Manufacturing and calibration of search coils

Part II – Calibration

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Contents

Introduction

Main assumption: **2D field** (i.e thin lens approximation, $L_{magnet} << \lambda_{betatron}$)

[see also: lecture by A. Wolski]

$$\Phi(\vartheta) = \Re\left(\sum_{n=1}^{\infty} \frac{\kappa_n}{r_{ref}^{n-1}} \mathbf{C}_n e^{in\vartheta} \right)$$

Measurement(s)

$\{\mathbf{C_n = B_n + iA_n}\}$

time

coil position

$\{\kappa_n\}$

$\Delta\Phi_{coil}$

FFT coefficients

Result analysis

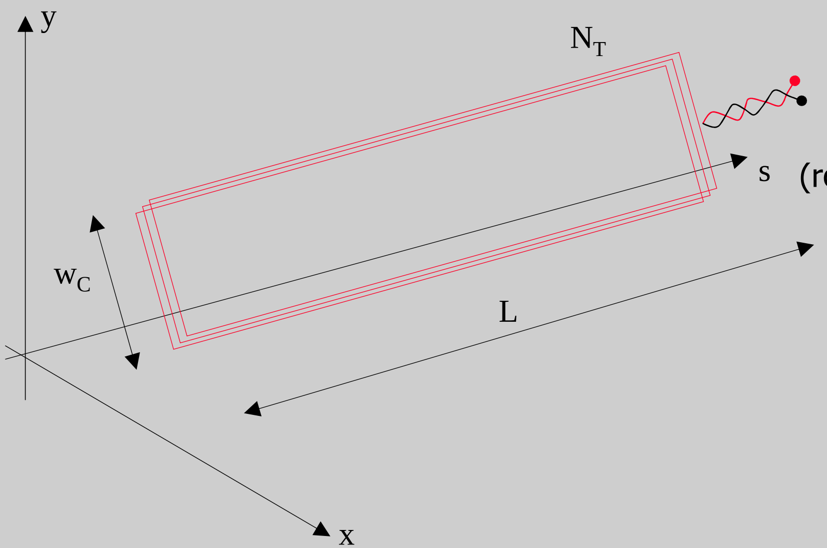
$$\mathbf{C}_n = \frac{2r_{ref}^{n-1}}{N} \frac{\Psi_{n+1}}{\kappa_n}, \; n = 1..\tfrac{N}{2}$$

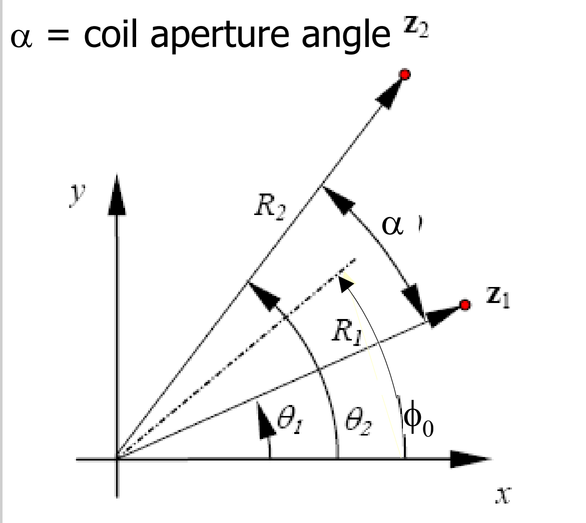
Azimuthal sampling over N points + inverse FFT

[L. Bottura, LHC-XMT-ES-0001]

Calibration

Finding the $\{\boldsymbol{\kappa}_n\}$ needed to infer the $\{\mathbf{C_n}\}$ from Φ_{coil} (within a certain accuracy)





Further assumptions:

- Perfect rectangular geometry
- Infinitesimally thin winding cross-section

$$\kappa_n = \frac{N_T L}{n}(z_2^n - z_1^n) = \frac{N_T L}{n}\left(R_2^n e^{in\frac{\alpha}{2}} - R_1^n e^{-in\frac{\alpha}{2}}\right)e^{in\varphi_0}$$

[A. Jain, HARMONIC COILS, CAS Measurement and Alignment of Accelerator and Detector Magnets 1997]

Finite winding size correction

if square cross-section ≤ 1 mm², $R_0 \geq 10$ mm, n≤6 ⇒

no correction for a dipole, correction << 10^{-4} for a quadrupole, << 10^{-3} for higher harmonics

The correction can be either ignored or computed from the nominal geometry (no special calibration necessary)

[L. Deniau, CERN LHC-MTA IN 98-026, Coils calibration correction factors for rectangular windings]

Harmonic coil coefficients: typical cases

n	radial coil $\varphi_0=0$	tangential coil $\varphi_0=\pi/2,\ w=2R_0\sin\alpha/2$	tangential coil $\varphi_0=\pi/2,\ \alpha\approx 0$
κ_1	$N_T L w$	$N_T L w$	$N_T L w$
κ_2	$\frac{1}{2} N_T L w R_0$	$i\cos\frac{\alpha}{2} N_T L w R_0$	$i N_T L w R_0$
κ_3	$N_T L w\left(\frac{w^2}{12}+R_0^2\right)$	$-\frac{1}{3}(1+2\cos\alpha) N_T L w R_0^2$	$-N_T L w R_0^2$
κ_4	$N_T L w R_0\left(\frac{w^2}{4}+R_0^2\right)$	$-i\cos\alpha\cos\frac{\alpha}{2} N_T L w R_0^3$	$-i N_T L w R_0^3$
κ_5	$N_T L w\left(\frac{w^4}{80}+\frac{w^2 R_0^2}{2}+R_0^4\right)$	$\frac{1}{5}(1+2\cos\alpha+2\cos 2\alpha) N_T L w R_0^4$	$N_T L w R_0^4$
κ_6	$N_T L w R_0\left(\frac{w^4}{16}+\frac{5}{6} w^2 R_0^2+R_0^4\right)$	$\frac{i}{3} N_T \cos\frac{\alpha}{2}\left(4\cos^2\alpha-1\right) L w R_0^5$	$i N_T L w R_0^5$

- All coefficients can be calculated from coil length L, width w and radius R_0
- All coefficients proportional to total coil area $A_c=N_T L w$
- All coefficents increase like radius R_0^{n-1}

NB: calibration coefficients can be used at any field level – **inherently linear sensor**
However: S/N at calibration gets better at high field

Geometry measurements

N_t, L, w

Coil geometry nominal= N_T,L,w,R_0,φ_0 +imperfections

compute from definition:

$$\kappa_n = \frac{N_T L}{n}\left(R_2^n e^{in\frac{\alpha}{2}} - R_1^n e^{-in\frac{\alpha}{2}}\right)e^{in\varphi_0}$$

$$A_c = \kappa_1, w_{eff} = \frac{\kappa_1}{N_t L}, R_0 = \frac{2\kappa_2}{A_c}$$

Coil factors $\{\kappa_n\}$

Magnetic measurements

compare directly measurements to known reference field:

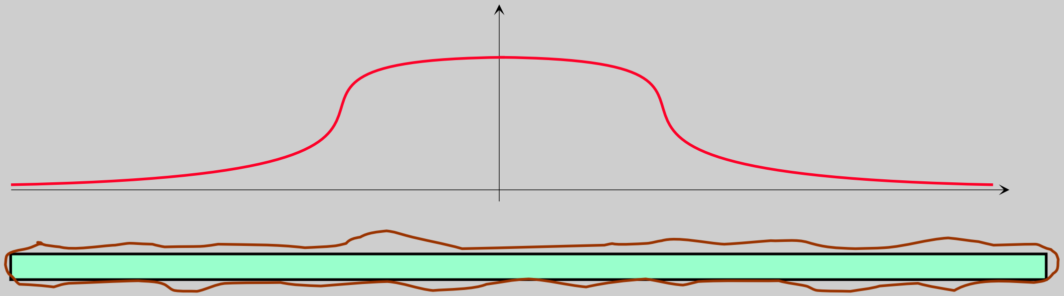
$$\kappa_n = \frac{2r_{ref}^{n-1}}{N}\frac{\Psi_{n+1}}{\mathbf{C}_n}$$

- Geometry measurement: practically only L is accurate enough (10^{-3}~10^{-4}). Moreover: if coil completely within or outside the magnet, precise determination of L not necessary.
- Geometry may be the only option when no suitable reference magnet is available (e.g. large or curved coils …)
- Magnetic measurement: best option when possible (all non-idealities automatically included in the result) Reference dipole and quadrupole are the norm; sextupole sometimes used
- Computation of κ_n from geometry: done for the orders where no reference comparison is available. In this case, accuracy is estimated from calculations, repeatability, cross-checks with different instruments

Finite Length Effects

$$\Phi = N_T \int_0^L w(s)B(s)ds = \overline{B}\underbrace{\overline{w}_{eff}L}_{A_{eff}} = \overbrace{\overline{B}L}^{Bd\ell}\,\overline{w}_{eff}$$

$$\begin{cases} \overline{B} = \dfrac{1}{L}\displaystyle\int_0^L B(s)ds \\ \overline{w}_{eff} = \dfrac{\displaystyle\int_0^L N_T w(s)B(s)ds}{\displaystyle\int_0^L B(s)ds} = \dfrac{\Phi}{\mathrm{B}d\ell} \\ A_{eff} = L\overline{w}_{eff} \end{cases}$$



- The flux corresponding to a given coil position can be obtained in various ways (flipping or rotating the coil, pulsing the field from zero)
- L can be considered as known from mechanical measurements
- General case: unless B(s) or w(s) are constant and can be taken out of the integral sign, the flux <u>cannot</u> be obtained from average width and average field:

$$\frac{\frac{1}{L}\int_0^L w(s)B(s)ds}{\frac{1}{L}\int_0^L B(s)ds\,\frac{1}{L}\int_0^L w(s)ds} \neq 1$$

- Define: **effective width** (N_T gets lumped in for convenience) = **average of width weighted with the field**

considerations made here for a dipole field hold true for other components as well

$$\Phi = \overline{w}_{eff} B d\ell = \overline{B} A_{eff}$$

error made if $B(s)_{calibration} \neq B(s)_{measurement}$

most favourable case: B=const. $error = \frac{\delta w}{w_0} \frac{\delta L}{L}$

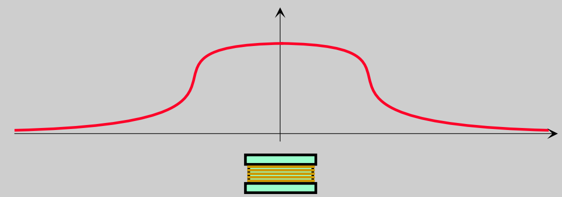
$L_c << L_M$ **local measurement**

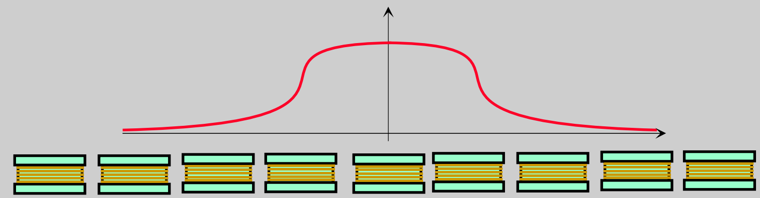
if we can assume B=constant (during both calibration and measurement), then the calibrated value A_{eff} can be used in any magnet to obtain the average field.

Usage:

- local field quality (diagnostic tool for SC coils)
- integral measurement by scanning

scanning required, w(z) unimportant, wavg sufficient



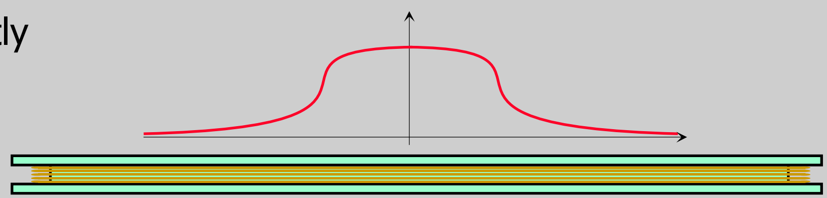


$L_c > L_M$ **integral measurement**

w_{eff} depends on both w(s) and B(s) – accurate absolute measurements cannot be done even if w(s) is known perfectly

Usage:

- precise and efficient determination of the field integral for relative measurements
- absolute cailbration may be done with another instrument (stretched wire) – to be repeated if B(s) changes

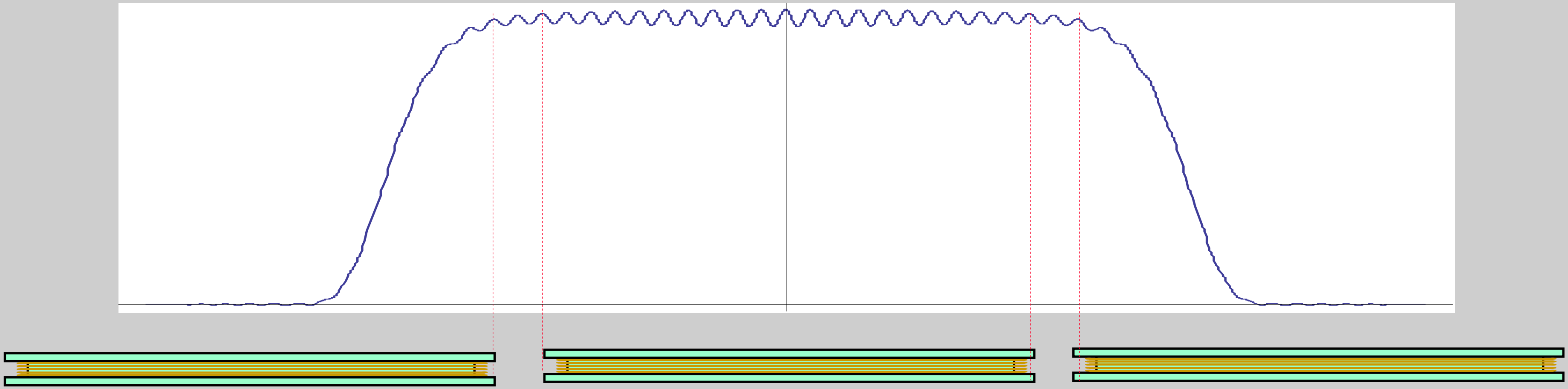


rectangular (default)	racetrack
• precise definition of coil length • best practical solution to reconstruct field integral from multiple longitudinal measurements (eg. gap between coils in long SC dipoles must be a multiple of twist pitch) • sharp corners spell wire problems: risk of breakage, bulge out, bond does not tak	• well-defined geometry facilitates winding • calculations more complicated • should be used only if the ends fall entirely within B=0 or B=const. region

Rounded ends: BdL can be measured precisely only if B is constant or linear over the end regions.

$$\Phi = N_T \int_{-w/2}^{0} 2\sqrt{\frac{w^2}{4} - s^2}\, B(s)ds + N_T w \int_{0}^{L} B(s)ds + N_T \int_{L}^{w/2} 2\sqrt{\frac{w^2}{4} - (s-L)^2}\, B(s)ds$$

Integral field of SC magnets



- superconducting magnets made with Rutherford cable develop a periodical field modulation, due to non-uniform interstrand current distribution, with period λ=twist pitch length and relative amplitude of order $\mathbf{10^{-3}}$
- to measure the integral correctly, coil length and gap must be **integer multiples of λ**
- to correctly fill in the gaps by interpolation/extrapolations, gaps must occur in the flat (central) field region

P. Pugnat, **INVESTIGATION OF THE PERIODIC MAGNETIC FIELD MODULATION INSIDE APERTURES OF LHC SUPERCONDUCTING DIPOLE MODELS, EPAC 2000**

Quality Control

Quality control = **acceptance test** to verify broad conformity with specifications
not necessarily a precise quantitative measurement (go/no go)
done routinely after manufacture and during operation (periodical or on-call checks)

Routine controls carried out at CERN:

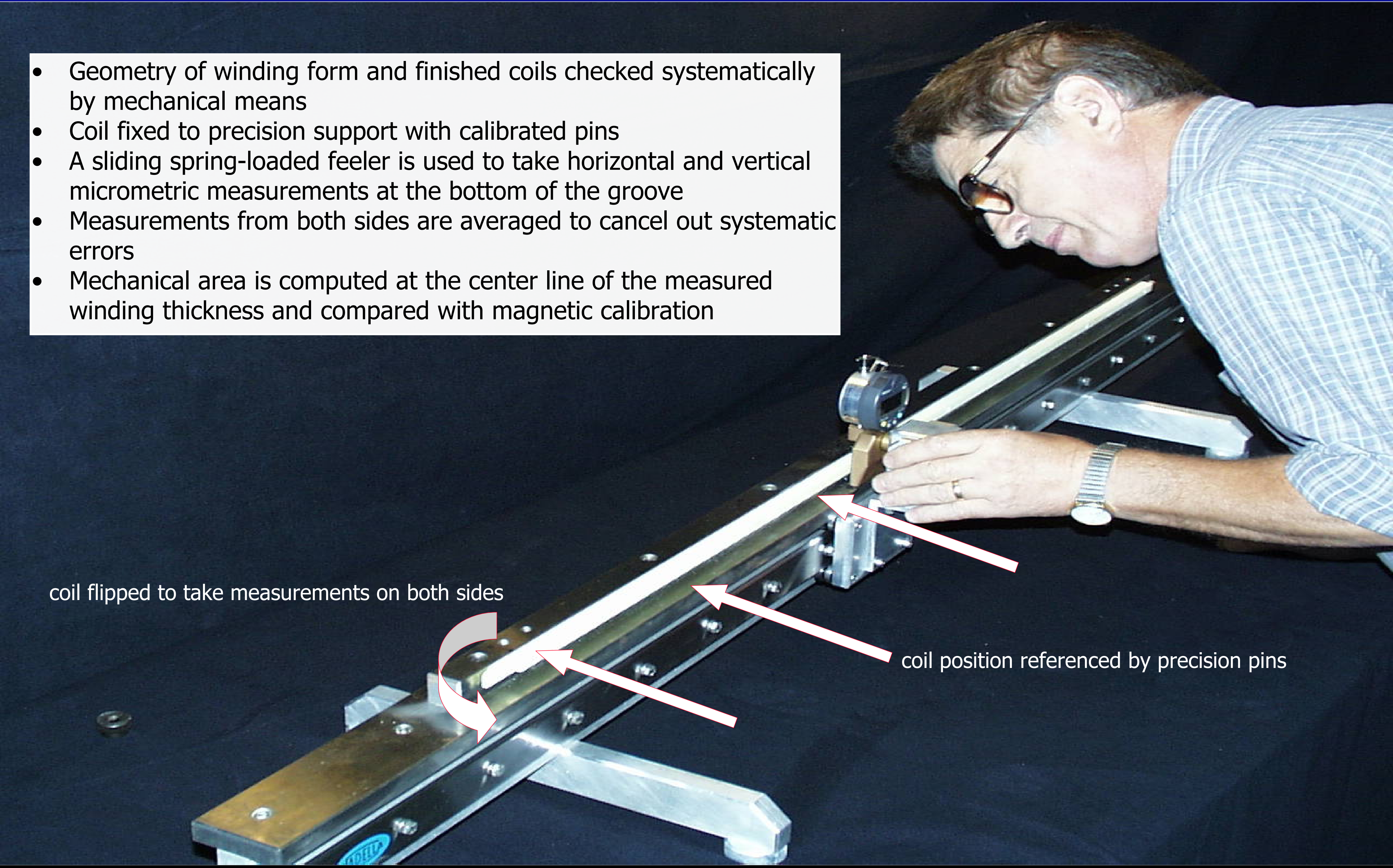
Visual Inspection

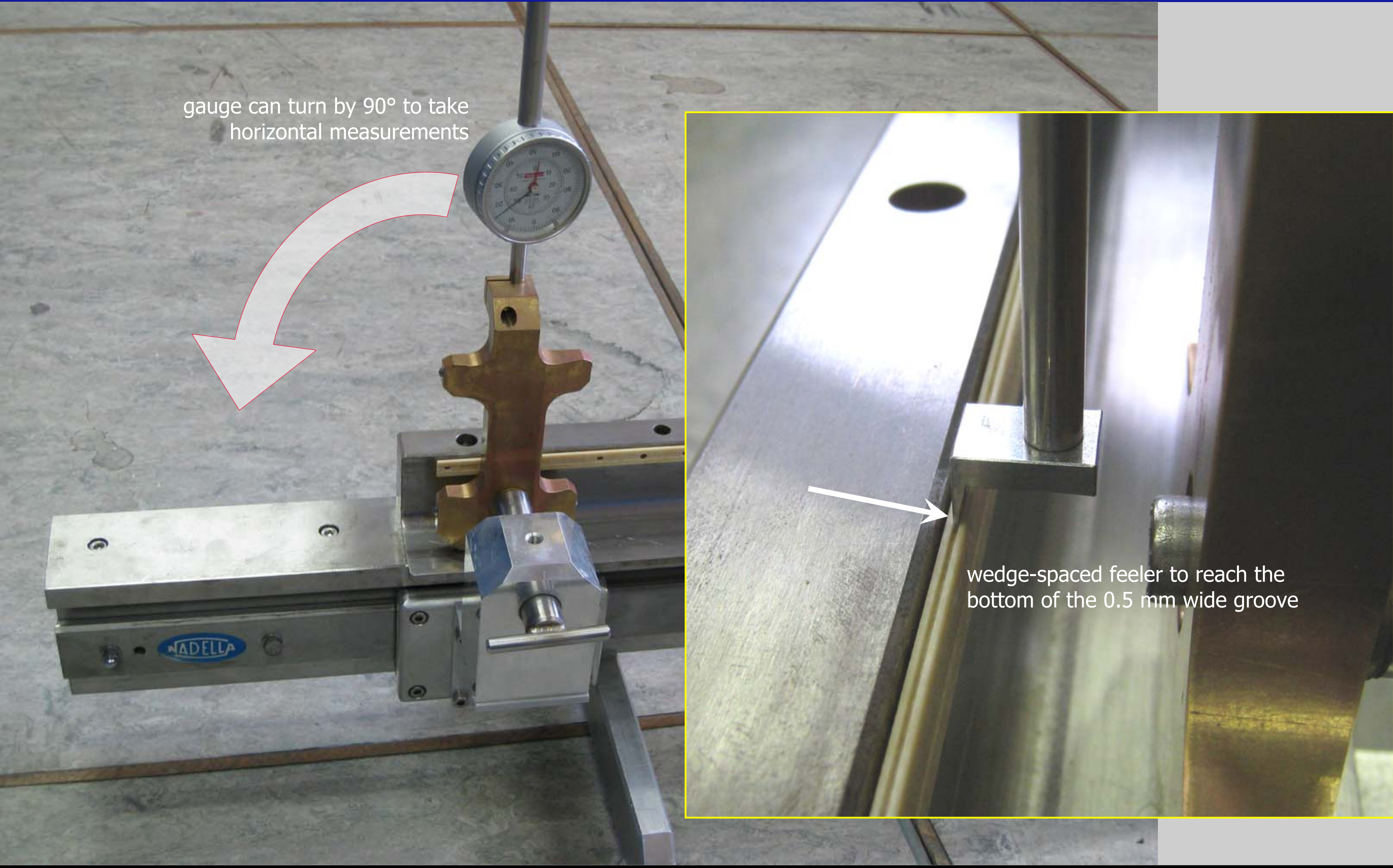
- Regular winding geometry
- Epoxy polymerized uniformly, no air bubbles, cracks or swellings

Geometrical Controls

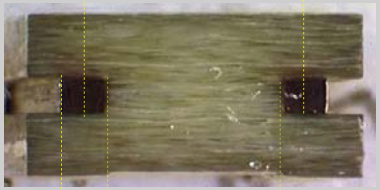
- Mechanical measurement of length, width of winding form/finished coil.
  Derived quantities: total coil area, winding thickness (including the epoxy bonding)
- Microphotogrammetry gives a lot of information, but it is destructive.
  Carried out occasionally to measure cross-section parameters to be used for the calculation of finite cross-section effects.
- Radiographic photogrammetry: might be useful (non-destructive), but is costly

- Geometry of winding form and finished coils checked systematically by mechanical means
- Coil fixed to precision support with calibrated pins
- A sliding spring-loaded feeler is used to take horizontal and vertical micrometric measurements at the bottom of the groove
- Measurements from both sides are averaged to cancel out systematic errors
- Mechanical area is computed at the center line of the measured winding thickness and compared with magnetic calibration



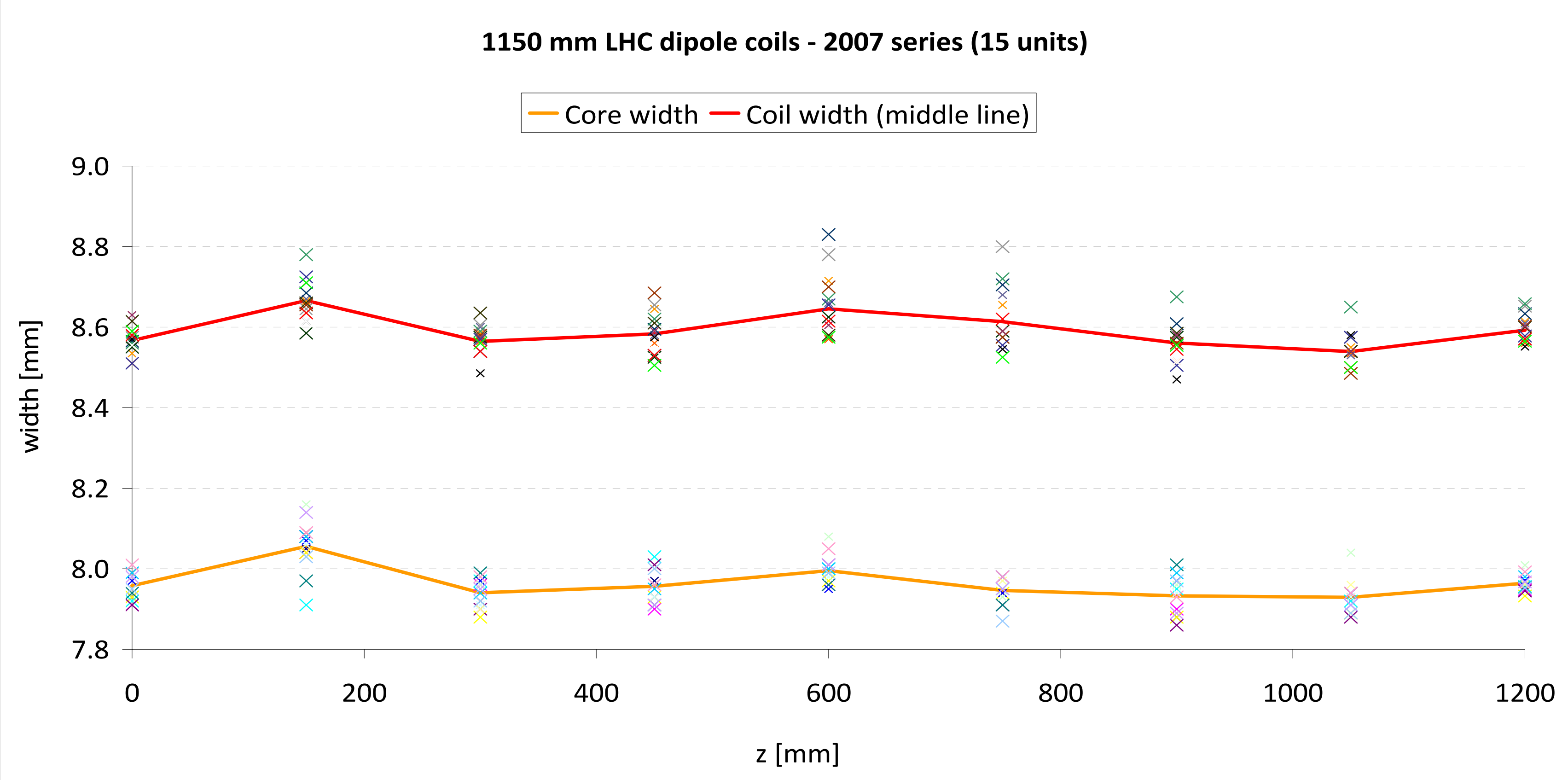

gauge can turn by 90° to take
horizontal measurements
NADELLA
wedge-spaced feeler to reach the
bottom of the 0.5 mm wide groove

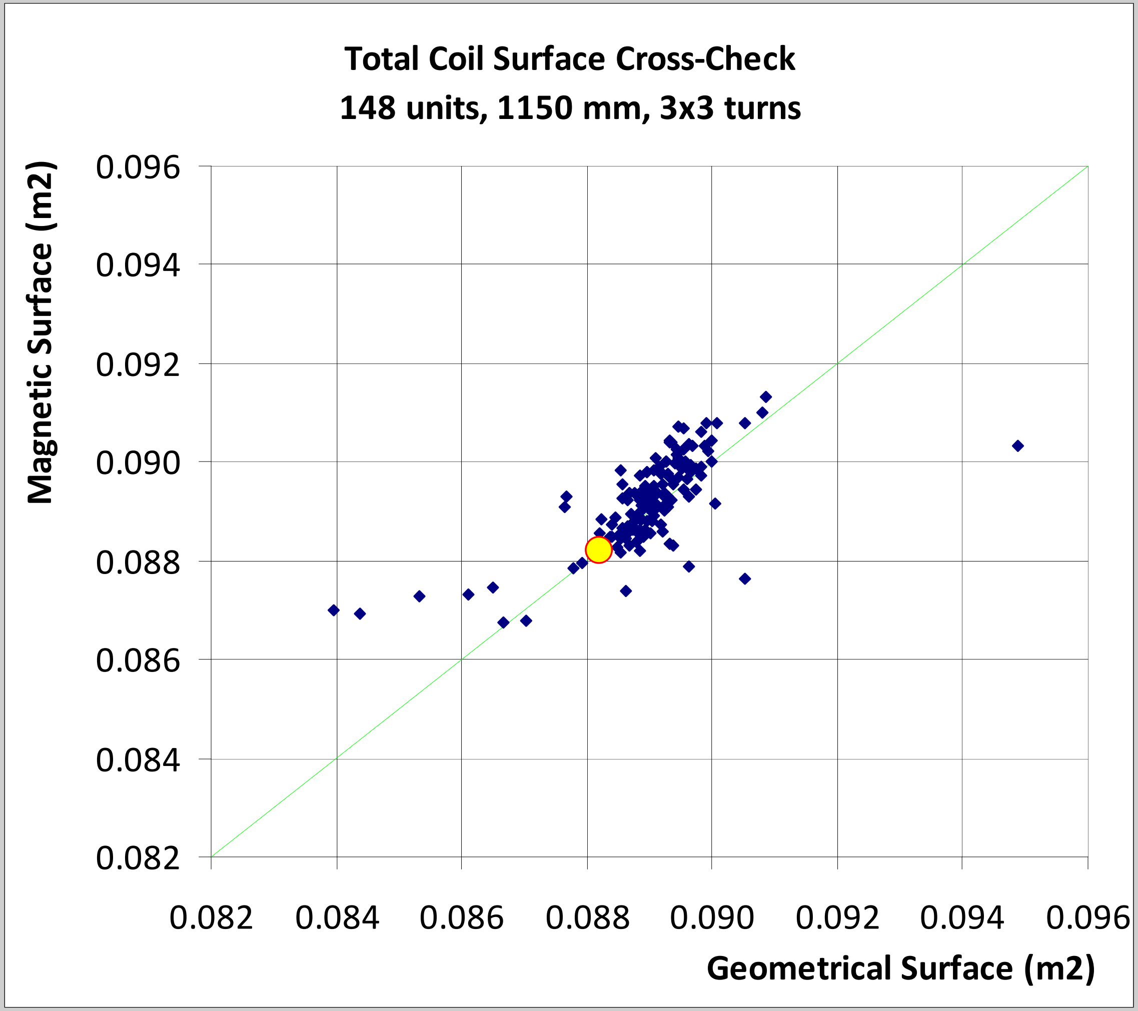
Coil width = 8.56 ± 0.07 mm



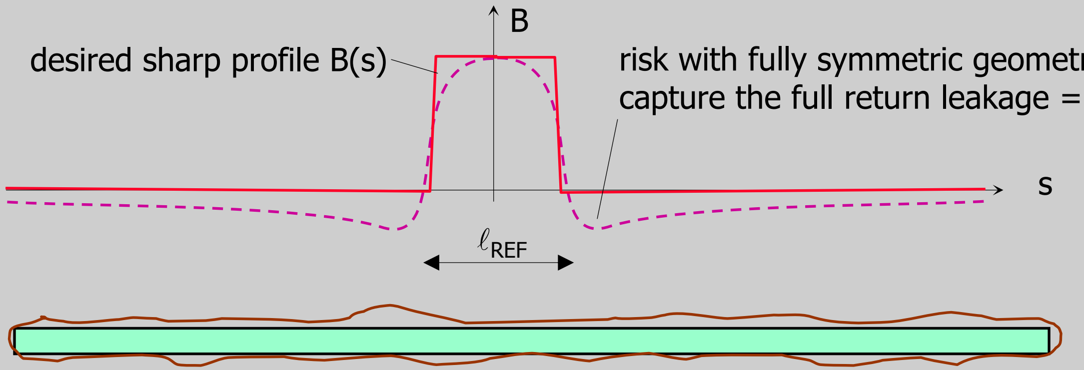
Winding width = 0.63 ± 0.06 mm
(variations in wire diameter, bonding thickness, ..)

Core width = 7.96 ± 0.05 mm





- Obtained area ~1% larger than specified on average
- Magnetic area 0.25% larger than geometric on average
- Spread (including outliers) is about **1%** of the nominal value
- Even excluding outliers, about 0.7% spread of the difference between geometrical and magnetic values is $\Rightarrow$ this kind of mechanical measurements **cannot be used as a reliable predictor of magnetic area**.

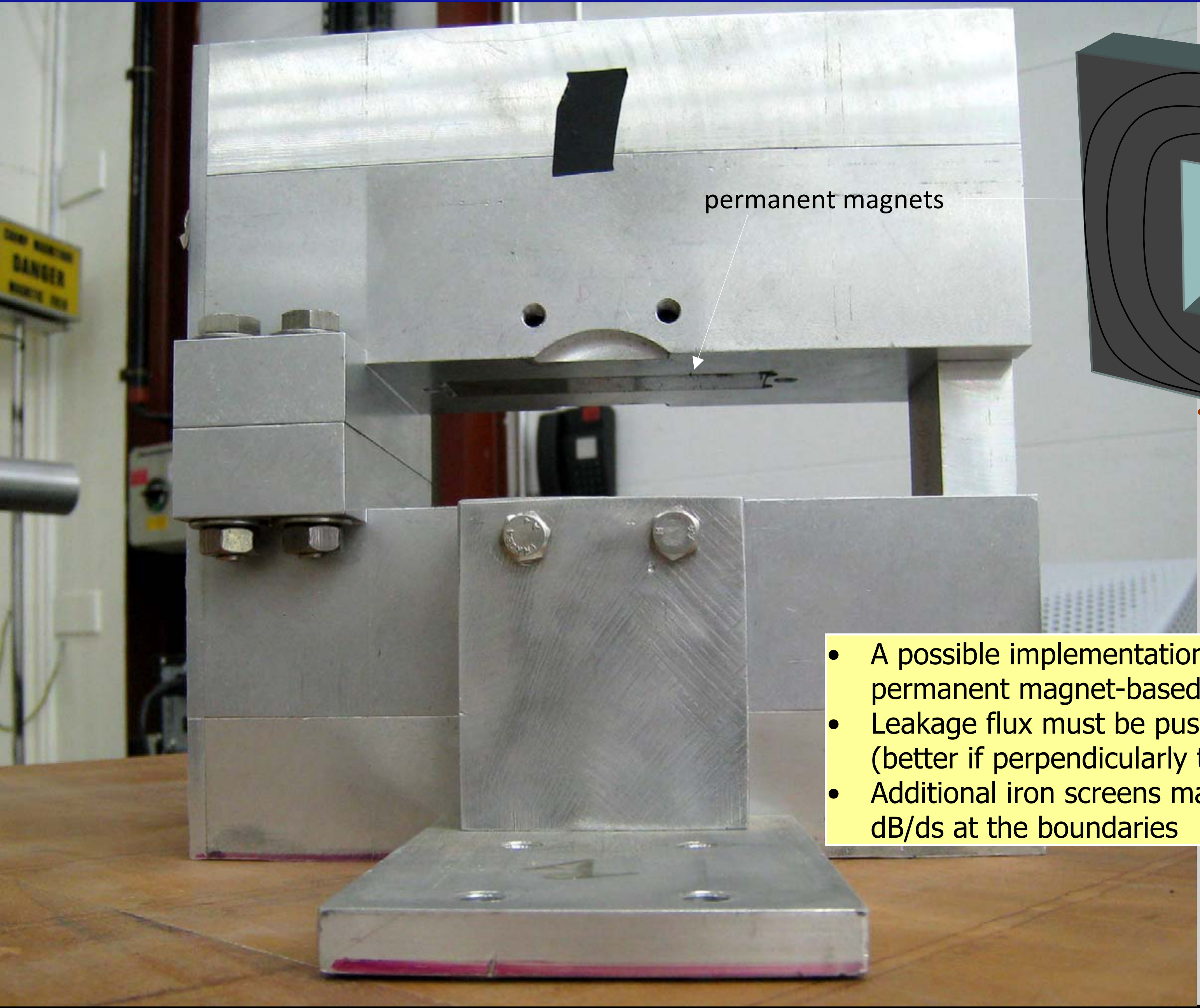


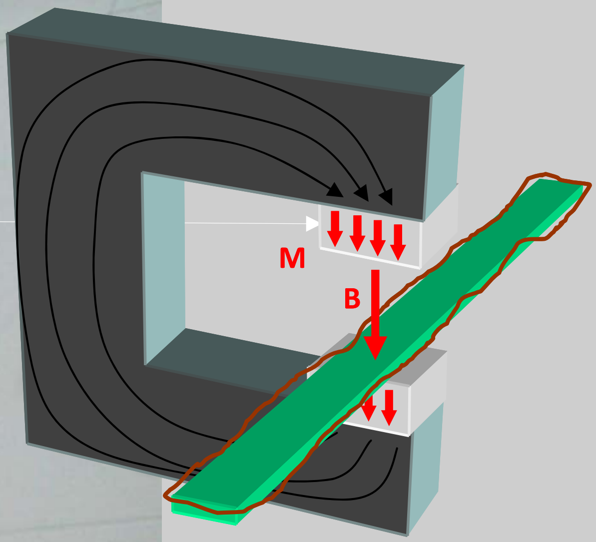
$$\Phi = N_T \int_0^L w(s)B(s)ds \approx N_T \int_{\bar{s}-\Delta/2}^{\bar{s}+\Delta/2} w(s)B(s)ds \approx N_T w(\bar{s}) \underbrace{\int_{\bar{s}-\Delta/2}^{\bar{s}+\Delta/2} B(s)ds}_{Bd\ell_{ref}}$$

$$\Rightarrow \quad \boxed{w(\bar{s}) = \frac{\Phi}{N_T Bd\ell_{ref}}}$$

- Apply a local magnetic field to obtain an indication of the width variations
- Relative measurement – absolute width may be obtained from average w calibrated in a reference dipole
- Potentially preferable to mechanical measurements – takes into account all non-idealities
- Flux signal can be obtained in different ways (flipping the coil in a constant field, translating the coil at constant velocity, applying a pulsed or AC field)

Quality Control- Magnetic determination of the local width





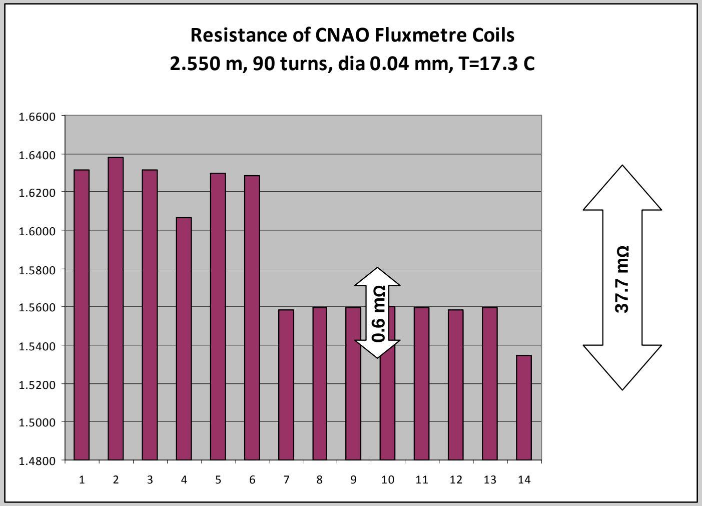
- A possible implementation of the idea: a simple permanent magnet-based dipole
- Leakage flux must be pushed as far away as possible (better if perpendicularly to the coil)
- Additional iron screens may improve the gradient dB/ds at the boundaries

Electrical Controls

- Continuity test
- Resistance measurement: difficult to detect single-turn shorts (e.g: 1 inter-turn short on a 400-turn coil = 0.25% R drop equivalent to 0.5 °C !!)

$$R_c = \gamma_R(\ell_c, w_c, \varnothing_w)\rho N_T$$

$$\frac{1}{R_c}\frac{\partial R_c}{\partial N_T} = \frac{1}{N_T}$$

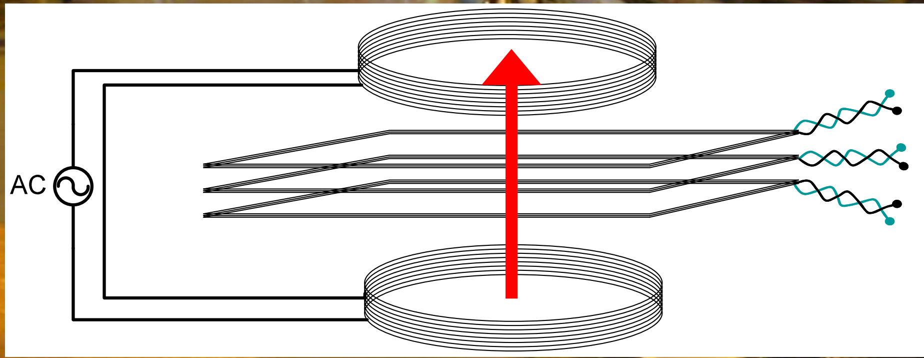


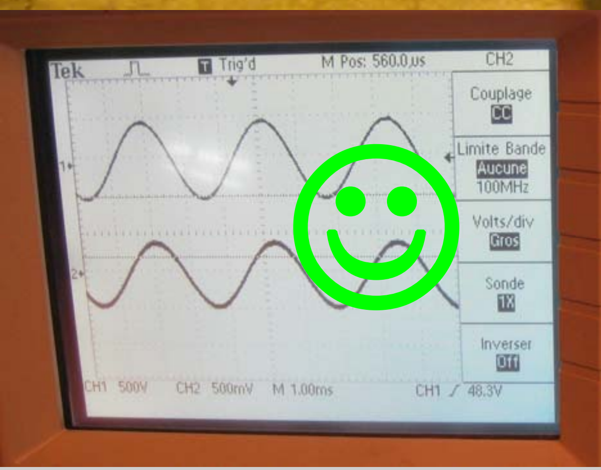
Example of R variability in a batch of equal coils

- Inductance measurement: no material property dependence, 2×sensitivity (actually better than that, as the shorted turns add a mutual inductance in phase opposition).

$$L_c = \gamma_L(\ell_c, w_c, \varnothing_w)\mu_0 N_T^2 \quad \Rightarrow \quad \frac{1}{L_c}\frac{\partial L_c}{\partial N_T} = \frac{2}{N_T}$$

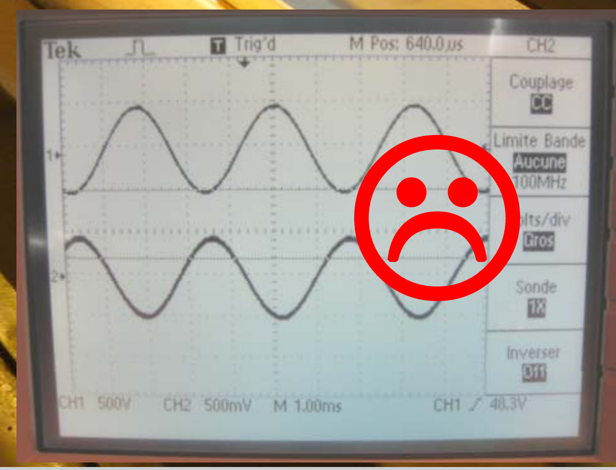
- Measurement of the time constant of the inductive discharge (τ^2=L/R) may be useful

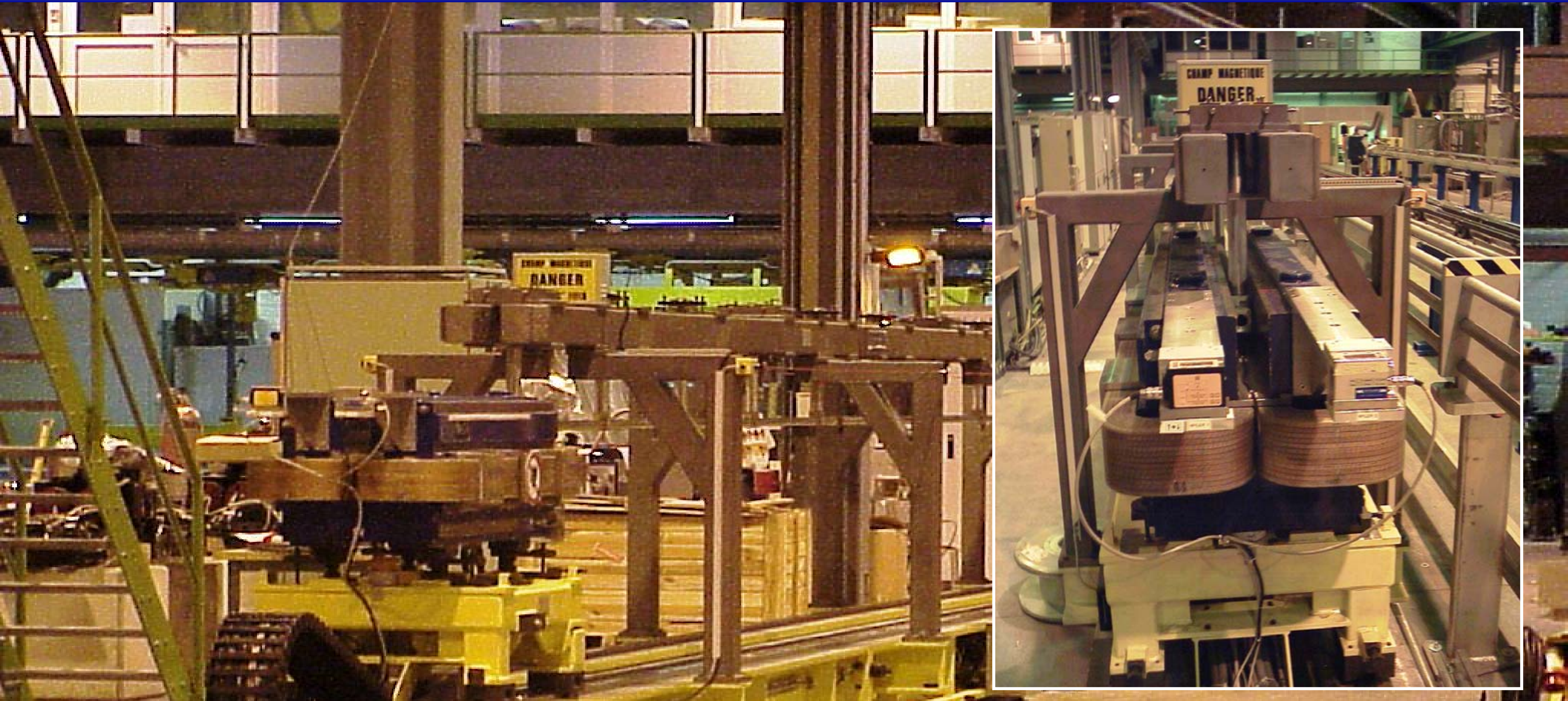




Polarity checker for coil assemblies

- up to ~20 connector assemblies in series – very easy to get inversions
- check phase of V induced in the coil vs. source

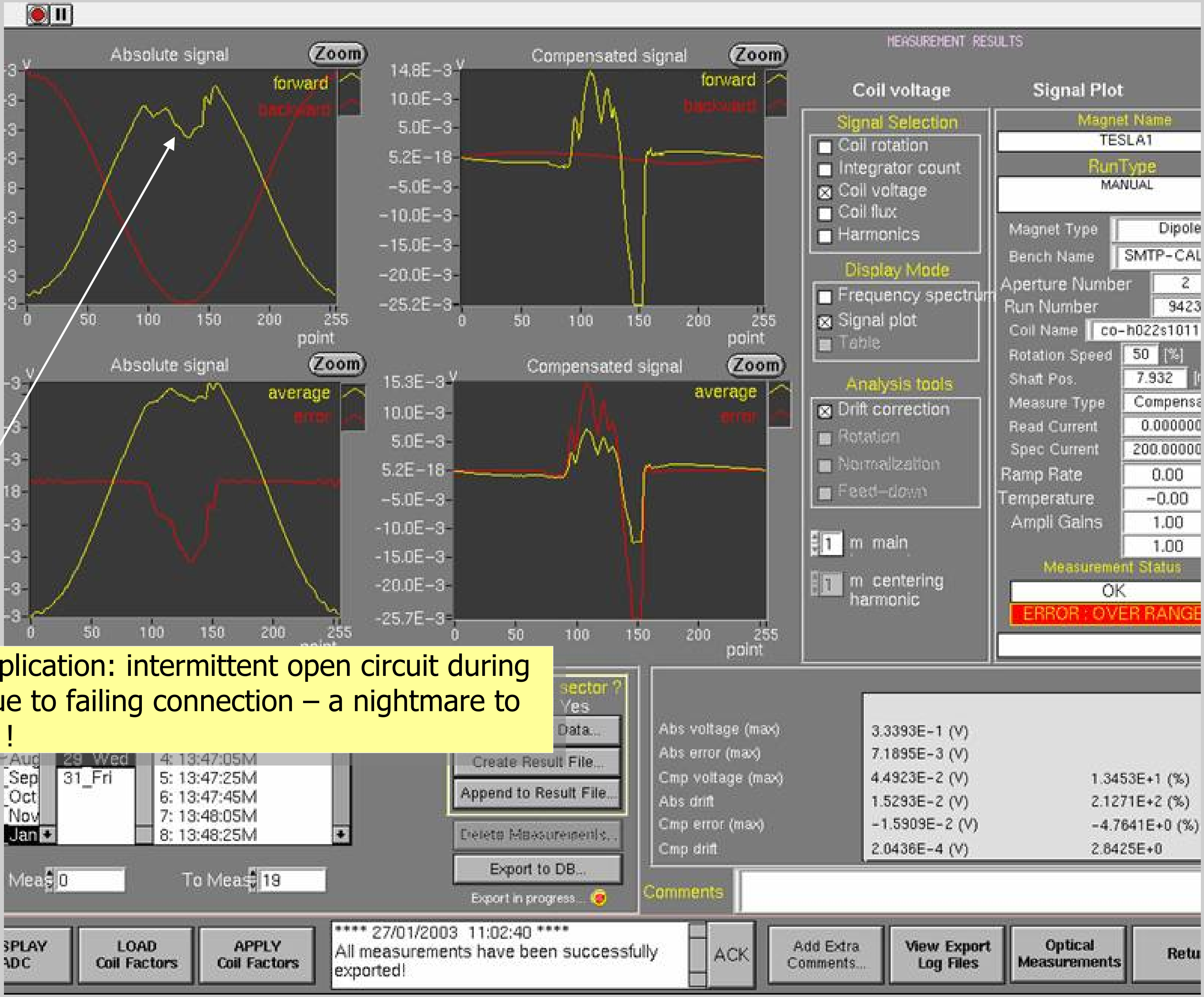




Example: calibration bench for long shaft at CERN (SM18)

- Calibration bench conceived for quick on-site magnetic and electrical verification of 15-m long coil shafts for LHC cryomagnets during the series test phase
- 0.5 T horizontal field over 1.5 m, very good uniformity (b3<2 units), travelling magnet
- Calibration of field direction for long shafts + verification of strength (→ coil area);
- realistic coil voltages for electrical checks

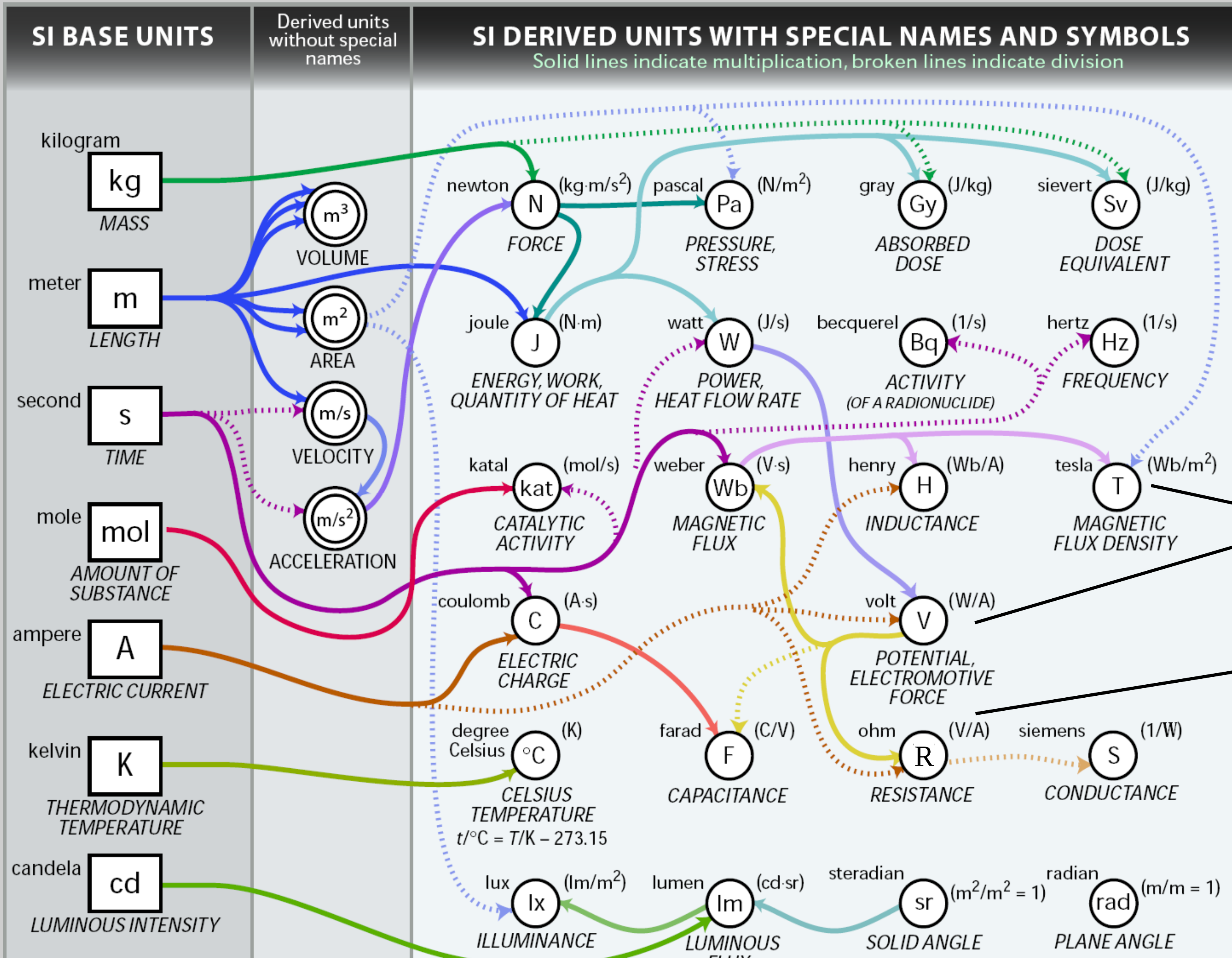
Magnetic Quality Control



Example of application: intermittent open circuit during the rotation due to failing connection – a nightmare to find otherwise !

Metrological Aspects

SI units



www.nist.gov

Josephson effect:
$V = nf\Phi_0$

Quantum hall effect:
$R = n\Phi_0/e$

Possible requirements

- **Absolute calibration**
  the "real" value of the magnetic field (according to some objective reference standard) is required to compare the result with an external source
  Examples:
  - field of a spectrometer (must provide absolute beam energy values)
  - matching the energy between chained accelerators

- **Relative calibration**
  the value of the magnetic field in comparison to a local reference fulfils the purpose of the measurement
  Examples:
  - integrated strength of the main magnets of a synchrotron (they must be equal to guarantee the same bending radius/focusing when they are powered in series)
  - field mapping to compute homogeneity/harmonics

relative measurements are both easier to do and more precise
differential measurements allow the cancellation of a number of error sources
null methods allow to work in the most linear range of the instrumentation

In both cases: open-loop (geometry-based) calibration does not well enough for harmonic coils ⇒ recourse to appropriate reference magnets is necessary

Also: applications where few % calibration is sufficient (e.g. measurment in pusled magnets, which are self-calibrated - relative)

Reference standards of any physical quantity must fulfill three basic requirements:

1. Realize the quantity in a useful range
2. $\partial/\partial t = 0$ (long term stability)
3. $\partial/\partial x = 0$ (transportability)

Physical Reference Standard

The instrument to be calibrated is checked directly or indirectly.

Reference Instrument

An instrument calibrated with a reference standard is used to charcterize a user field source, which can then be used as a reference
Coil + instrument = the reference

Any Field Source

Any Instrument

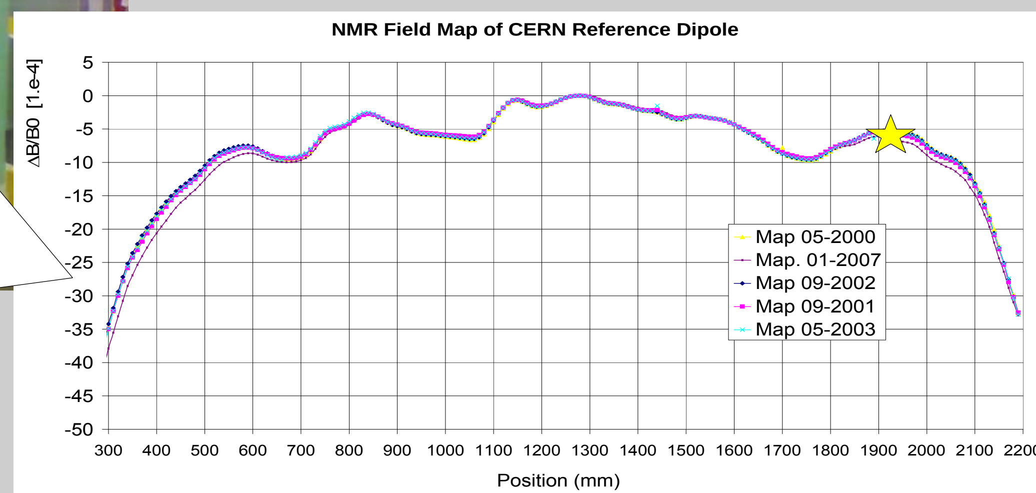
Measurement Results

In-situ calibration: using symmetries, the instrument and the magnet are calibrated simultaneously.

In practice: cross check of different instruments, confidence comes from redundance.

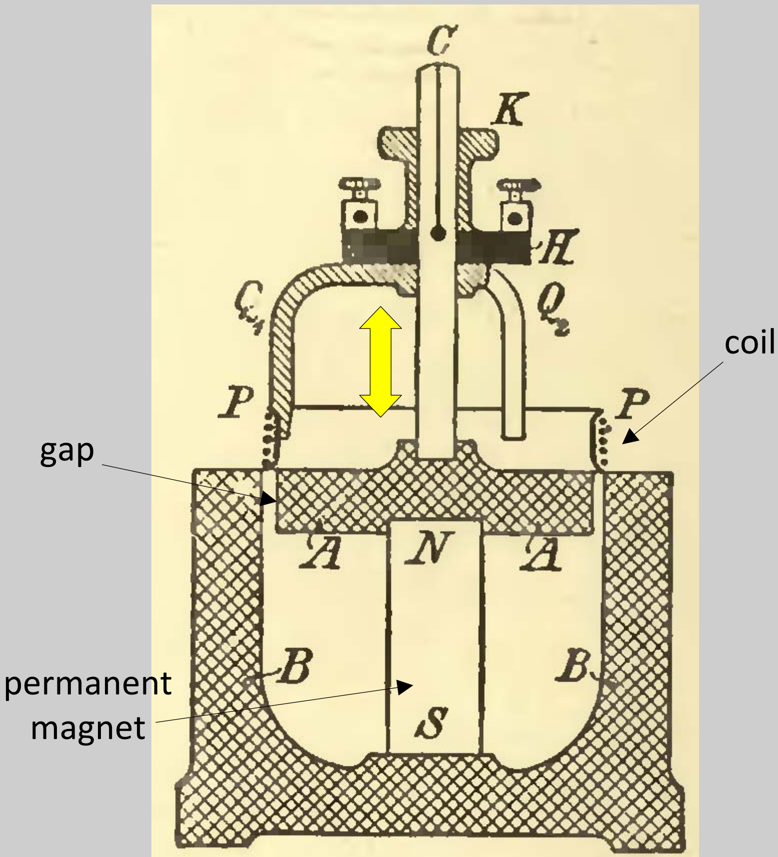
Flux Density $[B] = [M]^{1} [L]^{0}[T]^{-2}[I]^{-1}$ **1 Tesla = 1 N/Am= 1 Wb/m2**

No official primary standard. The established secondary standard is the proton gyromagnetic ratio γ_0=42.5759 MHz/T, at the basis of the NMR measurements (e.g. Metrolab teslameter). In practice, one has to realize their own reference by taking a dipole magnet and mapping it. Main limitations: absolute accuracy 5 ppm; tracking rate 1%/s; homogeneity 13 ppm/mm.



Magnetic Flux $[\Phi] = [M]^{1} [L]^{2} [T]^{-2} [I]^{-1}$ **1 Weber = 1 Vs = 1 HA = 1 Tm2 = 1 J/A**

No official primary standard. Secondary standards of flux and (equivalently) mutual inductance employed routinely as calibrators (e.g. Hibbert's standards, Campbell coils, or more modern calculable sources). The flux quantum is logically a candidate to replace the old standards, but for the moment it is impractical to use it directly.
NB: realizations of the Vs can be obtained with high precision, and are used to calibrate acquisition electronics; since they do not represent physically a magnetic flux, however, they cannot be used to calibrate directly a coil.



Historical Note: Hibbert's flux standard

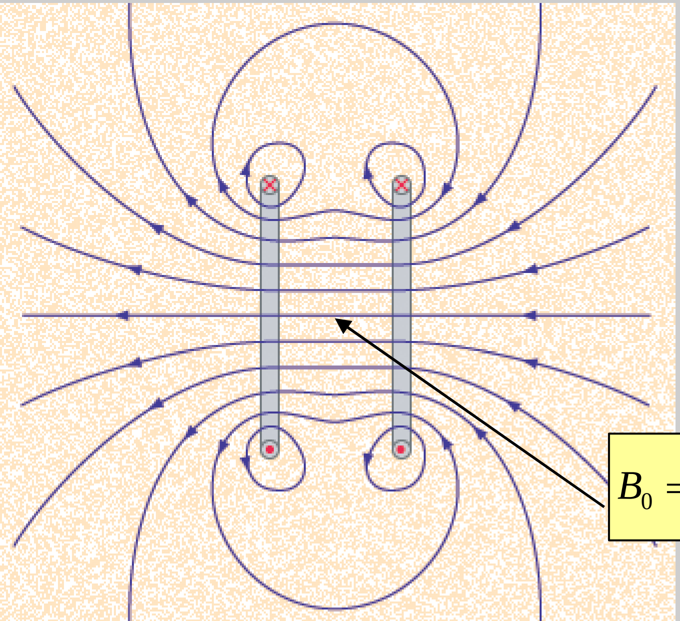
- Narrow annular gap in a permanent-magnet based iron circuit
- Coil can slide between mechanical stops on a rod into and out of the field
- A few ‰ reproducibility with artificial ageing to improve long-term stability

> . . . an exaggerated rough treatment, consisting of boilings, shocks, blows, being allowed to fall, repeated magnetisation, and the like; this treatment may be called *artificial ageing*; magnets thus treated usually keep better.

Du Bois, The magnetic Circuit in Theory and Practice, Longmans, 1896

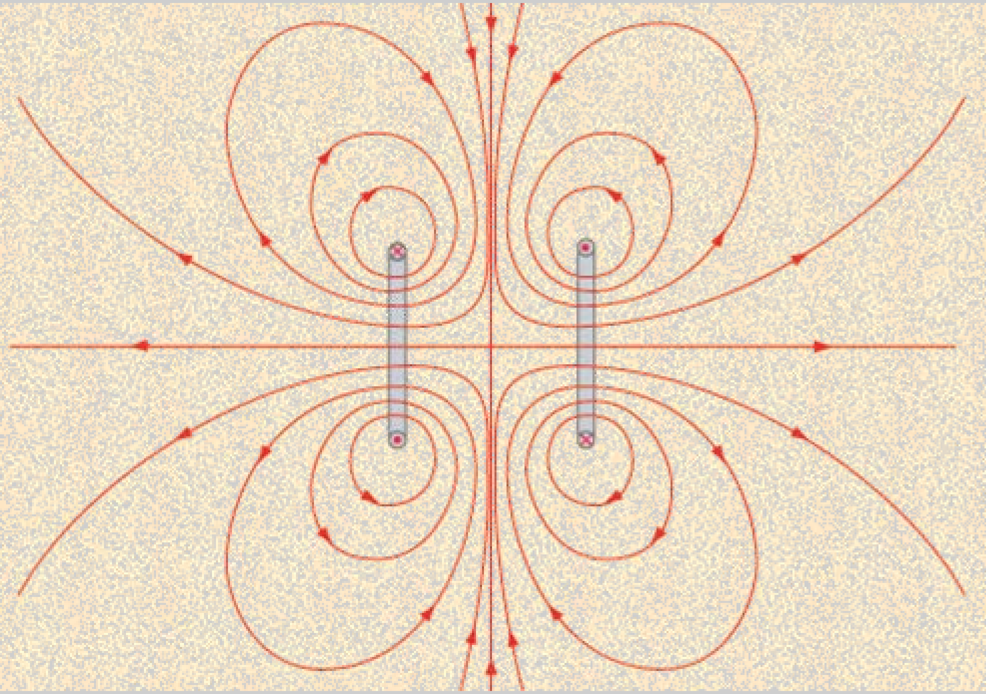
- Two circular coils of radius R separated by a distance R
- Zero sextupole at the center – 6% $\Delta B/B_0$ at z=R/2
- Excellent accessibility of the homogeneous field volume
- Can be used in reverse mode as a search coil (e.g. for measurement of small permanent magnets) – it will be sensitive mainly to the dipole component only
- Square arrangement of conductors carried small penalty
- Calculable system – suitable for use as a reference

3D Helmoltz coil system
http://www.laboratorio.elettrofisico.com



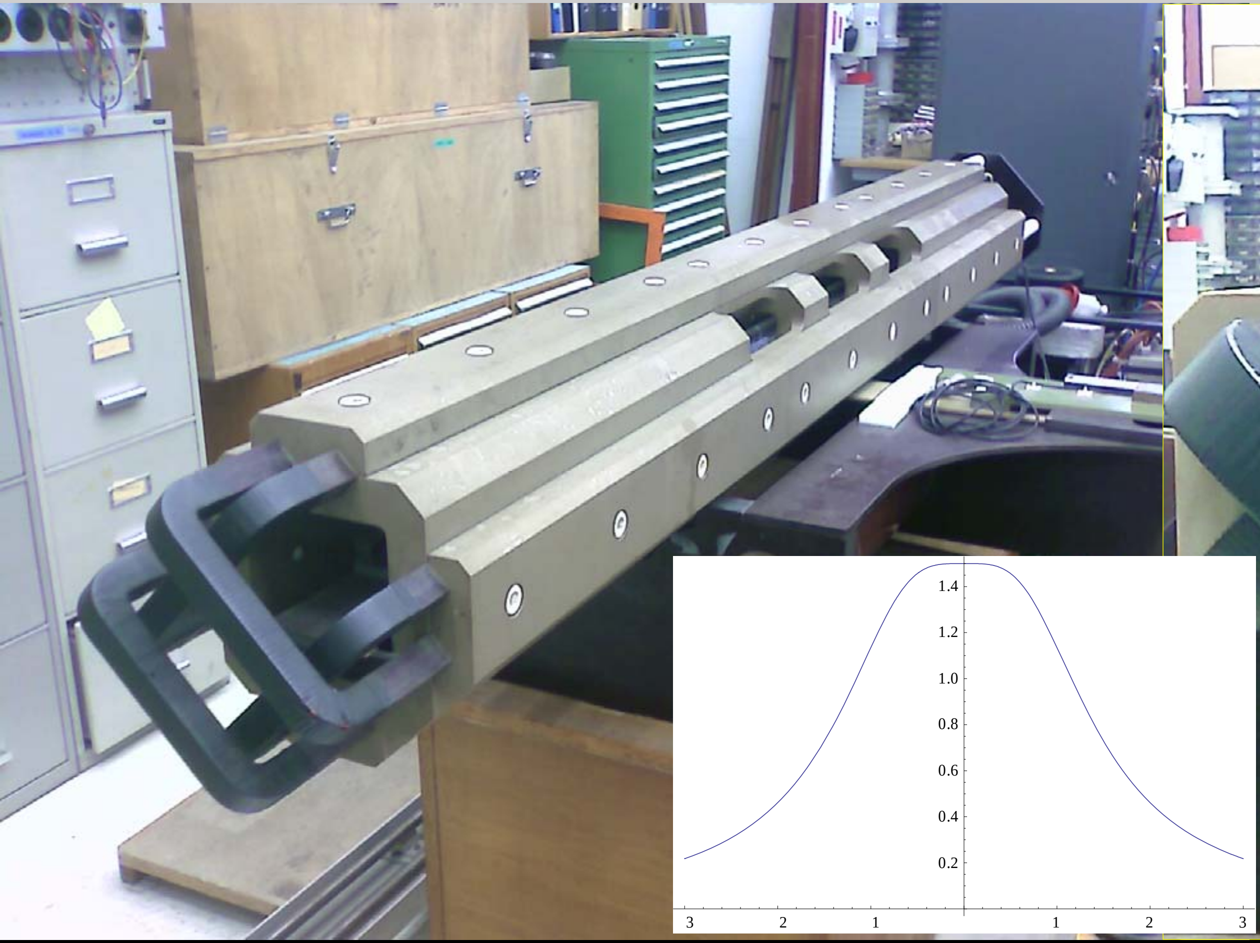
Magnetic field lines for Helmholtz coils.

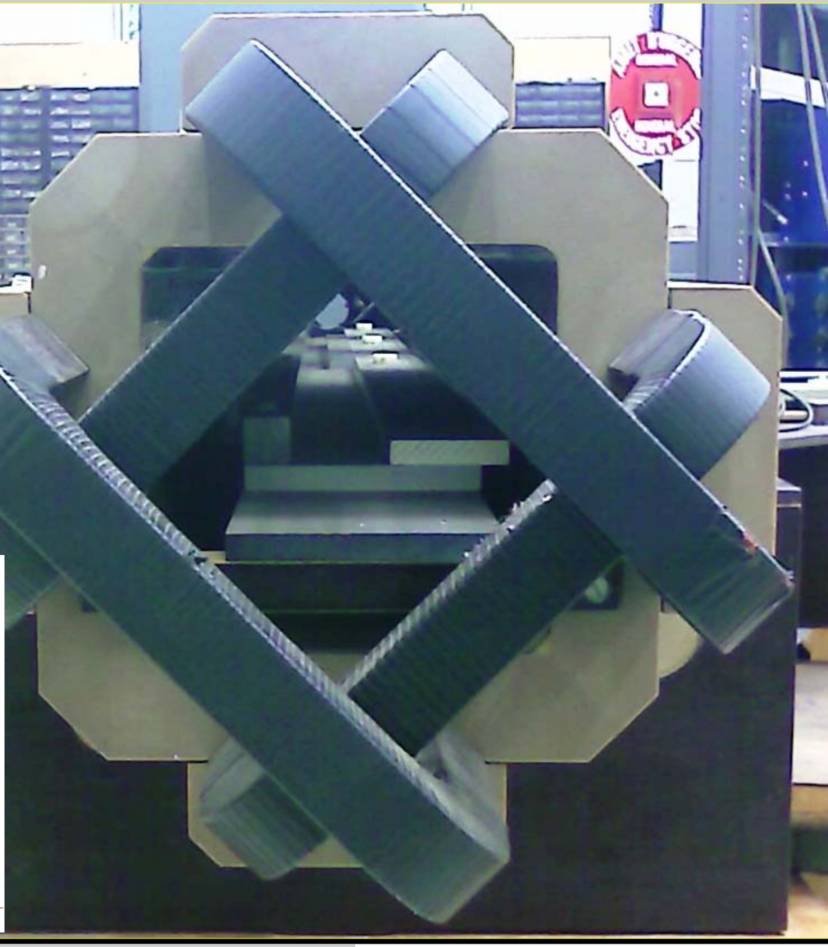
$$B_0 = \sqrt[3]{\frac{4}{5}}\frac{\mu_0 nI}{R} = 0.715\frac{\mu_0 nI}{R}$$



S. R. Trout, "Use of Helmholtz Coils for Magnetic Measurements", IEEE Trans Magnetics, V.24, No.4, Jul 88

- Two perpendicular Helmoltz coils can create combinations of normal and skew dipole + quadrupole field
- 2D geometry → optimal spacing between coils is $1/\sqrt{3}$ of the distance between conductors
- 2400×120×240 mm coils, 30×30 mm cross section, 150×
- 64 turns of 4 mm2 air-cooled Cu conductor, nominal current I=20 A ⇒ max. modulus field = 0.76 T
- Force on each conductor = 6.5 kN → needs a strong structure
- Must be kept far from metallic masses





Other Calculable Systems

- Many variations on the Helmoltz theme with different coil numbers, arrangement and optimization criteria
- Braunbeck Coils: 4 coils in series used to cancel Earth's field to better than 1 nT – used for instrument calibration inside a large volume.

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Calibration in a Dipole

Flip Coil Method

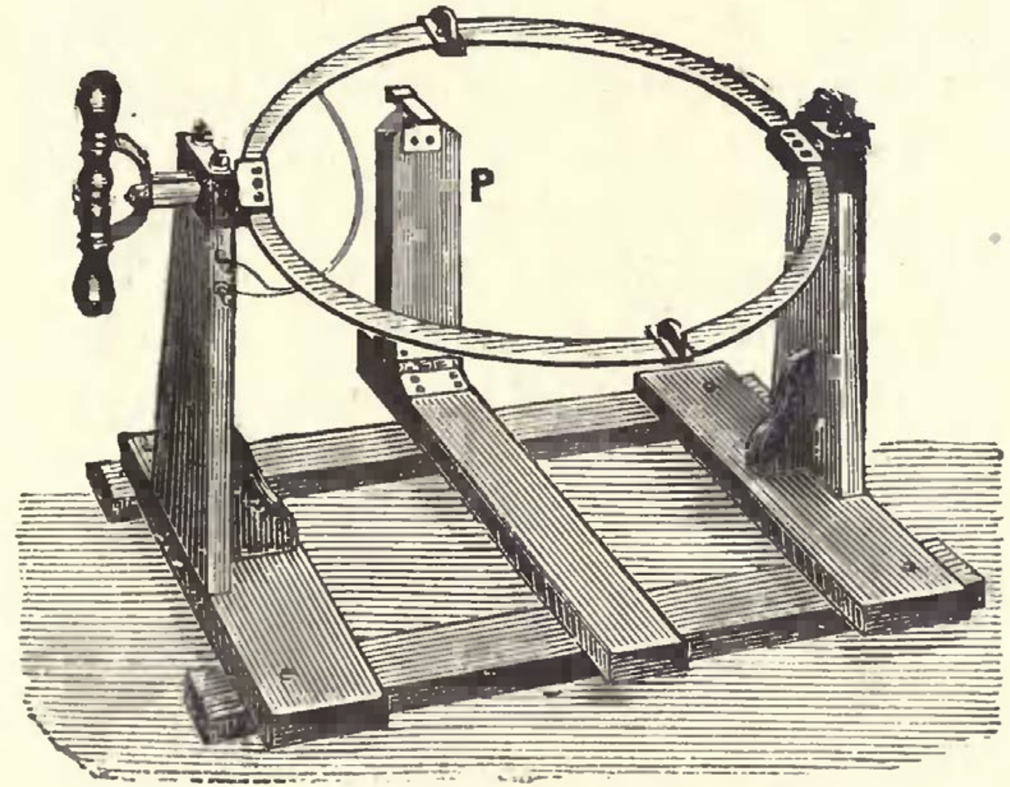


Fig. 22.—Earth Coil for use in Ballistic Measurements.

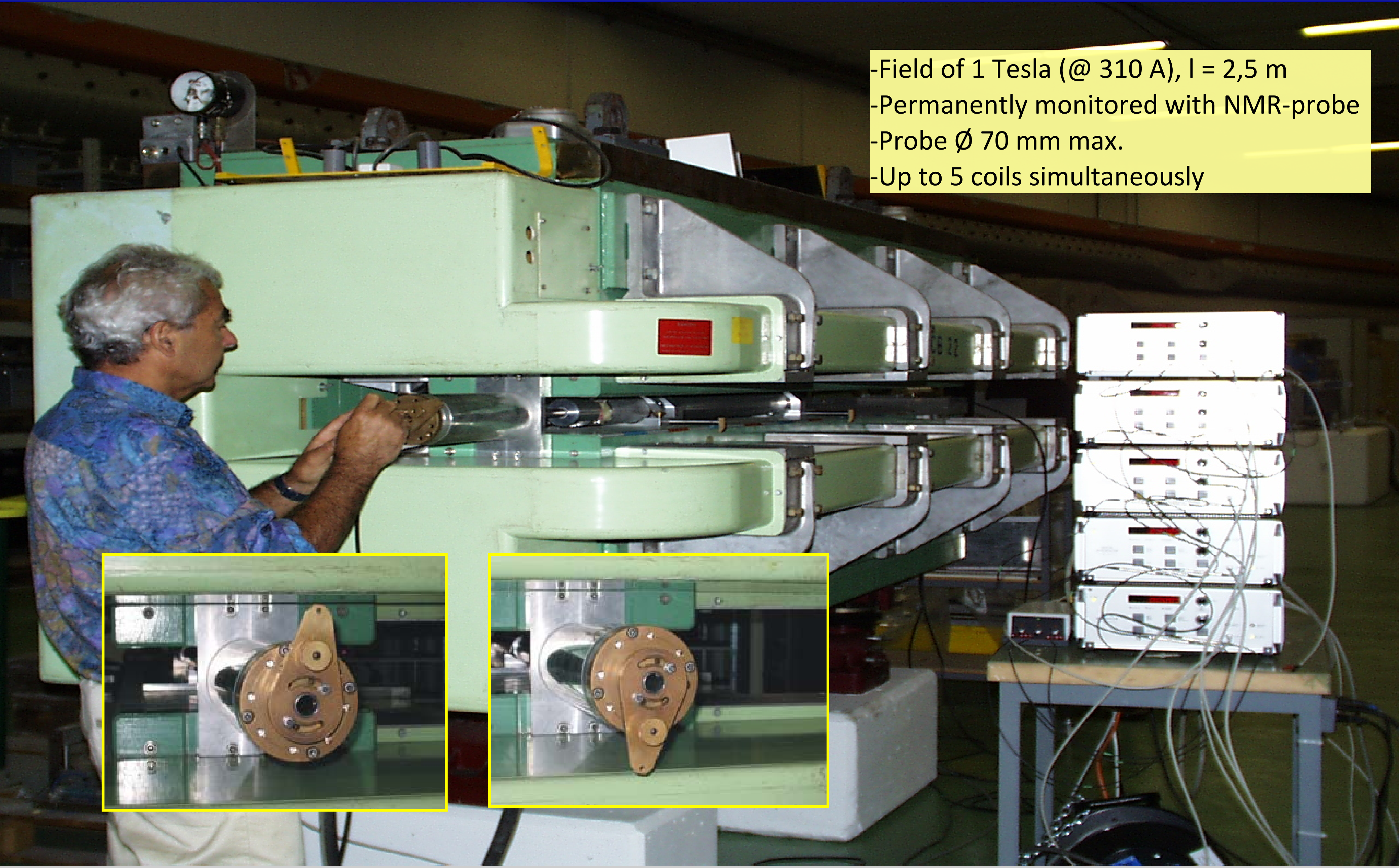
- Not a novel technique …
- Find the total flux linked in a given position
- Relates coil area and average field
- Returning back to the original position allows estimation and correction of integrator drift
- Equivalent to harmonic (rotating) coil measurement with two sampling points 180° apart
- Works in any field: use to calibrate (B→A) is possible only if B is known precisely …

$$-\int_0^t V_c dt = \Phi - (-\Phi) = 2A_C\overline{B}$$

Transform an **integral w.r.t. time** into another **w.r.t. position** (same as for harmonic coil method) guarantees insensitivity to speed and trajectory fluctuations

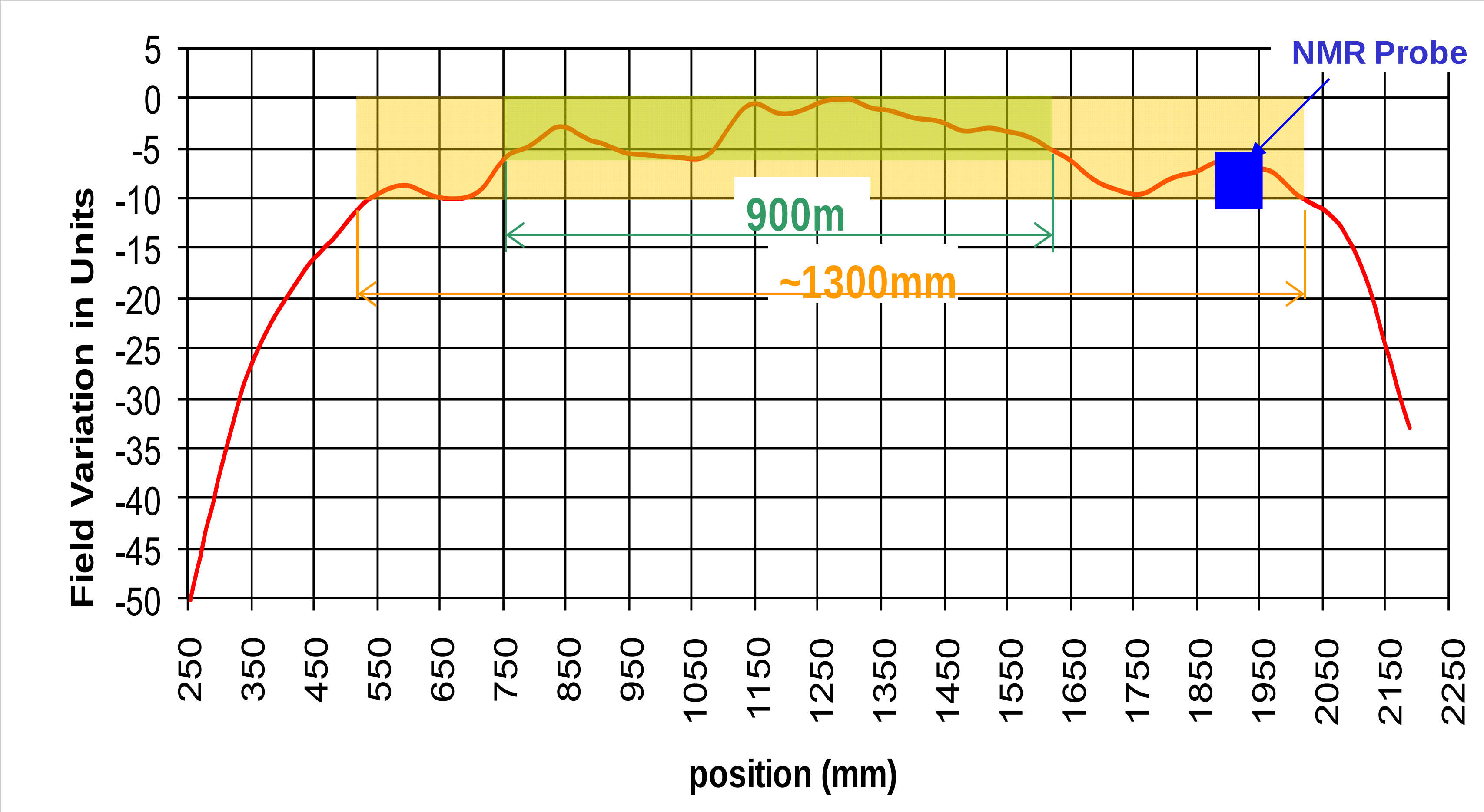
JA Ewing, « Magnetic Induction in Iron and Other Metals», Van Nostrand, London, 1900

CERN Reference Dipole



- -Field of 1 Tesla (@ 310 A), l = 2,5 m
- -Permanently monitored with NMR-probe
- -Probe Ø 70 mm max.
- -Up to 5 coils simultaneously

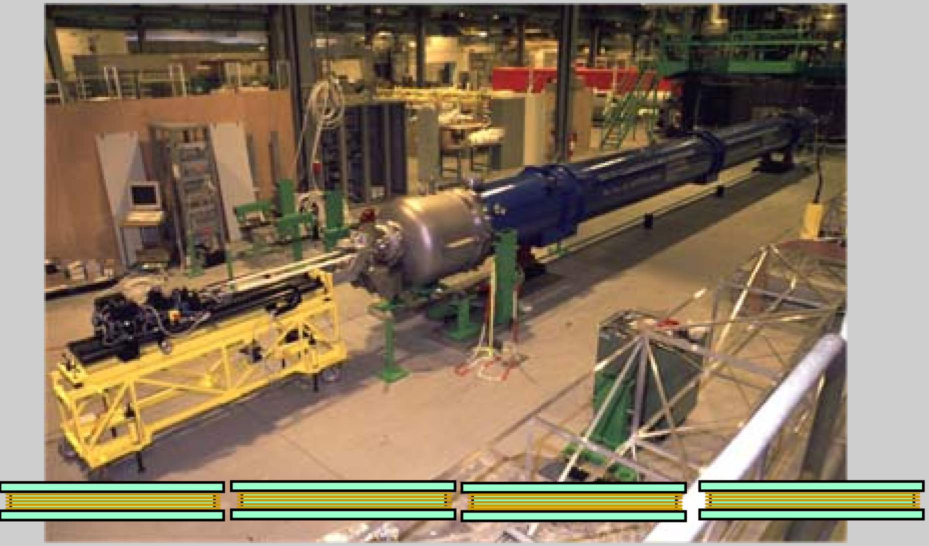
	☺	☹
Reference magnet	• best accuracy if parameters are consistently tracked (current, temperature, …) • best convenience – doubts can be dispelled by a quick check • can be done offline	• may require transportation of the instrument
in situ	• most accurate	• not always practical (geometry reasons) • adds time to series measurements

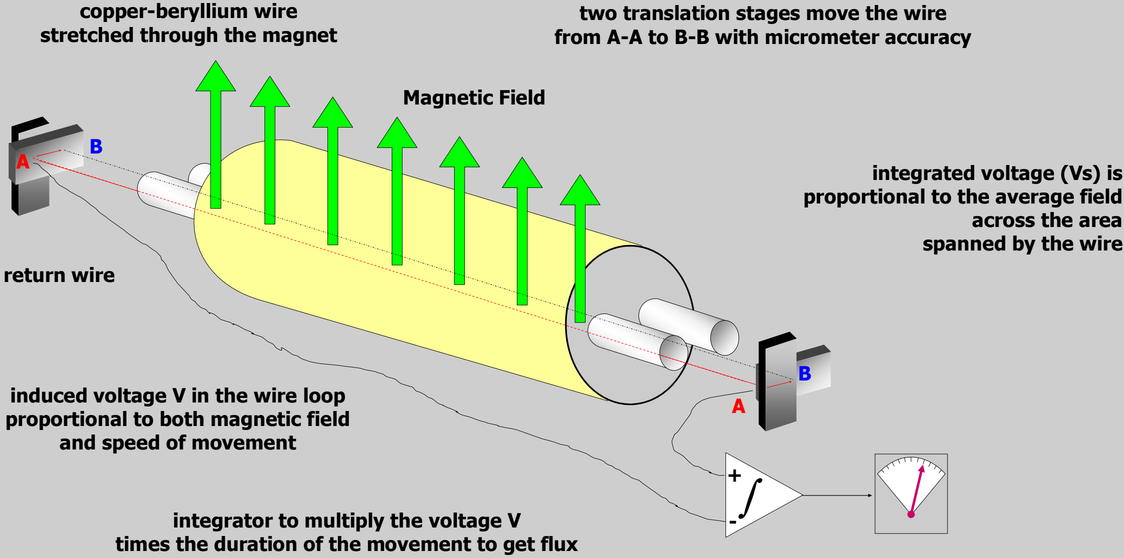


- Field mapped yearly and checked continuously with a local NMR (used for scaling)
- Monitor temperature and current during the measurements !

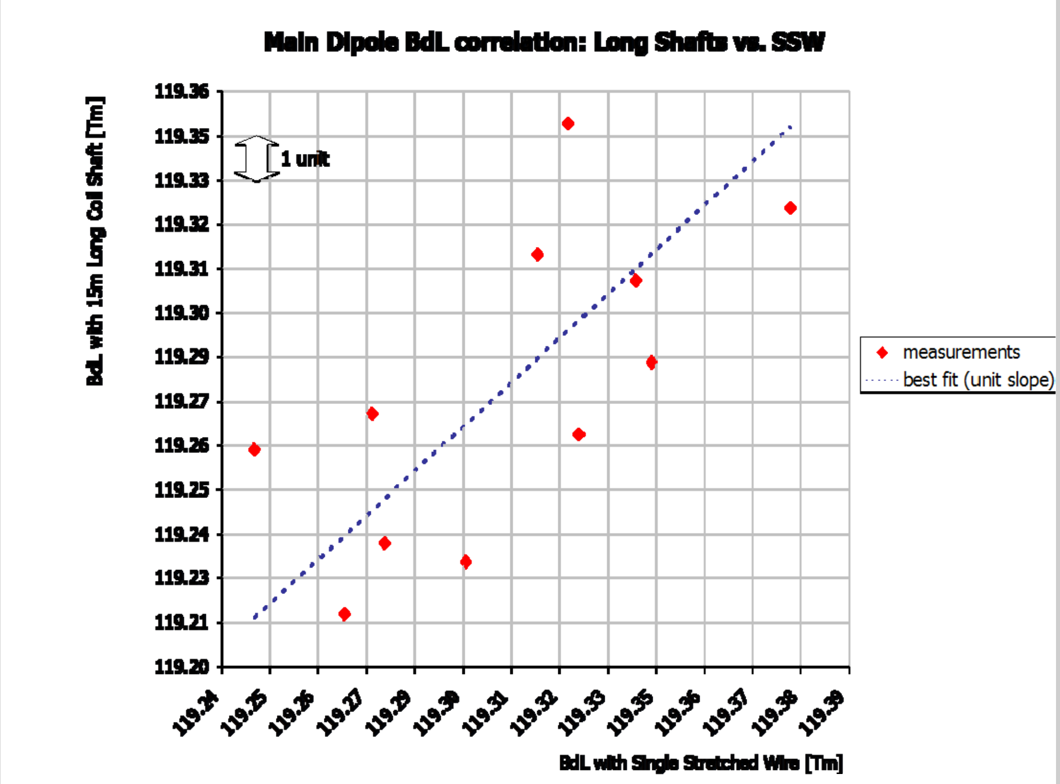
$$A_C = \frac{\Delta\Phi}{2\overline{B}_1}$$

- Individual coils are calibrated in a NMR-mapped reference dipole fringe fields are excluded
- LHC dipole field integral is measured in the same conditions with: 15 m coil shaft (12 coil sectors) + Single Stretched Wire (SSW)
- The SSW provides an independent calibration standard for integral fields only (precision comes from high-precision translation stages ~1 μm/10 mm. No direct comparison between SSW and NMR-mapped magnet is possible

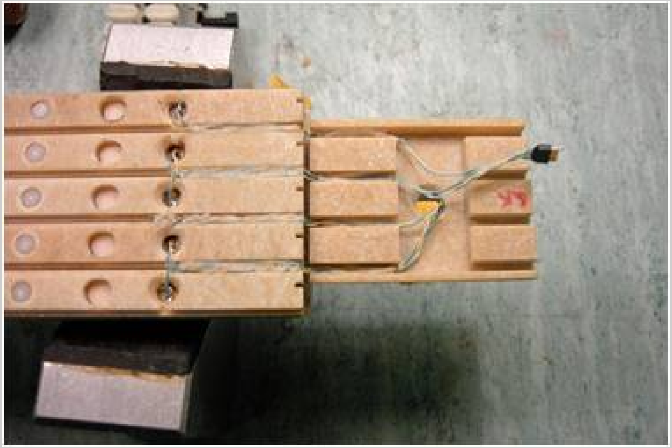


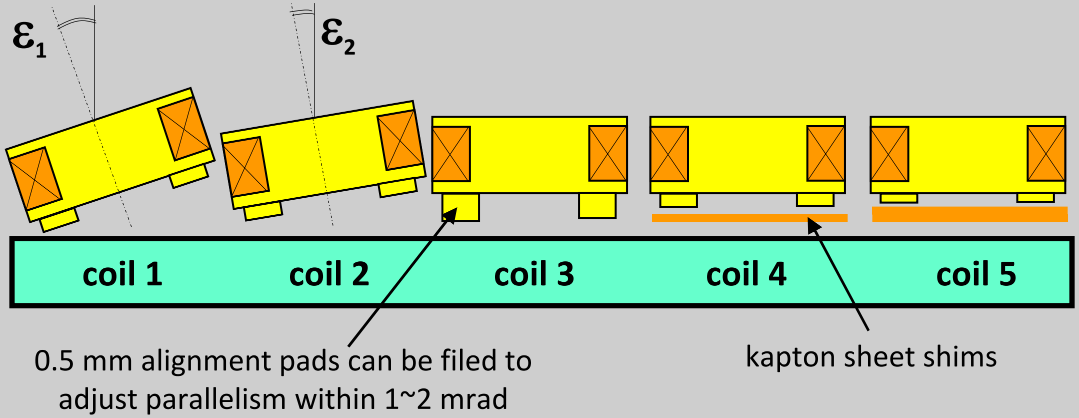


Comparison between SSW and harmonic coils gives an indication of absolute accuracy
In this particular case: RMS spread = **2·10^{-4}**



cross-checks with multiple instruments are an essential tool to get independent confirmation of calibration accuracy





- Connect coils with equal nominal area in series opposition

$$\kappa_1^1 = A_1(\cos\varepsilon_1 + i\sin\varepsilon_2) \approx A_1(1+i\varepsilon_2)$$
$$\kappa_1^2 \approx A_2(1+i\varepsilon_1)$$
$$\Rightarrow \quad \kappa_1^{diff} = \kappa_1^2 - \kappa_1^1 \approx \Delta A + iA\Delta\varepsilon$$

- Flip the array 180° to measure $\Delta\Phi$. Choose initial phase $\varphi_0 = \pi/2$ to be insensitive to area differences.

$$\Delta\Phi = 2\Re\left(\kappa_1^{diff} B_1 e^{in\varphi_0}\right) = 2B_1\left(\Delta A\cos\varphi_0 - A\Delta\varepsilon\sin\varphi_0\right)$$

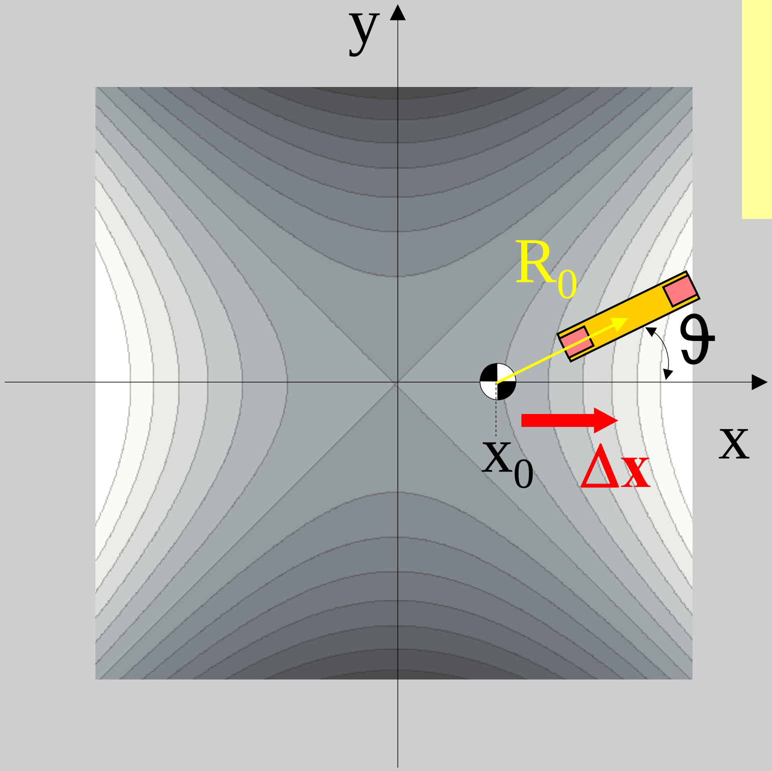
- Compute the tilt angle difference. File and/or shim, then iterate.

$$\Delta\varepsilon = \frac{\Delta\Phi}{2AB_1}$$

- NB: the tilt of the reference coil remains unknown (see field direction calibration later on)

Calibration in a Quadrupole

CERN Reference Quadrupole



Precisely controlled horizontal displacement to calibrate the gradient

$$B_2 = \frac{r_{ref}}{\Delta x}\frac{\Phi(0, x_0 + \Delta x) - \Phi(0, x_0)}{A_c}$$

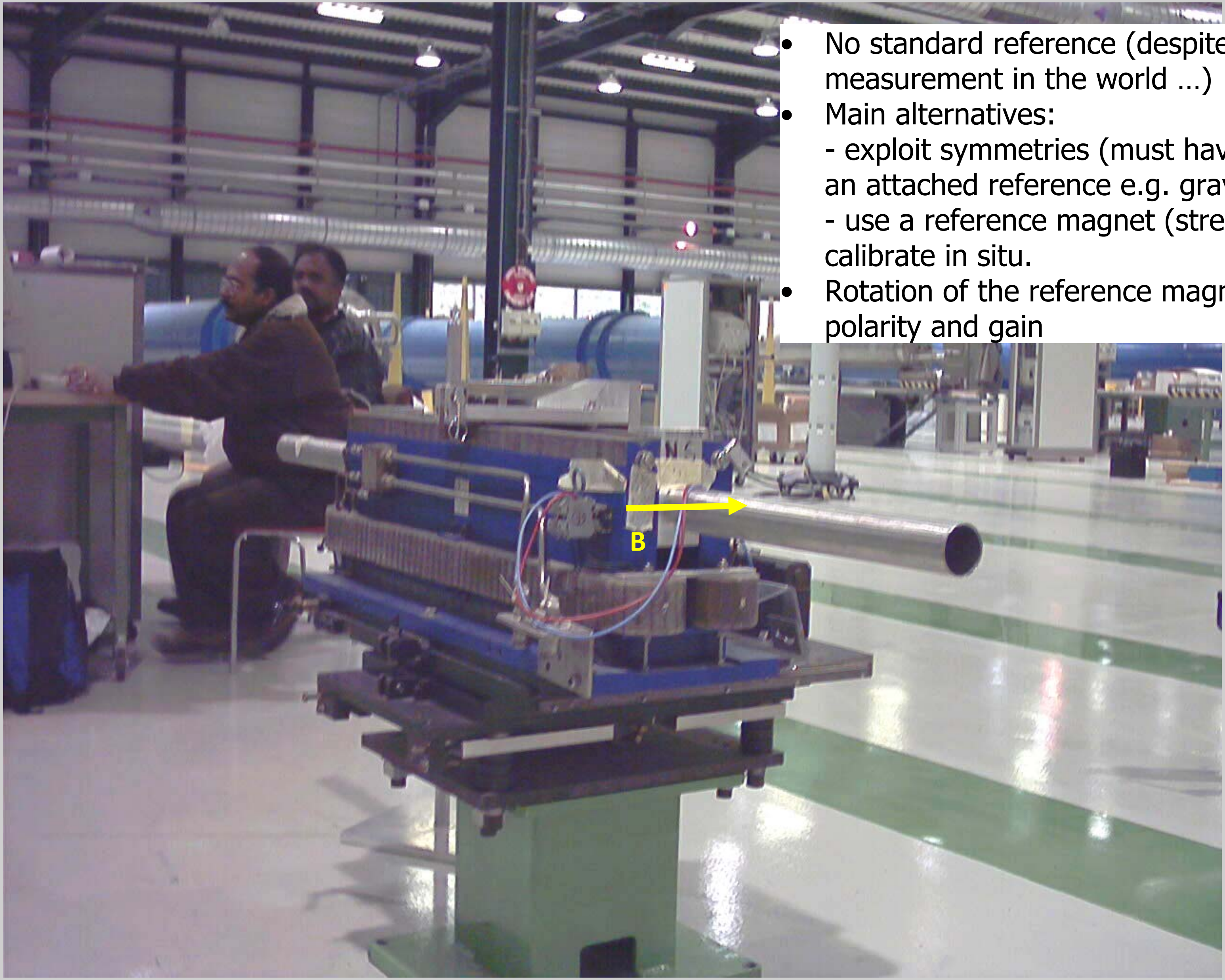
$$\Phi(\vartheta) = \Re\left(\kappa_1 B_1 e^{in\vartheta} + \kappa_2 B_2 e^{2in\vartheta}\right) = \frac{A_c B_2}{r_{ref}}\left(x_0 \cos\vartheta + \frac{1}{2} R_0 \cos 2\vartheta\right)$$

$$\begin{cases} \Phi(0) = \dfrac{A_c B_2}{r_{ref}}\left(x_0 + \dfrac{1}{2} R_0\right) \\ \Phi\left(\frac{\pi}{2}\right) = \dfrac{A_c B_2}{r_{ref}}\left(\qquad - \dfrac{1}{2} R_0\right) \\ \Phi(\pi) = \dfrac{A_c B_2}{r_{ref}}\left(-x_0 + \dfrac{1}{2} R_0\right) \end{cases} \Rightarrow \quad x_0 = r_{ref}\frac{\Phi(0) - \Phi(\pi)}{A_c B_2} \qquad R_0 = 2 r_{ref}\frac{\Phi(0) + \Phi(\pi)}{A_c B_2}$$

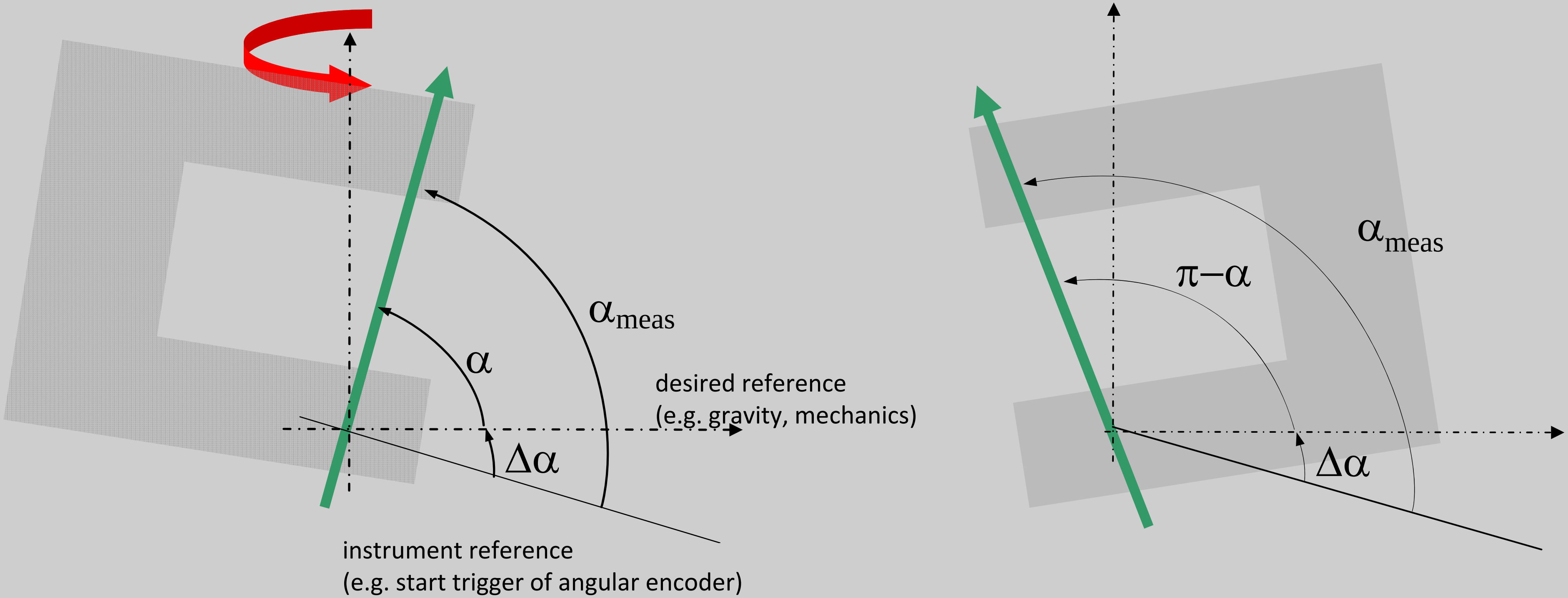
- Assume for simplicity a purely horizontal rotation axis offset
- It is essential that the rotation axis be the same during calibration and measurements (same ball bearings)
- Typical accuracy cross-checked via SSW about 17 units.
- No reference standard for field gradient: use a SSW validated previously with a NMR (Guy ?)
- SSW and rotating coil check: 1.7 units of difference on field integral

Calibration of field direction

Calibration of main field direction



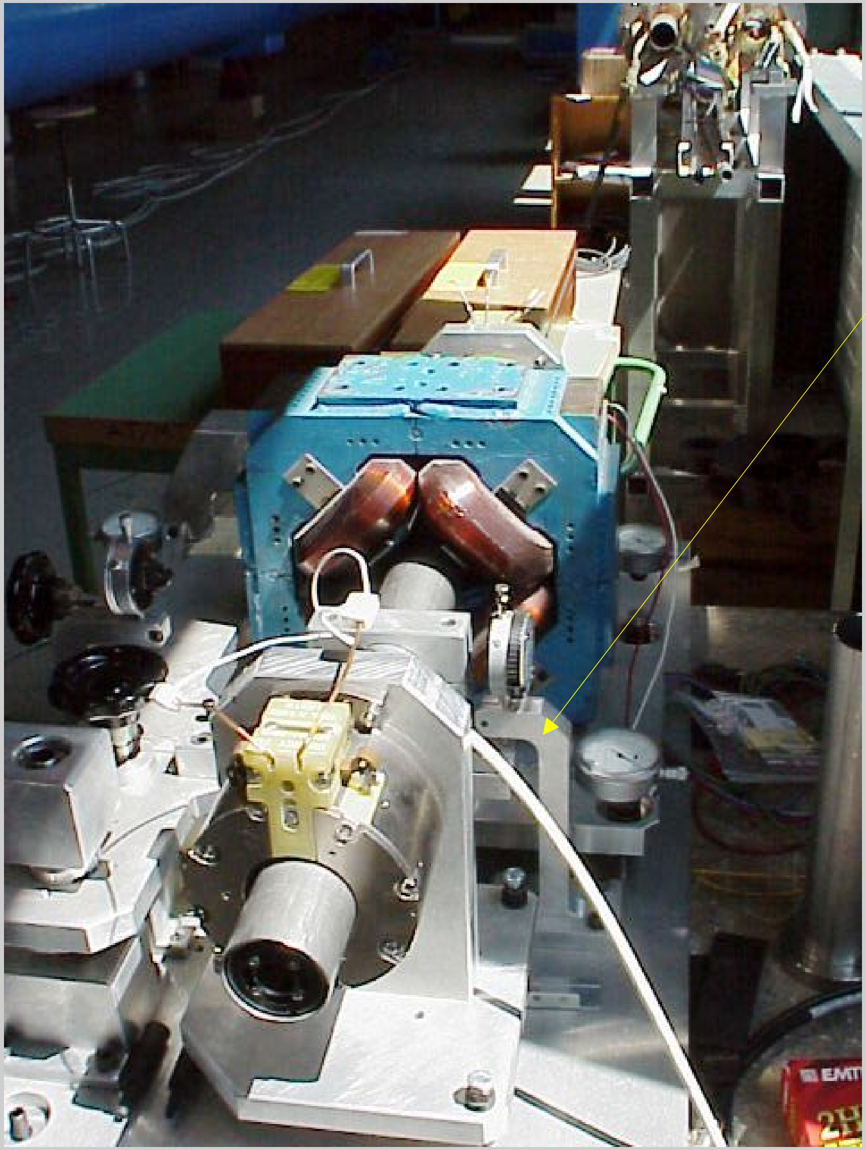
- No standard reference (despite being the oldest magnetic measurement in the world …)
- Main alternatives:
  - exploit symmetries (must have fixed rotating coil axis or an attached reference e.g. gravity)
  - use a reference magnet (stretched wire method) or re-calibrate in situ.
- Rotation of the reference magnet allows one to verify polarity and gain



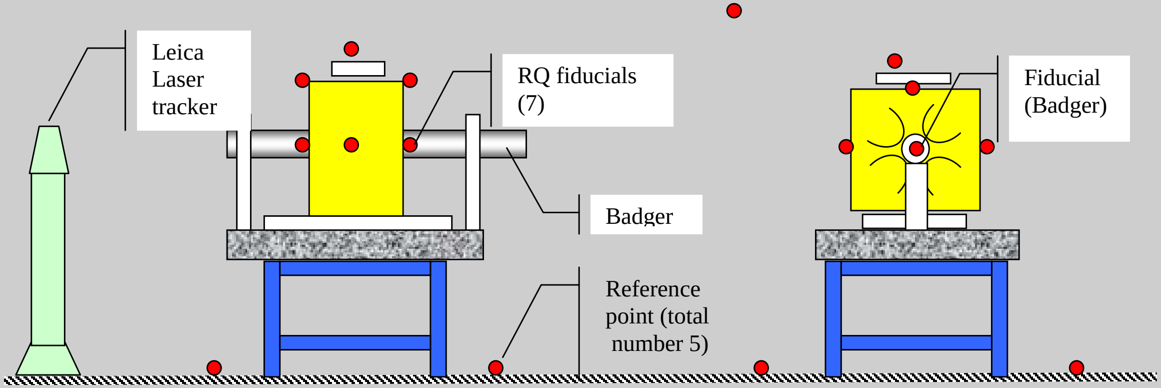
$$\begin{cases} \alpha^1_{meas} = \alpha + \Delta\alpha \\ \alpha^2_{meas} = \pi - \alpha + \Delta\alpha \end{cases} \Rightarrow \begin{cases} \alpha = \dfrac{\pi}{2} + \dfrac{\alpha^1_{meas} - \alpha^2_{meas}}{2} \\ \Delta\alpha = -\dfrac{\pi}{2} + \dfrac{\alpha^1_{meas} + \alpha^2_{meas}}{2} \end{cases}$$

- Measurement affected by unknown internal offset (e.g. mechanical tilt angle of inclinometer to encoder zero)
- Turning the instrument by 180° allows to measure the wanted angle + calibrate the offset

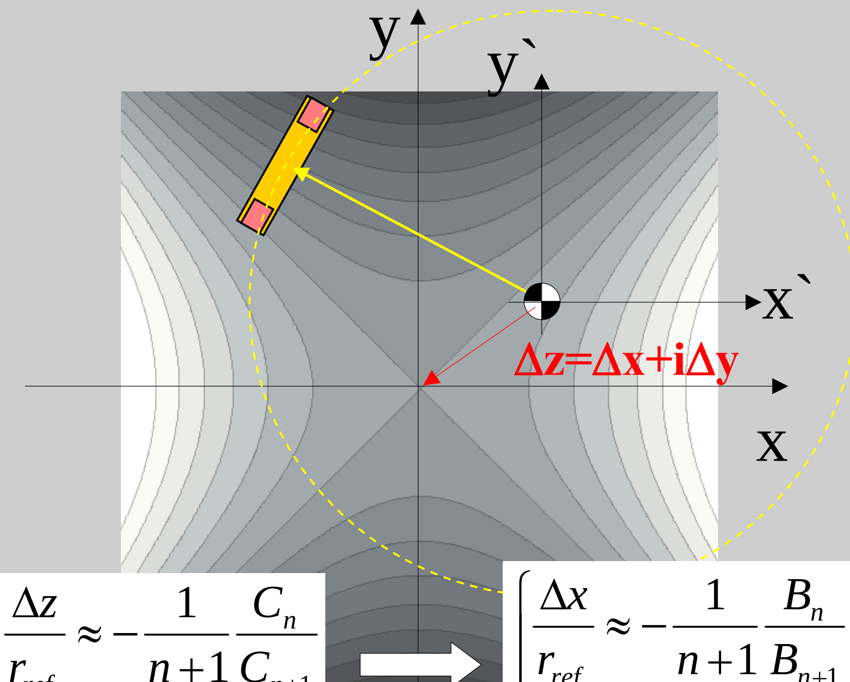
Calibration of axis



- No standard reference – ad-hoc techniques needed.
- Reference benches with quadrupole and higher multipole magnets
- Allow easy translation of the magnets for verification of polarity and gain
- Cross-check with multiple systems: rotating coils, AC coils, stretched wire



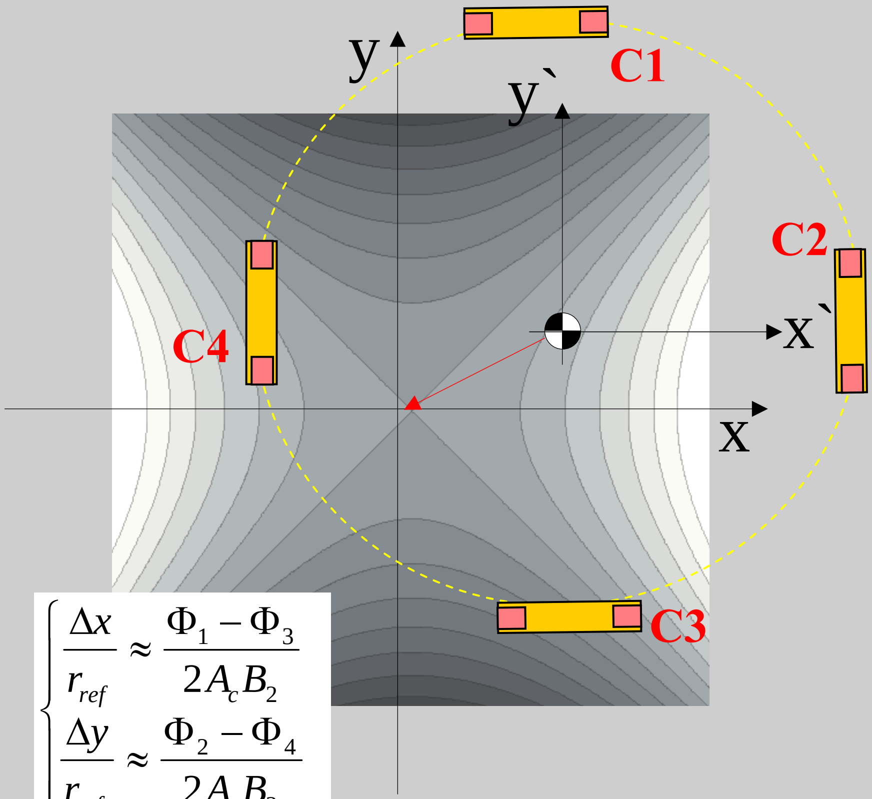
- Main axis measurement methods: harmonic feed-down from rotating coil data, morgan-coil type signal balance (AC or pulsed), stretched wire (AC or DC), Hall probe (null technique)
- Measured harmonics contain information about the null field region
- Assuming small offsets and one dominant harmonic, a linearized equation provides the center as a function of the strength of the dominant harmonic and the next lower one
- The computed Δz is relative to the coil rotation axis $\rightarrow$ this must be transferred (mechanically or optically) to an external frame of reference (magnet fiducial references).



$$\frac{\Delta z}{r_{ref}} \approx -\frac{1}{n+1}\frac{C_n}{C_{n+1}}$$

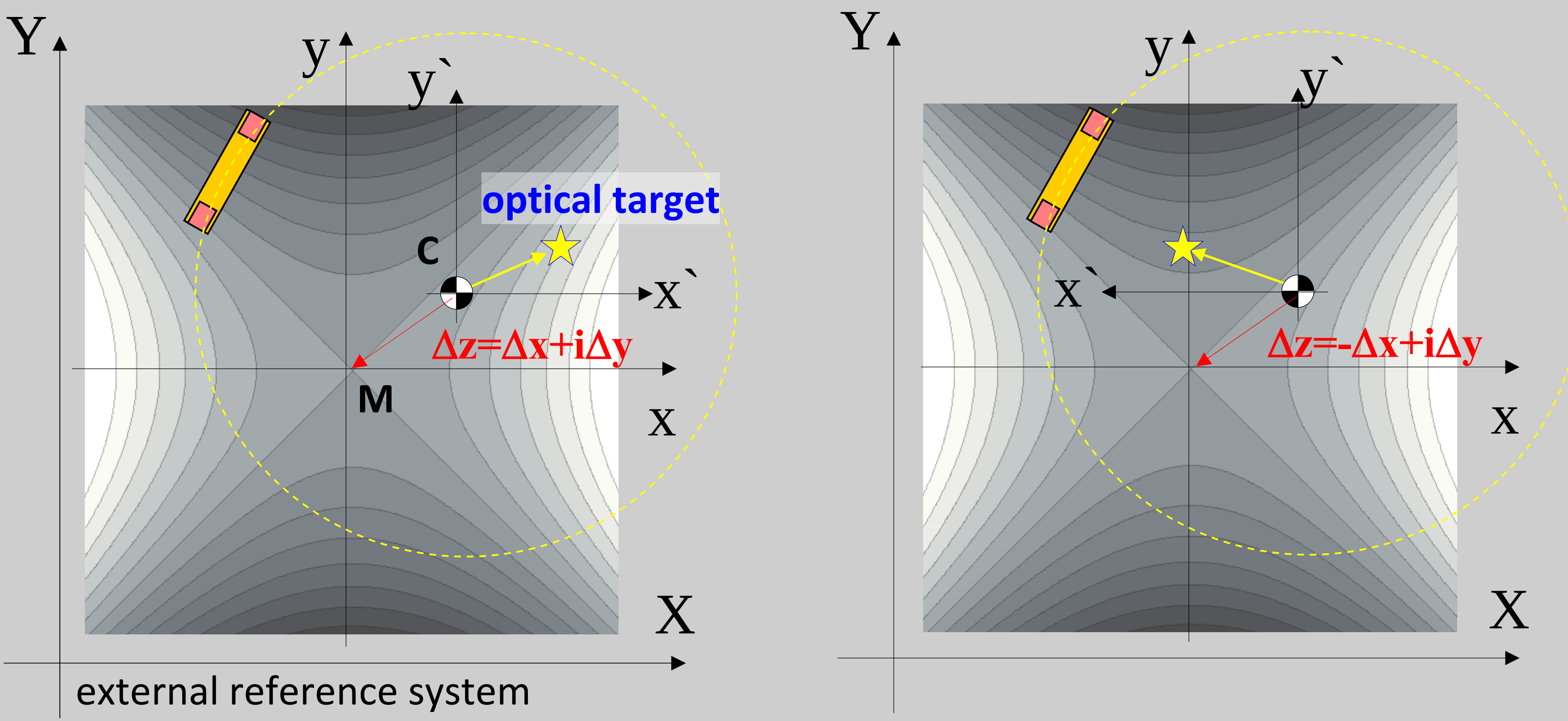
For a purely normal magnet:

$$\begin{cases} \dfrac{\Delta x}{r_{ref}} \approx -\dfrac{1}{n+1}\dfrac{B_n}{B_{n+1}} \\ \dfrac{\Delta y}{r_{ref}} \approx -\dfrac{1}{n+1}\dfrac{A_n}{B_{n+1}} \end{cases}$$

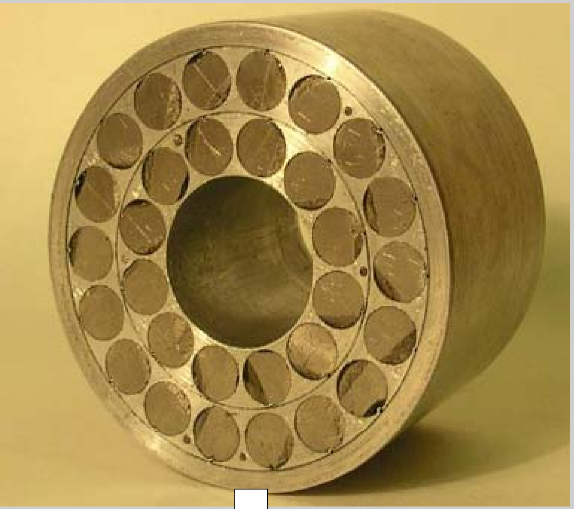


$$\begin{cases} \dfrac{\Delta x}{r_{ref}} \approx \dfrac{\Phi_1 - \Phi_3}{2A_c B_2} \\ \dfrac{\Delta y}{r_{ref}} \approx \dfrac{\Phi_2 - \Phi_4}{2A_c B_2} \end{cases}$$

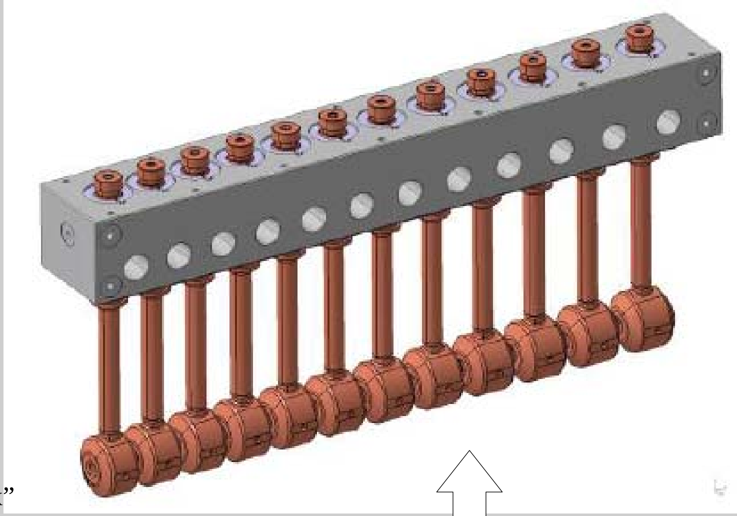
- Reverse by 180 one of the reference systems (the probe in the example below)
- As Δz is re-measured, the precise location of the vertical rotation axis is irrelevant
- Add an optical target (fixed w.r.t. the coil rotation radius) measured in an external reference (X,Y)



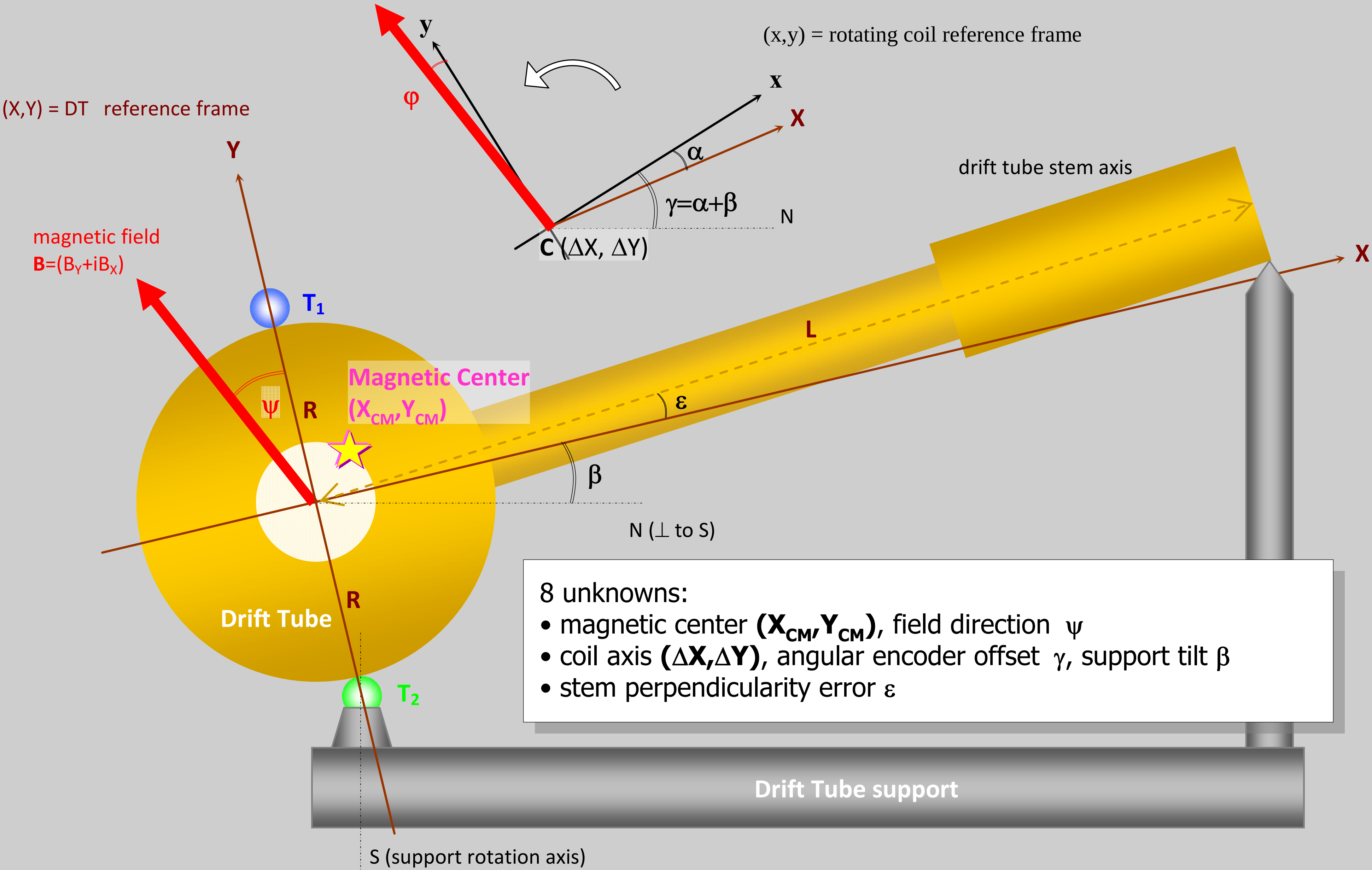
$$\begin{cases} X_{o1} = X_M + \Delta x_1 + x_0' \\ X_{o2} = X_M + \Delta x_2 - x_0' \end{cases}$$



- Linac4 currently in construction at CERN
- Drift tubes include permanent quads
- Must be aligned in each DFT tank to better than 0.08 mm



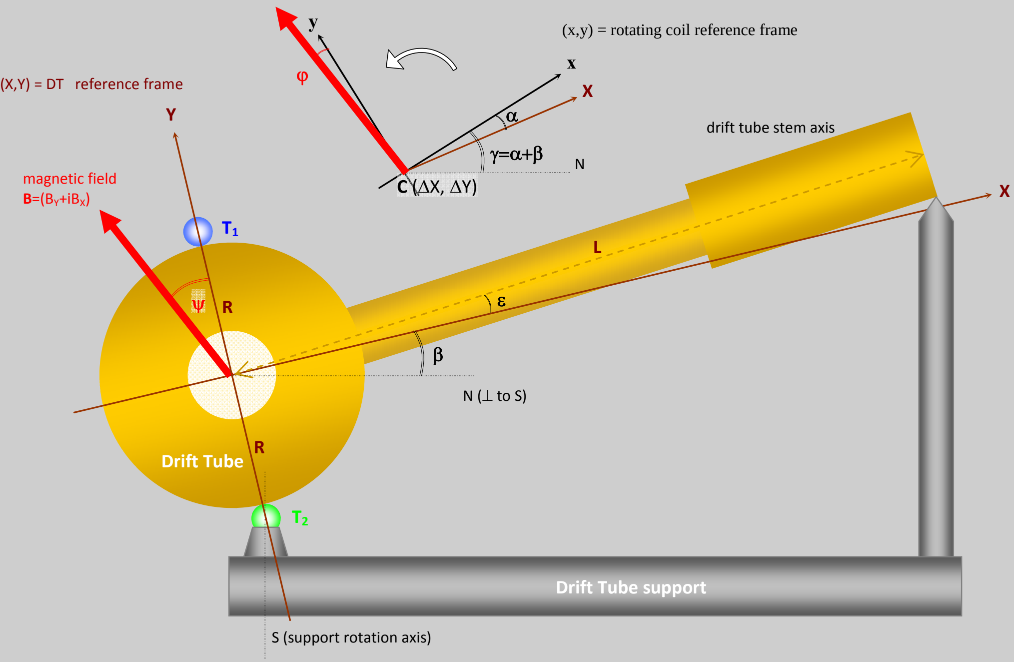
M. Vretenar, "Linac4 - a new linear accelerator for the CERN complex"

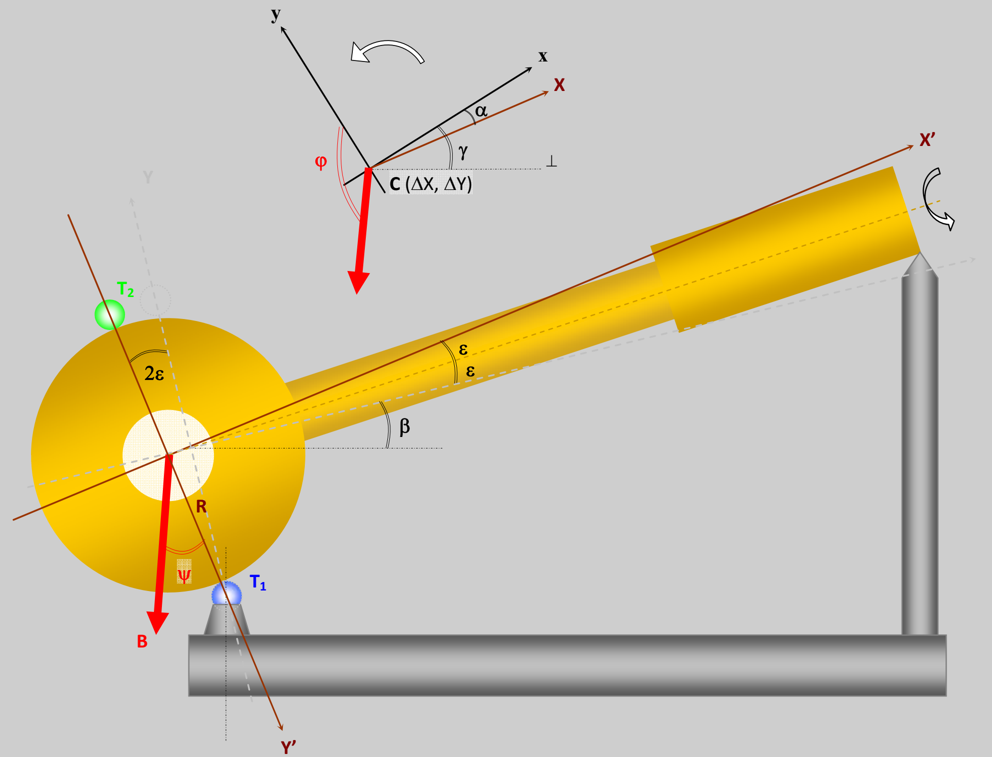


8 unknowns:

- magnetic center **(X_{CM},Y_{CM})**, field direction ψ
- coil axis **(ΔX,ΔY)**, angular encoder offset γ, support tilt β
- stem perpendicularity error ε

Drift Tube Measurement – Horizontal Flip





Position 0 - nominal

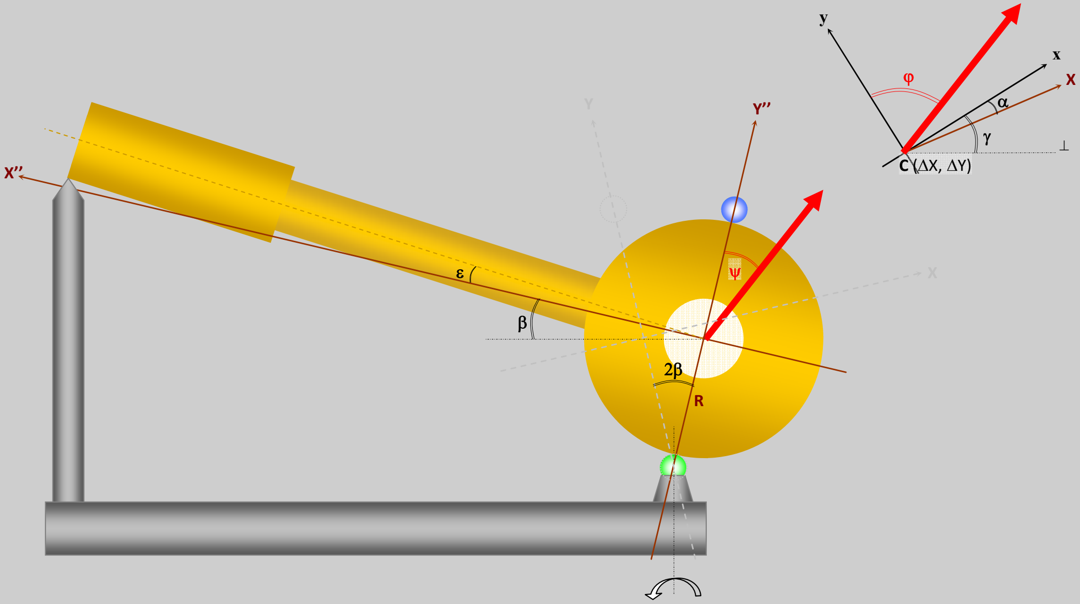
$$
\begin{cases} X = \Delta X + x - \alpha y \\ Y = \Delta Y + \alpha x + y \end{cases}
$$

$$
\psi = \varphi + \alpha
$$

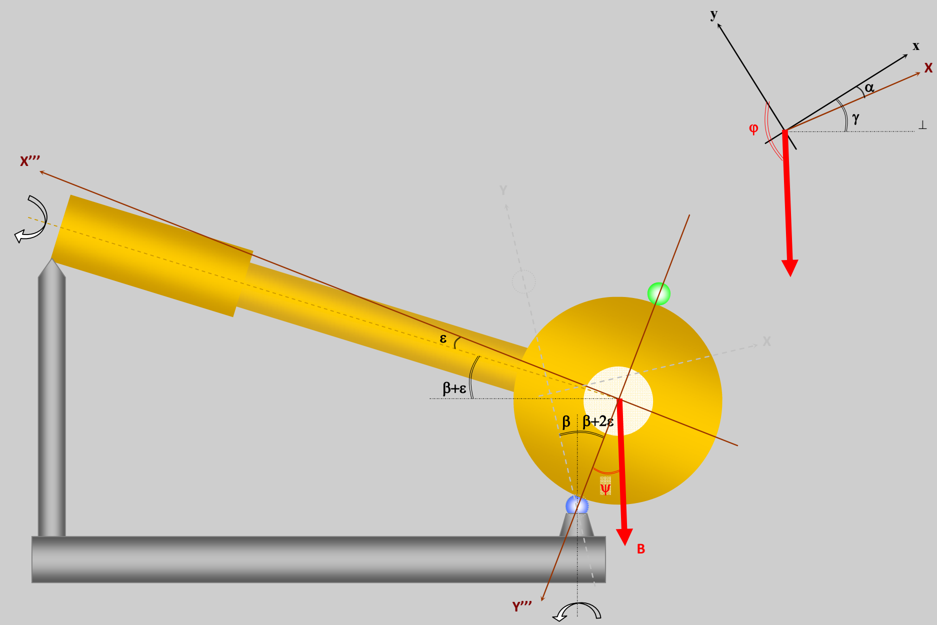
Position 1 – flip around X

$$
\begin{cases} X' = -2\varepsilon R + X + 2\varepsilon Y \\ Y' = -Y + 2\varepsilon X \end{cases}
$$

$$
\psi = \pi - \varphi - \alpha + 2\varepsilon
$$



Position 2 – flip around S



Position 3 – flip around X and S

$$\begin{cases} X'' = 2\beta R - X + 2\beta Y \\ Y'' = Y + 2\beta X \end{cases}$$

$$\psi = -\varphi + \alpha + 2\beta$$

$$\begin{cases} X''' = -2(\beta+\varepsilon)R - X + 2(\beta+\varepsilon)Y \\ Y''' = -2(\beta+\varepsilon)X - Y \end{cases}$$

$$\psi = -\pi + \varphi + \alpha + 2(\beta+\varepsilon)$$

Substitution of measurement results in position 0,1,2,3 and linearization of the equations:

$$
\begin{array}{lll}
Pos.0 & \begin{cases} X_{cm} = \Delta X + x_0 - \alpha y_0 \\ Y_{cm} = \Delta Y + \alpha x_0 + y_0 \end{cases} & \begin{array}{l} \approx \Delta X + x_0 \\ \approx \Delta Y + y_0 \end{array} \\
Pos.1 & \begin{cases} X_{cm} = -2\varepsilon R + (\Delta X + x_1 - \alpha y_1) + 2\varepsilon(\Delta Y + \alpha x_1 + y_1) \\ Y_{cm} = 2\varepsilon(\Delta X + x_1 - \alpha y_1) - (\Delta Y + \alpha x_1 + y_1) \end{cases} & \begin{array}{l} \approx -2\varepsilon R + \Delta X + x_1 \\ \approx -\Delta Y - y_1 \end{array} \\
Pos.2 & \begin{cases} X_{cm} = 2\beta R - (\Delta X + x_2 - \alpha y_2) + 2\beta(\Delta Y + \alpha x_2 + y_2) \\ Y_{cm} = 2\beta(\Delta X + x_2 - \alpha y_2) + (\Delta Y + \alpha x_2 + y_2) \end{cases} & \begin{array}{l} \approx 2\beta R - \Delta X - x_2 \\ \approx \Delta Y + y_2 \end{array} \\
Pos.3 & \begin{cases} X_{cm} = -2(\beta + \varepsilon)R - (\Delta X + x_3 - \alpha y_3) + 2(\beta + \varepsilon)(\Delta Y + \alpha x_3 + y_3) \\ Y_{cm} = -2(\beta + \varepsilon)(\Delta X + x_3 - \alpha y_3) - (\Delta Y + \alpha x_3 + y_3) \end{cases} & \begin{array}{l} \approx -2(\beta + \varepsilon)R - \Delta X - x_3 \\ \approx -\Delta Y - y_3 \end{array}
\end{array}
$$

$$
\begin{cases} \psi = \varphi_0 + \alpha \\ \psi = \pi - \varphi_1 - \alpha + 2\varepsilon \\ \psi = -\varphi_2 + \alpha + 2\beta \\ \psi = -\pi + \varphi_3 + \alpha + 2(\beta + \varepsilon) \end{cases}
$$

8 unknowns, 12 equations:

- magnetic center **(X_{CM},Y_{CM})**, field direction ψ
- coil axis **(ΔX,ΔY)**, angular encoder offset γ, support tilt β
- stem perpendicularity error ε

Redundant equations useful to double-check and estimate error bars !

Field direction measurements ⇒ calculation of calibrated angular parameters

$$\begin{Bmatrix} \psi \\ \alpha \\ \varepsilon \\ \beta \end{Bmatrix} = \frac{1}{2} \begin{bmatrix} -1 & -1 & -1 & 1 \\ 1 & -1 & 1 & 1 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & -1 & 1 \end{bmatrix} \begin{Bmatrix} -\varphi_0 \\ \varphi_1 - \pi \\ \varphi_2 \\ -\varphi_3 + \pi \end{Bmatrix}$$

ψ **magnetic field direction** in the DT reference frame

$\gamma=\alpha+\beta$ **offset angle** between encoder Ø and support
(this is a fixed quantity useful to check stability)

ε ⊥ **error** of stem w.r.t. fiducial line T_1-T_2
(can be cross-checked with geometry survey results)

Magnetic center measurements ⇒ calculation of calibrated linear parameters

$$\begin{cases} X_{cm} = \beta R + \dfrac{x_0 - x_2}{2} = -(\beta + 2\varepsilon)R + \dfrac{x_1 - x_3}{2} \\ Y_{cm} = \dfrac{y_0 - y_1}{2} = \dfrac{y_2 - y_3}{2} \end{cases}$$

$$\Delta X = 2\beta R - \frac{x_0 + x_2}{2}$$

$$\Delta Y = -\frac{y_1 + y_2}{2}$$

$$\begin{cases} N_C = -R\beta + \Delta X \\ S_C = R + \Delta Y \end{cases}$$

- (X_{CM},Y_{CM}) **magnetic axis position** in the DT reference frame
- (N_C,S_C) coordinates of the **coil rotation axis** in a frame fixed to the test bench
(another built-in parameter useful to check system stability)

Calibration of fixed coils

- In principle: calibration coefficients work whatever the source of $\partial\Phi/\partial t$, so all previous considerations apply
- Field mapping techniques require normally repeated measurements in different coil positions – one has to worry about the repeatability of the magnetic field:
  - stability of power supply (10^{-4}~10^{-5})
  - accuracy of current measurements
  - reproducibility of hysteresis cycle (dependence upon history, time, temperature, ramp rates, ripple of the power supply → minor hysteresis loops)
- Magnets must be stabilized into a reproducible magnetic state by suitable pre-cycling (a few to a few dozen cycles may be needed)
- Variable-field measurements with moving coils are possible, but the analysis is much more complicated [BNL]
- The start and stop currents must be chosen in a range where the power supply is stable – no ripple (unipolar power converters may be unstable close to zero current)
- Integration lengths should be integer multiples of the period of the dominant perturbations (50 Hz mains, converter ripple …)

$$\int_0^L B(s, I(t))ds - \int_0^L B(s, I_0)ds = -\frac{1}{\overline{w}_{eff}} \int_0^t V_c dt$$

Example: calibration of CNAO fluxmeter

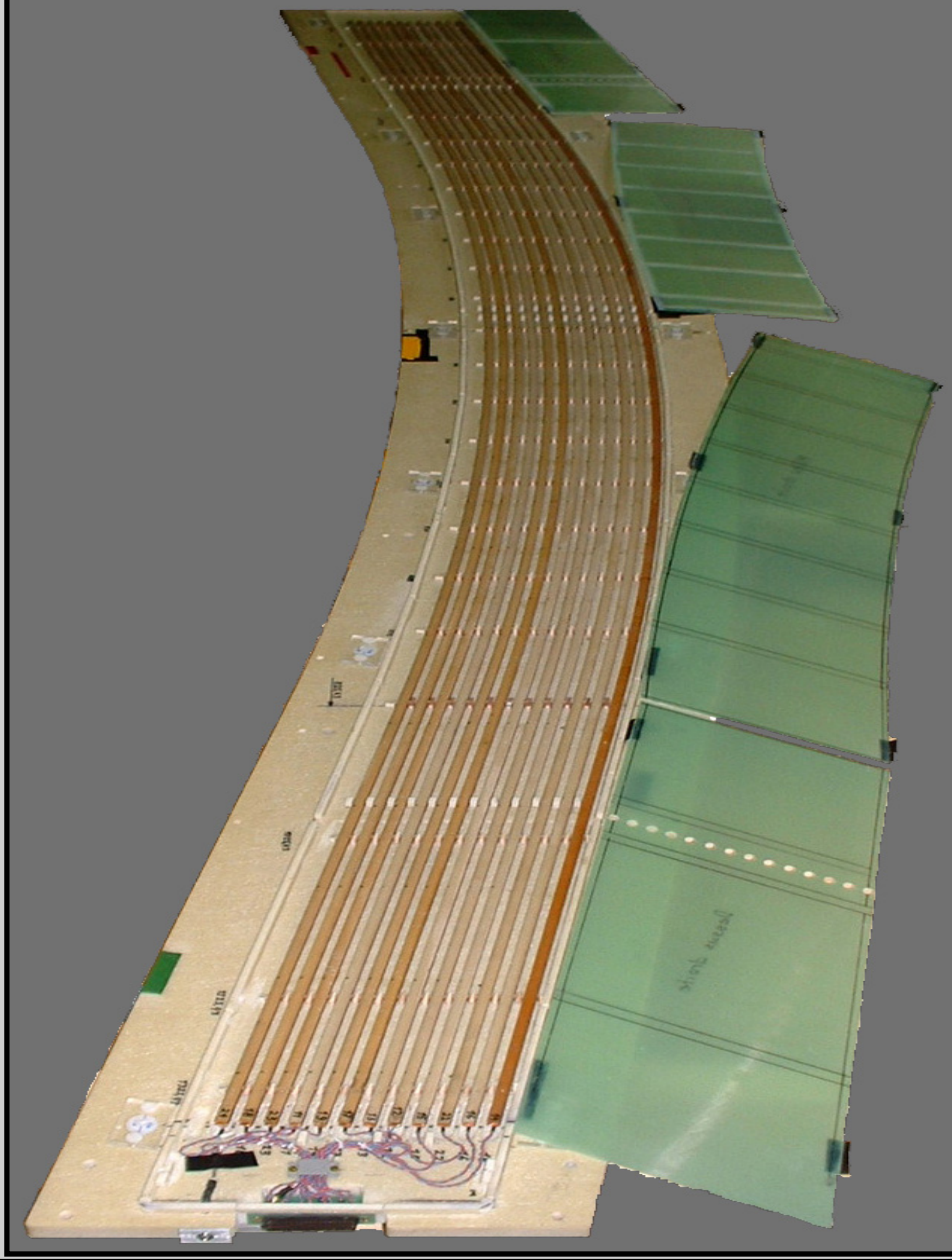
- Main problem: long curved coil shape → whole coil calibration inside a reference magnet impossible.
- Worse: in-situ comparison with a stretched wire systems also impossible !
- Solution: **relative measurements w.r.t. a reference coil**
- Surface area is expected to change ~10^{-3} due to the bending (wire is stretched outboard and compressed inboard). Area change should be systematic → coils sorted based on original areas
- Three best coils at the centre and extremities of the goos field region to minimize error. The reference chosen as the closest to the average.
- A caveat: moving the coil is difficult and likely to affect mechanical stability and calibration.

Coil ordering	Position in the support assembly (mm)	N° of the Coil	Surfaces (m^2)	Delta surfaces (‰)
13	-90			
12	-75	24	2.3674	-10.86
11	-60	18	2.3934	-0.01
10	-45	23	2.4191	10.74
9	-30	11	2.4117	7.67
8	-15	19	2.4041	4.49
7	0	17	2.3934	0.00
6	15	13	2.4015	3.42
5	30	12	2.3803	-5.44
4	45	15	2.4131	8.24
3	60	22	2.3923	-0.43
2	75	16	2.4216	11.79
1	90	14	2.3554	-15.86

GFR

Coil ordering	Reference support	N° of the Coil	Surface (m^2)	Delta surface (‰)
14	0	21	2.3991	2.40

Spare Coil	Surface (m^2)	Delta surface (‰)
20	2.3881	-2.18

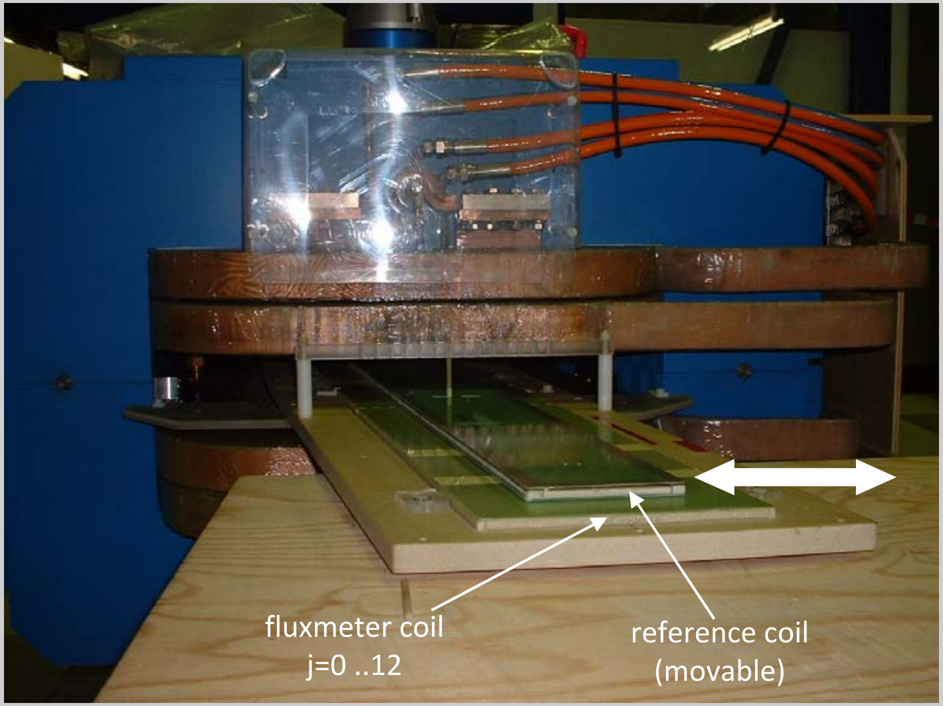


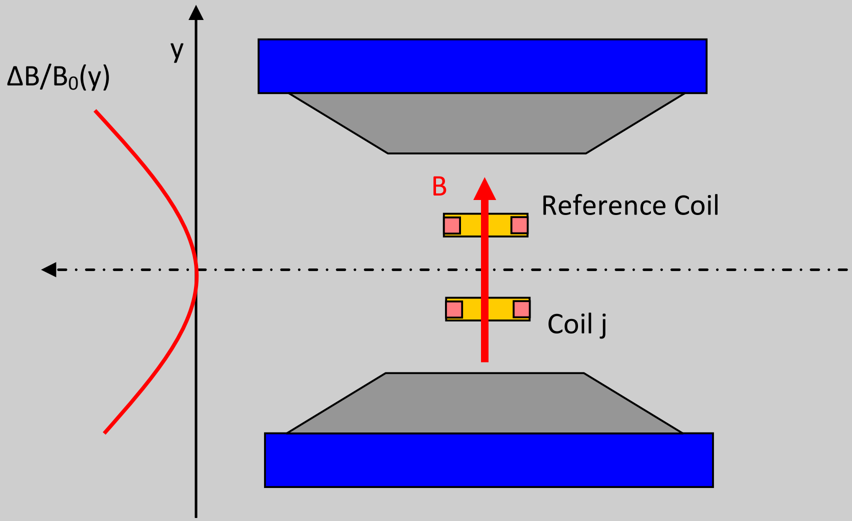
Relative calibration

- Reference and calibrated coils are placed symmetrically w.r.t. midplane – assuming allowed harmonics only, they see the same field (automatic compensation of power supply instability, magnetic history, drifts ..)
- A measurement in series opposition gives therefore:

$$\frac{\Phi^j}{\overline{w}_{eff}^{\,j}} = \frac{\Phi^{ref}}{\overline{w}_{eff}^{\,ref}} \quad \Rightarrow \quad \overline{w}_{eff}^{\,j} = \overline{w}_{eff}^{\,ref} \underbrace{\left(1 + \frac{\Delta\Phi^j}{\Phi^{ref}}\right)}_{k_j} \quad \Longrightarrow \quad \boxed{\frac{\Delta B d\ell^{\,j}}{B d\ell^{\,0}} = \frac{\Phi^j}{\Phi^0}\frac{\overline{w}_{eff}^{\,0}}{\overline{w}_{eff}^{\,j}} - 1 = \frac{\Phi^j}{\Phi^0}\frac{k_0}{k_j} - 1}$$

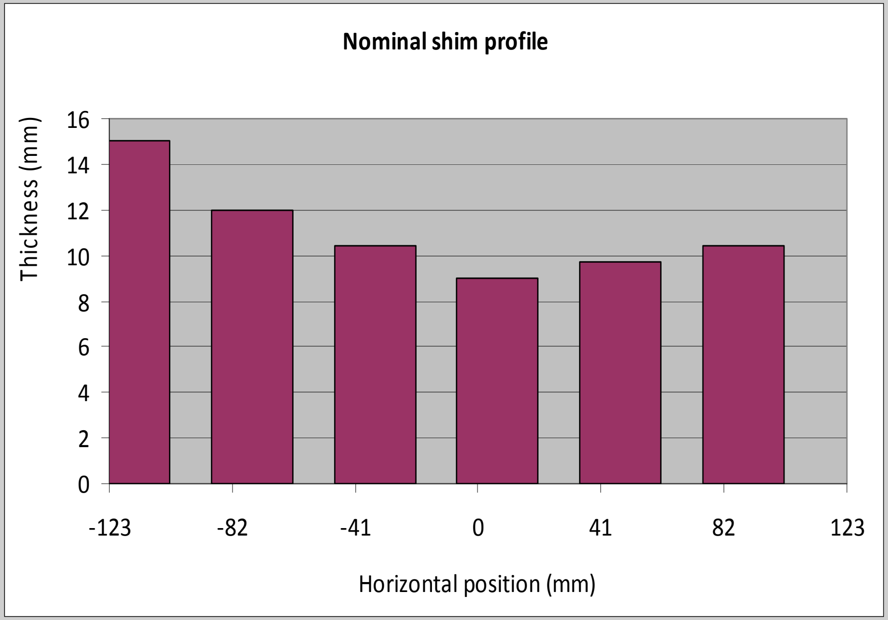
- Important advantage: each calibration takes into account the proper profile B(s) for each position ("in situ" concept: the reference follows the instrument)

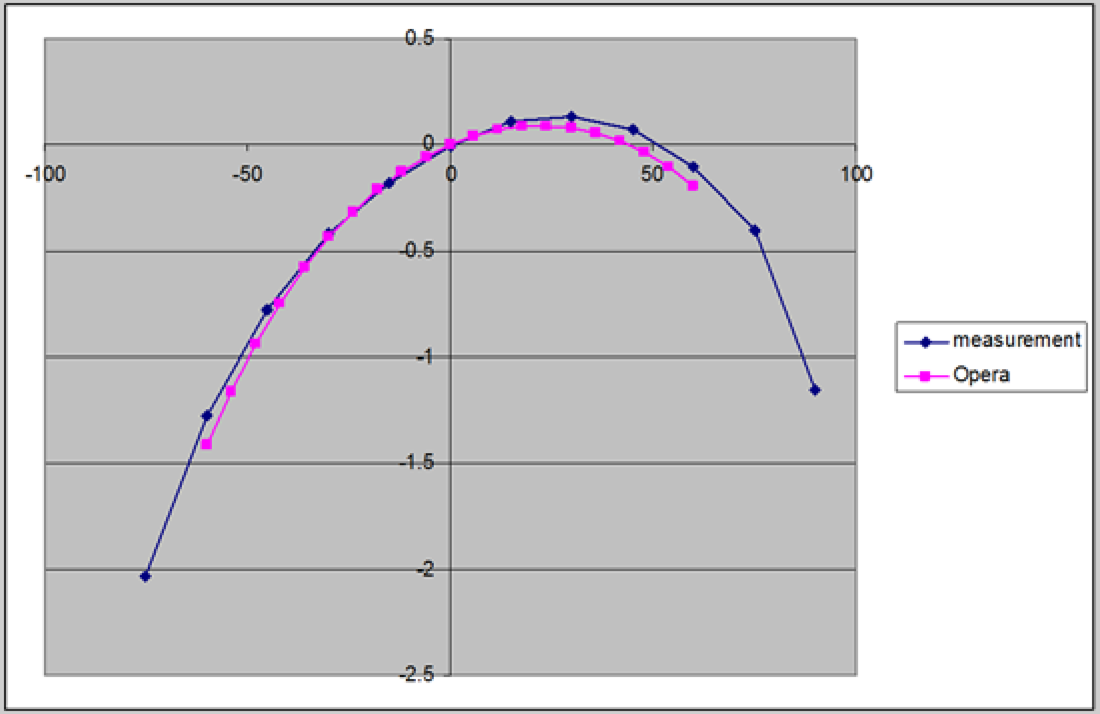




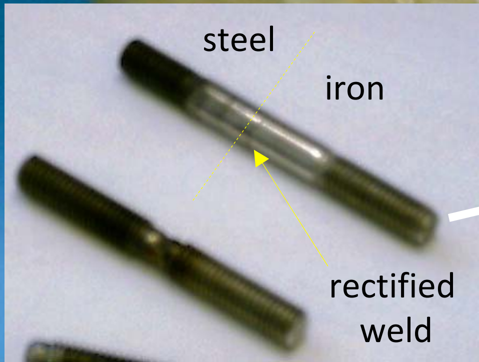
Calibrated measurement of field homogeneity – before shimming

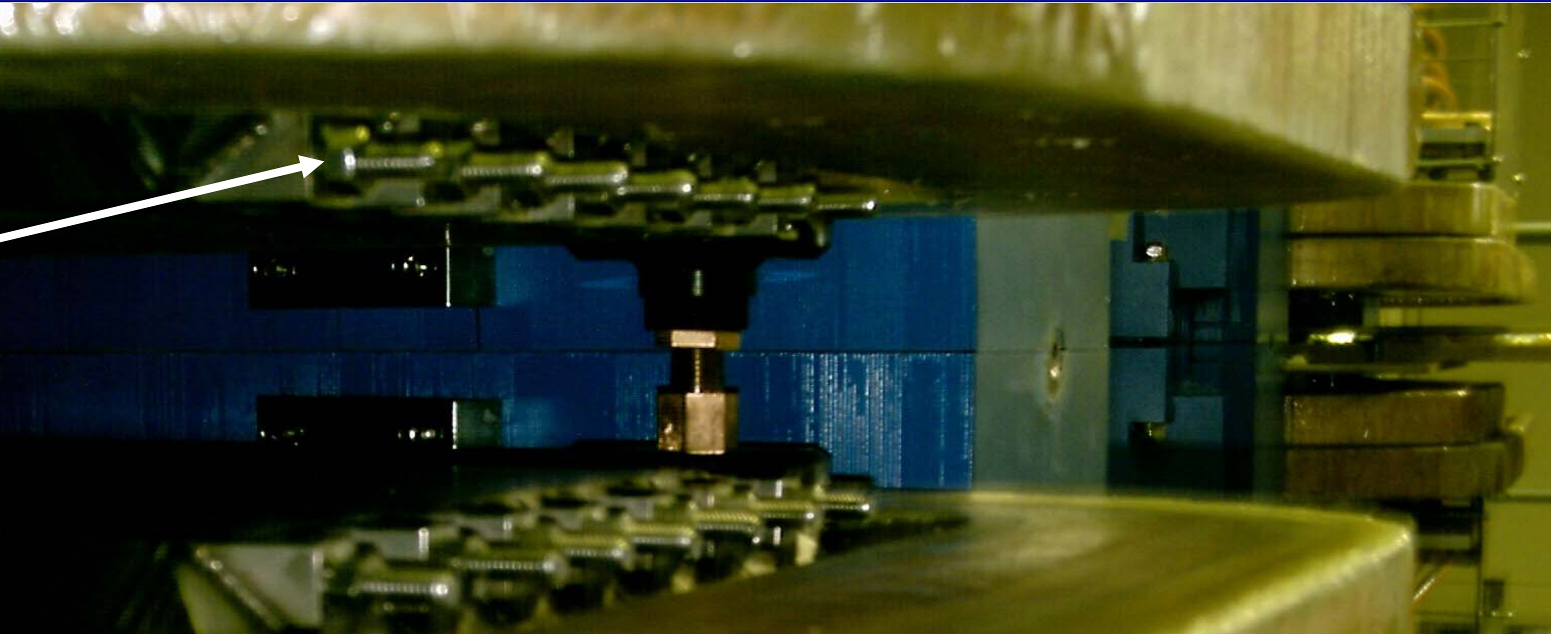
- Curved magnets with parallel end generate strong quadrupole component (fringe field ⊥ ends)
- Nominal shimming with linear + parabolic profile to offset quadrupole and sextupole



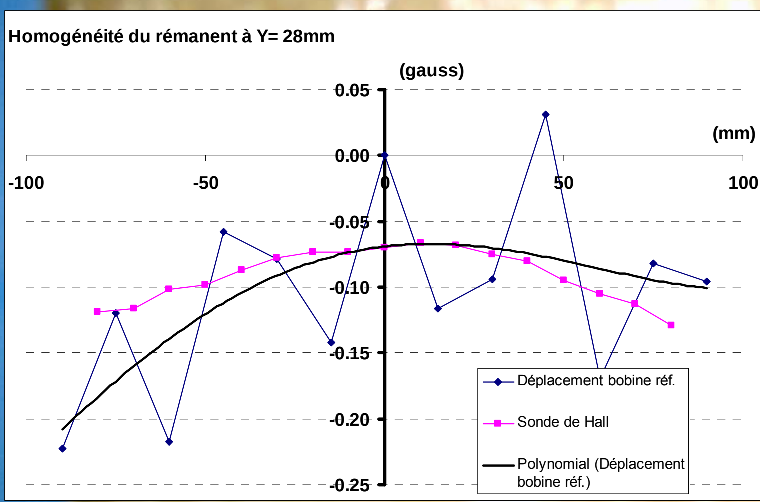


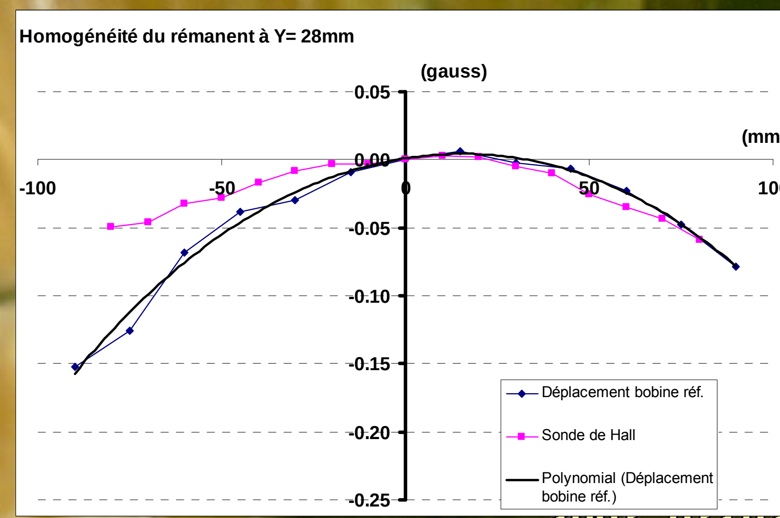
… why bother measuring if the simulation is good ?





- Nasty surprise – effect of stainless steel shim bolts on field uniformity curves
- Solved resorting to bi-metal screws – iron half engages into the yoke, steel half remains outside



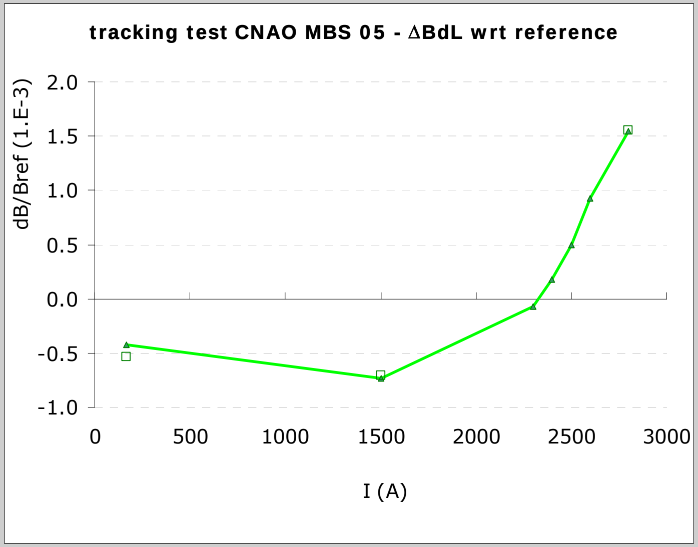


Tracking measurements

- Relative measurement of BdL w.r.t. reference magnets
- Objective: shim all main dipoles to ensure an uniform BdL (accuracy of the beam orbit, facilitates maintenance)
- Method used with long-term success in many other cases (e.g. SPS).

Fluxmeter inside the magnet to be shimmed

Reference inside the reference dipole



Final Remarks

Conclusions

- Like for a magnet, good measurements start with good mechanics
- The two biggest enemies of accuracy: time and temperature – do your best to control them
- Cross-checks with multiple systems are a good practical way to find out and eliminate systematic errors
- Carry out calibration and actual measurement changing conditions as little as possible (environment, electronics, magnet …)
- Good quality assurance practice is the hallmark of a professional work: track and record everything, who does what, when and with which equipment