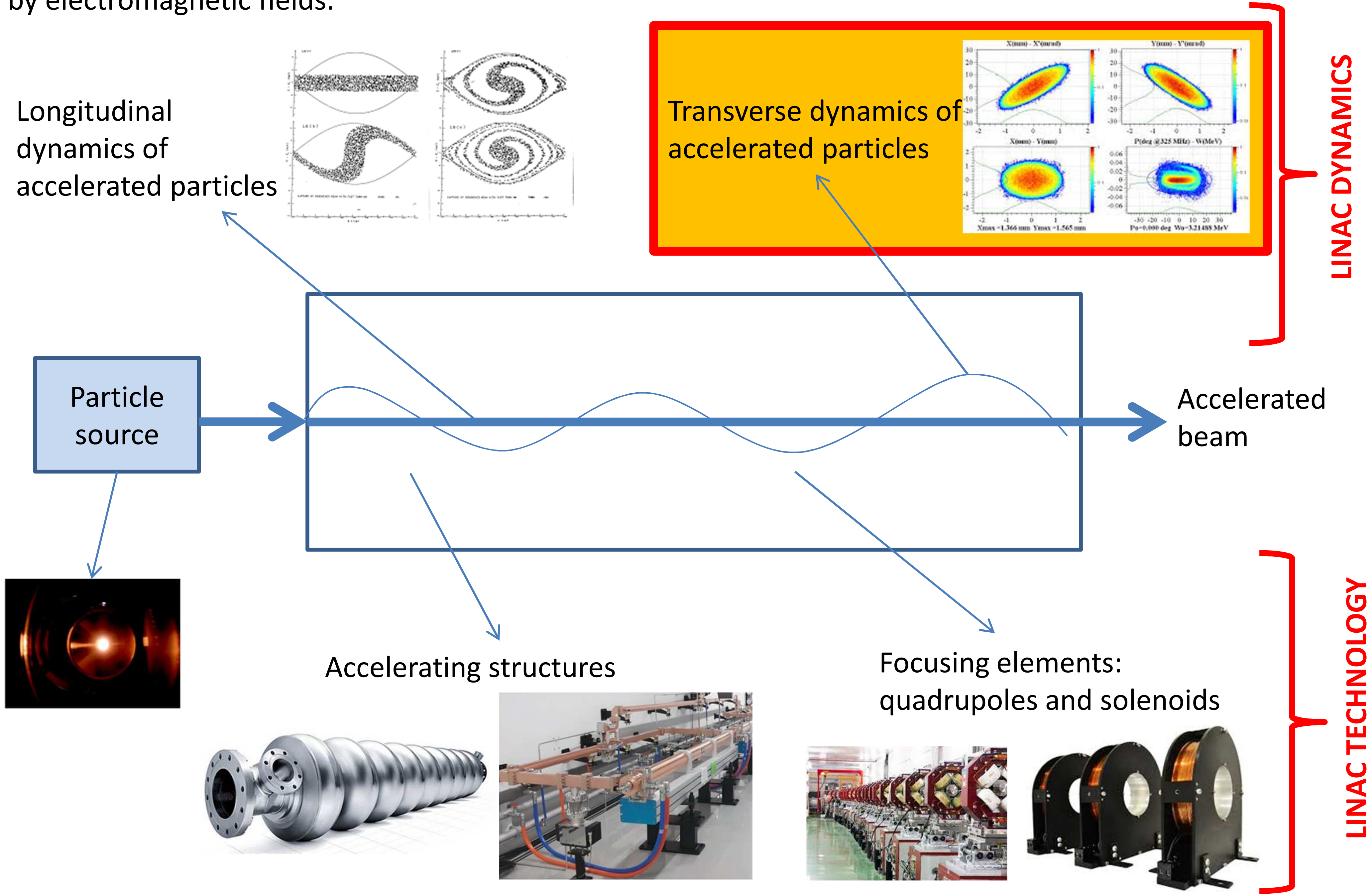


Useful for beam emittance compensation at low energies

Particles that enter into a solenoidal field with a transverse component of the velocity (divergence) start to **spiralize describing circular trajectories** .

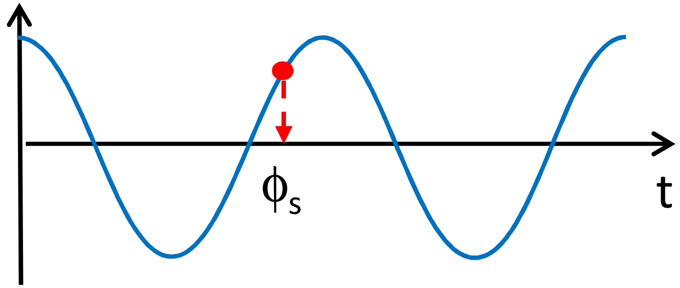
LINAC: BASIC DEFINITION AND MAIN COMPONENTS

LINAC (linear accelerator) is a **system that allows to accelerate charged particles through a linear trajectory** by electromagnetic fields.



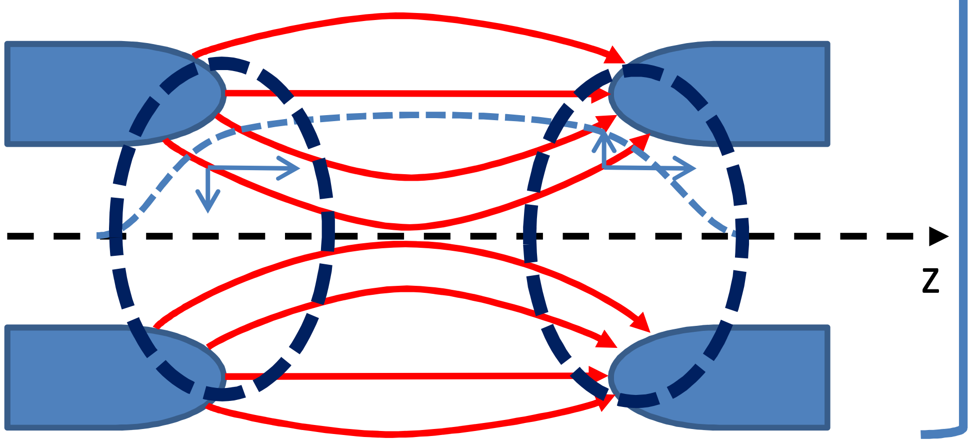
RF DEFOCUSING

⇒**Phase stability** principle requires that the accelerating voltage increase in time ($dV_{acc}/dt>0$)

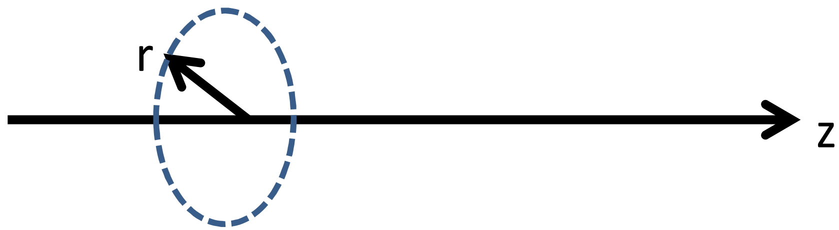


⇒According to Maxwell equations the **divergence of the field is zero** and this implies that in traversing one accelerating gap there is a focusing/defocusing term

$$\nabla\vec{E}=0 \Rightarrow \frac{1}{r}\frac{\partial(rE_r)}{\partial r}+\frac{\partial E_z}{\partial z}=0 \Rightarrow E_r=-\frac{1}{2}\frac{\partial E_z}{\partial z}r$$



⇒especially for ions there is a **velocity variation** along the gap that has to be taken into account to correctly evaluate the effect of the transverse focusing/defocusing field



Transverse momentum increase

Defocusing force since $\sin\phi<0$

Gap length

Defocusing effect

$$\Delta p_r = -\frac{\pi q E_{RF} L \sin\phi}{c\gamma^2\beta^2\lambda_{RF}} r$$

⇒transverse defocusing $\sim 1/\gamma^2$ **disappears at relativistic velocity**

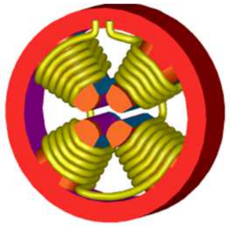
⇒for non relativistic particles this term represent a **coupling between the longitudinal and the transverse beam dynamics**

⇒Important consequence: in an **electron linac, transverse and longitudinal dynamics are decoupled**

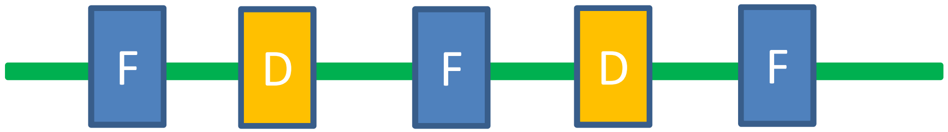
MAGNETIC FOCUSING CONFIGURATION IN A LINAC

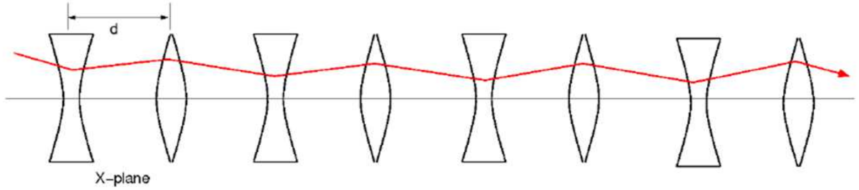
⇒**Defocusing RF forces** or the natural divergence (emittance) of the beam need to be **compensated** and controlled by **focusing forces**.

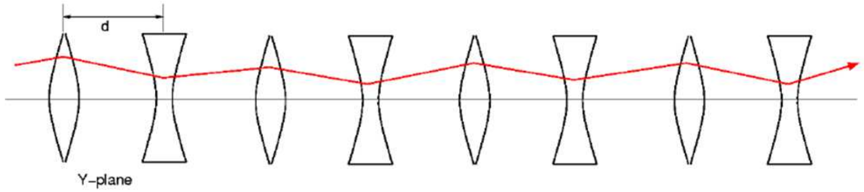
This is provided by **quadrupoles** along the beam line.



⇒As previously pointed out a quadrupole is focusing in one plane and defocusing on the other. It can be demonstrated that a global focalization is provides by **alternating quadrupoles** with opposite signs (doublets, triplets) like in an optical system.



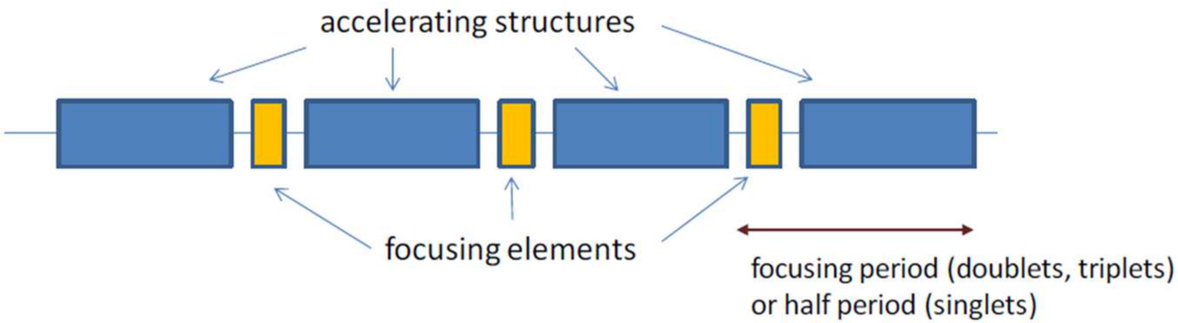




⇒A linac **alternates accelerating sections with focusing sections**.

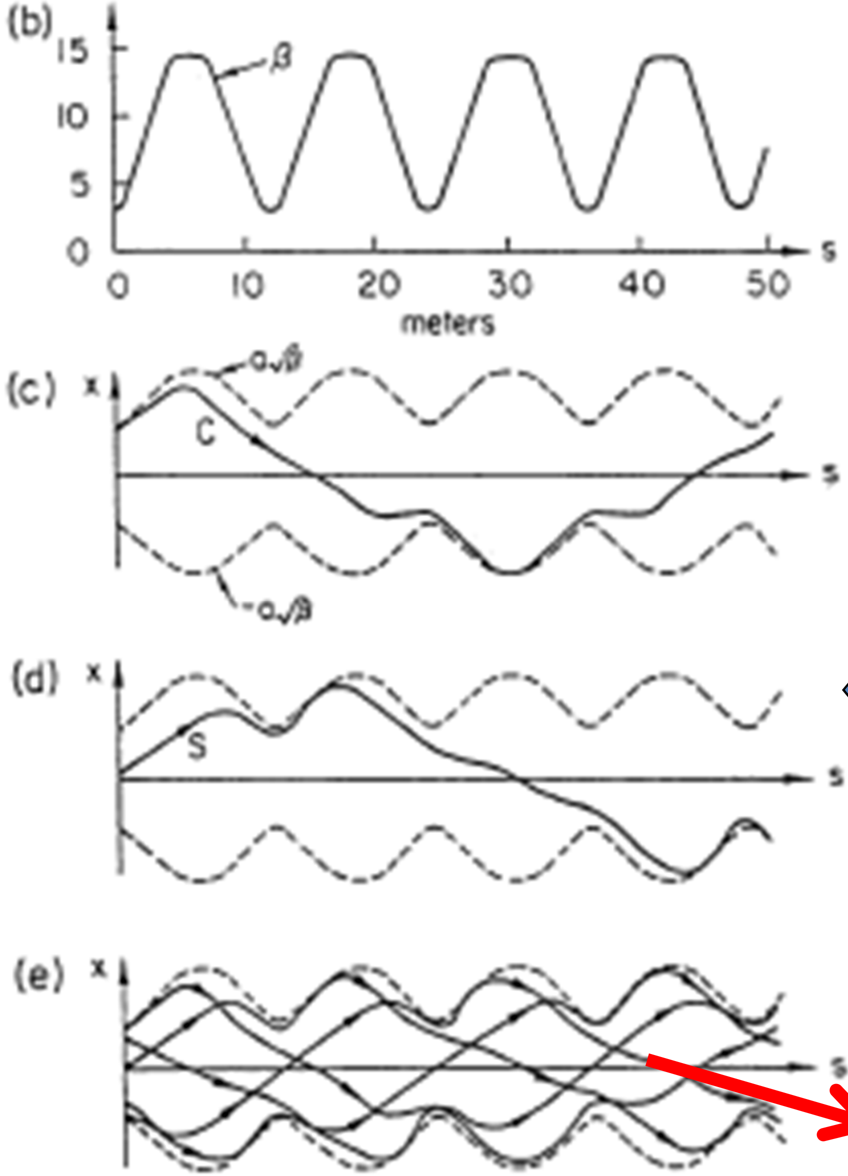
⇒By definition a **focusing period** is the length after which the structure is **repeated**.

⇒The **maximum allowed distance between focusing elements** depends on beam energy and current and change in the different linac sections (from only one gap in the DTL to one or more multi-cell cavities at high energies).



TRANSVERSE OSCILLATIONS AND BEAM ENVELOPE

Due to the **alternating quadrupole focusing system** each particle perform transverse oscillations along the LINAC.



The **equation of motion in the transverse plane** (as example x) is of the type:

Term depending on the magnetic configuration

RF defocusing term

$$\frac{d^2x}{ds^2}+\underbrace{\left[\kappa^2(s)-k_{RF}^2(s)\right]}_{K^2(s)}x=0$$

It is possible to demonstrate that the **single particle trajectory is a pseudo-sinusoid** described by the equation:

$$x(s)=\sqrt{\varepsilon_o\beta(s)}\cos\left[\int_{s_o}^{s}\frac{ds}{\beta(s)}+\phi_0\right]$$

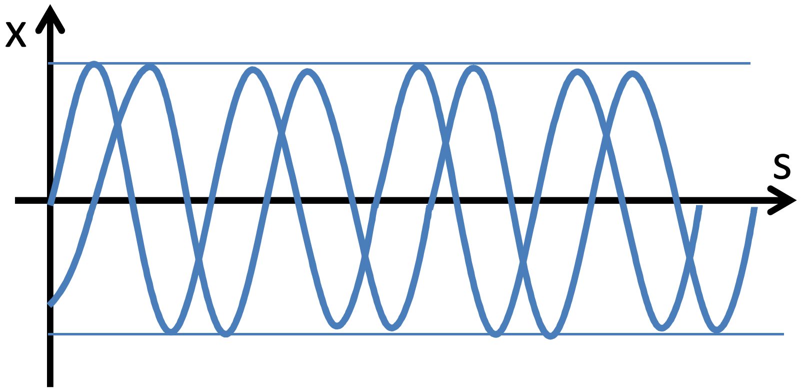
Characteristic function (Twiss β-function [m]) that depend on the magnetic and RF configuration

Depend on the initial conditions of the particle

The final transverse beam dimensions ($\sigma_{x,y}(s)$) varies along the linac and are contained within an **envelope**

SMOOTH APPROXIMATION, EXAMPLES

⇒In case of "**smooth approximation**" of the LINAC (we consider a sort of average quadrupole focusing and RF defocusing effect) we obtain a simple harmonic motion along s of the type (β is constant):



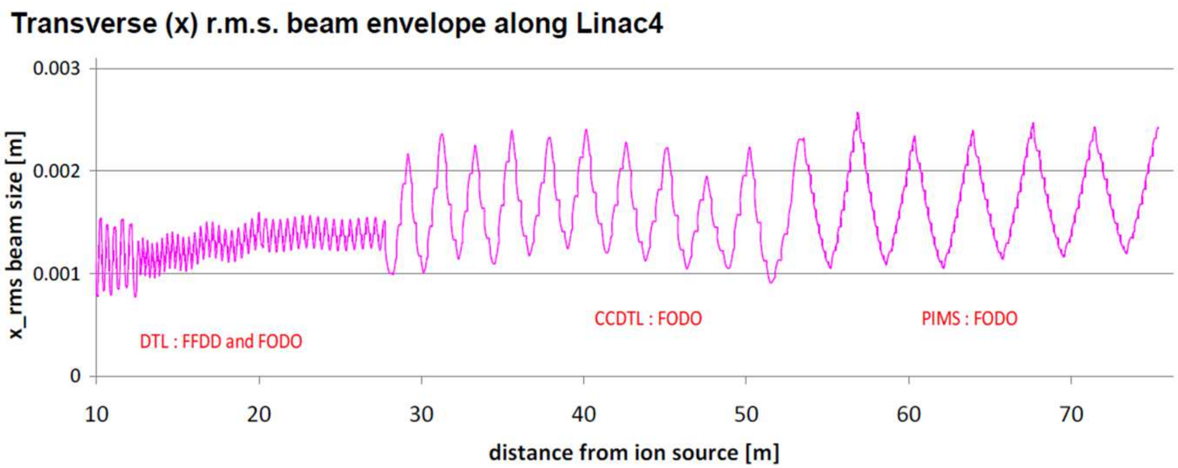
Phase advance per unit length

$$x(s) = A\cos(K_0 s + \phi_0)$$

$$K_0 = \sqrt{\left(\frac{qGl}{2mc\gamma\beta}\right)^2 - \frac{\pi q E_{RF}\sin(-\phi)}{mc^2\lambda_{RF}(\gamma\beta)^3}}$$

Magnetic focusing elements

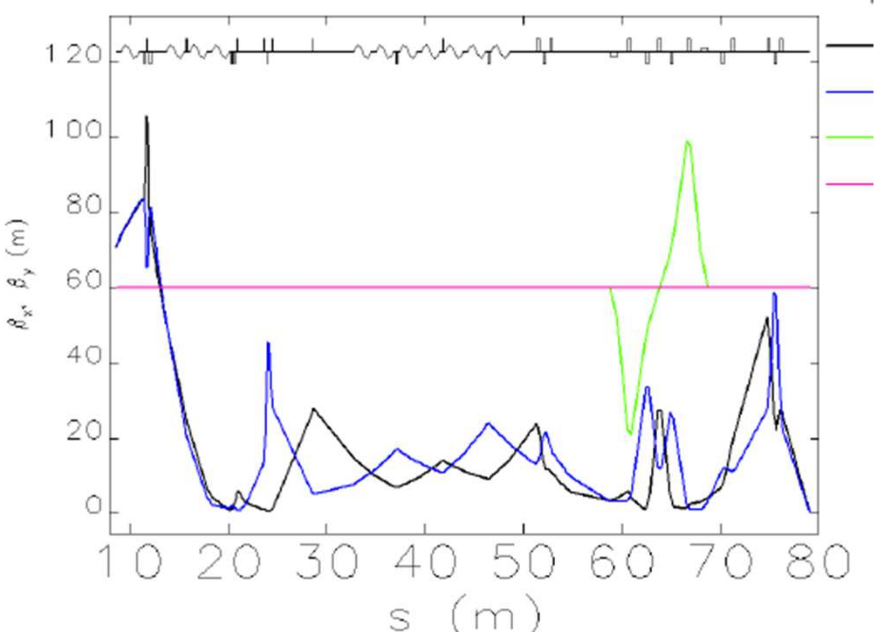
RF defocusing term



Courtesy A. Lombardi, M. Vretenar

ELI-NP LINAC (electron linac)

Courtesy C. Vaccarezza

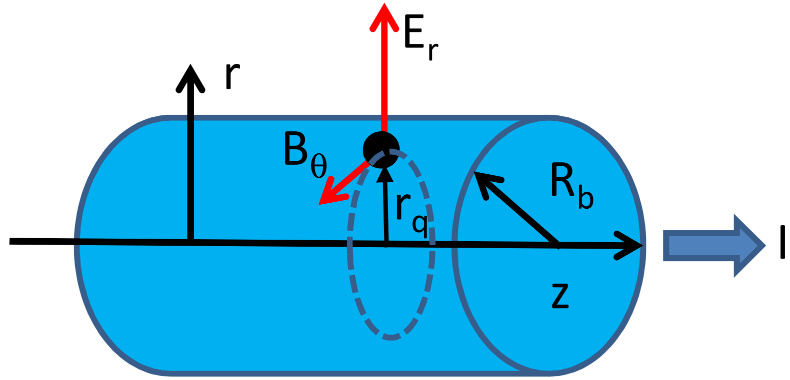


SPACE CHARGE EFFECTS

In the longitudinal and transverse beam dynamics we have neglected up to now the **effect of Coulomb repulsion between particles** (**space charge**).

These effects cannot be neglected especially at **low energy and at high current** because the space charge forces scales as $1/\gamma^2$ and with the current I.

Uniform and infinite cylinder of charge moving along z



$$\overrightarrow{F_{SC}} = q\left(E_r - vB_\theta\right)\hat{r} = q\frac{I}{2\pi\varepsilon_0 R_b^2 \beta c \gamma^2} r_q \hat{r}$$

The individual particles satisfy the equation

$$\frac{d^2x}{ds^2} + K^2(s)x - F_{SC} = 0$$

External forces (magnets+RF)

Space Charge forces that is in general **nonlinear and beam current dependent**

In case of smooth approximation and ellipsoidal beam we have:

$$K_0 = \sqrt{\left(\frac{qGl}{2mc\gamma\beta}\right)^2 - \frac{\pi q E_{RF}\sin(-\phi)}{mc^2\lambda_{RF}(\gamma\beta)^3} - \frac{3Z_0 q I \lambda_{RF}(1-f)}{8\pi mc^2\beta^2\gamma^3 r_x r_y r_z}}$$

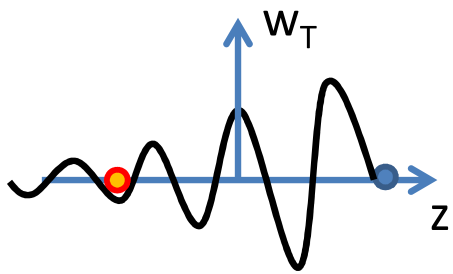
Space charge term

I= beam current
$r_{x,y,z}$=ellipsoid semi-axis
f= form factor
Z_0=free space impedance (377 Ω)

For ultrarelativistic **electrons** RF defocusing and space charge disappear and the external focusing is required to control the emittance and to stabilize the beam against instabilities.

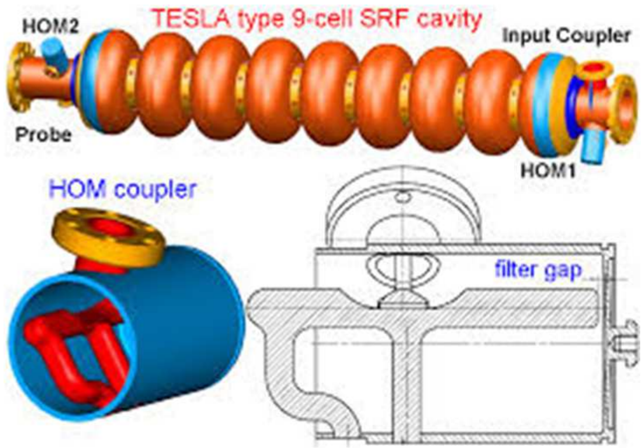
COLLECTIVE EFFECT: WAKEFIELDS

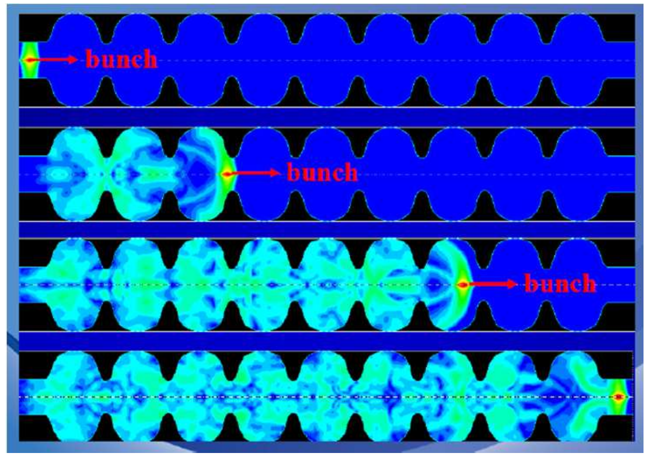
Collective effects are all effects related to the number of particles. We already mentioned the space charge one that can give an increase of beam dimensions, defocusing, etc…
The other effects are due to the **wakefield**. The passage of bunches through accelerating structures excites electromagnetic **field**. This field can have longitudinal and transverse components and, interacting with subsequent bunches (long range wakefield), **can affect the longitudinal and the transverse beam dynamics**. In particular the **transverse wakefields**, can drive an instability along the train called **multibunch beam break up** (BBU).

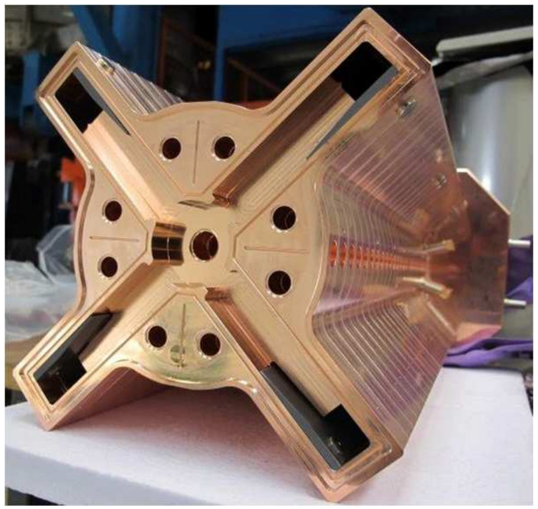


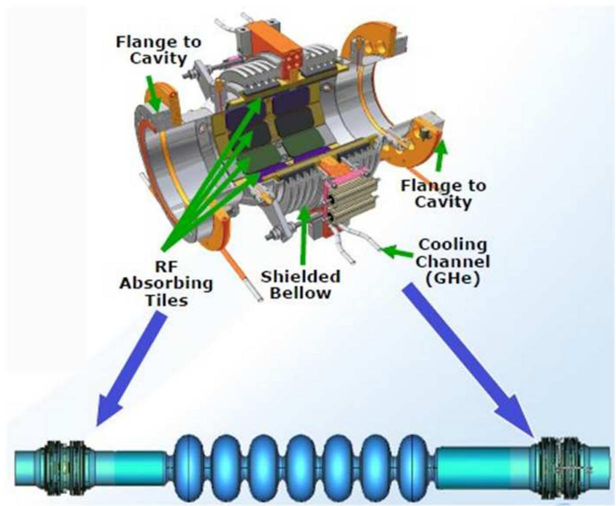
Several approaches are used to absorb these field :

- **Loop** couplers (several per cavity for different modes/orientations)
- **Waveguide** dampers
- Beam pipe **absorbers** (ferrite or ceramic)



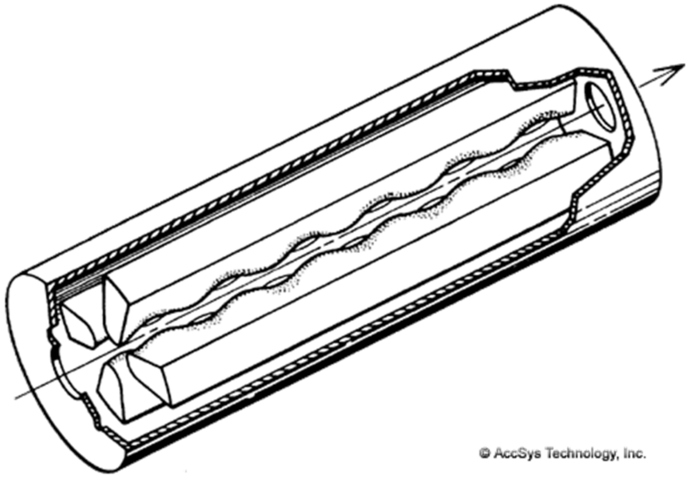


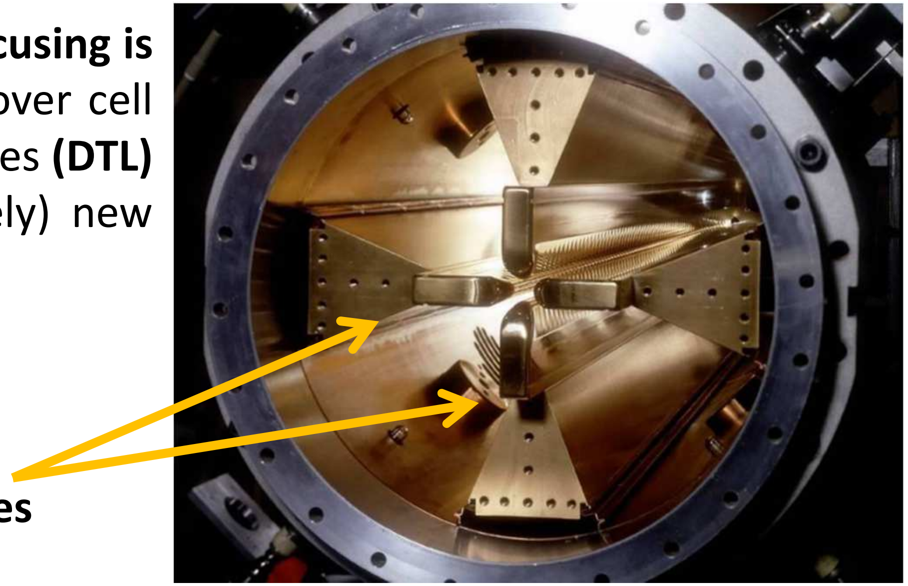




RADIO FREQUENCY QUADRUPOLES (RFQ)

At low proton (or ion) energies (β~0.01), **space charge defocusing is high and quadrupole focusing is not very effective**. Moreover cell length becomes small and conventional accelerating structures **(DTL) are very inefficient**. At this energies it is used a (relatively) new structure, the **Radio Frequency Quadrupole** (1970).

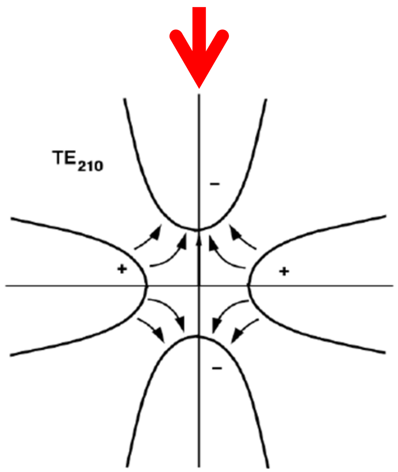




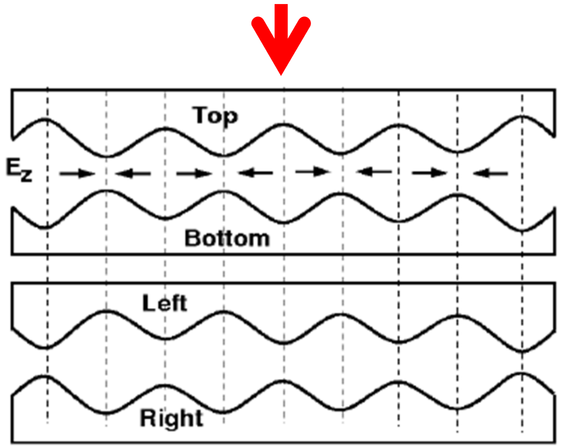
Courtesy M. Vretenar

These structures allow to simultaneously provide:

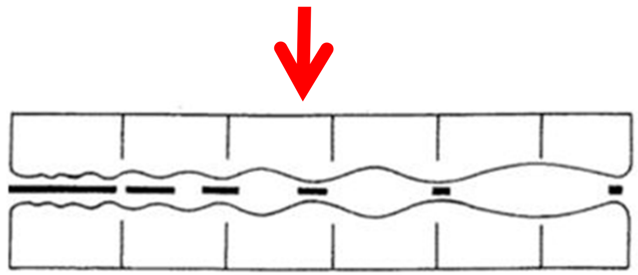
Transverse Focusing



Acceleration



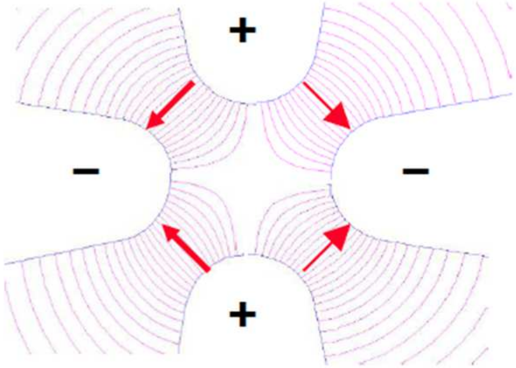
Bunching of the beam

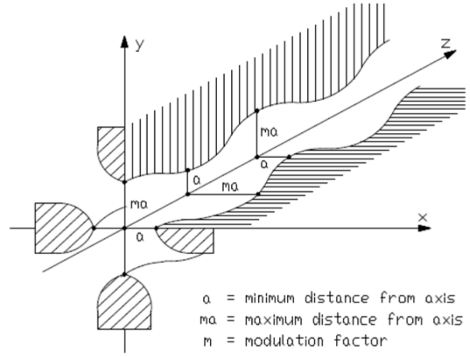


RFQ: PROPERTIES

1-Focusing

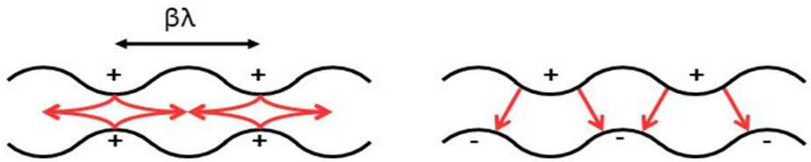
The resonating mode of the cavity (between the four electrodes) is a **focusing mode**: **Quadrupole mode (TE_{210})**. The alternating voltage on the electrodes produces an **alternating focusing channel** with the period of the RF (**electric focusing** does not depend on the velocity and is ideal at low β)





2-Acceleration

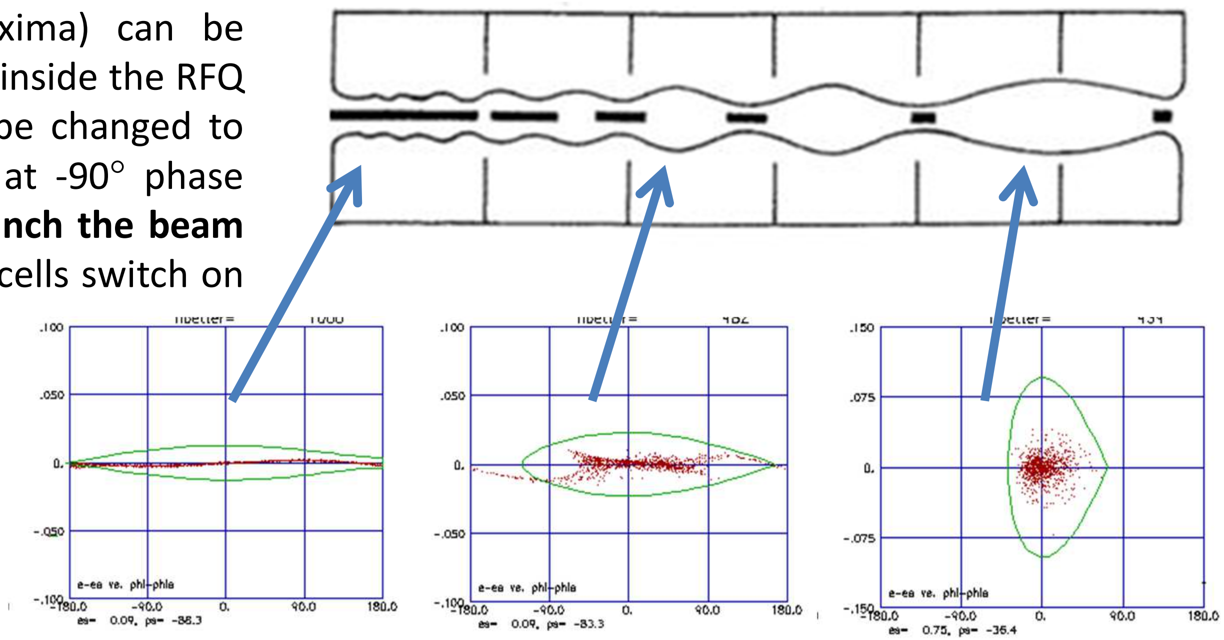
The vanes have a **longitudinal modulation** with period = $\beta\lambda_{RF}$ this creates a **longitudinal component of the electric field** that accelerate the beam (the modulation corresponds exactly to a series of RF gaps).

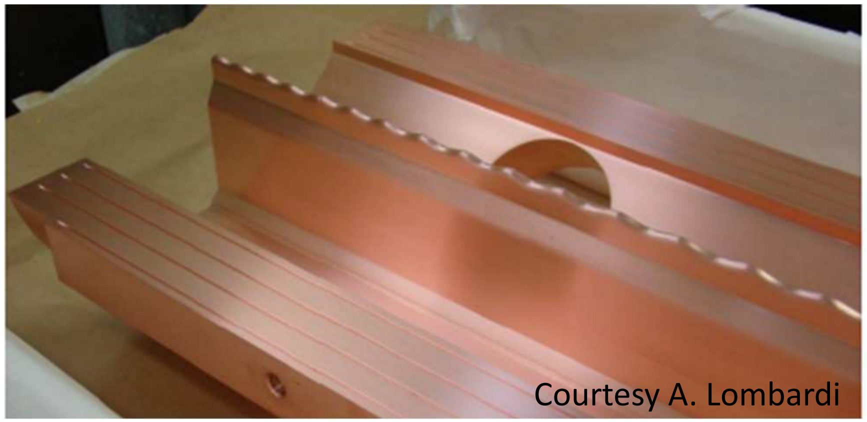


Opposite vanes (180°) Adjacent vanes (90°)

3-Bunching

The modulation period (distance between maxima) can be slightly adjusted to change the phase of the beam inside the RFQ cells, and the amplitude of the modulation can be changed to change the accelerating gradient. One can start at -90° phase (linac) with some **bunching cells**, progressively **bunch the beam** (adiabatic bunching channel), and only in the last cells switch on the **acceleration**.



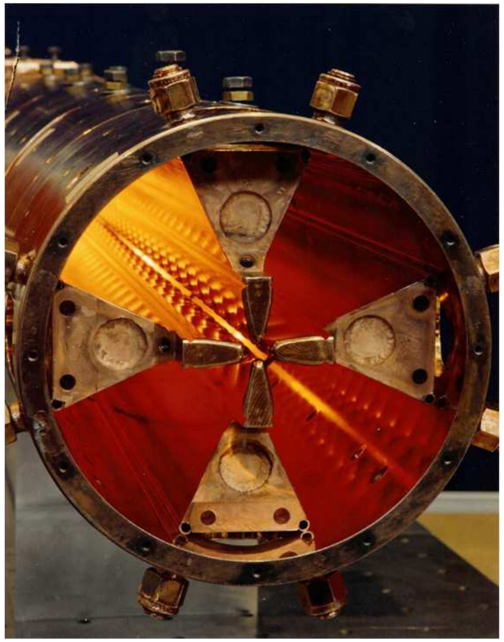


Courtesy A. Lombardi

The RFQ is the only linear accelerator that can accept a low energy continuous beam.

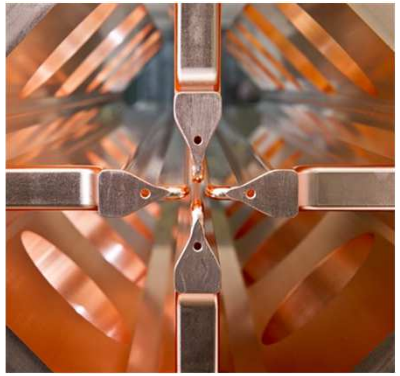
Courtesy M. Vretenar and A. Lombardi

RFQ: EXAMPLES

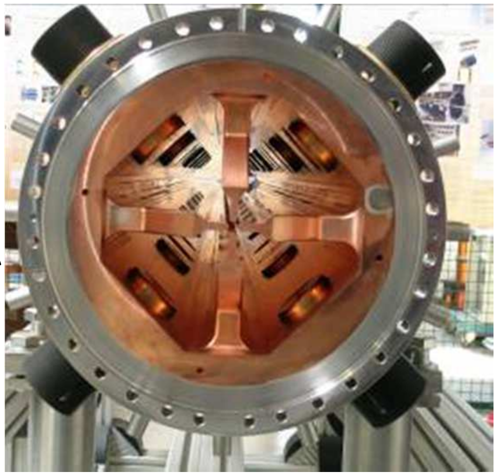


The 1st 4-vane RFQ, Los Alamos
1980: 100 KeV - 650 KeV, 30 mA , 425 MHz

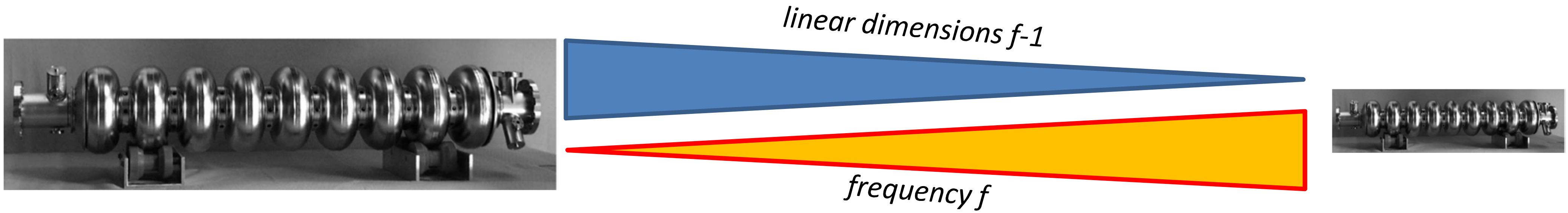
The CERN Linac4 RFQ
45 keV – 3 MeV, 3 m
80 mA H-, max. 10% duty cycle



TRASCO @ INFN Legnaro
Energy In: 80 keV
Energy Out: 5 MeV
Frequency 352.2 MHz
Proton Current (CW) 30 mA



THE CHOICE OF THE FREQUENCY



Structure dimensions	Scales with 1/f
Shunt impedance (efficiency) per unit length r	**NC structures** r increases and this push to adopt **higher frequencies** $\propto f^{1/2}$
	SC structures the power losses increases with f^2 and, as a consequence, r scales with 1/f this push to adopt **lower frequencies**
Power sources	At very high frequencies (>10 GHz) **power sources** are less available
Mechanical realization	Cavity fabrication at very high frequency requires **higher precision** but, on the other hand, at low frequencies one needs more material and **larger machines**/brazing oven
Bunch length	**short bunches** are easier with higher f (FEL)
RF defocusing (ion linacs)	Increases with frequency ($\propto$ f)
Cell length ($\beta\lambda$RF)	1/f
Wakefields	more critical at high frequency ($w_{//} \propto f^{2}$, $w_{\perp} \propto f^{3}$)

Electron linacs tend to use higher frequencies (1-12 GHz) than ion linacs.
SW SC: 500 MHz-1500 MHz
TW NC: 3 GHz-12 GHz

Proton linacs use lower frequencies (100-800 MHz), increasing with energy (ex.: 350–700 MHz): compromise between focusing, cost and size.
Heavy ion linacs tend to use even lower frequencies (30-200 MHz), dominated by the low beta in the first sections

THE CHOICE OF THE ACCELERATING STRUCTURE

In general **the choice of the accelerating structure depends on**:

⇒ **Particle type**: mass, charge, energy
⇒ **Beam current**
⇒ **Duty cycle** (pulsed, CW)
⇒ **Frequency**
⇒ **Cost** of fabrication and of operation

Moreover **a given accelerating structure has also a curve of efficiency** (shunt impedance) with respect to the particle energies and the choice of one structure with respect to another one depends also on this.

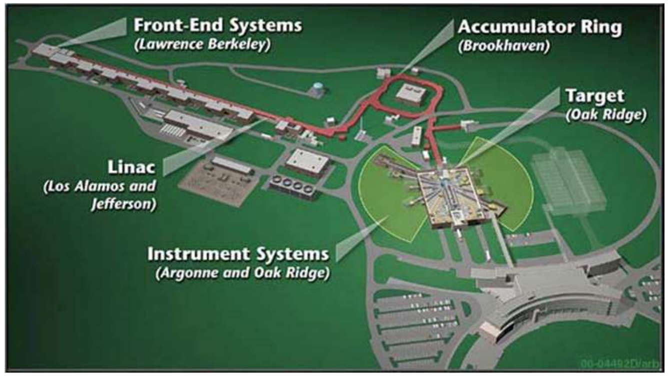
As example a very general scheme is given in the Table (absolutely not exhaustive).

Cavity Type	β Range	Frequency	Particles
RFQ	0.01– 0.1	40-500 MHz	Protons, Ions
DTL	0.05 – 0.5	100-400 MHz	Protons, Ions
SCL	0.5 – 1	600 MHz-3 GHz	Protons, Electrons
SC Elliptical	> 0.5-0.7	350 MHz-3 GHz	Protons, Electrons
TW	1	3-12 GHz	Electrons

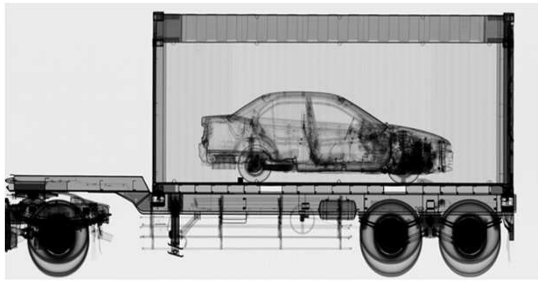
THANK YOU FOR YOUR ATTENTION

Medical applications

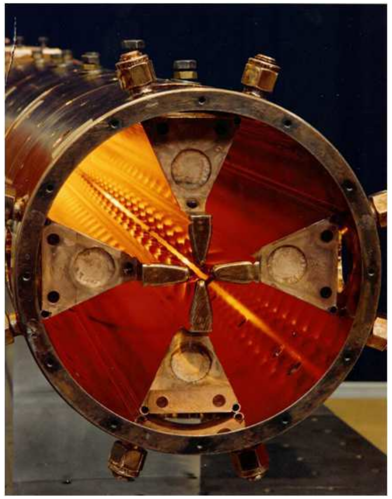
Neutron spallation sources



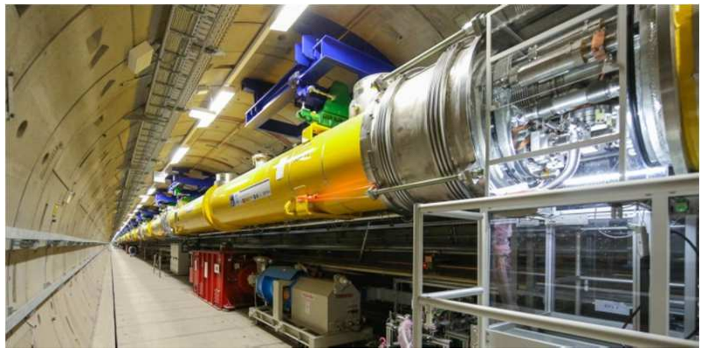
Security: Cargo scans

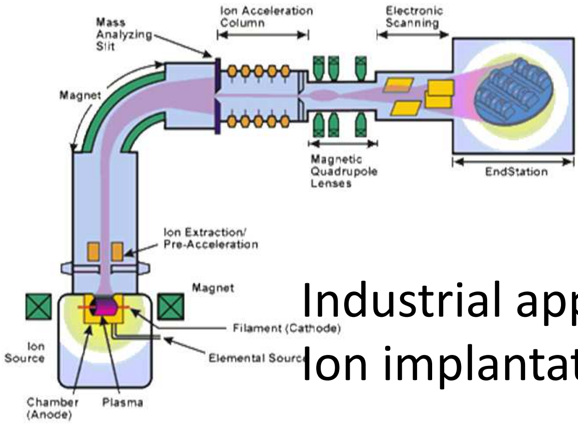


Injectors for colliders and synchrotron light sources



FEL





Industrial applications:
Ion implantation for semiconductors

APPENDIX: SEPARATRIX

To study the longitudinal dynamics at large oscillations, we have to consider the non linear system of differential equations without approximations. By neglecting particle energy and speed variations along the LINAC we obtain:

$$\frac{d^2\varphi}{dz^2} = -\frac{\omega_{RF} qE_{RF}}{cE_0\beta_s^3\gamma_s^3}\left[\cos(\phi_s+\varphi)-\cos\phi_s\right] = F$$

The restoring force ***F*** can not be considered purely elastic anymore and may by derived from a **potential function** according to the usual definition:

$$U = -\int_0^{\varphi} F d\varphi' = \frac{\omega_{RF}\, qE_{RF}}{cE_0\beta_s^3\gamma_s^3}\left[\sin(\phi_s+\varphi)-\varphi\cos\phi_s-\sin(\phi_s)\right]$$

With few simple passages we obtain an “energy conservation”–like law:

$$\frac{d}{dz}\left[\left(\frac{d\varphi}{dz}\right)^2\right] = 2\frac{d\varphi}{dz}\frac{d^2\varphi}{dz^2} = 2\frac{d\varphi}{dz}\cdot\left(-\frac{dU}{d\varphi}\right) = -2\frac{d}{dz}U \Rightarrow \frac{d}{dz}\left[\left(\frac{d\varphi}{dz}\right)^2+2U\right]=0 \;\Rightarrow\; \frac{1}{2}\left(\frac{d\varphi}{dz}\right)^2 + U = \text{cost}$$

$$\frac{d\varphi}{dz} = -\frac{\omega_{RF}}{cE_0\beta_s^3\gamma_s^3}w$$

$$\frac{1}{2}\left(\frac{\omega_{RF}}{cE_0\beta_s^3\gamma_s^3}\right)^2 w^2 + \frac{\omega_{RF}\, qE_{RF}}{cE_0\beta_s^3\gamma_s^3}\left[\sin(\phi_s+\varphi)-\varphi\cos\phi_s-\sin(\phi_s)\right] = \text{const} = \text{H}$$

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