Part II
Effect of Insertion Devices on the Electron Beam

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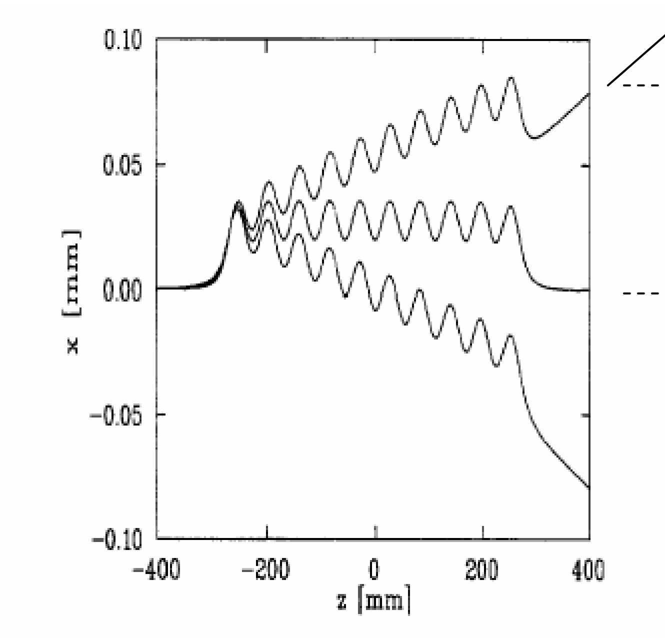
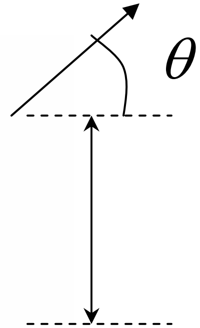
Effect of an Insertion Device on the electron beam of a storage ring

- Perturbation of the lattice functions
  - Affect the synchrotron radiation integrals:
    - Increase energy spread, length the bunch
    - Reduce emittance
  - The effect is in general weak with the exception of the damping wigglers.

- Deflection, linear and non linear focusing from:
  - Residual field errors
  - Nominal field

Closed Orbit Distortion from Residual Field Integrals

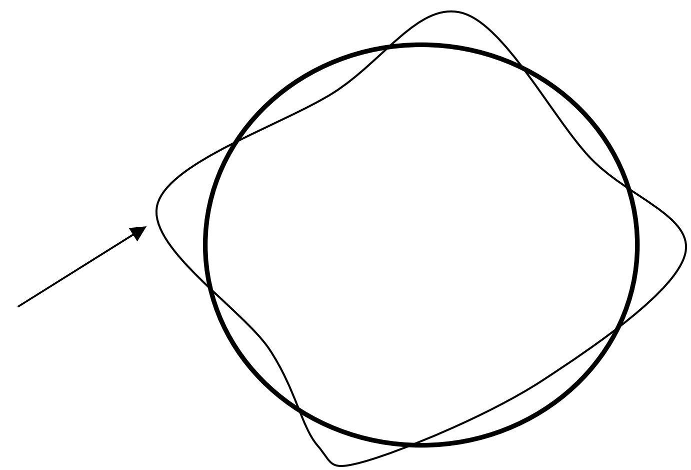




Angle induced
by field integral

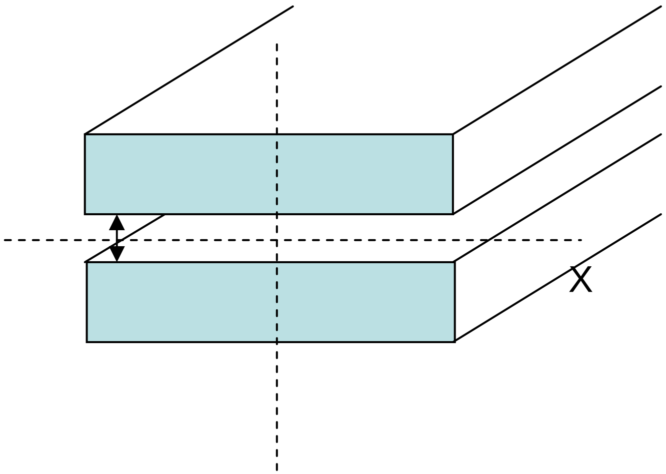
Displacement induced
By double field integral

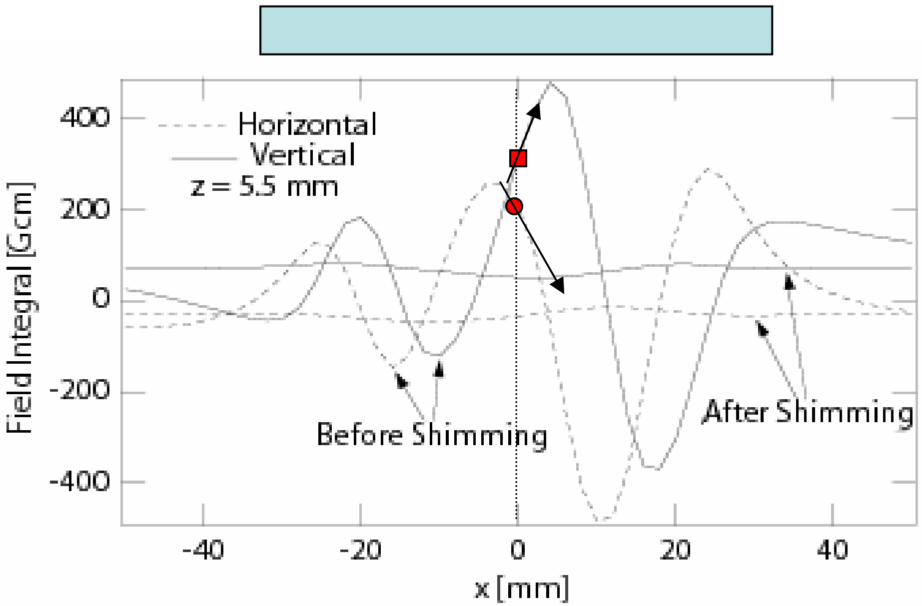
$$\delta x(s) = \theta \sqrt{\beta_x(s)\beta_x^{ID}} \frac{\cos(\pi \upsilon_x - \left|\phi_x(s) - \phi_x^{ID}\right|)}{2\sin(\pi \upsilon_x)}$$



$$\Rightarrow \quad \left|\int_{ID} B_x(s)ds\right|, \left|\int_{ID} B_z(s)ds\right| < 10 - 30\, Gcm$$

Magnetic Field Errors





$$\theta_x = \frac{e}{\gamma mc}\int_{ID} B_z(s)ds + o(\frac{1}{\gamma})$$

$$\theta_z = \frac{-e}{\gamma mc}\int_{ID} B_x(s)ds + o(\frac{1}{\gamma})$$

$$\frac{1}{F_x} = \frac{\partial\theta_x}{\partial x} = \frac{e}{\gamma mc}\frac{\partial}{\partial x}\int_{ID} B_z(s)ds$$

$$\frac{1}{F_z} = \frac{\partial\theta_z}{\partial z} = -\frac{e}{\gamma mc}\frac{\partial}{\partial z}\int_{ID} B_x(s)ds = -\frac{1}{F_x}$$

$$\frac{1}{F_c} = \frac{\partial\theta_x}{\partial z} = \frac{e}{\gamma mc}\frac{\partial}{\partial z}\int_{ID} B_z(s)ds$$

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Focusing Effect

An Insertion Device which presents a local focusing and introduces a tune shift
And beta beat which are dependent on the field of the Insertion Device

$$\delta\upsilon_x = \frac{1}{4\pi}\int_{ID} \beta_x K_x ds \simeq \frac{1}{4\pi}\frac{\overline{\beta}_x^{ID}}{F_x}$$

$$\delta\upsilon_z = \frac{1}{4\pi}\int_{ID} \beta_z K_z ds \simeq \frac{1}{4\pi}\frac{\overline{\beta}_z^{ID}}{F_z}$$

$$\delta\upsilon_c = \frac{1}{4\pi}\frac{\sqrt{\overline{\beta}_x^{ID}\overline{\beta}_x^{ID}}}{F_c}$$

and a Beta Beat :

$$\frac{\Delta\beta_x}{\beta_x} = \frac{2\pi\delta\upsilon_x}{\sin(2\pi\upsilon_x)} \quad , \quad \frac{\Delta\beta_z}{\beta_z} = \frac{2\pi\delta\upsilon_z}{\sin(2\pi\upsilon_z)}$$

$\Rightarrow$ Beam size variation along circumference

$\Rightarrow$ Stop band around half intereger resonances

=> Avoid large Beta functions in Insertion Devices

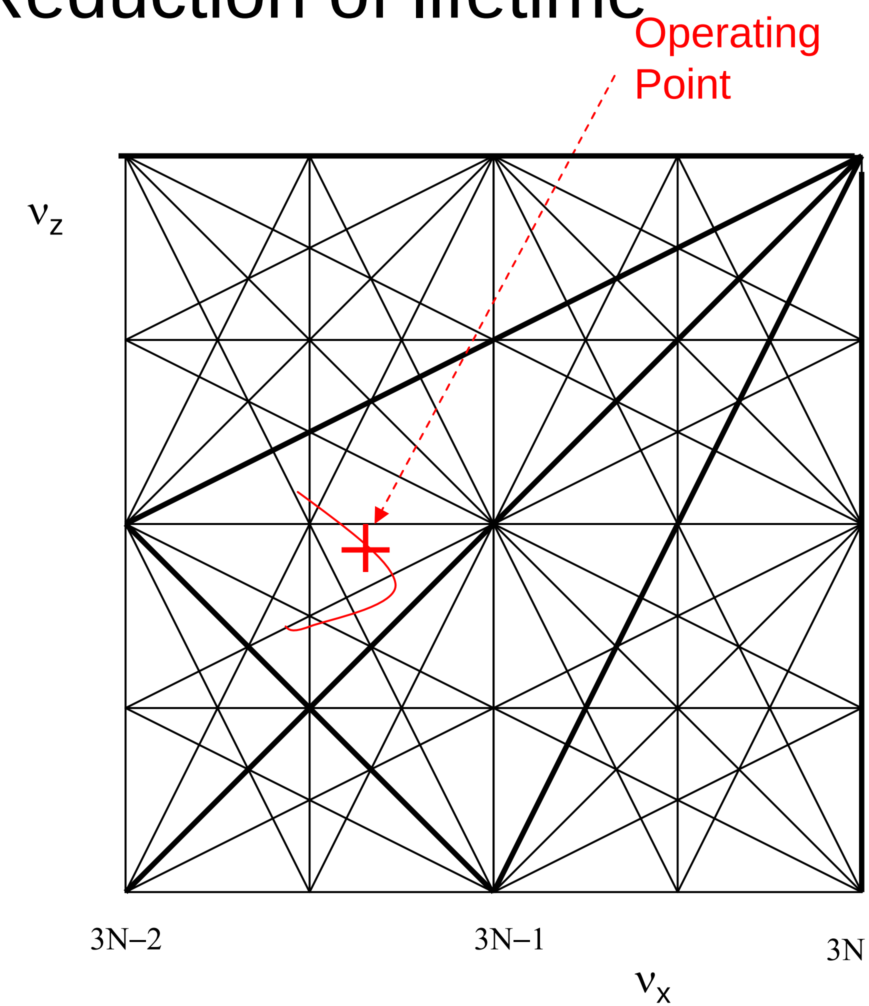
Reduction of Dynamic Aperture induced by Insertion Devices => Reduction of lifetime

- The linear focusing of an Insertion Devcie change the betatron function all over the circumference => break the N symmetry => excites non systematic resonances (normally very weak) which generate beam losses => important to locally correct the focusing and restore the beta functions

- The non linear focusing excites the non systematic resonances and may create additional losses.

- Both the nominal field and field errors can be responsible

- The effect can be serious on low energy rings with many Insertion Devices.



Systematic Resonances : $m\nu_x+p\nu_z= Nq$

Non Systematic Resonances : $m\nu_x+p\nu_z= q$

m.n,q : integers

$with$

$$K_x = \frac{e}{\gamma mc}\frac{\partial B_z(s)}{\partial x} + K_x^{2nd\ \ order} + o(\frac{1}{\gamma^2})$$

$$K_z = -\frac{e}{\gamma mc}\frac{\partial B_z(s)}{\partial x} + K_z^{2nd\ \ order} + o(\frac{1}{\gamma^2})$$

$$K_c = \frac{e}{\gamma mc}\frac{\partial B_x(s)}{\partial x} + K_c^{2nd\ \ order} + o(\frac{1}{\gamma^2})$$

Simple Theory of 2nd Order Undulator Focusing Applied to Planar Undulator

- A planar undulator presents a 2D magnetic field which in free space can be derived from a scalar potential satisfying : $\Delta\varphi(z,s)=0$

- A solution is : $\varphi(z,s)=\dfrac{B_0\lambda_0}{2\pi}\sinh(2\pi\dfrac{z}{\lambda_0})\cos(2\pi\dfrac{s}{\lambda_0})$

$$\boxed{\begin{aligned} B_z &= \frac{\partial\varphi(z,s)}{\partial z} = B_0\cosh(2\pi\frac{z}{\lambda_0})\cos(2\pi\frac{s}{\lambda_0}) \\ B_s &= \frac{\partial\varphi(z,s)}{\partial s} = -B_0\sinh(2\pi\frac{z}{\lambda_0})\sin(2\pi\frac{s}{\lambda_0}) \end{aligned}}$$

- The Lorentz Force equation in such a field remains to be solved

$$\gamma m\frac{d\vec{v}}{dt} = e\vec{v}\times\vec{B}$$

$$\frac{dv_x}{ds} = \frac{dt}{ds}\frac{e}{\gamma m}(v_z B_s - v_s B_z)$$

$$\frac{dv_z}{ds} = \frac{dt}{ds}\frac{e}{\gamma m}(v_s B_x - v_x B_s)$$

$$\frac{dv_s}{ds} = \frac{dt}{ds}\frac{e}{\gamma m}(v_x B_z - v_z B_x)$$

with

$$B_x = 0$$

$$B_z = B_0 \cosh(2\pi \frac{z}{\lambda_0}) \cos(2\pi \frac{s}{\lambda_0})$$

$$B_s = -B_0 \sinh(2\pi \frac{z}{\lambda_0}) \sin(2\pi \frac{s}{\lambda_0})$$

- 0 order in $1/\gamma$

$$\vec{v} = [0, 0, c]$$
$$s = ct$$

- 1st order in $1/\gamma$

$$\vec{v} = \left[\frac{-e}{\gamma m} B_0 \frac{\lambda_0}{2\pi} \cosh(2\pi \frac{z}{\lambda_0}) \sin(2\pi \frac{s}{\lambda_0}), 0, c \right]$$
$$s = ct$$

- 2nd order in $1/\gamma$

$$\frac{dv_x}{ds} = -\frac{e}{\gamma mc} v_s B_z = \frac{-e}{\gamma m} B_0 \cosh(2\pi \frac{z}{\lambda_0}) \cos(2\pi \frac{s}{\lambda_0})$$

$$\frac{dv_z}{ds} = -\frac{e}{\gamma mc} v_x B_s = -c\left(\frac{eB_0}{\gamma mc}\right)^2 \frac{\lambda_0}{2\pi} \cosh(2\pi \frac{z}{\lambda_0}) \sinh(2\pi \frac{z}{\lambda_0}) \sin^2(2\pi \frac{s}{\lambda_0})$$

$$\frac{dv_s}{ds} = \frac{e}{\gamma mc} v_x B_z = -c\left(\frac{eB_0}{\gamma mc}\right)^2 \frac{\lambda_0}{2\pi} \cosh^2(2\pi \frac{z}{\lambda_0}) \cos(2\pi \frac{s}{\lambda_0}) \sin(2\pi \frac{s}{\lambda_0})$$

- averaging over one period

$$\frac{dv_x}{ds} = 0$$

$$\frac{dv_z}{ds} = -\frac{c}{2}\left(\frac{eB_0}{\gamma mc}\right)^2 \frac{\lambda_0}{4\pi}\sinh(4\pi\frac{z}{\lambda_0})$$

$$\frac{dv_s}{ds} = 0$$

- or

$$\frac{d^2z}{ds^2} = \frac{d}{ds}(\frac{dz}{dt}\frac{dt}{ds}) = \frac{d}{ds}(\frac{v_z}{c}) \simeq -\frac{1}{2}\left(\frac{eB_0}{\gamma mc}\right)^2 (z + \frac{16\pi^2}{6\lambda_0^2}z^3 + ...) \cong -\frac{1}{2}\left(\frac{eB_0}{\gamma mc}\right)^2 z \quad \text{if} \quad z \ll \frac{\lambda_0}{4\pi}$$

Planar Undulators are vertically focusing with a focal length :

$$K_z = \frac{1}{2}\left(\frac{eB_0}{\gamma mc}\right)^2 \quad , \qquad \frac{1}{F_z} = \int_{ID} K_z ds = \frac{1}{2}\left(\frac{eB_0}{\gamma mc}\right)^2 L = \frac{e^2}{\gamma^2 m^2 c^2}\int B_z^2 ds$$

General Theory of 2nd Order Focusing

Start from the Lorentz Force Equation of motion of an electron in an arbitrary magnetic field expressed in a fixed Cartesian frame (Oxzs)

$$\frac{d\vec{v}}{dt} = \frac{e}{\gamma m}\vec{v}\times\vec{B} \quad \Longrightarrow$$

$$x'' = -\frac{e}{\gamma mc}\sqrt{1+x'^2+z'^2}\left[z'B_s-(1+x'^2)B_z+x'z'B_x\right]$$

$$z'' = \frac{e}{\gamma mc}\sqrt{1+x'^2+z'^2}\left[x'B_s-(1+z'^2)B_x+x'z'B_z\right]$$

$$with$$

$$y' = \frac{dy}{ds} \quad , \quad y'' = \frac{d^2y}{ds^2}$$

Solve these equations in power series of $1/\gamma$ making use of the Maxwell Equation :

$$\vec{\nabla}\vec{B} = 0 \quad , \quad \vec{\nabla}\times\vec{B} = 0$$

Insertion Device extends from s = 0 to L :

Field Errors Nominal Field

$$\frac{dx}{ds}(L) = \frac{dx}{ds}(0) + \boxed{\frac{e}{\gamma mc}\int_0^L B_z ds} \boxed{-\frac{1}{2}\left(\frac{e}{\gamma mc}\right)^2 \frac{\partial \Phi}{\partial x}} + o\left(\frac{1}{\gamma^2}\right)$$

$$\frac{dz}{ds}(L) = \frac{dz}{ds}(0) - \boxed{\frac{e}{\gamma mc}\int_0^L B_x ds} \boxed{-\frac{1}{2}\left(\frac{e}{\gamma mc}\right)^2 \frac{\partial \Phi}{\partial z}} + o\left(\frac{1}{\gamma^2}\right)$$

$$\text{with} \quad \Phi(x,z) = \int_0^L \left(\left(\int_0^s B_x(x,z,s')ds' \right)^2 + \left(\int_0^s B_z(x,z,s')ds' \right)^2 \right) ds$$

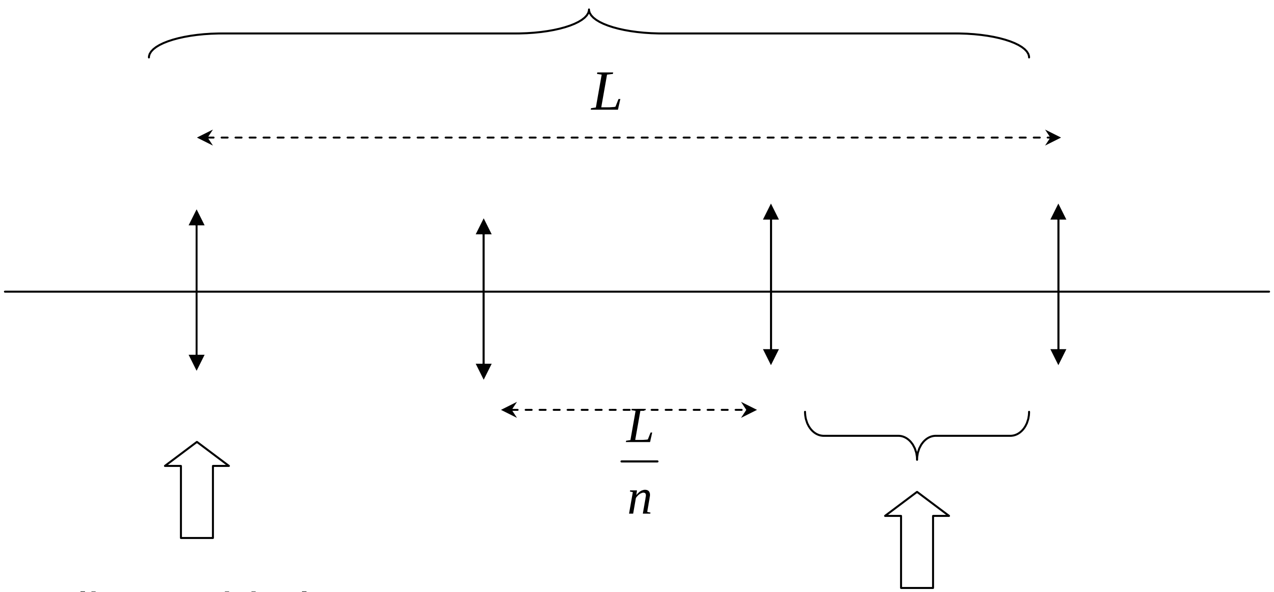
For a periodic field with period λ_0 :

$$\Phi(x,z) = N \int_0^{\lambda_0} \left(\left(\int_0^s B_x(x,z,s')ds' \right)^2 + \left(\int_0^s B_z(x,z,s')ds' \right)^2 \right) ds$$

The detailed deflection, focusing and non-linear focusing can be predicted From the function Φ(x,z) computed from the transverse field

Tracking of e- beam in an Insertion Device

Split the Undulator into n thin Lenses separated by drift spaces



Non linear thin lens

$$\Delta x'(x,z) = -\frac{1}{2n}\left(\frac{e}{\gamma mc}\right)^2 \frac{\partial}{\partial x}\Phi(x,z)$$

$$\Delta z'(x,z) = -\frac{1}{2n}\left(\frac{e}{\gamma mc}\right)^2 \frac{\partial}{\partial z}\Phi(x,z)$$

Drift Space

$$\Delta x = x + x'\frac{L}{n}$$

$$\Delta z = z + z'\frac{L}{n}$$

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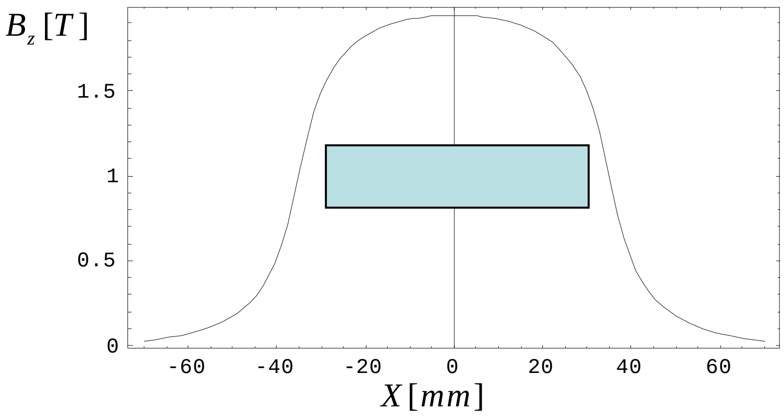
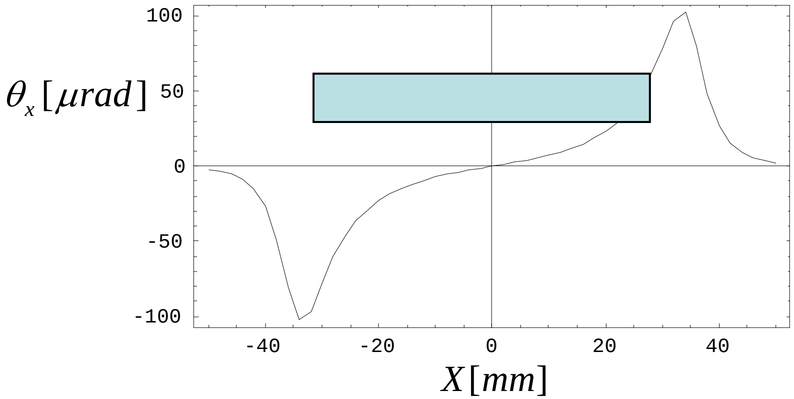
2nd Order Focusing from High Field Wigglers

Technology	PPM
Period [mm]	100
Gap [mm]	10
Length [m]	2.2
Peak Field [T]	2
Magnet Width [mm]	60
Energy [GeV]	6
Vertical Beta [m]	4

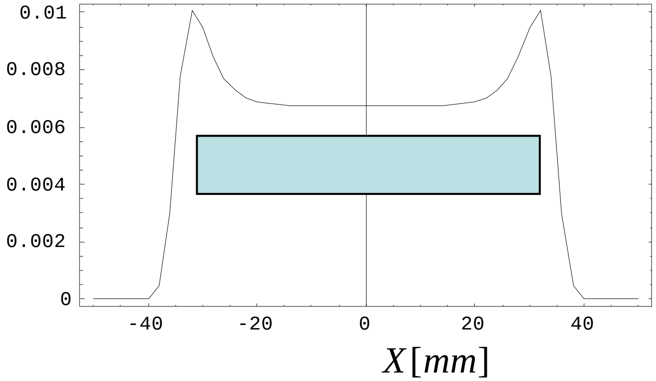
Vertical Field under Pole



Horizontal Deflection [micro-rad] vs Horizontal Position



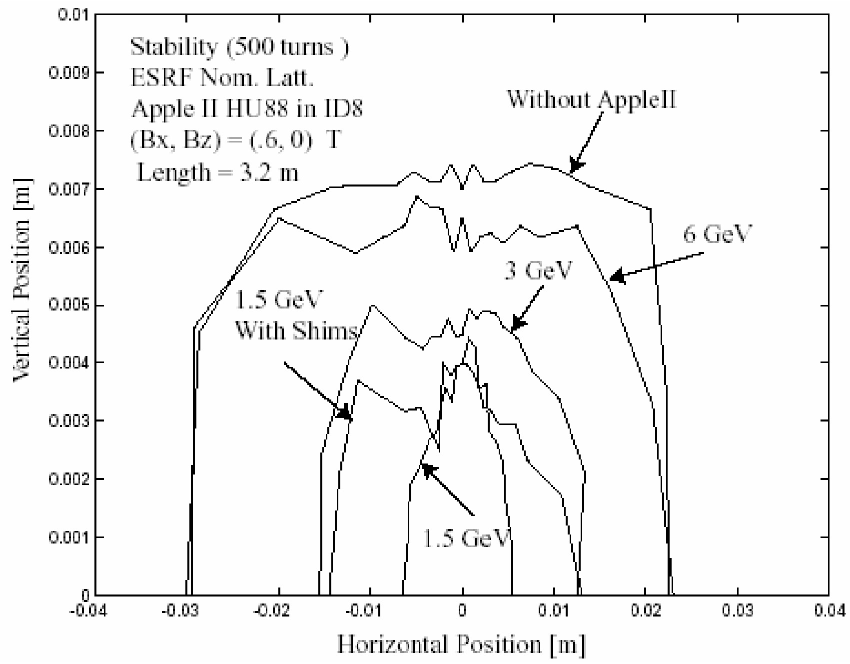
Vertical Tune Shift vs Horizontal Position

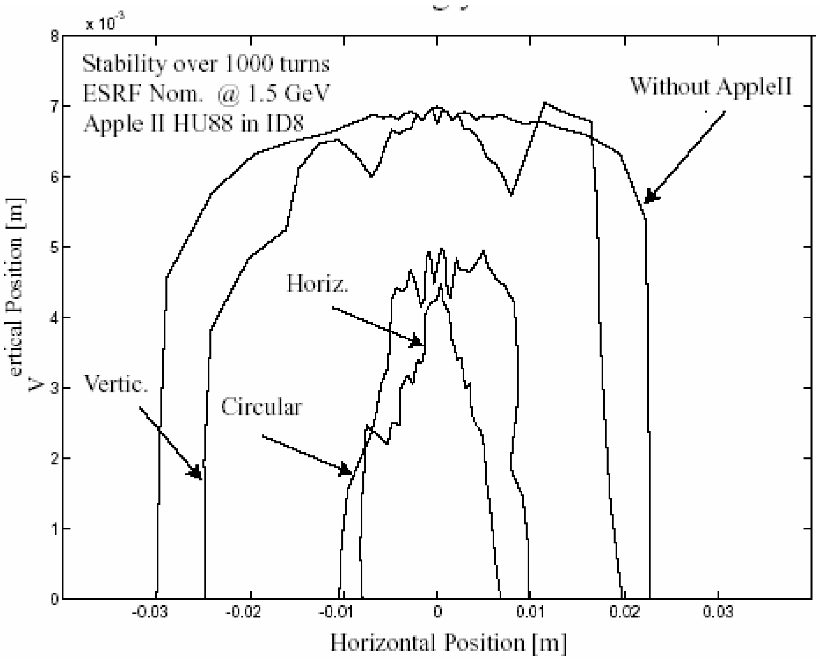


In extreme cases (high field Wiggler, Narrow Pole, Low Energy), one may not be able to inject in a Wiggler because of the horizontal non linearity . Example : SPEAR BL11 Wiggler

Reduction of Dynamic Aperture from Apple II

Period = 88 mm
Gap = 16 mm
Length = 3.2 m
Betax,z = 35, 2.5 m





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Conclusion

- Insertion devices may be the source of perturbations :
  - Closed Orbit distortion
  - Tune shift
  - Lifetime reduction

- The problem is most severe on low energy rings with many insertion devices.

- Nowadays the technique of field shimming allows to get rid of most of the perturbations induced by the residual field errors.

- For high field devices or complicated field geometries (Apple II), one may need local correctors.