

CERN Accelerator School

Bruges , Belgium June 16-25 , 2009

MAGNETS



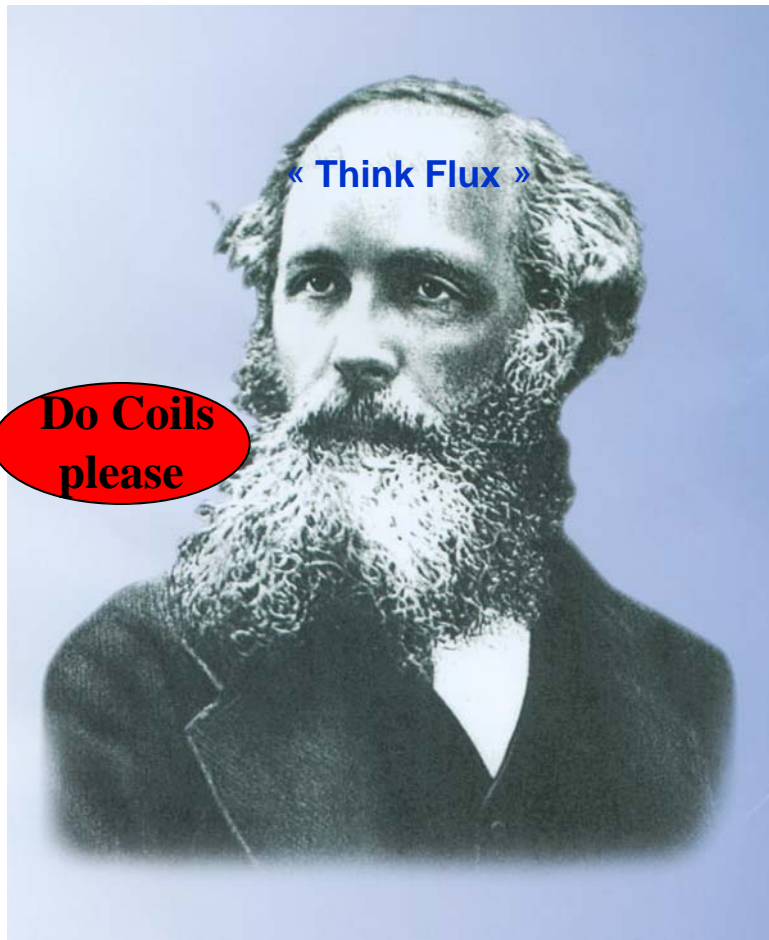
Solenoids

Antoine DAËL, CEA , Saclay, france

A few things that I know about solenoids....

- Flux tubes
- Formulas
- Homogeneity
- Bitter Magnet
- Solenoid lense
- CMS
- Iseult

Maxwell equations : think Flux



James Clerk MAXWELL

- The phenomenons are perfectly known.
- All the day we obey to Maxwell equations to create magnetic field.
- The representation of flux tubes is a very powerful method.
- Flux is going from North to south

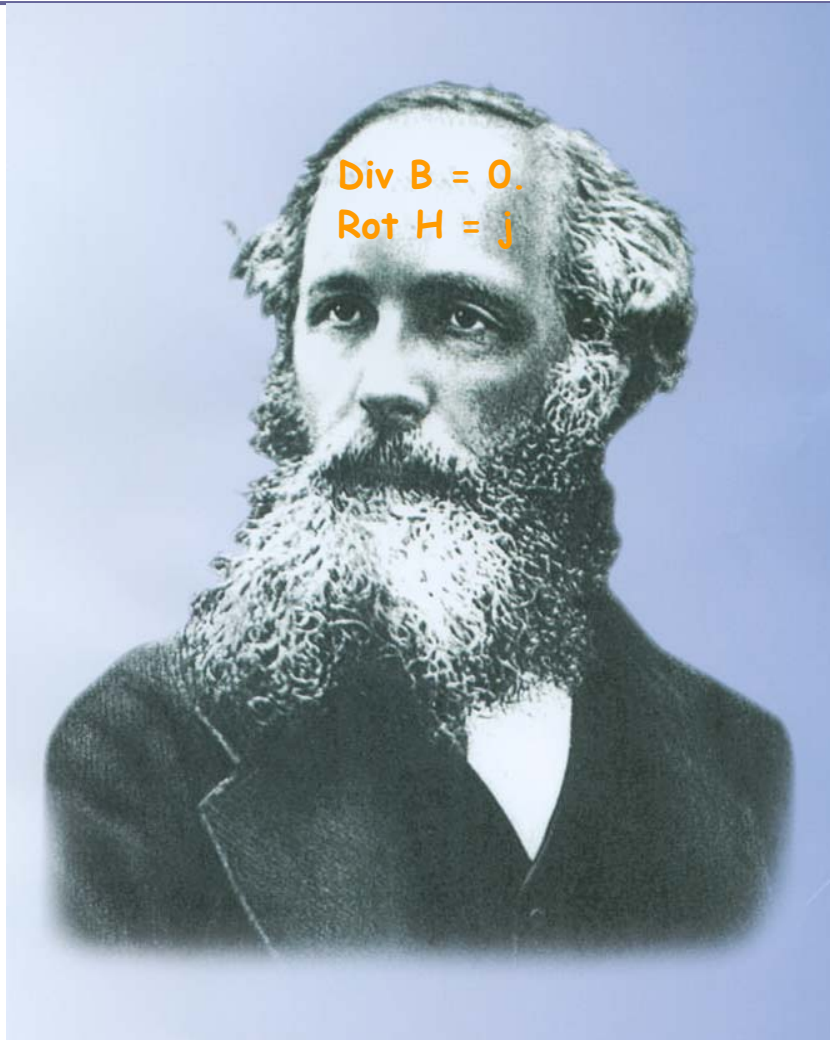
DISCUSSION 3.6: “Double-Pancake” vs. “Layer-Wound”

Of the two magnet winding techniques, one is commonly known as “double-pancake” or simply “pancake,” and the other as “layer-wound.” A double-pancake coil is generally wound with flat conductor, e.g., tape, and sometimes with “large” square- or rectangular-cross-sectioned conductor, e.g., CIC. Each is wound with a



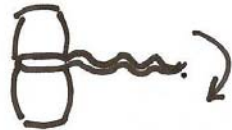
Fig. 3.23 Pictorial view of a double-pancake coil, with the top and bottom pancakes separated axially for clarity. The pancakes in this drawing are wound with a tape conductor. Points A and B indicate the ends of a continuous conductor, with Point C marking the approximate midpoint.

Maxwell's equations for magneto-statics

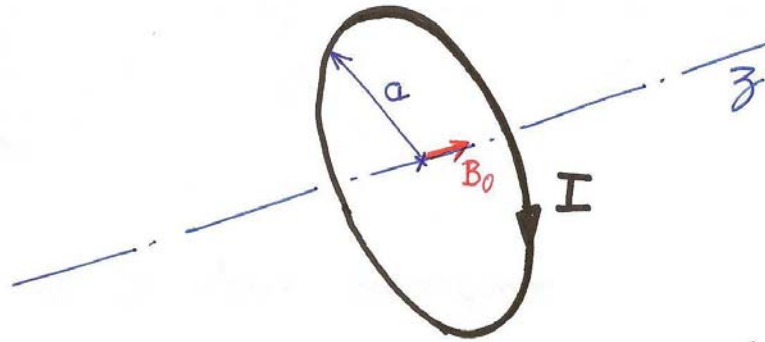


- Maxwell's equations have been solved by different methods.
- The precision request is in the range of **10-6**
- Sophisticated computer codes have been developed on a large scale due for RMN & MRI industry
- These codes need 3D analytical representation of the field and “double precision”

The Ring Coil



Center Field of a ring coil
of radius a



$$B_0 = \mu_0 \frac{I}{2a}$$

$$\mu_0 = 4\pi \cdot 10^{-7}$$

diameter

I	B	$2a$
1A	1mT	1.25mm
200A	0.8mT	300mm

Ring Coil

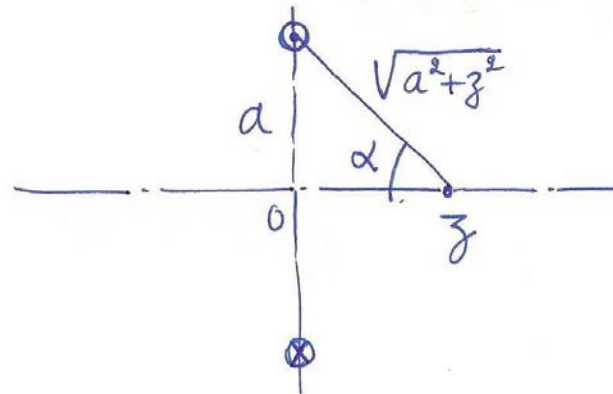
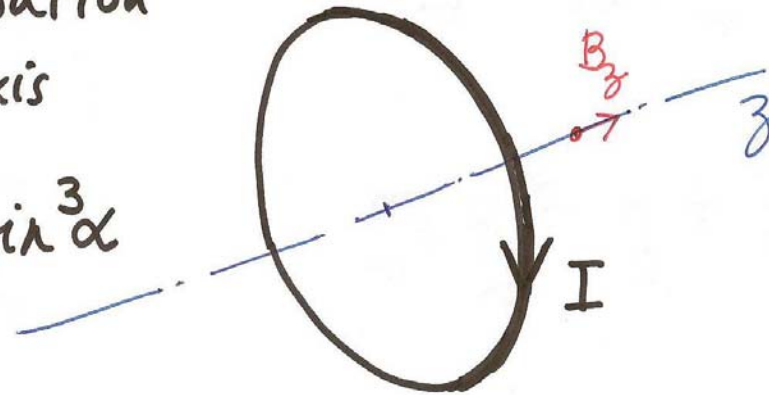
Field Distribution
along the axis

$$B_z(z) = B_0 \sin^3 \alpha$$

$$z = a$$

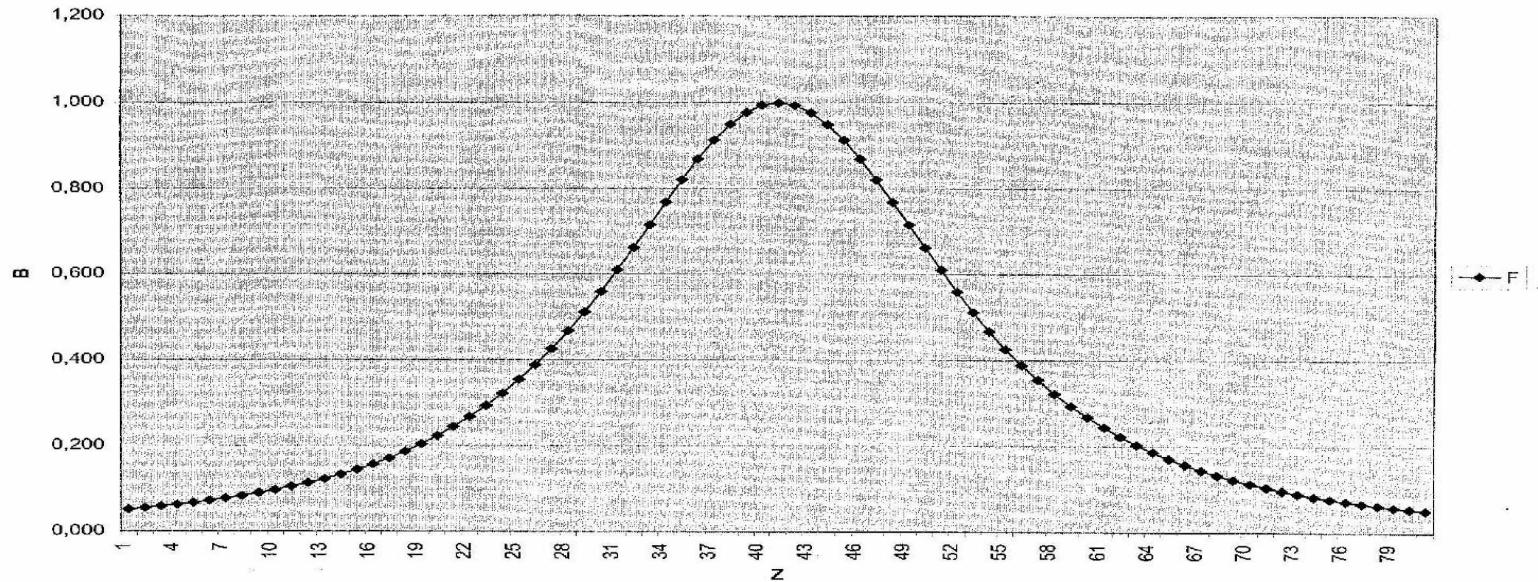
$$\alpha = \frac{\pi}{4}$$

$$B_z(a) \approx 0.35 B_0$$



Field distribution of a ring coil along axis

Champ d'une spire circulaire



APPLICATION du théorème d'Ampère

$$\int H dl = NI$$

$$\int_{-\infty}^{+\infty} B_z dz = \mu_0 NI$$

Equivalent length : $2*a$

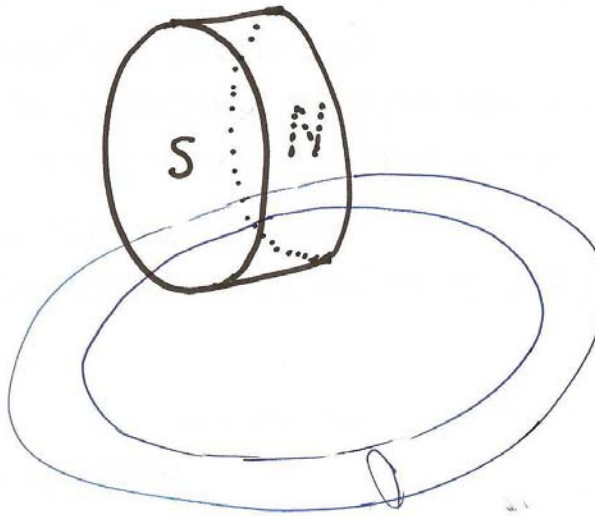
Inflexion at $a/2$

Long distance effect

Ring Coil

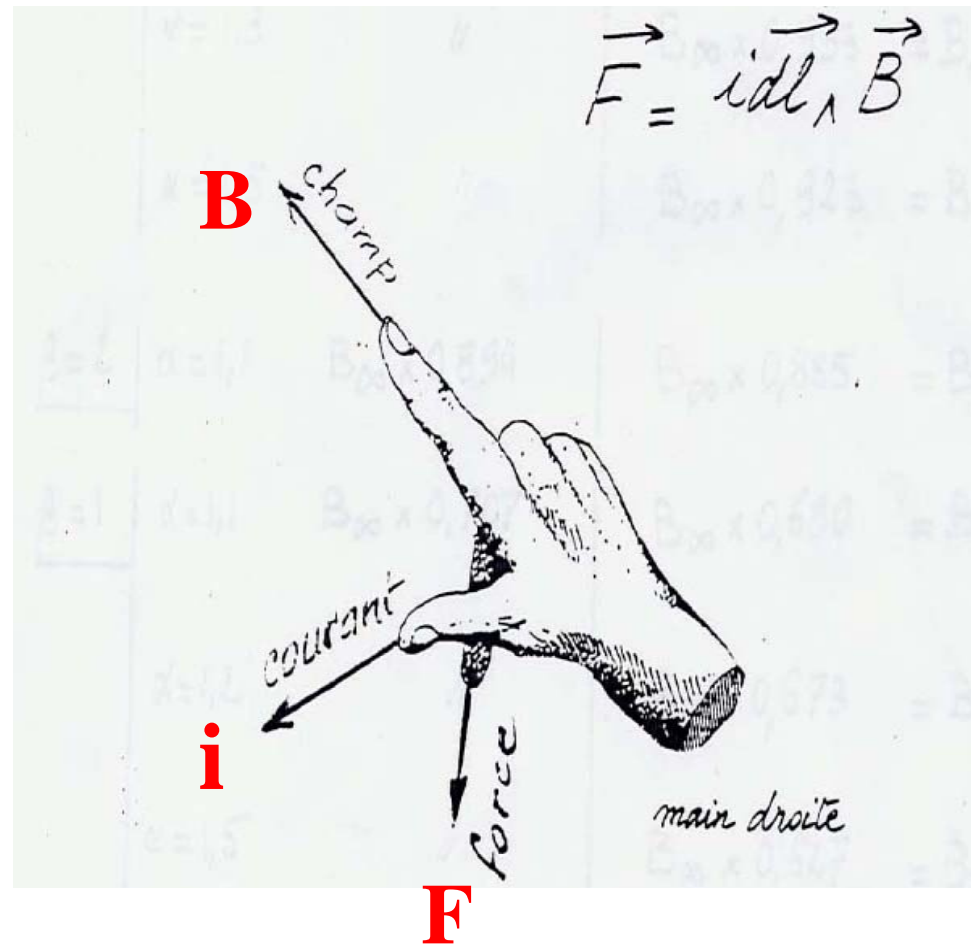
Equivalence with Permanent Magnet

*Flux is going
from North to
South outside
of the magnet*



Laplace's theorem: the 3 fingers of the right hand

$$\mathbf{F} = \mathbf{BiL}$$

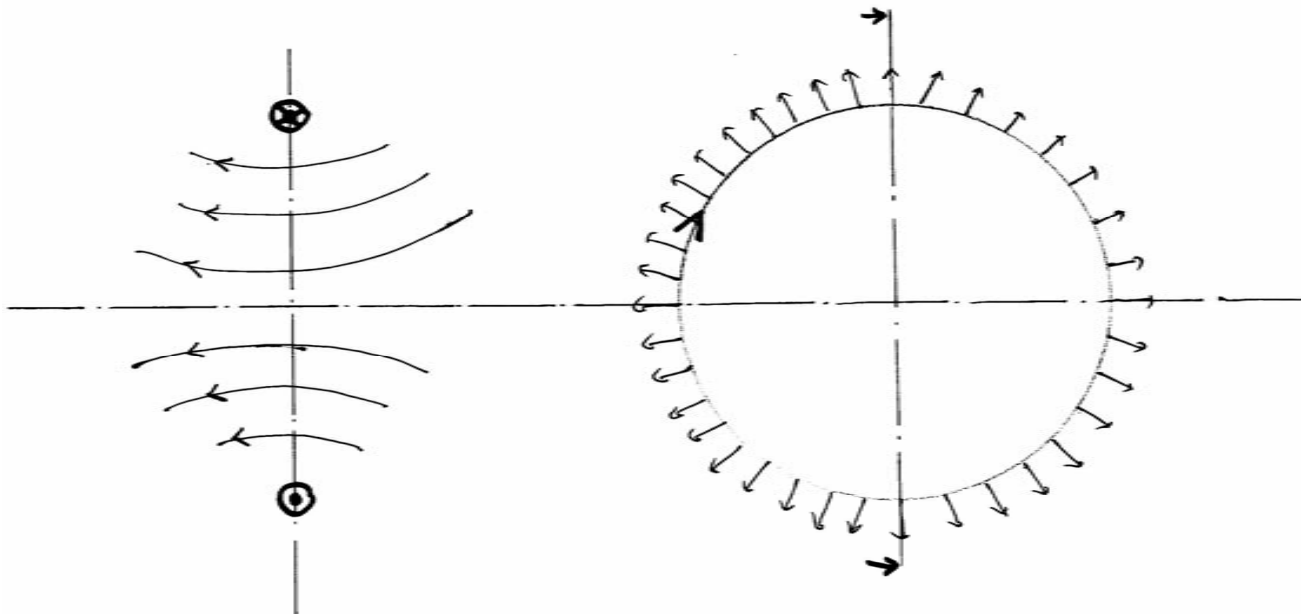


« Hoop stress »

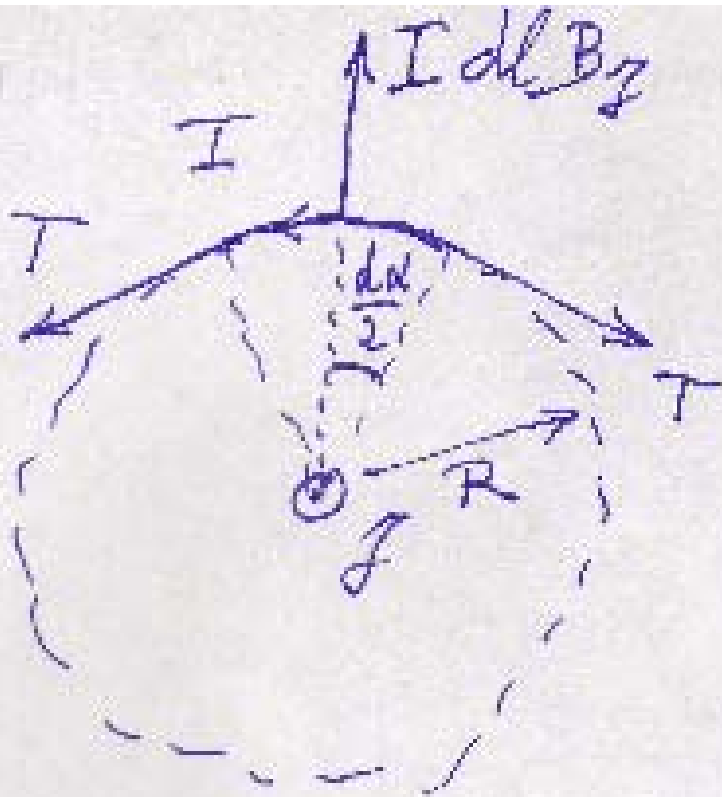
Forces sur les conducteurs

$$d\vec{F} = I d\vec{l} \wedge \vec{B}$$

cas de la spire circulaire



Hoop stress in the ring coil (manuscript of Pr Guy Aubert)



$$I dl B_z = 2T \sin \frac{d\alpha}{2}$$

$$dl = R d\alpha$$

$$T = I R B_z$$

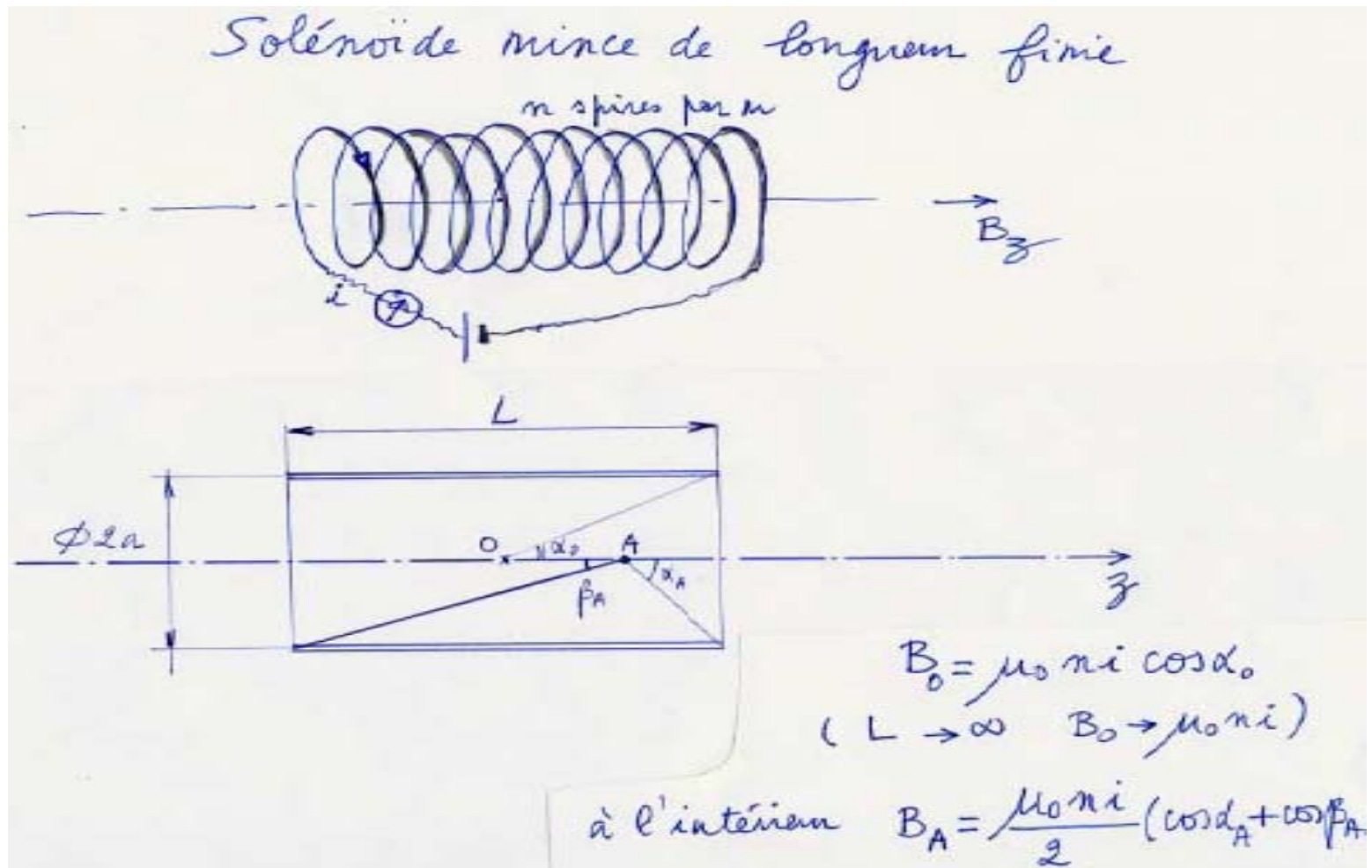
$$\text{Stress} = J * B * R$$

$$(\text{MPa}) = (\text{A/mm}^2) * \text{Tesla} * \text{m}$$

$$100 \text{ MPa} = 100 \text{ A/mm}^2 * 2 \text{ T} * 0.5 \text{ m}$$

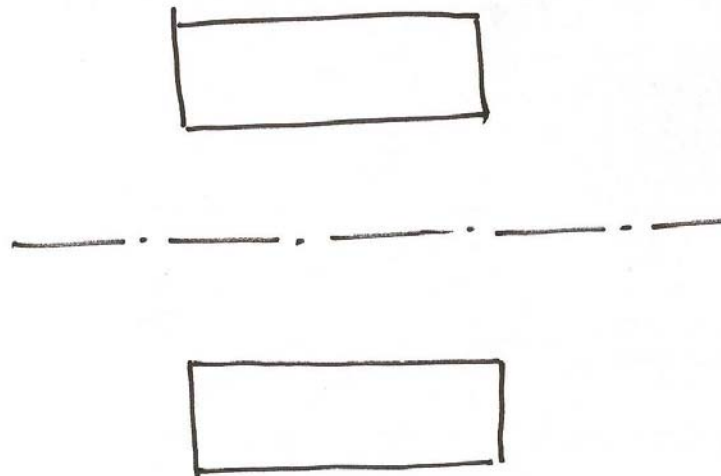
The radial force is trying to extend the radius of the ring coil and it is equilibrated by the tension force. The approximation is valid for a turn on itself and slightly pessimistic in solid coils

Field of a thin solenoid of finite length



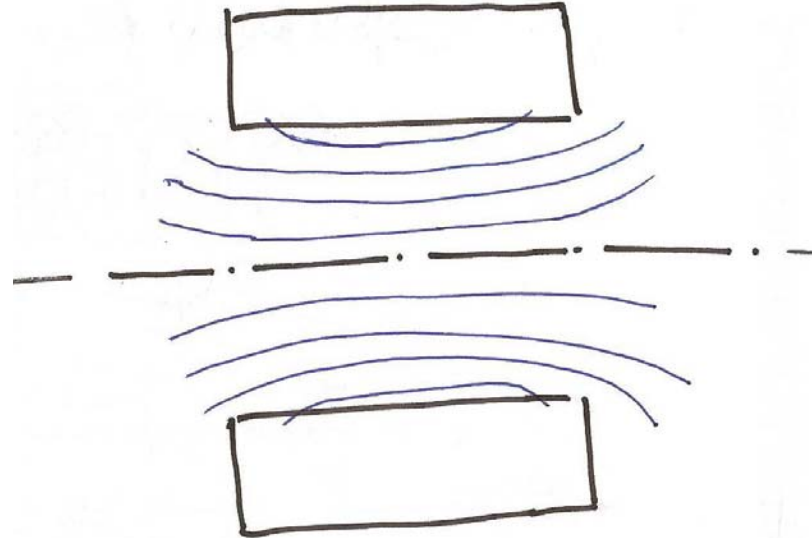
Plotting the flux lines of a thick air core solenoid

Flux tubes step1



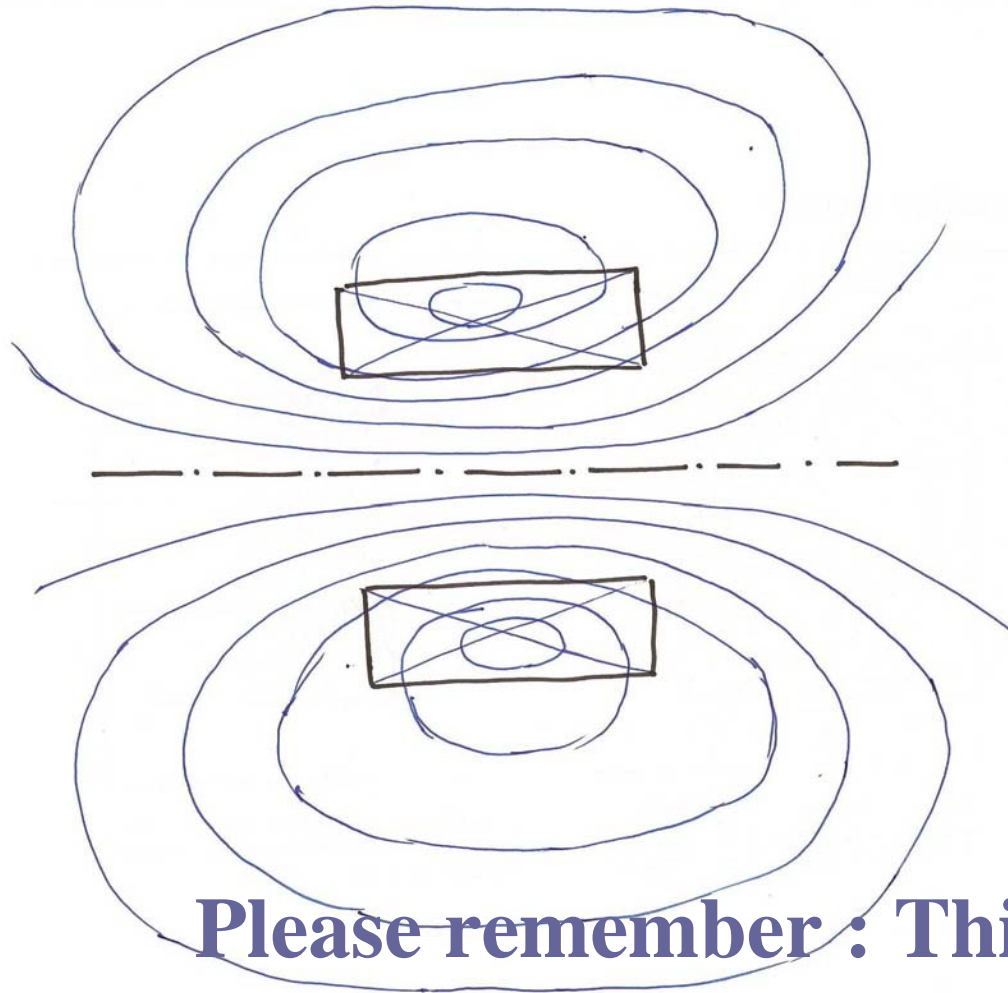
Plotting the flux lines of an air core solenoid

Flux tubes step2



Plotting the flux lines of an air core solenoid

Flux tubes step3



Plotting the flux lines of an air core solenoid

- ❑ In the middle , field is homogeneous , flux lines are parallel
- ❑ In the ends , field is roughly half , flux tubes should be twice larger
- ❑ Near the axis , flux tubes are going very far to close on them selves
- ❑ Close to the coil , flux is turning , creating a longitudinal compression,
- ❑ Near the median plane, part of the flux is returning inside the coil around a zero field point.
- ❑ The inner part of the coil is suffering an expanding effort
- ❑ The outer part of the coil is suffering a radial compression
- ❑ The total force is an expanding effort.

Thick solenoid

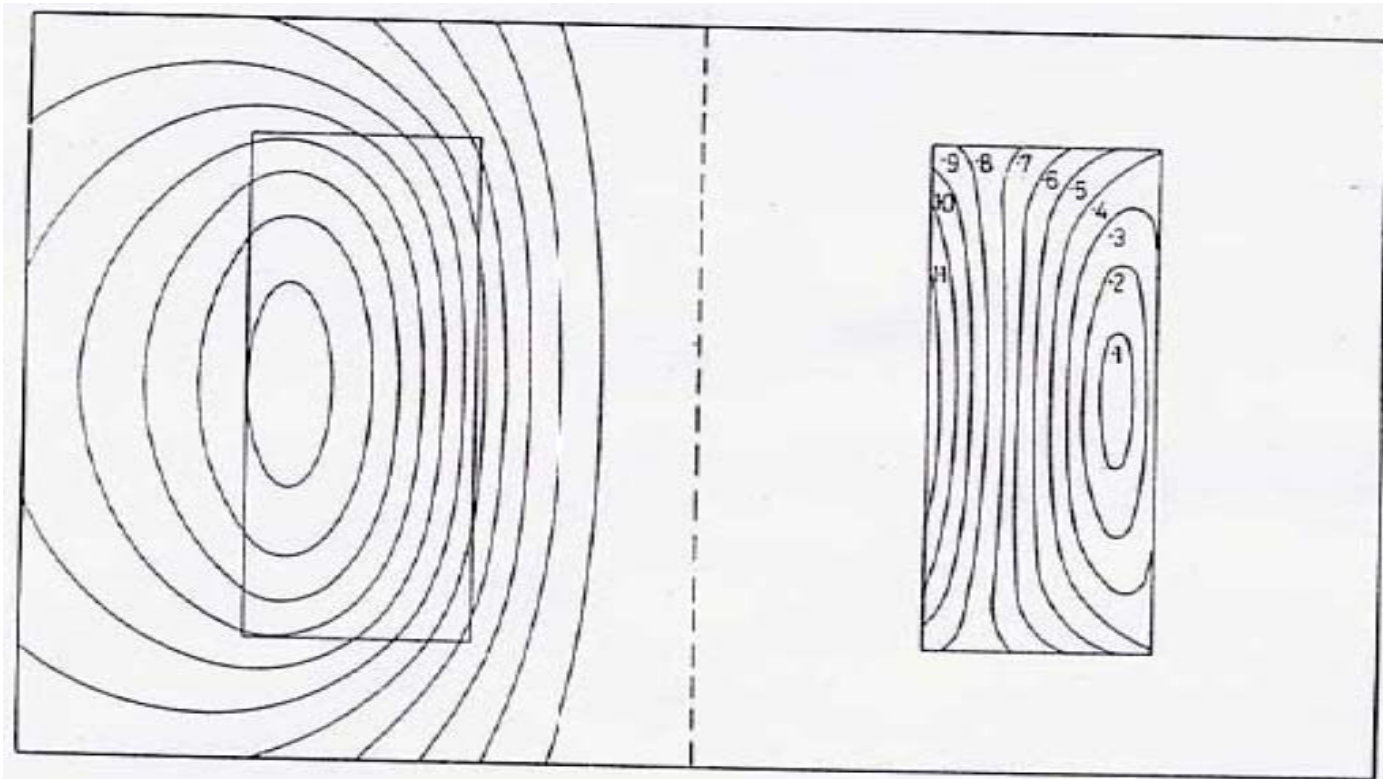
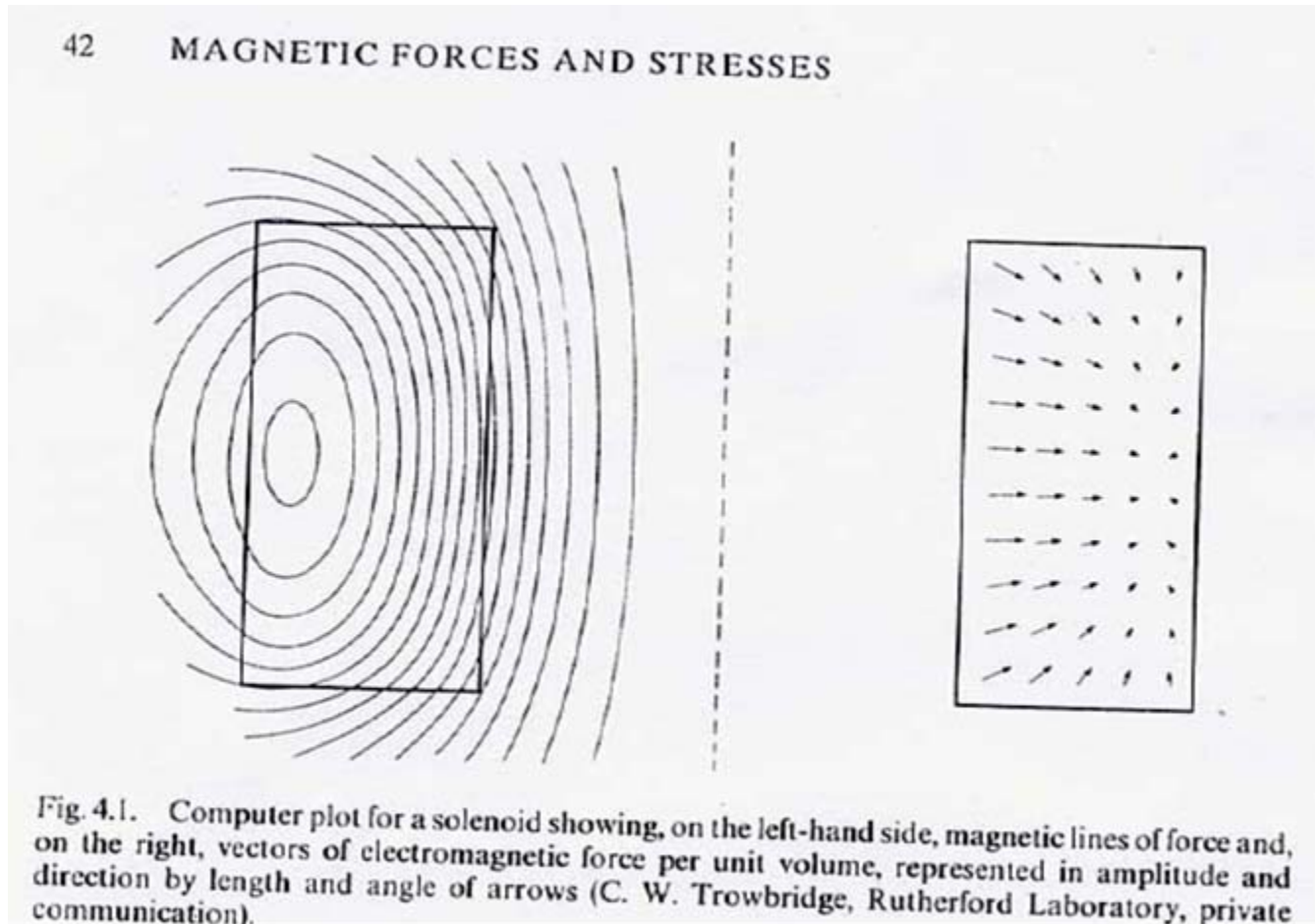


Fig. 3.6. Computer plot of the field in a simple solenoid showing, on the left hand, magnetic lines of force and, on the right hand contours of constant field intensity $|B|$ relative to the central field B_0 (C. W. Trowbridge, Rutherford Laboratory, private communication).

La compréhension des forces est primordiale



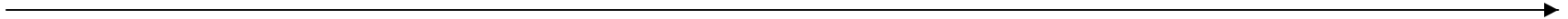
Thick solenoid

- Field formulas for thick solenoids are complicated
- Peak field
- Lehmann Point
- Internal magnetic forces
- Magnetostatic pressure is equivalent to stored energy density : $B^2/2\mu_0$

Field	Magnetic pressure
1.6Tesla	1MPa
4Tesla	6.25 MPa

Discussion on homogeneity SMC magnet parameters

- 2 M Ampere turns per meter are creating around 2.57 T



If a finite solenoid of 300mm in diameter
Is only 2 meters long the field drop at 0.1m is
already:

Ampere's Theorem

- Please remember:

Current	Field	Length
1A	1mT	1.25 mm

**It applies to
infinite
solenoids**

800*2500 = 2M Ampere turns i.e. for 500 Amps 4000 turns per meter	2.57 Tesla Factor 2500	1m Factor 800
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Even a long thin solenoid is not homogeneous.....

- $B_0 = \mu_0 n I \cos(\alpha_0)$
- $B_1 = \mu_0 n I (\cos\alpha_1 + \cos\alpha_2)$

$\alpha_0 = 8,530$ degrés $B_0 = 0,9941$ Tesla

At $z = 0,100$ m

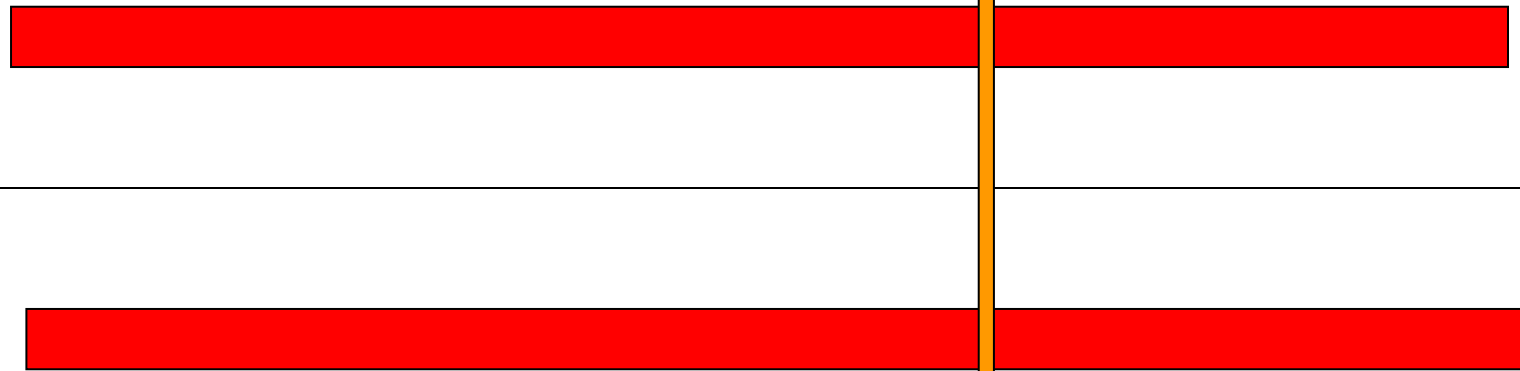
$\alpha_1 = 9,462$ degrés $\alpha_2 = 7,765$ degrés

$$B_1 = 0,98861205$$

$$\Delta B / B_0 = 0,005607 = 5 \cdot 10^{-3}$$

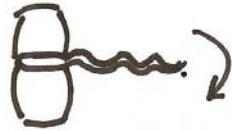
Long solenoid with small error gap in the winding

- 0.1 mm error gap is a negative ring coil of 2000 A

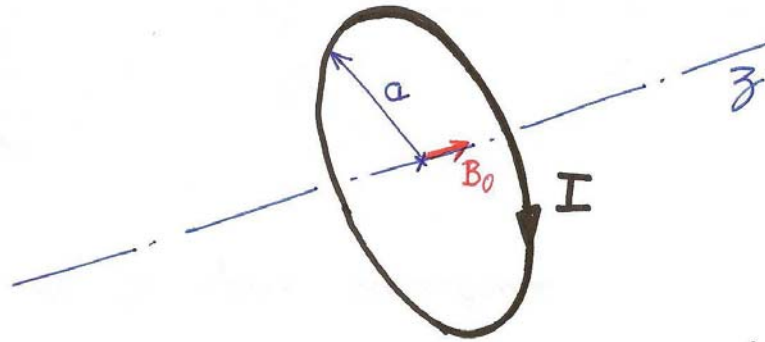


$\Delta B = -0.0008\text{T}$ in the range of 500 ppm
If you need a magnet with 25 ppm, it is huge

The Ring Coil



Center Field of a ring coil
of radius a



$$B_0 = \mu_0 \frac{I}{2a}$$

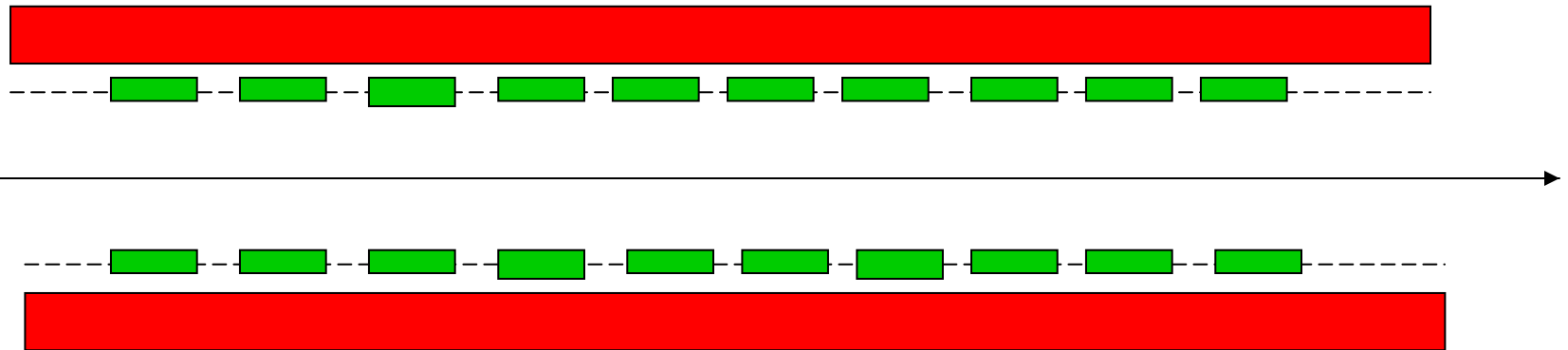
$$\mu_0 = 4\pi \cdot 10^{-7}$$

diameter

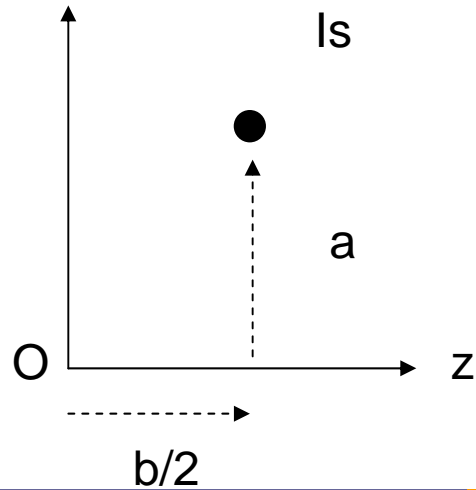
I	B	$2a$
1A	1mT	1.25mm
200A	0.8mT	300mm

Long solenoid with errors in the winding

- It is impossible to realise the winding of a long solenoid without a lot of distributed errors in the range of 0.1mm

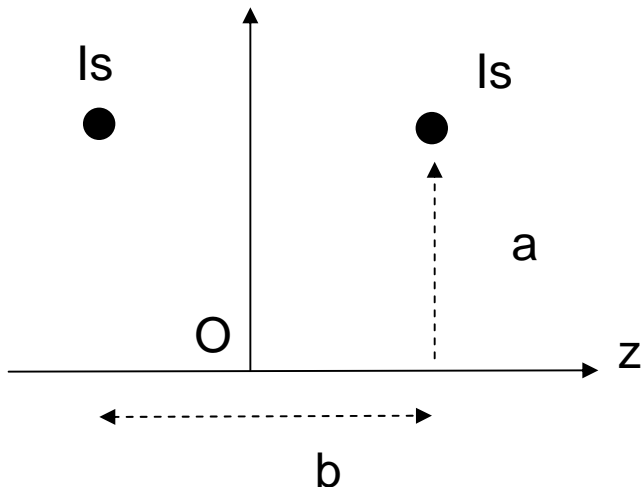


The error results in a superposition of ring coil curves randomly distributed. It must be corrected by a family of longitudinal trim coils independently supplied. They will also compensate for the natural drop



B in a point z of the axis created by a ring coil of radius a situated in b z=b/2 and with current I

$$B_z(z) = \frac{\mu_0 I_s^2 a^2}{2 \left(a^2 + \left(z - \frac{b}{2} \right)^2 \right)}$$



For two rings

$$B_z(z) = 0.5 \mu_0 I_s^2 a^2 \left(\frac{1}{a^2 + \left(z - \frac{b}{2} \right)^2} + \frac{1}{a^2 + \left(z + \frac{b}{2} \right)^2} \right)$$

For two ring coils

$$\frac{d^2 B_z}{dz^2} = \frac{3 \mu_0 I_s^2 a^2 (b^2 - a^2)}{\left(\frac{7}{2}\right)}$$

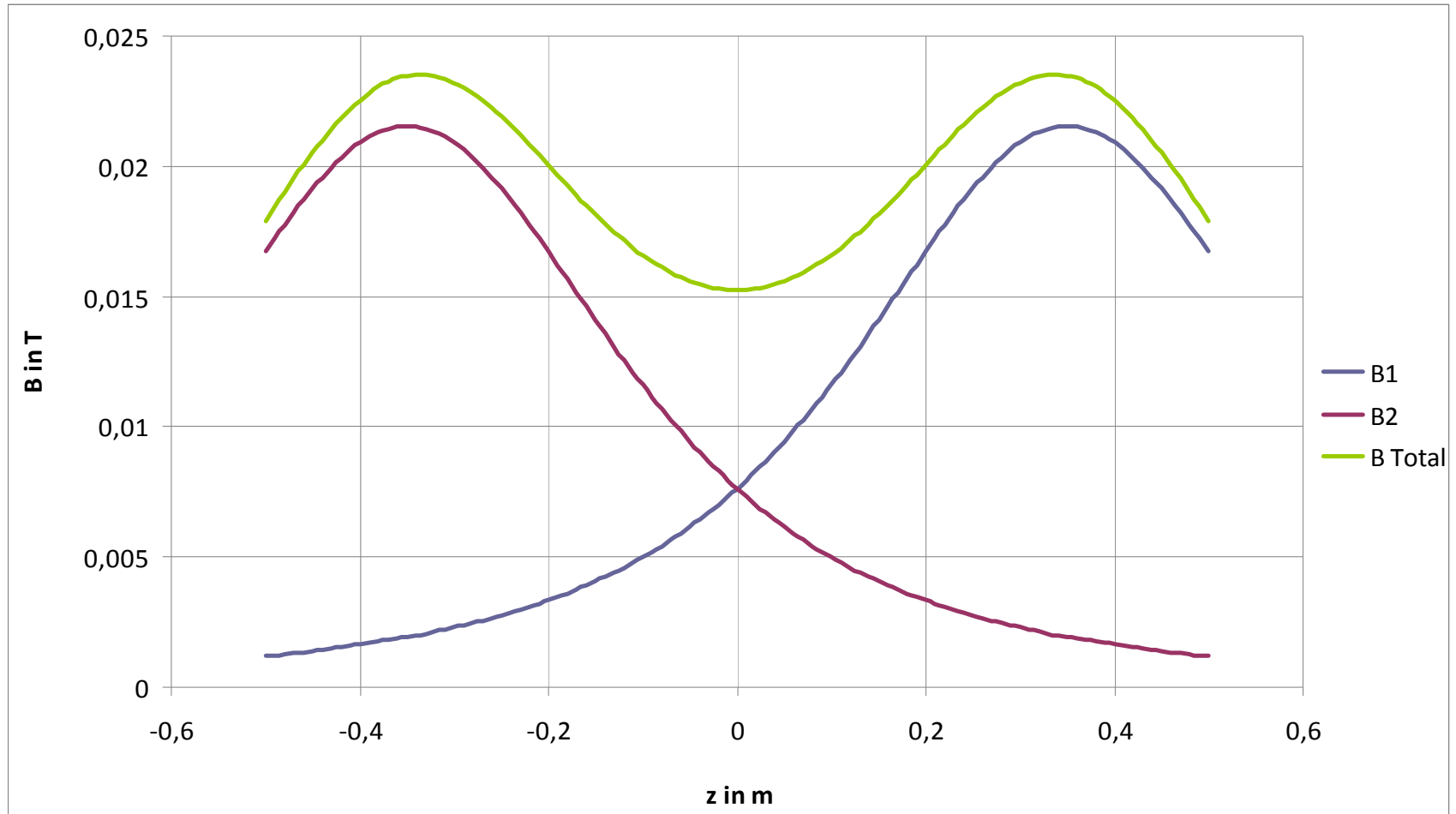
$$\left(a^2 + \frac{b^2}{4}\right)$$

$$\frac{d^4 B_z}{dz^4} = \frac{45 \mu_0 I_s^2 a^2 (8 a^4 - 24 a^2 b^2 + 4 b^4)}{\left(\frac{11}{2}\right)}$$
$$8 \left(a^2 + \frac{b^2}{4}\right)$$

$$\frac{d^2 B_z}{dz^2} = 0 \quad \text{conduit à} \quad a = b \quad (\text{position de Helmholtz})$$

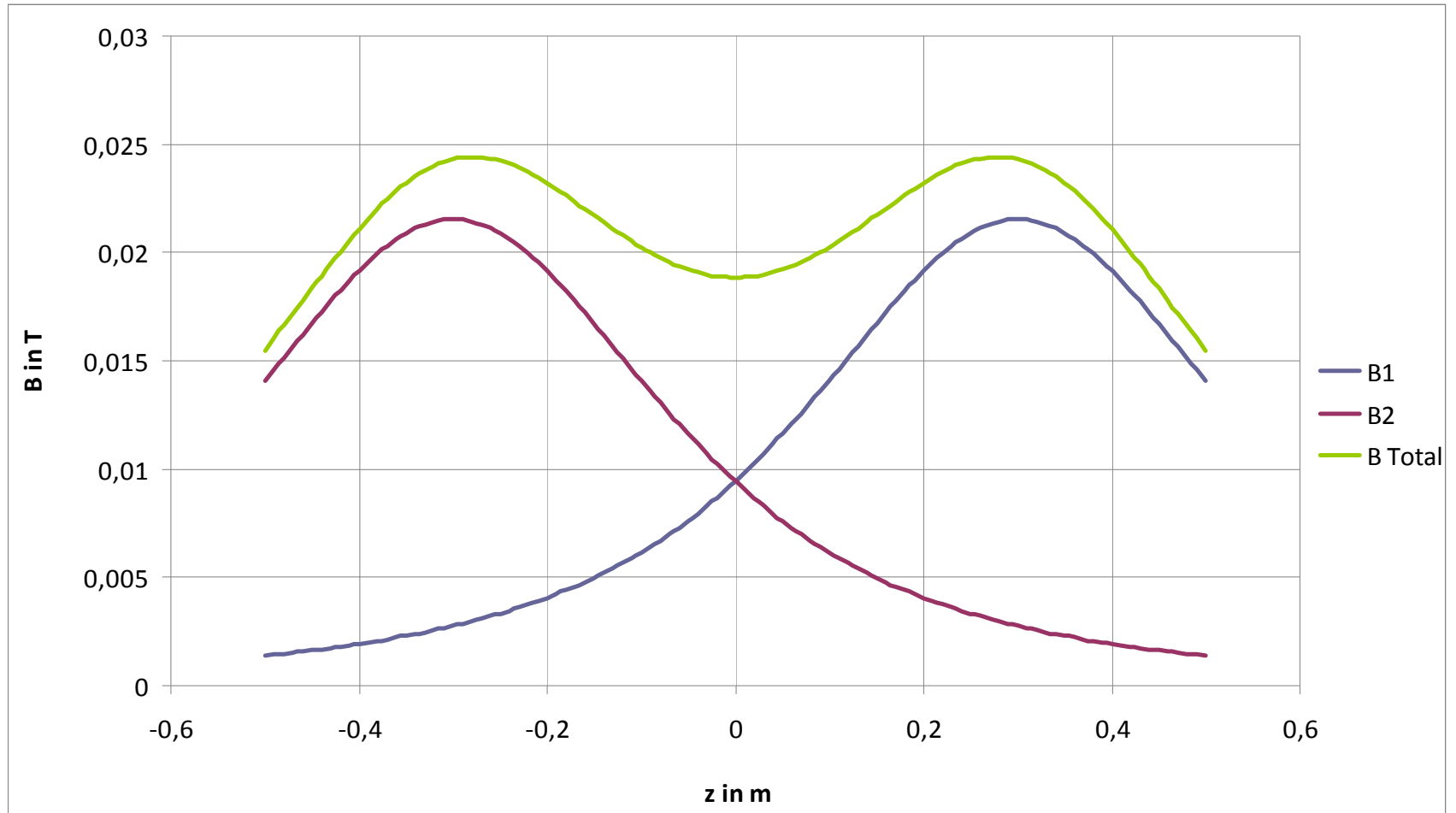
$a=0,350$ $2b=0,700$

Ordre 2: 2 Helmholtz coils (12000A)



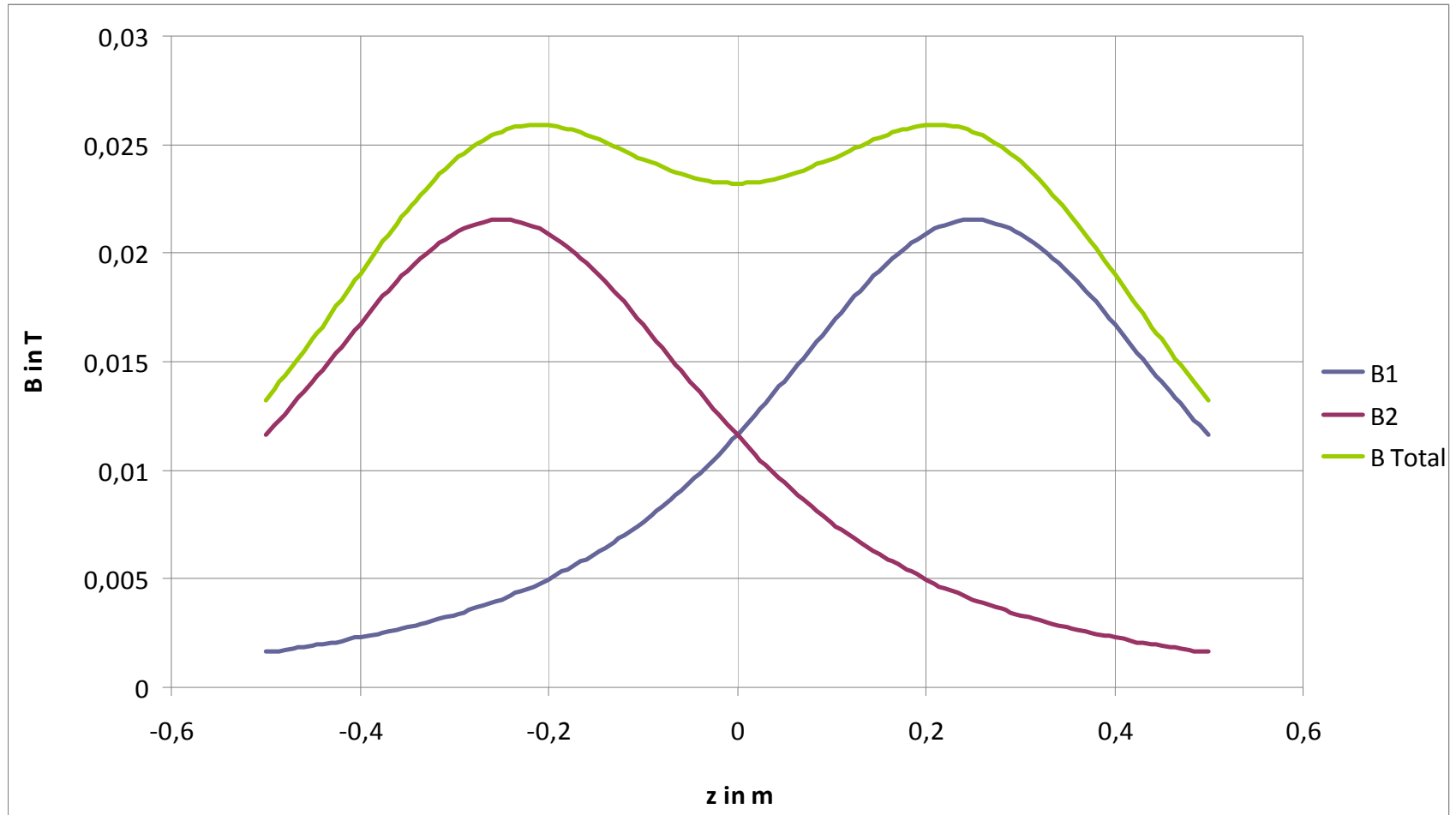
$a=0,350$ $2b=0,600$

Ordre 2: 2 Helmholtz coils (12000A)



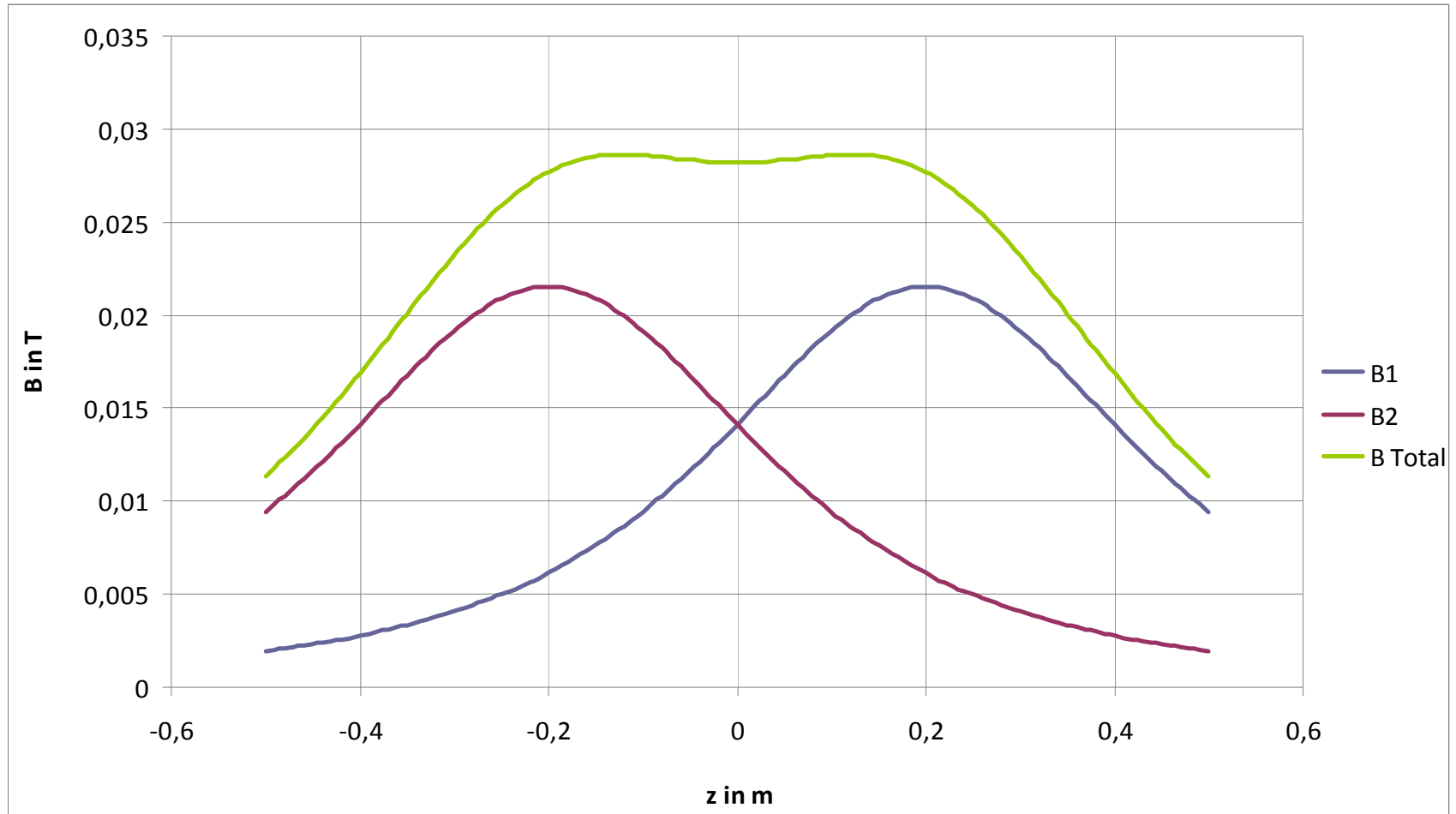
$a=0,350$ $2b=0,500$

Ordre 2: 2 Helmholtz coils (12000A)



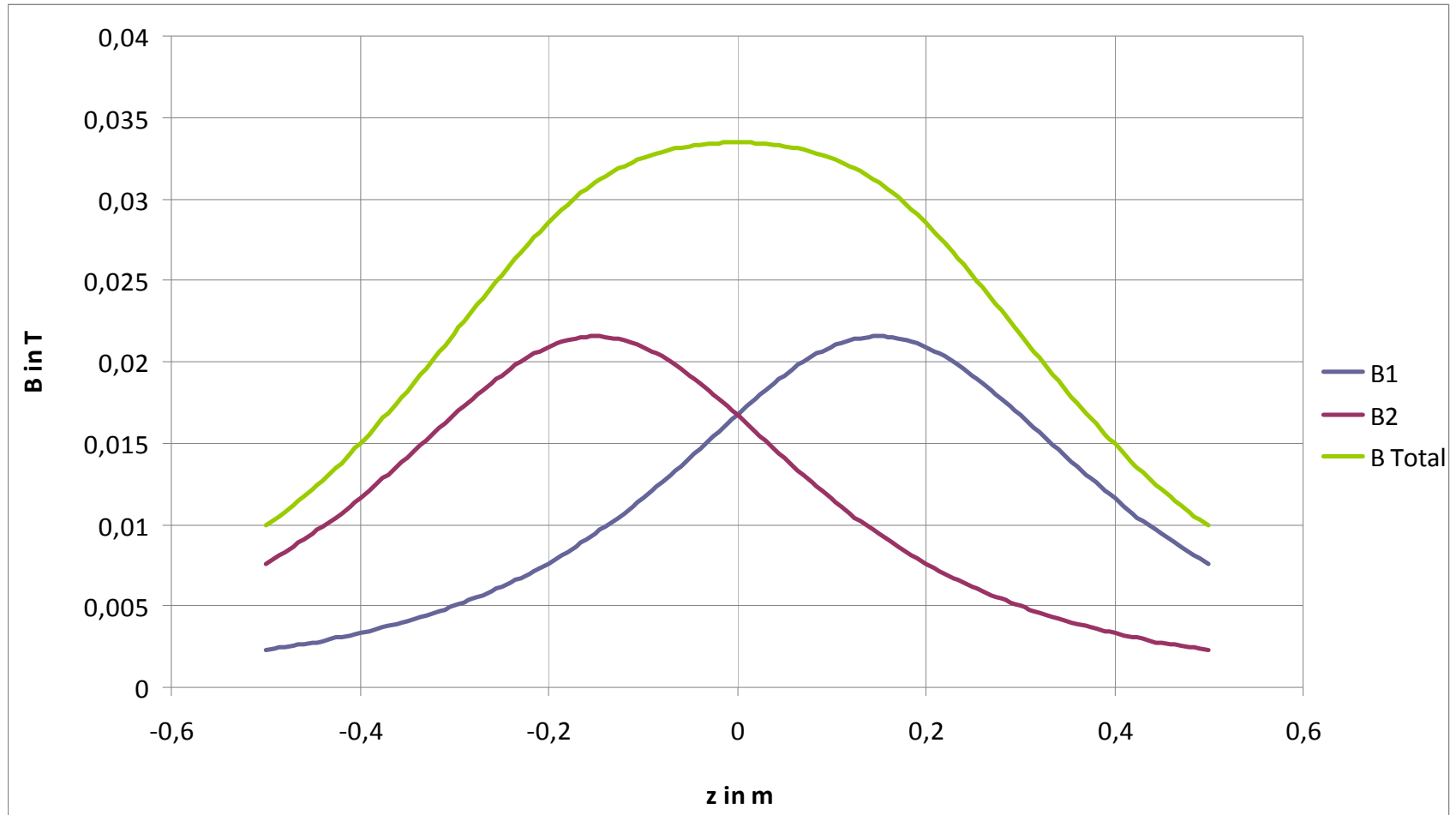
$a=0,350$ $2b=0,400$

Ordre 2: 2 Helmholtz coils (12000A)



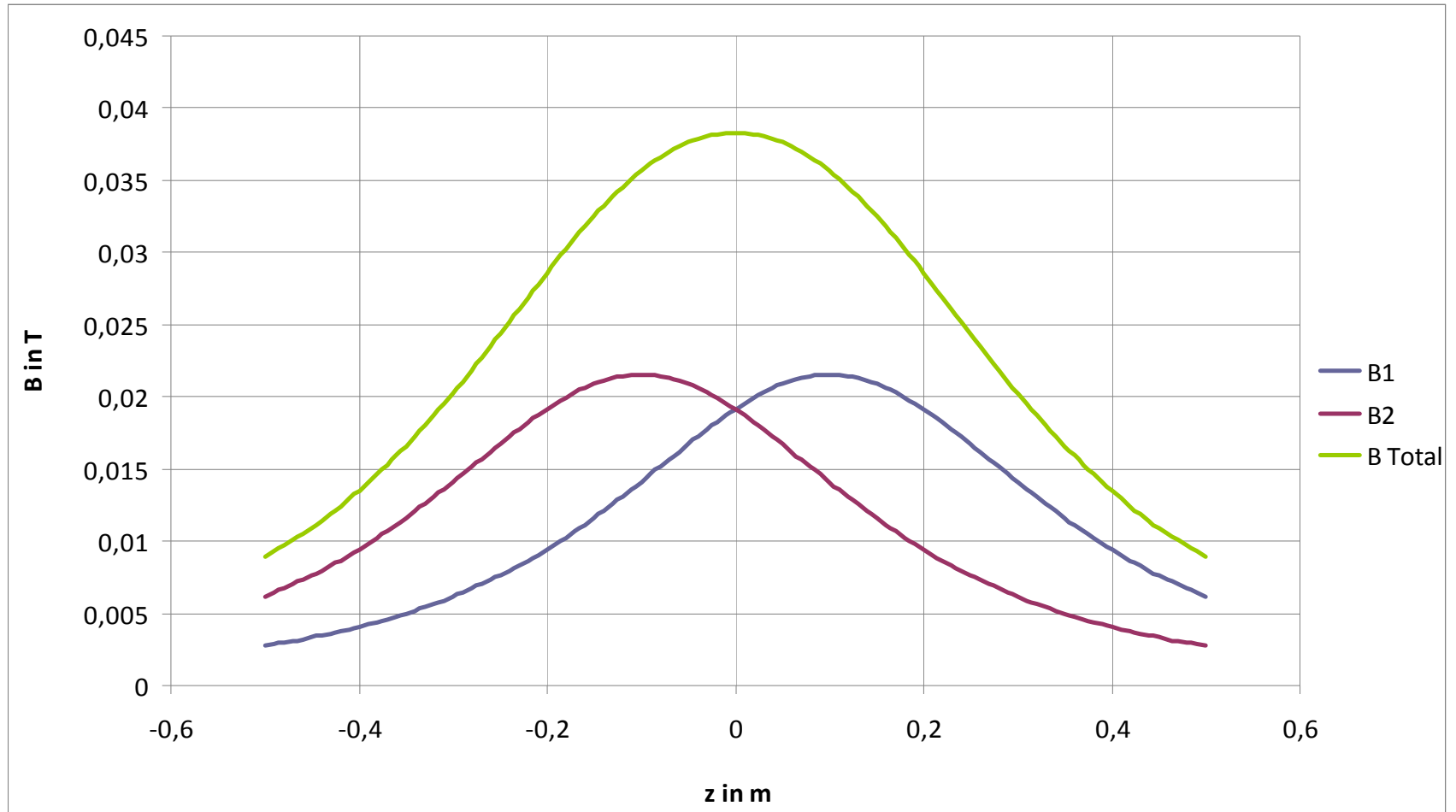
$a=0,350$ $2b=0,300$

Ordre 2: 2 Helmholtz coils (12000A)



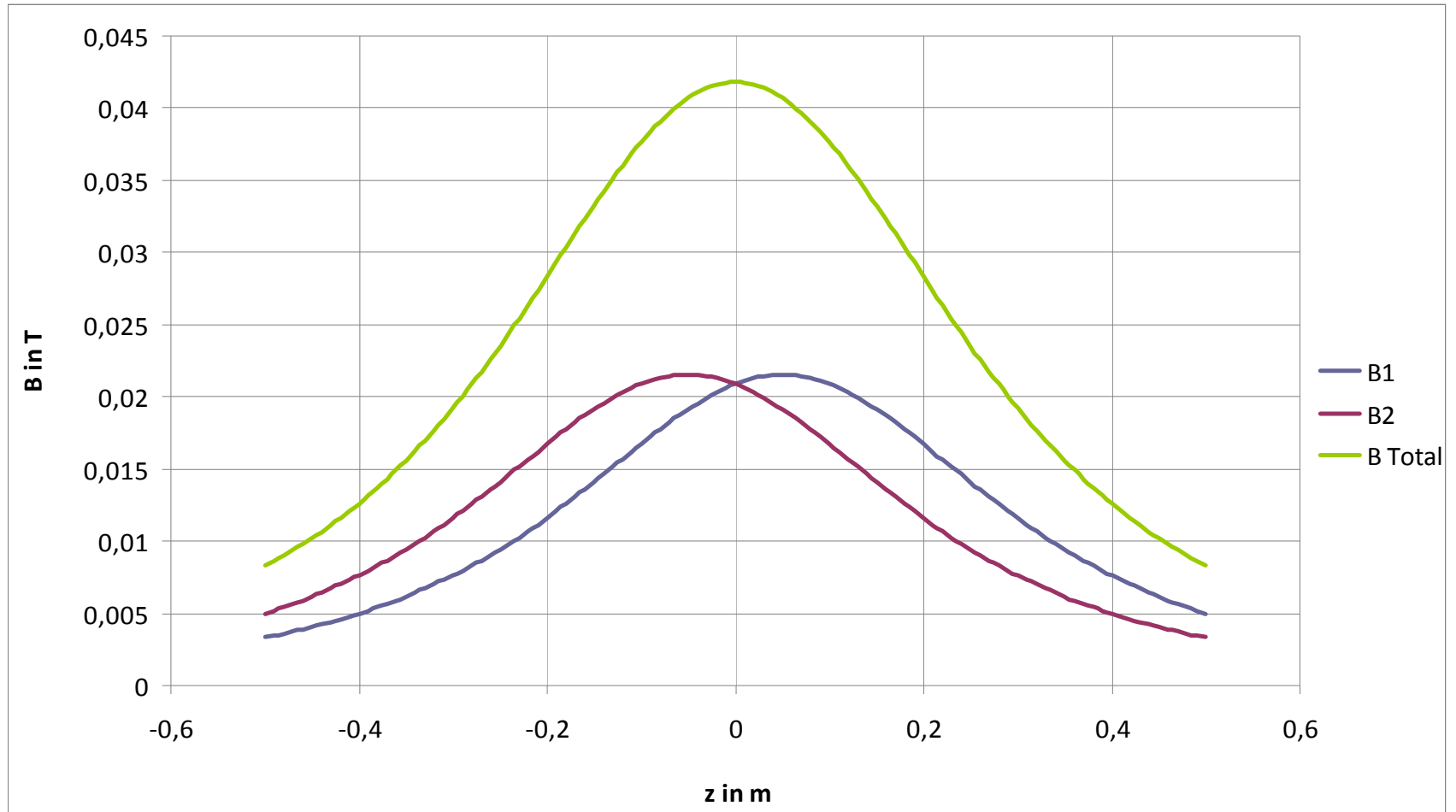
$a=0,350$ $2b=0,200$

Ordre 2: 2 Helmholtz coils (12000A)



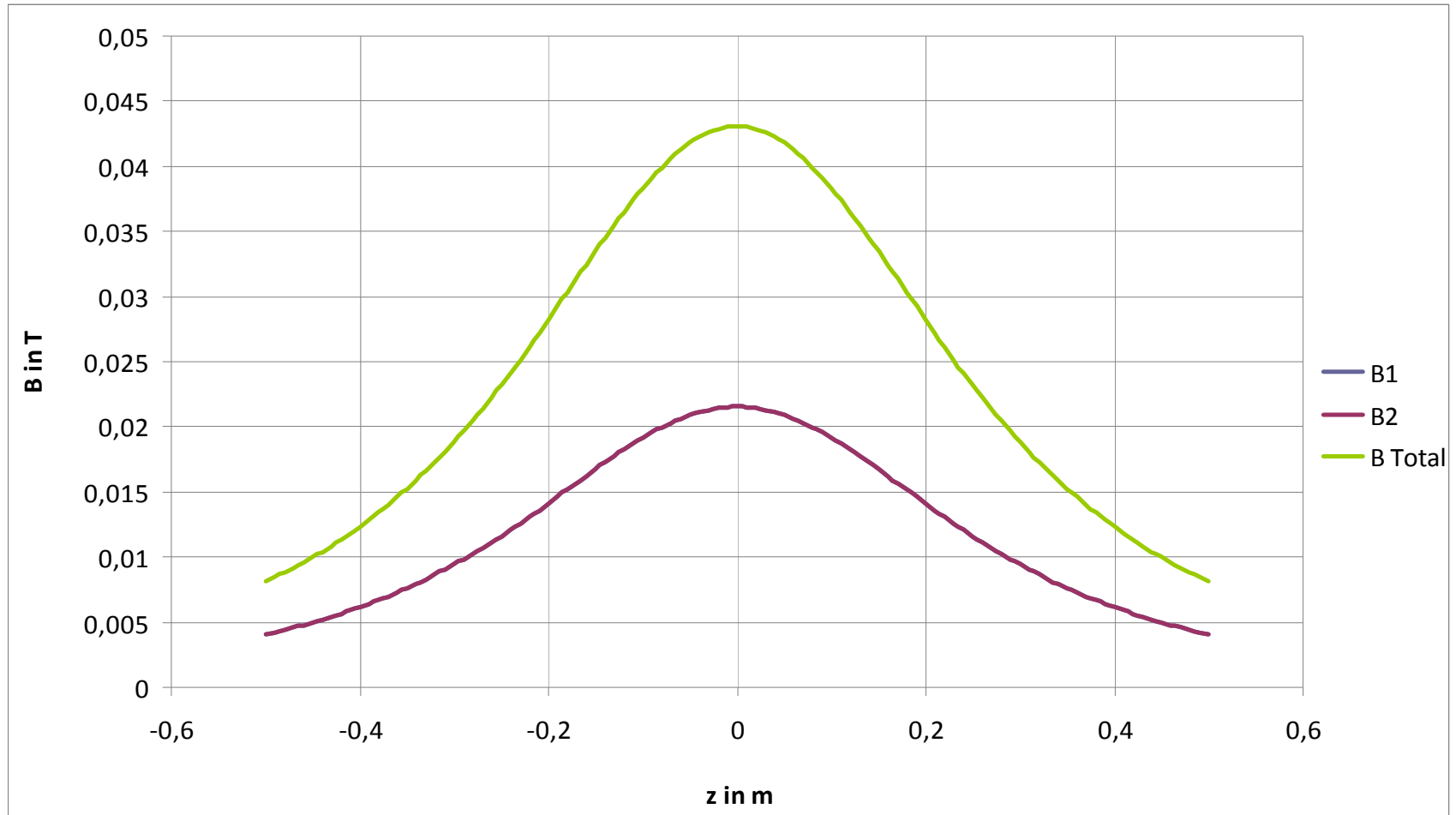
$a=0,350$ $2b=0,100$

Ordre 2: 2 Helmholtz coils (12000A)



$a=0,350$ $2b=0,000$

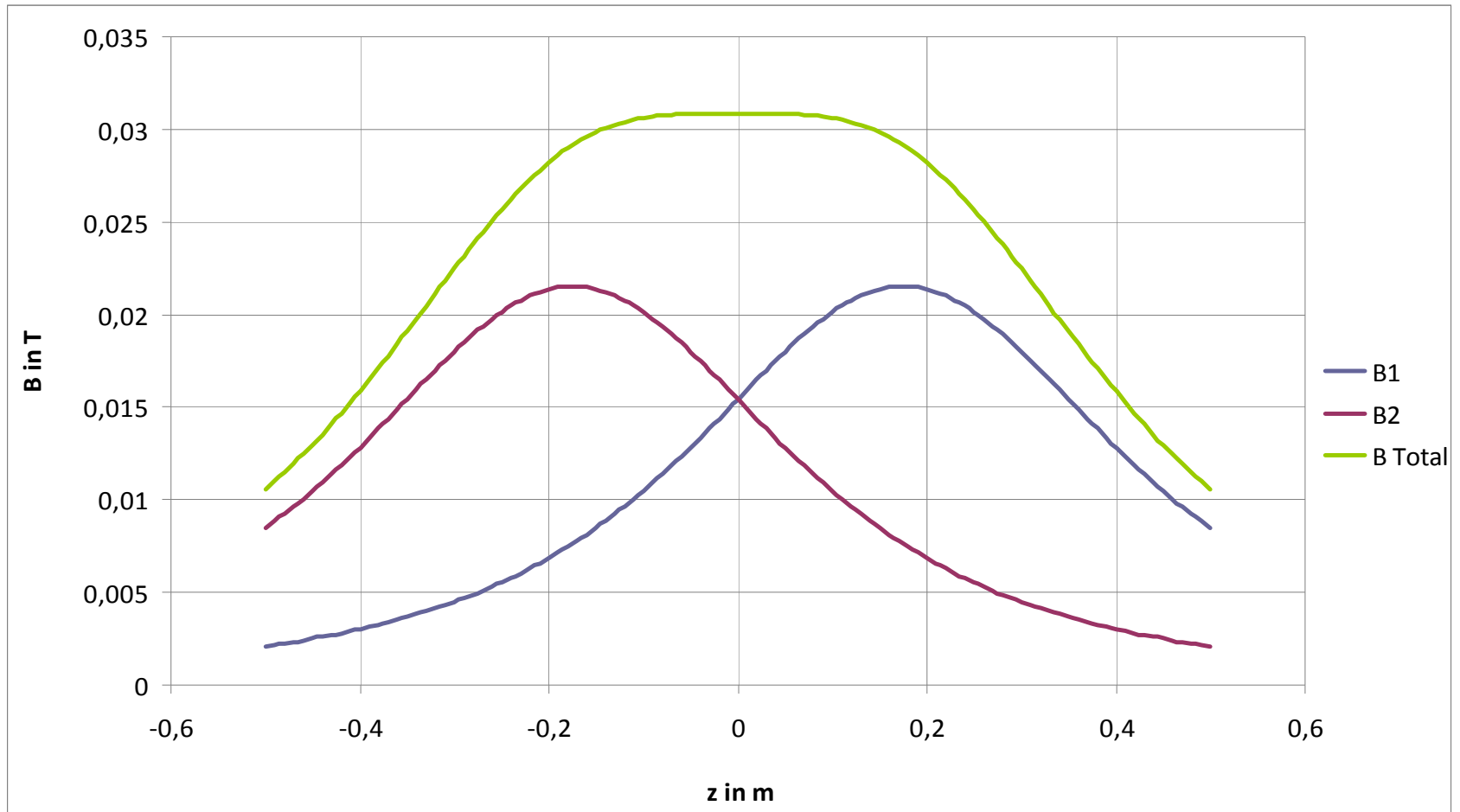
Ordre 2: 2 Helmholtz coils (12000A)



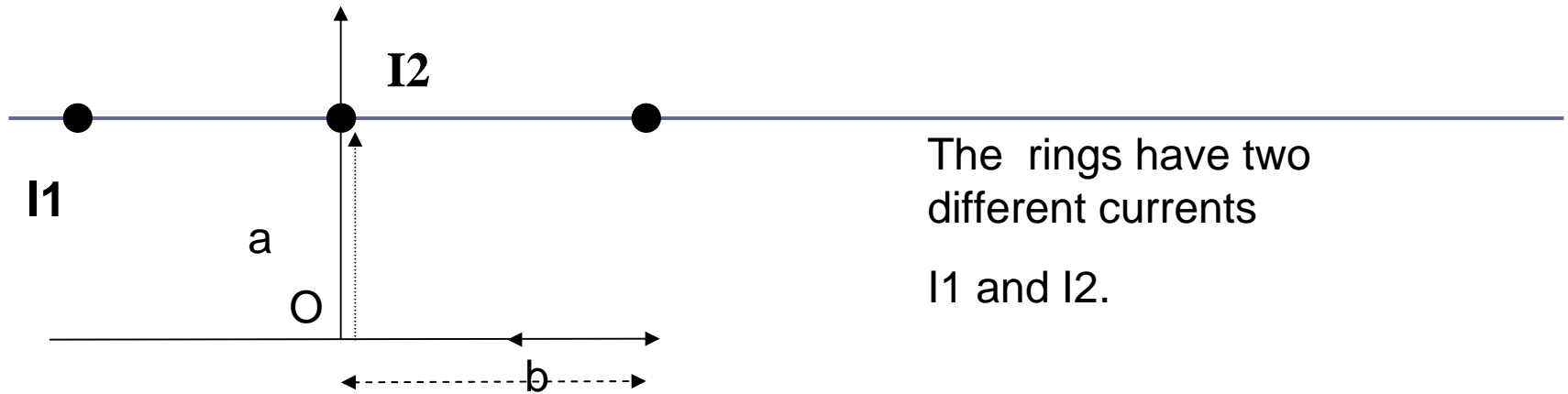
1000ppm in $\pm 65\text{mm}$

$a=0,350$ $2b=0,350$

Ordre 2: 2 Helmholtz coils (12000A)



Optimisation with 3 symmetric ring coils



The rings have two different currents

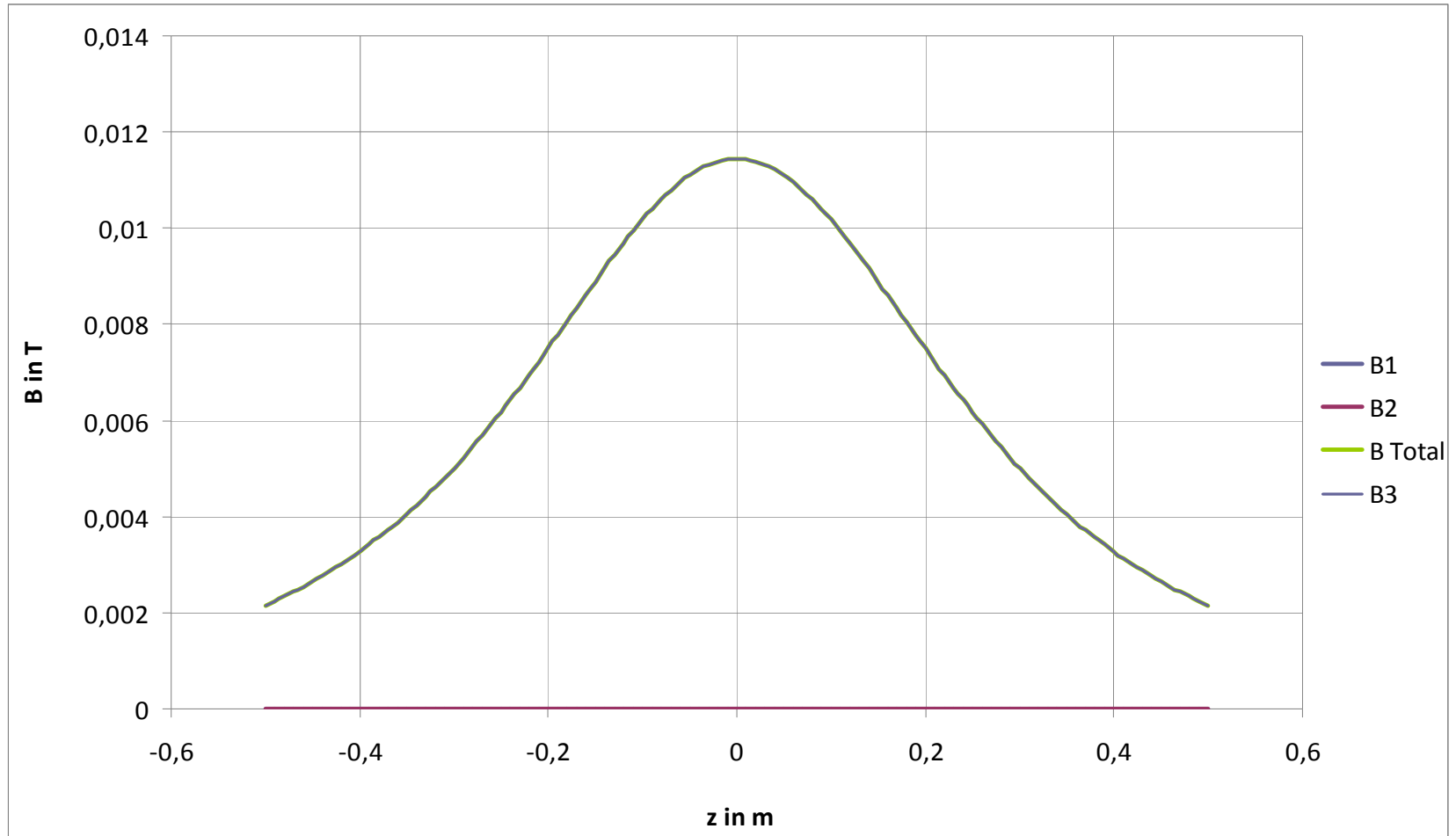
I1 and I2.

We look for:

$$\frac{d^2 B_z}{dz^2} = 0 \quad \text{and} \quad \frac{d^4 B_z}{dz^4} = 0$$

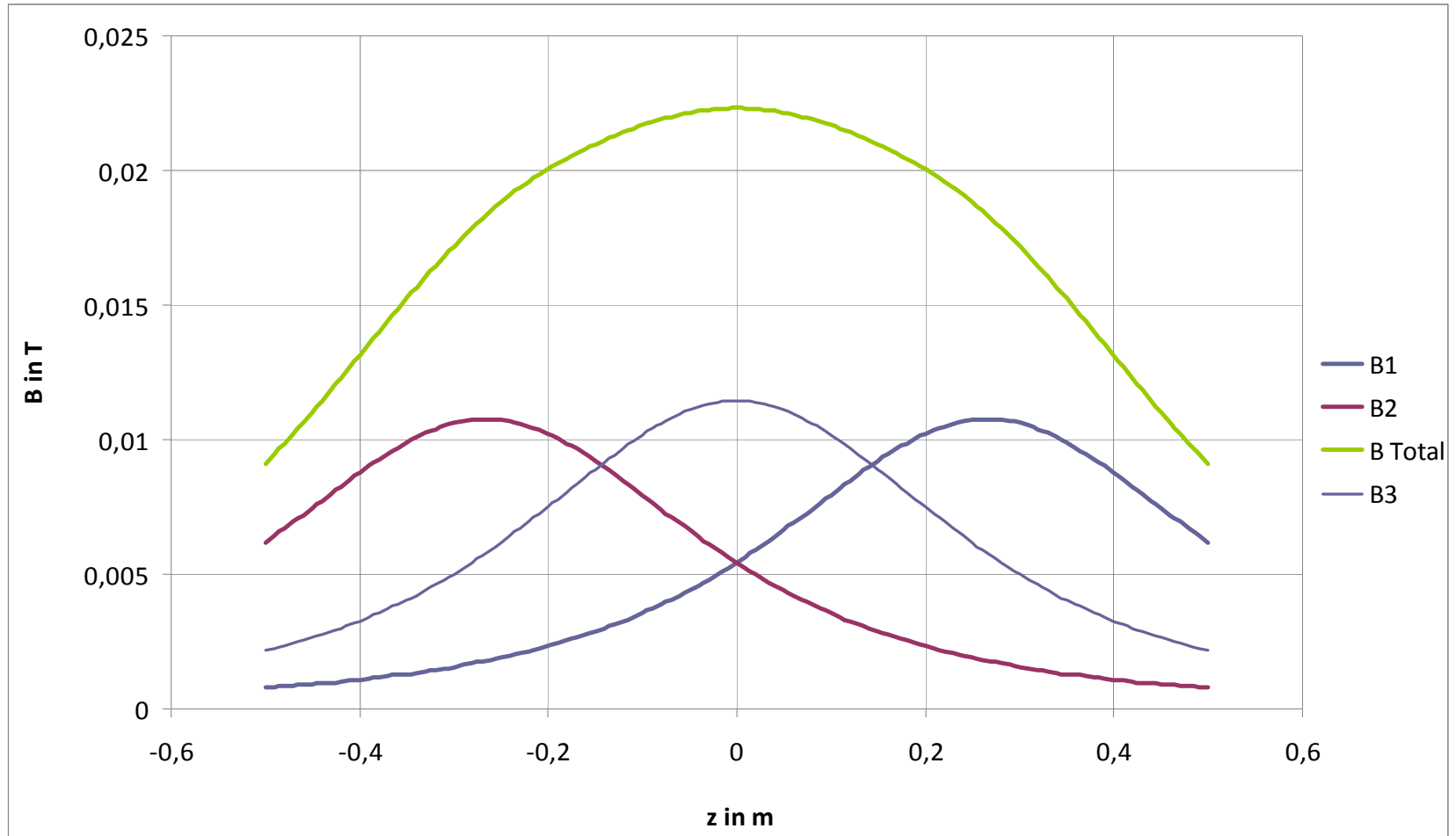
$I_1=0A$ $I_2=6377A$

Ordre 4: two symmetric coils ($2b=0.532$) I_1 & one central coil I_2



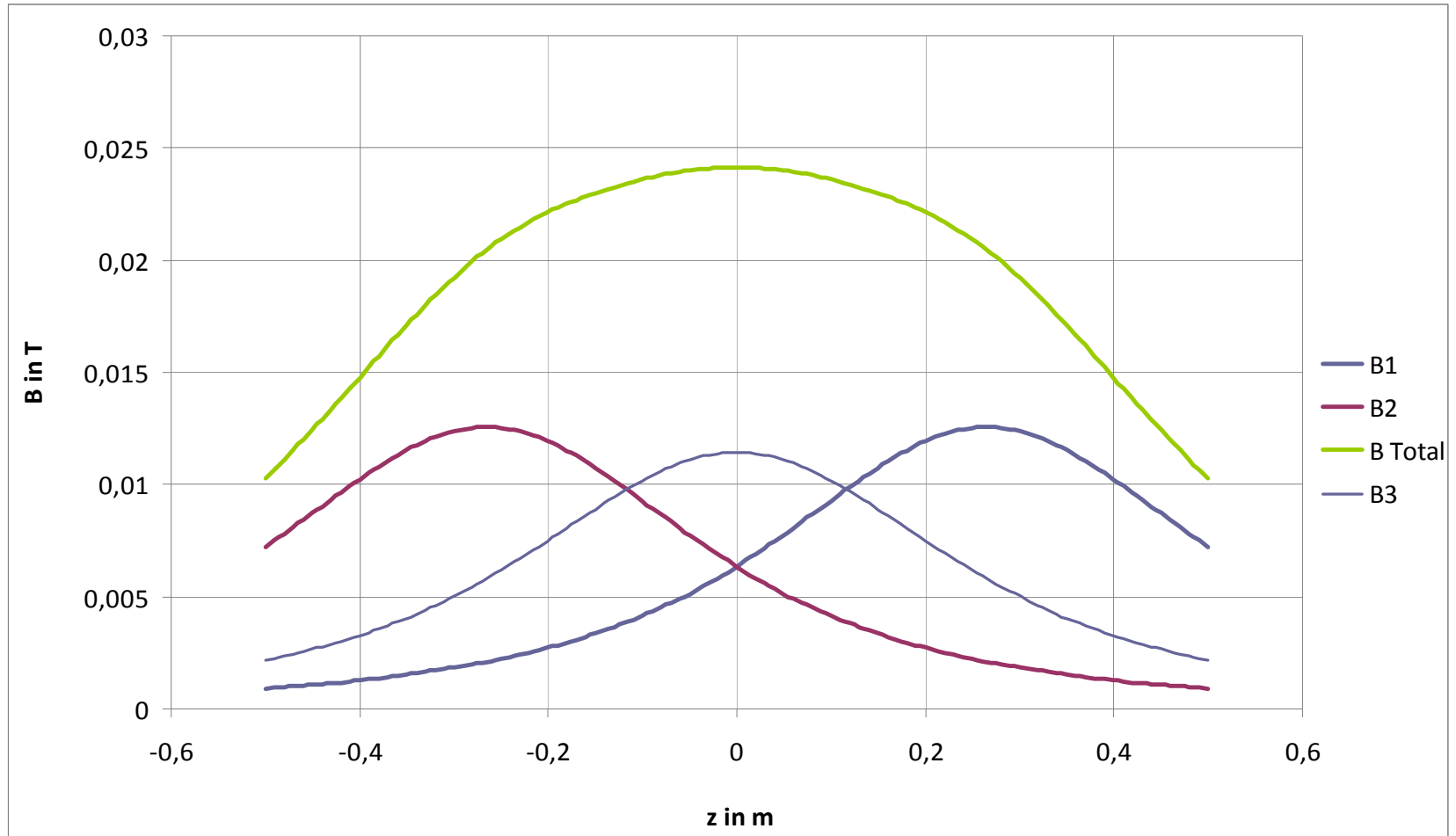
$I_1=6000A$ $I_2=6377A$

Ordre 4: two symmetric coils ($2b=0.532$) I_1 & one central coil I_2



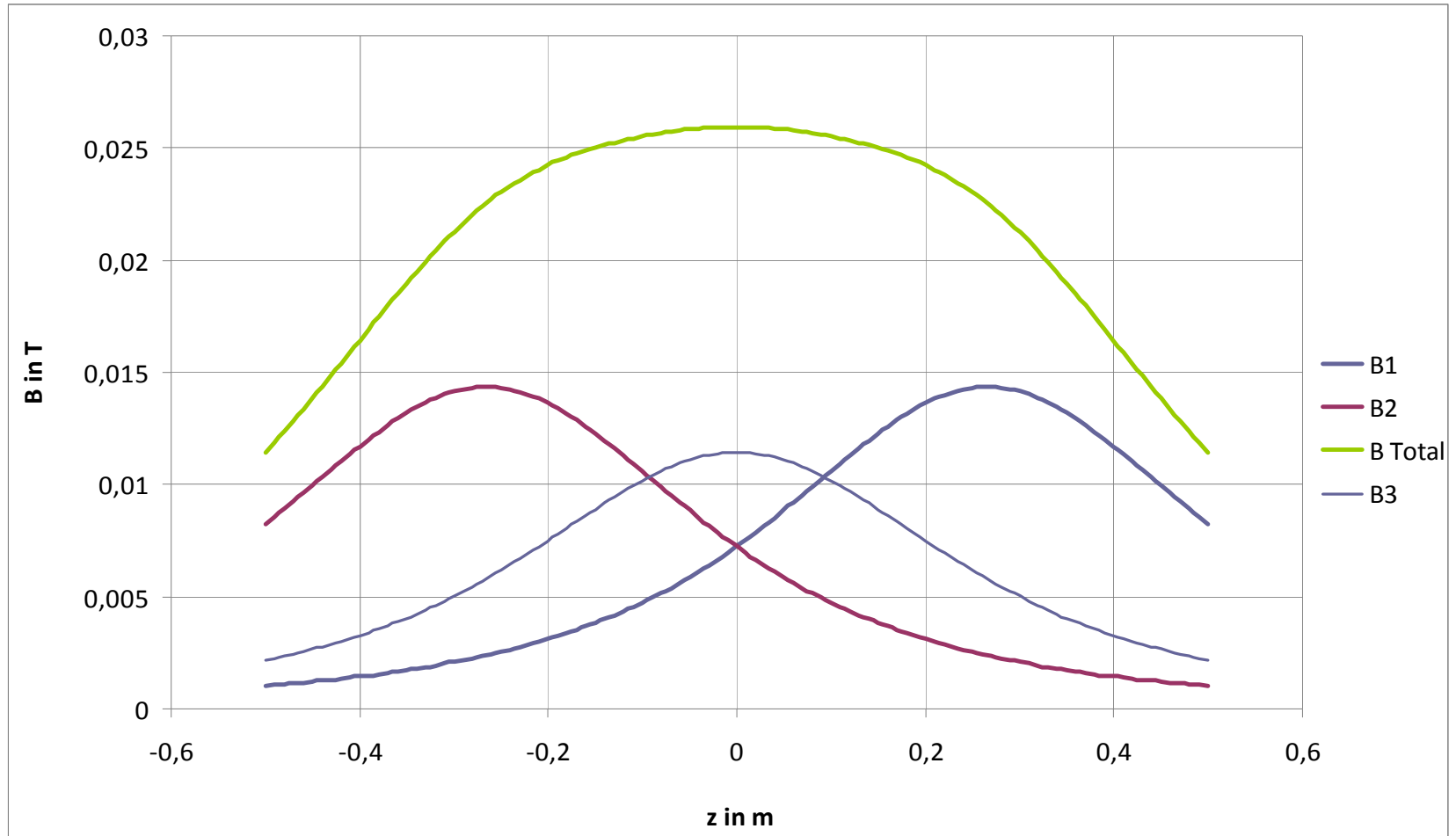
$I_1=7000A$ $I_2=6377A$

Ordre 4: two symmetric coils ($2b=0.532$) I_1 & one central coil I_2



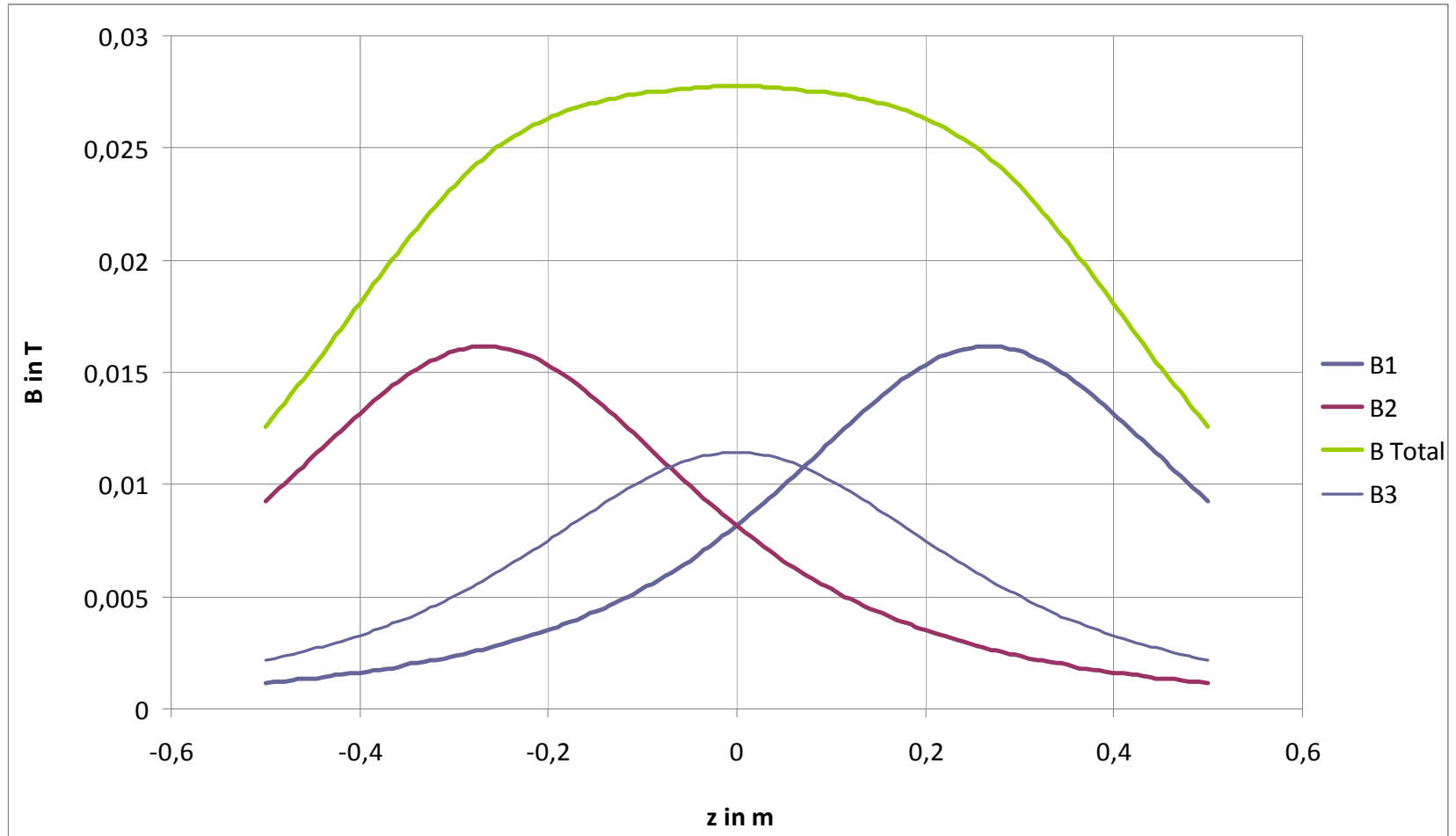
$I_1=8000A$ $I_2=6377A$

Ordre 4: two symmetric coils ($2b=0.532$) I_1 & one central coil I_2



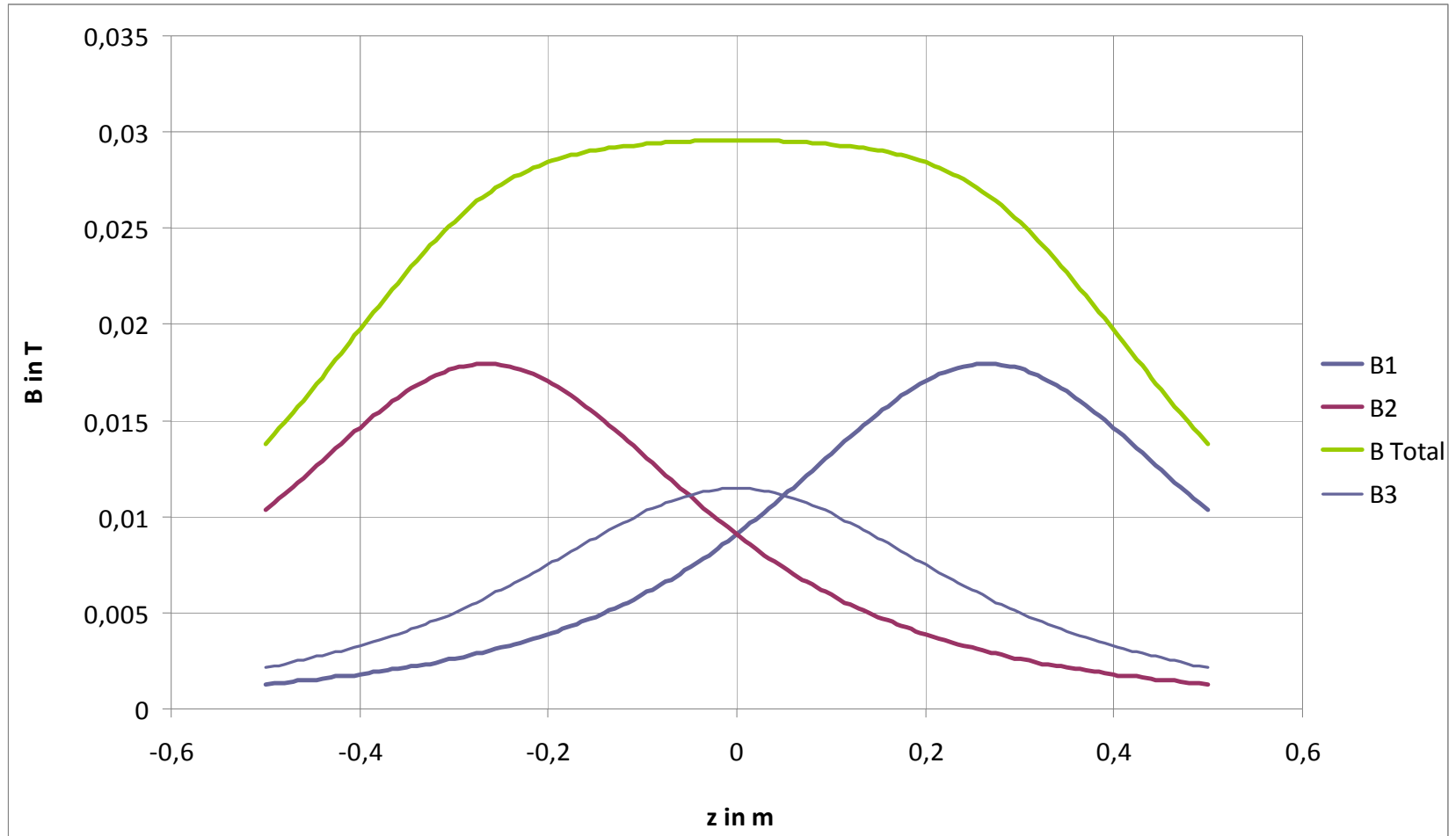
$I_1=9000A$ $I_2=6377A$

Ordre 4: two symmetric coils ($2b=0.532$) I_1 & one central coil I_2



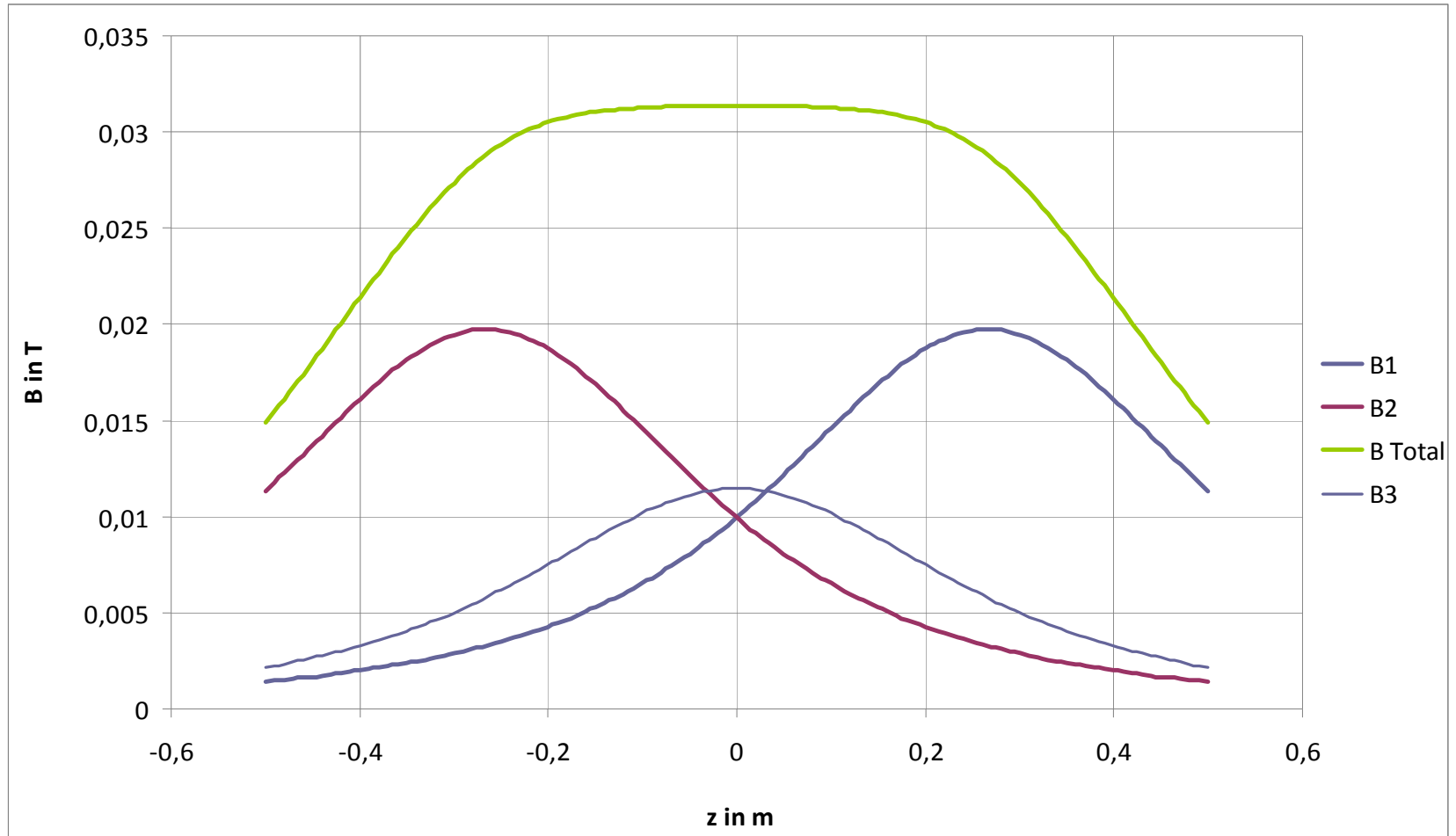
$I_1=10000A$ $I_2=6377A$

Ordre 4: two symmetric coils ($2b=0.532$) I_1 & one central coil I_2



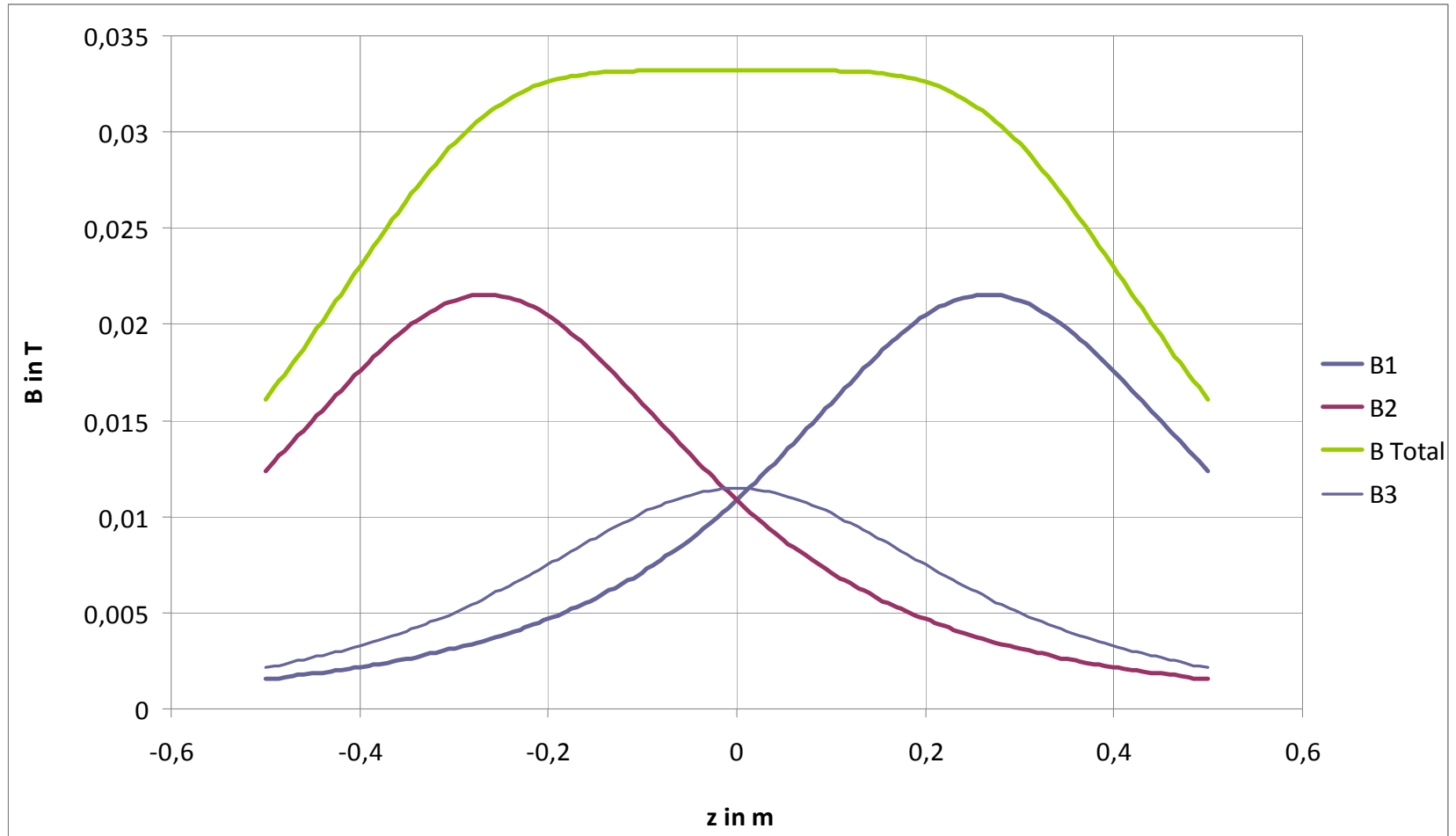
$I_1=11000A$ $I_2=6377A$

Ordre 4: two symmetric coils ($2b=0.532$) I_1 & one central coil I_2



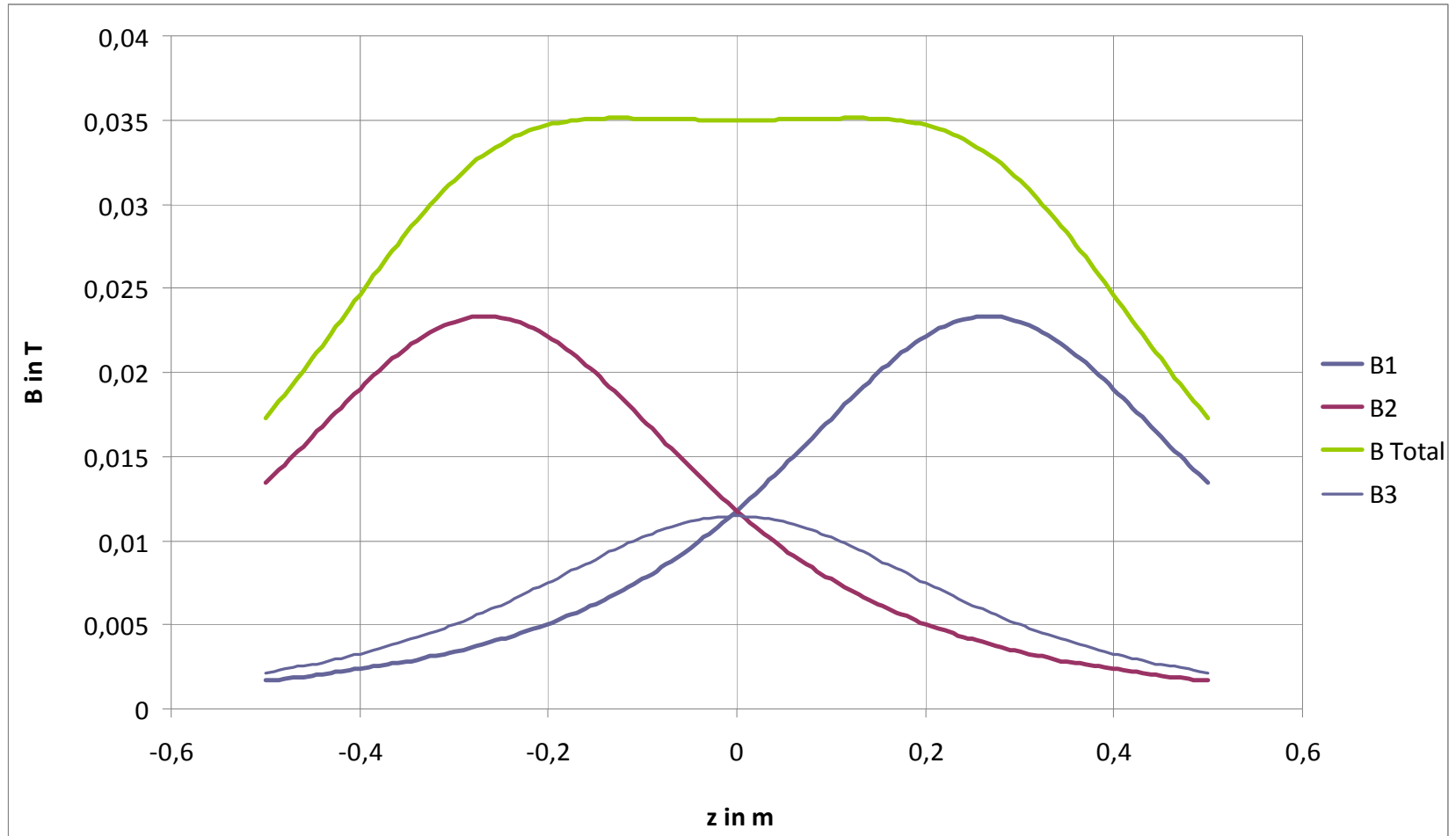
$I_1=12000A$ $I_2=6377A$

Ordre 4: two symmetric coils ($2b=0.532$) I_1 & one central coil I_2



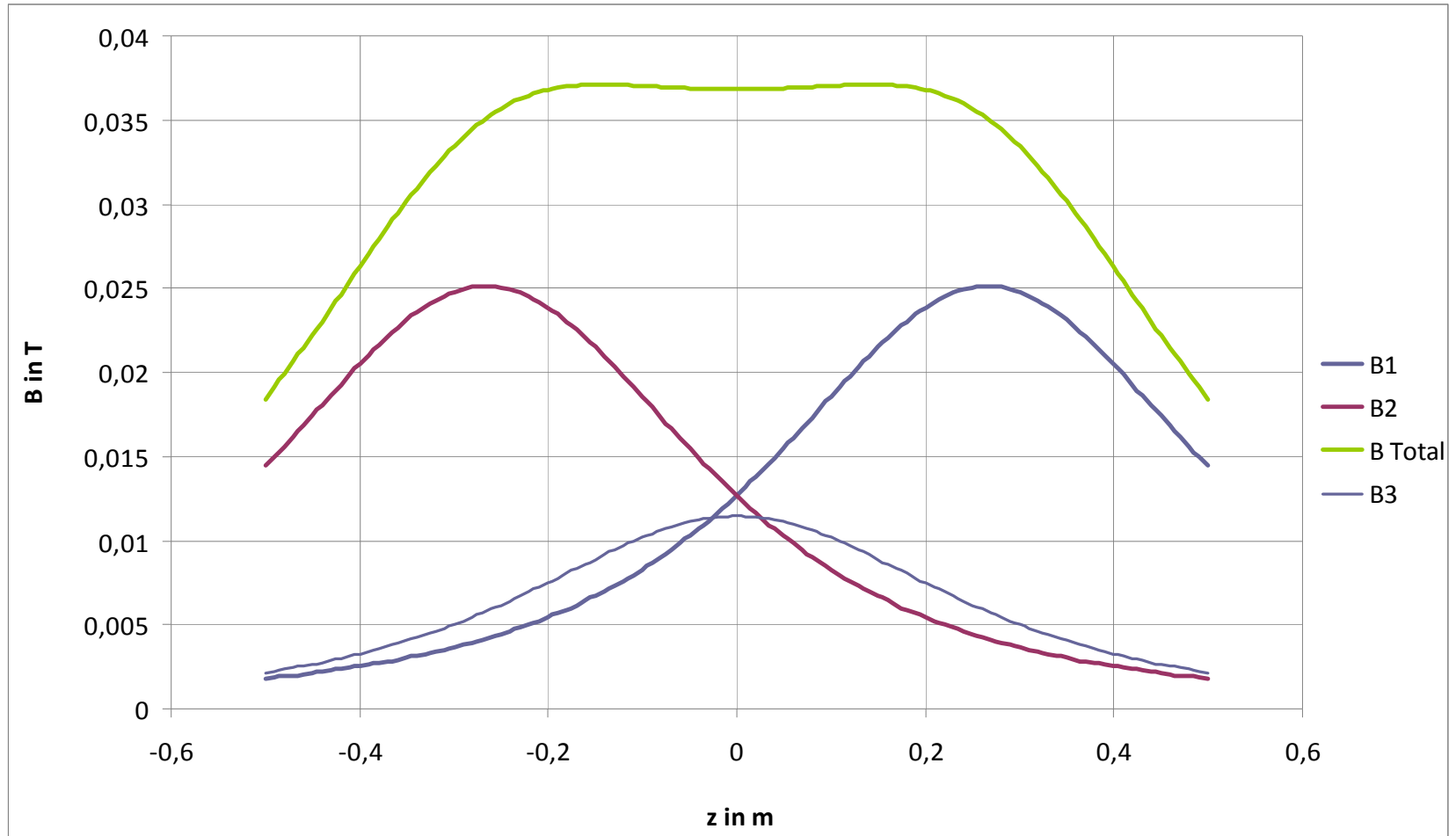
$I_1=13000A$ $I_2=6377A$

Ordre 4: two symmetric coils ($2b=0.532$) I_1 & one central coil I_2



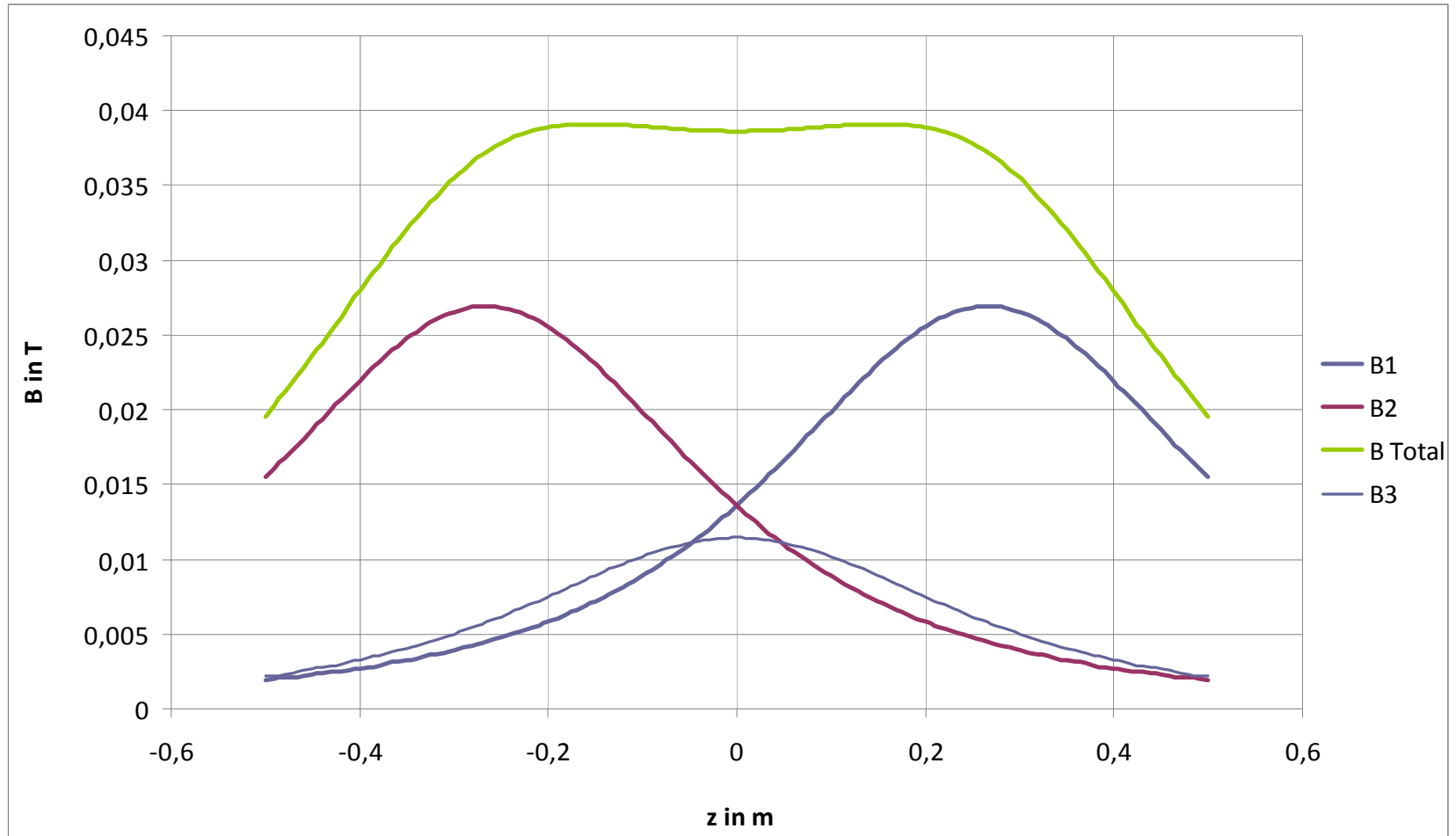
$I_1=14000A$ $I_2=6377A$

Ordre 4: two symmetric coils ($2b=0.532$) I_1 & one central coil I_2



$I_1=15000A$ $I_2=6377A$

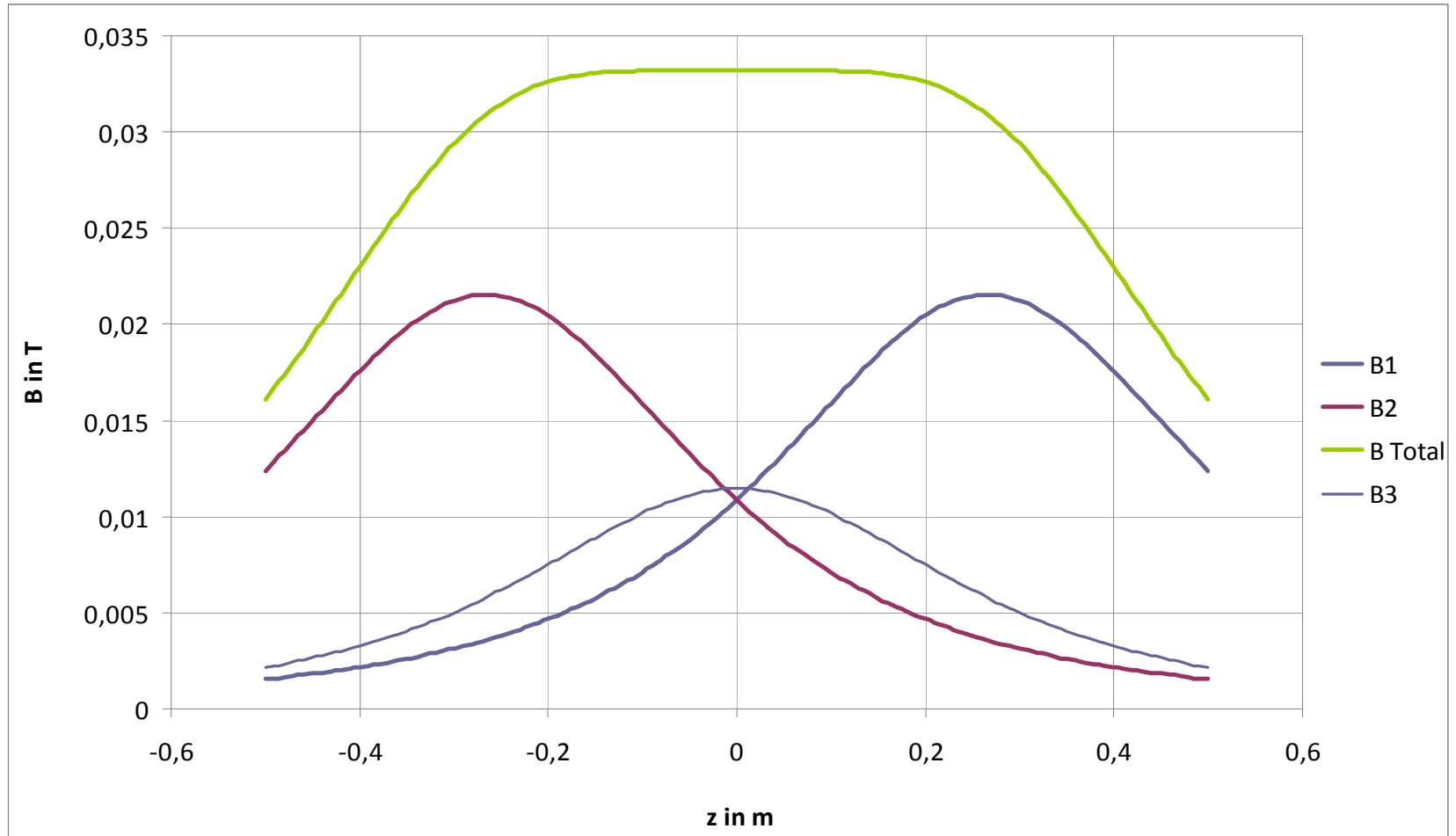
Ordre 4: two symmetric coils ($2b=0.532$) I_1 & one central coil I_2



1000 ppm in $\pm 115\text{mm}$

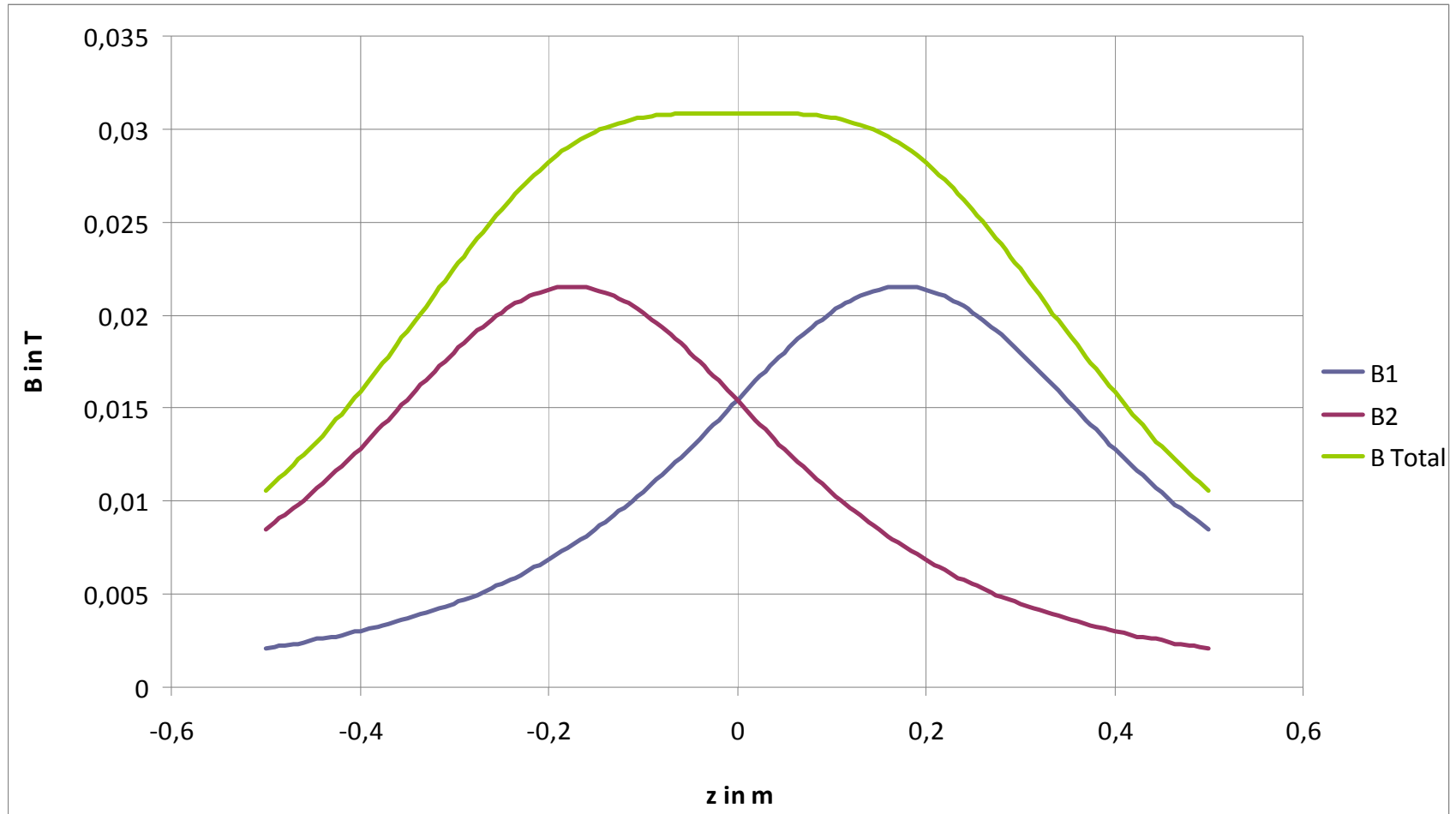
$I_1=12000\text{A}$ $I_2=6377\text{A}$

Ordre 4: two symmetric coils ($2b=0.532$) I_1 & one central coil I_2



$a=0,350$ $2b=0,350$

Ordre 2: 2 Helmholtz coils (12000A)



How to increase the field

My Magnets (Part 2 of 3 Parts)

—Passage from Francis Bitter's *Magnets: The Education of a Physicist*

... By using a variable-size wire in the construction of a coil, I found it possible to increase the magnetic field at the center by a factor of just 1.52 over the best design for a coil with a uniform winding. Therefore, by going into all kinds of practical complications, one could improve the performance of coils only one and a half times or a little more. However, this now was settled; there was no use worrying about it any more. In the end these calculations did show me a practical way of improving the performance of coils by an appreciable amount.

Bitter Magnet

Bitter's design employed a conductor in the form of a stack of annular plates, each with a slit and separated by a thin sheet of insulation except over a sector. The slit allows the bare sector to pressure-contact the next plate's bare sector, enabling the current to commute from one plate to the next in a quasi-helical path as it flows from one end of the stack to the other. Each "Bitter" plate is punched with hundreds of cooling holes. To generate a high field, tens of thousands of amperes of current are pushed through the electrically resistive stack, consuming megawatts of electrical power, which heat the stack. This heat is removed by water forced through the cooling holes at high velocity, ~ 20 m/s. A silhouette of two nested "Florida-Bitter" plates, developed in the 1990s at the National High Magnetic Field Laboratory (NHMFL), is shown in Fig. 3.18 [3.9]. A radial slit in each plate is clearly visible. Also note that each water passage hole is not circular as in Bitter's plates; the elongated—in the direction of current—shape was first developed by Weggel at M.I.T. in the 1970s. The outer plate of the set here is 148 mm in diameter; plate sizes have been more than 400 mm in diameter. The sixteen large holes are for axially clamping the plates with tie rods. A key feature of the Bitter magnet construction is that it is modular, consisting of many similar plates. Plate thickness, mechanical properties, and electrical properties can be tailored to the axial position to optimize magnet performance.

NHMFL Bitter Magnet

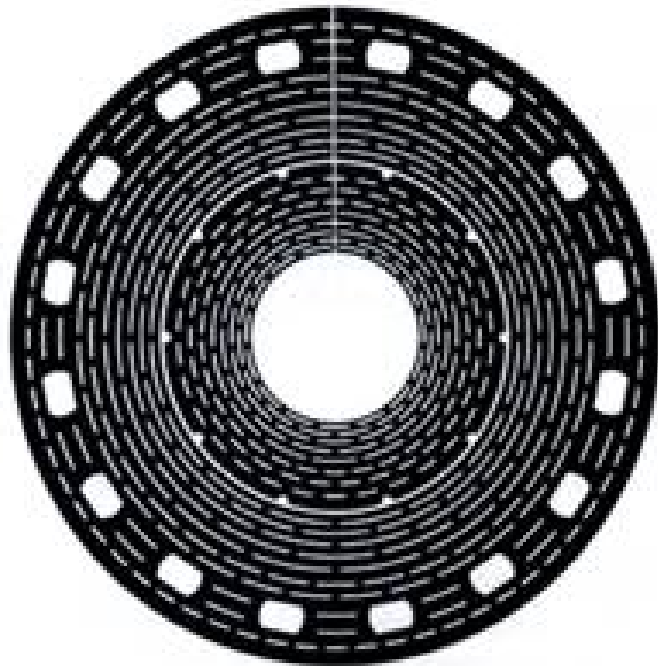


Fig. 3.18 Silhouette of two nested "Florida-Bitter" plates, with an outer plate of 140 mm in diameter, in "water" magnets at the National High Magnetic Field Laboratory [3.9].

NHMFL 45 T Hybrid Magnet

Figure 3.22 shows a cross-sectional view of the “water” magnet, SCM, and some auxiliary components of the 45-T hybrid magnet at the NHMFL [3.31]. The water magnet has four nested coils; it generates a center field of 31 T at 24 MW. The SCM, consisting of three coils, A, B, and C, operated at 1.8 K, initially generated 14 T but now operates at 11 T [3.30]; the water magnet has been redesigned to contribute 34 T at 30 MW. The system includes a superfluid helium supply cryostat, to which the SCM cryostat is connected by a pipe, shown truncated at the right, middle of the figure.

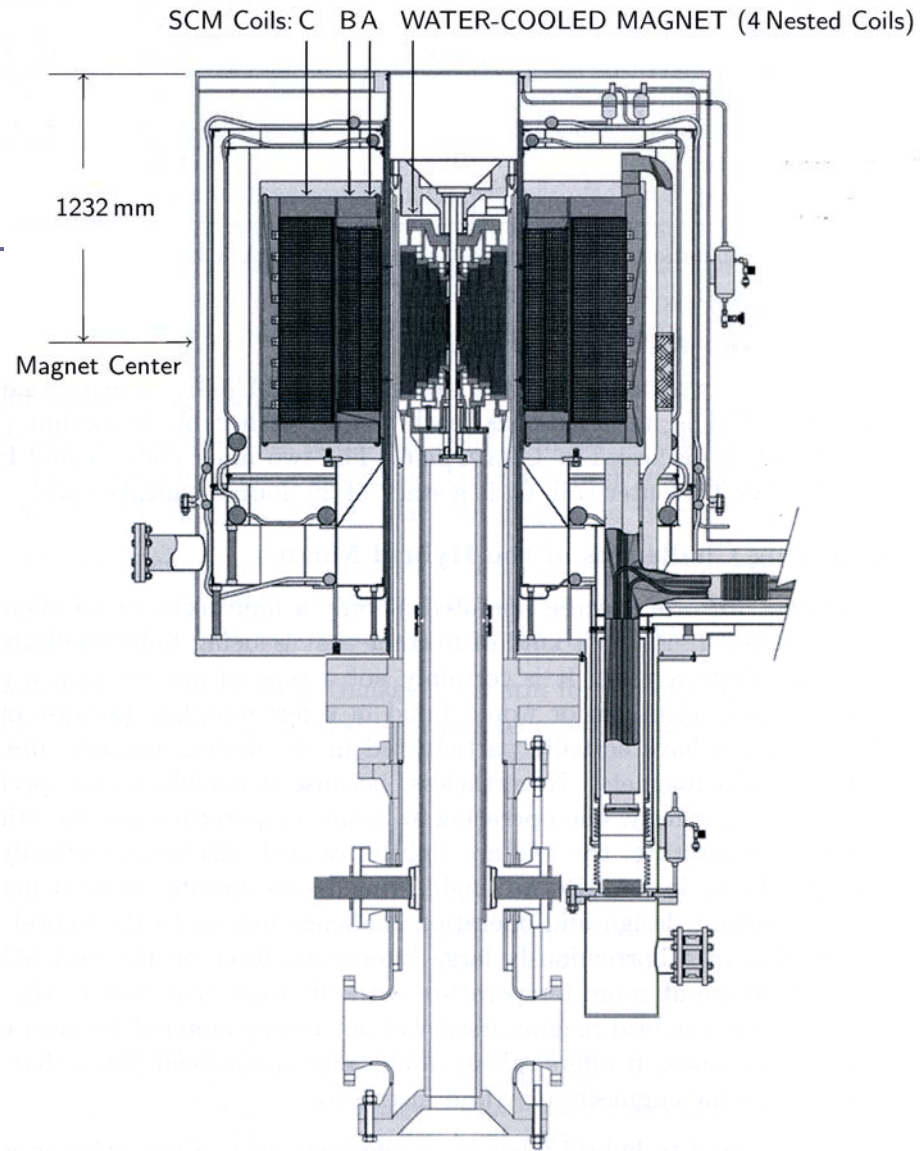
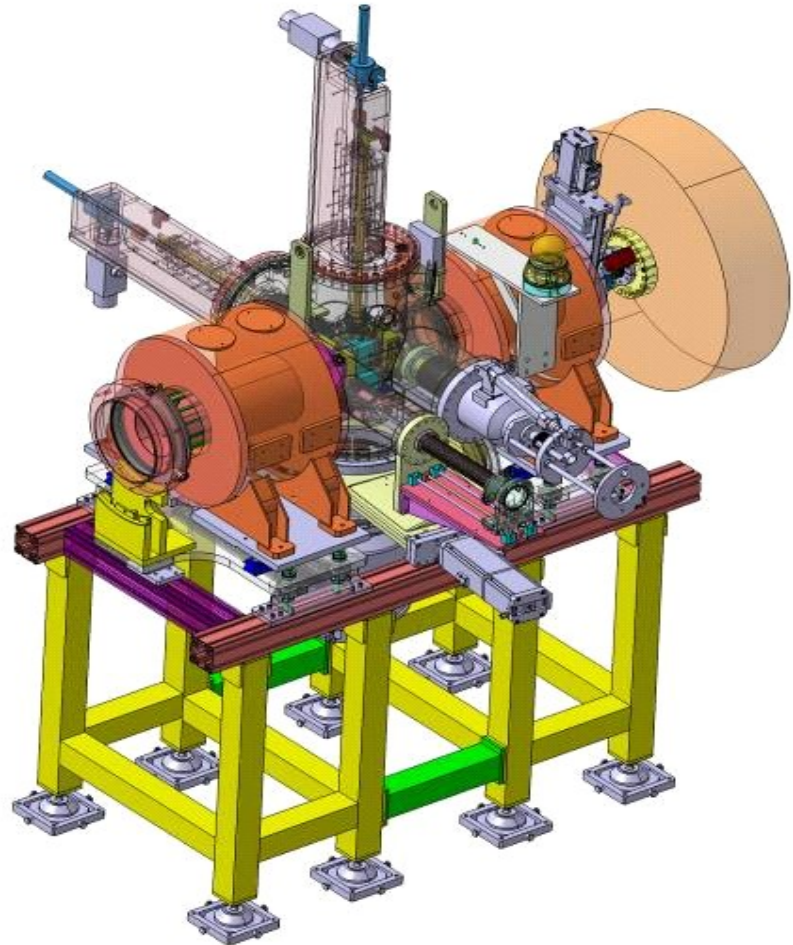
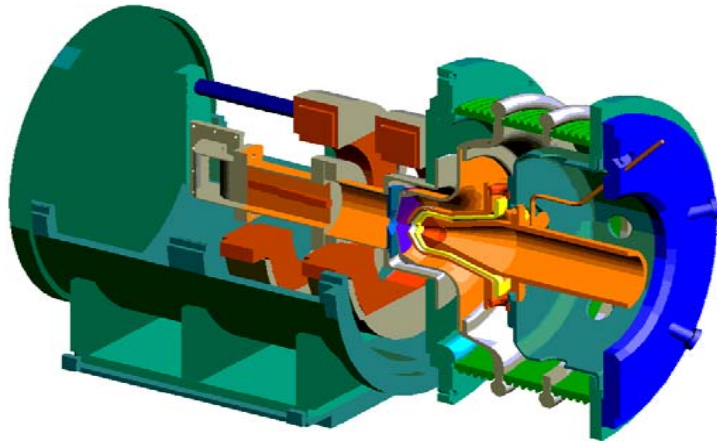
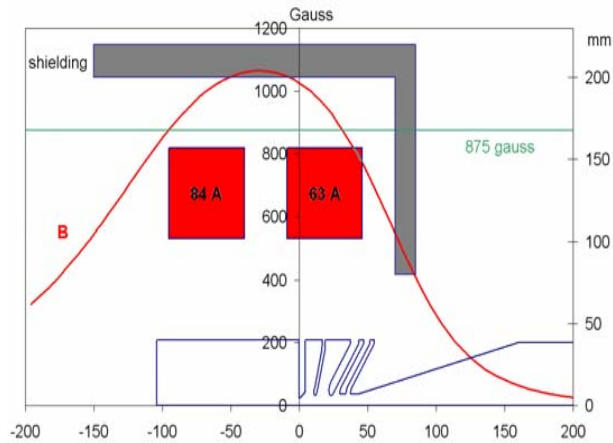


Fig. 3.22 Cross sectional view of the 45-T hybrid magnet at NHMFL [3.31].

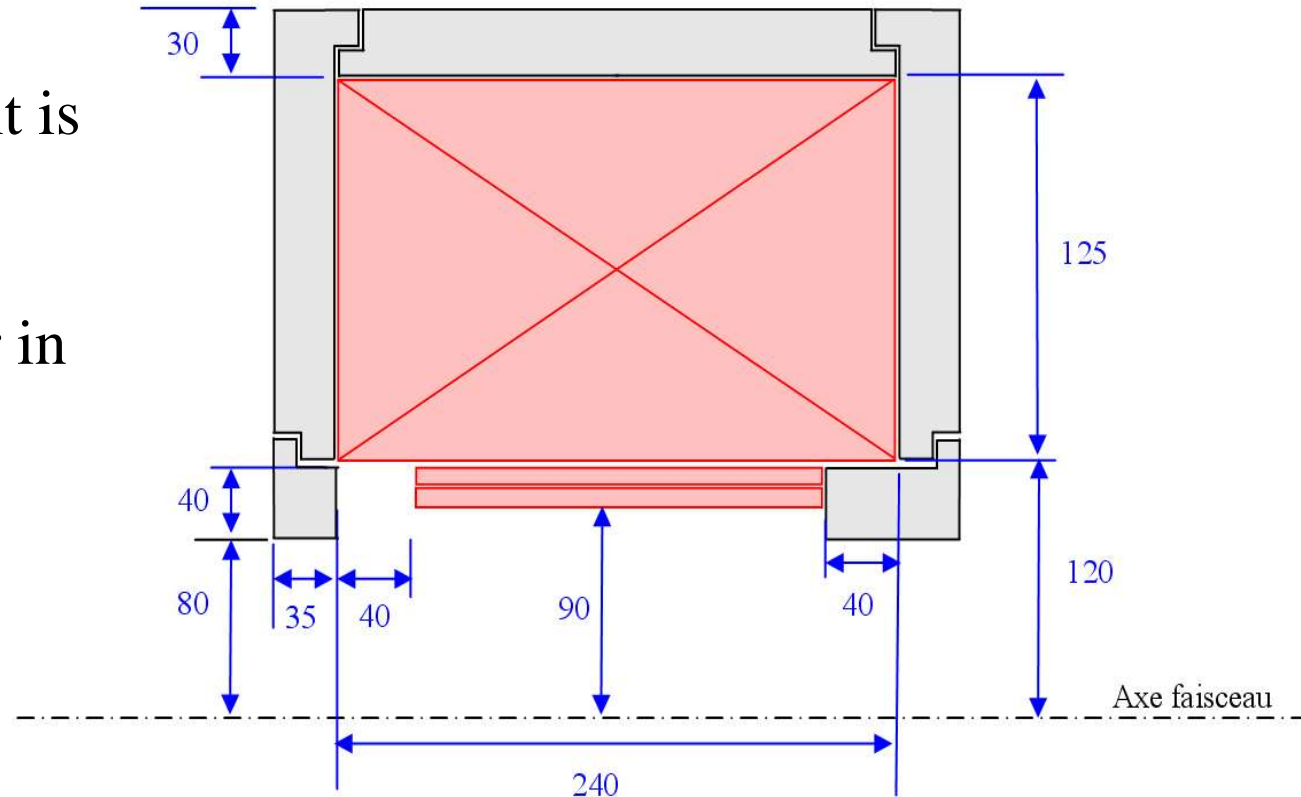
Solenoid Lens for Ion Beam source & LEBT



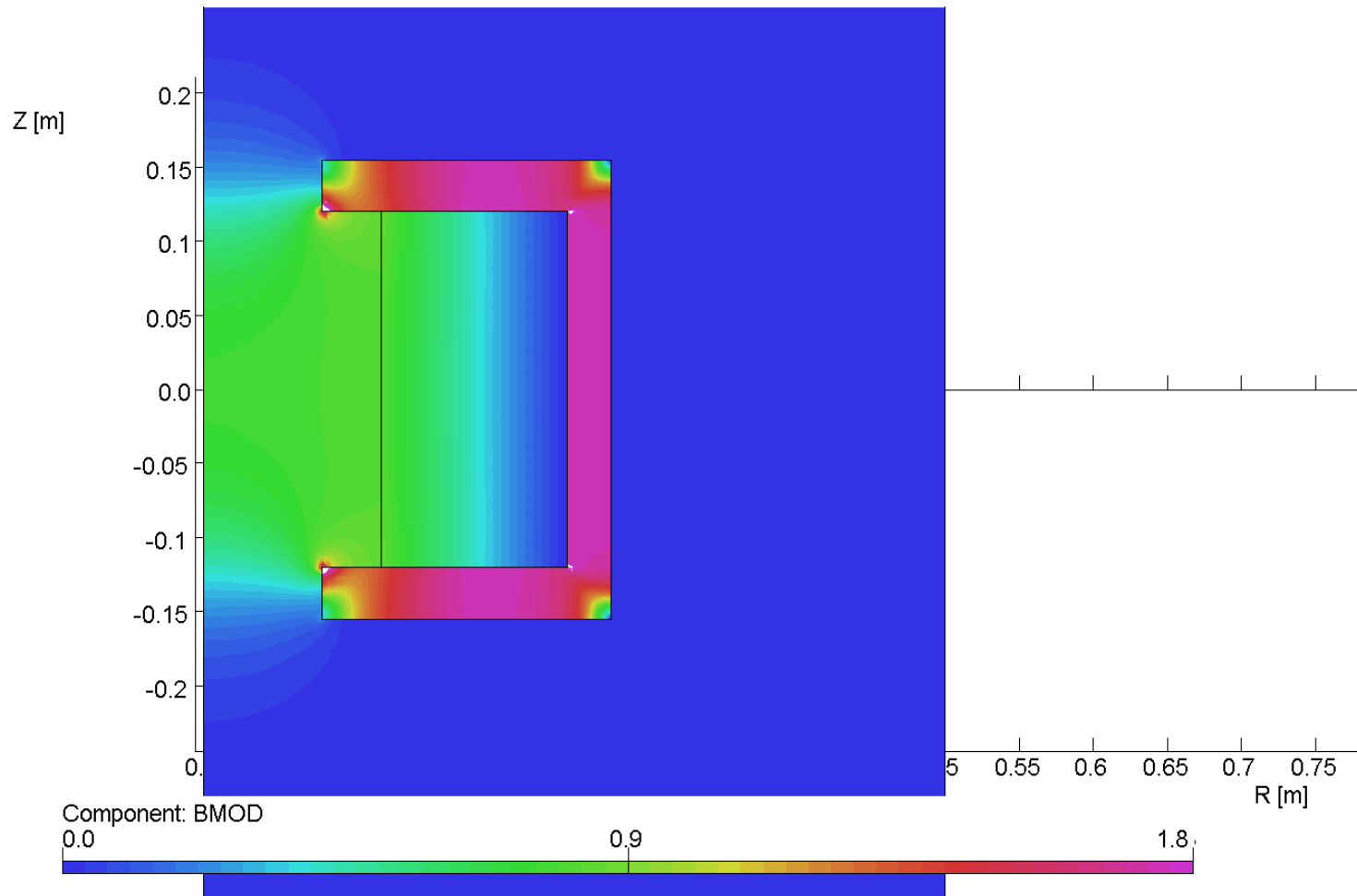
Coil parameters

1A	1mT	1.25mm
1A 160000At	1mT 0,800T	240 Factor 200

The magnetic circuit is saturated
 $J_{eng} = 10 \text{ A/mm}^2$
 An electrical Power in the range of 15kW is necessary



Field level computed by TOSCA



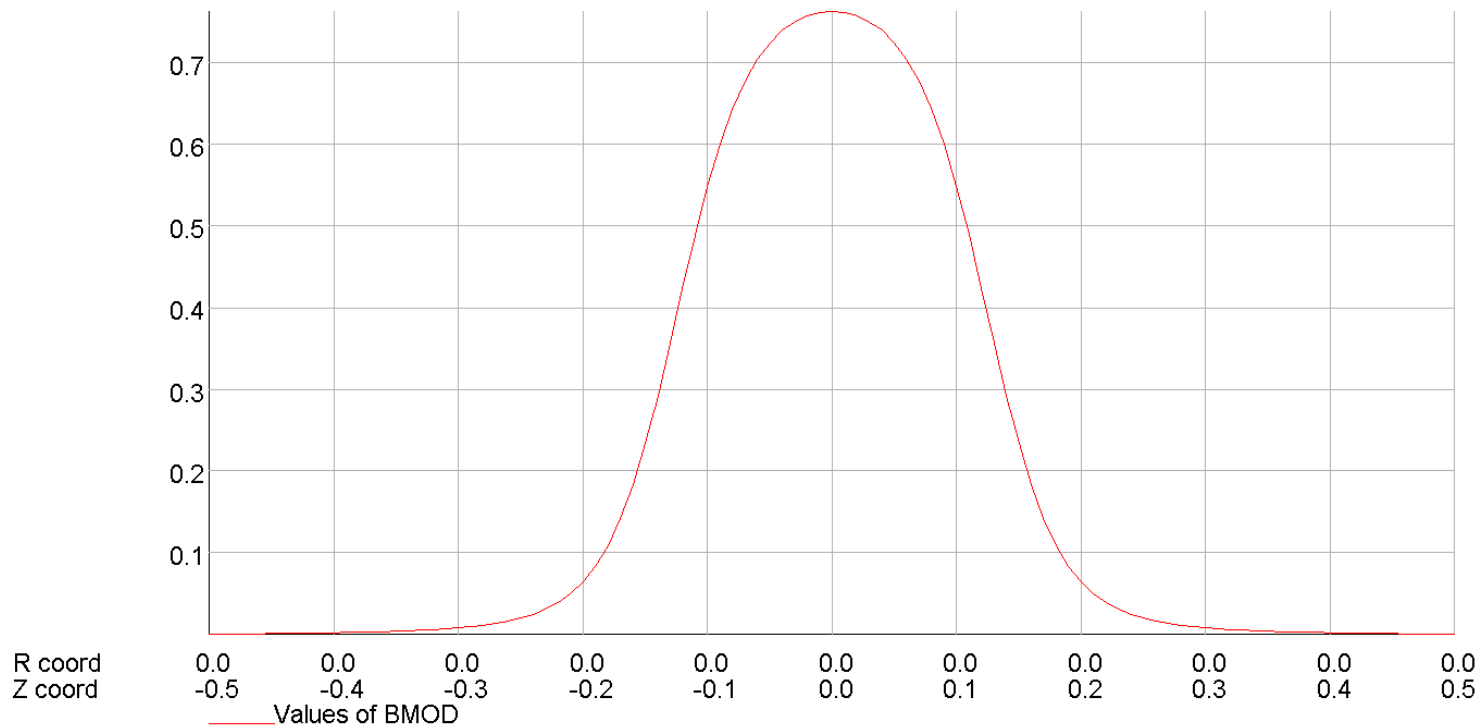
UNITS	
Length	: m
Flux density	: T
Field strength	: A m ⁻¹
Potential	: Wb m ⁻¹
Conductivity	: S m ⁻¹
Source density	: A m ⁻²
Power	: W
Force	: N
Energy	: J
Mass	: kg

PROBLEM DATA
D:\Users\delferni\VF\LB
E-IFMIF\Solen-LBE-IFM
IF\Solen-court-R105mm
-Rmax230mm\solen-co
urt-Ri125mm-Re245m
m-160000At-br40mm-b
z35mm.st
Quadratic elements
Axi-symmetry
Modified R*vec pot.
Magnetic fields
Static solution
Scale factor: 1.0
31544 elements
63989 nodes
15 regions

19/Nov/2008 13:56:43 Page 12

Vector Fields
software for electromagnetic design

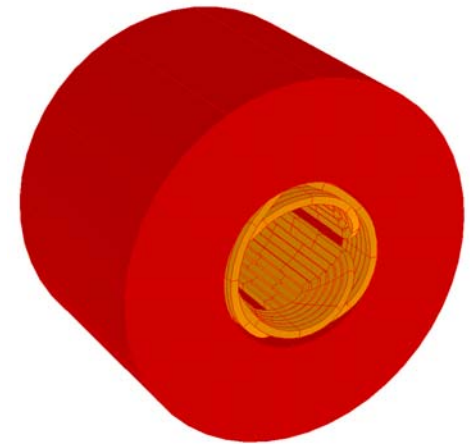
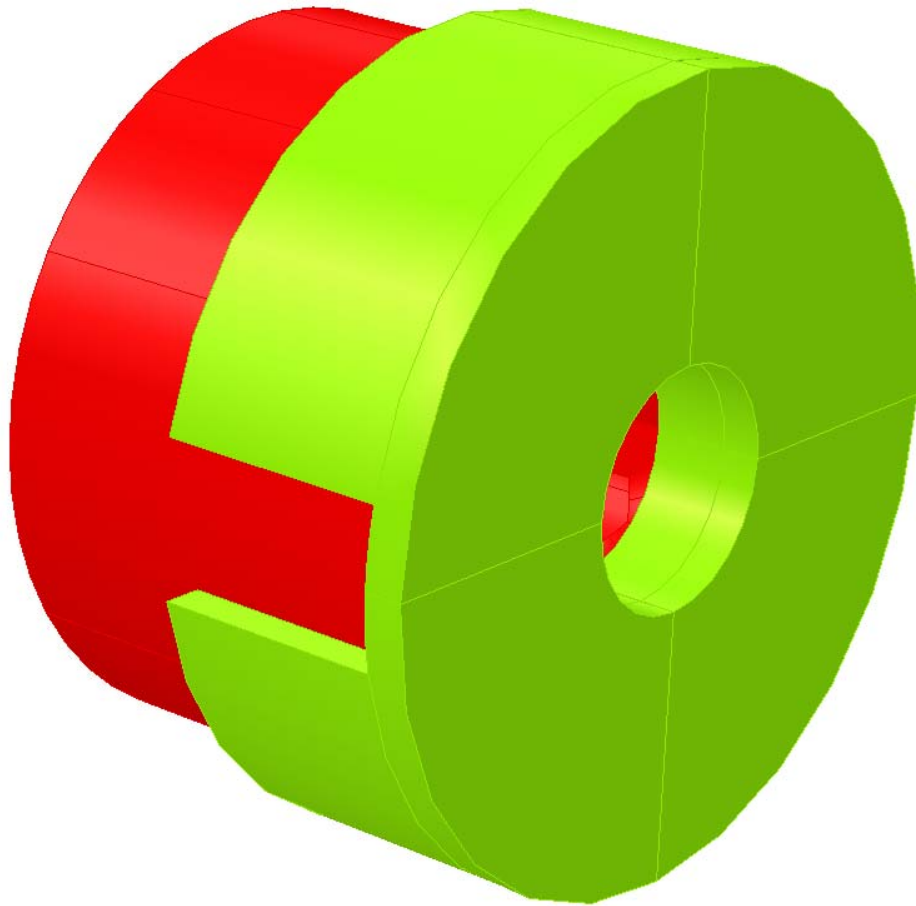
Field curve of a magnetic lens



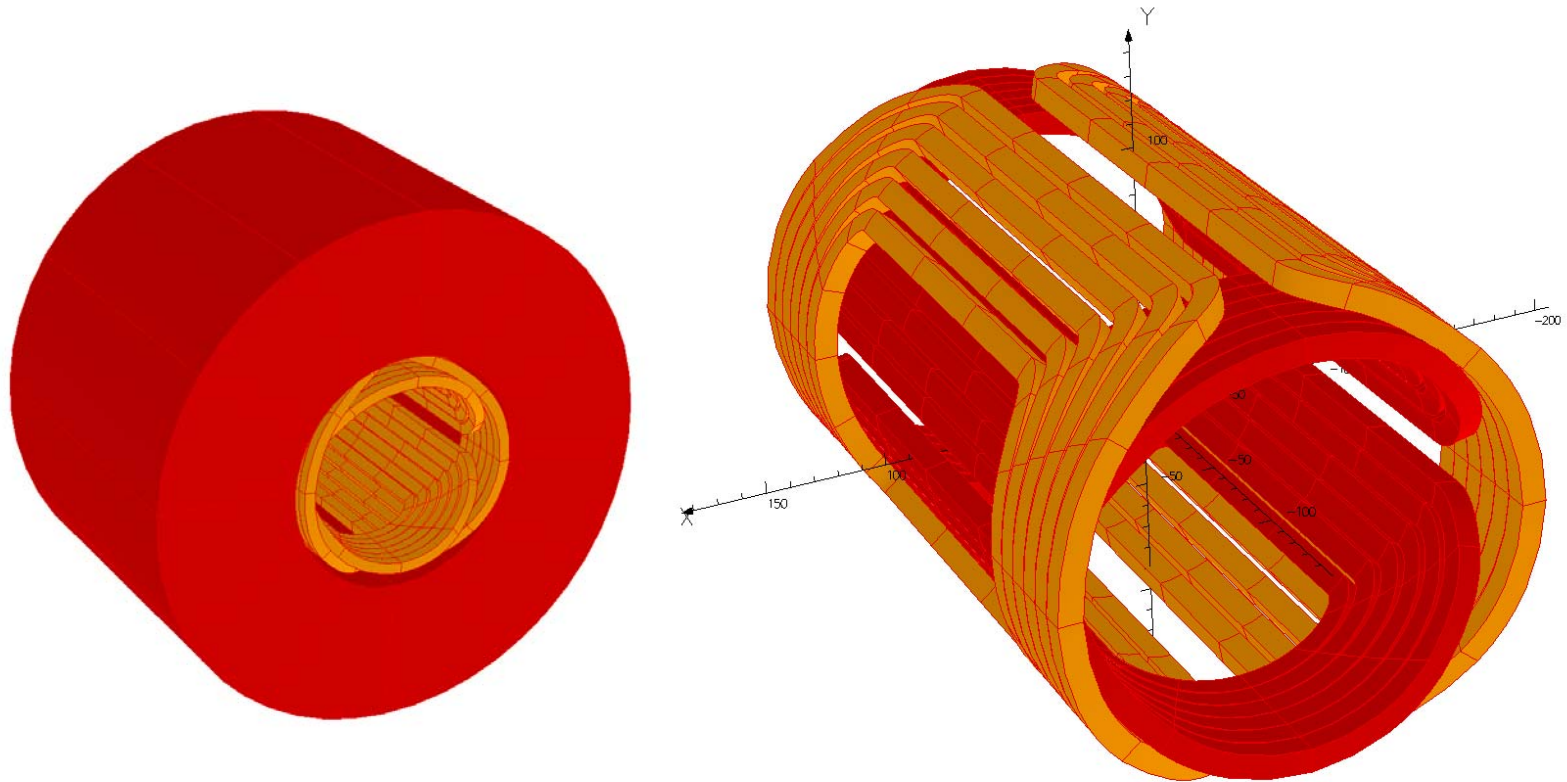
UNITS	
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Source density	: A m ⁻²
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Force	: N
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PROBLEM DATA
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Magnetic fields
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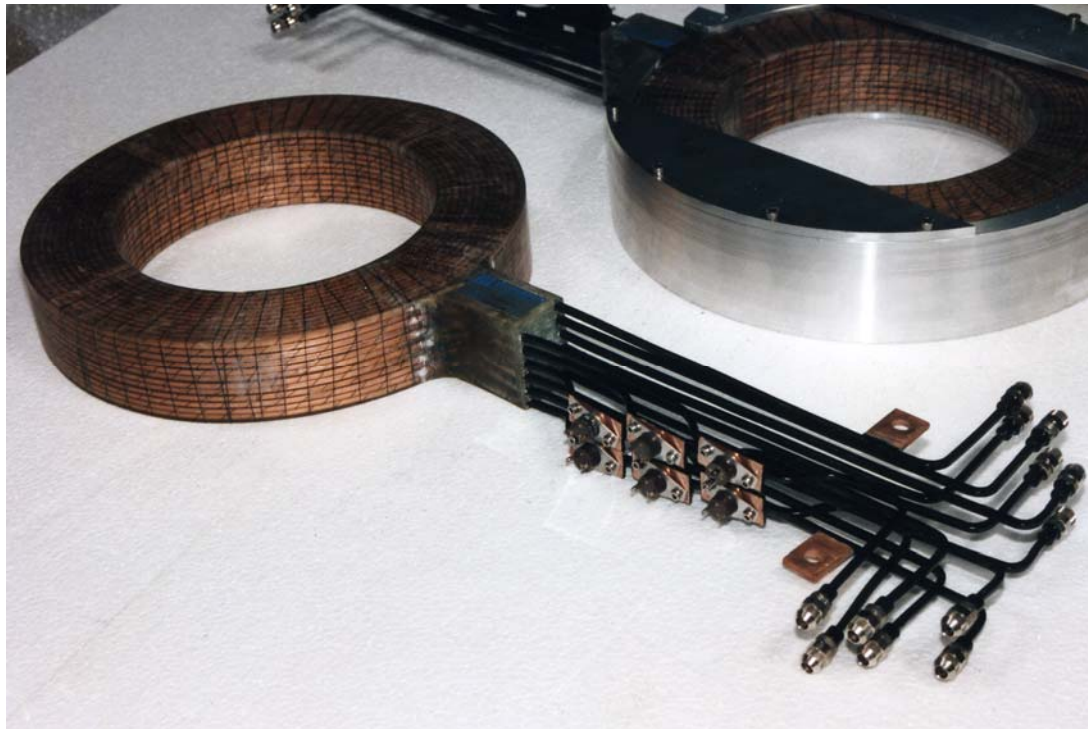
19/Nov/2008 13:56:11 Page 9



Steering coils are installed inside the solenoid to save space.



View of the coils: due to high power , the connections are space consuming.



In accelerators solenoids are used
For focusing in the low energy sections:
Electron guns
CLIC-CTF3 Probe beam LINAC



Structure 3 GHz nue



Califes: vue upstream

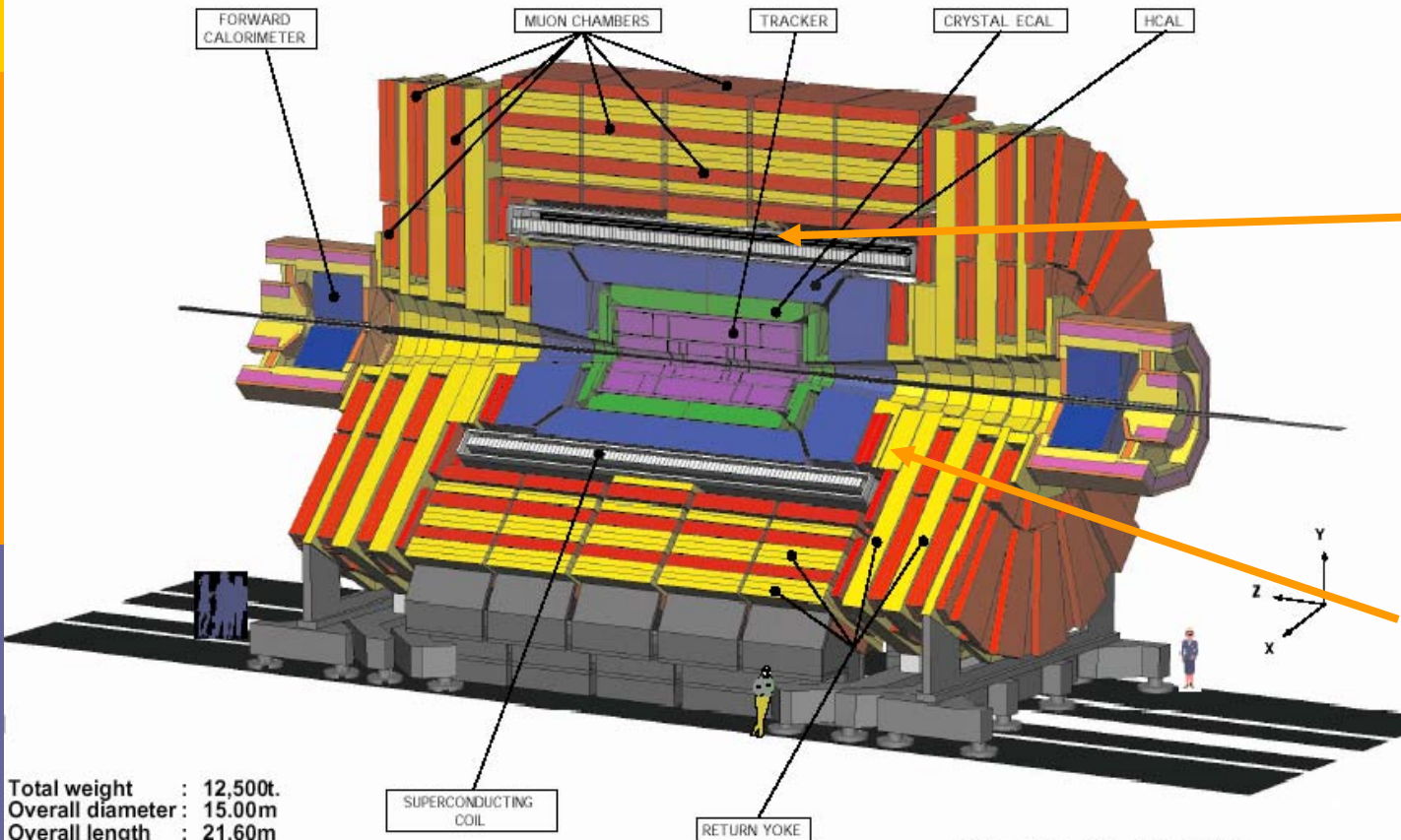
Structure 3 GHz avec solénoïdes



Vue downstream

Let's stay at CERN: CMS (Compact Muon Solenoid)

CMS A Compact Solenoidal Detector for LHC

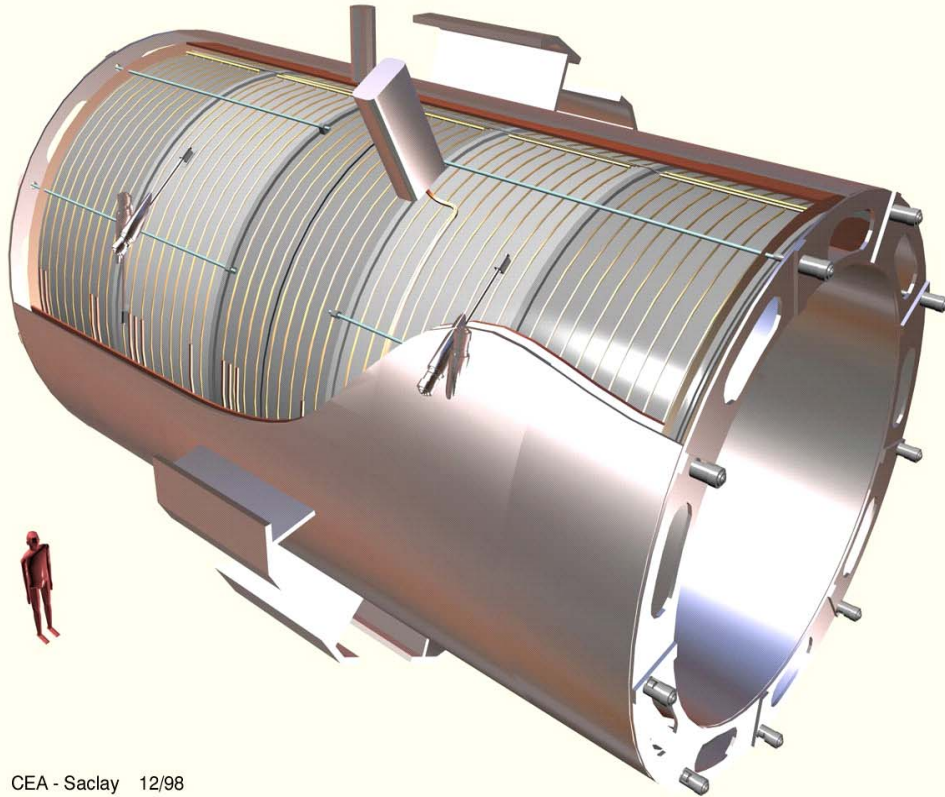


Total weight : 12,500t.
Overall diameter : 15.00m
Overall length : 21.60m
Magnetic field : 4 Tesla

CMS-PARA-001-11/07/97 JLB.PP

- Design compact
- Basé sur un solénoïde SC
- 6 m de diamètre
- 13 m de long
- Champ élevé 4T
- + culasse en fer doux

Du Virtuel au Réel : 1998-2006



CEA - Saclay 12/98
DSM DAPNIA STCM
K 0000 004

CMS Solenoïde

Champ central : 4 T
Courant nominal : 20 kA
Energie stockée : 2,6 GJ

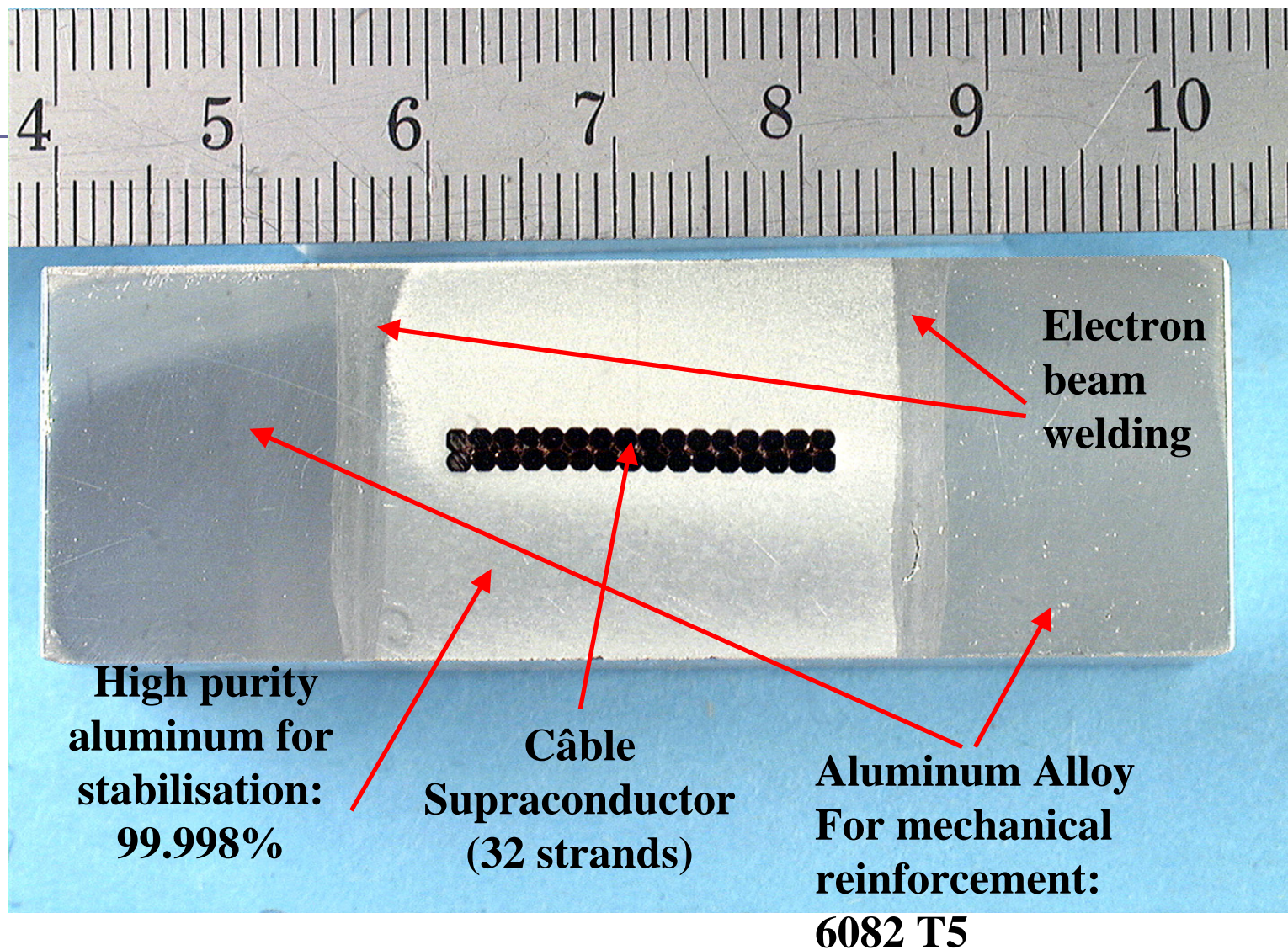
Masse froide

Longueur : 12,5 m
Diamètre interne : 6 m
Poids : 220 t

A few challenges of the CMS Magnet CMS

- Magnetic pressure is 6.4MPa
- Conductor 20 kA reinforced mechanically by aluminum alloy
- Stress=
- Winding in 5 modules, each of 4 layers chacun. The winding is practised in an outer mandrel
- Magnetic pressure 6.25 MPa
- Attraction force between modules: $6.25 \text{ MPa} \cdot 20 \text{ m}^2 = 12000 \text{ tons}$
- Stores energy 11,6 kJ/kg in the cold mass

CMS conductor

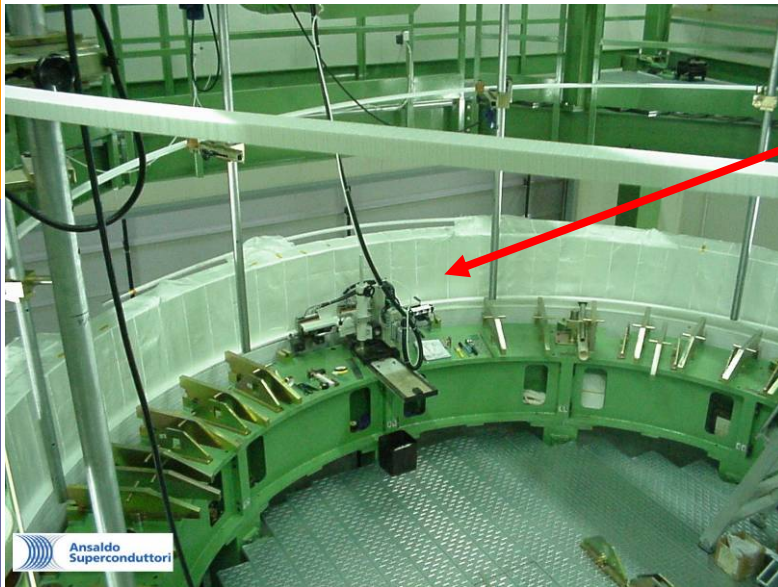
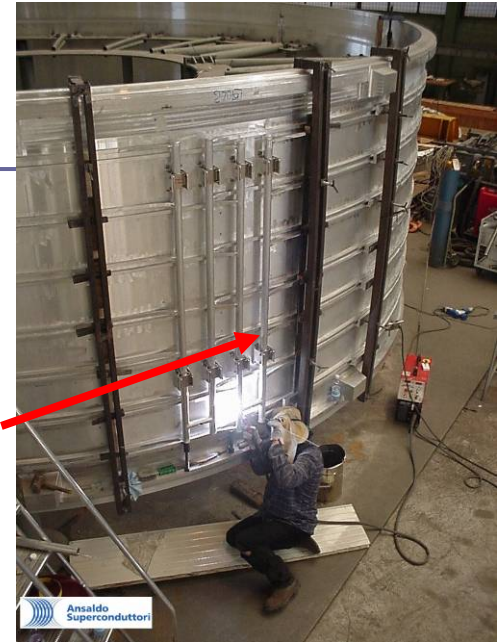


Manufacturing of modules (06/04)



Polymérisation
CB-1

Finition CBO

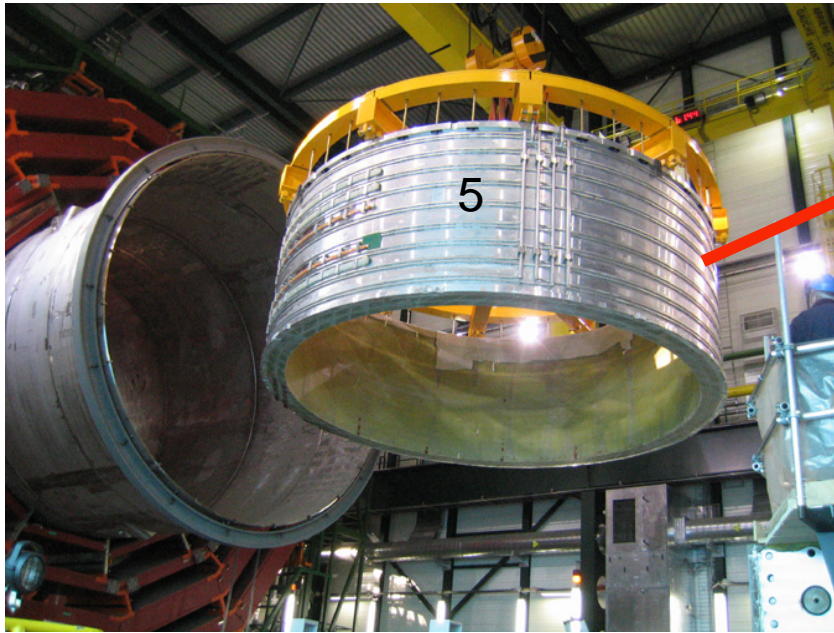


Winding CB+1

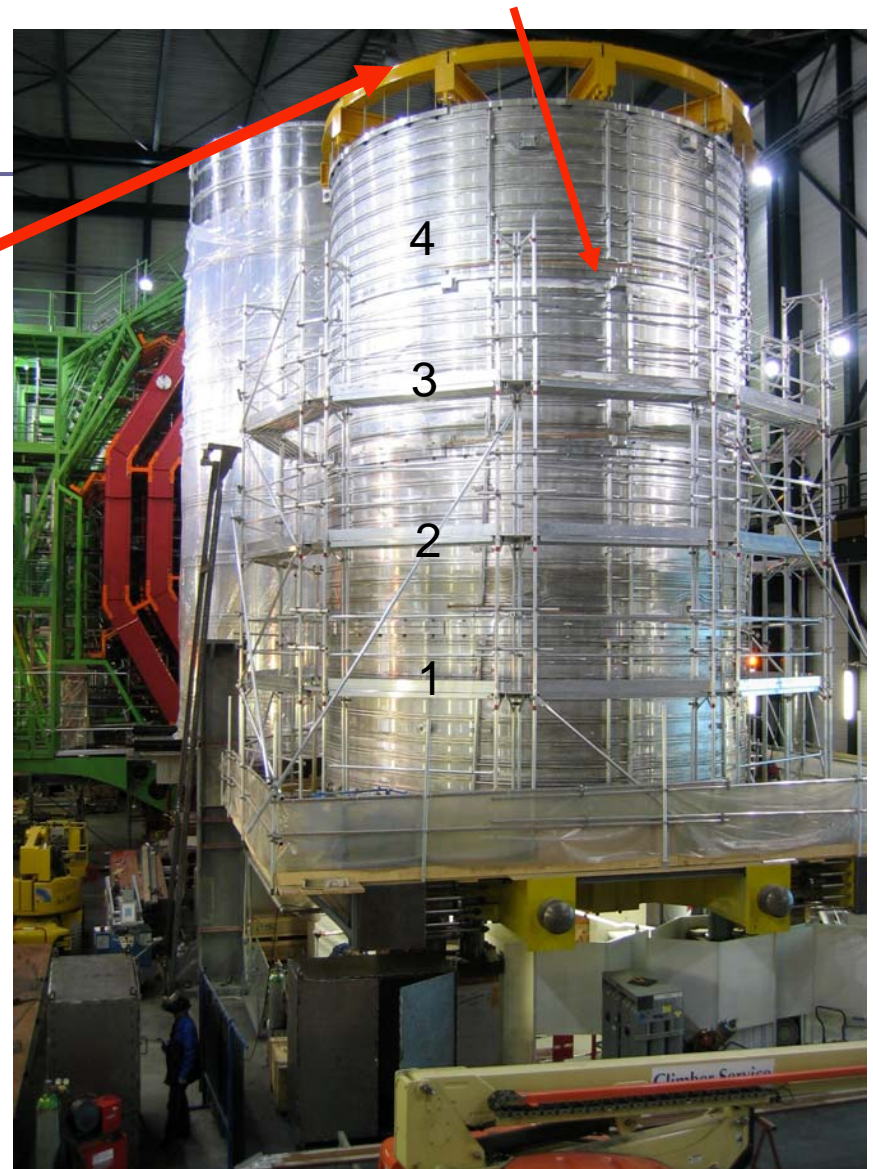
outer
cylinder
CB+2



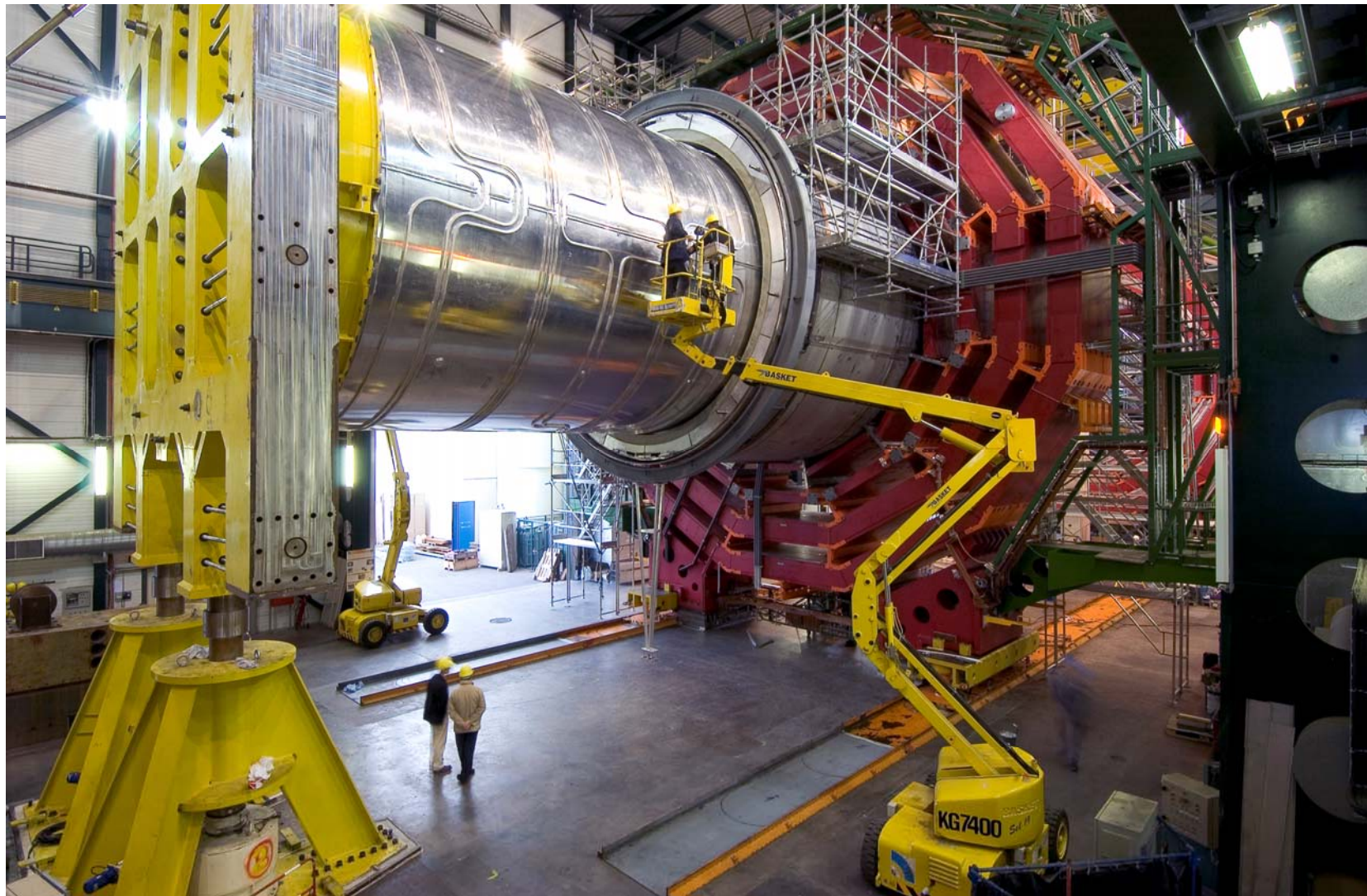
Assemblage de la bobine en vertical



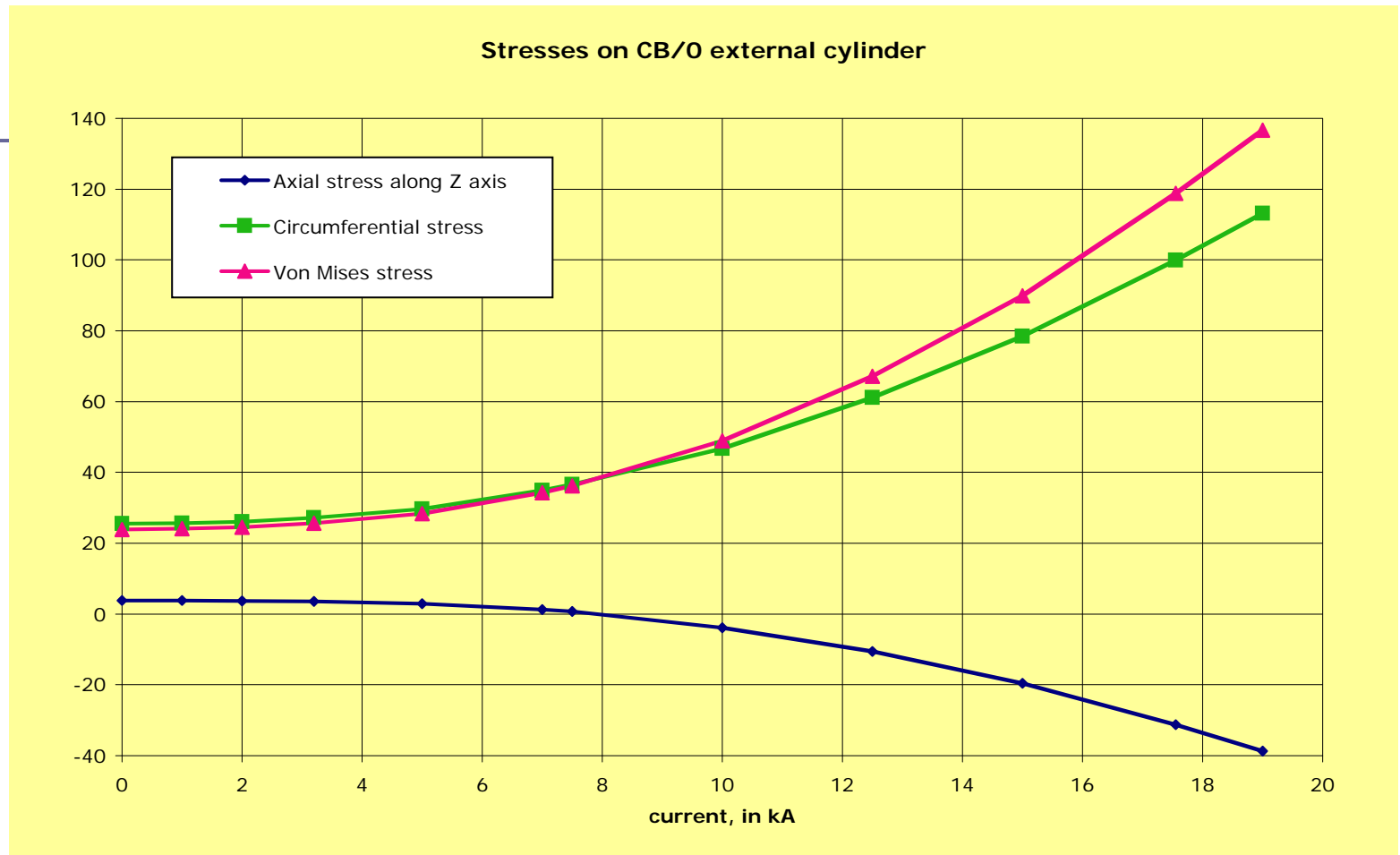
Rings between the modules are taking the axial attraction force or 12000 tons which is just magnetic pressure (6MPa) cross the section(20 m²)



Août 05 : insertion of cold mass in the vacuum vessel



Efforts on the cold mass



- Contrainte de Von Mises mesurée à 4T : 138 MPa
- En accord complet avec les calculs de 1998.

MRI limits



Magnet 1.5T (GE) SHFJ/CEA

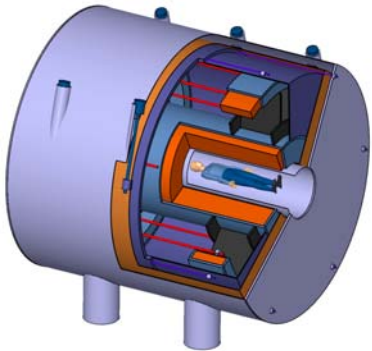


Magnet 3.0T (Bruker) SHFJ

1 mm
1s



Magnet 9.4 T GE 600 mm (USA)

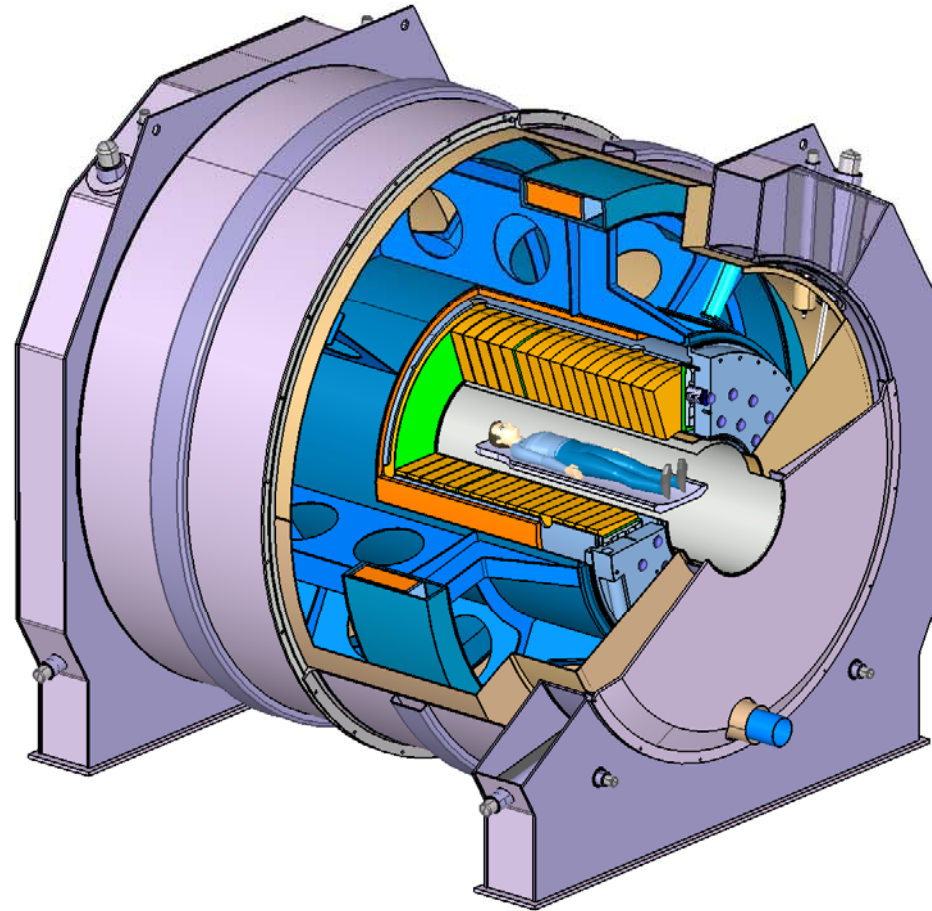


Iseult 11.7 T

0.1 mm
0.1s

Increase spatial and time resolution →

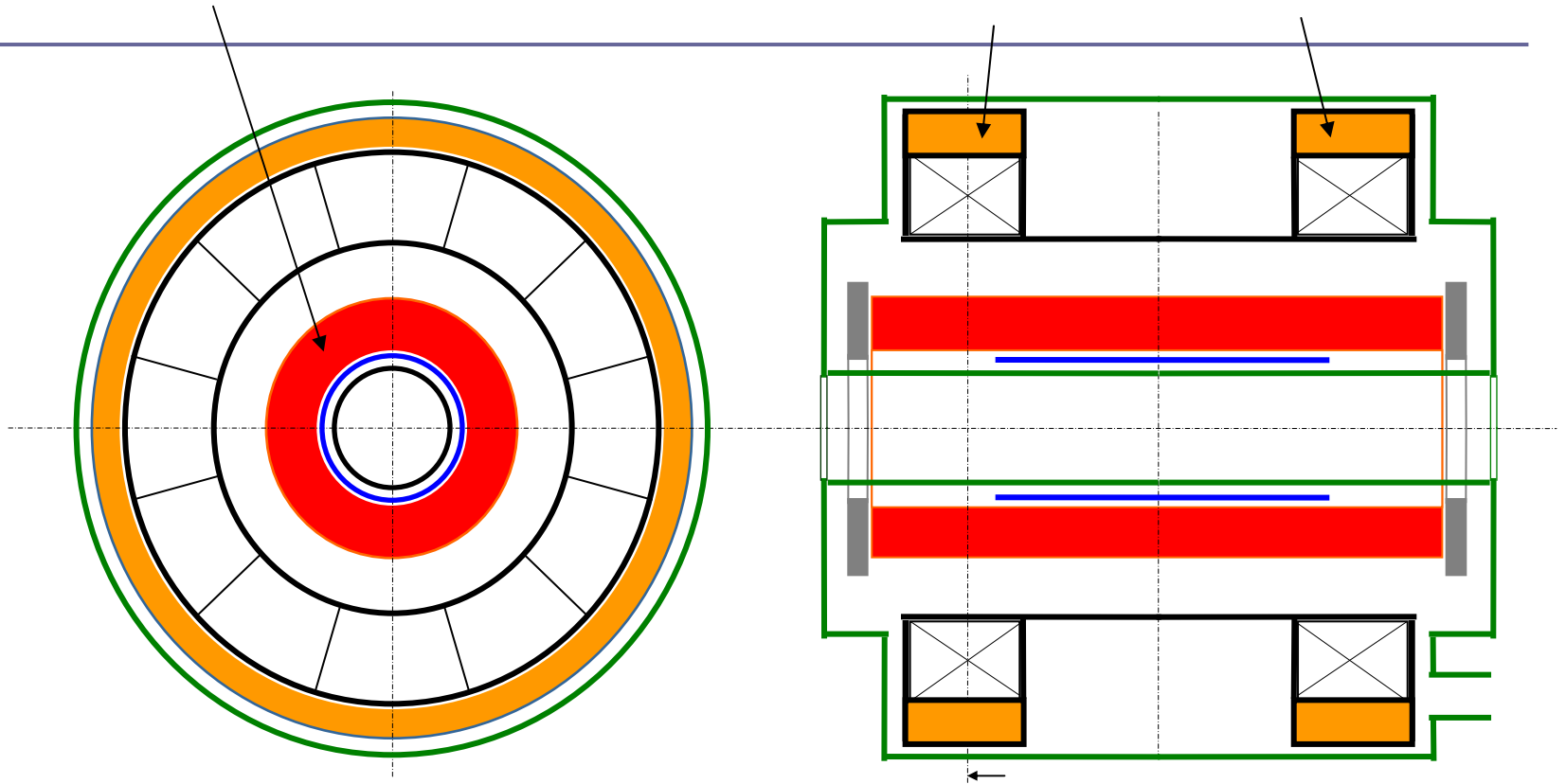
MRI Magnet architecture



Winding

Main Coil

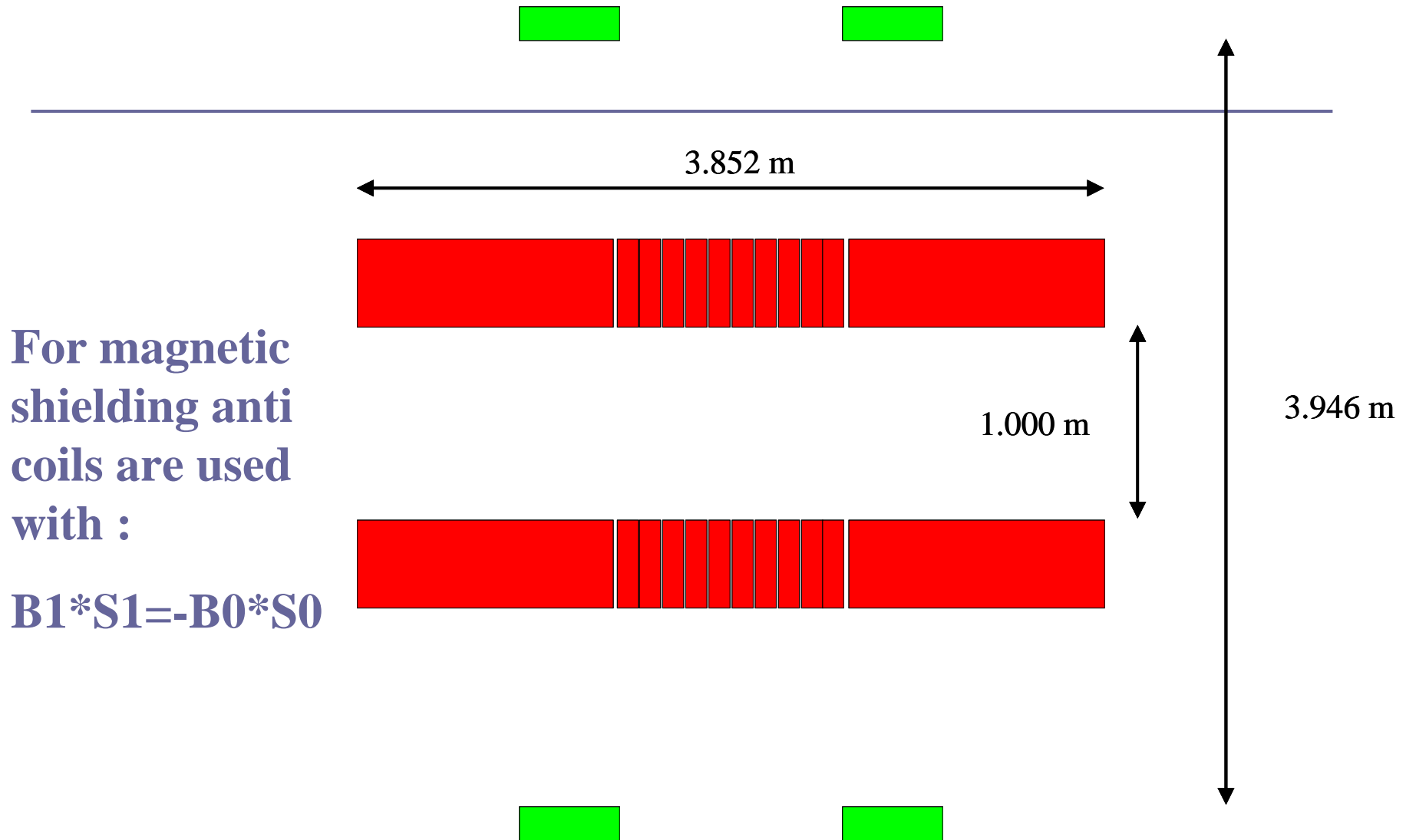
compensation coil



Main parameters of the ISEULT Magnet

- 
- **B0 / Warm Bore** 11.75 T / 900 mm
 - **Field stability** 0.05 ppm/hr
 - **Field Homogeneity** < 0.5 ppm over 22 cm DSV
 - **Stray field (5 G line)** 9.6 m axial, 5.1 m radial
 - **Stored Energy** 330 MJ
 - **Inductance** 301 H
 - **Winding Current Density** 28.2 A/mm²
 - **Temperature** 1.8 K
 - **Current** 1487 A
 - **Conductor size** 9.2 mm x 4.6 mm
 - **Conductor weight** 60 t of NbTi

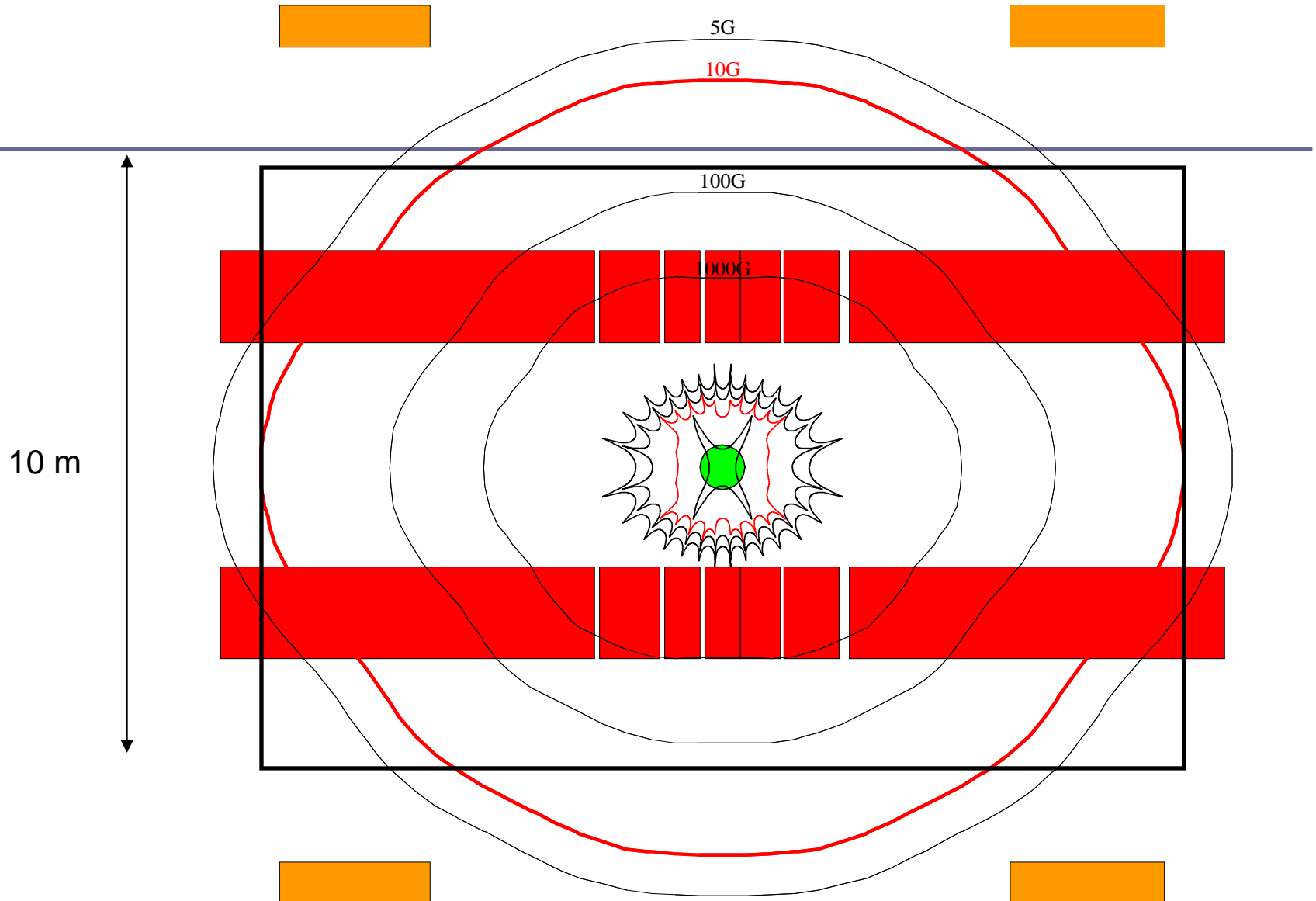
Winding pack design



For magnetic shielding anti coils are used with :

$$B1 * S1 = -B0 * S0$$

Rôle du blindage actif (2/2)

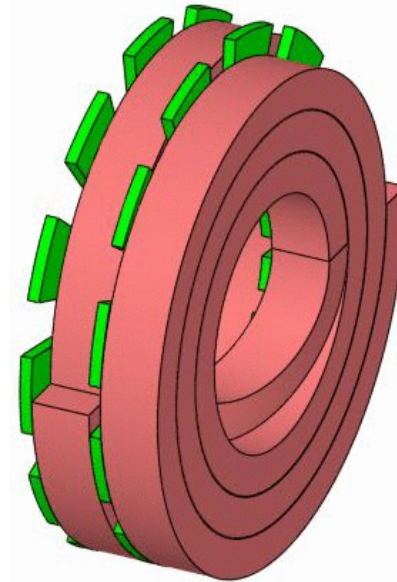
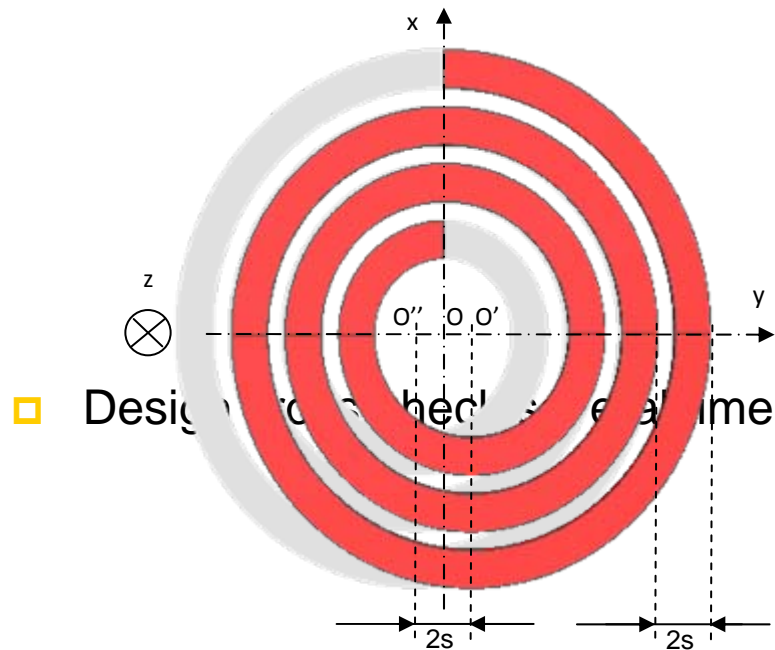


Winding pack design

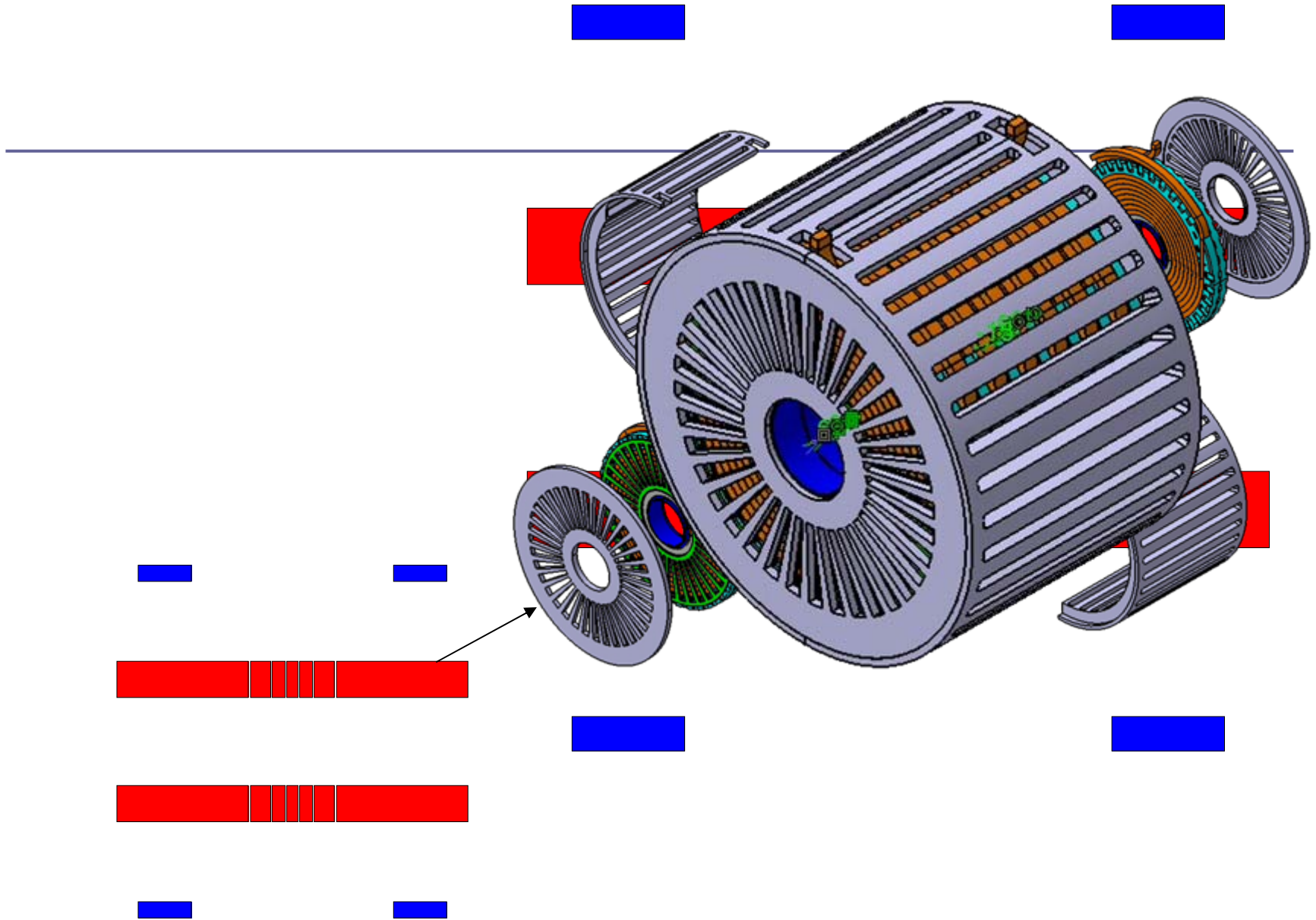
Original Double Pancake design

- The objective is to design a magnet theoretically **intrinsically** homogeneous

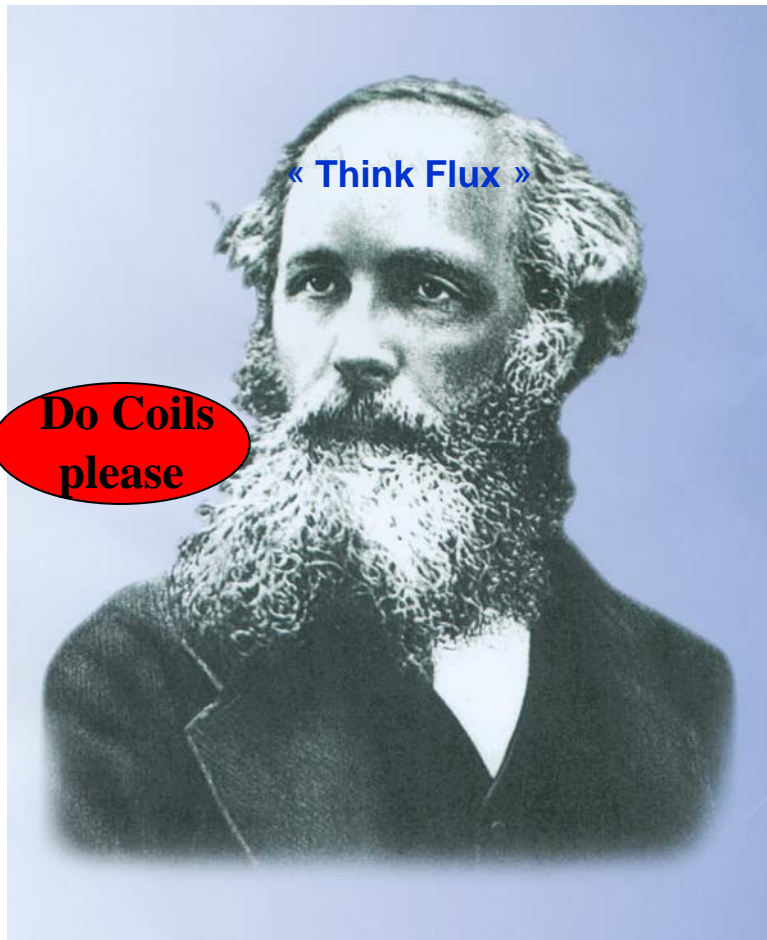
$$B_z(r, \theta, \varphi) = B_0 + \sum_{n=1}^{\infty} r^n \left[Z_n P_n(\cos \theta) + \sum_{m=1}^n \begin{pmatrix} X_n^m \cos m\varphi \\ Y_n^m \sin m\varphi \end{pmatrix} W_n^m P_n^m(\cos \theta) \right]$$



Assembly of the double pancakes

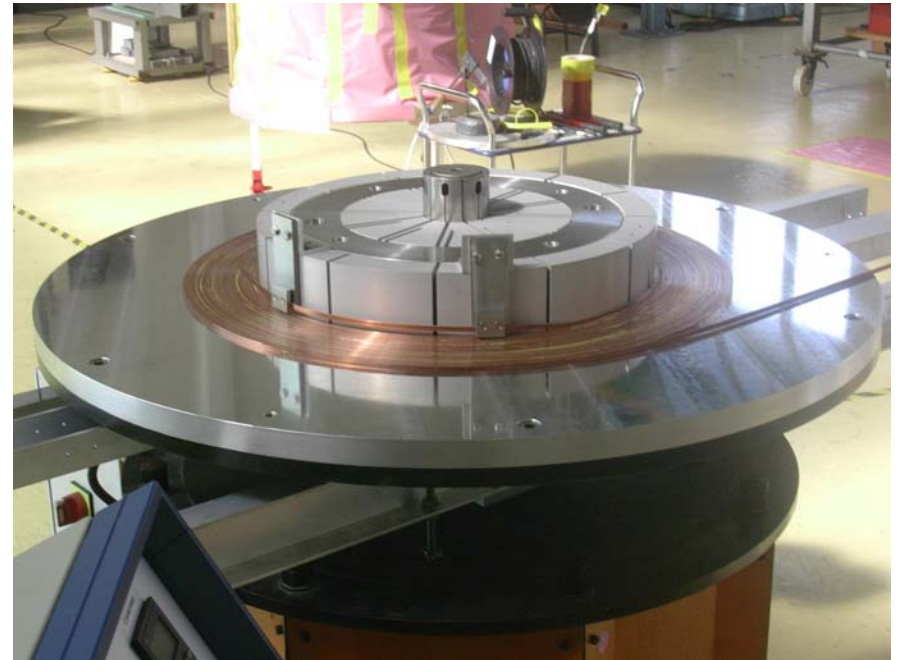


Summary and Conclusion



James Clerk MAXWELL

I hope I have given you a flavour the magic world of solenoids



Field Analysis of Solenoidal Coil

$$H_z(r, \theta, \varphi) = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} r^n (n+m+1) P_n^m(u) (A_n^m \cos m\varphi + B_n^m \sin m\varphi) \quad 3, 1,$$

Spherical Harmonics.

with $u = \cos\theta$

If $\frac{\partial}{\partial \varphi} = 0$

$$H_z(z) = \sum_{n=0}^{\infty} z^n (n+1) A_n^0$$

