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Traditio et Innovatio

HOM

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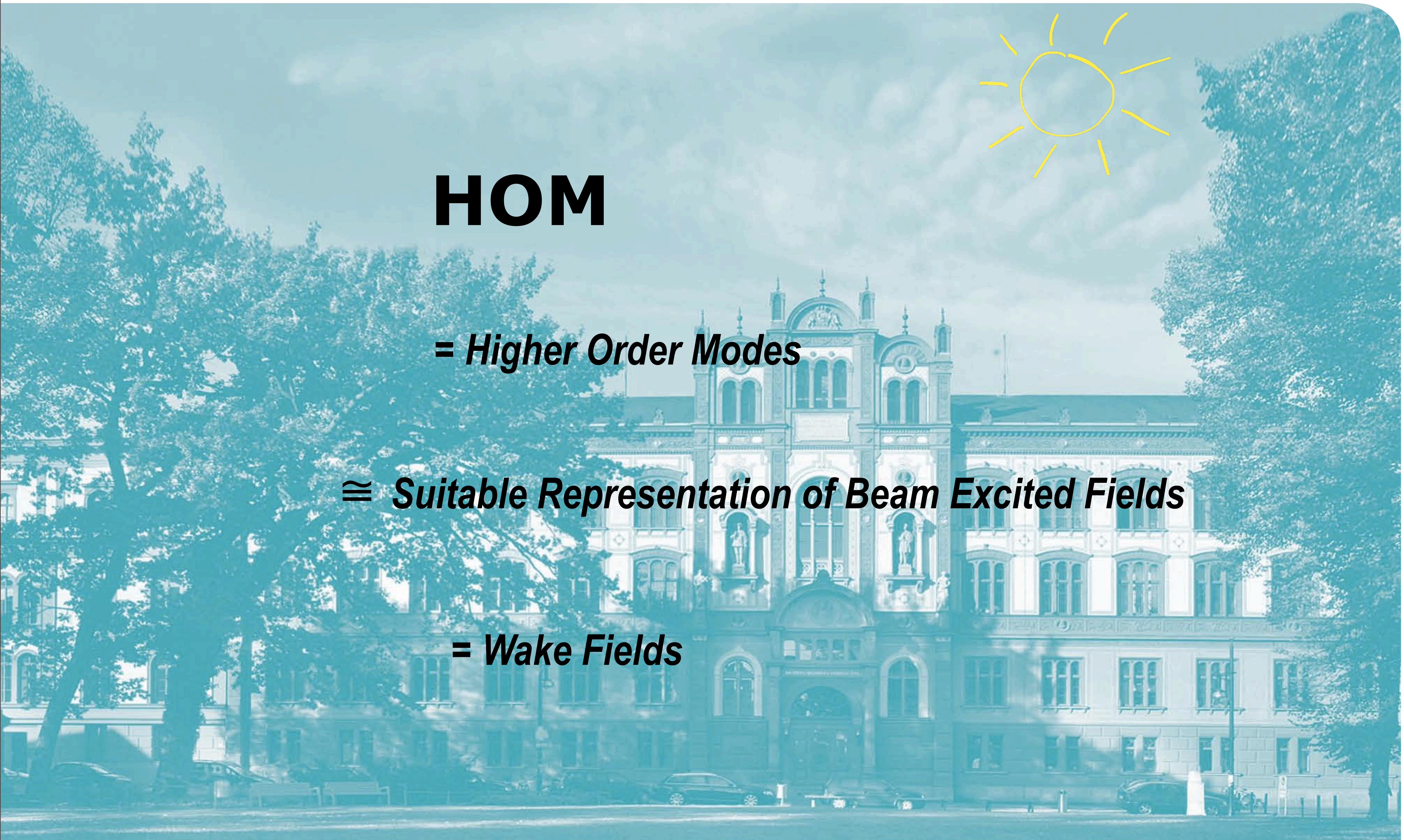
Universität Rostock - Institut für Allgemeine Elektrotechnik

27.5.2011

Cern Accelerator School: High Power Hadron Machines

24.5.-2.6.2011



HOM

= *Higher Order Modes*

≅ *Suitable Representation of Beam Excited Fields*

= *Wake Fields*

Overview (~50 years of research in 1 hour)

- Let's get acquainted: Showtime with beam.
- Tribute to Maxwell and linearity: From beam to fields to modes.
- Walking through the zoo: Modes in variations.
- Beat the beasts: Suppressing, canceling or ignoring.
- What I have not told you.

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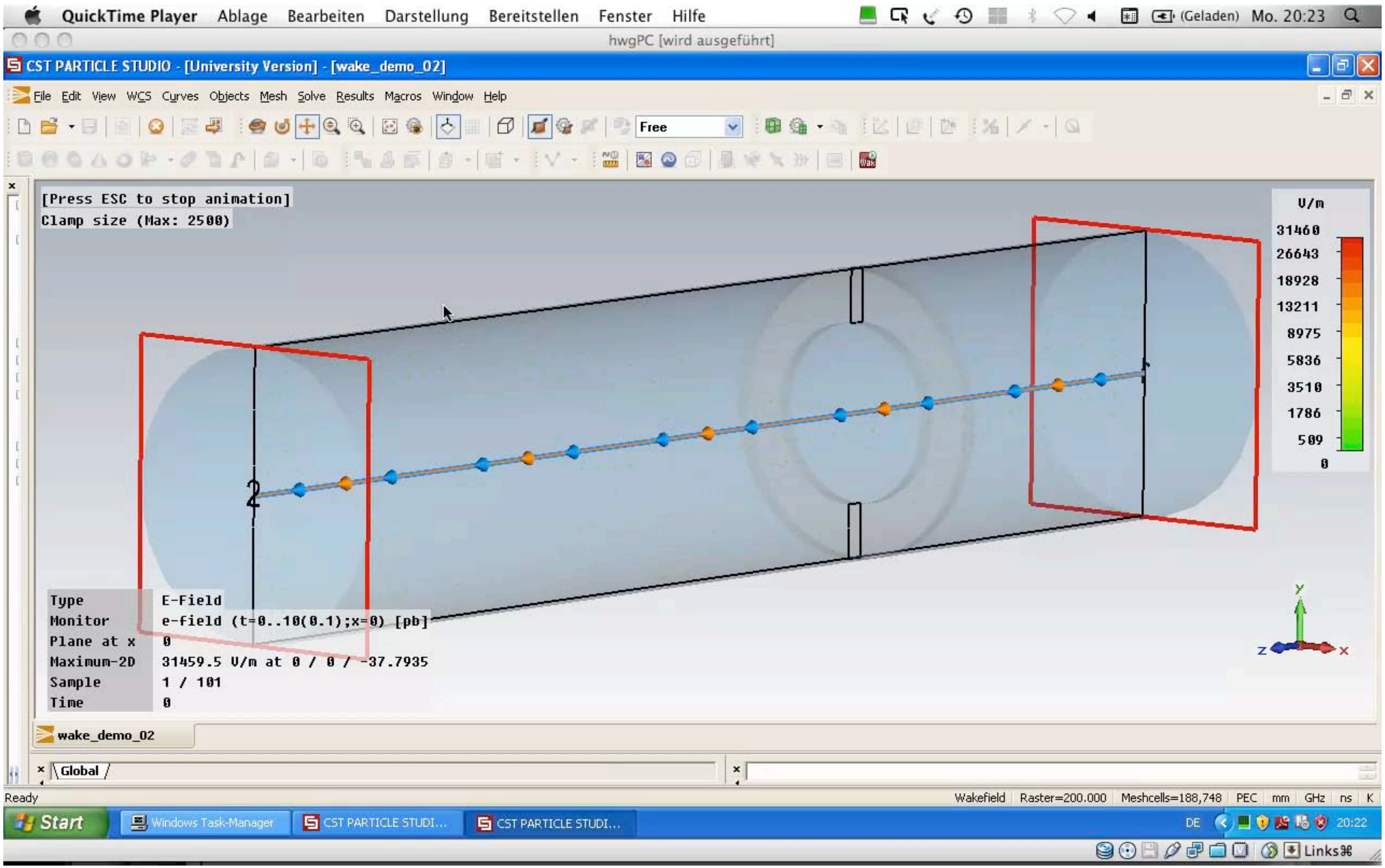
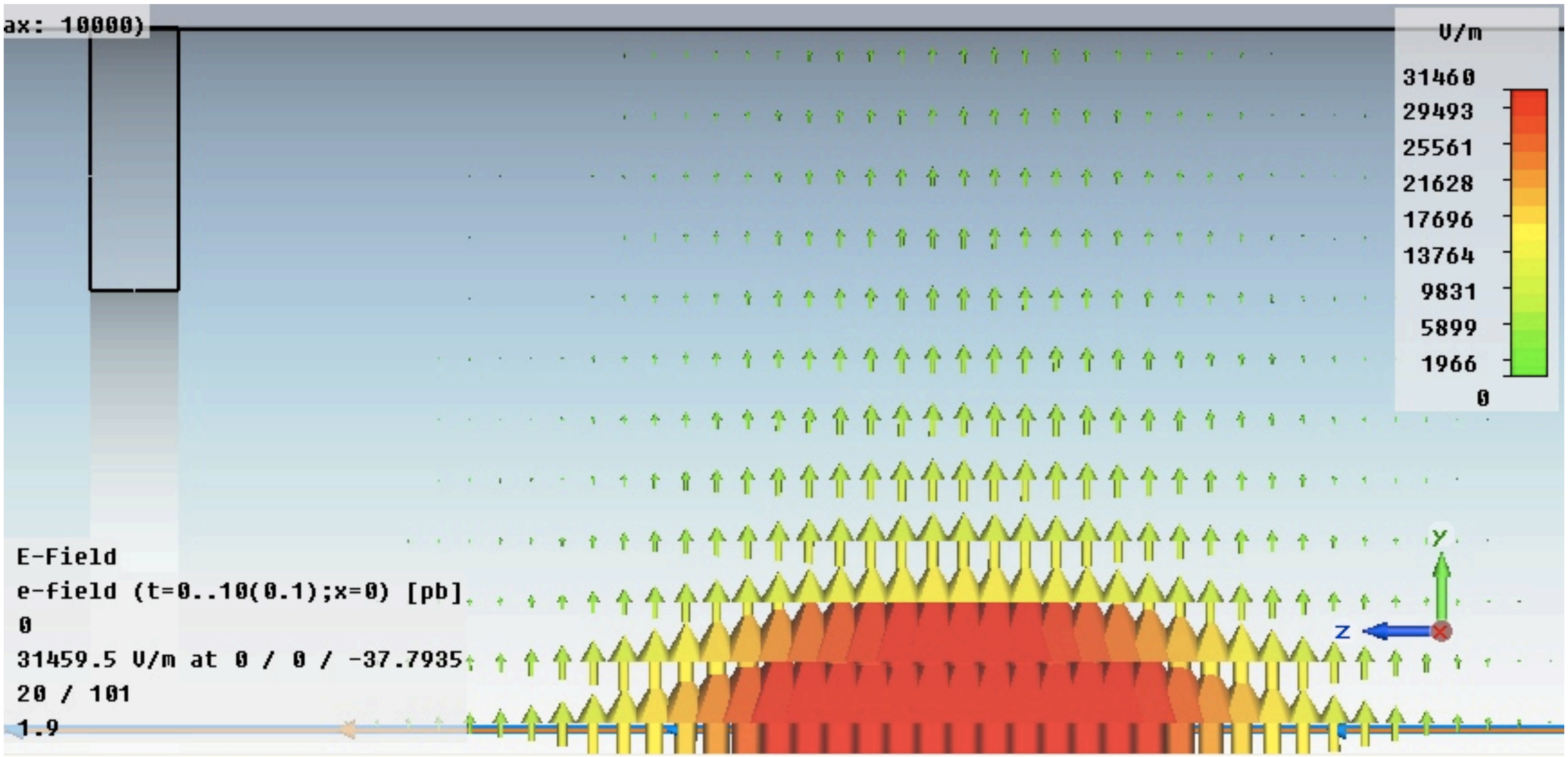
Showtime with beam.

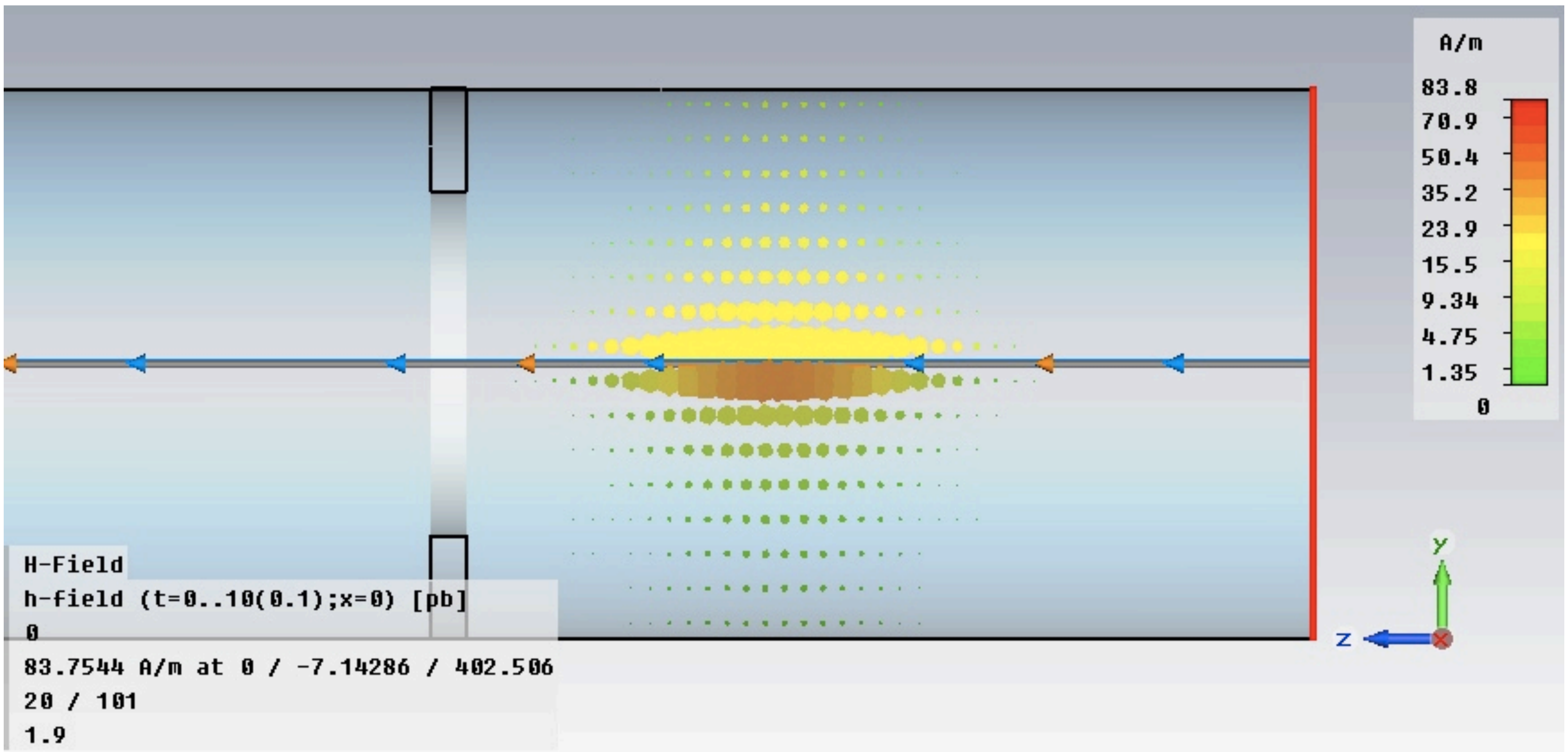


Phenomenologically I:



electric self-field directed radially outward, amplitude reflects Gaussian charge density in the bunch

Phenomenologically I:



H-field forms circles around the beam

How to illustrate this? (some in-between theory)

Lorentz-transformation of em-fields from co-moving to laboratory frame (movement with $v \cdot \mathbf{e_z}$):

$$\beta = v / c, \quad \gamma = \frac{1}{\sqrt{1-\beta^2}}$$

$$E_z = \bar{E}_z$$
$$\mathbf{E}_\perp = \gamma(\bar{\mathbf{E}} - \mathbf{v} \times \bar{\mathbf{B}})_\perp$$
$$B_z = \bar{B}_z$$
$$\mathbf{B}_\perp = \gamma(\bar{\mathbf{B}} + \frac{1}{c^2}\mathbf{v} \times \bar{\mathbf{E}})_\perp$$

i.e.:

- fields tangential to the movement's direction remain unaffected

- but transversal **E** and **B** mix up ...

- ... and experience amplification by the factor γ

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$$B_z = \bar{B}_z$$

$$\mathbf{B}_\perp = \gamma(\bar{\mathbf{B}} + \frac{1}{c^2}\mathbf{v} \times \bar{\mathbf{E}})_\perp$$

Now apply this to the pure space-charge field of a charge q, resting in the co-moving frame. Then:

$$\bar{\mathbf{B}} = 0$$

$$\bar{\mathbf{E}} = -\frac{1}{r^2}\frac{q}{4\pi\varepsilon_0}\mathbf{e_r} = -\frac{q}{4\pi\varepsilon_0}\frac{1}{(x^2+y^2+z^2)^{3/2}}\begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$E_z = -\frac{q\,z}{4\pi\varepsilon_0 r^3}\,; \quad B_z = 0\,; \quad \mathbf{E}_\perp = -\gamma\frac{q}{4\pi\varepsilon_0}\frac{1}{r^3}\begin{pmatrix} x \\ y \\ 0 \end{pmatrix}; \quad \mathbf{B}_\perp = \gamma\frac{q}{4\pi\varepsilon_0}\frac{1}{r^3}\frac{\beta}{c}\begin{pmatrix} y \\ -x \\ 0 \end{pmatrix}$$

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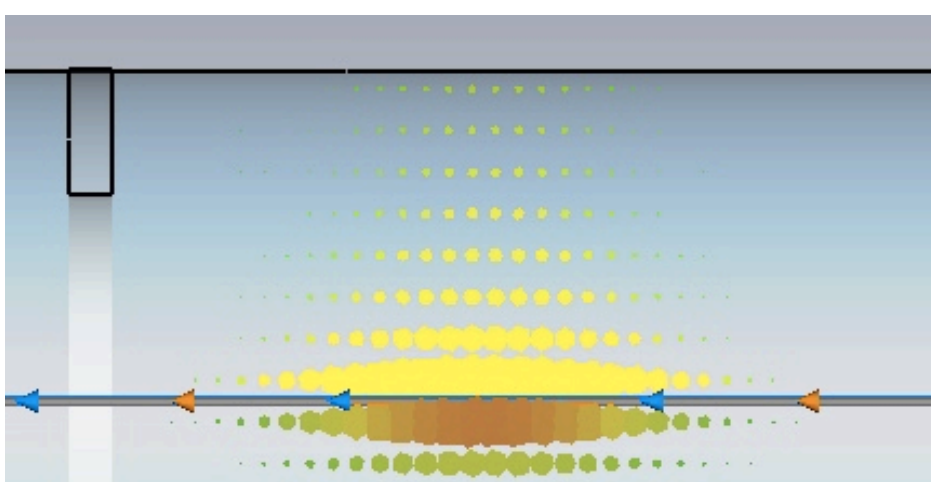
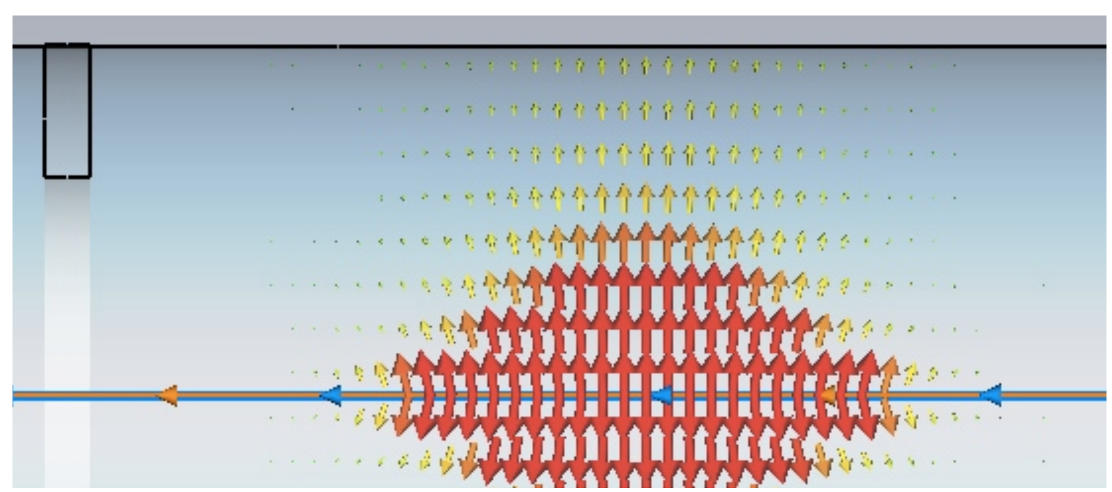
$$B_z = 0\,; \quad \mathbf{B}_\perp = \gamma \frac{q}{4\pi\varepsilon_0} \frac{1}{r^3} \frac{\beta}{c} \begin{pmatrix} y \\ -x \\ 0 \end{pmatrix};$$

$$E_z = -\frac{q}{4\pi\varepsilon_0} \frac{z}{r^3}\,; \quad \mathbf{E}_\perp = -\gamma \frac{q}{4\pi\varepsilon_0} \frac{1}{r^3} \begin{pmatrix} x \\ y \\ 0 \end{pmatrix}$$





here γ=2

i.e. p+ @ 1 GeV

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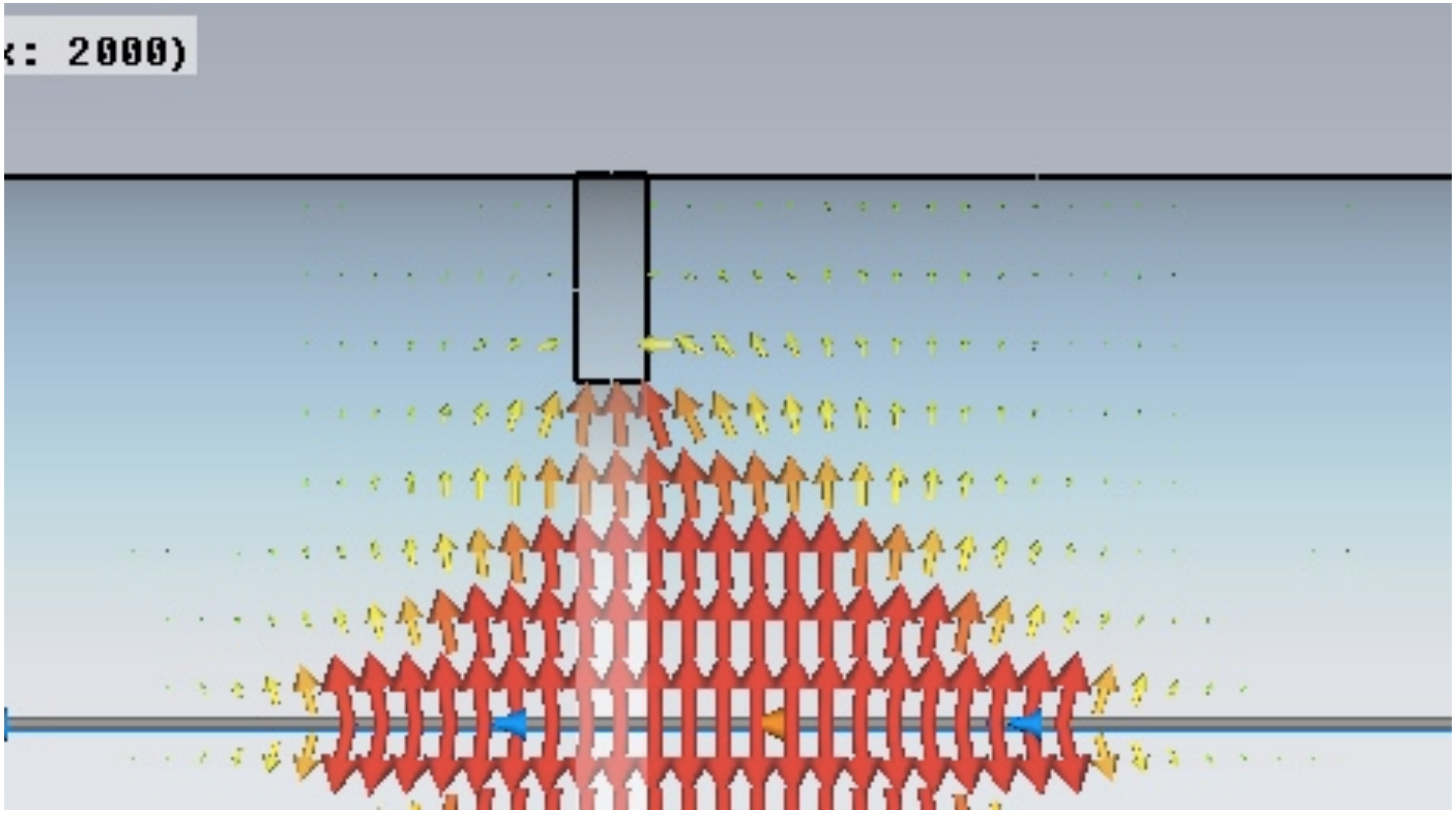
Phenomenologically II:

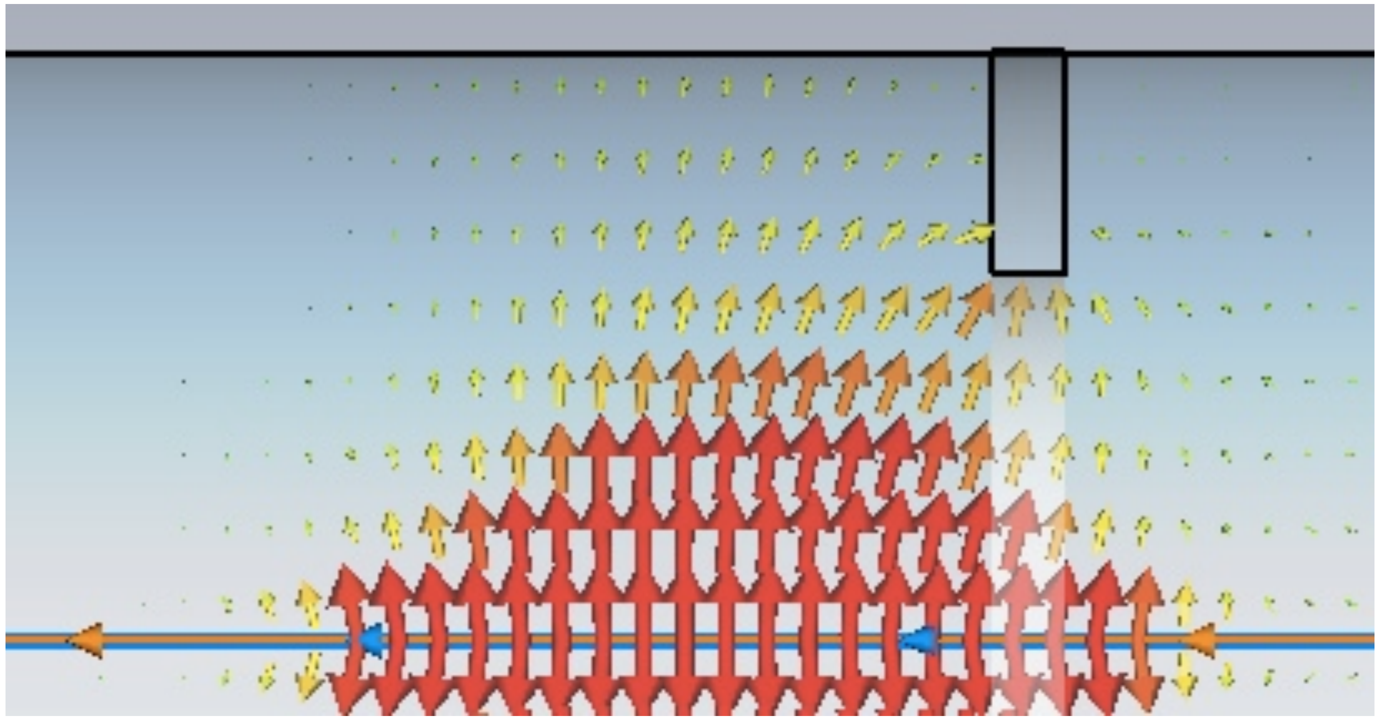
Relativistic bunch charge field propagate unperturbed in homogeneous beam pipe, ...



... until the cross section is changed. Then scattered fields are needed to fulfill boundary condition.

Phenomenologically III:

The original field shape is re-established some distance behind the obstacle:



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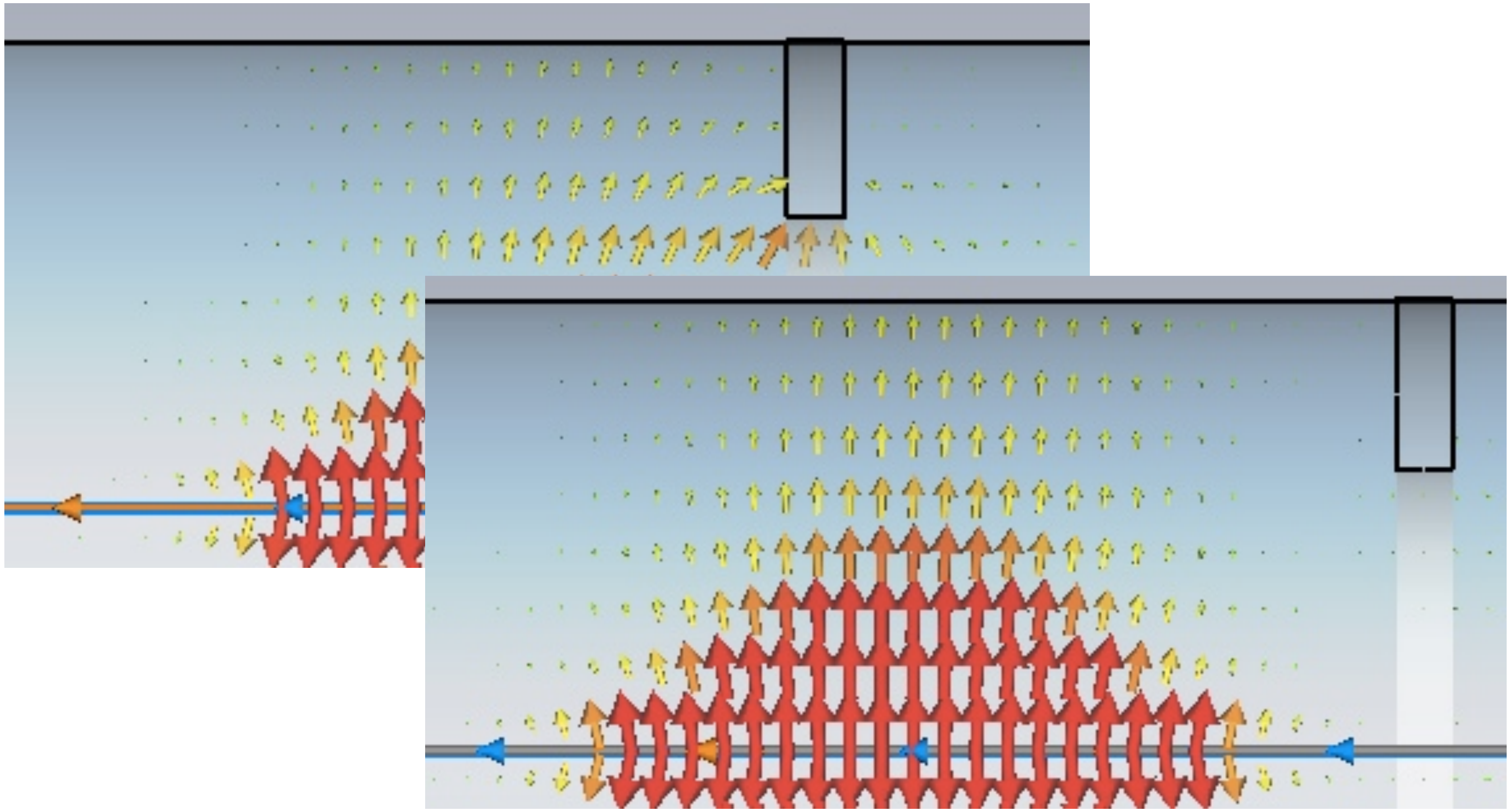

Phenomenologically III:

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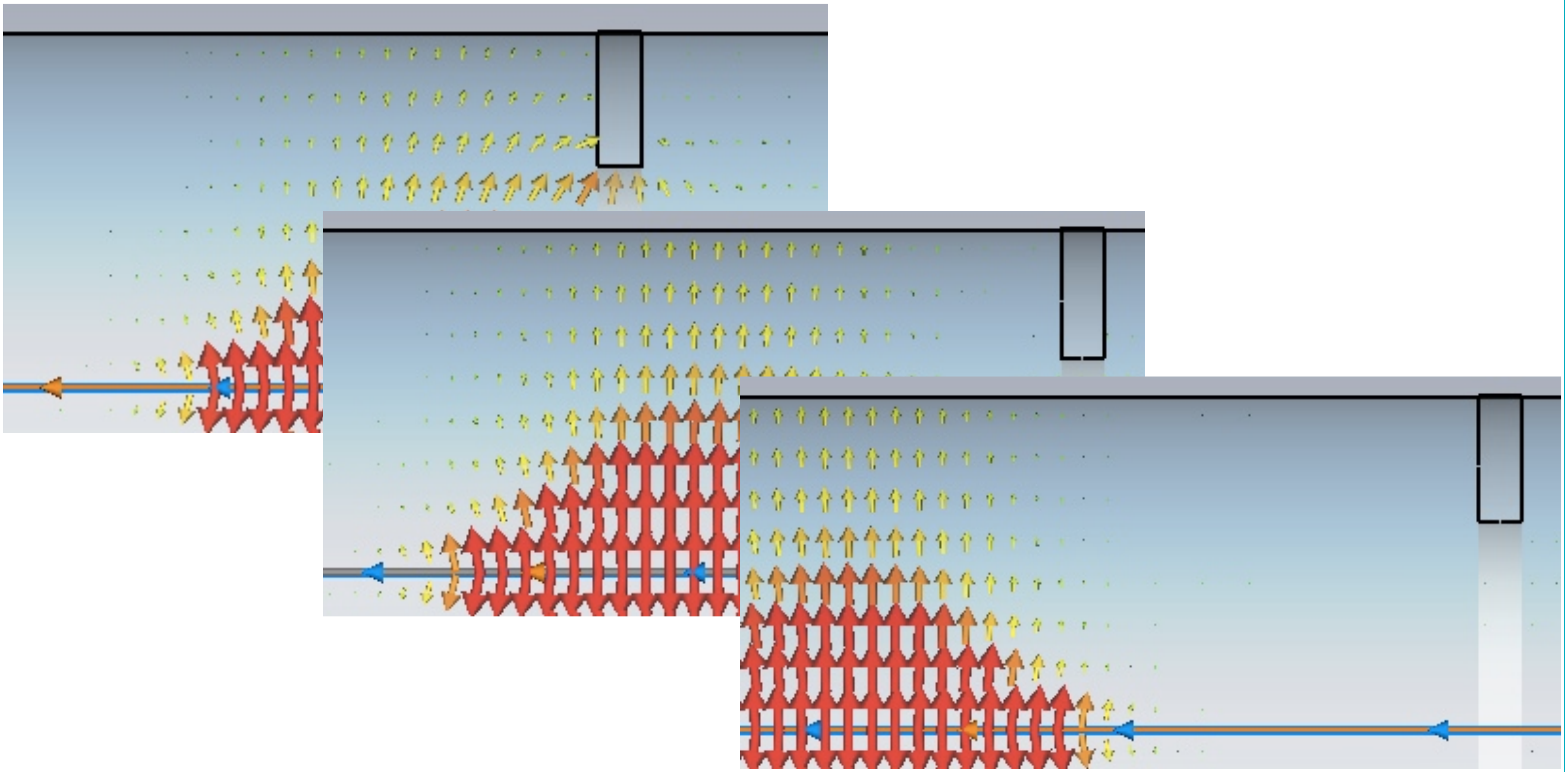

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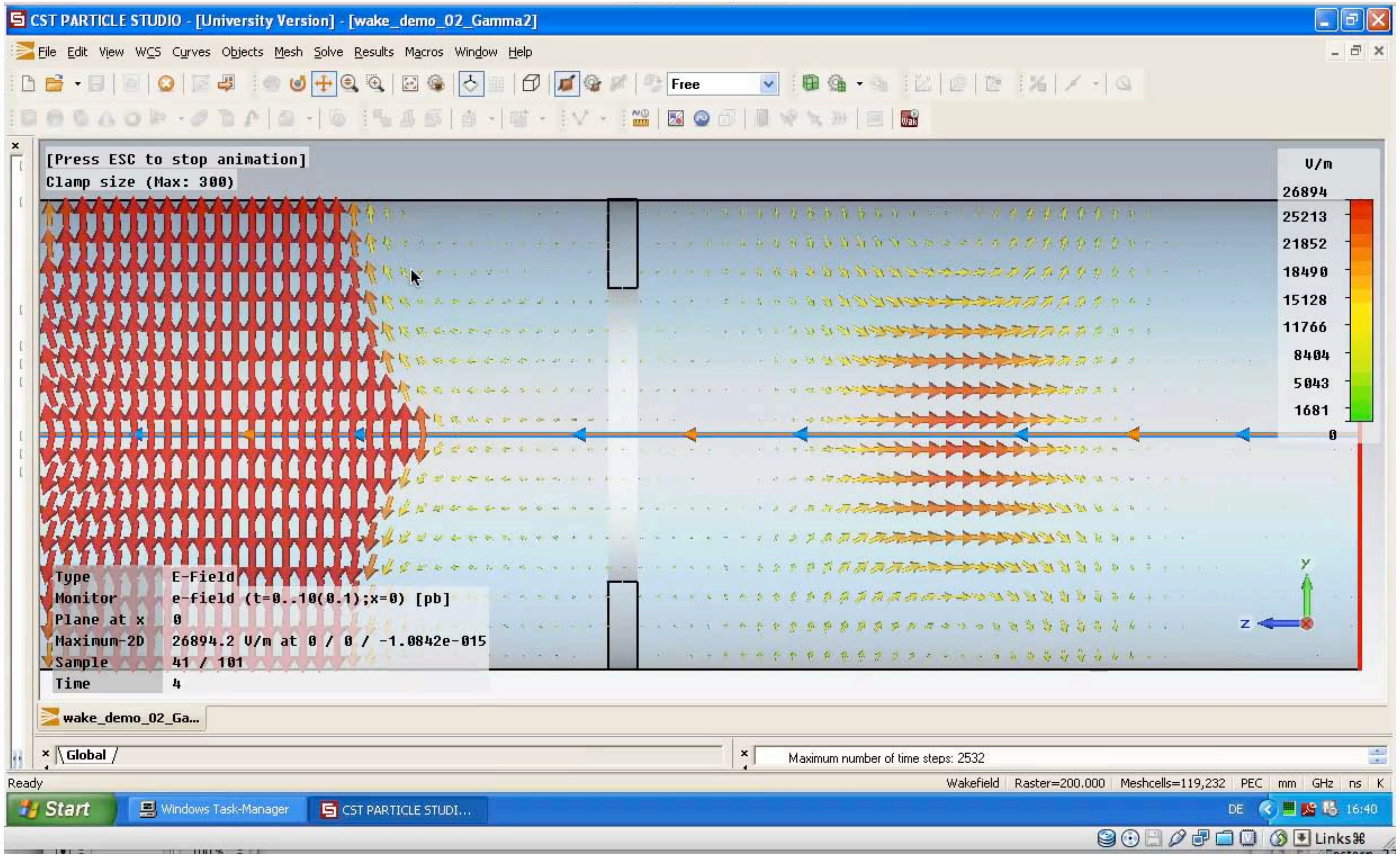
Phenomenologically IV:

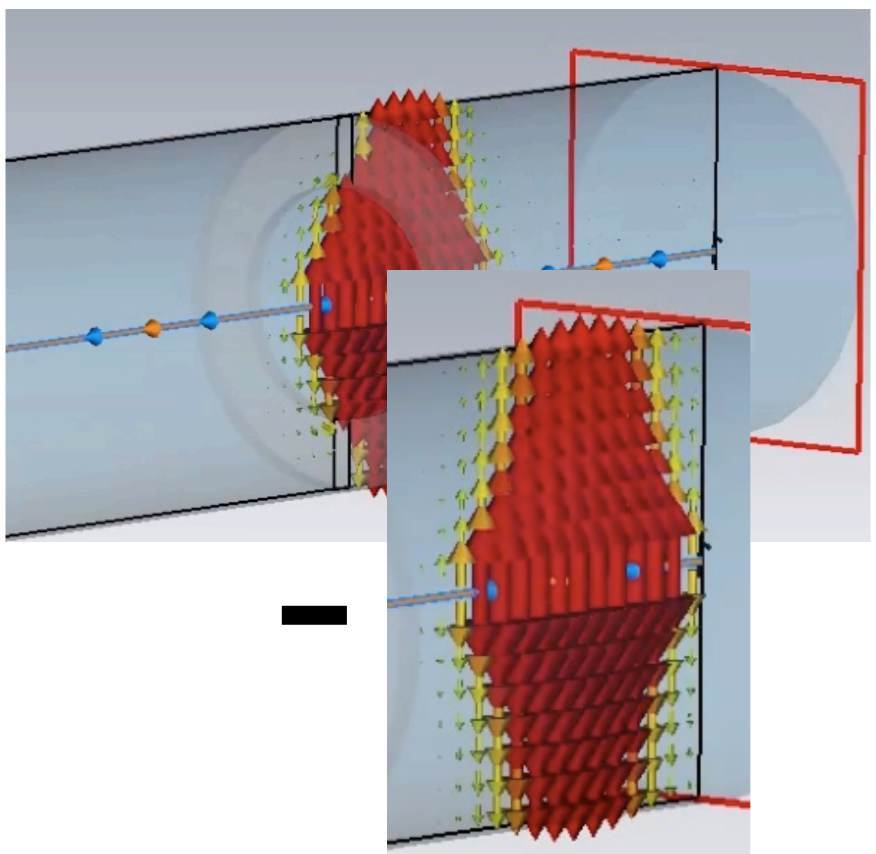
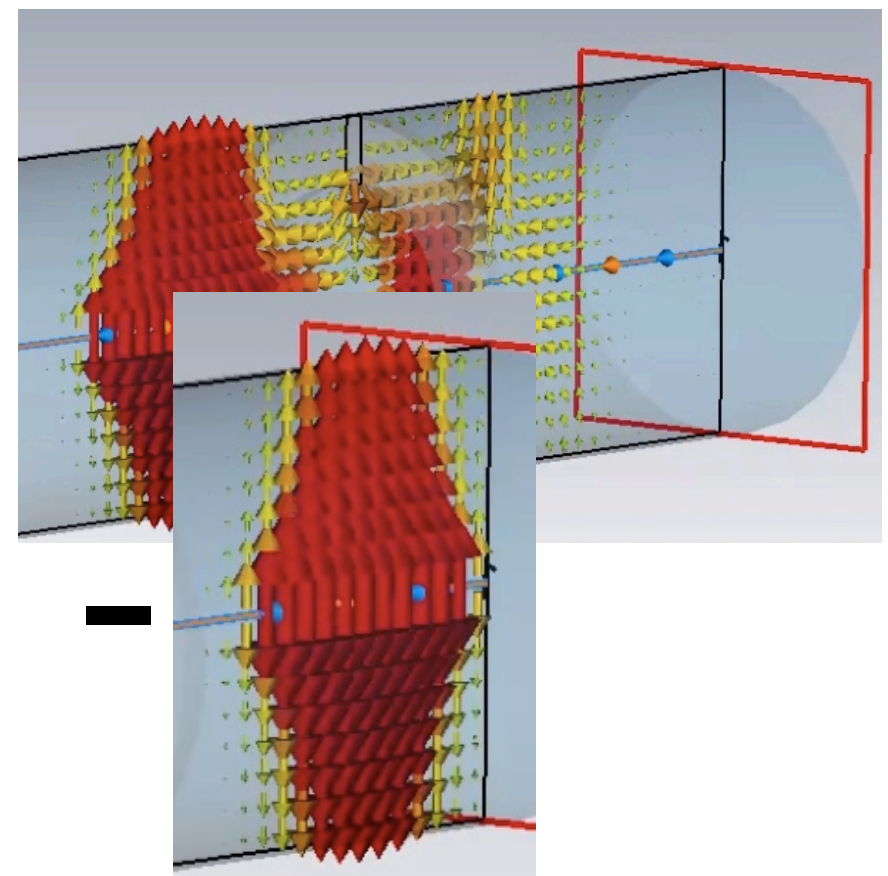
But there remains field at the obstacle,... (field scaling changed)



... continuously ringing after the bunch passage.

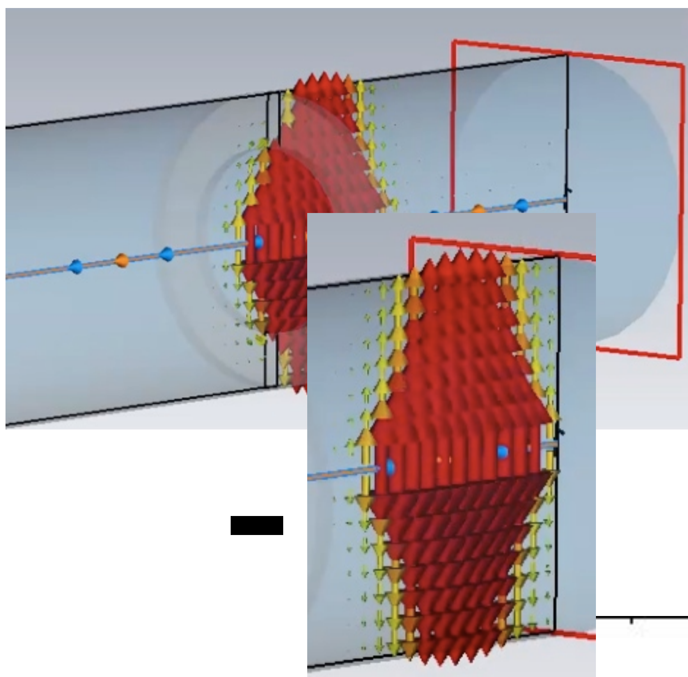
From beam to fields

Substract the unperturbed self-field of the bunch in a co-moving manner, i.e.:

• • •  • • •  • • •

From beam to fields

Substract the unperturbed self-field of the bunch in a co-moving manner, i.e.:



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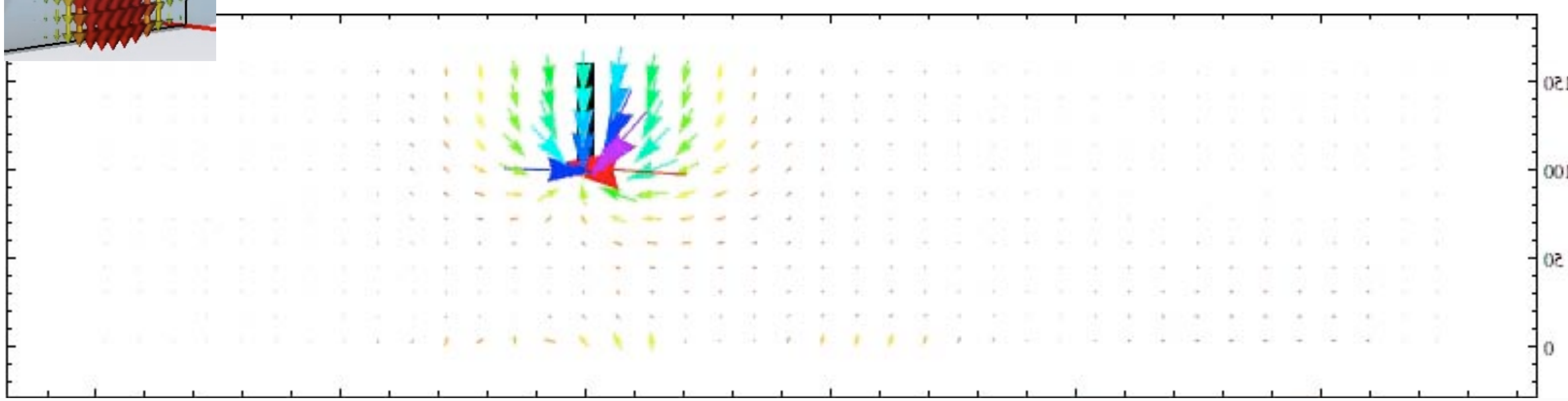
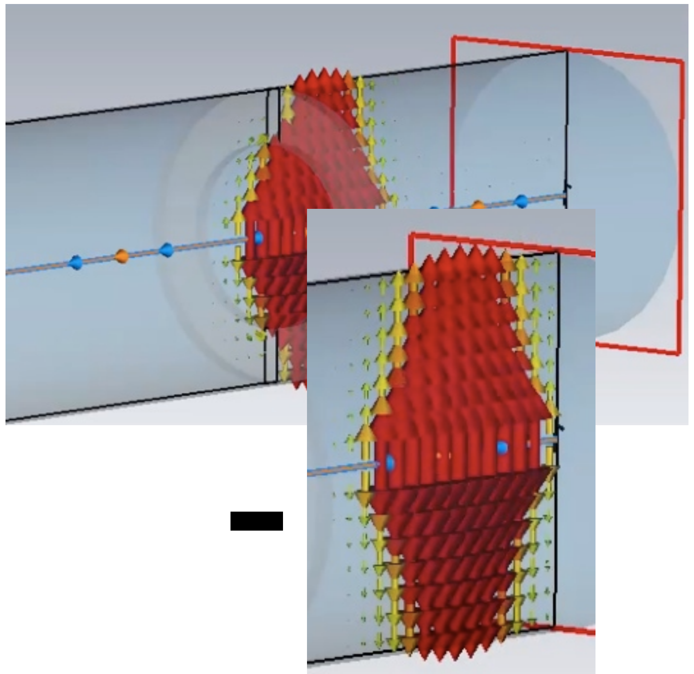
(in some time interval)



(Only a part of the cross section shown)

From beam to fields

Substract the unperturbed self-field of the bunch in a co-moving manner, i.e.:



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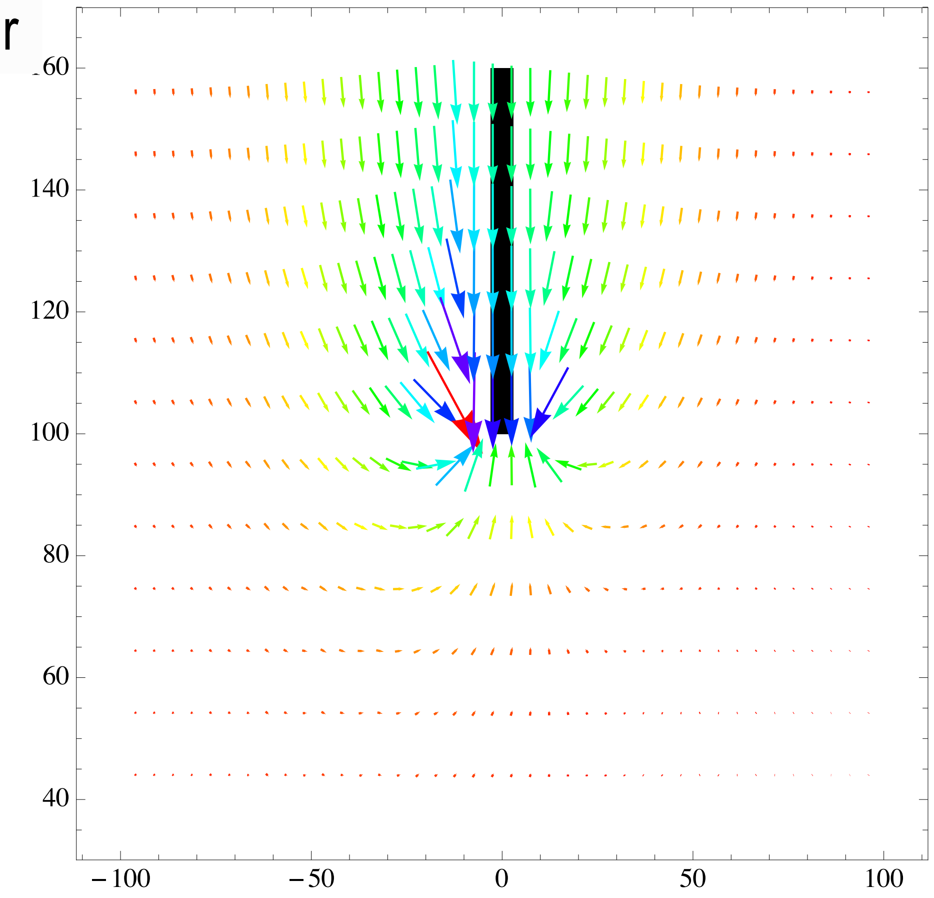
(at some certain time)



Wakefields are exited as compensation of the bunch self-field wherever the boundary surface has a radial component, i.e. at every change of the cross section.

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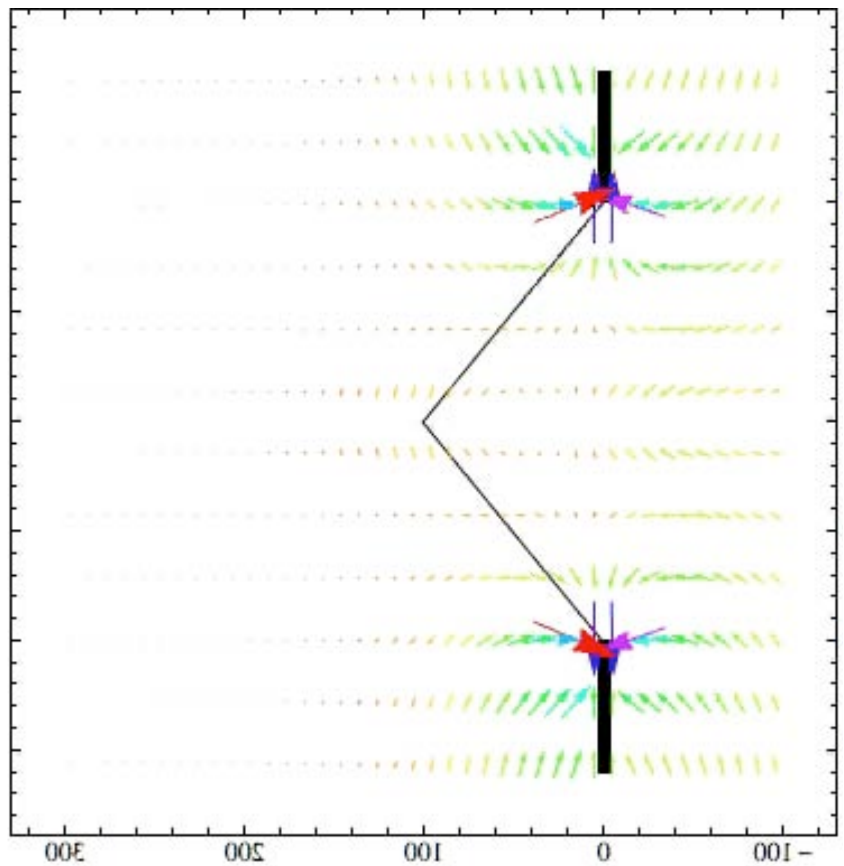
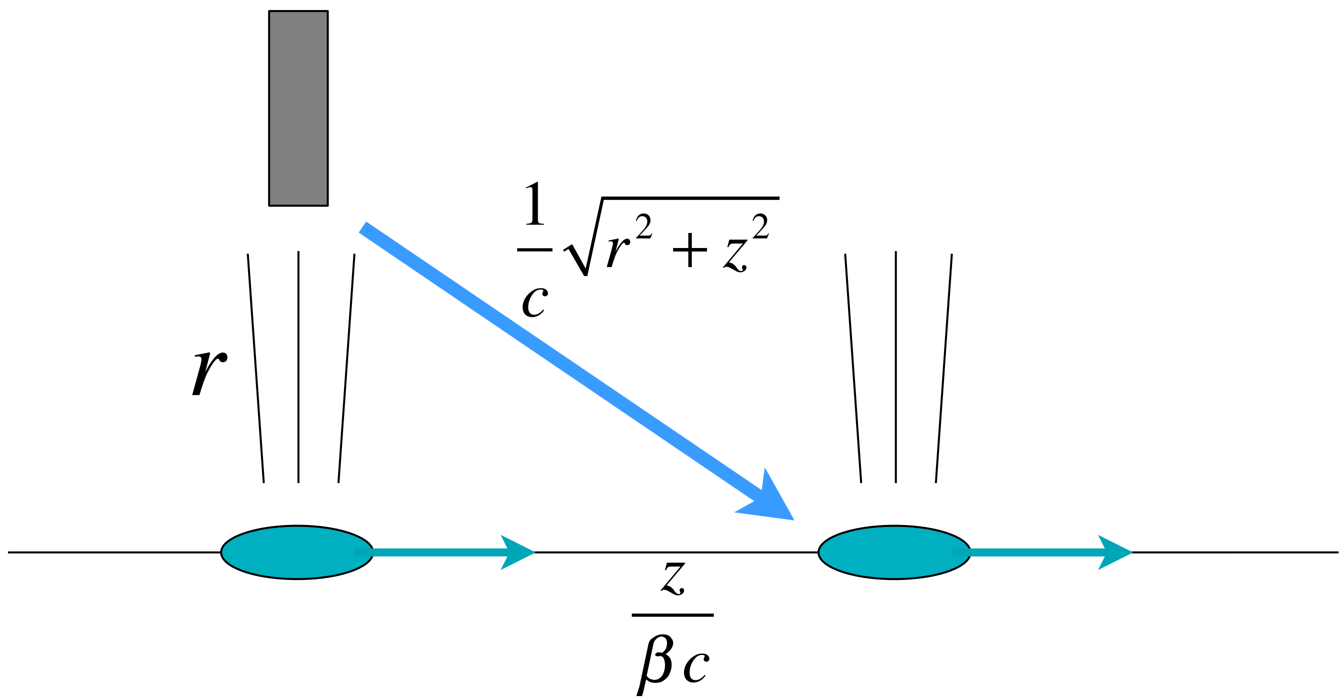
From beam to fields

Field energy of wakes first is taken from the co-moving field ...



... but then is fed back from the kinetic energy of the moving charges (thus re-establishing the radial self-field).

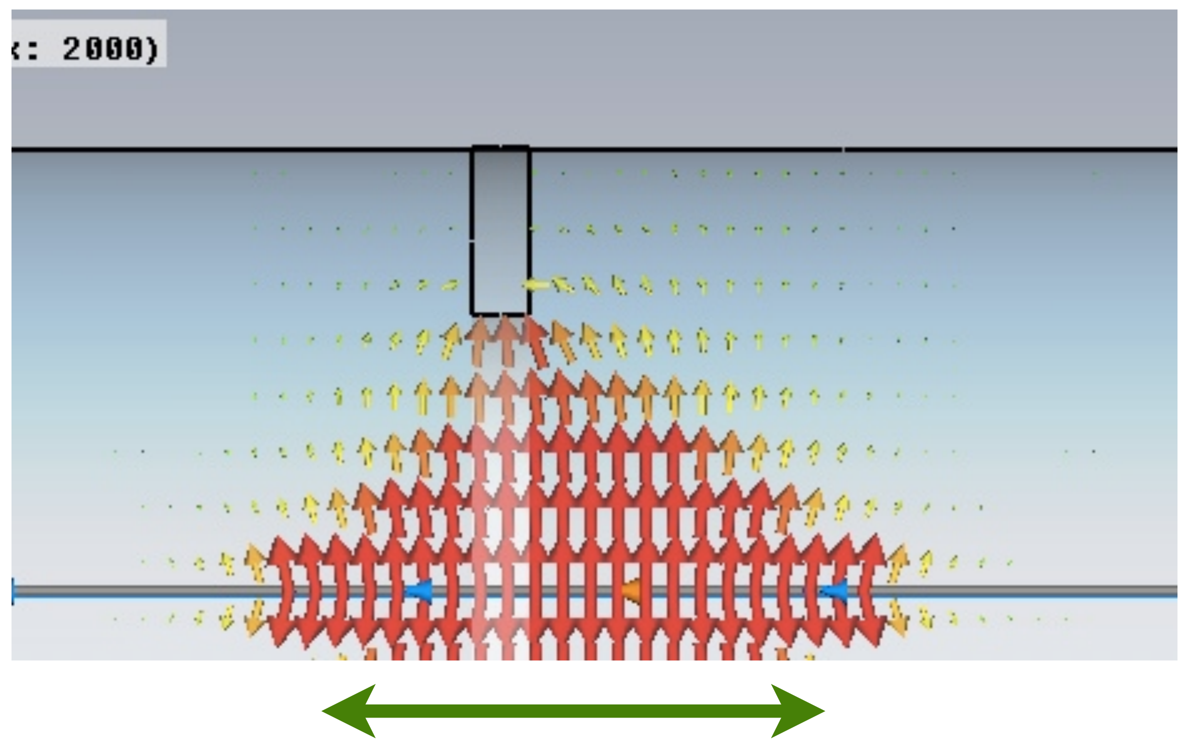
Where happens the "catch-up"?



$$\Rightarrow \quad \frac{z}{\beta c} = \frac{\sqrt{r^2+z^2}}{c} \quad \Rightarrow \quad z = r\beta\gamma \approx r\gamma$$

... which may be rather far away (assuming the bunch propagates on a straight line).

Let's briefly talk about frequencies:



e.g. bunch length $2\sigma = 100$ mm, $\beta=0.87$

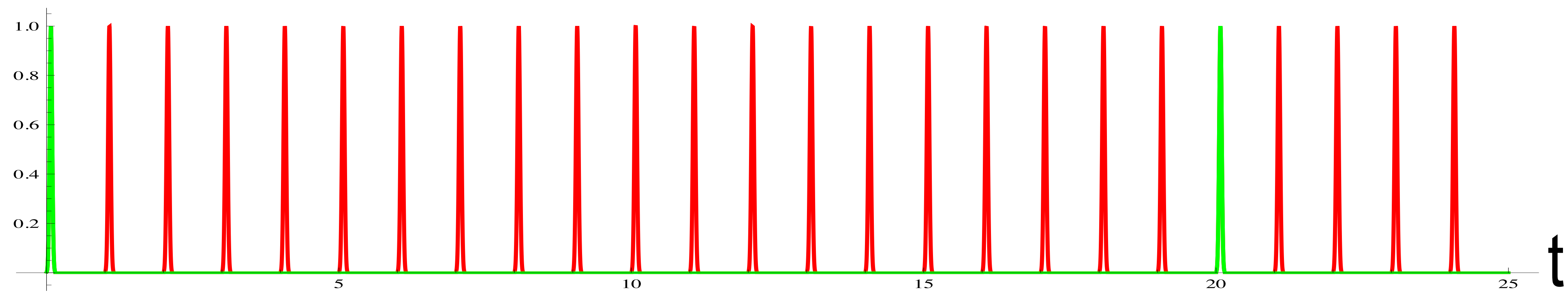
=> bunch passes obstacle in $\Delta t = 2\sigma/(\beta\ c) \approx 4 \cdot 10^{-10}$ s

=> Bunch/field contain Gaussian spectrum (centered @ 0 Hz) with characteristic bandwidth $1/\Delta t = (\beta\ c)/(2\sigma) = 2.6$ GHz

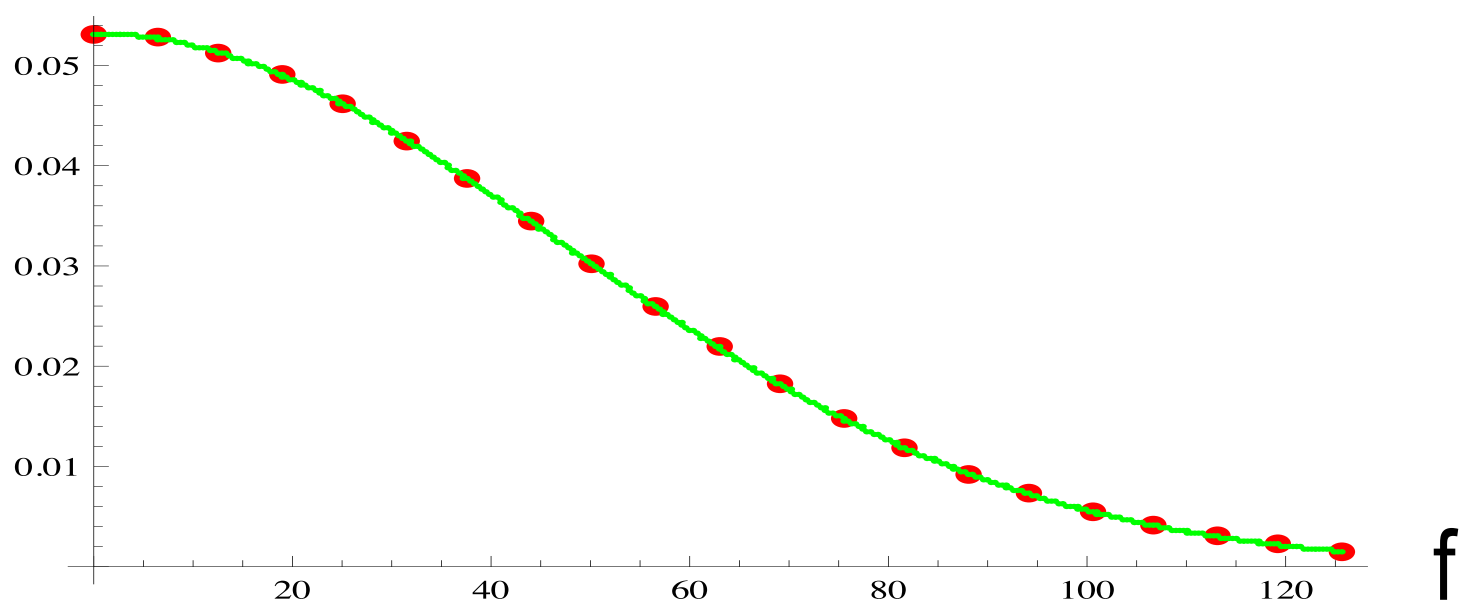
=> The shorter the bunch, the broader the spectrum!

Let's briefly talk about frequencies II:

Compare to similar beams with same bunch shape, but different repetition rate:



... in frequency domain:



=> The higher the repetition rate, the fewer lines in the spectrum!

Fields and modes

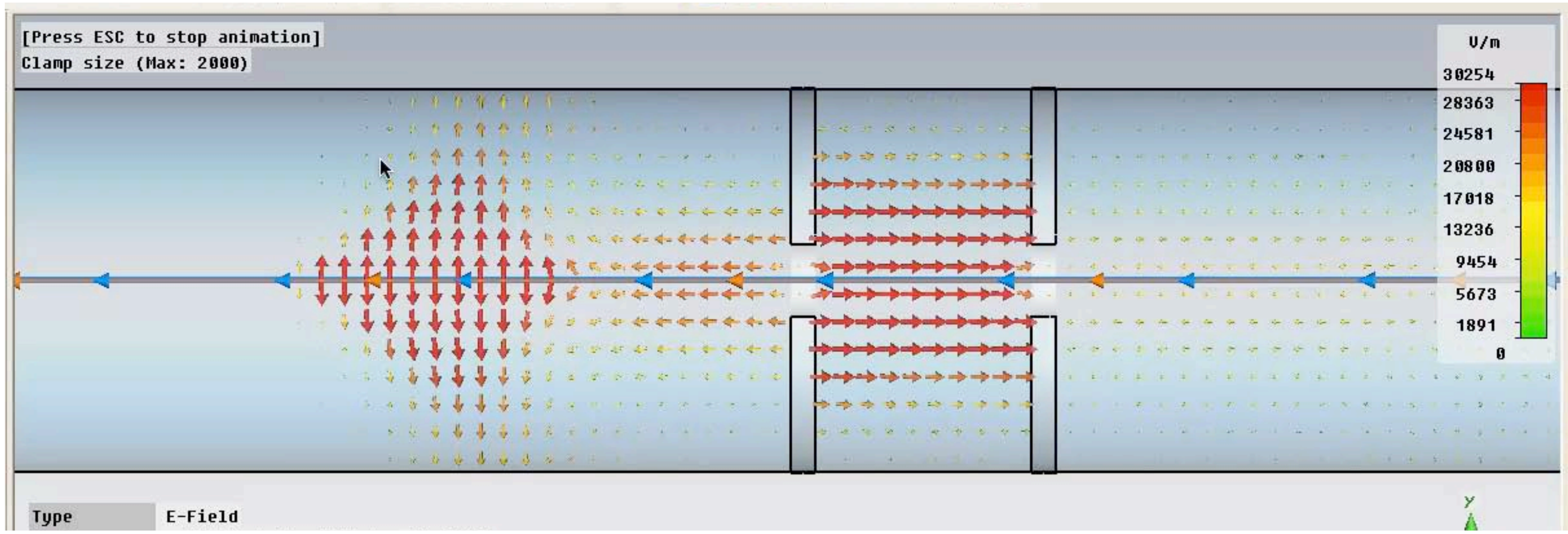
Analysis of Maxwell's equations shows*:

- ... that they are linear. Superposition of any two solutions again will be a solution.
- ... that imposing boundary conditions (i.e.: cavity/waveguide walls) lead to an infinite set of individual solutions, so-called ***modes***.
- ... that different modes (both cavity/waveguides) have distinct field patterns and resonance frequencies (cavities) / cut-off-frequencies <=> propagating constants (waveguides).
- ... that one has to pay attention in case of charges present inside the boundaries: There exist two sets of field patterns:
  - i) the "classical" modes being divergence-free;
  - ii) the rotation-free solutions of the scalar Helmholtz-equations

*: You may e.g. refer to one of the most complete and rigorous explanation of the topic, which is:
T. Weiland, R. Wanzenberg: Wakefields and Impedances, published as DESY-M-Report 91-6

Does this apply? - A first impression:

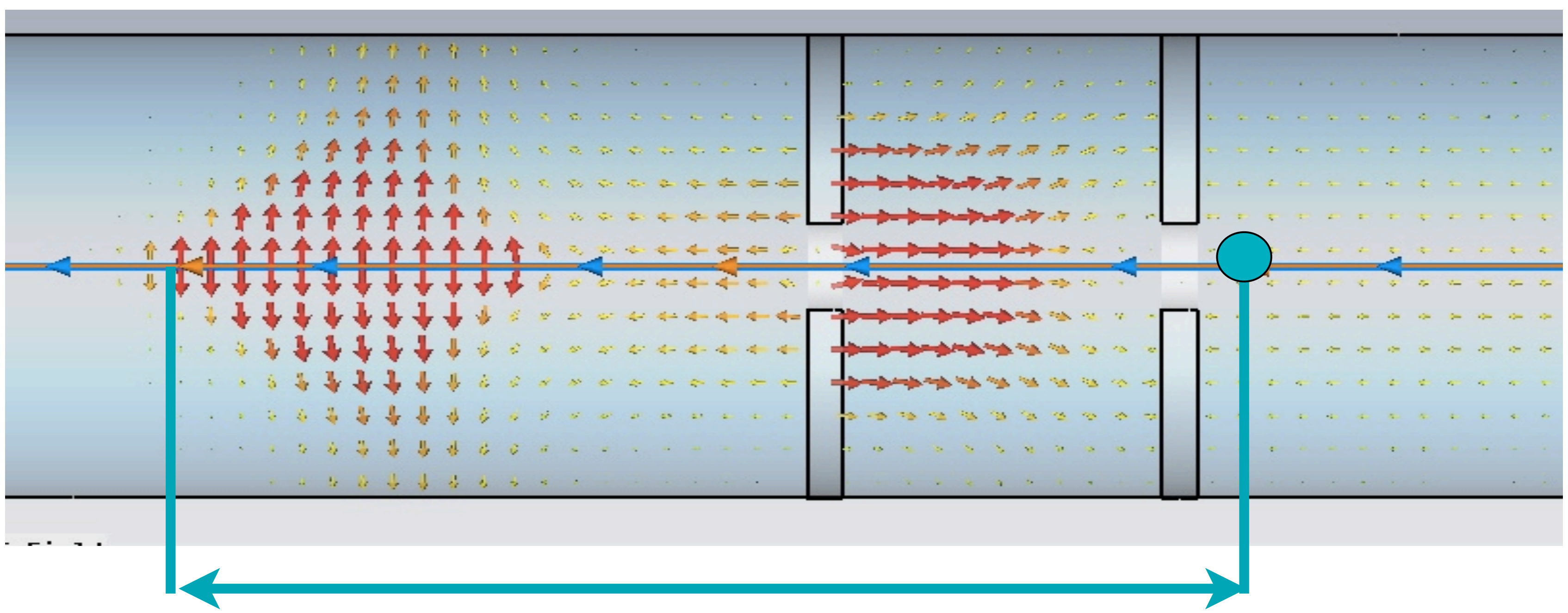
Introducing a second iris (and reducing their inner diameter) builds up a cavity-like structure:



Fields ring inside the "cavity" in a regular manner.

A new concept:

We might ask, which integrated force experiences a charge ...



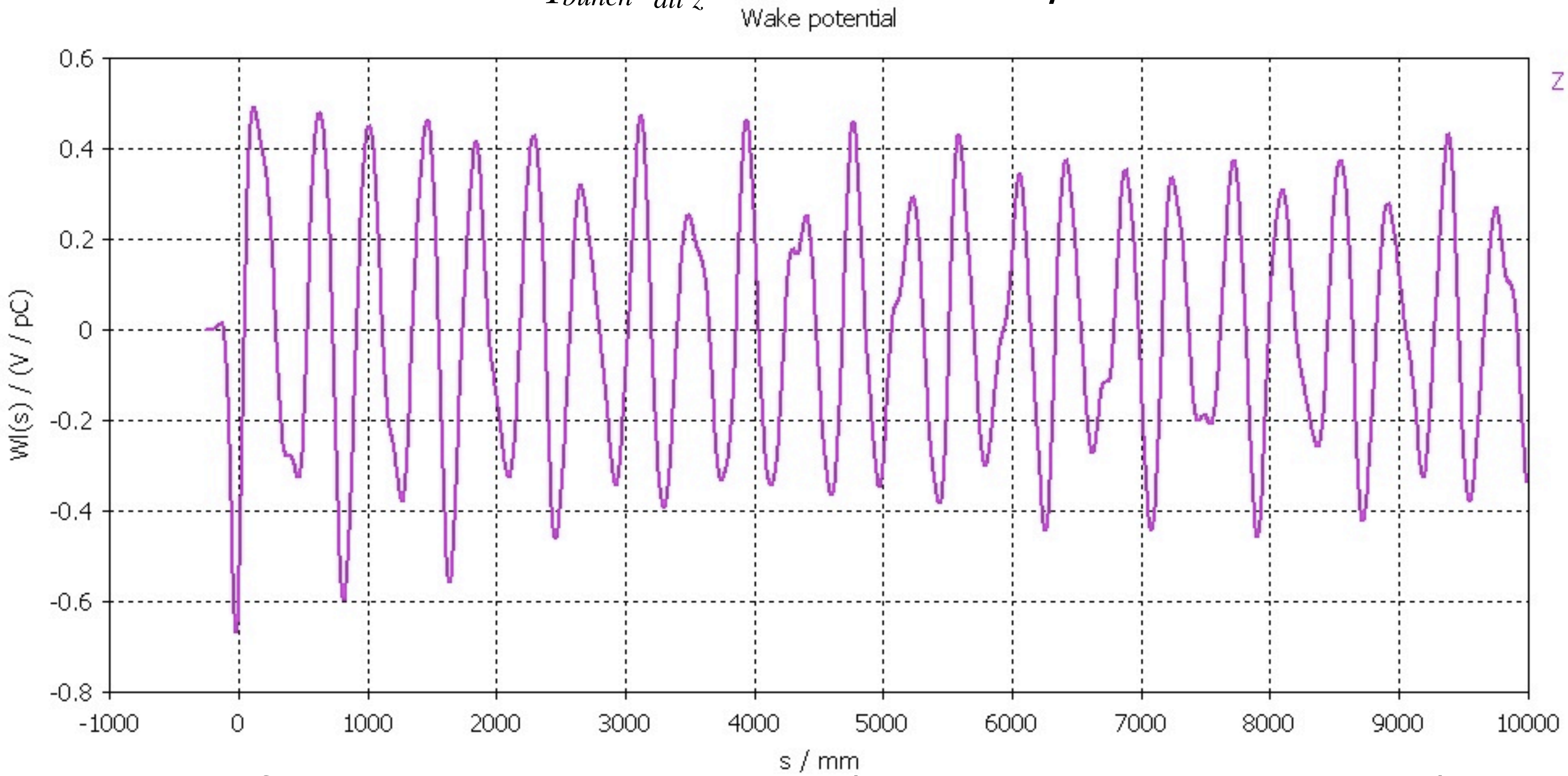
following the bunch @ z in a certain distance s

$$W_{long.}(x,y;s) = \frac{1}{q_{bunch}} \int_{all\ z} dz\, E_z(x,y;z,t=\frac{s+z}{\beta c})$$

$W_{long.}$ is the so called longitudinal wake-potential

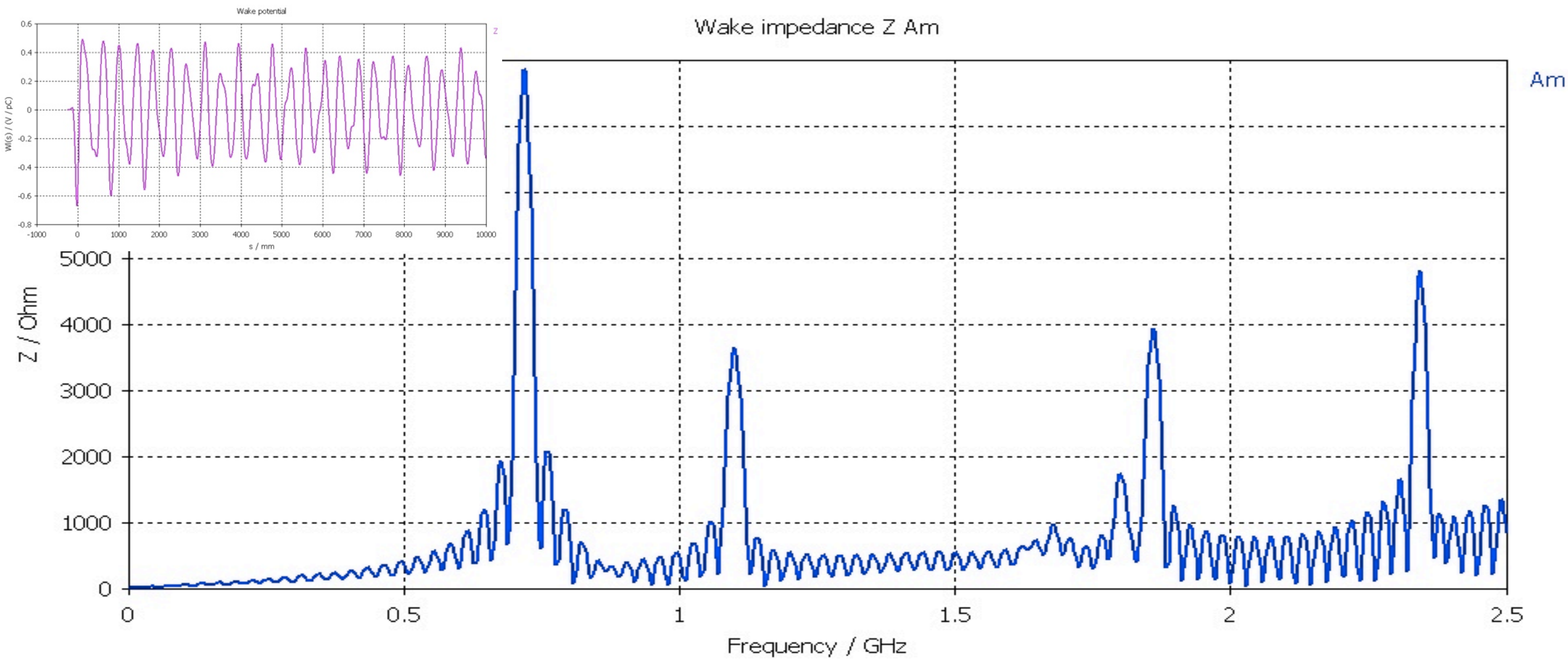
Longitudinal wake potential for our cavity:

$$W_{long.}(x,y;s) = \frac{1}{q_{bunch}} \int_{all\ z} dz\, E_z(x,y;z,t=\frac{s+z}{\beta c})$$



This shows: following charges in 10 m (extrapolate 20 m, 30 m ...) distance experience acceleration/deceleration: *Long range* wake

Next step: Perform a Fourier transform on $W_{long.}$ ...



... which is commonly denoted as *wake impedance.*

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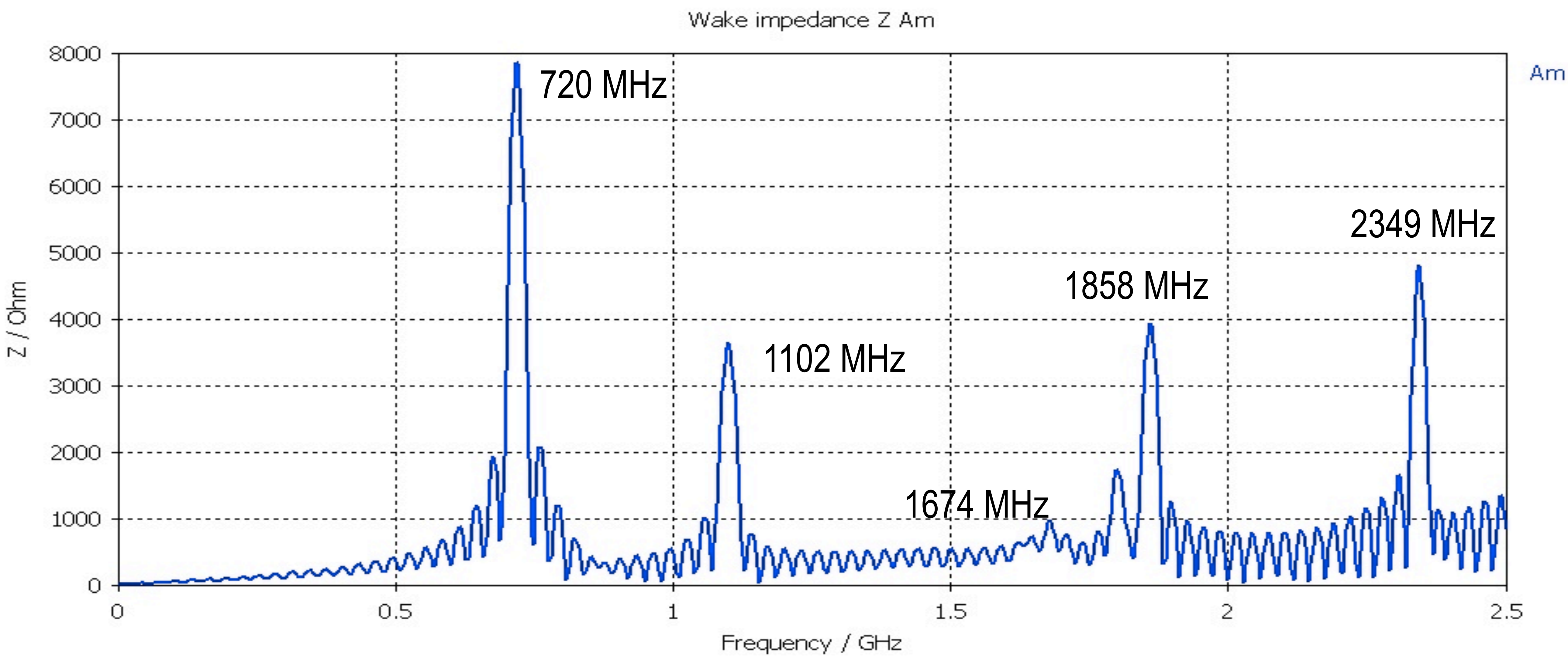
The frequencies of the impedance maxima ...



... can be identified by an eigenmode computation of the cavity:

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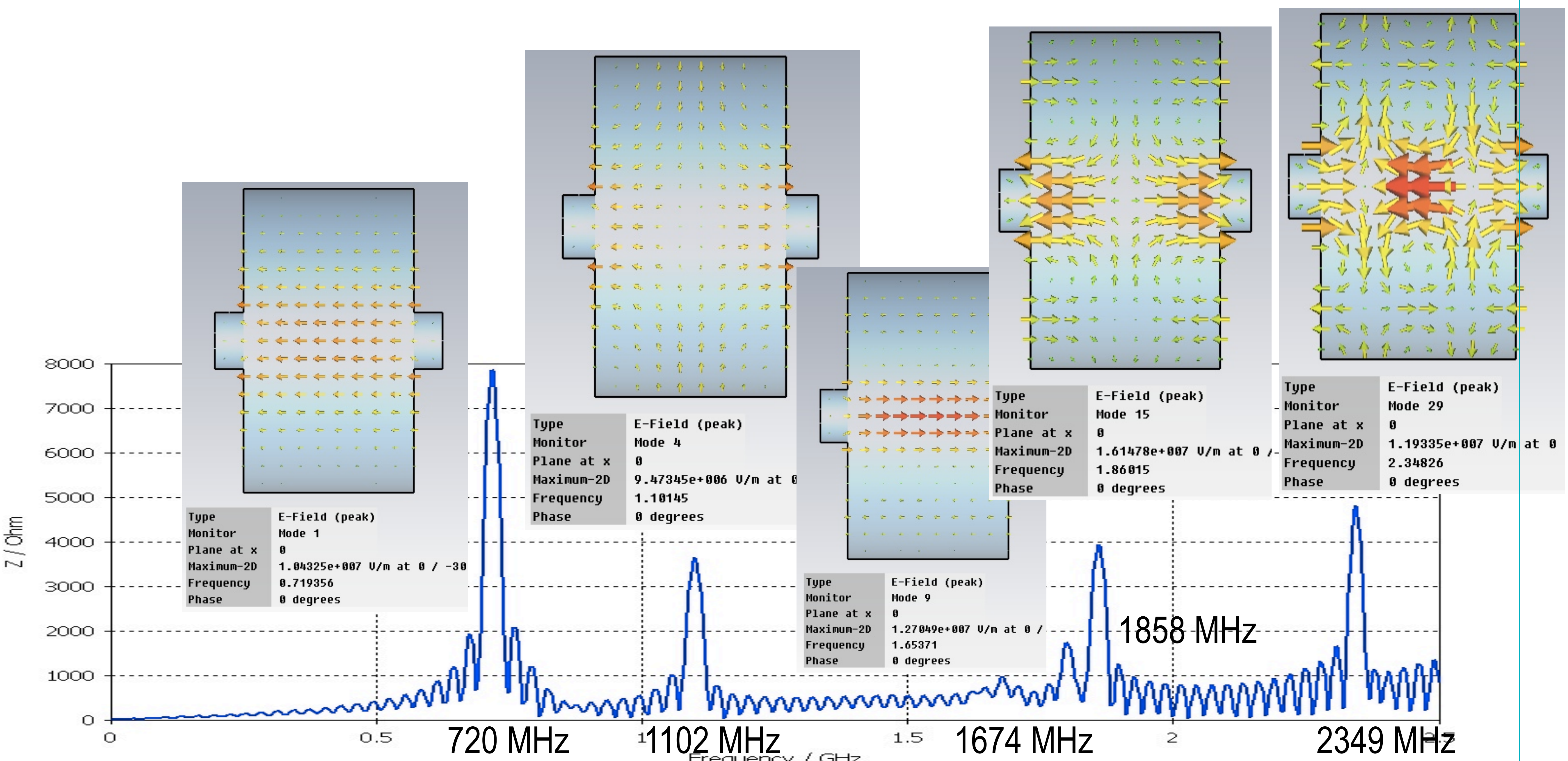
Eigenmodes, relevant for an on-axis-beam:

... can be identified by an eigenmode computation of the cavity:



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The other way round: which force experience a charge in a certain mode?

Take a given mode with a longitudinal field profile $E_z(z)$ along the beam axis, oscillating with frequency f (and some phase φ).
Then a particle with charge q and velocity βc exchanges the energy ΔU with the field:

$$\Delta U = q \int_{cavity} E_z(z) \cos\left(2\pi f \frac{z}{\beta c} + \varphi\right) dz$$

Therefore:

- Strong interaction not necessarily happens, if E_z takes high values.
- Either rather short areas of field interacts strongly (no oscillatory cos-weighting) ...
- ... or fields show (spatial) synchronism with cos-term (e.g. accelerating mode)
- Energy exchange is velocity-dependent!

Walking through the mode-zoo

- Circular cross sections: Be aware of azimuthal dependencies
- Chains of identical elements: understand passbands
- Chains of almost identical elements: traps for modes
- A real-world example

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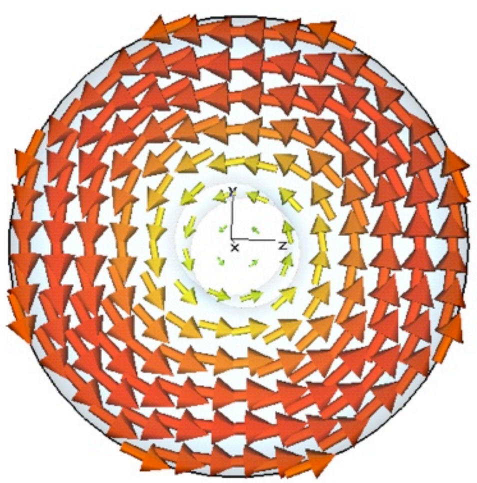
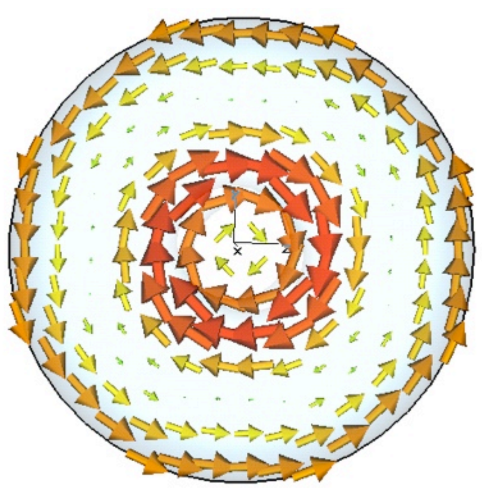
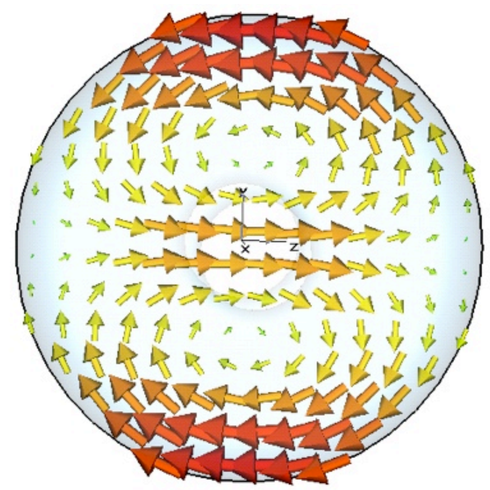
What are Monopole-, Dipole-, Quadrupol-Modes?

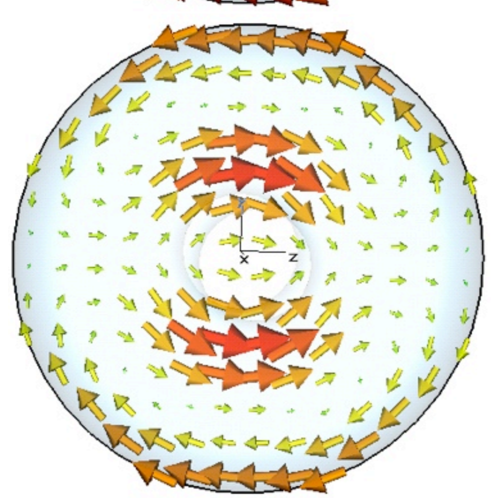
Consider structures of axial circular symmetry. Then ***all*** fields belong to classes with certain azimuthal dependencies:



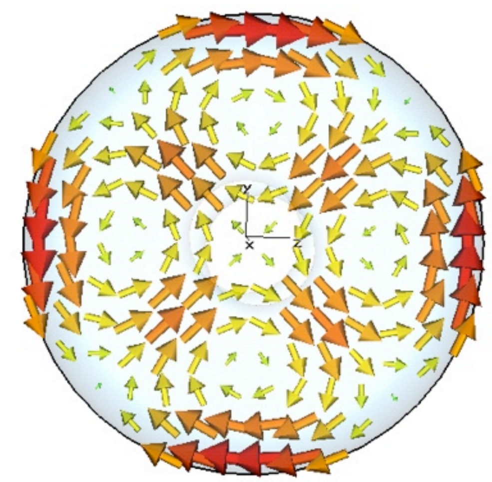


Monopol, Cos(0•α)

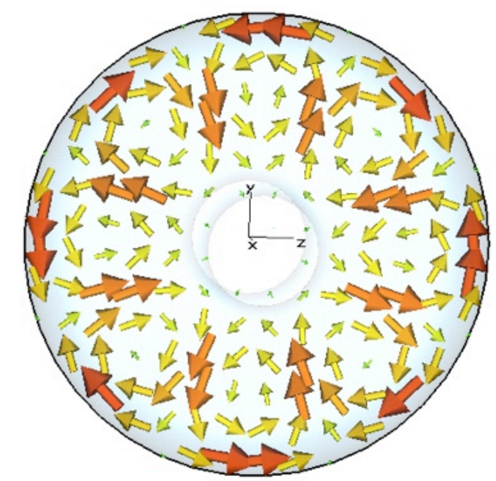




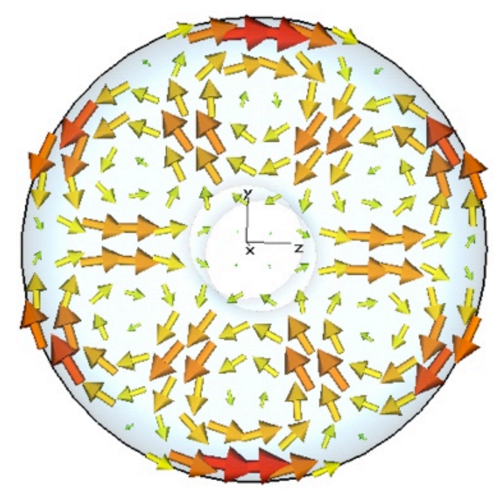
Dipol, Cos(1• α)



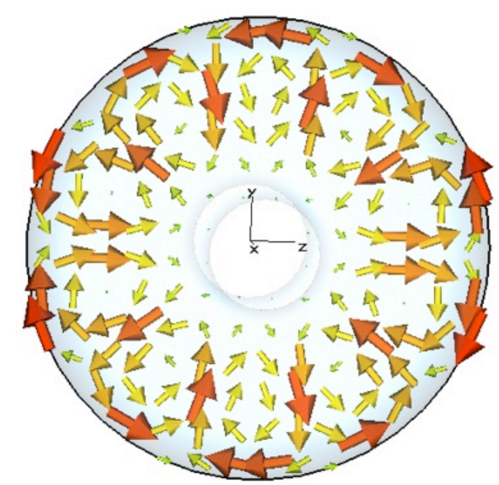
Quadrupol, Cos(2• α)



Oktupol, Cos(4• α)



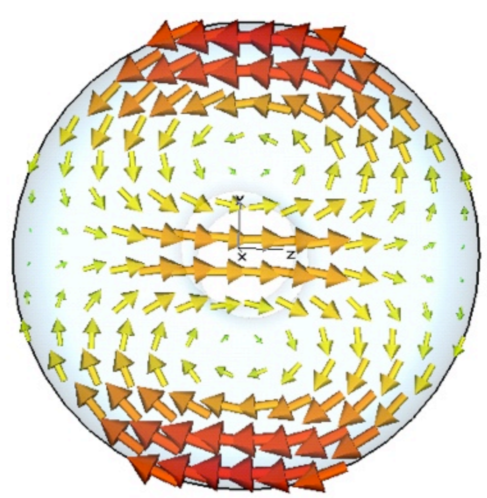
Sextupol, Cos(3• α)



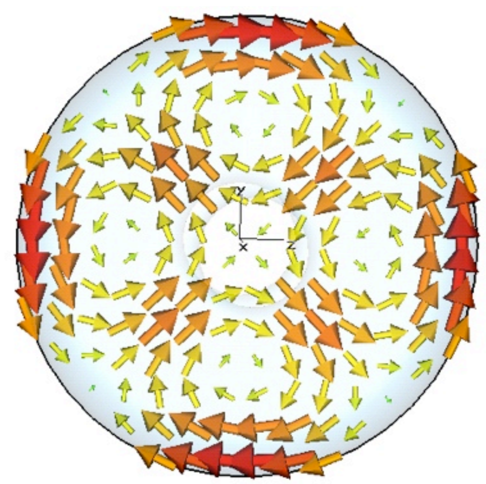
Dekapol, Cos(5• α)

All (but monopols) exist in two polarizations

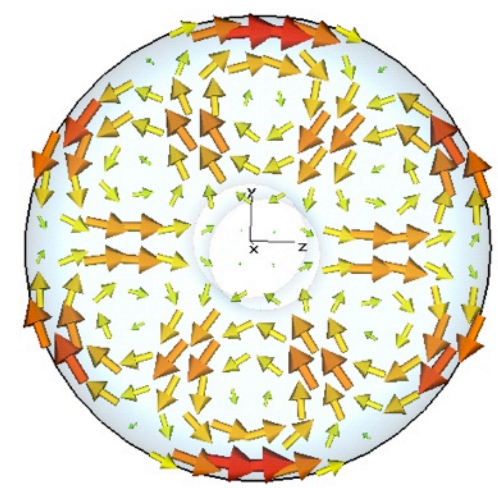
... which have to be considered as individual modes with identical resonance frequencies



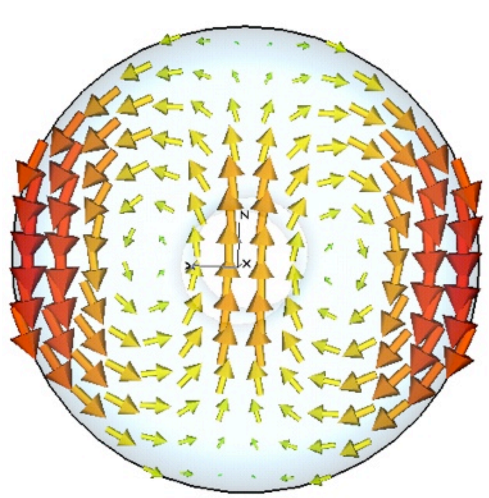
Dipol, Cos(1• α)



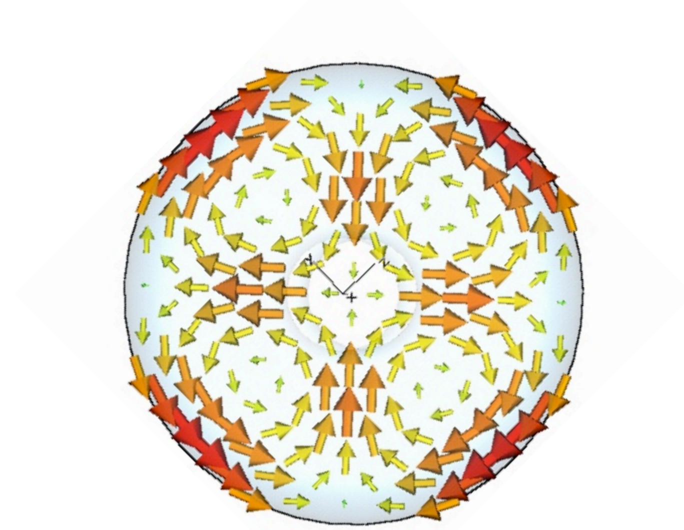
Quadrupol, Cos(2• α)



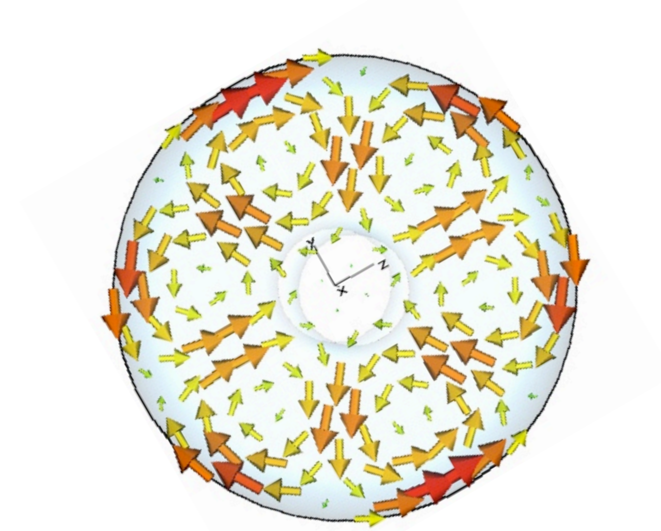
Sextupol, Cos(3• α)



Dipol, Cos(1• α + π/2)



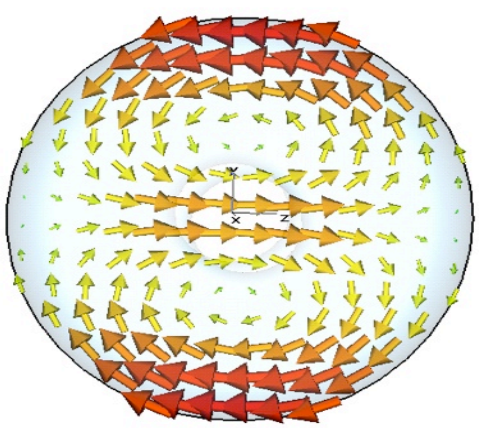
Quadrupol, Cos(2• α + π/4)



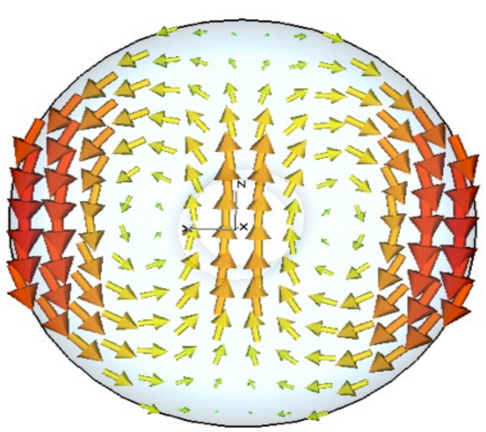
Sextupol, Cos(3• α + π/6)

Deviations from round shape define actual orientation

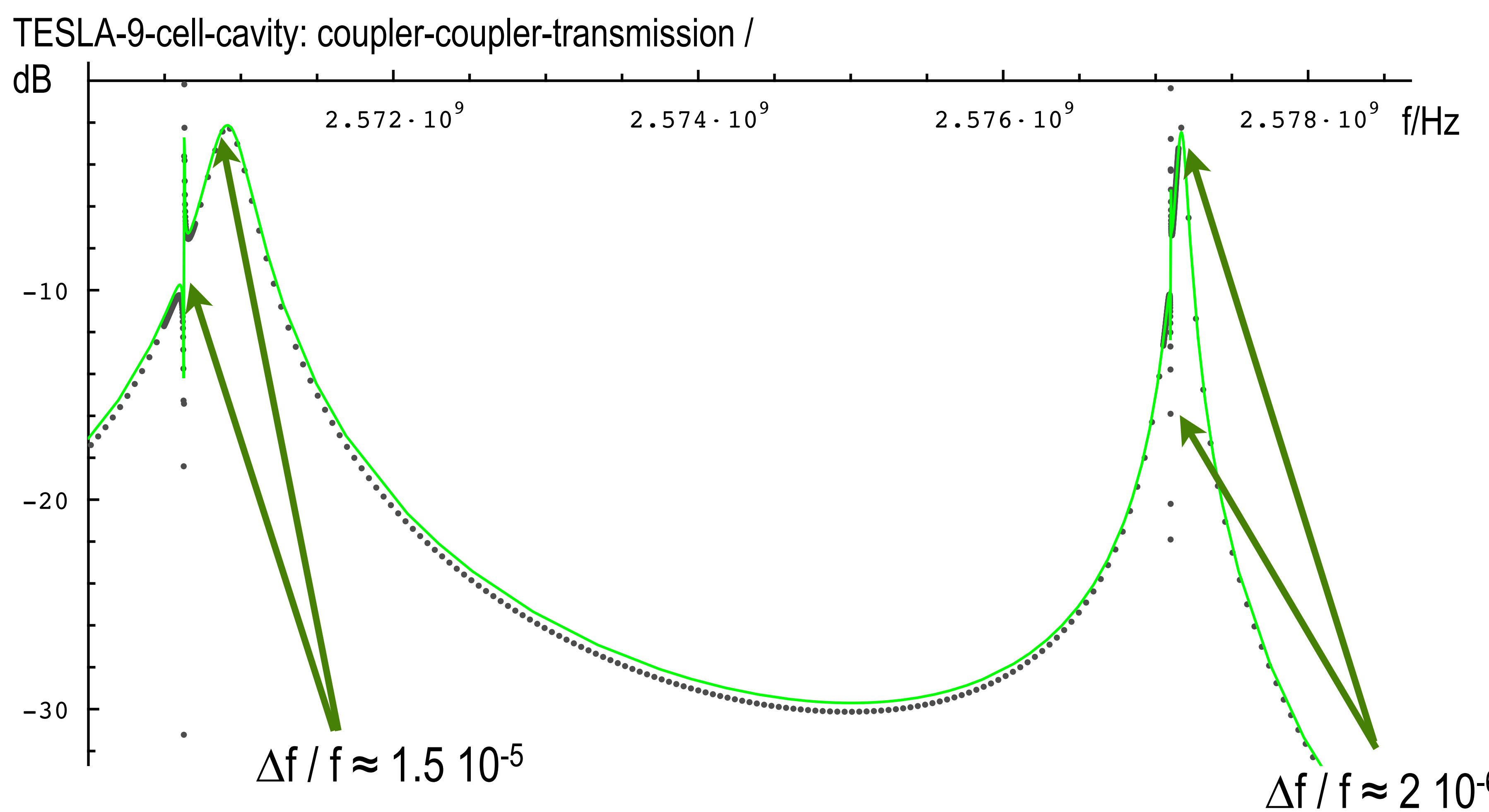
... which may be e.g. cavity deformations or attached couplers



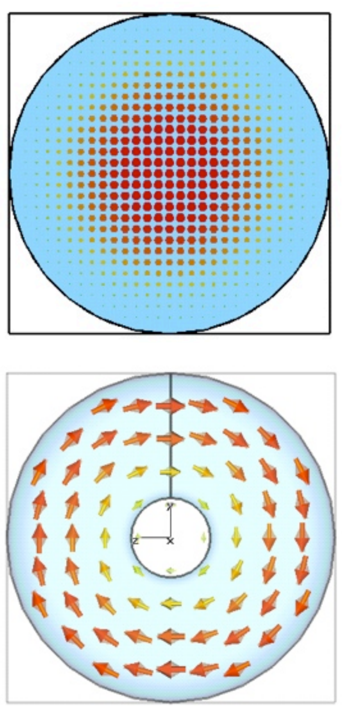
Dipol, Cos(1• α)



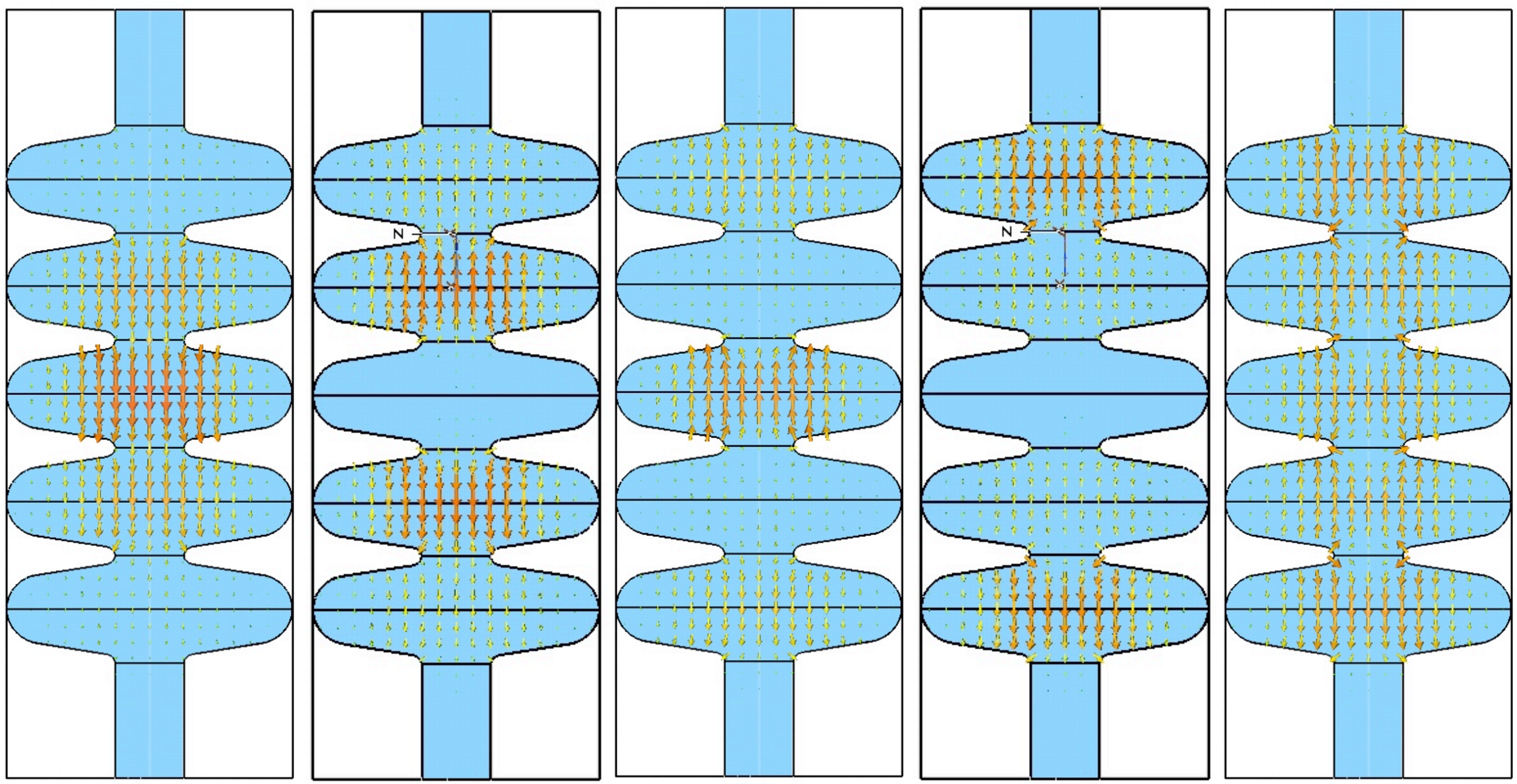
Dipol, Cos(1• α + π/2)



Passband fields - a 5-cell example



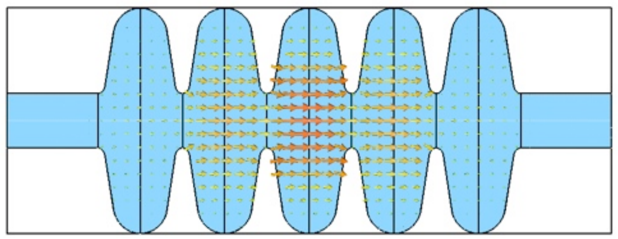
Monopol-type E- and H-field, common to all modes and cells



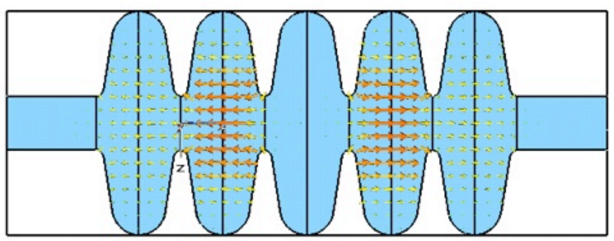
TM_{010}- π/5 (pseudo 0) | TM_{010}-2π/5 | TM_{010}-3π/5 | TM_{010}-4π/5 | TM_{010}-π (accelerating)

Passband fields and frequencies

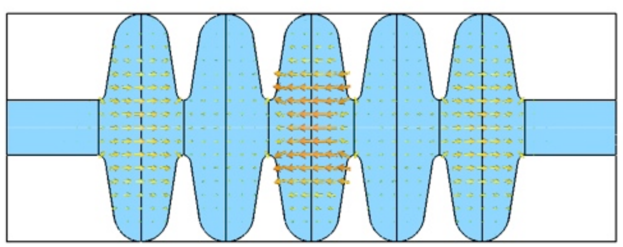
TM_{010}-$\pi/5$
700.40098 MHz



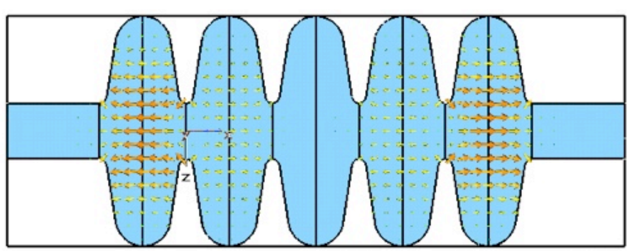
TM_{010}-$2\pi/5$
702.21446 MHz



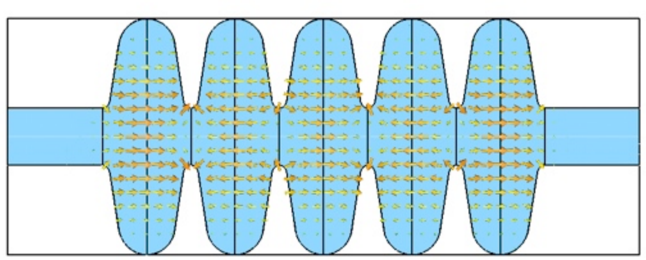
TM_{010}-$3\pi/5$
704.52333 MHz

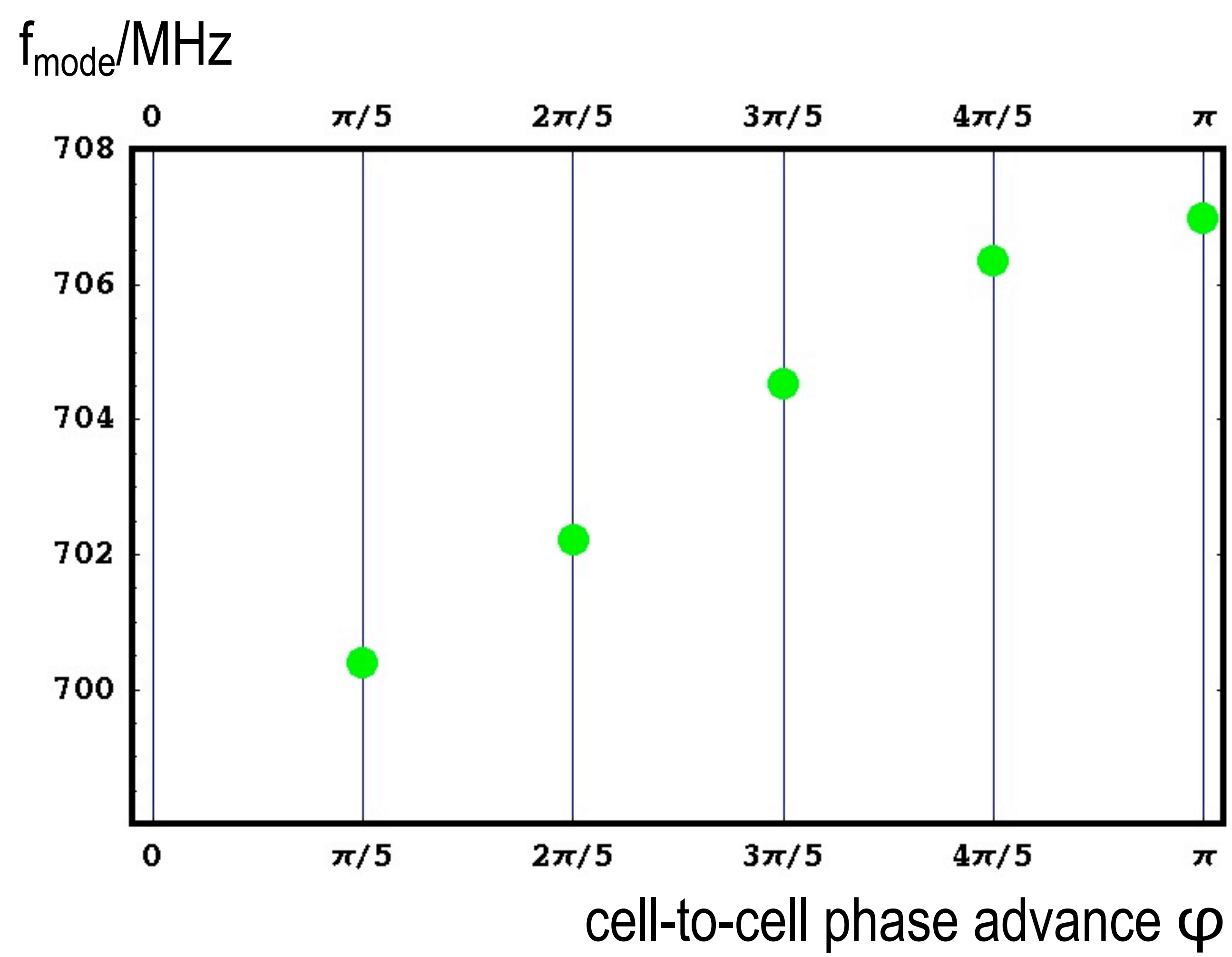


TM_{010}-$4\pi/5$
706.34471 MHz



TM_{010}-π
706.97466 MHz

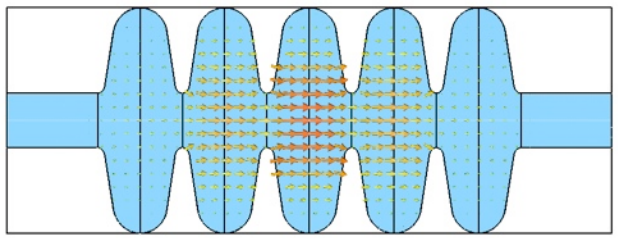




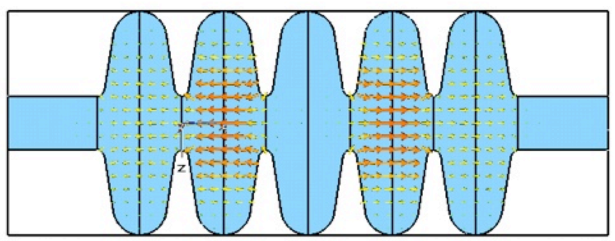
... which seems to obey some rule ?!

Passband fields and frequencies

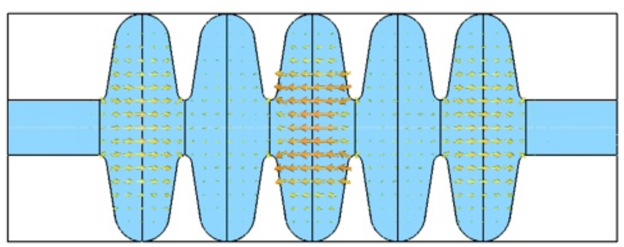
TM_{010}-π/5
700.40098 MHz



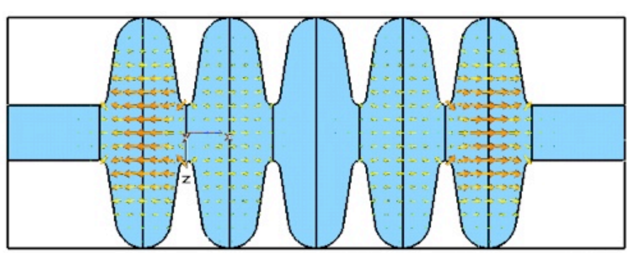
TM_{010}-2π/5
702.21446 MHz



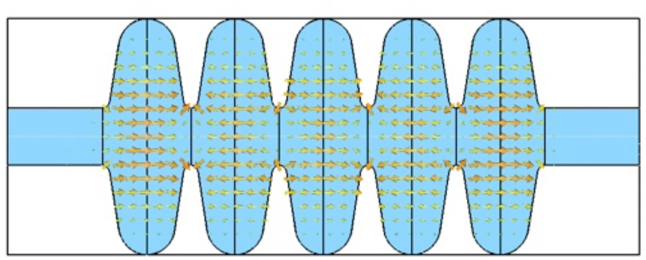
TM_{010}-3π/5
704.52333 MHz

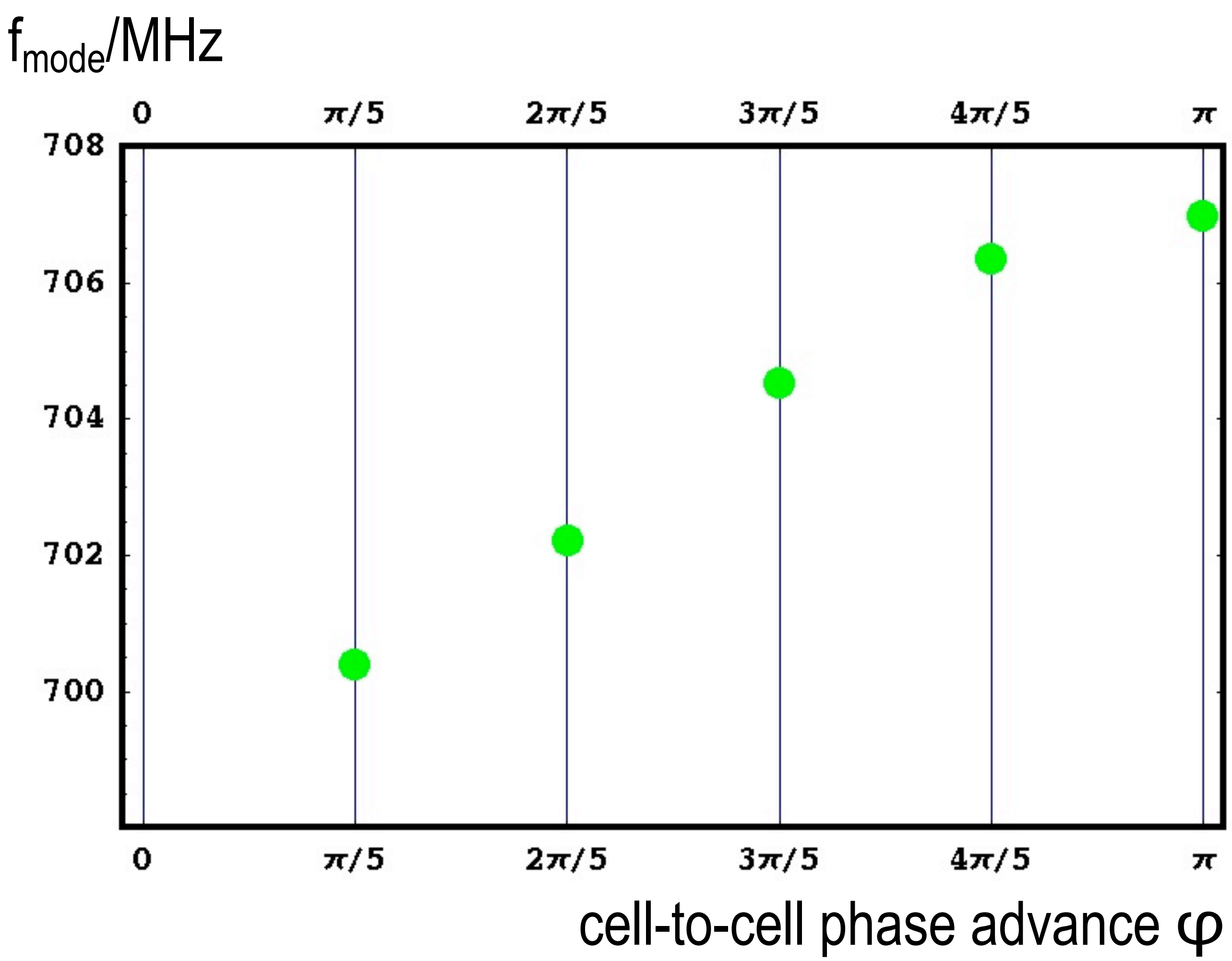


TM_{010}-4π/5
706.34471 MHz



TM_{010}-π
706.97466 MHz





In fact:

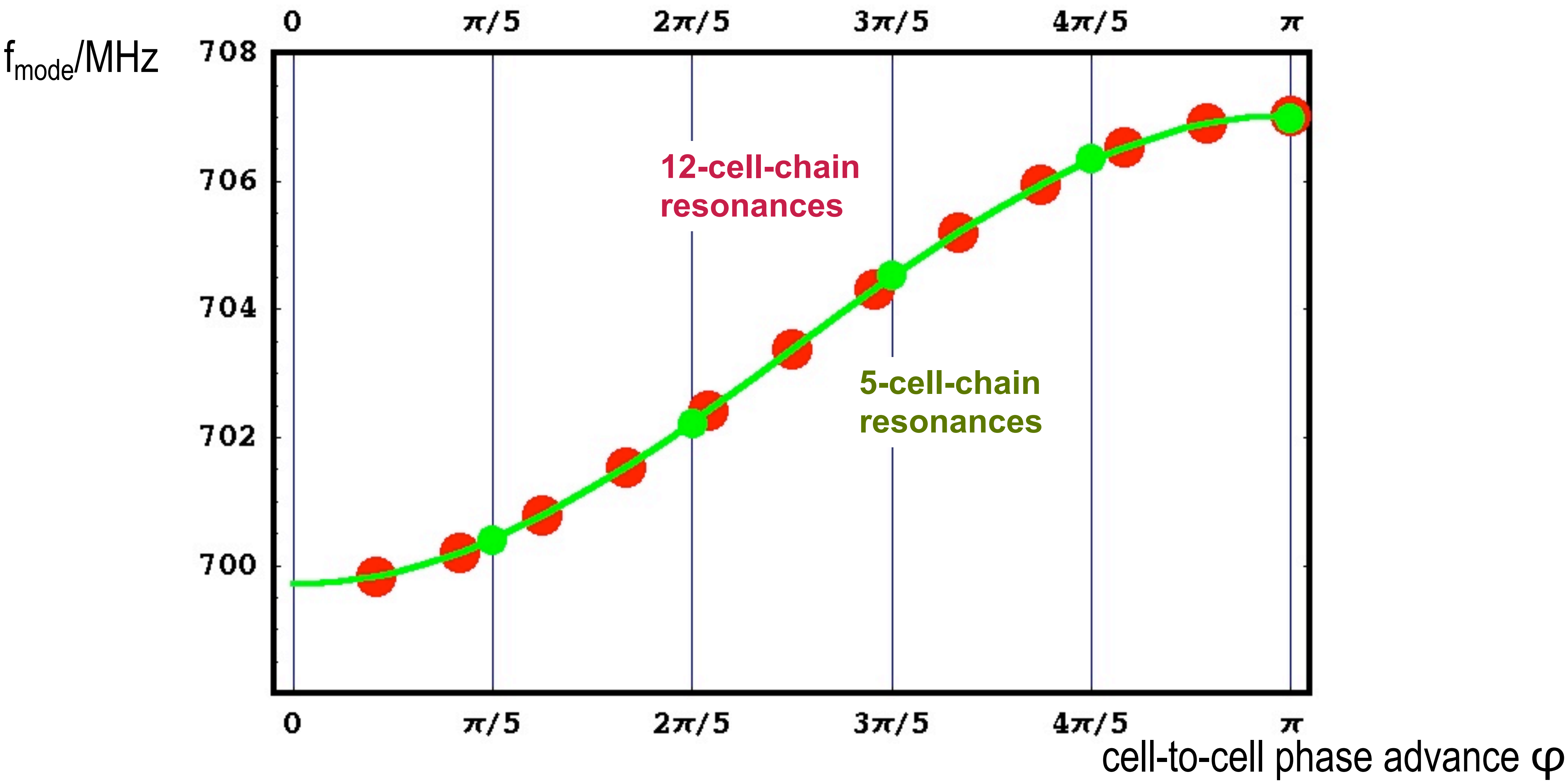
$$f_{mode} \approx \frac{f_0 + f_\pi}{2}\left[1 - \frac{\kappa_{cc}}{2}\cos(\varphi)\right]$$

κ_{cc} : cell-to-cell coupling

So, what are passbands?

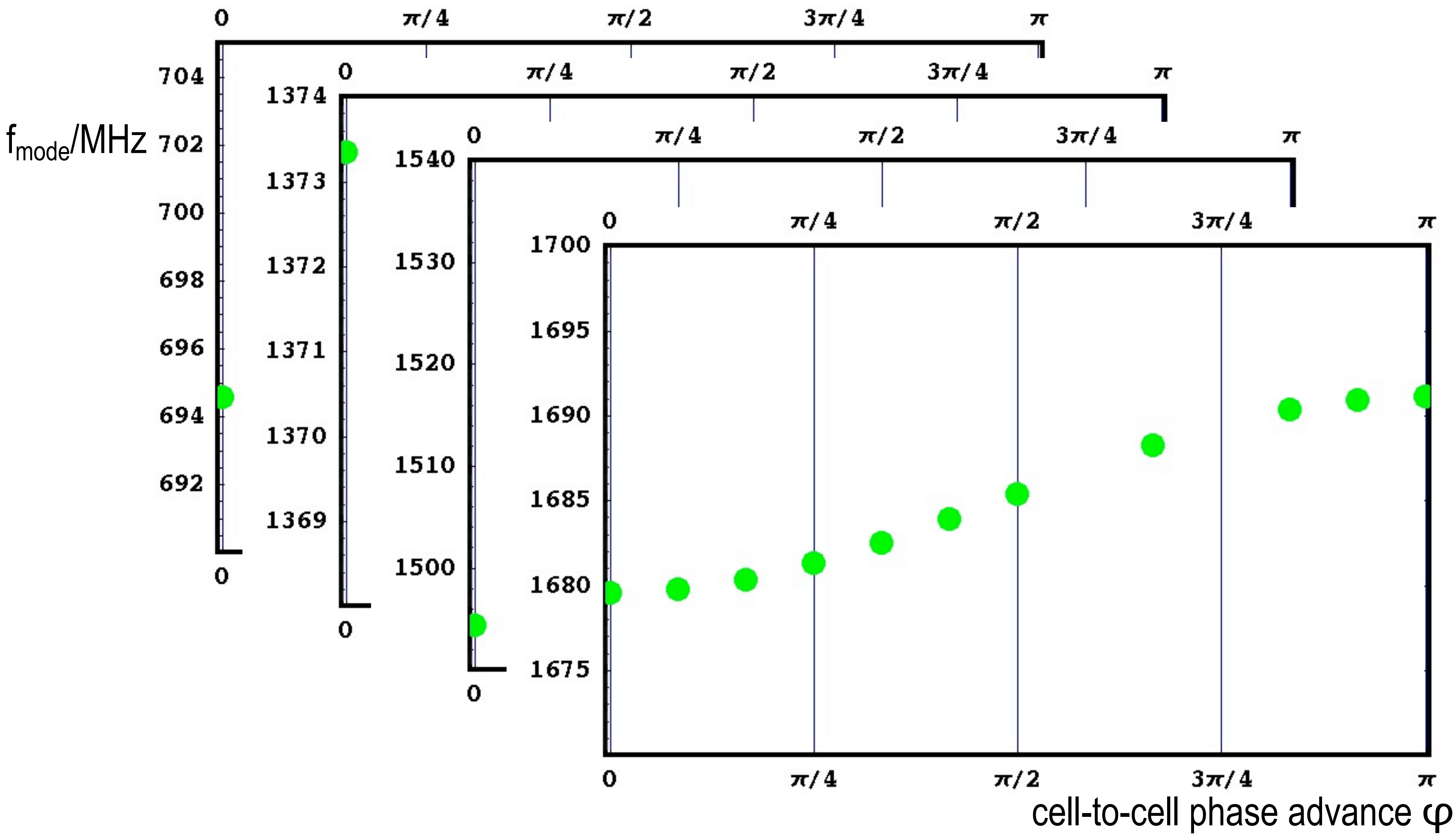
Cavities build up as chains of *identical cells* show resonances in certain frequency intervalls, called passbands, *determined only by the shape of the elementary cell*.

The distribution of resonances in the band depends on the number of cells in the chain:



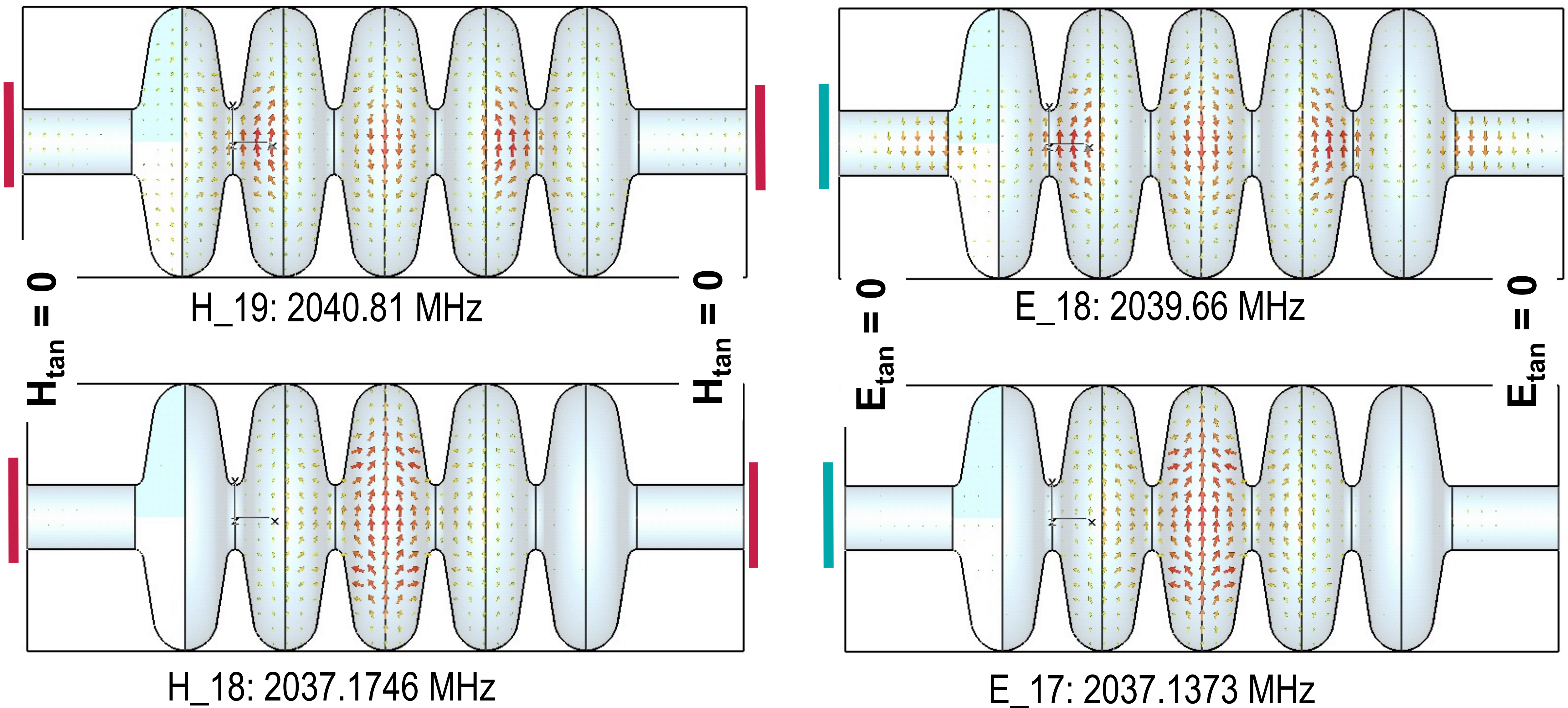
But there is an infinite number of passbands

(here computed as single cell with periodic boundaries, some phases missing)



Numerical trapped mode analysis

Search for strongly confined field distributions by simulating same structure with different waveguide terminations at beam pipe ends. Compare spectra! Small frequency shifts indicate weak coupling.

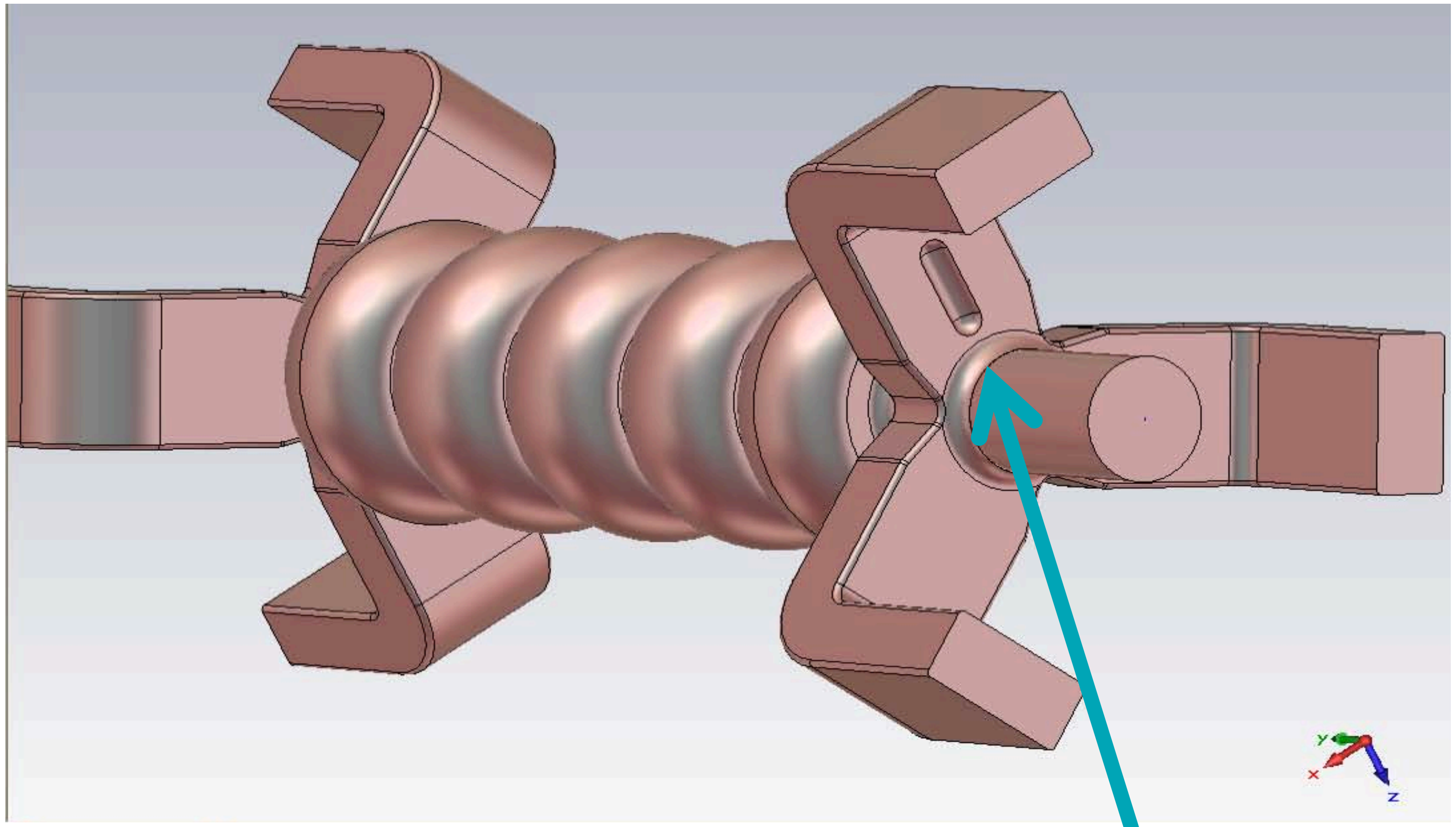


Remark: TE_{11}-cut off of beam pipe at 1953 MHz

A real-world example

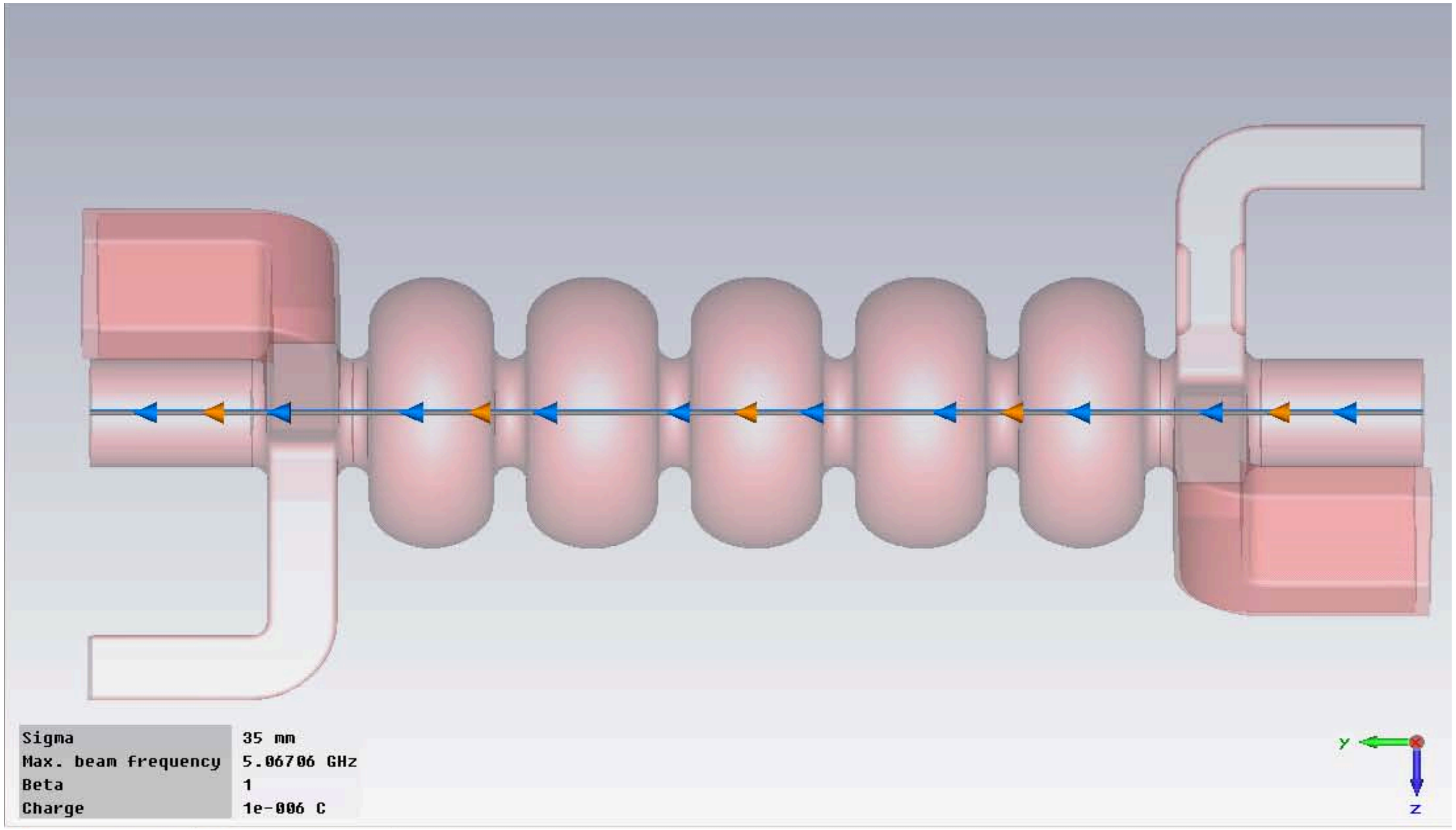
JLAB-5-cell 1.5 GHz resonator with waveguide couplers

(cavity shape and model courtesy F. Marhauser, Jefferson Lab.)



one waveguide also used as power coupler,
cross section reduction to adjust fundamental mode coupling

Direct beam-excited field computation

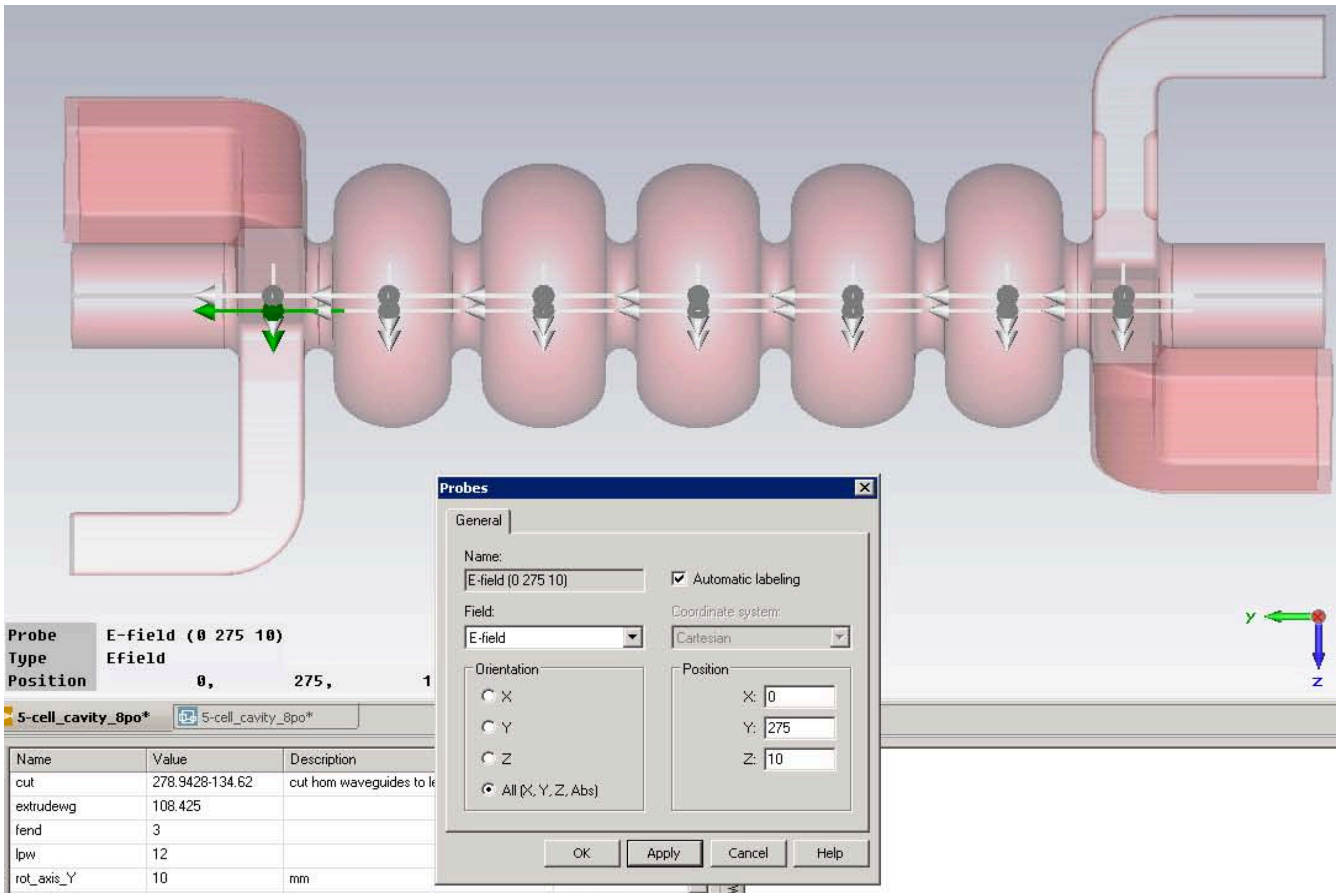


CST ParticleStudio©-simulation with 35mm-Gauss-Bunch (~ 0 ... 5 GHz, s = 0 ... 1440 m):

a) on axis => Monopol; b) off-axis (z= -10 mm) => dipole

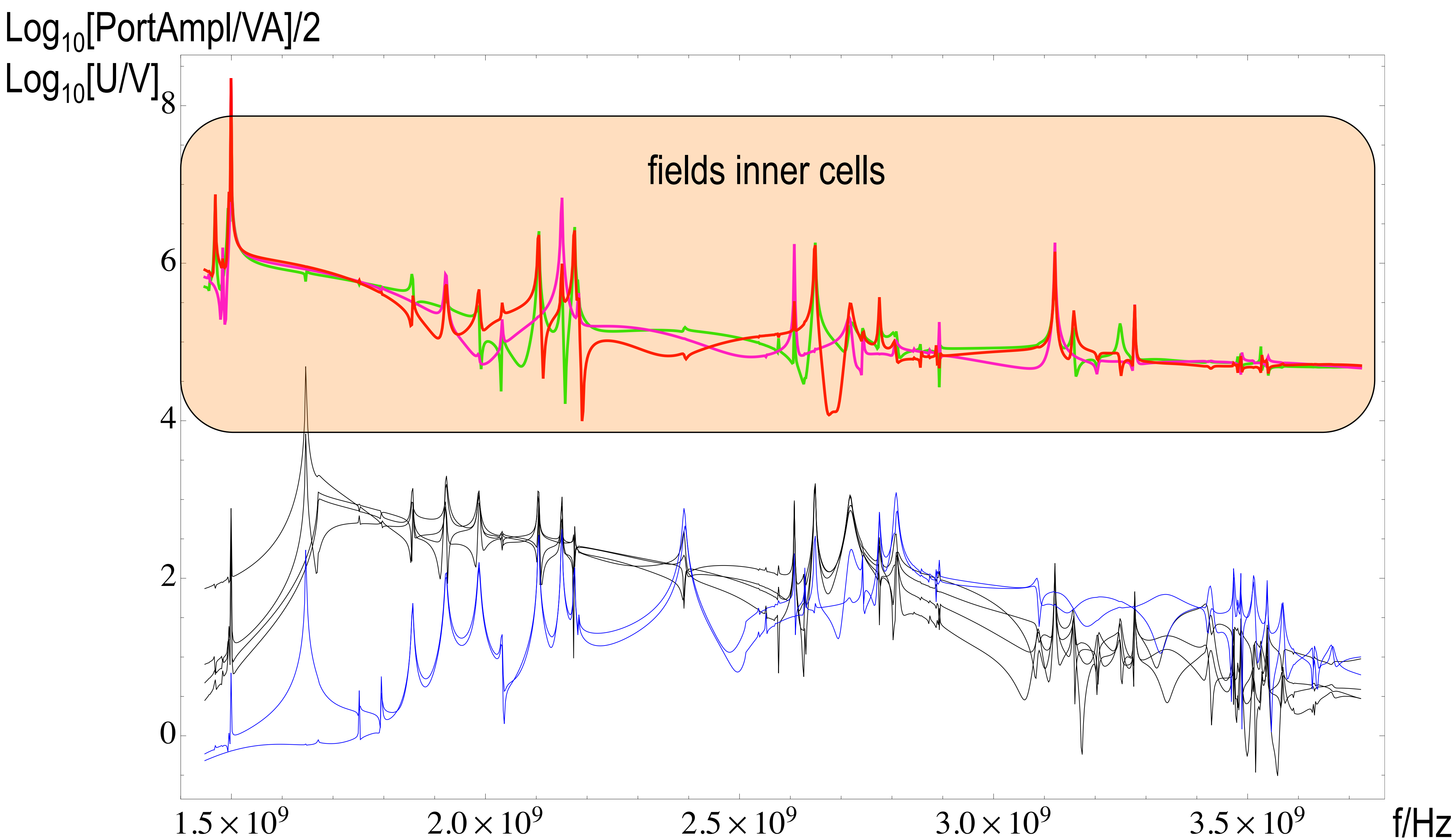
=> port-signalse(t), ...

... and additionally E-field-probes ...



... in all cells and both coupler sections => try to identify localized fields

Port-/probe signals with off-axis-beam after Fourier transform



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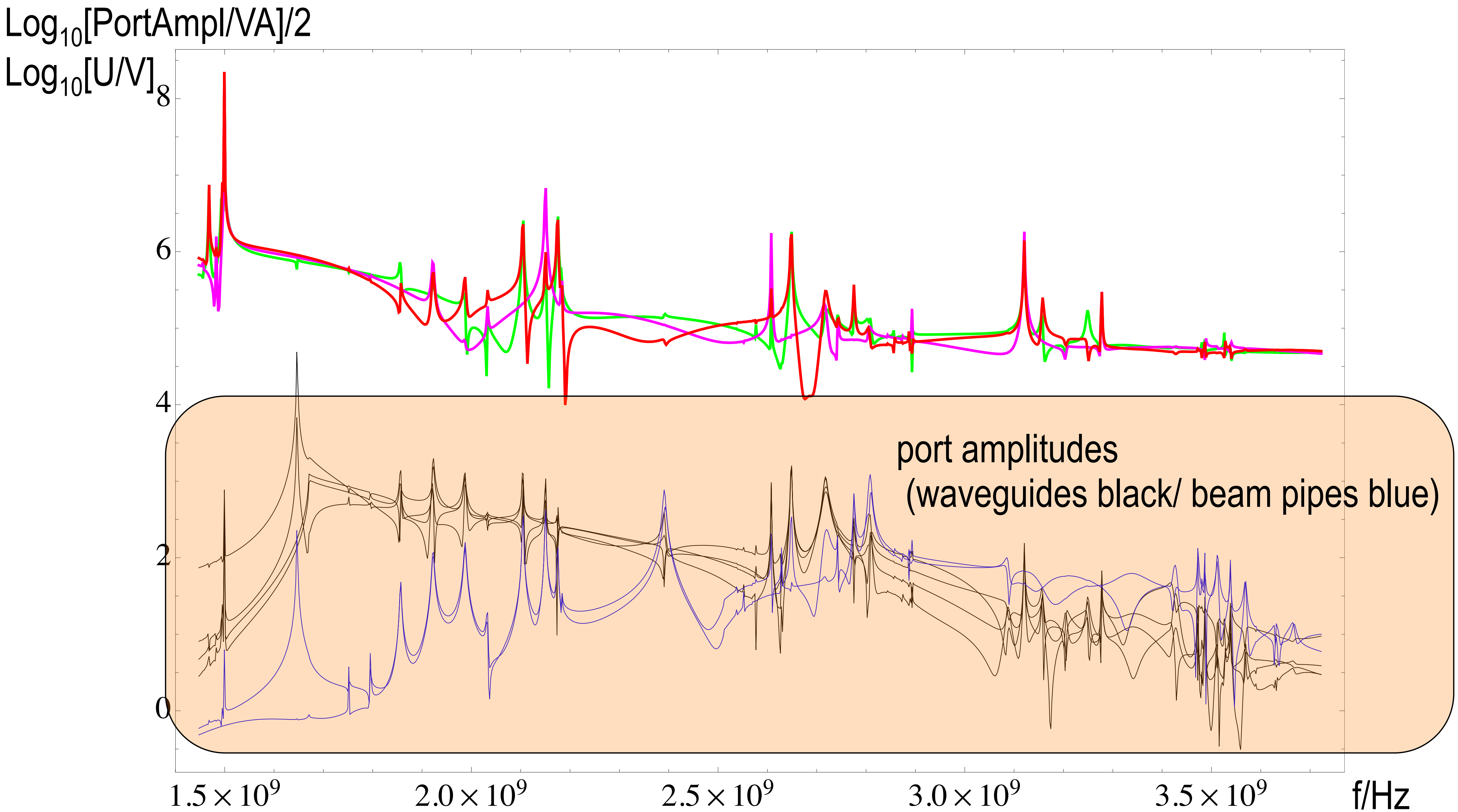
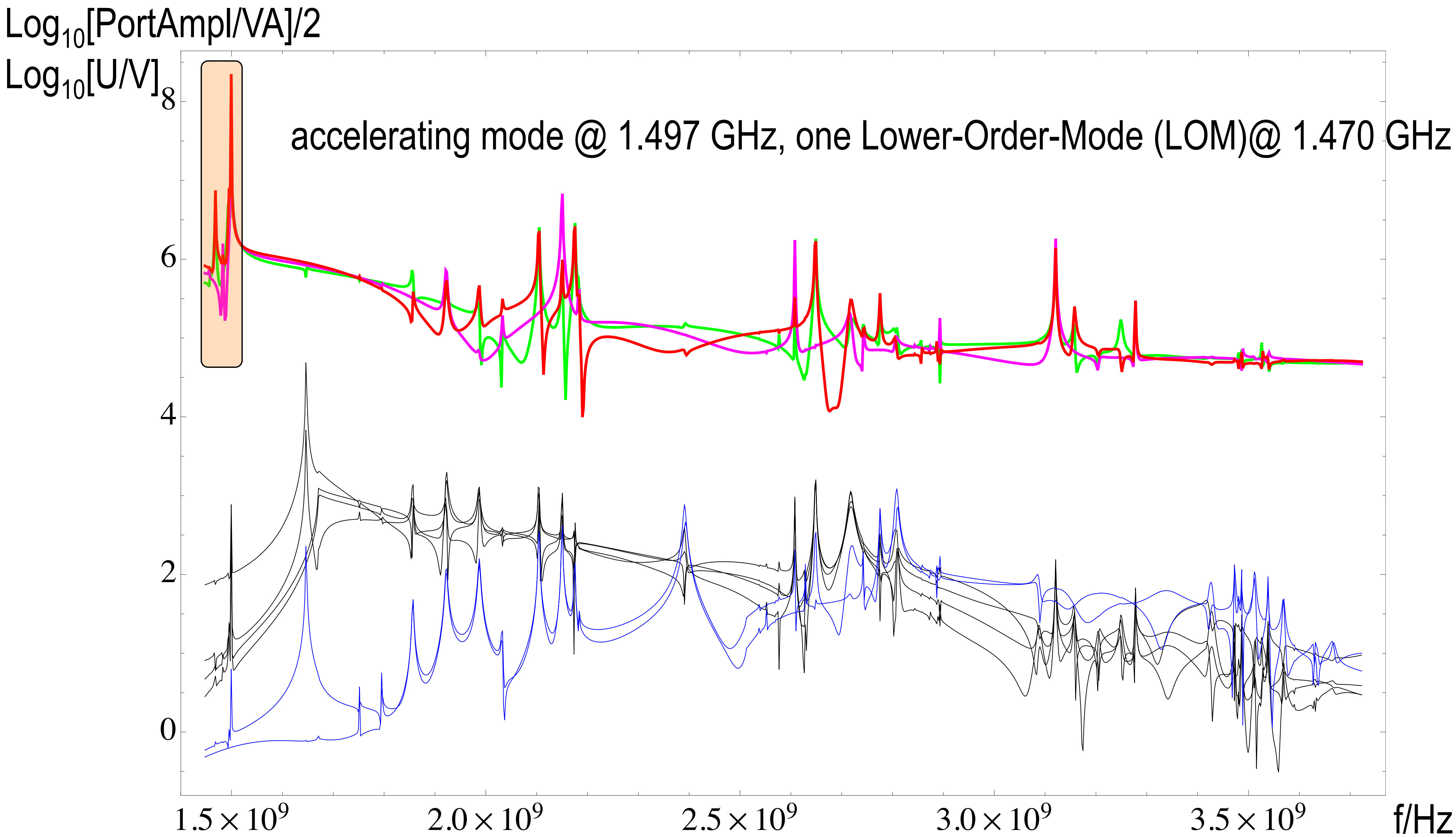
Port-/probe signals with off-axis-beam after Fourier transform



Port-/probe signals with off-axis-beam after Fourier transform



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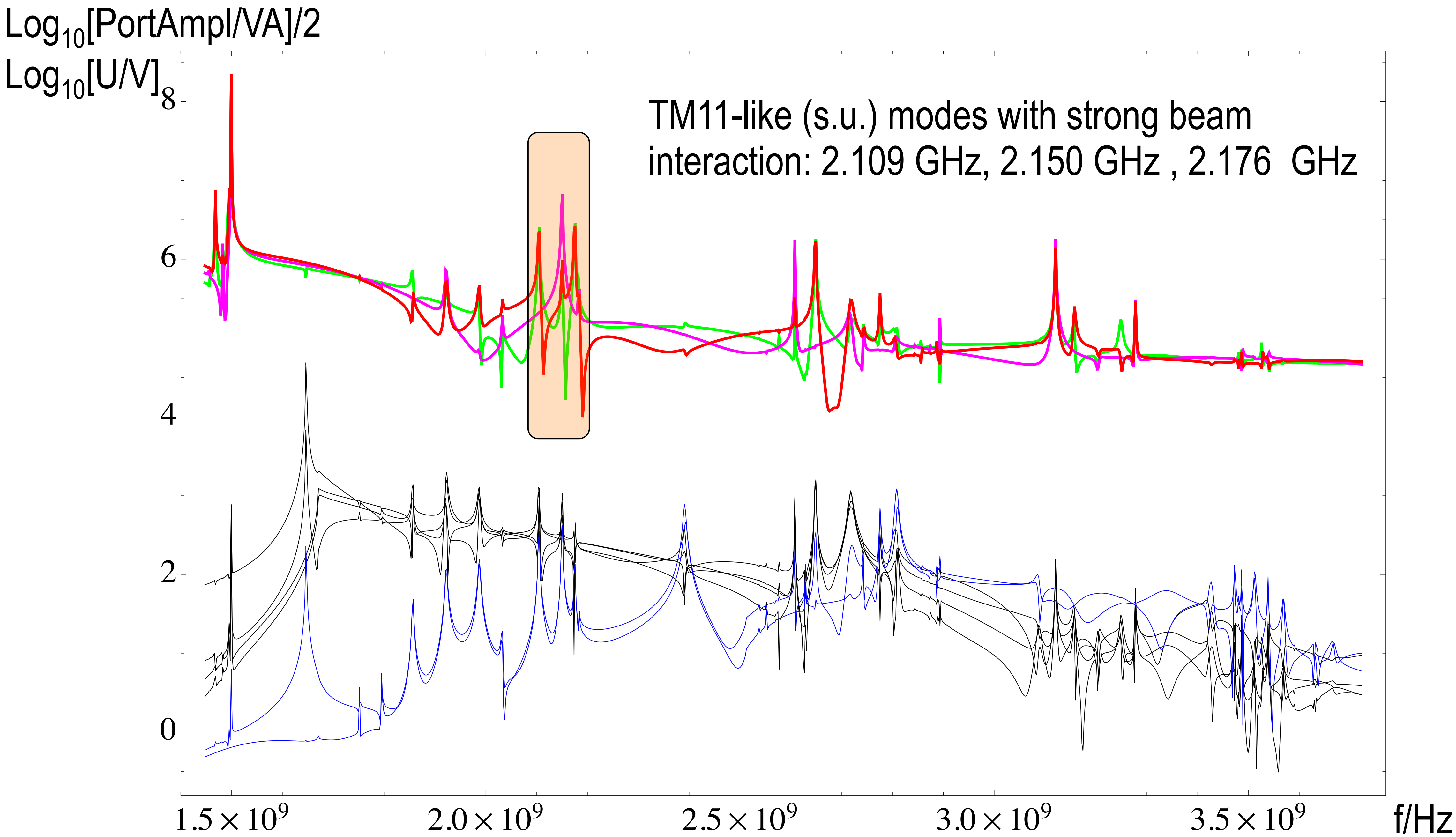
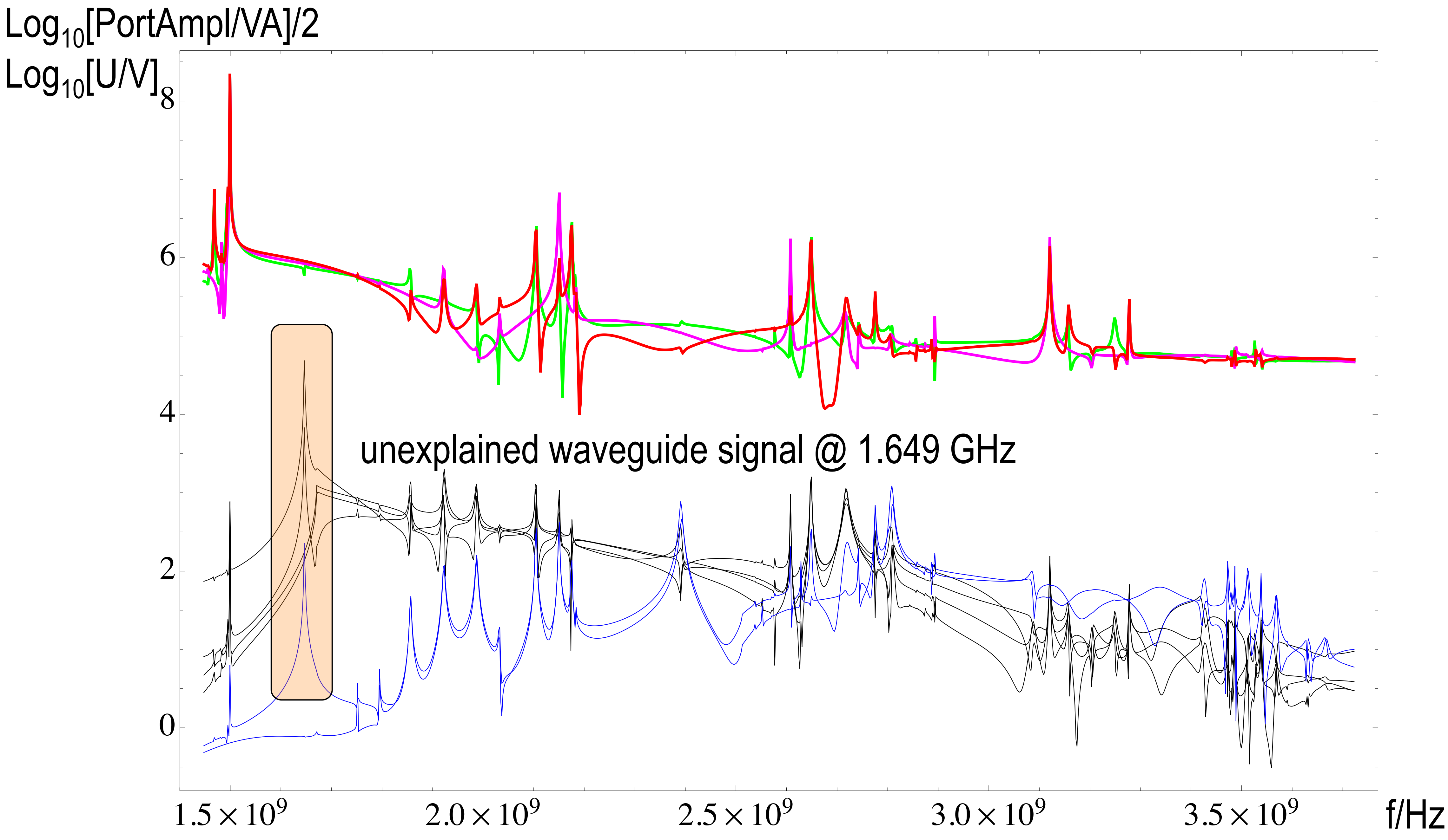
Port-/probe signals with off-axis-beam after Fourier transform



Port-/probe signals with off-axis-beam after Fourier transform



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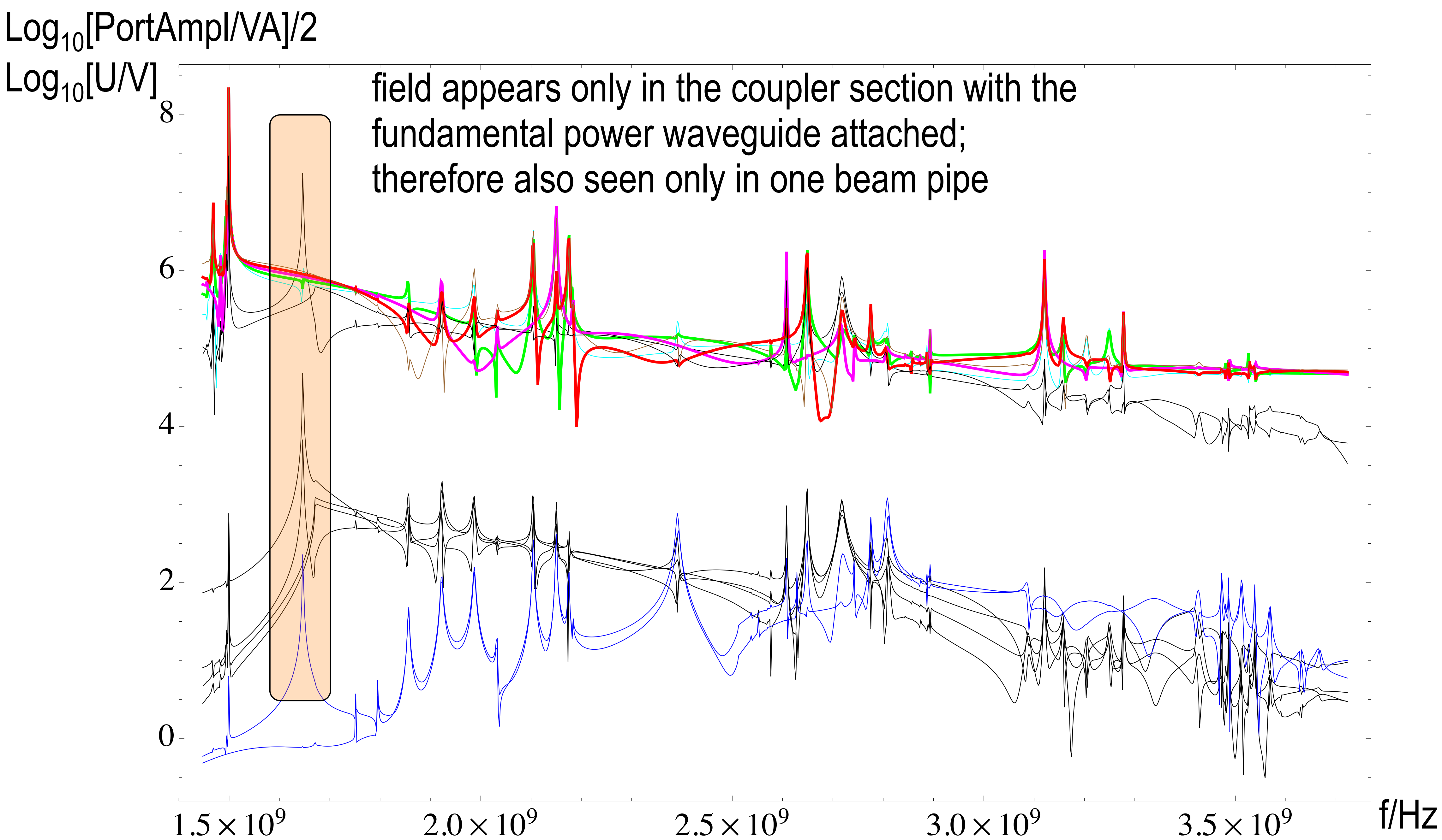
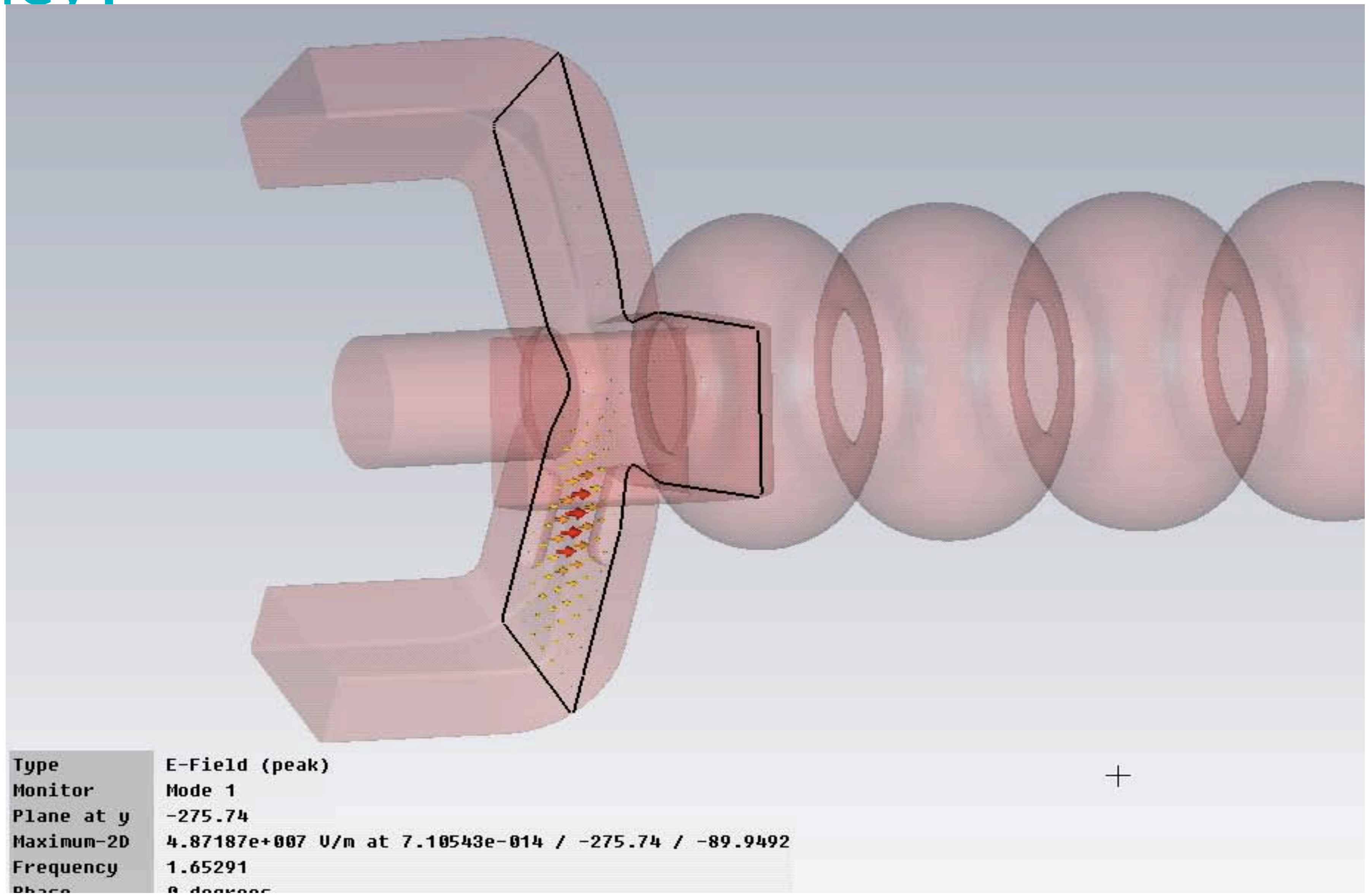
Port-/probe signals with off-axis-beam after Fourier transform

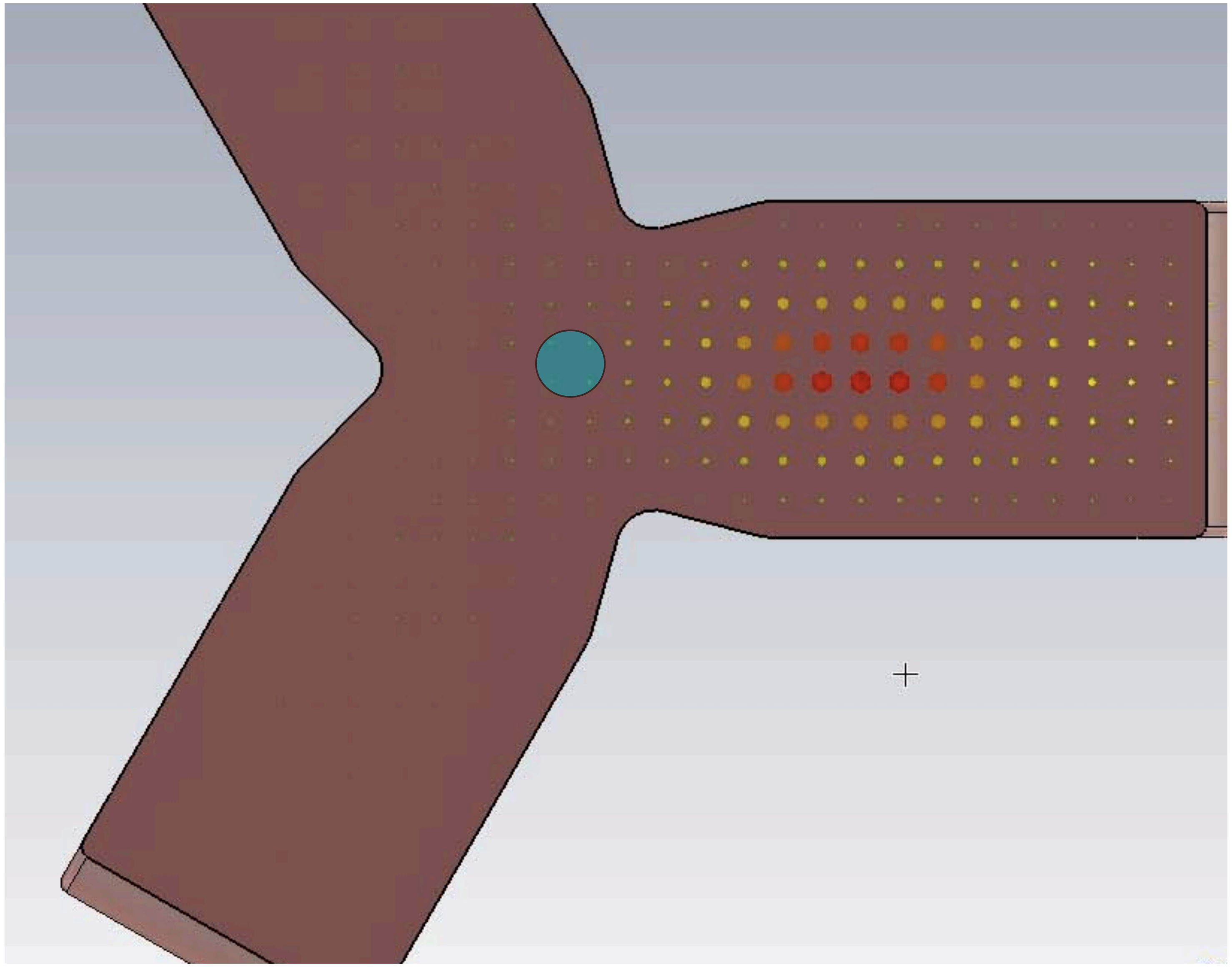


So look for eigenmodes close to the ambiguous frequency:



fields localized due to inhomogeneity in the fundamental power waveguide,
f_{res} = 1.653 GHz (expected @ 1.649 GHz)

... which shows some longitudinal E-field at the beam axis:



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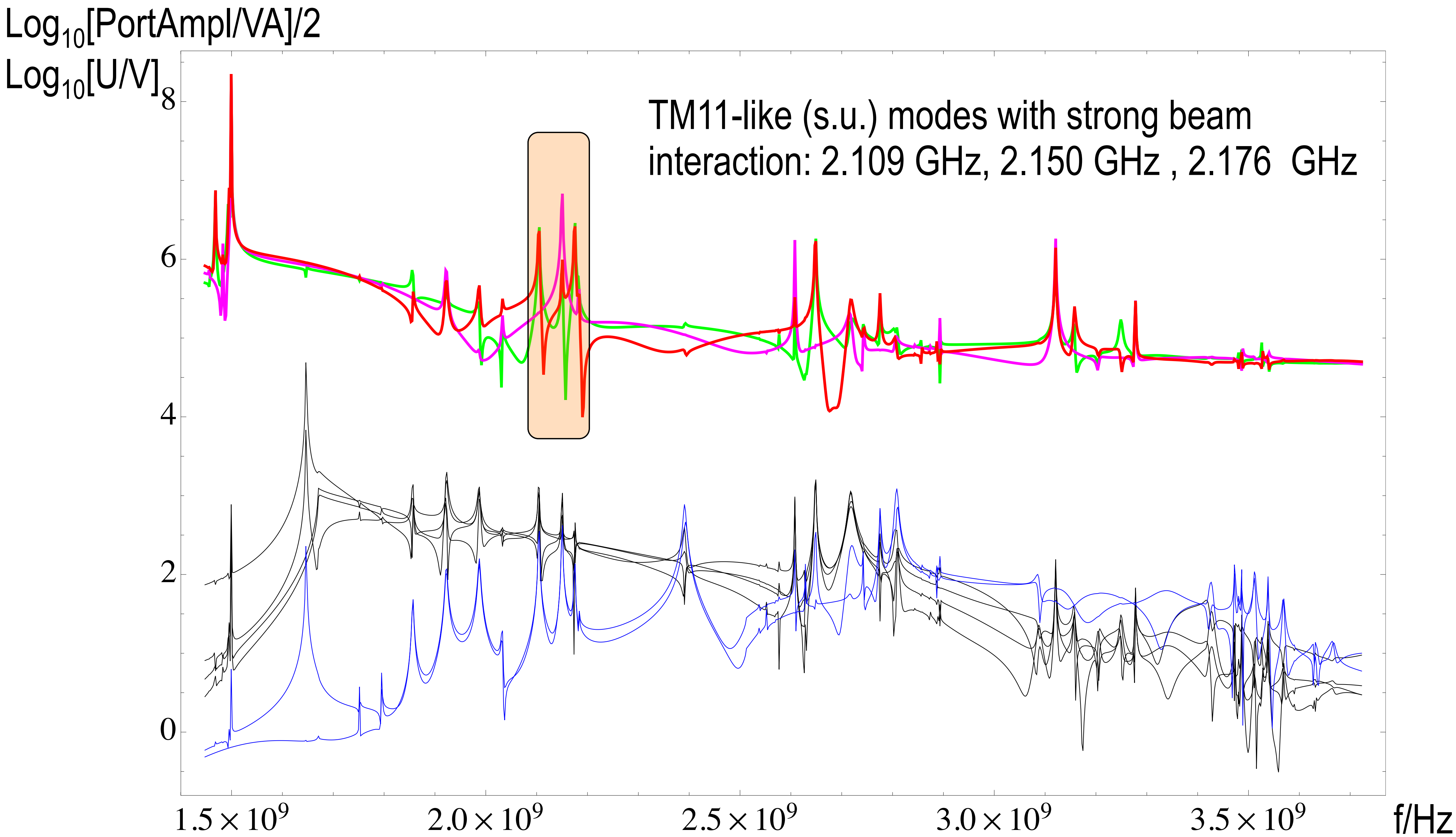
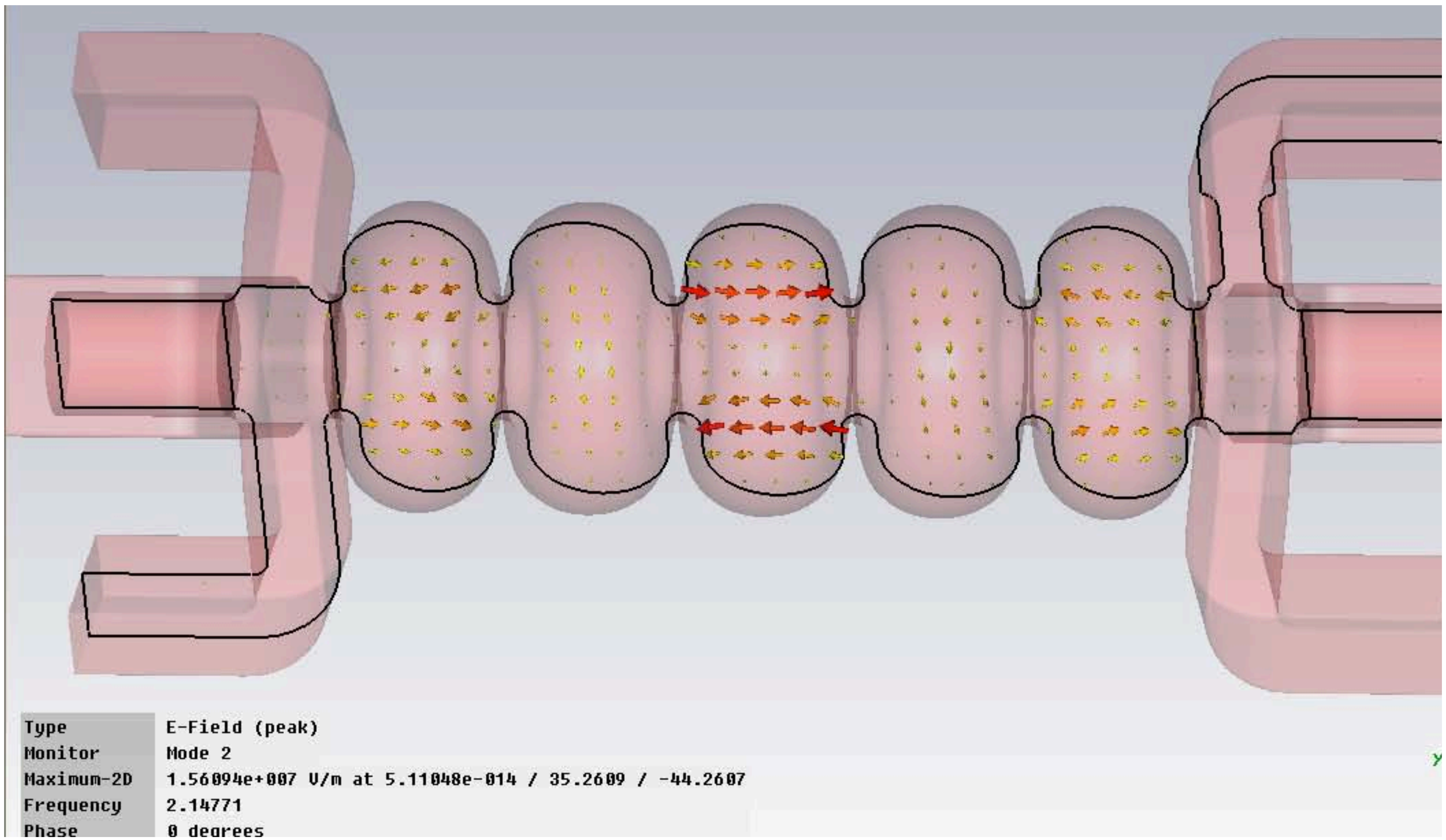
Port-/probe signals with off-axis-beam after Fourier transform

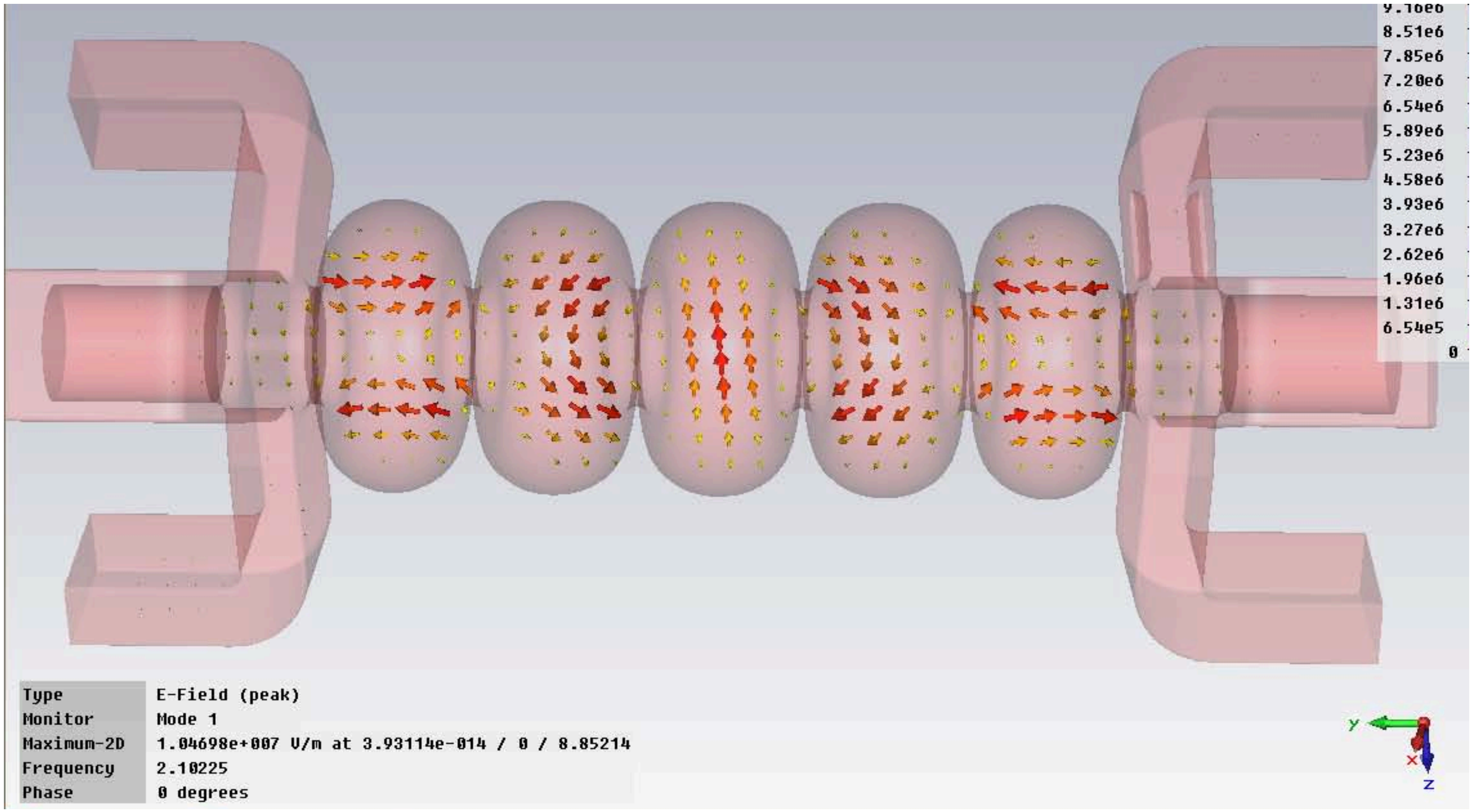


Three beam-relevant dipole modes I: f_{res} = 2.148 GHz



hybrid mode type: 1st, 3rd, 5th cell of TM_{11} character, 2nd and 4th cell TE_{11}-like

Three beam-relevant dipole modes II:f_{res} = 2.102 GHz



hybrid mode type: inner cell of pure TE_{11} character, end cells TM_{11}-like

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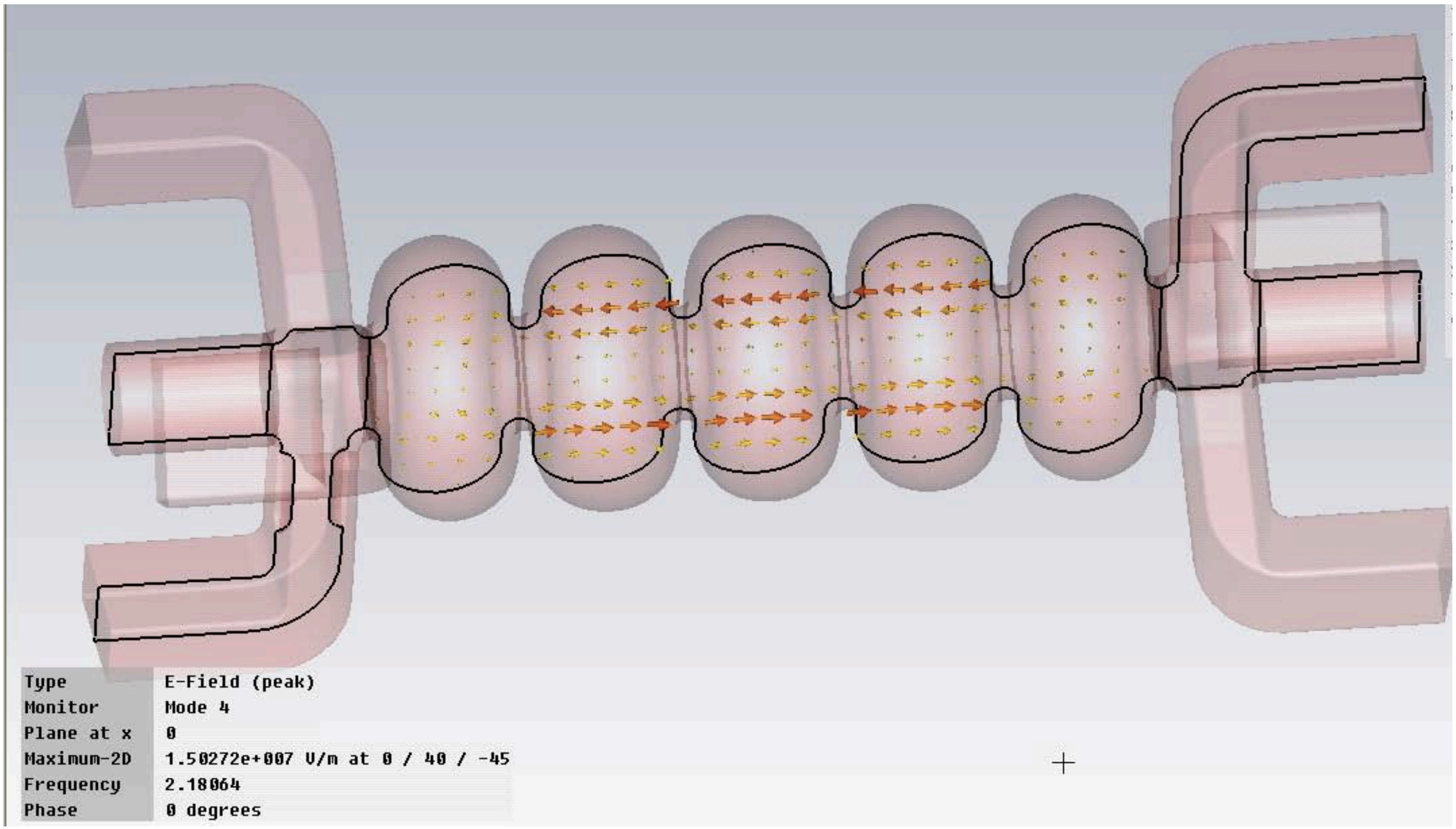
Three beam-relevant dipole modes III: f_{res} = 2.186 GHz



close to TM_{11}-0-mode, but strong unflatness because of coupler sections

Suppressing, canceling or ignoring

- Ignoring: May work, if your beam is stiff enough and your cooling power sufficient.
- Canceling: A complete field of research. Basic idea: de-cohere modes of individual cavities, making (or leaving) them slightly different.
- Suppressing - take HOM energy out of the cavity, using
  - internal absorbers
  - waveguide coupler
  - "coaxial" coupler

Q-value as a figure of merit

Starting from the common definition, putting resonance frequency f, stored field energy in the cavity W and dissipated power (averaged by a period) P in correlation:

$$Q := \frac{2\pi\ f\ W}{P}$$

one easily finds from the preservation of energy $P = \frac{dW}{dt}$

an exponential decay of W(t) following $W(t) = W(t=0)\ e^{-\frac{2\pi\ f}{Q}t}$

i.e.: The lower the quality factor Q, the faster decays a field inside the cavity.

=> If beam stability or cooling power is an issue, try to extract HOMs from the cavity

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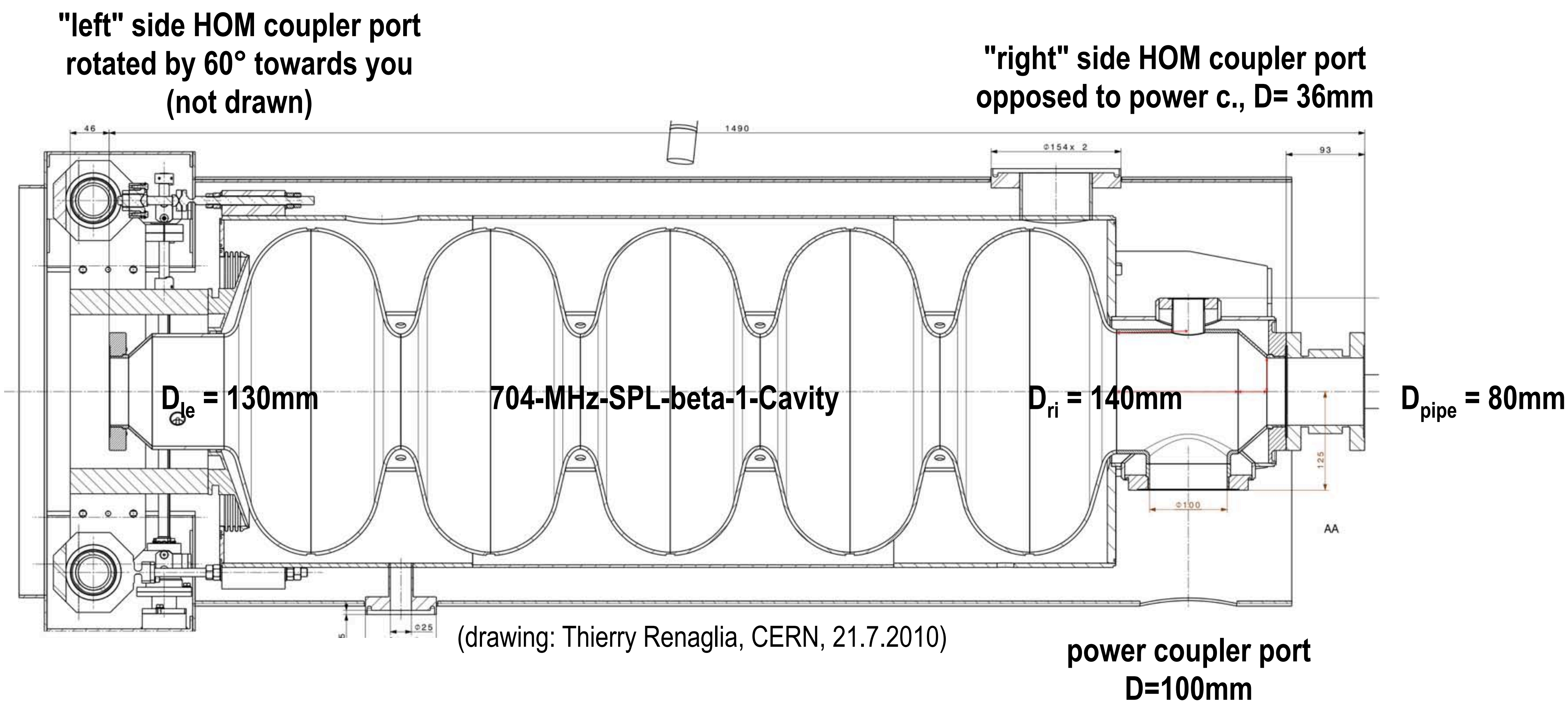
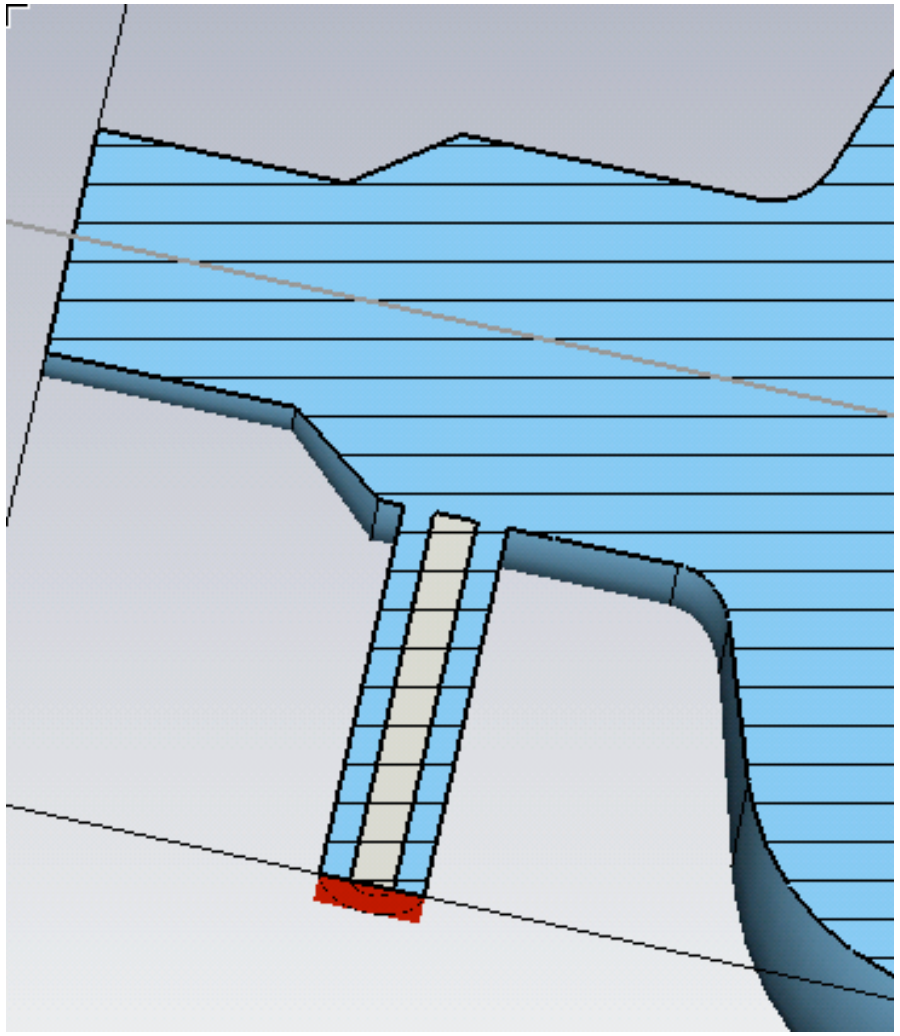
As a typical* example: SPL-5-cell cavity



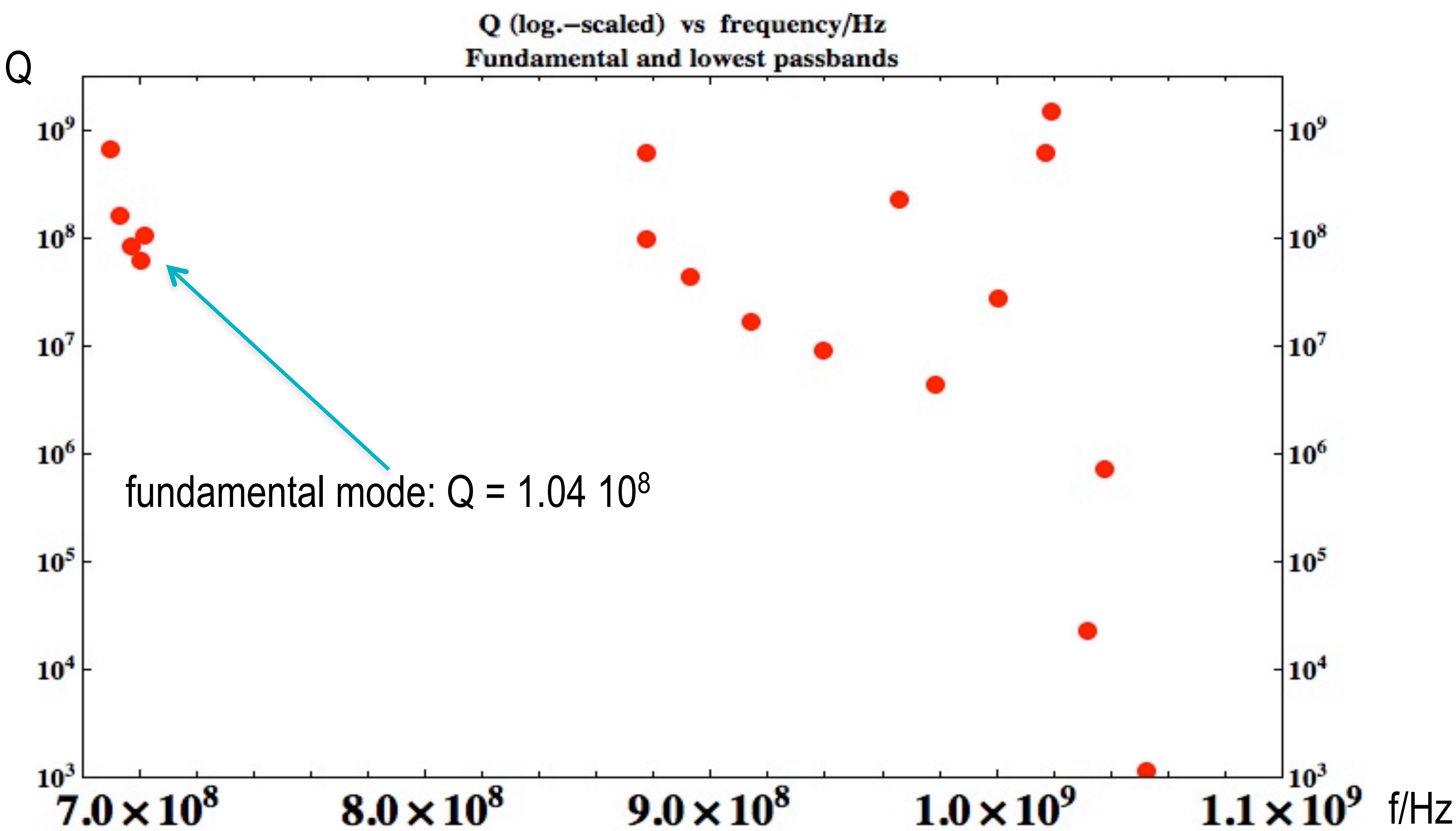
(drawing: Thierry Renaglia, CERN, 21.7.2010)

*typical: elliptical multicell, one fundamental power coupler, to HOM coupler ports at the beam pipes; different azimuthal orientation of HOM couplers

Consider such a coupler - keep it successfully simple?

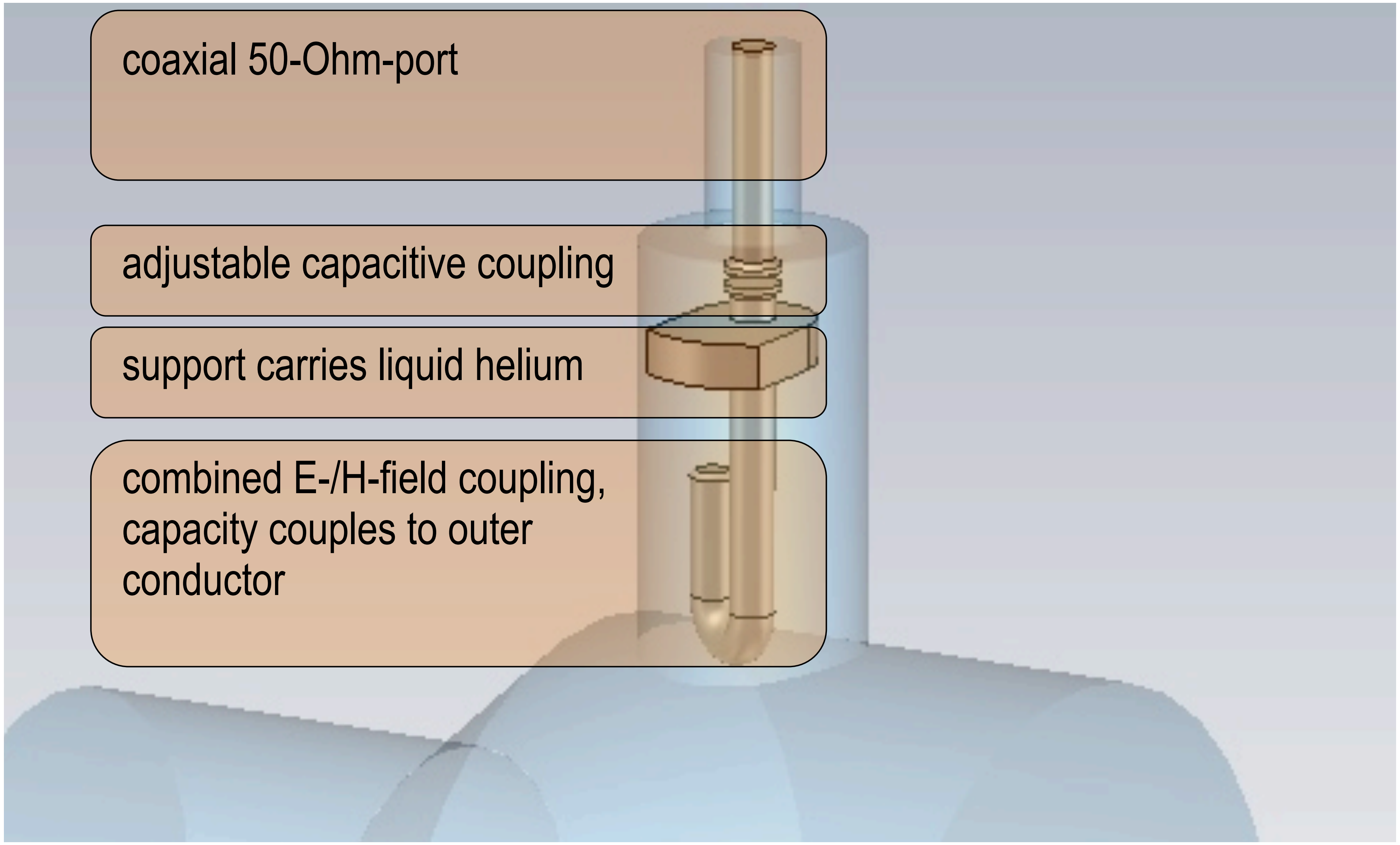


Q-value of lowest modes for 0 mm antenna depth:



Coaxial coupler *touches fundamental mode similar like HOMs*
=> extracts too much fundamental power

So we need a filter:



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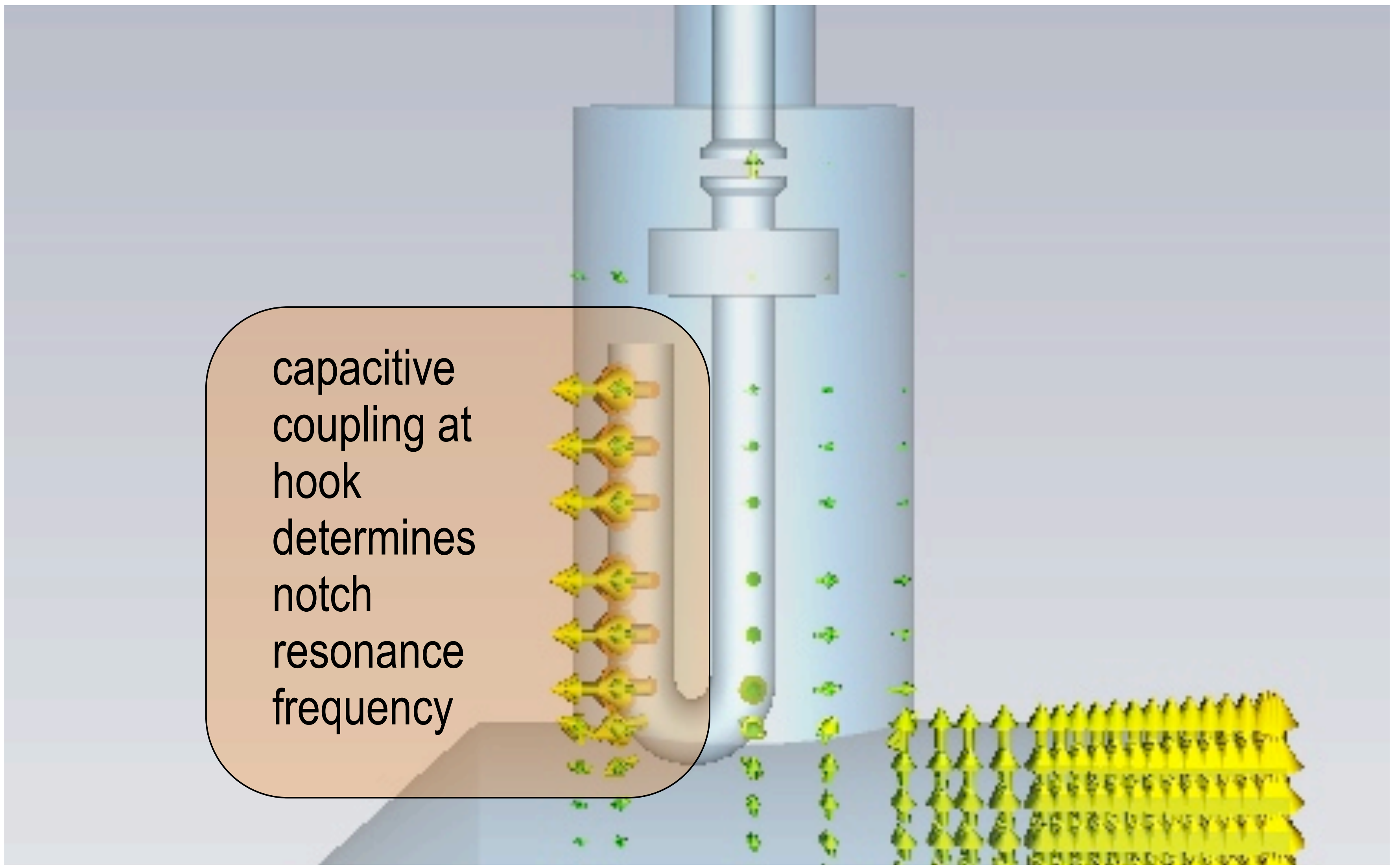
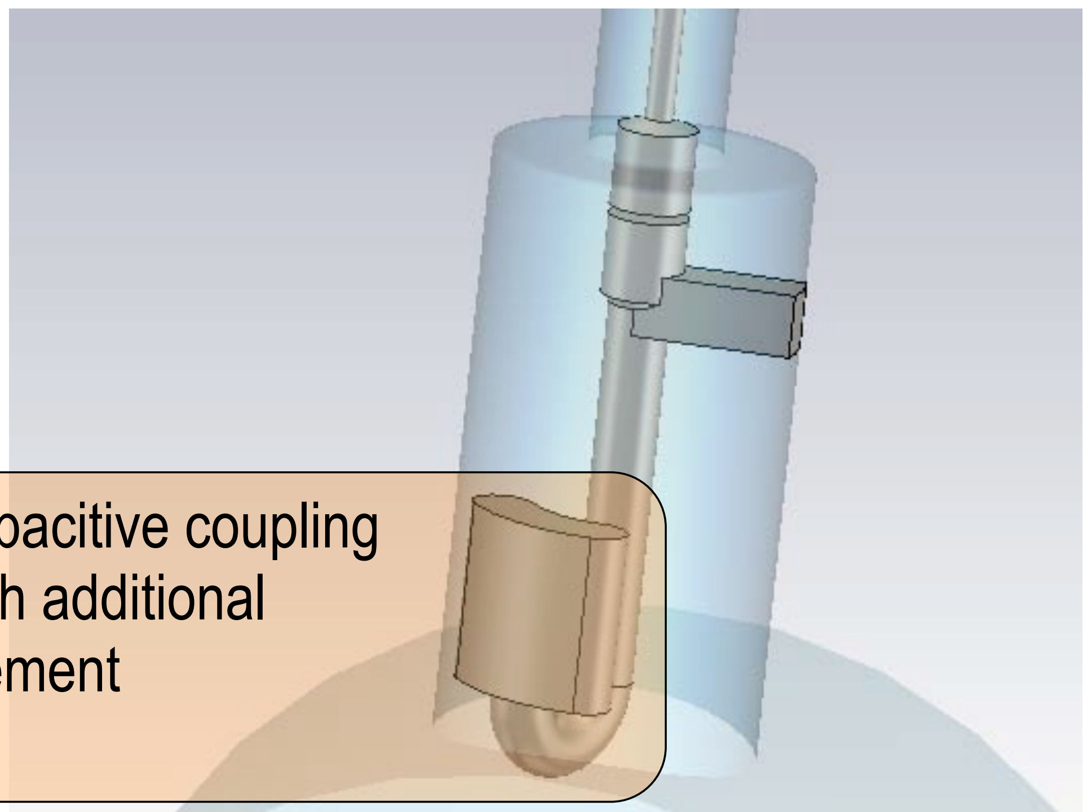
... which works like this:
E-field geometry @ 704 MHz

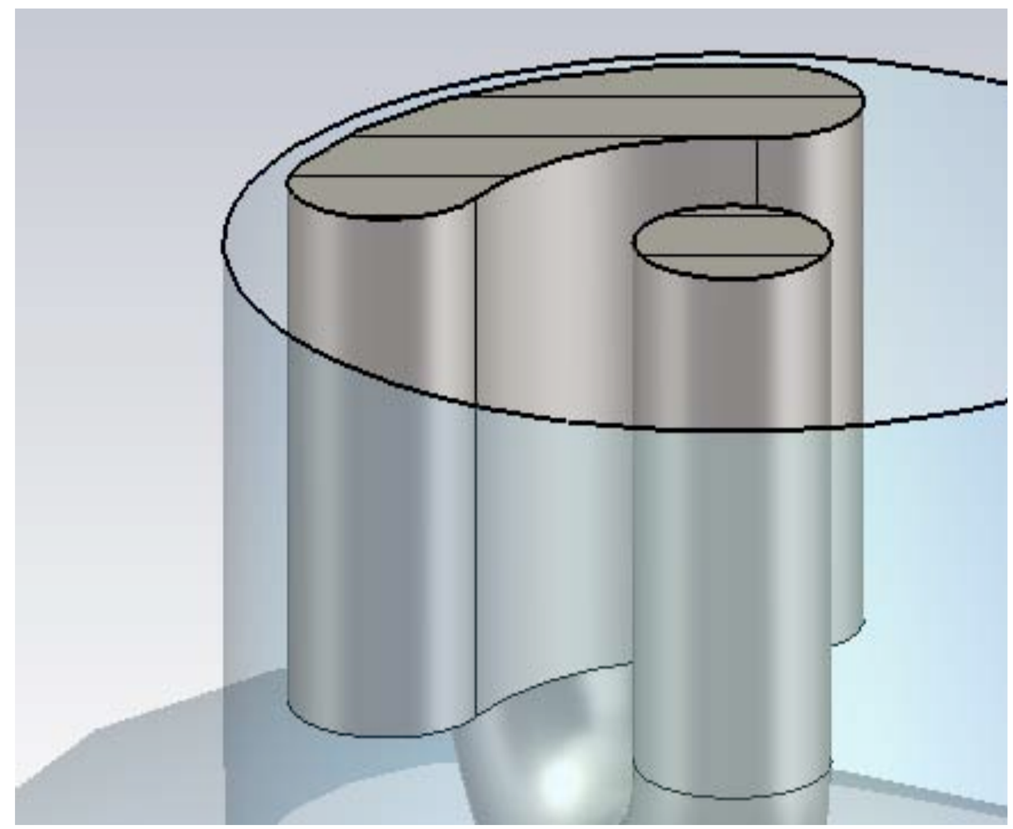


strong capacitive coupling between "hook" and outer conductor

Pure hook not tuneable for 704 MHz => Enlarge hook end capacity

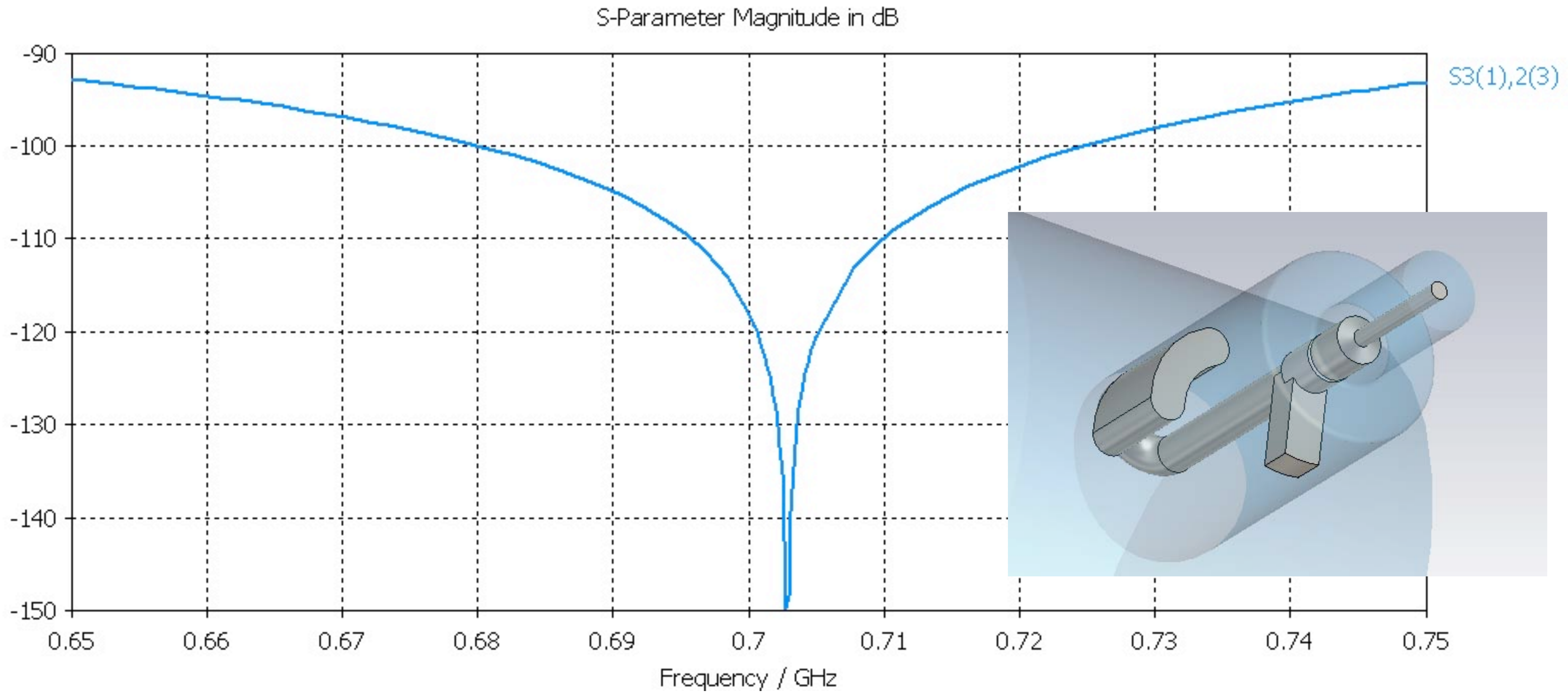


enlarge capacitive coupling at hook with additional surface element



Design inspired by LEP, TESLA-Saclay, LHC - couplers

Waveguide(TM_0)–Coax–Transmission blocked @ fundamental mode frequency => Tuning ok

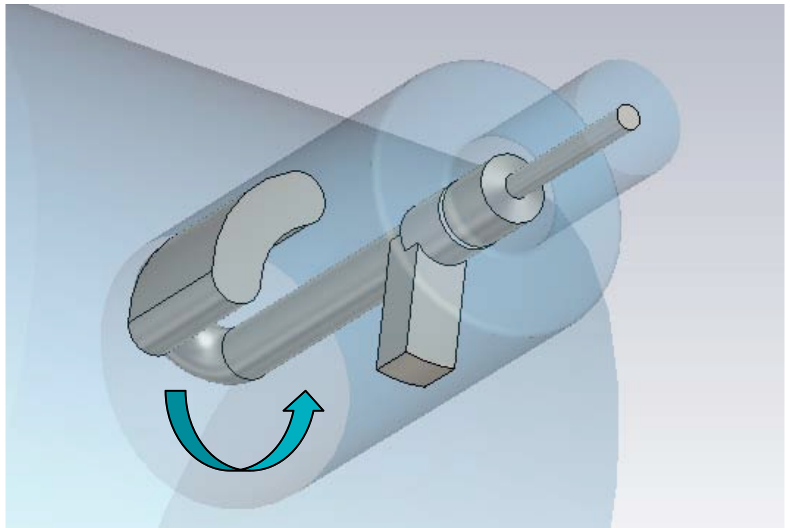


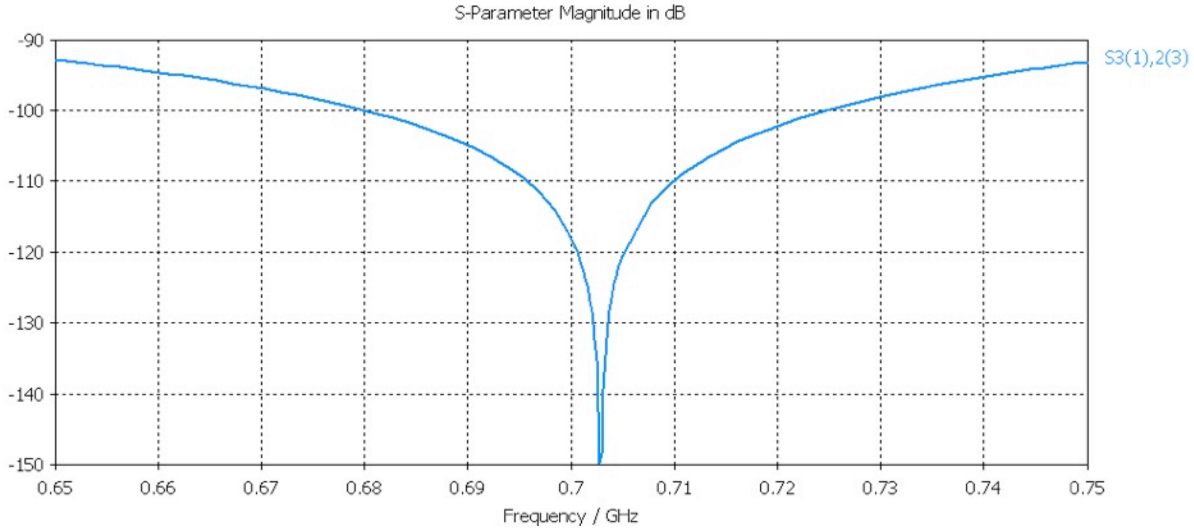
Remarks about fundamental mode notch filter:

1.) Tuning rather sensitive both against capacity surface AND rotation angle (~5 MHz/Degree ⇔ 30 dB/Degree)

2.) => notch filter understood as combination of resonance AND "directional coupler"-effect: certain E-H-correlation causes cancelation

3.) This demands for external re-tuning capability after mounting (e.g. rotation)





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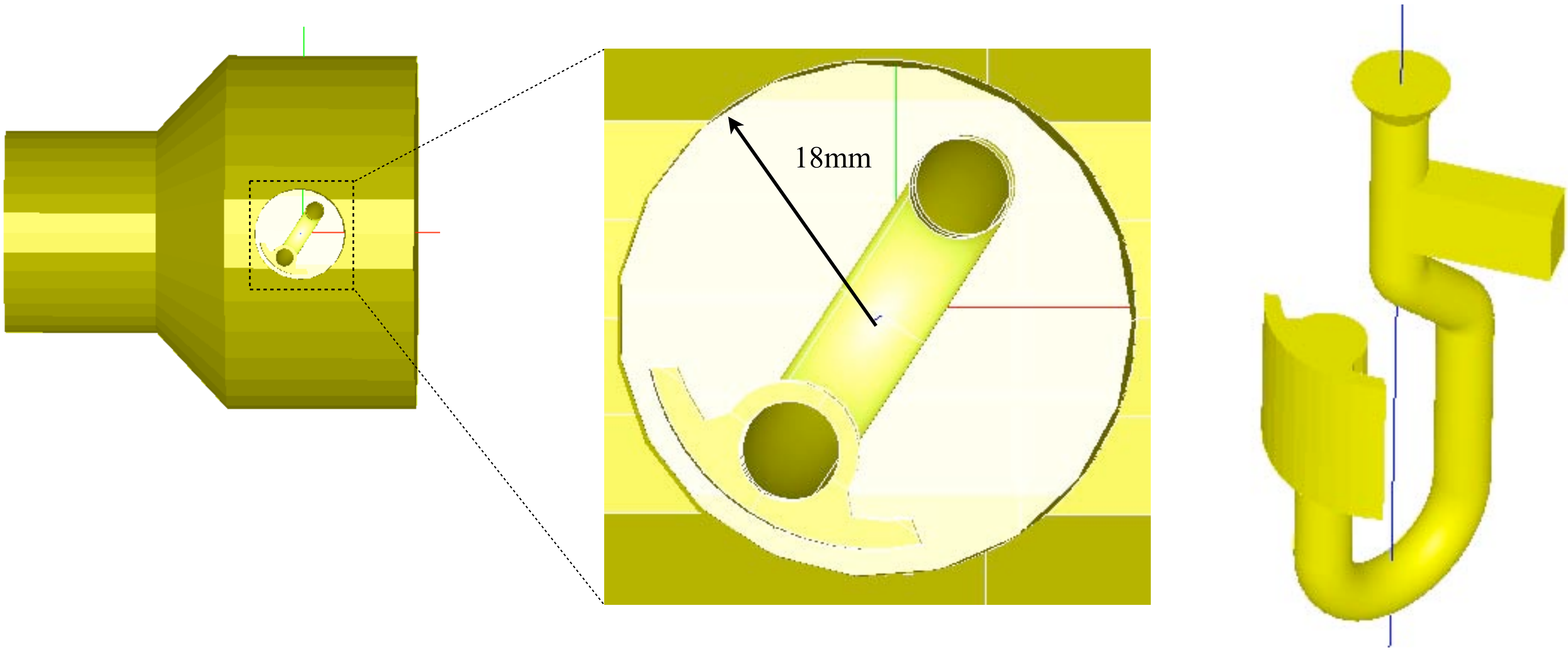
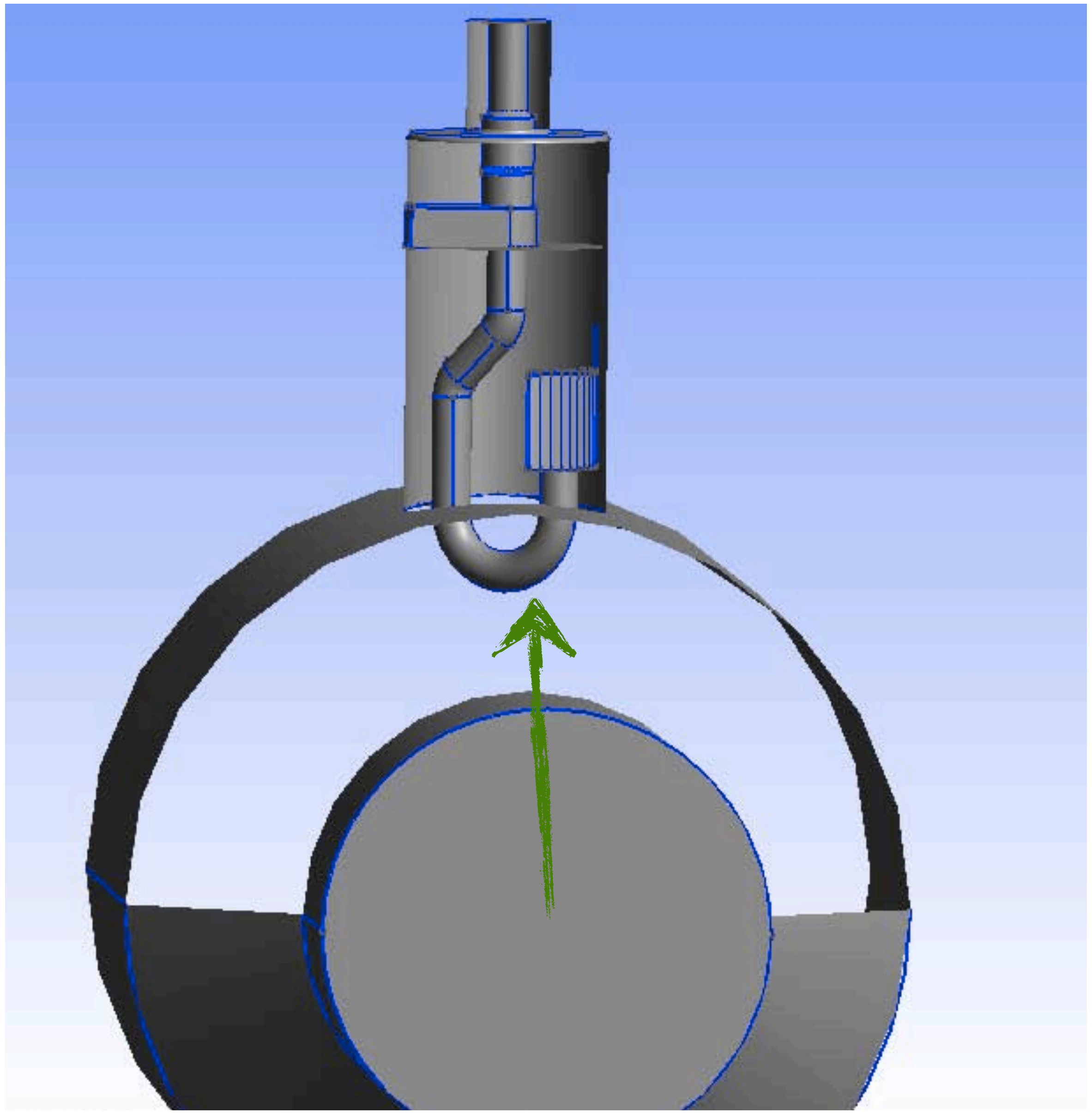
Current design for SPL HOM-coupler with increased hook



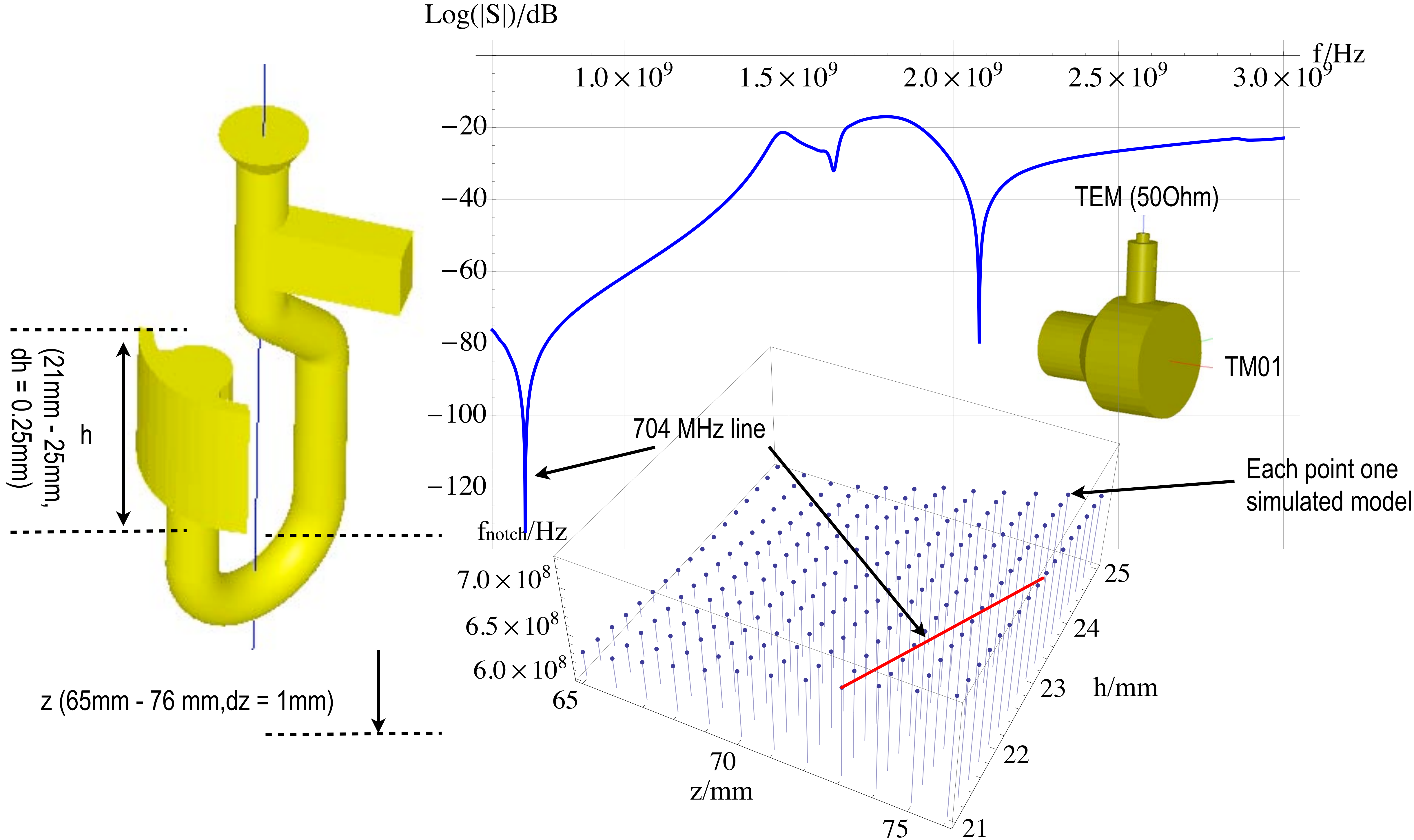
Q-analysis based on scattering properties of individual coupler section, concatenated afterwards with cavity and second coupler section.

Seen "through" the pipe



minimal **radius 53 mm**
(=> a = 70 mm)

Current Design for SPL HOM-Coupler



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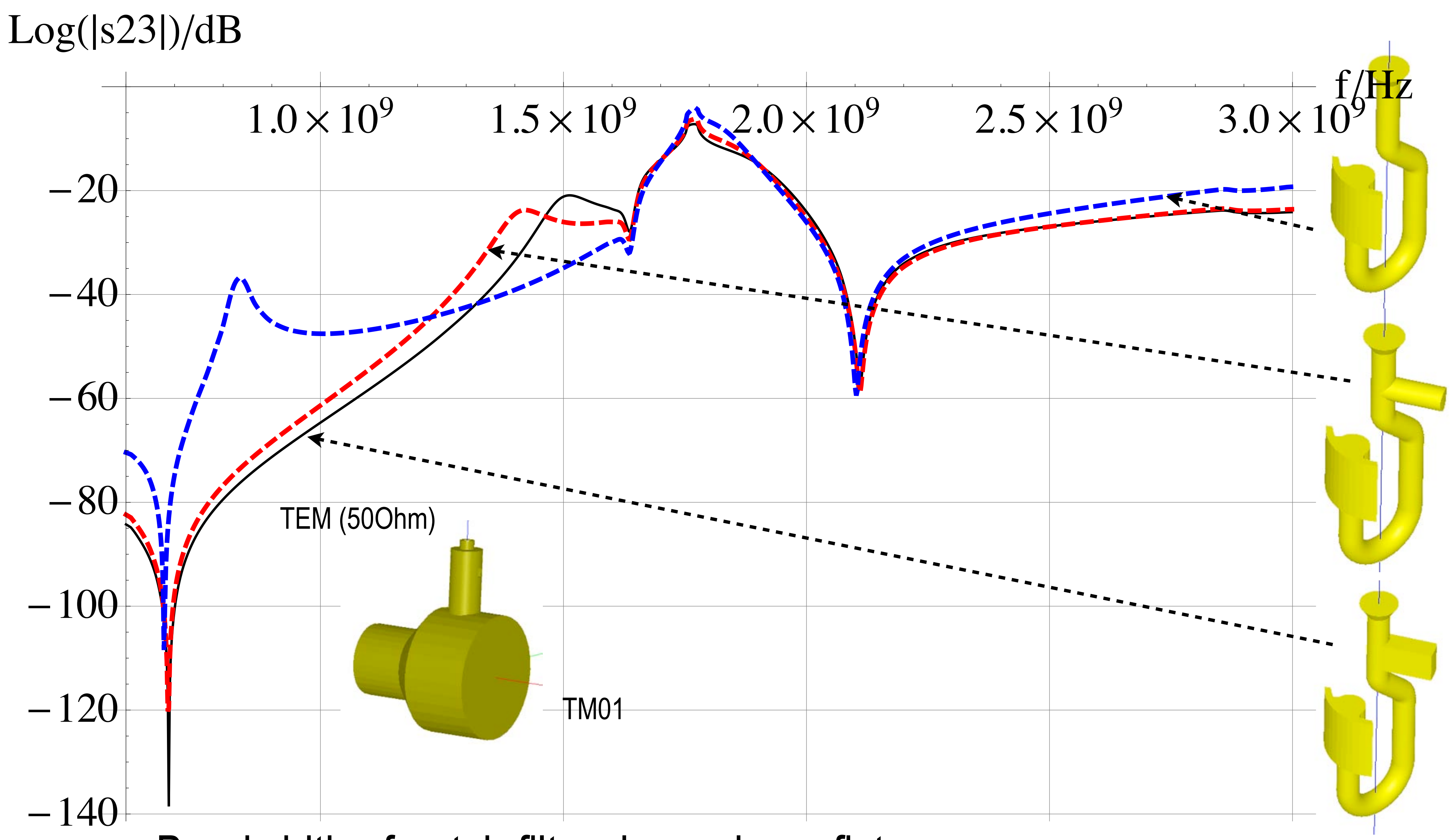
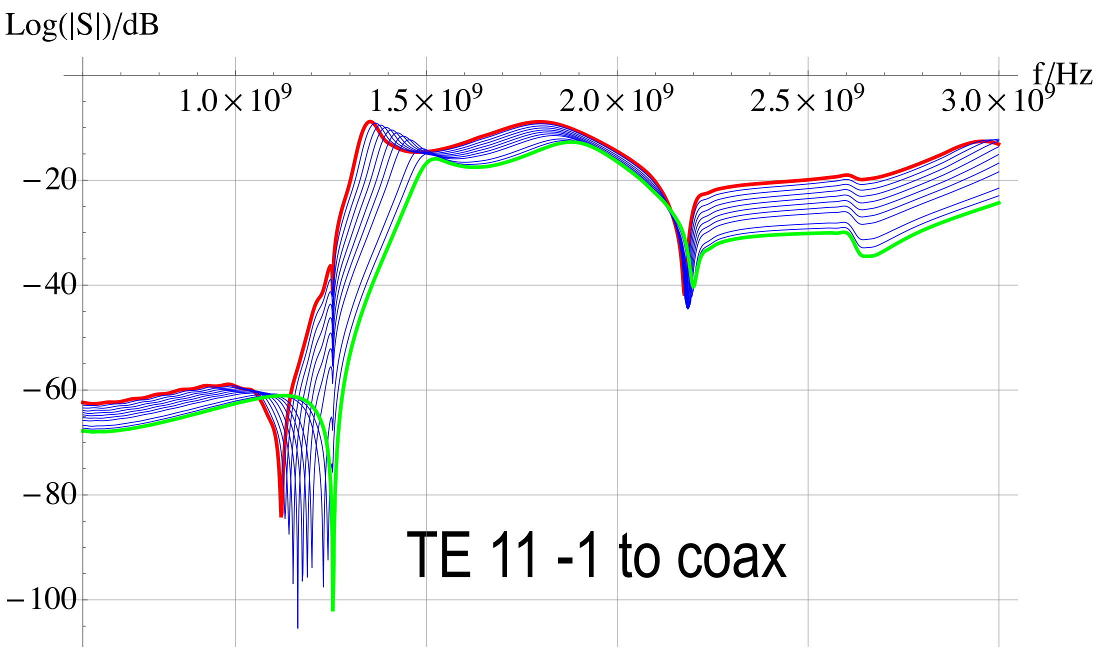
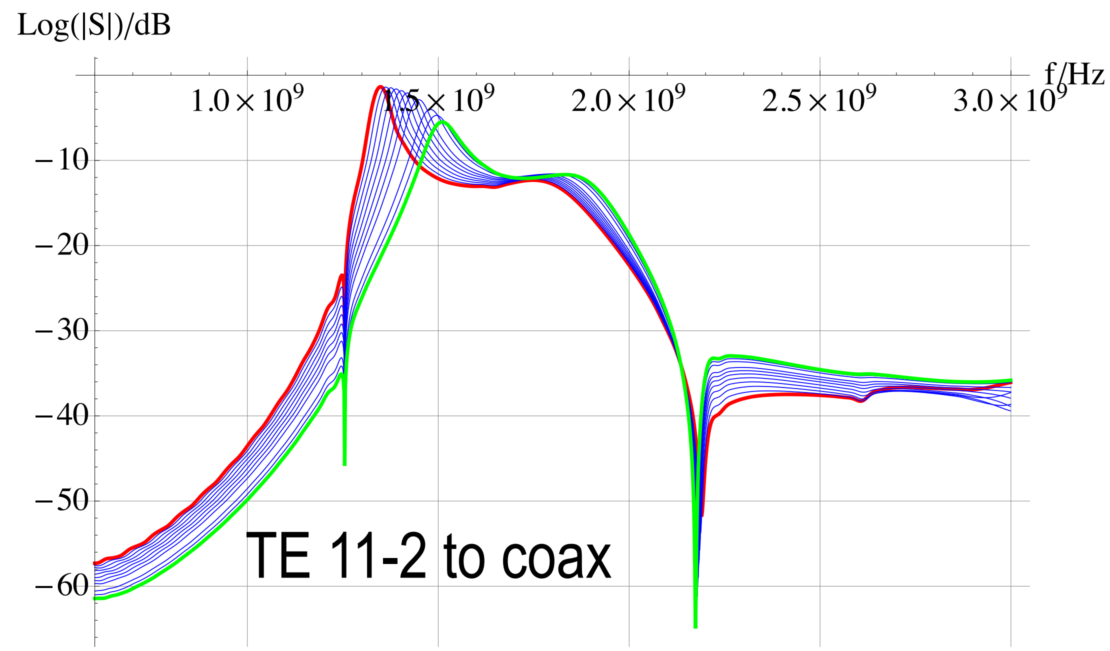
Influence of fixture shape on the RF properties

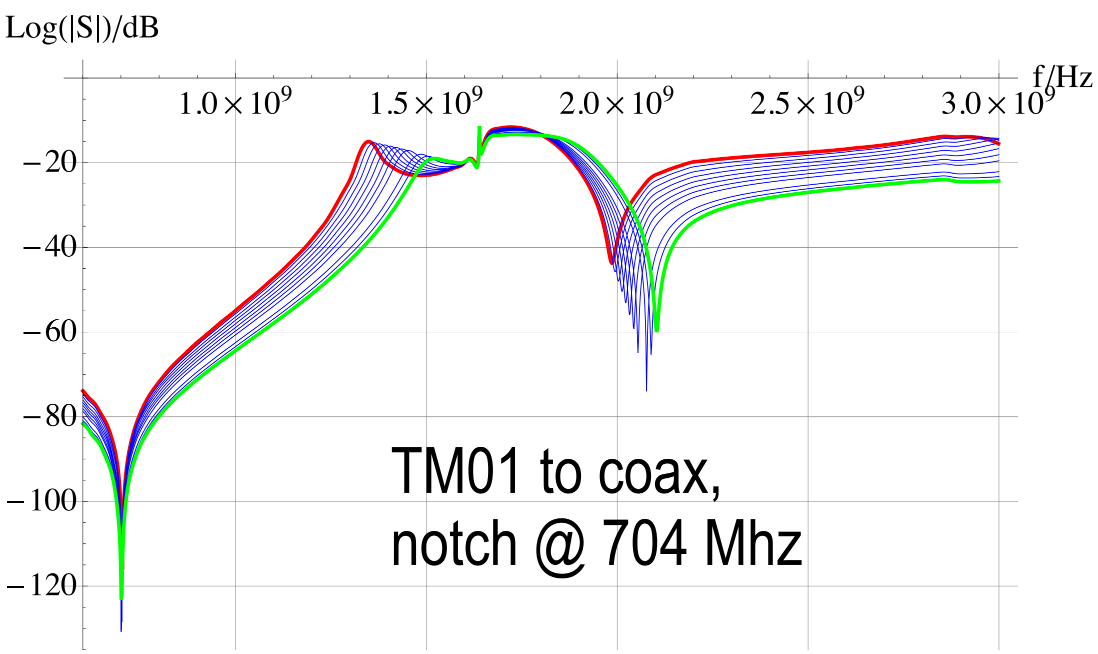


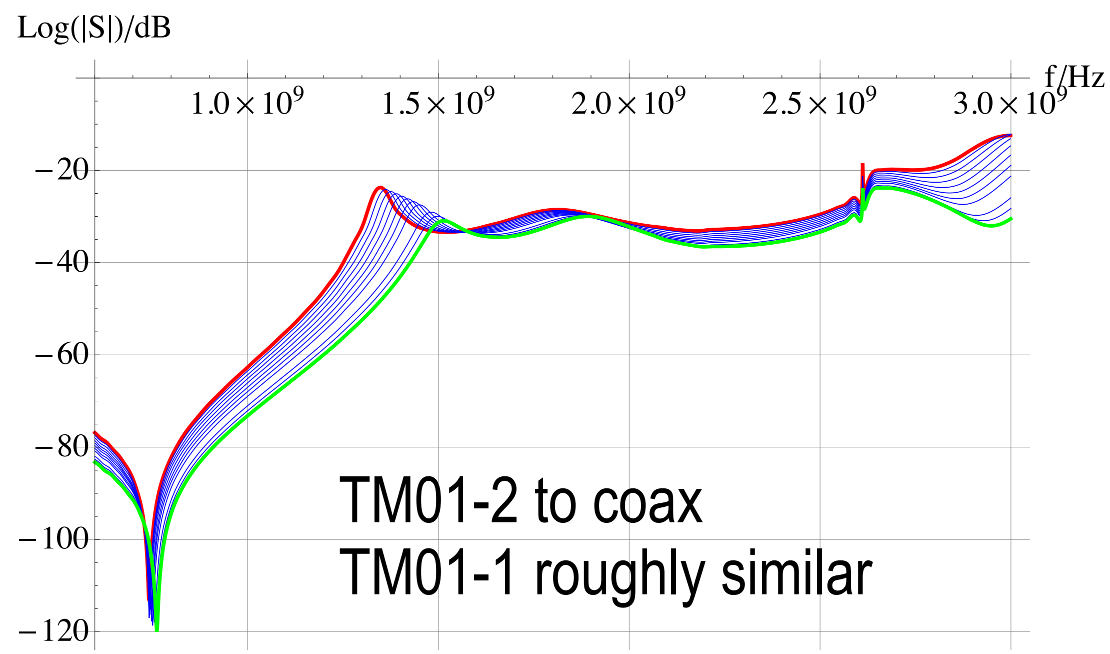
Bandwidth of notch filter depends on fixture

Tuning dependency on penetration depth









10 mm additional depth ~ very roughly +10dB coupling

What I have not told you:

- A LOT of formulas, e.g.:
- ... that there is also a transversal wake potential, which is directly correlated with the logitudinal one ("Panofsky-Wenzel-Theorem").
- the Fundamental Theorem of Beam Loading (c.f. e.g. P. Wilson et.al.)
- ... that you may integrate wake potentials in an indirect manner under certain conditions (c.f. e.g. Napoly, Zotter, Chin, Zagorodnov, Gjonaj, Weiland et.al.), exploiting certain mathematical properties of wake potentials.
- ... that wakefields also are caused by surface impedance, surface roughness, dielectric coating, ferrites etc.
- ... that there are dozens of programs for one or another aspect of HOM/wakefield computations.
- ... that HOMs may be used both for deflecting (c.f. Crab cavities) and diagnostics,
- ... etc. etc. - maybe enough for an Accelerator School

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Hope you nevertheless found something interesting.

Thank you for your attention

Spare Slides

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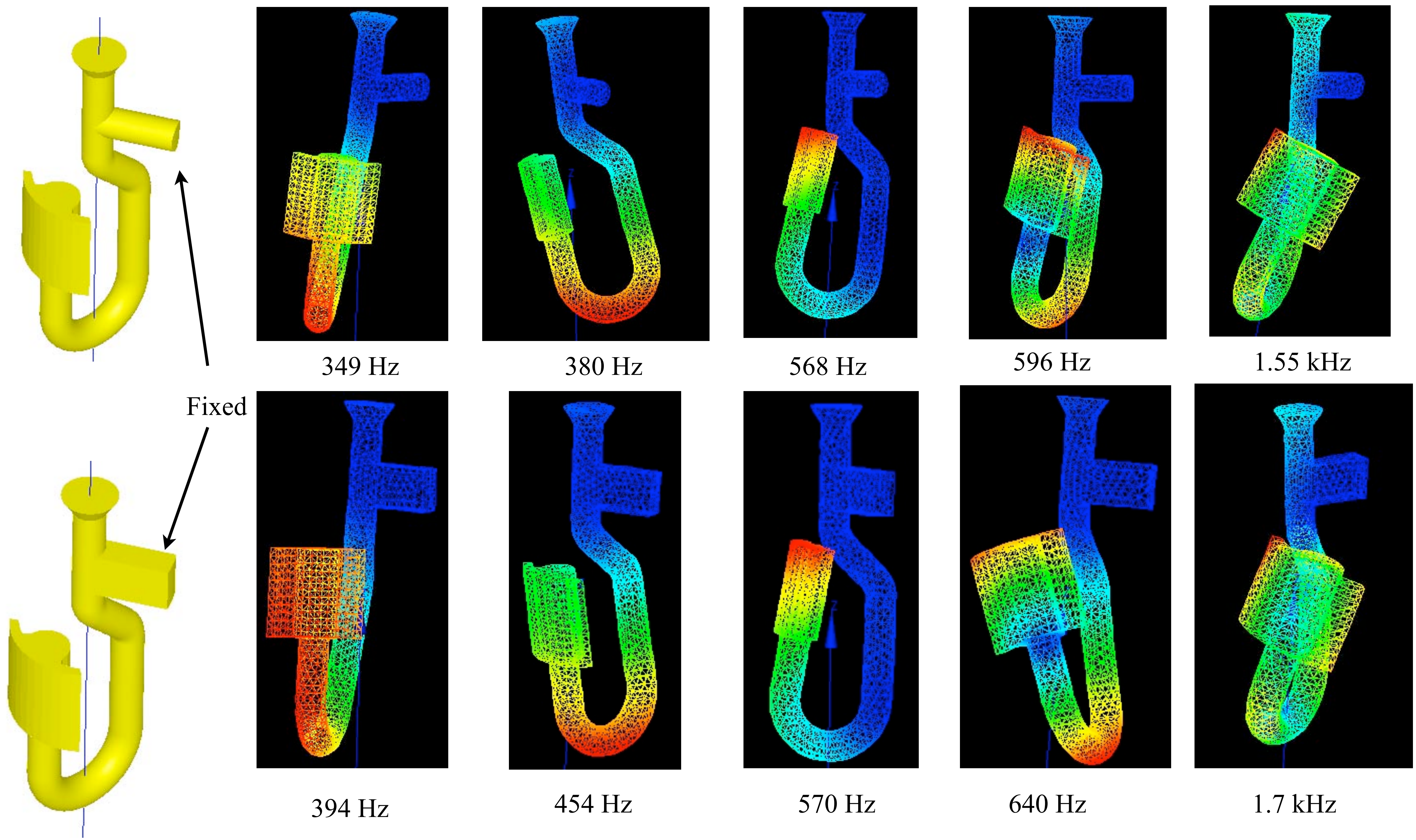
Mechanical properties - Eigenmodes based on fixture



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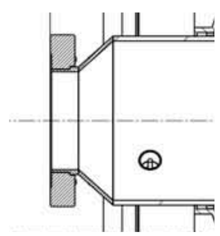
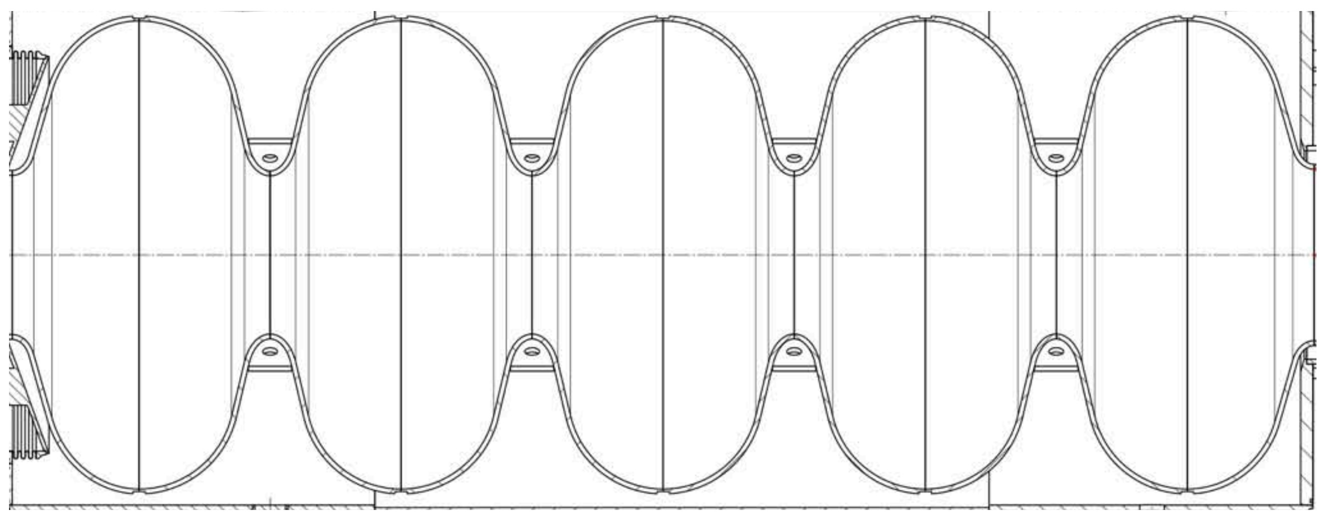
SPL-5-cell cavity - split

"left" side HOM coupler port
rotated by 60° towards you
(not drawn)

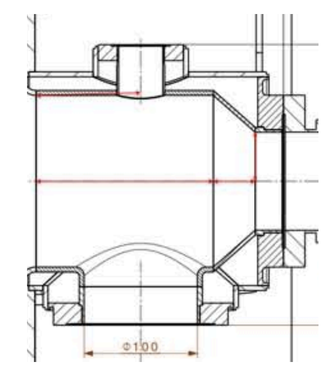


D_{le} = 130mm

704-MHz-SPL-beta-1-Cavity



"right" side HOM coupler port
opposed to power c., D= 36mm



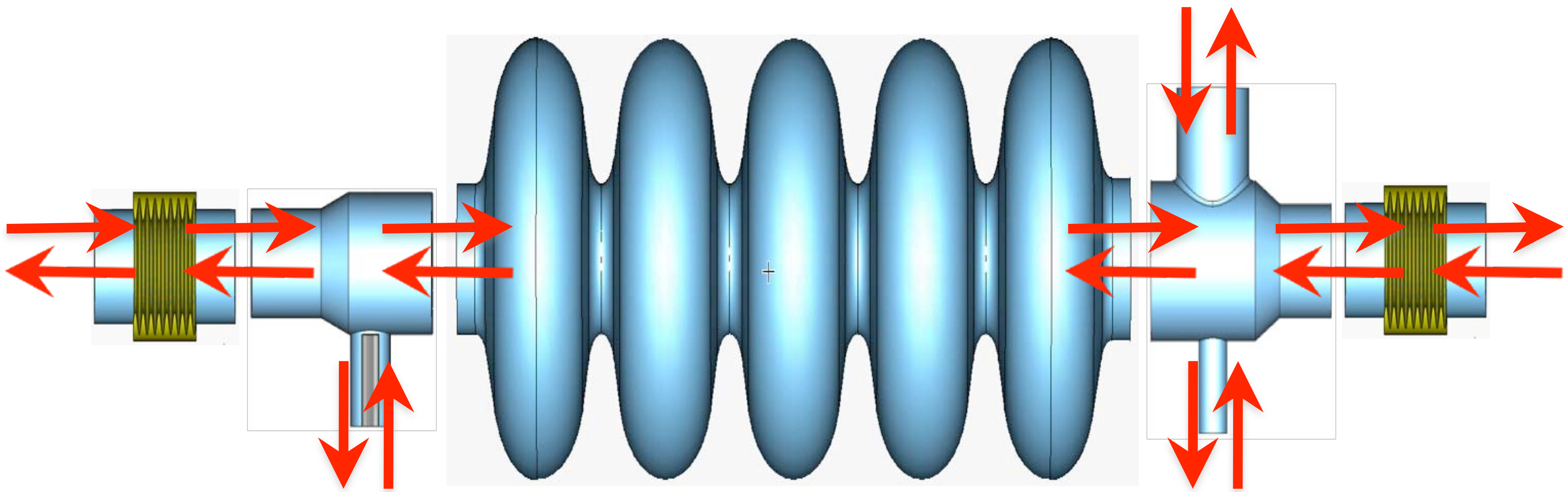
D_{ri} = 140mm
power coupler port
D=100mm

Try to separate different functional sections

=> **Save computational effort**

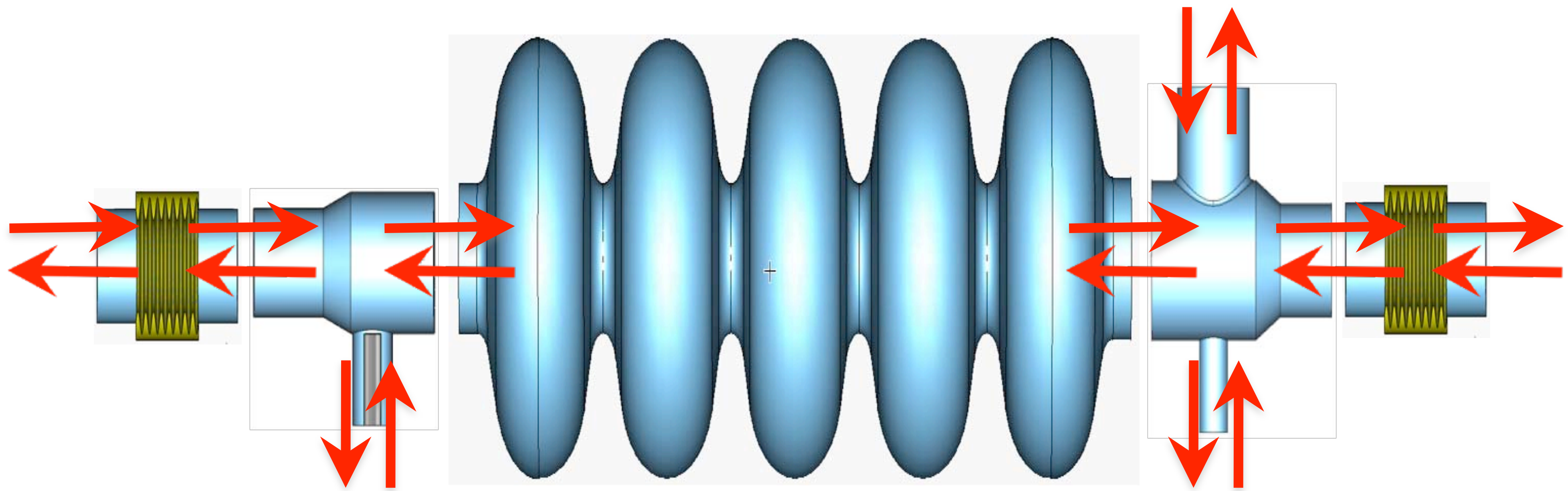
=> **especially in design optimization tasks**

Concatenation procedure based on scattering properties: Coupled S-Parameter Computation = CSC



- Split structure in sections
- Compute scattering (S-) parameters of all sections individually with appropriate solvers
- Compute overall S-parameters as function of f with special algorithm*, applicable to any structure topology and mode number
- *: **e.g.:** H.-W. Glock, K. Rothemund, U. van Rienen: "CSC - A System for Coupled S-Parameter Calculations", TESLA-Report 2001-25 **or** K. Rothemund, H.-W. Glock, U. van Rienen: "Eigenmode Calculation of Complex RF-Structures using S-Parameters", IEEE Transactions on Magnetics, Vol. 36, (2000): 1501-1503 and references therein

Concatenation procedure based on scattering properties: Coupled S-Parameter Computation = CSC



- Split structure in sections
- Compute scattering (S-) parameters of all sections individually with appropriate solvers
- Compute overall S-parameters as function of f with special algorithm*, applicable to any structure topology and mode number
- Derive loaded Q-values from S-parameter spectra