

# Beam Quality and Lifetime

Introduction

Brilliance

Machine Stability

Apertures and Acceptance

Lifetime

Beam Instabilities

## User Requirements

**Stable, long lived and tuneable brilliant light.**

These are conflicting requirements.

High brilliance means small photon spot sizes and reduced costs and tolerances on optical elements.

Long lifetime means constant heat-load on optical elements giving stability and improved quality of experiments.

Lifetime is a key issue since high brilliance leads to lower lifetimes. Brilliance is often compromised for increased lifetime in VUV and soft X-ray machines. This is less of a problem for high energy machines.

Third generation synchrotron radiation facilities are characterized by the use of insertion devices as their main source of photons.

These devices affect the dynamic aperture and impose small physical apertures, both of which affect the lifetime.

They also need stability of the electron beam to be fully exploited.

**Requires low emittance** - Conflicts with long lifetime  
Increased Touschek effect & Intra-Beam Scattering

**Low Emittance needs strong focusing quadrupoles**

Worsens the effects of machine misalignments (coupling, dispersion, COD's)  
Large chromatic effects and small dispersion which need strong sextupoles  
Tune dependence on amplitude  
Smaller momentum acceptance  
Reduction in dynamic aperture

**Increase Brilliance by using smaller insertion device gaps and longer devices**

Reduction in gas scattering lifetime  
Reduction in dynamic aperture and increased optics distortions  
Increase in resistive wall effect

**Increase current**

Easily reach thresholds for instabilities  
Increased out-gassing  
Reduced lifetime (Touschek and Gas scattering)  
Frequent refills

**Good closed orbit**

Needs good stability

## Long lifetime conflicts with Brilliance

### Reduce the Touschek effect and Bremsstrahlung scattering

Large momentum acceptance => Adequate RF and off momentum dynamic aperture

### Reduce Elastic scattering off residual gas

Good vacuum

Design vacuum chamber to reduce “out-gassing”

### Have a large dynamic aperture

Careful construction of linear elements

Reduce the non-linear effects of ID's =>

- Reduce construction errors

- Increase transverse width of poles

- Appropriate lattice functions

Use many families of sextupoles

### Eliminate ion/dust trapping

Good Vacuum, Careful design of ID vacuum chambers

### Combat Multi-bunch Instabilities

### Long lifetime also means increased component reliability

## Mechanical Stability

Appropriate Choice of Site.

Uncouple storage ring from experimental area.

Place magnets on supports which do not amplify ground vibrations.

## Thermal stability

Longer lifetime lessens heat load effects.

Temperature regulation of ring environment.

Temperature regulation of service area.

## Magnetic Stability

Full energy injection.

Tight tolerances on power supplies, both short and long term.

## Reduce closed orbit distortions when changing ID gaps

Good magnetic field performance of ID's

## Eliminate beam instabilities

Careful design of vacuum chamber and RF cavities

## Eliminate ion trapping

Good vacuum

or Use positrons (costly)

## Use Feedback systems

Transverse Global and Local

Longitudinal

# Brilliance and Flux

The brilliance is the phase space density of photon flux evaluated in the forward direction and at the centre of the source:

$$B = \left. \frac{d^4 F}{d\theta d\phi dx dy} \right|_0 \quad F = \frac{dN}{dt}$$

Integration in either angle or space gives the angular and spatial flux densities:

$$\frac{d^2 F}{d\theta d\phi} = \int \frac{d^4 F}{d\theta d\phi dx dy} \cdot dx dy \quad \text{Intensity/Brightness}$$

$$\frac{d^2 F}{dx dy} = \int \frac{d^4 F}{d\theta d\phi dx dy} d\theta d\phi$$

If we are interested in a small energy range about a given photon energy, then dividing by  $d\omega/\omega$  we obtain the **spectral brilliance** and **spectral flux**.

$$B = \frac{\text{Number of photons}}{[\text{sec}] [mm^2] [mrad^2] [0.1\% \text{ bandwidth}]}$$

$$F = \frac{\text{Number of photons}}{[\text{sec}] [0.1\% \text{ bandwidth}]}$$

A systematic approach to calculate the brilliance is to compute the intensity of the electric field of a single electron and convolute it with the probability distribution function of the electrons:

$$B = \int B_{e^-} \cdot \eta \, dV$$

Brilliance of a single  $e^-$ 
Probability distribution function of  $e^-$

Consider the photons to be particles rather than waves with a Gaussian distribution in spatial and angular coordinates, then:

$$B = \frac{1}{d\omega/\omega} \frac{F}{4\pi^2 \Sigma_x \Sigma_{x'} \Sigma_y \Sigma_{y'}}$$

↑ phase space volume of the photons
← Flux in the forward direction

$\Sigma$  are the spatial and angular standard deviations of the photons in the two planes.

$\Sigma$  are the convolution of the electron beam distribution  $\sigma_{xyx'y'}$  and that of the photon distribution from a single electron  $\sigma_{rr'}$

$$\Sigma_{xy} = \left( \sigma_{x,y}^2 + \sigma_r^2 \right)^{1/2} \quad \Sigma_{x'y'} = \left( \sigma_{x',y'}^2 + \sigma_{r'}^2 \right)^{1/2}$$



The brilliance is an invariant and can be considered as an emittance.  
It obeys Liouville's theorem and is conserved by linear transformations.  
Optical elements in the beam line propagate the brilliance from one point to another:

$$B_1 = \mathbf{M} B_0 \quad \mathbf{M} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix}$$

Emittance is area in phase space - the product of divergence and size

Objective: a small photon spot size on sample

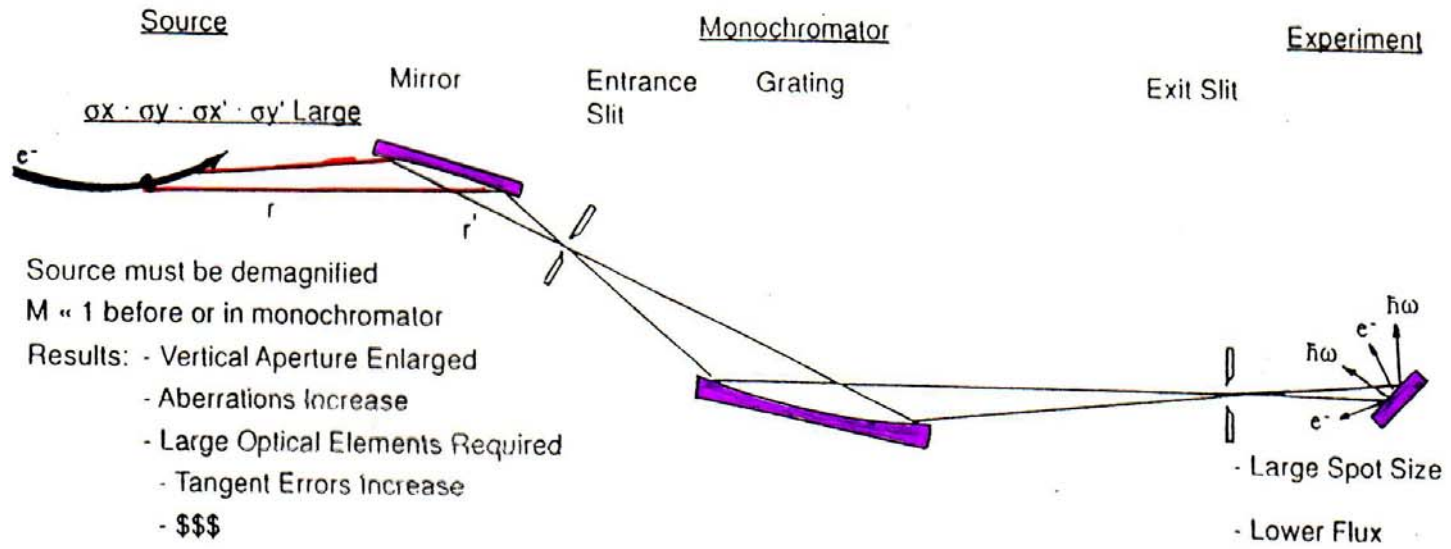
Large  $\varepsilon$   $\rightarrow$  Large photon beam divergence  $\rightarrow$  large optics  
(costs, aberrations, errors, etc..)

# The practical meaning of Brilliance

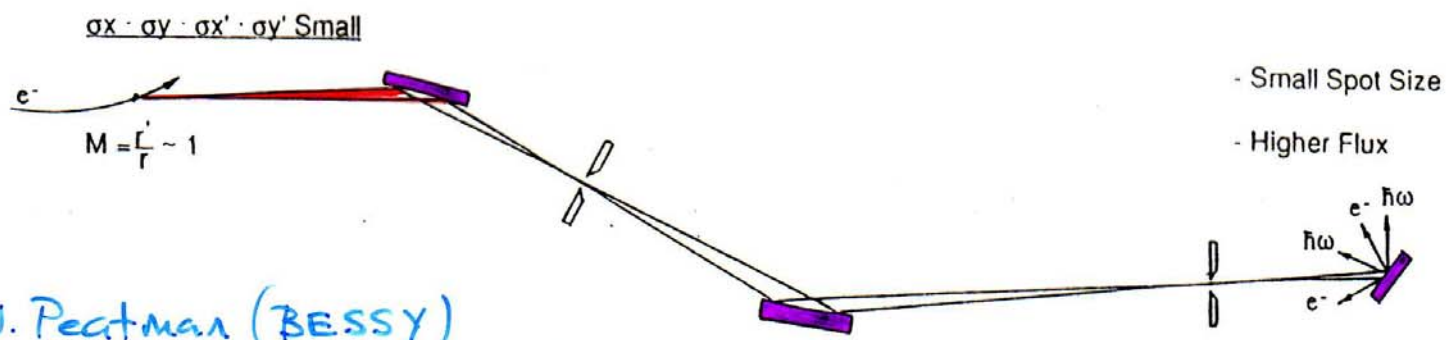


Figure 1.1.1: The Practical Meaning of Brilliance

low  $B$   
large  
 $\sigma_{xy}, \sigma_{x'y'}$   
Big  $E$



High  $B$   
Small  
 $\sigma_{xy}, \sigma_{x'y'}$   
Small  $E$



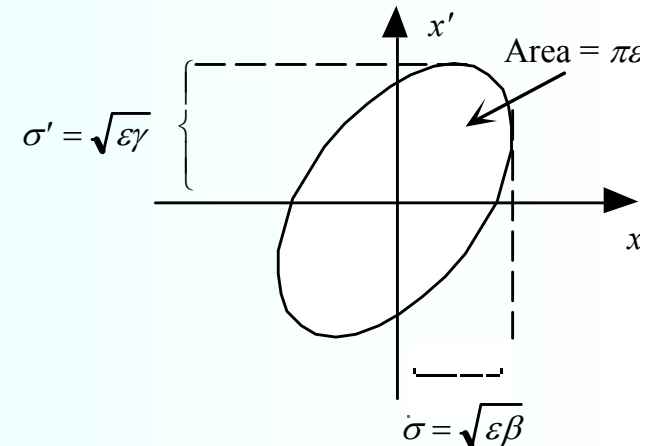
W. Peatman (BESSY)  
Talk given at ICTP (1995)

# Source Sizes - Electrons

The electron beam sizes and divergences are given by the beam emittance and the Courant-Snyder parameters (properties of the lattice).

$$\sigma_x = \sqrt{\beta_x \varepsilon_x} \quad \sigma_{x'} = \sqrt{\gamma_x \varepsilon_x} \quad \gamma_x = \frac{1 + \alpha^2}{\beta}$$

The natural electron beam emittance is determined by radiation damping, quantum emission and intra-beam scattering. Beam distribution is Gaussian in the six dimensions.



Storage rings are flat and dispersion exists in the horizontal plane.

$$\sigma_x = \sqrt{\beta_x \varepsilon_x + D^2 \left( \frac{\Delta p}{p} \right)^2} \quad \sigma_{x'} = \sqrt{\gamma_x \varepsilon_x + D'^2 \left( \frac{\Delta p}{p} \right)^2}$$

i.e., beam size is a convolution,

$$\sigma_x^2 = \sigma_\beta^2 + \sigma_{\Delta p/p}^2$$

# Source Sizes - Electrons



The vertical emittance for a lattice with no errors or coupling is  $\sim 10^{-4}$  nmrad.  
A real lattice has errors that couples the horizontal and vertical emittances.

Errors: quadrupole roll, vertical closed orbit displacements in sextupoles.

Define an emittance ratio  $g$ :  $g = \frac{\varepsilon_y}{\varepsilon_x}$  with  $\varepsilon_0 = \varepsilon_x + \varepsilon_y$

$$\Rightarrow \varepsilon_y = \frac{g}{1+g} \varepsilon_0 \quad \text{and} \quad \varepsilon_x = \frac{1}{1+g} \varepsilon_0$$

$\varepsilon_0$  is the emittance in the absence of coupling.

$g$  is typically 0.1 to 3% in 3rd generation light sources

Vertical beam size and divergence:  $\sigma_y = \sqrt{g\varepsilon_x\beta_y}$ ,  $\sigma_{y'} = \sqrt{g\varepsilon_x\gamma_y}$

Beamlines use the vertical plane as the dispersive plane - monochromators have better resolution. Important to conserve vertical emittance.

The size and divergence of the photon beam from a single electron satisfies the condition:

$$\sigma_r \sigma_{r'} = \frac{\lambda}{4\pi}$$

Where  $\lambda$  is the radiation wavelength.

The product is the diffraction limited emittance.

For a **bending magnet** the vertical opening angle is:

$$\sigma_{r'} = \frac{560}{\gamma} \left( \frac{\lambda}{\lambda_c} \right)^{0.43} \quad [\text{mrad}] \quad \text{For } 0.2\lambda_c < \lambda < 100\lambda_c$$

Since  $\Sigma_y = \sqrt{\sigma_y^2 + \sigma_r^2}$ ,

If  $\sigma_r > \sigma_y$  a there is no increase in brilliance from a reduction in electron beam emittance.

Plane **undulator** emits radiation at harmonic (k) frequencies:

$$\omega_k = \frac{\omega_0 2k\gamma^2}{\left(1 + K^2/2 + \gamma^2\theta^2\right)}$$

$$\omega_0 = \frac{2\pi c}{\lambda_0}$$

$\lambda_0$	length of undulator period
$\gamma$	relativistic factor $E/m_0c^2$
$K$	magnetic field strength parameter $0.934 B[\text{T}]\lambda_0[\text{cm}]$
$\theta$	observation angle with respect to the closed orbit

The intensity of the light at the end of the undulator is given by the interference of the beamlets from each deflection point.

The intensity has a sinc function dependence.

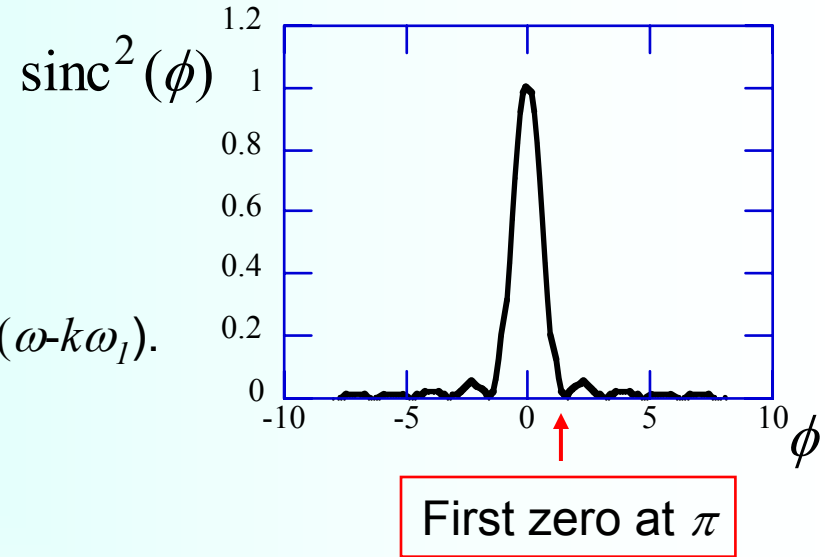
$$I \propto \text{sinc}^2 \left( N \pi \frac{\Delta \omega}{\omega_1(\theta)} \right)$$

$N$  is the number of periods

$\omega_1$  is the frequency of the first harmonic,  $\Delta \omega = (\omega - k \omega_1)$ .

Writing,

$$\frac{\omega}{\omega_1(\theta)} = \frac{\omega}{\omega_1(0)} \left( 1 + \frac{\gamma^2 \theta^2}{1 + K^2/2} \right)$$



Near the location of the  $k$ 'th harmonic, i.e., at  $\omega/\omega_1(0)=k$ ,  $\text{sinc}^2 \sim$  Gaussian.

$$\text{sinc}^2 \left( \frac{N \pi \omega}{\omega_1(\theta)} \right) \approx \text{sinc}^2 \left( \frac{N \pi k \gamma^2 \theta^2}{1 + K^2/2} \right) \approx \exp \left( \frac{-\theta^2}{2 \sigma_{r'}^2} \right)$$

To determine the opening angle of the radiation, fit  $\text{sinc}^2$  to the Gaussian to have the same integrated intensity.

$$\sigma_{r'} \sim \frac{1}{2\gamma} \sqrt{\frac{1+K^2/2}{kN}} = \sqrt{\frac{\lambda}{2L}} \Big|_{\substack{\text{forward direction} \\ \theta=0}}$$

Where  $L=N\lambda_0$  is the undulator length and  $\lambda$  the radiation wavelength.

The opening angle is about  $(kN)^{1/2}$  smaller than that from a bending magnet  $\theta \sim \gamma^{-1}$ .

The photon source size is.

$$\sigma_r \sigma_{r'} = \frac{\lambda}{4\pi} \Rightarrow \sigma_r = \frac{\sqrt{\lambda L}}{2\sqrt{2}\pi}$$

Taking the limit of  $N \rightarrow \infty$

$$\text{sinc}^2\left(\frac{N\pi\Delta\omega}{\omega_1(\theta)}\right) \rightarrow \frac{1}{N} \delta\left(\frac{\Delta\omega}{\omega_1(\theta)}\right) = \frac{1}{kN} \delta\left(\frac{\Delta\omega}{\omega - \Delta\omega}\right) \sim \frac{1}{kN} \delta\left(\frac{\Delta\omega}{\omega}\right)$$

Gives the natural line width for the  $k$ 'th harmonic,

$$\frac{\Delta\omega}{\omega} \Big|_{\theta=0} \sim \frac{1}{kN}$$



# Source Sizes - Photons

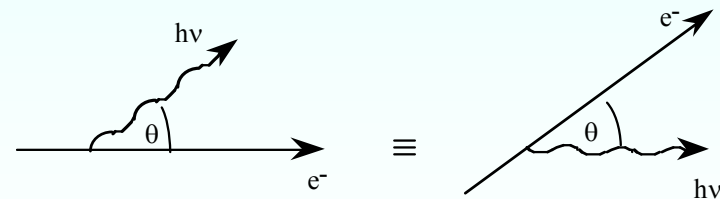
## Dependence on viewing angle and beam energy

$$\left. \frac{\Delta\omega}{\omega} \right|_{\theta \sim 0} \sim \frac{\gamma^2 \Delta\theta^2}{1 + K^2/2} \qquad \left. \frac{\Delta\omega}{\omega} \right|_{\theta \sim 0} = 2 \frac{\Delta\gamma}{\gamma}$$

Full bandwidth of k'th harmonic is convolution of these three effects

$$\left( \frac{\Delta\omega}{\omega} \right)_{\Delta\theta, \Delta\gamma} \sim \left[ \left( \frac{1}{kN} \right)^2 + \left( \frac{\gamma^2 \Delta\theta^2}{1 + K^2/2} \right)^2 + 4 \left( \frac{\Delta\gamma}{\gamma} \right)^2 \right]^{1/2}$$

For fixed  $\gamma$ , to keep linewidth comparable to the natural width  $\Delta\theta \sim \sigma_r$ ,  
However, not possible to distinguish the source of photons in  $\Delta\theta$ ,



$$\left( \frac{\Delta\omega}{\omega} \right)_{\Delta\sigma_{x,y}, \Delta\gamma} \sim \left[ \left( \frac{1}{kN} \right)^2 + \left( \gamma^2 (\sigma_{x'}^2 + \sigma_{y'}^2) \right)^2 + 4 \left( \frac{\Delta\gamma}{\gamma} \right)^2 \right]$$

Affected by closed orbit  
Stability and coupling

Affected by  
multibunch  
instabilities

# Motion and Apertures

The source point is the centre of an insertion device.

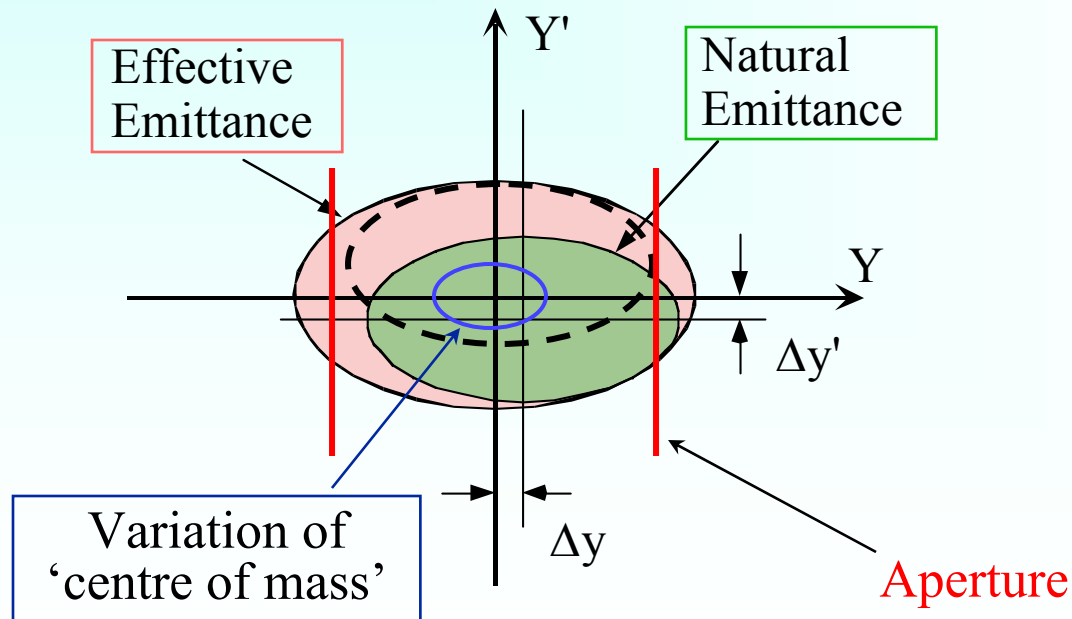
The lattice functions usually have a waist at that point and usually have zero dispersion.

Then  $\alpha=0$ ,  $\gamma=1/\beta$  and

$$\varepsilon_{x,y} = \sigma_{x,y} \sigma_{x',y'}$$

Beam movement  $\Delta y$ ,  $\Delta y'$  produces an effective increased source emittance.

Behaviour of the disturbance at the source is described by an invariant  $\varepsilon_{cm}$ .



A beamline is composed of masks, collimators, mirrors, slits, monochromators. These define an aperture for the photon beam.

Motion with respect to the beamline aperture can result in a reduction of intensity and/or energy resolution.

**Focused** beam: on slits or pinhole matched to the beam size or smaller.

Consider a symmetric 2D Gaussian photon intensity distribution,  $3\sigma \sim 99\%$  of photons.

Fraction of beam in radius  $r = 1 - \exp(-r^2 / 2\sigma^2)$

If aperture is  $\pm 3\sigma$  and beam is miss-steered by  $10\% \sigma$ , reduction in flux is  $0.01\%$

If aperture is  $\pm 2\sigma$  and beam is miss-steered by  $10\% \sigma$ , reduction in flux is  $0.1\%$

If beam size increases by  $10\%$  ( $\sim 20\%$  emittance), reduction in flux is  $1.3\%$  (circular aperture).

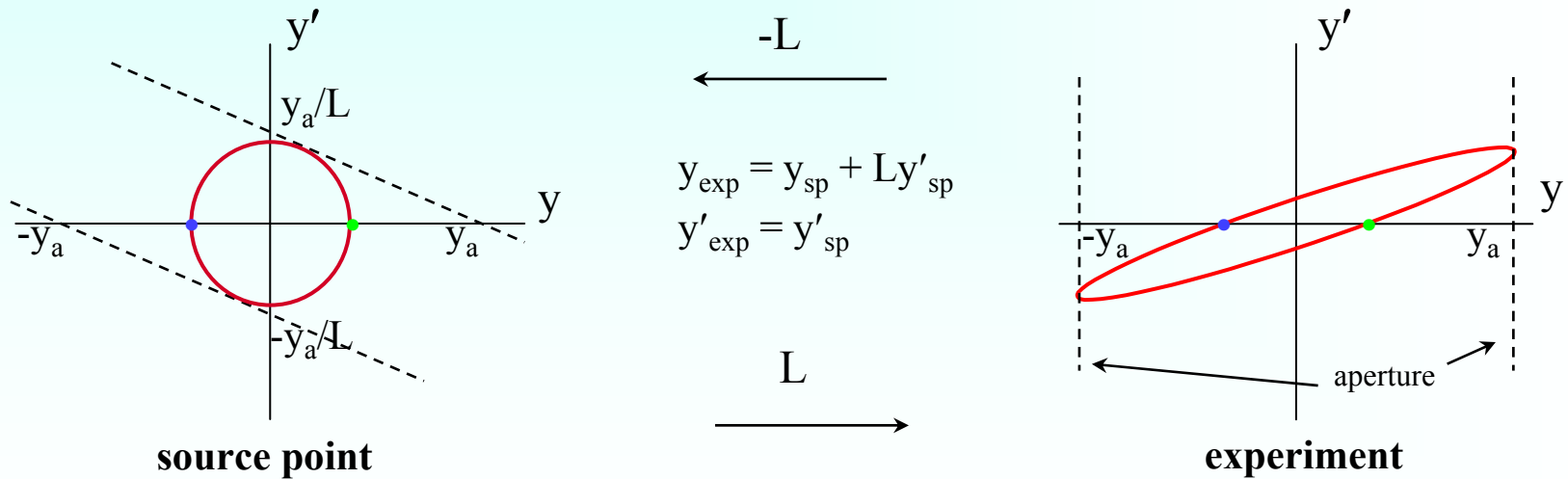
# Motion and Apertures

For an **unfocused** beam the divergence dominates.  
At a position L meters away from the source the beam size is,

$$\Sigma_y^2(L) = \Sigma_y^2(0) + L^2 \Sigma_{y'}^2(0)$$

10% change in angle gives 10% change in beam size.

Changes in angle changes the energy of photons transmitted by a monochromator.



From B. Hettel, SSILS, Shanghai, 2001

- If the time scale of the disturbance is greater than the time to perform an experiment, the result is a simple miss-steering.

This manifests as an energy calibration error and a loss of intensity.

The emittance increase is: 
$$\frac{\Delta\varepsilon}{\varepsilon} = 2\sqrt{\frac{\varepsilon_{cm}}{\varepsilon_0}}, \quad (\varepsilon_{cm} \ll \varepsilon_0)$$

- If the time scale of the disturbance is much faster than the time to perform the experiment, the beam is blurred and there is an effective emittance increase.

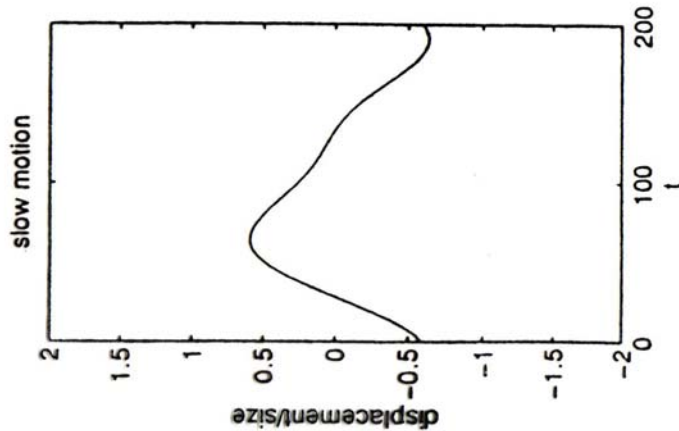
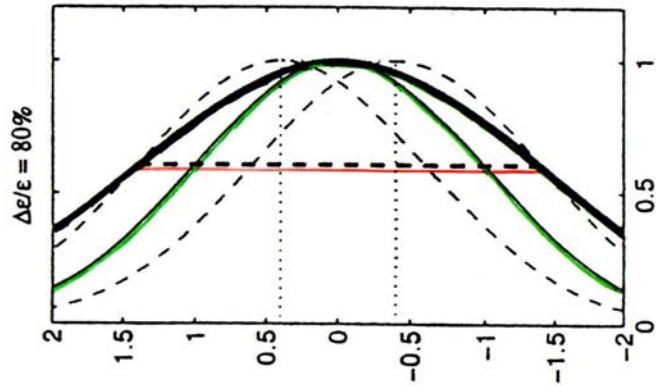
The emittance increase is: 
$$\frac{\Delta\varepsilon}{\varepsilon} = \frac{\varepsilon_{cm}}{\varepsilon_0}, \quad (\varepsilon_{cm} \ll \varepsilon_0)$$

The effect is more severe for slow (> 1 sec) motion: a 20%  $\Delta\varepsilon$  from  $\Delta_y$ ,  $\Delta_y \sim 10\%$   
For fast (< 1 sec) motion a 20%  $\Delta\varepsilon$  occurs for  $\Delta_y$ ,  $\Delta_y$  of  $\sim 45\%$

# Time Scales of Disturbance

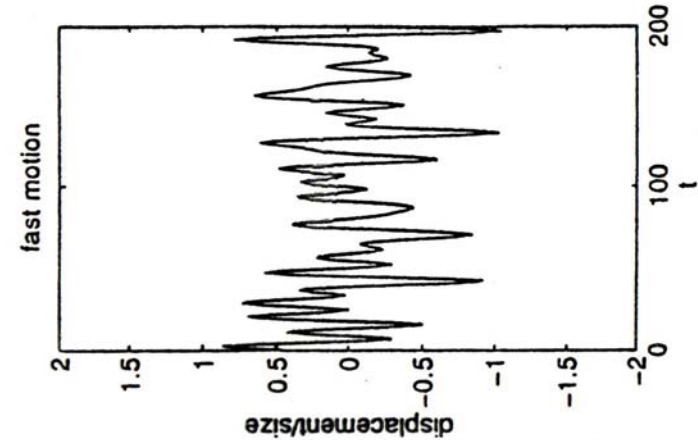
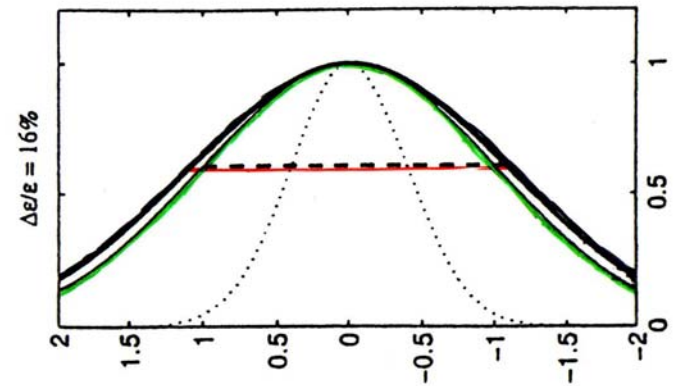
WORSE

Slow Motion



BETTER

Fast Motion



To maximise the brilliance we need to reduce the electron beam emittance.  
However since

$$\Sigma_{xy} = \left( \sigma_{x,y}^2 + \sigma_r^2 \right)^{1/2} \quad \Sigma_{x'y'} = \left( \sigma_{x',y'}^2 + \sigma_{r'}^2 \right)^{1/2}$$

If  $\sigma_r > \sigma_{x,y}$  and  $\sigma_{r'} > \sigma_{x',y'}$ , the brilliance is dominated by diffraction effects. This occurs for long wavelengths.

If  $\sigma_r < \sigma_{x,y}$  and  $\sigma_{r'} < \sigma_{x',y'}$ , the brilliance is dominated by the electron beam emittance. For most of the operating machines and those planned the brilliance is dominated by the electron beam.

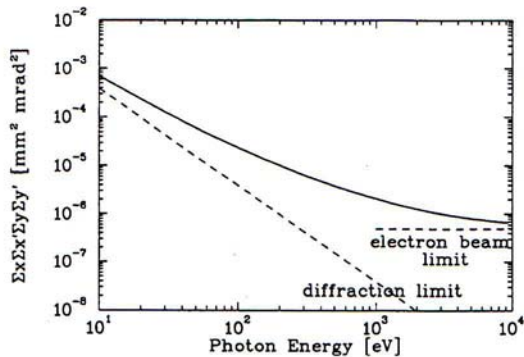


Fig. 13 Product of photon beam emittances in the case of ELETTRA

R.P.Walker, CAS 98-04, 129

For machines of comparable emittance the effect of finite emittance is greater for the higher energy machine since  $\sigma_y$  is independent of energy but  $\sigma_r \sim \gamma^{-1}$ .

Finite emittance and energy spread cause broadening of undulator lines and a reduction in flux at fixed frequency.

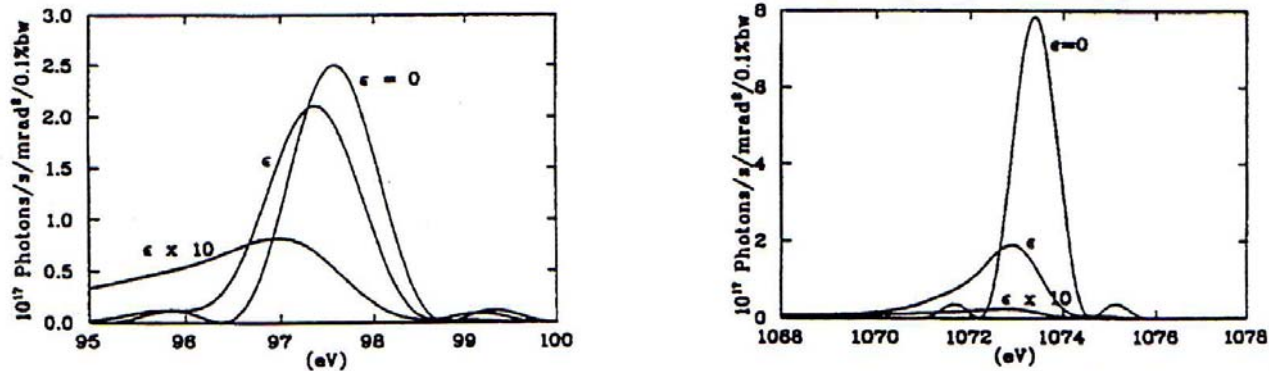


Fig. 17 Effect of electron beam emittance on the 1st (left) and 11th (right) harmonics for an undulator in ELETTRA;  $E = 2$  GeV,  $\lambda_0 = 56$  mm,  $N = 81$ ,  $K = 3.45$

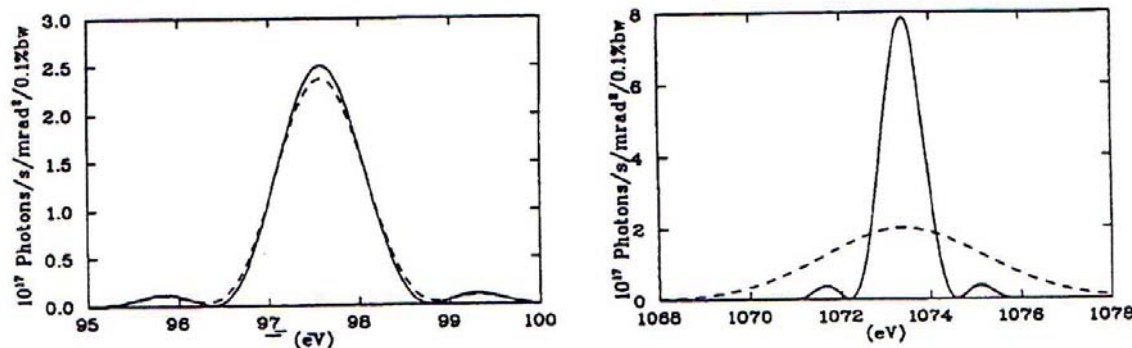


Fig. 18 Effect of electron beam energy spread on the 1st (left) and 11th (right) harmonics for an undulator in ELETTRA; parameters as Fig. 17



The photon beam intensity is proportional to the electron beam current.

Apart from electron beam stability, a loss of intensity at an aperture results from

- The beam lifetime
- Changes in beam size
  - coupling mis-alignments
  - spurious dispersion
  - resonances
    - driven by insertion devices
    - working point close to a coupling resonance
- Instabilities
  - multibunch, transverse and longitudinal
  - ions