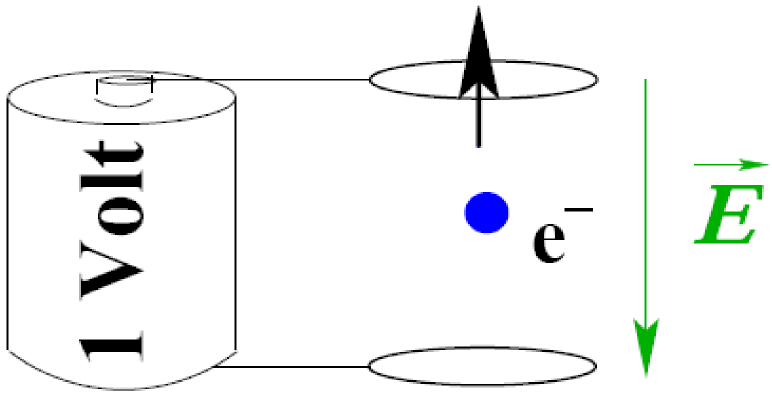
Beam Dynamics

D. Brandt, CERN

Some generalities …

Units: the electronvolt (eV)



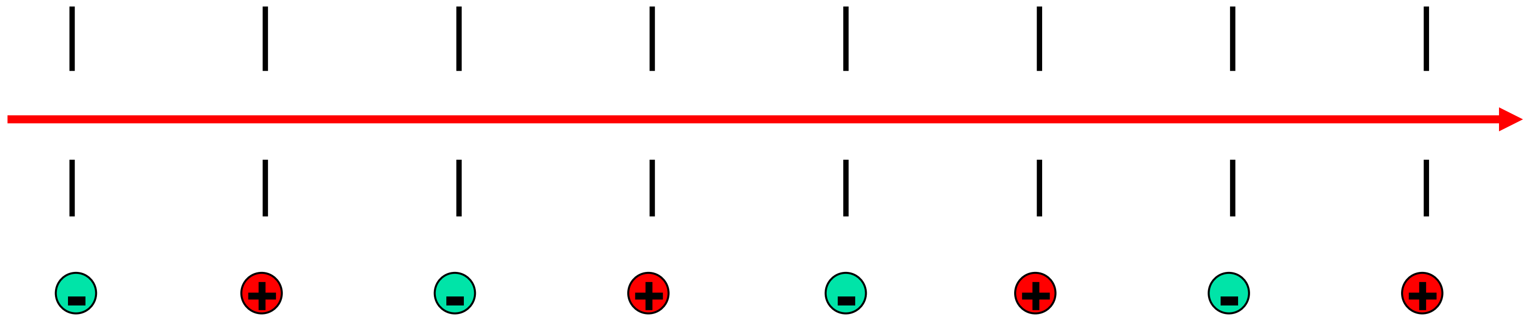
The electronvolt (eV) is the energy gained by an electron travelling, in vacuum, between two points with a voltage difference of 1 Volt. 1 eV = 1.602 10^{-19} Joule

We also frequently use the electronvolt to express masses from $E=mc^2$: 1 eV/c^2 = 1.783 10^{-36} kg

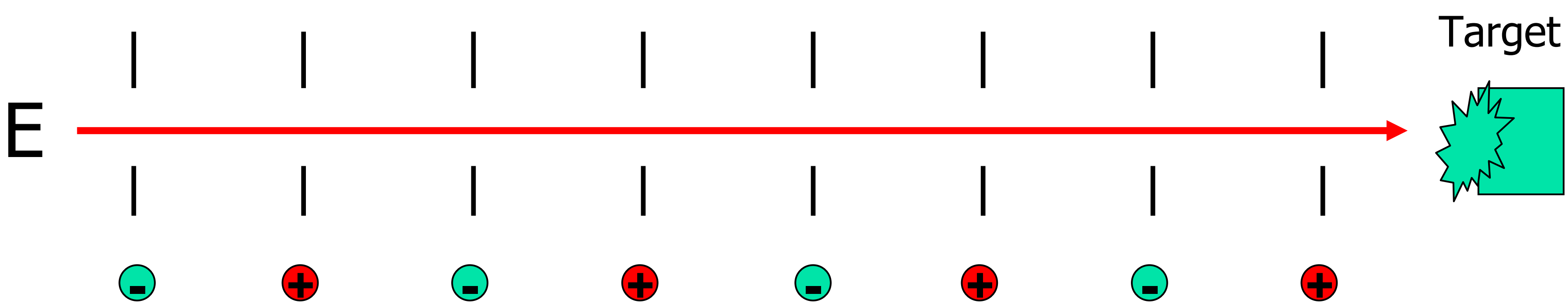
What is a Particle Accelerator?

➢ a machine to accelerate some particles ! How is it done ?

➢ Many different possibilities, but rather easy from the general principle:



Ideal linear machines (linacs)



Available Energy : $E_{c.m.} = m \cdot (2+2\gamma)^{1/2} = (2m.(m+E))^{1/2}$

with $\gamma = E/E_0$

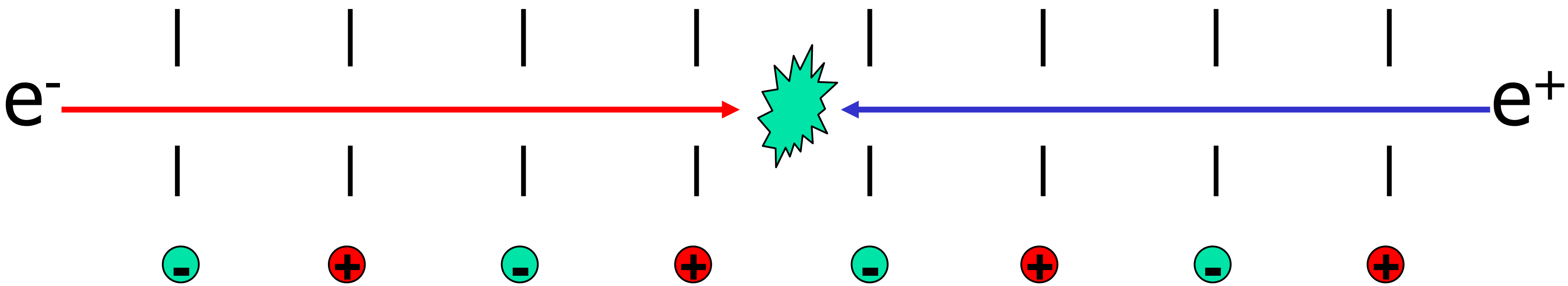
<u>Advantages</u>: Single pass

High intensity

<u>Drawbacks</u>: Single pass

Available Energy

Improved solution for $E_{c.m.}$



Available Energy : $E_{c.m.} = 2m\gamma = 2E$

with $\gamma = E/E_0$

Advantages: High intensity

Drawbacks: Single pass

Space required

Watch out !

The difference between fixed target and colliding mode deserves to be considered in some detail:

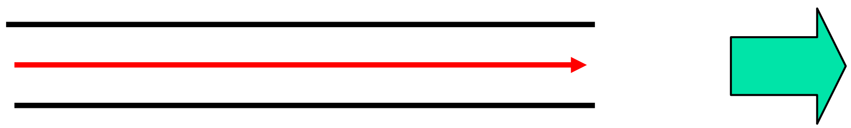
Fixed target mode:		Colliding mode:
$E_{c.m.} \propto (2mE)^{1/2}$	$\Leftrightarrow$	$E_{c.m.} \propto 2E$

What would be the required beam energy to achieve $E_{c.m.}=14$ TeV in fixed target mode ?

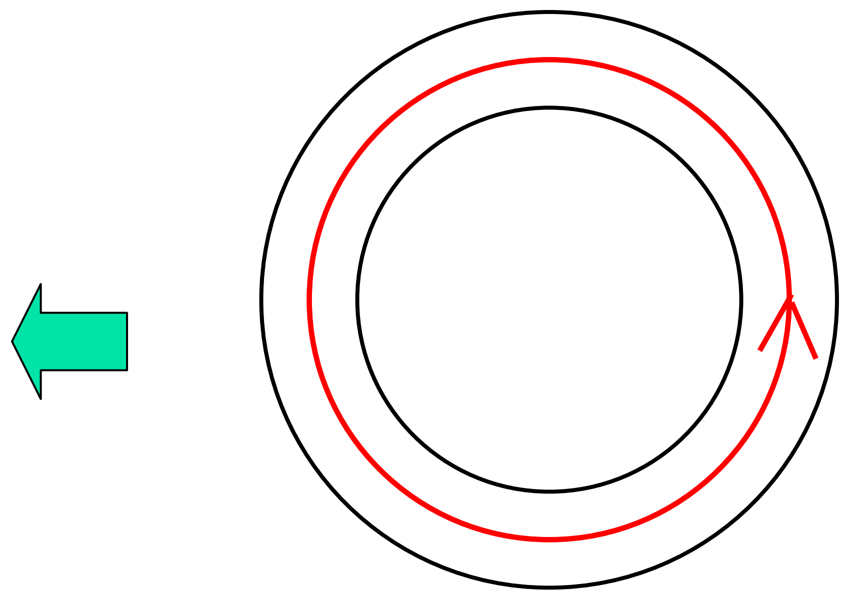
Keep particles: circular machines

Basic idea is to keep the particles in the machine for many turns.

Move from the linear design



To a circular one:

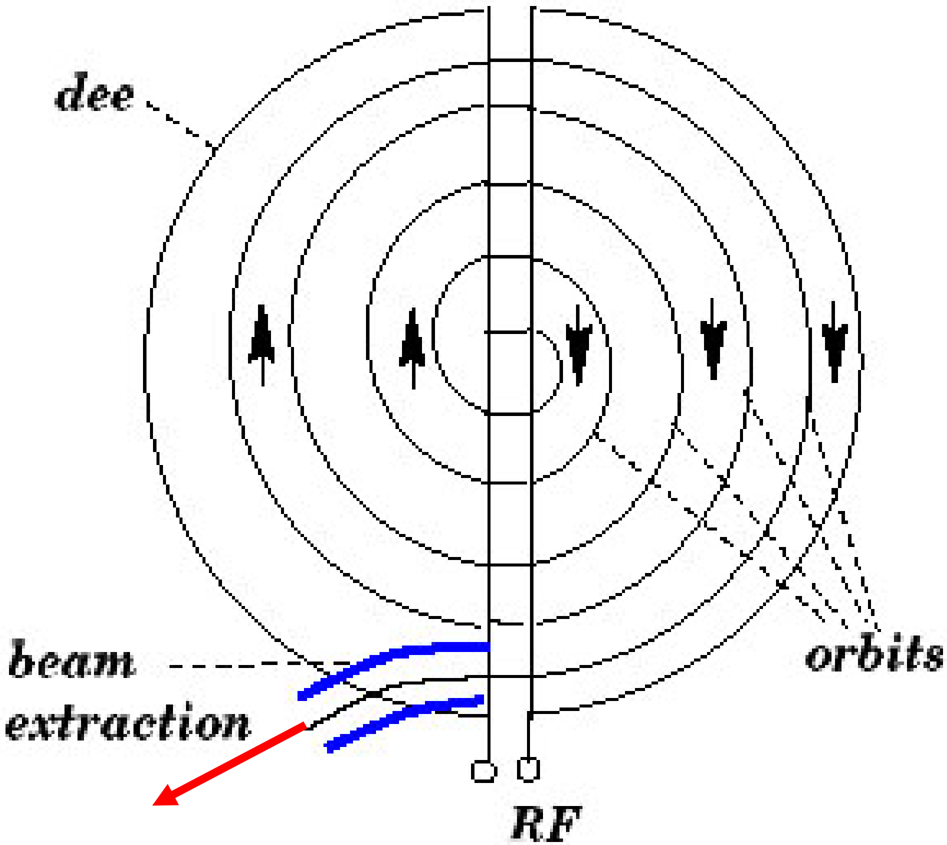


- Need Bending
- Need Dipoles!

Circular machines 1 ($E_{c.m.} \sim (mE)^{1/2}$)

fixed target:

cyclotron



huge dipole, compact design, B = constant , low energy, single pass.

Circular machines 2 ($E_{c.m.} \sim (mE)^{1/2}$)

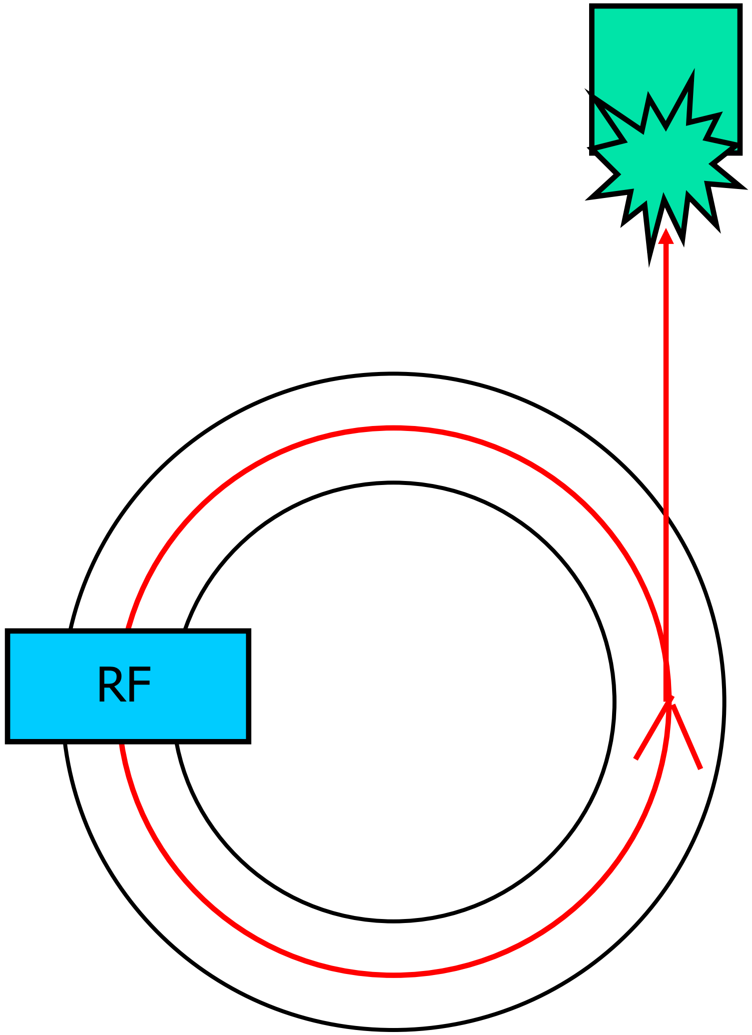
fixed target:

synchrotron

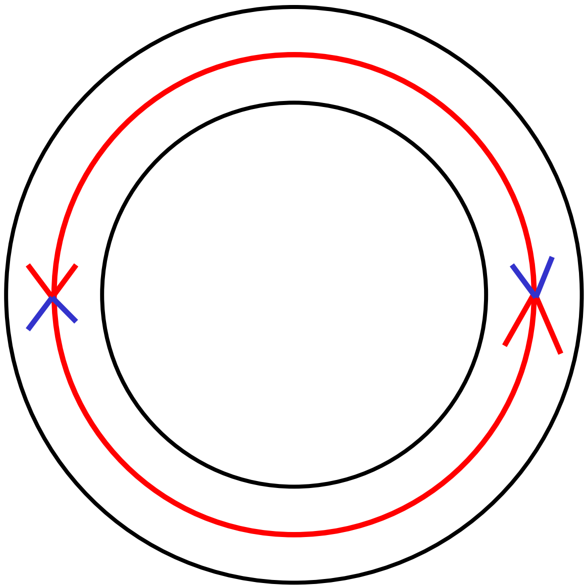
varying B

small magnets,

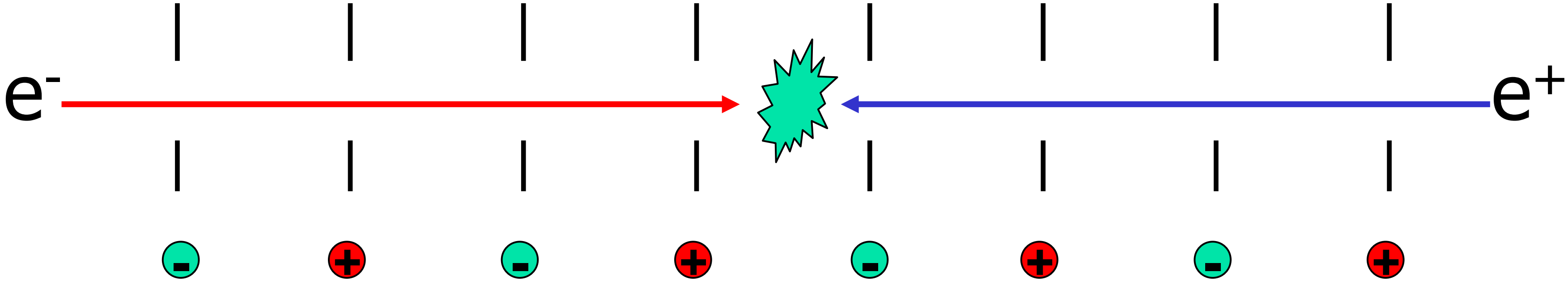
high energy



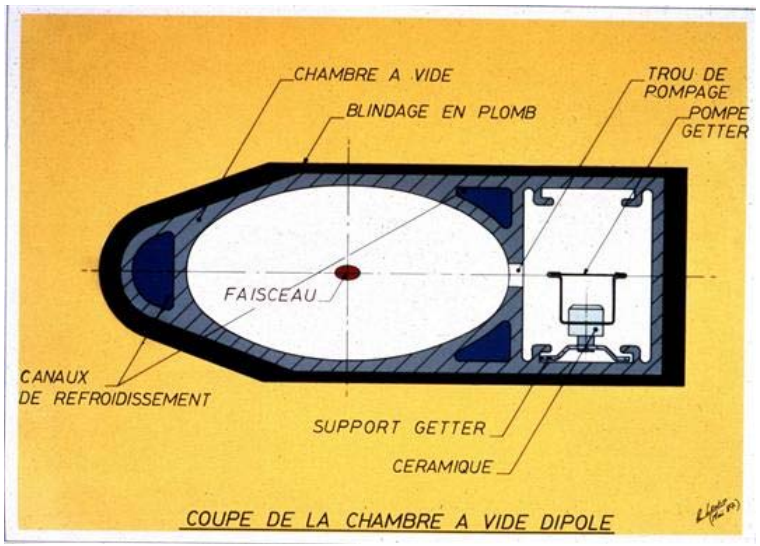
Colliders ($E_{c.m.}$=2E)



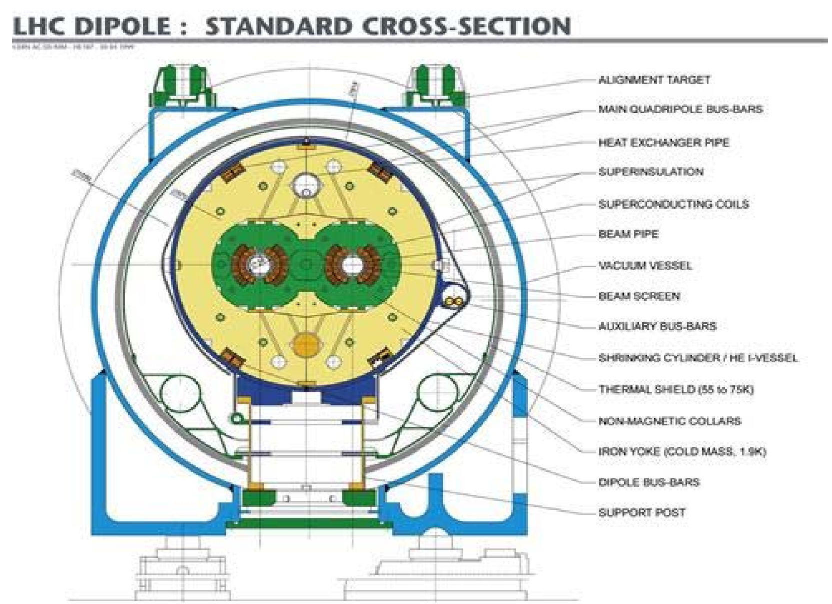
Colliders with the same type of particles (e.g. p-p) require two separate chambers. The beam are brought into a common chamber around the interaction regions



Colliders (e^+ - e^-) et (p – p)



LHC



LEP

Transverse Dynamics

$$F = e\,(E + v \times B)$$

Beam Dynamics (1)

<u>In order to describe the motion of the particles, each particle is characterised by:</u>

- Its azimuthal position along the machine: s
- Its momentum: p (or Energy E)
- Its horizontal position: x
- Its horizontal slope: x'
- Its vertical position: y
- Its vertical slope: y'

i.e. a sixth dimensional vector

(s, p, x, x', y, y')

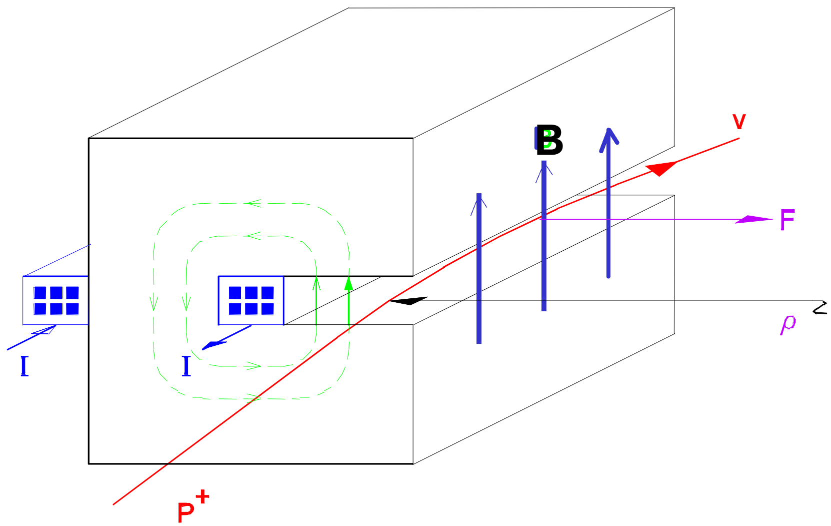
Beam Dynamics (2)

- In an accelerator designed to operate at the energy E_{nom}, all particles having $(s, E_{nom}, 0, 0, 0, 0)$ will happily fly through the center of the vacuum chamber without any problem. These are "ideal particles".

- The difficulties start when:

  - one introduces dipole magnets
  - the energy $E \neq E_{nom}$ or $(p - p_{nom}/p_{nom}) = \Delta p/p_{nom} \neq 0$
  - either of $x, x', y, y' \neq 0$

Circular machines: Dipoles



Classical mechanics:

Equilibrium between two forces

Lorentz force

$F = e.(\underline{v} \times \underline{B})$

Centrifugal force

$F = mv^2/\rho$

$evB = mv^2/\rho$

$p = m_0.c.(\beta\gamma)$

Magnetic rigidity:

$B\rho = mv/e = p/e$

Relation also holds for relativistic case provided the classical momentum mv is replaced by the relativistic momentum p

Why fundamental ?

Constraints:

E and **ρ** given ➡ Magnets defined (**B**)

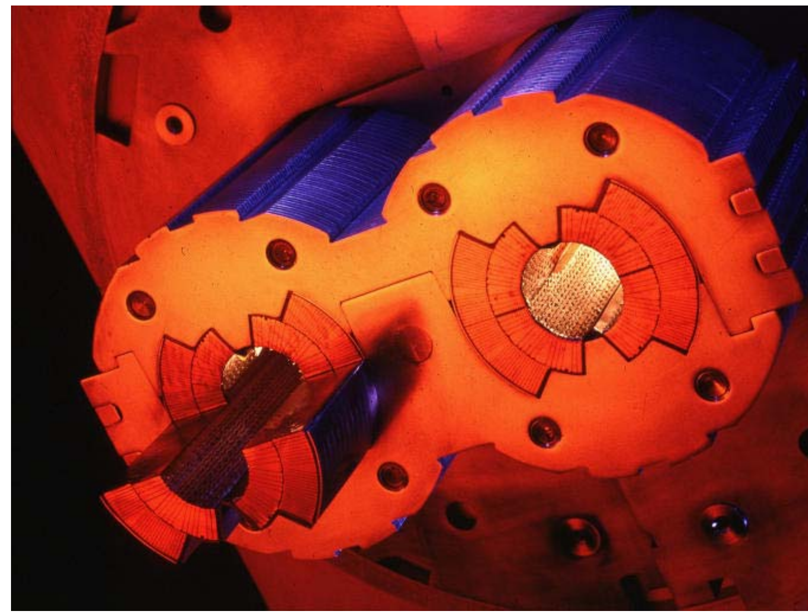
Constraints:

E and **B** given ➡ Size of the machine (**ρ**)

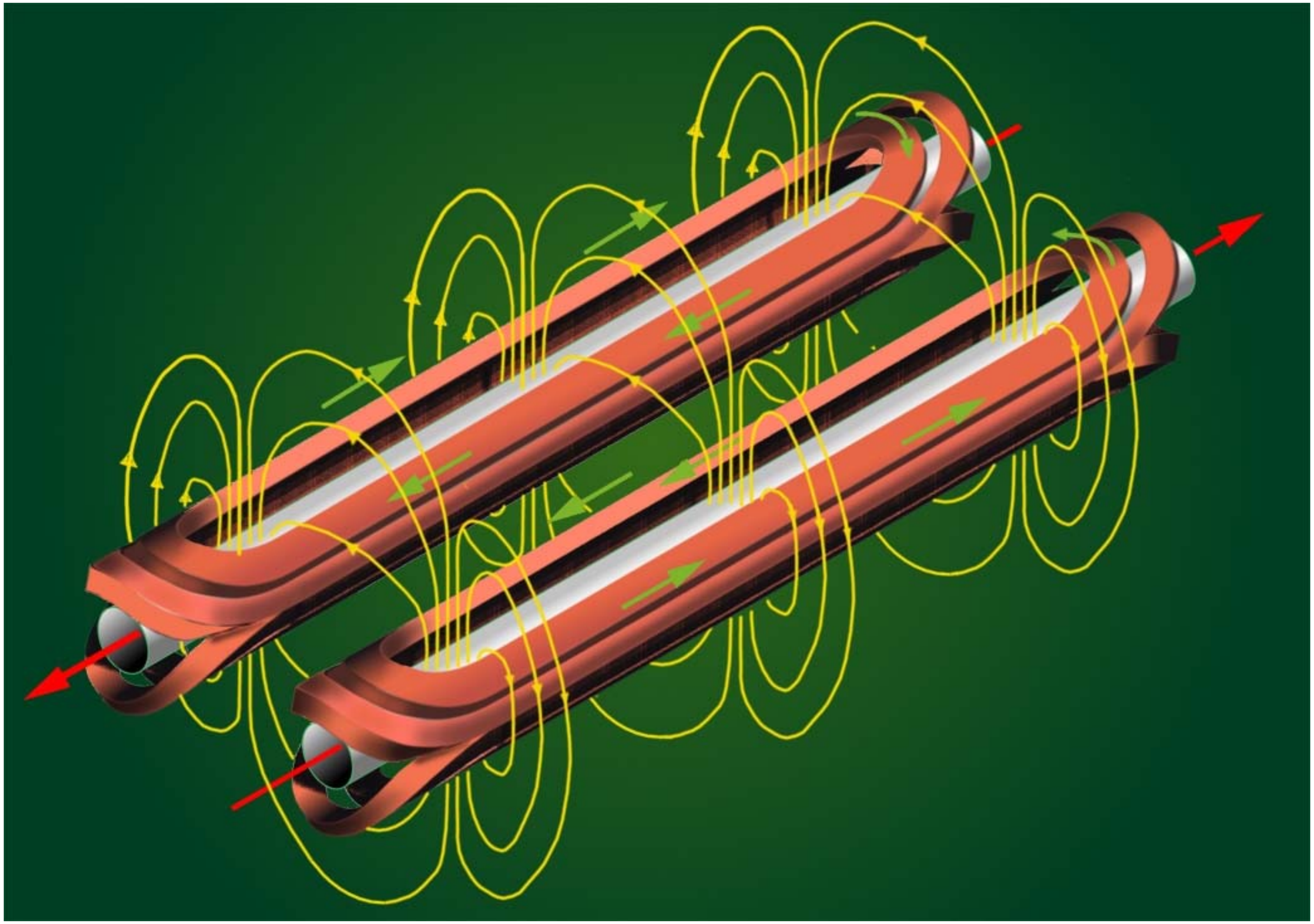
Constraints:

B and **ρ** given ➡ Energy defined (**E**)

Dipoles (1):



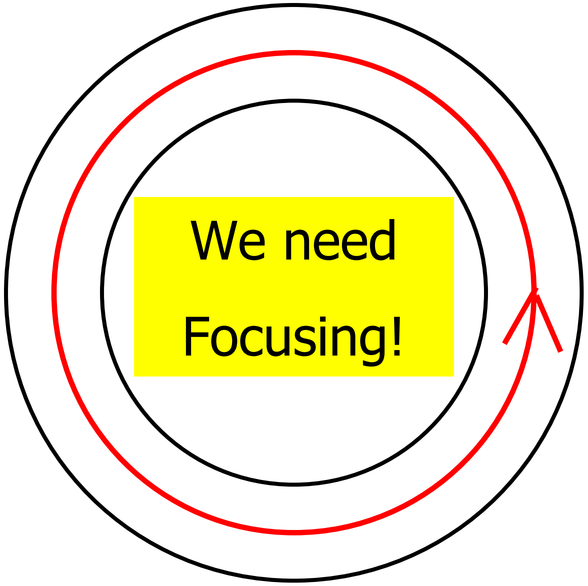
Dipoles (2):



Ideal circular machine:

- Neglecting radiation losses in the dipoles
- Neglecting gravitation

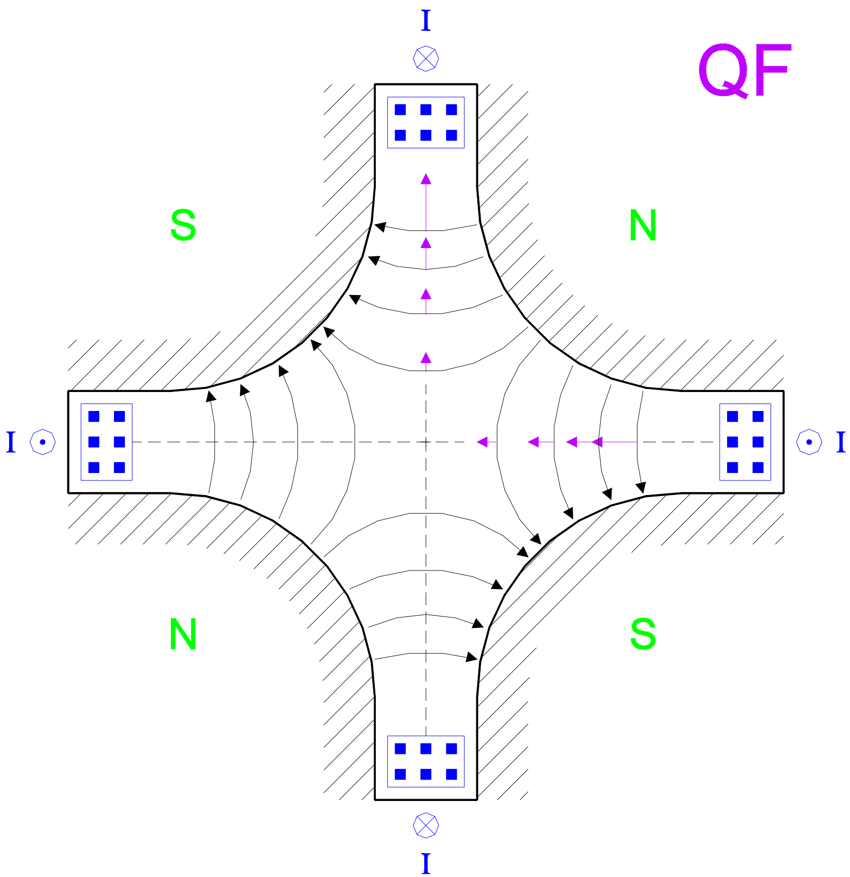
ideal particle would happily circulate on axis in the machine for ever!



Unfortunately: real life is different!

Gravitation: Δy = 20 mm in 64 msec!	
Alignment of the machine	Limited physical aperture
Ground motion	Field imperfections
Energy error of particles and/or $(x, x')_{inj} \neq (x, x')_{nominal}$	
Error in magnet strength (power supplies and calibration)	

Focusing with quadrupoles



$$F_x = -g.x$$

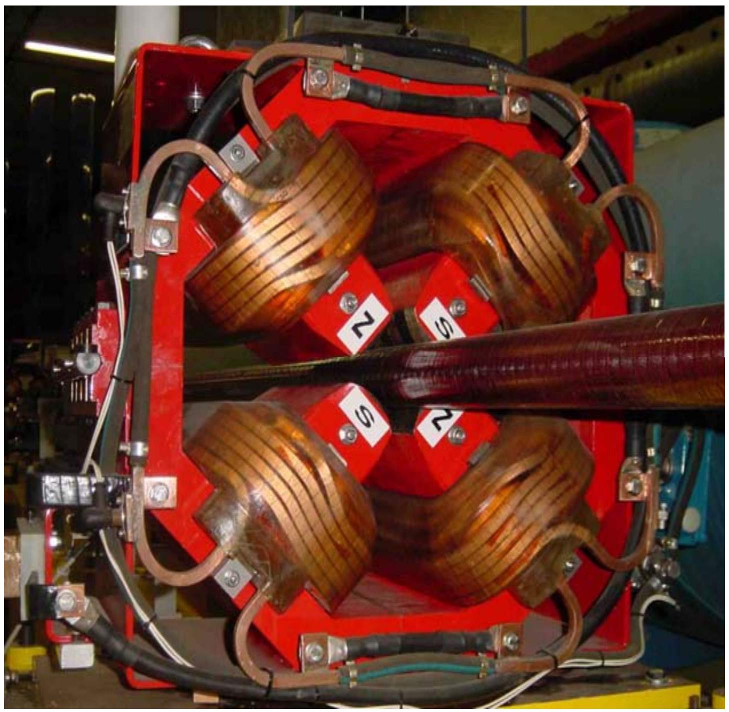
$$F_y = g.y$$

Force increases linearly with displacement.

Unfortunately, effect is opposite in the two planes (H and V).

Remember: this quadrupole is focusing in the horizontal plane but defocusing in the vertical plane!

Quadrupoles:



Focusing properties …

A quadrupole provides the required effect in one plane…

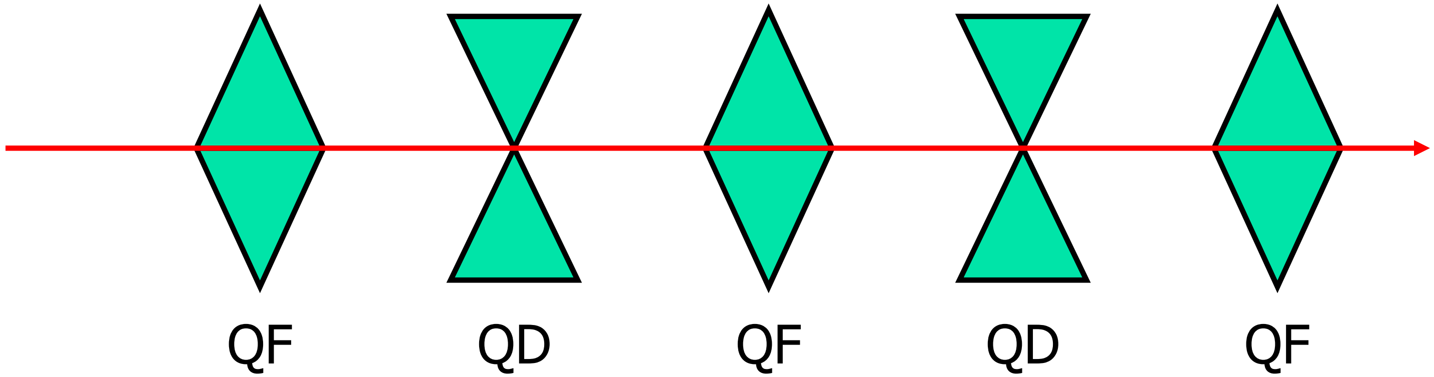
but the opposite effect in the other plane!

Is it really interesting ?

Alternating gradient focusing

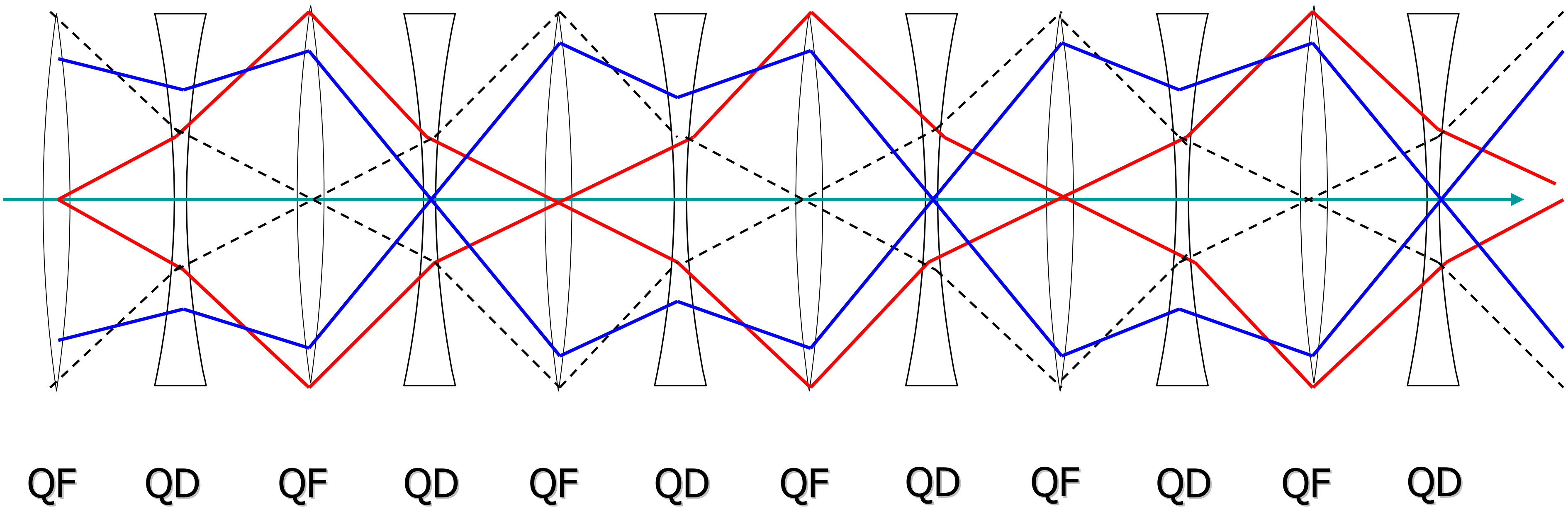
Basic new idea:

Alternate QF and QD



valid for one plane only (H or V) !

Alternating gradient focusing



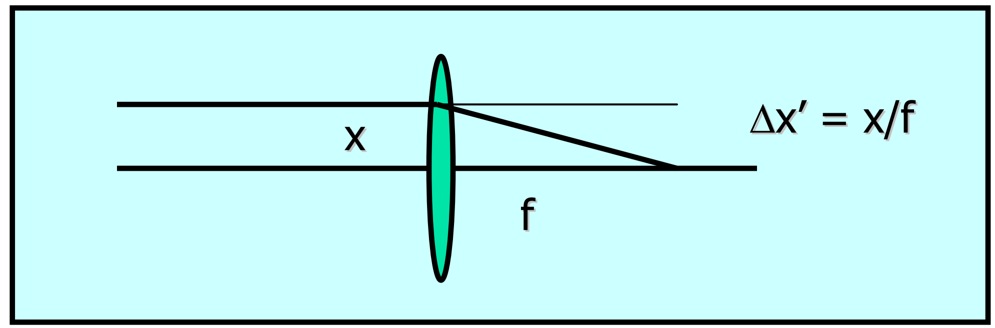
Alternating gradient focusing:

Particles for which $x, x', y, y' \neq 0$ thus oscillate around the ideal particle …

but the trajectories remain inside the vacuum chamber !

Thin lens analogy of AG focusing

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{out} = \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}_{in}$$



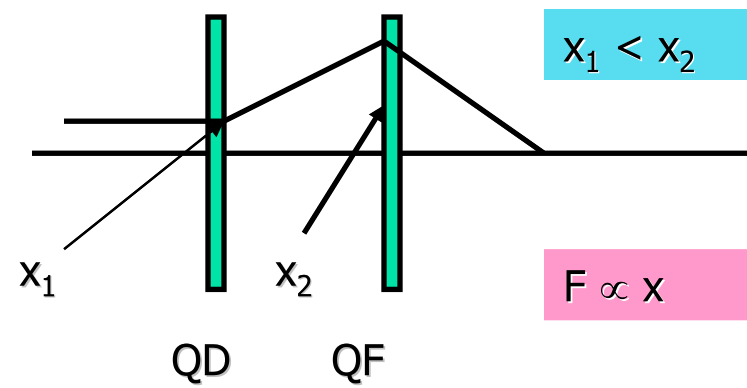
$$X_{out} = x_{in} + 0.x'_{in}$$
$$x'_{out} = (-1/f).x_{in} + x'_{in}$$

$$\text{Drift} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}$$

$$\text{QF-Drift-QD} = \begin{pmatrix} 1-L/f & L \\ -L/f^2 & 1+L/f \end{pmatrix}$$

Initial: $x = x_0$ and $L < f$
$x' = 0$

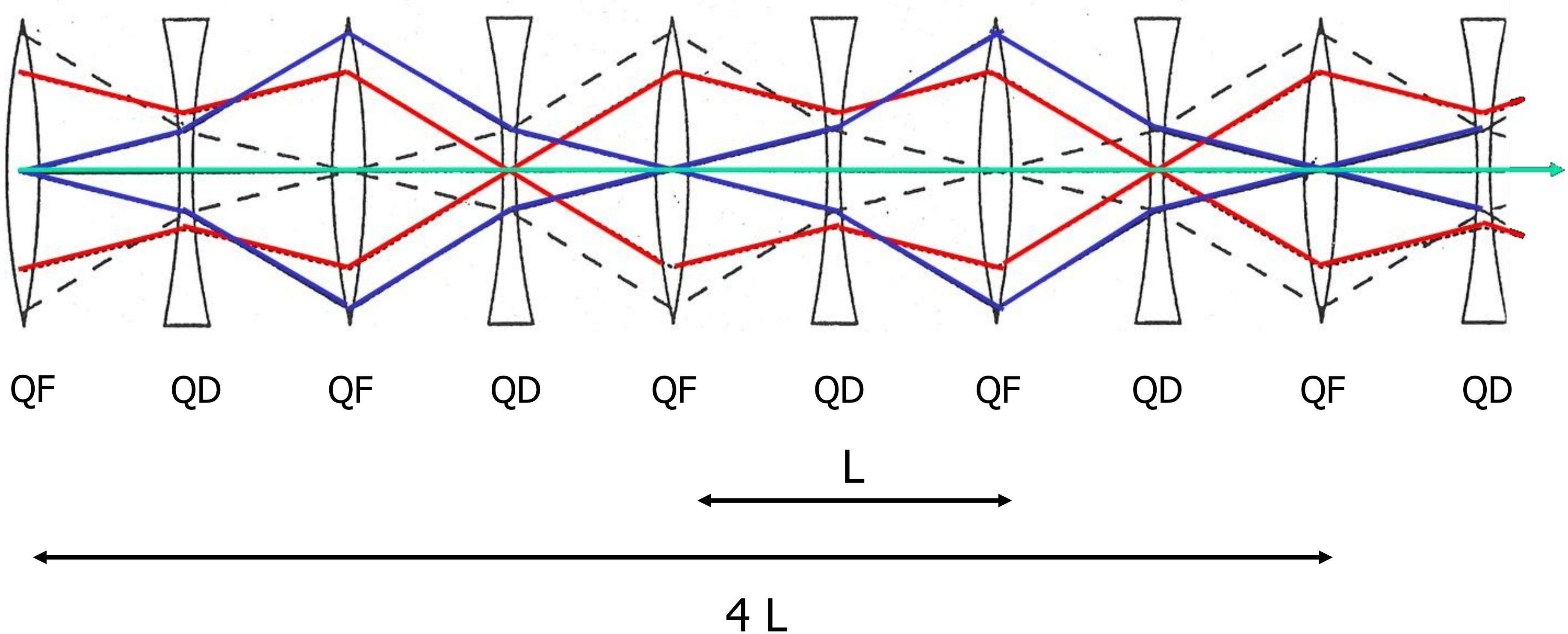
More intuitively:



$x_1 < x_2$

$F \propto x$

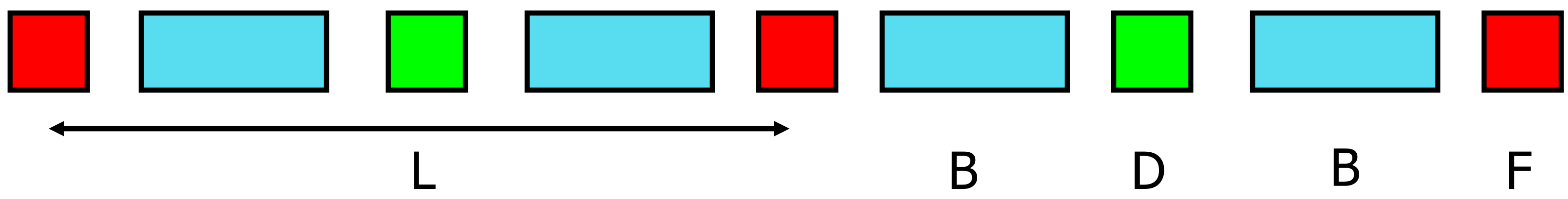
The concept of the « FODO cell »



One complete oscillation in 4 cells $\Rightarrow$ 90°/ cell $\Rightarrow$ **μ = 90°**

Circular machines (no errors!)

The accelerator is composed of a periodic repetition of cells:



➢ The phase advance per cell μ can be modified, in each plane, by varying the strength of the quadrupoles.

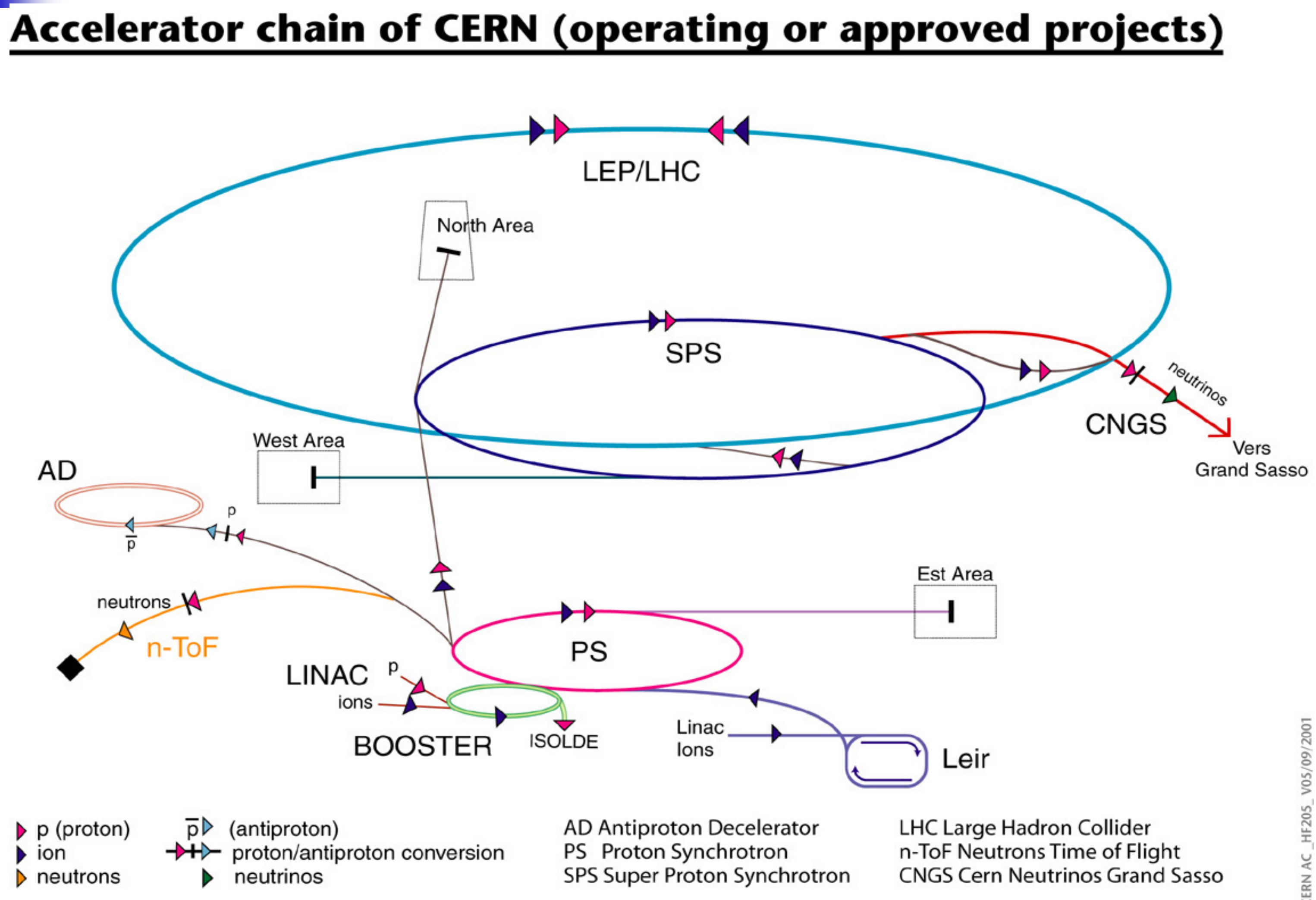
➢ The ideal particle will follow a particular trajectory, which closes on itself after one revolution: the closed orbit.

➢ The real particles will perform oscillations around the closed orbit.

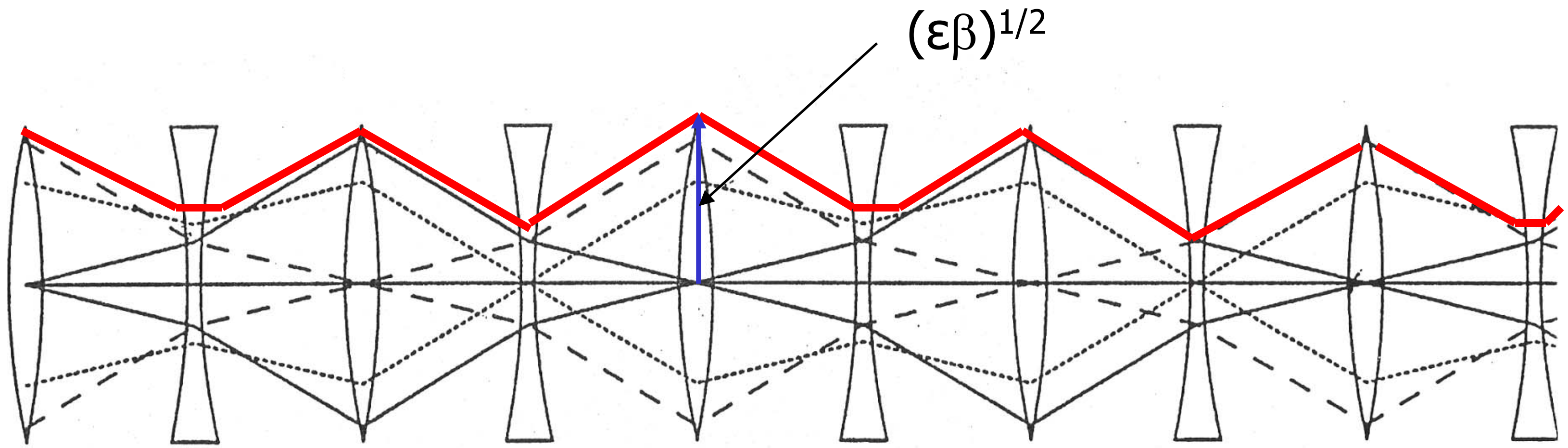
➢ The number of oscillations for a **complete revolution** is called the **Tune Q** of the machine (Qx and Qy).

Regular periodic lattice: The Arc

Synchrotrons …



The beta function $\beta(s)$



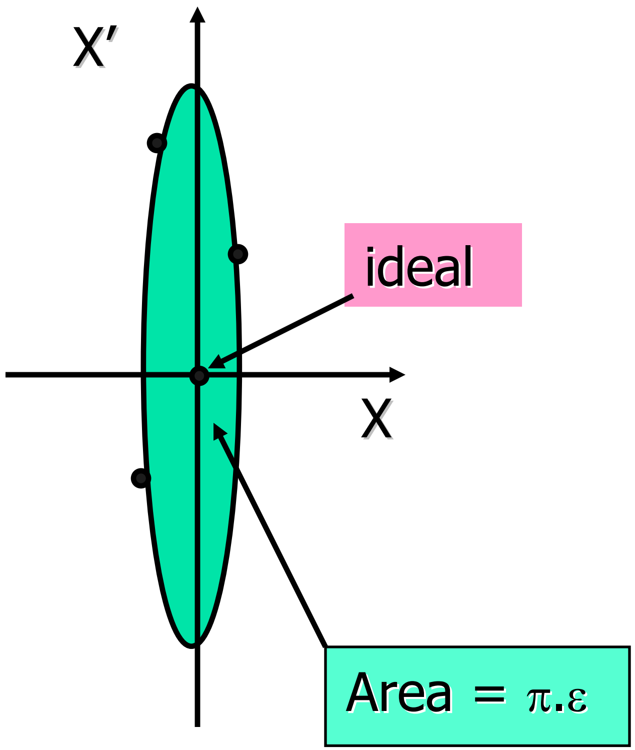
The β-function is the envelope around all the trajectories of the particles circulating in the machine.

The β-function has a minimum at the QD and a maximum at the QF, ensuring the net focusing effect of the lattice.

It is a periodic function (repetition of cells). The oscillations of the particles are called betatron motion or **betatron oscillations**.

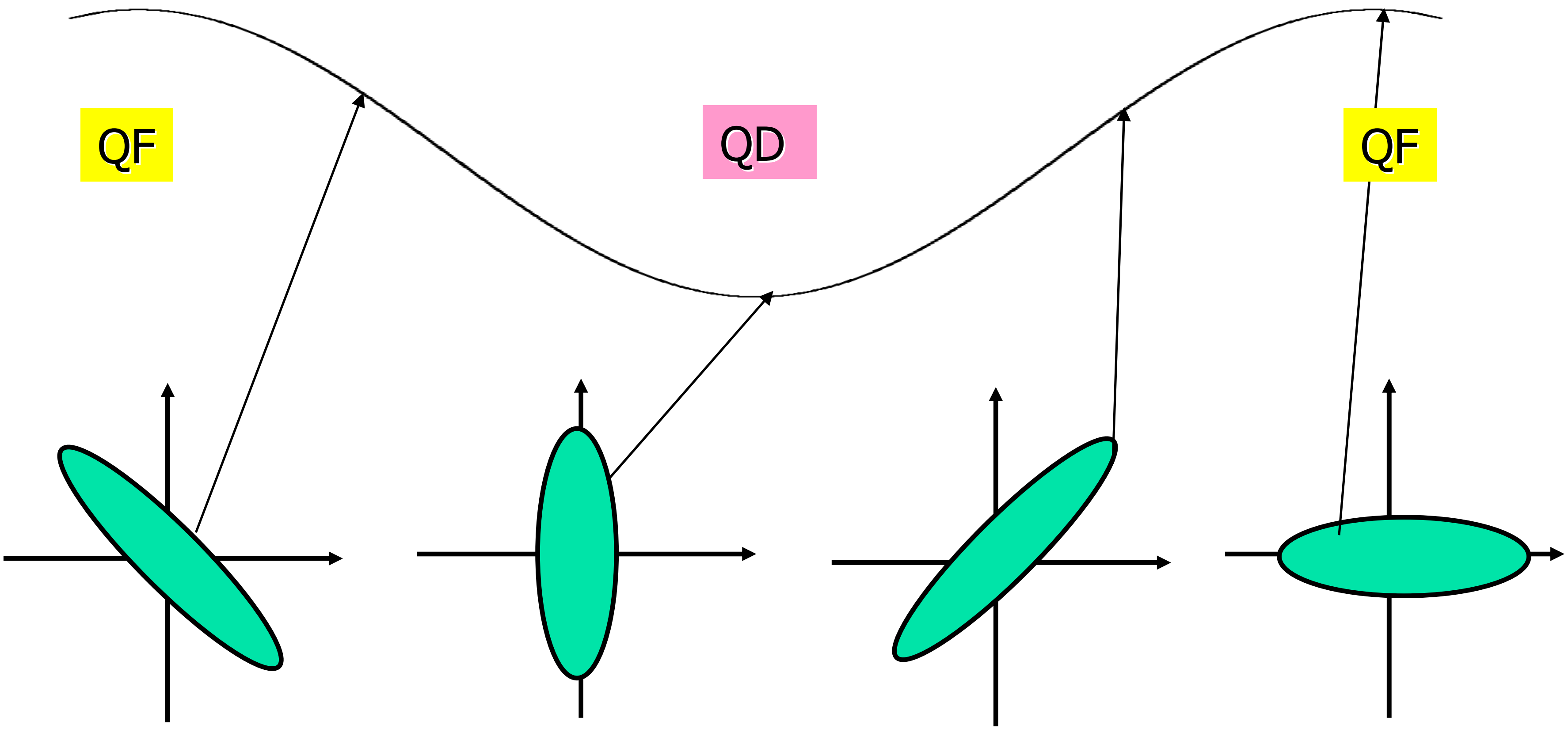
Phase space

- Select a particle in the beam being at 1 sigma (68%) of the distribution and plot its position vs. its phase (x vs. x') at some location in the machine for many turns.



- ε Is the emittance of the beam [mm mrad]
- ε describes the quality of the beam
- Measure of how much particle depart from ideal trajectory.
- β is a property of the machine (quadrupoles).

Emittance conservation



The shape of the ellipse varies along the machine, but its area (the emittance ε) remains constant at a given energy.

Why introducing these functions?

The β function and the emittance are fundamental parameters, because they are directly related to the beam size (measurable quantity !):

Beam size [m]

$$\sigma_{x,y}(s) = (\varepsilon.\beta_{x,y}(s))^{1/2}$$

σ (IP) = 17 μm

at 7 TeV (β=0.55 m)

The emittance ε characterises the quality of the injected beam (kind of measure how the particules depart from ideal ones). It is an invariant at a given energy.

ε = beam property

β = machine property (quads)

Off momentum particles:

- These are "non-ideal" particles, in the sense that they do not have the right energy, i.e. all particles with $\Delta p/p \neq 0$

What happens to these particles when traversing the magnets ?

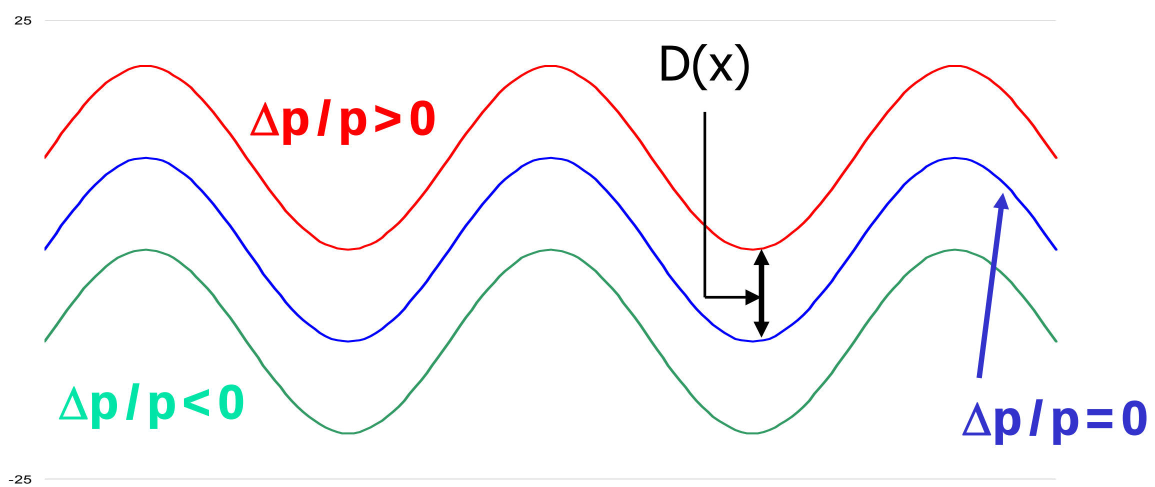
Off momentum particles ($\Delta p/p \neq 0$)

Effect from Dipoles

- If $\Delta p/p > 0$, particles are less bent in the dipoles ➔ should spiral out !
- If $\Delta p/p < 0$, particles are more bent in the dipoles ➔ should spiral in !

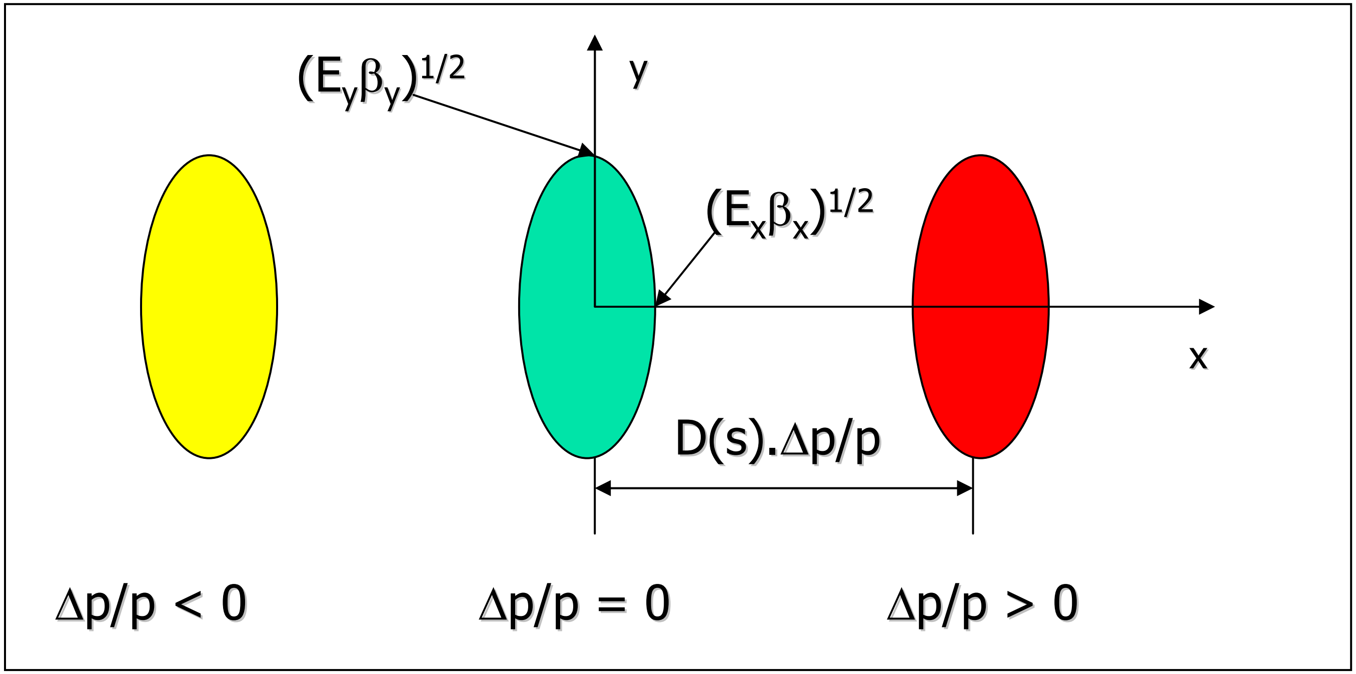
No!

There is an equilibrium with the restoring force of the quadrupoles



Dispersion

In general:



Only extreme values of Δp/p are shown.

The vacuum chamber must accomodate the full width.

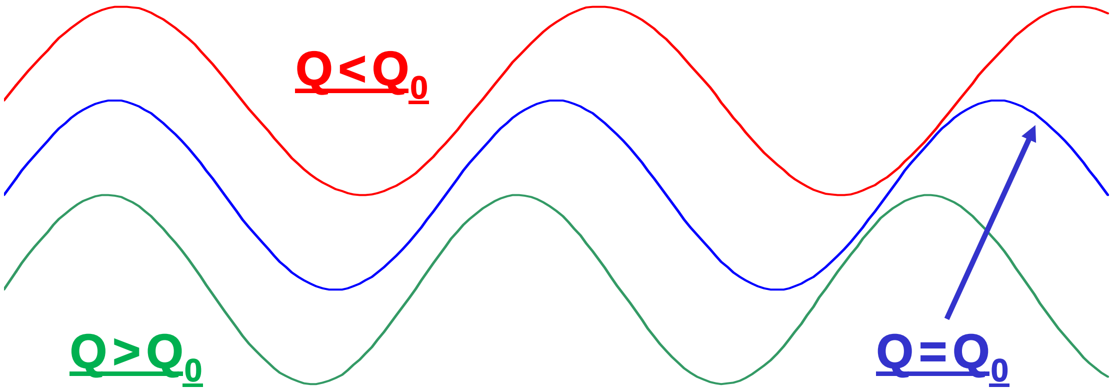
VH: $A_y(s) = (E_y\beta_y(s))^{1/2}$ and HW: $A_x(s) = (E_x\beta_x(s))^{1/2} + D(s).\Delta p/p$

Off momentum particles ($\Delta p/p \neq 0$)

Effect from Quadrupoles

- If $\Delta p/p > 0$, particles are less focused in the quadrupoles ➔ **lower Q !**
- If $\Delta p/p < 0$, particles are more focused in the quadrupoles ➔ **higher Q !**

Particles with different momenta would have a different betatron tune $Q=f(\Delta p/p)$!



The chromaticity Q'

Particles with different momenta ($\Delta p/p$) would thus have different tunes Q.
So what ?

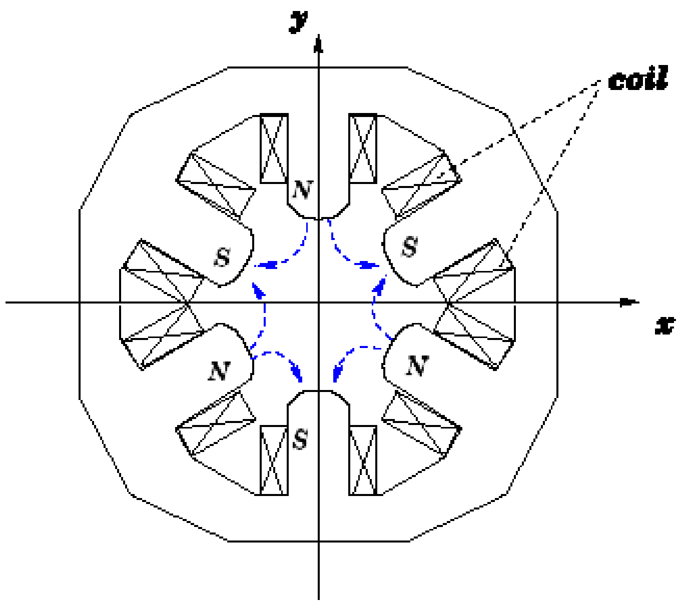
unfortunately

- The tune dependence on momentum is of fundamental importance for the stability of the machine. It is described by the chromaticity of the machine Q':

$$Q' = \Delta Q / (\Delta p/p)$$

The chromaticity has to be carefully controlled and corrected for stability reasons.

The sextupoles (SF and SD)

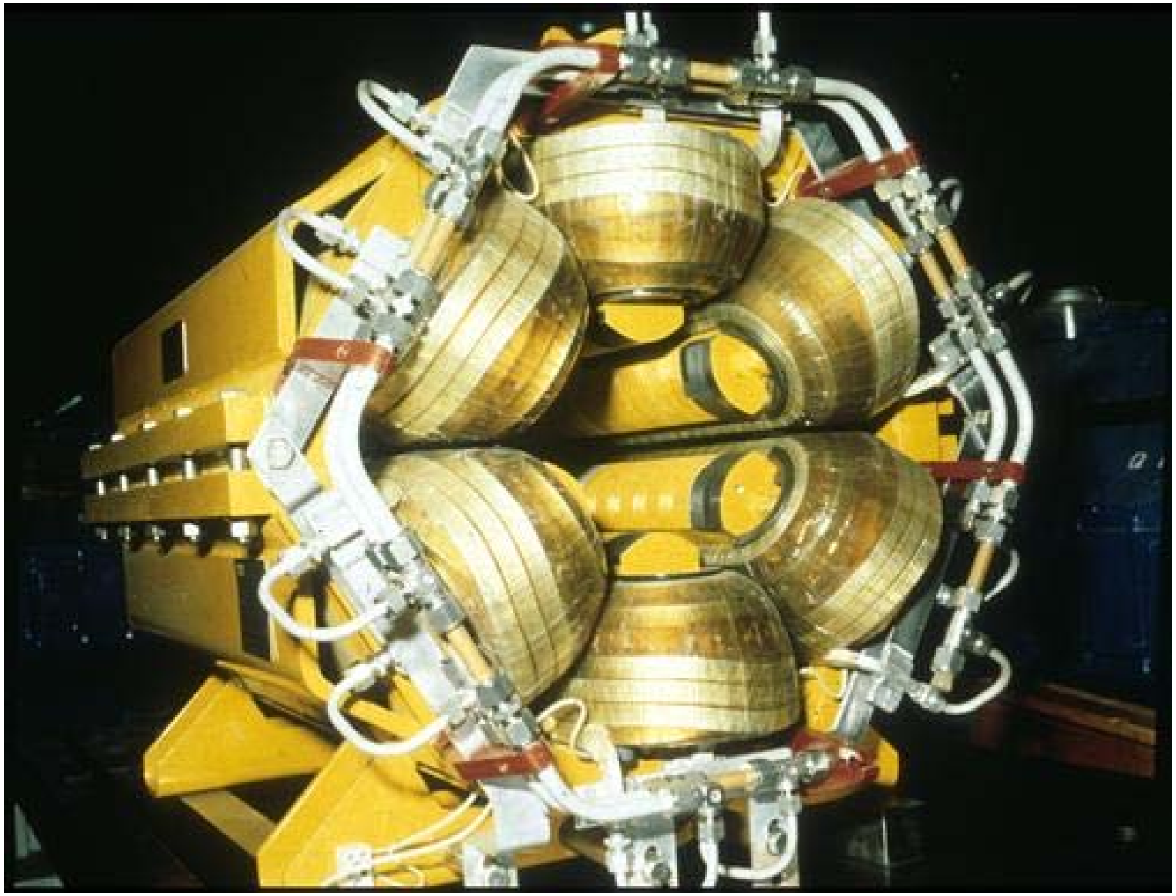


- $\Delta x' \propto x^2$

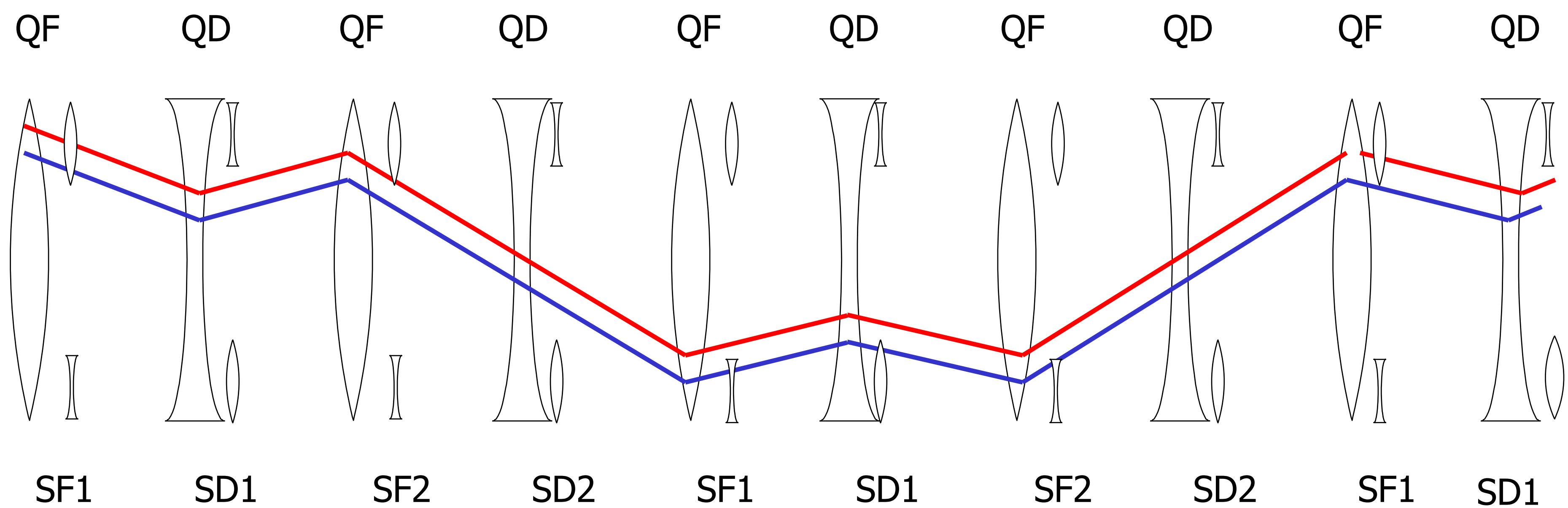
- A SF sextupole basically « adds » focusing for the particles with $\Delta p/p > 0$, and « reduces » it for $\Delta p/p < 0$.
- The chromaticity is corrected by adding a sextupole after each quadrupole of the FODO lattice.

Sextupoles:

SPS



Chromaticity correction



The undesired effect of sextupoles on particles with the nominal energy can be avoided by grouping the sextupoles into « families ».

Nr. of families:

$N = (k * 180\,°)/\mu$ = Integer

e.g. $180\,° / 90\,° = 2$

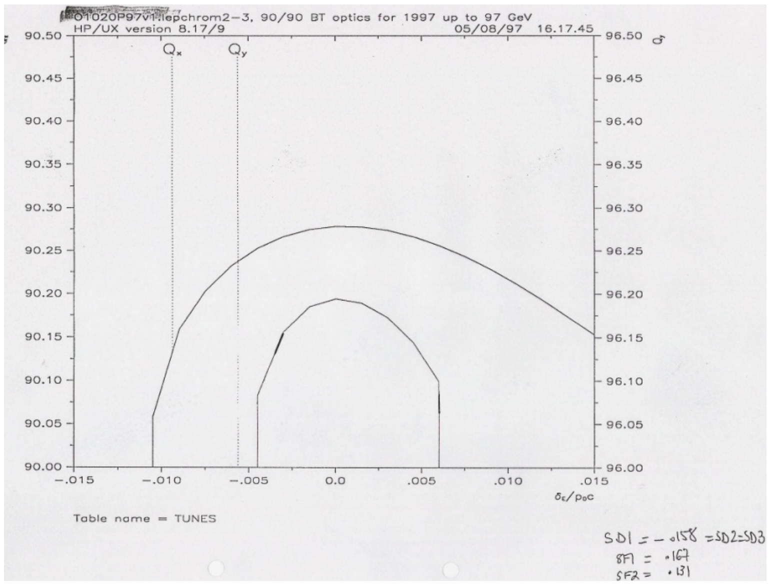
Natural chromaticity…

• Take a particle and slightly increase its momentum:
➔ $\Delta p/p > 0$ ➔ $\Delta Q < 0$ ➔ $Q' < 0$

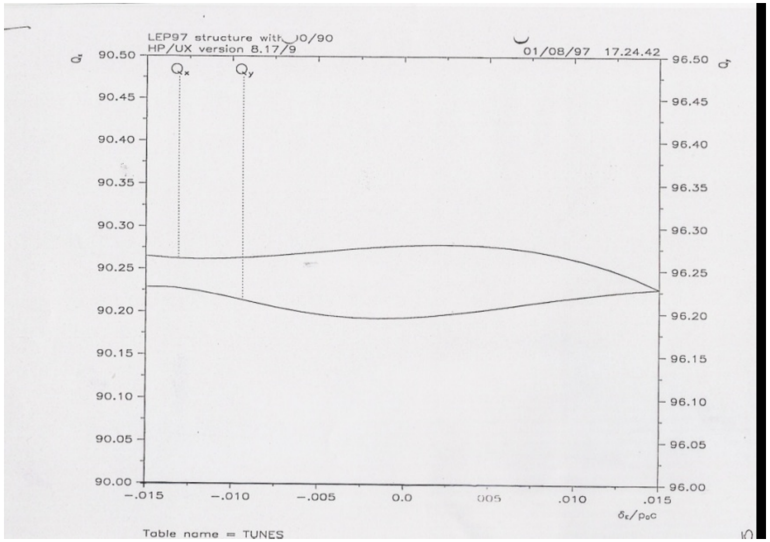
• Take a particle and slightly decrease its momentum:
➔ $\Delta p/p < 0$ ➔ $\Delta Q > 0$ ➔ $Q' < 0$

Q' is always negative !

Tune vs. momentum



Correction with 1 sextupole family: Bad!
Off momentum particles rapidly cross the integer (Qy!).



Correction with 2 sextupole families: Excellent!
Tunes remain almost constant over the whole range of momentum!

Transverse stability of the beam:

So, apparently, the tunes Q_x and Q_y have to be selected and controlled very accurately. Why this ?

LHC in collision:

$Q_x = 64.31$

$Q_y = 59.32$

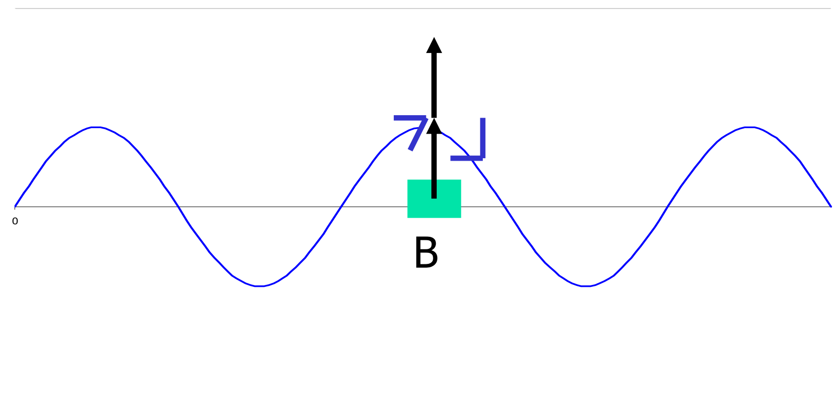
Forbidden values for Q

- An error in a dipole gives a kick which has always the same sign!

Integer Tune Q = N

Forbidden!

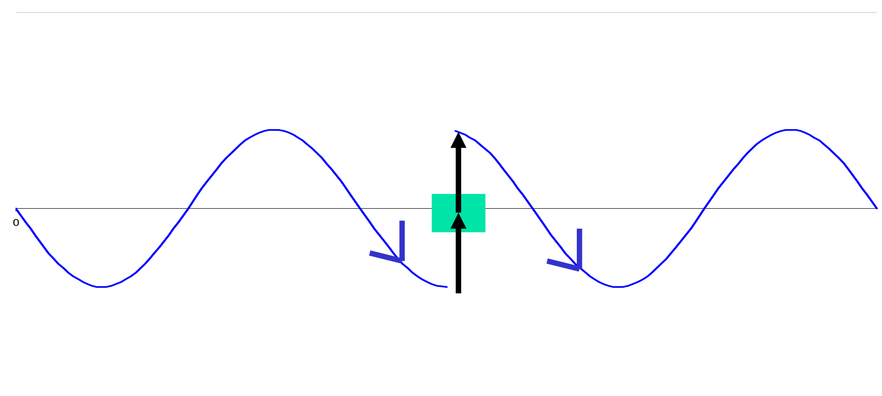
The perturbation adds up!



Half-integer Tune Q = N + 0.5

O.K. for an error in a dipole!

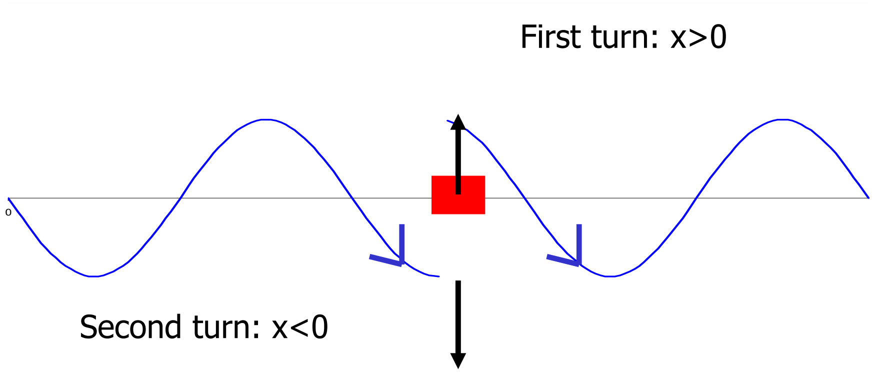
The perturbation cancels after each turn!



Forbidden values for Q

- An error in a quadrupole gives a kick whose sign depends on x ($F \propto x$)

Half-integer Tune Q = N + 0.5

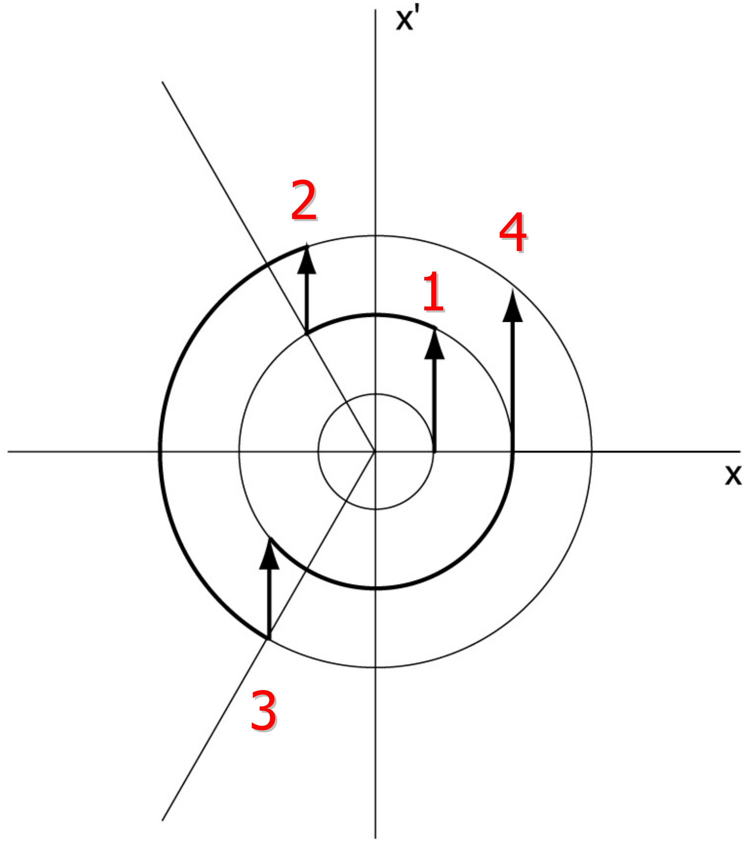


Forbidden !

The amplitude of the oscillation is steadily increasing!

What about a 1/3 integer tune ?

1/3 integer Tune

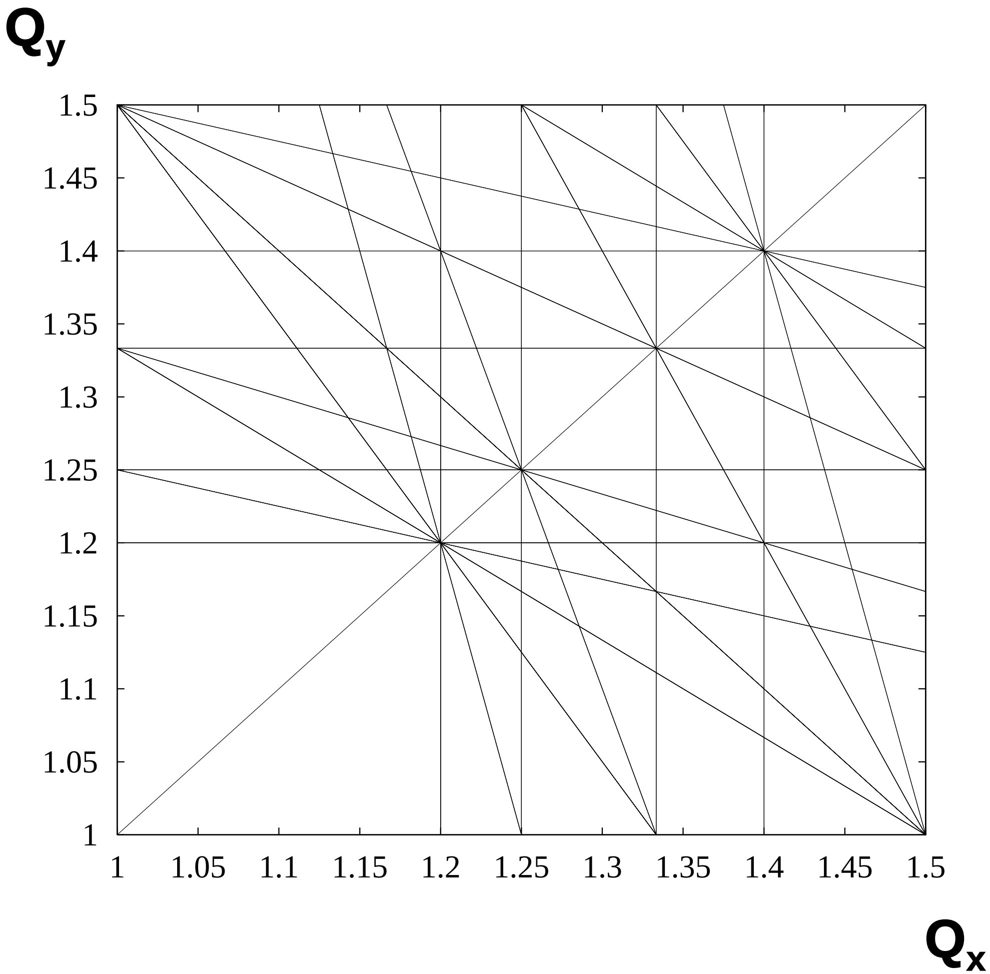


Tune Q = N + 0.33

Forbidden !
The amplitude of the oscillation increases every third turn!

One would come to similar conclusions for
Q = 1/4 , 1/5, …

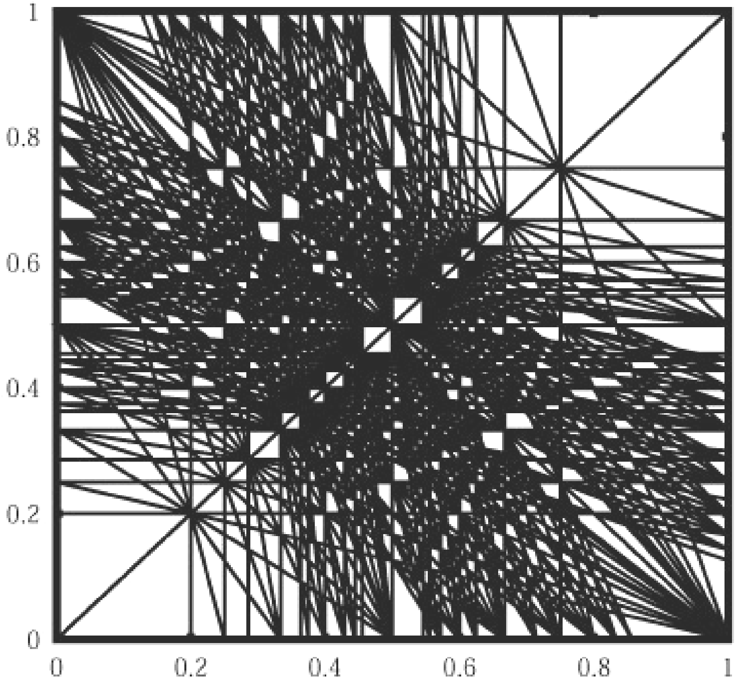
Tune diagram for leptons (5th order):



Tune values (Qx and/or Qy) which are forbidden in order to avoid resonances

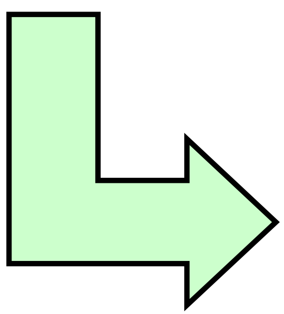
The lowest the order of the resonance, the most dangerous it is.

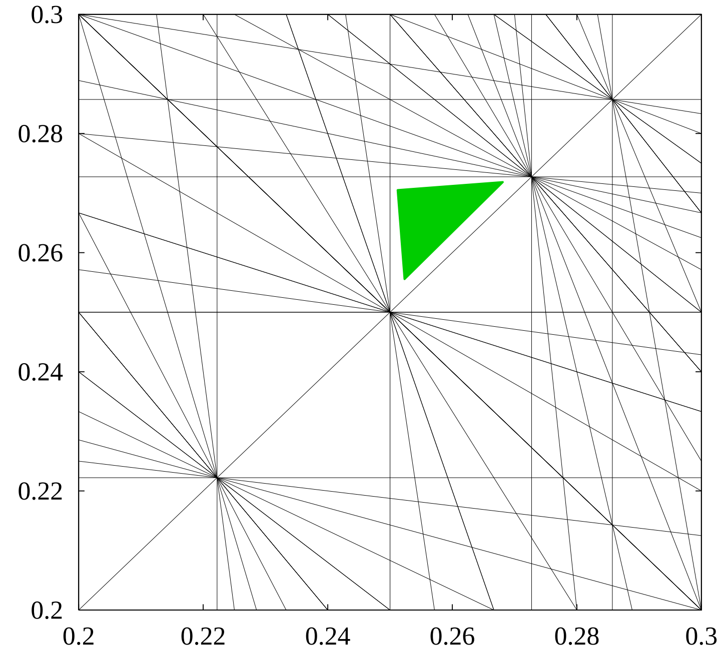
Tune diagram for protons



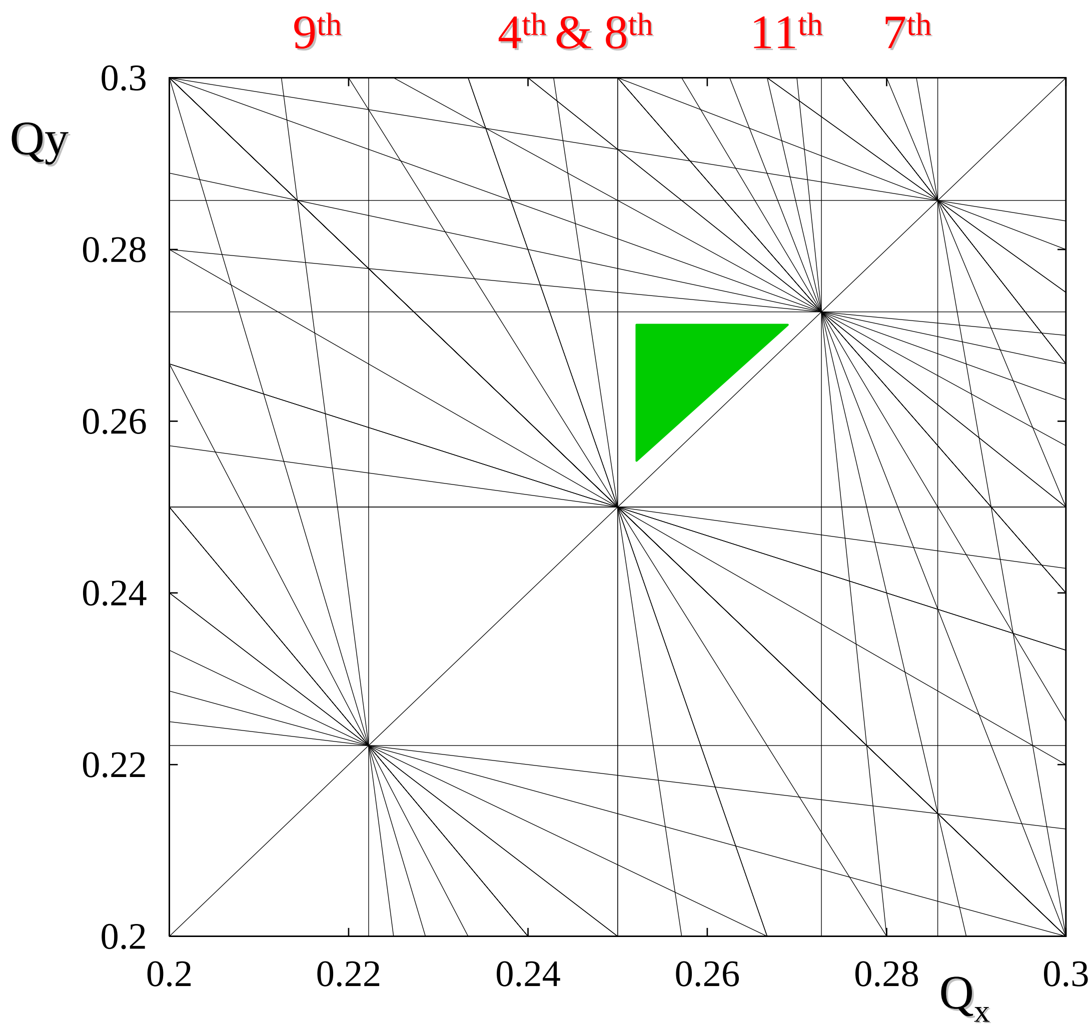
Due to the energy spread in the beam, we have to accommodate an « area » rather than a point!

LHC





In more details:



Different energy dependent effects (e.g. space charge) will modify the "area" when the energy is changed!

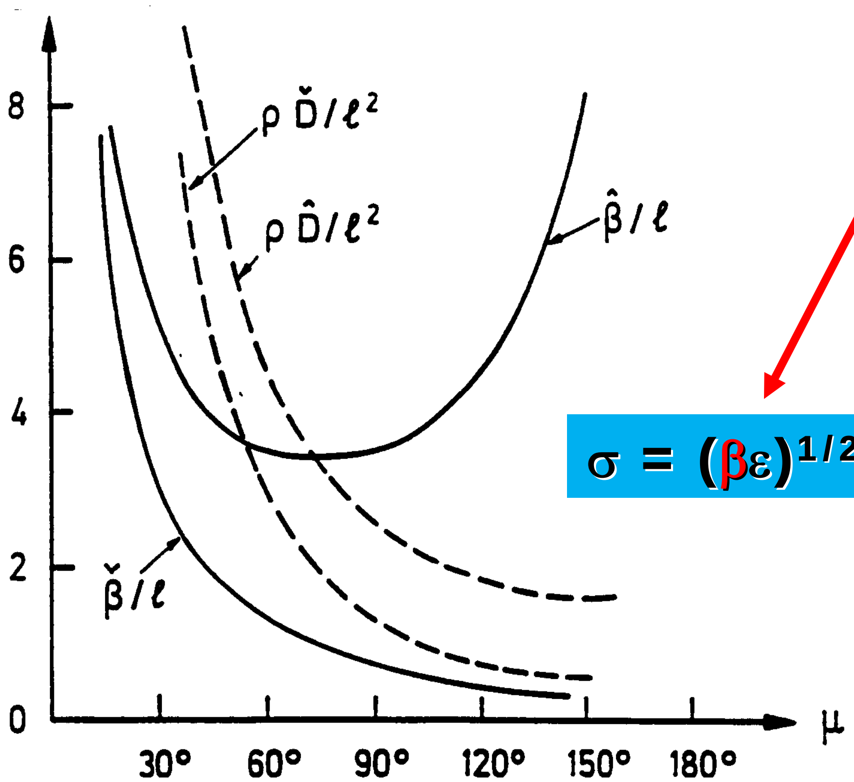
Choice of the lattice:

➢ If you are working on a conventional machine, then you are very likely to use a standard FODO lattice.

➢ If your synchrotron has insertions (injection, extraction, RF, low-β, experiments), then you will need an « Optics program » to adapt (match) these specific regions to the FODO/periodic cells.

➢ If you are working on a Synchrotron Light Source (very small emittance, insertion devices, FELs) you will opt for a special lattice. For such a case, the use of a dedicated « Optics program» is probably unavoidable.

The phase advance per cell μ



$\sigma = (\beta\varepsilon)^{1/2}$

E. Wilson's lecture, CAS Sesimbra 2002

- Aperture expensive → μ between 60 and 90 degrees.
- Closed orbit correction
- Chromaticity correction with a reasonable number of sextupole families

Some phase advances are advantageous for the lattice design (e.g. 60° or 90°)

$60° < \mu > 90°$

More general case …

- You will need a dedicated « Optics program » to compute the lattice of your machine (e.g. MAD-X). This applies for « special » synchrotrons like the Synchrotron Light Sources, where FODO lattices are not optimal as far as the required beam parameters are concerned. In any case…

Get the correct Optics

↓

- Match your insertions.
- Correct the chromaticity
- Compensate coupling

Predict the performance

↓

- Compute Tunes vs. Momentum
- Perform tracking with errors
- Evaluate the dynamic aperture

A few useful checks…

- Although the « Optics code » will provide you all the required parameters, it is always recommended to perform a few very basic checks (garbage IN, garbage OUT $\leftrightarrow$ the program does what YOU asked it to do).

Useful checks:

$<\beta> \approx R/Q$	$\alpha \approx 1/Q^2$
$<D> = \alpha R \approx R/Q^2$	$\gamma_{tr} \approx Q$

Summary for the transverse planes:

- A particle is described by its position and its slope (x, x') and (y, y')
- The circular trajectory is achieved with dipoles.
- The particles are kept together in the chamber with quadrupoles.
- The particles perform betatron oscillations around the closed orbit.
- The number of oscillations per turn (the tune Q) has to be carefully selected in order to avoid resonances.
- The phase advance per cell (μ) can be modified with quadrupoles.
- The natural chromaticity of the machine Q' (<0) is compensated with sextupoles.

What type of particles?

- The choice of the type of particles is intimately linked to the dedicated application. For high energy circular machines, synchrotron radiation and the available magnet strength will be the important parameters. Possible candidates:
  - Electrons and/or positrons (synchrotron radiation in circular machines)
  - Protons (magnet strength)
  - Antiprotons, neutrinos (available intensities)
  - Ions (sources)
  - Muons (future machines)

Where are we ?

We have covered the transverse beam dynamics and we have learned that, essentially, the machine is composed of a periodic repetition of dipoles, quadrupoles and sextupoles.

Is there still anything missing ?

A system to accelerate the particles

A system to get efficient collisions